Primordial tensor non-Gaussianities from general single-field inflation with non-Bunch-Davies initial states

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It has been found that the primordial non-Gaussianity of the curvature perturbation in the case of non-Bunch-Davies initial states can be enhanced compared with those in the case of the Bunch-Davies one due to the interactions among the perturbations on subhorizon scales. The purpose of the present paper is to investigate whether tensor non-Gaussianities can also be enhanced or not by the same mechanism. We consider general gravity theory in the presence of an inflaton, and evaluate the tensor auto-bispectrum and the cross-bispectrum involving one tensor and two scalar modes with the non-Bunch-Davies initial states for tensor modes. The crucial difference from the case of the scalar auto-bispectrum is that the tensor three-point function vanishes at the flattened momentum triangles. We point out that the cross-bispectrum can potentially be enhanced at nontrivial triangle shapes due to the non-Bunch-Davies initial states.

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I. INTRODUCTION

The validity of inflation [1–3] is almost beyond suspicion owing to both the theoretical consistency that nicely resolves the various problems in the standard big bang cosmology and the observational consistency with Planck and other data [4]. Of particular interest is therefore to know which inflation model is viable among a huge number of possibilities proposed so far. One of the powerful tools for filtering inflationary models is primordial non-Gaussianity of the curvature perturbation, which turned out to be not quite large according to the observed CMB fluctuations [4] and thus helped to rule out various models predicting large non-Gaussianity. Similarly to this scalar non-Gaussianity, the three-point functions involving tensor modes are expected to have rich information about the physics of the early universe and the interaction between gravity and the inflaton.

The inflationary perturbations have been studied mainly by assuming the Bunch-Davies initial state [5]. However, in principle, the initial state is not necessarily given by the Bunch-Davies one, and the validity of the assumptions on the initial state must be tested against observations in the end. Deviations from the Bunch-Davies state mean that the initial state is excited, i.e., there exist particles initially. In this case, the particles present initially can interact with each other at early times, leading possibly to the generation of non-Gaussianities on subhorizon scales. Therefore, assuming non-Bunch-Davies initial states would result in novel non-Gaussian signatures compared to the standard case of the Bunch-Davies initial state.

The nature of primordial perturbations from non-Bunch-Davies initial states has been explored so far in the literature [6–32]. In particular, it has been found that the non-Gaussianity of the curvature perturbation at the squeezed and flattened configurations can be enhanced compared with those in the case of the Bunch-Davies state [7–10,13,16,17,20,25,31]. It is therefore natural to ask whether or not the non-Gaussianities associated with the tensor modes can be enhanced as well. There have already been several studies regarding tensor non-Gaussianities from non-Bunch-Davies initial states [26,32]. To address this question in more detail, in this paper, we investigate the auto-bispectra of tensor modes and the cross-bispectra involving one tensor and two scalar modes in more general gravity theory than in the previous literature.

One naively expects that higher-derivative interactions have more impacts on non-Gaussianities due to non-Bunch-Davies initial states. Generalizing the underlying gravity theory yields such higher-derivative interactions. As a framework including higher-derivative interactions, we use an effective description of scalar-tensor gravity, writing down the operators composed of geometrical quantities such as extrinsic and intrinsic curvature tensors [33,34]. Based on this effective description, in the present paper, we will estimate the size of tensor non-Gaussianities from non-Bunch-Davies initial states in general single-field inflation models.

This paper is outlined as follows. In the next section, we consider general quadratic and cubic actions for tensor modes and introduce non-Bunch-Davies initial states from a Bogoliubov transform of the usual Bunch-Davies modes.

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In Sec. III, we calculate the auto-bispectrum of the tensor modes and investigate whether the enhanced non-Gaussian amplitudes can be obtained or not. We then study in Sec. IV the cross-bispectrum involving one tensor and two scalar modes and discuss how it can be enhanced compared with the case of the Bunch-Davies initial state. A summary of the present paper is given in Sec. V.

II. TENSOR MODES

A. General quadratic and cubic interactions

In the present paper, we investigate the properties of the tensor modes with non-Bunch-Davies initial states in order to see whether the tensor non-Gaussianities could be enhanced or not. Although we also study the cross-bispectrum with the scalar modes briefly, here we only summarize the quadratic and cubic interactions of the tensor modes.

To derive the generic action for the tensor modes during inflation, it is convenient to employ the Arnowitt-Deser-Misner (ADM) decomposition with uniform inflaton hypersurfaces as constant time hypersurfaces and write down the possible operators composed of the extrinsic curvature tensor K_{ij} and the intrinsic curvature tensor $R^{(3)}_{ij}$ of the constant time hypersurfaces. First, the operators having the dimension of mass squared are

$$\mathcal{L}_{\rm GR} \supset K_{ij}^2, \quad K^2, \quad R^{(3)}, \tag{1}$$

where K is the trace of K_{ij}. All these terms are present in general relativity. Then, one can consider the leading-order corrections to Eq. (1),

$$\mathcal{L}_{\rm cor} \supset K_{ij}^3, \quad KK_{ij}^2, \quad K^3, \quad K^{ij}R^{(3)}_{ij}, \quad KR^{(3)}. \tag{2}$$

One may anticipate that these corrections play a crucial role in the generation of non-Gaussianities. Therefore, in the present study, we consider the Lagrangian up to this order, and evaluate the contributions on the non-Gaussian amplitudes from these correction terms.

More specifically, we consider the following wide class of the ADM action:

$$S = \int {\rm d}t{\rm d}^3x\sqrt{\gamma}N\mathcal{L}, \tag{3}$$

where γ is the determinant of the spacial metric γ_{ij}, N is the lapse function, and

$$\begin{aligned}\mathcal{L} = &M_0^4(t,N) + M_1^3(t,N)K + M_2^2(t,N)(K^2 - K_{ij}^2)\\ &+ M_3^2(t,N)R^{(3)} + M_4(t,N)(K^3 - 3KK_{ij}^2 + 2K_{ij}^3)\\ &+ M_5(t,N)\left(K^{ij}R^{(3)}_{ij} - \frac{1}{2}KR^{(3)}\right),\end{aligned} \tag{4}$$

with $M_i(t,N)$ being a function having the dimension of mass. Here we have included the lower-order terms M_0^4 and M_1^3K, though they do not contribute to the action for the tensor modes. Equation (4) is nothing but the so-called Gleyzes-Langlois-Piazza-Vernizzi (GLPV) Lagrangian [33], and it includes the Horndeski theory as a subclass. By introducing a Stückelberg field ϕ, one can restore the full four-dimensional covariance.

The transverse and traceless tensor perturbations h_{ij} on top of a spatially flat Friedmann-Lemaître-Robertson-Walker background are defined by

$$ {\rm d}s^2 = -N^2(t){\rm d}t^2 + \gamma_{ij}{\rm d}x^i{\rm d}x^j, \qquad \gamma_{ij} = a^2(t)(e^h)_{ij}, \tag{5}$$

where

$$(e^h)_{ij} \coloneqq \delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h^k_j + \frac{1}{6}h_{ik}h^k_l h^l_j + \cdots. \tag{6}$$

At the level of the background, we may always reparametrize the time coordinate so that we hereafter take $N = 1$ and write $M_i(t, N(t)) = M_i(t)$. Since $\sqrt{\gamma}$ and the trace part K do not involve h_{ij}, the terms such as M_0^4, M_1^3K, and $M_2^2K^2$ in the Lagrangian do not contribute to the dynamics of the tensor perturbations.

Substituting the above metric into Eq. (4), the action for the tensor perturbations up to cubic order in h_{ij} can be obtained as [35,36]

$$S_h = S_h^{(2)} + S_h^{(3)}, \tag{7}$$

where

$$S_h^{(2)} = \int {\rm d}t{\rm d}^3x a^3\frac{M_T^2}{c_h^2}\left[\dot{h}_{ij}{}^2 - \frac{c_h^2}{a^2}(\partial_k h_{ij})^2\right], \tag{8}$$

$$\begin{aligned}S_h^{(3)} &= \int {\rm d}t{\rm d}^3x a^3\left[\frac{M_T^2}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)\partial_k\partial_l h_{ij}\right.\\ &\qquad\left. + \frac{M_4}{4}\dot{h}_{ij}\dot{h}_{jk}\dot{h}_{ki}\right]\\ &\eqqcolon -\int {\rm d}t H_{\rm int},\end{aligned} \tag{9}$$

with

$$M_T^2 \coloneqq 2M_3^2 + \dot{M}_5, \tag{10}$$

$$c_h^2 \coloneqq -2(M_2^2 + 3HM_4)/M_T^2. \tag{11}$$

Here, a dot stands for differentiation with respect to t and $H \coloneqq \dot{a}/a$. The interaction Hamiltonian $H_{\rm int}$ is introduced for later convenience. We assume that $M_T \sim M_{\rm Pl}$. The terms in the first line in Eq. (9) are present in general relativity, while the one in the second line is a new operator introduced as a result of the extension of general relativity with $M_4(t) \neq 0$. For example, this operator is obtained from the so-called G_5 term in the Horndeski theory [36]. One might think that the third line in Eq. (4) could also lead to a new cubic operator, but it turns out that this can be integrated by parts to yield the same terms as in the first line in Eq. (9).

B. Non-Bunch-Davies initial states

We now move to the Fourier domain,

$$h_{ij}(t,\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\tilde{h}_{ij}(t,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{12}$$

In the standard setup, one expands the quantized tensor modes as

$$\tilde{h}_{ij}(t,\mathbf{k}) = \sum_s \left[u_k(t)e_{ij}^{(s)}(\mathbf{k})a_{\mathbf{k}}^{(s)} + u_k^*(t)e_{ij}^{(s)*}(-\mathbf{k})a_{-\mathbf{k}}^{(s)\dagger}\right], \tag{13}$$

where $u_k(t)$ is the Bunch-Davies mode function. The creation and annihilation operators satisfy

$$\left[a_{\mathbf{k}}^{(s)}, a_{\mathbf{k}'}^{(s')\dagger}\right] = (2\pi)^3\delta_{ss'}\delta(\mathbf{k}-\mathbf{k}'), \tag{14}$$

$$\text{others} = 0, \tag{15}$$

and the subscript s denoting two helicity modes takes $s = \pm$. The polarization tensor, e_{ij}, satisfies the transverse and traceless conditions, $\delta_{ij}e_{ij}^{(s)}(\mathbf{k}) = 0 = k^i e_{ij}^{(s)}(\mathbf{k})$. This also satisfies $e_{ij}^{(s)}(\mathbf{k})e_{ij}^{(s')*}(\mathbf{k}) = \delta_{ss'}$ and $e_{ij}^{(s)*}(\mathbf{k}) = e_{ij}^{(-s)}(\mathbf{k}) = e_{ij}^{(s)}(-\mathbf{k})$.

The equation of motion for the mode function u_k is derived from Eq. (8) as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a^3M_T^2}{c_h^2}\dot{u}_k\right) + aM_T^2k^2u_k = 0. \tag{16}$$

We solve Eq. (16) under the assumption that $M_T^2, c_h^2 =$ const in the de Sitter background, $H =$ const. Then, the Bunch-Davies mode function is obtained as

$$u_k = \frac{\sqrt{\pi}}{a}\frac{c_h}{M_T}\sqrt{-\eta}H_{3/2}^{(1)}(-c_hk\eta), \tag{17}$$

where $H_{3/2}^{(1)}$ is the Hankel function of the first kind of order 3/2. We write the state annihilated by $\hat{a}_{\mathbf{k}}^{(s)}$ as $|0_a\rangle$: $\hat{a}_{\mathbf{k}}^{(s)}|0_a\rangle = 0$.

In this paper, we instead expand $\tilde{h}_{ij}$ as

$$\tilde{h}_{ij} = \sum_s \left[\psi_k^{(s)}e_{ij}^{(s)}(\mathbf{k})b_{\mathbf{k}}^{(s)} + \psi_k^{(s)*}e_{ij}^{(s)*}(-\mathbf{k})b_{-\mathbf{k}}^{(s)\dagger}\right], \tag{18}$$

where $\psi_k^{(s)}$ is a Bogoliubov transform of the Bunch-Davies modes,

$$\psi_k^{(s)} = \alpha_k^{(s)}u_k + \beta_k^{(s)}u_k^*. \tag{19}$$

The Bogoliubov coefficients are normalized as $|\alpha_k^{(s)}|^2 - |\beta_k^{(s)}|^2 = 1$ and the creation and annihilation operators satisfy

$$a_{\mathbf{k}}^{(s)} = \alpha_k^{(s)}b_{\mathbf{k}}^{(s)} + \beta_k^{(s)*}b_{-\mathbf{k}}^{(s)\dagger}, \tag{20}$$

$$b_{\mathbf{k}}^{(s)} = \alpha_k^{(s)*}a_{\mathbf{k}}^{(s)} - \beta_k^{(s)*}a_{-\mathbf{k}}^{(s)\dagger}. \tag{21}$$

We write the state annihilated by $b_{\mathbf{k}}^{(s)}$ as $|0_b\rangle$,

$$b_{\mathbf{k}}^{(s)}|0_b\rangle = 0. \tag{22}$$

Nonvanishing $\beta_k^{(s)}$ coefficients indicate that the tensor modes get excited from the Bunch-Davies vacuum, $a_{\mathbf{k}}^{(s)}|0_b\rangle \neq 0$.

Let us assume that the deviations from the Bunch-Davies initial states are characterized by some small, real parameters as

$$\beta_k^{(s)} = \delta_1^{(s)}(k) + i\delta_2^{(s)}(k), \tag{23}$$

$$\alpha_k^{(s)} = 1 + i\delta_3^{(s)}(k), \tag{24}$$

where $\delta_1^{(s)} \sim \delta_2^{(s)} \sim \delta_3^{(s)} \ll 1$. This is a reasonable assumption because the magnitude of $\beta_k^{(s)}$ has an upper bound in order for the inflationary background not to be spoiled by the excited tensor modes, which is typically given by $|\beta_k^{(s)}| \lesssim 10^{-6}$ as argued in the Appendix. The assumption on the form of $\alpha_k^{(s)}$ [Eq. (24)] follows from $|\alpha_k^{(s)}|^2 - |\beta_k^{(s)}|^2 = 1$.

C. Primordial power spectrum

The two-point correlation function is defined by

$$\langle 0_b|\tilde{h}_{ij}(\mathbf{k})\tilde{h}_{kl}(\mathbf{k}')|0_b\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\mathcal{P}_{ij,kl}, \tag{25}$$

where

$$\mathcal{P}_{ij,kl} \coloneqq \sum_{s,s'}\left[\psi_k^{(s)}\psi_k^{(s')*}e_{ij}^{(s)}(\mathbf{k})e_{kl}^{(s')*}(\mathbf{k})\right]. \tag{26}$$

Using Eqs. (17)–(19), we obtain the power spectrum $\mathcal{P}_h$ as

$$\mathcal{P}_h \coloneqq \frac{k^3}{2\pi^2}\mathcal{P}_{ij,ij} = \frac{1}{\pi^2}\frac{H^2}{M_T^2c_h}\sum_s\left|\alpha_k^{(s)} - \beta_k^{(s)}\right|^2, \tag{27}$$

evaluated at the time of horizon crossing, $c_hk = aH$. Its tilt is then derived as

$$n_t := \frac{\mathrm{d}\ln\mathcal{P}_h}{\mathrm{d}\ln k} \simeq -2\epsilon - s_h - 2m_T + \frac{\mathrm{d}}{\mathrm{d}\ln k}\sum_s \left|\alpha_k^{(s)} - \beta_k^{(s)}\right|^2, \tag{28}$$

where

$$\epsilon := -\frac{\dot{H}}{H^2}, \qquad s_h := \frac{\dot{c}_h}{Hc_h}, \qquad m_h := \frac{\dot{M}_T}{HM_T} \tag{29}$$

are assumed to be small. To leading order in $\beta_k^{(s)}$, we have $|\alpha_k^{(s)} - \beta_k^{(s)}|^2 \simeq 1\text{–}2\,\mathrm{Re}[\beta_k^{(s)}]$, and so

$$n_t \simeq -2\epsilon - s_h - 2m_T - 2\sum_s \frac{\mathrm{d}\,\mathrm{Re}[\beta_k^{(s)}]}{\mathrm{d}\ln k}. \tag{30}$$

This is a rather straightforward generalization of previous results, simultaneously taking into account the different effects on the spectral tilt: the time variation of the inflationary Hubble parameter, the speed of gravitational waves, and the effective Planck mass, as well as the k dependence of the Bogoliubov coefficients. Note that in principle the sign of each term in Eq. (30) is not constrained. In particular, a blue tensor spectrum can be obtained as a consequence of a time-dependent speed of gravitational waves [37–41] and/or k-dependent $\beta_k^{(s)}$ [42] even if the null energy condition is preserved, $\epsilon > 0$.

III. AUTO-BISPECTRUM

Let us now calculate the tensor three-point correlation functions with non-Bunch-Davies initial states. Since the cubic interaction (9) is composed of the two different contributions, i.e., the one present in general relativity and the new one beyond general relativity, we write the bispectrum as $\mathcal{B}^{s_1s_2s_3}_{(\mathrm{GR})} + \mathcal{B}^{s_1s_2s_3}_{(\mathrm{new})}$, where $\mathcal{B}^{s_1s_2s_3}_{(\mathrm{GR})}$ and $\mathcal{B}^{s_1s_2s_3}_{(\mathrm{new})}$ are originated from the former and the latter, respectively. The tensor bispectrum (evaluated at $t = t_f$) is defined by

$$\langle 0_b|\xi^{s_1}(t_f,\mathbf{k}_1)\xi^{s_2}(t_f,\mathbf{k}_2)\xi^{s_3}(t_f,\mathbf{k}_3)|0_b\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(\mathcal{B}^{s_1s_2s_3}_{(\mathrm{GR})} + \mathcal{B}^{s_1s_2s_3}_{(\mathrm{new})}), \tag{31}$$

with $\xi^s(t,\mathbf{k}) := \tilde{h}_{ij}(t,\mathbf{k})e_{ij}^{(s)*}(\mathbf{k})$, and the three-point correlation function can be calculated using the in-in formalism as

$$\langle 0_b|\xi^{s_1}(t_f,\mathbf{k}_1)\xi^{s_2}(t_f,\mathbf{k}_2)\xi^{s_3}(t_f,\mathbf{k}_3)|0_b\rangle = -i\int_{t_i}^{t_f}\mathrm{d}t'\langle[\xi^{s_1}(t_f,\mathbf{k}_1)\xi^{s_2}(t_f,\mathbf{k}_2)\xi^{s_3}(t_f,\mathbf{k}_3), H_{\mathrm{int}}(t')]\rangle. \tag{32}$$

Here, t_i is some time when the perturbation modes are deep inside the horizon, and t_f is the time at the end of inflation.

In terms of the conformal time defined by $\mathrm{d}\eta := \mathrm{d}t/a$, we take $\eta_f = 0$. As for the initial time, we do not simply take $\eta_0 \to -\infty$, but we keep it finite, $\eta_i = \eta_0 (< 0)$, where η_0 is associated with the cutoff scale M_* as $M_* = k/a(\eta_0) \simeq (-k\eta_0)H_{\mathrm{inf}}$, because the physical momentum k/a is larger than M_* for $\eta < \eta_0$.

Before moving to an explicit calculation of the bispectrum (32), we comment on the crucial difference between the calculation with the Bunch-Davies state and that with non-Bunch-Davies initial states. This difference explains the reason why we keep η_0 finite. Formally, Eq. (32) includes an integral of the form

$$S(\tilde{k}) := \int_{\eta_0}^{0}\mathrm{d}\eta(-\eta)^n e^{ic_h\tilde{k}\eta}, \tag{33}$$

where $n = 1$ for the standard cubic term with two spatial derivatives and $n = 2$ for the $\dot{h}_{ij}\dot{h}_{jk}\dot{h}_{ki}$ term.

In the case of the Bunch-Davies initial state in which there are only the positive-frequency modes participating in this integral, we have $\tilde{k} = k_t$ with

$$k_t := k_1 + k_2 + k_3 > 0 \tag{34}$$

and so

$$S(\tilde{k}) \propto \frac{1}{(ic_h\tilde{k})^{n+1}}, \tag{35}$$

because the exponential function rapidly oscillates for $|c_h\tilde{k}\eta| \gg 1$. In contrast to this standard case, in the case of non-Bunch Davies states, we have both positive and negative frequency modes in the integral, leading to $\tilde{k} = -k_m + k_{m+1} + k_{m+2}$ with m being defined modulo 3. Note that $\tilde{k}$ exactly vanishes at the flattened configuration, $k_m = k_{m+1} + k_{m+2}$. For this configuration, the exponential function no longer oscillates even for $\eta \sim \eta_0$, and thus the integral reads

$$S(\tilde{k}) \simeq \frac{(-\eta_0)^{n+1}}{n+1}, \tag{36}$$

which depends explicitly on η_0. For other configurations, the results of the integral are identical to Eq. (35). In this section, we therefore need to calculate the primordial bispectra in the two different cases separately, the nonflattened and flattened configurations.

Let us define

$$k_0 := k_1 - k_2 - k_3, \tag{37}$$

which appears frequently in the following discussion.

A. Nonflattened configurations ($k_0 \neq 0$)

We first focus on the nonflattened configurations, i.e., $k_0 \neq 0$. Assuming the de Sitter background and $M_T, c_h, M_4 = \mathrm{const}$, the two contributions in the bispectrum (31), respectively, read

$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{GR})} = \mathrm{Re}[\tilde{\mathcal{B}}^{s_1s_2s_3}_{(\mathrm{GR})}](s_1k_1 + s_2k_2 + s_3k_3)^2 F(s_i, k_i), \quad (38)$$

$$\mathcal{B}^{s_1s_2s_3}_{(\mathrm{new})} = \mathrm{Re}[\tilde{\mathcal{B}}^{s_1s_2s_3}_{(\mathrm{new})}] F(s_i, k_i), \quad (39)$$

where

$$\begin{aligned}\tilde{\mathcal{B}}^{s_1s_2s_3}_{(\mathrm{GR})} =& \frac{2H^4}{c_h^2 M_T^4}\frac{1}{k_1^3k_2^3k_3^3}\left[\Pi_i\left(\alpha^{(s_i)*}_{k_i} - \beta^{(s_i)*}_{k_i}\right)\right] \\ &\times\left\{\left(\alpha^{(s_1)}_{k_1}\alpha^{(s_2)}_{k_2}\alpha^{(s_3)}_{k_3} + \beta^{(s_1)}_{k_1}\beta^{(s_2)}_{k_2}\beta^{(s_3)}_{k_3}\right)\mathcal{I}_0(k_1,k_2,k_3)\right. \\ &+\left[\left(\alpha^{(s_1)}_{k_1}\alpha^{(s_2)}_{k_2}\beta^{(s_3)}_{k_3} + \beta^{(s_1)}_{k_1}\beta^{(s_2)}_{k_2}\alpha^{(s_3)}_{k_3}\right)\mathcal{I}_1(k_1,k_2,k_3)\right. \\ &\left.\left.+(k_1,s_1\leftrightarrow k_2,s_2) + (k_1,s_1\leftrightarrow k_3,s_3)\right]\right\}, \quad (40)\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{B}}^{s_1s_2s_3}_{(\mathrm{new})} =& \frac{192 M_4 H^5}{M_T^6}\frac{1}{k_1k_2k_3}\left[\Pi_i\left(\alpha^{(s_i)*}_{k_i} - \beta^{(s_i)*}_{k_i}\right)\right] \\ &\times\left\{\left(\alpha^{(s_1)}_{k_1}\alpha^{(s_2)}_{k_2}\alpha^{(s_3)}_{k_3} + \beta^{(s_1)}_{k_1}\beta^{(s_2)}_{k_2}\beta^{(s_3)}_{k_3}\right)\frac{1}{k_t^3}\right. \\ &-\left[\left(\alpha^{(s_1)}_{k_1}\beta^{(s_2)}_{k_2}\beta^{(s_3)}_{k_3} + \beta^{(s_1)}_{k_1}\alpha^{(s_2)}_{k_2}\alpha^{(s_3)}_{k_3}\right)\frac{1}{(-k_1+k_2+k_3)^3}\right. \\ &\left.\left.+(k_1,s_1\leftrightarrow k_2,s_2) + (k_1,s_1\leftrightarrow k_3,s_3)\right]\right\} \quad (41)\end{aligned}$$

and

$$\begin{aligned}F(s_i, k_i) \coloneqq& \frac{1}{64}\frac{k_t}{k_1^2k_2^2k_3^2}(s_1k_1 + s_2k_2 + s_3k_3)^2 \\ &\times(k_1 - k_2 - k_3)(k_1 + k_2 - k_3) \\ &\times(k_1 - k_2 + k_3), \quad (42)\end{aligned}$$

$$\mathcal{I}_0(k_1,k_2,k_3) \coloneqq -k_t + \frac{k_1k_2k_3}{k_t^2} + \frac{k_1k_2 + k_2k_3 + k_3k_1}{k_t}, \quad (43)$$

$$\begin{aligned}\mathcal{I}_1(k_1,k_2,k_3) \coloneqq& k_1 + k_2 - k_3 + \frac{k_1k_2k_3}{(k_1+k_2-k_3)^2} \\ &+ \frac{-k_1k_2 + k_2k_3 + k_1k_3}{k_1 + k_2 - k_3}. \quad (44)\end{aligned}$$

These expressions are a generalization of Ref. [36] and reproduce the previous results by taking the Bunch-Davies states ($\alpha^{(s)}_k = 1$ and $\beta^{(s)}_k = 0$). Note that we have derived the auto-bispectrum from the $\dot{h}_{ij}\dot{h}_{jk}\dot{h}_{ki}$ term for the first time in the context of the non-Bunch-Davies states.

Taking into account the smallness of $\beta^{(s)}_k$ [Eqs. (23) and (24)], the resultant bispectra to first order in $\beta^{(s)}_k$ are given by

$$\begin{aligned}\mathcal{B}^{s_1s_2s_3}_{(\mathrm{GR})} =& \frac{2H^4}{c_h^2M_T^4}\frac{1}{k_1^3k_2^3k_3^3}(s_1k_1 + s_2k_2 + s_3k_3)^2 F(s_i, k_i) \\ &\times\left\{\left(1 - \sum_i \mathrm{Re}[\beta^{(s_i)}_{k_i}]\right)\mathcal{I}_0(k_1,k_2,k_3)\right. \\ &\left.+[\mathrm{Re}[\beta^{(s_3)}_{k_3}]\mathcal{I}_1(k_1,k_2,k_3) + \cdots]\right\}, \quad (45)\end{aligned}$$

$$\begin{aligned}\mathcal{B}^{s_1s_2s_3}_{(\mathrm{new})} =& \frac{192M_4H^5}{M_T^6}\frac{F(s_i,k_i)}{k_1k_2k_3}\left\{\frac{1 - \sum_i \mathrm{Re}[\beta^{(s_i)}_{k_i}]}{k_t^3}\right. \\ &\left.-\left[\frac{\mathrm{Re}[\beta^{(s_1)}_{k_1}]}{(-k_1+k_2+k_3)^3} + \cdots\right]\right\}, \quad (46)\end{aligned}$$

where the ellipsis denotes permutations.

Let us consider the squeezed configuration with $k_L \coloneqq k_3 \ll k_S \coloneqq k_1 = k_2$. In the squeezed limit, the expressions in the curly brackets in Eqs. (45) and (46) are written, respectively, as

$$\{\cdots\} \simeq -\frac{3}{2}k_S\left(1 - \frac{4}{3}\mathrm{Re}\left[\beta^{(s_1)}_{k_S} + \beta^{(s_2)}_{k_S}\right]\frac{k_S}{k_L}\right) \quad (47)$$

and

$$\{\cdots\} \simeq \frac{1}{8k_S^3}\left(1 - 8\mathrm{Re}\left[\beta^{(s_1)}_{k_S} + \beta^{(s_2)}_{k_S}\right]\frac{k_S^3}{k_L^3}\right). \quad (48)$$

These equations show that the effect of nonvanishing $\beta^{(s)}_k$ could be enhanced and seen in the squeezed configuration. In particular, the generation of squeezed non-Gaussianity from the $\dot{h}_{ij}\dot{h}_{jk}\dot{h}_{ki}$ term is in contrast with the standard case of the Bunch-Davies state in which the bispectrum has a peak at the equilateral configuration [36].[1]

To see whether this enhancement effect is significant or not, let us take $k_S/k_L \sim 10^2$. The non-Bunch-Davies contributions in Eqs. (47) and (48) are then of $\mathcal{O}(10^2|\beta^{(s)}_{k_S}|)$ and $\mathcal{O}(10^6|\beta^{(s)}_{k_S}|)$, respectively. As argued in the Appendix, the upper bound on the Bogoliubov coefficients is obtained from the backreaction constraint, which depends on the ratio $M_*/M_T(\sim M_*/M_{\mathrm{Pl}})$. If one takes $M_* \sim M_T \sim M_{\mathrm{Pl}}$, one has $|\beta^{(s)}_{k_S}| \lesssim 10^{-6}$, so that the non-Bunch-Davies contribution in $\mathcal{B}^{s_1s_2s_3}_{(\mathrm{GR})}$ is small, $\sim 10^{-4}$, while that in $\mathcal{B}^{s_1s_2s_3}_{(\mathrm{new})}$ is of $\mathcal{O}(1)$. This can be larger if one assumes smaller M_*. For example, one gets $|\beta^{(s)}_{k_S}| \lesssim 10^{-2}$ if $M_* \sim 10^{-2}M_T \sim 10^{-2}M_{\mathrm{Pl}}$. In this case, the non-Bunch-Davies contribution in $\mathcal{B}^{s_1s_2s_3}_{(\mathrm{GR})}$ is of $\mathcal{O}(1)$ and that in $\mathcal{B}^{s_1s_2s_3}_{(\mathrm{new})}$ is as large as $\mathcal{O}(10^4)$. Therefore, tensor

[1] Squeezed tensor non-Gaussianities from the $\dot{h}_{ij}\dot{h}_{jk}\dot{h}_{ki}$ operator has been found also in the nonattractor inflation models [43] and bouncing models [44].

squeezed non-Gaussianity could be generated from the non-Bunch-Davies initial states, depending on the parameters.

B. Flattened configuration ($k_0 \to 0$)

So far, we have assumed that $k_0 = k_1 - k_2 - k_3 \neq 0$. Let us now investigate the flattened configuration, $k_0 \simeq 0$, using Eq. (36). In this case, $\tilde{\mathcal{B}}^{s_1 s_2 s_3}_{(\mathrm{GR})}$ and $\tilde{\mathcal{B}}^{s_1 s_2 s_3}_{(\mathrm{new})}$ in Eqs. (38) and (39) are given, respectively, by

$$\begin{aligned}\tilde{\mathcal{B}}^{s_1 s_2 s_3}_{(\mathrm{GR})} \simeq& \frac{2H^4}{c_h^2 M_T^4}\frac{1}{k_1^3 k_2^3 k_3^3}\left[\Pi_i\left(\alpha_{k_i}^{(s_i)*} - \beta_{k_i}^{(s_i)*}\right)\right] \\ &\times\left[\left(\alpha_{k_1}^{(s_1)}\alpha_{k_2}^{(s_2)}\alpha_{k_3}^{(s_3)} + \beta_{k_1}^{(s_1)}\beta_{k_2}^{(s_2)}\beta_{k_3}^{(s_3)}\right)\right. \\ &\times\mathcal{I}_0(k_1,k_2,k_3) - \frac{k_1k_2k_3}{2}c_h^2\eta_0^2 \\ &\left.\times\left(\beta_{k_1}^{(s_1)}\alpha_{k_2}^{(s_2)}\alpha_{k_3}^{(s_3)} + \alpha_{k_1}^{(s_1)}\beta_{k_2}^{(s_2)}\beta_{k_3}^{(s_3)}\right)\right], \qquad (49)\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{B}}^{s_1 s_2 s_3}_{(\mathrm{new})} \simeq& \frac{192 M_4 H^5}{M_T^6}\frac{1}{k_1k_2k_3}\left[\Pi_i\left(\alpha_{k_i}^{(s_i)*} - \beta_{k_i}^{(s_i)*}\right)\right] \\ &\times\left[\left(\alpha_{k_1}^{(s_1)}\alpha_{k_2}^{(s_2)}\alpha_{k_3}^{(s_3)} + \beta_{k_1}^{(s_1)}\beta_{k_2}^{(s_2)}\beta_{k_3}^{(s_3)}\right)\frac{1}{k_t^3}\right. \\ &\left.+\frac{i}{6}c_h^3\eta_0^3\left(\beta_{k_1}^{(s_1)}\alpha_{k_2}^{(s_2)}\alpha_{k_3}^{(s_3)} - \alpha_{k_1}^{(s_1)}\beta_{k_2}^{(s_2)}\beta_{k_3}^{(s_3)}\right)\right], \qquad (50)\end{aligned}$$

where we used $k_0 \ll k_i$, $|c_h k_i \eta_0| \gg 1$, and $|c_h k_0 \eta_0| \ll 1$. In Ref. [32], the flattened tensor non-Gaussianity has already been studied, but the interactions among the different polarization modes have not been considered.

Similarly to the nonflattened configurations, we express the resultant bispectra to first order in $\mathcal{O}(\beta_k^{(s)})$ as

$$\begin{aligned}\mathcal{B}^{s_1 s_2 s_3}_{(\mathrm{GR})} \simeq& \frac{2H^4}{c_h^2 M_T^4}\frac{1}{k_1^3k_2^3k_3^3}(s_1k_1 + s_2k_2 + s_3k_3)^2 F(s_i,k_i) \\ &\times\left\{\left(1 - \sum_i \mathrm{Re}[\beta_{k_i}^{(s_i)}]\right)\mathcal{I}_0(k_1,k_2,k_3)\right. \\ &\left.-\frac{k_1k_2k_3}{2}c_h^2\eta_0^2\mathrm{Re}[\beta_{k_1}^{(s_1)}]\right\}, \qquad (51)\end{aligned}$$

$$\begin{aligned}\mathcal{B}^{s_1 s_2 s_3}_{(\mathrm{new})} \simeq& \frac{192 M_4 H^5}{M_T^6}\frac{F(s_i,k_i)}{k_1k_2k_3} \\ &\times\left\{\frac{1 - \sum_i \mathrm{Re}[\beta_{k_i}^{(s_i)}]}{k_t^3} - \frac{c_h^3\eta_0^3}{6}\mathrm{Im}[\beta_{k_1}^{(s_1)}]\right\}. \qquad (52)\end{aligned}$$

Now, we see that the primordial bispectra always vanish at the exact flattened configurations, because $F(s_i,k_i) = 0$ for $k_0 = 0$. This universal feature can be understood intuitively from the viewpoint of angular momentum conservation [45]. Although the expressions in the curly brackets could be enhanced by powers of $k_i\eta_0$, it would be difficult to obtain large flattened non-Gaussianities due to this universal factor.[2] This is in sharp contrast to the result of the similar analysis for the curvature perturbation. However, this is not the case for the cross-interaction, as shown in the next section.

IV. CROSS-BISPECTRUM

In this section, we consider a scalar-scalar-tensor bispectrum, rather than a tensor-tensor-tensor bispectrum, and explore the possibility of enhancing it with nontrivial initial states of the tensor modes. The cross-bispectrum we will consider is defined by

$$\begin{aligned}&\langle 0_b|\tilde{\zeta}(0,\mathbf{k}_1)\tilde{\zeta}(0,\mathbf{k}_2)\xi^{(s)}(0,\mathbf{k}_3)|0_b\rangle \\ &\quad= (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{B}^s_{\zeta\zeta h}. \qquad (53)\end{aligned}$$

For the Lagrangian (4), the quadratic action for the curvature perturbation in the unitary gauge, ζ, takes the form [35]

$$S_\zeta^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\frac{a^3M_S^2}{c_s^2}\left[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\partial_i\zeta)^2\right], \qquad (54)$$

where we do not need the concrete expression for M_S and c_s in the present discussion. These are time-dependent functions in general, but in the inflationary universe we may assume that they are approximately constant. We assume that the Fourier component of the curvature perturbation, $\tilde{\zeta}(t,\mathbf{k})$, can be written as

$$\tilde{\zeta} = \psi_k a_{\mathbf{k}} + \psi_k^* a_{-\mathbf{k}}^\dagger, \qquad (55)$$

where

$$\psi_k = \frac{\sqrt{\pi}}{2\sqrt{2}a}\frac{c_s}{M_S}\sqrt{-\eta}H_{3/2}^{(1)}(-c_sk\eta) \qquad (56)$$

is the Bunch-Davies mode function and the initial state is in a vacuum state annihilated by $a_{\mathbf{k}}$. By assuming this, we focus on the effect of the excited tensor modes.

It has been found that the generic action [Eqs. (3) and (4)] introduces various cubic operators that are not present in the simple case where the inflaton is minimally coupled to gravity [46]. Among such operators, it is sufficient to consider one representative term that is expected to be a dominant source of the non-Gaussianities in order to see whether the bispectrum can be enhanced or not. Naively, operators with many derivatives are important for the generation of non-Gaussianities on subhorizon scales, and thus we focus on the following interaction Hamiltonian:

$$H_{\mathrm{int}}^{\zeta\zeta h} = -\int \mathrm{d}^3x\frac{M_S^2\Lambda_c}{ac_s^2H^2}\partial^2 h_{ij}\partial_i\zeta\partial_j\zeta, \qquad (57)$$

[2] A different conclusion was obtained in [32] because the overall factor $F(s_i,k_i)$ was overlooked.

where we assume that $\Lambda_c = \text{const}$. This term is indeed present in the general Horndeski class of theories [47].

Similarly to the auto-correlation function, the cross-correlation function includes the integral

$$S_c(\tilde{k}_c) \coloneqq \int_{\eta_0}^{0} \mathrm{d}\eta (-\eta)^3 e^{i\tilde{k}_c\eta}, \tag{58}$$

where

$$\tilde{k}_c \coloneqq c_h k_3 - c_s(k_1 + k_2). \tag{59}$$

For the configuration satisfying $\tilde{k}_c = 0$, the cross-bispectrum depends on η_0 and is enhanced by powers of $k_i\eta_0$ due to the excited tensor modes. Note that this configuration depends on the propagation speeds. For given $c_s/c_h(<1)$, one has a one-parameter family of different shapes satisfying $\tilde{k}_c = 0$ away from the flattened configuration.

In the same way as the previous calculations, we derive the cross-bispectrum to first order in $\beta_{k_3}^{(s)}$,

$$\begin{aligned}\mathcal{B}^s_{\zeta\zeta h} = \mathcal{B}^s_{\zeta\zeta h,(\mathrm{BD})}\big|_{\tilde{k}_c=0} \\ \times \Bigg\{1 - \mathrm{Re}[\beta_{k_3}^{(s)}] + \frac{(2/5)k_1k_2}{2k_1^2 + 5k_1k_2 + 2k_2^2} \\ \times c_s^4(k_1+k_2)^4\eta_0^4 \mathrm{Re}[\beta_{k_3}^{(s)}]\Bigg\},\end{aligned} \tag{60}$$

where $\mathcal{B}^s_{\zeta\zeta h}$ is the cross-bispectrum in the case of the Bunch-Davies initial state. This quantity is obtained in [47] as

$$\begin{aligned}\mathcal{B}^s_{\zeta\zeta h,(\mathrm{BD})} = \frac{H^4\Lambda_c}{M_S^2 M_T^2 c_s^4 c_h} \cdot \frac{k_t}{16k_1^3k_2^3k_3^3} \\ \times \frac{(k_1-k_2-k_3)(k_1+k_2-k_3)(k_1-k_2+k_3)}{[c_s(k_1+k_2)+c_hk_3]^4} \\ \times \{c_s^2[c_s(k_1+k_2)+4c_hk_3](k_1^2+3k_1k_2+k_2^2) \\ + c_h^2k_3^2[4c_s(k_1+k_2)+c_hk_3]\}.\end{aligned} \tag{61}$$

From the above result, we see that the non-Bunch-Davies contribution is of $\mathcal{O}(\beta_k^{(s)} c_s^4 k_i^4 \eta_0^4)$.

In the actual observables, we anticipate that this non-Bunch-Davies enhancement will be softened by (at least) one power of $|k\eta_0|$ due to the angular averaging [7]. Let us therefore estimate roughly how large $\beta_k^{(s)}(c_sk_i\eta_0)^n$ could be. As argued in the Appendix, the Bogoliubov coefficients have an upper bound from the backreaction constraint, which depends on the cutoff scale. We also have $|c_sk_i\eta_0| \lesssim c_sM_*/H_{\mathrm{inf}}$. Combining these, we find

$$\beta_k^{(s)}(c_sk_i\eta_0)^n \lesssim \frac{c_s^n}{c_h^{1/2}} \frac{M_{\mathrm{Pl}}M_*^{n-2}}{H_{\mathrm{inf}}^{n-1}}. \tag{62}$$

Even for $n = 2$, the upper bound is typically larger than $\mathcal{O}(1)$. We thus conclude that initially excited tensor modes can leave a potentially observable imprint in the cross-bispectrum.[3]

V. SUMMARY

In the present paper, we have considered primordial tensor perturbations with non-Bunch-Davies initial states. Employing a general scalar-tensor theory, we have described nonminimal couplings between gravity and the inflaton.

First, we evaluated the size of tensor three-point functions and showed that the squeezed non-Gaussianities in particular from the newly introduced operator in nonminimally coupled theories can potentially be enhanced. In contrast to the case of the scalar three-point functions [7], the tensor three-point function always vanishes at the flattened momentum triangles. This is as it should be, as can be seen from the momentum conservation argument [45].

Next, we have studied the cross-bispectrum involving one tensor and two scalar modes. We have found that the enhancement due to the non-Bunch-Davies effect can be large at nontrivial triangle shapes. It would therefore be interesting to investigate how such non-Gaussian signature is imprinted, e.g., on CMB bispectra [48], which we leave for further studies.

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APPENDIX: BACKREACTION CONSTRAINT ON $\beta_k^{(s)}$

If a scalar field is minimally coupled to gravity, the energy-momentum tensor of tensor perturbations is derived by expanding the Einstein tensor to second order in h_{ij}. Even if the scalar field is nonminimally coupled to gravity, one may proceed essentially in the same way and expand the field equations to second order in h_{ij} to estimate the energy density of tensor perturbations. This is how one can evaluate the backreaction of excited tensor modes to the homogeneous background. The effective energy density of subhorizon tensor perturbations is thus given by

[3]In the present paper, we have considered the scalar-scalar-tensor bispectrum, but initially excited scalar modes would be able to enhance the scalar-tensor-tensor bispectrum as well.

$$\rho_h \sim \frac{M_T^2}{a^2 c_h^2} h'_{ij}{}^2 \sim M_T^2 \frac{(\partial_i h_{jk})^2}{a^2}, \tag{A1}$$

where a dash stands for differentiation with respect to η. The backreaction can safely be ignored if

$$\langle 0_b | \hat{\rho}_h | 0_b \rangle \lesssim \bar{\mathcal{E}}, \tag{A2}$$

where $\bar{\mathcal{E}}$ is the homogeneous part of the field equation, which can be estimated naively as

$$\bar{\mathcal{E}} \sim M_{\rm Pl}^2 H_{\rm inf}^2, \tag{A3}$$

where $H_{\rm inf}$ is the inflationary Hubble parameter and $M_{\rm Pl} \sim M_T$.

The backreaction from the excited modes of tensor perturbations can be estimated at $\eta = \eta_0$ from

$$\begin{aligned}\langle 0_b | \hat{\rho}_h | 0_b \rangle &\sim \frac{M_T^2}{a^2 c_h^2} \langle 0_b | \hat{h}'^2_{ij} | 0_b \rangle \\ &\sim \frac{c_h}{a^4(\eta_0)} \int_0^{M_* a(\eta_0)} |\beta_k^{(s)}|^2 k^3 \mathrm{d}k, \end{aligned} \tag{A4}$$

where we discarded the vacuum energy. Then, by requiring that

$$\frac{c_h}{a^4(\eta_0)} \int_0^{M_* a(\eta_0)} |\beta_k^{(s)}|^2 k^3 \mathrm{d}k \lesssim M_{\rm Pl}^2 H_{\rm inf}^2, \tag{A5}$$

one can save the inflationary background from being spoiled by the backreaction.

To derive a more explicit constraint, we need to assume the momentum dependence of the Bogoliubov coefficients. Here, let us suppose that $\beta_k^{(s)}$ is of the form

$$\beta_k^{(s)} \sim \beta \exp\left[-\frac{k^2}{M_*^2 a^2(\eta_0)}\right] \tag{A6}$$

as a simple model, where β is a constant parameter. Substituting this into Eq. (A5), we obtain

$$|\beta|^2 \lesssim \frac{1}{c_h} \left(\frac{M_{\rm Pl}}{M_*}\right)^2 \left(\frac{H_{\rm inf}}{M_*}\right)^2. \tag{A7}$$

As is explained in the main text, the deviation of the tensor power spectrum from the standard Bunch-Davies result is at most of $\mathcal{O}(|\beta_k^{(s)}|) \ll 1$, and thus we may use $\mathcal{P}_h \sim H_{\rm inf}^2/(c_h M_T^2)$. Then, the constraint (A7) can be rewritten as

$$\begin{aligned}|\beta|^2 &\lesssim \mathcal{P}_h \frac{M_{\rm Pl}^2}{M_*^2} \frac{M_T^2}{M_*^2} \sim r \mathcal{P}_\zeta \frac{M_{\rm Pl}^2 M_T^2}{M_*^4} \\ &\lesssim 10^{-11} \frac{M_{\rm Pl}^2 M_T^2}{M_*^4}. \end{aligned} \tag{A8}$$

For example, if we take $M_* \sim M_{\rm Pl} \sim M_T$, then we have $|\beta| \lesssim 10^{-6}$, while if we assume that the cutoff scale is much smaller, say, $M_* \sim 10^{-2} M_{\rm Pl} \sim 10^{-2} M_T$, the bound is looser, $|\beta| \lesssim 10^{-2}$.

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