The Magnetic Moment of the Proton

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The magnetic moment of the proton has been measured by a method depending on the deflection of a beam of neutral hydrogen atoms in weak magnetic fields of sufficient inhomogeneity. When the field is of such magnitude that the nuclear spin is only partially decoupled from the electron spin the nuclear moment can be evaluated by measuring the atomic magnetic moments of the magnetic levels. Detection was effected on a molybdenum oxide coated plate and the results evaluated by the measurement of the widths and separation of the traces produced. A novel method of great convenience and simplicity for producing the proper magnetic field is described. The value of the proton moment obtained is 3.25±10 percent.

The discovery of the intrinsic magnetic moment of the electron was of first importance in obtaining an understanding of atomic structure. It may likewise be expected that a knowledge of the magnetic moment of the proton will play a similar rôle in the field of nuclear structure. Nuclear magnetic moments have been obtained principally from an investigation of the hyperfine structure of spectral lines. From the intervals between the hyperfine levels one can calculate the magnetic moment of the nucleus if sufficient knowledge of the wave functions is available. For hydrogen this optical method is inadequate. The Lyman lines are in the far ultraviolet and the h.f.s. has never been detected. However a more powerful method based on the use of molecular beams and proposed originally[1] for measuring nuclear spins may be modified to apply to such cases.

The experiments to be described consist in deflecting a narrow beam of neutral hydrogen atoms in the normal $^2S_{\frac{1}{2}}$ state by a weak magnetic field of sufficient inhomogeneity. When a hydrogen atom is in a magnetic field the electronic moment is under the influence of both the external field and the magnetic field due to the nuclear moment. Instead of taking on just the two orientations characterized by $m_s=+\frac{1}{2}, -\frac{1}{2}$ it takes four orientations since the nuclear spin is $\frac{1}{2}$. These four magnetic levels arise from

$$(m_s, m_i)=(\tfrac{1}{2}, \tfrac{1}{2}),\ (\tfrac{1}{2}, -\tfrac{1}{2}),\ (-\tfrac{1}{2}, \tfrac{1}{2}),\ (-\tfrac{1}{2}, -\tfrac{1}{2}).$$

In these levels the component of the magnetic moment of the atom in the direction of the field is given by

$$f_1=1,\quad f_2=x/(1+x^2)^{\frac{1}{2}},\quad f_3=-x/(1+x^2)^{\frac{1}{2}},\quad f_4=-1, \tag{1}$$

in units of the Bohr magneton. The parameter x is defined by

$$x=2\mu_0 H/hc\Delta\nu, \tag{2}$$

where H is the value of the magnetic field, and $\Delta\nu$ is the separation in wave numbers of the hyperfine levels $F=1$ and $F=0$ corresponding to the electron spin and the nuclear spin parallel and anti-parallel. Due to the precession the other components average out to zero and do not contribute to the force on the atom. The contribution of the magnetic moment of the nucleus to the total magnetic moment of the atom may be neglected to this approximation; its effect is only on the average orientation of the electron moment with respect to the field.

Our deflection experiment has as its object the measurement of f_2 and f_3, and hence of the unknown $\Delta\nu$. Our method therefore measures the same quantity as does the h.f.s. method when available. The quantity $\Delta\nu$ is related to the nuclear moment by[2]

$$\Delta\nu=(32\pi/3hc)\mu_P\mu_0\psi^2(0), \tag{3}$$

where μ_P is the unknown magnetic moment of the proton and $\psi(0)$ is the value of the Schrödinger wave function at the nucleus which is known exactly for hydrogen. The other symbols possess their usual meanings. If μ_P is expressed in units of $\mu_0/1838$[3] we have, putting in numerical values,

$$\mu_P=\Delta\nu/0.0169. \tag{4}$$

[1] Breit and Rabi, Phys. Rev. **38**, 2082 (1931); Rabi, Phys. Rev. **39**, 864 (1932); Rabi and Cohen, Phys. Rev. **43**, 582 (1933).

[2] Fermi, Zeits. f. Physik **60**, 320 (1930).

[3] A convenient unit of nuclear magnetic moment for which we propose the symbol μ_s.

In Fig. 1 is plotted the variation of the effective atomic moments f (in units of μ_0) with x. Measurements of the values of f at different fields yield independent relations for the calculation of $\Delta\nu$. For a given value of x a beam of atoms of homogeneous velocity will give a deflection pattern consisting of four lines corresponding to the four values of f in Eq. (1). For $x=0.354$ the four lines will be equidistant, and if $\mu_P=\mu_s$ (i.e., $\Delta\nu=0.0169$), the value of H will be 64.5 gauss. For larger values of μ_P the field will be proportionally larger.

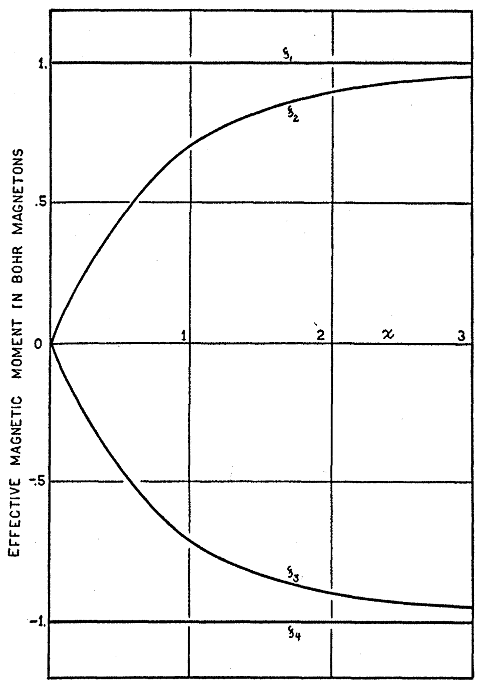


FIG. 1. Variation of the moments of the magnetic levels with magnetic field.

In these experiments no source of hydrogen atoms homogeneous in velocity was available; consequently these considerations have to be modified to take account of the Maxwellian distribution of velocities. Although all the atoms in a given magnetic level are under the influence of the same force, the fast atoms are deflected less than the slow. Each level thus gives rise to a band instead of a line. Since each level is equally probable the areas under the component bands are equal. The superposition of these bands yields instead of lines a deflection pattern as shown in Fig. 2. However a measurement of this pattern can still give results of sufficient precision under proper conditions.

APPARATUS

The hydrogen atoms are formed in a long (5 meter) Wood discharge tube. The pressure in this tube is maintained by a continuous circulation of moist hydrogen at 0.2 to 0.3 mm of mercury. Examination of the discharge with a spectroscope shows very little background to the Balmer lines. A side arm of 6 mm Pyrex tubing 20 cm long and cooled for part of its length leads from the middle of the discharge tube to the source slit S_1, Fig. 3. This slit, in reality a canal 0.02 mm wide and 4 mm high is made by sawing a 0.1 mm slot in the end of a closed Pyrex tube. The jaws of this slot are then collapsed on a thin copper foil which is subsequently dissolved out. The source

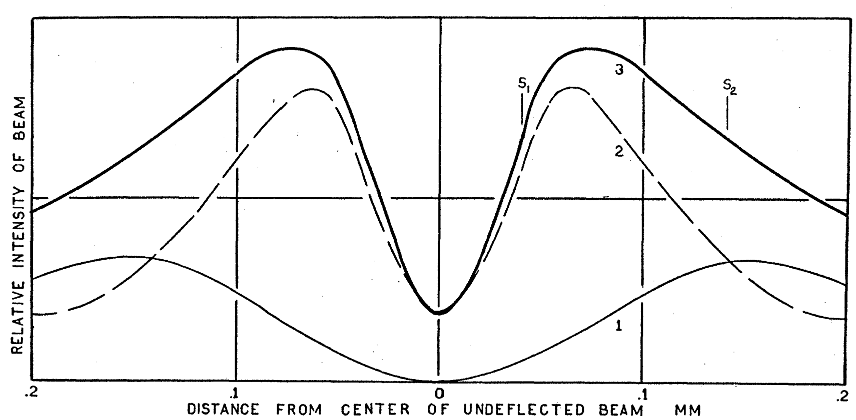


FIG. 2. Deflection pattern with $T=300$, $l_1^2+2l_1l_2=303$, $H=244$, $\partial H/\partial y=1980$, and an assumed proton moment of 2.6 units.

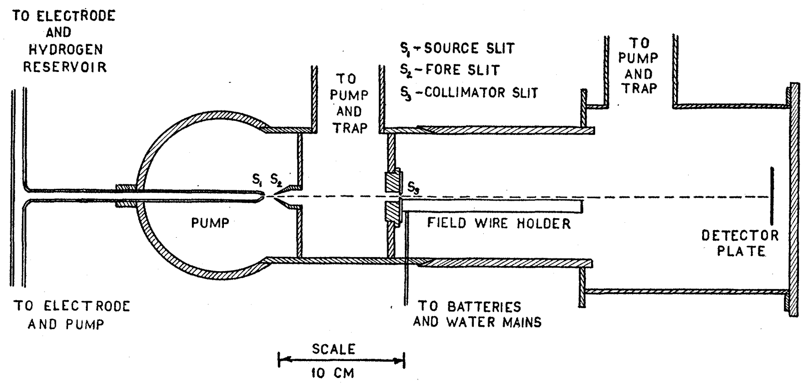


FIG. 3. Diagram of apparatus.

slit projects into the inlet of a high speed oil diffusion pump which maintains a pressure of 3×10^{-5} mm in the source chamber $S.C.$ To obtain high vacuum for the greater part of the beam path the source chamber is isolated by an adjustable steel fore slit S_2, 0.1 mm wide and 7 mm from S_1. A fast oil diffusion pump[4] with a liquid air trap maintains a pressure of 10^{-6} mm in the space between S_2 and the collimating slit S_3. This slit, formed by adjustable phosphor bronze jaws, is 0.02 mm wide and 2.4 mm high. The remaining beam path is maintained at a pressure of 5×10^{-7} mm by another fast oil pump with a liquid-air trap. This pressure is measured with an ionization gauge whereas all the others are measured with a McLeod.

Detection of the beam is effected on a layer of molybdenum oxide soot[5] deposited on a glass plate. This yellow oxide is reduced by the atomic hydrogen to the so-called "blue" oxide of molybdenum. The plate is carried on a slide which may be moved across the beam by a magnetic control. In addition the distance of the plate from the end of the field may also be varied. The detector is so sensitive that the trace of an undeflected beam 40 cm long appears after four minutes of exposure.

The three slits cannot be aligned by optical means alone. Since the source slit is a canal the atoms do not emerge according to the cosine law but are so strongly concentrated in the forward direction that a fairly sharp trace is obtained on a detecting plate placed between S_1 and S_2. The fore slit is set in the line of the beam with the aid of this trace. This precaution is very important if maximum intensity is to be obtained. A shift in the position of the fore slit of 0.1 mm would cause a reduction of intensity at the receiving end of 90 percent although optically the line up could still appear perfect.

A novel feature of this apparatus is the production of the magnetic field without the use of iron. The field must meet exacting requirements. As will be evident in the discussion of procedure, the magnetic field must be small (100 to 400 gauss), the gradient of the field must be sufficiently large to produce easily measurable deflections, both the magnitude of the field and the gradient must be constant over the height of the beam, and both must be accurately known. These conditions

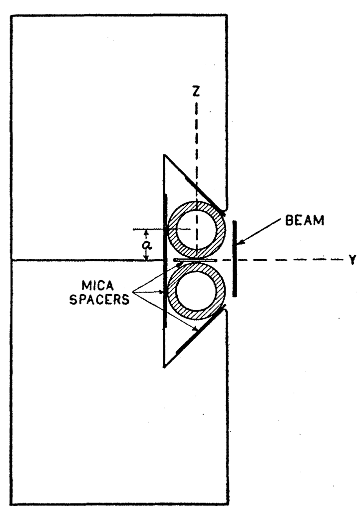


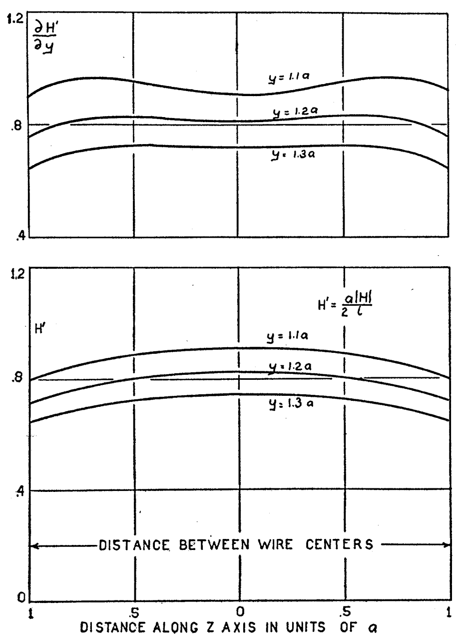
FIG. 4. Cross section showing field wire holder and field wires. The direction of the beam is perpendicular to the plane of the paper.

[4] After a design kindly sent us by Dr. R. M. Zabel.
[5] Phipps and Taylor, Phys. Rev. **29**, 309 (1927).

introduce only moderate difficulty if the height of the beam be severely limited, but since the intensity at the detector is proportional to the effective height of the beam the problem cannot be met in this way with the detectors in use at present. Fortunately the magnetic field in the neighborhood of two long parallel straight wires carrying currents in opposite directions meets these requirements. Since the field and gradient are completely calculable from the geometrical arrangement of the currents we avoid the very difficult and tedious measurement of these quantities. Fig. 4 shows in cross section the relation of such a pair of wires to the beam and to a set of coordinate axes yz. Graphs of the magnitude of the magnetic field and of the gradient are shown in Figs. 5 and 6. If $2a$ is the distance between the centers of the wires, then it is evident from these curves that for $y=1.2a$ both field and gradient are nearly constant over the region between $z=-0.7a$ and $z=+0.7a$. The relations giving the field and gradient at any point are

$$H=2I\frac{2a}{r_1r_2},\quad \frac{\partial H}{\partial y}=-2I\frac{2a}{r_1^3r_2^3}(r_1^2+r_2^2)y,$$

where I is the current in amperes/10 and r_1 and r_2



FIGS. 5–6. Plot of the magnetic field and gradient in the region of the beam.

the distances of the point in question from the centers of the two wires. In our apparatus with $a=1.23$ mm and $z=0$, a current of 100 amp. in each wire will produce a field of 133 gauss and a gradient of 1080 gauss per cm at $y=1.2a$. Such high current densities require the use of hollow water cooled copper tubes instead of wires. These tubes, 15 cm long, 0.241 cm in diameter, and separated by a mica spacer 0.005 cm thick, are held rigidly in an aluminum block. Current is supplied to these tubes by a bank of twelve 160 amp.-hr. storage batteries in parallel. The position of the field wires with respect to the beam is conveniently determined by the beam itself. A detecting plate is mounted with one edge in contact with the wires and a trace is formed. The distance from the edge of the plate to the middle of the trace is then measured with a travelling microscope.

PROCEDURE

The field current is adjusted to a suitable value and the beam turned on. The double trace corresponding to curve III, Fig. 2 appears on the detecting plate usually within 45 minutes, and after 6 to 8 hours is fairly intense. Fig. 7 is a photograph of a typical trace. The plate is then moved a few millimeters to one side and a trace of the direct beam without the field is made. This serves as a check on the width of the beam and on the general constancy of the alignment of the slits. The experimental results are evaluated from a measurement of the separation of the inside edges of the traces and their widths. These distances are measured with a microscope with a micrometer ocular or a micrometer stage. The aim of the measurement is to obtain four points on the traces of presumably equal intensity. As is evident from Fig. 7 the edges of the traces are rather fuzzy and somewhat difficult to measure. The precision of these experiments is limited by these conditions more than by any other factors. However, this method of evaluation first suggested by Stern,[6] has been applied by his school and others to experiments on a variety of atoms including hydrogen at high fields with satisfactory results.

[6] Stern, Zeits. f. Physik **41**, 563 (1927); Rabi, Zeits. f. Physik **54**, 190 (1929).

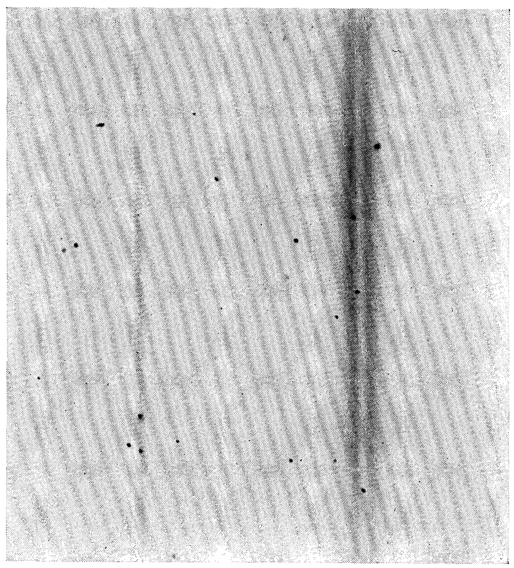


FIG. 7. Photograph of trace. Distance between center of split trace and direct trace is 2.06 mm.

The deflection of an atom in the direction y perpendicular to the beam is given by

$$s=\frac{f_j\mu_0}{4E}\frac{\partial H}{\partial y}(l_1^2+2l_1l_2),$$

where l_1 and l_2 are the distances the atom moves in the field and beyond the field, respectively, E is the kinetic energy of the atom and f_j is the magnetic moment of the atom in a given magnetic level (f_2 and f_3 are functions of H, f_1 and f_4 are independent of H, see Fig. 1). For an atom with kinetic energy $E=E_\alpha=\frac{1}{2}mv_\alpha^2=kT$, s will have the particular value

$$s_\alpha=\frac{f_j\mu_0}{4kT}\frac{\partial H}{\partial y}(l_1^2+2l_1l_2). \tag{5}$$

For atoms with $f_j=\pm 1$, $s_\alpha=\pm s_\alpha{}^0$ may be calculated directly since the quantities in this equation are then all known. For atoms with $f_j=\pm x/(1+x^2)^{\frac{1}{2}}$ (Eq. (1)) experimental evaluation of $s_\alpha=s_\alpha{}^i$ permits calculation of f_2 and f_3 and therefore of $\Delta\nu$ by Eq. (2).

Consider atoms which are deflected from an element du, Fig. 8, in the undeflected beam a distance w to the element ds at s. The form of the undeflected trace is trapezoidal as calculated from the geometry of the apparatus and can be

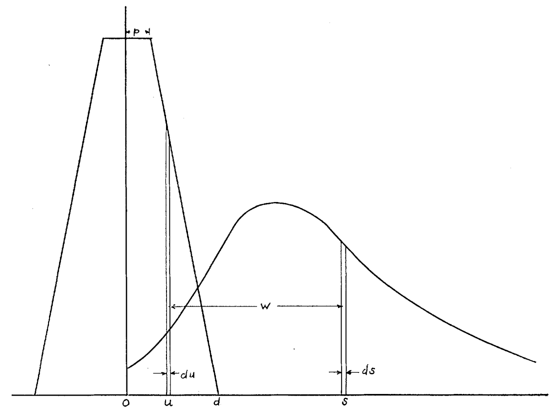


FIG. 8.

represented by the function $I(u)$. The deflection is a function of f_j and E. For atoms in a state with moment f_j and *a priori* probability w_j this number will be

$$dn_j=w_jI(u)due^{-v_w^2/v_\alpha^2}(2v_w^3/v_\alpha^4)dv_w$$
$$=w_jI(u)due^{-E_w/E_\alpha}(E_w/E_\alpha^2)dE_w.$$

Since all magnetic levels are equally probable, $w_j=\frac{1}{4}$. The intensity (number of atoms per unit width) at the position s is

$$dJ=\frac{dn}{ds}=w_jI(u)due^{-E_w/E_\alpha}\frac{E_w}{E_\alpha^2}\frac{dE_w}{ds}$$
$$=w_jI(u)e^{-E_w/E_\alpha}\frac{E_w}{E_\alpha^2}dE_w,$$

since $s=u+w$, $ds=dw=-du$. Since w is a function of E only, we can integrate this expression over the undeflected trace and obtain

$$J(s,s\alpha)=\{1/4(d-p)\}\{(s+d)e^{-s\alpha/s+d}+|(s-d)|e^{-s\alpha/|(s-d)|}-(s+p)e^{-s\alpha/(s+p)}-|(s-p)|e^{-s\alpha/|(s-p)|}\},$$

where the symbol $|\ |$ means that the absolute value of this quantity is used. This gives the relative intensities due to the component to the right of the undeflected trace. The deflection pattern is symmetrical with respect to the center. The total intensity is obtained by summing the two component intensities, $J(s, s_\alpha{}^0)+J(s, s_\alpha{}^i)$. Fig. 2 is a plot of the intensity curve obtained by this means. If s_1 and s_2 denote the position on the trace of two points of equal intensity, see Fig. 2, then

$$J(s_1, s_\alpha^0)+J(s_1, s_\alpha^i)=J(s_2, s_\alpha^0)+J(s_2, s_\alpha^i).$$

Solution of this equation yields s_α^i and therefore from Eqs. (5, 1, 2, 4) a value of μ_P.

The above procedure has to be somewhat modified to take account of the fact that both the field and the gradient vary by a few percent over the width of the deflected beam. This has been taken into account in the table of results given below.

These considerations tell us how to obtain a moment from a given set of measurements. To obtain maximum precision we should have f_2, f_3 which correspond to curve 2 Fig. 2 as different as possible from f_1, f_4 consistent of course with obtaining sufficient inner separation of the traces to measure comfortably. This circumstance separates to a large extent the contributions to the summation curve of the two sets of moments. It is easily shown that when this is not the case experimental errors (e.g. in T, l_1, l_2, $\partial H/\partial y$) are greatly magnified in the determination of f. A second consideration which is of equal importance is the fact that when x is small (in the region of 0.3) the error in the evaluation of the nuclear moment is practically equal to the error in the determination of f, since $\Delta\mu/\mu=(1+x^2)\Delta f/f$ but when x is large (greater than 1.5) the error in μ_P may become several times the experimental error in f. It is consequently of great advantage to be able to perform these experiments at sufficiently low field intensities.

Results

A tabulation of the results of the various runs is given in Table I. As can be seen the measurements were made under a great variety of conditions. In the different runs the width of the initial beam $2d$, was changed by a factor of 1.5, the field by a factor of 2, l_2 by a factor of 3, and the inside separation of the traces by a factor of 3. In addition the density of the traces themselves was greatly varied. The energy distribution of the atoms emerging from the source slit was taken to be characteristic of the temperature of the walls of the glass tube leading to the slit, i.e., room temperature. The proton moment obtained by averaging the results of the various runs is 3.25±10 percent. The sources of error which enter are chiefly those of measurement of the traces. These we consider as accidental errors in view of the experience of other observers with this type of measurement. Another possible source of error which would operate only toward reduction of the moment lies in the estimation of the temperature of the beam. However in the view of length of the communicating tube, the pressure in the discharge tube, and the fineness of the source slit the hydrogen atoms must have suffered a sufficient number of collisions to take on the temperature of the walls. Errors in the determination of the width of the direct beam which might be brought about by scattering of the beam or by error in the estimation of the slit widths have a very small effect on the calculated values of the moment. An assumed increase of 15 percent in d brings about an increase in the calculated nuclear moment of about 1.5 percent. These considerations and the general consistency of the results as presented in Table I make it seem unlikely that a constant error of any considerable magnitude has entered into the measurement. We consider 10 percent a fairly conservative estimate of the precision.

Table I.

H gauss	$\partial H/\partial y$ gauss/cm	$(l_1^2+2l_1l_2)$ cm²	d mm	s_1 mm	s_2 mm	f	μ_P
175	1422	652	0.07	0.037	0.152	0.299	3.06
208	1691	652	.07	.048	.195	.347	3.08
210	1709	473	.058	.040	.150	.358	3.00
236	1920	391	.052	.031	.148	.351	3.44
244	1985	303	.047	.028	.122	.367	3.38
272	2215	652	.07	.079	.275	.423	3.20
306	2490	652	.07	.094	.335	.456	3.26
306	2490	652	.07	.090	.303	.429	3.52
306	2490	652	.07	.093	.333	.451	3.32

Because of various corrections the values of s_1 and s_2 recorded are not the values which will lead exactly to the corresponding moment, but are included in order to give an idea of the magnitude of these quantities.

Conclusions

The value we obtain for the proton moment is 3.25±10 percent. This value differs considerably from the value 2.5±10 percent obtained by Stern, Estermann and Frisch[7] who measured the quantity by an entirely different method depend-

[7] Frisch and Stern, Zeits. f. Physik **85**, 4 (1933); Estermann and Stern, Zeits. f. Physik **85**, 17 (1933).

ing on the deflection of hydrogen molecules in a very strongly inhomogeneous field. The deflection in their experiments is due to the force exerted directly on the nuclear moment. The moment is evaluated from these deflections after various corrections are made for the moment due to the rotation of the molecule. At present the substantially fair agreement of the two results must be regarded as more important than the difference. There is as yet no sufficient theoretical basis to discuss the far reaching implications of this value of the proton moment. However it can be taken as certain that the proton is not describable by the simple type of Dirac wave equation which describes the electron. An important consequence of these experiments is that we can compare for the first time a measurement made on a nuclear moment directly and one which is fundamentally analogous to the h.f.s. method. The result establishes the h.f.s. method of measuring nuclear moments on firm experimental ground. When both methods are brought to a greater degree of precision we may hope to obtain further tests of the Dirac equation for the electron and perhaps detect other modes of interaction between electron spin and the proton.

The Magnetic Moment of the Deuton[1]

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The magnetic moment of the deuton was measured by the deflection of a beam of neutral deuterium atoms. The magnetic moment of the deuton is found to be 0.75 ± 0.2 nuclear units.

The deuton as the simplest atomic nucleus next to the proton is of particular interest in nuclear investigations. In view of the extreme likelihood that the deuton consists of a proton and a neutron, an evaluation of the magnetic moment of the deuton must yield important information with regard to the magnetic moment of the neutron.

In the experiments to be described the magnetic moment of the deuton was measured by the same method and in the same apparatus which we used for the measurement of the proton moment.[2] A narrow beam of neutral deuterium atoms from a Wood discharge tube is sent through a magnetic field weak enough not to decouple entirely the deuton spin from the electron spin. The field must be sufficiently inhomogeneous to produce easily measurable deflections. As with the H^1 detection was effected on a molybdenum oxide plate. It turned out that the chemical properties of deuterium are not sufficiently different to affect this reaction to any appreciable extent. The traces were measured in the same way as the traces produced by H^1.

The deuton is known to have a spin of 1,[3] consequently there are six magnetic levels corresponding to

$$(m_i,\ m_s)=(1,\ \tfrac{1}{2}),\ (-1,\ -\tfrac{1}{2}),\ (0,\ \tfrac{1}{2}),\ (0,\ -\tfrac{1}{2}),\ (-1,\ \tfrac{1}{2}),\ (1,\ -\tfrac{1}{2}), \quad (1)$$

which have effective magnetic moments of

$$f_1=\pm 1,\quad f_2=\pm x+\tfrac{1}{3}/(1+\tfrac{2}{3}x+x^2)^{\frac{1}{2}},\quad f_3=\pm x-\tfrac{1}{3}/(1-\tfrac{2}{3}x+x^2)^{\frac{1}{2}} \quad (2)$$

and x is defined by

$$x=\mu_0 H/hc\Delta\nu. \quad (3)$$

[1] Estermann and Stern, Phys. Rev. **45**, 761 (1934); Rabi, Kellogg and Zacharias, Phys. Rev. **45**, 769 (1934).

[2] See preceding paper, *The Magnetic Moment of the Proton*, page 157. The deuterium gas as well as a sufficient quantity of heavy water to poison the walls of our discharge tube was generously supplied to us by Professor H. C. Urey.

[3] Murphy and Johnston, Phys. Rev. **45**, 761 (1934).

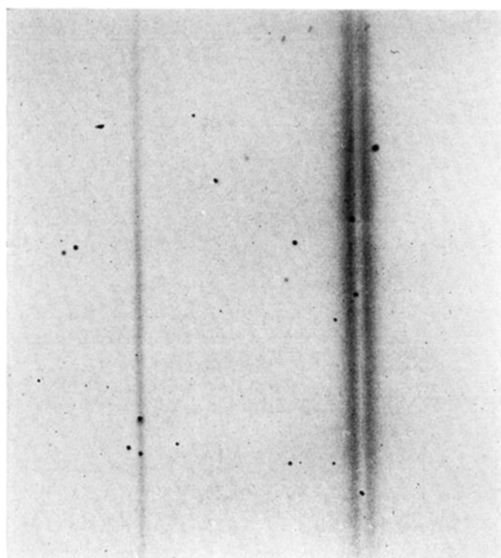

FIG. 7. Photograph of trace. Distance between center of split trace and direct trace is 2.06 mm.