Quantum Transport in Interacting Spin Chains: Exact Derivation of the Tracy-Widom Distribution

Kazuya Fujimoto and Tomohiro Sasamoto
Department of Physics, Institute of Science Tokyo, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan

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We theoretically study quantum spin transport in a one-dimensional folded XXZ model with an alternating domain-wall initial state via the Bethe ansatz technique, exactly demonstrating that a probability distribution of finding a leftmost up spin with an appropriate scaling variable converges to the Tracy-Widom distribution for the Gaussian unitary ensemble (GUE), which is a universal distribution for the largest eigenvalue of GUE under a soft-edge scaling limit. Our finding presented here offers a first exact derivation of the GUE Tracy-Widom distribution in the dynamics of the interacting quantum model not being mapped to noninteracting fermions via the Jordan-Wigner transformation. On the basis of the exact solution of the folded XXZ model and our numerical analysis of the XXZ model, we discuss a universal behavior for the probability of finding the leftmost up spin in the XXZ model.

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Introduction—Transport of a physical quantity is ubiquitous both for classical and quantum systems, having played pivotal roles in deepening our understanding of many-body dynamics over decades [1–4]. One of the notable achievements in classical transport is the establishment of the celebrated Kardar-Parisi-Zhang (KPZ) universality [5–9], which was originally developed in classical statistical mechanics for growing surface physics [10] and transport of stochastic processes [11,12]. When a stochastic system belongs to the KPZ universality, the integrated particle current is universally characterized by the Tracy-Widom distribution of random matrix theory, which is a universal distribution for the largest eigenvalue of random matrices [13–16]. Recently, such universal transport featuring random matrix theory and the KPZ universality is intensively explored in quantum regimes from theoretical [17–28] and experimental [29–31] perspectives, having been recognized as an important research subject in quantum many-body systems.

One of the intriguing exact results for quantum transport featuring random matrix theory is the emergence of the Tracy-Widom distribution in a one-dimensional XX model being equivalent to noninteracting fermions [32,33]. The previous works of Refs. [32,33] consider quantum spin transport starting from a domain-wall state, uncovering that a probability $P(x,t)$ for the farthest up spin at site x and time t obeys the Gaussian unitary ensemble (GUE) Tracy-Widom distribution [13,15,16], which is the universal distribution for the largest eigenvalue in the GUE of random matrix theory. After this finding, several numerical works [34,35] studied the impact of interactions on the quantum dynamics using a one-dimensional XXZ model, which is mapped into interacting fermions, and then reported a signature for absence of the GUE Tracy-Widom behavior. On the other hand, Bulchandani and Karrasch reported tendencies for the GUE Tracy-Widom behavior [36]. On the mathematical side, Saenz *et al.* conducted pioneering and laborious analysis for the probability $P(x,t)$ in the XXZ model via the Bethe ansatz [37,38], proposing an important conjecture concerning a scaling limit for $P(x,t)$ [33]. However, exact derivation of the GUE Tracy-Widom distribution in the XXZ model has yet to be completed. Therefore, determining whether the GUE Tracy-Widom behavior can survive in interacting quantum many-body systems has been elusive.

In this Letter, we present a first example for exactly deriving the GUE Tracy-Widom distribution in an interacting quantum spin model on a one-dimensional lattice, namely a folded XXZ model [39–44], which cannot be mapped into noninteracting fermions via the Jordan-Wigner transformation [45,46]. This theoretical model is originally derived as an effective Hamiltonian for the XXZ model with the large anisotropic interaction. Using the folded XXZ model, we theoretically study quantum spin transport starting from an alternating domain-wall state where the up spin occupies half of the system every other site as depicted in Fig. 1(a). We employ exact analysis based on the Bethe ansatz [37,38], analytically showing that the rescaled probability $P(x,t)$ of finding the leftmost up spin at site x and time t converges to a probability density function for the GUE Tracy-Widom distribution function $F_2(\bullet)$ in the long-time limit. Figure 1(b) schematically illustrates this result. Beyond the folded XXZ model, we numerically investigate the XXZ model by the time-evolving block decimation method (TEBD) [47–50], discussing signatures for a universal behavior of $P(x,t)$ in the XXZ model.

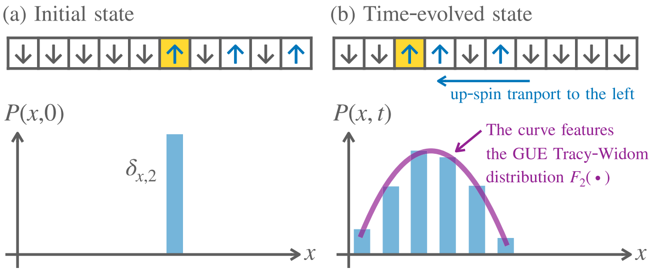

FIG. 1. Schematic illustration for the main result of this Letter. The prime quantity of our interest is a probability $P(x,t)$ of finding a leftmost up spin at site x and time t, which is emphasized by the yellow cell. (a) Initial spin configuration and probability $P(x,0)$. The initial state is an alternating domain-wall state, where the up spins occupy every other site in half of the system ($x \geq 2$). By definition, we have $P(x,0) = \delta_{x,2}$ [see Eqs. (2) and (3)]. (b) Time-evolved spin configuration and probability $P(x,t)$ at time t. After the unitary time evolution with the folded XXZ model, the up spins are transported to the left region ($x < 2$). In this Letter, we demonstrate that the rescaled probability $P(x,t)$ converges to a probability density function for the GUE Tracy-Widom distribution function $F_2(\bullet)$ [13,15,16] in the long-time limit.

Setup—We consider an infinite lattice, sites of which are labeled by $\mathbb{Z}$, and denote spin-1/2 operators at site $w \in \mathbb{Z}$ by $\hat{X}_w$, $\hat{Y}_w$, and $\hat{Z}_w$ in the x, y, and z directions, respectively. These operators satisfy SU(2) commutation relations, e.g., $[\hat{X}_x, \hat{Y}_y] = \mathrm{i}\hat{Z}_x\delta_{x,y}$, where we set the Dirac constant $\hbar$ to be unity. Under this setup, we consider the Hamiltonian of the folded XXZ model [39–44], which is defined by

$$\hat{H}_{\mathrm{fXXZ}} \coloneqq \sum_{x\in\mathbb{Z}} (\hat{X}_x\hat{X}_{x+1} + \hat{Y}_x\hat{Y}_{x+1}) \frac{1 + 4\hat{Z}_{x-1}\hat{Z}_{x+2}}{2}. \quad (1)$$

This is the effective Hamiltonian for the XXZ model with the large anisotropic interaction $\Delta \gg 1$. Here, the Hamiltonian for the XXZ model is given by $\hat{H}_{\mathrm{XXZ}} \coloneqq \sum_{x\in\mathbb{Z}} (\hat{X}_x\hat{X}_{x+1} + \hat{Y}_x\hat{Y}_{x+1} + \Delta\hat{Z}_x\hat{Z}_{x+1})$. We denote the quantum state by $|\phi(t)\rangle$ and assume that it obeys the Schrödinger equation, $\mathrm{i}d|\phi(t)\rangle/dt = \hat{H}_{\mathrm{fXXZ}}|\phi(t)\rangle$. The initial state mainly used in this Letter is the alternating domain-wall state defined by

$$|\phi(0)\rangle = \prod_{x=1}^{N} \hat{R}_{2x}|0\rangle, \quad (2)$$

with the vacuum $|0\rangle$ representing a state that all the spins are down, the raising operator $\hat{R}_x \coloneqq \hat{X}_x + \mathrm{i}\hat{Y}_x$, and the total number N of the up spins. Figure 1(a) displays the schematic illustration for this initial state. Since $\hat{H}_{\mathrm{fXXZ}}$ conserves the total up spin, we can expand the quantum state as $|\phi(t)\rangle = \sum_{\boldsymbol{x}:x_j<x_{j+1}} \Phi(x_1,\ldots,x_N,t)|x_1,\ldots,x_N\rangle$, where $x_j (j\in\{1,\ldots,N\})$ is a lattice site occupied by an up spin and $\Phi(x_1,\ldots,x_N,t)$ is the many-body wave function in the basis $|x_1,\ldots,x_N\rangle \coloneqq \prod_{j=1}^{N} \hat{R}_{x_j}|0\rangle$.

The quantity of our interest is the probability $P(x,t)$ of finding the leftmost up spin at site x and time t, which is defined by

$$P(x,t) \coloneqq \sum_{y_2=1}^{\infty} \cdots \sum_{y_N=1}^{\infty} \left| \Phi\left(x, x+y_2, \ldots, x + \sum_{j=2}^{N} y_j, t\right) \right|^2. \quad (3)$$

The corresponding complementary cumulative distribution function $F(x,t)$ is defined by

$$F(x,t) \coloneqq \sum_{y=x}^{\infty} P(y,t). \quad (4)$$

In what follows, we shall prove that $F(x,t)$ converges to the GUE Tracy-Widom distribution function in the long-time limit.

Determinantal formula of $F(x,t)$ via the Bethe ansatz—We shall derive a determinantal expression for $F(x,t)$ using the Bethe ansatz [37,38] because the folded XXZ model is Bethe-solvable [40–42].

We first derive an integral formula for $\Phi(x_1,\ldots,x_N,t)$ using the Bethe-ansatz method developed by Yudson [51,52], Schütz [53], and Tracy and Widom [54]. As described in Sec. I of Supplemental Material (SM) [55], we derive

$$\Phi(x_1,\ldots,x_N,t) = \int_{C_r} d\boldsymbol{\xi} \sum_{\sigma\in\mathbb{S}_N} A_\sigma(\boldsymbol{\xi}) \prod_{j=1}^{N} \xi_{\sigma_j}^{x_j-2\sigma_j-1} e^{-\mathrm{i}E_{\xi_j}t}, \quad (5)$$

with the set $\mathbb{S}_N$ for Nth permutations and the multiple complex integral $\int_{C_r} d\boldsymbol{\xi} \coloneqq (2\pi\mathrm{i})^{-N} \int_{C_r} d\xi_1 \cdots \int_{C_r} d\xi_N$, and $E_\xi \coloneqq (\xi + \xi^{-1})/2$. The contour C_r is a circle encircling the origin in the complex plane and its radius r is strictly smaller than unity. The coefficient $A_\sigma(\boldsymbol{\xi})$ is defined by $A_\sigma(\boldsymbol{\xi}) \coloneqq \prod_{(\sigma_j,\sigma_k)\in\mathcal{A}_\sigma} S(\xi_{\sigma_j},\xi_{\sigma_k})$, where $\mathcal{A}_\sigma$ is a set for (σ_j,σ_k) such that (σ_j,σ_k) is an inversion in a given element σ of $\mathbb{S}_N$. The scattering amplitude is given by $S(\xi_j,\xi_k) \coloneqq -\xi_j/\xi_k$ [40–42].

We next calculate $F(x,t)$ by using Eq. (5). By definition, we get the expression of $P(x,t)$ as

$$P(x,t) = \sum_{\sigma\in\mathbb{S}_N} \sum_{\mu\in\mathbb{S}_N} \int_{C_r} d\boldsymbol{\xi} \int_{C_r} d\boldsymbol{\eta} \frac{A_\sigma(\boldsymbol{\xi})A_\mu(\boldsymbol{\eta}) \prod_{j=1}^{N-1} \left(\xi_{\sigma_{j+1}}\eta_{\mu_{j+1}}\right)^j}{\prod_{j=1}^{N-1} \left(1 - \prod_{k=j+1}^{N} \xi_{\sigma_k}\eta_{\mu_k}\right)} \prod_{j=1}^{N} \left[(\xi_j\eta_j)^{x-2j-1} e^{-\mathrm{i}t(E_{\xi_j}-E_{\eta_j})}\right]. \quad (6)$$

To derive this expression, we use the fact that $\Phi(x_1,\ldots,x_N,t)$ vanishes if there exists a site label j such that $x_j = x_{j+1}+1$ is satisfied (see the derivation of this property in Sec. II of SM [55]). To compute the summations over σ and μ in Eq. (6), we note the following identity [33,57,58] related to the Izergin-Korepin determinant of the six-vertex model [59–62]:

$$\sum_{\sigma\in\mathbb{S}_N}\sum_{\mu\in\mathbb{S}_N}\frac{B_\sigma(\boldsymbol{\xi})B_\mu(\boldsymbol{\eta})\prod_{j=1}^{N-1}(\xi_{\sigma_{j+1}}\eta_{\mu_{j+1}})^j}{\prod_{j=1}^{N-1}(1-\prod_{k=j+1}^{N}\xi_{\sigma_k}\eta_{\mu_k})} = \frac{(1-\prod_{j=1}^{N}\xi_j\eta_j)\prod_{j,k=1}^{N}(\xi_j+\eta_k-2\Delta\xi_j\eta_k)}{\prod_{j<k}(1+\xi_j\xi_k-2\Delta\xi_j)(1+\eta_j\eta_k-2\Delta\eta_j)}D_N(\boldsymbol{\xi},\boldsymbol{\eta}), \tag{7}$$

where we define $D_N(\boldsymbol{\xi},\boldsymbol{\eta}) \coloneqq \det(d(\xi_j,\eta_k))_{j,k\in\{1,\ldots,N\}}$ with $d(\xi_j,\eta_k) \coloneqq (1-\xi_j\eta_k)^{-1}(\xi_j+\eta_k-2\Delta\xi_j\eta_k)^{-1}$. The function $B_\sigma(\boldsymbol{\xi})$ is defined by $B_\sigma(\boldsymbol{\xi}) \coloneqq \prod_{(j,k)\in\mathcal{A}_\sigma} S_{\mathrm{XXZ}}(\xi_j,\xi_k)$ with the scattering amplitude $S_{\mathrm{XXZ}}(\xi_j,\xi_k) \coloneqq -(1+\xi_j\xi_k-\Delta\xi_j)/(1+\xi_j\xi_k-\Delta\xi_k)$ for the XXZ model [37,38]. We can show $\lim_{\Delta\to\infty} B_\sigma(\boldsymbol{\xi}) = A_\sigma(\boldsymbol{\xi})$. Thus, taking the limit $\Delta\to\infty$ in Eq. (7), we derive

$$\sum_{\sigma\in\mathbb{S}_N}\sum_{\mu\in\mathbb{S}_N}\frac{A_\sigma(\boldsymbol{\xi})A_\mu(\boldsymbol{\eta})\prod_{j=1}^{N-1}(\xi_{\sigma_{j+1}}\eta_{\mu_{j+1}})^j}{\prod_{j=1}^{N-1}(1-\prod_{k=j+1}^{N}\xi_{\sigma_k}\eta_{\mu_k})} = \left(1-\prod_{j=1}^{N}\xi_j\eta_j\right)\det\left(\frac{(\xi_j)^{j-1}(\eta_k)^{k-1}}{1-\xi_j\eta_k}\right)_{j,k\in\{1,\ldots,N\}}. \tag{8}$$

Plugging Eq. (8) into Eq. (6) and taking the summation $\sum_{y=x}^{\infty}P(y,t)$, we get the following determinantal formula of $F(x,t)$:

$$F(x,t) = \det(K(x,t,j,k))_{j,k\in\{1,\ldots,N\}}, \tag{9}$$

where $K(x,t,j,k)$ is defined by $K(x,t,j,k) \coloneqq -\int_{C_r}d\eta\int_{C_r}d\xi\xi^{x-j-2}\eta^{x-k-2}e^{\mathrm{i}t(E_\eta-E_\xi)}/(4\pi(1-\xi\eta))$. This determinantal form of Eq. (9) is compatible with random matrix theory for GUE because many formulas for GUE are given by determinants [16,63].

Derivation of the GUE Tracy-Widom distribution—Using Eq. (9), we shall show that $F(x,t)$ converges to the GUE Tracy-Widom distribution function $F_2(\bullet)$ in the long-time limit. A determinant with a function being similar to $K(x,t,j,k)$ was investigated with techniques of Toeplitz operators [64] in Ref. [33]. Following the same techniques with $N\to\infty$, we obtain

$$F(x,t) = \det(1-K_{\mathrm{B}}(t,m,n))_{l^2(\{2-x,3-x,\ldots\})}, \tag{10}$$

with $K_{\mathrm{B}}(t,m,n) \coloneqq t(J_m(t)J_{n+1}(t)-J_{m+1}(t)J_n(t))/(2m-2n)$. Here, $J_n(t)$ represents the nth order Bessel function of the first kind. We apply the asymptotic analysis with a scaling variable s defined through $x = 2+\lfloor -t-s(t/2)^{1/3}\rfloor$, deriving

$$\lim_{t\to\infty}F(2+\lfloor -t-s(t/2)^{1/3}\rfloor,t) = F_2(s). \tag{11}$$

Here, the GUE Tracy-Widom distribution function is given by $F_2(s) = \mathrm{Det}(1-K_{\mathrm{Ai}}(x,y))_{\mathbb{L}^2(s,\infty)}$ with the Fredholm determinant $\mathrm{Det}[\bullet]$, the Airy kernel $K_{\mathrm{Ai}}(x,y) \coloneqq (\mathrm{Ai}(x)\mathrm{Ai}'(y)-\mathrm{Ai}'(x)\mathrm{Ai}(y))/(x-y)$, and the Airy function $\mathrm{Ai}(x)$. From Eq. (11) and the relation $P(x,t) = F(x,t)-F(x+1,t)$, we obtain for $t\gg 1$,

$$P(2+\lfloor -t-s(t/2)^{1/3}\rfloor,t) \simeq \left(\frac{2}{t}\right)^{1/3}\frac{dF_2(s)}{ds}. \tag{12}$$

Therefore, we analytically demonstrate the emergence of the GUE Tracy-Widom distribution in the folded XXZ model.

Finally, we comment on the dependence of $P(x,t)$ on the initial state. In End Matter, we numerically investigate this dependence by systematically varying the deviation of the initial state from the alternating domain-wall state. As shown in Fig. 3 of End Matter, we find that, in the incomplete alternating domain-wall state, $P(x,t)$ in the central and right-tail regions is well described by the GUE Tracy-Widom distribution whereas the left-tail region exhibits a deviation from it. However, this deviation of the left tail diminishes as the initial state is closer to the alternating domain-wall state.

Relation between exact many-body wave functions of the folded XXZ model and the XX model—We explain that the GUE Tracy-Widom distribution in the folded XXZ model is related to a many-body wave function of the XX model.

Let us consider the XX model, Hamiltonian, which is defined by $\hat{H}_{\mathrm{XX}} \coloneqq \sum_{x\in\mathbb{Z}}(\hat{X}_x\hat{X}_{x+1}+\hat{Y}_x\hat{Y}_{x+1})$ [37,38]. We denote the many-body wave function for this model at time t by $\Phi_{\mathrm{XX}}(x_1,\ldots,x_N,t)$ with a constraint $(x_j < x_{j+1})$ and assume that the initial state is the domain-wall state, $\Phi_{\mathrm{XX}}(x_1,\ldots,x_N,0) = \prod_{j=1}^{N}\delta x_j, j$. As explained in Sec. III of SM [55], the exact form of the many-body wave function reads

$$\Phi_{\mathrm{XX}}(x_1,\ldots,x_N,t) = \int_{C_r}d\boldsymbol{\xi}\det(\xi_k^{x_j-k-1}e^{-\mathrm{i}E_{\xi_j}t})_{j,k\in\{1,\ldots,N\}}. \tag{13}$$

On the other hand, the exact many-body wave function for the folded XXZ mode with the alternating domain-wall initial state is expressed by

$$\Phi(x_1,\ldots,x_N,t) = \int_{C_r}d\boldsymbol{\xi}\det(\xi_k^{x_j-j-k-1}e^{-\mathrm{i}E_{\xi_j}t})_{j,k\in\{1,\ldots,N\}}, \tag{14}$$

which is proved in Sec. II of SM [55]. The origin of this determinantal form in the presence of the interaction stems from the product structure of the scattering amplitude $S(\xi_j, \xi_k) \coloneqq -\xi_j/\xi_k$ in the folded XXZ model.

We find that the many-body wave function of Eq. (14) for the folded XXZ model has the determinantal structure similar to that of Eq. (13) for the XX model. The difference between Eqs. (13) and (14) lies in the power exponents of the rapidity ξ_k. This indicates that the effect of the interaction of the folded XXZ model is the shift of the site labels. As discussed in Ref. [33], the leftmost up spin for the XX model with the domain-wall initial state obeys the GUE Tracy-Widom distribution. Hence, a mathematical origin for the GUE Tracy-Widom distribution in the folded XXZ is similar to that for the XX model.

Numerical study for $P(x,t)$ of the XXZ model—So far, we have analytically studied the folded XXZ model, which serves as an effective description for the XXZ model with the large anisotropic interaction ($\Delta \gg 1$) because the Ising term becomes just a constant for our initial state. It therefore is natural and intriguing to explore the GUE Tracy-Widom distribution in the XXZ model itself from the theoretical viewpoint, and such exploration is important for discussing experimental possibilities of observing our theoretical prediction since the XXZ model has been experimentally realized [30,31,65–67]. In what follows, we present our numerical investigation of this issue.

The model used in the numerical simulation is the XXZ model with an open boundary condition. The Hamiltonian is defined by $\hat{H}^{(\mathrm{open})}_{\mathrm{XXZ}} \coloneqq \sum_{j=1}^{L-1} (\hat{X}_x\hat{X}_{x+1} + \hat{Y}_x\hat{Y}_{x+1} + \Delta\hat{Z}_x\hat{Z}_{x+1})$ with the total lattice number L. Here, we assume L to be a multiple of four. We denote the quantum state at time t by $|\psi(t)\rangle$ and the initial state is assumed to be the alternating domain-wall state $|\psi(0)\rangle = \prod_{x=L/4}^{L/2} \hat{R}_{2x}|0\rangle$. We numerically solve the Schrödinger equation using the TEBD method [47–50], computing the probability $P(x,t)$ of finding the leftmost up spin at site x and time t.

Figure 2 displays the numerical results of $P(x,t)$ with $\Delta = 5$, 10, and 20, where we rescale the abscissas and the ordinates by following Eq. (12) and a result for fast convergence described in Sec. IV of SM [55]. We find that deviations between the numerical data and the probability density function $dF_2(s)/ds$ become large when Δ is small. Thus, we speculate the absence of the GUE Tracy-Widom distribution for the XXZ model in the *long-time limit*.

We, however, find the signature that the probability $P(\lfloor -t - s(t/2)^{1/3} \rfloor, t)$ in the right region ($s > 0$) of Fig. 2 exhibits the universal curve being independent of Δ. This curve is characterized by the diagonal Airy kernel $K_{\mathrm{Ai}}(s,s)$ (see Sec. VI and Fig. S4 of SM [55]).

Discussion—We discuss two topics: (i) the universal behavior of $P(x,t)$ in the XXZ model and (ii) experimental possibilities of observing our theoretical prediction.

As to (i), we discuss the signature that the curve of $P(\lfloor -t - s(t/2)^{1/3} \rfloor, t)$ for large $s > 0$ is independent of Δ, as pointed out in Fig. 2. We here discuss its origin analytically in the two limiting cases, namely $\Delta = 0$ and large Δ. First we consider the case with $\Delta = 0$. As derived exactly in Sec. VI of SM [55], we have $\lim_{t\to\infty} F(2 + \lfloor -t - s(t/2)^{1/3} \rfloor, t) = F_{\mathrm{Ai}}(s, 1/2)$ with $F_{\mathrm{Ai}}(s,a) \coloneqq \mathrm{Det}(1 - aK_{\mathrm{Ai}}(x,y))_{\mathbb{L}^2(s,\infty)}$. Then, the distribution function is approximated to be $F_{\mathrm{Ai}}(s,1/2) \simeq 1 - \int_s^\infty dy K_{\mathrm{Ai}}(y,y)/2$ for large s because $K_{\mathrm{Ai}}(x,y)$ is small. Hence, the rescaled probability of finding the leftmost up spin is described by $dF_{\mathrm{Ai}}(s,1/2)/ds \simeq K_{\mathrm{Ai}}(s,s)/2$ (see

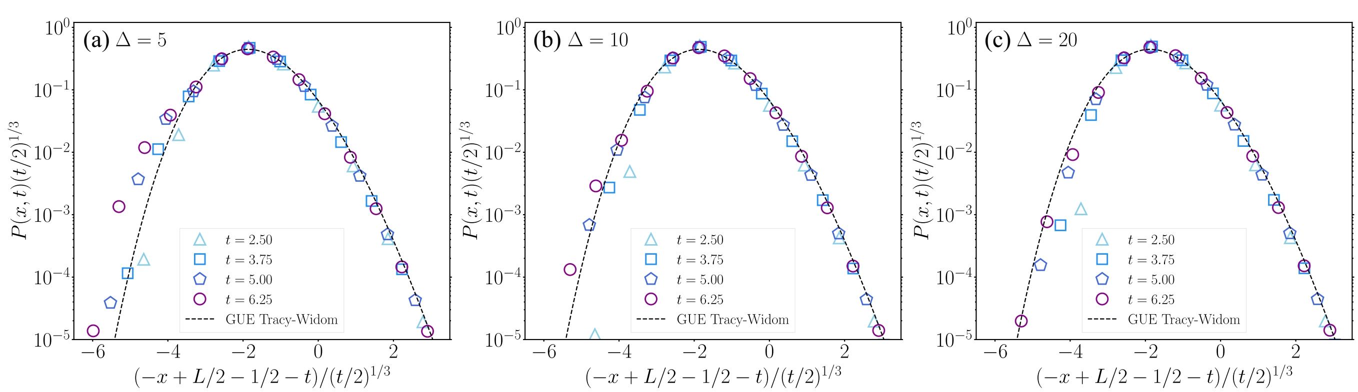

FIG. 2. Numerical results for the probability $P(x,t)$ of finding the leftmost up spin at site x and time t in the XXZ model with the alternating domain-wall initial state. The anisotropic interaction parameter is $\Delta =$ (a) 5, (b) 10, and (c) 20 and the system is $L = 100$. The time evolution of $P(x,t)$ is numerically computed by the TEBD method [47–50]. In order to compare the numerical data with the GUE Tracy-Widom distribution function $F_2(\bullet)$, the ordinate is divided by $(t/2)^{1/3}$ and the abscissa is rescaled by using a result for fast convergence [68], which is described in Sec. IV of SM [55]. The dashed lines in the panels indicate the probability density function $dF_2(s)/ds$ for the GUE Tracy-Widom distribution. The numerical convergences associated with truncation of a matrix product state are discussed in Sec. V of SM [55].

Fig. S4 of SM [55]). Second, we consider the folded XXZ model corresponding to large Δ. Using Eq. (11), we similarly approximate the probability distribution function as $F_2(s) \simeq 1 - \int_s^\infty dy K_{\mathrm{Ai}}(y, y)$ for positively large s, obtaining that the rescaled probability is characterized by $K_{\mathrm{Ai}}(s, s)$. Combining our analytical discussions and the numerical findings of Fig. 2, we find the signature that the probability of finding the leftmost up spin for positively large s is universally characterized by the diagonal Airy kernel $K_{\mathrm{Ai}}(s, s)$ regardless of Δ. Finally, we mention that the conjecture by Saenz, Tracy, and Widom for the XXZ model (see Conjecture 1 of Ref. [33]) may be useful for proving the diagonal Airy kernel in the dynamics of the XXZ model (see Sec. VII of SM [55]).

As to (ii) we discuss experimental possibilities of observing the GUE Tracy-Widom distribution function in quantum spin transport on one-dimensional systems. As shown in Fig. 2, the probability $P(x, t)$ in the XXZ model with $\Delta \gg 1$ shows the signature of the GUE Tracy-Widom behavior in the *finite time regions*, and its timescale is about 5 times spin flipping. On the experimental side, previous literature in cold atom experiments [30,65–67] realized the XXZ model as an effective description for a two-component Bose-Hubbard model under the hardcore limit [69–71], and also an experiment using superconducting qubits [31] simulates the XXZ model by the periodic applications of two-qubit unitary gates. These experiments accessed the spin transport where the spins flip more than 5 times. Taking these experimental achievements into account, we expect that signatures of the GUE Tracy-Widom distribution may be observable in state-of-the-art experiments.

Finally, we comment on how to observe $P(x, t)$ experimentally. From Eq. (3), $P(x, t)$ is computed by the probability $|\Phi(x_1, x_2, \ldots, x_N, t)|^2$ for a given up-spin configuration $(x_1, x_2, \ldots, x_N)$. At present, such a probability can be measured thanks to quantum gas microscopes, which allow detection of an up-spin configuration. Thus, the observation of $P(x, t)$ is feasible in state-of-the-art experiments with ultracold atoms.

Conclusions and future prospects—We theoretically studied the one-dimensional quantum spin transport described by the folded XXZ model with the alternating domain-wall initial state by focusing on the probability $P(x, t)$ of finding the leftmost up spin. Employing the exact method based on the Bethe ansatz, we exactly demonstrated that the rescaled probability converges to the probability density function for the GUE Tracy-Widom distribution function in the long-time limit. Beyond the folded XXZ model, we numerically studied the XXZ model via the TEBD method, discussing the universal behavior in terms of the diagonal Airy kernel.

As a future prospect, it will be intriguing to explore universal features of propagating-edge dynamics, particularly those involving the leftmost up spins, in generic quantum many-body dynamics because edges of physical observables often display universal behavior, as exemplified in random matrix theory [16,72,73] and surface growth phenomena [5–10]. Previous literature, including this Letter, has shown that in certain quantum many-body dynamics the leftmost up spins can be characterized by the GUE Tracy-Widom distribution. However, the extent of this universality in quantum many-body dynamics remains an open question. It is therefore important to investigate this distribution in other quantum many-body models. One possible candidate is the phase model, a strongly interacting bosonic model on a one-dimensional lattice [74–76]. This model is known to be integrable and has a product structure in its scattering amplitude analogous to that of the folded XXZ model. As demonstrated here, this structure is crucial for the exact derivation of the GUE Tracy-Widom distribution via the Bethe ansatz for an infinite system. A fundamentally important model is the XXZ model because its scattering amplitude is not given by a product form and thus the emergence of the Tracy-Widom distribution is highly nontrivial compared with the phase model. Saenz *et al.* have studied the leftmost up spins of the XXZ model via the Bethe ansatz for an infinite system, yet the emergence of the GUE Tracy-Widom distribution has remained elusive [33]. By developing the application of such Bethe ansatz, one can gain a deeper understanding of the GUE Tracy-Widom distribution in generic quantum many-body dynamics, and this potentially provides insights into the KPZ physics in quantum magnets because the Bethe ansatz for an infinite system has played a significant role in the establishment of the KPZ universality in classical systems.

As another prospect, it is fundamentally important to understand the universal behavior of the XXZ model analytically by using the generalized hydrodynamics [34,77–91] and ballistic macroscopic fluctuation theory [92,93].

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Data availability—The data that support the findings of this article are openly available [94]; embargo periods may apply.

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[55] See Supplemental Material at http://link.aps.org/supplemental/10.1103/lbl9-n9ws for (i) proof of Eq. (5) in the main text, (ii) determinantal formula for Eq. (5) in the main text, (iii) proof of Eq. (13) in the main text, (iv) asymptotic analysis to the GUE Tracy-Widom distribution with fast convergence, (v) numerical truncation in the TEBD method, (vi) probability distribution function for $\Delta = 0$, and (vii) conjecture for the XXZ model by Saenz, Tracy, and Widom, which includes Ref. [56].

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End Matter

We shall consider the probability $P(x,t)$ of finding the leftmost up spin at time t and site x in the folded XXZ model with an incomplete alternating domain-wall initial state, numerically investigating how the choice of the initial state affects the GUE Tracy-Widom distribution.

The initial state considered here is defined by the following mixed density matrix with a fixed up-spin number N:

$$\rho_{\mathrm{mix}} \coloneqq \sum_{\boldsymbol{y}\in\Lambda_N} P_{\mathrm{config}}(\boldsymbol{y})|\boldsymbol{y}\rangle\langle\boldsymbol{y}|, \tag{A1}$$

where an up-spin configuration $|\boldsymbol{y}\rangle = |y_1,\ldots,y_M\rangle$ with a positive integer M is generated by reversing several up spins of the alternating domain-wall state $|2,4,\ldots,2N\rangle$ with a reversing probability p for each up spin. Thus the probability $P_{\mathrm{config}}(\boldsymbol{y})$ of having a configuration $|\boldsymbol{y}\rangle$ is determined by p. We denote the probability of having no up spins $|0\rangle$ by $P_{\mathrm{config}}(0)$. The summation of Eq. (A1) is taken over the set $\Lambda_N \coloneqq \{(y_1,y_2,\ldots,y_M) : M\in\mathbb{Z}_{\geq 0}, M\leq N, y_j\in\{2,4,\ldots,2N\}, y_1<\ldots<y_j<y_{j+1}<\ldots<y_M\}$, where we define that the case with $M=0$ corresponds to $\boldsymbol{y}=0$. Let us consider the example with $N=2$. The alternating domain-wall state $|2,4\rangle$ is realized with a probability $P_{\mathrm{config}}(2,4)=(1-p)^2$, and other incomplete alternating domain-wall states $|2\rangle$, $|4\rangle$, and $|0\rangle$ occur with probabilities $P_{\mathrm{config}}(2)=p$, $P_{\mathrm{config}}(4)=p$, and $P_{\mathrm{config}}(0)=p^2$, respectively.

We can derive a formula for numerically calculating the probability $P(x,t)$ using the mixed initial state of Eq. (A1). Suppose that $\hat{\Pi}_x$ is a projector onto states with no up spins at sites smaller than $x-1$. Then, the complementary cumulative distribution function $F(x,t)$ corresponding to $P(x,t)$ becomes

$$F(x,t) = \mathrm{Tr}(e^{-\mathrm{i}\hat{H}_{\mathrm{fXXZ}}t}\rho_{\mathrm{mix}}e^{\mathrm{i}\hat{H}_{\mathrm{fXXZ}}t}\hat{\Pi}_x) \tag{A2}$$

$$= \sum_{\boldsymbol{y}\in\Lambda_N} P_{\mathrm{config}}(\boldsymbol{y})F(x,t;\boldsymbol{y}), \tag{A3}$$

with $F(x,t;\boldsymbol{y}) \coloneqq \langle\boldsymbol{y}|e^{\mathrm{i}\hat{H}_{\mathrm{fXXZ}}t}\hat{\Pi}_x e^{-\mathrm{i}\hat{H}_{\mathrm{fXXZ}}t}|\boldsymbol{y}\rangle$. Following the calculation for the derivation of Eq. (9) and using $1/(1-\xi\eta)=\sum_{m=0}^{\infty}(\xi\eta)^n$ for $|\xi\eta|<1$ and the integral formula for the Bessel function $J_n(x)$ of the first kind, we derive

$$F(x,t;\boldsymbol{y}) = \det\left(\sum_{l=0}^{\infty} J_{x-y_j+j-1+l}(t)J_{x-y_k+k-1+l}(t)\right)_{j,k\in\{1,2,\ldots,M\}}. \tag{A4}$$

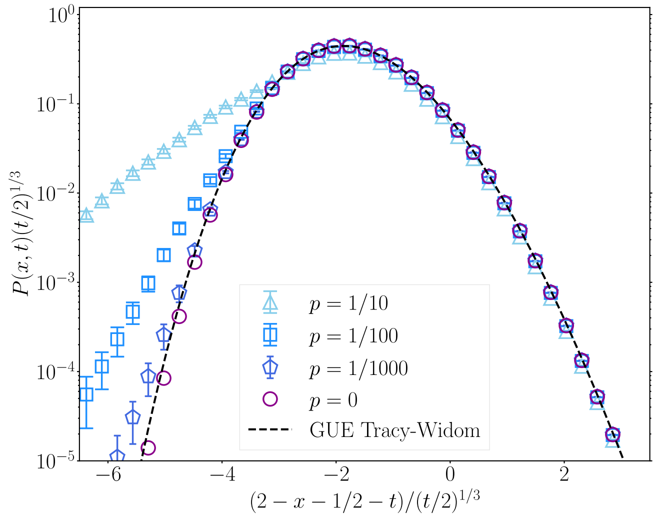

FIG. 3. Dependence of $P(x,t)$ at $t = 100$ on the initial states with the reversing probabilities $p = 0, 1/10, 1/100$, and $1/1000$. The data with $p = 0$ are obtained in the same way as the method used in Fig. S1 in SM [55]. The probability $P(x,t) = F(x,t) - F(x+1,t)$ for nonzero p is numerically obtained via Eqs. (A3) and (A4) by randomly sampling initial states and taking the ensemble average over 4000 samples. The error bars represent $6\sqrt{\sigma(x,t)^2}/\sqrt{4000}$, where $\sigma(x,t)^2$ is the variance calculated from the 4000 samples.

Thus, we can numerically calculate $F(x,t)$ of Eq. (A3) by computing (A4) with randomly chosen configurations $\boldsymbol{y}$ with the reversing probability p and taking the ensemble average over them.

Figure 3 displays the numerical results for $P(x,t)$ with $p = 0, 1/10, 1/100$, and $1/1000$. The numerical data with the smaller reversing probabilities show better agreement with the GUE Tracy-Widom distribution. Interestingly, we find that the right edge of Fig. 3 exhibits good agreement with the GUE Tracy-Widom distribution even when p is not so small. The similar behavior is discussed in the context of the XXZ model.