Determination of the $^8\text{B}(p,\gamma)^9\text{C}$ reaction rate from $^9\text{C}$ breakup

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The astrophysical factor of $^8\text{B}(p,\gamma)^9\text{C}$ at zero energy, $S_{18}(0)$, is determined from three-body model analysis of $^9\text{C}$ breakup processes. The elastic breakup reaction $^{208}\text{Pb}^8\text{B}(^9\text{C},p^8\text{B})^{208}\text{Pb}$ at 65 MeV/nucleon and the one-proton removal reaction of $^9\text{C}$ at 285 MeV/nucleon on C and Al targets are calculated with the continuum-discretized coupled-channels method (CDCC) and the eikonal reaction theory (ERT), respectively. The asymptotic normalization coefficient (ANC) of $^9\text{C}$ in the $p^8\text{B}$ configuration, $C_{p^8\text{B}}^\text{ANC}$, extracted from the two reactions shows good consistency, unlike in previous studies. As a result of the present analysis, $S_{18}(0) = 66 \pm 10$ eVb is obtained.

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Introduction. In low-metallicity supermassive stars, the proton capture reaction of $^8\text{B}$, $^8\text{B}(p,\gamma)^9\text{C}$ ignites explosive hydrogen burning [1]:

$^8\text{B}(p,\gamma)^9\text{C}(\alpha, p)^{12}\text{N}(p, \gamma)^{13}\text{O}(\beta^+\nu)^{13}\text{N}(p, \gamma)^{14}\text{O}$.

This process, called the hot $pp$ chain, is expected to be a possible alternative path to the synthesis of the CNO elements. Because of the difficulties in measuring the $^8\text{B}(p,\gamma)^9\text{C}$ cross section $\sigma_{pp}$ at very low energies, several alternative reactions have been proposed [2–4] to indirectly determine the astrophysical factor $S_{18}(\varepsilon)$:

$$S_{18}(\varepsilon) = \sigma_{pp}(\varepsilon) \exp[2\pi \eta].$$

(1)

Here, $\varepsilon$ is the relative energy between $p$ and $^8\text{B}$ in the center-of-mass (c.m.) frame and $\eta$ is the Sommerfeld parameter. Because an astrophysical factor has quite weak energy dependence, several previous studies have paid special attention to the evaluation of $S_{18}(\varepsilon)$ at zero energy, $S_{18}(0)$ [1–5].

The Coulomb dissociation method [4] is based on the assumption that elastic breakup of $^9\text{C}$ by a heavy target, for example, $^{208}\text{Pb}$, is essentially a one-step electric dipole ($E1$) transition to the $p + ^8\text{B}$ continuum. Then $\sigma_{pp}$ can be obtained by evaluating the cross section of the inverse process of the breakup reaction [6]. This assumption needs to be examined, since nuclear breakup, Coulomb dissociation with higher multipolarities, and multistep transitions can play non-negligible roles even in $E1$-dominated breakup processes [7]. In fact, an attempt to evaluate these higher order contributions was made in Ref. [4]; we return to this point later.

The asymptotic normalization coefficient (ANC) method [8], which is one of the most important techniques of indirect measurements, has been used in several studies [7,9–15] in order to determine astrophysical reaction rates. The basic idea of the ANC method is that only the tail of the overlap between the initial and final states contributes to a reaction at stellar energies. Thus, the purpose in the present case is to determine the ANC $C_{p^8\text{B}}^\text{ANC}$ of the $^9\text{C}$ wave function in the $p + ^8\text{B}$ configuration by using some alternative reactions. In Refs. [2] and [3], respectively, the $d(^8\text{B}, ^{10}\text{C})n$ reaction at 11.4 MeV/nucleon and the one-proton removal reaction of $^9\text{C}$ at 285 MeV/nucleon were analyzed to determine $C_{p^8\text{B}}^\text{ANC}$, and hence $S_{18}(0)$. One of the most important conditions for the ANC method is that a reaction used to determine the ANC must be peripheral. From this aspect, transfer reactions at low incident energies [9–14] and nucleon-removal reactions in wide range of energies [15] have been used as alternative reactions for the ANC method. In Ref. [16], it was demonstrated that an ANC can be extracted from an elastic breakup cross section (angular distribution) for which $E1$ breakup plays a dominant role. Later this method was carefully examined and justified [7]; important findings of the work are (i) $E1$-dominated breakup processes are peripheral with respect to the relative coordinate between the two fragments after the breakup, (ii) the breakup cross section in a coupled-channel framework is proportional to the square of the ANC to be determined, and (iii) if the two fragments are ejected in forward angles, which is the case in usual breakup experiments of unstable nuclei, dynamical excitation of each fragment during the breakup process has no essential effects on the ANC.

We show in Table I the $S_{18}(0)$ reported in the aforementioned indirect measurements [2–4], together with theoretical evaluations [1,5]. One sees that the two theoretical values have a large difference of about a factor of 3. Experimental results seem to support the $S_{18}(0)$ obtained by a cluster model calculation [5]. There is, however, still a significant discrepancy of about 50% between the $S_{18}(0)$ obtained by the Coulomb dissociation method [4] and the ANC method [2,3].

In this Rapid Communication, we reinvestigate the Coulomb dissociation [4] (elastic breakup) and the proton removal process [3] of $^9\text{C}$ by means of coupled-channel calculation with a three-body ($p + ^8\text{B} + \text{target}$) model. We adopt the continuum-discretized coupled-channels method (CDCC) [17–19] for the former and the eikonal reaction theory (ERT) [20,21] for the latter; we use the ANC method for both reactions. The main purpose of the present study is to show the consistency between the two values of $S_{18}(0)$ extracted from these two types of breakup and thereby determine $S_{18}(0)$ with high reliability.

Theoretical framework. In Fig. 1 we show a schematic illustration of the three-body $(p + ^8\text{B} + \text{target})$ system. The

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TABLE I. Astrophysical factors of $^8B(p,γ)^9C$ in previous studies.

<table>
<thead>
<tr>
<th>$S_{18}$ (eVb)</th>
<th>Method</th>
</tr>
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<tbody>
<tr>
<td>45 ± 13</td>
<td>ANC (transfer)</td>
</tr>
<tr>
<td>46 ± 6</td>
<td>ANC (proton removal)</td>
</tr>
<tr>
<td>77 ± 15</td>
<td>Coulomb dissociation</td>
</tr>
<tr>
<td>210</td>
<td>Shell model</td>
</tr>
<tr>
<td>72, 80</td>
<td>Cluster model</td>
</tr>
</tbody>
</table>

scattering between $^9C$ and a target nucleus $A$ is described by the Schrödinger equation

$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2_R + h + U(r_p, r_B) - E \right] \Psi(r, R) = 0,$$

(2)

where $\Psi(r, R)$ is the three-body wave function and $r(R)$ is the coordinate of $^8B$ ($^9C$) relative to $p$ ($A$). The reduced mass between $^9C$ and $A$ is denoted by $\mu$ and $E$ is the total energy of the three-body system in the c.m. frame. The internal Hamiltonian of $^9C$ is shown by $h$. The interaction $U(r_p, r_B)$ is given by

$$U(r_p, r_B) = V_p^N(r_p) + V_p^C(r_p) + V_B^N(r_B) + V_B^C(r_B),$$

(3)

where $V_p^N$ and $V_X^C$ are the nuclear and Coulomb interactions, respectively, between $X$ and $A$; $X$ represents a fragment particle of the projectile, that is, $p$ or $^8B$. Similarly, $r_X$ denotes the relative distance between $X$ and $A$.

In the present analysis of the elastic breakup of $^9C$, we solve Eq. (2) with eikonal-CDCC (E-CDCC) [7,22]. E-CDCC assumes eikonal approximation to the scattering wave between $^9C$ and $A$. As a result, the total wave function $\Psi(r, R)$ is expressed by

$$\Psi(r, R) = \sum_c \Phi_c(r)e^{-i(m-m_0)\phi_p}\psi_c(b, z)\phi_k^C(b, z),$$

(4)

where $\Phi_c(r)$ is the internal wave function of $^9C$ with $c$ the channel indices $\{i, \ell, S, I, m\}; i > 0 (i = 0)$ stands for the $i$th discretized-continuum (ground) state, and $\ell, S, I, and$ $m$ are, respectively, the orbital angular momentum, the channel spin, and the total angular momentum of the $p$ and $^8B$ system. $m$ is the projection of $I$ on the $z$ axis taken to be parallel to the incident beam; $m_0$ is the value of $m$ in the incident channel. $b$ is the impact parameter defined by $b = \sqrt{x^2 + y^2}$ with $R = (x, y, z)$ in the Cartesian representation. The use of the Coulomb incident wave $\phi_k^C(b, z)$ instead of the plane wave $exp(K_c \cdot R)$ in the eikonal approximation is one of the most important features of E-CDCC; $K_c$ is the asymptotic wave-number vector of $^9C$ in channel $c$ from $A$. In the actual calculation, we use an approximate asymptotic form of $\phi_k^C(b, z)$. E-CDCC is shown to work very well for describing both the nuclear and Coulomb breakup processes with high accuracy and computational speed [7,22].

The one-proton removal reaction, particularly its stripping component (see below), is analyzed by means of the eikonal reaction theory (ERT) [20,21], which can calculate an inclusive cross section, such as a nucleon-removal cross section, in the CDCC framework. ERT uses a formal solution (the scattering matrix $S$) to the coupled-channel equations of E-CDCC and makes adiabatic approximation to only the nuclear part of $S$. Then one can obtain the most important result of ERT, that is, the product form of $S$ [20]

$$S = S_0S_1,$$

(5)

where $S_0$ and $S_1$ show the contributions from the constituents $b$ and $c$ of the projectile, respectively. At this stage, however, this result can be derived only when $b$ or $c$ is chargeless, which is not the case for the $^9C$ projectile consisting of $p$ and $^8B$. Therefore, in the present study, we neglect the Coulomb breakup process in the one-proton removal process and replace the Coulomb interaction $V_p^C(r_p)$ with

$$V_p^C(r_p) \rightarrow V_p^C(R),$$

(6)

Then we can calculate the one-proton removal cross section $\sigma_{p-}$ with

$$\sigma_{p-} = \sigma_{b0} + \sigma_{a0},$$

(7)

as in Refs. [20,21]. In Eq. (7), $\sigma_{b0}$ and $\sigma_{a0}$ denote the elastic breakup cross section and the stripping cross section, respectively; ERT is used to evaluate $\sigma_{a0}$. The accuracy of the replacement of Eq. (6) can be examined by calculating $\sigma_{p-}$ with and without the Coulomb breakup. It is confirmed that the Coulomb breakup contributions to $\sigma_{p-}$ for $C$ and $A1$ targets by about 5%. Thus, we conclude that the Coulomb breakup by these two targets can be neglected with 5% errors. Below we include this amount in uncertainties of $S_{18}(0)$ extracted from $\sigma_{p-}$.

**Model setting.** For both the elastic breakup and one-proton removal processes, the $p-^8B$ wave function is calculated with the same Hamiltonian $h$. We include only the intrinsic spin of $p$. We adopt the standard Woods-Saxon central potential with the radial parameter $R_0 = 1.25 \times 8^{1/3}$fm and the diffuseness parameter $d_0 = 0.65$ fm. The Coulomb interaction between a point charge ($p$) and a uniformly charged sphere ($^8B$) with the charge radius of 2.5 fm is included. For the $p$-wave states, we add the Thomas-type spin-orbit interaction, with the same $R_0$ and $d_0$ as of the central part. The depth of the spin orbit is set to 4.40 MeV and that of the central part is determined to reproduce the proton separation energy $S_p = 1.30$ MeV in the $3/2^-$ state. With this potential, we have a resonance state at $\varepsilon = 0.915$ MeV with the width $\Gamma = 0.137$ MeV in the $1/2^-$ state, in good agreement with the experimental values, that is, $\varepsilon = 0.918 \pm 0.011$ MeV and $\Gamma = 100 \pm 20$ keV [23]. We include $s1/2^+, p1/2^-, p3/2^-, d3/2^+, d5/2^+, f5/2^-$, and $f7/2^-$ waves of the $p-^8B$ system in the coupled-channel calculations.

As for the nuclear part of the distorting potential $V_p^N(X = p or ^8B)$, we adopt the microscopic folding model [24,25].
with the Melbourne nucleon-nucleon $g$ matrix [26]. Nuclear densities of $^{8}\text{B}, ^{12}\text{C}, ^{27}\text{Al}$, and $^{208}\text{Pb}$ are calculated by Hartree-Fock (HF) method with the Gogny-D1S force [27,28]. The resulting microscopic proton optical potentials are found to reproduce, with no adjustable parameters, the elastic scattering cross sections for $p$-$^{208}\text{Pb}$ at 65 MeV [29] and the $p$-$^{12}\text{C}$ reaction cross sections at 200–400 MeV [30]. For $^{8}\text{B}$-$^{\alpha}$ scattering, however, it turns out that a fine tuning of the optical potential is necessary. This can be done by replacing the argument of both the real and imaginary parts of $V^{(N)}_{\alpha\nu}$ as

$$ r_B \to (1 + x)r_B, \quad (8) $$

which effectively increase the range of the potential. We set $x$ to 0.04 (0.03) for the $^{8}\text{B}, ^{12}\text{C} (^{8}\text{B}, ^{27}\text{Al})$ potential at 285 MeV/nucleon to reproduce the experimental data of the reaction cross section [31]. As for the $^{8}\text{B}$-$^{208}\text{Pb}$ reaction at 65 MeV/nucleon, since there is no experimental data, we calculate the reaction cross section by CDCC with a $p + ^{7}\text{Be} + ^{208}\text{Pb}$ three-body model, and $x = 0.10$ is obtained to reproduce the calculated value. The prescription of Eq. (8) can be understood as a modification of the HF density of $^{8}\text{B}$ to include a halo structure effectively.

The model space of the present CDCC calculation is summarized in Table II, where $k_{\text{max}}$ ($r_{\text{max}}$) is the maximum value of the relative wave number $k$ (coordinate $r$) between $p$ and $^{8}\text{B}$, and $\Delta k$ represents the width of the momentum bin. $R_{\text{max}}$ and $L_{\text{max}}$ are, respectively, the maximum values of the relative coordinate and the orbital angular momentum between $^{9}\text{C}$ and $A$. We have confirmed with the model space the convergence of the elastic breakup reaction in Fig. 2 for $\varepsilon \lesssim 1$ MeV and $\sigma_{\text{eff}}$ (Table III), both within 1%.

**Results and discussion.** First, we analyze the elastic breakup reaction $^{208}\text{Pb}(^{9}\text{C}, ^{8}\text{B})^{208}\text{Pb}$ at 65 MeV/nucleon. In Fig. 2, we show the breakup cross section as a function of the relative energy $\varepsilon$ between $p$ and $^{8}\text{B}$. We have included the experimental efficiency $e(\varepsilon)$ [32] and resolution $\Gamma$ in the calculation. We adopt $\Gamma = 0.23$ MeV extracted from the experimental breakup spectrum of $^{12}\text{C}(^{8}\text{B}, ^{8}\text{B})^{12}\text{C}$ at 65 MeV/nucleon [32]. In order to determine $C^{(N)}_{\rho B}$, we fit the theoretical result (dashed line) to the experimental data [4], and the solid line is obtained. The renormalization factor is 1.10, which results in $(C^{(N)}_{\rho B})^2 = 1.78$ fm$^{-1}$ and $S_{18}(0) = 67.3$ eVb.

In Fig. 2, our calculation describes well the breakup spectrum below $\varepsilon \sim 1.0$ MeV, that is, both the transition to the $1/2^-$ resonant state and breakup to low-energy nonresonant states of $^{9}\text{C}$. It should be noted that we treat the resonant and nonresonant breakup continua on the same footing in the CDCC calculation. In the $\varepsilon$ region higher than the resonance energy, however, the calculation significantly underestimates the experimental data. It is expected that this is due to incompleteness of our present framework. The back-coupling effects of three-body breakup states of $^{9}\text{C}$ to $p + p + ^{7}\text{Be}$ on the $p + ^{8}\text{B}$ state observed become important as $\varepsilon$ increases. In addition, a more accurate description of the $p + ^{8}\text{B}$ continua for higher partial waves with a proper $p$-$^{8}\text{B}$ interaction $V^{(N)}_{\rho B}$ will be needed. At low $\varepsilon$, these possible problems will not exist, because only the tail of the overlap between $^{9}\text{C}$ and $p$-$^{8}\text{B}$ contributes to the breakup process. For more detailed discussion on this point, see Ref. [7].

To examine the peripherality of the $^{208}\text{Pb}(^{9}\text{C}, ^{8}\text{B})^{208}\text{Pb}$ reaction, we see the dependence of $C^{(N)}_{\rho B}$ on the parameters of $V^{(N)}_{\rho B}$, both $R_0$ and $\alpha_0$ are changed by 20%. Note that we put a constraint on the depth of the central potential so that it must reproduce the proton-separation energy $S_p$. It is found that the uncertainty of $C^{(N)}_{\rho B}$ regarding $V^{(N)}_{\rho B}$ is 8%. This indicates that the present elastic breakup reaction proceeds peripherally with respect to $r$, as required by the ANC method.

Second, we analyze the one-proton removal reaction of $^{9}\text{C}$ at 285 MeV/nucleon on $^{12}\text{C}$ and $^{27}\text{Al}$ targets. As already mentioned, we neglect the Coulomb breakup of $^{9}\text{C}$ in this case. We calculate $\sigma_{\text{ba}}$ by CDCC and the stripping cross section $\sigma_{\text{str}}$ by ERT, and obtain the one-proton removal cross TABLE III. Results of the one-proton removal reactions with $^{12}\text{C}$ and $^{27}\text{Al}$ targets. The experimental data of $\sigma_{\text{eff}}$ are taken from Ref. [31].

<table>
<thead>
<tr>
<th>Target</th>
<th>$^{12}\text{C}$</th>
<th>$^{27}\text{Al}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{ba}}$ (mb)</td>
<td>2.7</td>
<td>4.7</td>
</tr>
<tr>
<td>$\sigma_{\text{str}}$ (mb)</td>
<td>44.9</td>
<td>48(8)</td>
</tr>
<tr>
<td>$(C^{(N)}_{\rho B})^2$ (fm$^{-1}$)</td>
<td>1.73</td>
<td>1.65</td>
</tr>
<tr>
<td>$S_{18}(0)$ (eVb)</td>
<td>65.2</td>
<td>62.2</td>
</tr>
</tbody>
</table>
section \(\sigma_{-p}\), as the sum of the two. Then we renormalize
the calculated \(\sigma_{-p}\) to fit the experimental value taken from
Ref. [31], which determines \((C_{pB}^{C})^{2}\) and hence \(S_{18}(0)\). These
values are summarized in Table III. One sees that the two
results of \(S_{18}(0)\), corresponding to \(^{12}\)C and \(^{27}\)Al targets, agree
well with each other. By taking an average of the two values, we
obtain \((C_{pB}^{C})^{2} = 1.69 \text{ fm}^{-1}\) and \(S_{18}(0) = 63.7 \text{ eVb}\). In order
to evaluate the uncertainty of the ANC for the one-proton removal
reactions, we take the same procedure as in the analysis of the
elastic breakup reaction; the uncertainty turns out to be
20%. By adding the aforementioned 5% uncertainty due to the
neglect of Coulomb breakup, we find the total uncertainty of
\(S_{18}(0)\) extracted from \(\sigma_{-p}\) to be 21%.

We here remark that in our three-body coupled-channel
analysis, the values of \(S_{18}(0)\) extracted from two different
breakup reactions, 67.3 eVb (elastic breakup) and 63.7 eVb
(proton removal), show very good agreement. This indicates
reliability of the present analysis and the result of \(S_{18}(0)\). As
a principal result of the present study, we obtain \((C_{pB}^{C})^{2} =
1.7 \pm 0.3 \text{ fm}^{-1}\), which corresponds to
\[ S_{18}(0) = 66 \pm 10 \text{ eVb}. \] (9)

In Fig. 3, the \(S_{18}(0)\) extracted by the present work is compared
with previous values. As mentioned above, previous results can
be categorized into two: one is around 80 eVb (Refs. [4,5]) and
the other is around 45 eVb (Refs. [2,3]). Our result exists in
between them, slightly favoring the former.

In Ref. [4], the \(E1\) contribution to the elastic breakup of
\(^{9}\)C by \(^{208}\)Pb at 65 MeV/nucleon was extracted by subtracting
the contributions of the nuclear and \(E2\) breakup processes
\((\sim 10\%)\) from the measured cross section, with a help of the
\(^{9}\)C breakup data by \(^{12}\)C at the same energy. The rather
good consistency between the present and previous results of
\(S_{18}(0)\) will indicate that the procedure for extracting the
\(E1\) contribution worked quite well. It was reported in Ref. [4],
however, that about 80% of the peak in the \(^{208}\)Pb\(^{(99.8\%)}\)\(^{12}\)C
breakup spectrum around \(\varepsilon = 0.9 \text{ MeV}\) was explained by
nonresonant \(E1\) breakup processes. On the other hand, in the
present analysis, the peak is found to be mainly generated by
the nuclear and \(E2\) transition to the \(1/2^{-}\) resonance state.
was carried out using the computer facilities at the Research Institute for Information Technology, Kyushu University. This research was supported in part by a Grant-in-Aid of the Japan Society for the Promotion of Science (JSPS).