Erratum: Axions and the strong *CP* problem [Rev. Mod. Phys. 82, 557 (2010)]

Jihn E. Kim and Gianpaolo Carosi

(published 30 December 2019)

DOI: 10.1103/RevModPhys.91.049902

With a small θ in Eq. (38) in Kim and Carosi (2010), we presented the *CP* violating coupling in the πNN vertex in Eq. (8). But a factor, arising from the matrix element between nucleon states $\langle N'|$ and $|N\rangle$, was missed in Eq. (8). Equation (8) should read

$$\overline{g_{\pi NN}} \simeq -\frac{\theta}{3} g_{\pi NN}. \tag{1}$$

The axial-vector isospin current is $J^{i5}_\mu = \bar{q}\gamma_\mu\gamma_5 T_i q$, where q is the quark doublet $(u\, d)^T$ and T_i $(i = 1, 2, 3)$ is the generator in the isospin space. The divergence of the axial-vector current between q' and q is

$$\partial^\mu J^{(q'q)5}_\mu = (m_q + m'_q)\bar{q}' i\gamma_5 q. \tag{2}$$

In the partially conserved axial-vector current, the divergence of axial-vector current J^{i5}_μ in the broken phase of the chiral symmetry is related to the creation of the Goldstone field π^i,

$$\frac{1}{f_\pi}\langle \pi^i(q)|\partial^\mu J^{j5}_\mu|0\rangle \simeq q^2\delta^{ij}, \tag{3}$$

where $q^2 = m^2_\pi$ is usually used. Considering the Goldstone pole propagator, thus the pion-nucleon coupling is given by (Georgi, 1984),

$$\frac{1}{f_\pi}\langle \pi^i(q), N'|\partial^\mu J^{i5}_\mu|N\rangle \simeq q^2 \frac{1}{q^2} g_{\pi NN} = g_{\pi NN}. \tag{4}$$

Now using the right-hand side of Eq. (2), the relation

$$\langle n|\frac{m_q + m'_q}{f_\pi}\bar{q}' i\gamma_5 q|n\rangle = \frac{M_n g_A}{f_\pi} \tag{5}$$

is required for the validity of the Goldberger-Treiman (GT) relation (Goldberger and Treiman, 1958a, 1958b),

$$M_N = \frac{f_\pi g_{\pi NN}}{g_A}. \tag{6}$$

If π^0 develops a vacuum expectation value, it breaks the *CP* invariance. In that case, Eq. (5) is multiplied by a phase $e^{i\theta_{\pi^0}}$. In Eq. (38) of Kim and Carosi (2010),

$$m_u \frac{\Lambda^2_u v^6}{K^5}$$

and

$$m_d \frac{\Lambda^2_d v^6}{K^5} \simeq 2Z m_d \frac{\Lambda^2_d v^6}{K^5}$$

arise from $\langle m_u \bar{u} i\gamma_5 u\rangle$ and $\langle m_d \bar{d} i\gamma_5 d\rangle$, respectively. For $\Lambda^2_u = \Lambda^2_d$ that is equivalent to $\langle \bar{q} i\gamma_5 q\rangle \equiv \langle \bar{u} i\gamma_5 u\rangle = \langle \bar{d} i\gamma_5 d\rangle$, $\theta_{\eta'} = 0$, and $c^u_2 = c^d_2 = 0$, i.e., in the Kim-Shifman-Vainshtein-Zakharov model, the last two lines of Eq. (38) give, for small θ_π and θ,

$$
\begin{aligned}
& - m_u \bar{u} i\gamma_5 u \cos(-\theta_\pi - \theta) - m_d \bar{d} i\gamma_5 d \cos(\theta_\pi - \theta) \\
& \simeq m_u \bar{u} i\gamma_5 u \sin\theta_\pi \sin\theta - m_d \bar{d} i\gamma_5 d \sin\theta_\pi \sin\theta \\
& \simeq (m_u + m_d) \bar{q} i\gamma_5 q \frac{Z - 2Z}{1+Z} \frac{\pi^0}{f_\pi} \theta \\
& \simeq -\frac{Z}{1+Z} \theta (m_u + m_d) \bar{q} i\gamma_5 q \frac{\pi^0}{f_\pi},
\end{aligned} \tag{7}
$$

where we neglected θ independent terms and $O(\theta^2)$ terms. Without the kinetic term, it describes the θ term. Using Eq. (5), we obtain

$$
\langle n | 2(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d) | n \rangle = M_N g_A. \tag{8}
$$

Therefore, from Eqs. (7) and (8), we have the following θ dependent term for a small θ:

$$
\langle n, \pi^0 | m_u \bar{u} i\gamma_5 u \cos(-\theta_\pi - \theta) + m_d \bar{d} i\gamma_5 d \cos(\theta_\pi - \theta) | n \rangle = -\frac{M_N g_A}{6 f_\pi} \theta = -\frac{g_{\pi NN}}{6} \theta. \tag{9}
$$

In view of two possibilities of inserting the *CP* violating vertex in Fig. 4(b), we note that a factor $g_{\pi NN}$ is missed in Eq. (8) of Kim and Carosi (2010). Using the 2018 bound on the upper limit on the neutron electric dipole moment, then we have the following bound on $\bar{\theta}$:

$$
|\bar{\theta}| \lesssim 2.8 \times 10^{-13}. \tag{10}
$$

Note further that, unlike in Crewther *et al.* (1979), this estimation on the bound on $|\bar{\theta}|$ is applicable even in the case without the s quark.

Crewther, R. J., P. Di Vecchia, G. Veneziano, and E. Witten, 1979, “Chiral Estimate of the Electric Dipole Moment of the Neutron in Quantum Chromodynamics,” Phys. Lett. B **88**, 123.

Georgi, H., 1984, *Weak Interactions and Modern Particle Theory* (Benjamin-Cummings Publishing Co., Inc., Menro Park, CA), Eq. (4.7.4).

Goldberger, M. L., and S. B. Treiman, 1958a, “Decays of the Pi Meson,” Phys. Rev. **110**, 1178;

Goldberger, M. L., and S. B. Treiman, 1958b, “Form Factors in β Decay and μ Capture,” Phys. Rev. **111**, 354.

Kim, J. E., and G. Carosi, 2010, “Axions and the Strong *CP* Problem,” Rev. Mod. Phys. **82**, 557.