Extended Inflationary Cosmology

Daile La and Paul J. Steinhardt
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 11 October 1988)

We present a new type of inflationary scenario based on metric formulations of gravity different from that of Einstein, e.g., a Brans-Dicke theory of gravity. Unlike previous inflation models, the inflationary phase transition can be completed via bubble nucleation. Hence, the fine tuning of an effective potential to obtain a slow-rollover transition is not required.

PACS numbers: 98.80.Cq, 04.50.+h

The inflationary-universe model is designed to resolve a number of cosmological puzzles, including the horizon, flatness, and monopole problems. The key feature is a brief, finite period of exponential expansion. Such expansion can occur if the Universe undergoes a strongly first-order phase transition. As the Universe supercools into a false-vacuum phase, the false-vacuum energy density acts as an effective cosmological constant which triggers an epoch of de Sitter (exponential) expansion. Resolution of the cosmological puzzles requires the Robertson-Walker scale parameter to increase by a factor of e^{60} or more before the de Sitter expansion ends.

The flaw in the original or "old inflation" model[1] is that the de Sitter expansion never ends. For a generic, strongly first-order phase transition, there is a large energy barrier separating the false from the true phase. Spontaneous nucleation of bubbles of true vacuum cannot keep pace with the exponentially expanding false phase.[2] The flaw is sometimes referred to as the "graceful exit" problem.

"New inflation"[3] achieves a graceful exit by fine tuning the parameters in the effective potential so that the barrier just disappears as the exponential expansion begins. The result is sometimes called a slow-rollover transition: The false phase becomes unstable and evolves continuously towards the true phase. If the effective potential is sufficiently flat near the false-phase extremum, the Universe undergoes further inflation during the initial evolution towards the true phase. Although the scenario is workable, the fine tuning of parameters to achieve a slow-rollover transition is an unattractive feature. Variants such a chaotic inflation[4] and quantum cosmology[5] require similar fine tuning and/or exotic quantum effects near the Planck scale that are not completely understood at present.

In this Letter, we suggest an "extended" inflationary (EI) model that, in some ways, restores the spirit of the old inflation model. The universe undergoes a generic, strongly first-order phase transition associated with some (unspecified) high-temperature particle-physics phenoenon (e.g., gauge-symmetry breaking). That is, the Universe supercools to a false phase that remains separated by a large energy barrier from the true vacuum phase. As in the old inflation model, the energy scale for the transition can be much lower than the Planck scale so that quantum gravity effects can be ignored. In the EI approach, though, graceful exit and successful inflation are attainable through the introduction of a metric theory of gravity different from that of Einstein. A strong link between inflation and modified gravity theories is thereby proposed, suggesting an added significance for inflationary cosmology.

In this Letter, we will restrict attention to the Brans-Dicke[6] (BD) theory of gravity. The action is given by

$$A=\int d^4x\sqrt{g}\left[-\Phi R+\omega\left(\frac{\partial_\mu\Phi\,\partial^\mu\Phi}{\Phi}\right)+16\pi L_{\rm matter}\right], \quad (1)$$

where Φ is the BD scalar field. We will presume that matter, described by $L_{\rm matter}$, includes a Higgs-type sector which undergoes a strongly first-order phase transition at high temperatures. During the transition, the vacuum energy density changes from ρ_f to ρ_t. Of course, the conventional BD model is highly constrained by experiment[7] to be very close to the Einstein form. The theory is parametrized by a dimensionless constant ω, where $\omega\rightarrow\infty$ as Brans-Dicke theory goes over to the Einstein theory. Present limits based on time-delay experiments[8] require $\omega>500\gg1$. Hence, one might suppose that any deviation between Brans-Dicke and Einstein gravity is negligible. However, our basic point is that this small difference can have a dramatic effect in the course of the strongly first-order phase transition, allowing a graceful exit from the inflationary false-vacuum phase. A value of $\omega>500$ does not obstruct the graceful exit at all.

Perhaps it is also worth noting that, upon compactification to four dimensions, string theories[9] and Kaluza-Klein theories[10] may lead to modified metric theories of gravity closely akin to BD theory. Typically, one finds $\omega\approx1$. For example, string theories produce one or massless fields, including the dilaton and the moduli, which appear to couple coherently to matter in a way very similar to the BD scalar coupling.

Furthermore, the EI scenario may ensue under a range of conditions for the scalar field. The analog of the BD scalar field may be free, as in BD theory; or there may be a nonzero scalar potential[11] [i.e., an added $V(\Phi)$ in Eq. (1)]. In the true-vacuum phase, the scalar potential may force the value of Φ to settle at some large expectation value [e.g., the value of the (Planck mass)^2 today]. In this case, Φ eventually ceases to vary with time in the true-vacuum phase and the modified gravity theory then becomes indistinguishable from the Einstein form. Alternatively, the scalar potential may be such that Φ continues to evolve with time *ad infinitum.*[9] In any case, it is sufficient for the EI model that the scalar field evolves with time during the inflationary transition (the false-vacuum phase). This can be easily arranged, for example, by choosing $\rho_f \gg V(\Phi)$, so that the vacuum energy dominates the equation of motion for Φ in the false phase. Hence, while we confine our discussion to the conventional BD theory in the remainder of this Letter, one can easily envisage a variety of scenarios which incorporate extended inflation.

The salient difference between old inflation and the EI model is the form of the cosmological expansion during the inflationary phase. In BD theory, the equations of motion for the Robertson-Walker scale factor $R(t)$ and the BD scalar $\Phi(t)$ are[12]

$$H^2 = \frac{8\pi\rho}{3\Phi} - \frac{k}{R^2} + \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 - H\left(\frac{\dot{\Phi}}{\Phi}\right), \tag{2}$$

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{8\pi(\rho - 3p)}{3+2\omega}, \tag{3}$$

where ρ is the (matter) energy density, p is the pressure, $H = \dot{R}/R$, and $k = 0$, $+1$, or -1 corresponds to a flat, closed, or open universe. As the Universe supercools in the false phase, the energy density approaches the false-vacuum energy density ρ_f, which acts as an effective cosmological constant. The conservation laws then imply that $p = -\rho_f$.

The solution to the BD equations when the equation of state is $p = -\rho = -\rho_f$ and $k = 0$ is[13]

$$\Phi = m_P^2 (1 + \chi t/\alpha)^2, \tag{4}$$

$$R(t) = (1 + \chi t/\alpha)^{\omega + 1/2}, \tag{5}$$

where $\chi^2 = 8\pi\rho_f/3m_P^2$ is the square of the Hubble constant in the Einstein theory, and $\alpha^2 = (3+2\omega)(5+6\omega)/12$ approaches ω^2 for $\omega \gg 1$. The constant m_P^2 is an arbitrary integration constant corresponding to the effective (Planck mass)^2 at the beginning of inflation, $t = 0$. The (Planck mass)^2 today is $\Phi(t_{\text{today}})$.

For short times, $\chi t < \alpha$, the BD solution approaches the usual Einstein–de Sitter solution: Φ is nearly constant and $R(t)$ grows exponentially with time, $R(t) \approx \exp(\chi t)$. If $\omega > 90$, one obtains the 60 e foldings of inflation necessary to solve the cosmological puzzles in this first stage of inflation. We do not require such a large value of ω, though, since significant expansion occurs during the second stage of inflation when $\chi t > \alpha$. In the second stage, $\Phi \approx m_P^2 (\chi t/\alpha)^2$ and $R(t) \approx (\chi t/\alpha)^{\omega + 1/2}$, a power-law rather than exponential expansion. (Here, power-law expansion comes from the extension of the de Sitter solution to BD theory. This differs from the "power-law" and "induced-gravity" inflation models discussed previously,[14] in which finely tuned slow-rollover transitions are required.)

The crossover from exponential to power-law expansion changes the rate at which bubble nucleation converts the Universe from false- to true-vacuum phase. As shown by Guth and Weinberg,[2] the probability of a point remaining in the false-vacuum phase during a bubble-nucleation process beginning at time t_B is

$$p(t) = \exp\left[-\int_{t_B}^{t} dt' \lambda(t') R^3(t') \frac{4\pi}{3}\left(\int_{t'}^{t} \frac{dt''}{R(t'')}\right)^3\right], \tag{6}$$

where $\lambda(t)$ is the nucleation rate per unit time per unit volume, approximately constant during the inflationary phase. For de Sitter expansion, the exponent is $\approx -\frac{4}{3}\pi\epsilon\chi(t - t_B)$, where $\epsilon \equiv \lambda/\chi^4$. The parameter ϵ is calculable from the effective potential and is typically quite small (10^{-100} is even plausible). For small ϵ, the filling rate cannot keep up with the exponential expansion of false vacuum, and one encounters the graceful exit problem.[2]

When the expansion crosses over to power law, the exponent in Eq. (6) becomes $(\pi/3)\epsilon\omega(y^4 - y_B^4)$, where $y \equiv \chi t/\omega > 1$. (For simplicity, we assume for the rest of this Letter that $\omega \gg 1$, so $\alpha \approx \omega$.) The filling of space with true vacuum still occurs exponentially in time (the exponent is quadratic rather than linear in time), but now the expansion is only power law. At $\chi t/\omega > (3/\pi\omega\epsilon)^{1/4}$, $p(t) \ll 1$, and $p(t)$ is decreasing much faster than the volume $[\propto R^3(t)]$ is increasing. The Universe is dominated by true vacuum, and exit from the false vacuum is achieved.[15]

When the bubbles are first nucleated, most of the false-vacuum energy is converted into bubble-wall energy.[16] As the bubbles fill the Universe, their walls collide. The coherent wall energy is rapidly converted into a thermal distribution of particles. The latent heat of the transition is thereby released, producing a thermal energy density roughly equal to the false-vacuum energy density during the transition. Since the Universe is now in the true-vacuum phase with zero cosmological constant, subsequent evolution proceeds as in an ordinary Friedmann-Robertson-Walker universe. From this point onwards, provided that ω is sufficiently large, the evolution is virtually indistinguishable from the usual hot-big-bang model based on ordinary gravity. In the matter-dominated era,[12] $\Phi(t) \propto t^{2/(4+3\omega)}$ (compared to $\Phi \propto t^2$ in the inflationary epoch), and $R(t) \propto t^{(2\omega+2)/(3\omega+4)} \approx t^{2/3}$.

Several comments may prove useful in considering this scenario:

(1) Reviewing the analysis, one observes that the crucial features of the EI model are the following: (a) Newton's constant G is replaced by a time-dependent scalar field, $\Phi(t)$; and (b) although $\Phi(t)$ varies very slowly during a radiation- or matter-dominated epoch, it varies rapidly (in this case, $\propto t^2$) during the inflationary epoch. The rapid variation of $\Phi(t)$ shows the inflation of $R(t)$ from exponential to power law. These properties probably occur in a broad range of theories.

(2) A large value of ω acts to increase the amount of inflation during the exponential expansion stage. However, even if ω is small, expansion of $R(t)$ by e^{60} can occur during the power-law inflation stage prior to graceful exit provided that ϵ is sufficiently small: $R(t_e)/R(t_B) \approx (3/\pi\omega\epsilon)^{\omega/4}$, where we have estimated the end of power-law inflation, t_e, to be the time when the exponent in Eq. (6) is of order unity. Hence, no strong constraint on ω is needed for inflation.

(3) We have only presented the solutions for $k=0$. To resolve the flatness problem, one should take $k\neq 0$ initially. However, it is clear that the solution rapidly converges to the $k=0$ solution. As $R(t)$ increases, the spatial curvature term in Eq. (2) decreases by $\Phi/R^2 \propto (1/t)^{2\omega-1}$ compared to the false-vacuum energy-density term. Once the transition starts, rapid expansion of $R(t)$ reduces the spatial curvature to a negligibly small value.

Nevertheless, $\Omega \equiv \rho/\rho_c \equiv 8\pi\rho/3\Phi H^2 \neq 1$. The kinetic energy of the BD scalar field in Eq. (2) makes a nonzero (constant) contribution to Ω. Substituting Eqs. (4) and (5) into Eq. (2), we find that (for $\omega \gg 1$)

$$\Omega = 1 + 4/3\omega\,, \tag{7}$$

at the end of the inflationary epoch; with use of the expressions for $\Phi(t)$ and $R(t)$ from the matter-dominated era,[12] the value today (for $\omega \gg 1$) is

$$\Omega = 1 + 5/6\omega\,. \tag{8}$$

In other words, if Φ continues to evolve with time today, the EI model predicts a difference between geometrical versus dynamical measurements of Ω. For the particular case of BD theory, ω is constrained to be very large, and hence $\Omega - 1$ is experimentally insignificant.

(4) In order to solve the monopole problem, the inflationary phase transition must occur strictly after the transition in which monopoles are produced. If they were the same transition, the orientation of the Higgs field in the bubbles would be uncorrelated, and a cosmologically unacceptable number of monopoles would be formed at the bubble intersections when they coalesced.

(5) Our initial analysis of gravitational wave fluctuations in the EI model is essentially identical to the case for power-law inflation models.[14] For large ω, the spectrum is nearly scale invariant and the induced-microwave background fluctuations are $\delta T/T \approx H_e/M_P$, where both the Hubble constant H_e and $(M_P)^2 = \Phi$ are evaluated near the end of inflation ($M_P \approx$ Planck mass today). This implies that $\delta T/T < 10^{-4}$ provided $\rho_f < (10^{17}\ \text{GeV})$.[4] However, there are the numerous new subtleties to consider. For example, how do we follow the production and evolution of fluctuations during the bubble-coalescence process? How do deviations from Einstein gravity affect the time evolution of fluctuations?

Our optimistic view is that this work suggests a new and perhaps compelling link between inflationary cosmology and theories of gravity that deviate from Einstein's formulation. There is even the possibility in some EI scenarios that the difference between geometrical and dynamical measurements of Ω can serve as a direct, cosmological probe to measure deviations from Einstein gravity.

We thank G. Segre, P. Langacker, M. Evans, and S. Coleman for their thoughtful comments. This work was supported in part by U.S. DOE by Grant No. DOE-EY-76-C-02-3071.

[1]A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[2]A. H. Guth and E. J. Weinberg, Nucl. Phys. **B212**, 321 (1983).

[3]A. D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[4]A. D. Linde, Phys. Lett. **129B**, 177 (1983).

[5]J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983).

[6]C. Brans and C. H. Dicke, Phys. Rev. **24**, 925 (1961).

[7]For a review of limits on metric gravity theories, see C. M. Will, Phys. Rep. **113**, 345 (1984).

[8]R. D. Reasenberg *et al.*, Astrophys. J. **234**, L219 (1979).

[9]See, for example, M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory: 2* (Cambridge Univ. Press, Cambridge 1987), pp. 326–330, 403–404.

[10]See, for example, P. G. O. Freund, Nucl. Phys. **B209**, 146 (1982).

[11]Theories with hidden sectors coupled to gravity (S. Coleman, private communication) and induced-gravity models (S. Barr and G. Segre, private communication) can be recast in this form.

[12]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), especially Sec. 16.4.

[13]See, for example, C. Mathiazhagan and V. B. Johri, Class. Quantum Grav. **1**, L29 (1984).

[14]Power-law inflation: L. F. Abbott and M. B. Wise, Nucl. Phys. **B244**, 541 (1984); F. Lucchin and S. Mataresse, Phys. Rev. D **32**, 1316 (1985), and Phys. Lett. **164B**, 282 (1985). Induced-gravity inflation: F. S. Accetta, D. J. Zoller, and M. S. Turner, Phys. Rev. D **31**, 3046 (1985); also by F. Lucchin, S. Matarresse, and M. D. Pollock, Phys. Lett. **167B**, 163 (1986).

[15]Bubble percolation and the percolation time are analyzed in D. La and P. Steinhardt, University of Pennsylvania report (to be published).

[16]S. Coleman, Phys. Rev. D **15**, 2929 (1977).