PHYSICAL REVIEW D **71,** 127505 (2005)

Correction of source mass effects in the HUST-99 measurement of *G*

Zhong-Kun Hu,* Jun-Qi Guo, and Jun Luo†

Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China

(Received 26 May 2005; published 27 June 2005)

Eccentricities of the mass center from the geometric center of two cylindrical source masses used in the HUST-99 measurement of the Newtonian gravitational constant G are determined by means of an electronic balance. Considering a linear density distribution of source masses as well as the effect of the air buoyancy, our value of G should be revised to be $6.6723(9) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, which is 0.036% larger than our previous published value in Phys. Rev. D **59**, 042001 (1999).

DOI: 10.1103/PhysRevD.71.127505 PACS numbers: 04.80.Cc, 06.20.Jr

The absolute value of the gravitational constant G plays an important role in physics, but it is the least precisely known among the fundamental physical constants. In 1998 the CODATA Task Group on Fundamental Constants decided to increase the uncertainty in the recommended value for G to 1500 ppm. Since then, four new values of G have been published with relative uncertainties below 50 ppm. These, with their assigned uncertainties, are the experiments carried out by Gundlach and Merkowitz (14 ppm) in 2000 [1], Quinn *et al.* (41 ppm) in 2001 [2], Schlamminger *et al.* (33 ppm) in 2002 [3], and Armstrong and Fitzgerald (40 ppm) in 2003 [4]. In 2002, the CODATA task group recommended for the Newtonian gravitational constant a value of 6.6742×10^{-11} m^3 kg^{-1} s^{-2} with the uncertainty of 150 ppm [5].

We have determined G to 105 ppm by means of the time-of-swing method with a high-Q torsion pendulum [6]. Following the publication of the HUST-99 result, it was decided to look further for systematic effects in the measurement. We have studied physical properties of torsion pendulums and found that the nonlinearity and thermoelasticity of the torsion fiber would only introduce an error less than 1 ppm in our experiments [7–9]. When we checked the density inhomogeneities and the mass of two cylindrical source masses, we found some error sources had been neglected in the HUST-99 measurement.

In our HUST-99 experiment, two 6.25 kg stainless steel cylinders, respectively, marked A and B on one face were used as source masses. They were placed on either side of the lower test mass with the marked faces close to the test mass, and the test mass lay on their axial symmetry. In this configuration, it is more sensitive to the axial position than the radial position of the cylinders. For example, a 1 μm axial uncertainty would introduce 14 ppm uncertainty in the value of G, while a 500 μm radial uncertainty would only introduce an 11 ppm uncertainty in G. The axial eccentricities of the cylindrical source masses are measured using a weighbridge with a beam balance, which is very similar to the method used in Ref. [10] to locate the center of mass of a test object with the precision of micrometers. The schematic view of the experiment is shown in Fig. 1. An aluminum alloy weighbridge with three brass braces is used in this measurement. The left brace is centered on the balance pan and the right two are put on a heavy brass cylinder. The balance (Mettler-Toledo PR2004) has a capacity of 2.3 kg, a resolution of 0.1 mg, and a repeatability of 0.3 mg. The cylindrical source mass is positioned on the U-shaped notch of the weighbridge with one face against a fixed micrometer gauge. If the cylinder M_A is placed on the weighbridge with the marked face on the left, the relation between the change in the reading of the balance M_L and that of the micrometer x must be

$$M_L = \frac{L_0 + e}{L} M_A + \frac{M_A}{L} x \equiv M_{L0} + K_L x, \qquad (1)$$

where L and L_0 are the distances of the linking line the left

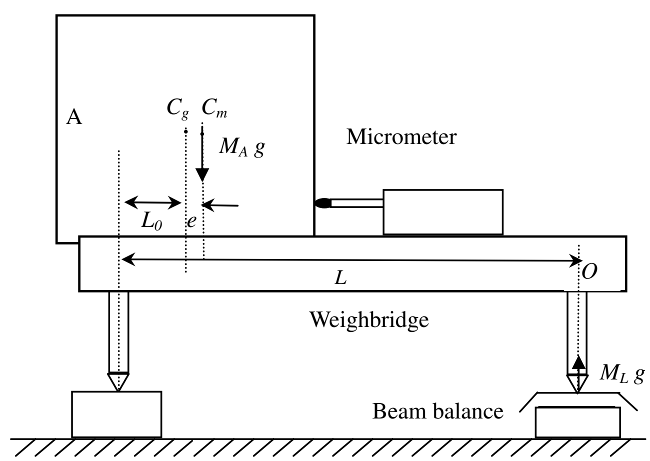

FIG. 1. Apparatus for measuring the eccentricity of the center of mass C_m of the cylinder A from its geometric center C_g. An aluminum alloy weighbridge with span L is put on a beam balance and a heavy brass cylinder. The cylindrical source mass A is positioned on the U-shaped notch of the weighbridge with one face against a fixed micrometer gauge.

*Presently at the Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA.
†To whom correspondence should be addressed.
Email address: junluo@mail.hust.edu.cn

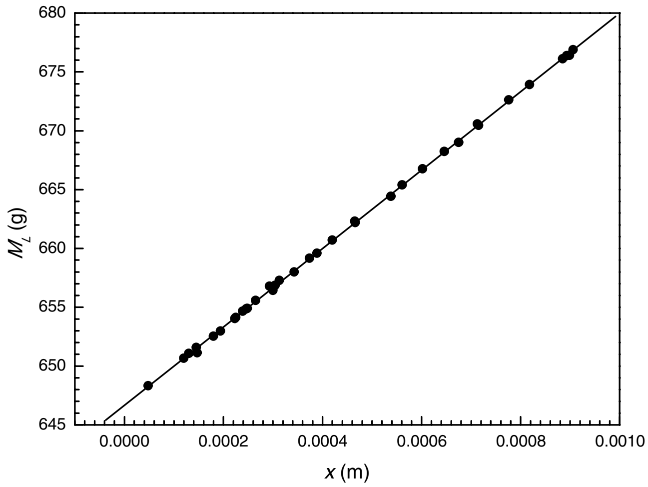

FIG. 2. The value of M_L versus the cylinder relative position x. Circle points represent the experimental data, and the solid line represents the linear fitting result of M_L varying with x.

pivots from the right pivot and from the geometric cylinder center as the case of the micrometer reading is zero, respectively, and e is the eccentricities of the mass center from the geometric center in the axial direction. M_{L0} and K_L are the intercept and the slope of the line of M_{L0} versus x, respectively. When the cylinder is put on the weighbridge with the marked face on the right, the change in the reading of the balance M_R varying with x can be written as

$$M_R = \frac{L_0 - e}{L} M_A + \frac{M_A}{L} x \equiv M_{R0} + K_R x. \quad (2)$$

According to the above equations, the value of e can be determined as

$$e_A = \frac{M_{L0} - M_{R0}}{K_R + K_L}. \quad (3)$$

Figure 2 shows the value of M_L versus the relative position of the cylinder indicated by the value of x. The circle points represent the experimental data, and the solid line represents the linear fitting result of M_L varying with x. The slope of the fitted line is $K_L = (33\,329 \pm 90)$ g/m, and the intercept is $M_{L0} = (646.657 \pm 0.046)$ g. With the marked face on the right, we obtain that $K_R = (33\,569 \pm 325)$ g/m, and $M_{R0} = (645.971 \pm 0.164)$ g. So the eccentricity of cylinder A is

$$e_A = (10.3 \pm 2.6)\ \mu\text{m}. \quad (4)$$

With the same method, the eccentricity of cylinder B is determined as

$$e_B = (6.3 \pm 3.7)\ \mu\text{m}. \quad (5)$$

The positive values of e_A and e_B mean that the centers of mass are shifted away from the marked faces, relative to the geometric centers. The marked faces of the two cylinders are closest to the test mass in the experiment. This means that the equivalent displacements between the test mass and the source masses are larger than those given in Ref. [6], tending to make our previous published value of G too small. To calculate this effect, we suppose that the density of the cylinder has a linear distribution along the axial direction. The value of G should be 210 ppm larger than our previous value HUST-99 due to this effect. It also will introduce an additional uncertainty in G of 78 ppm due to uncertainties of the source mass eccentricities. Furthermore, G is determined by comparing the period of a torsion pendulum with and without the source masses in position. When the source mass is moved out, it was replaced by air. The masses of the source mass given in Ref. [6] are the vacuum masses. The air density in the experimental environment is about 1.19 kg/m^3. This effect introduced another error of 150 ppm, which also tended to make the previous value of G smaller.

In summary, considering the eccentricities of the source masses as well as the effect of the air buoyancy, the value of G should be 360 ppm larger than our previous published value, and the relative uncertainty should be increased from 105 to 130 ppm. Finally, our value of the gravitational constant G should be revised to

$$G = (6.6723 \pm 0.0009) \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}. \quad (6)$$

Of course, different distributions of source mass density would introduce different system errors to the value of G. To improve the experimental precision, we can use more homogeneous source masses and more different orientations. In fact, spherical source masses, whose orientations may be more easily changed, have been used in our new experiment design for determination of G.

We would like to thank R. D. Newman of the University of California at Irvine, and T. J. Quinn and R. S. Davis of the Bureau International des Poids et Mesures for their valuable discussions. This work was supported by the National Natural Science Foundation of China under Grant No. 10121503, and the National Basic Research Program of China under Grant No. 2003CB716300.

[1] J. H. Gundlach and S. M. Merkowitz, Phys. Rev. Lett. **85**, 2869 (2000).
[2] T. J. Quinn *et al.*, Phys. Rev. Lett. **87**, 111101 (2001).
[3] S. Schlamminger, E. Holzschuh, and W. Kündig, Phys.

Rev. Lett. **89**, 161102 (2002).
[4] T. R. Armstrong and M. P. Fitzgerald, Phys. Rev. Lett. **91**, 201101 (2003).
[5] http://physics.nist.gov/cgi-bin/cuu/Value?bg
[6] J. Luo, Z. K. Hu, X. H. Fu, and S. H. Fan, Phys. Rev. D **59**, 042001 (1999).
[7] Z. K. Hu, J. Luo, and H. Hsu, Phys. Lett. A **264**, 112 (1999).
[8] J. Luo, Z. K. Hu, and H. Hsu, Rev. Sci. Instrum. **71**, 1524 (2000).
[9] J. Luo and Z. K. Hu, Classical Quantum Gravity **17**, 2351 (2000).
[10] R. S. Davis and T. J. Quinn, Meas. Sci. Technol. **6**, 227 (1995).