Quantum Fluctuations in Quasi-One-Dimensional Dipolar Bose-Einstein Condensates

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Recent experiments have revealed that beyond-mean-field corrections are much more relevant in weakly interacting dipolar condensates than in their nondipolar counterparts. We show that in quasi-one-dimensional geometries quantum corrections in dipolar and nondipolar condensates are strikingly different due to the peculiar momentum dependence of the dipolar interactions. The energy correction of the condensate presents not only a modified density dependence, but it may even change from attractive to repulsive at a critical density due to the surprising role played by the transversal directions. The anomalous quantum correction translates into a strongly modified physics for quantum-stabilized droplets and dipolar solitons. Moreover, and for similar reasons, quantum corrections of three-body correlations, and hence of three-body losses, are strongly modified by the dipolar interactions. This intriguing physics can be readily probed in current experiments with magnetic atoms.

Introduction.—Quantum fluctuations introduce a shift of the ground-state energy of a Bose gas, which at first order is given by the well-known Lee-Huang-Yang (LHY) correction [1]. However, in the weakly interacting regime, experiments on Bose-Einstein condensates are well described within the mean-field approximation. The situation may be crucially different in the presence of competing interactions, as recently discussed in the context of Bose-Bose mixtures [2]. In that scenario, the interplay between inter- and intra-species interactions results, at the verge of mean-field instability, in a dominant LHY correction well within the weakly interacting regime. The LHY correction may stabilize a collapsing condensate, resulting in the formation of quantum droplets, a novel ultradilute liquid whose surface tension is provided by purely quantum effects.

Dipolar condensates, formed by particles with large magnetic or electric dipolar moments, are also characterized by competing interactions, in this case short-range and dipole-dipole interactions. Indeed, recent experiments on highly magnetic atoms have revealed the crucial role played by quantum fluctuations at the mean-field instability, showing for the first time the formation of quantum droplets [3], which may remain self-bound even in the absence of external trapping [4]. Quantum stabilization and droplet formation have attracted wide theoretical and experimental attention [5–12], being a general phenomenon that is expected to characterize not only condensates of magnetic atoms, but the whole rapidly developing field of strongly dipolar gases [13,14].

In Bose-Bose mixtures and in dipolar condensates, quantum stabilization stems from the compensation between the attractive residual mean-field interaction, proportional to the 3D density $n_{3D}$, and the repulsive LHY correction, which in both systems is proportional to $n_{3D}^{3/2}$ [2,15]. As a result, there is a critical density at which both contributions compensate. Quantum fluctuations play an even more intriguing role in lower dimensions. In particular, droplets are stabilized for a sufficiently low density in 1D Bose-Bose mixtures [16], against melting rather than collapse, by the competition of a residual repulsive mean-field term, proportional to the 1D density $n_{1D}$, and the attractive LHY correction, proportional to $-n_{1D}^{1/2}$.

Whereas beyond-mean-field effects in 3D Bose-Bose mixtures and dipolar condensates are very similar due to the almost identical density dependence of the quantum correction, we show in this Letter that quantum fluctuations lead in quasi-1D dipolar condensates to a strikingly different physics compared to their nondipolar counterparts. This difference stems from the peculiar momentum dependence of the dipole-dipole interactions in quasi-1D geometries [17]. As a result, not only is the density dependence of the quantum corrections very different, but even its sign may change due to the remarkable role played by transversal directions in dipolar gases well within the 1D regime. The anomalous quantum corrections change the nature of quantum stabilization and strongly influence the physics of solitons. We also show that, whereas three-body correlations present the same density dependence in 3D dipolar and nondipolar condensates [18], they display in 1D a radically different dependence.

Dipolar interaction in 1D.—We consider bosons with mass $M$ and magnetic moment $\vec{\mu}_D$, although our results also apply for electric dipoles. The system is strongly confined on the $xy$ plane by an isotropic harmonic trap of frequency $\omega_\perp$, but it is untrapped along $z$. We assume that the chemical potential $\mu \ll \hbar \omega_\perp$, and hence the condensate, remains kinematically 1D such that its wave function splits
as \( \Psi(\mathbf{r}) = \psi(x,y)\phi(z) \), with \( \psi(x,y) = e^{-i(x^2+y^2)/2\hbar^2}/\sqrt{2\hbar} \) the ground state of the transversal trap, with \( \hbar = \hbar/M\omega_\perp \). After integrating over \( x \) and \( y \), the interaction between particles in the condensate acquires a momentum dependence \( k \) of the form
\[
\tilde{V}_{1D}(k) = g_{1D}\{1 + \epsilon_{dd}[3F(0,k^2\hbar^2/2) - 1]\},
\]
with \( F(j,\sigma) \equiv \sigma^{j+1}e^{j\Gamma(-j,\sigma)} \) [17], where \( \Gamma(-j,\sigma) \) is the incomplete Gamma function. Short-range interactions are characterized by the 1D coupling constant \( g_{1D} = g_{3D}/2\pi^2 \), where \( g_{3D} = 4\pi\hbar^2a/M \), with \( a > 0 \) the \( s \)-wave scattering length. Assuming \( \mu_D \) along \( z \), \( \epsilon_{dd} = \mu_D^2/3g_{3D} \) is the ratio between the strengths of the dipolar and contact interactions [19], with \( \mu_D \) the vacuum permeability. This 1D condition \( |\mu|/\hbar\omega_\perp \ll 1 \) demands \( |1 - \epsilon_{dd}| \ll 1/2n_{1D}a \), a condition satisfied in all the calculations in this Letter [20].

**LHY correction.**—Single particle excitations, \( (n_r, m, k) \), are characterized by their radial quantum number \( n_r \), angular momentum \( m \), and axial linear momentum \( k \). In 1D contact-interacting systems, transversal excitations, with \( (n_r, m, k) \neq (0,0,0) \), play a negligible role in beyond-mean-field corrections. Some may be crucially different in dipolar gases. In the weakly interacting regime, the main mean-field corrections. This may be crucially different in dipolar gases. The elementary excitations may be obtained for each \( m \) and \( k \) from the Bogoliubov–de Gennes equations
\[
\xi_{\nu}(\mathbf{v}) = \begin{pmatrix} \hat{E}_m(k) + \hat{U}_m(k) & \hat{U}_m(k) \\ -\hat{U}_m(k) & -\hat{E}_m(k) + \hat{U}_m(k) \end{pmatrix} \begin{pmatrix} \hat{v}_\nu \\ \hat{v}_\nu \end{pmatrix},
\]
where \( [\hat{E}_m(k)]_{n_r,n_r'} = E_{n_r,m}(k)\delta_{n_r,n_r'}, \) with \( E_{n_r,m}(k) = \hbar^2/2M + \hbar\omega_\perp(2n_r + m) \). Following a similar procedure as in Ref. [23], the LHY energy correction \( \Delta E_{\text{LHY}} \) may be obtained from the differential equation [21]
\[
\frac{\Delta E_{\text{LHY}}}{\hbar} = \frac{1}{2} \frac{d}{dn_{1D}} \left( \frac{\Delta E_{\text{LHY}}}{\hbar} \right) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \sum_{n_r} \sum_{m} \left[ E_{n_r,m}(k) - \xi_{\nu}(\mathbf{v})^2 \right],
\]
with \( L \) the quantization length [24]. Figure 2 shows the LHY correction of the chemical potential, \( \Delta \mu_{\text{LHY}} = \langle d/dn_{1D} \rangle (\Delta E_{\text{LHY}}/\hbar) \), for different \( g_{1D}n_{1D}/\hbar\omega_\perp = n_{1D}a \).

For \( n_{1D}a < 1 \), the effect of the transversal modes is, as expected, negligible, and the LHY correction remains attractive. However, whereas for contact interacting systems \( \Delta \mu_{\text{LHY}} \propto n_{1D}^{1/2} \) [16], the density dependence in dipolar condensates is radically different. For \( n_{1D}a \rightarrow 0 \), \( \Delta \mu_{\text{LHY}} \propto -n_{1D} \), whereas for growing \( n_{1D}a \), \( \Delta \mu_{\text{LHY}} \) departs from the linear dependence (top inset of Fig. 2). This is crucial for the physics of 1D droplets, as discussed below.

For \( n_{1D}a > 0.1 \), transversal excitations become significant. The LHY correction reaches a maximal negative value at \( n_{1D}a = 0.2 \) and then increases, becoming repulsive for

\[ \Lambda \equiv (\Delta \mu_{\text{LHY}}/\hbar\omega_\perp)/l_{1D}/a \] as a function of \( n_{1D}a \) for a homogeneous 1D dipolar condensate. The top inset depicts \( \log_{10}(\log_{10}(n_{1D}a)) \) against \( \log_{10}(n_{1D}a) \), showing that \( \Delta \mu_{\text{LHY}} \propto n_{1D} \) for \( n_{1D} \rightarrow 0 \). The bottom inset shows \( \Delta E_{\text{LHY}}/N \) as a function of \( n_{1D}a \).
with $\mu R_l$, the Gross-Pitaevskii equation is an approximation, i.e., substituting in Eq. (4) a scattering [25]. Neglecting quantum corrections, the interaction results as in Bose-Bose mixtures [16] in the formation of a soliton.

This radical change in the nature of the quantum correction occurs, the soliton density is significantly lower than that expected from mean-field theory (up to a factor of 2 in Fig. 4). Moreover, since for large-enough densities, $\Delta \mu_{LHY}$ and $\Delta \mu$ smoothly for increasing $n$, the system smoothly crosses over into the soliton regime. For $e_{dd} > 1$, the soliton density grows smoothly for increasing $e_{dd}$, and the LHY correction changes eventually from attractive to repulsive. When this occurs, the soliton density is significantly lower than that expected from mean-field theory (up to a factor of 2 in Fig. 4). Moreover, since for large-enough densities, $\Delta \mu_{LHY} \propto -n_{1D}^{1/2}$, the effect of the LHY correction remains relevant even far from the mean-field instability. This must be compared to the case of nondipolar Bose-Bose mixtures, where the $\Delta \mu_{LHY} \propto -n_{1D}^{1/2}$ dependence renders the LHY correction.

At $e_{dd} = 1$, the system smoothly crosses over into the soliton regime. For $e_{dd} > 1$, the soliton density grows smoothly for increasing $e_{dd}$, and the LHY correction changes eventually from attractive to repulsive. When this occurs, the soliton density is significantly lower than that expected from mean-field theory (up to a factor of 2 in Fig. 4). Moreover, since for large-enough densities, $\Delta \mu_{LHY} \propto -n_{1D}^{1/2}$, the effect of the LHY correction remains relevant even far from the mean-field instability. This must be compared to the case of Bose-Bose mixtures, where the $\Delta \mu_{LHY} \propto -n_{1D}^{1/2}$ dependence renders the LHY correction.
basically negligible within the soliton regime. Note that for sufficiently large \( \epsilon_{d3} > 1 \), eventually \( \mu \leq \hbar \omega_{l1} \), and the condensate crosses over into the 3D regime, where the repulsive LHY prevents collapse. This would correspond to the elongated 3D macro-droplet regime recently explored experimentally [12]. The description of this crossover, however, lies beyond the scope of this Letter.

Three-body correlations.—Whereas in mean-field approximation three-body correlations fulfill \( g^{(3)} = 1/n(\bar{r}^3) \bar{\Psi}(\bar{r})^3 = 1 \), quantum corrections may significantly correct its value, \( g^{(3)} = 1 + \Delta g^{(3)} \), and hence in turn the three-body loss rate. As for the LHY correction, the momentum dependence of the density dependence in dipolar condensates. The correction of the dipolar interactions leads to a markedly different situation is radically different in 1D. For a 1D nondipolar condensate \( \Delta g^{(3)} = -(6/\pi) \sqrt{\gamma} \) [28], with \( \gamma = 2a/\sqrt{n_{1D}} \ll 1 \). Three-body corrections are hence reduced by quantum effects, and the correction increases for a decreasing density, since, counterintuitively, 1D systems are more strongly interacting the more dilute they are. As for the LHY correction, the momentum dependence of the dipolar interactions leads to a markedly different density dependence in dipolar condensates. The correction of \( g^{(3)} \) averaged over the transversal degree of freedom [29] may be evaluated from the LHY correction using the Hellmann-Feynman theorem [21],

\[
\Delta g^{(3)} = \int \frac{d^3r}{L} \frac{\psi(x,y)^4}{n(\bar{r}^3)} \left( \frac{\bar{\Psi}(\bar{r})^3}{n(\bar{r})^3} - 1 \right) = \frac{6}{n_{1D}L} \frac{\partial E_{LHY}}{\partial g_{1D}} = -\frac{6}{\pi} \sqrt{\gamma} \beta(\epsilon_{d3}, n_{1D}a),
\]

where \( \beta(\epsilon_{d3}, n_{1D}a) \) is depicted in Fig. 5. For small \( n_{1D}a \), \( \Delta g^{(3)} \propto -n_{1D}^{-1/2} \), with \(-1/2 < \lambda < 0\). As for nondipolar condensates, \( \Delta g^{(3)} \) remains negative and increases with decreasing \( n_{1D} \), albeit with a significantly modified power law. In contrast, when \( n_{1D}a > 0.42 \), the growing role of the transversal modes results in a change in the sign of \( \Delta g^{(3)} \); i.e., three-body corrections remain in the 1D regime. For \( n_{1D}a \gg 1 \), \( \Delta g^{(3)} \propto n_{1D}^{1/2} \), as expected for 3D condensates. This nontrivial behavior of three-body correlations in quasi-1D dipolar condensates may be probed in on-going experiments with magnetic atoms using similar techniques as those applied in nondipolar quasi-1D condensates [26].

Conclusions.—The momentum dependence of the dipolar interactions leads to strikingly different quantum effects in quasi-1D dipolar condensates compared to their nondipolar counterparts. In contrast to Bose-Bose mixtures, quantum stabilization is disrupted in dipolar condensates at low densities due to the modified density dependence of the LHY correction. As a result quantum droplets only exist in a window of density values. Moreover, although the condensate remains one dimensional, the LHY may be crucially affected by transversal modes, which induce a change from attractive to repulsive LHY correction at a critical density. This change of character results in a significant reduction of the peak density of the soliton, as well as a modification of its shape. Hence quantum corrections should be carefully considered in future studies of dipolar solitons. Furthermore, the peculiar nature of quantum fluctuations is also reflected in the beyond-mean-field correction of three-body losses, which also changes its sign within the 1D regime for growing density. Our results open intriguing questions about 2D dipolar condensates, where we expect a similar nontrivial density dependence of the quantum corrections, as well as about the role of transverse modes in anharmonic transversal confinement.

This surprising physics of low-dimensional dipolar condensates can be readily probed in current experiments with magnetic atoms.

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For a general angle $\alpha$ between the dipole moment and the $z$ axis, $\epsilon_{dd} = \mu_0 \mu_0^z (3 \cos^2 \alpha - 1)/6 \theta_{3D}$. Hence, contrary to 3D systems, the crucial parameter $\epsilon_{dd}$ may be externally reduced in 1D by modifying $\alpha$. This may be particularly useful in the case of dysprosium, where $\epsilon_{dd} > 1$ for $\alpha = 0$, for which the crossover through $\epsilon_{dd} = 1$ may be realized without the need of a Feshbach resonance.

In a homogeneous quasi-1D dipolar condensate, $\mu = g_{1D} n_{1D} (1 - \epsilon_{dd})$. Hence $\mu/\hbar \omega_L = 2 n_{1D}(1 - \epsilon_{dd})$.


In the vicinity of $\epsilon_{dd} = 1$, which is our regime of interest, quantum corrections vary only very slightly with $\epsilon_{dd}$.

$g_{1D}$ acquires a more involved dependence when $l \sim a$ [M. Olshanii, Phys. Rev. Lett. 81, 938 (1998)]. For simplicity of the theoretical analysis, we do not consider this case, which would demand separate treatment.

Higher-order corrections could be evaluated following the procedure discussed in Ref. [23]. However, in the weakly interacting regime considered in this Letter, only the LHY correction plays a relevant role.

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g^{(3)} = 1 + (3/n_{3D} \int (d^3 k/(2\pi)^3) [(\hbar^2 k^2)/(2M E(k))] - 1 + [2M \hat{U}(k)n_{3D}/(\hbar^2 k^2)]$, where $\hat{U}(k) = g_{1D} [1 + \epsilon_{dd} (3 \cos^2 \theta - 1)]$, with $\theta$ the angle between $k$ and the dipole moment, and $E(k)^2 = (\hbar^2 k^2/2M) (\hbar^2 k^2/2M) + 2 \hat{U}(k)n_{3D}$.


Note that the three-body losses are proportional to $g^{(1)} n(\vec{r})^2$. Hence, the proper transversal average is performed by averaging over $\psi(x, y)^4$. 