Experimental Multiparticle Entanglement Swapping for Quantum Networking

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This Letter reports the first experimental demonstration of Greenberger-Horne-Zeilinger (GHZ) entanglement swapping. We start with three pairs of entangled photons. Upon projection of three single photons, each from an entangled pair, into a GHZ state, the other three originally independent photons are entangled in a GHZ state—creation of multiparticle entanglement without any direct interaction. This scheme may facilitate networks for quantum telephone exchange, multiparty quantum communication and distributed quantum computation.

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The creation and manipulation of entanglement lies at the heart of quantum information science. A particularly intriguing and useful method to create entangled states is entanglement swapping [1]. For a simple scenario of entanglement swapping, we start with two entangled pairs $\mathcal{A}_1\mathcal{A}_2$ and $\mathcal{B}_3\mathcal{B}_4$, which can be locally generated at two distant locations $\mathcal{A}$ and $\mathcal{B}$ respectively. We then jointly measure the particles $\mathcal{A}_1$ and $\mathcal{B}_3$ in the Bell basis; this will project $\mathcal{A}_2$ and $\mathcal{B}_4$ in an entangled state, although they may be far apart, have never interacted or share any common past. Because of its ability of entangling distant qubits, entanglement swapping has found a unique place in quantum networking and quantum repeater protocols [2].

A more interesting situation arises when entanglement swapping is exploited to manipulate multiparticle entanglement. Given a prior distribution of entangled pairs, Bose *et al.* [3] have proposed to use entanglement swapping to establish multiparticle entanglement among distant nodes in a quantum network. Such multiparticle entangled states are important physical resources in distributed quantum computation [4] and multiparty quantum communication schemes such as quantum cryptographic conference [3] and secret sharing [5].

So far, the protocol of entanglement swapping has been tested in experiments with photons [6–12], atom-photon entanglement [13,14] and on an ion-trap quantum computer [15]. The previous works have demonstrated the simplest case of entanglement swapping which involves only two-qubit entangled states. However, the realization of multiparticle entanglement swapping remained an experimental challenge. Here, the challenge not only lies in the control of increased number of qubits, but also in the joint projective measurements of multiparticle entanglement. In this Letter we overcome these difficulties and demonstrate the entanglement swapping of a three-photon Greenberger-Horne-Zeilinger (GHZ) state [16].

In the GHZ entanglement swapping, let us assume a quantum network that consists of a central exchange (Ex) and three users A, B, and C at different locations [3] (see Fig. 1). The Ex shares with the users A, B, and C three entangled pairs—$\mathcal{A}_1\mathcal{A}_2$, $\mathcal{B}_3\mathcal{B}_4$, and $\mathcal{C}_5\mathcal{C}_6$, which can be written in the form of $|\phi\rangle_{ij} = (1/\sqrt{2})(|0\rangle_i|0\rangle_j + |1\rangle_i|1\rangle_j)$. Now suppose A, B, and C wish to share a GHZ state for a certain quantum information task. To do so, the Ex can perform a joint measurement of the qubits 2, 4, and 6 at his hand, projecting them into a three-qubit GHZ state. We may write the wave function of the whole system as (coefficients omitted for clarity):

$$\begin{aligned}&|\phi\rangle_{12}\otimes|\phi\rangle_{34}\otimes|\phi\rangle_{56}\\&=|\phi^+\rangle_{135}|\phi^+\rangle_{246}+|\phi^-\rangle_{135}|\phi^-\rangle_{246}+|\psi^+\rangle_{135}|\psi^+\rangle_{246}\\&\quad+|\varphi^+\rangle_{135}|\varphi^+\rangle_{246}+|\varphi^-\rangle_{135}|\varphi^-\rangle_{246}+|\psi^-\rangle_{135}|\psi^-\rangle_{246}\\&\quad+|\chi^+\rangle_{135}|\chi^+\rangle_{246}+|\chi^-\rangle_{135}|\chi^-\rangle_{246},\end{aligned}$$

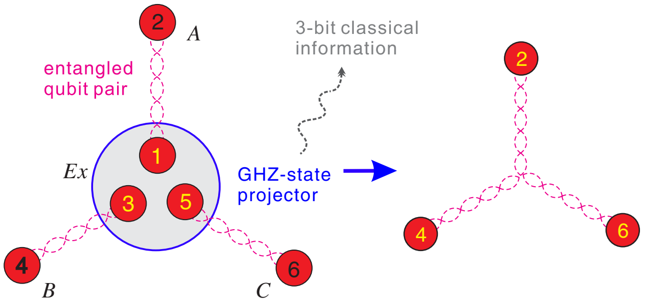

FIG. 1 (color online). Configuration of a multiparty quantum network and GHZ entanglement swapping. Initially, users A, B, and C share entangled qubit pairs with the central exchange Ex. If Ex projects the three particles, 1, 3, and 5, into a GHZ state, the other three particles, 2, 4, and 6 belonging to A, B, and C respectively, will be entangled into a GHZ state by entanglement swapping.

where

$$
\begin{aligned}
|\phi^{\pm}\rangle_{ijk} &= (|0\rangle_i|0\rangle_j|0\rangle_k \pm |1\rangle_i|1\rangle_j|1\rangle_k),\\
|\psi^{\pm}\rangle_{ijk} &= (|0\rangle_i|0\rangle_j|1\rangle_k \pm |1\rangle_i|1\rangle_j|0\rangle_k),\\
|\varphi^{\pm}\rangle_{ijk} &= (|0\rangle_i|1\rangle_j|0\rangle_k \pm |1\rangle_i|0\rangle_j|1\rangle_k),\\
|\chi^{\pm}\rangle_{ijk} &= (|0\rangle_i|1\rangle_j|1\rangle_k \pm |1\rangle_i|0\rangle_j|0\rangle_k),
\end{aligned}
\tag{1}
$$

are the eight orthogonal GHZ states spanning the whole three-qubit Hilbert space. It is clear from above that, the three distant qubits belonging to A, B, and C will be subsequently entangled into one of the eight GHZ states according to Ex's measurement result. Thus the generation and distribution of multiparticle entangled states is achieved, although the qubits 2, 4, and 6 have never directly interacted. As an application, this network configuration can work as a quantum telephone exchange [3]. By sharing entangled pairs between the Ex and each of its users, the establishment of biparticle or multiparticle entanglement between any subset of the users can be done flexibly when the necessity arises; no prior arrangement is required.

We now proceed with the experimental demonstration. The experimental setup is illustrated in Fig. 2. A femtosecond ultraviolet laser is successively passed through three β-barium borate (BBO) crystals. Three entangled photon pairs are produced by type-II spontaneous parametric down-conversion [17] and prepared in the form of $|\phi\rangle = (1/\sqrt{2})(|H\rangle|H\rangle + |V\rangle|V\rangle)$ [H (V) denotes horizontal (vertical) polarization]. We observed an average two-photon coincidence count rate of ~100 KHz and a mean visibility of 93% in the H/V basis and 91% in the $+/-$ ($\pm = H \pm V$) basis.

Next, we aim to perform the joint measurement on photons 1, 3, and 5 to project them into a GHZ state. This implies nonlinear interactions among the three qubits, which would normally need two controlled-NOT gates (or equivalent). For the photonic qubits that interact weakly, it is convenient to exploit a partial GHZ-state projector using linear optics and post-selection [18]. We superpose photons 1 and 3 on a polarizing beam splitter (PBS1), and combine one of its output with photon 5 on the PBS2 (see Fig. 2, dashed box). Fine adjustments of the delays ($\Delta d1$, $\Delta d2$) between the different paths are made so that the photons arrive at the PBSs simultaneously. Furthermore, the photons are spectrally filtered and detected by single-mode fiber-coupled single-photon detectors to ensure good temporal and spatial overlap, thus making the original independent photons 1, 3, and 5 now indistinguishable.

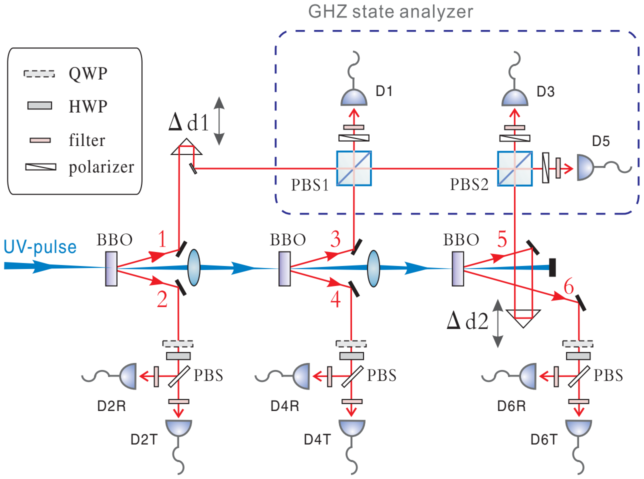

FIG. 2 (color online). Experimental setup for entanglement swapping of a three-photon GHZ state. Ultraviolet laser pulses (with a central wavelength of ~394 nm, a pulse duration of ~120 fs, and a repetition rate of ~76 MHz) are focused on three BBO crystals, producing entangled photon pairs emitted into spatial modes 1–2, 3–4, and 5–6. Photons 1, 3, and 5 are projected into a GHZ state (dashed box, see text and Ref. [18]), and the photons 2, 4, and 6 are analyzed by a combination of a quarter-wave plate (QWP), a half-wave plate (HWP) and a PBS. The photons are spectrally filtered by narrow-band filters ($\Delta\lambda_{\mathrm{FWHM}} = 3.2$ nm) and monitored by fiber-coupled silicon avalanche single-photon detectors (D1, D2T, $\cdots$, D6R). The multiphoton events are registered by a laser clocked multichannel coincidence unit.

As the PBSs transmit H and reflect V polarizations, a coincidence detection of the three outputs can only originate from either the case that all photons are transmitted (if the input state is $|H\rangle|H\rangle|H\rangle$) or all reflected (if the input state is $|V\rangle|V\rangle|V\rangle$)—two cases quantum mechanically indistinguishable if the photons are perfectly overlapped spatially and temporally. Thus, the two GHZ states $|\phi^{\pm}\rangle = (|H\rangle|H\rangle|H\rangle \pm |V\rangle|V\rangle|V\rangle)$ can be distinguished out of the overall set of eight [see Eq. (1)]. These two GHZ states $|\phi^{\pm}\rangle$ can be further separated by placing a polarizer after each PBS, setting at the $+/-$ basis. In this basis, while the state $|\phi^{+}\rangle$ leads to a coincidence event $+++$, $+--$, $-+-$, or $--+$, $|\phi^{-}\rangle$ results in an event $++-$, $+-+$, $-++$, or $---$. The overall success probability of the GHZ analyzer is thus $1/4$. Nevertheless, it is sufficient to demonstrate the working principle of multiparticle entanglement swapping. Using ancilla photons and teleportation-assisted linear-optical gates, the efficiency of the GHZ-state analyzer can, in principle, be improved to near unity [19].

The GHZ-state analyzer requires the three independent photons 1, 3, and 5 to be indistinguishable. Figure 3 shows a step-by-step verification of the indistinguishability as a function of temporal delays. First we consider the setup with the photons 1, 2, 3, and 4 and register the fourfold coincidences at the $+/-$ basis [see Fig. 3(a)]. At zero delay where the photons are optimally overlapped in time, the four-photon events $|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4$ and $|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4$ become indistinguishable and form a superposition state. Therefore, the counts of $|+\rangle_1|+\rangle_2|+\rangle_3|+\rangle_4$ show an enhancement due to the Hong-Ou-Mandel-Shih-Alley [20] and Rarity-Zeilinger [21] type interference, while the $|+\rangle_1|+\rangle_2|+\rangle_3|-\rangle_4$ events show a dip. The increase of the delay gradually destroys the quantum indistinguishability so the state becomes a classical mixture. Thus, at large delays the counts appear flat. This type of interferometer is sensitive only to length changes on the order of the coherence length of the detected photons (≈160 μm) and stay stable for days. Similarly, the setup and data for another four-photon interferometer 1, 2, 5, and 6 is presented in Fig. 3(b), where the photon 2 is used as a trigger and the photon 1 is prepared in the $|+\rangle$ state to be combined with the entangled pair 5–6 on the PBS2. The visibilities of the raw data in Figs. 3(a) and 3(b) are 0.726 ± 0.032 and 0.732 ± 0.041, respectively. Finally, the interference involving all three-photon pairs is shown in Fig. 3(c), where the six-photon coincidence is registered in the basis of $|+\rangle_1|+\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5|+\rangle_6$ and $|+\rangle_1|+\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5|-\rangle_6$ as a function of the delay ($\Delta d1$). A visibility of 0.537 ± 0.093 is observed, which is limited by double photon-pair emission and partial distinguishability of the independent photons [22].

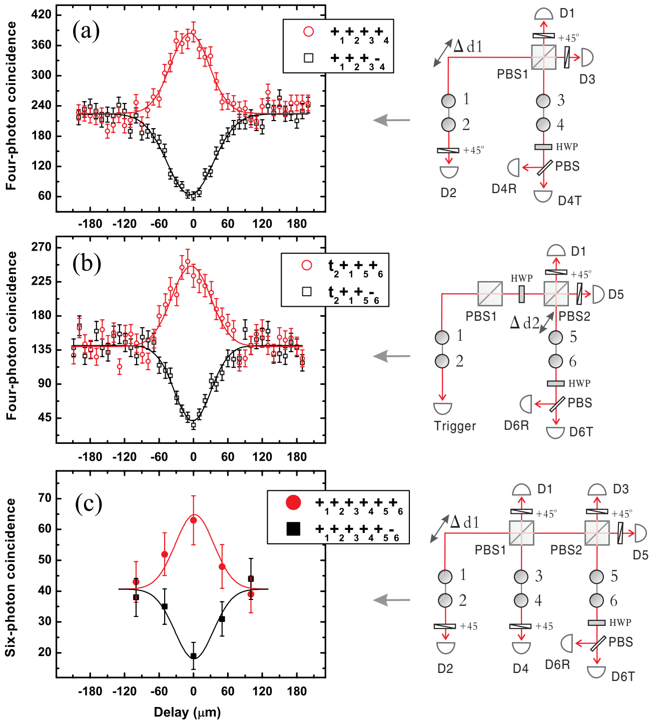

FIG. 3 (color online). Step-by-step observation of the Hong-Ou-Mandel-Shih-Alley [20] and Rarity-Zeilinger [21] type multiphoton interferences as a function of temporal delay. The right insets show briefly the setup employed for each step. (a) The photon pairs 1–2 and 3–4 are prepared in the $|\phi\rangle$ state. After combining the photons 1 and 3 on the PBS1, the four photons are analyzed in the $+/-$ polarization setting as a function of the delay ($\Delta d1$). (b) The photon 2 is used as a trigger. The photon 1 is directed through the PBS1, prepared in the $|+\rangle$ state using a HWP, and then superposed on the PBS2 with the photon 5. (c). The photons 1, 3, and 5 are superposed on the two PBSs as required by the GHZ-state analyzer (see text) and the six-photon coincidence in the $|\pm\rangle$ basis is registered, showing constructive and destructive interference. The accumulation time for the multiphoton coincidence counts in (a),(b), and (c) is 60 s, 40 s, and 18 h, respectively. Gaussian curves are to guide the eyes.

With the delays set at zero, we now project the photons 1, 3, and 5 into a GHZ state by passing them through the PBSs and detecting them in the $+/-$ basis. We choose to distinguish the state $|\phi^+\rangle_{135}$ by the coincidence event of $|+\rangle_1|+\rangle_3|+\rangle_5$; the total efficiency of entanglement swapping is thus $1/8 \times 1/4 = 1/32$. Having done the GHZ-state projection, the other three photons 2, 4, and 6 should be entangled into the GHZ state $|\phi^+\rangle_{246}$. To verify this, we need to determine the fidelity of the three-photon state and detect the presence of genuine triplet entanglement [23]. The fidelity is defined as the overlap of the experimentally produced state with the ideal one: $F_{\phi^+} = \langle\phi^+|\rho_{\mathrm{exp}}|\phi^+\rangle$. The density matrix $|\phi^+\rangle\langle\phi^+|$ can be decomposed as [25]

$$\begin{aligned}|\phi^+\rangle\langle\phi^+| = (1/2)[(&|HHH\rangle\langle HHH| + |VVV\rangle\langle VVV|)\\ &+ (1/3)(-\sigma_{-1}\sigma_{-1}\sigma_{-1} + \sigma_0\sigma_0\sigma_0\\ &- \sigma_1\sigma_1\sigma_1)],\end{aligned}$$

where $\sigma_\kappa = \cos(\kappa\pi/3)\sigma_x + \sin(\kappa\pi/3)\sigma_y$ (σ_x and σ_y are the Pauli matrices). The first term indicates the population

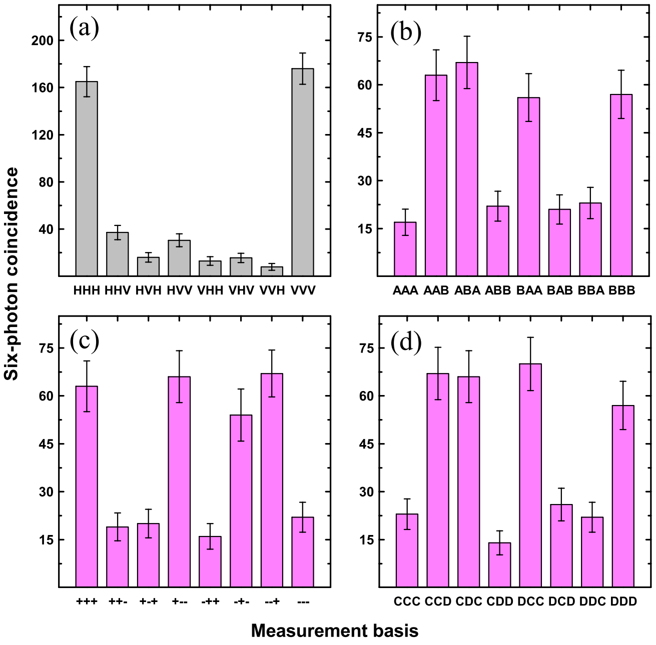

FIG. 4 (color online). Sixfold coincidence in the measurement basis of: (a) H/V, (b) A/B, (c) $+/-$, and (d) C/D for witnessing the genuine entanglement of the three emerging photons 2, 4, and 6. The accumulation time for each data set is 24 h in (a) and 18 h in (b),(c), and (d). The error bars represent 1 standard deviation deduced from Poissonian counting statistics of the raw detection events.

of $|HHH\rangle$ and $|VVV\rangle$ over all possible 8 combinations, the results for which are displayed in Fig. 4(a). The second term is a manifestation of the coherence of the GHZ state, essentially referring to the far off-diagonal elements of the GHZ-state density matrix. Corresponding measurements are performed in the basis of $A/B = H \pm e^{-i\pi/3}V$, $+/-$, and $C/D = H \pm e^{i\pi/3}V$ for $\kappa = -1,0,1$, respectively, with the results showing in Figs. 4(b)–4(d). From these data the fidelity of the three-photon GHZ state can be extracted: $F_{\phi^+} = 0.624 \pm 0.017$. A fidelity above 0.5 for the GHZ state is sufficient to witness the presence of genuine entanglement [24]. This confirms that the photons 2, 4, and 6 have been truly entangled, and thus proves the working principle of GHZ entanglement swapping.

In conclusion, we have completed the first experimental demonstration of multiparticle entanglement swapping, where a three-photon GHZ state is created without direct interaction. In practical quantum networks, this method can be used to flexibly establish bi- or multipartite entanglement among distant nodes, and thus may find applications in quantum telephone exchange, multiparty cryptography and distributed quantum computing. From the perspective of optical one-way quantum computation, our experiment can be seen as growing a triggered three-qubit cluster state [26] from Bell states by photon fusion [27]—signaled by a successful GHZ-state projection—which in principle works in an "event-ready" fashion and does not require photon-number-discriminating detectors [28].

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[23] The notion of genuine multipartite entanglement characterizes that the experimentally created N-qubit state shows entanglement effects which cannot be produced by $N-1$ qubits and all parties must have participated in the creation of the state. Technically, a pure state is called biseparable, whenever a grouping of the N parties into two groups can be found, such that the state is written as a product state of them. A mixed state is called biseparable, if it is a mixture of biseparable pure states. A state which is not biseparable is called genuine multipartite entangled. See also Refs. [24].

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