Efficient Scheme for Two-Atom Entanglement and Quantum Information Processing in Cavity QED

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A scheme is proposed for the generation of two-atom maximally entangled states and realization of quantum logic gates and teleportation with cavity QED. The scheme does not require the transfer of quantum information between the atoms and cavity. In the scheme the cavity is only virtually excited and thus the requirement on the quality factor of the cavities is greatly loosened.

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Entanglement is one of the most striking features of quantum mechanics. If two subsystems are entangled, the whole state vector cannot be separated into a product of the states of the two subsystems. In this case the two subsystems are no longer independent even if they are far spatially separated. A measurement on one subsystem not only gives information about the other subsystem, but also provides possibilities of manipulating it. The generation of entangled states for two or more particles is fundamental to demonstrate quantum nonlocality [1,2]. Recently, two-particle entangled states have been realized in both cavity QED [3] and ion traps [4].

On the other hand, quantum entanglement is useful in quantum information processing, such as quantum cryptography [5], computer [6], and teleportation [7]. For the implementation of quantum computer and teleportation the main ingredient is the conditional quantum dynamics, in which one subsystem undergoes a coherent evolution depending on the state of another system. In cavity QED, schemes have been proposed for realizing quantum logic gates [8] and teleportation [9]. The ion trap is also a good system for quantum information processing [10]. Quantum logic gates have been demonstrated in cavity QED [11], ion trap [12], and NMR [13] experiments. On the other hand, quantum teleportation has been demonstrated using optical systems [14] and NMR [15].

In most of the previous schemes for quantum information processing in cavity QED and ion traps, the cavity and ion motion act as memories, which store the information of an electronic system and then transfer back to this electronic system after the conditional dynamics. Thus one of the main obstacles for the implementation of quantum information in cavity QED is the decoherence of the cavity field, while that in ion traps is the difficulty to achieve the joint ground state of the ion motion and the heating of the ions. Recently, Sørensen and Mølmer [16] have proposed a novel scheme for realizing quantum computation in ion traps via virtual vibrational excitations. The scheme does not use the motional mode as the data bus and is insensitive to the vibrational states. The same authors [17] have also proposed a scheme for the generation of multiparticle entangled states in ion traps also without the requirement of the full control of the ion motion. The idea of Ref. [16] can also be used to teleport the state of a trapped ion [18]. In this Letter we propose a scheme for the generation of two-atom maximally entangled states and realization of quantum logic gates and teleportation in cavity QED. The distinct advantage of the proposed scheme is that during the operation the cavity is only virtually excited and thus the efficient decoherence time of the cavity is greatly prolonged.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity field. The interaction Hamiltonian in the interaction picture is

$$H_i = g \sum_{j=1,2} (e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+), \tag{1}$$

where $S_j^+ = |e_j\rangle\langle g_i|$ and $S_j^- = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ ($j = 1,2$) being the excited and ground states of the jth atom, a^+ and a are, respectively, the creation and annihilation operators for the cavity mode, g is the atom-cavity coupling strength, and δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω. In the case $\delta \gg g$, there is no energy exchange between the atomic system and the cavity. Then the effective Hamiltonian is given by

$$H = \lambda \Bigg[\sum_{j=1,2} (|e_j\rangle\langle e_j| a a^+ - |g_j\rangle\langle g_j| a^+ a) + (S_1^+ S_2^- + S_1^- S_2^+) \Bigg], \tag{2}$$

where $\lambda = g^2/\delta$. The first and second terms describe the photon-number dependent Stark shifts, and the third and fourth terms describe the dipole coupling between the two atoms induced by the cavity mode. If we assume the cavity field is initially in the vacuum state the Hamiltonian reduces to

$$H = \lambda \Bigg[\sum_{j=1,2} |e_j\rangle\langle e_j| + (S_1^+ S_2^- + S_1^- S_2^+) \Bigg]. \tag{3}$$

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Assume the atoms are initially in the state $|e_1\rangle|g_2\rangle$. Then the state evolution of the system is given by

$$|e_1\rangle|g_2\rangle \rightarrow e^{-i\lambda t}[\cos(\lambda t)|e_1\rangle|g_2\rangle - i\sin(\lambda t)|g_1\rangle|e_2\rangle]. \tag{4}$$

With the choice of $\lambda t = \pi/4$, we obtain the maximally entangled two-atom state, i.e., Einstein-Podolsky-Rosen pair (EPR pair) [19],

$$|\psi_{\mathrm{EPR}}\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle - i|g_1\rangle|e_2\rangle), \tag{5}$$

where we have discarded the common phase factor $\pi/4$.

Now we show how we can use the idea to realize quantum controlled-not (CNOT) gates. In order to do so we use ladder-type three-level atoms, whose states are denoted by $|g\rangle$, $|e\rangle$, and $|i\rangle$. The transition frequency between the states $|e\rangle$ and $|i\rangle$ is highly detuned from the cavity frequency and thus the state $|i\rangle$ is not affected during the atom-cavity interaction. The quantum information is stored in the state $|e\rangle$ and $|g\rangle$. Assume atom 1 acts as the control bit and atom 2 acts as the controlled bit. We first let atom 2 cross two classical fields tuned to the transitions $|g\rangle \rightarrow |e\rangle$ and $|e\rangle \rightarrow |i\rangle$, respectively. Choose the amplitudes and phases of the classical fields appropriately so that this atom undergoes the transition

$$|e_2\rangle \rightarrow \frac{1}{\sqrt{2}}(|e_2\rangle + |g_2\rangle) \rightarrow \frac{1}{\sqrt{2}}(|i_2\rangle + |g_2\rangle), \tag{6}$$

$$|g_2\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_2\rangle - |e_2\rangle) \rightarrow \frac{1}{\sqrt{2}}(|g_2\rangle - |i_2\rangle). \tag{7}$$

Then we let atoms 1 and 2 simultaneously enter a single-mode cavity. The interaction Hamiltonian takes no effect on states $|g_1\rangle|g_2\rangle$ and $|g_1\rangle|i_2\rangle$ but induces a phase factor $e^{-i\lambda t}$ to the state $|e_1\rangle|i_2\rangle$. The state $|e_1\rangle|g_2\rangle$ undergoes the transition of Eq. (4). After an interaction time π/λ, we obtain

$$|e_1\rangle|i_2\rangle \rightarrow -|e_1\rangle|i_2\rangle, \tag{8}$$

$$|e_1\rangle|g_2\rangle \rightarrow |e_1\rangle|g_2\rangle. \tag{9}$$

Then atom 2 crosses two classical fields tuned to the transitions $|e\rangle \rightarrow |i\rangle$ and $|g\rangle \rightarrow |e\rangle$, respectively. Choose the amplitudes and phases of the classical fields appropriately so that this atom undergoes the transition

$$|g_2\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_2\rangle + |e_2\rangle), \tag{10}$$

$$|i_2\rangle \rightarrow |e_2\rangle \rightarrow \frac{1}{\sqrt{2}}(|e_2\rangle - |g_2\rangle). \tag{11}$$

Thus we obtain the transformation

$$\begin{aligned} |g_1\rangle|g_2\rangle &\rightarrow |g_1\rangle|g_2\rangle, \\ |g_1\rangle|e_2\rangle &\rightarrow |g_1\rangle|e_2\rangle, \\ |e_1\rangle|g_2\rangle &\rightarrow |e_1\rangle|e_2\rangle, \\ |e_1\rangle|e_2\rangle &\rightarrow |e_1\rangle|g_2\rangle. \end{aligned} \tag{12}$$

This transformation corresponds to the quantum CNOT gate operation, in which if and only if atom 1 is in the state $|e\rangle$ atom 2 flips its state.

We note the idea can be further used to teleport an unknown atomic state. Assume atom 1 is initially in a superposition state

$$|\phi_1\rangle = c_e|e_1\rangle + c_g|g_1\rangle, \tag{13}$$

where c_e and c_g are unknown coefficients. Atom 2 and atom 3 (to receive the teleported state) are prepared in the entangled state

$$\frac{1}{\sqrt{2}}(|e_2\rangle|g_3\rangle - i|g_2\rangle|e_3\rangle). \tag{14}$$

The state for the whole system can be expanded as

$$|\psi\rangle = \tfrac{1}{2}[|\Psi^+\rangle(c_e|e_3\rangle + c_g|g_3\rangle) + |\Psi^-\rangle(c_e|e_3\rangle - c_g|g_3\rangle) + |\Phi^+\rangle(c_e|g_3\rangle - c_g|e_3\rangle) + |\Phi^-\rangle(c_e|g_3\rangle + c_g|e_3\rangle)], \tag{15}$$

where $|\Psi^\pm\rangle$ and $|\Phi^\pm\rangle$ are the Bell states [20]

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(-i|e_1\rangle|g_2\rangle \pm |g_1\rangle|e_2\rangle), \tag{16}$$

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle|e_2\rangle \pm i|g_1\rangle|g_2\rangle). \tag{17}$$

We then let atom 2 cross two classical fields tuned to the transitions $|e\rangle \rightarrow |i\rangle$ and $|g\rangle \rightarrow |e\rangle$, respectively, undergoing the transitions of Eqs. (6) and (7). Then atoms 1 and 2 are sent through a cavity, undergoing the transitions of Eqs. (8) and (9). This leads to

$$|\Psi^\pm\rangle \rightarrow \tfrac{1}{2}(-i|e_1\rangle \pm |g_1\rangle)(|g_2\rangle + |i_2\rangle), \tag{18}$$

$$|\Phi^\pm\rangle \rightarrow \tfrac{1}{2}(|e_1\rangle \pm i|g_1\rangle)(|g_2\rangle - |i_2\rangle). \tag{19}$$

We then let atom 1 cross a classical field tuned to the transition $|e\rangle \rightarrow |g\rangle$, undergoing the transition

$$|e_1\rangle \rightarrow \frac{1}{\sqrt{2}}(|e_1\rangle - i|g_1\rangle), \tag{20}$$

$$|g_1\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_1\rangle - i|e_1\rangle). \tag{21}$$

Atom 2 crosses two classical fields tuned to the transitions $|e\rangle \rightarrow |i\rangle$ and $|g\rangle \rightarrow |e\rangle$, respectively, undergoing the transition of Eqs. (10) and (11). Thus the evolution for the Bell states is given by

$$|\Psi^{\pm}\rangle \to \begin{cases} -i|e_1\rangle|e_2\rangle \\ -|g_1\rangle|e_2\rangle \end{cases}, \tag{22}$$

$$|\Phi^{\pm}\rangle \to \begin{cases} |e_1\rangle|g_2\rangle \\ -i|g_1\rangle|g_2\rangle \end{cases}. \tag{23}$$

Hence, the joint measurement can be achieved by detecting atoms 1 and 2 separately. The outcome of the joint measurement on atoms 1 and 2 is transmitted to the receiver, who can apply an appropriate rotation to atom 3 to reconstruct the initial state of atom 1.

It is necessary to give a brief discussion on the experimental matters. For the Rydberg atoms with principal quantum numbers 49, 50, and 51, the radiative time is about $T_r = 3 \times 10^{-2}$ s, and the coupling constant is $g = 2\pi \times 24$ kHz [21]. With the choice $\delta = 10g$, the required atom-cavity-field interaction time is on the order of $\pi\delta/g^2 \simeq 2 \times 10^{-4}$ s. Then the time needed to complete the whole procedure is on the order of 10^{-3} s, much shorter than T_r. A cavity with a quality factor $Q = 10^8$ is experimentally achievable [21]. In the present case the cavity field frequency is about 50 GHz. The corresponding photon lifetime is $T_c = Q/2\pi\nu \simeq 3.0 \times 10^{-4}$ s. In the present scheme the cavity has only a small probability, about 0.01, of being excited during the passage of the atoms through the cavity. Thus the efficient decay time of the cavity is about 3.0×10^{-2} s, on the order of the atomic radiative time. Therefore, based on cavity QED techniques presently or soon to be available the proposed scheme might be realizable.

The present scheme requires that two atoms be simultaneously sent through a cavity; otherwise there will be an error. We here estimate such an error. Assume that during the generation of a two-atom maximally entangled state the atom initially in the excited state enters the cavity $0.01t$ sooner than another atom, with t being the time of each atom staying in the cavity. Then, the two atoms are finally prepared in the state

$$|\psi\rangle = [\cos(0.99\lambda t)\,|e_1\rangle|g_2\rangle - ie^{-0.01i\lambda t}\sin(0.99\lambda t)\,|g_1\rangle|e_2\rangle] \tag{24}$$

with the common phase factor λt being discarded. Choosing $t = \pi/(4\lambda)$ we have

$$|\langle\psi_{\mathrm{EPR}}\,|\,\psi\rangle|^2 \simeq 0.99, \tag{25}$$

where $|\psi_{\mathrm{EPR}}\rangle$ is given by Eq. (5). In this case the operation is only slightly affected.

In summary, we have proposed a scheme to generate two-atom maximally entangled states and realize quantum controlled-not gates and quantum teleportation with dispersive cavity QED. Unlike previous schemes, the present one does not require the transfer of quantum information between the cavity and atom. During the passage of the atoms the cavity is only virtually excited; thus the requirement on the quality factor of the cavity is greatly loosened. The present scheme opens a new prospect for quantum entanglement and quantum information processing.

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