

Temporal steering of entanglement decay with single-shot control

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Entanglement in open quantum systems can vanish abruptly through entanglement sudden death (ESD) under dissipative evolution, posing a challenge for quantum technologies. Here, we show that the timing of a single local unitary operation can deterministically steer the trajectory of entanglement decay. This introduces temporal steering of dissipative quantum dynamics as a control resource. We develop a time-dependent open-system framework in which a time-shift operator generates a family of effective conditional maps parametrized by $x \in [0, 1]$, connecting independent and correlated amplitude damping. Within this framework, we derive analytic conditions for ESD and show that a single intermediate local σ_x operation can avoid, delay, or hasten ESD by redirecting the dissipative trajectory rather than modifying the environment. We further prove that this single-shot protocol is optimal within a broad class of local control strategies. Experimentally, a displaced Sagnac interferometer realizes the correlated-damping-like regime and, for the first time in one photonic platform, demonstrates avoidance, delay, and hastening of the separability transition. Our results establish temporal steering of open-system dynamics as a practical paradigm for decoherence control, and demonstrate operational advantages in use cases such as teleportation, illustrating that in dissipative quantum systems, knowing when to act can be as important as knowing which operation to perform.

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I. INTRODUCTION

Quantum systems inexorably lose coherence through environmental interactions, with entanglement—the quintessential quantum resource—being particularly fragile. This decoherence problem is universal: it limits qubit lifetimes in superconducting processors [1], degrades quantum memories [2], and constrains communication distances in quantum networks [3]. One particularly striking manifestation is entanglement sudden death (ESD), where entanglement vanishes in finite time rather than decaying asymptotically [4,5]. Unlike gradual decoherence, ESD represents a threshold phenomenon that poses fundamental questions about the nature of open quantum evolution and presents practical challenges for quantum technologies that rely on maintaining entanglement over extended periods. These challenges motivate the search for control strategies that can shape the trajectory of dissipative evolution, raising a central question: in open quantum systems, can knowing when to intervene be as important as knowing which operation to perform?

Here we show that temporal steering of open-system dynamics via a single local unitary operation provides a deterministic mechanism for controlling entanglement decay, with timing being a crucial element. We consider a continuous family of amplitude-damping channels, introducing timing as a

control resource for steering dissipative quantum trajectories. We report three principal advances.

First, we demonstrate experimentally that a single local NOT operation, applied at a chosen time during amplitude damping, can deterministically achieve three qualitatively distinct outcomes: complete ESD avoidance (avoidance of finite-time ESD), significant delay (extending the damping parameter range by approximately 30%), or hastening (accelerating ESD onset). The regime is selected purely by the operation timing, characterized by a parameter p representing the strength of damping experienced before the NOT, with no modification to the environmental noise or coupling strength. Second, we introduce a displaced Sagnac interferometer architecture where intrinsic path delays naturally suppress the cross-components $|HV\rangle$ and $|VH\rangle$ of the two-photon state, H and V denoting polarization of the photons. These delays, which would be detrimental in conventional interferometric implementations, instead produce behavior closely mimicking correlated amplitude damping—a form of collective decoherence where both qubits flip together or not at all, rather than independently. This realization connects idealized theoretical models to realistic photonic systems without requiring the engineering of global system-bath coupling. Third, we develop a time-dependent framework that captures this physics through a single tunable parameter $x \in [0, 1]$ arising from the temporal structure of photon detection. This parameter interpolates continuously between standard amplitude damping ($x = 1$, where qubits decohere independently) and correlated

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amplitude damping ($x = 0$, where decoherence is collective). Intermediate values of x describe novel damping channels that are neither standard nor fully correlated, revealing a rich landscape of engineerable noise models accessible through simple geometric control of optical path delays.

Current strategies to mitigate decoherence fall into three broad categories, each with distinct operational requirements and fundamental limitations. Dynamical decoupling (DD) [1,2,6–8] applies sequences of control pulses designed to time-average environmental noise to zero. While effective in many contexts, DD requires precise pulse timing that becomes increasingly challenging as system complexity grows. Moreover, each pulse introduces errors that accumulate over the sequence, and the approach becomes perturbative for strong system-environment coupling. The quantum Zeno effect (QZE) [9–12] employs frequent measurements that effectively freeze quantum evolution through repeated projection onto a target subspace. However, achieving the Zeno regime demands measurement rates faster than environmental correlation times, which often proves experimentally prohibitive. Quantum weak measurement reversal (QWMR) [13–19] attempts probabilistic state recovery through a combination of weak measurements and reversal operations, but by its nature succeeds only when postselected on favorable measurement outcomes, making it unsuitable for deterministic protocols.

Despite their different mechanisms, all these approaches share a common feature: they require multiple operations—pulse trains, repeated measurements, or statistical averaging—to suppress decoherence. This multiplicity has both practical and fundamental limitations. Practically, each additional operation introduces overhead and potential for error. Fundamentally, the multi-operation requirement suggests that existing approaches primarily fight decoherence through brute force averaging or projection, rather than exploiting the underlying structure of the open-system dynamics.

Here, we demonstrate a fundamentally different paradigm based on a simple observation: quantum decay exhibits a fundamental temporal asymmetry. Excited states decay to ground states, never reverse. For a two-qubit entangled state $|\psi\rangle = \alpha|HH\rangle + \beta|VV\rangle$ under amplitude damping, the $|VV\rangle$ component (both qubits excited) preferentially decays toward $|HH\rangle$ (both ground), $|HV\rangle$, and $|VH\rangle$ (single excitations). Now consider inserting a local NOT operation at an intermediate time t_1 during this evolution. The NOT swaps the qubit populations: $|H\rangle \leftrightarrow |V\rangle$. Critically, this population inversion means the subsequent evolution from t_1 onward follows a completely different trajectory because the system now “sees” inverted populations—what was stable becomes unstable and vice versa. This raises a striking question: can the timing of a single local NOT operation—with no change to the noise itself—completely control decoherence?

Early theoretical work by Rau, Ali, and Alber [20,21] showed that appropriately timed local operations can manipulate the onset of entanglement sudden death (ESD). These studies revealed that the timing of a local operation during dissipative evolution can influence whether ESD is hastened, delayed, or avoided. However, these analyses were formulated for specific noise models and did not provide a

continuous channel framework describing how such control emerges across a broader family of dissipative dynamics. Subsequent work by Singh *et al.* [22] proposed an optical realization of this idea using cascaded amplitude-damping channels, suitably adapting H and V for polarization instead of energy excitation. In this case, experimental realization faced a fundamental barrier: the proposed scheme required two separate interferometers implementing sequential damping channels, with path-length matching at the nanometer scale over meter-long setups—a requirement far beyond practical feasibility. This motivates the search for an architecture that captures the underlying physics while remaining experimentally feasible. Our work addresses this challenge through a key insight: rather than eliminating the intrinsic path delays that arise in cascaded optical channels, we exploit them. In a displaced Sagnac interferometer, these delays naturally suppress the cross-components $|HV\rangle$ and $|VH\rangle$, producing behavior closely mimicking correlated amplitude damping.

The mechanism underlying our protocol is conceptually straightforward. Amplitude damping preferentially drives the transition $|V\rangle \rightarrow |H\rangle$ but not the reverse. When applied to $|\psi\rangle = \alpha|HH\rangle + \beta|VV\rangle$ with $\alpha < \beta$, damping causes the $|VV\rangle$ component to decay, generating mixed-state components $|HV\rangle$ and $|VH\rangle$ that destroy entanglement. This leads to a redistribution of the original $|VV\rangle$ amplitude among four components in the $|H\rangle, |V\rangle$ basis of the two-qubit system. A local NOT operation swaps the state to $|\psi'\rangle = \beta'|HH\rangle + \alpha'|VV\rangle + \text{cross terms}$, where $\beta' < \beta$ and $\alpha' > \alpha$ reflect partial decay that has already occurred during the first damping stage. When population inversion leads to $\alpha' < \beta'$, subsequent damping attacks can prevent the entanglement from ever reaching zero. This is not quantum error correction: the protocol does not restore the initial state. Rather, we perform trajectory engineering, exploiting decay asymmetry to steer evolution onto paths where entanglement survives longer or dies faster, depending on when we intervene.

This work establishes that temporal control—when to apply operations—can be as important as operational control—which operations or how many—in determining open-system dynamics. The single-shot, nonperturbative nature of our protocol is relevant to any platform where local gates are fast, but gate sequences accumulate errors, such as superconducting qubits with coherence-limited gate fidelities approaching fundamental bounds. The tunable parameter x connecting idealized and realistic noise models suggests routes to engineer custom decoherence channels by controlling temporal correlations, applicable beyond photonics to circuit QED and trapped ions, where bath spectral properties can be tailored. Our Hamiltonian formulation reveals connections to dynamical decoupling through a custom “parity operator,” but operating nonperturbatively rather than in the weak-coupling limit where standard DD applies.

The remainder of this paper is organized as follows: Section II develops the theoretical framework, introducing standard and correlated amplitude-damping models, deriving the phase diagram showing avoidance, delay, and hastening regimes, presenting the time-dependent formalism with parameter x , and proving that local NOT is the unique optimal operation. Section III presents the Hamiltonian formulation connecting our protocol to dynamical decoupling. Section IV

TABLE I. Summary of ESD manipulation regimes for $\alpha = 0.55$.

Regime	Condition	Effect of NOT
Avoidance	$p \lesssim 0.17$	$P_{\text{ESD}} \rightarrow 1$
Delay	$0.17 < p < 0.28$	$\Delta P \approx 0.3$
Hastening	$p \gtrsim 0.28$	ESD accelerated

describes the experimental implementation using a displaced Sagnac interferometer. Section V demonstrates complete ESD manipulation across all three regimes. Section VI discusses the physical mechanism, platform generalization, and error analysis. We conclude in Sec. VII with implications and open questions.

The principal theoretical and experimental results are as follows:

(1) We develop a time-dependent open-system framework for a continuous family of amplitude-damping-like channels [(CADC-NOT) n -ADCs] and correlated amplitude-damping-like channels (CADCs). A physically motivated time-shift operator induces an effective correlation parameter $x \in [0, 1]$ that interpolates between independent ADC-like ($x = 1$) and a CADC-like limit ($x \approx 0$), and we obtain closed-form expressions for the concurrence and the associated ESD boundary as functions of the damping parameters (p, P) of the first and second damping channels, respectively, and the time delay parameter x .

(2) Within this family, we analytically classify all possible single-shot control behaviors and prove the optimality of a single local NOT. We show that a single local σ_x applied at an intermediate time can avoid, delay, or hasten ESD by steering the dissipative trajectory, and we demonstrate that cascaded (CADC-NOT) n protocols and a broad class of general SU(2) controls cannot outperform this single-shot scheme.

(3) We experimentally implement the framework in a displaced Sagnac interferometer and realize a CADC-like channel with a tunable x , fully characterize both ADC and CADC-like behavior, and perform new control experiments with and without the NOT, thereby observing all three regimes of ESD manipulation (see Table I) in a single platform with quantitative agreement between theory and experimental data.

Taken together, these results present a unified theoretical and experimental demonstration that temporal steering of open-system dynamics via a single local NOT operation can provide deterministic control of entanglement loss within a widely relevant noise family.

II. THEORETICAL FRAMEWORK

We consider a two-qubit state $\rho_{\text{in}} = |\psi\rangle\langle\psi|$ with $|\psi\rangle = \alpha|HH\rangle + \beta|VV\rangle$ evolving through two cascaded ADCs with an intermediate local NOT operation to steer the ESD trajectory. The damping channels are characterized by parameters p and P . These parameters represent the probability that an excited state $|V\rangle$ decays to the ground state $|H\rangle$ during each damping stage. The evolution, described by Kraus operators, depends crucially on whether a local NOT operation is inserted between the two channels. The initial state ρ_{in} evolves through the damping channel through the action of Kraus

operators $\mathbb{K}_i$ to give

$$\rho_{\text{out}} = \sum_i \mathbb{K}_i \rho_{\text{in}} \mathbb{K}_i^\dagger, \text{ subject to } \sum_i \mathbb{K}_i^\dagger \mathbb{K}_i = \mathbb{I}. \quad (1)$$

One of the most widely used measures of quantum entanglement between two qubits is the concurrence [23]. For an arbitrary bipartite quantum state ρ , the concurrence is defined as

$$C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (2)$$

where the quantities λ_i are the eigenvalues in decreasing order of the matrix:

$$R = \rho(\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y).$$

Here, ρ^* is the complex conjugate of the matrix representing the input state ρ and σ_y is the Pauli matrix,

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

The detailed derivation of the Kraus operators from the unitary evolution of the system and environment is given in Appendix A. For standard ADC, each qubit couples independently to its local environment. The single-photon Kraus operators are $K_1 = \text{diag}(1, \sqrt{1-p})$ and $K_2 = \text{off-diag}(0, \sqrt{p})$, where we use shorthand notation for the 2×2 matrices. The two-photon evolution involves tensor products of these operators. Without a NOT operation between channels, the concurrence is

$$C(p, P) = 2 \max[0, \{|\alpha\beta| - |(p(1-P) + P)\beta^2|\} \times (1-p)(1-P)]. \quad (3)$$

With a local NOT operation $\sigma_x \otimes \sigma_x$ applied after the first channel but before the second, and for our choice of parameters α and β , the output becomes an X state [4,5,20,22] with the concurrence

$$C(p, P) = 2 \max[0, \{1 - p(1-P)\}(1-P)|\alpha\beta| - (1-P)[P - (1-p)\{P - p(1-P)\}\beta^2]]. \quad (4)$$

The key observation is that NOT fundamentally changes which density-matrix elements compete in determining the concurrence.

CADC represents a qualitatively different decoherence process where the two qubits couple to a common environment, leading to collective rather than independent decay. The Kraus operators act on the full two-qubit Hilbert space, with the transformation $|VV\rangle \rightarrow \sqrt{p}|HH\rangle + \sqrt{1-p}|VV\rangle$ being correlated—both qubits flip together or not at all. This suppresses the single-excitation states $|HV\rangle$ and $|VH\rangle$. For a CADC first channel followed by a standard ADC second channel, without NOT, the concurrence is

$$C(p, P) = 2 \max[0, \{|\alpha\beta| - \sqrt{1-p}P\beta^2\}\sqrt{(1-p)(1-P)}]. \quad (5)$$

With NOT between channels:

$$C(p, P) = 2 \max[0, \sqrt{1-p}|\alpha\beta| - P(\alpha^2 + p\beta^2)](1-P). \quad (6)$$

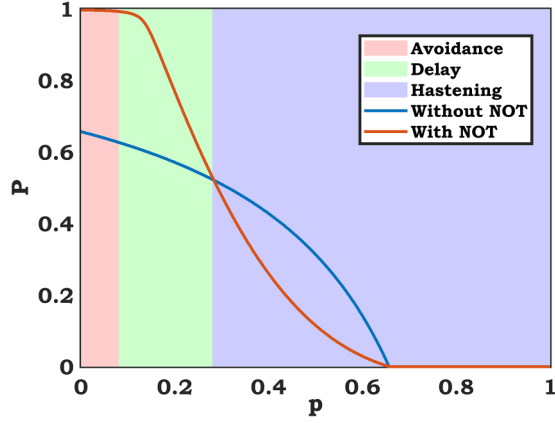


FIG. 1. A contour plot showing the ESD line $C(p, P) = 0$ for $x = 1$ (standard ADC) with initial state $|\Psi\rangle = \alpha|HH\rangle + \beta|VV\rangle$, where $\alpha = 0.55$. The (p, P) parameter space is divided into three regions: avoidance (red), delay (green), and hastening (blue). The blue and red curves denote the ESD boundaries without and with the NOT operation, respectively.

The ESD line—defined as the set of (p, P) values where $C(p, P) = 0$ for the first time—partitions the parameter space into regions with qualitatively different behavior. For ADC-NOT-ADC:

$$[1 - p(1 - P)]|\alpha\beta| = \{P - (1 - p)[P - p(1 - P)]\}\beta^2. \quad (7)$$

For CADC-NOT-ADC:

$$\sqrt{1 - p}|\alpha\beta| = P(\alpha^2 + p\beta^2). \quad (8)$$

These ESD lines depend sensitively on the initial state parameter α . Figures 1 and 2 show the resulting phase diagrams for $\alpha = 0.55$. The (p, P) plane divides into three distinct regions. In the avoidance regime (small $p \lesssim 0.17$), the first damping barely affects the state before the NOT. Population inversion then places the larger coefficient on the stable $|HH\rangle$ state, preventing ESD. The delay regime (intermediate $0.17 < p < 0.28$) represents partial degradation before NOT. The inversion reduces but does not eliminate the asymmetry driving ESD, postponing it by $\Delta P \approx 0.3$. The hastening regime (large

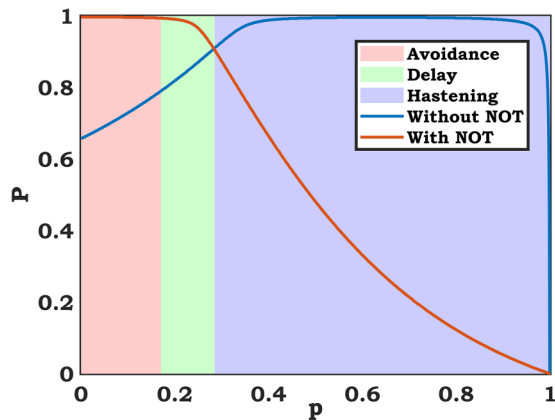


FIG. 2. ESD lines for $x = 0$ (CADC-like) with $\alpha = 0.55$. The three regions retain the same physical interpretation but the “without NOT” ESD line has a positive slope, opposite to the ADC case.

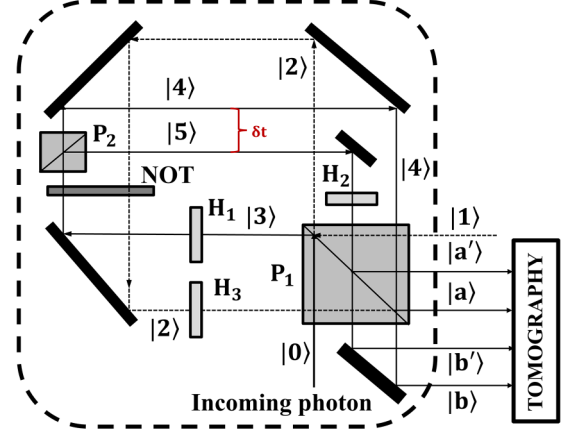


FIG. 3. Detailed schematic diagram of each interferometric setup. The time delay between the two paths of the second polarizing beam-splitter P_2 is demarcated by δt in red. The delay δt between the two paths of the second polarizing beam splitter controls the suppression of the cross components $|HV\rangle$ and $|VH\rangle$ within the coincidence window.

$p \gtrsim 0.28$) exhibits counterintuitive behavior where the NOT accelerates or induces ESD because the state is already too degraded. Detailed derivations are done in Appendix B.

The experimental challenge is implementing these different damping channels. Previous proposals [22] required two separate interferometers with nanometer path matching—experimentally infeasible. Our solution exploits intrinsic path delays. In a displaced Sagnac interferometer, photon paths have different lengths, creating a time delay δt , as illustrated in Fig. 3. If δt exceeds the coincidence window Δt , pairs with one photon in each path are suppressed. Crucially, the $|HV\rangle$ and $|VH\rangle$ components populate such path combinations, while $|HH\rangle$ and $|VV\rangle$ do not. This naturally suppresses cross terms, mimicking CADC.

We model the input state as a time-ensemble:

$$|\psi_{in}\rangle = \sum_t [a(t)\bar{a}(t)|H_0H'_0\rangle + b(t)\bar{b}(t)|V_0V'_0\rangle], \quad (9)$$

where $a(t)$, $\bar{a}(t)$ and $b(t)$, $\bar{b}(t)$ are the temporal amplitude envelopes for the $|H\rangle$ and $|V\rangle$ polarized photons along paths $|0\rangle$, $|0'\rangle$, respectively, and define a time-shift operator through

$$x(\delta t) = \left| \frac{\int d\omega a(\omega)\bar{a}(-\omega)e^{-i\omega\delta t}}{\int d\omega a(\omega)\bar{a}(-\omega)} \right|, \quad (10)$$

See Appendix C for details. The parameter $x \in [0, 1]$ measures the effective temporal overlap of delayed photon wave packets within the coincidence window: $x = 1$ corresponds to the ADC-like limit in which the cross terms survive, whereas $x = 0$ corresponds to the CADC-like limit in which the cross terms are suppressed.

Importantly, the maps generated in this way are effective conditional maps associated with the detected coincidence ensemble. At $x = 1$ the map reduces to ADC-like which is fully completely positive and trace-preserving (CPTP). However, for $x < 1$ the cross components $|HV\rangle$ and $|VH\rangle$ are progressively suppressed by the coincidence window, and the

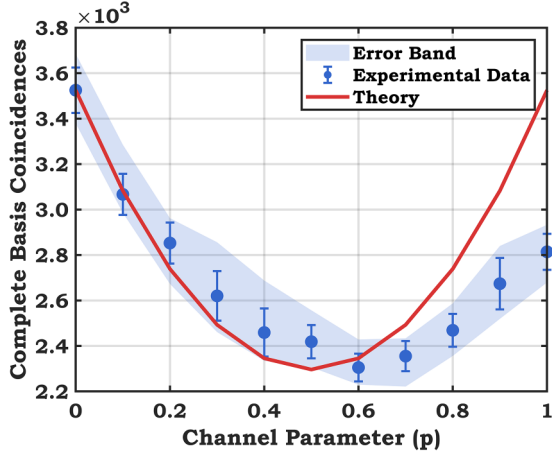


FIG. 4. Experimental coincidence counts (blue markers) compared against theoretical counts (red line) $\eta_{\text{post}} C_{\text{ref}}$ as a function of the channel parameter p . Here C_{ref} is the initial coincidence count at $p = 0$, and η_{post} is the theoretically admitted-event fraction given by the normalization factor in Eq. (11).

output state is no longer normalized. The normalization factor

$$\eta_{\text{post}}(p, x; \alpha, \beta) = N(p, x; \alpha, \beta) = 1 - 2p(1-p)(1-x^2)|\beta|^2, \quad (11)$$

tracks precisely how much probability is carried by the suppressed cross components at each value of x and as a function of the damping parameter p . The $x = 0$ limit, realized in the present experiment, corresponds to complete suppression of the cross components and is directly verified in Fig. 9.

For the main experimental state, $\alpha = 0.55$, and the CADC-like regime $x \approx 0$, η_{post} gives a minimum admitted-branch weight of approximately 0.65 at $p = 0.5$. This shows that the detected branch is not a vanishing-probability component of the dynamics. We compare the theoretically expected coincidence counts using the admitted event fraction η_{post} multiplied with the initial counts C_{ref} ($p = 0$), with the experimentally observed coincidence counts in Fig. 4. The difference in the experimental tail from theoretical expectations is attributed to variable coupling efficiencies at different detectors. As p transitions from $p < 0.5$ to $p > 0.5$, more photons are directed toward the $L_2 R_2$ couplers, which has lower coupling efficiency in comparison to $L_1 R_1$, causing a drop in coincidence counts (see Fig. 6).

With an intermediate local NOT, the output conditional state takes the form

$$\rho_{\text{out}} = \frac{1}{N} \begin{pmatrix} A & 0 & 0 & \mathcal{X} \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ \mathcal{X}^* & 0 & 0 & D \end{pmatrix}, \quad (12)$$

where

$$\begin{aligned} A &= (1-p)^2 \beta^2 + P[P - (1-p)\{P - p(2x^2 - P)\} \beta^2], \\ B &= C = (1-P)[P - (1-p)\{P - p(x^2 - P)\} |\beta|^2], \\ D &= (1-P)^2 (|\alpha|^2 + |\beta|^2 p^2), \quad \mathcal{X} = \alpha^* \beta (1-p)(1-P), \\ N &= 1 - 2p(1-p)(1-x^2) |\beta|^2. \end{aligned} \quad (13)$$

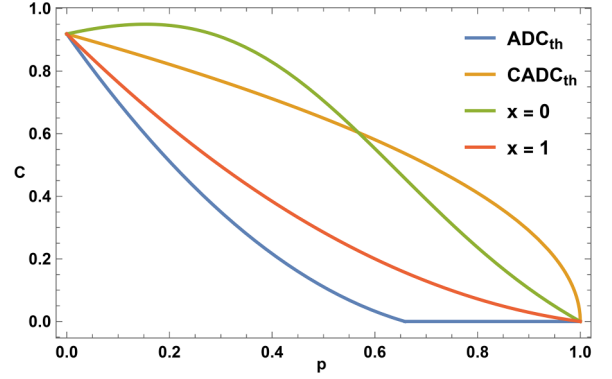


FIG. 5. Comparison of the effective conditional-map framework with the conventional ADC (blue) and CADC (orange) for $\alpha = 0.55$. The time-dependent formalism continuously interpolates between the ADC-like ($x = 1$) (red) and CADC-like ($x = 0$) (green) limits, which are different from the conventional damping channels.

The associated concurrence is

$$C(p, P) = \frac{2}{N} \max[0, (1-p)|\alpha\beta| - \{P - (1-p) \times (P - p(x^2 - P))|\beta|^2\}](1-P). \quad (14)$$

Figure 5 illustrates the behavior of a conventional ADC (blue curve) and CADC (orange curve) channel. In the same figure, we also plot the limiting cases $x = 1$ (red curve) and $x = 0$ (green curve) (see Appendix B for the derivation of the concurrence for the time-dependent case). This demonstrates the difference between ADC-like and CADC-like limits. Equation (14) approaches the apparatus-defined ADC-like at $x = 1$ and CADC-like limit at $x = 0$. Intermediate values of x correspond physically to partial overlap of delayed photon wave packets and modulate the suppression of the cross terms $|HV\rangle$ and $|VH\rangle$. This provides a compact description of how geometry and detection timing reshape dissipative entanglement dynamics.

To verify that NOT is optimal, we test the general $SU(2)$ operator $\vec{\sigma} = n_0 I + n_x \sigma_x + n_y \sigma_y + n_z \sigma_z$. Analysis (Appendix D) shows NOT ($n_x = 1$, others zero) maximizes the concurrence for delaying ESD. Population inversion is the key physical ingredient, not arbitrary unitary scrambling.

III. HAMILTONIAN FORMULATION AND CONNECTION TO DYNAMICAL DECOUPLING

The Hamiltonian perspective reveals why single shot protocol performs better in comparison to traditional approaches such as Dynamical Decoupling [8]. The total unitary $U = \exp(-iH)$ consists of $H = H_{\text{free}} + H_{\text{int}}$, where H_{free} produces no system-environment entanglement and H_{int} creates correlations responsible for decoherence. $H_{\text{free}} = H_A^{\text{free}} \otimes H_B^{\text{free}}$ where A, B are the respective single qubit subsystems such that

$$\begin{aligned} H_{\text{free}}(\alpha|HH\rangle + \beta|VV\rangle) \otimes |00\rangle \\ \rightarrow (\alpha'|HH\rangle + \beta'|VV\rangle) \otimes |kk\rangle. \end{aligned} \quad (15)$$

Thus, H_{int} is solely responsible for system-environment correlations. Dynamical decoupling seeks a parity operator Π satisfying $[\Pi, H_{\text{free}}] = 0$ and $[\Pi, H_{\text{int}}] = 0$. Inserting the par-

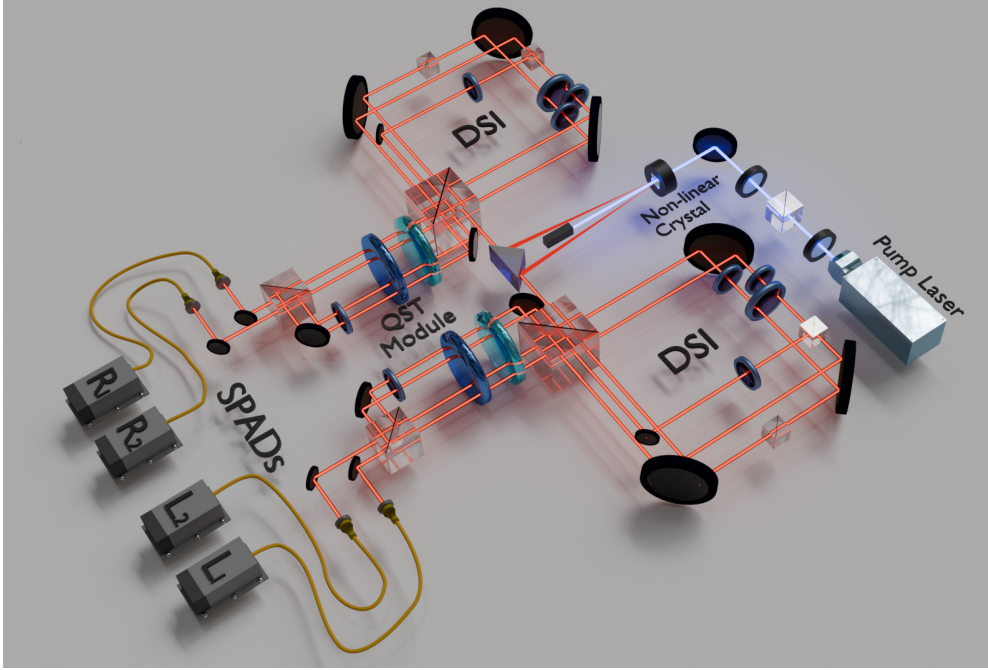


FIG. 6. Experimental schematic overview showing entangled photon source (left), displaced Sagnac interferometer implementing cascaded damping protocol (center), and quantum state tomography module (right). DSI: Displaced Sagnac Interferometer, QST: Quantum State Tomography, SPAD: Single-Photon Avalanche Diode.

ity kicks between the total unitary U results in

$$(\Pi U \Pi)U = U_{\text{free}}^2. \quad (16)$$

Hence, repeated operations of the iterative kicks (n times) cancel the environmental interaction in the following sense:

$$\text{n terms} \dots \underbrace{(\Pi U \Pi)U}_{\text{1st term}} = \exp(-2inH_{\text{free}}) = U_{\text{free}}^{2n}. \quad (17)$$

Note that the parity operator for the system is then given by $\Pi = \Pi_A \otimes \Pi_B$, the tensor product of the operators for each subsystem. To determine Π_A , we write H_A^{free} in its eigenbasis, (see the Supplemental Material [24]) and decompose,

$$H_A^{\text{int}} = \sqrt{p} \sum_{i \neq j} \mu_{ij} |\lambda_i\rangle \langle \lambda_j| + O(p). \quad (18)$$

Therefore, to first order, we can write the interacting Hamiltonian as

$$H_{\text{int}} = \sqrt{p} (H_A^{\text{int}} \otimes H_B^{\text{free}} + H_A^{\text{free}} \otimes H_B^{\text{int}}) + O(p). \quad (19)$$

The parity operator $\Pi_A = \sum_i s_i |\lambda_i\rangle \langle \lambda_i|$ by construction, implies $s_i = \pm 1 \forall i$. Of these $s_i = -1$ for $i = 6, 7, 16, 17$ while the rest are 1. The parity operator is thus

$$\Pi_A = I - 2(|\lambda_6\rangle \langle \lambda_6| + |\lambda_7\rangle \langle \lambda_7| + |\lambda_{16}\rangle \langle \lambda_{16}| + |\lambda_{17}\rangle \langle \lambda_{17}|), \quad (20)$$

where the explicit forms of the relevant eigenvectors are given in Appendix E. In terms of the computational basis, i.e., polarization and path states, we can write Π_A as

$$\begin{aligned} \Pi_A = & |H\rangle \langle H| \otimes (|1\rangle \langle 1| - |3\rangle \langle 3| - |4\rangle \langle 4| - |9\rangle \langle 9|) \\ & + |V\rangle \langle V| \otimes (|0\rangle \langle 0| - |3\rangle \langle 3| - |5\rangle \langle 5| - |8\rangle \langle 8|) \\ & - |H\rangle \langle V| \otimes (|1\rangle \langle 0| + |3\rangle \langle 3| + |4\rangle \langle 5| + |9\rangle \langle 8|) \\ & - |V\rangle \langle H| \otimes (|0\rangle \langle 1| + |3\rangle \langle 3| + |5\rangle \langle 4| + |8\rangle \langle 9|), \end{aligned} \quad (21)$$

where $|H\rangle$ and $|V\rangle$ are the polarization states of the photon and $|i\rangle$ denote the path states. Π_B is of the exact same form as Π_A with appropriate renaming of the path variables for subsystem B. Equation (21) demonstrates the need for custom set of offset pulses on various paths applied in parallel, to cancel the effective system-environment interaction.

Table II compares our approach to other methods operating in weak coupling that require pulse sequences with $\Omega_{\text{pulse}} \gg \gamma_{\text{decay}}$ and precise timing. Our protocol is nonperturbative, requires only one operation with coarse timing tolerance, and avoids error accumulation. The parity operator projects onto system-environment eigenstates aligned with the NOT-induced population swap. A single application redirects the trajectory rather than averaging over many cycles. This clarifies why single-shot works: we perform steering, not averaging. Appendix F illustrates that cascading $n > 1$ stages degrades performance, confirming that single-shot is optimal.

IV. EXPERIMENTAL IMPLEMENTATION

To realize time-delay-induced CADC-like behavior, we designed an architecture around four requirements: cascaded amplitude damping with tunable p and P , intermediate NOT operation without destroying coherence, controllable path delays engineering parameter x , and interferometric stability over meter scales. Previous proposals [22] envisioned two separate Sagnac interferometers cascaded in series—experimentally infeasible due to nanometer path-matching requirements. Our solution recognizes that path delays, traditionally detrimental, can be exploited. By embedding both damping channels within a single displaced Sagnac interferometer (DSI), intrinsic path differences naturally suppress $|HV\rangle$ and $|VH\rangle$ components.

TABLE II. Comparison of existing methods with our protocol.

Feature	DD	QZE	WMR	This work
Coupling	Weak	Weak	N/A	Nonperturbative
Operational	Pulse sequence	Multiple	Pre- or postselection	Single shot
Characteristics		measurements		
Control sequences	$n \gg 1$	$n \gg 1$	Weak interaction + postselection	One-time unitary evolution
Timing precision	$\omega\tau = \pi$	N/A	Coarse	Coarse ($\Delta p \approx 0.05$)
State recovery	Iterative	Iterative and continuum	Probabilistic	Deterministic
Hardware error	Accumulates iteratively	Accumulates iteratively	Limited by single weak interaction	Limited by the single unitary

Figure 6 provides the system overview. The entangled photon source uses Type-I spontaneous parametric down conversion (SPDC) with two sandwiched β -BBO (Beta Barium Borate) crystals pumped by a 405 nm laser (100 mW). Crystals oriented with perpendicular optic axes produce the state $|\Psi\rangle = \cos\alpha_0|HH\rangle - \sin\alpha_0|VV\rangle$, where pump polarization is controlled by a combination of polarization beam splitter (PBS) and a half-wave plate (HWP), tunes $\alpha = \cos\alpha_0$ from 0 to 1. After down-conversion at 810 nm, photons are spectrally filtered (3 nm bandwidth) and spatially filtered (using pinhole apertures) before fiber coupling. The typical single count rates were approximately $4 \times 10^4 \text{ s}^{-1}$ per coupler, with the maximum coincidence count rate between different detector pairs reaching the order of $5 \times 10^3 \text{ s}^{-1}$.

Figure 7 shows source details. The source is aligned to maximize coincidence rates and verify high-quality entanglement ($C > 0.95$ for maximally entangled states) before ESD manipulation experiments.

Figure 8 shows the complete DSI architecture. Each photon enters PBS P_1 , which splits by polarization into clockwise (cw) and counterclockwise (CCW) paths. On the CCW path [3], HWP H_1 at angle θ implements polarization rotation mimicking spontaneous emission: $|VV\rangle \rightarrow \sqrt{p}|HH\rangle + \sqrt{1-p}|VV\rangle$ with $p = \sin^2(2\theta)$. PBS P_2 reflects $|V\rangle$ to path

$|5\rangle$ and transmits $|H\rangle$ to path $|4\rangle$. The cw path bypasses both H_1 and P_2 . HWP H_4 at 45° acts on both paths implementing σ_x : $|H\rangle \leftrightarrow |V\rangle$. HWPs H_2 (on cw) and H_3 (on CCW path $|5\rangle$), both at angle ϕ , implement the second ADC with $P = \sin^2(2\phi)$. Paths recombine at PBS P_1 .

The crucial innovation. PBS P_2 creates spatially separated paths with length difference $\Delta L > 30 \text{ cm}$, corresponding to time delay $\delta t > 1 \text{ ns}$. This is comparable to coherence time $\tau_c > 1 \text{ ns}$ and larger than coincidence window $\Delta t = 1 \text{ ns}$. For $|HH\rangle$ and $|VV\rangle$ components, both photons follow similar paths experiencing negligible relative delay. For cross-components $|HV\rangle$ and $|VH\rangle$, photons follow different paths with relative delay δt . When δt becomes comparable to Δt , these are partially excluded from detected state. This is the physical origin of x from Eq. (10). Our configuration gives $\delta t/\Delta t \geq 1$, yielding $x \approx 0$.

After the DSI, QST requires projective measurements onto 36 basis states. Our QST module uses a quarter-wave plate Q_1 (0° or 45°), half-wave plate H_5 (0° , 22.5° , 45° , or 67.5°), and

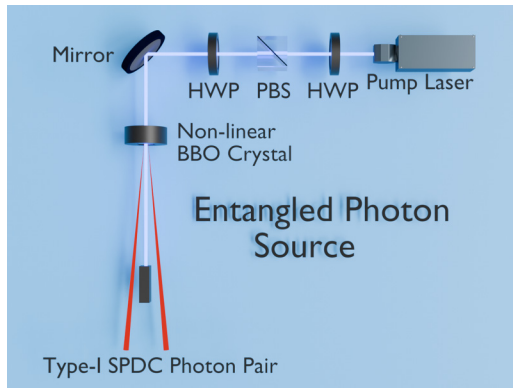


FIG. 7. Type-I SPDC entangled photon source. HWP H_0 controls pump polarization and entanglement parameter α . Two 1 mm β -BBO crystals produce collinear degenerate pairs at 810 nm. DM: dichroic mirrors, BPF: bandpass filters, PH: pinholes, HWP: Half-Wave Plate, PBS: Polarizing Beam Splitter, BBO: Beta Barium Borate.

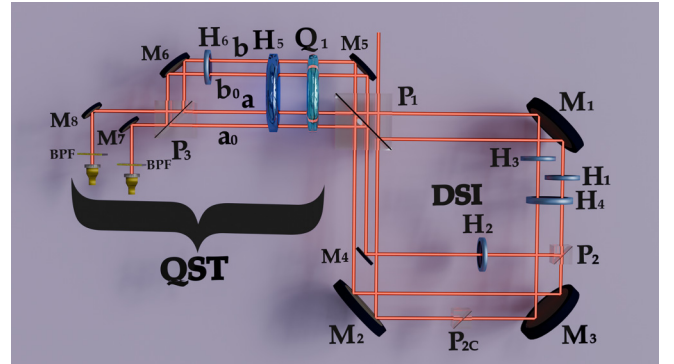


FIG. 8. Detailed displaced Sagnac interferometer and QST schematic. PBS P_1 splits photons into clockwise (cw, path $|2\rangle$) and counterclockwise (CCW, path $|3\rangle$). On CCW path, HWP H_1 at angle θ implements first ADC with $p = \sin^2(2\theta)$, then PBS P_2 reflects $|V\rangle$ to path $|5\rangle$ and transmits $|H\rangle$ to path $|4\rangle$. The CW path bypasses H_1 and P_2 . HWP H_4 at 45° implements NOT on both paths. HWPs H_2 , H_3 at angle ϕ implement second ADC with $P = \sin^2(2\phi)$. Paths $|4\rangle$ and $|5\rangle$ have different spatial lengths creating time delay $\delta t \approx 1 \text{ ns}$ (for $\Delta L \approx 30 \text{ cm}$). Combined with 1 ns coincidence window, this produces $x \approx 0$. The QST module uses Q_1 , H_5 , H_6 , PBS P_3 for 16 projective measurements onto SPADs L1, L2, R1, R2. M_1 – M_8 : mirrors, BPF: bandpass filter.

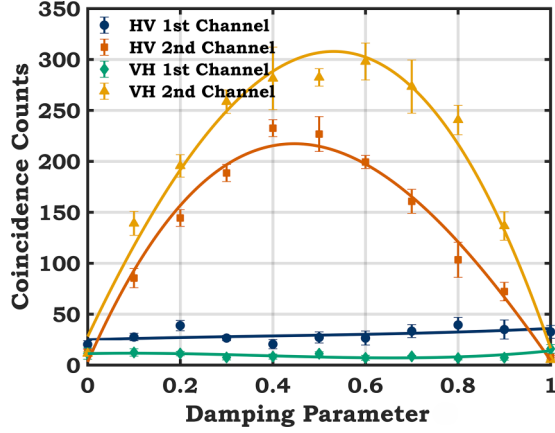


FIG. 9. Experimental verification of CADC-like behavior. Blue circles and green diamonds with respective error bars: measured coincidence counts in $|HV\rangle$ and $|VH\rangle$ bases versus the first ADC parameter. Orange squares and yellow triangles with respective error bars: measured coincidence counts in $|HV\rangle$ and $|VH\rangle$ bases versus the second ADC parameter. Solid curves fitting with measured data showing at least 85% suppression. Strong deviation from ADC confirms CADC-like characteristics.

half-wave plate H_6 (45°) combined with PBS P_3 separating H and V . Detection uses four SPADs with a 1 ns coincidence window. For each projection, we integrate coincidence counts over 10 s, yielding counts that vary with projection and damping strength. The density matrix ρ is reconstructed via maximum-likelihood estimation with positivity constraint. The concurrence of a two-qubit density matrix ρ is estimated using Eq. (2) as $C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$. Fidelities with the maximally entangled state are computed to be $\mathcal{F} > 0.95$ for high-quality states ($p, P = 0$).

To verify CADC-like behavior of the first channel, we measure diagonal elements $\rho_{|HV\rangle\langle HV|}$ and $\rho_{|VH\rangle\langle VH|}$ of the two-photon output polarization state. Standard ADC predicts coincidences varying as $p(1-p)$ and peaking at $p = 0.5$ whereas CADC predicts zero. Our implementation with CADC-like implementation ($x = 0$) should show suppression but not complete elimination in practice.

Figure 9 shows that measured HV and VH counts for the first channel deviate strongly from the normal ADC prediction, following the $x = 0$ characteristics with a minimum of 85% suppression. The observed 15% occurrence of coincidence counts primarily results from accidental coincidences caused by noise in the system. This confirms that our DSI realizes CADC-like physics through path-delay-induced cross-term suppression without requiring engineered global system-bath coupling.

In the following section, for the results pertaining to the ESD manipulation, implementing the without-NOT configuration directly within the displaced Sagnac interferometer is experimentally challenging. Removal of the NOT operation is impractical because it leads to a complete interferometric realignment for the second ADC realization. A detailed description of the strategy adopted for the implementation of with-NOT and without-NOT configurations is provided in Appendix G. Briefly, the post-ADC1/post-NOT state was reproduced at the source by tuning the pump polarization,

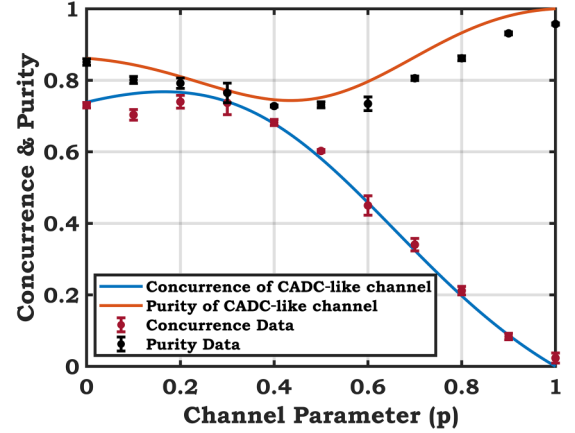


FIG. 10. First channel characterization. Initial state prepared with $p = 0$ (HWP H_1 with its fast axis aligned with the vertical direction). Parameter p varied 0 to 1 while keeping the second channel disabled ($P = 0$) with NOT active. The red and black circles with error bars represent the measured concurrence and purity data, respectively. Both quantities vary systematically with p , consistent with the expected CADC-like behavior as predicted by the blue and red solid lines.

with the reconstructed state matching the measured with-NOT input concurrence to within 4%, ensuring the second-channel evolution faithfully represents the without-NOT dissipative trajectory.

V. RESULTS

We demonstrate complete ESD manipulation across all three regimes, comparing experimental data to theoretical predictions from Eq. (14) using $x = 0$ implementation and prepared initial state with $\alpha = 0.55$ and $\beta = 0.835$. This state exhibits ESD for both ADC and CADC-like configurations without NOT, making it ideal for testing manipulation.

We first characterize each damping channel independently to validate our model and establish baseline ESD behavior.

In the first run, the second channel is disabled, and the damping parameter p of the first channel is varied while the NOT operation remains active, allowing us to examine the evolution of Concurrence and purity under the first damping channel. In the second run, the first channel is disabled, and the parameter P of the second channel is varied, starting from the initially prepared entangled state. Figures 10 and 11 show channel characterizations confirming that our implementation behaves as modeled.

Figure 12 shows the avoidance regime ($p = 0$). Without NOT, ESD occurs at $P = 0.53$, agreeing with theory. With NOT, Concurrence remains positive across the entire range to $P = 1$, demonstrating complete ESD avoidance. At $p = 0$, the NOT swaps $0.55|HH\rangle + 0.835|VV\rangle \rightarrow 0.835|HH\rangle + 0.55|VV\rangle$, placing the larger coefficient on stable $|HH\rangle$. Subsequent damping depletes the smaller $|VV\rangle$ component, which never dominates, and ESD is prevented.

Figure 13 shows the delay regime ($p = 0.22$). Without NOT, ESD at $P = 0.62$; with NOT, postponed to $P = 0.93$. The delay $\Delta P = 0.31$ represents a 50% extension. This is

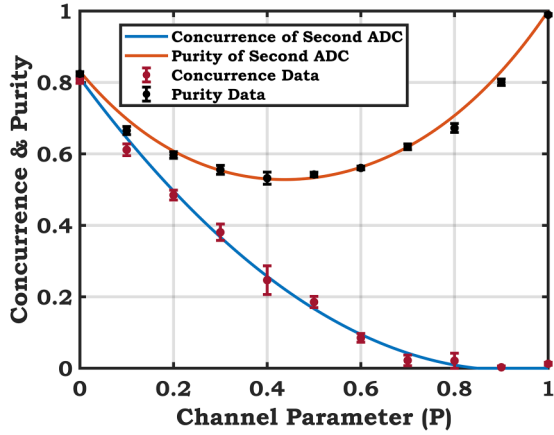


FIG. 11. Second channel characterization. First channel disabled ($p = 0$) gives initial $C \approx 0.82$, purity $> 80\%$. Parameter P varied from 0 to 1. The experimentally measured Concurrency and purity are shown as red and black circles with error bars, respectively, whereas the solid blue and orange curves represent the corresponding theoretical predictions. ESD occurs at $P \approx 0.85$ and the channel behavior agrees with the ADC prediction.

another practically relevant regime, as the delay mechanism also provides a significant extension of the coherence time.

From an operational perspective, the shift in P_{ESD} can be translated into the performance of concrete tasks such as teleportation. Using the standard relation between the concurrence and average teleportation fidelity, our delayed-ESD configuration keeps the fidelity above the classical threshold for a longer interval in P (and hence in t) than the uncontrolled case (Appendix H). While we do not optimize protocol parameters for any specific architecture here, teleportation fidelity illustrates how temporal steering of the dissipative trajectory can be directly linked to the useful lifetime of entanglement as a resource.

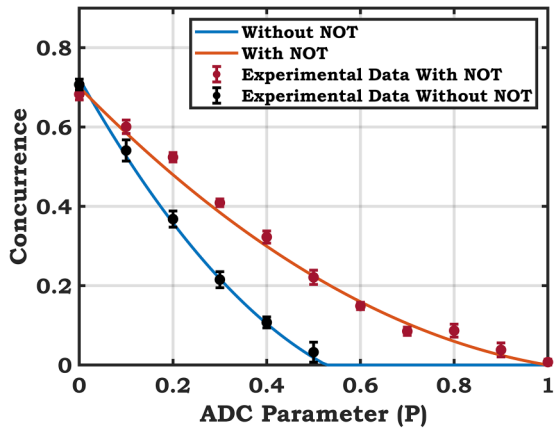


FIG. 12. ESD avoidance for $p = 0$. Black circles: without NOT, ESD at $P = 0.53$ matches with the theory (blue solid line). With NOT, Concurrency remains nonzero until $P = 1$ (red circles with error bars and the red solid line presenting experimental data and theory, respectively). The avoidance window $\Delta P = 0.47$ demonstrates complete ESD prevention within the accessible range of P .

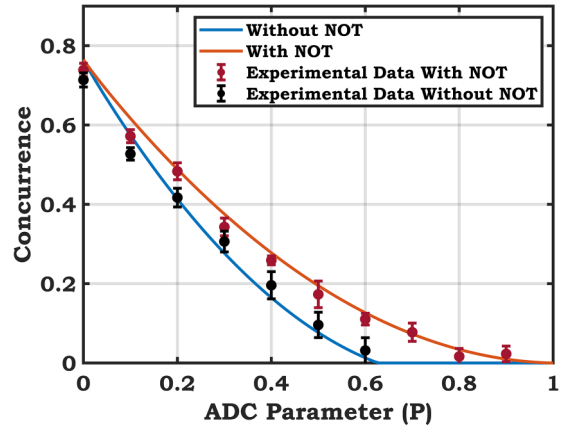


FIG. 13. Delay of ESD. The initial state $|\Psi\rangle$ was created with $\alpha = 0.55$. The first damping channel is kept active with damping strength $p = 0.22$. In the absence of the NOT operation, we observe ESD at a damping parameter $P = 0.62$ (black circles with error bars represent experimental data, and the blue solid line denotes the theoretical prediction). With NOT applied, we observe ESD delayed to the damping parameter $P = 0.93$ (red circles with error bars represent experimental data, and the red solid line corresponds to the theoretical prediction). The observed delay of $\Delta P = 0.31$ represents 50% extension in the damping parameter range.

Figure 14 shows the counterintuitive hastening regime ($p = 0.43$). Without NOT, Concurrency decays asymptotically, becoming zero at $P = 1$. With NOT, ESD is induced at $P \approx 0.6$. After strong first-channel damping, the state is highly mixed (purity < 0.7). The NOT introduces additional asymmetry that pushes the system across the separability threshold rather than away from it. This demonstrates that population inversion is not universally beneficial; rather, its effectiveness is strongly dependent on the timing of its implementation.

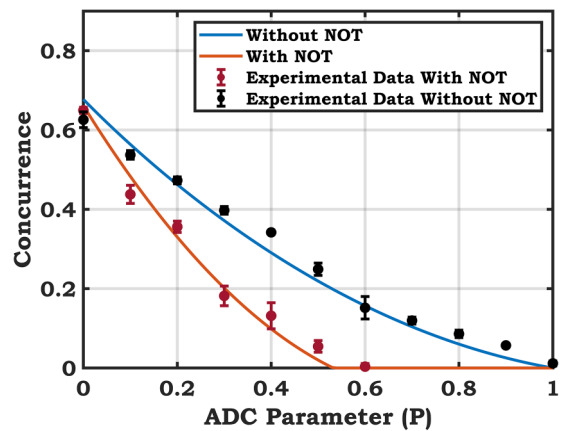


FIG. 14. Hastening of ESD. The initial state $|\Psi\rangle$ was created with $\alpha = 0.55$. The first damping channel parameter is set at 0.43. Without the NOT operation, we observe an avoidance of ESD (black circles with error bars: represent experimental data, blue solid line: prediction from the theoretical model). With NOT applied, we observe ESD occurring at the damping parameter $P = 0.6$ (red circles with error bars: represent experimental data, red solid line: theoretical prediction). Hastening was observed with $\Delta P = 0.4$.

TABLE III. Summary of experimental ESD manipulation results for $\alpha = 0.55$. First damping channel implemented with $x = 0$.

Regime	p	P_{ESD}		ΔP
		without NOT	with NOT	
Avoidance	0	0.53	No ESD ($\rightarrow 1$)	≈ 0.47
Delay	0.22	0.62	0.93	≈ 0.31
Hastening	0.43	No ESD ($\rightarrow 1$)	0.60	-0.40

Table III summarizes all three regimes. Agreement between experiment and theory is excellent (within 0.02–0.04 for Concurrence), strongly validating our theoretical framework and confirming that our DSI implementation realizes the predicted physics.

To demonstrate the robustness of our single-shot protocol, we present additional experimental results in the Supplemental Material [24] for initial-state parameters $\alpha = 0.45$ and $\alpha = 0.5$, complementing the primary experimental run at $\alpha = 0.55$ reported in the main text. These results confirm that our key findings on manipulation of ESD are not specific to a single state parameter but persist across a range of input states. Porting the protocol to different initial conditions demonstrates its versatility and validates the theoretical predictions across varying entanglement regimes, further underscoring the robustness of the experimental implementation.

VI. DISCUSSION

The experimental results confirm that appropriately timed single NOT operations can completely control entanglement dynamics by exploiting quantum decay asymmetry. Amplitude damping has directional asymmetry: $|V\rangle \rightarrow |H\rangle$ occurs but not the reverse. For $|\psi\rangle = \alpha|HH\rangle + \beta|VV\rangle$ with $\alpha < \beta$, the larger β on $|VV\rangle$ preferentially decays, populating mixed components $|HV\rangle$ and $|VH\rangle$ that eventually dominate coherence, causing ESD.

The NOT performs population inversion: $|H\rangle \leftrightarrow |V\rangle$. Applied at intermediate time with parameter p , it swaps to $\beta'|HH\rangle + \alpha'|VV\rangle$ + cross terms, where $\beta' < \beta$. If $\alpha' < \beta'$, the population asymmetry is inverted and ESD is avoided. This is not quantum error correction—we do not restore the initial state. We perform trajectory engineering: using decay asymmetry to steer evolution onto paths avoiding the ESD threshold despite ongoing decoherence.

Timing determines the state's condition when the operation acts. Early application (small p) catches the state before degradation, allowing effective redirection. Late application (large p) finds the state badly damaged; redirection becomes counterproductive, as hastening demonstrates. Comparison to other methods is instructive: for example, DD averages away the interaction Hamiltonian H_{int} through many operations requiring weak coupling and precise timing. Our approach applies one operation that redirects the trajectory, working for strong coupling with coarse timing tolerance but sacrificing generality.

While demonstrated in photonics, the physics applies to any platform with amplitude-damping-dominated decoherence, fast local gates, and the ability to delay gate application.

Superconducting qubits have 20–50 ns gates and 50–150 μs T_1 with dominant energy relaxation. Implementation would engineer one damping stage via tunable resonator couplings and a σ_x operation on each qubit. Trapped ions offer 1–10 μs gates and >1 ms motional lifetimes, with cascaded damping via sequential motional mode coupling. NV centers have 100 ns manipulation and 1–10 ms T_1 , although engineering independent damping channels is challenging. The tunable parameter x is more platform-specific. In photonics, x arises naturally from path geometry. For other platforms, achieving analogous tuning would require engineering temporal correlations in system-bath coupling through resonator quality factors, multimode cavities, or modulated laser coupling.

Our measured concurrences fall below theory by approximately 1%–14%. We identify three dominant error sources: PBS extinction ratio ($\approx 10^{-2}$ – 10^{-3}), HWP angle precision ($\delta\theta$), and path mismatch contributes to ΔC . A detailed first-order quantitative analysis of systematic imperfections affecting the concurrence measurement is presented in Appendix I. We find the PBS extinction to dominate the error budget.

VII. CONCLUSION AND OUTLOOK

We have demonstrated complete, deterministic control of entanglement sudden death using a single local NOT operation whose timing determines the outcome. Three regimes—avoidance, delay, and hastening—are achieved by varying the damping parameter when the NOT operation is applied, without modifying the noise itself. This establishes temporal control—knowing when to act—as a practical and underexplored resource for shaping open-system dynamics, complementing operational control over what gates to apply or how many.

The single-shot paradigm is relevant to platforms where gate sequences accumulate errors. The tunable parameter x suggests routes to engineer custom decoherence channels by controlling temporal structure, extending beyond photonics to circuit QED and trapped ions. Our approach performs trajectory steering rather than averaging, operating nonperturbatively, unlike DD or QZE.

Open questions include extending the time-delay framework to phase-damping or depolarizing channels, dynamically tuning x for error compensation, and understanding multi-qubit population inversion for graph states. This work opens a research direction: temporal engineering of decoherence. As quantum devices scale, knowing when to act may be as important as knowing what to do.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this paper. Requests for further information or data should be sent to the corresponding author.

APPENDIX A: KRAUS OPERATOR DERIVATION

The polarizing beam-splitter $\mathcal{P}_1$ is given by

$$\mathcal{P}_1 = |H_2\rangle\langle H_0| + |V_3\rangle\langle V_0| + \dots, \quad (\text{A1})$$

where we neglect contributions orthogonal to the input path $|00\rangle$. The half-wave plate on path $|3\rangle$ is

$$\mathcal{H}_1(\theta) = H(\theta) \otimes |3\rangle\langle 3| + \mathbb{I} \otimes |2\rangle\langle 2|, \quad (\text{A2})$$

where the HWP matrix is

$$\begin{aligned} H(\theta) &= -\cos 2\theta |H\rangle\langle H| + \cos 2\theta |V\rangle\langle V| \\ &\quad + \sin 2\theta (|H\rangle\langle V| + |V\rangle\langle H|), \\ \mathbb{I} &= |H\rangle\langle H| + |V\rangle\langle V|. \end{aligned} \quad (\text{A3})$$

The NOT operation is

$$U_{\text{NOT}} = \sigma_x \otimes (|3\rangle\langle 3| + |2\rangle\langle 2|), \quad \sigma_x = |H\rangle\langle V| + |V\rangle\langle H|. \quad (\text{A4})$$

The second PBS with time-delay operator X is

$$\mathcal{P}_2(X) = |H_4\rangle\langle H_3| + X|V_5\rangle\langle V_3| + |H_2\rangle\langle H_2| + |V_2\rangle\langle V_2| + \dots, \quad (\text{A5})$$

and the second HWP is

$$\mathcal{H}_2(\phi) = H(\phi) \otimes (|5\rangle\langle 5| + |2\rangle\langle 2|) + \mathbb{I} \otimes |4\rangle\langle 4|. \quad (\text{A6})$$

Finally, the recombination PBS is

$$\begin{aligned} \mathcal{P}_1^\dagger &= |H_a\rangle\langle H_2| + |V_b\rangle\langle V_2| + |H_b\rangle\langle H_4| \\ &\quad + |V_a\rangle\langle V_5| + |H_b\rangle\langle H_5| + |V_a\rangle\langle V_4|. \end{aligned} \quad (\text{A7})$$

The total unitary operator can be written as $U^A = |F\rangle\langle H_0| + |G\rangle\langle V_0| + \dots$, and for two photons:

$$\begin{aligned} U_T &= U^A \otimes U^B = |GG\rangle\langle V_0V_0| + |FF\rangle\langle H_0H_0| \\ &\quad + |FG\rangle\langle H_0V_0| + |GF\rangle\langle V_0H_0| + \dots \end{aligned} \quad (\text{A8})$$

Applying to the input path $|00\rangle$ gives

$$\begin{aligned} \tilde{U} = U|00\rangle &= |FF\rangle\langle HH| + |GG\rangle\langle VV| \\ &\quad + |GF\rangle\langle VH| + |FG\rangle\langle HV|, \end{aligned} \quad (\text{A9})$$

which can be expanded in terms of Kraus operators:

$$\tilde{U} = \sum_{ij} \mathbb{K}_{ij} |ij\rangle, \quad |ij\rangle \in |\text{output paths}\rangle. \quad (\text{A10})$$

The explicit forms of the Kraus operators for with and without NOT cases for cascaded damping channels are given in the Supplemental Material [24]. In the next two sections, we consider cascading damping channels in detail with and without the NOT operation and calculate the expressions of the concurrence in each cases.

APPENDIX B: DETAILS OF PHASE DIAGRAM: ESD LINE

1. Time-independent formalism

In this case, the Kraus operators are that corresponding to traditional ADC and CADC. We discuss the cascades of these damping channels with and without NOT operator below.

2. Cascading ADC

We have two cascaded amplitude-damping channels connected either with or without a NOT operator. The single-photon Kraus operators are 2×2 matrices given by

$$K_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \quad \text{and} \quad K_2 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}. \quad (\text{B1})$$

The Kraus operators for the two-photon state are given by

$$\mathbb{K} = (K_1 \otimes K_1, K_1 \otimes K_2, K_2 \otimes K_1, K_2 \otimes K_2). \quad (\text{B2})$$

For the cascaded ADCs on the initial state in (1), the final output state is obtained by applying (1) in succession first without the NOT operation so that

$$\rho_{\text{out}} = \sum_j \mathbb{K}_j(P) \left(\sum_i \mathbb{K}_i(p) \rho_{\text{in}} \mathbb{K}_i(p)^\dagger \right) \mathbb{K}_j(P)^\dagger, \quad (\text{B3})$$

where p and P are the parameters characterizing the first and second ADC. ρ_{out} is the form of an X state given by

$$\rho_{\text{out}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (\text{B4})$$

where

$$\begin{aligned} \rho_{11} &= \alpha^2 + (p + P - pP)^2 \beta^2, \\ \rho_{14} &= \rho_{41} = (1-p)(1-P)\alpha\beta, \\ \rho_{22} &= \rho_{33} = (1-p)(1-P)(p(1-P) + P)\beta^2, \\ \rho_{44} &= (1-p)^2(1-P)^2\beta^2. \end{aligned} \quad (\text{B5})$$

The concurrence of the output state is [20]

$$\begin{aligned} C(p, P) &= 2 \max[0, \{|\alpha\beta| - |(p(1-P) + P)\beta^2|\} \\ &\quad \times (1-p)(1-P)]. \end{aligned} \quad (\text{B6})$$

Including the NOT operation in between the two damping channels gives rise to a different scenario. In this case, starting from the total unitary operator for the cascade, given by

$$U_{\text{TOT}} = U_{\text{ADC}}(P) U_{\text{NOT}} U_{\text{ADC}}(p), \quad (\text{B7})$$

we get a total of nine Kraus operators. The output state is

$$\rho_{\text{out}} = \sum_{i=1}^9 \mathbb{K}_i \rho_{\text{in}} \mathbb{K}_i^\dagger = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (\text{B8})$$

where

$$\begin{aligned} \rho_{11} &= P^2 + (1-p)(1-P)\{1 + P - p(1-P)\}\beta^2, \\ \rho_{14} &= \rho_{41} = \{1 - p(1-P)\}(1-P)\alpha\beta, \\ \rho_{22} &= \rho_{33} = (1-P)[P - (1-p)\{P - p(1-P)\}\beta^2], \end{aligned}$$

$$\begin{aligned}\rho_{23} &= \rho_{32} = 2\sqrt{p(1-p)P(1-P)}\alpha\beta, \\ \rho_{44} &= (1-P)^2(\alpha^2 + p\beta^2),\end{aligned}\quad (\text{B9})$$

with Concurrence

$$C(p, P) = 2 \max(0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}). \quad (\text{B10})$$

Since the quantity $|\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}$ is negative for the entire range of state and channel parameters, it does not contribute to the concurrence. So,

$$\begin{aligned}C(p, P) &= 2 \max(0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}) \\ &= 2 \max[0, (1-p(1-P))(1-P)|\alpha\beta| \\ &\quad - (1-P)(P - (1-p)\{P - p(1-P)\}\beta^2)].\end{aligned}\quad (\text{B11})$$

3. Cascading CADC with ADC

Here we consider a CADC channel with channel parameter p and with or without local NOT operation on each qubit, and finally an ADC channel with channel parameter q . First, we consider the CADC + ADC channel scenario. The CADC channel acts globally on both photonic polarization states. The Kraus operators are

$$\begin{aligned}K_1 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{1-p} \end{pmatrix} \quad \text{and} \\ K_2 &= \begin{pmatrix} 0 & 0 & 0 & \sqrt{p} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.\end{aligned}\quad (\text{B12})$$

After evolution under the CADC channel, the state evolves under an ADC channel with channel parameter q , and the final output state becomes

$$\rho_{\text{out}} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (\text{B13})$$

where

$$\begin{aligned}\rho_{11} &= \alpha^2 + \{p + (1-p)P^2\}\beta^2, \\ \rho_{14} &= \sqrt{(1-p)}(1-P)\alpha\beta, \\ \rho_{22} &= \rho_{33} = (1-p)(1-P)P\beta^2, \\ \rho_{44} &= (1-p)(1-P)^2\beta^2.\end{aligned}\quad (\text{B14})$$

The concurrence of the output state is

$$C(p, P) = 2 \max[0, \{|\alpha\beta| - \sqrt{1-p}P\beta^2\}]\sqrt{(1-p)}(1-P). \quad (\text{B15})$$

Finally, for a CADC followed by a NOT and then an ADC, we get a total of seven nonzero Kraus operators. The output

state is

$$\rho_{\text{out}} = \sum_{i=1}^7 \mathbb{K}_i \rho_{\text{in}} \mathbb{K}_i^\dagger = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (\text{B16})$$

where

$$\begin{aligned}\rho_{11} &= \alpha^2 P^2 + \{1 - p(1 - P^2)\}\beta^2, \\ \rho_{14} &= \rho_{41} = \sqrt{(1-p)}(1-P)\alpha\beta, \\ \rho_{22} &= \rho_{33} = (1-P)P(\alpha^2 + p\beta^2), \\ \rho_{44} &= (1-P)^2(\alpha^2 + p\beta^2),\end{aligned}\quad (\text{B17})$$

with Concurrence

$$\begin{aligned}C(p, P) &= 2 \max[0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}] \\ &= 2 \max[0, \{\sqrt{1-p}|\alpha\beta| - P(\alpha^2 + p\beta^2)\}(1-P)].\end{aligned}\quad (\text{B18})$$

4. Time-dependent formalism

In this case, the tunable parameter x controls whether the channel is ADC or CADC-like. Hence in this case, we keep the x general and group the observation under the categories of whether a local NOT operation is considered or not in the cascade of our custom damping channels. For both cases, the Kraus operators are derived in Appendix A.

5. Setup with NOT

The output density matrix is given by

$$\rho_{\text{out}} = \sum_{ij} \mathbb{K}_{ij} \rho_{\text{in}} \mathbb{K}_{ij}^\dagger = \frac{1}{N} \begin{pmatrix} A & 0 & 0 & \mathcal{X} \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ \mathcal{X}^* & 0 & 0 & D \end{pmatrix}, \quad (\text{B19})$$

where

$$\begin{aligned}A &= (1-p)^2\beta^2 + P[P - (1-p)\{P - p(2x^2 - P)\}\beta^2], \\ B &= C = (1-P)[P - (1-p)\{P - p(x^2 - P)\}|\beta|^2], \\ D &= (1-P)^2(|\alpha|^2 + |\beta|^2 p^2), \quad \mathcal{X} = \alpha^* \beta (1-p)(1-P), \\ N &= 1 - 2p(1-p)(1-x^2)|\beta|^2.\end{aligned}\quad (\text{B20})$$

The parameters $p = \sin^2 2\theta$ associated with HWPs $\mathcal{H}_1$ and $P = \sin^2 2\phi$ with $\mathcal{H}_2, \mathcal{H}_3$ for the first and second damping channel, respectively, are real and satisfy $0 \leq p, q \leq 1$. The concurrence of the output state is

$$\begin{aligned}C(p, P) &= \frac{2}{N} \max[0, [(1-p)|\alpha\beta| - \{P - (1-p) \\ &\quad \times [P - p(x^2 - P)]|\beta|^2\}](1-P)].\end{aligned}\quad (\text{B21})$$

The corresponding ESD line is given by

$$(1-p)|\alpha\beta| - \{P - (1-p)[P - p(x^2 - P)]|\beta|^2\} = 0. \quad (\text{B22})$$

6. Setup without NOT

In this case, the unitary operator U^A acting on each photonic state is given by

$$U^A = \mathcal{P}_1^\dagger \mathcal{H}_2(\phi) \mathcal{P}_2 \mathcal{H}_1(\theta) \mathcal{P}_1. \quad (\text{B23})$$

The output state becomes

$$\rho_{\text{out}} = \sum_{ij} \mathbb{K}_{ij} \rho_{\text{in}} \mathbb{K}_{ij}^\dagger = \frac{1}{N} \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix}, \quad (\text{B24})$$

where

$$\begin{aligned} \rho_{11} &= P^2 + p[2P(x^2 - P) + p(1 - 2Px^2 + P^2)]|\beta|^2, \\ \rho_{22} &= \rho_{33} = (1 - P)[P + p\{x^2(1 - p) - P(2 - p)\}|\beta|^2], \\ \rho_{44} &= (1 - P)^2[1 - p(2 - p)|\beta|^2], \\ \rho_{14} &= p(1 - P)\alpha^*\beta, \\ N &= 1 - 2p(1 - p)(1 - x^2)|\beta|^2. \end{aligned} \quad (\text{B25})$$

The concurrence of the output state is

$$C(p, P) = \frac{2}{N} \max[0, [p|\alpha\beta| - \{P + p(x^2(1 - p) - P(2 - p))|\beta|^2\}](1 - P)]. \quad (\text{B26})$$

Consequently, the ESD line will be

$$p|\alpha\beta| - \{P + p(x^2(1 - p) - P(2 - p))|\beta|^2\} = 0. \quad (\text{B27})$$

APPENDIX C: THE INTERPRETATION OF X

To understand the action of X on $a(t)$ note that

$$\mathcal{P}_2(X)\mathcal{P}_2(X)^\dagger = \mathbb{I} \quad (\text{C1})$$

implies that $XX^\dagger = 1$. This forces the form of the operator $X = \exp(-i\chi)$. The operator X acting on a function $f(t)$ induces a time delay of δt . Mathematically,

$$Xf(t)X^\dagger = f(t + \delta t), \quad (\text{C2})$$

which enforces the condition,

$$[\chi, f] = i\delta t \partial_t f. \quad (\text{C3})$$

Now we analyze the action of $\mathcal{P}_2(X)$ and/or $\mathcal{P}_2(\bar{X})$ on a product $\sum_t a(t)\bar{a}(t)$. To preserve generality, we take distinct $\chi_1 \equiv i\delta t_1 \partial_t$ and $\chi_2 \equiv i\delta t_2 \partial_t$. The entire action of $\mathcal{P}_2(X) \otimes \mathcal{P}_2(\bar{X})$ boils down to

$$\begin{aligned} X\bar{X} \sum_t a(t)\bar{a}(t) &= \sum_t (Xa(t)X^\dagger) \otimes (\bar{X}\bar{a}(t)\bar{X}^\dagger), \\ X\bar{X} |\text{pol}_1, \text{pol}_2\rangle &= |\text{pol}_1, \text{pol}_2\rangle. \end{aligned} \quad (\text{C4})$$

Next, we take the mode decomposition of $a(t)$,

$$a(t) = \frac{1}{2\pi} \int d\omega a(\omega) e^{-i\omega t}, \quad (\text{C5})$$

and similarly for $\bar{a}(t)$. Using (C2), we can write

$$\begin{aligned} Xa(t)X^\dagger &= \frac{1}{2\pi} \int d\omega a(\omega) X e^{-i\omega t} X^\dagger \\ &= \frac{1}{2\pi} \int d\omega a(\omega) e^{-i\omega(t+\delta t_1)}, \end{aligned} \quad (\text{C6})$$

and similarly for $\bar{a}(t)$. Thus (C4) becomes

$$X\bar{X} \sum_t a(t)\bar{a}(t) = \sum_t \int d\omega_1 d\omega_2 a(\omega_1) \bar{a}(\omega_2) \times e^{-i(\omega_1+\omega_2)t} e^{-i\omega_1\delta t_1 - i\omega_2\delta t_2}.$$

Approximating [25]

$$\sum_t e^{-i(\omega_1+\omega_2)t} \approx 2\pi \delta(\omega_1 + \omega_2), \quad (\text{C7})$$

we can write

$$X\bar{X} \sum_t a(t)\bar{a}(t) = \int d\omega a(\omega) \bar{a}(-\omega) e^{-i\omega\tau} = I(\tau), \quad (\text{C8})$$

where $\tau = \delta t_1 - \delta t_2$ is the relative optical time delay. We assume that $a(t)$ and $\bar{a}(t)$ are Gaussian real amplitudes, so that $f(\omega) = a(\omega)\bar{a}(-\omega)$ is real, positive, and integrable. The phase factor $\exp(-i\omega\tau)$ oscillates with a period of $2\pi/\tau$ in ω and the oscillations increase with increasing τ . Hence the positive and negative contributions progressively cancel. To see this more clearly, we integrate by parts, the integrand with respect to ω ,

$$I(\tau) = -\frac{i}{\tau} \int d\omega f'(\omega) e^{-i\omega\tau}, \quad (\text{C9})$$

where the total integral vanishes outside the domain of $f(\omega)$. Taking the modulus and applying the integral bound $|\int d\omega g(\omega)| \leq \int d\omega |g(\omega)|$,

$$|I(\tau)| \leq \frac{1}{\tau} \int d\omega |f'(\omega)| = \frac{C}{\tau}, \quad (\text{C10})$$

where C is a finite number, independent of τ . Thus $|I(\tau)| \rightarrow 0$ as $\tau \rightarrow \infty$. As an example, for Gaussian profile centered around $\omega = \omega_0$ with width σ , $f(\omega) = \exp[-(\omega - \omega_0)^2/\sigma^2]$ and

$$I(\tau) \sim \exp(-\tau^2\sigma^2/4) \exp(-i\omega_0\tau). \quad (\text{C11})$$

We can thus define

$$x(\tau) = \left| \frac{I(\tau)}{I(0)} \right|, \quad (\text{C12})$$

so that $x(\tau)$ is real, positive, and bounded between $[0, 1]$ where $x(0) = 1$ and $x(\tau \rightarrow \infty) \rightarrow 0$. We can write the general form of the action as

$$X\bar{X} \left(\sum_t a(t)\bar{a}(t) \right) = x(\tau) \sum_t a(t)\bar{a}(t), \quad x(\tau) \in [0, 1]. \quad (\text{C13})$$

Note however, that in a practical setup, the coincidence window (Δt) behaves as a hard-cutoff such that $x(\tau > \Delta t) = 0$. The ability to distinguish $\tau \leq \Delta t$ or $\tau > \Delta t$ depends on the spectral profile of the photon wave packets. $x(\tau)$ quantifies the fraction of the joint spectral distribution of the two photons, separated by τ , that is captured within the coincidence window Δt .

Gaussian profiles for photons give soft cutoffs i.e., there is no clear boundary where $x(\tau) = 0$. To get the hard cutoff τ_c ,

such that $x(\tau > \tau_c) = 0$, we start with $a(t) = \bar{a}(t) = \Theta(T - |t|)$ which gives

$$f(\omega) = 4T^2 \text{sinc}^2(\omega T), \text{ where } \text{sinc}(x) = \frac{\sin x}{x}. \quad (\text{C14})$$

Correspondingly,

$$I(\tau) = 2\pi \max(0, 2T - \tau), \text{ with } I(0) = 4\pi T. \quad (\text{C15})$$

Consequently,

$$x(\tau) = \begin{cases} 1 - \tau/(2T), & \tau \leq 2T \\ 0, & \tau > 2T. \end{cases} \quad (\text{C16})$$

$2T = \Delta t$ is the natural cutoff, i.e., the coincidence window connected with the detector. The particular functional form of $x(\tau)$ used here corresponds to the chosen input wave-packet profile and is used as a representative model satisfying $x(0) = 1$, $x(1) = 0$, and monotonic decay with increasing delay. The broader framework does not depend on this specific profile: different spectral envelopes would modify the detailed form of

$x(\tau)$ while preserving its role as an effective temporal-overlap parameter.

APPENDIX D: SU(2) GENERALIZATION ANALYSIS

To determine whether the NOT operation is uniquely optimal for ESD manipulation, we analyze the most general local SU(2) operation. The general single-qubit SU(2) operator can be parametrized as

$$U_{\text{SU}(2)} = n_0 I + in_x \sigma_x + in_y \sigma_y + in_z \sigma_z, \quad (\text{D1})$$

with normalization, $|n_0|^2 + |n_x|^2 + |n_y|^2 + |n_z|^2 = 1$, ensuring unitarity.

We reparameterize using $n_{\pm} = n_x \pm in_y$, $\bar{n}_0 = n_0 + in_z$, and $\bar{n}_3 = n_0 - in_z$. Applying this general operation between the two damping channels and computing the output concurrence yields

$$C = 2 \max(0, |D| - |B|), \quad (\text{D2})$$

where

$$D = \bar{n}_3(1 - P)^2[\alpha^2 + p^2\beta^2] + n_+n_-(1 - P)^2p(1 - p)\beta^2, \quad (\text{D3})$$

and

$$\begin{aligned} B = & \frac{1}{4} \{ \beta^2 P(1 - P) [\bar{n}_0(\bar{n}_3 + 2in_+\sqrt{p(1 - p)}) + n_-(n_+ - 2i\bar{n}_3\sqrt{p(1 - p)}) + (\bar{n}_0\bar{n}_3 - n_+n_+)(1 - 2p)]^2 \\ & - 4\alpha^2 [-n_+(n_-\sqrt{P(1 - P)} + i(2P - 1)\bar{n}_0) + \sqrt{P(1 - P)}(\bar{n}_0^2 - \bar{n}_3\bar{n}_0 + 1) + n_+^2\sqrt{P(1 - P)}] \\ & \times [-in_-(2P - 1)\bar{n}_3 + \sqrt{P(1 - P)}(-\bar{n}_3^2 + \bar{n}_0\bar{n}_3 - 1) - n_-^2\sqrt{P(1 - P)} + n_+n_-\sqrt{P(1 - P)}] \\ & + \beta^2(1 - P)x[i(n_+\bar{n}_0 - n_-\bar{n}_3)(1 - 2p) - in_+\bar{n}_0 - in_-\bar{n}_3 - 2\sqrt{p(1 - p)} + 4n_+n_-\sqrt{p(1 - p)}] \\ & \times [i(n_+\bar{n}_0 - n_-\bar{n}_3)(1 - 2p) + in_+\bar{n}_0 + in_-\bar{n}_3 - 2\sqrt{p(1 - p)} + 4n_+n_-\sqrt{p(1 - p)}] \}. \end{aligned} \quad (\text{D4})$$

Analyzing these expressions reveals the following:

Case 1. NOT operation ($n_x = 1$, others = 0): This gives $n_+ = 1$, $n_- = 1$, $\bar{n}_3 = 0$, maximizing $|D| - |B|$ for all parameter values (p, P, α). Note, however, there is no difference in Concurrence behavior for $\sigma_x \leftrightarrow \sigma_y$. Our expressions for Concurrence for $n_x = 1$ and $n_y = 1$ are identical reflecting this behavior. The reason is attributed to the indifference of amplitude damping channels to relative phases.

Case 2. Identity ($n_0 = 1$, others = 0): This gives $n_{\pm} = 0$, yielding the baseline “without-NOT” case. For this case as well, $n_0 = 1$ or $n_3 = 1$ gives expressions identical to the without-NOT case.

Case 3. Hadamard-type operations: For operations like $(\sigma_x + \sigma_z)/\sqrt{2}$, the mixed coefficients lead to smaller values of $|D| - |B|$ and can even make the concurrence negative (forced to zero by the max operation), effectively destroying entanglement. Hadamard-type operations:

The mathematical analysis confirms that among all local SU(2) operations, the NOT operation uniquely maximizes the concurrence for delaying or avoiding ESD. The physical reason is clear: NOT performs population inversion, directly addressing the decay asymmetry ($|V\rangle \rightarrow |H\rangle$ but not reverse) that drives ESD. Other unitaries create superpositions, rotate phases, or distribute populations in ways that do not exploit this asymmetry and often worsen entanglement dynamics.

APPENDIX E: HAMILTONIAN EIGENVECTORS

The parity operator given in Eq. (20) involves specific eigenvectors $|\lambda_6\rangle$, $|\lambda_7\rangle$, $|\lambda_{16}\rangle$, and $|\lambda_{17}\rangle$ among the set of 24 eigenvectors for H_{free} denoted by $|\lambda_i\rangle$ for $i = 1, 2, \dots, 24$. These eigenvectors are a linear combination of the tensor product of polarization and path states, viz. $|\lambda_i\rangle = |H\rangle \otimes \sum a_m^i |m\rangle + |V\rangle \otimes \sum b_n^i |n\rangle$, where $|m\rangle$, $|n\rangle$ are the path states in the computational basis. These correspond to system-environment states where the photon polarization (system) is entangled with the spatial path (environment), creating correlations that drive decoherence. The generic form of the $|\lambda_i\rangle$ where $i = 6, 7, 16, 17$ is given by

$$|\lambda_i\rangle = |H\rangle \otimes |P_i\rangle + |V\rangle \otimes |Q_i\rangle, \quad (\text{E1})$$

where the path-states $|P_i\rangle$ and $|Q_i\rangle$ are given by

$$\begin{aligned} |P_i\rangle &= a_1|1\rangle + p\frac{i}{2\sqrt{2}}|3\rangle - a_1^*|4\rangle + \frac{1}{2\sqrt{2}}|9\rangle, \\ |Q_i\rangle &= a_1|0\rangle + p\frac{i}{2\sqrt{2}}|3\rangle - a_1^*|5\rangle + \frac{1}{2\sqrt{2}}|8\rangle, \end{aligned} \quad (\text{E2})$$

where the corresponding coefficients (a_1, p) for the relevant eigenvectors are given in Table IV. The parity operator Π in Eq. (20) projects onto the subspace orthogonal to these four states. Physically, this means Π removes the

TABLE IV. Values of the parameters (a_1, p) for eigenvectors $|\lambda_6\rangle$, $|\lambda_7\rangle$, $|\lambda_{16}\rangle$, and $|\lambda_{17}\rangle$ from the full set $\{|\lambda_i\rangle\}$ of eigenvectors for $i = 1, \dots, 24$ for H_{free} , contributing to the parity operator in (20).

Eigenvector	Parameter (a_1, p)
λ_6	$((1 - i)/4, +)$
λ_7	$((1 + i)/4, -)$
λ_{16}	$(-(1 + i)/4, -)$
λ_{17}	$(-(1 - i)/4, +)$

system-environment correlations that cause decoherence. A single application of Π (via the NOT operation) redirects the state onto eigenvectors where the system and environment remain factorizable, preventing further entanglement loss between the system and the bath.

This explains the connection to dynamical decoupling: DD repeatedly applies Π to average away H_{int} , while our protocol applies it once at an optimal time to steer onto a favorable eigenspace trajectory. The full set of eigenvectors for H_{free} is given in the Supplemental Material [24].

APPENDIX F: CASCADED (CADC-NOT)ⁿ CHANNELS

We examined repeated cascades of CADC-like blocks interspersed with NOT operations. For an n -cascade, the output state is

$$\rho_{\text{out}} = \sum_{i,j,k,\dots} (\dots (K_k(K_j(K_i\rho_{\text{in}}K_i^\dagger)K_j^\dagger)K_k^\dagger) \dots), \quad (\text{F1})$$

where K_i are the Kraus operators for individual blocks and $\rho_{\text{in}} = |\psi\rangle\langle\psi|$ with $|\psi\rangle = \alpha|HH\rangle + \beta|VV\rangle$. The maximum value for the concurrence $C_1(\alpha)$ for $n = 1$ and $0 < p_1 < 1$, $0 < \alpha \leq 1/\sqrt{2}$ is

$$C_1^{\text{max}}(\alpha) = \frac{\alpha}{\sqrt{2} - \sqrt{1 - \alpha^2}} \quad \text{for} \quad p_1^* = 1 - \frac{1}{\sqrt{2(1 - \alpha^2)}}. \quad (\text{F2})$$

For $\alpha = 0.55$, $C_1^{\text{max}} = 0.949$ which occurs at $p_1^* = 0.153$. For $n = 2$ and higher identical cascades, the attainable concurrence remains below $C_1^{\text{max}}(\alpha)$, as shown in Table V. Although the values are not strictly monotonic for the fixed choice $p_i = p_1^*$, none of the cascaded protocols exceeds the single-shot value. This behavior is illustrated in Fig. 15. Note that we have kept the channel strength identical for all the cascades $p_i = p_1$ to compare the effects as we apply the cascades in sequence.

Lemma. For any fixed initial state parameter $\alpha \in (0, 1/\sqrt{2})$, the concurrence achievable by an n -fold cascade of identical [(CADC-like)-NOT] blocks satisfy $C_n(\alpha) \leq C_1^{\text{max}}(\alpha)$ for all $n \geq 1$ with equality for $n = 1$. In particular, within the family of cascaded [(CADC-like)-NOT]ⁿ protocols

TABLE V. Concurrence for $n > 1$ cascade for $\alpha = 0.55$ and $p_i = p_1^*$ in comparison to $C_1^{\text{max}}(\alpha)$.

n	1	2	3	4	5	6	7
Concurrence	0.949	0.895	0.923	0.871	0.896	0.848	0.8711

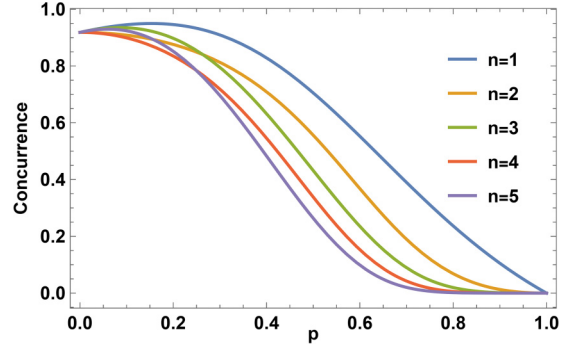


FIG. 15. We show the effect of cascading identical (CADC-like)-NOT blocks for $n = 1, 2, 3, 4, 5$. The blue curve is for $n = 1$ which has the highest concurrence.

considered here, a single-shot NOT is optimal for maximizing the concurrence and delaying ESD.

The single-shot protocol is therefore not merely economical in implementation; it is optimal within the analyzed family of local strategies.

APPENDIX G: WITH AND WITHOUT NOT IMPLEMENTATION FOR ESD CONTROL

In our experiment, we consider the bipartite entangled state $|\Psi\rangle = \alpha|HH\rangle + \beta|VV\rangle$, with $\alpha = 0.55$, which evolves through a custom ADC of strength p , followed by a local NOT operation and a second ADC of strength P . By tuning the first damping parameter p , we realize the three distinct regimes of ESD control: Avoidance, Delay, and Hastening.

1. Experimental strategy for the without-NOT configuration

Within our displaced Sagnac interferometric architecture, the NOT gate is intrinsically coupled with the second PBS and the wave plates that implement the second ADC. Therefore, direct physical removal of the NOT operation is impractical, as it would necessitate removal of multiple optical components and a complete realignment of the interferometer before switching back to the “with-NOT” configuration. To preserve interferometric stability and avoid systematic misalignment errors, we adopted an emulation strategy for the without-NOT case.

Instead of physically removing the NOT gate, we reproduced at the source the quantum state that would exist immediately after the first ADC and the NOT operation. For each initial state corresponding to $P = 0$ in Figs. 12, 13, and 14, we first extracted the parameters (α, β) from the experimentally measured with-NOT input state, which already incorporates the effects of the first damping (p) and the NOT operation.

Using these extracted parameters, we tuned the pump polarization to $(\alpha|H\rangle + \beta|V\rangle)$ via the pump wave plates. This produced, through SPDC at the crystal, the entangled state

$$(\beta|HH\rangle + \alpha|VV\rangle).$$

This prepared state faithfully mirrors the post-ADC1, post-NOT condition. The concurrence of this reconstructed state matched that of the measured with-NOT input state within

TABLE VI. Comparison of measured Concurrence between the input states for with-NOT and without-NOT configurations across the three ESD control regimes.

Regime	Configuration	Mean concurrence	Std. Dev.
Avoidance	With-NOT	0.68	0.01
	Without-NOT	0.71	0.01
Delay	With-NOT	0.74	0.02
	Without-NOT	0.71	0.02
Hastening	With-NOT	0.65	0.01
	Without-NOT	0.64	0.02

a 4% deviation, confirming the accuracy of the emulation procedure. The fidelities were computed by applying an additional NOT operation numerically to the measured initial state corresponding to the without-NOT case and comparing it to the measured with-NOT input state.

Once prepared, this state was propagated through the second ADC with all interferometer settings unchanged. The resulting evolution corresponds effectively to the without-NOT configuration. The measured data for this case are represented as black markers with corresponding error bars in the Figs. 12, 13, and 14.

2. Input-state characterization

To validate the correctness of state preparation in both configurations, we measured the concurrence of the input states for the with-NOT and without-NOT cases over five independent experimental runs in each ESD regime. The mean concurrence values and their standard deviations are summarized in Table VI.

The close agreement between the concurrence values for the two configurations confirms that the prepared states accurately represent the intended theoretical conditions. The small standard deviations demonstrate good reproducibility across independent runs. Because the fidelities and the concurrence values are well-matched in both scenarios, we were able to demonstrate complete and controlled ESD manipulation experimentally across all three regimes.

APPENDIX H: TEMPORAL STEERING EXTENDS THE OPERATIONAL LIFETIME OF ENTANGLEMENT

To connect temporal steering to an operational quantum-information task, we evaluate the average teleportation fidelity of the evolving two-qubit state. The relevant benchmark is the classical threshold for the average fidelity $\mathcal{F}_{\text{avg-cl}} = 2/3$. The starting point is the output state,

$$\rho = \mathcal{N}^{-1} \begin{pmatrix} a & 0 & 0 & d \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ d & 0 & 0 & f \end{pmatrix}, \quad \text{with } \mathcal{N} = a + 2b + f. \quad (\text{H1})$$

This form of ρ is an X -state. To compute the maximum attainable average teleportation fidelity for the X state, we refer to

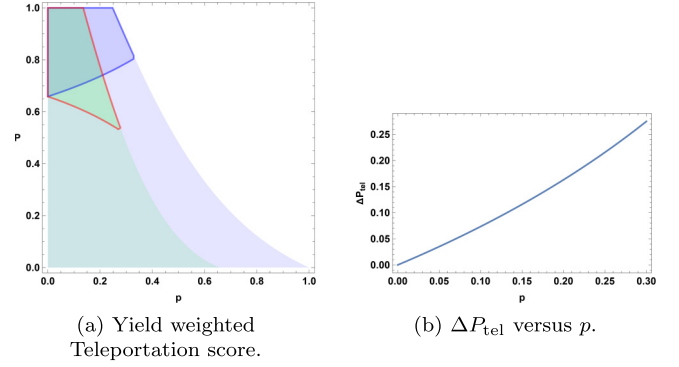


FIG. 16. Operational consequence of temporal steering corresponding to input state $\alpha = 0.55$. Figure 16(a) shows the yield-weighted score, where $\mathcal{U}_{\text{tel}}(p, P) > 0$ for the (ADC-like)-ADC cascade (green region) and (CADC-like)-ADC cascade (blue region). The regions bounded by red and blue boundaries mark the advantage of the local NOT operation for the ADC-like and CADC-like cascades, respectively. Figure 16(b) plots ΔP_{tel} as a function of p at the boundary of the with-NOT configuration for both the cascades. As p increases until the critical value $p_* \approx 0.3$, ΔP_{tel} also increases, showing the advantage of the (CADC-like)-ADC cascade over the (ADC-like)-ADC cascade.

Horodecki's formula [26,27],

$$\mathcal{F}_{\text{avg}} = \frac{1}{2} \left(1 + \frac{1}{3} \text{tr} \sqrt{T^\dagger T} \right), \quad (\text{H2})$$

where $\text{tr}(T^\dagger T)^{1/2}$ is the trace norm of $T = \{t_{mn}\}$ and $t_{mn} = \text{tr}(\rho(\sigma_m \otimes \sigma_n))$ where σ_i are Pauli matrices for $(m, n) = 1, 2, 3$. For the coincidence-conditioned implementation, the appropriate rate-aware quantity $\eta_{\text{post}} \mathcal{F}_{\text{avg}}$ is not a new fidelity but a yield-weighted operational advantage. For the (ADC-like)-ADC case, $\eta_{\text{post}} = 1$, and the conditional teleportation fidelity itself determines the useful region. For the (CADC-like)-ADC case, the coincidence-conditioned map has a nontrivial admitted-branch weight, so we also consider a yield-weighted operational figure of merit. We define

$$\mathcal{U}_{\text{tel}}(p, P) = \eta_{\text{post}}(p) \max \left[0, \mathcal{F}_{\text{avg}}(p, P) - \frac{2}{3} \right], \quad (\text{H3})$$

which weights the teleportation advantage above the classical threshold by the admitted-event fraction of the conditional map. Superscripts indicate whether the local NOT operation is applied. Equation (H3) should not be interpreted as defining a new teleportation fidelity; rather, it measures the rate-weighted usefulness of the entangled resource within the coincidence-detected ensemble. As shown in Fig. 16(a) for input state corresponding to $\alpha = 0.55$, the (CADC-like)-ADC cascade and the (ADC-like)-ADC cascade marked by blue and green regions admit the net yield-weighted score $\mathcal{U}_{\text{tel}}(p, P) > 0$. The regions bounded by the red and blue boundaries respectively denote the advantage of the local NOT operation for the (ADC-like)-ADC and (CADC-like)-ADC cascade. A global measure is the integrated teleportation advantage given by the total area enclosed by the condition $\eta_{\text{post}}(p) \max[0, \mathcal{F}_{\text{avg}}(p, P) - 2/3]$.

For the coincidence-conditioned implementation, the appropriate rate-aware quantity is given by

$$\mathcal{A}_{\text{cond}} = \int_{\mathbf{R}} dP dp [\mathcal{U}_{\text{tel}}^{\text{NOT}}(p, P) - \mathcal{U}_{\text{tel}}(p, P)], \quad (\text{H4})$$

which quantifies the yield-aware area above the classical threshold. The area enclosed by the regions bounded by the red and blue boundaries in Fig. 16(a) is where “with NOT” dominates without-NOT. Comparing

$$\mathcal{A}_{\text{cond}}^{\text{ADC-like}} = 0.0746, \quad \mathcal{A}_{\text{cond}}^{\text{CADC-like}} = 0.0704, \quad (\text{H5})$$

we can see that the (ADC-like)-ADC cascade appears to offer slightly more maneuverability. Note, however, that the global measure is not sufficient to quantify the usefulness of teleportation fidelity. This is because the area for the (CADC-like)-ADC channel is suppressed by $\eta_{\text{post}}(p)$. To further compare the usefulness of the cascades, we quantify the extension of the useful damping range by

$$\Delta P_{\text{tel}} = P_{\text{crit}}^{\text{NOT}} - P_{\text{crit}}^{\text{no NOT}}, \quad (\text{H6})$$

where P_{crit} is defined by the curve $\mathcal{F}_{\text{avg}}(p, P_{\text{crit}}) = 2/3$. We plot ΔP_{tel} as a function of p in Fig. 16(b) to demonstrate that ΔP_{tel} increases with p up to the critical $p = p_*$, $P = P_*$ such that

$$\mathcal{F}_{\text{avg}}^{\text{NOT}}(p_*, P_*) = 2/3 = \mathcal{F}_{\text{avg}}(p_*, P_*). \quad (\text{H7})$$

A positive ΔP_{tel} , therefore, indicates that the NOT-assisted trajectory preserves teleportation-useful entanglement over a larger range of the second damping parameter for the (CADC-like)-ADC cascade over its counterpart.

Note that since the p_* for both cascades are slightly apart, we take the smaller p_* for the (ADC-like)-ADC cascade for meaningful comparison, since after that the advantage of the (ADC-like)-ADC cascade ceases, as is clearly visible from Fig. 16(a). ΔP_{tel} increases with p and hits a ceiling of $\Delta P_{\text{tel}} \approx 0.3$ beyond the p_* for ADC-like cascade. The findings of (H5) and (H6) are not conflicting since we must remember the suppression factor in the global measure. These two findings together imply that the total events where we see teleportation fidelity above the classical threshold may be lower in the case of the CADC-like cascade; however, the subset of such events will sustain the fidelity for a longer duration in comparison to the ADC-like cascade for nonzero damping.

This preliminary analysis and collective illustration from Fig. 16 show that teleportation fidelity can be used as an operational figure of merit for optimizing temporal steering protocols. In particular, it suggests a hardware-agnostic optimization problem: choosing the intervention time and the effective damping structure so as to maximize the teleportation-useful lifetime of entanglement, while accounting for the admitted-event fraction of the conditional map.

APPENDIX I: SYSTEMATIC ERROR ANALYSIS AND FIRST-ORDER CORRECTION

We analyze systematic errors arising from imperfect optical components affecting the concurrence measurement. The total unitary implemented in the experiment can be written as

$$U_T = U^A \otimes U^B, \quad (\text{I1})$$

where

$$U^A = P_1^\dagger \cdot H_2(\phi) \cdot P_2 \cdot U_{\text{NOT}} \cdot H_1(\theta) \cdot P_1. \quad (\text{I2})$$

Here, P_i denote polarizing beam splitters, H_i half-wave plates, and U_{NOT} is the Pauli-X operation implemented using an HWP at 45° .

The experimentally observed reduction in the concurrence arises from three dominant sources: (i) state-preparation errors, (ii) PBS leakage and extinction limitations, and (iii) finite angular precision in wave-plate rotations implementing amplitude damping and NOT operations.

Error in state preparation. We prepare the bipartite entangled state

$$|\psi\rangle = \alpha|HH\rangle + \beta|VV\rangle. \quad (\text{I3})$$

The pump polarization is set using a half-wave plate at angle ϕ , producing

$$|\psi_{\text{pump}}\rangle = \alpha|V\rangle + \beta|H\rangle. \quad (\text{I4})$$

Due to the finite least count $\delta\phi$ of the rotation mount (typically 2°), the parameter α acquires an uncertainty

$$\delta\alpha = 2 \cos(2\phi) \delta\phi. \quad (\text{I5})$$

Thus, the prepared state amplitudes become $(\alpha \pm \delta\alpha)$, introducing a small deviation in the initial concurrence.

Imperfect PBSs. PBS leakage is the dominant source of systematic error. An ideal PBS transmits $|H\rangle$ and reflects $|V\rangle$ perfectly. Real devices exhibit finite extinction ratios, leading to cross-coupling between orthogonal polarizations.

We model the imperfect PBS as

$$P_{\text{real}} = \begin{pmatrix} 1 - \delta & 0 & \delta & 0 \\ 0 & \delta' & 0 & 1 - \delta' \\ \delta_1 & 0 & 1 - \delta_1 & 0 \\ 0 & 1 - \delta'_1 & 0 & \delta'_1 \end{pmatrix}. \quad (\text{I6})$$

For high-quality PBSs under standard operation, $\delta \approx 10^{-3}$. However, in our displaced Sagnac geometry, PBS P_1 is driven from three different input ports, and calibration reveals extinction ratios spanning

$$\delta_i \approx 10^{-3} \text{ to } 10^{-2}.$$

Wave-plate rotation errors in ADC and NOT operations. The amplitude-damping parameter is related to HWP angle θ via

$$p = \sin^2(2\theta). \quad (\text{I7})$$

The angular uncertainty $\delta\theta$ produces

$$\delta p = 2 \sin(4\theta) \delta\theta. \quad (\text{I8})$$

The NOT gate implemented with $\theta = \pi/4$ yields a first-order error

$$\text{Err}_{\text{NOT}} = \left. \frac{\partial}{\partial \theta} \begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix} \right|_{\theta=\pi/4} \delta\theta = -2\sigma_z \delta\theta. \quad (\text{I9})$$

The faulty NOT operator therefore becomes

$$U_{\text{NOT}} = (\sigma_x - 2\mu\sigma_z) \otimes (|1\rangle\langle 1| + |2\rangle\langle 2|), \quad \mu = \delta\theta. \quad (\text{I10})$$

Similar corrections are incorporated for all HWP rotations implementing amplitude damping.

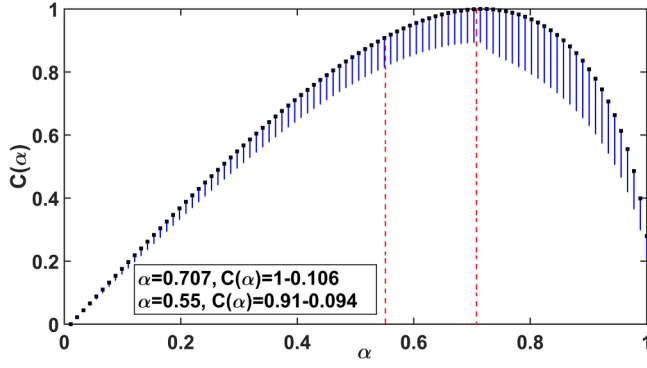


FIG. 17. Systematic errors affecting the concurrence measurement. Two representative points corresponding to $\alpha = 0.55$ and $\alpha = 1/\sqrt{2} \approx 0.707$ —common in experimental runs—are highlighted. The plot shows the maximum measurable concurrence for these values of α , illustrating the impact of systematic errors.

First-order Kraus expansion. Including all imperfections, the system evolves as

$$\rho_{\text{out}} = \sum_i \mathbb{K}_i \rho_{\text{in}} \mathbb{K}_i^\dagger, \quad \sum_i \mathbb{K}_i^\dagger \mathbb{K}_i = \mathbb{I}. \quad (\text{I11})$$

Expanding

$$\mathbb{K}_i = \mathbb{K}_i^0 + \delta \mathbb{K}_i,$$

we obtain

$$\rho_{\text{out}} = \rho_{\text{out}}^0 + \sum_i (\mathbb{K}_i^0 \rho_{\text{in}} \delta \mathbb{K}_i^\dagger + \delta \mathbb{K}_i \rho_{\text{in}} \mathbb{K}_i^{0\dagger}). \quad (\text{I12})$$

Concurrence is computed from the eigenvalues of

$$\tilde{\rho}_{\text{out}} = \rho_{\text{out}}(\sigma_y \otimes \sigma_y) \rho_{\text{out}}^*(\sigma_y \otimes \sigma_y).$$

The first-order correction to eigenvalues λ_i is

$$\Delta \lambda_i = \langle V_i | \tilde{\rho}_{\text{out}} | V_i \rangle.$$

Thus, the first-order correction to the concurrence is

$$\Delta C = \frac{1}{2} \left(\frac{\Delta \lambda_1}{\sqrt{\lambda_1}} - \frac{\Delta \lambda_2}{\sqrt{\lambda_2}} - \frac{\Delta \lambda_3}{\sqrt{\lambda_3}} - \frac{\Delta \lambda_4}{\sqrt{\lambda_4}} \right). \quad (\text{I13})$$

In the error analysis leading to Fig. 17, the least count of the HWP implementing the NOT operation is denoted by μ , while the uncertainties in the damping parameters p and P , arising from the finite angular resolution of the HWPs, are denoted by δp and δP , respectively. These quantities are given by

$$\mu = \frac{\pi}{180}, \quad \delta p = \sqrt{p(1-p)} \frac{\pi}{90}, \quad \delta P = \sqrt{P(1-P)} \frac{\pi}{90},$$

and $\delta_i \approx 10^{-3}$ – 10^{-2} , we obtain

$$\delta_i \approx 10^{-3} : \Delta C = 1.57\% \pm 0.91\%, \quad (\text{I14})$$

$$\delta_i \approx 10^{-2} : \Delta C = 9.8\% \pm 4.5\%. \quad (\text{I15})$$

For $\alpha = 1/\sqrt{2}$, the total concurrence error lies in the range

$$\Delta C \approx 1\% \text{--} 9\%.$$

The experimentally observed 1%–14% deviation from the ideal concurrence is quantitatively explained by the first-order error model. Systematic errors discussed above lead to lower entanglement than theoretically expected. Figure 17 incorporates all such experimentally measured sources of error and highlights representative values of the concurrence for selected values of α used during data acquisition. We conclude, however, that the dominant limiting error arises from PBS imperfections. Further details are given in the Supplemental Material [24].

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