UV sensitive one-loop matter power spectrum in degenerate higher-order scalar-tensor theories

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We study matter density perturbations up to third order and the one-loop matter power spectrum in degenerate higher-order scalar-tensor (DHOST) theories beyond Horndeski. We systematically solve gravitational field equations and fluid equations order by order and find three novel shape functions characterizing the third-order solution in DHOST theories. A complete form of the one-loop matter power spectrum is then obtained using the resultant second- and third-order solutions. We confirm the previous result that the convergence condition of the loop integrals in the infrared limit becomes more stringent than that of the standard one in general relativity. We show that also in the ultraviolet limit the convergence condition becomes more stringent and the one-loop matter power spectrum is thus sensitive to the short-wavelength behavior of the linear power spectrum.

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I. INTRODUCTION

Scalar-tensor theories are exciting candidates of the origin of the accelerated cosmic expansion at late time [1–3]. While the scalar degree of freedom causes the accelerated expansion on large cosmological scales, its effect on small scale gravity experiments must be highly suppressed in successful theories to evade existing stringent tests such as in the Solar System. To achieve this, screening mechanisms are implemented to elaborated scalar-tensor theories. For example, the Vainshtein mechanism hides the scalar-mediated force very effectively through the nonlinear derivative interaction of the scalar degree of freedom [4]. In the Horndeski family of theories [5–7], which spans the most general scalar-tensor theory having second-order field equations and hence is a class of theories free of Ostrogradsky's ghost [8,9], the Vainshtein screening mechanism is implemented naturally as the Lagrangian contains powers of second derivatives of the scalar field [10–12]. Extending the Horndeski theory even further, degenerate higher-order scalar-tensor (DHOST) theories have been developed recently [13–15] (see Refs. [16–18] for a review). The field equations in such theories are apparently of higher order, but a careful counting of the degrees of freedom shows that there in fact are one scalar and two tensor degrees of freedom due to the degeneracy of the theories, and consequently Ostrogradsky's ghost is removed. New types of nonlinear derivative interactions arise in DHOST theories beyond Horndeski. Their effect on the screening mechanism has been discussed in [19–25], emphasizing that partial breaking of Vainshtein screening (first discovered in Ref. [19]) occurs in the presence of matter [26–41]. The nonstandard interactions between scalar and gravitational-wave degrees of freedom in DHOST theories result also in the decay of gravitons [42,43].

In this paper, we study the impact of the nonlinear derivative interactions of the scalar field in DHOST theories on nonlinear evolution of matter density perturbations. It has been found that the matter bispectrum in a class of DHOST theories shows a distinct feature at the equilateral and folded limits compared with that in general relativity (GR) [44]. References [45,46] have investigated the bi- and trispectra of the matter density perturbations and also the one-loop matter power spectrum in the context of DHOST theories. They have, in particular, focused on the behaviors at the infrared (IR) limit: the squeezed limit for the bispectrum, the double soft limit for the trispectrum, and the IR contributions in the loop integrals for the one-loop power spectrum, and studied a consistency relation for large scale structure and its violation. So far, in DHOST theories, the third-order solution for the matter density

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perturbations has been obtained only at the IR limit as done in Refs. [45,46], and the complete form of the one-loop matter power spectrum has not been derived yet. The goal of this paper is therefore to derive the third-order solution for the matter density perturbations and to investigate the one-loop matter power spectrum in its complete form.

This paper is organized as follows. In the next section, we introduce quadratic DHOST theories which we focus on in this paper and derive basic equations for cosmological perturbations under the quasistatic approximation. Then, we review the solutions for the matter density perturbations up to second order. In Sec. III, we obtain the third-order solution to calculate the complete form of the one-loop matter power spectrum. In Sec. IV, we derive the one-loop matter power spectrum and investigate the asymptotic behavior of the loop integrals at the IR and ultraviolet (UV) limits. In particular, we emphasize that the convergence condition of the loop integrals in the UV limit becomes more stringent. Finally, we draw the conclusion of the present paper in Sec. V.

II. LARGE SCALE STRUCTURE IN QUADRATIC DHOST THEORIES

A. Quadratic DHOST theories

The action of the quadratic DHOST theories [13] is given by

$$\begin{aligned} S = \int \mathrm{d}^4x\sqrt{-g}[&G_2(\phi,X) - G_3(\phi,X)\Box\phi + F(\phi,X)R \\ &+ a_1(\phi,X)\phi_{\mu\nu}\phi^{\mu\nu} + a_2(\phi,X)(\Box\phi)^2 \\ &+ a_3(\phi,X)(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu + a_4(\phi,X)\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu \\ &+ a_5(\phi,X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2], \end{aligned} \tag{1}$$

where $\phi_\mu = \nabla_\mu\phi$, $\phi_{\nu\rho} = \nabla_\rho\nabla_\nu\phi$, and $X = -\phi_\mu\phi^\mu/2$. To avoid the Ostrogradsky's ghosts, we impose degeneracy conditions among the functions F and $a_i (i = 1,\ldots,5)$, and hence not all these are independent. We focus on the class Ia DHOST theories, as they are basically healthy and can be free from instabilities on a cosmological background [47,48]. The class Ia degeneracy conditions [14] are summarized as

$$\begin{aligned} &a_1 + a_2 = 0, \qquad \beta_2 = -6\beta_1^2, \\ &\beta_3 = -2\beta_1[2(1+\alpha_\mathrm{H}) + \beta_1(1+\alpha_\mathrm{T})], \end{aligned} \tag{2}$$

where

$$\begin{aligned} &M^2 = 2(F + 2Xa_1), \qquad M^2\alpha_\mathrm{T} = -4Xa_1, \\ &M^2\alpha_\mathrm{H} = -4X(F_X + a_1), \qquad M^2\beta_1 = 2X(F_X - a_2 + Xa_3), \\ &M^2\beta_2 = 4X[a_1 + a_2 - 2X(a_3 + a_4) + 4X^2a_5], \\ &M^2\beta_3 = -8X(F_X + a_1 - Xa_4). \end{aligned} \tag{3}$$

Here and hereafter, we use the notation $F_X = \partial F/\partial X$. We thus have three free functions in addition to G_2 and G_3. In the Horndeski theory [5–7], we have $\alpha_\mathrm{H} = \beta_1 = \beta_2 = \beta_3 = 0$, while in the Gleyzes-Langlois-Piazza-Vernizzi theory [49–51], we have $\beta_1 = \beta_2 = \beta_3 = 0$.

To keep generality, in this paper, we do not impose any other constraints among the functions. In particular, we do not take into account seriously the constraint on scalar-tensor gravity from the propagation speed of gravitational waves, as the energy scale observed at LIGO is close to the cutoff scale of the effective theory if applied to dark energy [52]. One should note also that in principle the gravitational-wave constraints are irrelevant to the high-redshift universe. For these reasons, it is fair to say that there still is a room for general DHOST theories as viable dark energy models, and it is important to seek for independent cosmological constraints on DHOST theories.

B. Perturbation equations

We consider a spatially flat, homogeneous, and isotropic background universe, and the cosmological perturbations in the Newtonian gauge. The perturbed metric is given by

$$\mathrm{d}s^2 = -[1 + 2\Phi(t,\boldsymbol{x})]\mathrm{d}t^2 + a^2(t)[1 - 2\Psi(t,\boldsymbol{x})]\mathrm{d}\boldsymbol{x}^2, \tag{4}$$

and the perturbed scalar field is

$$\phi(t,\boldsymbol{x}) = \phi(t) + \pi(t,\boldsymbol{x}). \tag{5}$$

We introduce a dimensionless variable $Q := H\pi/\dot\phi$, where $H = \dot a/a$ and a dot denotes differentiation with respect to t. The matter density perturbation is defined by

$$\rho_\mathrm{m}(t,\boldsymbol{x}) = \bar\rho_\mathrm{m}(t)[1 + \delta(t,\boldsymbol{x})]. \tag{6}$$

We consider irrotational dust as a matter content and therefore have only a scalar mode in the velocity field u^i, which is characterized by $\theta = \partial_i u^i/aH$.

We study the quasistatic behavior of those perturbations deep inside the horizon. Substituting Eqs. (4) and (5) to the action (1) and expanding it in terms of the perturbations, one obtains the action for the perturbations [22,24,25,44,45,53]. In doing so, we take the quasistatic approximation and neglect time derivatives compared to spatial derivatives. Note, however, that in DHOST theories we have mixed derivatives such as $\partial^2\dot Q$ which cannot be simply ignored [19]. We thus arrive in the end at the action for the quasistatic perturbations, which we vary to derive the following equations of motion (EoMs) in Fourier space:

$$\begin{aligned} &(1+\alpha_\mathrm{T})\Psi - (1+\alpha_\mathrm{H})\Phi + b_2 Q + \alpha_\mathrm{H}\frac{\dot Q}{H} \\ &\quad = -\frac{\alpha_\mathrm{T} - 4\alpha_\mathrm{H}}{4a^2H^2p^2}\mathcal{S}_\gamma[t,\boldsymbol{p};Q,Q] - \frac{\alpha_\mathrm{H}}{a^2H^2p^2}\mathcal{S}_{\alpha_s}[t,\boldsymbol{p};Q,Q], \end{aligned} \tag{7}$$

$$(1+\alpha_{\rm H})\Psi-\frac{\beta_3}{2}\Phi+b_1Q+\frac{2\beta_1+\beta_3}{2}\frac{\dot{Q}}{H}+\frac{a^2}{2M^2p^2}\bar{\rho}_{\rm m}\delta=\frac{d_2+2(2\beta_1+\beta_3)}{2a^2H^2p^2}\mathcal{S}_\gamma[t,\boldsymbol{p};Q,Q]-\frac{2\beta_1+\beta_3}{2a^2H^2p^2}\mathcal{S}_{\alpha_s}[t,\boldsymbol{p};Q,Q], \quad (8)$$

$$\begin{aligned}&c_1\Phi+c_2\Psi+b_3Q+4\alpha_{\rm H}\frac{\dot{\Psi}}{H}-2(2\beta_1+\beta_3)\frac{\dot{\Phi}}{H}+b_4\frac{\dot{Q}}{H}+2(4\beta_1+\beta_3)\frac{\ddot{Q}}{H^2}\\&=\frac{d_1}{a^2H^2p^2}\mathcal{S}_\gamma[t,\boldsymbol{p};Q,Q]+\frac{2\alpha_{\rm T}}{a^2H^2p^2}\mathcal{S}_\gamma[t,\boldsymbol{p};Q,\Psi]+\frac{4d_2}{a^2H^2p^2}\mathcal{S}_\gamma[t,\boldsymbol{p};Q,\Phi]\\&-\frac{4\alpha_{\rm H}}{a^2H^2p^2}\mathcal{S}_\alpha[t,\boldsymbol{p};\Psi,Q]+\frac{2(2\beta_1+\beta_3)}{a^2H^2p^2}\mathcal{S}_\alpha[t,\boldsymbol{p};\Phi,Q]-\frac{2(4\beta_1+\beta_3)}{a^2H^3p^2}\mathcal{S}_\alpha[t,\boldsymbol{p};Q,\dot{Q}]\\&-\frac{4(4\beta_1+\beta_3)}{a^2H^3p^2}(\mathcal{S}_{\alpha_s}[t,\boldsymbol{p};Q,\dot{Q}]-\mathcal{S}_\gamma[t,\boldsymbol{p};Q,\dot{Q}])-\frac{b_4}{a^2H^2p^2}(\mathcal{S}_{\alpha_s}[t,\boldsymbol{p};Q,Q]-\mathcal{S}_\gamma[t,\boldsymbol{p};Q,Q])\\&+\frac{2d_2+\alpha_{\rm T}}{a^4H^4p^2}\mathcal{T}_\xi[t,\boldsymbol{p};Q,Q,Q]-\frac{2(4\beta_1+\beta_3)}{a^4H^4p^2}\mathcal{T}_\zeta[t,\boldsymbol{p};Q,Q,Q], \end{aligned} \quad (9)$$

where $\boldsymbol{p}$ denotes a comoving wave vector in Fourier space and $p=|\boldsymbol{p}|$. Here, $\mathcal{S}_\Sigma[t,\boldsymbol{p};Y,Z](\Sigma=\alpha,\alpha_s,\gamma)$ and $\mathcal{T}_\Upsilon[t,\boldsymbol{p};X,Y,Z](\Upsilon=\xi,\zeta)$ are, respectively, second- and third-order contributions with respect to the metric and scalar field perturbations, and are defined by

$$\mathcal{S}_\Sigma[t,\boldsymbol{p};Y,Z]=\frac{1}{(2\pi)^3}\int {\rm d}^3k_1{\rm d}^3k_2\delta_{\rm D}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{p})k_1^2k_2^2\Sigma(\boldsymbol{k}_1,\boldsymbol{k}_2)Y(t,\boldsymbol{k}_1)Z(t,\boldsymbol{k}_2), \quad (10)$$

$$\mathcal{T}_\Upsilon[t,\boldsymbol{p};X,Y,Z]=\frac{1}{(2\pi)^6}\int {\rm d}^3k_1{\rm d}^3k_2{\rm d}^3k_3\delta_{\rm D}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3-\boldsymbol{p})k_1^2k_2^2k_3^2\Upsilon(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)X(t,\boldsymbol{k}_1)Y(t,\boldsymbol{k}_2)Z(t,\boldsymbol{k}_3), \quad (11)$$

where

$$\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2)=1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1^2}, \quad (12)$$

$$\alpha_s(\boldsymbol{k}_1,\boldsymbol{k}_2)=\frac{1}{2}[\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2)+\alpha(\boldsymbol{k}_2,\boldsymbol{k}_1)], \quad (13)$$

$$\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2)=1-\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}, \quad (14)$$

$$\xi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)=1-3\frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_2^2k_3^2}+2\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)(\boldsymbol{k}_3\cdot\boldsymbol{k}_1)}{k_1^2k_2^2k_3^2}, \quad (15)$$

$$\begin{aligned}\zeta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)=&\frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_2^2k_3^2}+2\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_3)(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_1^2k_2^2k_3^2}+\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_2^2}\\&+\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2+\boldsymbol{k}_3\cdot\boldsymbol{k}_1)(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)}{k_1^2k_2^2}, \end{aligned} \quad (16)$$

and $\delta_{\rm D}(\boldsymbol{k})$ is the Dirac delta function. The explicit expressions for the time-dependent coefficients c_1, c_2, c_3, b_1, b_2, b_3, d_1, and d_2 in terms of the functions in the DHOST action are presented in Appendix A. The terms involving $\alpha_{\rm H}$, β_1, and β_3 are specific to theories more general than Horndeski. (Here, β_2 is removed by using one of the degeneracy conditions (2), but β_3 is retained because using β_3 leads to simpler expressions.) In GR, the above set of equations corresponds to the Poisson equation and $\Phi=\Psi$ that are used to express the matter density perturbations δ in terms of the metric potentials. In scalar-tensor theories, the Poisson equation is modified and anisotropic stress induced by the scalar field changes the relation between Φ and Ψ. Nonlinear self-interactions of the scalar field also come into play in the equations. In particular, $\mathcal{S}_\alpha$ and $\mathcal{T}_\zeta$ newly appear in DHOST theories beyond Horndeski, and it turns out that these terms lead to the more stringent convergence conditions of the loop integrals in the one-loop matter power spectrum as we will see in Sec. IV B.

We assume that the matter is minimally coupled to gravity. Then, fluid equations are the same as the standard ones in GR,

$$\frac{1}{H}\frac{\partial\delta(t,\boldsymbol{p})}{\partial t}+\theta(t,\boldsymbol{p})=-\frac{1}{(2\pi)^3}\int {\rm d}^3k_1{\rm d}^3k_2\delta_{\rm D}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{p})\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2)\theta(t,\boldsymbol{k}_1)\delta(t,\boldsymbol{k}_2), \quad (17)$$

$$\frac{1}{H}\frac{\partial\theta(t,\boldsymbol{p})}{\partial t}+\left(2+\frac{\dot{H}}{H^2}\right)\theta(t,\boldsymbol{p})-\frac{p^2}{a^2H^2}\Phi(t,\boldsymbol{p}) = -\frac{1}{(2\pi)^3}\int \mathrm{d}^3k_1\mathrm{d}^3k_2\delta_{\mathrm{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{p}) \times[\alpha_s(\boldsymbol{k}_1,\boldsymbol{k}_2)-\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2)]\theta(t,\boldsymbol{k}_1)\theta(t,\boldsymbol{k}_2). \tag{18}$$

Although these are the same as the standard ones, the effects of modified gravity participate through the gravitational potential Φ which is determined by the EoMs (7)–(9). The nonlinear terms on the right-hand side also modify the higher-order solutions from the standard ones in GR because these are induced by the linear solution that already contains the effects of gravity modification.

C. Solving the perturbation equations

A solution to the EoMs (7)–(9) and the fluid equations (17) and (18) can be expressed as a perturbative series,

$$\delta=\sum_{n=1}\delta^{(n)},\qquad \Phi=\sum_{n=1}\Phi^{(n)},\qquad \ldots, \tag{19}$$

where $\delta^{(n)},\Phi^{(n)},\cdots[=\mathcal{O}(\epsilon^n)]$ are the nth order quantities with $\delta^{(1)}$ being $\mathcal{O}(\epsilon)$ a quantity.

Let us now describe the systematic procedure to obtain the nth order solution. At nth order, Eqs. (7) and (8) are schematically written as

$$\mathsf{M}\begin{pmatrix}\Psi^{(n)}\\ \Phi^{(n)}\end{pmatrix}=\mathsf{N}\begin{pmatrix}Q^{(n)}\\ \dot{Q}^{(n)}/H\end{pmatrix}-\frac{a^2\bar{\rho}_{\mathrm{m}}}{2M^2p^2}\begin{pmatrix}0\\ \delta^{(n)}\end{pmatrix} -\frac{a^2H^2}{p^2}\mathsf{O}^{(n)}\begin{pmatrix}W^{(n)}_{\Pi_1}\\ \vdots\\ W^{(n)}_{\Pi_m}\end{pmatrix}, \tag{20}$$

where M and N are 2×2 matrices, $\mathsf{O}^{(n)}$ is a $2\times m$ matrix with m being different for different n, and $W^{(n)}_{\Pi_1},\ldots,W^{(n)}_{\Pi_m}$ are the nth order functions with respect to the initial density field, δ_{L}, which comes from the higher-order contributions in Eqs. (7), (8), (9), (17), and (18). Here, Π represents the shape of the kernel of the nonlinear mode coupling. We define $W^{(n)}_{\Pi}$ as

$$W^{(n)}_{\Pi}(\boldsymbol{p}) \coloneqq \int\frac{\mathrm{d}^3k_1\cdots\mathrm{d}^3k_n}{(2\pi)^{3(n-1)}}\delta_{\mathrm{D}}(\boldsymbol{k}_1+\cdots+\boldsymbol{k}_n-\boldsymbol{p}) \times\Pi(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta_{\mathrm{L}}(\boldsymbol{k}_n), \tag{21}$$

where $\delta_{\mathrm{L}}(\boldsymbol{k})$ denotes the initial density field. The matrices M and N are independent of n, whose explicit forms are presented in Appendix B 1. Inverting M, we obtain the solutions for $\Psi^{(n)}$ and $\Phi^{(n)}$ as follows:

$$\begin{pmatrix}\Psi^{(n)}\\ \Phi^{(n)}\end{pmatrix}=\mathsf{M}^{-1}\mathsf{N}\begin{pmatrix}Q^{(n)}\\ \dot{Q}^{(n)}/H\end{pmatrix}-\frac{a^2\bar{\rho}_{\mathrm{m}}}{2M^2p^2}\mathsf{M}^{-1}\begin{pmatrix}0\\ \delta^{(n)}\end{pmatrix} -\frac{a^2H^2}{p^2}\mathsf{M}^{-1}\mathsf{O}^{(n)}\begin{pmatrix}W^{(n)}_{\Pi_1}\\ \vdots\\ W^{(n)}_{\Pi_m}\end{pmatrix}. \tag{22}$$

Substituting these and their time derivatives into the nth order part of Eq. (9), we obtain the solution for $Q^{(n)}$,

$$Q^{(n)}=-\frac{a^2H^2}{p^2}\left(\nu_Q\frac{\dot{\delta}^{(n)}}{H}+\kappa_Q\delta^{(n)}\right)-\frac{a^2H^2}{p^2}\sum_{\Pi\in U_n}\tau^{(n)}_{Q,\Pi}W^{(n)}_{\Pi}, \tag{23}$$

where $U_n=\{\Pi_1\cdots,\Pi_m\}$ denotes the set of the kernels to describe the nth order mode coupling. At this step, $\dot{Q}^{(n)}$ and $\ddot{Q}^{(n)}$ are all canceled thanks to the degeneracy conditions, so that the equation can be solved algebraically for $Q^{(n)}$. The explicit forms of the coefficients ν_Q and κ_Q are given in Appendix B 1. It should be noted that $\dot{\delta}^{(n)}$ appears in Eq. (23) when one considers theories more general than Horndeski [19,24,25,44,45,53,54]. Given the concrete form of $\mathsf{O}^{(n)}$, it is straightforward to write the explicit form of $\tau^{(n)}_{Q,\Pi}$. Turning back to Eq. (22), now one can eliminate $Q^{(n)}$ and $\dot{Q}^{(n)}$ on the right-hand side to express $\Psi^{(n)}$ and $\Phi^{(n)}$ in terms of $\delta^{(n)}$ and $W^{(n)}_{\Pi}$ as

$$\Psi^{(n)}=-\frac{a^2H^2}{p^2}\left(\mu_\Psi\frac{\ddot{\delta}^{(n)}}{H^2}+\nu_\Psi\frac{\dot{\delta}^{(n)}}{H}+\kappa_\Psi\delta^{(n)}\right) -\frac{a^2H^2}{p^2}\sum_{\Pi\in U_n}\tau^{(n)}_{\Psi,\Pi}W^{(n)}_{\Pi}, \tag{24}$$

$$\Phi^{(n)}=-\frac{a^2H^2}{p^2}\left(\mu_\Phi\frac{\ddot{\delta}^{(n)}}{H^2}+\nu_\Phi\frac{\dot{\delta}^{(n)}}{H}+\kappa_\Phi\delta^{(n)}\right) -\frac{a^2H^2}{p^2}\sum_{\Pi\in U_n}\tau^{(n)}_{\Phi,\Pi}W^{(n)}_{\Pi}. \tag{25}$$

The explicit expressions for the coefficients of the homogeneous solutions, $\mu_\Psi,\nu_\Psi,\cdots$, are also given in Appendix B 1. We also show the coefficients of the second- and third-order mode couplings in Appendixes B 2–B 4.

Having thus obtained $\Psi^{(n)}$, $\Phi^{(n)}$, and $Q^{(n)}$ expressed in terms of the matter density perturbations, we use the fluid equations (17), (18), and (25) to obtain the evolution equation for $\delta^{(n)}$,

$$\frac{\partial^2\delta^{(n)}}{\partial t^2}+(2+\varsigma)H\frac{\partial\delta^{(n)}}{\partial t}-\frac{3}{2}\Omega_{\rm m}\Xi_\Phi H^2\delta^{(n)}=H^2\sum_{\Pi\in U_n}S_\Pi^{(n)}W_\Pi^{(n)}, \tag{26}$$

where

$$\varsigma:=\frac{2\mu_\Phi-\nu_\Phi}{1-\mu_\Phi},\qquad \frac{3}{2}\Omega_{\rm m}\Xi_\Phi:=\frac{\kappa_\Phi}{1-\mu_\Phi},\qquad \Omega_{\rm m}:=\frac{\bar{\rho}_{\rm m}}{3M^2H^2}. \tag{27}$$

We assume that there is no intrinsic nonlinearity at an initial time $t_{\rm i}$, that is, $\delta^{(2)}(t_{\rm i})=\delta^{(3)}(t_{\rm i})=\cdots=0$. Then, the first-order solution is obtained by solving the homogeneous equation, and the higher-order solutions are given solely by the inhomogeneous solution.

The first- and second-order solutions for the matter density perturbations in DHOST theories have already been obtained in the literature [45,46], but here for completeness we replicate the previous discussion. The third-order solution, which is obtained for the first time in this paper, is presented in the next section.

1. *First-order solution*

From Eq. (26), by setting $W_\Pi^{(n)}=0$, we see that the first-order evolution equation for the density perturbation is given by

$$\frac{\partial^2\delta^{(1)}}{\partial t^2}+(2+\varsigma)H\frac{\partial\delta^{(1)}}{\partial t}-\frac{3}{2}\Omega_{\rm m}\Xi_\Phi H^2\delta^{(1)}=0. \tag{28}$$

This equation has growing and decaying solutions, denoted, respectively, as $D_+(t)$ and $D_-(t)$. Discarding the decaying solution, we write the solution as

$$\delta^{(1)}(t,\boldsymbol{p})=D_+(t)\delta_{\rm L}(\boldsymbol{p}). \tag{29}$$

The linear growth rate, f, is convenient for characterizing the growth of the matter density perturbations and is defined by

$$f=\frac{{\rm d}\ln D_+}{{\rm d}\ln a}. \tag{30}$$

Substituting the solution (29) into the continuity equation (17), we obtain

$$\theta^{(1)}(t,\boldsymbol{p})=-f(t)\delta^{(1)}(t,\boldsymbol{p}). \tag{31}$$

2. *Second-order solution*

In order to obtain the second-order solution of the matter density perturbations, we need to substitute the first-order solutions (29) and (31) into the right-hand side of Eqs. (17) and (18). The relevant kernel to describe the second-order mode-coupling functions are α and γ defined in Eqs. (12) and (14), namely, $U_2=\{\alpha,\gamma\}$. With these nonlinear mode-coupling functions (21), the second-order evolution equation is given by

$$\frac{\partial^2\delta^{(2)}}{\partial t^2}+(2+\varsigma)H\frac{\partial\delta^{(2)}}{\partial t}-\frac{3}{2}\Omega_{\rm m}\Xi_\Phi H^2\delta^{(2)}=H^2\sum_{\Pi=\alpha,\gamma}S_\Pi^{(2)}W_\Pi^{(2)}. \tag{32}$$

The coefficients of the second-order mode coupling are given by

$$(1-\mu_\Phi)S_\alpha^{(2)}=\tau_{\Phi,\alpha}^{(2)}+\frac{1}{a^2H^2D_+}(a^2HD_+^3f)^{\cdot}, \tag{33}$$

$$(1-\mu_\Phi)S_\gamma^{(2)}=\tau_{\Phi,\gamma}^{(2)}-D_+^2f^2, \tag{34}$$

where the explicit form of $\tau_{\Phi,\Pi}^{(2)}$ is given in Appendixes B 2 and B 3.

The second-order solution is obtained as

$$\delta^{(2)}(t,\boldsymbol{p})=D_+^2(t)\left[\kappa(t)W_\alpha^{(2)}(\boldsymbol{p})-\frac{2}{7}\lambda(t)W_\gamma^{(2)}(\boldsymbol{p})\right], \tag{35}$$

where

$$\kappa(t)=\frac{1}{D_+^2(t)}L[H^2S_\alpha^{(2)}],\qquad \lambda(t)=-\frac{7}{2D_+^2(t)}L[H^2S_\gamma^{(2)}], \tag{36}$$

and we defined the functional L acting on a function s of time as

$$L[s]:=\int_0^t{\rm d}T\,\frac{D_+(T)D_-(t)-D_+(t)D_-(T)}{D_+(T)\dot{D}_-(T)-\dot{D}_+(T)D_-(T)}s(T). \tag{37}$$

In the Einstein–de Sitter universe in GR, we have $\mu_\Phi=\varsigma=\tau_{\Phi,\alpha}^{(2)}=\tau_{\Phi,\gamma}^{(2)}=0$ and $\Xi_\Phi=1$. We then see that $\lambda=1$ and $\kappa=1$. In the Horndeski theory, λ can deviate much from the standard value, $\lambda\neq1$, but κ still takes the standard value, $\kappa=1$ [55,56]. In DHOST theories beyond Horndeski, not only λ but also κ can deviate from 1 [44–46]. For κ away from 1, the matter bispectrum is altered at the folded configuration in momentum space [44,45].

Substituting $\delta^{(1)}$, $\delta^{(2)}$, and $\theta^{(1)}$ into Eq. (17), we can also obtain the second-order velocity divergence,

$$\theta^{(2)}(t,\boldsymbol{p})=-D_+^2f\left[\kappa_\theta(t)W_\alpha^{(2)}(\boldsymbol{p})-\frac{4}{7}\lambda_\theta(t)W_\gamma^{(2)}(\boldsymbol{p})\right], \tag{38}$$

where we defined

$$\kappa_\theta(t) = 2\kappa - 1 + \frac{\dot{\kappa}}{fH}, \tag{39}$$

$$\lambda_\theta(t) = \lambda + \frac{\dot{\lambda}}{2fH}. \tag{40}$$

In the Einstein–de Sitter universe in GR, $\lambda_\theta = 1$ and $\kappa_\theta = 1$. As λ and κ can, in DHOST theories both λ_θ and κ_θ can also deviate much from 1 and they are sensitive to the time derivative of λ and κ, respectively.

III. THIRD-ORDER SOLUTION

Having reviewed the first- and second-order solutions, now let us proceed to derive the third-order solution for the matter density perturbations in DHOST theories. The procedure is basically the same as in the case of the second-order solution. Before going to the detailed analysis, we need to discuss the kernels of the nonlinear mode couplings of the third-order solution. There are several choices of the shape functions to describe the third-order solutions for Ψ, Φ, and Q. Since the mode couplings in Eqs. (7) and (8) are determined by α_s and γ, the relevant kernels can be straightforwardly chosen to be $\alpha\alpha$, $\alpha\gamma$, $\gamma\alpha$, and $\gamma\gamma$, which are defined in Eqs. (B22)–(B25). Moreover, in order to include the effect of the antisymmetric part of the kernel α appearing in Eqs. (9) and (17), the additional two kernels $\alpha\alpha_\ominus$ and $\alpha\gamma_\ominus$ defined in Eqs. (B26) and (B27) are needed. In addition to these six kernels, we further consider the kernels ξ and ζ defined in Eqs. (15) and (16) to take into account the mode couplings from the three-point self-interaction terms of the scalar field perturbations in Eq. (9). In summary, the set of the relevant third-order shape kernels is given by $U_3 = \{\alpha\alpha, \alpha\gamma, \gamma\alpha, \gamma\gamma, \alpha\alpha_\ominus, \alpha\gamma_\ominus, \xi, \zeta\}$.

It then follows that the third-order evolution equation is given by

$$\ddot{\delta}^{(3)} + (2+\varsigma)H\dot{\delta}^{(3)} - \frac{3}{2}\Omega_{\rm m}\Xi_\Phi H^2\delta^{(3)} = H^2\sum_{\Pi\in U_3} S_\Pi^{(3)} W_\Pi^{(3)}. \tag{41}$$

The coefficients of the third-order mode couplings are

$$(1-\mu_\Phi)S_{\alpha\alpha}^{(3)} = \tau_{\Phi,\alpha\alpha}^{(3)} + 2D_+^3 f^2\kappa_\theta + \frac{1}{a^2H^2}[a^2HD_+^3 f(\kappa+\kappa_\theta)]^{\cdot}, \tag{42}$$

$$(1-\mu_\Phi)S_{\alpha\gamma}^{(3)} = \tau_{\Phi,\alpha\gamma}^{(3)} - \frac{8}{7}D_+^3 f^2\lambda_\theta - \frac{2}{7a^2H^2}[a^2HD_+^3 f(\lambda+2\lambda_\theta)]^{\cdot}, \tag{43}$$

$$(1-\mu_\Phi)S_{\gamma\alpha}^{(3)} = \tau_{\Phi,\gamma\alpha}^{(3)} - 2D_+^3 f^2\kappa_\theta, \tag{44}$$

$$(1-\mu_\Phi)S_{\gamma\gamma}^{(3)} = \tau_{\Phi,\gamma\gamma}^{(3)} + \frac{8}{7}D_+^3 f^2\lambda_\theta, \tag{45}$$

$$(1-\mu_\Phi)S_{\alpha\alpha\ominus}^{(3)} = \tau_{\Phi,\alpha\alpha\ominus}^{(3)} + \frac{1}{a^2H^2}[a^2HD_+^3 f(\kappa-\kappa_\theta)]^{\cdot}, \tag{46}$$

$$(1-\mu_\Phi)S_{\alpha\gamma\ominus}^{(3)} = \tau_{\Phi,\alpha\gamma\ominus}^{(3)} - \frac{2}{7a^2H^2}[a^2HD_+^3 f(\lambda-2\lambda_\theta)]^{\cdot}, \tag{47}$$

$$(1-\mu_\Phi)S_\xi^{(3)} = \tau_{\Phi,\xi}^{(3)}, \tag{48}$$

$$(1-\mu_\Phi)S_\zeta^{(3)} = \tau_{\Phi,\zeta}^{(3)}, \tag{49}$$

where the explicit expression of $\tau_{\Phi,\Pi}^{(3)}$ is shown in Appendixes B 2 and B 4. Using the following relation:

$$W_{\gamma\alpha} = \frac{1}{2}(W_{\alpha\gamma} + W_{\alpha\gamma\ominus}) + W_{\gamma\gamma} - \frac{1}{2}W_\xi, \tag{50}$$

one can remove $W_{\gamma\alpha}$ and absorb its coefficients into $W_{\alpha\gamma}$, $W_{\alpha\gamma\ominus}$, $W_{\gamma\gamma}$, and W_ξ.

Following the same step as the second-order solution, we thus arrive at the third-order solution,

$$\delta^{(3)} = D_+^3\Big[d_{\alpha\alpha}W_{\alpha\alpha}^{(3)} - \frac{4}{7}d_{\alpha\gamma}W_{\alpha\gamma}^{(3)} - \frac{2}{21}d_{\gamma\gamma}W_{\gamma\gamma}^{(3)} + \frac{1}{9}d_\xi W_\xi^{(3)} + d_{\alpha\alpha\ominus}W_{\alpha\alpha\ominus}^{(3)} + d_{\alpha\gamma\ominus}W_{\alpha\gamma\ominus}^{(3)} + d_\zeta W_\zeta^{(3)}\Big], \tag{51}$$

where

$$d_{\alpha\alpha} = \frac{1}{D_+^3}L[H^2S_{\alpha\alpha}^{(3)}], \quad d_{\alpha\gamma} = -\frac{7}{4D_+^3}L\left[H^2\left(S_{\alpha\gamma}^{(3)} + \frac{1}{2}S_{\gamma\alpha}^{(3)}\right)\right],$$
$$d_{\gamma\gamma} = -\frac{21}{2D_+^3}L[H^2(S_{\gamma\gamma}^{(3)} + S_{\gamma\alpha}^{(3)})],$$
$$d_\xi = \frac{9}{D_+^3}L\left[H^2\left(S_\xi^{(3)} - \frac{1}{2}S_{\gamma\alpha}^{(3)}\right)\right],$$
$$d_{\alpha\alpha\ominus} = \frac{1}{D_+^3}L[H^2S_{\alpha\alpha\ominus}^{(3)}], \quad d_{\alpha\gamma\ominus} = \frac{1}{D_+^3}L\left[H^2\left(S_{\alpha\gamma\ominus}^{(3)} + \frac{1}{2}S_{\gamma\alpha}^{(3)}\right)\right],$$
$$d_\zeta = \frac{1}{D_+^3}L[H^2S_\zeta^{(3)}], \tag{52}$$

and $L[\cdots]$ has already been defined in Eq. (37). In the limit of the Einstein–de Sitter universe in GR, it is easy to show that $d_{\alpha\alpha}$, $d_{\alpha\gamma}$, $d_{\gamma\gamma}$, and d_ξ reduce to unity, while the other three, $d_{\alpha\alpha\ominus}$, $d_{\alpha\gamma\ominus}$, and d_ζ, vanish. In the case where gravity is described by the Horndeski family, we still have $d_{\alpha\alpha} = 1$ and $d_{\alpha\alpha\ominus} = d_{\alpha\gamma\ominus} = d_\zeta = 0$, but now $d_{\alpha\gamma}$, $d_{\gamma\gamma}$, and d_ξ deviate from unity [57,58]. The present analysis shows that in DHOST theories all of these seven quantities can have nonstandard values in general. In particular, $d_{\alpha\alpha} \neq 1$, $d_{\alpha\alpha\ominus} \neq 0$, $d_{\alpha\gamma\ominus} \neq 0$, and $d_\zeta \neq 0$ are specific to theories beyond Horndeski.

IV. ONE-LOOP POWER SPECTRUM

We now calculate the one-loop power spectrum for the matter density perturbations. The one-loop matter power spectrum has been discussed in the context of modified gravity theories [59–62] and in particular in the context of the Horndeski theory [57,58] and DHOST theories [45,46]. In Refs. [45,46], the one-loop matter power spectrum in the IR limit of the loop integrals has been investigated and the third-order solution has been obtained only in the IR limit. However, the complete form of the one-loop matter power spectrum including the UV contribution of the loop integrals has not been derived yet in the context of DHOST theories. In the present paper, we calculate the complete form of the one-loop matter power spectrum by using the third-order solution derived in Sec. III.

The power spectrum for δ is given in terms of the two-point correlation function as

$$\langle\delta(t,\boldsymbol{k}_1)\delta(t,\boldsymbol{k}_2)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)P_{\delta\delta}(t,\boldsymbol{k}_1). \quad (53)$$

In this paper, we focus on the autopower spectrum for δ; the one-loop cross-power spectrum between matter density perturbation and velocity divergence, and the autopower spectrum for velocity divergence has the same structure as that of the autopower spectrum for the matter density perturbation.

A. One-loop matter power spectrum

Using the solution of the matter density perturbations up to third order, $\delta(t,\boldsymbol{k}) = \delta^{(1)}(t,\boldsymbol{k}) + \delta^{(2)}(t,\boldsymbol{k}) + \delta^{(3)}(t,\boldsymbol{k})$, and assuming that the initial density field obeys the Gaussian statistics, one can write the one-loop matter power spectrum as

$$P_{\delta\delta}(t,\boldsymbol{k}) = D_+^2(t)P_{\rm L}(k) + D_+^4(t)[P_{\delta\delta}^{(22)}(t,\boldsymbol{k}) + 2P_{\delta\delta}^{(13)}(t,\boldsymbol{k})], \quad (54)$$

where $P_{\delta\delta}^{(22)}$ and $P_{\delta\delta}^{(13)}$ are one-loop corrections to the linear power spectrum due to the second- and third-order solutions defined by

$$\langle\delta^{(2)}(t,\boldsymbol{k}_1)\delta^{(2)}(t,\boldsymbol{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2)D_+^4(t)P_{\delta\delta}^{(22)}(t,\boldsymbol{k}_1), \quad (55)$$

$$\langle\delta^{(1)}(t,\boldsymbol{k}_1)\delta^{(3)}(t,\boldsymbol{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2)D_+^4(t)P_{\delta\delta}^{(13)}(t,\boldsymbol{k}_1), \quad (56)$$

and $P_{\rm L}(k)$ is the linear power spectrum for the initial density field $\delta_{\rm L}$. It follows from Eqs. (35) and (51) that the second- and third-order solutions can be written in terms of the kernels as

$$\delta^{(2)}(t,\boldsymbol{k}) = \frac{D_+^2(t)}{(2\pi)^3}\int \mathrm{d}^3p_1\mathrm{d}^3p_2\delta_{\rm D}(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{k}) \times F_2(t,\boldsymbol{p}_1,\boldsymbol{p}_2)\delta_{\rm L}(\boldsymbol{p}_1)\delta_{\rm L}(\boldsymbol{p}_2), \quad (57)$$

$$\delta^{(3)}(t,\boldsymbol{k}) = \frac{D_+^3(t)}{(2\pi)^6}\int \mathrm{d}^3p_1\mathrm{d}^3p_2\mathrm{d}^3p_3\delta_{\rm D}(\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3-\boldsymbol{k}) \times F_3(t,\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3)\delta_{\rm L}(\boldsymbol{p}_1)\delta_{\rm L}(\boldsymbol{p}_2)\delta_{\rm L}(\boldsymbol{p}_3), \quad (58)$$

with

$$F_2(t,\boldsymbol{p}_1,\boldsymbol{p}_2) = \kappa(t)\alpha_s(\boldsymbol{p}_1,\boldsymbol{p}_2) - \frac{2}{7}\lambda(t)\gamma(\boldsymbol{p}_1,\boldsymbol{p}_2), \quad (59)$$

$$F_3(t,\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) = d_{\alpha\alpha}(t)\alpha\alpha(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) - \frac{4}{7}d_{\alpha\gamma}(t)\alpha\gamma(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) - \frac{2}{21}d_{\gamma\gamma}(t)\gamma\gamma(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) + \frac{1}{9}d_{\xi}(t)\xi_c(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) + d_{\alpha\alpha\ominus}(t)\alpha\alpha_{\ominus}(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) + d_{\alpha\gamma\ominus}(t)\alpha\gamma_{\ominus}(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) + d_{\zeta}(t)\zeta_c(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3), \quad (60)$$

where the explicit forms of the mode-coupling kernels are shown in Eqs. (B22)–(B29). Substituting these into Eqs. (55) and (56) and using Wick's theorem, we obtain

$$P_{\delta\delta}^{(22)}(t,\boldsymbol{k}) = \frac{2}{(2\pi)^3}\int \mathrm{d}^3pF_2^2(t,\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p})P_{\rm L}(p)P_{\rm L}(|\boldsymbol{k}-\boldsymbol{p}|), \quad (61)$$

$$P_{\delta\delta}^{(13)}(t,\boldsymbol{k}) = \frac{3}{(2\pi)^3}P_{\rm L}(k)\int \mathrm{d}^3pF_3(t,\boldsymbol{k},\boldsymbol{p},-\boldsymbol{p})P_{\rm L}(p). \quad (62)$$

Performing the integrals, we arrive at the final form of the one-loop corrections,

$$P_{\delta\delta}^{(22)}(t,\boldsymbol{k}) = \frac{k^2}{(2\pi)^2}\int_0^\infty \mathrm{d}p\mathcal{P}_{22}(p), \tag{63}$$

$$P_{\delta\delta}^{(13)}(t,\boldsymbol{k}) = \frac{k^2}{(2\pi)^2}\int_0^\infty \mathrm{d}p\mathcal{P}_{13}(p). \tag{64}$$

Here we have defined the kernel functions as

$$\mathcal{P}_{22}(p) = \frac{P_{\mathrm{L}}(p)}{98}\int_{-1}^1 \mathrm{d}xP_{\mathrm{L}}((k^2+p^2-2kpx)^{1/2})\frac{[(7\kappa-4\lambda)p+7\kappa kx-2px^2(7\kappa-2\lambda)]^2}{(k^2-2kpx+p^2)^2}, \tag{65}$$

$$\mathcal{P}_{13}(p) = \frac{P_{\mathrm{L}}(k)P_{\mathrm{L}}(p)}{12}\left[\frac{2}{7}d_{\gamma\gamma}\frac{k^2}{p^2}+4\left(\frac{3}{4}\mathcal{D}-\frac{4}{21}d_{\gamma\gamma}-d_{\alpha\alpha}-d_{\alpha\alpha\ominus}-2d_\zeta\right)\right.$$
$$\left.+8\left(\mathcal{D}-\frac{1}{28}d_{\gamma\gamma}-d_{\alpha\alpha\ominus}-d_\zeta\right)\frac{p^2}{k^2}+3\mathcal{D}\frac{p^4}{k^4}+\frac{3}{2}\left(\mathcal{D}\frac{p^2}{k^2}+\frac{2}{21}d_{\gamma\gamma}\right)\frac{(k^2-p^2)^3}{k^3p^3}\ln\left(\frac{k+p}{|k-p|}\right)\right], \tag{66}$$

where x denotes the directional cosine between $\boldsymbol{k}$ and $\boldsymbol{p}$ defined as $x = \boldsymbol{k}\cdot\boldsymbol{p}/kp$, and we have introduced

$$\mathcal{D} \coloneqq d_{\alpha\alpha} - \frac{4}{7}d_{\alpha\gamma} - \frac{2}{21}d_{\gamma\gamma} - d_{\alpha\alpha\ominus} - d_{\alpha\gamma\ominus}. \tag{67}$$

Given a concrete model of modified gravity and a linear power spectrum, it is now straightforward to calculate the one-loop matter power spectrum using Eqs. (63) and (64).

B. Asymptotic behaviors of the loop integrals

In order to study the one-loop contributions to the matter power spectrum in the context of DHOST theories, we would like to examine their asymptotic behavior of the short- and long-wavelength limits in the loop integrals as done in Ref. [63]. To do this, let us divide the one-loop contributions into that from the momentum integration for $p \gg k$ (UV region) and that from the integration for $p \ll k$ (IR region), for fixed k. It was shown in Ref. [63] that, when assuming GR and the standard linear matter power spectrum,[1] the leading terms from $\mathcal{P}_{22}$ and $\mathcal{P}_{13}$ in the IR limit are exactly canceled out and the loop integrals in both the IR and UV regions are convergent. In this section, we extend their analysis to DHOST theories, and in particular, we investigate the asymptotic behavior of the matter power spectrum and the condition for their convergence.

[1] Assuming the scale-invariant primordial curvature perturbations, the standard scale dependence of the linear power spectrum is roughly given by

$$P_{\mathrm{L}}(k) \propto kT^2(k) \propto \begin{cases} k & (k \ll k_{\mathrm{eq}}), \\ k^{-3} & (k \gg k_{\mathrm{eq}}), \end{cases} \tag{68}$$

where $T(k)$ is the transfer function and k_{eq} is the wave number at the matter-radiation equality time.

1. IR limit

Let us first consider the long-wavelength contribution in the IR limit, namely, $p/k \to 0$. In the naive $p \to 0$ limit of Eq. (65), we have

$$\mathcal{P}_{22} \to \frac{1}{3}\kappa^2 P_{\mathrm{L}}(k)P_{\mathrm{L}}(p). \tag{69}$$

However, as pointed out in Ref. [63], since the second-order kernel $F_2(t,\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p})$ is symmetric between $\boldsymbol{p}$ and $\boldsymbol{k}-\boldsymbol{p}$, we also have to take into account the $|\boldsymbol{k}-\boldsymbol{p}| \to 0$ limit so that the integrand in the appropriate limit is twice larger than Eq. (69). Hence, the appropriate IR limit of (65) is given by

$$\mathcal{P}_{22} \to \frac{2}{3}\kappa^2 P_{\mathrm{L}}(k)P_{\mathrm{L}}(p). \tag{70}$$

On the other hand, the same limit of Eq. (66) yields

$$\mathcal{P}_{13} \to -\frac{1}{3}(d_{\alpha\alpha}+d_{\alpha\alpha\ominus}+2d_\zeta)P_{\mathrm{L}}(k)P_{\mathrm{L}}(p). \tag{71}$$

We find from Eqs. (70) and (71) that in the IR limit the sum of the kernel functions in the one-loop correction, $\mathcal{P}_{22}+2\mathcal{P}_{13}$, is canceled out within the Horndeski family of theories, $\kappa = d_{\alpha\alpha} = 1$ and $d_{\alpha\alpha\ominus} = d_\zeta = 0$, and the convergence condition is the same as those in GR [58]. However, once we consider DHOST theories, this cancellation does not occur, and the convergence condition of the loop integrals seems to become more stringent than that of standard one in GR. This phenomenon has been already reported in Refs. [45,46] (see Ref. [58] in the context of effective field theory of large scale structure) while we described the explicit forms of the functions d, which determine the asymptotic behavior of $\mathcal{P}_{22}+2\mathcal{P}_{13}$, in terms of parameters characterizing DHOST theories.

We anticipate that the more stringent convergence condition of the loop integrals in the IR region originates from strong correlations between short and long modes in such theories. In order to support this argument, we revisit the matter trispectrum in DHOST theories.

The trispectrum for δ is given in terms of the four-point correlation function as

$$\langle\delta(t,\boldsymbol{k}_1)\delta(t,\boldsymbol{k}_2)\delta(t,\boldsymbol{k}_3)\delta(t,\boldsymbol{k}_4)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)T(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4). \quad (72)$$

Since the linear density field is assumed to be Gaussian, the matter trispectrum is given to leading order by

$$T(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \simeq D_+^6(t)[T^{(1122)}(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) + T^{(1113)}(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)], \quad (73)$$

where

$$\begin{aligned} T^{(1122)} = {}& 4P_{\rm L}(k_1)P_{\rm L}(k_2)[P_{\rm L}(|\boldsymbol{k}_1+\boldsymbol{k}_3|)F_2(t,\boldsymbol{k}_1,-\boldsymbol{k}_1-\boldsymbol{k}_3)F_2(t,\boldsymbol{k}_2,\boldsymbol{k}_1+\boldsymbol{k}_3) \\ &+ P_{\rm L}(|\boldsymbol{k}_1+\boldsymbol{k}_4|)F_2(t,\boldsymbol{k}_1,-\boldsymbol{k}_1-\boldsymbol{k}_4)F_2(t,\boldsymbol{k}_2,\boldsymbol{k}_1+\boldsymbol{k}_4)] \\ &+ 5\,\text{perms.}, \end{aligned} \quad (74)$$

$$T^{(1113)} = 6P_{\rm L}(k_1)P_{\rm L}(k_2)P_{\rm L}(k_3)F_3(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + 3\,\text{perms.}. \quad (75)$$

As we are interested in the interactions between short and long modes, we take the double soft limit in which two wave vectors are taken to be much smaller than the other two. Let us look at the dimensionless reduced trispectrum defined by

$$Q(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q}_1,\boldsymbol{q}_2) = \frac{T(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q}_1,\boldsymbol{q}_2)}{D_+^6(t)[P_{\rm L}(k_1)P_{\rm L}(k_2)P_{\rm L}(q_1) + 3\,\text{perms.}]}. \quad (76)$$

In the double soft limit, $q_1, q_2 \ll k_1, k_2$ with $\boldsymbol{k}_1 \approx -\boldsymbol{k}_2$ and $\boldsymbol{q}_1 \approx -\boldsymbol{q}_2$, Eqs. (74) and (75) reduce to

$$T^{(1122)} \to 8P_{\rm L}(k_1)P_{\rm L}^2(q_1)\kappa^2(t)\alpha_s^2(\boldsymbol{q}_1,\boldsymbol{k}_1), \quad (77)$$

$$\begin{aligned} T^{(1113)} \to {}& 12P_{\rm L}(k_1)P_{\rm L}^2(q_1)[d_{\alpha\alpha}(t)\alpha\alpha(\boldsymbol{k}_1,\boldsymbol{q}_1,\boldsymbol{q}_2) \\ &+ d_{\alpha\alpha\ominus}(t)\alpha\alpha_\ominus(\boldsymbol{k}_1,\boldsymbol{q}_1,\boldsymbol{q}_2) + d_\zeta(t)\zeta_c(\boldsymbol{k}_1,\boldsymbol{q}_1,\boldsymbol{q}_2)], \end{aligned} \quad (78)$$

and hence the reduced trispectrum reads

$$Q(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q}_1,\boldsymbol{q}_2) \to \frac{P_{\rm L}(q_1)}{P_{\rm L}(k_1)}(\kappa^2 - d_{\alpha\alpha} - d_{\alpha\alpha\ominus} - 2d_\zeta)\left(\frac{\boldsymbol{q}_1\cdot\boldsymbol{k}_1}{q_1^2}\right)^2. \quad (79)$$

In the Horndeski theory, we have $\kappa = d_{\alpha\alpha} = 1$ and $d_{\alpha\alpha\ominus} = d_\zeta = 0$, so that the above would-be leading contribution vanishes. However, in theories more general than Horndeski, the above expression does not vanish in general. We thus see that in the trispectrum there is a non-negligible contribution in the double soft limit that appears for the first time in DHOST theories beyond Horndeski. This result is consistent with the more stringent convergence condition of the loop integrals in the IR limit and we reproduce the results of Ref. [45,46]. We conclude that the more stringent convergence condition of the loop integrals in the IR limit originates from the strong correlations between short and long modes.

2. UV limit

Let us move on the short-wavelength contribution in the UV limit, namely, $p/k \to \infty$. Hereafter, we assume that the linear power spectrum $P_{\rm L}(p)$ in the UV regions behaves asymptotically in proportion to p^n. In the $p/k \to \infty$ limit, Eq. (65) reduces to

$$\mathcal{P}_{22} \to \frac{343\kappa^2 - 336\kappa\lambda + 128\lambda^2}{735}\frac{k^2}{p^2}[P_{\rm L}(p)]^2 \propto p^{2(n-1)}. \quad (80)$$

This expression can be rewritten as

$$\frac{P_{\delta\delta}^{(22)}(k)}{P_{\rm L}(k)} \to \frac{343\kappa^2 - 336\kappa\lambda + 128\lambda^2}{735(2\pi)^2} \times \frac{k^4}{P_{\rm L}(k)}\int_{p\gtrsim k} {\rm d}p\, p^{-2}[P_{\rm L}(p)]^2. \quad (81)$$

Thus, this term is convergent for $n \le 1/2$, which is the same as the convergence condition in GR. We then investigate the same limit of Eq. (66),

$$\mathcal{P}_{13} \to -\frac{2}{3}(d_{\alpha\alpha\ominus} + d_\zeta)\frac{p^2}{k^2}P_{\rm L}(k)P_{\rm L}(p) \propto p^{n+2}. \quad (82)$$

Hence, we have

$$\frac{P_{\delta\delta}^{(13)}(k)}{P_{\rm L}(k)} \to -\frac{2(d_{\alpha\alpha\ominus}+d_\zeta)}{3(2\pi)^2}\int_{p\gtrsim k}\mathrm{d}p\,p^2P_{\rm L}(p), \quad (83)$$

which immediately leads to that the integration with the UV regions in $P_{\delta\delta}^{(13)}$ is separately convergent only for $n\le -3$. We then find that its leading dependence on p is stronger and the condition of its convergence becomes more stringent than that of in GR. On the other hand, in the case of the Horndeski theory, the coefficient of the leading term vanishes and the next-to-leading term is given by

$$\mathcal{P}_{13}^{\rm Horn}\to\frac{147-144\lambda-64d_{\gamma\gamma}}{315}P_{\rm L}(k)P_{\rm L}(p)\propto p^n, \quad (84)$$

implying that the convergence condition reduces to that in GR, $n\le -1$. Therefore, we conclude that in DHOST theories beyond Horndeski the linear power spectrum should be required to be redder than that in the case of the Horndeski theory and GR for the convergence of the one-loop correction. An important observation is that the standard linear power spectrum which behaves as $P_{\rm L}(k)\propto k^{-3}$ for short wavelengths is on the edge of the convergence in DHOST theories. Note that the coefficient of the leading term, $d_{\alpha\alpha\ominus}+d_\zeta$, does not vanish even in the viable DHOST theory evading gravitational-wave constraints [42,64].

Before closing the section, let us suggest some possibilities to resolve this UV sensitive behavior of the one-loop matter power spectrum in DHOST theories. The first possibility is, as we have already discussed, to consider the linear power spectrum with the power-law index being $n\le -3$ for short wavelengths. The second is to introduce the cutoff scale in the matter power spectrum, which depends on the nature of dark matter [65]. One may also have another, rather different, possibility that one eliminates the UV terms at the level of the integrand, namely, one imposes the additional condition $d_{\alpha\alpha\ominus}+d_\zeta=0$, which can be used to add the constraint on the combination of the parameters, on the basis of the assumption that this UV divergent behavior would be spurious and must vanish.

V. CONCLUSIONS

In this paper, we have studied the third-order solution of the matter density perturbations and the one-loop matter power spectrum in the context of the DHOST theories. We have solved the field equations for the gravitational potentials and scalar field perturbation order by order under the quasistatic approximation and obtained the formal solutions at all order. We then explicitly presented the second- and third-order nonlinear terms appearing in the evolution equation for the density perturbation. The second- and third-order solutions can be characterized, respectively, by two and seven functions describing the nonlinear mode couplings. In particular, we found that at third order there appear three new shape functions in the momentum space [Eqs. (B26), (B27), and (B29)] in DHOST theories beyond Horndeski, which could yield the unique signature of this new class of scalar-tensor theories.

Furthermore, by using the resultant second- and third-order solutions of the matter density perturbations, we calculated the one-loop matter power spectrum and investigated their asymptotic behavior in the short- and long-wavelength limits in the loop integrals. Although as far as the Horndeski theory is concerned, the asymptotic behavior both in IR and UV limits is basically the same as that in general relativity; we have shown that in DHOST theories the behavior of the loop integrals can be drastically changed. At the IR limit, the leading terms in $P_{\delta\delta}^{(22)}$ and $P_{\delta\delta}^{(13)}$ do not cancel and the condition for the IR convergence is thus more stringent than the standard one in general relativity. Even though this feature has been already discussed in Refs. [45,46], we derive the complete expressions for the leading terms in terms of the functions characterizing the theories and it can make the origin of this distinctive IR behavior in DHOST theories clearer. By evaluating the matter trispectrum in the double soft limit, we argue that the more stringent convergence condition of the loop integrals in the IR limit is the consequence of strong correlations between short and long modes in such theories. For the UV limit, we have shown that the loop integral related to the third-order solution in DHOST theories has logarithmic divergence in the case of the standard linear power spectrum. Hence, we conclude that the one-loop contributions to the matter power spectrum would be sensitive to the short-wavelength behavior of the linear power spectrum as long as gravity is described by DHOST theories beyond Horndeski.

The more stringent convergence conditions of the loop integrals could be interpreted from the point of view of quantum field theory. As usually discussed in quantum field theory, symmetry protects loop corrections of correlation functions. As reported in Refs. [45,46], Horndeski theories have the accidental symmetry which is related to the Friedmann-Lemaître-Robertson-Walker symmetry and shift symmetry in terms of fields (see Refs. [45,46] as the detailed discussion) while operators in DHOST theories beyond Horndeski violate that. So, this violation may be related to the more stringent convergence conditions of the loop integrals in DHOST theories beyond Horndeski. To support this argument, it is important to investigate whether divergent terms in the loop integrals vanish thanks to the above accidental symmetry in light of effective field theory of large scale structure [66]. Or, moving to the Einstein frame, the coupling between matter and the scalar degree of freedom could be large, so that the prediction based on perturbation theory might not be reliable. We hope to come back to these issues in the future.

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APPENDIX A: DEFINITION OF THE COEFFICIENTS IN THE EQUATIONS OF MOTION

In this section, we summarize the definition of the effective field theory parameters and the coefficients in the equations of motion. In addition to the parameters that appear in the class Ia degeneracy conditions (2) and (3), one can characterize cosmological perturbations in DHOST theories by introducing α_B, α_M, and α_V defined by

$$M^2 H\alpha_M = \frac{d}{dt}M^2, \tag{A1}$$

$$\begin{aligned} M^2H\alpha_B = {}& M^2H\alpha_V - 3M^2H\beta_1 \\ &+ \dot{\phi}(-XG_{3X} + G_{4\phi} + 2XG_{4\phi X}) \\ &+ \dot{\phi}\ddot{\phi}[2X(G_{4XX} - a_{2X} + Xa_{3X} - a_4 \\ &+ 2Xa_5) + 3(G_{4X} - a_2 + Xa_3)], \end{aligned} \tag{A2}$$

$$M^2\alpha_V = 4X(G_{4X} - 2a_2 - 2Xa_{2X}). \tag{A3}$$

These parameters appear within Horndeski theories. Note that we have yet another parameter which is often denoted as α_K, but it does not appear in the equations under the quasistatic region (i.e., on subhorizon scales).

The explicit expressions of the coefficients in Eqs. (7)–(9) are given by

$$c_1 = -4\left[\alpha_B - \alpha_H + \frac{\beta_3}{2}(1+\alpha_M) + \frac{\dot{\beta}_3}{2H}\right], \tag{A4}$$

$$c_2 = 4\left[\alpha_H(1+\alpha_M) + \alpha_M - \alpha_T + \frac{\dot{\alpha}_H}{H}\right], \tag{A5}$$

$$\begin{aligned} c_3 = -2\Bigg\{&\left(1+\alpha_M+\frac{\dot{H}}{H^2}\right)(\alpha_B-\alpha_H) + \frac{\dot{\alpha}_B - \dot{\alpha}_H}{H} \\ &+ \frac{3\Omega_m}{2} + \frac{\dot{H}}{H^2} + \alpha_T - \alpha_M \\ &+ \left[-2\frac{\dot{H}}{H^2}\beta_1 + \frac{\beta_3}{4}(1+\alpha_M) + \frac{\dot{\beta}_3}{2H}\right]\left(1+\alpha_M - \frac{\dot{H}}{H^2}\right) \\ &- 2\frac{\dot{H}}{H^2}\frac{\dot{\beta}_1}{H} + \left(\frac{\dot{H}}{H^2}\right)^2\frac{\beta_3}{2} + \frac{\dot{\alpha}_M}{H}\frac{\beta_3}{4} + \frac{\ddot{\beta}_3}{4H^2}\Bigg\}, \end{aligned} \tag{A6}$$

$$b_1 = \frac{c_1}{4} + \frac{1}{2}(1+\alpha_M)(2\beta_1+\beta_3) + \frac{1}{2}\frac{d}{dt}\left(\frac{2\beta_1+\beta_3}{H}\right), \tag{A7}$$

$$b_2 = -\frac{c_2}{4} + (1+\alpha_M)\alpha_H + \left(\frac{\alpha_H}{H}\right)^{\cdot}, \tag{A8}$$

$$\begin{aligned} b_3 = {}& 2c_3 + \left[\left(1+\alpha_M - \frac{\dot{H}}{H^2}\right)(1+\alpha_M) + \frac{\dot{\alpha}_M}{H}\right](4\beta_1+\beta_3) \\ &+ 2(1+\alpha_M)\left(\frac{4\beta_1+\beta_3}{H}\right)^{\cdot} + \left(\frac{2\beta_1+\beta_3}{H^2}\right)^{\cdot\cdot}, \end{aligned} \tag{A9}$$

$$b_4 = 2\left[\left(1+\alpha_M - \frac{\dot{H}}{H^2}\right)(4\beta_1+\beta_3) + \left(\frac{4\beta_1+\beta_3}{H}\right)^{\cdot}\right], \tag{A10}$$

$$\begin{aligned} d_1 = -\Bigg[&\alpha_V + 3(\alpha_H - \alpha_T) - 4\alpha_B + \alpha_M(2 - \alpha_V + \alpha_H + 8\beta_1) \\ &+ 2(4\beta_1+\beta_3)\frac{\dot{H}}{H^2} - \frac{\dot{\alpha}_V - \dot{\alpha}_H - 8\dot{\beta}_1}{H}\Bigg], \end{aligned} \tag{A11}$$

$$d_2 = \frac{1}{2}(\alpha_V - \alpha_H - 4\beta_1), \tag{A12}$$

where Ω_m was defined in Eq. (27).

APPENDIX B: COEFFICIENTS OF FIRST-, SECOND-, AND THIRD-ORDER SOLUTIONS

In this section, we summarize the coefficients of first-, second- and third-order solutions.

1. Homogeneous solutions

The components of the matrices M and N in Eq. (20) are read off from Eqs. (7) and (8) as

$$\mathsf{M} = (\mathsf{M})_{ab} = \begin{pmatrix} \mathsf{M}_{\Psi\Psi} & \mathsf{M}_{\Psi\Phi} \\ \mathsf{M}_{\Phi\Psi} & \mathsf{M}_{\Phi\Phi} \end{pmatrix} = \begin{pmatrix} 1+\alpha_T & -(1+\alpha_H) \\ 1+\alpha_H & -\beta_3/2 \end{pmatrix}, \tag{B1}$$

$$\mathsf{N} = \begin{pmatrix} \mathsf{N}_{\Psi Q} & \mathsf{N}_{\Psi\dot{Q}} \\ \mathsf{N}_{\Phi Q} & \mathsf{N}_{\Phi\dot{Q}} \end{pmatrix} = \begin{pmatrix} -b_2 & -\alpha_H \\ -b_1 & -(2\beta_1+\beta_3)/2 \end{pmatrix}, \tag{B2}$$

with a, b stand for Ψ and Φ. The coefficients in (24) and (25) can be written in terms of above quantities and the coefficient of Eq. (23) as

$$\mu_a = (\mathsf{M}^{-1}\mathsf{N})_{a\dot{Q}}\nu_Q, \tag{B3}$$

$$\nu_a = (\mathsf{M}^{-1}\mathsf{N})_{aQ}\nu_Q + (\mathsf{M}^{-1}\mathsf{N})_{a\dot{Q}}\left[\kappa_Q + \frac{(a^2H\nu_Q)^{\cdot}}{a^2H^2}\right], \tag{B4}$$

$$\kappa_a = \frac{3}{2}\Omega_m(\mathsf{M}^{-1})_{a\Phi} + (\mathsf{M}^{-1}\mathsf{N})_{aQ}\kappa_Q + (\mathsf{M}^{-1}\mathsf{N})_{a\dot{Q}}\frac{(a^2H^2\kappa_Q)^{\cdot}}{a^2H^3}. \tag{B5}$$

Substituting these back into Eq. (9), we obtain the explicit forms of the coefficients in Eq. (23) as

$$\nu_Q = -\frac{3}{2}\frac{\Omega_{\rm m}}{Z}[4\alpha_{\rm H}(\mathsf{M}^{-1})_{\Psi\Phi} - 2(2\beta_1+\beta_3)(\mathsf{M}^{-1})_{\Phi\Phi}], \tag{B6}$$

$$\kappa_Q = -\frac{3}{2}\frac{\Omega_{\rm m}}{Z}\left[c_2(\mathsf{M}^{-1})_{\Psi\Phi} + c_1(\mathsf{M}^{-1})_{\Phi\Phi} + \frac{4aM^2\alpha_{\rm H}}{H}\left[\frac{1}{aM^2}(\mathsf{M}^{-1})_{\Psi\Phi}\right]^{\cdot} - \frac{2aM^2(2\beta_1+\beta_3)}{H}\left[\frac{1}{aM^2}(\mathsf{M}^{-1})_{\Phi\Phi}\right]^{\cdot}\right], \tag{B7}$$

where

$$Z = b_3 + c_2(\mathsf{M}^{-1}\mathsf{N})_{\Psi Q} + c_1(\mathsf{M}^{-1}\mathsf{N})_{\Phi Q} + \frac{4\alpha_{\rm H}}{H}[(\mathsf{M}^{-1}\mathsf{N})_{\Psi Q}]^{\cdot} - \frac{2(2\beta_1+\beta_3)}{H}[(\mathsf{M}^{-1}\mathsf{N})_{\Phi Q}]^{\cdot}. \tag{B8}$$

2. General expression of coefficients of higher-order solutions

We show that the nth order coefficient with the shape Π in Eqs. (24) and (25) is generally written in terms of the nth order coefficient of $Q^{(n)}$ and the matrix components of M, N, and $\mathsf{O}^{(n)}$ as

$$\tau^{(n)}_{a,\Pi} = (\mathsf{M}^{-1}\mathsf{O}^{(n)})_{a\Pi} + (\mathsf{M}^{-1}\mathsf{N})_{aQ}\tau^{(n)}_{Q,\Pi} + (\mathsf{M}^{-1}\mathsf{N})_{a\dot{Q}}\frac{(a^2H^2\tau^{(n)}_{Q,\Pi})^{\cdot}}{a^2H^3}. \tag{B9}$$

Substituting the nth order solutions of Ψ, Φ and Eq. (23) into Eq. (9), we then obtain the form of $\tau^{(n)}_{Q,\Pi}$ as

$$\tau^{(n)}_{Q,\Pi} = \frac{1}{Z}\left[\mathsf{O}^{(n)}_{Q,\Pi} - c_2(\mathsf{M}^{-1}\mathsf{O}^{(n)})_{\Psi\Pi} - c_1(\mathsf{M}^{-1}\mathsf{O}^{(n)})_{\Psi\Pi} - \frac{4\alpha_{\rm H}}{a^2H^3}[a^2H^2(\mathsf{M}^{-1}\mathsf{O}^{(n)})_{\Psi\Pi}]^{\cdot} + \frac{2(2\beta_1+\beta_3)}{a^2H^3}[a^2H^2(\mathsf{M}^{-1}\mathsf{O}^{(n)})_{\Phi\Pi}]^{\cdot}\right], \tag{B10}$$

where Z was defined in Eq. (B8). Here, the coefficient $\mathsf{O}^{(n)}_{Q,\Pi}$ EoM of Q and the coefficient of $W^{(n)}_{\Pi}$. Therefore, once the lower-order solutions and the nth order matrix components of $\mathsf{O}^{(n)}$ are given, we can straightforwardly derive the nth order solution of Ψ, Φ, and Q.

3. Second-order solutions

To derive the second-order coefficients in Eqs. (23)–(25), we need to write down the reduced first-order solution. When substituting Eqs. (28) and (30) into the first-order solution of Eqs. (23)–(25), $\Psi^{(1)}$, $\Phi^{(1)}$, and $Q^{(1)}$ can be rewritten as

$$\Psi^{(1)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^2(t)}{p^2}K_\Psi(t)D_+(t)\delta_{\rm L}(\boldsymbol{p}), \tag{B11}$$

$$\Phi^{(1)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^2(t)}{p^2}K_\Phi(t)D_+(t)\delta_{\rm L}(\boldsymbol{p}), \tag{B12}$$

$$Q^{(1)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^2(t)}{p^2}K_Q(t)D_+(t)\delta_{\rm L}(\boldsymbol{p}), \tag{B13}$$

$$\dot{Q}^{(1)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^3(t)}{p^2}K_{\dot{Q}}(t)D_+(t)\delta_{\rm L}(\boldsymbol{p}). \tag{B14}$$

At the second order, the relevant shape functions to describe the solutions are shown to be $\alpha_s(\boldsymbol{k}_1,\boldsymbol{k}_2)$ and $\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2)$, which are defined in Eqs. (13) and (14). Since the nonlinear interaction in Eqs. (7) and (8) are determined by Q, the matrix components of O in Eq. (20) at the second order, that is, $\mathsf{O}^{(2)}$, can be written in terms of the first order solution of Q. We then have

$$\mathsf{O}^{(2)} = \begin{pmatrix} \mathsf{O}^{(2)}_{\Psi,\alpha} & \mathsf{O}^{(2)}_{\Psi,\gamma} \\ \mathsf{O}^{(2)}_{\Phi,\alpha} & \mathsf{O}^{(2)}_{\Phi,\gamma} \end{pmatrix} = \frac{1}{4}D_+^2K_Q^2\begin{pmatrix} 4\alpha_{\rm H} & \alpha_{\rm T}-4\alpha_{\rm H} \\ 2(2\beta_1+\beta_3) & -2(d_2+2\beta_1+\beta_3) \end{pmatrix}. \tag{B15}$$

Moreover, with the use of K_Ψ, K_Φ, and K_Q, and the shape functions, the coefficient in Eq. (B10) is given by

$$\mathsf{O}^{(2)}_{Q,\alpha} = D_+^2K_Q\{4\alpha_{\rm H}K_\Psi - 2(2\beta_1+\beta_3)K_\Phi + b_4K_Q + 6(4\beta_1+\beta_3)K_{\dot{Q}}\}, \tag{B16}$$

$$\mathsf{O}^{(2)}_{Q,\gamma} = -D_+^2K_Q\{2\alpha_{\rm T}K_\Psi + 4d_2K_\Phi + (d_1+b_4)K_Q + 4(4\beta_1+\beta_3)K_{\dot{Q}}\}. \tag{B17}$$

4. Third-order solutions

Following the same step as the previous subsection, to derive the third-order solutions, it is useful to introduce the reduced second-order solutions. Substituting the second-order solution Eq. (35) into Eqs. (23)–(25), we obtain

$$\Psi^{(2)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^2(t)}{p^2}[\tilde{\tau}_{\Psi,\alpha}(t)W_\alpha(\boldsymbol{p}) + \tilde{\tau}_{\Psi,\gamma}(t)W_\gamma(\boldsymbol{p})], \tag{B18}$$

$$\Phi^{(2)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^2(t)}{p^2}[\tilde{\tau}_{\Phi,\alpha}(t)W_\alpha(\boldsymbol{p}) + \tilde{\tau}_{\Phi,\gamma}(t)W_\gamma(\boldsymbol{p})], \tag{B19}$$

$$Q^{(2)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^2(t)}{p^2}[\tilde{\tau}_{Q,\alpha}(t)W_\alpha(\boldsymbol{p}) + \tilde{\tau}_{Q,\gamma}(t)W_\gamma(\boldsymbol{p})], \tag{B20}$$

$$\dot{Q}^{(2)}(t,\boldsymbol{p}) = -\frac{a^2(t)H^3(t)}{p^2}[\tilde{\tau}_{\dot{Q},\alpha}(t)W_\alpha(\boldsymbol{p}) + \tilde{\tau}_{\dot{Q},\gamma}(t)W_\gamma(\boldsymbol{p})], \tag{B21}$$

where $\tilde{\tau}_{*,\Pi}$ can be described by the lower-order solutions and $\tau_{*,\Pi}$ itself.

Let us consider the kernels that describe the nonlinear mode coupling of the third-order solutions. We first define the kernels that are generated by α_s and γ as

$$\alpha\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{3}[\alpha_s(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3)\alpha_s(\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}], \tag{B22}$$

$$\alpha\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{3}[\alpha_s(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3)\gamma(\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}], \tag{B23}$$

$$\gamma\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{3}[\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3)\alpha_s(\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}], \tag{B24}$$

$$\gamma\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{3}[\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3)\gamma(\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}]. \tag{B25}$$

In solving Eqs. (9), (17), and (18), we need to introduce the kernels that are generated by the asntisymmetric part of α as well as α_s and γ. Hence, we define

$$\alpha\alpha_\ominus(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{6}\{[\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3) - \alpha(\boldsymbol{k}_2+\boldsymbol{k}_3,\boldsymbol{k}_1)]\alpha_s(\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}\}, \tag{B26}$$

$$\alpha\gamma_\ominus(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{6}\{[\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2+\boldsymbol{k}_3) - \alpha(\boldsymbol{k}_2+\boldsymbol{k}_3,\boldsymbol{k}_1)]\gamma(\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}\}. \tag{B27}$$

In addition to these six kernels, we have to take into account the mode couplings from the three-point self-interaction terms of Q in Eq. (9), that is, ξ and ζ. We then define the following cyclic-symmetrized mode-coupling functions as

$$\xi_c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{3}\{\xi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}\}, \tag{B28}$$

$$\zeta_c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{3}\{\zeta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + 2\,\text{perms.}\}. \tag{B29}$$

In summary, we need to consider the set of the eight kernels, $U_3 = \{\alpha\alpha, \alpha\gamma, \gamma\alpha, \gamma\gamma, \alpha\alpha_\ominus, \alpha\gamma_\ominus, \xi_c, \zeta_c\}$. With these kernels, the matrix components of $\mathsf{O}^{(3)}$ can be written as

$$\mathsf{O}^{(3)} = \begin{pmatrix} \mathsf{O}^{(3)}_{\Psi,\alpha\alpha} & \mathsf{O}^{(3)}_{\Psi,\alpha\gamma} & \mathsf{O}^{(3)}_{\Psi,\gamma\alpha} & \mathsf{O}^{(3)}_{\Psi,\gamma\gamma} & \mathsf{O}^{(3)}_{\Psi,\alpha\alpha\ominus} & \mathsf{O}^{(3)}_{\Psi,\alpha\gamma\ominus} & \mathsf{O}^{(3)}_{\Psi,\xi} & \mathsf{O}^{(3)}_{\Psi,\zeta} \\ \mathsf{O}^{(3)}_{\Phi,\alpha\alpha} & \mathsf{O}^{(3)}_{\Phi,\alpha\gamma} & \mathsf{O}^{(3)}_{\Phi,\gamma\alpha} & \mathsf{O}^{(3)}_{\Phi,\gamma\gamma} & \mathsf{O}^{(3)}_{\Phi,\alpha\alpha\ominus} & \mathsf{O}^{(3)}_{\Phi,\alpha\gamma\ominus} & \mathsf{O}^{(3)}_{\Phi,\xi} & \mathsf{O}^{(3)}_{\Phi,\zeta} \end{pmatrix}$$
$$= D_+K_Q\begin{pmatrix} 2\alpha_{\rm H}\tilde{\tau}_{Q,\alpha} & 2\alpha_{\rm H}\tilde{\tau}_{Q,\gamma} & (\alpha_{\rm T}-4\alpha_{\rm H})\tilde{\tau}_{Q,\alpha}/2 & (\alpha_{\rm T}-4\alpha_{\rm H})\tilde{\tau}_{Q,\gamma}/2 & 0 & 0 & 0 & 0 \\ (2\beta_1+\beta_3)\tilde{\tau}_{Q,\alpha} & (2\beta_1+\beta_3)\tilde{\tau}_{Q,\gamma} & -(d_2+4\beta_1+2\beta_3)\tilde{\tau}_{Q,\alpha} & -(d_2+4\beta_1+2\beta_3)\tilde{\tau}_{Q,\gamma} & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{B30}$$

Substituting the reduced lower-order solutions Eqs. (B11)–(B14) and (B18)–(B21) into Eq. (9), we can extract the correspondence between the coefficient $\mathsf{O}^{(3)}_{Q,\Pi}$ and other parameters, which are given by

$$\mathsf{O}^{(3)}_{Q,\alpha\alpha} = 2D_+\{4\alpha_{\rm H}K_{(\Psi} - 2(2\beta_1+\beta_3)K_{(\Phi} + b_4K_{(Q} + 6(4\beta_1+\beta_3)K_{(\dot{Q}}\}\tilde{\tau}_{Q),\alpha}, \tag{B31}$$

$$\mathsf{O}^{(3)}_{Q,\alpha\gamma} = 2D_+\{4\alpha_{\rm H}K_{(\Psi} - 2(2\beta_1+\beta_3)K_{(\Phi} + b_4K_{(Q} + 6(4\beta_1+\beta_3)K_{(\dot{Q}}\}\tilde{\tau}_{Q),\gamma}, \tag{B32}$$

$$\mathsf{O}^{(3)}_{Q,\gamma\alpha} = -2D_+\{2\alpha_{\rm T}K_{(\Psi} + 4d_2K_{(\Phi} + (d_1+b_4)K_{(Q} + 4(4\beta_1+\beta_3)K_{(\dot{Q}}\}\tilde{\tau}_{Q),\alpha}, \tag{B33}$$

$$\mathsf{O}^{(3)}_{Q,\gamma\gamma} = -2D_+\{2\alpha_{\rm T}K_{(\Psi} + 4d_2K_{(\Phi} + (d_1+b_4)K_{(Q} + 4(4\beta_1+\beta_3)K_{(\dot{Q}}\}\tilde{\tau}_{Q),\gamma}, \tag{B34}$$

where we have used the round bracket as the symmetrized symbol defined as $K_{(A}\tilde{\tau}_{B),\Pi} = (K_A\tilde{\tau}_{B,\Pi} + K_B\tilde{\tau}_{A,\Pi})/2$. Introducing the antisymmetric symbol $K_{[A}\tilde{\tau}_{B],\Pi} = (K_A\tilde{\tau}_{B,\Pi} - K_B\tilde{\tau}_{A,\Pi})/2$, the remaining coefficients can be given by

$$\mathsf{O}^{(3)}_{Q,\alpha\alpha\ominus} = Z\tau_{Q,\alpha\alpha\ominus} = 4D_+\{2\alpha_{\rm H}K_{[\Psi} - (2\beta_1+\beta_3)K_{[\Phi} - (4\beta_1+\beta_3)K_{[\dot{Q}}\}\tilde{\tau}_{Q],\alpha}, \tag{B35}$$

$$\mathsf{O}^{(3)}_{Q,\alpha\gamma\ominus} = Z\tau_{Q,\alpha\gamma\ominus} = 4D_+\{2\alpha_{\rm H}K_{[\Psi} - (2\beta_1+\beta_3)K_{[\Phi} - (4\beta_1+\beta_3)K_{[\dot{Q}}\}\tilde{\tau}_{Q],\gamma}, \tag{B36}$$

$$\mathsf{O}^{(3)}_{Q,\xi} = Z\tau^{(3)}_{Q,\xi} = D_+^3 K_Q^3(2d_2+\alpha_T), \tag{B37}$$

$$\mathsf{O}^{(3)}_{Q,\zeta} = Z\tau^{(3)}_{Q,\zeta} = -2D_+^3 K_Q^3(4\beta_1+\beta_3). \tag{B38}$$

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