Radiative level shifts of an accelerated hydrogen atom and the Unruh effect in quantum electrodynamics

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The contribution of vacuum fluctuations and the radiation reaction to the energy-level shifts of a hydrogen atom, moving with uniform acceleration and interacting with the electromagnetic field, is considered. It is found that the reaction field contribution is not affected by the acceleration, whereas the vacuum fluctuations' contribution depends on the acceleration. The differences with previous results for an accelerated two-level atom interacting with a scalar field are discussed in detail; in particular, it is shown that the effect of electromagnetic vacuum fluctuations on atomic level shifts, contrarily to the scalar field case, is not totally equivalent to that of a thermal field. It is argued that this lack of equivalence should be observable. [S1050-2947(98)06203-9]

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I. INTRODUCTION

The physical origin of the radiative level shifts of atoms has been studied in detail since the beginning of quantum electrodynamics. Radiative level shifts may be ascribed to vacuum fluctuations [1,2], or to the radiation reaction field [3], or to a combination of them [4]. This ambiguity in physical interpretation is usually considered as a sort of complementarity which mathematically arises from different possible choices of the ordering of commuting operators. The subject was reviewed many times until very recently [5–7]. In a classic paper, Dalibard, Dupont-Roc, and Cohen-Tannoudji [8] showed that the requirement that the contributions of vacuum fluctuations and of the reaction field to radiative corrections should be separately represented by Hermitian operators entails the adoption of symmetric ordering, thereby resolving the mentioned ambiguity. They also used symmetric ordering in more general cases such as in the interaction of a (small) system with a resevoir [9].

More recently, Audretsch and Muller extended the formalism of Ref. [8] to evaluate vacuum fluctuations and reaction field contributions to the spontaneous emission rate [10], and to the radiative level shifts [11] of an accelerated two-level atom interacting with a scalar field. Their results are consistent with the Unruh effect, i.e., the spontaneous excitation of an accelerated atom detector due to the fact that the latter perceives vacuum fluctuations as a thermal field with a temperature proportional to its acceleration [12]. In an accelerated frame the vacuum is populated by the so-called Rindler particles [13,14]. The Lamb shift of a two-level atom interacting with a scalar field in an arbitrary stationary trajectory was also studied [15].

In this paper, using the general method of Ref. [11], we consider the contributions of vacuum fluctuations and the reaction field to the level shifts af an accelerated hydrogen atom interacting with the quantum electromagnetic field. This is a more realistic system than that considered in Ref. [11], and we shall show that it displays nontrivial features. These features are essentially related to two facts. The first is that from the point of view of an accelerated observer, scalar and electromagnetic fields behave quite differently. In particular, the vacuum correlation functions of the electromagnetic field in an accelerated system are not equivalent to those of a thermal field, contrarily to the scalar field case [16]; we show that this is indeed relevant to the description of radiative level shifts of an atom in an accelerated system. Second, we may expect different results and different physical interpretations of radiative corrections when a realistic multilevel system such as a hydrogen atom is considered in place of a simple two-level system, due to the well-known limitations of the two-level model in the description of vacuum processes [17]. An example of this inadequacy is that the calculation of the Lamb shift for a (inertial) two-level atom does not require mass renormalization because of accidental cancellation of divergent contributions when energy differences are considered [18]; such a cancellation does not occur for a hydrogen atom.

II. ATOM-FIELD INTERACTION AND EFFECTIVE HAMILTONIANS

We consider an accelerated hydrogen atom interacting with the quantum electromagnetic radiation field. For simplicity, we assume a linear interaction between atom and field. The nonrelativistic QED interaction also contains quadratic terms, both in the minimal coupling and in the multipolar coupling schemes [7], but we shall neglect them, as it is often done in the calculation of radiative shifts.

The Hamiltonian that describes the time evolution of our system with respect to the proper time τ, in the multipolar coupling scheme, is

$$H(\tau)=H_{\text{atom}}(\tau)+H_{\text{field}}(\tau)+H_{\text{int}}(\tau), \tag{2.1}$$

where

$$H_{\text{atom}}(\tau)=\hbar\sum_n \omega_n\sigma_{nn}(\tau) \quad (\sigma_{nn}=|n\rangle\langle n|), \tag{2.2}$$

$$H_{\rm field}(\tau)=\hbar\sum_k \omega_k a_k^\dagger a_k \frac{dt}{d\tau}, \tag{2.3}$$

$$H_{\rm int}(\tau)=-e\mathbf{r}(\tau)\cdot\mathbf{E}(x(\tau))=-e\sum_{mn}\mathbf{r}_{mn}\cdot\mathbf{E}(x(\tau))\sigma_{mn}(\tau), \tag{2.4}$$

where k denotes the wave vector and polarization of the field modes, e is the electron electric charge, $e\mathbf{r}$ the atomic electric dipole moment, $x=(t,\mathbf{x})$ the space-time coordinate of the atom, and n denotes a complete set of atomic states with energy $\hbar\omega_n$. We assume that ω_n includes any direct change (not radiative shift) of the transition frequency caused by the acceleration.

In the formal solution of the Heisenberg equations for the field variables, we can separate the ''free'' and ''source'' parts

$$a_k(t(\tau))=a_k^f(t(\tau))+a_k^s(t(\tau)), \tag{2.5}$$

where

$$a_k^f(t(\tau))=a_k(t(\tau_0))e^{-i\omega_k(t(\tau)-t(\tau_0))} \tag{2.6}$$

$$a_k^s(t(\tau))=\frac{ie}{\hbar}\sum_{mn}\mathbf{r}_{mn}\cdot\int_{\tau_0}^{\tau}d\tau'\,e^{-i\omega_k(t(\tau)-t(\tau'))}\times[\mathbf{E}(x(\tau')),a_k(t(\tau'))]\sigma_{mn}(\tau'). \tag{2.7}$$

An analogous separation can be performed for the atomic variables.

The equations of motion in the interaction representation for an arbitrary atomic observable $O(t)$, using symmetric ordering, can now be separated in the vacuum fluctuations and the reaction field contributions [9]

$$\frac{dO(\tau)}{d\tau}=\left(\frac{dO(\tau)}{d\tau}\right)_{\rm vf}+\left(\frac{dO(\tau)}{d\tau}\right)_{rr}, \tag{2.8}$$

where

$$\left(\frac{dO(\tau)}{d\tau}\right)_{\rm vf}=-\frac{ie}{2\hbar}\left(\mathbf{E}^f(x(\tau))\cdot\sum_{mn}\mathbf{r}_{mn}[\sigma_{mn}(\tau),O(\tau)]+\sum_{mn}\mathbf{r}_{mn}\cdot[\sigma_{mn}(\tau),O(\tau)]\mathbf{E}^f(x(\tau))\right), \tag{2.9}$$

$$\left(\frac{dO(\tau)}{d\tau}\right)_{rr}=-\frac{ie}{2\hbar}\left(\mathbf{E}^s(x(\tau))\cdot\sum_{mn}\mathbf{r}_{mn}[\sigma_{mn}(\tau),O(\tau)]+\sum_{mn}\mathbf{r}_{mn}\cdot[\sigma_{mn}(\tau),O(\tau)]\mathbf{E}^s(x(\tau))\right). \tag{2.10}$$

The average values of the equations of motion (2.9) and (2.10) on the vacuum state $|0\rangle$, using the same general procedure as in Refs. [9,11] and apart from non-Hermitian terms connected to relaxation, can be expressed in terms of effective Hamiltonians

$$H_{vf}^{eff}=\frac{ie^2}{2\hbar}\int_{\tau_0}^{\tau}d\tau'\,C_{ij}^F(x(\tau),x(\tau'))[r_j(\tau'),r_i(\tau)], \tag{2.11}$$

$$H_{rr}^{eff}=-\frac{e^2}{2}\int_{\tau_0}^{\tau}d\tau'\,\chi_{ij}^F(x(\tau),x(\tau'))\{r_j(\tau'),r_i(\tau)\}, \tag{2.12}$$

where $\{\cdots,\cdots\}$ denotes the anticommutator, and we defined the statistical functions (symmetric correlation function and linear susceptibility) of the field in the vacuum state

$$C_{ij}^F(x(\tau),x(\tau'))=\tfrac{1}{2}\langle 0|\{E_i(x(\tau)),E_j(x(\tau'))\}|0\rangle, \tag{2.13}$$

$$\chi_{ij}^F(x(\tau),x(\tau'))=\frac{i}{\hbar}\langle 0|[E_i(x(\tau)),E_j(x(\tau'))]|0\rangle. \tag{2.14}$$

The vacuum fluctuations and the radiation reaction contributions to the radiative shift of the atomic level $|b\rangle$ at order e^2 are given by the expectation values of the effective Hamiltonians on the state $|b\rangle$

$$(\delta E_b)_{vf}=-\frac{e^2}{2}\int_{\tau_0}^{\tau}d\tau'\,C_{ij}^F(x(\tau),x(\tau'))(\chi_{ij}^A)_b(\tau,\tau'), \tag{2.15}$$

$$(\delta E_b)_{rr}=-\frac{e^2}{2}\int_{\tau_0}^{\tau}d\tau'\,\chi_{ij}^F(x(\tau),x(\tau'))(C_{ij}^A)_b(\tau,\tau'), \tag{2.16}$$

where the statistical functions of the atom in state $|b\rangle$, i.e., symmetric correlation function and linear susceptibility, are defined as

$$(C_{ij}^A)_b(\tau,\tau')=\tfrac{1}{2}\langle b|\{r_i(\tau),r_j(\tau')\}|b\rangle, \tag{2.17}$$

$$(\chi_{ij}^A)_b(\tau,\tau')=\frac{i}{\hbar}\langle b|[r_i(\tau),r_j(\tau')]|b\rangle. \tag{2.18}$$

It should be noted that, at order e^2, all operators appearing in the atomic and field statistical functions are free operators.

III. ENERGY-LEVEL SHIFTS OF THE ACCELERATED ATOM

The trajectory of the hydrogen atom, uniformly accelerated along the x direction, is [19]

$$t(\tau)=\frac{c}{a}\sinh\frac{a\tau}{c},\quad x(\tau)=\frac{c^2}{a}\cosh\frac{a\tau}{c},\quad y(\tau)=z(\tau)=0, \tag{3.1}$$

where a is the proper acceleration.

The field statistical functions in the accelerated frame can be obtained from the general results of Takagi for the field correlation function [20]

$$\langle 0|E_i(x(\tau))E_j(x(\tau'))|0\rangle=\delta_{ij}\frac{\hbar}{4\pi c^7}\frac{a^4}{\sinh^4\frac{a}{2c}(\tau-\tau'-i\epsilon)}, \tag{3.2}$$

where $\epsilon\to+0$. From Eq. (3.2), we obtain the symmetric correlation function

$$C^F_{ij}(x(\tau),x(\tau'))=\delta_{ij}\frac{\hbar}{4\pi c^7}P\frac{a^4}{\sinh^4\frac{a}{2c}(\tau-\tau')} \tag{3.3}$$

(P denotes the principal part), and the linear susceptibility

$$\chi^F_{ij}(x(\tau),x(\tau'))=\delta_{ij}\frac{4}{3c^3}\delta'''(\tau-\tau'), \tag{3.4}$$

where $\delta'''(\tau)$ is the third derivative of the Dirac delta function.

For our purposes, it is convenient to express Eqs. (3.3) and (3.4) in terms of frequency integrations. In order to express the symmetric correlation function in such a way, we first note that

$$\int_0^\infty d\omega\ \omega^3\coth\left(\frac{\pi\omega c}{a}\right)(e^{-i\omega\tau}+e^{i\omega\tau}) = \frac{a^4}{4c^4}\left(\frac{3}{\sinh^4\frac{a\tau}{2c}}+\frac{2}{\sinh^2\frac{a\tau}{2c}}\right), \tag{3.5}$$

$$\int_0^\infty d\omega\ \omega\coth\left(\frac{\pi\omega c}{a}\right)(e^{-i\omega\tau}+e^{i\omega\tau})=-\frac{a^2}{2c^2}\frac{1}{\sinh^2\frac{a\tau}{2c}}. \tag{3.6}$$

The proof of these equations involves some lengthy but straightforward algebra: the integrals on the left-hand side, after having added an appropriate infinitesimal imaginary part to τ that provides convergence in the upper integration limit, yield combinations of the polygamma function, which finally lead to Eqs. (3.5) and (3.6) [21]. Then, using Eqs. (3.5) and (3.6) in Eq. (3.3), we obtain

$$C^F_{ij}(x(\tau),x(\tau'))=\delta_{ij}\frac{\hbar}{3\pi c^3}\int_0^\infty d\omega\ \omega^3\left(1+\frac{a^2}{c^2\omega^2}\right)\times\coth\left(\frac{\pi c\omega}{a}\right)(e^{i\omega(\tau-\tau')}+e^{-i\omega(\tau-\tau')}). \tag{3.7}$$

The antisymmetric correlation function (3.4) can be expressed as a frequency integration in the following form:

$$\chi^F_{ij}(x(\tau),x(\tau'))=-\delta_{ij}\frac{2i}{3\pi c^3}\int_0^\infty d\omega\ \omega^3(e^{i\omega(\tau-\tau')}-e^{-i\omega(\tau-\tau')}). \tag{3.8}$$

It is interesting to compare the symmetric function (3.7) with the analogous function for a stationary observer in a thermal field at temperature T [16],

$$\tfrac{1}{2}\langle 0|\{E_i(\mathbf{x},t),E_j(\mathbf{x},t')\}|0\rangle_T =\delta_{ij}\frac{\hbar}{3\pi c^3}\int_0^\infty d\omega\ \omega^3\coth\left(\frac{\hbar\omega}{2k_BT}\right)\times(e^{i\omega(t-t')}+e^{-i\omega(t-t')}), \tag{3.9}$$

where k_B is the Boltzmann constant. By inspection, the symmetric correlation function in an accelerated frame is different from the analogous function obtained for an intertial observer in a thermal field with the Unruh temperature $T=\hbar a/2\pi ck_B$, due to the term proportional to a^2 in Eq. (3.7). The presence of this term is a direct consequence of the $a^4\sinh^{-4}[a(\tau-\tau')/2c]$ behavior of the symmetric correlation function in Eq. (3.3), as our derivation of Eq. (3.7) shows. In the case of the scalar field the symmetric correlation function is $\sim a^2\sinh^{-2}[a(\tau-\tau')/2c]$ and no term proportional to a^2 appears, as it is possible to show from Eq. (3.6). Therefore the vacuum of the electromagnetic field in an accelerated frame is not equivalent to a thermal field, contrarily to the scalar field case [16].

The symmetric and antisymmetric correlation functions for the accelerated hydrogen atom are easily obtained in the forms

$$(C^A_{ij})_b(\tau,\tau')=\frac{1}{2}\sum_d\ [\langle b|r_i(0)|d\rangle\langle d|r_j(0)|b\rangle e^{i\omega_{bd}(\tau-\tau')}+\langle b|r_j(0)|d\rangle\langle d|r_i(0)|b\rangle e^{-i\omega_{bd}(\tau-\tau')}], \tag{3.10}$$

$$(\chi^A_{ij})_b(\tau,\tau')=\frac{i}{\hbar}\sum_d\ [\langle b|r_i(0)|d\rangle\langle d|r_j(0)|b\rangle e^{i\omega_{bd}(\tau-\tau')}-\langle b|r_j(0)|d\rangle\langle d|r_i(0)|b\rangle e^{-i\omega_{bd}(\tau-\tau')}]. \tag{3.11}$$

Substituting Eqs. (3.7), (3.8), (3.10), and (3.11) into Eqs. (2.15) and (2.16), and taking the limits $\tau_0\to-\infty,\tau\to\infty$, after some algebra yields

$$(\delta E_b)_{vf}=\frac{e^2}{3\pi c^3}\sum_d\ |\langle b|\mathbf{r}(0)|d\rangle|^2\int_0^\infty d\omega\ \omega^3\left(1+\frac{a^2}{c^2\omega^2}\right)\times\coth\left(\frac{\pi c\omega}{a}\right)P\left(\frac{1}{\omega+\omega_{bd}}-\frac{1}{\omega-\omega_{bd}}\right) \tag{3.12}$$

$$(\delta E_b)_{rr} = -\frac{e^2}{3\pi c^3}\sum_d |\langle b|\mathbf{r}(0)|d\rangle|^2 \int_0^\infty d\omega\ \omega^3 \times P\left(\frac{1}{\omega+\omega_{bd}} + \frac{1}{\omega-\omega_{bd}}\right). \tag{3.13}$$

The vacuum fluctuations contribution can be also written in the form

$$(\delta E_b)_{\mathrm{vf}} = \frac{e^2}{3\pi c^3}\sum_d |\langle b|\mathbf{r}(0)|d\rangle|^2 \int_0^\infty d\omega\ \omega^3 \left(1+\frac{a^2}{c^2\omega^2}\right) \times \left(1+\frac{2}{e^{2\pi c\omega/a}-1}\right) P\left(\frac{1}{\omega+\omega_{bd}} - \frac{1}{\omega-\omega_{bd}}\right). \tag{3.14}$$

Equations (3.12), (3.13), and (3.14) are the main results of this paper. Let first discuss the radiation reaction term (3.13). The same result can be obtained for an atom at rest (in the multipolar coupling scheme) [9], as expected because the linear susceptibility of the field is not affected by the uniform acceleration [see Eq. (3.8)]; therefore, in our case, similarly to the case of the two-level atom and the scalar field [11], a uniform acceleration does not change the part of the energy shift due to radiation reaction. However, contrarily to the two-level case [11], this contribution does not cancel out when relative energy shifts (not renormalized) are considered. In a multilevel system, both for inertial and accelerated atoms, there is no ''accidental'' cancellation of the radiation reaction shifts for energy differences. This is a clear example in which the two-level atom is not a good representation of a real multilevel atom (for the accelerated as well for the inertial case). The radiation reaction part of the shift is, however, essentially the part that is subtracted when mass renormalization is performed, but we shall not discuss this point here.

On the other hand, acceleration affects significantly the contribution of vacuum fluctuations (3.14). For an inertial atom in the multipolar coupling scheme used in this paper,

$$(\delta E_b)_{\mathrm{vf}}^{\mathrm{inertial}} = \frac{e^2}{3\pi c^3}\sum_d |\langle b|\mathbf{r}(0)|d\rangle|^2 \int_0^\infty d\omega\ \omega^3 \times \left(\frac{1}{\omega+\omega_{bd}} - \frac{1}{\omega-\omega_{bd}}\right). \tag{3.15}$$

Comparison with Eq. (3.12) shows that, while in the scalar field case the effect of acceleration is only a ''thermal'' correction with the Unruh temperature $T=\hbar a/2\pi c k_B$ due to the factor coth($\pi c\omega/a$) [11], in the present case (electromagnetic field with multipolar coupling) there is an extra correction proportional to a^2. This extra term is not in the form of a thermal effect. In other words, the equivalence between uniform acceleration and thermal fields is lost in QED. The order of magnitude of the ''nonthermal'' correction can be estimated from the value of $a^2/c^2\omega^2$ in the ω integral in Eq. (3.12). For the Lamb shift, the relevant portion of the field spectrum is that around $\omega \sim \omega_0$, where ω_0 represents a typical transition frequency of the atom. Thus the order of magnitude of the correction is $a^2/c^2\omega_0^2$. For a ground-state hydrogen atom, $\omega_0 \sim 10^{15}\ \mathrm{s}^{-1}$. This shows that thermal and nonthermal contributions to the level shift are comparable for $a\sim c\omega_0 \sim 10^{25}\ \mathrm{cm/s}^2$. Although this acceleration is extremely high, it is of the same order of the acceleration necessary to observe the Unruh effect in atomic systems. Therefore, we expect that in the cases in which radiative effects due to acceleration might be observed (for example, for electrons in large storage rings, as suggested in Refs. [11,22]), the ''nonthermal'' part of the vacuum fluctuations level shifts should not be negligible compared to the thermal one.

Finally, we mention that the term proportional to a^2 discussed above is apparently absent if the Hamiltonian is taken in the minimal coupling scheme neglecting the A^2 term in the interaction, due to the different form of the statistical functions for the vector potential. It is not possible, however, to compare the two cases in the framework of the formalism of this paper, since the gauge equivalence of the multipolar and minimal couplings requires accounting for the quadratic terms that have been neglected in this paper. We shall discuss this point in a subsequent paper.

IV. CONCLUSIONS

In this paper we considered, using a symmetric ordering of operators and a multipolar form of the atom-field coupling, contributions of vacuum fluctuations and the radiation reaction to the energy-level shifts of a hydrogen atom interacting with the electromagnetic field and moving with a uniform acceleration. We showed that the radiation reaction contribution is not affected by the acceleration, whereas the vacuum fluctuations' contribution depends on the acceleration of the atom. We discussed the latter contribution in detail, and showed that the effect of vacuum fluctuations on the energy-level shifts of the accelerated atom is not equivalent to that of a thermal field, with the Unruh temperature $T=\hbar a/2\pi c k_B$. The order of magnitude of the nonthermal contribution was estimated in comparison to the thermal one, and we argued that it should be observable. These results were also compared with recent results in the literature concerning an accelerated two-level atom interacting with a scalar field.

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