Long-range forces from two-neutrino exchange reexamined

Stephen D. H. Hsu*
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

Pierre Sikivie
Department of Physics, University of Florida, Gainesville, Florida 32611
(Received 7 December 1992; revised manuscript received 8 October 1993)

The exchange of two massless neutrinos gives rise to a long-range force which couples to weakly charged matter. The form of this force was first computed by Feinberg and Sucher using dispersion-theoretic techniques. Here we give a simple derivation using Fourier transforms. Our result agrees with that of Feinberg and Sucher.

PACS number(s): 13.15.+g, 12.15.Ji

The prospect of discovering a new long-range force coupling to ordinary matter is exciting from both the theoretical and experimental points of view [1]. Since long-range forces require the existence of a massless particle, a logical place to look in the electroweak theory is at the effect due to neutrinos. The exchange of a single neutrino (or in general a single fermion) cannot give rise to a force since the interaction changes the angular momentum of the sources involved. However, the exchange of two neutrinos can leave the quantum numbers of the sources unchanged, and hence can lead to a long-range force.

One might guess on the basis of dimensional analysis that the potential for this interaction could take the form $V_\nu(r) \sim G_F^2 m^2/r^3$, where m is the mass of the source particle. Feynman considered this form when contemplating neutrinos as the mediators of a gravitylike interaction [2]. If this were the interaction, the effects of such a force might be observable in laboratory tests [3]. At $r \sim cm$, the resulting force on normal matter would be roughly 10^{-6} times smaller than that due to gravity if m is the electron mass, and comparable to that of gravity if m is a nucleon mass. The current limit on deviations from a $1/r$ potential at $r \sim cm$ is of order 10^{-4} [3]. If the two neutrino force were measurable, it would provide experimental information on neutrino masses which is complementary to that obtained from standard particle physics experiments—long-range force experiments are sensitive to extremely small masses.

Unfortunately, the form given above for $V_\nu(r)$ is incorrect. The correct behavior, which we will derive below, is $1/r^5$. This yields a much smaller effect. The exact form of the interaction is therefore somewhat academic, but seems to us worth computing. The two-neutrino force has previously been investigated by Feinberg and Sucher [4] and by De Rújula, Georgi, and Glashow (unpublished). In fact, all of the results which appear in this paper have been previously noted by Feinberg and Sucher. However, we feel that our method of computation is sufficiently simple to warrant exposition.

Consider the diagrams shown in Fig. 1. Since we are interested in a long distance effect, and correspondingly low momentum exchange, it is a good approximation to combine the effects of W and Z exchange into four-Fermi operators involving neutrinos and weakly charged source particles. The resulting operator can be Fierz transformed into the form

$$\mathcal{O}_4 = \frac{G_F}{\sqrt{2}} [\bar{\nu}\gamma_\mu(1-\gamma_5)\nu][\bar{u}\gamma^\mu(a-b\gamma_5)u] \ , \tag{1}$$

where a and b depend on the fermion u. If u is an electron, both W and Z exchange contribute, yielding $a = 2\sin^2\theta_W + \frac{1}{2}$ and $b = \frac{1}{2}$. For a nucleon, only Z exchange contributes, yielding $a = -\frac{1}{2}, b = -g_A/2$ for the neutron and $a = -2\sin^2\theta_W + \frac{1}{2}, b = g_A/2$ for the proton. g_A is the isotriplet axial vector form factor, with a value of order 1.25. W exchange alone produces $a = b = 1$ for electrons.

It remains to compute the diagram which results from two insertions of the operator $\mathcal{O}_4$. The resulting S matrix element can be directly related to the interaction potential in the nonrelativistic limit. We will use momentum space Feynman rules. However, working directly in coordinate space allows one to quickly deduce the $1/r^5$ form of the potential. In coordinate space massless fermion propagators behave like $1/x^3$, and one integrates over all time to obtain the potential. Thus we have[1]

$$V_\nu(r) \sim G_F^2 \int dt\, x^{-6} \sim G_F^2 x^{-5} \ . \tag{2}$$

The momentum space amplitude for Fig. 1 is

*Electronic address: Hsu@HUHEPL.bitnet, Hsu@HSUNEXT.Harvard.edu

[1]We thank S. Coleman for pointing out this result. It had been noted previously by J. Sucher in [5].

0556-2821/94/49(9)/4951(3)/$06.00

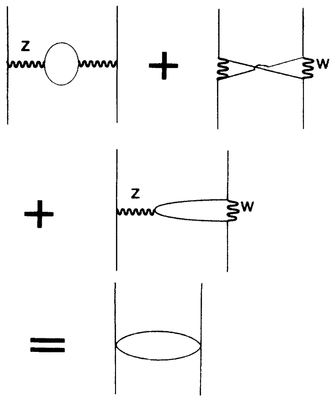

FIG. 1. Two-neutrino exchange in four-Fermi effective theory.

$$T(q) = iG_F^2[\bar{u}(p_4)\gamma^\mu(a - b\gamma_5)u(p_3)] \times [\bar{u}(p_2)\gamma^\nu(a' - b'\gamma_5)u(p_1)]\Pi_{\mu\nu}(q) , \tag{3}$$

where $q = p_2 - p_1 = p_4 - p_3$ is the momentum transfer. $\Pi_{\mu\nu}(q)$ is the vacuum polarization tensor which results from the nonchiral loop and can be found in standard textbooks [6] on QED. By current conservation $\Pi_{\mu\nu}(q) = A(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)$, where $A(q^2)$ is a dimensionless function of q^2. To obtain $V_\nu(r)$ it is necessary to take the Fourier transform

$$V_\nu(r) = i\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} T(q)|_{q^0=0} . \tag{4}$$

The piece of the vacuum polarization which leads to a long-range interaction must depend on a logarithm of q^2. This is because the Fourier transform of a function which is a polynomial in q_μ can always be written as derivatives of a delta function $[\partial_\mu \delta(r)]$, which yield only contact interactions. The Fourier transform of a polynomial $\times$ log function of q can be shown to lead to $1/r^k$ type terms.

The relevant piece of $\Pi_{\mu\nu}(q)$ is readily extracted. This yields

$$\Pi_{\mu\nu}(q) = -\frac{1}{12\pi^2}(q_\mu q_\nu - g_{\mu\nu}q^2)\ln(q^2) + \text{polynomial} . \tag{5}$$

Substituting this into the expression for T yields several terms. Consider the terms which result from the contraction of q_μ with $\gamma^\mu(a - b\gamma_5)$. The $q_\mu\gamma^\mu$ term is zero by the equations of motion, while the $q_\mu\gamma^\mu\gamma_5$ term yields (in the nonrelativistic approximation) $\phi^\dagger(\sigma\cdot q)\phi$, where ϕ is a two component Pauli spinor. The $g_{\mu\nu}$ terms yield the monopole-monopole term along with more complicated spin-dependent terms.

The Fourier transform we need to extract the potentials from T is given by

$$I_{ij}(r) \equiv \int \frac{d^3q}{(2\pi)^3} \ln q q_i q_j e^{iq\cdot r} = \frac{1}{4\pi}\left[15\frac{r_i r_j}{r^7} - 3\frac{\delta_{ij}}{r^5}\right] . \tag{6}$$

Using the above equation, we have the following result for $V_\nu(r)$:

$$V_\nu(r) = \frac{G_F^2}{4\pi^3 r^5}\{aa' - bb'[\tfrac{3}{2}\sigma_1\cdot\sigma_2 - \tfrac{5}{2}(\hat{\mathbf{r}}\cdot\sigma_1)(\hat{\mathbf{r}}\cdot\sigma_2)] + \cdots\} , \tag{7}$$

where the ellipsis denotes more complicated terms which are proportional to velocity and are suppressed in the nonrelativistic limit, or which are of order $1/r^6$ or higher. The result agrees with that obtained by Feinberg and Sucher[2] [4]. Note that the monopole-monople interaction is repulsive between two electrons. The sign of the interaction is changed if one of the particles is replaced by its antiparticle.

Let us briefly consider the phenomenological relevance of the above interaction. The two-neutrino force discussed here, which couples to both nucleons and electrons, is unscreened in normal matter. However, as mentioned previously, a $1/r^5$ potential is extremely weak. The adjective "long range" is perhaps misleading as even large volumes are not capable of enhancing the effect of the force. For two nucleons $V_\nu(r)$ becomes comparable to the gravitational potential energy only at separations $r \sim 10^{-6}$ cm. Detection of the two-neutrino force would therefore require laboratory tests of gravity on distance scales far smaller and at sensitivities far greater than thus far realized. It seems highly unlikely that this effect will be observed in the forseeable future.

It is straightforward to see that the exchange of two massless fermions can at best lead to a $1/r^5$ potential for fermionic matter. Consider a general four-Fermi interaction given by

$$\mathcal{O}_d = \frac{1}{M^{d+2}}\bar{u}\Gamma_1 u\bar{\psi}\Gamma_2\psi . \tag{8}$$

Here the ψ's are the massless fermions mediating the force, Γ_1, Γ_2 are generic Lorentz tensors of total dimension d (they may contain derivatives), and M is some heavy mass scale. By dimensional analysis the amplitude T resulting from two insertions of $\mathcal{O}_d$ must have the form

$$\frac{1}{M^{2d+4}}A(q^2)(q\cdots q)\bar{u}\bar{\Gamma}_1 u\bar{u}\bar{\Gamma}_1 u , \tag{9}$$

where the q's are contracted with the $\bar{\Gamma}_1$ tensors, which are now dimensionless, and $A(q^2)$ is at most logarithmically dependent on q^2. Taking the Fourier transform of the above amplitude results in a potential which falls as $1/r^{5+2d}$.

In the most general case of an arbitrary four-fermion vertex and a *massive* fermion it is possible to generate an interaction potential of the form $V(r) \sim m^2 e^{-2mr}/M^4 r^3$,

[2]An earlier version of this note disagreed with [4]. We thank Joe Sucher for pointing out our error.

where M is the large mass scale suppressing the four-fermion interaction and m is the neutrino mass. Note that m^2e^{-2mr} has replaced $1/r^2$ in the massive case. However, it is easy to see that even for a fermion mass which is fine-tuned to some inverse distance scale r_*, the strength of the massive interaction cannot be significantly enhanced over the massless one.

The authors would like to thank the Aspen Center for Physics, where this work was begun, for its kind hospitality. While at the Aspen Center, the authors benefited from conversations with D. Caldwell, S. Coleman, G. Gelmini, B. Kayser, A. Kostelecky, P. Lanou, P. Rosen, and L. Wolfenstein. The authors are grateful to Joe Sucher for pointing out an error in an earlier draft. S.D.H. acknowledges support from the National Science Foundation under Grant No. NSF-PHY-87-14654, the state of Texas under Grant No. TNRLC-RGFY106, the Milton Fund of the Harvard Medical School, and from the Harvard Society of Fellows. S.D.H. would also like to thank the Physics Department at Iowa State University for its hospitality while this work was completed. P.S. acknowledges support from the DOE under Contract No. DE-FG05-86ER40272.

[1] See, e.g., E. Adelberger *et al.*, Annu. Rev. Nucl. Part. Sci. **41**, 269 (1991), and references therein.

[2] R. P. Feynman, lectures on gravitation, Caltech, 1962–63 (unpublished).

[3] R. Spero *et al.*, Phys. Rev. Lett. **44**, 1645 (1980).

[4] G. Feinberg and J. Sucher, Phys. Rev. **166**, 1638 (1968); G. Feinberg, J. Sucher, and C.k. Au, Phys. Rep. **180**, 83 (1989).

[5] J. Sucher, Lectures at the 1972 Cargese Summer Institute, Cargese Series Vol. 7 (unpublished).

[6] J. Bjorken and S. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965); P. Ramond, *Field Theory: A Modern Primer* (Benjamin-Cummings, Redwood City, CA, 1991).