Letter

Complementary *CP* violation induced by *T*-odd and *T*-even correlations

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In this Letter, we demonstrate that *T*-odd and *T*-even correlations, with *T* representing time reversal, when satisfying specific conditions, result, respectively, in cosine and sine strong phase dependencies of the corresponding *CP* violation. Additionally, we identify pairs of these *CP* violation observables in hadron decays depend on precisely the same strong phases within the helicity amplitude scheme. This complementarity could reduce the strong phase reliance in the study of *CP* violation while also mitigating the risk of suppressed *CP* violation due to exceptionally small strong phases. Moreover, our proposal holds broad application prospects in further investigating *CP* violation in baryon decays, which has recently been discovered in the LHCb experiment.

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Introduction. Understanding the asymmetry between baryons and antibaryons in the Universe is a significant challenge in modern particle physics and cosmology. This puzzle can be addressed by satisfying three conditions known as the Sakharov criteria [1]: baryon number violation, *C* and *CP* violation (CPV), and departure from equilibrium. In the Standard Model (SM) of particle physics, the only confirmed CPV source is the weak phase in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix mechanism [2,3] as proposed in [3]. However, the level of CPV in the SM is not adequate to account for the matter-dominated Universe, as observed in [4], suggesting the presence of additional CPV sources. Furthermore, precise CPV measurements are crucial for determining the elements of the CKM matrix, which is essential for testing the unitarity of the CKM matrix required by the SM. Therefore, CPV serves as a promising avenue for exploring new physics beyond the SM.

In flavor physics, extensive research has been conducted on CPV in meson decays and mixing [5–9]. Despite the accumulation of more data and higher-order calculations, precision tests of CPV observables in most decay channels still face challenges in reconciling theory and experiment, thus hindering the search for nonstandard CPV sources. This difficulty is particularly evident in the case of direct *CP* asymmetry, which is proportional to the sine of the strong phase. Theoretical calculations of strong phases often introduce significant uncertainties. To tackle this issue, new CPV observables have emerged [10–15]. Some of these observables exhibit a cosine dependence on strong phases, including CPV induced by triple products, Lee-Yang asymmetries, and more general *T*-odd correlations [15–27]. This characteristic potentially allows for the cancellation of strong phase dependence if two CPV observables depend on the sine and cosine of exactly the same strong phase [17–20]. We refer to this phenomenon as true complementarity.[1] For instance, with additional constraints, a CPV observable constructed from Lee-Yang parameters [16] can be approximately independent of strong phases [28]. Despite these advancements, there remain significant unanswered questions from previous investigations of *T*-odd correlations and complementarity. The unanswered questions can be summarized as follows:

(i) What is the underlying reason for the cosine dependence of strong phases in *T*-odd correlation induced CPV?

(ii) Are true complementarities widespread, and if so, how can we identify them?

By this work, we clarify the aforementioned questions by offering two proofs: (1) We provide a strict proof showing that the *T*-odd correlation induced CPV indicates

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[1]Conversely, if two CPV observables are proportional to the sine and cosine of different strong phases, one cannot conclude that they are complementary to each other.

a cosine dependence on the strong phase under certain conditions, and that the corresponding T-even correlation induced CPV indicates a sine dependence. (2) We present the criteria for true complementarity between pairs of T-odd and T-even CPV observables in two-body decays with helicity amplitude frameworks. Based on the proofs, we propose the feasibility of simultaneously measuring a pair of CPV observables that exhibit dependencies on $\sin\delta_s$ and $\cos\delta_s$ relative to the same strong phase difference δ_s. Our proof will also provide a systematic way to build this type of complementary observation and thus can lead to a blanket search for the complementary T-odd and T-even CPV observables.

We emphasize that the complementarity holds broad application prospects for baryonic CPV, which was recently discovered in $\Lambda_b \to pK^-\pi^+\pi^-$ by LHCb [29]. Complementary CPV observables provide an avenue for detecting CPV regardless of the involved strong phase values, since either $\sin\delta_s$ or $\cos\delta_s$ exceeds $\sqrt{2}/2$. Finally, we demonstrate the feasibility of our proposal experimentally through a specific example of $\Lambda_b^0 \to N^*(1520)K^*(892)$ with $N^*(1520) \to p\pi$ and $K^*(892) \to K\pi$ and analyze its potential applications in other decays involving baryons. It becomes evident that appropriate angular-distribution observables in this decay precisely give rise to the complementary T-odd and T-even observables, which are experimentally measurable.

Strong phase dependence. As the first step, we prove that the CPV $a_{\mathrm{CP}}^{Q_-}$ induced by a subset of T-odd correlations Q_- are proportional to the cosine of the involved strong phase differences, $\cos\delta_s$. The T-odd property of Q_- indicates its transformation under the time reversal $\mathcal{T}$ as

$$\mathcal{T}Q_- = -Q_-\mathcal{T}. \tag{1}$$

It is important to note that not all Q_- can generate CP asymmetries proportional to $\cos\delta_s$ (see, e.g., [23]). We propose that a qualified Q_- satisfies the following conditions: (i) In the Hilbert space of the final states of a physical process of interest, with a properly chosen basis $\{|\psi_n\rangle; n = 1, 2, \ldots\}$, there exists a unitary transformation $\mathcal{U}$ that transforms $\mathcal{T}|\psi_n\rangle$ back to $|\psi_n\rangle$ up to a universal phase factor, i.e., $\mathcal{U}\mathcal{T}|\psi_n\rangle = e^{i\alpha}|\psi_n\rangle$. (ii) Q_- is symmetric under this unitary transformation, i.e., $\mathcal{U}Q_-\mathcal{U}^\dagger = Q_-$. The proof of $a_{\mathrm{CP}}^{Q_-}$ being proportional to $\cos\delta_s$ is as follows.

The Q_- expectation value of the final state $|f\rangle \equiv S|i\rangle$ of a process, with S being the S-matrix operator, can be expressed in terms of the transition amplitudes from the initial state to basis vectors $A_n \equiv \langle\psi_n|S|i\rangle$ as

$$\begin{aligned}\langle f|Q_-|f\rangle &= \langle i|S^\dagger Q_- S|i\rangle \\ &= \sum_{m,n}\langle\psi_i|S^\dagger|\psi_m\rangle\langle\psi_m|Q_-|\psi_n\rangle\langle\psi_n|S|\psi_i\rangle \\ &= \sum_{m,n}A_m^* A_n\langle\psi_m|Q_-|\psi_n\rangle.\end{aligned} \tag{2}$$

The dynamics are now coded in A_n's, and $\langle\psi_m|Q_-|\psi_n\rangle$'s consist only of kinematics. Then it can be shown that the matrix element $\langle\psi_m|Q_-|\psi_n\rangle$ is purely imaginary by

$$\begin{aligned}\langle\psi_m|Q_-|\psi_n\rangle &= \langle\psi_m|\mathcal{T}^\dagger\mathcal{T}Q_-|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger Q_-\mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger\mathcal{U}^\dagger\mathcal{U}Q_-\mathcal{U}^\dagger\mathcal{U}\mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger\mathcal{U}^\dagger Q_-\mathcal{U}\mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|Q_-|\psi_n\rangle^*,\end{aligned} \tag{3}$$

where in the first step the antiunitarity of $\mathcal{T}$ is used. Consequently, only the imaginary part of the amplitude interference $\mathrm{Im}(A_m^* A_n)$ contributes because $\langle f|Q_-|f\rangle$ must be real. This conclusion holds true for both perturbative and nonperturbative dynamics, and for diverse physical systems such as beauty, charm, strange, top, and even Higgs physics.

The CP asymmetry induced by a T-odd correlation Q_- is defined as

$$a_{CP}^{Q_-} \equiv \langle f|Q_-|f\rangle - \langle\bar{f}|\bar{Q}_-|\bar{f}\rangle, \tag{4}$$

where $|\bar{f}\rangle \equiv S(CP)|i\rangle$ and $\bar{Q}_- \equiv (CP)Q_-(CP)^{-1}$. By inserting a complete basis of $|\psi_n\rangle$ and $|\bar{\psi}_n\rangle \equiv CP|\psi_n\rangle$, we obtain

$$a_{CP}^{Q_-} \propto \sum_{m,n} i\ \mathrm{Im}(A_m^* A_n - \bar{A}_m^*\bar{A}_n)\langle\psi_m|Q_-|\psi_n\rangle, \tag{5}$$

where the relation $\langle\psi_m|Q_-|\psi_n\rangle = \langle\bar{\psi}_m|\bar{Q}_-|\bar{\psi}_n\rangle$ independent of dynamics has been utilized. In quark-flavor processes whose CPV is induced by the CKM mechanism, the imaginary CP differences $\mathrm{Im}(A_m^* A_n - \bar{A}_m^*\bar{A}_n)$ must be proportional to the sine of the weak phase difference $\sin\delta_w$, and hence the cosine of the relevant strong phase difference $\cos\delta_s$.

Analogously, if a T-even correlation Q_+ satisfies conditions (i) and (ii), the right-hand side of (3) flips the sign such that the Q_+ expectation depends on the real part of amplitude interferences and, of course, on the possible modulo terms. Therefore, its induced CPV will be proportional to the sine of the strong phase difference. In fact, direct CPV is induced by a T-even correlation, which can be defined by $|f_d\rangle\langle f_d|$, with $|f_d\rangle$ the desired final state, so they have the sine dependence on δ_s. If a pair of $\langle Q_-\rangle$ and

$\langle Q_+\rangle$ pick the $\mathrm{Im}(A_m^* A_n)$ and $\mathrm{Re}(A_m^* A_n)$ contributions, respectively, with the same weights, we will prove that they give rise to *CP* asymmetries proportional to the cosine and sine of the same strong phase in the subsequent section. From this perspective, they exhibit an exact complementary relationship with each other.

It is important to note that the above proposition is not limited to time reversal but instead applies universally to any antiunitary transformation, such as the combined transformation of spatial and time reversals $\mathcal{PT}$. In addition, the condition (i) can be slightly relaxed: it is sufficient that $\langle\psi_m|\mathcal{T}^\dagger\mathcal{U}^\dagger Q_-\mathcal{UT}|\psi_n\rangle = \langle\psi_m|Q_-|\psi_n\rangle$ instead of requiring $\mathcal{UT}|\psi_n\rangle = e^{i\alpha}|\psi_n\rangle$.

Our prescription can be easily applied to two-body hadron decays involving at least two nonzero spin particles. The *T*-odd correlation Q_- can be selected as an odd-multiple product of spin and momentum vectors of the particles involved, such as the triple product $(\vec{s}_1\times\vec{s}_2)\cdot\vec{p}$, where the particle spins are defined in the rest frame of each respective particle. Correspondingly, the unitary transformation $\mathcal{U}$ is chosen as the spatial rotation, and the basis vectors $|\psi_n\rangle$ are selected as the helicity eigenstates. The final-state helicity eigenstates are denoted by $|J,M;\lambda_1,\lambda_2\rangle$, where J is the final-state angular momentum, M is its z-direction component, which is determined by the initial state, and λ_1 and λ_2 are the helicities of the two final-state particles. Following the convention of [30], the time reversal $\mathcal{T}$ and the rotation about the y axis by π, $\mathcal{U}=e^{-i\pi J_y}$, both transform $|J,M;\lambda_1,\lambda_2\rangle$ to $(-1)^{J-M}|J,-M;\lambda_1,\lambda_2\rangle$. Therefore, the condition (i) is satisfied, with

$$\mathcal{UT}|J,M;\lambda_1,\lambda_2\rangle = (-1)^{2J}|J,M;\lambda_1,\lambda_2\rangle. \tag{6}$$

Furthermore, the triple products, being spatial-SO(3) scalars, remain invariant under spatial rotations, thus fulfilling condition (ii). Subsequently, we will delve into further details regarding this type of decay process, elucidating the genuine complementarity of the *CP* violation observables involved.[2]

Criteria for a complementary observable. As demonstrated earlier, *T*-odd and *T*-even correlations satisfying conditions (i) and (ii) induce CPV observables with cosine and sine dependences on strong phases, respectively. However, a critical question remains as to how to determine whether two observables are exactly complementary. Providing a general answer to this question would be quite challenging. Instead, we will limit ourselves to two-body decays and select the final-state bases to be the helicity eigenstates [27]. In this context, we introduce a criterion within the helicity framework.

(i) *Criterion.* If two observables exhibit dependencies on the real and imaginary parts of the same interference term under the helicity amplitude scheme, then they will induce exactly complementary CPV observables.

Proof. In the helicity bases, the expression of $\langle Q_-\rangle$ (2) is composed of helicity amplitude interferences and $\langle Q_+\rangle$ is analogous. Consider the simplest case where two operators $\mathcal{O}_+$ and $\mathcal{O}_-$ have expectations given by

$$\begin{aligned}\langle\mathcal{O}_+\rangle &= \mathrm{Re}\,(\mathcal{H}_{\lambda_i,\lambda_j}\mathcal{H}^*_{\lambda_m,\lambda_n} + \mathcal{H}_{-\lambda_i,-\lambda_j}\mathcal{H}^*_{-\lambda_m,-\lambda_n}),\\ \langle\mathcal{O}_-\rangle &= \mathrm{Im}\,(\mathcal{H}_{\lambda_i,\lambda_j}\mathcal{H}^*_{\lambda_m,\lambda_n} + \mathcal{H}_{-\lambda_i,-\lambda_j}\mathcal{H}^*_{-\lambda_m,-\lambda_n}),\end{aligned} \tag{7}$$

where $\lambda_{i,j}$ and $\lambda_{m,n}$ are general helicity indices of the final-state particles. This can be fulfilled when the operators have only nonzero matrix elements $\langle\lambda_m,\lambda_n|\mathcal{O}_\pm|\lambda_i,\lambda_j\rangle$ and $\langle-\lambda_m,-\lambda_n|\mathcal{O}_\pm|-\lambda_i,-\lambda_j\rangle$. Note that $\langle\mathcal{O}_+\rangle$ and $\langle\mathcal{O}_-\rangle$ constitute two terms linked by the parity transformation. This choice is reasonable because observables that we are interested in invariably manifest specific symmetries under spatial inversion, such as triple products [17,23] and asymmetry parameters [16]. Here, one can ensure that both of them are parity even. This proof is also applicable to the parity-odd cases. The *CP* asymmetries induced by $\mathcal{O}_\pm$ are defined by

$$a_{CP}^{\mathcal{O}_+} = \langle\mathcal{O}_+\rangle - \langle\bar{\mathcal{O}}_+\rangle, \qquad a_{CP}^{\mathcal{O}_-} = \langle\mathcal{O}_-\rangle - \langle\bar{\mathcal{O}}_-\rangle, \tag{8}$$

where $\langle\bar{\mathcal{O}}_\pm\rangle$ are the corresponding charge conjugations. They can be further normalized to make them dimensionless.

A helicity amplitude can be decomposed into tree and penguin contributions as

$$\mathcal{H}_{\lambda_i,\lambda_j} = H^t_{i,j}e^{i\phi_t}e^{i\delta^t_{i,j}} + H^p_{i,j}e^{i\phi_p}e^{i\delta^p_{i,j}}, \tag{9}$$

where $H^{t(p)}_{\lambda_i,\lambda_j}$, $\delta^{t(p)}_{i,j}$, and $\phi_{t(p)}$ are the magnitude, the strong phase, and the weak phase of the tree (penguin) amplitude, respectively. Its *CP* conjugation partner $\bar{\mathcal{H}}_{\lambda_i,\lambda_j}$ can correspondingly be expressed as

$$\bar{\mathcal{H}}_{-\lambda_i,-\lambda_j} = H^t_{i,j}e^{-i\phi_t}e^{i\delta^t_{i,j}} + H^p_{i,j}e^{-i\phi_p}e^{i\delta^p_{i,j}} \tag{10}$$

by flipping the weak phase signs. It can be distinguished by an overall minus sign depending on the *CP* transformation conventions of the initial and final states, which does not change the physics. A similar relation holds between their parity partners. This leads to a comprehensive complementary observation,

[2]It is worth noting that the *T*-odd triple product $(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_3$ consisting of three-momenta in four-body decays cannot satisfy conditions (i) and (ii) simultaneously. The $\mathcal{T}$ transformation flips all the particle momenta, so condition (i) requires that $\mathcal{U}$ flips the momenta back. Then, we must have $\mathcal{U}(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_3\mathcal{U}^\dagger = -(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_3$, and thus condition (ii) is not satisfied. Therefore, the corresponding CPV is not necessarily proportional to $\cos\delta_s$ [23].

$$a_{CP}^{\mathcal{O}_+} \propto [-H^t_{i,j}H^p_{m,n}\sin(\delta^t_{i,j}-\delta^p_{m,n})+H^p_{i,j}H^t_{m,n}\sin(\delta^p_{i,j}-\delta^t_{m,n})]\sin\Delta\phi+(i,j,m,n\to -i,-j,-m,-n),$$
$$a_{CP}^{\mathcal{O}_-} \propto [-H^t_{i,j}H^p_{m,n}\cos(\delta^t_{i,j}-\delta^p_{m,n})+H^p_{i,j}H^t_{m,n}\cos(\delta^p_{i,j}-\delta^t_{m,n})]\sin\Delta\phi+(i,j,m,n\to -i,-j,-m,-n), \quad (11)$$

where $\Delta\phi \equiv \phi_t - \phi_p$. It can be observed that $a_{CP}^{\mathcal{O}_+}$ and $a_{CP}^{\mathcal{O}_-}$ are dependent on the identical set of strong phase differences and are thus exactly complementary to each other. This establishes the complementarity under the helicity scheme. Note that the complementarity exists between $a_{CP}^{\mathcal{O}_+}$ and $a_{CP}^{\mathcal{O}_-}$ rather than between $a_{CP}^{\mathcal{O}_-}$ and the direct *CP* asymmetry. The direct *CP* asymmetry characterizes the difference between the total widths Γ and $\bar{\Gamma}$ consisting of the modulo squared of distinct helicity configurations, while the *T*-odd *CP* asymmetry, as in (5), consists of interference terms, so they rely on different strong phases.

The discussions presented above are focused on two-body decays. However, the situation becomes more complex in the case of multibody systems owing to the presence of intricate intermediate resonances. Consequently, the applicability of the aforementioned proof might be compromised in such scenarios. Nevertheless, it is worth noting that the amplitude $\mathcal{H}_{\lambda_i,\lambda_j}$ can be directly extracted in experiments by employing the partial wave analysis method in multibody decays [31,32]. In this context, our proposal retains its value and practicality, providing a useful framework for analyzing and interpreting experimental results in multibody systems.

Application in the baryon sector. Our proposal has a wide range of applications in decay processes involving baryons. Given that the helicity information of the final-state particles undergoing subsequent decays is manifested in the angular distribution of their decay products, the helicity amplitudes of a cascade decay can be derived from its angular distribution. As an illustration, we analyze the decay channel $\Lambda_b^0 \to N^*(1520)K^*$ with the spin-3/2 $N^*(1520)\to p\pi$ and the spin-1 $K^*(892)\to K\pi$ [33]. The results apply directly to the similar $\Lambda_b^0\to N^*(1520)\rho$ decay with $\rho\to\pi^+\pi^-$. With unpolarized Λ_b^0, the complementary part of angular distribution is formulated as

$$\frac{d\Gamma}{d\mathrm{c}_L d\mathrm{c}_R d\phi} \ni -\frac{\mathrm{s}_L^2\mathrm{s}_R^2}{\sqrt{3}}\mathrm{Im}[\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}}+\mathcal{H}_{+1,+\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}]\sin 2\phi+\frac{\mathrm{s}_L^2\mathrm{s}_R^2}{\sqrt{3}}\mathrm{Re}[\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}}+\mathcal{H}_{+1,+\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}]\cos 2\phi$$
$$-\frac{4\mathrm{s}_L\mathrm{c}_L\mathrm{s}_R\mathrm{c}_R}{\sqrt{6}}\mathrm{Im}[\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}}+\mathcal{H}_{0,-\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}]\sin\phi+\frac{4\mathrm{s}_L\mathrm{c}_L\mathrm{s}_R\mathrm{c}_R}{\sqrt{6}}\mathrm{Re}[\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}}+\mathcal{H}_{0,-\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}]\cos\phi, \quad (12)$$

where $\mathrm{s}_{L,R}=\sin\theta_{L,R}$ and $\mathrm{c}_{L,R}=\cos\theta_{L,R}$. The angular variables $\theta_{L,R}$ represent the polar angles of the proton and K meson in the rest frames of $N^*(1520)$ and K^*, respectively, and ϕ denotes the angle between the decay planes of $N^*(1520)$ and K^*, as depicted in Fig. 1. The amplitudes $\mathcal{H}_{\lambda_1,\lambda_2}$ parametrize the dynamics of the $\Lambda_b^0\to N^*(1520)K^*$ decay, with λ_1 and λ_2 being helicity symbols of K^* and $N^*(1520)$, respectively. Here, we define two *T*-odd parameters with respect to $\sin\phi$ and $\sin 2\phi$ as $\mathcal{A}_{T,1}\equiv \mathrm{Im}(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}}+\mathcal{H}^*_{-1,-\frac{3}{2}}\mathcal{H}_{0,-\frac{1}{2}})$ and $\mathcal{A}_{T,2}\equiv \mathrm{Im}(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}}+\mathcal{H}^*_{-1,-\frac{3}{2}}\mathcal{H}_{+1,+\frac{1}{2}})$. These parameters can be obtained by integrating the differential decay width using the expression

$$\mathcal{A}_{T,i}\propto\int\frac{d\Gamma}{d\mathrm{c}_L d\mathrm{c}_R d\phi_R}W_i\, d\mathrm{c}_L\, d\mathrm{c}_R\, d\phi, \quad (13)$$

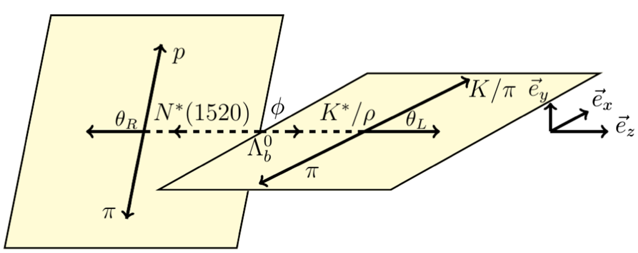

FIG. 1. The decay process of $\Lambda_b^0\to N^*(1520)K^*/\rho$ with $N^*(1520)\to p\pi^-$, $K^*(\rho)\to K(\pi)\pi$ decay. The angles ϕ, θ_L, and θ_R are defined in the rest frames of Λ_b, $K^*(\rho)$, and $N^*(1520)$, respectively.

with the weight functions $W_1=\sin\phi\mathrm{c}_L\mathrm{c}_R$ and $W_2=\sin 2\phi\mathrm{s}_L\mathrm{s}_R$. The expectations of corresponding *T*-even correlations $\mathcal{B}_{T,i}$ are also defined through the angular distribution with respect to $\cos\phi$ and $\cos 2\phi$, $\mathcal{B}_{T,1}\equiv\mathrm{Re}(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}}+\mathcal{H}^*_{-1,-\frac{3}{2}}\mathcal{H}_{0,-\frac{1}{2}})$, and $\mathcal{B}_{T,2}\equiv \mathrm{Re}(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}}+\mathcal{H}^*_{-1,-\frac{3}{2}}\mathcal{H}_{+1,+\frac{1}{2}})$ and can be analogously extracted. Subsequently, the induced *CP* asymmetries are given by the differences between $\mathcal{A}(\mathcal{B})_{T,i}$ and their charge conjugations,

$$a_{CP}^{i} = \mathcal{A}_{T,i} - \bar{\mathcal{A}}_{T,i} b_{CP}^{i} = \mathcal{B}_{T,i} - \bar{\mathcal{B}}_{T,i}. \quad (14)$$

It is important to emphasize that a_{CP}^{i} and b_{CP}^{i} are proportional to the cosine and sine of identical strong phases, as previously demonstrated. Experimentally, the momentum of each final-state particle must be measured, enabling the above angular analysis to be performed without additional cost. As demonstrated in (13), the corresponding weight function can extract the desired quantity, allowing the entire data sample to be utilized efficiently without requiring excessively large statistics.

We have noticed that the first evidence of baryonic CPV was detected in $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$ [34], which shares the same final states as the aforementioned decay channel. The observable they analyzed is constructed from the asymmetry of the triple product $\vec{p}_p \cdot (\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})$, which is proportional to $\mathrm{sgn}(\sin\phi s_L s_R)$. Evaluating its expectation through phase space integration of (12) consistently yields zero, indicating that the $N^*(1520)K^*$ resonance contributions to this observable always vanish.[3] Consequently, we infer that the CPV observable utilized by LHCb [34] eliminates the possibility of significant CP-violating sources stemming from resonances, rendering it an unsuitable observable for CPV investigations. Conversely, measurements of the complementary CPV observables defined in (14) effectively capture the $N^*(1520)K^*$ contributions. This finding can be tested in the $\Lambda_b \to pK^-\pi^+\pi^-$ channel, whose global CPV has been discovered [29], by measuring the CPV observables constructed from the asymmetry of the triple product and the ones defined in (14).

We anticipate that our proposal will offer advantages for the study of CPV in baryonic processes. In addition to the previously analyzed channels, we recommend conducting analogous angular-distribution analyses for other b-baryon decays in experiments, such as $\Lambda_b^0 \to \Lambda^*(1520)\rho/\phi$ and $\Lambda_b^0 \to pa_1$ [27,33,35–39]. Furthermore, similar complementary CP asymmetries are expected in baryonic meson decays, such as $B^0 \to \Lambda_c^+\bar{\Lambda}_c^-, \bar{\Xi}_c^-\Lambda_c^+, \Lambda\bar{\Lambda}$, warranting further investigation.

[3]It can be verified that this conclusion remains valid if a spin-1/2 excited particle is substituted for $N^*(1520)$.

Moreover, if an initially polarized baryon in a decay is considered, the angular analysis becomes more intricate, leading to the emergence of more complementary CP asymmetries [27,35]. However, the polarization of b-baryons produced in pp collisions at LHC is negligible, rendering it ineffective for phenomenological analyses [40–43]. Fortunately, the charm and strange baryons produced at lepton colliders are found to have sizable polarization [44–48], thus allowing for a more comprehensive angular analysis.

Summary. In this Letter, we have addressed the questions surrounding conventional T-odd correlation discussions by providing two proofs. Our findings reveal that the flavor CPV observables induced by T-odd correlations, under specific conditions, are directly proportional to the cosine of strong phases, while the corresponding T-even correlations give rise to strong-phase-sine CPV. Furthermore, within the helicity representation framework, we have demonstrated a true complementary dependence of strong phases between CPV observables induced by pairs of T-odd and T-even correlations whose expectations are proportional to the imaginary and real parts of the same helicity amplitude interferences. This provides a strong basis and could be utilized to reduce the strong phase reliance of CPV, especially in baryon decays. A detailed analysis of practical examples involving b-baryon decays demonstrates that the proposed CPV observables can be extracted by measuring the angular distribution of the decay products. As the amplitude analysis method continues to develop and be applied in experimental settings, these complementary forms of CPV will offer insights that will help inform future researches.

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