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Weak scale measurement constraints on string models

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The result of the modular-invariant one-loop string effective coupling constant for a large class of models is used to discuss the weak scale measurement constraints on superstring models. Superstring models with intermediate gauge symmetry breaking are proposed to account for the grand gauge unification scale $M_{\rm GUT} = 10^{16}$ GeV, which is deduced from the precision weak scale measurement in the minimal SUSY unification model. A gaugino-condensation-induced Hosotani mechanism is suggested for the intermediate gauge symmetry breaking. In this scheme, supergravity, gauge symmetry, and space-time compactification are induced by one nonperturbative dynamics: gaugino condensation. The string scale μ_s is calculated in intermediate SU(5) string models. A relation between the intermediate gauge symmetry-breaking scale $M_I = M_{\rm GUT}$ and string scale is found to be $\mu_s^3 = M_I m_P^2$. It is argued that this relation is a general result for all the superstring models with intermediate gauge symmetry breaking. The level one minimal SUSY left-right string models are shown to be excluded by weak scale measurement. The constraints on the ratio of affine levels is worked out. It is demonstrated that string unification is more restricted by the experiments than any other unification schemes are.

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I. INTRODUCTION

In this work, the constraints and implications from the weak scale measurements on string models are examined. In an explicitly worked out string model, where the matter content is specified and the vacuum expectation values (VEV's) of the dilaton and the moduli are determined [1], the string threshold correction can be calculated and the weak scale coupling constants can be derived. But at the present stage of string phenomenology, we are far from making these kinds of predictions. We are more interested in "taming the zoo of string models" with the help of low energy phenomenology.

We use the modular-invariant one-loop string effective gauge coupling constant derived in [2] and the analyzing method developed in [3]. The computation in [2] indicates that the string scale μ_s is the scale of two-loop gauge coupling unification, up to the possible additional modular-dependent threshold corrections. Although this result is computed explicitly for E_8 orbifold models, I argue that, except for the explicit form of the string threshold corrections, it covers a large class of models including orbifold models, free fermion models, and Calabi-Yau models with arbitrary gauge interaction content, since the derivation is quite general which does not depend on the gauge interaction content in a model and the form of Kähler potential used for calculation is not just limited to orbifold models.

It has been found [3] that the constraints from the low energy measurements on string models are more restrictive than on any other unification models. So far minimal superstring unification and especially the contribution from the string threshold correction to the gauge coupling unification have been discussed extensively [4–7,3]. In [6] the constraints on the modular weights of the quark, lepton, and Higgs superfields has been worked out. In our previous paper [3], we worked out the restriction on the particle spectrum in a supersymmetric (SUSY) minimal string model without string threshold corrections. Our method of analysis developed in [3] is different from [6] in that we identify string scale $\mu_s = \alpha \tilde{m}_P$ with the unification scale $g_a^{-2}(\mu_s) = k_a g^{-2}$ for a class of models for example Z_3, Z_7 orbifold models and our analysis is to second order from the quantum field theory point of view. Our second-order analysis is not very effective for studying the constraints on models with string threshold corrections Δ_a since in this case the "unification" relation $g_a^{-2}(\frac{1}{2}g^2 M_P^2) = k_a g^{-2} - \Delta_a/4\pi$ ($M_P = 1/\sqrt{8\pi G_N}$ is called the reduced Planck scale) is too complicated. But to first order, one can recognize [3] $\tilde{m}_P = m_P/2e^{1/2}$ ($m_P = 1/\sqrt{G_N}$ is the Planck scale) as the first-order unification scale. If one assumes that string threshold corrections Δ_a are larger than two-loop corrections, our analysis arrives at the relation

$$\alpha_{a(\mu)}^{-1} = k_a \alpha^{-1} + \frac{b_a}{2\pi} \ln \frac{\tilde{m}_P}{\mu} + \Delta_a \, . \tag{1}$$

These put much stronger constraints on a string model.

In the following, I will first review previous discussions on the minimal SUSY string models and then I try to make a complete discussion on the different string models that can give rise to $M_{\rm GUT} = 10^{16}$ GeV (the result deduced from the precision weak scale measurement in the minimal SUSY unification model). Superstring models with intermediate gauge symmetry breaking are proposed in which SU(5), SO(10), or E_6 gauge symmetry is broken at an intermediate scale $M_I = M_{\rm GUT} = 10^{16}$ GeV. The string scale μ_s is calculated from $M_{\rm GUT} = 10^{16}$ GeV and $\alpha_{(M_{\rm GUT})} \simeq 25$ for the minimal SU(5) superstring models and a relation between string scale, intermediate gauge symmetry breaking scale, and Planck scale is found to be $\mu_s^3 = M_I m_P^2$. It is argued that this relation is a general result for superstring models with intermediate gauge symmetry breaking regardless of the dynamical gauge symmetry-breaking scheme or the gauge unification group. We also suggest a possible dynamic gauge symmetry-breaking scheme for this kind of model: the gaugino-condensation-induced Hosotani mechanism. In this scheme, supergravity breaking, spacetime compactification, and gauge symmetry breaking may be triggered by one nonperturbative effect: gaugino condensation. It is also found that there is some gaugino-condensation-induced dynamical SUSY-breaking model may give rise to $M_{\rm GUT} = 10^{16}$ GeV and even $M_{\rm SUSY} = 1$ TeV. Then the minimal SUSY left-right string models (MSLRSM's) and the minimal left-right SUSY string models (MLRSSM's) are discussed. By the minimal left-right model, we mean the model restores left-right symmetry at some intermediate scale M_R and it has the standard fermion content plus right-handed neutrino and two types of Higgs bosons similar to the model in [8]. In the MSLRSM, the SUSY-breaking scale $M_{\rm SUSY}$ is less than M_R. In the MLRSSM, $M_{\rm SUSY}$ is greater than M_R. We find that just like in the level 1 minimal SUSY string unification models discussed in [3], in the level 1 MSLRSM with an arbitrary number of Higgs bosons the weak scale measurements require extra heavy triplet fermions in the model. But for level 1 MLRSSM there is no such requirement. This result indicates that the level 1 left-right model constructed in [9] should be a left-right SUSY string model.

II. IMPLICATIONS OF $M_{\rm GUT} = 10^{16}$ GeV FOR SUPERSTRING MODELS

The precision weak scale measurement seems to indicate that the minimal SUSY grand unification model leads to a good agreement with a single unification scale of $M_{\rm GUT} = 10^{16\pm 0.3}$ GeV [10] and a best fit for $M_{\rm SUSY}$ around 1 TeV. It is amazing that this SUSY grand unification idea works perfectly and predicts correctly the present experimental value for $\sin^2\theta_W(m_Z)$, once one sets $M_{\rm SUSY}$ around the TeV scale and use the actual values for $\alpha_s(M_Z)$ and $\alpha_{\rm em}(M_Z)$. The bottom to τ mass ratio is also predicted correctly; the proton lifetime is predicted to be above the present experimental limits. This analysis gives $\alpha^{-1}_{(M_{\rm GUT})} \simeq 25$. This does not agree with the string unification condition $g_a^{-2}(\frac{1}{2}g^2M_P^2) = k_a g^{-2} - \Delta_a/4\pi$ unless one tunes the gauge coupling constants and string threshold correction to 10^{-6} if we identify μ_s with $M_{\rm GUT}$. This straightforward analysis has been considered to be in contradiction with string phenomenology. Many kinds of string models have been explored to show that by putting in heavy fermions and string threshold corrections or by considering higher affine level string models, string models can confront weak scale measurements [3, 4, 11, 12]. But in these string models, one spoils the beautiful experimental indication of gauge interaction unification at $M_{\rm GUT} = 10^{16}$ GeV. There are a few kinds of string models one can come up with to account for this experimental result from the string phenomenology point of view. For example, one can assume for some string models that at the string scale one has different gauge couplings $g_a^{-2}(\frac{1}{2}g^2M_P^2) = k_a g^{-2} - \Delta_a/4\pi$, and when they evolve down to 10^{16} GeV the coupling constants become equal accidently. For this kind of models, one gets three constraints:

$$k_a\alpha^{-1} + \frac{b_a}{2\pi}\ln\frac{\tilde{m}_P}{M_{\rm GUT}} + \Delta_a + \delta_a^{(2)} = \alpha_{a(M_{\rm GUT})} \simeq 25\ . \tag{2}$$

One can also have some string models that have large gauge symmetry group, for example, SU(5), SO(10), or E_6 at string scale, and it is broken by some kind of dynamics at $M_{\rm GUT} = 10^{16}$ GeV. So in this kind of models, $M_{\rm GUT}$ is the intermediate gauge symmetry breaking scale instead of the grand gravity and gauge interaction unification scale, i.e., string scale. In the Higgs mechanism of gauge symmetry breaking, for the string SU(5) model to have an intermediate gauge symmetry-breaking scale, it has to be of a higher affine algebra level, since the **24** representation of scalars which is necessary to break SU(5) is not allowed in the level 1 SU(5) string model, or one has to appeal to other gauge symmetry-breaking mechanism as will be discussed later. The building of higher affine level string models has been discussed in [13], but no such model has been worked out explicitly. Here we discuss the simplest minimal supersymmetric string SU(5) models which have only three generations of representation **5** and representation **10** fermions and one representation **24** Higgs field (Φ) and one representation **5** Higgs field (H) above the intermediate scale 10^{16} GeV. In this case,

$$b_5 = -15 + 2n_g + 5n_H + \frac{1}{2}n_\Phi = -3.5. \tag{3}$$

Here n_g, n_H, and n_Φ are the number of generations of the fermions, the number of H Higgs fields, and the number of Φ Higgs fields, respectively:

$$b_{55} = 46.4\,n_g + 150\,n_H + 9.8\,n_\Phi - 150 = 149. \tag{4}$$

Using the running coupling constant formula derived in [3], one gets the relation

$$\alpha_5^{-1} + \frac{1}{4\pi}(b_5 + b_{55}b_5^{-1})\ln\alpha_5$$
$$= \alpha^{-1}_{(M_{\rm GUT})} - \frac{b_5}{2\pi}\ln\frac{\tilde{m}_P}{M_{\rm GUT}} + \frac{C_5^G}{2\pi}\ln 2 + \frac{1}{4\pi}b_{55}b_5^{-1}\ln\alpha_{(M_{\rm GUT})} - \frac{b_5}{4\pi}\ln k - \Delta\ , \tag{5}$$

where k is the affine level and α_5 is the SU(5) gauge coupling constant at the string scale. Putting $\alpha^{-1}_{(M_{\rm GUT})} = 25$ and $M_{\rm GUT} = 10^{16}$ GeV, one gets

$$\alpha_5^{-1} - 3.67\ln\alpha_5 = 39.74 + 0.28\ln k - \Delta\ . \tag{6}$$

In the case $k = 2$ and with no string threshold correction, i.e., $\Delta = 0$, one gets $\alpha_5^{-1} = 27.74$ and $\mu_s = 0.99\times 10^{18}$ GeV. So under the assumption that the intermediate

SU(5) gauge symmetry-breaking scale is the unification scale deduced from the experiment in the minimal SUSY SU(5) model, the string scale can be calculated and is around $\mu_s = 10^{18}$ GeV. For the level 3 model, $k = 3$, the string scale is calculated to be $\mu_s = 1.21 \times 10^{18}$ GeV, for $k = 4$, $\mu_s = 1.40 \times 10^{18}$ GeV. An interesting relation observed in these models is $\mu_s^3 = M_I m_P^2$. This identity holds approximately for the intermediate SU(5) models with different affine levels. For the string models with the string threshold correction, the relation still roughly holds. Here we only discuss a very special case. Suppose the string threshold correction cancels out all the renormalization effects on the coupling constant from $M_{\rm GUT}$ to μ_s, that is $\alpha_{(M_{\rm GUT})} = \alpha_5 = k^{-1}\alpha$; then the string scale for $k = 2$ is:

$$\mu_s = \tilde{m}_P\sqrt{\alpha} = 1.05 \times 10^{18}\ \text{GeV}$$

and for $k = 3$, $\mu_s = 1.28 \times 10^{18}$ GeV. From Eq. (29), one gets

$$\Delta = 39.74 - \alpha_5^{-1} + 3.67 \ln \alpha_5 + 0.28 \ln k. \tag{7}$$

For $k = 2$, $\Delta = 3.12$. Assuming that Δ takes the form

$$\Delta \simeq \ln\left[|\eta(T)|^4(T + \bar{T})\right] , \tag{8}$$

then the modulus is of the order of $T \simeq 1.77$.

It has been shown that the left-right unification scheme [8] leads to the unification scale $10^{15.20\pm0.25}$ GeV. In the same way intermediate left-right string models can be constructed. The same discussion can be carried on this kind of models. It is not hard to see that the relation $\mu_s^3 = M_I m_P^2$ still roughly holds here. Because of the relation $\mu_s = \tilde{m}_P\sqrt{\alpha}$, μ_s is always at the order of 10^{18} GeV. It is resonable to conclude that the relation $\mu_s^3 = M_I m_P^2$ holds approximately for all the superstring models with intermediate gauge symmetry breaking regardless of the gauge unification symmetry and symmetry-breaking dynamics. This relation may give some hint at the dynamics of the symmetry-breaking scheme in this kind of model. We will discuss about this possibility in the last part of this section.

To avoid the unnaturalness of high affine level models and some unattractiveness of the Higgs mechanism, one can appeal to dynamical gauge symmetry-breaking schemes. In the string phenomenology, the nontrivial spacetime topology (the Wilson line mechanism) [14] is introduced to obtain the realistic gauge interactions. It was shown in [15] that in the multiply connected spacetime, given the boundary conditions, the physically realized Wilson lines and so the gauge symmetries should be dynamically determined. The problem of dynamically determining gauge symmetries and gauge symmmetry-breaking scales has hardly been discussed in the string phenomenology so far. More work needs to be done in this respect. Here we just propose a simplistic gauge symmetry-breaking dynamics. In the Hosotani mechanism discussed in [16], it was shown that in a system of non-Abelian gauge fields and fermions with minimal gauge interaction, if part of the space-time is compact and there exist sufficiently heavy fermions, the gauge symmetry is dynamically broken for some special cases. If one argues that gaugino condensates may force space-time to be compactified [17, 18] and the onset of space-time compactification breaks SU(5), SO(10), or E_6 grand unification gauge interaction to the standard model at the gaugino condensation scale by the Hosotani mechanism, one can relate $M_{\rm GUT} = 10^{16}$ GeV to the gaugino condensates scale which may hopefully be determined from stringy dynamics. It is interesting to note that in this kind of models one can accomplish gauge symmetry breaking, supergravity breaking, and compactification of spacetime by one scheme. It also has the advantage of relating stringy dynamics directly to the "experimental result" $M_{\rm GUT} = 10^{16}$ GeV.

It is very encouraging to see that there has been some stringy dynamical model, for example, the string-inspired supergravity model at one loop constructed in [19] that is capable of generating gaugino condensation scale around 10^{16} GeV. The model contains three generations of matter in the untwisted sector, a consistent parametrization of gaugino condensation effects, and string-loop threshold effects needed to maintain modular invariance. It was found that the scale degeneracy of the vacuum is lifted at the one-loop level, allowing a determination of the fundamental parameters of the effective low energy theory. The numerical results for E_8 (in Ref. [19], it is stated as E_6, which is a misprint) hidden gauge symmetry gives gaugino condensates scale, $\Lambda_R = 1.30 \times 10^{17}$ GeV, and the coupling constant at the string scale, $\alpha^{-1} = 23.9$, which are very close to the result we want. In the case of E_6 hidden gauge symmetry, the numerical result will be closer. If one follows the argument in [20], the soft SUSY-breaking scale at 1 TeV is very easy to obtain. It has been shown in [20] that in some superstring models supersymmetry is broken in a hidden sector but remains globally conserved in the observable sector of quarks and gluons because of space-time duality. The soft supersymmetry is broken by anomalies and gaugino masses is generated through two steps of radiative corrections. In this way, the large hierarchy of scales is generated. These show some promise that string dynamics might produce $M_{\rm GUT} = 10^{16}$ GeV and $M_{\rm SUSY} = 1$ TeV for the superstring models with intermediate gauge symmetry breaking proposed here. But to give a serious thought about this result, a lot more work is needed. We will pursue this line of ideas in the future.

III. SUSY LEFT-RIGHT MODEL AND LEFT-RIGHT SUSY MODEL

In this part, I extend our discussion in [3] from minimal superstring models to left-right superstring models. The SUSY left-right models are another route of unification in which parity is restored at some intermediate scale M_R. Apart from this appealing feature of restoration of parity symmetry, the large mass differences between the left- and right-handed neutrinos can be accommodated in a natural way through the seesaw mechanism. A $Z(3)$ orbifold SUSY left-right string model has been built explicitly in [9], which is a level 1 model without string threshold corrections. In the following we will discuss the minimal SUSY left-right string models (MSLRSM's) and the minimal left-right SUSY string models (MLRSSM's) without string threshold corrections. By the minimal model we mean that the model has the standard fermion content plus the right-handed neutrino and two types of Higgs bosons: $\Delta_L = (0102)$ and $H = (0\frac{1}{2}\frac{1}{2}0)$ which is similar to the left-right model in [8]. For the MSLRSM we assume $M_{\rm SUSY} = M_T \equiv 1$ TeV $< M_R \equiv M_X$, and for the MLRSSM we suppose $M_R = M_T \equiv 1$ TeV $< M_{\rm SUSY} \equiv M_X$. In either case, the running coupling constants can be written as

$$\alpha^{-1}_{a(M_Z)} = k_a\alpha^{-1} + \frac{b_a^{(\rm I)}}{2\pi}\ln\frac{M_T}{M_Z} + \frac{b_a^{(\rm II)}}{2\pi}\ln\frac{M_X}{M_T} + \frac{b_a^{(\rm III)}}{2\pi}\ln\frac{\tilde{m}_p}{M_X} + \delta_a^{(2)} + \Delta_a. \tag{9}$$

Our analysis given in the Appendix computes the constraints on affine levels; it shows that for the minimal left-right SUSY model $\alpha^{-1} < 30.77$, $\frac{k_2}{k_3} < 1.31$, and for minimal SUSY left-right model $\alpha^{-1} < 21.98$, $\frac{k_2}{k_3} < 2.26$. It also indicates that the weak scale measurements exclude the level 1 MSLRSM. Extra heavy triplet fermions have to be added to make this kind of models work, just as in the case of the level 1 minimal SUSY string unification model discussed in [3]. But for the MLRSSM no such requirement is forced. All these results are done for an arbitrary number of Higgs bosons.

IV. CONCLUSION

Different superstring models that can account for $M_{\rm GUT} = 10^{16}$ GeV are discussed. Superstring models with intermediate gauge symmetry breaking are proposed. The string scale is calculated for minimal superstring models with SU(5) intermediate gauge symmetry breaking. It is found that the relation $\mu_s^3 = M_I m_P^2$ holds approximately for all kinds of superstring models with intermediate gauge symmetry breaking. In string phenomenology, symmetry breaking should happen dynamically and all the scales are related to each other and can be determined from one or two parameters. The experimentally deduced relation we find will put strong constraints on the possible underlying symmetry-breaking dynamics in these types of models. It is also argued that gaugino condensation may serve as a scheme to break gauge symmetry through the Hosotani mechanism in this kind of model and give $M_{\rm GUT} = 10^{16}$ GeV. It is encouragingly noticed that some dynamical string models may generate $M_{\rm GUT} = 10^{16}$ GeV and $M_{\rm SUSY} = 1$ TeV. Further detailed discussion will be given elsewhere. The constraints on the ratio of affine levels are worked out for minimal SUSY left-right string model (MSLRSM) and minimal left-right SUSY string models (MLRSSM). For the level one model, we find that the MSLRSM is excluded for an arbitrary number of Higgs bosons. This analysis indicates that the level one left-right string model in [9] should be the MLRSSM; i.e., the SUSY-breaking scale should be larger than the left-right symmetry-breaking scale. All the analysis is carried out using one-loop modular-invariant string effective coupling constant computed in [2]. Because of the relation between the string unification scale μ_s and the coupling constant at string scale, we find that string unification models are more restricted than any other unification schemes by low energy measurements.

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APPENDIX

The analysis starts with the running coupling constants

$$\alpha^{-1}_{a(M_Z)} = k_a\alpha^{-1} + \frac{b_a^{(\rm I)}}{2\pi}\ln\frac{M_T}{M_Z} + \frac{b_a^{(\rm II)}}{2\pi}\ln\frac{M_X}{M_T} + \frac{b_a^{(\rm III)}}{2\pi}\ln\frac{\tilde{m}_p}{M_X} + \delta_a^{(2)} + \Delta_a. \tag{A1}$$

Here

$$\delta_a^{(2)} = \frac{b_a^{(\rm III)}}{4\pi}\ln\alpha - \frac{C_a^G}{2\pi}\ln 2 + \frac{1}{4\pi}\sum_c b_{ac}^{(\rm I)}b_c^{(\rm I)-1}\ln\frac{\alpha_{c(M_T)}}{\alpha_{c(M_Z)}} + \frac{1}{4\pi}\sum_c b_{ac}^{(\rm II)}b_c^{(\rm II)-1}\ln\frac{\alpha_{c(M_X)}}{\alpha_{c(M_T)}} + \frac{1}{4\pi}\sum_c b_{ac}^{(\rm III)}b_c^{(\rm III)-1}\ln\frac{\alpha_{c(\mu_s)}}{\alpha_{c(M_X)}} \tag{A2}$$

and

$$\Delta_a = -2\pi\sum_{\rm I}\beta_a^{\rm I}\ln[\eta^2(it^{\rm I})\overline{\eta^2(it^{\rm I})}(t+\bar{t})^{\rm I}]. \tag{A3}$$

Using the lowest order expression for $\ln[\alpha_{a(M_T)}/\alpha_{a(M_Z)}]$, $\ln[\alpha_{a(M_X)}/\alpha_{a(M_T)}]$, and $\ln[\alpha_{a(\mu_s)}/\alpha_a(M_X)]$, Eq. (A1) can be written in the form

$$k_a\alpha^{-1}+b_a x+c_a\ln\alpha=B_a\ . \tag{A4}$$

Here

$$b_a\equiv b_a^{(\mathrm{II})}-b_a^{(\mathrm{III})}+\frac{1}{4\pi}\sum_c(b_c^{(\mathrm{II})-1}b_{ac}^{(\mathrm{II})}-b_c^{(\mathrm{III})-1}b_{ac}^{(\mathrm{III})})b_c^{(\mathrm{II})}\alpha_{c(M_{\mathrm{SUSY}})}$$
$$\simeq b_a^{(\mathrm{II})}-b_a^{(\mathrm{III})}, \tag{A5}$$

$$c_a\equiv\frac{1}{4\pi}(b_a^{(\mathrm{III})}+\sum_c b_c^{(\mathrm{III})-1}b_{ac}^{(\mathrm{III})}), \tag{A6}$$

$$B_a\equiv\alpha^{-1}_{a(M_Z)}-\frac{b'^{(\mathrm{I})}}{2\pi}\ln\frac{M_T}{M_Z}-\frac{b^{(\mathrm{III})}}{2\pi}\ln\frac{\tilde{M}_P}{M_T}+\frac{1}{4\pi}\sum_c b_{ac}^{(\mathrm{III})}b_c^{(\mathrm{III})-1}\ln\,\alpha_{c(M_T)}+\frac{c_a^G}{2\pi}\ln 2+\frac{1}{4\pi}\sum_c b_{ac}^{(\mathrm{III})}b_c^{(\mathrm{III})-1}\ln\,k_c\ , \tag{A7}$$

$$x\equiv\frac{1}{2\pi}\ln\frac{M_X}{M_T}\ . \tag{A8}$$

For the MSLRSM with Δ number of Δ Higgs bosons and H number of H Higgs bosons, we get three restrictions:

$$k_1\alpha^{-1}+(0.6-9\Delta)x+\frac{1}{4\pi}\left(6+9\Delta+\frac{7+54\Delta}{6+9\Delta}+\frac{18+72\Delta}{H+2\Delta}-\frac{8}{3}\right)\ln\alpha$$
$$=57.25\pm0.11-5.71\times(6+9\Delta)-\left(0.32\frac{7+54\Delta}{H+2\Delta}+0.27\frac{18+72\Delta}{H+2\Delta}-0.19\times\frac{8}{3}\right), \tag{A9}$$

$$k_2\alpha^{-1}+(1-H-2\Delta)\,x+\frac{1}{4\pi}\left(H+2\Delta+\frac{3+12\Delta}{6+9\Delta}+\frac{18+10H+24\Delta}{H+2\Delta}-8\right)\ln\,\alpha$$
$$=30.84\pm0.11-5.71\times(H+2\Delta)+0.22-\left(0.32\frac{3+12\Delta}{6+9\Delta}+0.27\frac{18+10H+24\Delta}{H+2\Delta}-0.19\times8\right), \tag{A10}$$

$$k_3\alpha^{-1}+\frac{1}{4\pi}\left(-3+\frac{1}{6+9\Delta}+\frac{18}{H+2\Delta}-\frac{14}{3}\right)\ln\,\alpha=11.23\pm0.79+5.71\times3+0.33$$
$$-\left(0.32\frac{1}{6+9\Delta}+0.27\frac{18}{H+2\Delta}-0.19\times\frac{14}{3}\right)\ , \tag{A11}$$

and for the MLRSSM the constraints are

$$k_1\alpha^{(-1)}+(1-9\Delta)\,x+\frac{1}{4\pi}\left(6+9\Delta+\frac{7+54\Delta}{6+9\Delta}+\frac{18+72\Delta}{H+2\Delta}-\frac{8}{3}\right)\ln\alpha$$
$$=57.25\pm0.11-5.71\times(6+9\Delta)-\left(0.32\frac{7+54\Delta}{6+9\Delta}+0.27\frac{18+72\Delta}{H+2\Delta}-0.19\times\frac{8}{3}\right)\ , \tag{A12}$$

$$k_2\alpha^{(-1)}+(-7/3-H-2\Delta)x+\frac{1}{4\pi}\left(H+2\Delta+\frac{3+12\Delta}{6+9\Delta}+\frac{18+10H+24\Delta}{H+2\Delta}-8\right)\ln\alpha$$
$$=30.84\pm0.11-5.71\times(H+2\Delta)+0.22-\left(0.32\frac{3+12\Delta}{6+9\Delta}+0.27\frac{18+10H+24\Delta}{H+2\Delta}-0.19\times8\right)\ , \tag{A13}$$

$$k_3\alpha^{(-1)}-4x+\frac{1}{4\pi}\left(-3+\frac{1}{6+9\Delta}+\frac{18}{H+2\Delta}-\frac{14}{3}\right)\ln\alpha$$
$$=11.23\pm0.79+5.71\times3+0.33-\left(0.32\frac{1}{6+9\Delta}+0.27\frac{18}{H+2\Delta}-0.19\times\frac{14}{3}\right)\ . \tag{A14}$$

First we work out the general constraints on the unification coupling constant α and the affine levels. For an arbitrary number of Higgs bosons, we have

$$\frac{1}{2} \leq \frac{3+12\Delta}{6+9\Delta} \leq \frac{4}{3} ,$$
$$10 \leq \frac{10H+24\Delta}{H+2\Delta} \leq 12 ,$$

and also

$$0 \leq x \leq \frac{1}{4\pi} \ln \alpha + 5.71 .$$

Using these inequalities, we get, for the minimal left-right SUSY model,

$$k_2\,\alpha^{-1} < 30.77, \quad \alpha^{-1} < 30.77, \quad k_3\,\alpha^{-1} > 23.52, \quad \frac{k_2}{k_3} < 1.31 ,$$

and, for minimal SUSY left-right model,

$$k_2\,\alpha^{-1} < 49.67, \quad \alpha^{-1} < 49.67, \quad k_3\,\alpha^{-1} > 21.98, \quad \frac{k_2}{k_3} < 2.26 .$$

Next we discuss the level 1 left-right models, i.e., models with $k_2 = k_3 = 1$. Taking Eqs. (A10) and (A9), one gets

$$\begin{aligned}(H+2\Delta-1)\,x = {}& -2.37 \pm 0.90 + 5.71 \times (H+2\Delta) + \frac{1}{4\pi}\left(H+2\Delta+\frac{2+12\Delta}{6+9\Delta}+\frac{10H+24\Delta}{H+2\Delta}-\frac{1}{3}\right)\ln\alpha \\ &+\left(0.32\frac{2+12\Delta}{6+9\Delta}+0.27\frac{10H+24\Delta}{H+2\Delta}-0.19\times\frac{10}{3}\right) .\end{aligned} \tag{A15}$$

Since there is at least one Higgs boson, the left-hand side of the equation is equal to or greater than zero. It is not hard to work out that the right-hand side is always less than zero. This means that within the experimental error for an arbitrary number of Higgs bosons, the MSLRSM would not work. Extra SU(3) triplet fermions have to be added. In fact one can calculate the constraints on the corrections to the running coupling constant equation from the extra heavy fermions Π_a:

$$\begin{aligned}\Pi_3 - \Pi_2 = {}& -(H+2\Delta-1)\,x - 2.37 \pm 0.90 + 5.71 \times (H+2\Delta) \\ &+\frac{1}{4\pi}\left(H+2\Delta+\frac{2+12\Delta}{6+9\Delta}+\frac{10H+24\Delta}{H+2\Delta}-\frac{1}{3}\right)\ln\alpha+\left(0.32\frac{2+12\Delta}{6+9\Delta}+0.27\frac{10H+24\Delta}{H+2\Delta}-0.19\times\frac{10}{3}\right) \\ &> -2.37-0.9+5.71+\frac{1}{4\pi}\left(\frac{4}{3}+12+1-8+3+\frac{14}{3}\right)\ln\alpha\left(0.32\times\frac{1}{3}+0.27\times 10-0.19\times\frac{10}{3}\right) \\ &> 4.61 + 1.11 \ln\alpha \\ &> 4.61 - 1.11 \ln 30.77 ,\end{aligned}$$

and so $\Pi_3 - \Pi_2 > 0.81$. For the MLRSSM, there is not such an inequality. No extra fermions have to be added to satisfy the weak scale measurement in this kind of models. Applying this analysis to the level one left-right model in [9] one can conclude that in this model the left-right symmetry-breaking scale is smaller than the SUSY-breaking scale.

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