High-order harmonic generation in atomic and molecular system provides a well-established technique to achieve a great deal of important applications in attosecond science [1–5]. Recently, with the advancement of the long wavelength and the ultrashort duration laser source, it has become relevant to study the highly nonlinear response of solids to the optical field in the case of preventing material damage due to multiphoton absorption and tunnel ionization. As an important and interesting strong-field process, HHG has been successfully observed in various condensed matters, including bulk crystals of ZnO [6], GaSe [7,8], MgO [9] and SiO$_2$ [10], rare-gas solids of Ar and Kr [11], and two-dimensional (2D) materials of MoS$_2$ [12] and graphene [13]. The used driving pulse covers a wide range of wavelengths from the terahertz and midinfrared to the near-infrared region. The HHG in solids has also been demonstrated to become an efficient spectroscopic tool which can encode band structure [14], lattice symmetry [8,12], and quasiparticle collisions [15].

The properties of the strong correlation, the many-body interaction, and the periodical structure pose a challenge to the HHG process from solids to the optical field in the case of preventing material damage due to multiphoton absorption and tunnel ionization. For the atomic and molecular HHG, the elliptically polarized light are less sensitive to the ellipticity compared with the HHG from interband transition [16,17], the time-dependent Schrödinger equation in the Bloch-Bloch [18,19] or in the Wannier-Bloch [20] representation, and the semiconductor Bloch equation [21–25] describing the evolution of the density matrix with the advantage of easily incorporating real material parameters and considering relaxation processes phenomenologically. All these approaches can be successfully applied in the respective situation, depending on the crystal nature and the laser parameter. An $ab$ initio approach based on a time-dependent density-functional framework is also used to analyze the HHG with partial inclusion of multielectron and exchange-correlation effects [26].

The polarization state of the driving-laser pulse provides an efficient parameter degree of freedom for controlling and steering the electron motion in the fundamental light-matter interaction. For the atomic and molecular HHG, the elliptically polarized light can dramatically reduce the harmonic yield due to the fact that the electron lateral shift induced by the ellipticity would modulate the recollision process within one laser cycle. This property has been successfully employed to constitute a temporal gating scheme for the reconstruction of the subcycle dynamics in atoms [27,28] and to achieve a chirality detection of molecular enantiomers on an electronic timescale [29].

Compared with the atomic and molecular case, HHG in solids will exhibit distinct characteristics for its dependence on the driving polarization, since the solid is an infinite periodical system where the electron moves like a delocalized plane wave. It was demonstrated by the first experimental observation [6] that the emitted harmonics from bulk ZnO driven by the elliptically polarized light are less sensitive to the ellipticity in comparison of HHG from the gas-phase medium. The result is further theoretically analyzed by solving the two-band semiconductor Bloch equation [30] and by performing the first-principles simulation [31]. Both models show that the interband and the intraband harmonics, which are a frequently used concept for interpreting the mechanism of HHG from solids, are differently affected by the ellipticity of the driving pulse. Additionally, the similar ellipticity dependence of solid HHG characterized by the monotonic decrease of the harmonic yield with increasing the ellipticity has also been experimentally observed from rare-gas solids [11] and monolayer MoS$_2$ [12].

It has been demonstrated that the ellipticity dependence of HHG from the ZnO model can be used to monitor the variation...
of the band structure [30], implying that solid harmonics generated by the elliptically polarized laser might exhibit the complicated feature that is dependent on solid species. Indeed, recent experimental work shows that another different behavior of the ellipticity dependence appears in bulk MgO [9] and zero-gap graphene [13], where the harmonic yield can be enhanced at a finite laser ellipticity. The enhancement mechanism is interpreted by the classical electron trajectory that connects the neighboring atomic sites in the case of MgO and by the semimetal regime in the case of graphene.

The nonlinear response of the graphene to the laser field has been widely investigated in Refs. [31–36]. The important frequency upconversion process known as HHG is also discussed in the terahertz regime [37–40]. The previous theoretical studies are mainly focused on the linearly polarized driving field.

In this work, we perform a detailed investigation of HHG from zero-gap graphene upon scanning the ellipticity of the midinfrared driving pulse, with a particular emphasis on the role of both the dephasing time and the optical transition dipole in the ellipticity-dependent harmonic yield. The graphene is treated by the tight-binding approximation and its interaction with the external strong field is calculated by the two-band semiconductor Bloch equation in the Houston basis, taking into account relaxation processes phenomenologically. We find that the dephasing time is the key parameter for making the simulation result match well with the experimental observation [13] in which the seventh- and the ninth-harmonic order reach the maximal yield at a particular ellipticity. The different dependence on the ellipticity for the interband and intraband harmonic emission, and the influence of the graphene orientation, are discussed. Moreover, we also offer an alternative interpretation to the origin of this unique ellipticity dependence by analyzing the transition dipole form around the Dirac point.

II. THEORETICAL METHOD

A. Graphene structure

Graphene is a monolayer material completely composed of carbon atoms arranged in a two-dimensional (2D) hexagonal honeycomb lattice structure, as shown in Fig. 1(a), where the labels “A” and “B” represent the two inequivalent sublattices. The two primitive lattice vectors are denoted by \( \mathbf{a}_1 = \frac{a}{2}(\sqrt{3}, 1) \) and \( \mathbf{a}_2 = \frac{a}{2}(\sqrt{3}, -1) \), with the lattice constant \( a = 2.46 \) Å. The corresponding reciprocal lattice forms a hexagonal Brillouin zone (BZ). Figure 1(b) shows the first BZ of graphene, together with two inequivalent and high-symmetry points referred to as Dirac points: \( K = \left( \frac{\pi}{\sqrt{3}a}, \frac{\pi}{\sqrt{3}a} \right) \) and \( K' = \left( \frac{\pi}{\sqrt{3}a}, -\frac{\pi}{\sqrt{3}a} \right) \). The \( K \) and \( K' \) points are degenerate in terms of energy. For the undoped graphene, the conduction and valence bands touch each other at these points which coincide with the Fermi surface and produce the zero energy gap.

In order to obtain the electronic band structure, we use the \( p_z \) orbit of each atomic site to form the Bloch wave function, and consider the nearest-neighbor coupling and tight-binding approximation. Therefore, the corresponding Hamiltonian [41] can be written as

\[
H_0 = \begin{pmatrix}
0 & -\gamma f(k) \\
-\gamma^* f^*(k) & 0
\end{pmatrix},
\]

where \( \gamma = 3.03 \) eV is the hopping integral and \( f(k) \) has the form

\[
f(k) = \exp \left( i \frac{a k_x}{\sqrt{3}} \right) + 2 \exp \left( -i \frac{a k_x}{2\sqrt{3}} \right) \cos \left( \frac{a k_x}{2} \right).
\]

The diagonalization of the \( H_0 \) matrix can yields energy eigenvalues, which describe the valence band \( E_v(k) = -\gamma |f(k)| \) and the conduction band \( E_c(k) = \gamma |f(k)| \). The resulting energy eigenvectors are found to be

\[
u_n(k) = \frac{|E_c(k)|}{\sqrt{2}} \left( \frac{1}{\pm e^{-i\phi(k)}} \right),
\]

where \( n = v, c \) denotes the valence (v) and the conduction (c) band, and \( \phi(k) = \text{Arg}[f(k)] \) is the phase of \( f(k) \). The \( u_v(k) \) and \( u_c(k) \) correspond, respectively, to the negative and the positive sign in the symbol “±.”

B. Semiconductor Bloch equation

The interaction of graphene with the laser field has been widely investigated by solving the length-gauge optical semiconductor Bloch equation based on the field-free Bloch basis function [42–44]. In our simulation, we choose the Houston functions [45–47] as the basis, providing an advantage of decoupling of the states with different values of crystal momentum. Unless otherwise indicated, atomic units (a.u.) are used throughout: \( e = \hbar = m_e = 1 \), where \( e \) and \( m_e \) are the electron charge and mass, respectively. The resulting dynamic equation for the temporal evolution of density-matrix elements reads

\[
\frac{d}{dt} \rho_{mn}(k,t) = -i \left( E_m^{k+A(t)} - E_n^{k+A(t)} \right) \rho_{mn}(k,t) - i F(t) \cdot \left[ \hat{d}(k + A(t)), \hat{\rho} \right]_{mn} - \frac{\rho_{mn}}{T_2} (1 - \delta_{mn}),
\]

where the symbol “[ ]” is defined as \( \{A, B\} = AB-BA \) and \( \hat{\rho} \) is the density matrix with the element \( \rho_{mn}(k,t) \). The diagonal element \( \rho_{mm}(k,t) \) represents the population probability in band \( m \), while the off-diagonal element \( \rho_{mn}(k,t) (m \neq n) \) is related to the interband polarization between band \( m \) and band \( n \). The \( \hat{d}(k + A(t)) \) denotes the matrix comprised of the dipole

![Fig. 1.](image-url)
transition element,
\[ d_{mn}^{k+\Delta t} = i \langle u_m | k + A(t) \rangle \nabla_k | u_n | k + A(t) \rangle, \] (5)
where \( u_m \) (\( u_n \)) is the periodical part of the Bloch wave function for the band \( m \) (\( n \)). Here, the \( m \) band energy \( E_m^{k+\Delta t} \) and the \( d_{mn}^{k+\Delta t} \) are calculated at the crystal momentum \( k \) displaced by the time-dependent laser vector potential \( A(t) \), which is related to the electric field \( \mathbf{F}(t) = -dA(t)/dt \). The last term on the right side of Eq. (4) describes the decoherence arising from the scattering effect beyond the mean-field approximation. An interband dephasing time \( T_{1} \) independent on \( k \) is introduced to consider the relaxation process. For \( T_{2} \) less than half the optical cycle of the driving pulse, the dephasing time can efficiently suppress the electron trajectory of multiple returns in the HHG process so that the well-defined harmonic structure in the spectrum is obtained. We have neglected longitudinal relaxation time \( T_{1} \) associated with each band population relaxing back to the initial distributions, since \( T_{1} \gg T_{2} \) [48–51], and we find \( T_{1} \) does not significantly affect the ellipticity dependence of HHG in graphene if \( T_{1} \) is comparable to or larger than the pump pulse duration.

C. Explicit matrix element and current density
We only consider the valence and conduction band in our simulation, i.e., \((m,n) \in (v,c)\) in Eq. (4), which leads to three independent coupled equations among density-matrix elements \( \rho_{cc}, \rho_{cc}, \) and \( \rho_{cv} \). By incorporating Eqs. (3) and (5), the explicit expression for \( \mathbf{d}_{cv}(k) \) is given by

\[ \mathbf{d}_{cv}(k) = \frac{a}{2|f(k)|^2} \left\{ \frac{1}{\sqrt{3}} \left[ \cos \left( \frac{\sqrt{3}}{2}ak_x \right) \cos \left( \frac{1}{2}ak_y \right) \mathbf{e}_x \right] + \mathbf{e}_y \right\}. \] (6)

After propagating the elements of the density matrix in Eq. (4), the time-dependent current density is given by

\[ \mathbf{J}(t) \propto \sum_{(m,n)\in(v,c)} \int_{BZ} \mathbf{p}_{mn}[k + A(t)]\rho_{mn}(k,t)d^2k, \] (7)
which can be written as the sum of the intraband current,

\[ \mathbf{J}_{iv}(t) \propto \sum_{m=c,v} \int_{BZ} \mathbf{p}_{mm}[k + A(t)]\rho_{mm}(k,t)d^2k, \] (8)
and the interband current,

\[ \mathbf{J}_{cv}(t) \propto \int_{BZ} \mathbf{p}_{cv}[k + A(t)]\rho_{cv}(k,t)d^2k + c.c., \] (9)
where \( \mathbf{p}_{mn} \) is the matrix element of the momentum operator. Its diagonal element represents the band velocity obtained by the gradient of the band energy with respect to \( k \), and is explicitly given by

\[ \mathbf{p}_{vv}(k) = -\mathbf{p}_{cv}(k) = ay f(k) \left\{ \sqrt{3} \sin \left( \frac{\sqrt{3}}{2}ak_x \right) \cos \left( \frac{ak_y}{2} \right) \mathbf{e}_x \right\} + \left\{ \cos \left( \frac{\sqrt{3}}{2}ak_x \right) \sin \left( \frac{ak_y}{2} \right) + \sin \left( \frac{ak_y}{2} \right) \mathbf{e}_y \right\}. \] (10)

The off-diagonal element \( \mathbf{p}_{cv}(k) \) is related to \( \mathbf{d}_{cv}(k) \) by

\[ \mathbf{p}_{cv}(k) = i[E_c(k) - E_v(k)]\mathbf{d}_{cv}(k). \] (11)

The resulting harmonic spectrum is given by \( I(\omega) \propto |\omega T_{FF}[\mathbf{J}(t)]|^2 \), where \( T_{FF} \) is the fast Fourier transform algorithm.

Equation (4) is numerically solved for each independent \( k \) by the classical fourth-order Runge-Kutta method combining with an adaptive step-size routine. We sample the Brillouin zone with a uniform grid along two nonorthogonal directions of the reciprocal lattice vectors. The equal \( k \)-grid spacing is employed for the two directions. The crystal momentum grid is fixed to \( \delta k = 0.003 \text{ a.u.} \). The evolution time grid is initially set as \( \delta t = 0.1 \text{ a.u.} \), which could be shortened by the adaptive algorithm in order to meet the required accuracy within \( \varepsilon = 10^{-8} \). We have examined and confirmed the simulation convergence by the comparison of results obtained with decreasing the \( \delta k \) and \( \varepsilon \) values.

III. RESULTS AND DISCUSSION
A. Calculation of ellipticity dependence
We have firstly calculated the harmonic yield from graphene upon scanning the ellipticity of the driving light, with the central wavelength 4700 nm and the pulse duration 160 fs. The electric field of the elliptically polarized pulse is expressed as

\[ E_{c}(t) = \frac{1}{\sqrt{1 + \epsilon^2}} F_0 \exp \left[ -2 \ln 2 \left( \frac{t}{t_d} \right)^2 \right] \cos \left( \omega_0 t + \phi \right), \] (12)

\[ E_{v}(t) = \frac{\varepsilon}{\sqrt{1 + \epsilon^2}} F_0 \exp \left[ -2 \ln 2 \left( \frac{t}{t_d} \right)^2 \right] \sin \left( \omega_0 t + \phi \right), \] (13)
where \( F_0 \) is the peak amplitude, \( \varepsilon \) controls the ellipticity satisfying \( |\varepsilon| \leq 1 \), \( t_d \) is the pulse duration (full width at half maximum), \( \omega_0 \) is the angular frequency, and \( \phi \) is the carrier-envelope phase. \( \phi = 0 \) is used and the major axis of the polarization ellipse is fixed along \( k_x \) axis shown in Fig. 1(b). We use the Fermi-Dirac distribution at room temperature (300 K) for the conduction and valence band as the initial condition to solve the density-matrix equation. From the point of view of the experiment, it is difficult to exactly estimate the electric field amplitude \( F_0 \) inside the graphene due to the reflection from the substrate and the irregular focal spot. Another simulation
The parameter that significantly affects the harmonic generation is the dephasing time $T_2$, whose value is also difficult to know in advance. For these reasons, the ellipticity-dependent harmonic yield is calculated by searching the 2D parameters space ($F_0$, $T_2$) in order to minimize the difference between the experimental [13] and the calculated result, which finally gives the fitted parameter $F_0 = 8.2348 \times 10^6$ V/cm and $T_2 = 35$ fs. The corresponding normalized harmonic yield of several harmonic orders versus the ellipticity is shown in Fig. 2(a), where the yield is defined by integrating each harmonic peak and normalized to the one obtained for the linearly polarized field ($\varepsilon = 0$). Figure 2(b) shows the clear and discrete high-harmonic (HH5–HH9) spectrum for graphene excited by the linearly polarized pulse ($\varepsilon = 0$).

One can see from Fig. 2(a) that the fifth-harmonic yield decreases monotonically with the ellipticity, while the seventh- and the ninth-harmonic yield increases until the maximal value occurring at the ellipticity $\varepsilon_{\text{opt}} = 0.32$ and then decreases monotonically. All harmonic radiation is efficiently switched off for the circularly polarized pulse. These features are in excellent agreement with the recent experimental observation, thus demonstrating the validity of our theoretical model.

It is worth pointing out that the procedure of searching the parameter might suggest an all-optical way to estimate the dephasing time in the laser-graphene interaction. The typical dephasing time in graphene is found to be in the 10–100 fs range [49–51], which supports our finding of $T_2 = 35$ fs. The effect of the dephasing time on the ellipticity-dependent harmonic yield will be further discussed later.

Additionally, we further find that if the few-cycle driving pulse is used, it fails to reproduce quantitatively the experimental observation, so that it is not appropriate to explore the mechanism of ellipticity-dependent harmonics using the few-cycle pulse in this work. Hence, we choose a multicycle driving pulse (ten to 11 cycles) in order to reveal the essentials and avoid the possible complexity induced by the nonadiabatic effect.

HHG in solids is usually interpreted in terms of the interband and the intraband current that contribute to the harmonic spectrum. Different mechanisms are associated with the interband and intraband emission. The intraband current describes field-driven carrier motion along a band, resulting in harmonic emission mainly through the nonsinusoidal dynamics of the electron and hole. The interband current is related to the excitation dynamics and the resulting polarization effect is the dominant contribution to the harmonics. Therefore, the interband and the intraband processes can be differently affected by the elliptically polarized laser.

We choose HH7 as a representative to observe the different ellipticity dependences exhibited by the interband and the intraband harmonic emission. Figure 3 shows the normalized yield of HH7 as a function of the ellipticity, calculated for the interband (blue solid) and intraband (red dashed) emission. Here both harmonic yields are normalized to the intraband value obtained for the linearly polarized pulse ($\varepsilon = 0$). It is found from Fig. 3 that the interband harmonics are stronger than the intraband one throughout the whole ellipticity range. The interband emission is enhanced and reaches the maximum at the ellipticity $\varepsilon_{\text{opt}} = 0.32$, which is consistent with Fig. 2(a). However, the maximal yield for the intraband emission is obtained in the case of the linearly polarized driving field.
local maximum at the ellipticity $\varepsilon_{\text{opt}} = 0.32$. As a result of the comparison, it can be concluded that the unique ellipticity dependence of HHG predominately originates from the interband harmonics.

### B. Interpretation in terms of transition dipole properties

The interband process describes the electron transition from the valence to the conduction band, which is quantitatively determined by the optical transition dipole $d_{\nu\sigma}(\mathbf{k})$. The analysis of the function form of $d_{\nu\sigma}(\mathbf{k})$ in the BZ would be helpful to obtain a deeper insight into the ellipticity dependence of HHG in graphene.

Figure 4 shows the magnitude (filled color) and the direction (white arrow) of the vector field $d_{\nu\sigma}(\mathbf{k})$ as a function of the crystal momentum $\mathbf{k}$. One of the important characteristics of $d_{\nu\sigma}(\mathbf{k})$ is the singularity at the two degenerate Dirac points, as denoted by $K$ and $K'$ in Fig. 4, where the transition amplitude tends to infinity. Consequently, once the field-driven electron and hole move through the Dirac point, the population transfer between the valence and conduction band is generated with the largest probability.

For the transition occurring near the Dirac point, to identify the region that can determine the harmonic ellipticity dependence, we artificially set the value of $d_{\nu\sigma}(\mathbf{k})$ equal to zero if $\mathbf{k}$ is outside the region $|\mathbf{k} - \mathbf{k}_D| = \Delta k$, where $\Delta k$ is the distance from $\mathbf{k}$ to the nearest Dirac point $\mathbf{k}_D$. This is equivalent to only considering the transition within the $\Delta k$ region around the Dirac point and neglecting the transition at other regions. In this case, Eq. (4) is propagated and the high-harmonic spectrum is calculated to obtain the normalized yield. We find that with increasing $\Delta k$ until $\Delta k = 0.2$ a.u., the main feature of ellipticity dependence in comparison of Fig. 2(a) is captured for HH5–HH9, as shown in Fig. 5. The calculation confirms the fact that the dipole transition $d_{\nu\sigma}(\mathbf{k})$ within the Brillouin zone of 0.2 a.u. around the Dirac point is responsible for producing the particular ellipticity dependence in graphene.

Additionally, we find $d_{\nu\sigma}(\mathbf{k}) \approx \pm |e_\nu|/\Delta k$ for $\mathbf{k}$ located around the Dirac point, where $e_\nu$ is the polar unit vector perpendicular to $\mathbf{k} - \mathbf{k}_D$ and “+” corresponds to the different Dirac points. The approximation expression implies another evident characteristic of the vector field $d_{\nu\sigma}(\mathbf{k})$, i.e., the circular vortex structure around the Dirac point (see Fig. 4), with the opposite rotation direction for the $K$ and $K'$ points. The importance of the circular vortex structure in the formation of the HHG ellipticity dependence can be confirmed by artificially varying the vortex curvature. Then, we observe how the ellipticity-dependent harmonic yield correspondingly changes.

The procedure is implemented as follows: For a smaller region $|\mathbf{k} - \mathbf{k}_D| \leq \Delta k = 0.1$ a.u., we rotate counterclockwise the direction of the vector field $d_{\nu\sigma}(\mathbf{k})$ by angle $\theta$ and the magnitude of $d_{\nu\sigma}(\mathbf{k})$ is fixed, which means that only the direction of $d_{\nu\sigma}(\mathbf{k})$ within $\Delta k = 0.1$ a.u. region around the Dirac point is varied and $d_{\nu\sigma}(\mathbf{k})$ remains unchanged elsewhere. In this case, for several different $\theta$ values we can calculate the harmonic yield of representative HH7 as a function of the ellipticity. Figure 6 shows the normalized HH7 yield versus ellipticity, calculated for $\theta = 0$ (black solid), $\theta = 0.1^\circ$ (red dashed), $\theta = 0.4^\circ$ (blue dot), and $\theta = 1.0^\circ$ (olive dashed dot). The $\mathbf{k}_D$ represents the nearest Dirac point relative to $\mathbf{k}$. The other simulation parameters are the same as in Fig. 2.
dashed), \( \theta = 0.4^\circ \) (blue dot), and \( \theta = 1.0^\circ \) (olive dashed dot). Note that \( \theta = 0 \) represents the original case directly extracted from Fig. 2(a). One can see that although increasing gradually the angle \( \theta \) from \( \theta = 0 \) to a small value \( \theta = 1.0^\circ \), the enhancement effect of ellipticity dependence at \( \varepsilon_{\text{opt}} = 0.32 \) becomes weaker and weaker and eventually disappears. This result reveals that the circular vortex structure has a strong influence on the unique ellipticity dependence behavior. The underlying mechanism can be attributed to the Rabi frequency \( \Omega \) defined by the dot product of the vector \( \mathbf{F}(t) \) and \( \mathbf{d}_{cv} \) involved in Eq. (4). We analyze the Rabi frequency associated with the interband transition around the Dirac point, since it makes the dominant contribution to the ellipticity dependence behavior. According to Eq. (4), when the carrier moves to the nearby region of the Dirac point, the corresponding Rabi frequency can be approximately expressed as

\[
\Omega \approx \frac{1}{|\Delta k(t)|^2} |\mathbf{F}(t) \cdot \mathbf{R}_{\pi/2}[\mathbf{k} + \mathbf{A}(t) - \mathbf{k}_D]|, \tag{14}
\]

where \( \mathbf{R}_\phi \) represents the operator that rotates counterclockwise the vector \( \mathbf{v} \) by the angle \( \phi \), and \( \Delta k(t) = |\mathbf{k} + \mathbf{A}(t) - \mathbf{k}_D| \) denotes the distance between the carrier and the nearest Dirac point to which the carrier is very close. For the counterclockwise rotation of \( \mathbf{d}_{cv} \) by the angle \( \theta \), the Rabi frequency is changed to

\[
\Omega' \approx \frac{1}{|\Delta k(t)|^2} |\mathbf{F}(t) \cdot \mathbf{R}_{\pi/2+\theta}[\mathbf{k} + \mathbf{A}(t) - \mathbf{k}_D]|. \tag{15}
\]

For the sufficiently small \( \theta \), the variation of the Rabi frequency is determined mainly by

\[
\delta \Omega(\theta) \approx \frac{1}{|\Delta k(t)|^2} |\mathbf{F}(t) \cdot \mathbf{R}_{\pi/2+\theta}[\mathbf{A}(t)] - \mathbf{F}(t) \cdot \mathbf{R}_{\pi/2}[\mathbf{A}(t)]| \\
\approx \frac{1}{|\Delta k(t)|^2} |\mathbf{F}(t)| \cdot |\mathbf{A}(t)| \sin \theta. \tag{16}
\]

We can see from Eq. (16) that although the \( \theta \) value is small, the Rabi frequency variation \( \delta \Omega(\theta) \) can be enlarged by the factor \( |\mathbf{F}(t)| \cdot |\mathbf{A}(t)| / |\Delta k(t)|^2 \) in the strong-field regime and in the neighborhoods of the Dirac point. The small rotation of \( \mathbf{d}_{cv} \) leads to the evident variation of \( \Omega \), which controls the excitation process, so that the interband emission is affected and the ellipticity dependence is correspondingly changed.

C. Effect of dephasing time and crystal orientation

The dephasing time \( T_2 \) is phenomenally included in the semiconductor Bloch equation. We have already shown that the careful choice of \( T_2 \) is the prerequisite for reproducing the experimental HHG result in graphene driven by the elliptically polarized pulse. By comparing the simulation and the experiment, it is possible to achieve the all-optical measurement of the dephasing time \( T_2 \). The feasibility should be analyzed by further considering the dephasing time effect on the ellipticity-dependent harmonic yield.

Figure 7 shows the normalized yield of HH7 as a function of the laser polarization ellipse. The rotation of the crystal axis with respect to the major axis of the laser polarization ellipse is easily implemented in the experiment. We further analyze how the ellipticity dependence of HHG varies with the orientation of the graphene. Figure 8 shows the normalized yield of HH7 as a function of the laser ellipticity and the graphene orientation, which is plotted in the polar coordinate with the radial direction representing the ellipticity and the polar angle representing the orientation. We find that the main feature of the ellipticity-dependent harmonic yield is almost independent of the orientation, since it is predominantly determined by the interband transition around...

![FIG. 7](image-url) The ellipticity-dependent normalized yield of HH7 as a function of the dephasing time \( T_2 \). For a fixed \( T_2 \), the red cycle shows the corresponding ellipticity \( \varepsilon_{\text{opt}} \) at which the HH7 yield reaches the maximum. The red dashed line is a guide for the eye, describing the dependence of \( \varepsilon_{\text{opt}} \) on \( T_2 \). The other simulation parameters are the same as in Fig. 2.

![FIG. 8](image-url) The ellipticity-dependent normalized yield of HH7 as a function of the graphene crystal orientation. The dominant isotropy and the tiny sixfold symmetry is presented. The simulation parameters are the same as in Fig. 2.

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the Dirac cone that has an isotropic nature. Nevertheless, the tiny dependence on the orientation, representing the sixfold symmetry of the graphene lattice structure, is still visible. This is due to the fact that the interband transition far away from the Dirac cone is small, but it cannot be completely ignored. The BZ outside the Dirac cone is not isotropic, leading to the small orientation dependence.

IV. CONCLUSIONS

In summary, the effect of the elliptical polarization of the driving pulse on the HHG in graphene is investigated by solving the semiconductor Bloch equation in the Houston basis. The graphene is modeled by the tight-binding approximation based on the nearest-neighbor coupling, which can give the analytical expression for the energy band and the dipole element. We find the harmonic yield of HH7 and HH9 can be enhanced at a particular ellipticity $\epsilon_{\text{opt}}$, which is dependent on the dephasing time $T_2$. Upon scanning $T_2$, we demonstrate that the excellent agreement between the calculated ellipticity-dependent harmonic yield and the experimental result can be obtained by choosing $T_2 = 35$ fs, where the largest change rate of $\epsilon_{\text{opt}}$ with respect to $T_2$ arrises, suggesting a feasible all-optical method for the measurement of the dephasing time in the graphene. The interband harmonic emission arising from the dipole transition around the Dirac point is confirmed as the dominant contribution to the formation of the unique ellipticity dependence. Moreover, we offer an interpretation of the underlying mechanism, which can be attributed to both the large magnitude and the circular vortex structure exhibited by the dipole transition element near the Dirac point. This physical picture is supported by the calculation based on the rotation of the crystal axis of graphene. It is shown that the ellipticity-dependent harmonic yield is stable against the crystal rotation due to the isotropy of the Dirac cone.

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