Creating a noninteracting Bose gas in equilibrium at finite temperature

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Experiments with ultracold atoms involve interatomic interactions, which are essential for cooling the atoms. The *s*-wave interaction between atoms can be tuned via the Feshbach resonance, potentially enabling the creation of a noninteracting system when the interaction reaches its vanishing limit. Although feasible at zero temperature, eliminating the interaction at a finite temperature in an isolated system prevents the system from reaching equilibrium. In this study, we used a Bose-Fermi mixture to create equilibrated noninteracting Bose gas at a finite temperature. First, we used the Bose-Fermi superfluid mixture of a dilute lithium gas in an optical dipole trap to determine the zero crossing of the boson-boson interaction at near-zero temperature. Thereafter, we showed that the noninteracting Bose gas created at a finite temperature represents an ideal Bose gas under the canonical description. The results of this study provides an avenue for experimental investigations of the fundamental properties of an ideal Bose gas in a harmonic trap.

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Elastic collisions are required for rethermalization during the cooling of systems comprising ultracold atoms with bosons. This implies that most experimental systems are interactive. Feshbach resonance is a widely used tool to manipulate interatomic interactions, which can be induced via an external magnetic field or optical radiation [1]. It can significantly strengthen the interaction for the exploration of the unitary regime [2–5]. The ability of a weakly interactive system to follow the tuned interaction depends on its temperature and the interaction strength. At zero temperature, the interaction can be tuned across the zero crossings to examine the properties of both the repulsive Bose-Einstein condensate (BEC) and attractive BEC solitons [6–13]. In this case, the system is purely quantum; moreover, it can continue evolving based on the Gross-Pitaevskii equation, even in the absence of interaction [14]. However, removing the interaction from an isolated system at a finite temperature stops the thermalization process in classical gases. This hinders the creation of a thermal Bose gas in thermal equilibrium without the interaction; thus, investigating an ideal Bose gas becomes difficult. Consequently, previous experiments on an ideal Bose gas were performed considering a certain amount of interaction, without reaching the vanishing limit [15–18]. On the other hand, one of the original works of Einstein suggests that BEC phase transition can occur without the aid of interaction [19]; hence, it is of fundamental importance to demonstrate atomic BEC on the vanishing interaction regime and understand its properties.

The Bose-Fermi superfluid mixture was first obtained in 2014 [20], and some properties of the superfluid mixture have been experimentally studied [21–23]. The advantage of a mixed system is that the interspecies interaction can be used to allow one component to cool the other component; this technique is known as sympathetic cooling [24–27]. If the interaction between a target bosonic species is eliminated while it is in thermal contact with an external heat bath, an ideal Bose gas in the canonical ensemble can be acquired. This provides an opportunity to explore the nature of an ideal Bose gas.

We created a mixture of ^{7}Li bosons and ^{6}Li fermions in a harmonic trap. We used an experimental setup described in previous studies [28,29]. The temperatures shown in this study were measured using the time-of-flight method with bosons. ^{6}Li was in a spin-balanced state occupying the two hyperfine states of $F = 1/2$ and was the dominant species inside the trap. The *s*-wave interaction between the fermions was near-resonant across all the conditions employed and was considered diverged. Therefore, fermions served as a heat bath for bosons. The scattering length between the bosons and fermions has a constant value of $a_{bf} \sim 40a_0$ [20], which was evaluated theoretically. The ^{7}Li bosons were in the $|F = 1, m_F = -1\rangle$ state where a possible zero crossing for the s-wave scattering length was expected to be approximately around a magnetic field of 900 G [30]. We found approximately 3×10^5 fermions and 5×10^4 bosons in the trap for a typical thermal mixture prepared at approximately 300 nK, which is about twice the critical temperature of BEC and more than thrice of the Fermi temperature.

We determined the effective scattering length of bosons a, which includes the *s*-wave scattering and other possible interactions, by cooling the gas to near-zero temperature across a magnetic field of 832–900 G. Under this condition, almost all the thermal components were absent, and the upper bound of the temperature was estimated to be 30 nK judging from

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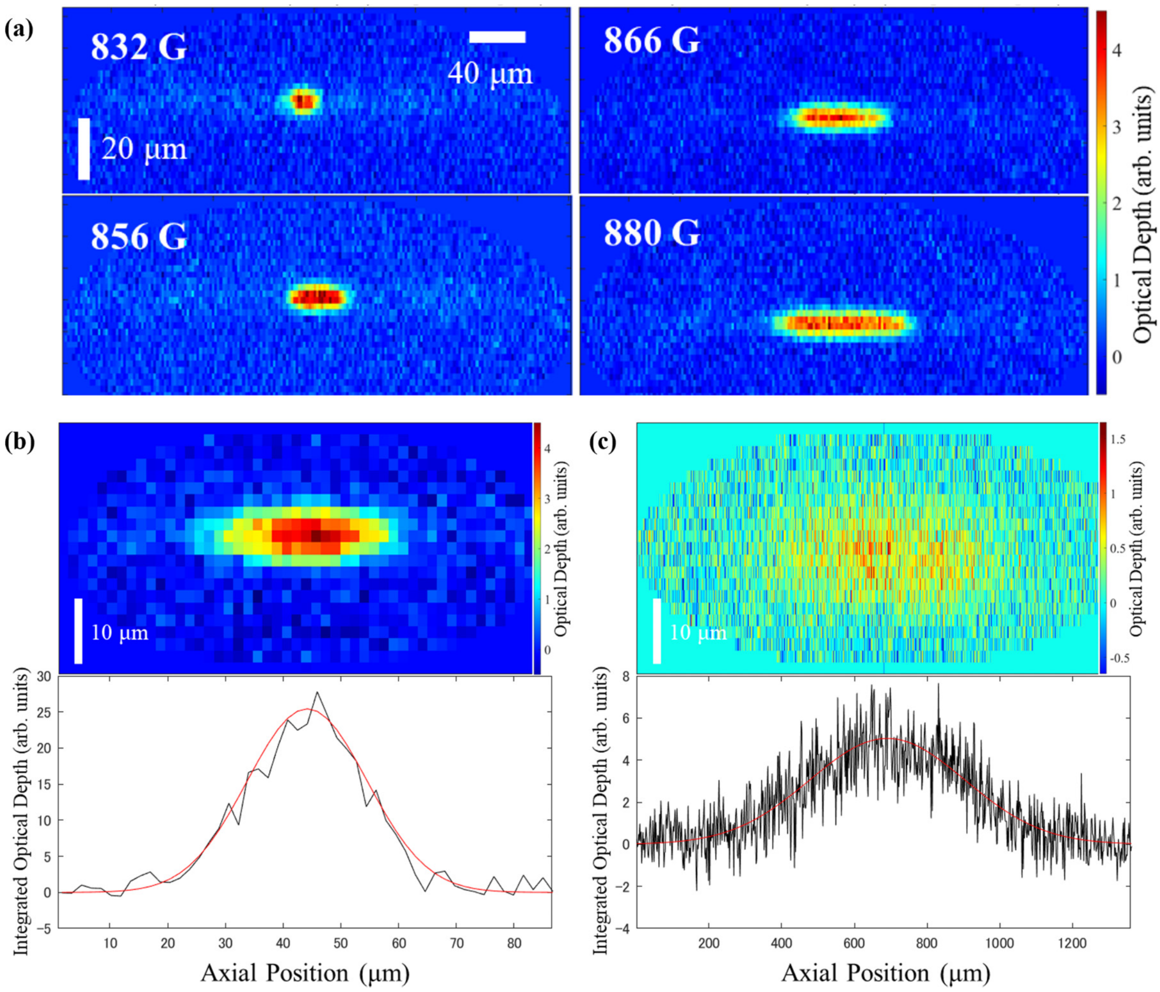

FIG. 1. *In situ* absorption images of the ground state BEC under different magnetic fields. (a) Sample BECs from weakly attractive at 832 G to weakly repulsive at 880 G. (b) Example of BEC at the measured noninteracting limit of 850.5 G and its corresponding radially integrated optical depth. The red curve represents a Gaussian fit. (c) Example of the fermion inside the mixture and its corresponding radially integrated optical depth, measured under the same condition as in (b). The red curve corresponds to a Gaussian fit.

time-of-flight measurements with conditions where higher temperatures are expected. At such a low temperature, the system became a superfluid mixture. The cooling period lasted 25 sec as the intensity of the trapping laser was slowly decreased to evaporatively cool the fermion component. We note that a ramp too fast leads to additional boson losses. Subsequently, we held the gas for 5 sec to ensure that the system would be fully relaxed. The radial and axial trapping frequencies, in this case, were measured to be $\omega_\rho = 2\pi \times 251$ Hz and $\omega_z = 2\pi \times 6.7$ Hz, respectively. We captured *in situ* absorption images and applied Gaussian fitting to evaluate the Gaussian width of the ground state BEC, as shown in Fig. 1. Note that the vertical scales of the images are enlarged to improve visibility, and that we repeated the procedures described above after each absorption imaging. We paid particular attention to the systematic errors that can occur when light absorption is strong [29]. Only the weakly confined axial direction was used in the measurement. Figure 1(a) shows that the interaction, characterized by the axial size of the BEC, gradually increases as the magnetic field becomes stronger. The system behaved as attractive BEC [31] at 832 G and became repulsive around 850 G. Furthermore, Fig. 1(b) shows an example of the Gaussian fit constructed using data recorded at 850.5 G, where the zero crossing was located. Therefore, the fitted Gaussian width matched the harmonic oscillator length of the trap. Moreover, Fig. 1(c) presents one of the two fermion hyperfine states inside the mixture, measured under the same conditions as in Fig. 1(b). Approximately 4.4×10^4 atoms are present in Fig. 1(c). We did not identify any immiscibility effect during this experiment [21,32]. Moreover, the scattering length was calculated by solving the stationary variational equation using the Gaussian approximation [33]

$$\upsilon_{0z}\upsilon_{0\rho}^4 = \upsilon_{0z} + P,$$
$$\left(\frac{\omega_z}{\omega_\rho}\right)^2 \upsilon_{0z}^4 = 1 + \frac{P\upsilon_{0z}}{\upsilon_{0\rho}^2}. \quad (1)$$

The following new variables are introduced for simplicity: $a_t = \sqrt{\hbar/(m_7\omega_\rho)}$, $P = \sqrt{2}\pi^{-1/2}Na/a_t$, and $w_{0\rho} = a_t\upsilon_{0\rho}$, $w_{0z} = a_t\upsilon_{0z}$, where N denotes the boson atom number and m_7 represents the mass of the ^{7}Li boson.

Figure 2 shows the measurement results for the scattering length. Specifically, Fig. 2(a) shows the remaining number

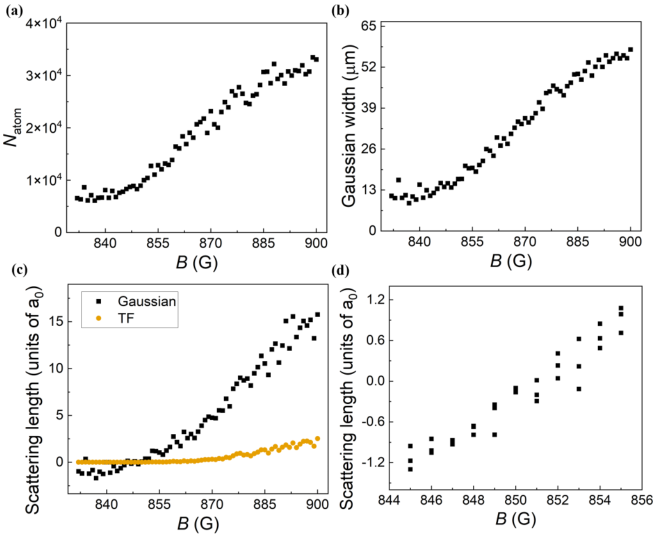

FIG. 2. Effective scattering length of boson $m_F = -1$ measured in a Bose-Fermi superfluid mixture across a magnetic field range of 832–900 G. Each data point was obtained in a single shot measurement. (a) Number of atoms N_{atom} inside the ground state BEC. (b) Axial Gaussian width of the BEC. (c) Scattering length estimated using Gaussian approximation and TF approximation. TF fitting is inaccurate for such small interactions. (d) Estimated scattering length near the zero crossing from a detailed measurement.

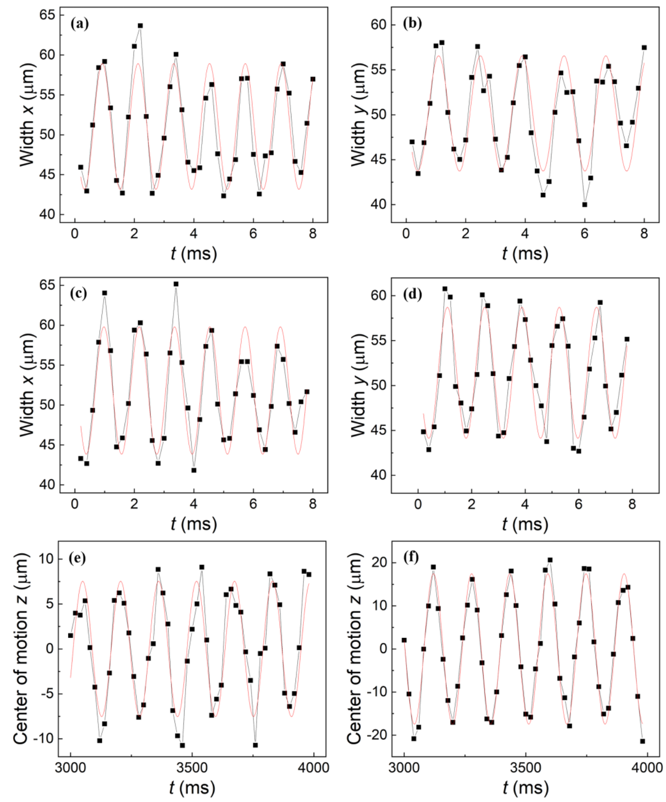

FIG. 3. Oscillation modes of the bosons. (a), (b) Collective oscillation in the x and y directions in the presence of fermions. The oscillation frequencies of the oscillating cloud width were 832.6 ± 4.7 Hz and 712.0 ± 5.2 Hz, respectively. (c), (d) Collective oscillation in the x and y directions without fermions. The oscillation frequencies were 843.2 ± 5.0 Hz and 718.6 ± 4.8 Hz, respectively. (e) Dipole oscillation in the z direction in the presence of fermions, which was 6.408 ± 0.036 Hz. (f) Dipole oscillation in the z direction without fermions, which was 6.382 ± 0.015 Hz.

of atoms in the BEC. Each sample was subjected to magnetic fields having strengths with an interval of 1 G. The corresponding axial Gaussian width is shown in Fig. 2(b). Furthermore, Fig. 2(c) shows the estimated scattering length calculated using Eq. (1). Additionally, results of Thomas-Fermi (TF) approximation, obtained via hyperbolic fitting, are displayed in Fig. 2(c) for comparison. Note that for a near-noninteracting gas, the kinetic energy of the system cannot be ignored. Therefore, the routinely used TF approximation is not appropriate and does not provide a correct assessment. Empirically, the TF approximation fits well with our system when the scattering length is larger than $50a_0$. Moreover, Fig. 2(d) shows another data set obtained near the zero crossing at approximately 850 G. The value closest to the vanishing limit was $a^{(-1)}_{850.5\text{G}} = 0.08 \pm 0.15a_0$ under a magnetic field of 850.5 G, as obtained by averaging more than 28 data points. We used this scattering length to represent a noninteracting Bose gas in the canonical ensemble.

We investigated two phenomena affecting the evaluation of the scattering length between bosons to show that the fermions behave as simple heat baths. As shown in Fig. 1, the interspecies interaction, a_{bf}, is too weak for observable phase separation effects to occur. Therefore, we considered the mixture to be effectively mixable. In this case, the bosons can be considered as impurities inside the dominant fermions. The fermions can also act as an additional antitrapping potential for bosons, which has been measured previously [20]. The amount of induced harmonic frequency change on bosons, $\tilde{\omega}_b$, can be evaluated as

$$\frac{\omega_b - \tilde{\omega}_b}{\omega_b} = \frac{13k_F a_{bf}}{7\pi\xi^{5/4}}, \tag{2}$$

where $\xi = 0.38$ denotes the Bertsch parameter and $k_F = \sqrt{2m_6\bar{\omega}_f(3N_f)^{1/3}/\hbar}$ is the Fermi momentum. Moreover, N_f and $\bar{\omega}_f$ represent the number of atoms and the geometric mean trapping potential for the fermions, respectively. Equation (2) reveals an approximately 1% change in the trapping frequency under our prescribed experimental conditions.

This effect can be measured using collective oscillation. To measure the oscillation, we chose the repulsive $|F = 1, m_F = 0\rangle$ state at 832.18 G with $a^{(0)}_{832.18\text{G}} = 70a_0$ whose interaction with the fermions is the same as $m_F = -1$. Figure 3 shows the oscillation modes of bosons measured with and without fermions. Specifically, Figs. 3(a) and 3(b) show the radial collective oscillations measured at a finite temperature of approximately 144 nK inside the mixture, whose oscillation frequencies in the x and y directions were 832.6 ± 4.7 Hz and 712.0 ± 5.2 Hz, respectively. The oscillation was excited by turning off the trap for 30 μs and then retrapping the atoms. Time-of-flight images were captured to examine the change in gas size after a certain holding time. The oscillation frequency of this mode corresponded to $\sqrt{5}\omega_x$ and $\sqrt{5}\omega_y$ [34]. The red curves were obtained by fitting the sine function; and were used to evaluate the trap frequencies.

Figures 3(c) and 3(d) show the radial collective oscillation, which was excited after removing the fermions from the system; the oscillation frequencies in the x and y directions were 843.2 ± 5.0 Hz and 718.6 ± 4.8 Hz, respectively. The mean values were approximately 1% higher than those for the case with the fermions. However, the difference was within the fitting error in the y direction. Figure 3(e) shows the axial dipole oscillation measured at near-zero temperature inside the mixture, which was 6.408 ± 0.036 Hz. The oscillation was excited by briefly adding another trapping beam into the existing beam to displace the atoms from the trap center. After turning off the additional beam, the BEC underwent dipole movement based on the axial (z direction) trapping frequency. Moreover, Fig. 3(f) shows the axial dipole oscillation excited after removing the fermions; its value was 6.382 ± 0.015 Hz. In this case, the oscillation frequencies coincided with the fitting error. However, we could not rule out the systematic error caused by a slight change in the optical path or intensity of the trapping laser owing to the substantially low oscillation frequency. Therefore, we could not confirm the 1% difference in the axial direction at near-zero temperature. Based on Eq. (1), a 1% difference in the trapping frequencies was determined to have changed the size of the BEC by approximately 0.07 μm, which affected the evaluation of the scattering length by a negligible amount of less than $0.02a_0$. Moreover, the change in size was negligible compared with our imaging resolution of 1.7 μm.

Another possible effect introduced by the superfluid mixture was the recently discovered Ruderman-Kittel-Kasuya-Yosida type interaction induced between bosons [23]. This effect changes the effective interaction by $g_{\text{eff}} = g_{bb} - (3\xi g_{bf}^2 n_f)/(2E_F)$, where $g_{bb} = 4\pi\hbar^2 a/m_7$, $g_{\text{eff}} = 4\pi\hbar^2 a_{\text{eff}}/m_7$, and $g_{bf} = 2\pi\hbar^2 a_{bf} m_6 m_7/(m_6 + m_7)$. Moreover, n_f represents the density of fermions, and $E_F = \hbar^2 k_F^2/2m_6$ is the Fermi energy. Under our experimental conditions, this effect is expected to change the interaction by less than $0.05a_0$, which is within the error of the measured scattering length. Furthermore, the additional interaction conserves the form of the Hamiltonian. Therefore, it does not significantly affect the properties of the bosons and should be automatically included while measuring the effective scattering length.

The noninteracting Bose gas with $0.08 \pm 0.15a_0$ represents a close analog to the textbook ideal gas in the canonical ensemble. However, the equivalence of ensembles is slightly questionable for real boson systems [18,35]. We performed the following demonstration by comparing the equation of state (EOS) of the Bose gas with various amounts of fermions to further address the problem and link our results to the textbook grand canonical scenario and other isolated experimental systems in a microcanonical ensemble.

Figure 4 shows the results obtained for three different temperatures, with the mix rate, N_6/N_7, calculated by dividing the number of single-state fermions by the boson number. The fermions were partially removed after evaporation using resonant radio frequency pulse and a probe beam. The number of bosons was unaffected by the process. Then, we applied a 5-sec holding time before imaging. Note that if the fermions are removed before the evaporative cooling is complete, the bosons remain in high momentum states and disappear from the trap because there is not enough interaction to rethermalize. For example, we confirmed that if we remove the fermions in about 10 sec of the 25-sec evaporative cooling

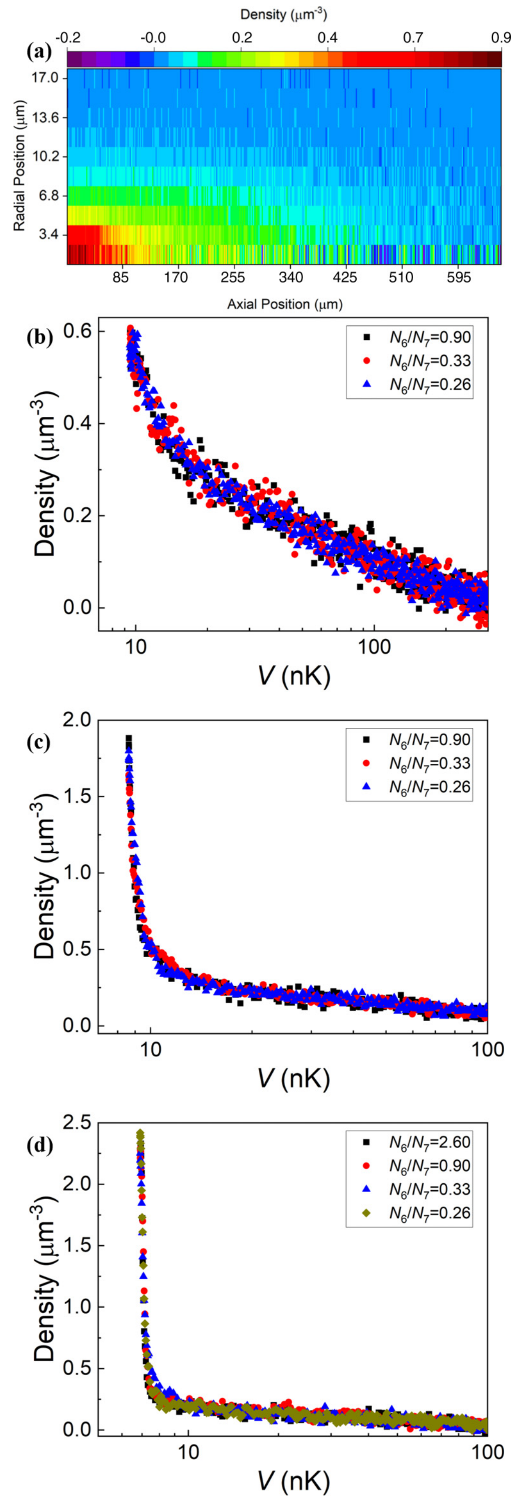

FIG. 4. Equation of state of the bosons with scattering length $0.08 \pm 0.15a_0$ under different boson-to-fermion ratios. The mix rate N_6/N_7 was obtained by dividing a single-state fermion number by the boson number. (a) Typical local density distribution at $\sim$ 156 nK with $N_6/N_7 = 0.90$. We adopted the second row from the bottom to obtain the EOS. (b) Mixture at $\sim$ 156 nK. (c) Mixture at $\sim$ 132 nK. (d) Mixture at $\sim$ 83 nK.

process, the bosons barely remain in the trap when the final trap depth is set. We averaged over 30 data points for each condition and used the inverse Abel transform to generate the EOS of the gas [29]. The local density determined from the transformation [Fig. 4(a) shows an example] was mapped onto the trapping potential using the local density approximation (LDA) to obtain the EOS. Only the second row of the EOS is shown to avoid the effect of the LDA violation introduced by the BEC, which is a known technical problem [36]. We did not detect any change in the EOS of the bosons for any of the temperatures with all boson-to-fermion ratios. This implies an equivalence between the canonical ensemble and the microcanonical ensemble. We can compare the fluctuation introduced by the energy exchange with the heat bath with the total internal energy using $[\langle H^2\rangle - \langle H\rangle^2]/\langle H\rangle^2 \propto C_V/N^2 \propto 1/N = 2\times 10^{-5}$, for a typical number of boson atoms, which is 5×10^4 [37]. Therefore, the energy fluctuation introduced by the heat bath is negligible. The result suggests that the equivalence of ensembles is preserved for our conditions without observing other unexpected phenomena, such as a possible difference between ensembles predicted for systems with hundreds of atoms [38,39]. Note that when using deeply degenerate fermions as heat baths, Pauli blocking can potentially decelerate the thermalization process, similar to the suppression in the fermion-photon scattering [40–42]. However, for the conditions presented in Fig. 4, the fermions were thermal and the Fermi statistics were not expected to affect bosons.

In conclusion, we measured the effective scattering length of ^{7}Li $|F=1, m_F=-1\rangle$ bosons over a magnetic field ranging from 832 to 900 G and identified a zero crossing at 850.5 G. The minimum interacting scattering length was measured to be $a^{(-1)}_{850.5\mathrm{G}} = 0.08 \pm 0.15 a_0$. We used this condition to represent a noninteracting Bose gas. Additionally, we showed that the Bose-Fermi mixture does not significantly change the properties of bosons and that fermions can be considered as simple heat baths or coolants. Moreover, we demonstrated that the equivalence of ensembles is applicable to our system; hence, our results are extendable to other ultracold atom systems. This paper presented a method for creating a noninteracting Bose gas in thermal equilibrium at a finite temperature. Using this method, a noninteracting BEC was spontaneously created. This provides experimental evidence corroborating Einstein's original theory that BEC phase transition can occur as a statistical manifestation without interaction. It has a significance not seen in previous approaches that prepare ideal BECs from interacting BECs. The proposed technique can be used to explore fundamental problems such as the critical behavior of an ideal BEC, thermal saturation, and quantum thermalization in a harmonic trap.

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