Erratum: Evolution of single-particle structure near the $N = 20$ island of inversion [Phys. Rev. C 104, L051301 (2021)]

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Since the publication of the original paper, related work has brought to light two errors. One concerns the quoted uncertainty in the absolute scale of the cross sections published in the paper. This does not change the conclusions of the paper because the interpretation in terms of spectroscopic factors (SFs) and centroids of strength relies on the *relative* values of the cross sections. The second was an incorrect assumption about how the reaction-model code handled unbound states. To investigate, we repeated the reaction modeling in different ways and found that our original conclusions are not so sensitive to the treatment of unbound states. Mistakes should, however, be corrected to avoid propagating errors more widely. We give more details below.

More recent measurements have since highlighted that deuterated polyethylene targets can contain non-negligible amounts of protons that appear to be associated with the use of a xylene solvent during manufacture. With a solenoidal spectrometer, both elastically scattered protons and deuterons are transported by the magnetic field to the monitor detector that is used to set the scale of the cross sections. These particles have different center-of-mass angles, but very similar energies, making it difficult to identify them as an unresolved doublet. For example, a detailed study of the ^{27}Na(d,p) reaction [1] found ≈14(2)% protons in terms of relative target thickness. Our experiment used similar targets, but the possibility of proton contamination had not been considered. Integrated elastic-scattering cross sections using optical potentials for protons and deuterons of Ref. [2] only differ by around 10% for the relevant angles, so the possible effect is largely in the uncertainty of the effective thickness. We therefore estimate that the absolute cross sections are subject to an additional error of around 15% beyond that originally quoted.

In the original publication, we stated that the finite-range code DWUCK5 [3] used a prescription by Vincent and Fortune [5] to avoid potential convergence issues with radial integrals for unbound states in the distorted-wave Born approximation (DWBA). This is incorrect. While that code constructs an unbound eigenfunction using a Woods-Saxon potential and a boundary condition of matching to a Coulomb function, no special treatment is used for the radial integral. As a check, we have the reaction modeling in two ways. First, with the zero-range code DWUCK4 [4], which does employ the complex integration suggested by Ref. [5] (referred to here as ZR+VF). Second, for unbound states above the neutron separation energy of 3.663 MeV, we employed the commonly used weak-binding approximation performed with DWUCK5 (FR+WB), where a bound state just below threshold is used instead of an unbound state. The resulting SF are shown in Table I compared with those from our original paper (referred to as FR).

For the bound states, small differences are found between exact finite-range and zero-range calculations, which are generally comparable to the statistical errors. The ground and first-excited states are exceptions since they form an unresolved doublet that was analyzed by fitting two DWBA distributions to data. Small differences in the predicted shapes of both ℓ transfers affect the fit, although finite-range calculations give a better overall reproduction.

For the unbound states up to ≈4 MeV, there are differences in SF for p states, as might be expected. However, the associated changes in the *centroids* of observed strength are relatively small, increasing the binding energy of the $p_{1/2}$ centroid by 90 keV for ZR+VF and by 170 keV for FR+WB. The corresponding numbers for the $p_{3/2}$ orbital are 200 and 280 keV respectively. Figure 4 is an updated version of the one in the original paper, where results using all three reaction models are shown for ^{29}Mg; their variation is small compared with the changes in binding energy of the $N = 17$ isotones. Moreover, the differences in SF for these unbound states do not make a significant difference to the overall comparison with shell-model results (see Fig. 3 of the original paper).

Although firm assignments above 5 MeV were not possible, in our original paper, we associated the weakly observed transitions with possible shell-model counterparts to consider the effect of unassigned strength. Reaction modeling is problematic as the eigenfunction search described above fails for p states at these energies. Calculations were instead performed at a lower binding energy of +300 keV to provide estimates of SF. Since predicted cross sections increase with excitation above the

TABLE I. Spectroscopic factors (SF) for states populated in the ^{28}Mg(d,p) ^{29}Mg reaction using three different approaches to the reaction modeling and normalized using the procedure of the original publication. Those labeled FR are taken from that reference. The value of SF for the ground state and state at 55 keV are deduced from a combined fit of angular distributions, which is sensitive to small changes in DWBA distributions. For the weak states above 5 MeV, SF are estimated from integrated cross sections for $J^\pi = 1/2^-, 3/2^-$, or $5/2^-$. The SF of the likely shell-model counterpart is underlined. As described in the text for the two possible p states above 5 MeV, these are upper limits.

E (keV)	ℓ	J^π	SF (FR)	SF (ZR+VF)	SF (FR+WB)
0	2	$3/2^+$	0.37(4)	0.28(2)	
55(1)	0	$1/2^+$	0.56(3)	0.81(5)	
1092(3)	1	$3/2^-$	0.35(2)	0.39(3)	
1432(2)	3	$7/2^-$	0.43(2)	0.36(3)	
2270(18)	1	$1/2^-$	0.17(1)	0.19(3)	
2501(6)	2	$3/2^+$	0.24(1)	0.21(3)	
2900(32)	3	$5/2^-$	0.02(1)	0.02(1)	
3220(16)	2	$5/2^+$	0.07(1)	0.07(2)	
3906(13)	1	$1/2^-, 3/2^-$	<u>0.24(5)</u>, 0.12(2)	<u>0.34(3)</u>, 0.19(1)	<u>0.42(2)</u>, 0.22(1)
4045(22)	1	$1/2^-, 3/2^-$	0.24(5), <u>0.13(3)</u>	0.35(2), <u>0.20(1)</u>	0.43(2), <u>0.22(1)</u>
4360(10)	3	$7/2^-$	0.18(1)	0.16(3)	0.21(4)
			$SF_{1/2^-}$, $SF_{3/2^-}$, $SF_{5/2^-}$	$SF_{1/2^-}$, $SF_{3/2^-}$, $SF_{5/2^-}$	$SF_{1/2^-}$, $SF_{3/2^-}$, $SF_{5/2^-}$
5623(9)			<u>0.20(2)</u>, 0.11(1), 0.02(1)	<u>0.76(7)</u>, 0.46(4), 0.06(1)	<u>0.71(7)</u>, 0.37(4), 0.06(1)
5811(11)			0.07(2), <u>0.04(1)</u>, 0.01(1)	0.27(1), <u>0.16(4)</u>, 0.02(1)	0.27(1), <u>0.14(3)</u>, 0.02(1)
6043(11)			0.15(3), 0.08(1), <u>0.01(1)</u>	0.45(8), 0.27(5), <u>0.03(1)</u>	0.45(8), 0.38(7), <u>0.03(1)</u>

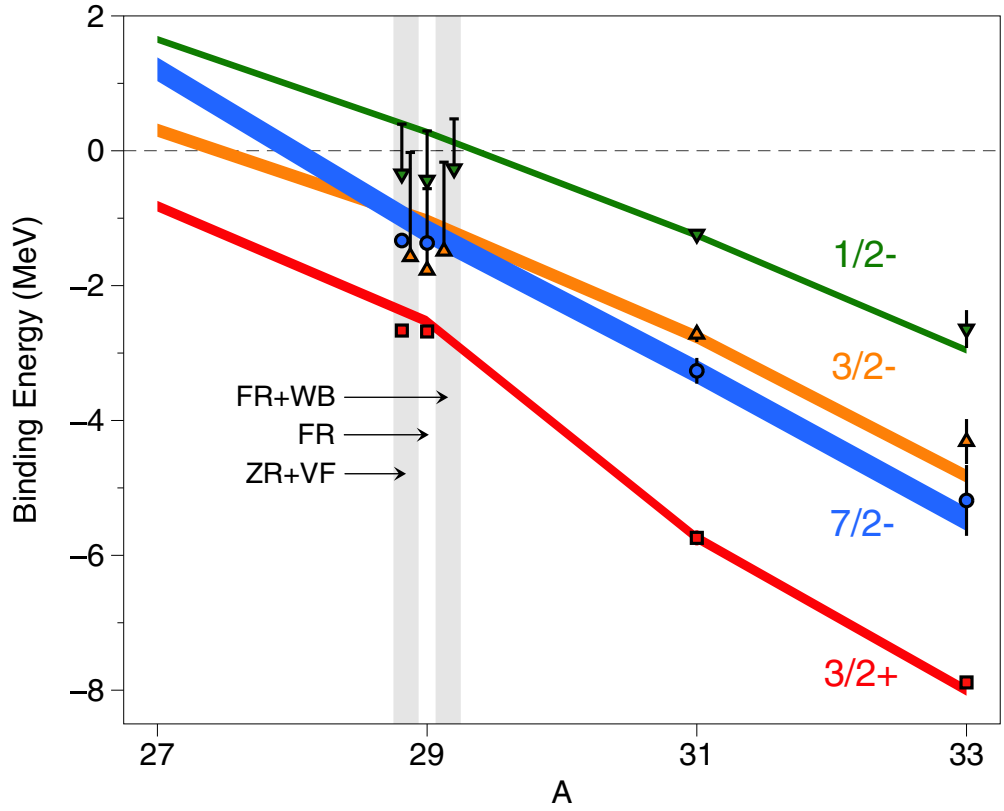

FIG. 4. Binding energies for the observed centroids of strength for the $N = 17$ isotones. This is the same figure as in the original paper but redrawn to show values for ^{29}Mg deduced using three different reaction models for unbound states (points are offset in A for clarity). Colored bands represent the predictions of the FSU shell-model interaction. The original results are labeled FR while FR+WB and ZR+VF represent alternative reaction modeling, see text for details.

separation energy, these estimates give an upper bound for associated SF and therefore for any deduced centroid. Indeed, possible shell-model values are much smaller. These limits are illustrated in Fig. 4 as an upper bar for $1/2^-$ and $3/2^-$ strength to illustrate possible effects of unassigned strength, which are similar for all three approaches to the unbound states. Clearly, though, if information beyond such estimation was needed in the future, a proper description of such weak unbound states would warrant more sophisticated reaction models than DWBA.

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