CP violation in the B_s^0 system

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Experimental and theoretical studies of CP violation in the B_s^0 system are reviewed. Updated predictions for the mixing parameters of the B_s^0 mesons expected in the standard model (SM) are given, namely, the mass difference $\Delta M_s^{\rm SM} = 18.3 \pm 2.7\ {\rm ps}^{-1}$, the decay rate difference $\Delta\Gamma_s^{\rm SM} = 0.085 \pm 0.015\ {\rm ps}^{-1}$, and the flavor-specific CP asymmetry $a_{\rm fs}^{s,\rm SM} = (2.22 \pm 0.27) \times 10^{-5}$ and the equivalent quantities in the B^0 sector. Current experimental values of ΔM_s and $\Delta\Gamma_s$ agree with remarkable precision with theoretical expectations. This agreement supports the applicability of theoretical tools such as the heavy quark expansion to these decays. CP-violating studies in the B_s^0 system provide essential information to test the SM expectations and to unveil a possible contribution of the new physics (NP). NP effects on ΔM_s of the order of 15% are still possible. The CP phase ϕ_s due to CP violation in interference of decays and mixing can accommodate effects of the order of $\mathcal{O}(100\%)$. The semileptonic CP asymmetry $a_{\rm sl}^s$ due to CP violation in mixing could still be a factor of 130 larger than its robust SM expectation and thus provides a very clean observable for NP searches. Theoretical improvements that are necessary to make full use of the experimental precision are discussed.

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CONTENTS

I. INTRODUCTION

The phenomenon of CP violation, discovered more than 50 years ago (Christenson *et al.*, 1964), is an essential ingredient to explain the apparent imbalance between matter and antimatter in the Universe (Sakharov, 1967). Consequently, this topic attracts a lot of attention. In the standard model (SM) (Glashow, 1961; Weinberg, 1967; Salam, 1968) CP violation arises in the Yukawa sector via quark mixing and it is described by a complex parameter in the Cabibbo-Kobayashi-Maskawa (CKM) matrix (Cabibbo, 1963; Kobayashi and Maskawa, 1973). Intensive studies of CP violation, especially at the e^+e^- B factories [see, e.g., Bevan *et al.* (2014) for a comprehensive review], provide convincing evidence that the main source of CP violation is the phase in the CKM matrix. More precisely, a vast body of measurements performed in different experimental conditions, such as accelerators, energies of operation, and detectors, confirm the unitarity of the CKM matrix; see Amhis *et al.* (2014).

The CKM phase accounts for all the observed CP-violating phenomena, but it is too small to account for the abundance of matter in the Universe. Thus additional sources of CP violation must be found. A recent discussion of this problem can be found in Bambi and Dolgov (2015). The quest for a broader understanding of CP violation is strongly motivated and may provide hints on the path toward a more complete physics picture of the elementary particles and their interactions.

In particular, the study of CP violation in the B_s^0 system offers an excellent opportunity to uncover new physics (NP). SM predictions for several B_s^0 meson observables have achieved reasonable precision. In addition, SM CP-violating effects are expected to be more highly suppressed than in B^0 meson decays. Therefore, even a relatively small contribution of new physics effects could be clearly visible in the B_s^0 system; see, e.g., Dunietz, Fleischer, and Nierste (2001). More precisely, the angle β[1] describing CP violation in interference of decay and mixing in the B^0 system is predicted to be of the order of 22°. The corresponding angle β_s in the B_s^0 system is expected to be about 1°. Thus the sensitivity to new physics is potentially enhanced. Unfortunately, the contribution of the so-called penguin effects to the measured value of β_s can also be $\sim 1°$. Thus a more precise determination of penguin contributions is mandatory (Aaij *et al.*, 2015e). On the contrary, solid conclusions about the existence of new physics could be drawn from the investigation of CP violation in mixing extracted from semileptonic charge asymmetries. Here a measured value of about 2 or 3 times the value of the SM predictions would be an unambiguous signal for new physics.

The study of B_s^0 mesons at e^+e^- B factories is possible only by running at the $\Upsilon(10860)$ center-of-mass energy. The typical center-of-mass energy of both the *BABAR* and Belle experiments corresponds to the $\Upsilon(4S)$ mass and is not sufficient to produce $B_s^0\bar{B}_s^0$ pairs. The Belle experiment took some data at the higher energy and obtained several interesting results, notably some branching fractions of B_s^0 decays; see, e.g., Olive *et al.* (2014). However, their statistical accuracy is not sufficient to study CP violation observables. Thus, the main source of information on B_s^0 mesons comes from hadron collider experiments at the Tevatron (CDF, D0) and the Large Hadron Collider (LHC) (ATLAS, CMS, LHCb). In particular, LHCb, the first experiment designed to study beauty and charm decays at the LHC, has produced an impressive body of data on CP violation in B_s^0-$\bar{B}_s^0$ mixing and decay.

This paper summarizes the current experimental knowledge of CP violation in the B_s^0 system as well as the theoretical implication of these data. It is organized as follows. Section II describes the main properties of the B_s^0 system, such as its mass and width difference, and the time evolution of the B_s^0 system. Section III reports on studies of CP violation in B_s^0-$\bar{B}_s^0$ mixing. CP violation in interference of B_s^0 mixing and decay is discussed in Sec. IV with a detailed review of penguin contributions. Section V reviews studies of CP violation in B_s^0 decays as well as methods to derive the CKM angle γ from B_s^0 decays. Section VI examines the data reported in this review in the context of NP searches. Model-independent constraints on NP contributions inferred from B_s^0 data reported here are presented. Finally, Sec. VII gives an outlook of future developments. The Appendixes give details of the numerical updates of the standard model predictions for the mixing quantities.

[1] Instead of the notation α, β, and γ for the angles of the unitarity triangle, ϕ_2, ϕ_1, and ϕ_3 are also commonly used.

II. THE B_s^0 SYSTEM

A. Theory: Basic mixing quantities, time evolution of the B_s^0 system, and the heavy quark expansion

1. Mixing observables

The quantum mechanical time evolution of a decaying particle B with mass m_B and lifetime $\tau_B = 1/\Gamma_B$ is given as

$$|B(t)\rangle = e^{-im_B t-(\Gamma_B/2)t}|B(0)\rangle, \tag{1}$$

where Γ_B denotes the total decay width of the B particle. We now consider the system of neutral B_s^0 mesons, defined by their quark flavor content $|B_s^0\rangle = |(\bar{b}s)\rangle$, and their antiparticles $|\bar{B}_s^0\rangle = |(b\bar{s})\rangle$. Its time evolution is described by the following simple differential equation for a two-state system:

$$i\frac{d}{dt}\begin{pmatrix}|B_s^0(t)\rangle\\|\bar{B}_s^0(t)\rangle\end{pmatrix} = \left(\hat{M}^s - \frac{i}{2}\hat{\Gamma}^s\right)\begin{pmatrix}|B_s^0(t)\rangle\\|\bar{B}_s^0(t)\rangle\end{pmatrix}. \tag{2}$$

Naively one expects the diagonal entries of the 2×2 matrix $\hat{M}^s$ to be equal to the mass of the B_s^0 meson $M_{B_s^0}$, the diagonal entries of $\hat{\Gamma}^s$ to be equal to the decay rate of the B_s^0 meson Γ_s, and all nondiagonal entries to vanish. However, because of the weak interaction, the flavor eigenstate B_s^0 can transform into its antiparticle $\bar{B}_s^0$ and vice versa. This transition is governed by the so-called box diagrams, depicted in Fig. 1, and it gives rise to the off-diagonal elements M_{12}^s in $\hat{M}^s$ and Γ_{12}^s in $\hat{\Gamma}^s$. These box diagrams include contributions from virtual internal particles, denoted by M_{12}^s and contributions from internal on-shell particles, denoted by Γ_{12}^s. Only internal charm and up quarks are involved in Γ_{12}^s, while M_{12}^s is sensitive to all possible internal particles, and, in principle, also to heavy new physics particles.[2] Because of the CKM structure both M_{12}^s and Γ_{12}^s can be complex:

$$M_{12}^s = |M_{12}^s|e^{i\phi_M}, \tag{3}$$

$$\Gamma_{12}^s = |\Gamma_{12}^s|e^{i\phi_\Gamma}. \tag{4}$$

The CKM phases ϕ_M and ϕ_Γ are not physical, but depend on the phase convention used in the CKM matrix. Later on we see that

$$e^{i\phi_M} = \frac{V_{ts}^* V_{tb}}{V_{ts} V_{tb}^*}. \tag{5}$$

No such simple relation exists for ϕ_Γ, because Γ_{12}^s depends on three different CKM structures in the standard model.

In order to obtain the physical eigenstates of the mesons with a definite mass and decay rate, the matrices $\hat{M}^s$ and $\hat{\Gamma}^s$ have to be diagonalized. This gives the meson eigenstates $|B_{s,H}\rangle$ ($H =$ heavy) and $|B_{s,L}\rangle$ ($L =$ light) as linear combinations of the flavor eigenstates:

[2] There can also be new physics contributions to Γ_{12}^s, for example, by modified tree-level operators or by new $bs\tau\tau$ operators, as discussed in Sec. III.

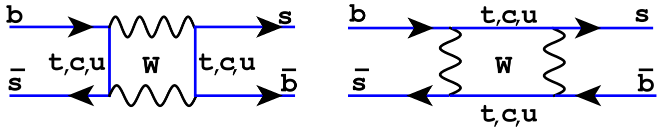

FIG. 1. Standard model diagrams for the transition between B_s^0 and $\bar{B}_s^0$ mesons. The contribution of internal on-shell particles (only the charm and the up quark can contribute) is denoted by Γ_{12}^s; the contribution of internal off-shell particles (all depicted particles can contribute) is denoted by M_{12}^s.

$$|B_{s,L}\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \tag{6}$$

$$|B_{s,H}\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle, \tag{7}$$

which are in general not orthogonal. The complex coefficients p and q fulfill $|p|^2 + |q|^2 = 1$ and the corresponding masses and decay rates of these states are denoted by M_L^s, M_H^s and Γ_L^s, Γ_H^s. The mass eigenstates of the B_s^0 mesons are almost CP eigenstates. Using the same conventions as, e.g., Dunietz, Fleischer, and Nierste (2001) for the CP properties and defining

$$CP|B_s^0\rangle = -|\bar{B}_s^0\rangle, \tag{8}$$

we get for the CP eigenstates of the B_s^0 meson

$$|B_s^{\text{even}}\rangle = \frac{1}{\sqrt{2}}(|B_s^0\rangle - |\bar{B}_s^0\rangle), \tag{9}$$

$$|B_s^{\text{odd}}\rangle = \frac{1}{\sqrt{2}}(|B_s^0\rangle + |\bar{B}_s^0\rangle). \tag{10}$$

In the absence of CP violation in mixing, which is a very small effect,[3] the heavy eigenstate is CP odd ($|B_{s,H}\rangle \approx |B_s^{\text{odd}}\rangle$) and the light one is CP even ($|B_{s,L}\rangle \approx |B_s^{\text{even}}\rangle$); in this case one has $p = 1/\sqrt{2}$ and $q = -1/\sqrt{2}$.

If we expand[4] the eigenvalues of $\hat{M}^s$ and $\hat{\Gamma}^s$ in powers of $|\Gamma_{12}^s/M_{12}^s| \approx 5 \times 10^{-3}$ in the SM, we can express the mass and decay rate differences as

$$\begin{aligned}\Delta M_s &:= M_H^s - M_L^s \\ &= 2|M_{12}^s|\left(1 - \frac{|\Gamma_{12}^s|^2 \sin^2\phi_{12}^s}{8|M_{12}^s|^2} + \cdots\right),\end{aligned} \tag{11}$$

$$\begin{aligned}\Delta\Gamma_s &:= \Gamma_L^s - \Gamma_H^s \\ &= 2|\Gamma_{12}^s| \cos\phi_{12}^s\left(1 + \frac{|\Gamma_{12}^s|^2 \sin^2\phi_{12}^s}{8|M_{12}^s|^2} + \cdots\right),\end{aligned} \tag{12}$$

with the mixing phase

$$\phi_{12}^s := \arg\left(-\frac{M_{12}^s}{\Gamma_{12}^s}\right) = \pi + \phi_M - \phi_\Gamma. \tag{13}$$

In contrast to ϕ_M and ϕ_Γ, this phase difference is physical. We follow here the definition given by Aaij *et al.* (2013e). In some references, for example, Anikeev *et al.* (2001) and Lenz and Nierste (2007), ϕ_{12}^s is denoted as ϕ_s. However, in the literature the notation ϕ_s is often used for different quantities, also related to CP violation in interference. We define the phase that appears in interference in Sec. IV. The correction factor $1/8|\Gamma_{12}^s/M_{12}^s|^2 \sin^2\phi_{12}^s$ in Eqs. (11) and (12) is of the order of 6×10^{-11} in the standard model and the current experimental bound for this factor is smaller than 5×10^{-5}, thus it can be safely neglected. Diagonalization of $\hat{M}^s$ and $\hat{\Gamma}^s$ also gives

$$\begin{aligned}\frac{q}{p} &= -e^{-i\phi_M}\left[1 - \frac{1}{2}\frac{|\Gamma_{12}^s|}{|M_{12}^s|}\sin\phi_{12}^s + \mathcal{O}\left(\frac{|\Gamma_{12}^s|^2}{|M_{12}^s|^2}\right)\right] \\ &= -\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}}\left[1 - \frac{a_{\text{fs}}^s}{2}\right] + \mathcal{O}\left(\frac{|\Gamma_{12}^s|^2}{|M_{12}^s|^2}\right),\end{aligned} \tag{14}$$

with the notation

$$a_{\text{fs}}^s = \frac{|\Gamma_{12}^s|}{|M_{12}^s|}\sin\phi_{12}^s. \tag{15}$$

Later on, in Sec. III, we will see that a_{fs}^s equals the so-called flavor-specific CP asymmetry. From Eq. (14) it follows also that, in the absence of CP violation in mixing, $q/p = -1$. In Eq. (14) again all terms of order $|\Gamma_{12}^s|^2/|M_{12}^s|^2$ can be discarded, many times also the term of order a_{fs}^s is not necessary.

2. Time evolution of neutral mesons

We now consider the time evolution of the flavor eigenstates of the B_s^0 mesons.[5] $|B_s^0(t)\rangle$ denotes a meson at time t that was produced as a B_s^0 meson at time $t = 0$. At a later time t, $|B_s^0(t)\rangle$ will have components both of $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$:

$$|B_s^0(t)\rangle = g_+(t)|B_s^0\rangle + \frac{q}{p}g_-(t)|\bar{B}_s^0\rangle, \tag{16}$$

$$|\bar{B}_s^0(t)\rangle = \frac{p}{q}g_-(t)|B_s^0\rangle + g_+(t)|\bar{B}_s^0\rangle, \tag{17}$$

with the coefficients

$$\begin{aligned}g_+(t) &= e^{-iM_s t}e^{-(1/2)\Gamma_s t} \\ &\times\left[\cosh\frac{\Delta\Gamma_s t}{4}\cos\frac{\Delta M_s t}{2} - i\sinh\frac{\Delta\Gamma_s t}{4}\sin\frac{\Delta M_s t}{2}\right],\end{aligned} \tag{18}$$

$$\begin{aligned}g_-(t) &= e^{-iM_s t}e^{-(1/2)\Gamma_s t} \\ &\times\left[-\sinh\frac{\Delta\Gamma_s t}{4}\cos\frac{\Delta M_s t}{2} + i\cosh\frac{\Delta\Gamma_s t}{4}\sin\frac{\Delta M_s t}{2}\right].\end{aligned} \tag{19}$$

[3] CP violation in mixing is expected to be of the order of 2×10^{-5} in the SM.

[4] Such an expansion does not hold in the charm system, because there $\Delta\Gamma$ and ΔM are of a similar size.

[5] A more detailed discussion of the B_s^0 mixing system and its time evolution can be found in Anikeev *et al.* (2001).

Here we used the averaged mass $M_{B_s^0}$ and decay rate Γ_s:

$$M_s = \frac{M_H^s + M_L^s}{2}, \qquad \Gamma_s = \frac{\Gamma_H^s + \Gamma_L^s}{2}. \tag{20}$$

Next we consider the time evolution of the decay rate for a B_s^0 meson that was initially (at time $t = 0$) tagged as a B_s^0 flavor eigenstate into an arbitrary final state f,

$$\Gamma[B_s^0(t) \to f] = N_f |\mathcal{A}_f|^2 (1 + |\lambda_f|^2) \times e^{-\Gamma t} \left\{ \frac{\cosh(\frac{\Delta\Gamma_s}{2} t)}{2} + \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \frac{\cos(\Delta M_s t)}{2} - \frac{2\Re(\lambda_f)}{1 + |\lambda_f|^2} \frac{\sinh(\frac{\Delta\Gamma_s}{2} t)}{2} - \frac{2\Im(\lambda_f)}{1 + |\lambda_f|^2} \frac{\sin(\Delta M_s t)}{2} \right\}. \tag{21}$$

Here N_f denotes a time-independent normalization factor, which includes phase space effects. The decay amplitude describing the transition of the flavor eigenstate B_s^0 in the final state f is denoted by $\mathcal{A}_f$; for the decay of a $\bar{B}_s^0$ state into f we use the notation $\bar{\mathcal{A}}_f$:

$$\mathcal{A}_f = \langle f | \mathcal{H}_{\text{eff}} | B_s^0 \rangle, \qquad \bar{\mathcal{A}}_f = \langle f | \mathcal{H}_{\text{eff}} | \bar{B}_s^0 \rangle. \tag{22}$$

The flavor changing weak quark transitions are described by an effective Hamiltonian including also perturbative and nonperturbative QCD effects. $\mathcal{H}_{\text{eff}}$ is described in more detail in Sec. IV.A. The amplitudes $\mathcal{A}_f$ and $\bar{\mathcal{A}}_f$ are typically governed by hadronic effects and they are very difficult to be calculated reliably in theory. In Sec. IV.A it is also shown that CP symmetries are governed by a single quantity λ_f, which is given by

$$\lambda_f = \frac{q}{p} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \approx - \frac{V_{ts} V_{tb}^*}{V_{ts}^* V_{tb}} \frac{\bar{A}_f}{A_f} \left[1 - \frac{a_{\text{fs}}^s}{2} \right]. \tag{23}$$

For the terms appearing on the right-hand side of Eq. (21) the following definitions are typically used:

$$\mathcal{A}_{CP}^{\text{dir}} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \tag{24}$$

$$\mathcal{A}_{CP}^{\text{mix}} = - \frac{2\Im(\lambda_f)}{1 + |\lambda_f|^2}, \tag{25}$$

$$\mathcal{A}_{\Delta\Gamma} = - \frac{2\Re(\lambda_f)}{1 + |\lambda_f|^2}. \tag{26}$$

$\mathcal{A}_{CP}^{\text{dir}}$ describes effects related to direct CP violation, which is described in Sec. V. This can be seen by neglecting CP violation in mixing, i.e., assuming $|q/p| = 1$ and considering the decay into a final state f, that is a CP eigenstate, i.e., $\bar{f} = \eta_{CP} f$. With these assumptions we get $|\lambda_f| = |\bar{\mathcal{A}}_{\bar{f}}| / |\mathcal{A}_f|$. A nonvanishing value for $\mathcal{A}_{CP}^{\text{dir}}$ is obtained for $|\lambda_f| \neq 1$ and this corresponds now to $|\bar{\mathcal{A}}_{\bar{f}}| \neq |\mathcal{A}_f|$, which is equivalent to direct CP violation. $\mathcal{A}_{CP}^{\text{mix}}$ encodes effects due to interference between mixing and decay, which is discussed in Sec. III and $\mathcal{A}_{\Delta\Gamma}$ is a correction factor, due to a finite value of the decay rate difference $\Delta\Gamma_s$. $\mathcal{A}_{\Delta\Gamma}$ also appears in the definition of the effective lifetimes τ^{eff}[6]:

$$\tau^{\text{eff}} = \tau_{B_s^0} \frac{1}{1 - y_s^2} \left(\frac{1 + 2\mathcal{A}_{\Delta\Gamma} y_s + y_s^2}{1 + \mathcal{A}_{\Delta\Gamma} y_s} \right) \tag{27}$$

with

$$\tau_{B_s^0} = \frac{1}{\Gamma_{B_s^0}}, \qquad y_s = \frac{\Delta\Gamma_s}{2\Gamma_{B_s^0}}. \tag{28}$$

Such lifetimes can also be used to determine $\Delta\Gamma_s$; examples of theoretical derivation can be found in Dunietz (1995), Hartkorn and Moser (1999), and Dunietz, Fleischer, and Nierste (2001) and are discussed in Sec. II.B. In general $\mathcal{A}_{CP}^{\text{dir}}$, $\mathcal{A}_{CP}^{\text{mix}}$, and $\mathcal{A}_{\Delta\Gamma}$ are governed by nonperturbative effects and there are no simple expressions for these quantities in terms of basic standard model parameters. These three quantities are, however, not independent and the following relation holds:

$$(\mathcal{A}_{CP}^{\text{dir}})^2 + (\mathcal{A}_{CP}^{\text{mix}})^2 + (\mathcal{A}_{\Delta\Gamma})^2 = 1. \tag{29}$$

Under certain circumstances, we get, however, simplified expressions for $\mathcal{A}_{CP}^{\text{dir}}$, $\mathcal{A}_{CP}^{\text{mix}}$, and $\mathcal{A}_{\Delta\Gamma}$:

(1) In the case of flavor-specific decays that are discussed in Sec. III, we have $\bar{\mathcal{A}}_f = 0$ and thus $\lambda_f = 0$, hence we get

$$\mathcal{A}_{CP}^{\text{fs,dir}} = 1, \qquad \mathcal{A}_{CP}^{\text{fs,mix}} = 0, \qquad \mathcal{A}_{\Delta\Gamma}^{\text{fs}} = 0, \tag{30}$$

$$\tau^{\text{fs,eff}} = \tau_{B_s^0} \frac{1 + y_s^2}{1 - y_s^2}. \tag{31}$$

(2) In Sec. IV we introduce the so-called golden modes, which have only one contributing CKM structure and one considers the decay into a CP eigenstate f. In that case we have $|\lambda_f| = 1$ and thus the simple relations

$$\mathcal{A}_{CP}^{\text{dir}} = 0, \qquad \mathcal{A}_{CP}^{\text{mix}} = -\Im(\lambda_f), \qquad \mathcal{A}_{\Delta\Gamma} = -\Re(\lambda_f). \tag{32}$$

Moreover the real and imaginary parts of λ_f are now given by simple combinations of CKM elements, which will be discussed in Sec. IV.

After discussing the decay of a B_s^0 meson into the final state f, we consider next the time evolution of the decay rate for a $\bar{B}_s^0$ meson into the same final state f. It is given by

[6]The total lifetime of the B_s^0 mesons is defined as $\tau(B_s^0) = 1/\Gamma_s = 2/(\Gamma_H^s + \Gamma_L^s)$. But, the decay of a B_s^0 meson is actually a superposition of a decay of a B_H meson and a B_L meson. Fitting such a decay with only one exponential probability density function (PDF) leads to the effective lifetime, which differs from the total lifetime.

$$\Gamma[\bar{B}_s^0(t)\to f] = N_f|\mathcal{A}_f|^2(1+|\lambda_f|^2)(1+a_{\rm fs}^s) \times e^{-\Gamma t}\left\{\frac{\cosh\left(\frac{\Delta\Gamma_s}{2}t\right)}{2} - \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\frac{\cos(\Delta M_s t)}{2} - \frac{2\Re(\lambda_f)}{1+|\lambda_f|^2}\frac{\sinh\left(\frac{\Delta\Gamma_s}{2}t\right)}{2} + \frac{2\Im(\lambda_f)}{1+|\lambda_f|^2}\frac{\sin(\Delta M_s t)}{2}\right\}. \quad (33)$$

The common prefactors, i.e., N_f and $|\mathcal{A}_f|^2(1+|\lambda_f|^2)$, typically cancel in CP asymmetries and we do not need to know their value. This is advantageous because the hadronic quantity $\mathcal{A}_f$ is notoriously difficult to calculate. Nevertheless, a dependence on the parameter λ_f will still be left in CP asymmetries. As stated, in general this parameter cannot be calculated from first principles. However, making some additional assumptions, such as neglecting penguin effects, a theoretical prediction for λ_f can be made, which enables then an extraction of fundamental standard model parameters (i.e., a combination of CKM elements) from the measurement of a CP asymmetry.

For completeness we also consider the decay of B_s^0 and $\bar{B}_s^0$ mesons into the CP conjugate of f, denoted by $\bar{f}$,

$$|\bar{f}\rangle = CP|f\rangle. \quad (34)$$

With the definitions

$$\bar{\mathcal{A}}_{\bar{f}} = \langle\bar{f}|\mathcal{H}_{\rm eff}|\bar{B}_s^0\rangle, \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{\mathcal{A}}_{\bar{f}}}{\mathcal{A}_{\bar{f}}}, \quad (35)$$

and assuming $N_f = N_{\bar{f}}$ we get for the time evolution of the decay rates

$$\Gamma[B_s^0(t)\to\bar{f}] = N_f|\bar{\mathcal{A}}_{\bar{f}}|^2(1+|\lambda_{\bar{f}}|^{-2})(1-a_{\rm fs}^s) \times e^{-\Gamma t}\left\{\frac{\cosh\left(\frac{\Delta\Gamma_s}{2}t\right)}{2} - \frac{1-|\lambda_{\bar{f}}|^{-2}}{1+|\lambda_{\bar{f}}|^{-2}}\frac{\cos(\Delta M_s t)}{2} - \frac{2\Re\left(\frac{1}{\lambda_{\bar{f}}}\right)}{1+|\lambda_{\bar{f}}|^{-2}}\frac{\sinh\left(\frac{\Delta\Gamma_s}{2}t\right)}{2} + \frac{2\Im\left(\frac{1}{\lambda_{\bar{f}}}\right)}{1+|\lambda_{\bar{f}}|^{-2}}\frac{\sin(\Delta M_s t)}{2}\right\}, \quad (36)$$

$$\Gamma[\bar{B}_s^0(t)\to\bar{f}] = N_f|\bar{\mathcal{A}}_{\bar{f}}|^2(1+|\lambda_{\bar{f}}|^{-2}) \times e^{-\Gamma t}\left\{\frac{\cosh\left(\frac{\Delta\Gamma_s}{2}t\right)}{2} + \frac{1-|\lambda_{\bar{f}}|^{-2}}{1+|\lambda_{\bar{f}}|^{-2}}\frac{\cos(\Delta M_s t)}{2} - \frac{2\Re\left(\frac{1}{\lambda_{\bar{f}}}\right)}{1+|\lambda_{\bar{f}}|^{-2}}\frac{\sinh\left(\frac{\Delta\Gamma_s}{2}t\right)}{2} - \frac{2\Im\left(\frac{1}{\lambda_{\bar{f}}}\right)}{1+|\lambda_{\bar{f}}|^{-2}}\frac{\sin(\Delta M_s t)}{2}\right\}. \quad (37)$$

These formulas can be used to extract the observables ΔM_s, $\Delta\Gamma_s$, and $a_{\rm fs}^s$ from experiment, which can then be compared with the theory predictions. According to Eqs. (11) and (12) these three observables are related to the matrix elements Γ_{12}^s and M_{12}^s, thus a standard model calculation of the three mixing observables requires a calculation of the box diagrams in Fig. 1.

3. Theoretical determination of M_{12}^s

The calculation of the standard model value for M_{12}^s is straightforward. In principle there are nine different combinations of internal quarks in the box diagrams; thus we get

$$M_{12}^s \propto \lambda_u^2 F(u,u) + \lambda_u\lambda_c F(u,c) + \lambda_u\lambda_t F(u,t) + \lambda_c\lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c\lambda_t F(c,t) + \lambda_t\lambda_u F(t,u) + \lambda_t\lambda_c F(t,c) + \lambda_t^2 F(t,t), \quad (38)$$

with the CKM structures $\lambda_q = V_{qs}^* V_{qb}$. The functions $F(x,y)$ depend on the masses of the internal quarks x and y normalized to the W boson mass. Using CKM unitarity, i.e., $\lambda_u + \lambda_c + \lambda_t = 0$, we get

$$M_{12}^s \propto \lambda_c^2[F(c,c) - 2F(u,c) + F(u,u)] + 2\lambda_c\lambda_t[F(c,t) - F(u,t) - F(u,c) + F(u,u)] + \lambda_t^2[F(t,t) - 2F(u,t) + F(u,u)]. \quad (39)$$

From this equation one sees the arising GIM cancellation (Glashow, Iliopoulos, and Maiani, 1970) in all three terms: if all masses would be equal, each of the three terms would vanish. As a result any constant term in the functions $F(x,y)$ also cancels in M_{12}^s and only the mass dependent terms will survive. An explicit calculation shows that $F(x,y)$ strongly grows with the masses [see Eq. (42)], thus there is a severe GIM cancellation in the first two terms (m_u/M_W and m_c/M_W can be very well approximated by zero), while the third term will give a sizable contribution ($m_t/M_W > 1$). Since the CKM structures all have similar size [$\lambda_c \propto \lambda^4 \propto \lambda_t$, with the Wolfenstein parameter λ (Wolfenstein, 1983)] we get to a good approximation

$$M_{12}^s \propto \lambda_t^2[F(t,t) - 2F(u,t) + F(u,u)] \quad (40)$$

$$\propto \lambda_t^2 S_0\left(\frac{m_t^2}{M_W^2}\right), \quad (41)$$

where S_0 denotes the Inami-Lim function (Inami and Lim, 1981):

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x\ln x}{2(1-x)^2}. \quad (42)$$

In that respect it is sometimes stated that only the top quark contributes to M_{12}^s. Formally the process of calculating M_{12}^s can be viewed as performing an operator product expansion (OPE) by integrating out the heavy W boson and the heavy top quark. Since both of these masses are far above the hadronic scale and the b quark mass, there is no doubt in the applicability of the OPE. This will change in the discussion of Γ_{12}^s. The complete calculation of M_{12}^s yields

$$M_{12}^s = \frac{G_F^2}{12\pi^2}\lambda_t^2 M_W^2 S_0(x_t) B f_{B_s}^2 M_{B_s^0}\hat{\eta}_B, \quad (43)$$

with simple prefactors: the Fermi constant G_F, the masses of the W boson M_W, and of the B_s meson $M_{B_s^0}$, and the normalization factor $1/12\pi^2$. As seen earlier there is only one CKM structure contributing $\lambda_t = V_{ts}^* V_{tb}$. The CKM elements are the only place in Eq. (43) where an imaginary part can arise. By writing

$$\lambda_t^2 = |\lambda_t^2| \frac{\lambda_t}{\lambda_t^*} = |\lambda_t^2| e^{i\phi_M} \tag{44}$$

we get the explicit dependence of the phase ϕ_M on CKM parameters, which was already stated in Eq. (5). As discussed, the result of the one-loop diagrams given in Fig. 1 is denoted by the Inami-Lim function $S_0(x_t = [\bar{m}_t(\bar{m}_t)]^2/M_W^2)$, where $\bar{m}_t(\bar{m}_t)$ is the $\overline{\mathrm{MS}}$ mass (Bardeen *et al.*, 1978) of the top quark. Perturbative two-loop QCD corrections are compressed in the factor $\hat{\eta}_B \approx 0.84$; they were calculated by Buras, Jamin, and Weisz (1990). Performing the calculation of M_{12}^s one gets a spinor operator for each external quark in the box diagram. Together with the arising Dirac matrices they form the four quark $\Delta B = 2$ operator

$$Q = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1-\gamma_5) b^\beta. \tag{45}$$

α and β are the color indices of the b and s quark spinors. All hadronic effects that describe the binding of the quarks into meson states as well as the nonperturbative QCD effects contributing to the transition of the B_s^0 meson into the $\bar{B}_s^0$ meson and vice versa are encoded in the hadronic matrix element of the operator Q. The hadronic matrix element[7] of this operator is parametrized in terms of a decay constant f_{B_s} and a bag parameter B:

$$\langle Q \rangle \equiv \langle \bar{B}_s^0 | Q | B_s^0 \rangle = \tfrac{8}{3} M_{B_s^0}^2 f_{B_s}^2 B(\mu). \tag{46}$$

The factor $8/3 = 2(1 + 1/N_c)$ stems from the color structure. It ensures that the bag parameter B obtains the value 1 in vacuum insertion approximation (VIA).[8] We also indicated the renormalization scale dependence of the bag parameter; in our analysis we take $\mu = m_b$.

Sometimes a different notation for the QCD corrections and the bag parameter is used in the literature [e.g., by the Flavour Lattice Averaging Group (FLAG) (Aoki *et al.*, 2014)], $(\eta_B, \hat{B})$ instead of $(\hat{\eta}_B, B)$ with

$$\hat{\eta}_B B =: \eta_B \hat{B} \tag{47}$$

$$= \eta_B \alpha_s(\mu)^{-6/23} \left[1 + \frac{\alpha_s(\mu)}{4\pi} \frac{5165}{3174} \right] B, \tag{48}$$

$$\hat{B} = 1.515\,99 B. \tag{49}$$

The parameter $\hat{B}$ has the advantage of being renormalization scale and scheme independent. A commonly used standard model prediction of ΔM_s was given by (Lenz and Nierste, 2011)

$$\Delta M_s^{\mathrm{SM},2011} = 17.3 \pm 2.6\ \mathrm{ps}^{-1}. \tag{50}$$

Using the most recent numerical inputs [G_F, M_W, M_{B_s}, and m_b from the Particle Data Group (PDG) (Olive *et al.*, 2014), the top quark mass from ATLAS, CDF, CMS, and D0 Collaborations (2014), the nonperturbative parameters from FLAG (Aoki *et al.*, 2014), and CKM elements from the CKMfitter group (Charles *et al.*, 2005)] [similar values can be taken from the UTfit group (Bona *et al.*, 2006b)], we predict the mass difference of the neutral B_s^0 mesons to be

$$\Delta M_s^{\mathrm{SM},2015} = 18.3 \pm 2.7\ \mathrm{ps}^{-1}. \tag{51}$$

Here the dominant uncertainty comes from the lattice predictions for the nonperturbative parameters B and f_{B_s}, giving a relative error of 14%. This input did not change compared to the 2011 prediction from Lenz and Nierste (2011). The uncertainty in the CKM elements contributes about 5% to the error budget. The CKM parameters were determined assuming unitarity of the 3×3 CKM matrix. For some new physics models this assumption might have to be given up, leading to larger CKM uncertainties. The uncertainties due to m_t, m_b, and α_s can be safely neglected at the current stage. A detailed discussion of the input parameters and the error budget is given in Appendix B.

There is, however, a word of caution: in the theoretical prediction (51) we use the nonperturbative value from FLAG $f_{B_s}\sqrt{B} = 216 \pm 15$ MeV[9] (with $N_f = 2+1$ active flavors in the lattice simulations). However, only one number [from the HPQCD Collaboration (Gamiz *et al.*, 2009)] is included in the FLAG average. It would of course be advantageous to have more numbers from different collaborations and there are currently some more (mostly preliminary) numbers on the market:

$$f_{B_s}\sqrt{B} \approx 200\ \mathrm{MeV} \Rightarrow \Delta M_s^{\mathrm{HPQCD}} \approx 15.7\ \mathrm{ps}^{-1}, \tag{52}$$

$$f_{B_s}\sqrt{B} \approx 211\ \mathrm{MeV} \Rightarrow \Delta M_s^{\mathrm{ETMC}} \approx 17.4\ \mathrm{ps}^{-1}, \tag{53}$$

$$f_{B_s}\sqrt{B} \approx 227\ \mathrm{MeV} \Rightarrow \Delta M_s^{\mathrm{Fermilab}} \approx 20.2\ \mathrm{ps}^{-1}. \tag{54}$$

HPQCD updated their results in Dowdall *et al.* (2014) and for our numerical estimate in Eq. (52) we had to read off the numbers from Fig. 3 in their proceedings

[7]Throughout this review we use the conventional relativistic normalization for the B_s^0 meson states, i.e., $\langle \bar{B}_s^0 | B_s^0 \rangle = 2EV$ (E is energy, V is volume).

[8]The matrix element in Eq. (46) can be rewritten by inserting a complete set of states between the two currents of the operator Q, given in Eq. (45). Next this expression is equated to the contribution of the vacuum state only times a correction factor B (bag factor) that corrects for the neglect of all higher states in the sum. Setting the bag parameter to 1 corresponds to the vacuum insertion approximation. Many lattice evaluations show that this assumption seems to be well justified (Bazavov *et al.*, 2016). The remaining matrix elements of the form $\langle B_s^0 | \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha | 0 \rangle$ are proportional to $f_{B_s} p_\mu$, where p_μ is the four-momentum of the B_s^0 meson.

[9]This value is derived from the FLAG value of $f_{B_s}\sqrt{\hat{B}}$. It is by accident equal to the value of $f_{B_d}\sqrt{\hat{B}}$ quoted from FLAG.

(Dowdall *et al.*, 2014). Their investigations suggest a possible error of about 5% for $f_{B_s}^2 B$ in the near future, which would be a major improvement. The European Twisted Mass Collaboration (ETMC) number stems from Carrasco *et al.* (2014); it is obtained with only two active flavors in the lattice simulation. The Fermilab-MILC number is an update for the LATTICE 2015 conference of Bouchard *et al.* (2011).[10] The range of numbers seems to be nicely covered by the current FLAG average, but it would of course be interesting to have final numbers and an average for the values given in Eqs. (52), (53), and (54). There is also a large value from RBC-UKQCD presented at LATTICE 2015, $f_{B_s}\sqrt{B} = 262$ MeV [update of Aoki *et al.* (2015)]. However, this number is obtained in the static limit and currently missing $1/m_b$ corrections are expected to be very sizable. Thus we do not give a value of ΔM_s for this lattice value. For our numerical analysis, we use only the value from FLAG. In summary, an uncertainty of about ±5% might be feasible for the theory prediction of ΔM_s taking future lattice improvements into account.

4. Heavy quark expansion

The calculation of the decay rate difference $\Delta\Gamma_s$ is more involved. In the box diagrams depicted in Fig. 1, we have to take into account now only the internal up and charm quarks. Integrating out all heavy particles (in this case only the W boson) we are not left with a local $\Delta B = 2$ operator as in the case of M_{12}^s, but with a bilocal object depicted in Fig. 2. To get to the level of local operators, which is needed for being able to make a theory prediction, a second operator product expansion is required. The second OPE relies on the smallness of the parameter Λ/m_b, where Λ is expected to be of the order of the hadronic scale $\Lambda_{\rm QCD}$ and m_b is the b quark mass. More precisely the HQE is an expansion in Λ normalized to the momentum release of the decay given by $\sqrt{M_i^2 - M_f^2}$, with the initial mass M_i and the final state masses M_f. For massless final states an expansion in Λ/m_b is generally expected to converge, while for a transition like $b \to c\bar{c}s$ it is not *a priori* clear, whether $\Lambda/\sqrt{m_b^2 - 4m_c^2}$ is small enough to get a converging series. Thus the validity of this so-called heavy quark expansion (HQE) has to be tested by comparisons of experiment and theory. The formulation of the HQE is based on work by Voloshin and Shifman (Khoze and Shifman, 1983; Shifman and Voloshin, 1985; Bigi and Uraltsev, 1992; Bigi, Uraltsev, and Vainshtein, 1992; Blok and Shifman, 1993a, 1993b) and in detail described by Lenz (2014).[11] The HQE also applies for lifetimes and totally inclusive decay rates of heavy hadrons. Historically there had been several discrepancies between experiment and theory that questioned the validity of the HQE such as the following:

- In the mid-1990s the *missing charm puzzle* [see, e.g., Lenz (2000) for a brief review], a disagreement between experiment and theory about the average number of charm quarks produced per b decay, was a hot topic. A possible interpretation could be new physics, but a violation of quark hadron duality, i.e., a violation of the validity of the HQE, was also considered to solve this discrepancy, in particular, in the decay $b \to c\bar{c}s$. This issue has now been resolved, by more precise data and improved theory predictions (Krinner, Lenz, and Rauh, 2013), leading to agreement between experiment and theory within uncertainties.
- For a long time the measured Λ_b lifetime was considerably shorter than its predicted value [according to estimates of the HQE; see, e.g., Bigi, Shifman, and Uraltsev (1997) and Voloshin (2000)]. This issue has been resolved by recent measurements, mostly by the LHCb Collaboration (Aaij *et al.*, 2013i, 2014j, 2014k) but also by the Tevatron experiments (Aaltonen *et al.*, 2014). The history of the Λ_b lifetime—HFAG quoted 2003 a value of $\tau_{\Lambda_b}^{\rm HFAG\,2003} = 1.229 \pm 0.080$ ps, which is about 3 standard deviations away from the 2015 average of $\tau_{\Lambda_b}^{\rm HFAG\,2015} = 1.466 \pm 0.010$ ps—and also (sometimes embarrassing) theoretical attempts to obtain low theory values are discussed in detail in the review of Lenz (2014). The low experimental values reported in the early measurements are mostly determined using semileptonic decays with an undetectable neutrino (Stone, 2014), while new measurements use nonleptonic decays with fully reconstructed final states. The large range in the theory predictions for the Λ_b lifetime stems from our missing knowledge about the size of the hadronic matrix elements. Some theory groups tried to create some extraordinary large enhancements of these matrix elements in order to describe the experimental data, while other groups, including, for example, Bigi and Uraltsev, stuck to theory estimates that were in conflict with the old measurements, but agree perfectly with the new ones. The current status of lifetimes is depicted in Fig. 3. No lifetime puzzle exists anymore. The theoretical precision is strongly limited by a lack of up-to-date values for the arising nonperturbative parameters. For the Λ_b baryon the most recent lattice numbers stem from 1999 (Di Pierro, Sachrajda, and Michael, 1999) and for the B mesons the most recent numbers are from 2001 (Becirevic, 2001). This lack of theoretical investigations also limits our current knowledge about the intrinsic precision of the HQE.

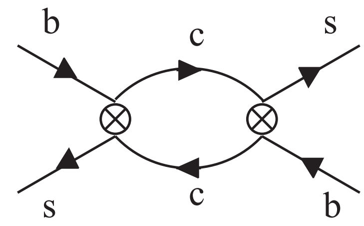

FIG. 2. To Γ_{12}^s only the box diagrams with internal up and charm quarks are contributing in the standard model; see Fig. 1. Integrating out the heavy W boson, we are left with a bilocal object, which is shown here for internal charm and anticharm quarks.

[10]During submission, Fermilab-MILC presented final results in Bazavov *et al.* (2016). The numerical effect of these new inputs on mixing observables was studied by Jubb *et al.* (2016).

[11]See, e.g., Bigi *et al.* (1989) and Bigi, Shifman, and Uraltsev (1997) for early reviews.

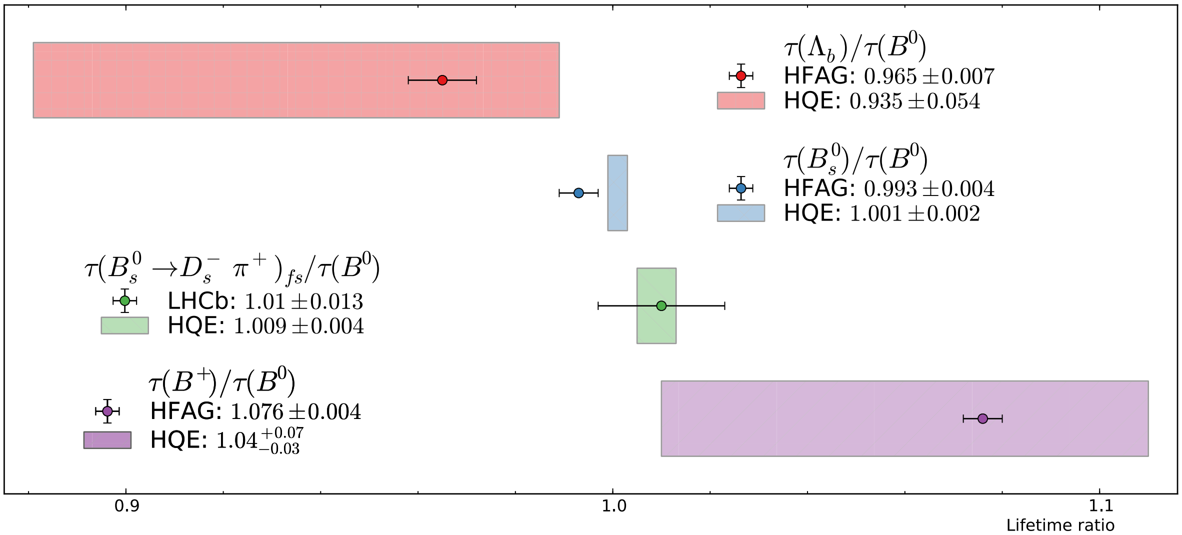

FIG. 3. Comparison of HQE predictions for lifetime ratios of heavy hadrons with experimental values. The theory values are taken from Lenz (2014). Experimental numbers are taken from (fall 2014) HFAG (Amhis *et al.*, 2014).

- Since $\Delta\Gamma_s$ is dominated by a $b \to c\bar{c}s$ transition, the applicability of the HQE was, in particular, questioned for $\Delta\Gamma_s$; see, e.g., Ligeti *et al.* (2010) and the discussion in Lenz (2011) and references therein. In the last years this was also related to the unexpected measurement of a large value of the dimuon asymmetry by the D0 Collaboration (Abazov *et al.*, 2010a, 2010b, 2011, 2014). In 2012 the issue of $\Delta\Gamma_s$ was solved experimentally by a direct measurement of this quantity by the LHCb Collaboration. The current HFAG (Amhis *et al.*, 2014) average, combining values from LHCb, ATLAS, CMS, D0, and CDF, is in perfect agreement with the HQE prediction from Lenz and Nierste (2011), which is based on the calculations of Beneke, Buchalla, Greub *et al.* (1999), Beneke *et al.* (2003), Ciuchini *et al.* (2003), and Lenz and Nierste (2007). This is discussed in detail next.

All in all the HQE has been experimentally proven to be very successful and one could try next to test its applicability also for charm physics; see, e.g., Bobrowski *et al.* (2010) and Lenz and Rauh (2013) for some recent investigations. Charm studies would be helpful for assessing the intrinsic uncertainties of the HQE. Having more confidence in the validity of HQE, it can now also be applied to quantities that are sensitive to new physics, in particular, to the semileptonic CP asymmetries, which are discussed in Sec. III. A recent study of the possible size of duality-violating effects (i.e., deviations from the HQE expectations) can be found in Jubb *et al.* (2016).

5. Theoretical determination of Γ_{12}^s

According to the HQE, the off-diagonal element Γ_{12}^s of the B_s^0 mixing matrix can be expanded as a power series in the inverse of the heavy b-quark mass m_b and the strong coupling α_s:

$$\Gamma_{12}^s = \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{s,(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{s,(1)} + \cdots\right) + \frac{\Lambda^4}{m_b^4}(\Gamma_4^{s,(0)} + \cdots) + \cdots. \tag{55}$$

Λ denotes a hadronic scale, which is assumed to be of the order of $\Lambda_{\rm QCD}$, but its actual value has to be determined by a nonperturbative calculation. Each of the $\Gamma_i^{s,(j)}$ is a product of perturbative Wilson coefficients and nonperturbative matrix elements. In Γ_3^s these matrix elements arise from dimension 6 four quark operators, in Γ_4^s from dimension 7 operators, and so on.

The leading term in Eq. (55) $\Gamma_3^{s,(0)}$ was calculated already quite long ago by Ellis *et al.* (1977), Hagelin (1981), Franco, Lusignoli, and Pugliese (1982), Chau (1983), Buras, Slominski, and Steger (1984), and Khoze *et al.* (1987). Here three different four quark operators arise; besides Q from Eq. (45) these are

$$Q_S = \bar{s}^\alpha(1+\gamma_5)b^\alpha \times \bar{s}^\beta(1+\gamma_5)b^\beta, \tag{56}$$

$$\tilde{Q}_S = \bar{s}^\alpha(1+\gamma_5)b^\beta \times \bar{s}^\beta(1+\gamma_5)b^\alpha. \tag{57}$$

The general structure of the leading term Γ_3^s has three ($uc = cu$) different CKM contributions

$$\Gamma_3^s = -\sum_{x=u,c}\sum_{y=u,c}\lambda_x\lambda_y\Gamma_{12}^{s,xy} \tag{58}$$

and each factor $\Gamma_{12}^{s,xy}$ has contributions of the three operators Q, Q_S, and $\tilde{Q}_S$:

$$\Gamma_{12}^{s,xy} = \Gamma_{xy}^{s,Q}\langle Q\rangle + \Gamma_{xy}^{s,Q_S}\langle Q_S\rangle + \Gamma_{xy}^{s,\tilde{Q}_S}\langle\tilde{Q}_S\rangle. \tag{59}$$

The matrix elements of the newly arising operators are typically parametrized as

$$\langle Q_S\rangle \equiv \langle\bar{B}_s|Q_S|B_s\rangle = -\tfrac{5}{3}M_{B_s^0}^2 f_{B_s}^2 B_S', \tag{60}$$

$$\langle\tilde{Q}_S\rangle \equiv \langle\bar{B}_s|\tilde{Q}_S|B_s\rangle = \tfrac{1}{3}M_{B_s^0}^2 f_{B_s}^2 \tilde{B}_S', \tag{61}$$

with the modified bag parameters

$$B'_X = \frac{M^2_{B_s^0}}{[\bar{m}_b(\bar{m}_b) + \bar{m}_s(\bar{m}_b)]^2} B_X \approx 1.577\,06 B_X. \quad (62)$$

In the vacuum insertion approximation, the unmodified bag parameters are equal to 1. More reliable values can be obtained by using nonperturbative methods such as QCD sum rules[12] or lattice QCD. Q, Q_S, and $\tilde{Q}_S$ were determined by several lattice groups, who actually determined all five operators of the so-called supersymmetric (SUSY) basis.[13] Becirevic *et al.* (2002), Carrasco *et al.* (2014), and Dowdall *et al.* (2014) used the notation O_1, O_2, and O_3 for these three operators:

$$Q \equiv O_1, \qquad Q_S \equiv O_2, \qquad \tilde{Q}_S \equiv O_3. \quad (63)$$

In the case of Bouchard *et al.* (2011) there is also an additional factor of 4 present:

$$Q \equiv 4O_1, \qquad Q_S \equiv 4O_2, \qquad \tilde{Q}_S \equiv 4O_3. \quad (64)$$

Becirevic *et al.* (2002) and Carrasco *et al.* (2014) used the same definitions of the bag parameters as we do:

$$B \equiv B_1, \qquad B_s \equiv B_2, \qquad \tilde{B}_S \equiv B_3, \quad (65)$$

while Dowdall *et al.* (2014) and Bouchard *et al.* (2011) used the modified bag parameters

$$B \equiv B_1, \qquad B'_s \equiv B_2, \qquad \tilde{B}'_S \equiv B_3. \quad (66)$$

It was found that these three operators are not independent (Beneke, Buchalla, and Dunietz, 1996) and that the following relation holds:

$$R_0 = Q_S + \alpha_1 \tilde{Q}_S + \frac{\alpha_2}{2} Q = 0 + \mathcal{O}\left(\frac{\Lambda}{m_b}\right), \quad (67)$$

with the coefficients [obtained by Beneke, Buchalla, Greub *et al.* (1999) using the renormalization scheme described there]

$$\alpha_1 = 1 + \frac{\alpha_s(\mu)}{3\pi}\left(12 \ln\frac{\mu}{m_b} + 6\right), \quad (68)$$

$$\alpha_2 = 1 + \frac{\alpha_s(\mu)}{3\pi}\left(6 \ln\frac{\mu}{m_b} + \frac{13}{2}\right). \quad (69)$$

With the help of Eq. (67) one can substitute one of the three operators; historically $\tilde{Q}_S$ was eliminated, obtaining

$$\Gamma_{12}^{s,xy} = \left[\Gamma_{xy}^{s,Q} - \frac{1}{2}\frac{\alpha_2}{\alpha_1}\Gamma_{xy}^{s,\tilde{Q}_S}\right]\langle Q\rangle + \left[\Gamma_{xy}^{s,Q_S} - \frac{1}{\alpha_1}\Gamma_{xy}^{s,\tilde{Q}_S}\right]\langle Q_S\rangle + \mathcal{O}\left(\frac{\Lambda}{m_b}\right), \quad (70)$$

which was denoted in the literature as

$$\Gamma_{12}^{s,xy} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}[G^{s,xy}\langle Q\rangle - G_S^{s,xy}\langle Q_S\rangle] + \Gamma_{12,1/m_b}^{s,xy} = \frac{G_F^2 m_b^2 f_{B_s}^2 M_{B_s^0}}{24\pi}\left[\frac{8}{3}G^{s,xy}B + \frac{5}{3}G_S^{s,xy}B'_S\right] + \Gamma_{12,1/m_b}^{xy}, \quad (71)$$

where the Wilson coefficients $G^{s,xy}$ and $G_S^{s,xy}$ contain the result of the calculation of the box diagrams with internal on-shell up and/or charm quarks; $xy \in \{uu, uc, cc\}$. Neglecting the mass of the charm quark and penguin contributions, $G^{s,xy}$ and $G_S^{s,xy}$ read in leading order (LO) QCD

$$G^{s,xy} = 3C_1^2 + 2C_1C_2 + \tfrac{1}{2}C_2^2, \quad (72)$$

$$G_S^{s,xy} = -(3C_1^2 + 2C_1C_2 - C_2^2), \quad (73)$$

where $C_{1,2}$ denote the $\Delta B = 1$ Wilson coefficients of the effective Hamiltonian describing b quark decays (in our notation C_2 corresponds to the color allowed operator). Early LO QCD estimates of $G^{s,xy}$ and $G_S^{s,xy}$ can be found in Ellis *et al.* (1977), Hagelin (1981), Franco, Lusignoli, and Pugliese (1982), Chau (1983), Buras, Slominski, and Steger (1984), and Khoze *et al.* (1987). Next-to-leading order (NLO) QCD corrections, i.e., $\Gamma_3^{s,(1)}$ in Eq. (55), were done for the first time by Beneke, Buchalla, Greub *et al.* (1999), and they turned out to be quite large. This work was also a proof of the IR safety of the HQE by direct calculation. The corresponding NLO QCD diagrams are shown in Fig. 4. General arguments for such a proof were given already in the seminal paper of Bigi and Uraltsev (1992), which resolved the theoretical issues that were prohibiting a systematic expansion in the inverse of the heavy b quark mass. Five years later the calculation of the QCD corrections was confirmed and also subleading CKM structures were included by Beneke *et al.* (2003) and Ciuchini *et al.* (2003). In these papers the full expressions for $G^{s,xy}$ and $G_S^{s,xy}$ are given; they also include contributions from the QCD penguin operators $Q_1 - Q_6$ and the chromomagnetic penguin operator Q_8. Beneke *et al.* (2002) found that the use of $\bar{m}_c(\bar{m}_b)$ (charm mass at the bottom mass scale) instead of $\bar{m}_c(\bar{m}_c)$ sums up large logs of the form $m_c^2/m_b^2 \ln m_c^2/m_b^2$ to all orders; we thus use the parameter $\bar{z}$ in our numerical analysis, given by

$$\bar{z} = \left(\frac{\bar{m}_c(\bar{m}_b)}{\bar{m}_b(\bar{m}_b)}\right)^2. \quad (74)$$

In Eq. (71) the term $\Gamma_{12,1/m_b}^{s,xy}$ denotes subleading $1/m_b$ corrections to Γ_{12}^s—in Eq. (55) these terms were called $\Gamma_4^{s,(0)}$. Such subleading $1/m_b$ corrections were first calculated

[12] A QCD sum rule determination of $\langle Q\rangle$ is given by Korner *et al.* (2003). However, we will not use the number obtained there in our analysis.

[13] In the standard model only Q contributes to ΔM_s, while in extensions of the standard model additional contributions of new operators can appear. The whole set of these operators is called a SUSY basis and is typically denoted by $O_1 \cdots O_5$. It turns out, however, that all five operators are also needed for a precise standard model prediction of $\Delta\Gamma_s$.

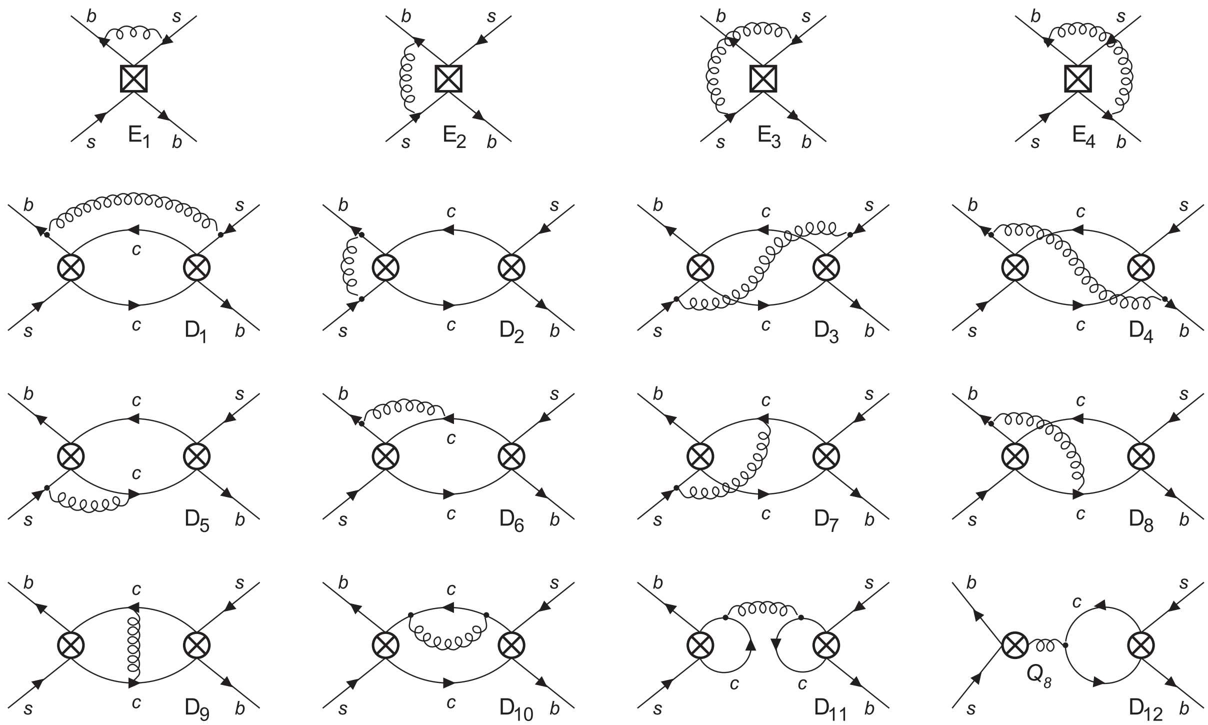

FIG. 4. Standard model diagrams contributing to Γ_{12}^s at NLO QCD, i.e., $\Gamma_3^{s,(1)}$. For obtaining the NLO QCD Wilson coefficients one has to calculate one-loop corrections to the $\Delta B = 2$ operators (E_1–E_4) and also two-loop corrections to the double insertion of $\Delta B = 1$ operators (D_1–D_{12}). An explicit cancellation of all infrared singularities in the matching was shown by Beneke, Buchalla, Greub *et al.* (1999) and later by Beneke *et al.* (2003) and Ciuchini *et al.* (2003). Such an IR safety is crucial for the consistency of the HQE. The next future steps will be the determination of $\Gamma_4^{s,(1)}$ and $\Gamma_3^{s,(2)}$. For that one has to take into account in these diagrams a nonvanishing strange quark momentum and one has to add a further gluon in these diagrams.

by Beneke, Buchalla, and Dunietz (1996) and they also turned out to be quite sizable. The operators arising in $\Gamma_4^{s,(0)}$ are of dimension 7 (e.g., four quark operators with one derivative), they are denoted by R_0, R_1, R_2, and R_3, as well as the color-rearranged counterparts $\tilde{R}_1$, $\tilde{R}_2$, and $\tilde{R}_3$; see, e.g., Lenz and Nierste (2007) for more details. The operators R_0, R_1, and $\tilde{R}_1$ can be reduced to four quark operators [see, e.g., the definition of R_0 in Eq. (67)] and thus they can be studied with current lattice technologies; their results can be deduced from Becirevic *et al.* (2002), Bouchard *et al.* (2011), Carrasco *et al.* (2014), and Dowdall *et al.* (2014), who were calculating the full five-dimensional SUSY basis of $\Delta B = 2$ operators. All of those five independent operators (Q, Q_S, $\tilde{Q}_S$, R_1, and $\tilde{R}_1$) contribute to Γ_{12}^s. The genuine dimension 7 operators R_2, R_3, $\tilde{R}_2$, and $\tilde{R}_3$ are considerably more complicated. For the corresponding matrix elements currently no lattice determination is available, so we have to rely on vacuum insertion approximation, i.e., the bag parameters B_{R_2}, B_{R_3}, $B_{\tilde{R}_2}$, and $B_{\tilde{R}_3}$ are set to 1. First steps toward a nonperturbative determination of these matrix elements within the framework of QCD sum rules have been done by Mannel, Pecjak, and Pivovarov (2007, 2011). Here a more complete study would be very desirable, because as seen later these parameters currently give the dominant uncertainty to Γ_{12}^s.

The precision of the theory prediction can be further improved by using ratios of theoretical expressions and by choosing an optimal operator basis:

- Γ_{12}^s depends on $f_{Bs}^2 B$, which is currently not very well known. Thus, it might be advantageous to consider the ratio Γ_{12}^s/M_{12}^s, where the decay constant cancels. One gets from this ratio

$$\mathrm{Re}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) = -\frac{\Delta\Gamma_s}{\Delta M_s}, \qquad \mathrm{Im}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) = a_{fs}^s. \tag{75}$$

The ratio Γ_{12}^s/M_{12}^s can be further modified by using the CKM unitarity ($\lambda_u + \lambda_c + \lambda_t = 0$):

$$-\frac{\Gamma_{12}^s}{M_{12}^s} = \frac{\lambda_c^2\Gamma_{12}^{s,cc} + 2\lambda_c\lambda_u\Gamma_{12}^{s,uc} + \lambda_u^2\Gamma_{12}^{s,uu}}{\lambda_t^2\tilde{M}_{12}^s} \tag{76}$$

$$= \frac{\Gamma_{12}^{s,cc}}{\tilde{M}_{12}^s} + 2\frac{\lambda_u}{\lambda_t}\frac{\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}}{\tilde{M}_{12}^s} + \left(\frac{\lambda_u}{\lambda_t}\right)^2\frac{\Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc} + \Gamma_{12}^{s,uu}}{\tilde{M}_{12}^s} \tag{77}$$

$$= -10^{-4}\left[c + a\frac{\lambda_u}{\lambda_t} + b\left(\frac{\lambda_u}{\lambda_t}\right)^2\right], \tag{78}$$

where $\tilde{M}_{12}^s$ is defined in such a way that only the CKM dependence of M_{12}^s in Eq. (43) is split off. Equation (78) introduces the a, b, and c notation of Beneke *et al.* (2003). In the ratios $\Gamma_{12}^{s,xy}/\tilde{M}_{12}^s$ which are the building blocks of the parameters a, b, and c many quantities cancel, in particular, the decay constant f_{B_s}, the mass of the B_s meson, and the Fermi constant. We get

$$\frac{\Gamma_{12}^{s,xy}}{\tilde{M}_{12}^s} = \frac{\pi m_b^2 [8G^{s,xy} + 5G_S^{s,xy} B_S'/B + \mathcal{O}(1/m_b)]}{6 M_W S_0(x_t) \hat{\eta}_B}. \tag{79}$$

Now the first term in Eq. (79), proportional to $G^{s,xy}$, is completely free of any nonperturbative contribution. It can be completely determined in perturbative QCD. Because of all these cancellations a, b, and c are theoretically quite clean and they are also almost identical for B_d and B_s mesons, except for differences in the primed bag factors and in the $1/m_b$ corrections. The way of writing Γ_{12}^s/M_{12}^s in Eqs. (77) and (78) can be viewed as a Taylor expansion in the small ratio of CKM parameters λ_u/λ_t, for which we get the following numerical values:

$$\frac{\lambda_u}{\lambda_t} = -8.0486 \times 10^{-3} + 1.81082 \times 10^{-2} I, \tag{80}$$

$$\left(\frac{\lambda_u}{\lambda_t}\right)^2 = -2.63126 \times 10^{-4} - 2.91491 \times 10^{-4} I. \tag{81}$$

Moreover, a pronounced GIM (Glashow, Iliopoulos, and Maiani, 1970) cancellation is arising in the coefficients a and b in Eq. (78). With the newest input parameters described in Appendix A, we get for the numerical values of a, b, and c:

$$c = -48.0 \pm 8.3 \ (-49.5 \pm 8.5), \tag{82}$$

$$a = +12.3 \pm 1.4 \ (+11.7 \pm 1.3), \tag{83}$$

$$b = +0.79 \pm 0.12 \ (+0.24 \pm 0.06). \tag{84}$$

The numbers in brackets denote the corresponding values for the B^0 system. Putting all this together, we see that the real part of Γ_{12}^s/M_{12}^s is absolutely dominated by the coefficient c, while for the imaginary part only a and to a lesser extent b are contributing. We get

$$\Re\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) = 10^{-4}\left(c + a\Re\left[\frac{\lambda_u}{\lambda_t}\right] + b\Re\left[\frac{\lambda_u^2}{\lambda_t^2}\right]\right)$$
$$\Rightarrow \frac{\Delta\Gamma_s}{\Delta M_s} \approx -10^{-4} c, \tag{85}$$

$$\Im\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) = 10^{-4}\left(a\Im\left[\frac{\lambda_u}{\lambda_t}\right] + b\Im\left[\frac{\lambda_u^2}{\lambda_t^2}\right]\right)$$
$$\Rightarrow a_{\rm fs}^s \approx 10^{-4} a \Im\left[\frac{\lambda_u}{\lambda_t}\right]. \tag{86}$$

So for a determination of only $\Delta\Gamma_s$ (or also $\Delta\Gamma_d$) to a good approximation the first term of Eq. (77) or equivalently the coefficient c is sufficient.

- Unfortunately it turned out after the calculation of the NLO QCD and the subleading $1/m_b$ corrections that $\Delta\Gamma_s$ is not very well behaved (Lenz, 2004): all corrections are quite large and they have the same sign. Surprisingly this problem could be solved to a large extent by using Q and $\tilde{Q}_S$ as the two independent operators instead of Q and Q_S, which is just a change of the operator basis (Lenz and Nierste, 2007). As an illustration of the improvement we discuss the real part of the ratio Γ_{12}^s/M_{12}^s and split up the terms according to Eq. (79). We leave only the ratio of bag parameters as free parameters, while we insert all standard model parameters according to the values given in Appendix A. We get now for $\Delta\Gamma_s/\Delta M_s$ in the old (operators Q and Q_S) and the new basis (operators Q and $\tilde{Q}_S$)

$$\frac{\Delta\Gamma_s}{\Delta M_s}^{\rm Old} = 10^{-4} \times \left[2.6 + 69.7\frac{B_S}{B} - 24.3\frac{B_R}{B}\right], \tag{87}$$

$$\frac{\Delta\Gamma_s}{\Delta M_s}^{\rm New} = 10^{-4} \times \left[44.8 + 16.4\frac{\tilde{B}_S}{B} - 13.0\frac{B_R}{B}\right], \tag{88}$$

where B_R is an abbreviation for all seven bag parameters of the dimension 7 operators. In the old basis the first term, which has no dependence on nonperturbative lattice parameters, is almost negligible. The second term that depends on the ratio of the matrix elements of the operators Q_S and Q is by far dominant, and the third term that describes $1/m_b$ corrections gives an important negative contribution. In the new basis the first term, being completely free of any nonperturbative uncertainties, is numerical dominant. The second term is subleading and the $1/m_b$ corrections became smaller and undesired cancellations therein are less pronounced. Thus the second formulation has a much weaker dependence on the poorly known bag parameters, and also on the dimension 7 ones. If all bag parameters were precisely known, then such a change of basis has no effect, but since B_R is unknown and the ratios B_S'/B and $\tilde{B}_S{}'/B$ are much less known compared to the exact value one (stemming from B/B), now a basis, where the coefficients of B_R/B and $\tilde{B}_S{}'/B$ are small, gives results with much better theoretical control. For more details, see Lenz and Nierste (2007).

$1/m_b$ corrections for the subleading CKM structures in Γ_{12}^s (Dighe *et al.*, 2002) and $1/m_b^2$ corrections for $\Delta\Gamma_s$ (Badin, Gabbiani, and Petrov, 2007) were also determined; their numerical effect is small. A commonly used standard model prediction for $\Delta\Gamma_s$ was given by (Lenz and Nierste, 2011)

$$\Delta\Gamma_s^{\rm SM,2011} = 0.087 \pm 0.021 \ {\rm ps}^{-1}. \tag{89}$$

We take the most recent numerical inputs from the following sources: G_F, M_W, M_{B_s}, and m_b from the PDG (Olive *et al.*, 2014), the top quark mass from ATLAS, CDF, CMS, and D0 Collaborations (2014), the nonperturbative parameters from FLAG (Aoki *et al.*, 2014), and $\tilde{B}_S/B$, B_{R_0}, B_{R_1}, and $B_{\tilde{R}_1}$ from Becirevic *et al.* (2002), Bouchard *et al.* (2011), Carrasco *et al.* (2014), and Dowdall *et al.* (2014), and CKM elements from CKMfitter (Charles *et al.*, 2005)—similar values can be taken from UTfit (Bona *et al.*, 2006b). With these new values we predict the decay rate difference of the neutral B_s mesons to be

$$\Delta\Gamma_s^{\mathrm{SM},2015} = 0.088 \pm 0.020\ \mathrm{ps}^{-1}. \tag{90}$$

The dominant uncertainty stems from the dimension 7 bag parameter B_{R_2} (about 15%), closely followed by $f_{B_s}\sqrt{B}$ (about 14%) and the renormalization scale dependence, which contributes about 8% to the error budget. A detailed listing of all the contributing uncertainties can be found in Appendix B. In order to reduce the theory uncertainty to a value between 5% and 10%, a nonperturbative determination of B_{R_2}, a calculation of next-to-next-to-leading order (NNLO) QCD corrections [denoted by $\Gamma_3^{s,(2)}$ in Eq. (55), a first step in this direction has been done by Asatrian, Hovhannisyan, and Yeghiazaryan (2012) and by $\Gamma_4^{s,(1)}$] and more precise values of the matrix elements of the operators Q, Q_S, and $\tilde{Q}_S$ are mandatory. All of this seems to be feasible in the next few years.

In the discussion of the dimuon asymmetry in Sec. III we also need several mixing quantities from the B^0 sector. Their calculation within the standard model is analogous to the one in the B_s^0 sector. We present here numerical updates of the predictions given by Lenz and Nierste (2011). The input parameters are identical to the ones in the B_s^0 system, except $f_{B_d}\sqrt{B}$, $\tilde{B}_S/B$, M_{B^0}, and m_d, which can found in the same literature as the values for the B_s^0 system. Our new predictions are

$$\Delta M_d^{\mathrm{SM},2015} = 0.528 \pm 0.078\ \mathrm{ps}^{-1}, \tag{91}$$

$$\Delta\Gamma_d^{\mathrm{SM},2015} = (2.61 \pm 0.59) \times 10^{-3}\ \mathrm{ps}^{-1}, \tag{92}$$

$$\left(\frac{\Delta\Gamma_d}{\Gamma_d}\right)^{\mathrm{SM},2015} = (3.97 \pm 0.90) \times 10^{-3}, \tag{93}$$

$$\Re\left(\frac{\Gamma_{12}^d}{M_{12}^d}\right)^{\mathrm{SM},2015} = (-49.4 \pm 8.5) \times 10^{-4}. \tag{94}$$

A detailed error analysis is given in Appendix B.

B. Experiment: Mass and decay rate difference ΔM_s and $\Delta\Gamma_s$

Experimental studies of ΔM_s and $\Delta\Gamma_s$ and their comparison with the theoretical predictions of Eqs. (51) and (90) constitute an important SM test. In addition, ΔM_s together with the mass difference ΔM_d of the B^0 meson can be used to evaluate the ratio of the CKM parameters $|V_{ts}/V_{td}|$. These elements are not likely to be measurable with high precision in tree-level decays involving a top quark, because the top quark is too short lived to form a hadron (Olive *et al.*, 2014), but the ratio between ΔM_d and ΔM_s provides a theoretically clean and precise constraint. Using the results discussed next, and unquenched lattice calculations, Olive *et al.* (2014) quoted

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.216 \pm 0.001 \pm 0.011, \tag{95}$$

where the first error stems from experiment and the second from theory. Therefore, the measurement of ΔM_s, although not directly related to *CP* violation, contributes significantly to the test of the unitarity of the CKM matrix (Amhis *et al.*, 2014).

The measurement of ΔM_s and $\Delta\Gamma_s$ eluded experimentalists for a very long time. A relatively large value of $|V_{ts}|$ results in a high oscillation frequency of B_s^0 mesons and numerous transitions from a particle to an antiparticle during its lifetime. Therefore, a high precision of the proper decay length measurement is required to be sensitive to ΔM_s. On the other side, the measurement of $\Delta\Gamma_s$ is also challenging because $\Delta\Gamma_s/\Gamma_s = \mathcal{O}(10\%)$.

The measurement of ΔM_s was attempted by many experiments during more than 20 years; the CDF Collaboration at Fermilab first succeeded in performing it with a statistical significance exceeding 5 standard deviations (Abulencia *et al.*, 2006).

From a technical point of view, the measurement of ΔM_s requires these essential components:

- identification of the flavor of the B_s^0 meson at the time of production,
- identification of the flavor of the B_s^0 meson when it decays, and
- measurement of its proper lifetime.

To measure the final state of the B_s^0 meson decay, a flavor-specific transition is used. The simplest flavor-specific state is the semileptonic decay $B_s^0 \to D_s^- \mu^+ \nu_\mu$ since the muon usually provides an excellent possibility for an efficient selection of such decays during both the data taking and the subsequent analysis. However, the precision of the proper lifetime measurement in this decay mode is rather poor because of the missing neutrino taking some part of the B_s^0 momentum. Figure 5 shows the proper decay time resolution for different decay modes as a function of the B_s^0 proper decay time in the CDF measurement. The resolution in the semileptonic decay channel deteriorates very quickly with an increase of the proper time. Therefore, the ability of an experiment to reconstruct hadronic B_s^0 decays such as $B_s^0 \to D_s^- \pi^+$ plays a crucial role in the ΔM_s measurement.

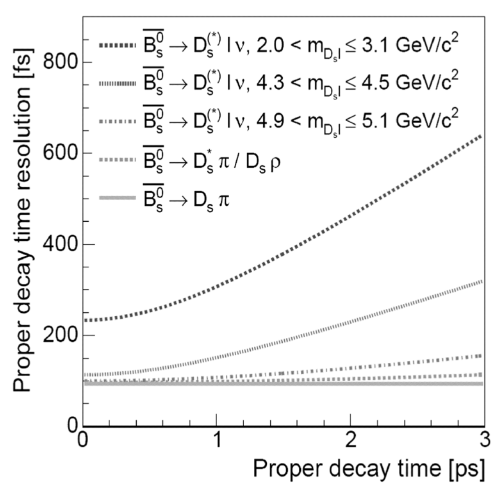

FIG. 5. The proper decay time resolution measured by the CDF Collaboration. From Abulencia *et al.*, 2006.

The identification of the B_s^0 initial state, also known as the *initial flavor tagging* (IFT), was first developed and used at hadron colliders by the CDF (Abulencia *et al.*, 2006) and D0 (Abazov *et al.*, 2006b) experiments at the Tevatron. In the LHCb implementation of the IFT (Aaij *et al.*, 2012g, 2013f, 2015a), the special capabilities of the detector, such as the particle identification and efficient reconstruction of secondary decays, are extensively used.

Technically, the IFT is divided into opposite-side (OS) and same-side (SS) tagging. At the LHC, where the gluon splitting dominates the $b\bar{b}$ production and the b quarks are considerably boosted, the "opposite side" is actually not "opposite" at all. Therefore, the naming of the two IFT methods is nowadays largely historical and does not reflect the actual topology of the $b\bar{b}$ events. The OS tagging is based on the correlation of the flavors of two produced B hadrons, while the SS tagging exploits the correlation of the flavor of the B_s^0 meson and the charge of additional particles produced in the hadronization of the initial b quark. The performance of the IFT is quantified by the *tagging power* P, which is expressed as $P = \epsilon(1-2w)^2$, where ϵ is the tagging efficiency and w is the wrong-tag probability. The tagging power multiplied by the total number of events in the analysis corresponds to the effective statistics used to measure ΔM_s.

The performance of the IFT in different experiments is presented in Table I. It includes the results of the ATLAS (Aad *et al.*, 2016) and CMS (Khachatryan, 2015) Collaborations, who use the IFT for the measurement of CP violation. It can be seen that the tagging power never exceeds a few percent meaning that a large statistics should be collected to obtain the significant measurement of ΔM_s. In general, the tagging power improves with a better understanding of the underlying event and with the refinement of multivariate tagging methods.

The period of oscillation of the B_s^0 meson corresponding to $\Delta M_s = 17.76\ \text{ps}^{-1}$ is $T = 2\pi/\Delta M_s \simeq 350$ fs. To measure it reliably and thus extract ΔM_s, the precision of the proper lifetime measurement should be at least 4 times better. The precision of the proper decay length measurement in the CDF experiment was about 100 fs, while for the LHCb experiment it is about 44 fs. This excellent performance together with large statistics collected by the LHCb experiment in the LHC Run I results in a much better precision of the ΔM_s measurement. They also succeeded to obtain a clear oscillation pattern in the proper decay length distribution, which is shown in Fig. 6.

TABLE I. Performance of the initial flavor tagging in different experiments. The numbers shown correspond to the same-side (SS) or the opposite-side (OS) tagging power (P). The uncertainty shown is the combination of the statistical and systematic uncertainties. In general, the same-side flavor tagging depends upon the mode being investigated. CDF finds a SS tagging power of $P = (4.8 \pm 1.2)\%$ (Abulencia *et al.*, 2006) in the semileptonic decay sample.

Experiment	Method	P (%)	Reference
CDF	OS	1.8 ± 0.1	Abulencia *et al.* (2006)
CDF	SS	3.7 ± 0.9	Abulencia *et al.* (2006)
D0	OS	2.48 ± 0.22	Abazov *et al.* (2006b)
LHCb	OS	2.55 ± 0.14	Aaij *et al.* (2013f)
LHCb	SS	1.26 ± 0.17	Aaij *et al.* (2013f)
ATLAS	OS	1.49 ± 0.02	Aad *et al.* (2016)
CMS	OS	1.307 ± 0.032	Khachatryan (2015)

The first double sided bound at 90% C.L. on the ΔM_s value was obtained by the D0 Collaboration (Abazov *et al.*, 2006a). Soon after that the CDF Collaboration reported the actual measurement of this quantity (Abulencia *et al.*, 2006)

$$\Delta M_s^{\text{CDF}} = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{syst})\ \text{ps}^{-1}. \tag{96}$$

Later, the LHCb Collaboration performed the most precise single-experiment measurement of ΔM_s (Aaij *et al.*, 2013h)

$$\Delta M_s^{\text{LHCb}} = 17.768 \pm 0.023(\text{stat}) \pm 0.006(\text{syst})\ \text{ps}^{-1}. \tag{97}$$

The combination of all ΔM_s measurements by the HFAG (Amhis *et al.*, 2014) gives

$$\Delta M_s^{\text{HFAG 2015}} = 17.757 \pm 0.021\ \text{ps}^{-1}. \tag{98}$$

The currently most precise measurement of $\Delta\Gamma_s$ consists of the simultaneous study of the proper decay length and angular distributions of the decay $B_s^0 \to J/\psi K^+K^-$ which mainly includes the $B_s^0 \to J/\psi\phi$ final state. For simplicity, this study is denoted as the $B_s^0 \to J/\psi\phi$ channel in the following discussion, although it should be remembered that the addition of the nonresonant (NR) contribution is required for an appropriate analysis of data. Both the CP-even and CP-odd B_s^0 states contribute in this decay mode and therefore its properties are sensitive to both the B_s^0 width difference and the phase ϕ_s (defined in Sec. IV.A) describing CP violation in the interference of decay and mixing.

All collider experiments at the Tevatron and the LHC perform the measurement of $\Delta\Gamma_s$ in the $B_s^0 \to J/\psi\phi$ decay. The first results were obtained by the CDF (Aaltonen *et al.*, 2012) and D0 (Abazov *et al.*, 2012a) Collaborations, who largely developed the measurement technique. The ATLAS (Aad *et al.*, 2016), CMS (Khachatryan, 2015), and LHCb (Aaij *et al.*, 2015h) Collaborations continue this study at the LHC, where a significantly larger statistics is collected and much more data are expected in the future.

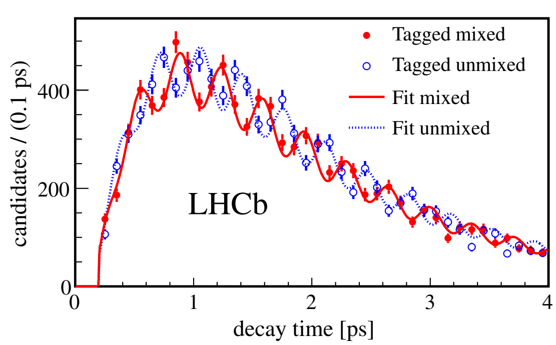

FIG. 6. Proper decay time distribution for the selected B_s^0 decay candidates tagged as mixed (different flavor at decay and production; red, solid line) or unmixed (same flavor at decay and production; blue, dotted line). The data and the fit projections are plotted in a signal window around the reconstructed B_s^0 mass of 5.32–5.55 GeV/c^2. From Aaij *et al.*, 2013h.

TABLE II. Measurements of $\Delta\Gamma_s$ in $B_s^0 \to J/\psi\phi$ decay. The last line gives the world average value obtained by HFAG.

Experiment	$\Delta\Gamma_s$ (ps^{-1})	Γ_s (ps^{-1})	Reference
CDF	$0.068 \pm 0.026 \pm 0.009$	$0.654 \pm 0.008 \pm 0.004$	Aaltonen *et al.* (2012)
D0	$0.163^{+0.065}_{-0.064}$	$0.693^{+0.018}_{-0.017}$	Abazov *et al.* (2012a)
ATLAS	$0.083 \pm 0.011 \pm 0.007$	$0.677 \pm 0.003 \pm 0.003$	Aad *et al.* (2016)
CMS	$0.095 \pm 0.013 \pm 0.007$	$0.6704 \pm 0.0043 \pm 0.0051$	Khachatryan (2015)
LHCb	$0.0805 \pm 0.0091 \pm 0.0033$	$0.6603 \pm 0.0027 \pm 0.0015$	Aaij *et al.* (2015h)
HFAG 2015	0.079 ± 0.006	0.6649 ± 0.0022	Amhis *et al.* (2014)

As for ΔM_s, the measurement of $\Delta\Gamma_s$ in $B_s^0 \to J/\psi\phi$ decay requires IFT and the proper decay length of the B_s^0 meson. In addition, the study of the angular distributions of the B_s^0 decay products is needed. This is the reason why this analysis is sensitive to the quality of the data description by the simulation. All experiments succeed in achieving an excellent understanding of their detectors.

The measurements of $\Delta\Gamma_s$ using $J/\psi\phi(K^+K^-)$ are summarized in Table II. It also includes the world average value obtained by the HFAG (Amhis *et al.*, 2014), which is found to be

$$\Delta\Gamma_s = 0.079 \pm 0.006 \text{ ps}^{-1} \quad (B_s^0 \to J/\psi\phi). \tag{99}$$

An alternative approach to determine $\Delta\Gamma_s$ relies upon the direct measurement of the effective lifetime of B_s^0 decays to pure CP eigenstates. The extraction of $\Delta\Gamma_s$ with this method is discussed in detail by Fleischer and Knegjens (2011a).

To first order in $y_s \equiv \Delta\Gamma_s/(2\Gamma_s)$, we have (Amhis *et al.*, 2014)

$$\tau_{\text{single}}(B_s^0 \to CP\text{-even}) \approx \frac{1}{\Gamma_L}\left(1 + \frac{(\phi_s)^2 y_s}{2}\right), \tag{100}$$

$$\tau_{\text{single}}(B_s^0 \to CP\text{-odd}) \approx \frac{1}{\Gamma_H}\left(1 - \frac{(\phi_s)^2 y_s}{2}\right), \tag{101}$$

where τ_{single} is the effective lifetime of the B_s^0 decaying to a specific CP-eigenstate state f. This formula assumes that $\mathcal{A}^{\Delta\Gamma}_{CP\text{-EVEN}} = \cos\phi_s$ and $\mathcal{A}^{\Delta\Gamma}_{CP\text{-ODD}} = -\cos\phi_s$, where the mixing angle ϕ_s will be defined in Sec. IV.A. Thus, the decay width measured in the CP-even final state, such as $B_s^0 \to K^+K^-$ and $B_s^0 \to D_s^+D_s^-$, is approximately equal to $1/\Gamma_L(s)$. Similarly, the CP-odd decay modes $B_s^0 \to J/\psi K_S^0$ and $B_s^0 \to J/\psi f_0(980)$ provide measurements of $1/\Gamma_H(s)$; thus $\Delta\Gamma_s$ can be obtained as the difference of these two quantities. There are several subtleties that need to be taken into account when using this method to measure $\Delta\Gamma_s$. For example, the decays $B_s^0 \to K^+K^-$ and $B_s^0 \to J/\psi K_S^0$ may suffer from CP violation due to interfering tree and loop amplitudes. Thus Amhis *et al.* (2014) used only the effective lifetimes obtained for $D_s^+D_s^-$ (CP-even), and $J/\psi f_0$, $J/\psi\pi\pi$ (CP-odd) decays to obtain

$$\tau_{\text{single}}(B_s^0 \to CP\text{-even}) = 1.379 \pm 0.031 \text{ ps}, \tag{102}$$

$$\tau_{\text{single}}(B_s^0 \to CP\text{-odd}) = 1.656 \pm 0.033 \text{ ps}. \tag{103}$$

Table III summarizes the current values as well as the average values of $1/\Gamma_L^s$ and $1/\Gamma_H^s$ reported by Amhis *et al.* (2014). Note that the effective lifetimes measured in $B_s^0 \to K^+K^-$ and $B_s^0 \to J/\psi K_S^0$ have not been used in these averages because of the difficulty in quantifying the penguin contribution in these modes. These effective lifetimes correspond to

$$\Delta\Gamma_s = 0.121 \pm 0.020. \tag{104}$$

This value is higher by 2 standard deviations than the one shown in Eq. (99). However, this difference should be considered with caution. The value in Eq. (104) is obtained with theoretical assumptions and external input on weak phases and hadronic parameters.

Using these data in conjunction with the $J/\psi\phi(K^+K^-)$ determinations of $\Delta\Gamma_s$, the current experimental average is (Amhis *et al.*, 2014)

$$\Delta\Gamma_s^{\text{HFAG 2015}} = 0.083 \pm 0.006 \text{ ps}^{-1}. \tag{105}$$

The comparison of different lifetime measurements of CP eigenstates, which can be used to extract $\Delta\Gamma_s$, is presented in Fig. 7.

To conclude this section we compare the experimental and theoretical numbers for the mass difference and the decay rate difference. For the experimental value of the mass difference we take the value from Eq. (98) and for the value of the decay

TABLE III. The B_s^0 width difference can be extracted from lifetime measurements in different channels with a definite CP quantum number.

Quantity	Source	Channel	Result (ps)
$1/\Gamma_L^s$	LHCb (Aaij *et al.*, 2014a)	$B_s^0 \to K^+K^-$	$1.407 \pm 0.016 \pm 0.007$
	LHCb (Aaij *et al.*, 2014b)	$B_s^0 \to D_s^+D_s^-$	$1.379 \pm 0.026 \pm 0.017$
$1/\Gamma_H^s$	CDF (Aaltonen *et al.*, 2011)	$B_s^0 \to J/\psi f_0(980)$	$1.70^{+0.12}_{-0.11} \pm 0.03$
	LHCb (Aaij *et al.*, 2012d)	$B_s^0 \to J/\psi f_0(980)$	$1.700 \pm 0.040 \pm 0.026$
	LHCb (Aaij *et al.*, 2013g)	$B_s^0 \to J/\psi K_S^0$	$1.75 \pm 0.12 \pm 0.07$
	LHCb (Aaij *et al.*, 2013f)	$B_s^0 \to J/\psi\pi^+\pi^-$	$1.652 \pm 0.024 \pm 0.024$

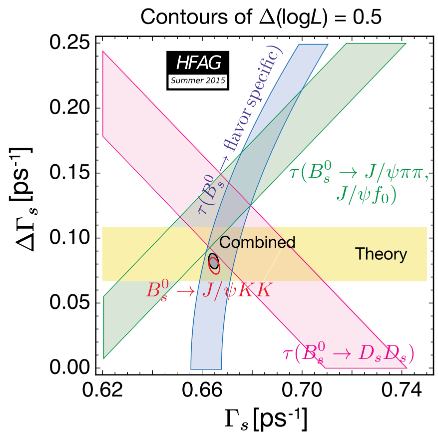

FIG. 7. The average of all the $B_s^0 \to J/\psi\phi$ and $B_s^0 \to J/\psi K^+K^-$ results is shown as the red contour, and the constraints given by the effective lifetime measurements of B_s^0 to flavor-specific [see Eq. (31)], pure CP-odd and pure CP-even final states are shown as the blue, green, and purple bands, respectively. The average taking all constraints into account is shown as the gray-filled contour. The yellow band is the theory prediction given in Eq. (89) that assumes no new physics in B_s^0 mixing. From Amhis *et al.*, 2014.

width difference we take Eq. (105). For the theory value, we take the more precise prediction of the ratio $\Delta\Gamma_s/\Delta M_s$. We find good agreement for experiment and theory

$$\frac{(\Delta\Gamma_s/\Delta M_s)^{\rm Exp}}{(\Delta\Gamma_s/\Delta M_s)^{\rm SM}} = \frac{0.004\,67(1\pm 0.072)}{0.004\,81(1\pm 0.173)} \tag{106}$$

$$= 0.97 \pm 0.07 \pm 0.17. \tag{107}$$

In the last line the first error is the experimental and the second is the theoretical. This result proves that the heavy quark expansion is working in the B sector with a precision of at least 20%, also for the decay channel $b \to c\bar{c}s$, which seems to be most sensitive to violations of quark hadron duality. Assuming that there are no new physics effects in ΔM_s and taking into account that the ratio $\Delta\Gamma_s/\Delta M_s$ is theoretically cleaner than $\Delta\Gamma_s$ alone, we get an improved prediction for $\Delta\Gamma_s$:

$$\Delta\Gamma_s^{\rm SM,2015b} = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm SM} \cdot \Delta M_s^{\rm Exp}$$
$$= 0.085 \pm 0.015\ {\rm ps}^{-1}. \tag{108}$$

This is the most precise theory value for $\Delta\Gamma_s$ that can currently be obtained. In the future this theory uncertainty might be improved by a factor of up to 3 as explained in Sec. II.

III. *CP* VIOLATION IN MIXING

A. Theory: HQE

CP violation in mixing is described by the weak mixing phase ϕ_{12}^s defined in Eq. (13). It can be measured directly via CP asymmetries of so-called flavor-specific decays. A flavor-specific decay $B_s^0 \to f$ is defined by the following properties:

- The decays $\bar{B}_s^0 \to f$ and $B_s^0 \to \bar{f}$ are forbidden. This reads in our notation

$$\bar{\mathcal{A}}_f = 0 = \mathcal{A}_{\bar{f}} \tag{109}$$

and thus

$$\lambda_f = 0 = \frac{1}{\lambda_{\bar{f}}}. \tag{110}$$

Hence the time evolution of these decays is quite simple, compared to the general case.
- No direct CP violation arises in the decay, i.e., $|\langle f|\mathcal{H}_{\rm eff}|B_s^0\rangle| = |\langle \bar{f}|\mathcal{H}_{\rm eff}|\bar{B}_s^0\rangle|$, which again reads in our notation

$$|\mathcal{A}_f| = |\bar{\mathcal{A}}_{\bar{f}}|. \tag{111}$$

Examples for such decays are, e.g., $B_s^0 \to D_s^-\pi^+$ or $B_s^0 \to Xl\nu$; therefore the corresponding asymmetries in semileptonic decays are also called *semileptonic CP asymmetries*. The CP asymmetry for flavor-specific decays is defined as

$$a_{\rm fs}^s = \frac{\Gamma(\bar{B}_s^0(t) \to f) - \Gamma(B_s^0(t) \to \bar{f})}{\Gamma(\bar{B}_s^0(t) \to f) + \Gamma(B_s^0(t) \to \bar{f})} \equiv a_{\rm sl}^s. \tag{112}$$

Inserting the time evolution of the B_s^0 mesons, given in Eqs. (33) and (36), the flavor-specific CP asymmetry $a_{\rm fs}^s$ can be further simplified[14] as

$$a_{\rm fs}^s = -2\left(\left|\frac{q}{p}\right| - 1\right)$$
$$= \Im\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) = \left|\frac{\Gamma_{12}^s}{M_{12}^s}\right| \sin\phi_{12}^s. \tag{113}$$

For the SM prediction of the flavor-specific asymmetries we can now simply use our determination of the ratio of the matrix elements M_{12}^s and Γ_{12}^s from the previous section, in particular, we need only the coefficient a (b gives only a small correction) defined in Eq. (78) to get

$$a_{\rm fs}^s \approx \Im\left(\frac{\lambda_u}{\lambda_t}\right) a \times 10^{-4}. \tag{114}$$

The coefficient a was given by the difference of the internal charm-charm loop and the internal up-charm loop. Using the exact expression for $\Im(\Gamma_{12}^s/M_{12}^s)$ the standard model prediction of $a_{\rm fs}^s$ was given by (Lenz and Nierste, 2011)

$$a_{\rm fs}^{s,\rm SM,2011} = (1.9 \pm 0.3) \times 10^{-5}. \tag{115}$$

With the most recent numerical inputs [G_F, M_W, M_{B_s}, and m_b from the PDG (Olive *et al.*, 2014), the top quark mass from ATLAS, CDF, CMS, and D0 Collaborations (2014), the nonperturbative parameters from FLAG (Aoki *et al.*, 2014) and $\tilde{B}_S/B$, B_{R_0}, B_{R_1}, and $B_{\tilde{R}_1}$ from Becirevic *et al.* (2002),

[14]This result was already used in Eq. (15).

Bouchard *et al.* (2011), Carrasco *et al.* (2014), and Dowdall *et al.* (2014) and CKM elements from CKMfitter (Charles *et al.*, 2005)] [similar values can be taken from UTfit (Bona *et al.*, 2006b)] we predict the flavor-specific CP asymmetries of the neutral B_s^0 mesons to be

$$a_{\rm fs}^{s,{\rm SM},2015} = (2.22 \pm 0.27) \times 10^{-5}. \qquad (116)$$

The dominant uncertainty stems from the renormalization scale dependence, with 9%, followed by the CKM dependence with 5%, and the charm quark mass dependence with 4%. A detailed discussion of the uncertainties is given in Appendix B. Because of this small value and the proven validity of the HQE, the flavor-specific asymmetries represent a nice null test, as any sizable experimental deviation from the prediction in Eq. (116) is a clear indication for new physics; see Jubb *et al.* (2016) for a more detailed discussion of this point. In addition we obtain the following SM prediction for the mixing phase ϕ_{12}^s:

$$\phi_{12}^{s,{\rm SM},2015} = (4.6 \pm 1.2) \times 10^{-3}\ {\rm rad} \qquad (117)$$

$$= 0.26° \pm 0.07°. \qquad (118)$$

In the discussion of the dimuon asymmetry we also need the semileptonic CP asymmetry from the B^0 sector. Its calculation within the SM is analogous to the one of $a_{\rm sl}^s$. We update the predictions given by Lenz and Nierste (2011), by using the same input parameters as for the B_s^0 system, except using M_{B^0}, m_d, and $\tilde{B}_S/B$. We get as new standard model values

$$a_{\rm fs}^{d,{\rm SM},2015} = (-4.7 \pm 0.6) \times 10^{-4}, \qquad (119)$$

$$\phi_{12}^{d,{\rm SM},2015} = -0.096 \pm 0.025\ {\rm rad} = -5.5° \pm 1.4°. \qquad (120)$$

A more detailed analysis of the uncertainties can be found in Appendix B. Measurements of the dimuon asymmetry triggered a lot of interest in B^0 and B_s^0 mixing, because early measurements seemed to indicate large new physics effects (Abazov *et al.*, 2010a, 2010b, 2011, 2014). Originally, the dimuon asymmetry A_{CP} was considered to be given by a linear combination of the semileptonic CP asymmetries in the B^0 and the B_s^0 system (Abazov *et al.*, 2010a, 2010b, 2011)

$$A_{CP} = C_d a_{\rm sl}^d + C_s a_{\rm sl}^s, \qquad (121)$$

with C_d and C_s being roughly equal. The large deviation of the measured value of A_{CP} from the calculated values of the linear combination of $a_{\rm sl}^d$ and $a_{\rm sl}^s$ seemed to be a hint for large new physics effects in the semileptonic CP asymmetries. Borissov and Hoeneisen (2013) found that there is actually also an additional contribution from indirect CP violation. This led to the following new interpretation [also used in Abazov *et al.* (2014)]:

$$A_{CP} = C_d a_{\rm sl}^d + C_s a_{\rm sl}^s + C_{\Delta\Gamma_d} \frac{\Delta\Gamma_d}{\Gamma_d}. \qquad (122)$$

Because of the small value of $\Delta\Gamma_d$ in the SM [see Eqs. (92) and (93)] the additional term did not solve the discrepancy. It was pointed out (Nierste, 2014) that the relation should be further modified to

$$A_{CP} = C_d a_{\rm sl}^d + C_s a_{\rm sl}^s + \alpha C_{\Delta\Gamma_d} \frac{\Delta\Gamma_d}{\Gamma_d}, \qquad (123)$$

where $\alpha \leq 1/2$. An interesting feature of this new interpretation is that a large enhancement of $\Delta\Gamma_d$ by new physics effects could explain the experimental value of the dimuon asymmetry, while large enhancements of the semileptonic CP asymmetries are disfavored by direct measurements; see the next section. The investigation of Bobeth *et al.* (2014) has further shown that enhancements of $\Delta\Gamma_d$ by several hundred percent are not excluded by any other experimental constraint—such an enhancement could bring the dimuon asymmetry in agreement with experiment. One possible enhancement mechanism would be new $bd\tau\tau$ transitions. Since two tau leptons are lighter than a B^0 meson such a new operator could contribute to Γ_{12}^d. This possibility can be tested by investigating $bd\tau\tau$ transitions directly. In Fig. 8 we show the possible enhancement of $\Delta\Gamma_d$ due to new scalar [left-hand side (lhs)] and new vector [right-hand side (rhs)] $bd\tau\tau$ operators. Currently enhancements within the shaded (yellow) regions are allowed. In the case of vector operators $\Delta\Gamma_d$ can be enhanced to about 3.5 the SM value of $\Delta\Gamma_d$. The connection between a direct measurement of or a bound on $B^0 \to \tau^+\tau^-$ is given by the solid (red) line. From Fig. 8 one can read off that a bound on $B^0 \to \tau^+\tau^-$ of the order of 10^{-3} would limit the enhancement of $\Delta\Gamma_d$ to about 15% of the SM value in the case of scalar new physics operators and to about 50% of the SM value in the case of vector new physics operators. Similar relations between a possible enhancement of $\Delta\Gamma_d$ and a direct search for $B^0 \to X_d\tau^+\tau^-$ and $B^+ \to \pi^+\tau^+\tau^-$ are indicated by the long-dashed (blue) and dashed (green) lines.

Another enhancement mechanism would be new physics effects in tree-level decays, which are typically neglected. Such studies were performed systematically by Bobeth, Gorbahn, and Vickers (2015), Bobeth *et al.* (2014), and Brod *et al.* (2015) and could also lead to sizable enhancements of $\Delta\Gamma_d$. Here a more precise measurement of $\Delta\Gamma_d$ would of course be very helpful.

B. Experiment: Semileptonic asymmetries $a_{\rm sl}^s$ and $a_{\rm sl}^d$, the dimuon asymmetry

The measurement of the flavor-specific charge asymmetry is conceptually simple. Essentially, it is given by the asymmetry between flavor-specific decays $B_s^0 \to f$ and $\bar{B}_s^0 \to \bar{f}$. As the expected value of the asymmetry is small, great care needs to be taken to assess any potential source of asymmetry, for example, production dynamics, background sources, or detection asymmetry. The final state typically used for this measurement is the semileptonic decay $B_s^0 \to D_s^{(*)-}\mu^+\nu$, where the notation $(*)$ denotes the production of either D_s^-, D_s^{*-}, or D_{sJ} states. The published results consider only the decay $D_s \to \phi\pi$ with $\phi \to K^+K^-$. The initial flavor of the B_s^0 meson is not determined and the measured quantity is

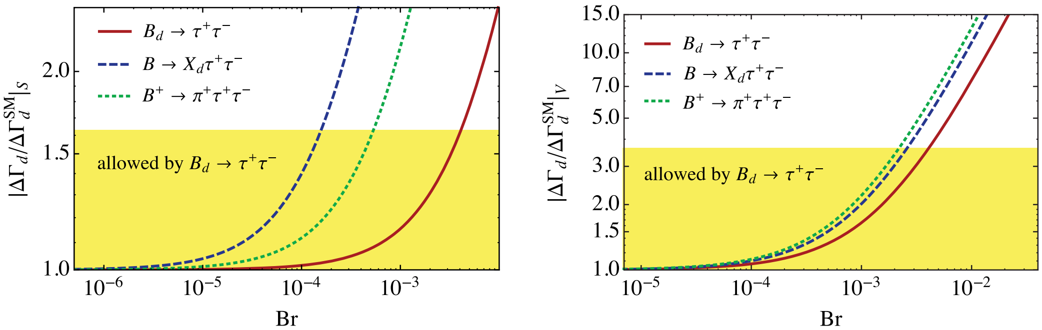

FIG. 8. The possible enhancement factors of $\Delta\Gamma_d$ by new scalar (lhs) and vector (rhs) $bd\tau\tau$ operators are indicated by the shaded (yellow) regions. In the case of a scalar operator $\Delta\Gamma_d$ can still be enhanced to about 1.6 times of the SM values. In the case of a vector operator $\Delta\Gamma_d$ can even be enhanced to about 3.5 times of the SM values. More precise bounds on $B^0 \to \tau^+\tau^-$, $B^0 \to X_d\tau^+\tau^-$, and $B^+ \to \pi^+\tau^+\tau^-$ could further shrink the allowed enhancement factor. The relation between the bounds $B^0 \to \tau^+\tau^-$, $B^0 \to X_d\tau^+\tau^-$, and $B^+ \to \pi^+\tau^+\tau^-$ and the possible enhancement factor of $\Delta\Gamma_d$ is given by the solid (red), long-dashed (blue) and dashed (green) lines.

$$A_{\rm meas} = \frac{N(D_s^-\mu^+) - N(D_s^+\mu^-)}{N(D_s^-\mu^+) + N(D_s^+\mu^-)}, \tag{124}$$

where $N(f)$ ($f = D_s^-\mu^+$ or $D_s^+\mu^-$) is the number of reconstructed events in the final state f. It can be expressed as

$$N(f) \propto \int_0^{+\infty} [\sigma(B_s^0)\Gamma(B_s^0(t) \to f) + \sigma(\bar{B}_s^0)\Gamma(\bar{B}_s^0(t) \to f)]\epsilon(f,t)dt. \tag{125}$$

This expression takes into account the absence of the initial flavor tagging, the possible difference in the production cross sections $\sigma(B_s^0)$ and $\sigma(\bar{B}_s^0)$, and time-dependent reconstruction efficiency $\epsilon(f,t)$ of the final state f. The most important instrumental charge asymmetries are related to differences between $\mu^+ - \mu^-$ and $\pi^+ - \pi^-$ detection efficiencies. The two opposite-charge kaons from ϕ decay have almost the same momentum spectrum, and thus charge-dependent detection effects do not influence the measured asymmetry.

Using the expressions of the time evolution of B_s^0 mesons, assuming that the ratio of the reconstruction efficiencies $r_\epsilon \equiv \epsilon(D_s^-\mu^+,t)/\epsilon(D_s^+\mu^-,t)$ does not depend on time, and neglecting the second order terms, the semileptonic charge asymmetry $a_{\rm sl}^s$ is related to $A_{\rm meas}$ as

$$A_{\rm meas} = \frac{a_{\rm sl}^s}{2} - \frac{1-r_\epsilon}{2} + \left(a_{\rm P} - \frac{a_{\rm sl}^s}{2}\right)I,$$
$$I \equiv \frac{\int_0^{+\infty} e^{-\Gamma_s t}\cos(\Delta M_s t)\epsilon(t)dt}{\int_0^{+\infty} e^{-\Gamma_s t}\cosh(\Delta\Gamma_s t/2)\epsilon(t)dt}. \tag{126}$$

Here $a_{\rm P}$ is the production asymmetry of the B_s^0 meson defined as

$$a_{\rm P} = \frac{\sigma(B_s^0) - \sigma(\bar{B}_s^0)}{\sigma(B_s^0) + \sigma(\bar{B}_s^0)}. \tag{127}$$

The asymmetry $a_{\rm P}$ is zero at a $p\bar{p}$ collider, while it does not exceed a few percent for the B_s^0 production at the LHC (Norrbin and Vogt, 2000; Aaij *et al.*, 2012f, 2013d). Because of the large value of ΔM_s, the value of I is about 0.2%. As a result, the value of the third term in Eq. (126) is of the order of 10^{-4} and can be safely neglected. Thus, the main experimental task in the measurement of the $a_{\rm sl}^s$ is the determination of r_ϵ.

Measurements of the asymmetry $a_{\rm sl}^s$ have been reported by the D0 (Abazov *et al.*, 2013) and LHCb (Aaij *et al.*, 2014c) Collaborations. Both D0 and LHCb collected large statistics using semileptonic B_s^0 decays. The number of reconstructed signal events in the D0 measurement is $215\,763 \pm 1467$. The corresponding $\mu^\pm\phi\pi^\mp$ mass distribution is shown in Fig. 9. Recently, LHCb updated the measurement of $a_{\rm sl}^s$, using 3 fb^{-1} and including all the possible D_s decays to the $K^+K^-\pi^+$ final state. The corresponding mass distribution is shown in Fig. 10.

The important feature of both experiments is a regular reversal of the magnet polarities. In the D0 experiment, the polarities of the toroidal and solenoidal magnetic fields (Abazov *et al.*, 2006c) were reversed on average every two weeks so that the four solenoid-toroid polarity combinations are exposed to approximately the same integrated luminosity. D0 reported only results averaged over all the magnet polarities. The 1 fb^{-1} LHCb sample comprises approximately 40% of data taken with the magnetic field up, oriented along the positive y axis in the LHCb coordinate system, and the rest

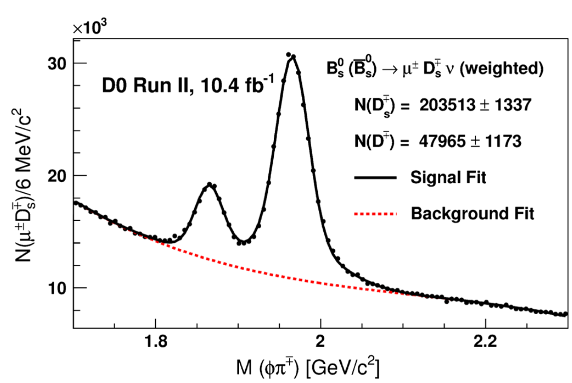

FIG. 9. The weighted $K^+K^-\pi^\mp$ invariant mass distribution for the $\mu\phi\pi^\mp$ sample. The solid line represents the result of the fit and the dashed line shows the background parametrization. The lower mass peak is due to the decay $D^\mp \to \phi\pi^\mp$ and the second peak is due to the $D_s^\mp$ meson decay. Note the suppressed zero on the vertical axis. From Abazov *et al.*, 2013.

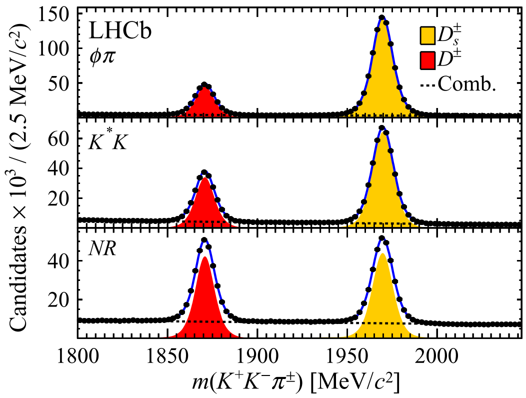

FIG. 10. Invariant mass distributions of $K^+K^-\pi^+$ and $K^+K^-\pi^-$ in the three Dalitz regions studied in the LHCb analysis, summed over both magnet polarities and data taking periods. Overlaid are the results of the fits, with signal and combinatorial background components as indicated in the legend. From Aaij *et al.*, 2016.

with the opposite down polarity. The 2 fb^{-1} sample comprises equal amounts of data with the two magnet polarities. LHCb analyzes data with magnetic field up and down separately, to allow a quantitative assessment of charge-dependent asymmetries. Figure 11 shows their measurement of the ratio r_ϵ for two magnet polarities and the two data sets. It can be seen that the majority of the detection asymmetry changes sign with the reversal of the magnet polarity, and thus the final average of the two samples is much less sensitive to detection asymmetry.

The resulting values of a_{sl}^s obtained by the two experiments as well as their average are

$$a_{sl}^{s\,\mathrm{D0}} = (-1.12 \pm 0.74 \pm 0.17) \times 10^{-2}, \tag{128}$$

$$a_{sl}^{s\,\mathrm{LHCb}} = (+0.39 \pm 0.26 \pm 0.20) \times 10^{-2}, \tag{129}$$

$$a_{sl}^{s\,\mathrm{average}} = (+0.17 \pm 0.30) \times 10^{-2}. \tag{130}$$

Both results are consistent with the standard model expectation (116), albeit the uncertainty is still a factor of about 130 larger than the central value in the standard model.

The *BABAR*, Belle, D0, and LHCb Collaborations perform the independent measurement of the asymmetry a_{sl}^d. Their

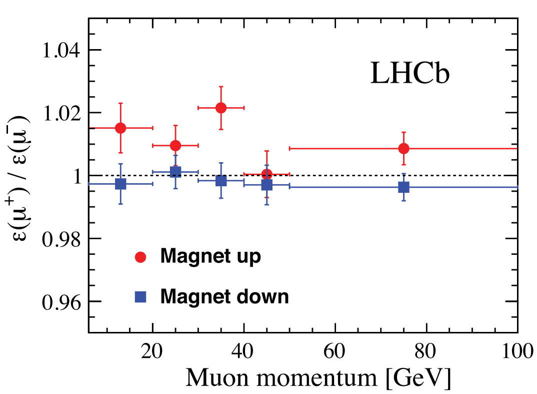

FIG. 11. Relative muon efficiency as a function of the muon momentum. From Aaij *et al.*, 2014c.

TABLE IV. The most recent measurements of the CP violation parameter a_{sl}^d.

Experiment	Measured a_{sl}^d (%)	Reference
LHCb $D^{(*)}\mu\nu X$	$-0.02 \pm 0.19 \pm 0.30$	Aaij *et al.* (2015f)
D0 $D^{(*)}\mu\nu X$	$+0.68 \pm 0.45 \pm 0.14$	Abazov *et al.* (2012b)
BABAR $D^*\ell\nu X$	$+0.29 \pm 0.84^{+1.88}_{-1.61}$	Lees *et al.* (2013)
BABAR $\ell\ell$	$-0.39 \pm 0.35 \pm 0.19$	Lees *et al.* (2015b)

results are summarized in Table IV. The world average value of a_{sl}^d is

$$a_{sl}^d(\mathrm{HFAG}) = 0.0001 \pm 0.0020. \tag{131}$$

The D0 experiment also reports a complementary measurement related to the semileptonic asymmetries of B_s^0 and B^0 mesons (Abazov *et al.*, 2014). It performs the simultaneous study of the inclusive semileptonic charge asymmetry and of the like-sign dimuon charge asymmetry. These quantities are defined as

$$a = \frac{n^+ - n^-}{n^+ + n^-}, \tag{132}$$

$$A = \frac{N^{++} - N^{--}}{N^{++} + N^{--}}. \tag{133}$$

Here n^+ and n^- are the number of events with the reconstructed positive or negative muon, respectively. N^{++} and N^{--} are the number of events with two positive or two negative muons, respectively. The asymmetries a and A are cast as

$$a = a_{CP} + a_{\mathrm{bkg}}, \tag{134}$$

$$A = A_{CP} + A_{\mathrm{bkg}}. \tag{135}$$

Here a_{CP} and A_{CP} are the asymmetries due to the genuine CP-violating processes, such as CP violation in mixing of B^0 and B_s^0 mesons. The asymmetries a_{bkg} and A_{bkg} are produced by the background processes not related to CP violation. The main source of these asymmetries is the difference in the interaction cross section of the positive and negative charged particles with the detector material. The main challenge in the D0 analysis is the accurate estimate of the background asymmetries a_{bkg} and A_{bkg} and the extraction of the values of a_{CP} and A_{CP}.

The asymmetries a_{CP} and A_{CP} depend on both a_{sl}^d and a_{sl}^s. Since the oscillation period of B^0 and B_s^0 mesons is significantly different, the contribution of a_{sl}^d and a_{sl}^s strongly depends on the decay time of collected B mesons. This decay time is not measured in the inclusive analysis. Instead, the D0 experiment measures the asymmetries a_{CP} and A_{CP} in subsamples containing the muons with different muon impact parameters. The division into the subsamples according to the muon impact parameter is used to estimate the contribution of a_{sl}^d and a_{sl}^s. In addition, the asymmetry A_{CP} is sensitive to the width difference $\Delta\Gamma_d$ of B^0 meson (Borissov and Hoeneisen,

2013) and this quantity is also obtained in the D0 analysis. Their result is

$$a_{\rm sl}^d = (-0.62 \pm 0.43) \times 10^{-2}, \tag{136}$$

$$a_{\rm sl}^s = (-0.82 \pm 0.99) \times 10^{-2}, \tag{137}$$

$$\frac{\Delta\Gamma_d}{\Gamma_d} = (+0.50 \pm 1.38) \times 10^{-2}. \tag{138}$$

The correlations between the fitted parameters are

$$\rho_{d,s} = -0.61, \qquad \rho_{d,\Delta\Gamma} = -0.03, \\ \rho_{s,\Delta\Gamma} = +0.66. \tag{139}$$

Although the central values of all three quantities are consistent with the SM prediction within the uncertainties, a deviation from the SM prediction by 3 standard deviations is reported because of the correlation between these observables.

The world knowledge of CP-violating parameters in B_s^0 and B^0 mixing is summarized in Fig. 12 that shows that the individual measurements of $a_{\rm sl}^d$ and $a_{\rm sl}^s$ are consistent with the standard model. Only the D0 dimuon result suggests a deviation from standard model expectations in CP violation in neutral B^0 oscillations. Since this measurement is inclusive, other unknown effects not directly related to CP violation in B_s^0 mixing could contribute to it.

The LHCb experiment is currently taking data and is expected to collect an additional sample of ~6 fb^{-1} in the current LHC run, and at least 50 fb^{-1} with an upgraded detector to be installed in 2020. Moreover Belle II will start taking data in a time scale comparable to the expected start of the LHCb upgraded detector. Therefore the prospects for increased precision in CP-violating asymmetries in neutral B meson decays are excellent.

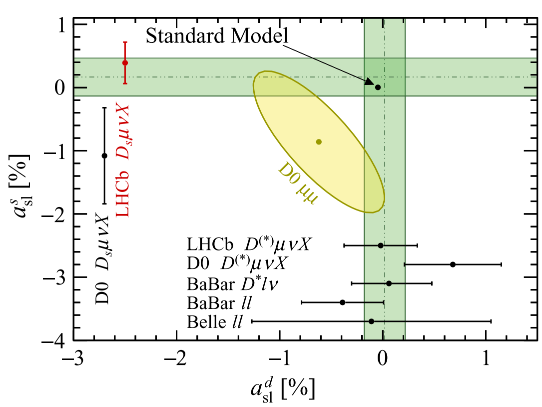

FIG. 12. Overview of measurements in the $a_{\rm sl}^d$-$a_{\rm sl}^s$ plane. Direct measurements of $a_{\rm sl}^s$ and $a_{\rm sl}^d$ listed in Table IV (B^0 average as the vertical band, B_s^0 average as the horizontal band, D0 dimuon result as the yellow ellipse). The black point close to (0; 0) is the standard model prediction reported in this paper with error bars multiplied by 10. From Aaij *et al.*, 2016.

IV. CP VIOLATION IN INTERFERENCE

A. Theory

In this section we discuss CP-violating effects that arise from interference between mixing and decay, which is also called *mixing-induced CP violation*. Therefore we consider a final state f in which in principle both the B_s^0 meson and the $\bar{B}_s^0$ meson can decay. The corresponding decay amplitudes are denoted by $\mathcal{A}_f$ and $\bar{\mathcal{A}}_f$, defined in Eq. (22). These amplitudes can have contributions from different CKM structures; their general structure looks like

$$\mathcal{A}_f = \sum_j \mathcal{A}_j e^{i(\phi_j^{\rm strong} + \phi_j^{\rm CKM})}, \tag{140}$$

where j sums over the different CKM contributions, $\phi_j^{\rm CKM}$ denotes the corresponding CKM phase, and $\mathcal{A}_j e^{i\phi_j^{\rm strong}}$ encodes the whole nonperturbative physics as well as the moduli of the CKM elements. The calculation of the strong amplitudes and phases from first principles is a nontrivial problem, for which a general solution has not yet been developed. Currently several working tools are available in order to investigate this nonperturbative problem: QCD factorization [QCDF, e.g., Beneke, Buchalla, Neubert, and Sachrajda (1999), Beneke *et al.* (2000, 2001), and Beneke and Neubert (2003)], soft collinear effective theory [SCET, e.g., Bauer *et al.* (2001, 2004, and Bauer, Pirjol, and Stewart (2002)], light cone sum rules [LCSR, e.g., Balitsky, Braun, and Kolesnichenko (1989), Khodjamirian (2001), and Khodjamirian, Mannel, and Melic (2003)], and perturbative QCD [pQCD, e.g., Li and Yu (1996) and Yeh and Li (1997)]. Considering the CP conjugate decay $\bar{B}_s^0 \to \bar{f}$, one finds

$$\bar{\mathcal{A}}_{\bar{f}} = -\sum_j \mathcal{A}_j e^{i(\phi_j^{\rm strong} - \phi_j^{\rm CKM})}, \tag{141}$$

so only the CKM phase has changed its sign, while the strong amplitude and the strong phase remain unmodified. The overall sign is due to the CP properties of the B_s^0 mesons, defined in Eq. (8) and $\bar{f}$ defined in Eq. (34). In some CP asymmetries the hadronic amplitudes cancel to a good approximation in the ratios of decay rates. The corresponding decay modes are the so-called *gold-plated modes*, which were introduced by Carter and Sanda (1981) and Bigi and Sanda (1981). Later on we see that gold-plated modes will appear, when the decay amplitude is governed by a single CKM structure. This could be the case in a decay like $\bar{B}_s^0 \to J/\psi\phi$, which is governed on a quark level by a $b \to c\bar{c}s$ transition. Such a transition has a large tree-level contribution and a suppressed penguin contribution; see Fig. 13. To a good first approximation the penguin contributions can be neglected and we have a gold-plated mode, with a precise relation of the corresponding CP asymmetry to fundamental standard model parameters, including the CKM couplings. In view of the dramatically increased experimental precision in recent years it turns out, however, that it is necessary to investigate the possible size of penguin effects, the so-called penguin pollution. This is discussed later. Let us return to the general

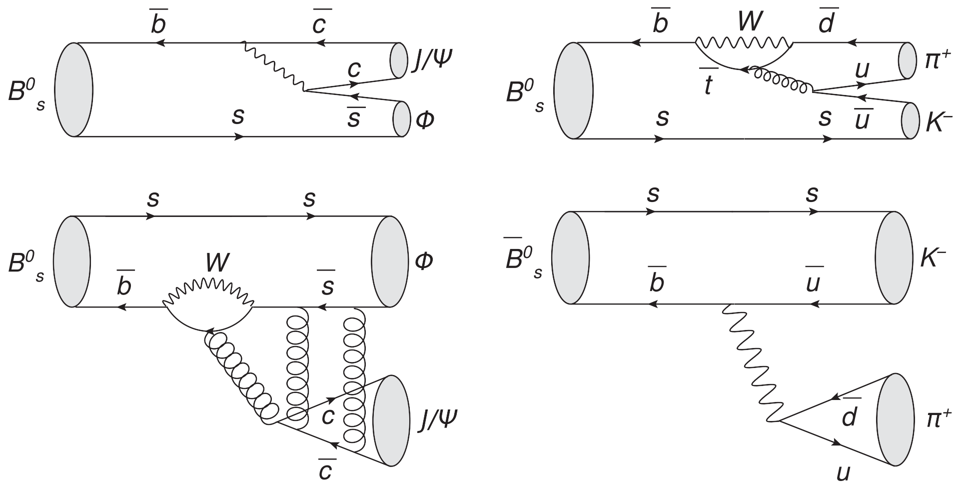

FIG. 13. Different decay topologies contributing to the decays $B_s^0 \to J/\psi\phi$ (lhs) and $B_s^0 \to K^-\pi^+$ (rhs). The top row shows the color suppressed topologies (in the case of $B_s^0 \to J/\psi\phi$ this is the tree-level contribution and in the case of $B_s^0 \to K^-\pi^+$ this is the penguin contribution) and the lower row shows the color allowed topologies (in the case of $B_s^0 \to J/\psi\phi$ this is now the penguin contribution and in the case of $B_s^0 \to K^-\pi^+$ this is the tree-level contribution). Since the J/ψ meson is color neutral, we have to add additional gluons for the penguin contribution to $B_s^0 \to J/\psi\phi$.

case and consider the following time-dependent CP asymmetry for a $B_s^0 \to f$ transition without any approximations concerning the structure of the decay amplitude:

$$A_{CP,f}(t) = \frac{\Gamma(\bar{B}_s^0(t) \to f) - \Gamma(B_s^0(t) \to f)}{\Gamma(\bar{B}_s^0(t) \to f) + \Gamma(B_s^0(t) \to f)}. \quad (142)$$

Inserting the time evolution given in Eqs. (21) and (33) one finds[15]

$$A_{CP,f}(t) = -\frac{\mathcal{A}_{CP}^{\rm dir}\cos(\Delta M_s t) + \mathcal{A}_{CP}^{\rm mix}\sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s t/2) + \mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_s t/2)} + \mathcal{O}(a_{fs}^s), \quad (143)$$

with $\mathcal{A}_{CP}^{\rm dir}$, $\mathcal{A}_{CP}^{\rm mix}$, and $\mathcal{A}_{\Delta\Gamma}$ being defined in Eqs. (24), (25), and (26). We can rewrite two of those definitions as

$$\mathcal{A}_{CP}^{\rm mix} = -\frac{2|\lambda_f|}{1+|\lambda_f|^2}\sin[\arg(\lambda_f)] = +\frac{2|\lambda_f|}{1+|\lambda_f|^2}\sin[\phi_s], \quad (144)$$

$$\mathcal{A}_{\Delta\Gamma} = -\frac{2|\lambda_f|}{1+|\lambda_f|^2}\cos[\arg(\lambda_f)] = -\frac{2|\lambda_f|}{1+|\lambda_f|^2}\cos[\phi_s], \quad (145)$$

with the phase ϕ_s to be defined as

$$\phi_s = -\arg(\lambda_f) = -\arg\left(\frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}\right) \quad (146)$$

$$= -\pi + \phi_M - \arg\left(\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}\right). \quad (147)$$

[15] A more detailed derivation can be found in Anikeev *et al.* (2001).

This is the most general definition of the phase that appears in interference. However, in this form a measurement of ϕ_s does not enable us to connect the phase with fundamental parameters of the underlying theory. To do so, either we find some simplifications for the decay amplitudes or we have to evaluate the ratio of amplitudes nonperturbatively. Before discussing a particular simplification, we note that sometimes a different notation [S_f for the coefficient that is arising in Eq. (143) with the term $\sin(\Delta M_s t)$ and C_f or A_f for the coefficient that is arising with the term $\cos(\Delta M_s t)$ up to signs] is used

$$-A_f = C_f \equiv \mathcal{A}_{CP}^{\rm dir}, \quad (148)$$

$$-S_f \equiv \mathcal{A}_{CP}^{\rm mix}. \quad (149)$$

BABAR uses C_f and Belle uses A_f. Expanding the hyperbolic functions in Eq. (143) for small arguments, i.e., small decay rate differences and/or short times, we can express the time-dependent CP asymmetry $A_{CP,f}(t)$ as

$$A_{CP,f}(t) \approx \frac{S_f\sin(\Delta M_s t) - C_f\cos(\Delta M_s t)}{1 + \mathcal{A}_{\Delta\Gamma}\frac{\Delta\Gamma_s}{2\Gamma_s}\frac{t}{\tau_s} + \frac{1}{2}\left(\frac{\Delta\Gamma_s}{2\Gamma_s}\frac{t}{\tau_s}\right)^2}. \quad (150)$$

This formula holds in general, and no approximation on the corresponding decay amplitudes has been made yet. In this general case, the quantities $\mathcal{A}_{CP}^{\rm dir}$, $\mathcal{A}_{CP}^{\rm mix}$, and $\mathcal{A}_{\Delta\Gamma}$ are unknown hadronic contributions that are very difficult to be determined in theory. In the following we discuss the simplified case of the gold-plated modes. Here we consider the final state f to be a CP eigenstate, i.e., $f = f_{CP} = \eta_{CP}\bar{f}$, and we assume that only one CKM structure is contributing to the decay amplitude by, e.g., neglecting penguins. In this special case we get

$$\mathcal{A}_{f_{CP}} = \mathcal{A}_j e^{i(\phi_j^{\rm strong} + \phi_j^{\rm CKM})}, \quad (151)$$

$$\bar{\mathcal{A}}_{f_{CP}} = \eta_{CP}\bar{\mathcal{A}}_{\bar{f}_{CP}} = -\eta_{CP}\mathcal{A}_j e^{i(\phi_j^{\rm strong}-\phi_j^{\rm CKM})}, \quad (152)$$

$$\Rightarrow \frac{\bar{\mathcal{A}}_{f_{CP}}}{\mathcal{A}_{f_{CP}}} = -\eta_{CP} e^{-2i\phi_j^{\rm CKM}}. \quad (153)$$

In the case of gold-plated modes all hadronic uncertainties cancel exactly in the ratio of the two decay amplitudes in Eq. (153) and one is left with a pure weak CKM phase. Thus the parameter λ_f, which triggers the CP asymmetries, is given by

$$\lambda_{f_{CP}} = \frac{q}{p}\frac{\bar{\mathcal{A}}_{f_{CP}}}{\mathcal{A}_{f_{CP}}} = \eta_{CP}\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} e^{-2i\phi_j^{\rm CKM}}. \quad (154)$$

Therefore we have in the case of only one contributing CKM structure $|\lambda_{f_{CP}}| = 1$ and thus

$$\mathcal{A}_{CP}^{\rm dir} = 0, \quad (155)$$

$$\mathcal{A}_{CP}^{\rm mix} = +\sin(\phi_s), \quad (156)$$

$$\mathcal{A}_{\Delta\Gamma} = -\cos(\phi_s), \quad (157)$$

leading to the simplified formula for the asymmetry

$$A_{CP,f}(t) \approx \frac{\sin\phi_s \sin(\Delta M_s t)}{\cos\phi_s \sinh(\Delta\Gamma_s t/2) - \cosh(\Delta\Gamma_s t/2)}. \quad (158)$$

This formula holds in the case of only one contributing CKM structure to the whole decay amplitude and the final state being a CP eigenstate.

If the corresponding decay is triggered by a $b \to c\bar{c}s$ transition on the quark level, as in the case of $B_s^0 \to J/\psi\phi$, we get

$$\phi_s = -\arg\left(\eta_{CP}\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}}\frac{V_{cs}^*V_{cb}}{V_{cs}V_{cb}^*}\right). \quad (159)$$

Thus a measurement of the mixing phase ϕ_s gives us direct information about the phases, i.e., the amount of CP violation, of the CKM elements. If in addition the final state has a CP eigenvalue $\eta_{CP} = +1$, then we get

$$\phi_s = -2\beta_s, \quad (160)$$

with the commonly used notation

$$\beta_s = -\arg\left[-\frac{V_{ts}^*V_{tb}}{V_{cs}^*V_{cb}}\right] \quad (161)$$

$$= 0.0183 \pm 0.0010 = (1.05 \pm 0.05)^\circ. \quad (162)$$

Here we used a definition for β_s that ensures that its numerical value is positive; sometimes a different sign is used. If there is only a modest experimental precision available, the penguin contributions can be neglected, to a first approximation, for the tree-level dominated $b \to c\bar{c}s$ decays like $B_s \to J/\psi\phi$ penguin terms, and we can use simplified formulas such as Eq. (160).

However, we see below that the current experimental precision in the determination of ϕ_s is of the order of $\pm 2^\circ$, which equals the SM expectation of $\phi_s^{\rm SM} = (2.1 \pm 0.1)^\circ$. In view of this high experimental precision, it seems mandatory to determine the possible size of penguin contributions, in order to make profound statements about new physics effects in these CP asymmetries.

Let us examine the general expression for the decay amplitude without neglecting penguin contributions. Examples for decays with both tree-level and penguin contributions are $B_s \to J/\psi\phi$ or $B_s \to K^-\pi^+$. The former is governed on the quark level by a $b \to c\bar{c}s$ transition, and the latter by a $b \to u\bar{u}d$ transition. The tree-level components and penguin contributions to these decays are shown in Fig. 13.

A naive dimensional estimate (the size of CKM couplings, the number of strong couplings, and color counting) gives a small penguin contribution in the case of $B_s \to J/\psi\phi$ and a large penguin contribution in the case $B_s \to K^-\pi^+$. Thus $B_s \to J/\psi\phi$ is a prime candidate for a gold-plated mode, while in the case of $B_s \to K^-\pi^+$ direct CP violation, i.e., CP violation directly in the decay might be visible; this is further discussed in Sec. V.

To become more quantitative, we take a closer look at the general structure of the decay amplitude of a $b \to c\bar{c}s$ transition. Using the effective Hamiltonian for $\Delta B = 1$ transitions [see, e.g., Buras (1998) for an introduction] we get for the amplitude

$$\mathcal{A}_f(B_s^0 \to f) = \langle f|\mathcal{H}_{\rm eff}|B_s^0\rangle, \quad (163)$$

with the effective SM Hamiltonian for $b \to c\bar{c}s$ transitions

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\lambda_u(C_1Q_1^u + C_2Q_2^u) + \lambda_c(C_1Q_1^c + C_2Q_2^c) + \lambda_t\sum_{i=3}^{6}C_iQ_i\right] + {\rm H.c.} \quad (164)$$

The CKM structure is given as before by $\lambda_q \coloneqq V_{qb}V_{qs}^*$; the decay $b \to c\bar{c}s$ proceeds via the current-current operators Q_1^c, Q_2^c and the QCD penguin operators $Q_3, \ldots, Q_6$. $C_1, \ldots, C_6$ are the corresponding Wilson coefficients. When the current-current operators Q_1^u and Q_2^u are inserted in a penguin diagram in the effective theory, they also contribute to $b \to c\bar{c}s$. Electroweak penguin contributions are neglected.

Therefore we have the following structure of the amplitude $\mathcal{A}_f(B_s^0 \to f)$:

$$\mathcal{A}_f = \frac{G_F}{\sqrt{2}}\left[\lambda_u\sum_{i=1,2}C_i\langle Q_i^u\rangle^P + \lambda_c\sum_{i=1,2}C_i\langle Q_i^c\rangle^{T+P} + \lambda_t\sum_{i=3}^{6}C_i\langle Q_i\rangle^T\right]. \quad (165)$$

$\langle Q\rangle^T$ denotes the tree-level insertion of the local operator Q, and $\langle Q\rangle^P$ denotes the insertion of the operator Q in a penguin diagram. Using further the unitarity of the CKM matrix $\lambda_t = -\lambda_u - \lambda_c$, we can rewrite the amplitude in a form where only two different CKM structures are appearing:

$$\mathcal{A}_f = \frac{G_F}{\sqrt{2}}\lambda_c\left[\sum_{i=1,2} C_i\langle Q_i^c\rangle^{T+P} - \sum_{i=3}^{6} C_i\langle Q_i\rangle^T + \frac{\lambda_u}{\lambda_c}\left(\sum_{i=1,2} C_i\langle Q_i^u\rangle^P - \sum_{i=3}^{6} C_i\langle Q_i\rangle^T\right)\right] \tag{166}$$

$$= \mathcal{A}_f^{\rm Tree} + \mathcal{A}_f^{\rm Peng}. \tag{167}$$

In the last line we separately defined a tree-level amplitude and a penguin amplitude. They are given by

$$\mathcal{A}_f^{\rm Tree} = \frac{G_F}{\sqrt{2}}\lambda_c\left[\sum_{i=1,2} C_i\langle Q_i^c\rangle^{T+P} - \sum_{i=3}^{6} C_i\langle Q_i\rangle^T\right] = |\mathcal{A}_f^{\rm Tree}|e^{i[\phi_{\rm Tree}^{\rm QCD}+\arg(\lambda_c)]}, \tag{168}$$

$$\mathcal{A}_f^{\rm Peng} = \frac{G_F}{\sqrt{2}}\lambda_u\left[\sum_{i=1,2} C_i\langle Q_i^u\rangle^{P} - \sum_{i=3}^{6} C_i\langle Q_i\rangle^T\right] = |\mathcal{A}_f^{\rm Peng}|e^{i[\phi_{\rm Peng}^{\rm QCD}+\arg(\lambda_u)]}. \tag{169}$$

Here we split up the amplitudes into their modulus and their phase. Sometimes it is advantageous to split off the explicit dependence on the modulus of the CKM structure:

$$|\mathcal{A}_f^{\rm Tree}| = \frac{G_F}{\sqrt{2}}|\lambda_c||\tilde{\mathcal{A}}_f^{\rm Tree}|, \tag{170}$$

$$|\mathcal{A}_f^{\rm Peng}| = \frac{G_F}{\sqrt{2}}|\lambda_u||\tilde{\mathcal{A}}_f^{\rm Peng}|. \tag{171}$$

The strong amplitudes and the strong phases are in principle unknown. A first naive estimate of the size of the modulus can be done by investigating what $\Delta B = 1$ Wilson coefficients are contributing. In the case of $B_s^0 \to J/\psi\phi$ the tree-level amplitude is enhanced by the CKM elements in λ_c and the tree-level contribution of the large Wilson coefficients C_1 and C_2; the penguin amplitude is suppressed by λ_u and further either by small penguin Wilson coefficients $C_{3\cdots6}$ or by a loop.

In general, without any approximations concerning the size of the hadronic effects, we get the ratio of decay amplitudes

$$\frac{\bar{\mathcal{A}}_{\bar f}}{\mathcal{A}_f} = -e^{-2i\arg(\lambda_c)}\left[\frac{1+re^{-i\arg(\lambda_u/\lambda_c)}}{1+re^{+i\arg(\lambda_u/\lambda_c)}}\right] \tag{172}$$

with r defined as

$$r = \left|\frac{\lambda_u}{\lambda_c}\right|\left|\frac{\tilde{\mathcal{A}}_f^{\rm Peng}}{\tilde{\mathcal{A}}_f^{\rm Tree}}\right|e^{i(\phi_{\rm Peng}^{\rm QCD}-\phi_{\rm Tree}^{\rm QCD})}. \tag{173}$$

In the case of $B_s^0 \to J/\psi$ the CKM part of r is very small; it is given by $|\lambda_u/\lambda_c| \approx 0.02$. The hadronic part of r is a nonperturbative quantity that is currently not calculated from first principles. Before we turn to some quantitative investigations in the literature, we look at naive estimates: $\tilde{\mathcal{A}}_f^{\rm Peng}$ and $\tilde{\mathcal{A}}_f^{\rm Tree}$ contain Wilson coefficients from the effective Hamiltonian. The penguin Wilson coefficients $|C_{3,\ldots,6}|$ are typically smaller than 0.04; therefore one can neglect them in comparison to the Wilson coefficient $C_2 \approx 1$; see, e.g., Buchalla, Buras, and Lautenbacher (1996) for numerical values. Thus we are left with the tree-level insertion of the operator Q_2 in the case of $\tilde{\mathcal{A}}_f^{\rm Tree}$ and with the penguin insertion of the operator Q_2 in the case of $\tilde{\mathcal{A}}_f^{\rm Peng}$. Since we do not know the relative size of these two, we take the analogy of inclusive b-quark decays as a first indication of its size. For the inclusive decay $b \to c\bar{c}s$ it was found (Bagan *et al.*, 1995; Lenz, Nierste, and Ostermaier, 1997; Krinner, Lenz, and Rauh, 2013) that $\langle Q\rangle^P \leq 0.05\langle Q\rangle^T$. Taking this value as an indication for the size of $\tilde{\mathcal{A}}_f^{\rm Peng}/\tilde{\mathcal{A}}_f^{\rm Tree}$ we get an estimate of r of about $|r| \approx 0.001$. One should be aware, however, that this naive estimate can easily be off by a factor of 10 and we also cannot quantify the size of the strong phase in this approach. Using the same methods for the decay $B_s^0 \to K^-\pi^+$ we get a value of $r^{B_s^0\to K^-\pi^+}$ of about 0.1, so roughly 100 times larger than in the case of $B_s^0 \to J/\psi\phi$. $B_s^0 \to K^-\pi^+$ is thus a prime candidate for decays where we are looking for large penguin effects, e.g., if we want to measure a direct CP asymmetry in the B_s^0 system. Our naive estimate does not take into account that these two channels proceed via different topologies; hence the factor 100 might have to be modified considerably.

Nevertheless, it seems that r is a small number in the case of $B_s^0 \to J/\psi\phi$ and we can make a Taylor expansion in Eq. (172) to obtain

$$\frac{\bar{\mathcal{A}}_{\bar f}}{\mathcal{A}_f} \approx -e^{-2i\arg(\lambda_c)}\left\{1 - 2ir\sin\left[\arg\left(\frac{\lambda_u}{\lambda_c}\right)\right]\right\}. \tag{174}$$

Further investigating Eq. (174) or (172), we see that the first term on the rhs gives rise to $-2\beta_s$ in the CP asymmetry in Eq. (158). The second term (proportional to r) corresponds to the SM penguin pollution, which we denote by $\delta^{\rm Peng,SM}$. Therefore the experimentally measured phase ϕ_s has the following two contributions in the standard model:

$$\phi_s = -2\beta_s + \delta^{\rm Peng,SM}, \tag{175}$$

where the standard model penguin is given by

$$e^{i\delta^{\rm Peng,SM}} \approx 1 - 2ir\sin\left[\arg\left(\frac{\lambda_u}{\lambda_c}\right)\right]e^{i(\phi_{\rm Peng}^{\rm QCD}-\phi_{\rm Tree}^{\rm QCD})}. \tag{176}$$

Inserting these approximations for $B_s^0 \to J/\psi\phi$ we get as a very rough estimate of the penguin pollution

$$e^{i\delta^{\rm Peng,SM}} \approx 1 - 0.002ie^{i(\phi_{\rm Peng}^{\rm QCD}-\phi_{\rm Tree}^{\rm QCD})} \Rightarrow \delta^{\rm Peng,SM} \leq \pm0.002 = \pm0.1°. \tag{177}$$

Thus naively we expect a penguin pollution of at most $\pm0.1°$ in the case of $B_s^0 \to J/\psi\phi$. This very rough estimate could, however, be easily modified by a factor of 10, due to nonperturbative effects and then we would be close to the current experimental uncertainties. Thus more theoretical work has to be done to quantify the size of penguin contributions. There are now several strategies to achieve this point.

(1) Measure ϕ_s in different decay channels: assuming that the penguin contributions are negligible, different determinations should give the same value for the mixing phase. Until now we have focused on the extraction of the phase ϕ_s from the decay $B_s^0 \to J/\psi\phi$. This final state is an admixture of CP-even and CP-odd components. To extract information on $\Delta\Gamma_s$ and ϕ_s an angular analysis is required; see the discussion in Secs. II.B and IV.B or Dighe *et al.* (1996), Dighe, Dunietz, and Fleischer (1999), and Dunietz, Fleischer, and Nierste (2001). Moreover, the $J/\psi\phi(\to K^+K^-)$ final state can be investigated for nonresonant K^+K^- contributions in order to increase the statistics. The phase ϕ_s has also been determined in different $b \to c\bar{c}s$ channels, such as $B_s^0 \to J/\psi\pi^+\pi^-$ [including $B_s^0 \to J/\psi f_0$, see, e.g., Stone and Zhang (2009, 2013), Colangelo, Fazio, and Wang (2011); Fleischer, Knegjens, and Ricciardi (2011a), and Zhang and Stone (2013)], $B_s^0 \to J/\psi\eta^{(\prime)}$ [see, e.g., Dunietz, Fleischer, and Nierste (2001), Fleischer, Knegjens, and Ricciardi (2011b), and Di Donato, Ricciardi, and Bigi (2012)], and $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ [see, e.g., Dunietz, Fleischer, and Nierste (2001) and Fleischer (2007b)] as a cross-check. Here $B_s^0 \to J/\psi f_0$ is a CP-odd final state, $J/\psi\eta^{(\prime)}$ and $D_s^+D_s^-$ are CP-even final states, and $D_s^{*+}D_s^{*-}$ is again an admixture of different CP components. Getting different values for ϕ_s from different decay modes points toward different and large penguin contributions in the individual channels. The different experimental results are discussed in the next section: they show no significant deviations within the current experimental uncertainties, but there is also plenty of space left for some sizable differences.

(2) Measure the phase ϕ_s for different polarizations of the final states in $B_s^0 \to J/\psi\phi$: potential differences might originate from the penguin contributions, which in general contribute differently to different polarizations (Fleischer, 1999a; Faller, Fleischer, and Mannel, 2009). Such an analysis was done by Aaij *et al.* (2015h) and within the current experimental uncertainties no hint for a polarization dependence of ϕ_s was found:

$$\phi_{s,\|} - \phi_{s,0} = -(1.03 \pm 2.46 \pm 0.52)^\circ, \qquad (178)$$

$$\phi_{s,\perp} - \phi_{s,0} = -(0.80 \pm 2.01 \pm 0.34)^\circ. \qquad (179)$$

On the other hand, one sees that effects of the order of 2°, which would be as large as the whole SM prediction for ϕ_s, are not ruled out yet. Further discussion of this result was given by De Bruyn and Fleischer (2015).

(3) Compare the decay $B_s^0 \to J/\psi\phi$ to a decay with a similar hadronic structure, but a CKM enhanced penguin contribution: differences in the phase ϕ_s extracted from $B_s^0 \to J/\psi\phi$ and from the new decay might then give experimental hints for the size of the penguin contribution.

Exchanging the s-quark line in Fig. 13 with a d-quark line one arrives at decays such as $B_s^0 \to J/\psi K_S$ (Fleischer, 1999b) or $B_s^0 \to J/\psi\bar{K}^*(892)$ (Dunietz, Fleischer, and Nierste, 2001; Faller, Fleischer, and Mannel, 2009; De Bruyn and Fleischer, 2015). In the first decay there is only one vector particle in the final state, while in the latter case we have two [$\bar{K}^*(892)$ is a vector meson] as in the case of $B_s^0 \to J/\psi\phi$. Thus we consider here only the decay $B_s^0 \to J/\psi\bar{K}^*(892)$. The analogous modes in B^0 decays are $B^0 \to J/\psi K_S \Leftrightarrow B^0 \to J/\psi\pi^0$. The extraction of penguin pollution via this relation was discussed by Ciuchini, Pierini, and Silvestrini (2005, 2011) and Faller *et al.* (2009). To get an idea of the size of the penguin uncertainties, we note that Ciuchini, Pierini, and Silvestrini (2011) found a possible standard model penguin pollution of about $\pm 1.1^\circ$ in the gold-plated mode $B^0 \to J/\psi K_S$.

Returning to the B_s^0 system, the relative size of the penguin contributions in the decays $B_s^0 \to J/\psi K_S$ and $B_s^0 \to J/\psi\bar{K}^*(892)$, compared to the tree-level components, are larger by a factor of about $1/\lambda^2 \approx 25$ than in $B_s^0 \to J/\psi\phi$. This enhancement of the penguin contribution might manifest itself in different values for the extracted values of the phase ϕ_s. A disadvantage of these decays is that they are more difficult to measure, because they proceed on a quark level via $b \to c\bar{c}d$, whose branching ratio is suppressed by a factor of about $\lambda^2 \approx 1/25$ compared to $B_s^0 \to J/\psi\phi$. This is the reason why the CP asymmetries in $B_s^0 \to J/\psi K_S$ and the one in $B_s^0 \to J/\psi\bar{K}^*(892)$ have been determined only recently with large uncertainties by Aaij *et al.* (2015g, 2015d). The corresponding branching ratios were measured earlier by the LHCb Collaboration (Aaij *et al.*, 2012c, 2012e). De Bruyn and Fleischer (2015) discussed some strategies to extract the size of penguin pollution without having the full knowledge about these CP asymmetries. A further drawback of this method is that the size of the hadronic effects due to the exchange of a ϕ meson with a $\bar{K}^*(892)$ meson cannot be quantified from first principles. Finally there are also penguin annihilation and weak-exchange topologies contributing to $B_s^0 \to J/\psi\phi$ that are not present in the $B_s^0 \to J/\psi\bar{K}^*(892)$ case; see, e.g., Faller, Fleischer, and Mannel (2009). Whether it is justified to neglect such contributions can be tested by decays such as $B^0 \to J/\psi\phi$ that proceed only via weak-exchange and annihilation topologies. Experimental constraints on $B^0 \to J/\psi\phi$ from Belle (Liu *et al.*, 2008), *BABAR* (Lees *et al.*, 2015a), and LHCb (Aaij *et al.*, 2013c) indicate, however, no unusual enhancement of annihilation or weak-exchange contributions.

(4) Compare the decay $B_s^0 \to J/\psi\phi$ with a decay which is related to it via a symmetry of QCD: having now a

symmetry might add confidence in obtaining some control over the effect of exchanging the initial and final state mesons with other mesons. Such a symmetry is the flavor symmetry $SU(3)_F$, i.e., a symmetry of QCD under the exchange of u, d, and s quarks. Application of these symmetries is quite widespread; see, e.g., Fleischer (1999b), Ciuchini, Pierini, and Silvestrini (2005, 2011), Faller, Fleischer, and Mannel (2009), Jung (2012), Bhattacharya, Datta, and London (2013), De Bruyn and Fleischer (2015), and Ligeti and Robinson (2015) for some examples related to B-meson decays. Again a word of caution: it is currently not clear how well the $SU(3)_F$ symmetry is working and how large the corrections are; see, e.g., Faller, Fleischer, and Mannel (2009) and Frings, Nierste, and Wiebusch (2015) for some critical comments. On the other hand, a comparison of experimental data finds that $SU(3)_F$ might work quite well for some of these decay channels; see, e.g., De Bruyn and Fleischer (2015).

A subgroup of $SU(3)_F$, which is supposed to work particularly well, is the so-called *U-spin symmetry*, i.e., the invariance of QCD under the exchange of the s quark with a d quark (Fleischer, 1999a, 1999b; De Bruyn and Fleischer, 2015). Substituting the s and $\bar{s}$ quarks on the lhs of Fig. 13 with down-type quarks one gets (Fleischer, 1999a)

$$B_s^0 \to J/\psi\phi \Leftrightarrow B^0 \to J/\psi\rho^0, J/\psi\pi^0. \qquad (180)$$

The decay $B^0 \to J/\psi\rho^0$ also has enhanced penguin contributions and a similar structure as $B^0 \to J/\psi\pi^0$ and $B_s^0 \to J/\psi K_S$; tree and penguin contributions to $B^0 \to J/\psi\pi^+\pi^-$, which contains $B^0 \to J/\psi\rho^0$, are depicted in Fig. 14. $B^0 \to J/\psi\rho^0$ is discussed further by De Bruyn and Fleischer (2015) and there are also first measurements of the mixing induced *CP* asymmetries by the LHCb Collaboration (Aaij *et al.*, 2015e). In this decay we have again two vector mesons in the final state, as in the case $B_s^0 \to J/\psi\phi$. Thus here we do not consider the decay $B^0 \to J/\psi\pi^0$ any further. However, this decay gave important constraints on the penguin pollution in $B^0 \to J/\psi K_S$ as explained earlier.

Applying U-spin symmetry to the B^0 system one gets

$$B^0 \to J/\psi K_S \Leftrightarrow B_s^0 \to J/\psi K_S. \qquad (181)$$

The decay $B_s^0 \to J/\psi K_S$ was already mentioned for estimating penguin uncertainties in $B_s^0 \to J/\psi\phi$. It is, however, much better suited for the decay $B^0 \to J/\psi K_S$ (Fleischer, 1999b; Faller *et al.*, 2009; De Bruyn and Fleischer, 2015). Further experimental studies of this decay were performed by Aaij *et al.* (2013g).

Currently symmetry considerations put a quite strong bound on the penguin pollution; De Bruyn and Fleischer (2015) [see also Fleischer (2015)] get for the decay $B_s^0 \to J/\psi\phi$ the following possible size of penguin pollution:

$$\delta_{J/\psi\phi}^{\text{Peng,SM}} = [0.08^{+0.56}_{-0.72}(\text{stat})^{+0.15}_{-0.13}(\text{SU(3)})]^\circ. \qquad (182)$$

This bound is currently dominated by statistical uncertainties stemming from experiment and it will thus be getting stronger in the future by improved measurements, even without theoretical improvements.

(5) Investigate purely penguin induced decays: an example for a decay that has no tree-level contribution is $B_s^0 \to \phi\phi$, which is governed by a $b \to s\bar{s}s$-quark level transition. Traditionally such decays are considered to be most sensitive to new physics effects. The decay $B_s^0 \to \phi\phi$ has contributions from a u, c, and t penguin. Its amplitude reads

$$\mathcal{A}_f(B_s^0 \to \phi\phi) = \frac{G_F}{\sqrt{2}}\left[\lambda_u \sum_{i=1,2} C_i \langle Q_i^u\rangle^P + \lambda_c \sum_{i=1,2} C_i \langle Q_i^c\rangle^P + \lambda_t \sum_{i=3}^{6} C_i \langle Q_i\rangle^T\right]. \qquad (183)$$

Using again the unitarity of the CKM matrix, we can rewrite the amplitude in a form where only two different CKM structures appear:

$$\mathcal{A}_f = \frac{G_F}{\sqrt{2}}\lambda_c\left[\sum_{i=1,2} C_i \langle Q_i^c\rangle^P - \sum_{i=3}^{6} C_i \langle Q_i\rangle^T + \frac{\lambda_u}{\lambda_c}\left(\sum_{i=1,2} C_i \langle Q_i^u\rangle^P - \sum_{i=3}^{6} C_i \langle Q_i\rangle^T\right)\right]. \qquad (184)$$

Neglecting the second term, proportional to λ_u/λ_c, we get the same result as in the case of the gold-plated mode $B_s^0 \to J/\psi\phi$: the measured mixing phase is $\phi_s = -2\beta_s$. In the case of $B_s^0 \to \phi\phi$ this might, however, not be a very good approximation. Our leading term is now given by the difference between the charm penguin and top penguin contributions, which will give a small contribution compared to the large tree-level term in the case of $B_s^0 \to J/\psi\phi$. The subleading term is suppressed by λ_u/λ_c, which is a

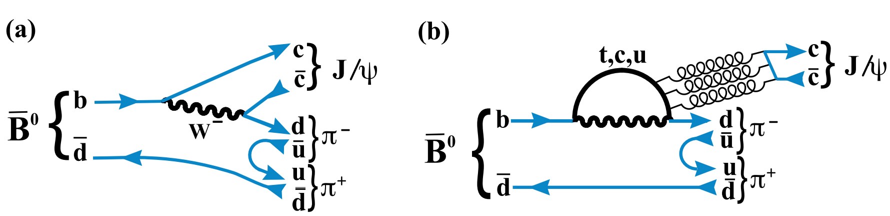

FIG. 14. (a) Tree level and (b) penguin diagrams for $\bar{B}^0$ decays into $J/\psi\pi^+\pi^-$, which also contains $\bar{B}^0 \to J/\psi\rho^0$.

small number, but the hadronic part is now the difference between the up penguin and top penguin contributions, which is of a similar size as the leading term. Thus deviations of the measured phase in $B_s^0 \to \phi\phi$ from $-2\beta_s$ might tell us something about unexpected nonperturbative enhancements of the up quark penguins compared to the charm quark penguin contributions. More advanced theory investigations have been given by Beneke, Rohrer, and Yang (2007), Bartsch, Buchalla, and Kraus (2008), Cheng and Chua (2009), and Datta, Duraisamy, and London (2012). First measurements (Aaij *et al.*, 2014e) have still a sizable uncertainty, but they show no significant deviation of the mixing phase in $B_s^0 \to \phi\phi$ from $-2\beta_s$.

(6) Try to do a calculation from first principles. Very recently penguin effects were estimated in that manner by Frings, Nierste, and Wiebusch (2015) by proofing the infrared safety of the penguin contributions in a factorization approach. This study suggests that penguin contributions to ϕ_s in the case of $B_s \to J/\psi\phi$ should be smaller than about 1°. First steps in such a direction have been performed by Boos, Mannel, and Reuter (2004) and they were pioneered by Bander, Silverman, and Soni (1979). In the framework of pQCD this was attempted recently by Liu, Wang, and Xie (2014).

Most of the current investigations point toward a maximal size of SM penguin effects of about $\pm 1°$, which is unfortunately very close to the current experimental precision of about $\pm 2°$. Thus more theoretical work has to be done in that direction. Note that the LHCb constraint from the study of the decay $B^0 \to J/\psi\rho$ (Aaij *et al.*, 2015e) gives a limit on penguin effects at about 1°.

B. Experiment

The experimental study of the CP-violating phase ϕ_s was pursued vigorously and considerable experimental progress was achieved. The main channels used are $B_s^0 \to J/\psi h^+ h^-$, where the h^+h^- system in general may comprise many states with different angular momenta. Many studies focus on the "golden mode" $B_s^0 \to J/\psi\phi$, discussed in Sec. II.B, which also contains the references to the latest experimental results. The analysis of this final state provides the constraint on both $\Delta\Gamma_s$ and ϕ_s and is therefore presented as a two-dimensional confidence level contour.

The determination of ϕ_s requires the CP-even and CP-odd components to be disentangled by analyzing the differential distribution $d\Gamma/dtd\Omega$, where $\Omega \equiv (\cos\theta_h, \cos\theta_\mu, \chi)$, defined as (a) θ_h is the angle between the h^+ direction in the h^+h^- rest frame with respect to the direction of the h^+h^- pair in the B_s^0 rest frame, (b) θ_μ is the angle between the μ^+ direction in the J/ψ frame with respect to the J/ψ direction in the B_s^0 rest frame, and (c) χ is the angle between the J/ψ and the h^+h^-, as shown in Fig. 15.

The decay $B_s^0 \to J/\psi K^+K^-$ proceeds predominantly via $B_s^0 \to J/\psi\phi$ with the ϕ meson decaying subsequently to K^+K^-. In this case, the B_s^0 decays into two vector particles, and the K^+K^- pair is in a P-wave configuration. The final state is then the superposition of CP-even and CP-odd states, depending upon the relative orbital angular momentum of the J/ψ and the ϕ. The same final state can be produced also with K^+K^- pairs in an S-wave configuration, as pointed out by Stone and Zhang (2009). This S-wave component is CP odd.

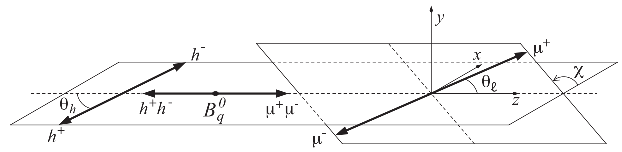

FIG. 15. Definition of the helicity angles. For details see text. The angle θ_μ is denoted as θ_l. From Zhang and Stone, 2013.

The decay width can be expressed in terms of four time-dependent complex amplitudes $A_i(t)$. Three of them arise from the P-wave configuration and correspond to the relative orientation of the linear polarization vectors of the J/ψ and ϕ mesons $(0, \perp, \parallel)$ (Aaij *et al.*, 2015f), and one of them corresponds to the S-wave configuration. The distributions of decay angles and time for a B_s^0 meson produced at time $t = 0$ can be expressed in terms of ten terms, corresponding to the four polarization amplitudes and their interference terms. The expressions for the decay rate $d\Gamma(B_s^0)/dtd\Omega$ are invariant under the transformation

$$(\phi_s, \Delta\Gamma_s, \delta_0, \delta_\parallel, \delta_\perp, \delta_S) \to (\pi - \phi_s, -\Delta\Gamma_s, -\delta_\parallel, \pi - \delta_\perp, -\delta_S). \quad (185)$$

Here the convention $\delta_0 = 0$ is chosen. Thus in principle there is a twofold ambiguity in the results. This is removed by performing fits in bins of m_{hh} (Xie *et al.*, 2009). Thus the LHCb Collaboration performed the fit to the distribution $dn/dtd\Omega$ in bins of m_{hh} to resolve this ambiguity. The projections of the decay time and angular distributions obtained from the analysis of the 3 fb^{-1} LHCb data set are shown in Fig. 16, and the corresponding fit parameters are summarized in Table V. Note that the mixing parameter ΔM_s is not constrained from other measurements in this fit and is consistent with world averages.

This decay mode was also studied in the general purpose detectors at the Tevatron (Aaltonen *et al.*, 2012; Abazov *et al.*, 2012a) and the LHC (Aad *et al.*, 2014; Khachatryan, 2015). The analysis method is similar to the one described before. Figure 17 shows the fit projections obtained with the recent CMS measurements reported by Khachatryan (2015).

Another channel (Stone and Zhang, 2009) was recognized to provide complementary information on ϕ_s, namely, $B_s^0 \to J/\psi f_0$, with $f_0 \to \pi^+\pi^-$. The original appeal of this mode is that it was assumed to be predominantly an S-wave decay and thus not in need of the complex multidimensional fit just described. The study of the Dalitz plot of $B_s^0 \to J/\psi\pi^+\pi^-$ (Aaij *et al.*, 2012a, 2014i) revealed a more complex resonant structure. A combination of five resonant states is required to described the data (Aaij *et al.*, 2014i): $f_0(980)$, $f_0(1500)$, $f_0(1790)$, $f_0(1270)$, and $f_2'(1525)$. The data are compatible with no additional NR components, as well as a combination of these five resonances plus significant NR components, with a fit fraction of $(5.9 \pm 1.4)\%$. The latter

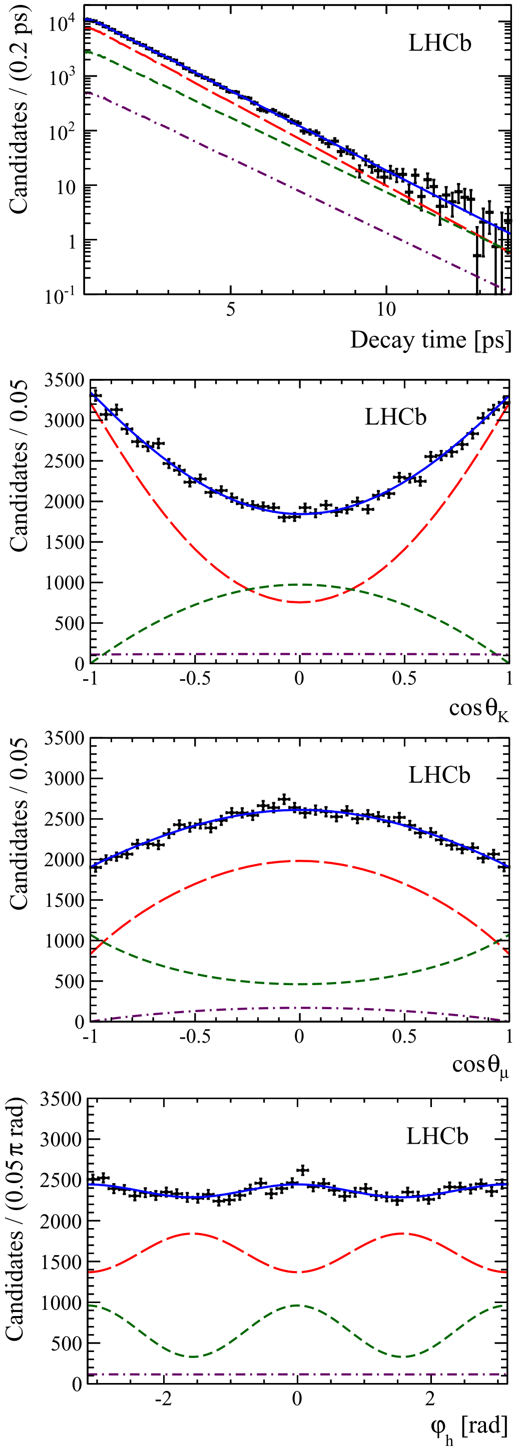

FIG. 16. Decay time and helicity-angle distributions for $B_s^0 \to J/\psi K^+K^-$ decays (data points) with the one-dimensional fit projections overlaid. The solid blue lines show the total signal contributions, which are composed of CP-even (long-dashed, red), CP-odd (short-dashed, green) and S-wave (dot-dashed, purple) contributions. From Aaij *et al.*, 2015h.

solution is shown in Fig. 18. Thus the most recent study of CP violation in $B_s^0 \to J/\psi\pi^+\pi^-$ uses the formalism developed by Zhang and Stone (2013). Their approach is to couple the three-body Dalitz formalism applied to the final state $J/\psi\pi^+\pi^-$ with the time-dependent CP-violation analysis,

TABLE V. Values of the principal physics parameters determined from the LHCb polarization-independent analysis of $B_s^0 \to J/\psi\phi$ in Aaij *et al.* (2015h).

Parameter	Value
Γ_s (ps^{-1})	$0.6603 \pm 0.0027 \pm 0.0015$
$\Delta\Gamma_s$ (ps^{-1})	$0.0805 \pm 0.0091 \pm 0.0032$
$\lvert A_\perp\rvert^2$	$0.2504 \pm 0.0049 \pm 0.0036$
$\lvert A_0\rvert^2$	$0.5241 \pm 0.0034 \pm 0.0067$
$\delta_\parallel$ (rad)	$3.26^{+0.10+0.06}_{-0.17-0.07}$
$\delta_\perp$ (rad)	$3.08^{+0.14}_{-0.15} \pm 0.06$
ϕ_s (rad)	$-0.058 \pm 0.049 \pm 0.006$
$\lvert\lambda\rvert$	$0.964 \pm 0.019 \pm 0.007$
ΔM_s (ps^{-1})	$17.711^{+0.055}_{-0.057} \pm 0.011$

by splitting the final state into CP-even and CP-odd components. They perform an unbinned maximum likelihood fit to the $J/\psi\pi^+\pi^-$ invariant mass m, the decay time t, the dipion invariant mass, the three helicity angles Ω, along with flavor information of the decay hadron, namely, whether it was produced as a B_s^0 or $\bar{B}_s^0$. Assuming the absence of direct CP violation, the result is

$$\phi_s = 75 \pm 67 \pm 8 \text{ mrad},$$

while allowing for direct CP violation they obtain

$$\phi_s = 70 \pm 68 \pm 8 \text{ mrad}, \qquad |\lambda| = 0.89 \pm 0.05 \pm 0.01.$$

Another channel that provides an independent constraint on ϕ_s, investigated by the LHCb experiment, is $B_s^0 \to D_s^+D_s^-$. This decay mode is particularly appealing because it is a CP-even final state and, including two pseudoscalar mesons in the final state, does not require an angular analysis. They obtain $\phi_s = 0.02 \pm 0.17 \pm 0.02$ rad (Aaij *et al.*, 2014g).

The combination of all the ϕ_s measurements performed by HFAG (Amhis *et al.*, 2014) in the spring 2016 gives

$$\phi_s = -0.033 \pm 0.033 \text{ rad} \tag{186}$$

$$= (-1.89 \pm 1.89)^\circ. \tag{187}$$

Individual experimental results are summarized in Table VI and in Fig. 19. The current experimental uncertainty in ϕ_s is commensurate with the central value of the SM prediction given in Eqs. (160) and (161).

The average value of ϕ_s is consistent with the standard model, but subtle effects produced by diagrams mediated by new particles may yet be uncovered. The level of precision required to improve upon current status requires the consideration of effects neglected so far, such as the penguin contributions described previously. Thus experiments have started to investigate decays that may constrain such contributions. The first such measurement is the study of the decay $B^0 \to J/\psi\pi^+\pi^-$. This mode has both penguin and tree diagrams shown in Fig. 14. Theoretical models predict that in this case the penguin diagram is greatly enhanced with respect

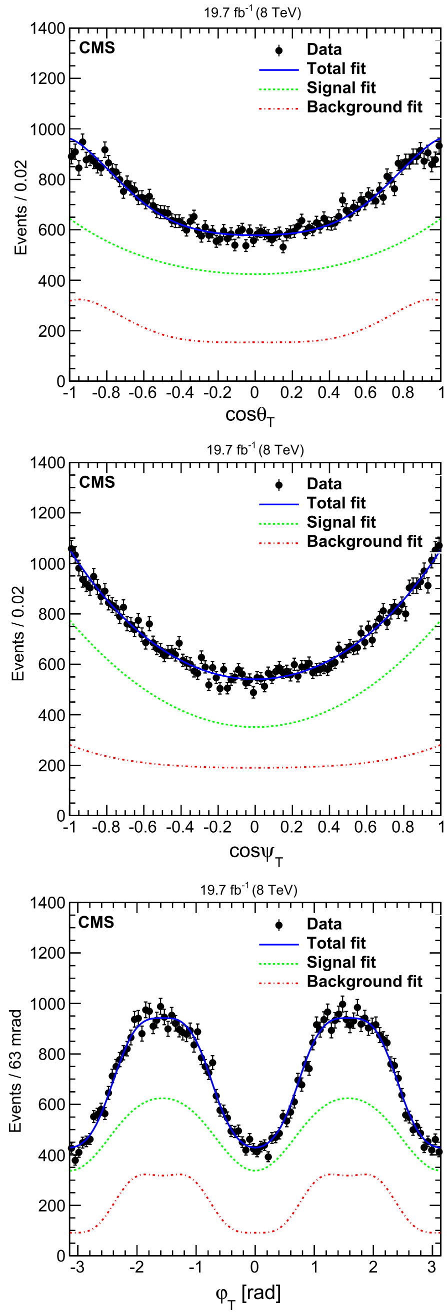

FIG. 17. The angular distributions ($\cos\theta_T$, $\cos\psi_T$, and φ_T) of the B_s^0 candidates from data (solid markers). The angles θ_T and ψ_T are the polar and azimuthal angles, respectively, of the μ^+ in the rest frame of the J/ψ, where the x axis is defined by the direction of the ϕ meson in the J/ψ rest frame, and the x-y plane is defined by the decay plane of the $\phi \to K^+K^-$ decay. The helicity angle ψ_T is the angle of the K^+ in the ϕ rest frame with respect to the negative J/ψ momentum direction. The solid line is the result of the fit, the dashed line is the signal fit, and the dot-dashed line is the background fit. From Khachatryan, 2015.

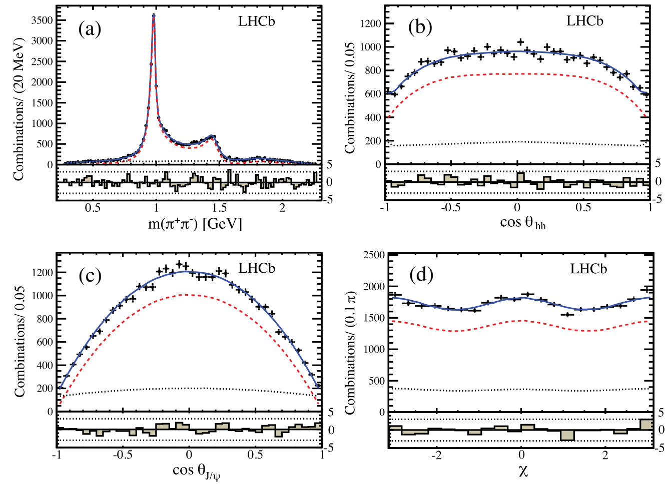

FIG. 18. Projections of (a) $m(\pi^+\pi^-)$, (b) $\cos\theta_{\pi^+\pi^-}$, (c) $\cos\theta_{J/\psi}$, and (d) χ for the solution with the five resonance discussed in the text. The points with error bars represent data. The dashed (red) lines represent the signals, the dotted (black) lines represent the backgrounds, and the solid (blue) lines represent the total fit. From Aaij *et al.*, 2014f.

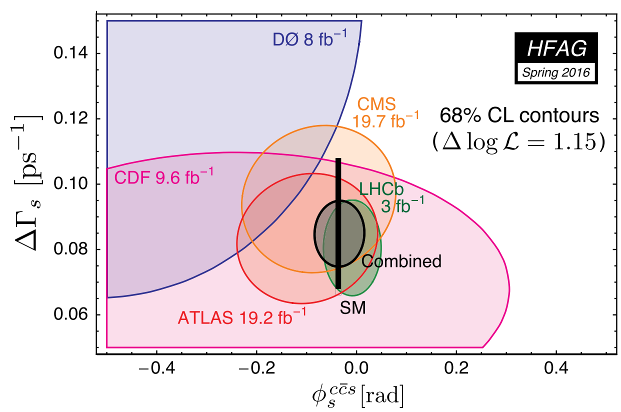

FIG. 19. 68% C.L. regions in B_s^0 width difference $\Delta\Gamma_s$ and weak phase ϕ_s obtained from individual and combined CDF, D0, ATLAS, CMS, and LHCb likelihoods of $B_s^0 \to J/\psi\phi$, $B_s^0 \to J/\psi K^+K^-$, and $B_s^0 \to J/\psi\pi^+\pi^-$. The expectation within the standard model is shown as the black rectangle. From Amhis *et al.*, 2014.

to the decay $B^0 \to J/\psi K_S$. The two decays $B^0 \to J/\psi\rho$ and $B_s^0 \to J/\psi\phi$ are related by $SU(3)$ symmetry if we also assume that the difference between the ϕ being mostly a singlet state and the ρ an octet state causes negligible breaking. If we assume equality between the penguin amplitudes and the strong phases in the two decays and neglecting higher order diagrams (Aaij *et al.*, 2015e), LHCb finds the penguin phase to be $\delta^{\mathrm{Peng}} = (0.05 \pm 0.56)^\circ = 0.9 \pm 9.8$ mrad. At 95% C.L., the penguin contribution in the $B_s^0 \to J/\psi\phi$ decay is within the interval $(-1.05, +1.18)$. Relaxing these assumptions changes the limits on the possible penguin induced shift. Figure 20 shows how δ^{Peng} varies as a function of the difference in strong phases between the two decays $\theta - \theta'$, indicating that the 95% C.L. limit on penguin pollution can increase to at most 1.2°. The phase δ^{Peng} is proportional to the ratio between penguin amplitudes a/a'. As this ratio varies

TABLE VI. Measurements of the mixing phase ϕ_s in different $b \to c\bar{c}s$ channels, such as $B_s^0 \to J/\psi\phi$, $B_s^0 \to J/\psi K^+K^-$, $B_s^0 \to J/\psi\pi^+\pi^-$, $B_s^0 \to J/\psi h + h^-$, and $B_s^0 \to D_s^+ D_s^-$. The standard model expectation (neglecting penguin contributions) for the phase ϕ_s reads -0.0366 ± 0.0020.

Experiment	Mode	ϕ_s (rad)	Reference
CDF	$J/\psi\phi$	$[-0.60, 0.12]$, 68% C.L.	Aaltonen *et al.* (2012)
D0	$J/\psi\phi$	$-0.55^{+0.38}_{-0.36}$	Abazov *et al.* (2012a)
ATLAS	$J/\psi\phi$	$+0.12 \pm 0.25 \pm 0.05$	Aad *et al.* (2014)
ATLAS	$J/\psi\phi$	$-0.123 \pm 0.089 \pm 0.041$	Aad *et al.* (2016)
CMS	$J/\psi\phi$	$-0.075 \pm 0.097 \pm 0.031$	Khachatryan (2015)
LHCb	$J/\psi K^+K^-$	$-0.058 \pm 0.049 \pm 0.006$	Aaij *et al.* (2015h)
LHCb	$J/\psi\pi^+\pi^-$	$+0.070 \pm 0.068 \pm 0.008$	Aaij *et al.* (2014h)
LHCb	$J/\psi h^+h^-$	-0.010 ± 0.039(tot)	Aaij *et al.* (2015h)[a]
LHCb	$D_s^+D_s^-$	$+0.02 \pm 0.17 \pm 0.02$	Aaij *et al.* (2014g)
All combined (HFAG 2016)		-0.033 ± 0.033	

[a]LHCb combination of $J/\psi K^+K^-$ (Aaij *et al.*, 2015h) and $J/\psi\pi^+\pi^-$ (Aaij *et al.*, 2014h).

over the interval 0.5 to 1.5, the limit on the penguin shift spans the range $(\pm 0.9, \pm 1.8)$, even allowing for maximal breaking between θ and θ'.

A complementary approach is based on the study of the polarization-dependent decay amplitudes of the decay $B_s^0 \to J/\psi \bar{K}^{\star 0}$ (Aaij *et al.*, 2015d).The results of Aaij *et al.* (2015e, 2015d) are combined to produce the limits on penguin pollution shown in Fig. 21.

Finally, the decay $B_s^0 \to \phi\phi$ is analogous to $B_s^0 \to J/\psi\phi$, but is forbidden at tree level. It proceeds mostly via the $b \to s s\bar{s}$ penguin, thus providing an excellent probe for the manifestation of interference of new physics particles with the penguin loop. CP violation in this decay was studied by LHCb (Aaij *et al.*, 2013a). They performed an unbinned maximum likelihood fit to $d\Gamma/(dt d\cos\theta_1 d\cos\theta_2 d\Phi)$, where t is the decay time and $\theta_{1,2}$ is the angle between the K^+ track momentum in the $\phi_{1,2}$ meson rest frame and the $\phi_{1,2}$ meson parent momentum in the B_s^0 rest frame, and Φ is the angle between the two ϕ decay planes. The background is taken into account by assigning a weight to each candidate derived with an sPlot technique (Pivk and Diberder, 2005), using the

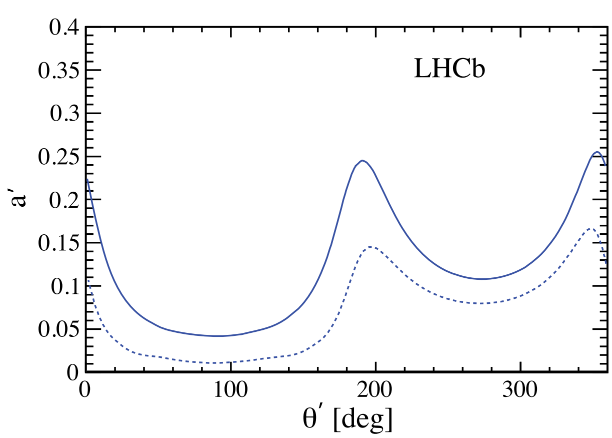

FIG. 20. Contours corresponding to 68% (dashed) and 95% (solid) confidence levels for the number of degrees of freedom (ndf) of 2, respectively, for the penguin amplitude parameters a' and θ'. From Aaij *et al.*, 2015e.

invariant mass of the four K system as a discriminating variable. The resulting fit projections are shown in Fig. 22. The CP-violating phase is found to be in the interval $[-2.46, -0.76]$ rad at 68% confidence level. The p value of the SM prediction is 16%. The precision of the ϕ_s determination is dominated by the statistical uncertainty and is expected to improve with more data. The current results are based on a sample of 1 fb^{-1}.

V. *CP* VIOLATION IN DECAYS AND DIRECT MEASUREMENTS OF γ

A. Theory

CP violation in decays, also called *direct CP violation*, can arise if we have $|\mathcal{A}_f| \neq |\bar{\mathcal{A}}_{\bar{f}}|$. In that case we expect the following CP asymmetry:

$$A_{\mathrm{dir}.CP,f}(t) = \frac{\Gamma(\bar{B}_s^0(t) \to \bar{f}) - \Gamma(B_s^0(t) \to f)}{\Gamma(\bar{B}_s^0(t) \to \bar{f}) + \Gamma(B_s^0(t) \to f)}, \qquad (188)$$

to give a nonvanishing value. Inserting the time evolution for the decay rates from Eqs. (21) and (37), we get a complicated expression that vanishes; however, for $|\mathcal{A}_f| = |\bar{\mathcal{A}}_{\bar{f}}|$, $|\bar{\mathcal{A}}_f| = |\mathcal{A}_{\bar{f}}|$ and neglecting terms of order a_{fs}^s. Neglecting mixing in a first step, i.e., setting ΔM_s and $\Delta\Gamma_s$ equal to zero, we get the simplified expression

$$A_{\mathrm{dir}CP,f}(t) = \frac{|\bar{\mathcal{A}}_{\bar{f}}|^2 - |\mathcal{A}_f|^2}{|\bar{\mathcal{A}}_{\bar{f}}|^2 + |\mathcal{A}_f|^2}. \qquad (189)$$

Using the definitions in Eqs. (167), (168), and (169) we can write the two amplitudes as

$$\mathcal{A}_f = |\mathcal{A}_f^{\mathrm{Tree}}| e^{i[\phi_{\mathrm{Tree}}^{\mathrm{QCD}} + \arg(\lambda_c)]} + |\mathcal{A}_f^{\mathrm{Peng}}| e^{i[\phi_{\mathrm{Peng}}^{\mathrm{QCD}} + \arg(\lambda_u)]}, \quad (190)$$

$$\bar{\mathcal{A}}_{\bar{f}} = |\mathcal{A}_f^{\mathrm{Tree}}| e^{i[\phi_{\mathrm{Tree}}^{\mathrm{QCD}} - \arg(\lambda_c)]} + |\mathcal{A}_f^{\mathrm{Peng}}| e^{i[\phi_{\mathrm{Peng}}^{\mathrm{QCD}} - \arg(\lambda_u)]}, \quad (191)$$

and we find for $A_{\mathrm{dir}CP,f}(t)$ the following expression:

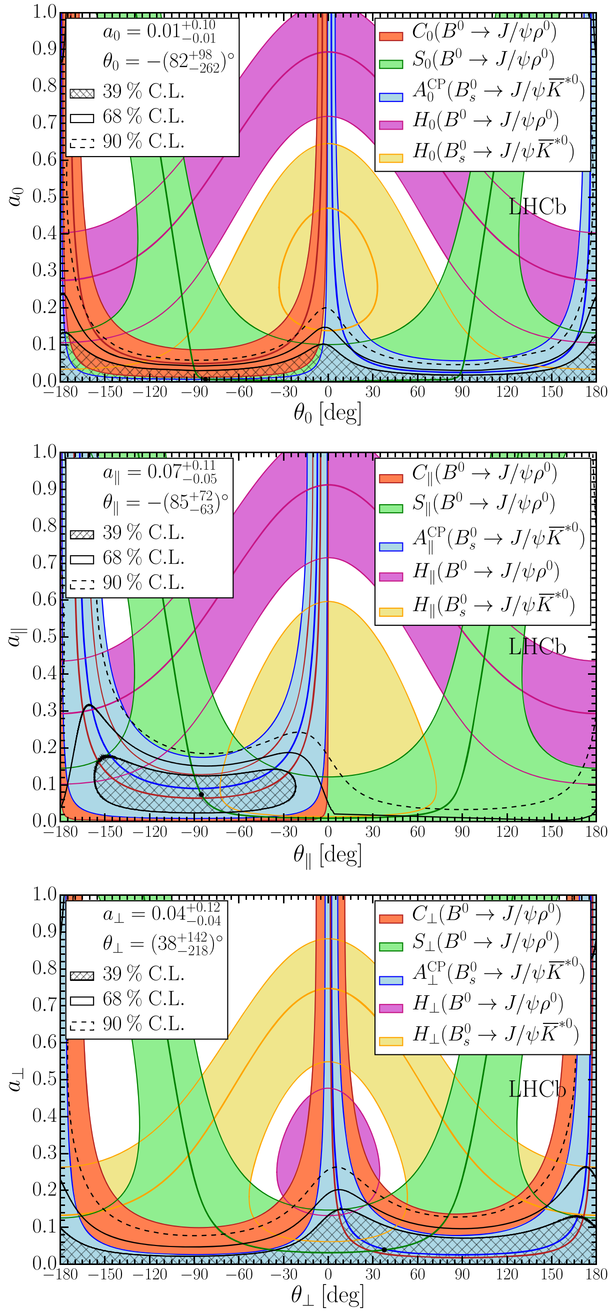

FIG. 21. Limits on the penguin parameters a_i and θ_i obtained from intersecting contours derived from the CP asymmetries and branching fraction information in $B_s^0 \to J/\psi \bar{K}^{*0}$ and $B^0 \to J/\psi\rho^0$. Superimposed are the confidence level contours obtained from a χ^2 fit to the data. The longitudinal (top), parallel (middle), and perpendicular (bottom) polarizations are shown. From Aaij *et al.*, 2015d.

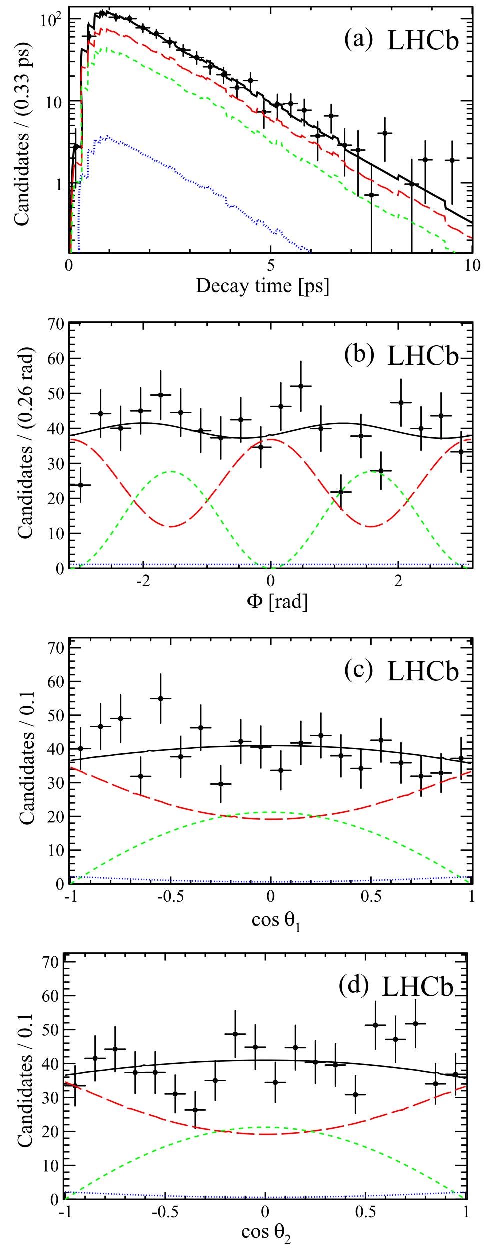

FIG. 22. One-dimensional projections of the $B_s^0 \to \phi\phi$ fit for (a) decay time, (b) helicity angle ϕ, and (c), (d) cosines of the helicity angles θ_1 and θ_2, respectively. The data are represented as points, with the one-dimensional fit projections overlaid. The solid blue line shows the total signal contribution, which is composed of CP-even (long-dashed, red), CP-odd (short-dashed, green) and S-wave (dotted, blue) contributions. From Aaij *et al.*, 2013a.

$$A_{\mathrm{dir}CP,f}(t) = \frac{2|\mathcal{A}_f^{\mathrm{Tree}}||\mathcal{A}_f^{\mathrm{Peng}}|\sin(\phi_{\mathrm{Peng}}^{\mathrm{QCD}} - \phi_{\mathrm{Tree}}^{\mathrm{QCD}})\sin[\arg(\lambda_u) - \arg(\lambda_c)]}{|\mathcal{A}_f^{\mathrm{Tree}}|^2 + |\mathcal{A}_f^{\mathrm{Peng}}|^2 + 2|\mathcal{A}_f^{\mathrm{Tree}}||\mathcal{A}_f^{\mathrm{Peng}}|\cos(\phi_{\mathrm{Peng}}^{\mathrm{QCD}} - \phi_{\mathrm{Tree}}^{\mathrm{QCD}})\cos[\arg(\lambda_u) - \arg(\lambda_c)]}$$

$$= \frac{2|r|\sin(\phi_{\mathrm{Peng}}^{\mathrm{QCD}} - \phi_{\mathrm{Tree}}^{\mathrm{QCD}})\sin[\arg(\lambda_u) - \arg(\lambda_c)]}{1 + |r|^2 + 2|r|\cos(\phi_{\mathrm{Peng}}^{\mathrm{QCD}} - \phi_{\mathrm{Tree}}^{\mathrm{QCD}})\cos[\arg(\lambda_u) - \arg(\lambda_c)]}, \quad (192)$$

where $|r|$ gives the modulus of the ratios of the penguin amplitude and the tree amplitude, analogous to Eq. (173). This simplified formula, that holds only in the absence of mixing, shows that we can have a direct CP violation in decay only, if we have at least two different CKM contributions with different weak and different strong phases. The size of the CP asymmetry is also proportional to the modulus of the penguin contributions normalized to the tree contributions. Thus such a asymmetry could in principle arise in the decays $B_s^0 \to K^-\pi^+$ and $\bar{B}_s^0 \to K^+\pi^-$ (see Fig. 13), where we expected large penguin contributions. Using the definition of the CKM angle γ

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right), \tag{193}$$

we can write to a very good approximation

$$A_{\mathrm{dir}CP,f}(t) = \frac{2|r|\sin(\phi_{\mathrm{Peng}}^{\mathrm{QCD}} - \phi_{\mathrm{Tree}}^{\mathrm{QCD}})\sin\gamma}{1 + |r|^2 - 2|r|\cos(\phi_{\mathrm{Peng}}^{\mathrm{QCD}} - \phi_{\mathrm{Tree}}^{\mathrm{QCD}})\cos\gamma}. \tag{194}$$

If $|r|$ and the strong phases were known, this direct CP asymmetry could be used to determine the CKM angle γ. We already pointed out several times the difficulty of determining these hadronic parameters from a first principles calculation. Further strategies to determine γ are discussed next. On the other hand, using a measured value of γ, the direct CP asymmetry can give indications about the size of hadronic parameters, which is a useful input in the investigation of penguin pollution. Another possibility in the search for direct CP violation is the investigation of final states that are common to B_s^0 and $\bar{B}_s^0$, as in $B_s^0 \to J/\psi\phi$ or $B_s^0 \to K^+K^-$. According to the definition of the asymmetry in Eq. (142) the coefficient of $\cos(\Delta M_s t)$ will be proportional to A_{CP}^{dir}, which describes direct CP violation and which is nonzero if $|\lambda_f| \neq 1$. Here again the ratio r will be the crucial parameter.

It is also worth mentioning that B_s^0 decays provide information about the CKM phase γ, which was defined in Eq. (193). This phase is directly proportional to the amount of CP violation in the SM. Thus any measurement of γ is a measurement of CP violation.

In the case of the tree-level decay $B_s^0 \to D_s^\pm K^\mp$ the extraction of γ was discussed by Dunietz and Sachs (1988), Aleksan, Dunietz, and Kayser (1992), Fleischer (2003), Fleischer and Ricciardi (2011), Gligorov (2011), and De Bruyn *et al.* (2013). $B_s^0 \to D_s^+K^-$ proceeds via a color-allowed $\bar{b} \to \bar{u}c\bar{s}$ transition and $B_s^0 \to D_s^-K^+$ proceeds via a color-allowed $\bar{b} \to \bar{c}u\bar{s}$ transition; see Fig. 23. Doing a naive counting of powers of the Wolfenstein parameter λ one expects that both amplitudes have a similar size, while the phase difference is given by the CKM angle γ, which is more or less the phase of the CKM element V_{ub}. From the diagrams in Fig. 23 one sees that both the B_s^0 and $\bar{B}_s^0$ mesons can decay into the same final state. Thus an interference between mixing and decay can arise, and in the end the value of $\phi_s + \gamma$ can be extracted from measuring CP asymmetries. Such an extraction of γ became very popular, using $B^- \to DK^-$, because tree-level decays are supposed to not be affected by new physics effects. In view of the increasing experimental precision this assumption should, however, be challenged. A recent study (Brod *et al.*, 2015) found that current experimental bounds on different flavor observables that are dominated by tree-level effects allow beyond SM effects to be as large as about ± 0.1 in the tree-level Wilson coefficients C_1 and C_2. A new physics contribution to the imaginary part of C_1 of about 0.1 would modify the measurement of γ from tree-level decays by about 4° (Brod *et al.*, 2015), which is smaller than the current experimental uncertainty of γ [about 7° according to Eq. (198)], but larger than the expected future uncertainty of about 1° (Abe *et al.*, 2010; LHCb Collaboration, 2011). Here clearly more studies are necessary in order to constrain the possible space for new physics effects in tree-level decays. Currently $B_s^0 \to D_s^\pm K^\mp$ decays lead to a value of $\gamma = 115^{+28°}_{-43}$ (Aaij *et al.*, 2014d), which is not competitive. An extraction of this angle from $B^0 \to \pi^+\pi^-$, $B_s^0 \to K^+K^-$, and $B_{d,s} \to \pi^\pm K^\mp$ decays, which have also loop contributions was discussed by Fleischer (1999c, 2007a), Fleischer and Knegjens (2011b), and Ciuchini *et al.* (2012). Assuming the SM value for β_s and neglecting standard model penguin contributions one gets a very precise value of $\gamma = 63.5^{+7.2°}_{-6.7}$ (Aaij *et al.*, 2013e). For this decay the usual argument about the theoretical cleanliness of the extraction does, however, not hold. Finally Bhattacharya and London (2015) also discussed the extraction of the CKM angle γ from three-body decays $B^0, B_s^0 \to K_S h^+h^-$ (with $h = K, \pi$).

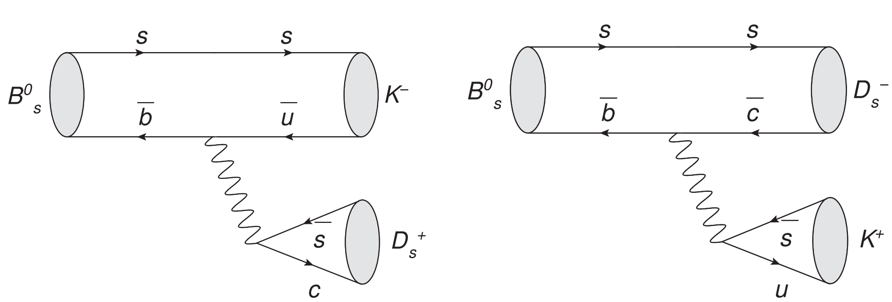

FIG. 23. Tree-level contribution to the decays $B_s^0 \to D_s^+K^-$ and $B_s^0 \to D_s^-K^+$. Both diagrams are color allowed and their CKM structure is similar in size, although the difference of the CKM phases is given by the CKM angle γ.

B. Experiment

The discovery of CP violation in charmless two-body decays of B^0 and B^+ mesons by the *BABAR* and Belle experiments provides very interesting data, whose impact is difficult to ascertain in view of the challenges in precisely determining the hadronic matrix element relating the observed asymmetries with fundamental phases. The first observables of interest are the direct CP asymmetries. So far flavor $SU(3)$ symmetry has been used to provide at least a theoretical framework to related such asymmetries measured in different decays. First principles calculations of the hadronic matrix elements involved will enable one to fully exploit these measurements to test SM predictions. The study of direct CP asymmetries in B_s^0 decays provides valuable additional constraints.

The LHCb Collaboration measured CP violation asymmetries in $B_s^0 \to K^-\pi^+$ (Aaij *et al.*, 2012b) and $B_s^0 \to K^+K^-$ (Aaij *et al.*, 2013e). These measurements share the same level of complexity as the measurements of asymmetries mediated by the interference between B_s^0-$\bar{B}_s^0$ mixing and CP violation in direct decays: they require a determination of the flavor of the decaying B_s^0, a time-dependent analysis to disentangle A_{CP} from the B_s^0 production asymmetry, in addition to a careful determination of all the instrumental asymmetries discussed before. An important advantage that enables the LHCb experiment to perform these measurements with high precision is the excellent hadron identification efficiency and purity provided by the ring imaging Cherenkov (RICH) detectors (Adinolfi *et al.*, 2013). As an illustration, Fig. 24 shows the invariant mass spectra for different species of $B \to hh$ final states. There is excellent separation between different particle species.

Using the formalism of Aaij *et al.* (2013d), the CP asymmetry is related to the raw asymmetry through

$$A_{CP} = A_{\rm raw} - A_\Delta \tag{195}$$

with

$$A_\Delta(B_s^0 \to K^+\pi^-) = -A_D(K^+\pi^-) + \kappa_s A_P(B_s^0), \tag{196}$$

where A_D represents the detection efficiency asymmetry that is derived from raw asymmetries measured for decays with known A_{CP}, $\kappa_s = -0.033 \pm 0.003$ (Aaij *et al.*, 2012b), and A_P is the B_s^0-$\bar{B}_s^0$ production asymmetry, derived from a fit to the time-dependent measured asymmetry. The parameter κ_s accounts for the dilution of the effect of the production asymmetry due to the fast B_s^0 oscillations and is given by

$$\kappa_s = \frac{\int_0^\infty e^{-\Gamma_s t}\cos(\Delta m_s t)\epsilon(B_s^0 \to K\pi, t)dt}{\int_0^\infty e^{-\Gamma_s t}\cosh(0.5\Delta\Gamma_s t)\epsilon(B_s^0 \to K\pi, t)dt}. \tag{197}$$

A_P introduced an oscillatory component that makes it possible to measure the production asymmetry unambiguously. Note that A_P has a very marginal effect on A_{CP}, because the fast flavor oscillations greatly diminish the correlation between the flavor at decay time with the flavor at production time. The final result is $A_{CP}(B_s^0 \to K^-\pi^+) = 0.27 \pm 0.04 \pm 0.01$, where the first error is statistical and the second systematic.

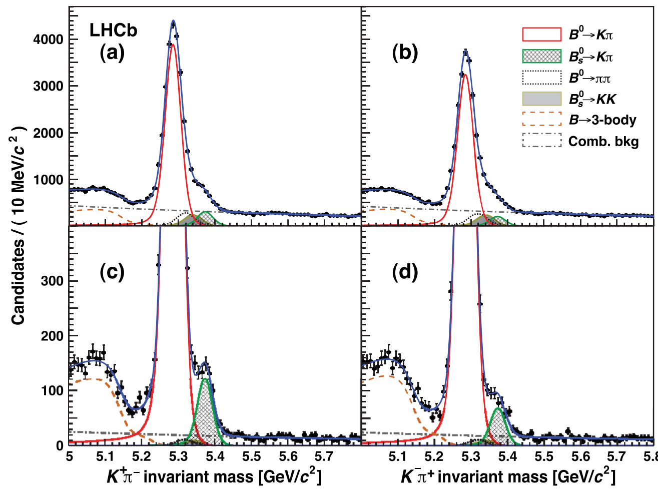

FIG. 24. (a), (b) The invariant mass spectra obtained using the event selection adopted for the best sensitivity to $A_{CP}(B^0 \to K^+\pi^-)$; (c), (d) the invariant mass spectra obtained using the event selection adopted for the best sensitivity to $A_{CP}(B_s^0 \to K^+\pi^-)$. (a), (c) The $K^+\pi^-$ invariant mass is shown, while (b), (d) show the $K^-\pi^+$ invariant mass. From Aaij *et al.*, 2013d.

The study of the CP asymmetry and branching fraction of the decay $B_s^0 \to K^+K^-$, combined with the knowledge of the corresponding observables in $B^0 \to \pi^+\pi^-$, can in principle be used to determine the CKM angle γ, defined in Eq. (193), or $-2\beta_s$, defined in Eq. (161), if U spin is a valid symmetry of the strong interaction. The LHCb Collaboration, using their measurements of CPV observables in $B_s^0 \to K^+K^-$, performed two analyses to determine either γ or β_s (Aaij *et al.*, 2015b). Here we quote the first analysis that used the measured value of β_s (and neglecting standard model penguins) to derive

$$\gamma = (63.5^{+7.2}_{-6.7})^\circ. \tag{198}$$

This value is consistent with the γ value derived from tree-level decays. Further understanding of U-spin symmetry breaking as well as penguin pollution is needed to assess the impact of this measurement.

The decay $B_s^0 \to D_s K^-$ is sensitive to the angle γ of the Cabibbo-Kobayashi-Maskawa matrix. This is an example of a determination of γ from a tree-level process, and thus, in principle, not sensitive to effects induced by most new physics models currently considered. Other such determinations of γ from tree-level mediated processes have been performed at the B factories and LHCb, through the study of $B^0 \to D^-\pi^+$ and $B^0 \to D^-K^+$ decays. In these decays, the ratio $r_{D^{(*)}} \equiv \mathcal{A}(B^0 \to D^{(*)-}\pi^+)/B^0 \to D^{(*)+}\pi^-/\mathcal{A}(B^0 \to D^{(*)+}\pi^-)$ is small, $r_{D^{(*)}} \approx 0.02$, while in the case of $B_s^0 \to D_s^+K^-$ the interfering amplitudes are of the same order of magnitude. Moreover, the decay width difference in the B_s^0 system $\Delta\Gamma_s$ is nonzero, which allows a determination of $\gamma - 2\beta_s$ from the

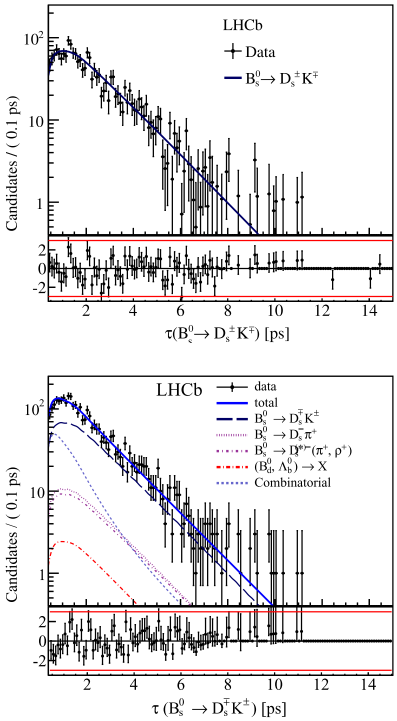

FIG. 25. Result of the decay time (top) s fit and (bottom) c fit to the $B_s^0 \to D_s K$ candidates. From Aaij *et al.*, 2014d.

sinusoidal and hyperbolic terms in the decay time evolution, up to a twofold ambiguity.

The measurement is sensitive to the combination $\gamma - 2\beta_s$, and, as we have seen that β_s is now measured with great precision from the study of $B_s^0 \to J/\psi h^+ h^-$, if standard model penguin contributions are neglected, it can be directly translated into a measurement of γ. This decay has been studied by the LHCb Collaboration using 1 fb^{-1} of data and the measurement requires a fit to the decay time distribution of the selected $B_s^0 \to D_s^- K^+$ candidates. It is a very challenging measurement because it requires the determination of the time-dependent efficiency, as well as the determination of the flavor of the decaying B_s. The kinematically similar mode $B_s^0 \to D_s^- \pi^+$ helps in constraining the time-dependent efficiency and the flavor tagging performance. In order to derive the CP-violating parameters, two different approaches have been pursued: the first, labeled s fit (Xie, 2009), consists of a statistical method to subtract background in the maximum likelihood fit, without relying on any separate sideband or simulation for background modeling, whereas the second, labeled c fit separates signal from background by fitting for these two components with separate PDFs. Figure 25 shows the results of the time-dependent fits, and Table VII shows the fitted values of the CP observables in this decay.

The study of B_s^0 decays into two vector particles ($B_s^0 \to V_1 V_2$), with the vector particles decaying into two pseudoscalar mesons, has three helicity states that are allowed by angular momentum conservation, with amplitudes identified as H_{+1}, H_{-1}, and H_0. It is convenient to map these amplitudes in terms of three transversity states to be considered, identified as "longitudinal" (0), "perpendicular" ($\perp$), and "parallel" ($\parallel$). They are related as

TABLE VII. Fitted values of the CP observables to the $B_s^0 \to D_s K$ time distribution for (left) s fit and (right) c fit, where the first uncertainty is statistical and the second is systematic. All parameters other than the CP observables are constrained in the fit. From Aaij *et al.*, 2014d.

Parameter	s fit fitted value	c fit fitted value
C_f	$0.52 \pm 0.25 \pm 0.04$	$0.53 \pm 0.25 \pm 0.04$
$A_f^{\Delta\Gamma}$	$0.29 \pm 0.42 \pm 0.17$	$0.37 \pm 0.42 \pm 0.20$
$A_{\bar{f}}^{\Delta\Gamma}$	$0.14 \pm 0.41 \pm 0.18$	$0.20 \pm 0.41 \pm 0.20$
S_f	$-0.90 \pm 0.31 \pm 0.06$	$-1.09 \pm 0.33 \pm 0.08$
$S_{\bar{f}}$	$-0.36 \pm 0.34 \pm 0.06$	$-0.36 \pm 0.34 \pm 0.08$

$$A_0 = H_0,$$
$$A_\perp = \frac{H_{+1} - H_{=1}}{\sqrt{2}},$$
$$A_\parallel = \frac{H_{+1} + H_{=1}}{\sqrt{2}}. \quad (199)$$

Two such decays have been studied at LHCb: $B_s^0 \to \phi\phi$ (discussed in the previous section), and $B_s^0 \to K^{*0}\bar{K}^{*0}$.

The study of the CP asymmetries and polarization fractions in $B_s^0 \to K^{*0}\bar{K}^{*0}$ (Aaij *et al.*, 2015c) takes a somewhat different approach. In view of the limited statistics, rather than trying to implement a flavor tagged time-dependent analysis, a study of the triple product and direct CP violation asymmetries is performed with a time-integrated analysis of $B_s^0 \to K^{*0}\bar{K}^{*0}$, without determining the flavor of the decaying B_s^0. In B meson decays there are two possible triple products

$$T_1 = (\hat{n}_{V_1} \times \hat{n}_{V_2}) \cdot \hat{p}_{V_1} = \sin\phi,$$
$$T_2 = 2(\hat{n}_{V_1} \cdot n_{V_2})(\hat{n}_{V_1} \times n_{V_2}) \cdot \hat{p}_{V_1} = \sin 2\phi. \quad (200)$$

They are found to be compatible with the standard model.

VI. MODEL-INDEPENDENT CONSTRAINTS ON NEW PHYSICS

Indirect searches for new physics effects can be performed by assuming certain extensions of the SM and calculating then the contribution of this model to different flavor observables, e.g., $M_{12}^{s,\mathrm{NP}}$. Combining these calculations with the SM contributions (e.g., $M_{12}^{s,\mathrm{SM}}$) one gets a theory prediction for flavor observables that depends on unknown parameters $x, y, \ldots$ of the considered new physics model. Currently a comparison of experimental numbers and these new theory predictions enables one to bound the parameter space of new physics models, e.g.,

$$\Delta M_s^{\mathrm{Exp}} = 2|M_{12}^{s,\mathrm{NP}}(x, y, \ldots) + M_{12}^{s,\mathrm{SM}}|. \quad (201)$$

In the future, this program could lead to a discovery of new physics effects, provided there is sufficient control over the theoretical uncertainties. But also if physics beyond the SM will be first found by direct detection of new particles, these investigations will be crucial in order to determine the flavor couplings of the new model. There is much literature determining contributions of specific new physics models to the observables discussed in this review, in particular, B_s^0 mixing. We present here some examples, but not an exhaustive list: supersymmetric contributions were discussed by Kifune, Kubo, and Lenz (2008), Kawashima, Kubo, and Lenz (2009), Kubo and Lenz (2010), Wang *et al.* (2010, 2011), Buras, Nagai, and Paradisi (2011), Crivellin *et al.* (2011), Endo and Yokozaki (2011), Endo, Shirai, and Yanagida (2011), Girrbach *et al.* (2011), Ishimori *et al.* (2011), Kaburaki *et al.* (2011), Altmannshofer and Carena (2012), and Hayakawa *et al.* (2012); contributions of two Higgs-double models were discussed by Urban *et al.* (1998), Dutta *et al.* (2012), and Chang, Li, and Li (2015); extra dimensions were discussed by Datta, Duraisamy, and Khalil (2011) and Goertz and Pfoh (2011); L-R symmetric models were discussed by Lee and Nam (2012) and Bertolini, Maiezza, and Nesti (2014); extended gauge sectors were discussed by Alok, Baek, and London (2011), Chang *et al.* (2011), Fox *et al.* (2011), Sahoo, Das, and Maharana (2011), Kim, Kim, and Shin (2013), Li *et al.* (2012), Sahoo, Kumar, and Banerjee (2013), and Chang, Li, and Yang (2014); and additional fermions[16] which were discussed by Alok and Gangal (2012) and Botella, Branco, and Nebot (2012).

In order to minimize the risk of betting on the wrong model, we discuss here a little more in detail the model-independent approach, where one tries to identify new physics effects without assuming a specific model. To start, it seems to be reasonable to assume that new physics only acts in mixing, in particular, in M_{12}, but not in tree-level decays. For simplicity we also assume no penguin contributions. Later on we soften these restrictions. Thus we postulate a general modification of M_{12}^s by an *a priori* arbitrary complex parameter Δ_s, while Γ_{12}^s is just given by the SM prediction,

$$M_{12}^s = M_{12}^{s,\mathrm{SM}} |\Delta_s| e^{i\phi_s^\Delta}, \tag{202}$$

$$\Gamma_{12}^s = \Gamma_{12}^{s,\mathrm{SM}}. \tag{203}$$

Such a modification changes the mixing observables in the following way[17]:

$$\Delta M_s^{\mathrm{Exp}} = 2|M_{12}^{s,\mathrm{SM}}| \cdot |\Delta_s|, \tag{204}$$

$$\Delta\Gamma_s^{\mathrm{Exp}} = 2|\Gamma_{12}^{s,\mathrm{SM}}| \cos(\phi_{12}^{s,\mathrm{SM}} + \phi_s^\Delta), \tag{205}$$

$$a_{\mathrm{sl}}^{s,\mathrm{Exp}} = \frac{|\Gamma_{12}^{s,\mathrm{SM}}|}{|M_{12}^{s,\mathrm{SM}}|} \cdot \frac{\sin(\phi_{12}^{s,\mathrm{SM}} + \phi_s^\Delta)}{|\Delta_s|}. \tag{206}$$

Also the phases ϕ_{12}^s and ϕ_s will get new contributions

$$\phi_{12}^{s,\mathrm{Exp}} = \phi_{12}^{s,\mathrm{SM}} + \phi_s^\Delta, \tag{207}$$

$$\phi_s^{\mathrm{Exp}} = -2\beta_s + \phi_s^\Delta. \tag{208}$$

Now a comparison of experimental numbers and SM predictions can be used to obtain the bounds on the complex parameter Δ_s. If there is no new physics present, the comparison should result in $\Delta_s = 1 + 0 \times i$. For a specific new physics model the parameter Δ_s can also be explicitly calculated in dependence on the new physics parameters $x, y, \ldots$. One gets

$$\Delta_s = \frac{M_{12}^{s,\mathrm{NP}}(x, y, \ldots) + M_{12}^{s,\mathrm{SM}}}{M_{12}^{s,\mathrm{SM}}}. \tag{209}$$

General model-independent investigations, using the earlier introduced notation, were done by Lenz and Nierste (2007), Lenz *et al.* (2011, 2012), and Charles *et al.* (2014, 2015). Next we discuss different approaches. Early investigations actually pointed toward large deviations from the SM. Unfortunately more data brought the extracted value for Δ_s in perfect agreement with the SM. The most recent result of such a investigation is depicted in Fig. 26.[18] For completeness we show also the result for the B_d^0 system. The constraint from the mass difference, Eq. (204), is denoted by the orange ring. The finite size of the ring is mostly due to the theory uncertainty of ΔM_q. In the case of B^0 mesons we have two rings, due to two different values for the CKM parameters ρ and η in the CKM fit. The purple (dark shaded wedge on the right-hand side) region stems from the measurement of the phase ϕ_s. According to Eq. (208) this constrains also ϕ_s^Δ. One has to keep in mind that this assumes no sizable SM penguins and also no new physics penguins. The dark-gray area stems from the semileptonic asymmetries. Here we are currently limited by the experimental precision. The overlap region of all experimental bounds is plotted in red. All in all we find in both mixing systems a perfect agreement with the SM, but there is still some sizable space (of the order of 10% in $|\Delta_q|$ and several degrees in the phase ϕ_s^Δ) for new physics effects in B_d^0 and B_s^0 mixing. It is entertaining and may be instructive, in the view of the currently discussed deviations of experiment and SM, to show the corresponding plots from 2010 (Lenz *et al.*, 2011) in Fig. 27. Here a quite clear hint for new physics effects can be seen, actually in both mixing systems, which unfortunately vanished completely in the last years.

Similar investigations had been performed by Fox *et al.* (2008) and the UTfit group [see, e.g., the web update of Bona *et al.* (2006a, 2008) and Bevan *et al.* (2013). In their notation one has

$$C_{B_s^0} e^{2i\phi_{B_s^0}} = \Delta_s, \tag{210}$$

$$C_{B_s^0} = |\Delta_s|, \tag{211}$$

$$\phi_{B_s^0} = \tfrac{1}{2}\phi_s^\Delta. \tag{212}$$

[16] A sequential, chiral, perturbative fourth generation of fermions is already excluded by experiment; see, e.g., Buchkremer, Gerard, and Maltoni (2012), Djouadi and Lenz (2012), Eberhardt *et al.* (2012a, 2012b), Eberhardt, Lenz *et al.* (2012), and Kuflik, Nir, and Volansky (2013). This exclusion holds, however, not for vectorlike quarks or a combination of a fourth chiral family with an additional modification of the SM; see, e.g., Lenz (2013).

[17] The correction factor $1/8|\Gamma_{12}^{s,\mathrm{SM}}/M_{12}^{s,\mathrm{SM}}|^2 |1/\Delta_s|^2 \sin\phi_{12}^s$ in Eqs. (11) and (12) still stays small.

[18] These plots are taken from the CKMfitter web page [Summer 2014; see Charles *et al.* (2005)].

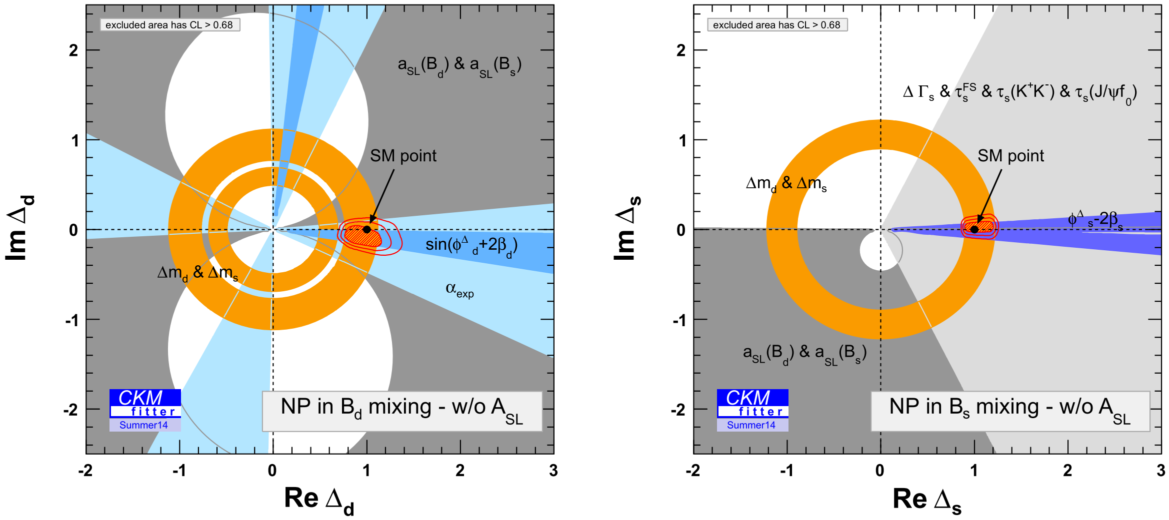

FIG. 26. Current bounds (Summer 2014) on the complex parameters Δ_d (left) and Δ_s (right) from different mixing observables. The point $\Delta_q = 1 + 0i$ corresponds to the SM—no deviation from the SM is visible. From Charles *et al.*, 2005.

Having only two parameters $C_{B_s^0}$ and $\phi_{B_s^0}$ for parametrizing new physics effects in B_s^0 mixing corresponds to making the same assumptions as before: no new physics effects in Γ_{12}^s and neglecting penguin contributions. Investigating all available mixing observables UFfit finds the following preferred parameter ranges:

$$C_{B_s^0} = 1.052 \pm 0.084, \tag{213}$$

$$\phi_{B_s^0} = 0.72° \pm 2.06°. \tag{214}$$

Again, everything seems to be perfectly consistent with the SM, while leaving room for sizable new physics effects, i.e., of the order of 10% in $C_{B_s^0}$ and of the order of a factor of 10 in the phase $\phi_{B_s^0}$. The corresponding allowed parameter regions for the B^0 system read

$$C_{B^0} = 1.07 \pm 0.17, \tag{215}$$

$$\phi_{B^0} = -2.0° \pm 3.2°, \tag{216}$$

yielding similar conclusions as in the B_s^0 system.

Sometimes these bounds are transferred into bounds on a hypothetical new physics scale. Charles *et al.* (2014) used the following notation for a deviation of M_{12}^s from its SM value:

$$M_{12}^s = M_{12}^{s,\mathrm{SM}}(1 + h_s e^{2i\sigma_s}),$$
$$1 + h_s e^{2i\sigma_s} = |\Delta_s| e^{i\phi_s^\Delta}. \tag{217}$$

Assuming further the operator

$$\frac{C_{ij}^2}{\Lambda^2}(\bar{q}_{i,L}\gamma^\mu q_{j,L})^2 \tag{218}$$

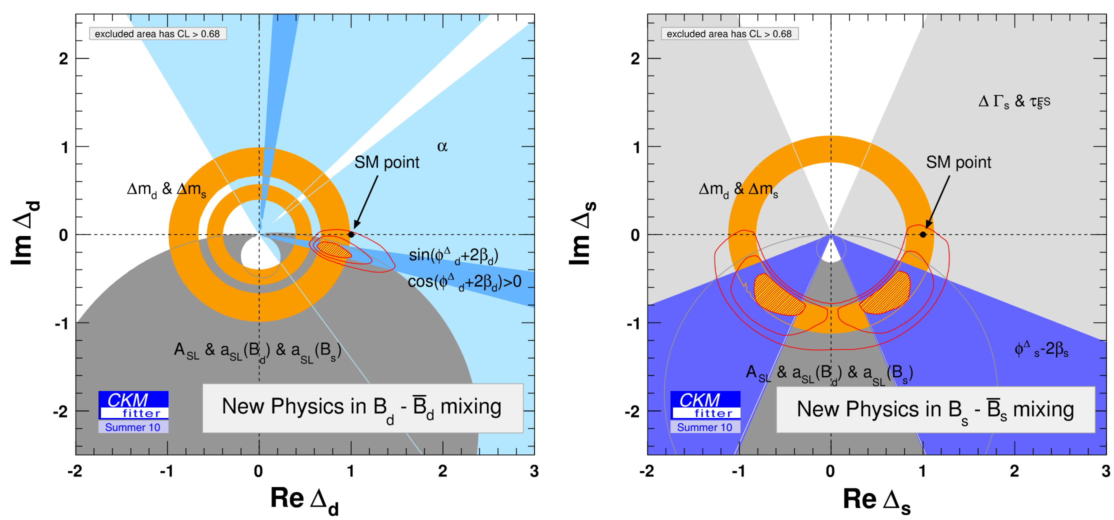

FIG. 27. Bounds on the complex parameters Δ_d (left) and Δ_s (right) from different mixing observables with data available till 2010. The point $\Delta_q = 1 + 0i$ corresponds to the SM. Unfortunately this quite clear hint for new physics effects has completely vanished by now. From Lenz *et al.*, 2011.

to describe the new physics contribution to B_s^0 mixing (i.e., $i = s$ and $j = b$), they found

$$h_s \approx \frac{C_{sb}^2}{\lambda_{sb}^2}\left(\frac{4.5\ \mathrm{TeV}}{\Lambda}\right)^2, \tag{219}$$

$$\sigma_s = \arg(C_{sb}\lambda_{sb}^*). \tag{220}$$

Here C_{ij} denotes the size of the new physics couplings and Λ is the mass scale of new physics. Both of these parameters are *a priori* unknown, because new physics has not been detected yet and an investigation of current experimental bounds on B_s^0 mixing gives information only about the ratio C_{ij}^2/Λ^2, but not about the individual values of the couplings and of the scale. $\lambda_{sb} = V_{ts}^* V_{tb}$ denotes the CKM structure of the SM contribution to B_s^0 mixing.

To make some statements about the new physics scale additional assumptions have to be made. Assuming that the coefficients C_{sb} have the same size as the CKM couplings, i.e., $C_{sb} = \lambda_{sb}$, Charles *et al.* (2014) got a new physics scale λ of about 19 TeV. Assuming instead $C_{sb} = 1$ the new physics scale increases to roughly 500 TeV. In particular, the second scale is far above the direct reach of the LHC and thus B_s^0 mixing could in principle probe new physics scales that are far from being accessible via direct measurements. On the other hand, one should not forget that the assumption about the size of the coupling is in principle arbitrary. If the new physics couplings are very small then also the new physics scale that can be probed is very low. In order to fulfill our final goal of unambiguously disentangling hypothetical new effects from mixing observables strict control over uncertainties is mandatory. Also the assumptions made have to be challenged. First we have to include penguin contributions, both from the SM and from new sources; this will modify the phase ϕ_s to

$$\phi_s = -2\beta_s + \phi_s^\Delta + \delta^{\mathrm{Peng,SM}} + \delta^{\mathrm{Peng,NP}}. \tag{221}$$

SM penguin contributions are expected to contribute at most up to 1°, while new physics penguin contributions are less constrained. General new physics effects in M_{12}^s will be treated as

$$M_{12}^s = M_{12}^{s,\mathrm{SM}}|\Delta_s|e^{i\phi_s^\Delta}. \tag{222}$$

In addition we also allow new effects in Γ_{12}^s, encoded by the parameter $\tilde{\Delta}_s$

$$\Gamma_{12}^s = \Gamma_{12}^{s,\mathrm{SM}}|\tilde{\Delta}_s|e^{-i\tilde{\phi}_s^\Delta}, \tag{223}$$

resulting in a modified mixing phase ϕ_{12}^s:

$$\phi_{12}^s = \phi_{12}^{s,\mathrm{SM}} + \phi_s^\Delta + \tilde{\phi}_s^\Delta. \tag{224}$$

New contributions to Γ_{12}^s can be due to new penguin and/or new contributions to tree-level decays. For a long time new physics effects in tree-level decays were considered to be negligible. Because of the dramatically improved experimental precision, this possibility has, however, to be considered. Taking only experimental constraints into account and no bias from model building, first studies performed by Bobeth *et al.* (2014), Bobeth, Gorbahn, and Vickers (2015), and Brod *et al.* (2015) found that the tree-level Wilson coefficients C_1 and C_2 can easily be affected by new effects of the order of 0.1. Such a deviation can sometimes have dramatic effects, e.g., a modification of the imaginary part of C_1 by about 0.1 would modify the extracted value of the CKM angle γ by about 4° (Brod *et al.*, 2015), which is larger than the expected future experimental uncertainty. Thus these possibilities should be taken into account for quantitative studies about new physics effects in mixing. For a future disentangling of new effects in mixing, penguin, and tree-level decays clearly more theoretical work has to be done.

The modification of M_{12}^s [see Eq. (222)] and Γ_{12}^s [see Eq. (223)] changes the mixing observables in the following way[19]:

$$\Delta M_s = 2|M_{12}^{s,\mathrm{SM}}| \cdot |\Delta_s|, \tag{225}$$

$$\Delta\Gamma_s = 2|\Gamma_{12}^{s,\mathrm{SM}}||\tilde{\Delta}_s| \cos(\phi_{12}^{s,\mathrm{SM}} + \phi_s^\Delta + \tilde{\phi}_s^\Delta), \tag{226}$$

$$a_{\mathrm{sl}}^s = \frac{|\Gamma_{12}^{s,\mathrm{SM}}|}{|M_{12}^{s,\mathrm{SM}}|} \cdot \frac{|\tilde{\Delta}_s|}{|\tilde{\Delta}_s|} \sin(\phi_{12}^{s,\mathrm{SM}} + \phi_s^\Delta + \tilde{\phi}_s^\Delta). \tag{227}$$

First steps in that direction haven been done in the analysis of Lenz *et al.* (2012), where in scenario IV new physics in Γ_{12}^s was introduced by the parameter δ_s:

$$\delta_q = \frac{\Gamma_{12}^s / M_{12}^s}{\Re(\Gamma_{12}^{s,\mathrm{SM}}/M_{12}^{s,\mathrm{SM}})}. \tag{228}$$

This parameter is related to mixing observables in the following way:

$$\Re(\delta_s) = \frac{\Delta\Gamma_s/\Delta M_s}{\Delta\Gamma_s^{\mathrm{SM}}/\Delta M_s^{\mathrm{SM}}}, \qquad \Im(\delta_s) = \frac{-a_{\mathrm{sl}}^s}{\Delta\Gamma_s^{\mathrm{SM}}/\Delta M_s^{\mathrm{SM}}}. \tag{229}$$

In 2012 the fit of Lenz *et al.* (2012) seemed to prefer some deviations of $\Im(\delta_s)$ and $\Im(\delta_s)$, which were mostly triggered by an interpretation of the dimuon asymmetry, which was commonly accepted at that time, but turned out to be incomplete.

In the future these model-independent investigations should include new physics effects in M_{12}^s, Γ_{12}^s, and penguin contributions. Doing the latter might also include a combination of $\Delta B = 2$ and $\Delta B = 1$ observables.

VII. CONCLUSION AND OUTLOOK

The study of CP violation phenomena in the B_s^0 system has been the focus of experimental and theoretical efforts. It was started by the Tevatron experiments CDF and D0, who made the first measurements in this system. Among their main achievements are the measurement of the B_s^0 meson mass difference ΔM_s (Abulencia *et al.*, 2006) and the study of the

[19]Again, the correction factor $1/8|\Gamma_{12}^{s,\mathrm{SM}}/M_{12}^{s,\mathrm{SM}}|^2|\tilde{\Delta}_s/\Delta_s|^2 \sin\phi_{12}^s$ stays small.

semileptonic charge asymmetry of the B_s^0 meson $a_{\rm sl}^s$ (Abazov *et al.*, 2012b, 2013, 2014). The measured value of $a_{\rm sl}^s$ based on the study of $B_s^0 \to D_s^+ \mu^- \bar{\nu}_\mu$ is still contributing to the average with the LHCb result, based on one-third of the run 1 data. The Tevatron experiments also initiated the studies of other *CP*-violating phenomena, such as the mixing phase ϕ_s in the $B_s^0 \to J/\psi\phi$ decay, albeit with large uncertainties.

The pioneering work of the Tevatron experiment is continued and refined at the LHC, with a new level of precision allowed by high statistics, improved detector performance, and new analysis techniques. In particular, the LHCb experiment has performed the most precise measurement of all types of *CP* violation (Aaij *et al.*, 2012b, 2014c, 2015h), as well as that of ΔM_s and $\Delta\Gamma_s$ (Aaij *et al.*, 2013h). They measured the CKM angle γ not only in B^0 decays previously studied by the e^+e^- B factories, but also in B_s^0 decays both in tree-mediated processes and in loop-mediated processes. Finally, they observed direct *CP* violation in several B_s^0 channels.

The current data do not confirm *CP* violation in the B_s^0 system in excess of the SM prediction, as was originally hoped for. Still, some room for new physics manifestations remains. In *CP* violation in the interference of decays and mixing quantified by the angle ϕ_s the experimental uncertainty is getting very close to the SM central value. In this respect, the emphasis on understanding small corrections such as penguin pollution is a field of active investigation in the theoretical and experimental communities. The theory prediction for *CP* violation in mixing is still orders of magnitude smaller than the experimental uncertainty. The level of understanding of the SM expectations for mixing observables and *CP*-violating phenomena in the B_s^0 system is now very advanced. Experimental studies have not only proven the CKM mechanism to be the primary source of quark mixing and *CP* violation, but they have also confirmed the validity of theoretical approaches such as the HQE to an unprecedented accuracy.

The uncertainty on the theory prediction for the mass difference ΔM_s is about $\pm 15\%$, thus allowing for new effects of the same order in this observable. To improve the accuracy in ΔM_s further, more precise lattice evaluations of bag parameters and decay constants are mandatory. In this respect, an uncertainty of about $\pm 5\%$ seems to be achievable in the next years.[20] The calculation of the width difference according to the HQE seems on less solid theoretical grounds. The assumption of quark hadron duality was questioned many times; see, e.g., Ligeti *et al.* (2010) or the discussion by Lenz (2011), and deviations of more than 100% were discussed. Such a failure of the HQE is now clearly ruled out. The measurement of the width difference $\Delta\Gamma_s$ has shown that the HQE works also in the most challenging channel, $b \to c\bar{c}s$, with an accuracy of at least 20%.[21] For further independent tests of the precision of the HQE, lattice determinations of the matrix elements that arise in lifetime difference of different b hadrons, such as $\tau(B^+)/\tau(B^0)$, $\tau(B_s^0)/\tau(B^0)$, and $\tau(\Lambda_b)/\tau(B^0)$ are urgently needed; see the detailed discussion by Lenz (2014). Here it might also be insightful to study the charm sector, in particular, the ratios $\tau(D^+)/\tau(D_0)$ and $\tau(D_s^+)/\tau(D_0)$. To reduce the uncertainty on the theory prediction of $\Delta\Gamma_s$ a nonperturbative determination of dimension 7 matrix elements is first needed, i.e., the bag parameters B_{R_0}, B_{R_2}, B_{R_3}, $B_{\tilde{R}_2}$, and $B_{\tilde{R}_3}$. Currently, these parameters contribute the largest individual uncertainty. Next, more precise lattice values of the complete SUSY basis of $\Delta B = 2$ four quark operators are needed.[22] In parallel to these nonperturbative improvements, NNLO QCD corrections[23] have to be calculated (i.e., $\Gamma_3^{s,(2)}$ and $\Gamma_4^{s,(1)}$ in our notation). Having all these improvements at hand, a final accuracy of about 5% for the $\Delta\Gamma_s$ prediction might also be feasible in the next years.[24] The current experimental uncertainty on $a_{\rm sl}^s$ is still about a factor of 130 larger than the small central value of the standard model expectations, thus still allowing plenty of room for new physics effects. Turning to indirect *CP* violation, we find that the current experimental precision is coming close to the SM central value and also to the intrinsic theoretical uncertainties due to penguin contributions. In principle the weak phase ϕ_s measured in $B_s^0 \to J/\psi\phi$ is a null test similar to the semileptonic asymmetries. In practice the theory prediction of the latter one is much more robust than the one for ϕ_s. To fully exploit the improving experimental precision extended studies of penguin effects and a quantification of them are mandatory.

All LHC experiments expect to continue data taking at least up to 2030. The LHCb Collaboration is currently engaging in a detector upgrade that should increase its sensitivity by a factor of 10, with a combination of operating at higher instantaneous luminosity, and the implementation of a purely software based trigger system, which will have to process the full 30 MHz of inelastic collisions delivered by the LHC. The physics opportunities offered by such an upgrade have been quantified by (LHCb Collaboration, 2014) assuming a total integrated luminosity of 50 fb^{-1}. Several key measurements have been studied. Table VIII summarizes the prospects for some of the observables described in this paper.

The plans of other LHC collaborations are less ambitious. For example, the ATLAS experiment projects to measure the value of ϕ_s with the precision of 0.022 by 2030 (ATLAS Collaboration, 2013). The precision of the LHC measurements will allow one to achieve the SM level in this quantity and to perform unprecedented tests of the contribution of new models beyond the SM. The large statistics, which will become available during the next ten years, will also allow one to measure the *CP*-violating phenomena in other channels like $B_s^0 \to J/\psi\eta$. Advancement in theory, in particular, in lattice

[20]Here we assume an accuracy of lattice values for dimension 6 operators considerably below 5%.

[21]For very recent estimates of the possible size of duality-violating effects, see Jubb *et al.* (2016).

[22]While preparing this paper a new study of the Fermilab Lattice and MILC Collaborations was made public (Bazavov *et al.*, 2016).

[23]See Asatrian, Hovhannisyan, and Yeghiazaryan (2012) for the first step in that direction.

[24]Here we assume an accuracy of lattice values for dimension 6 operators considerably below 5%, an accuracy of about 10% for the bag parameters B_R of the dimension 7 operators, and a reduction of the renormalization scale dependence by at least a factor of 2 due to NNLO QCD corrections.

TABLE VIII. Statistical sensitivity of the LHCb upgrade to key observables discussed in this paper. For each observable the projected sensitivity at the end of run II and with a luminosity of 50 fb^{-1} (phase I upgrade) are given. For a comparison we show also the current theory uncertainty of the standard model predictions, given in Eqs. (116), (160), and (161). The theory error in ϕ_s holds only for neglecting penguins.

Observable	LHCb 2018	Upgrade	Theory uncertainty
ϕ_s ($B_s^0 \to J/\psi\phi$)	0.025	0.009	≈0.002
a_{sl}^s (10^{-3})	1.4	0. 5	0.003
$\phi_s^{\mathrm{eff}}(B_s^0 \to \phi\phi)$	0.10	0.018	0.02
$\gamma(B_s^0 \to D_s K)$	11°	2.0°	Negligible

QCD and other approaches to constrain the hadronic matrix elements needed to access fundamental quantities are expected to follow a similar path. Thus, a new exciting era of B_s^0 meson studies is ahead of us.

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APPENDIX A: NUMERICAL INPUT FOR THEORY PREDICTIONS

In this Appendix we list all input parameters that were used for our numerical updates of several standard model predictions. We start with listing some well-known parameters in Table IX that are mostly taken from the PDG (Olive *et al.*, 2014). Next we list in Table X some not so well-determined input parameters. For lattice values our standard reference is FLAG (Aoki *et al.*, 2014). In the case of $\tilde{B}_S/B$ FLAG did not provide an average, so we took the values from Becirevic *et al.* (2002), Bouchard *et al.* (2011), Carrasco *et al.* (2014), and Dowdall *et al.* (2014) and did our own naive average. For B_{R_0} we took the preliminary value that can be read off the plots given by Dowdall *et al.* (2014). B_{R_1} and $B_{\tilde{R}_1}$ can be deduced from Becirevic *et al.* (2002), Bouchard *et al.* (2011), Carrasco *et al.* (2014), and (Dowdall *et al.* (2014). The operators R_1 and $\tilde{R}_1$ are denoted by O_4 and O_5 in the lattice literature

$$R_1 \equiv \frac{m_s}{m_b} O_4, \qquad \tilde{R}_1 \equiv \frac{m_s}{m_b} O_5. \tag{A1}$$

The Fermilab-MILC Collaboration (Bouchard *et al.*, 2011) used again an additional factor of 4

TABLE IX. List of precisely known input parameters needed for an update of the theory prediction of different mixing observables.

Parameter	Value	Reference
M_W	80.385(15) GeV	PDG 2015
M_Z	91.1876(21) GeV	PDG 2015
G_F	$1.1663787(6)10^{-5}$ GeV^{-2}	PDG 2015
$\hbar$	$6.58211928(15)10^{-25}$ GeV s	PDG 2015
$M_{B_s^0}$	5.3667(4) GeV	PDG 2015
$\bar{m}_b(\bar{m}_b)$	4.18(3) GeV	PDG 2015
$\bar{m}_c(\bar{m}_c)$	1.275(25) GeV	PDG 2015
$\bar{m}_s(2\ \mathrm{GeV})$	0.0935(25) GeV	PDG 2015
$\alpha_s(M_Z)$	0.1185(6)	PDG 2015

TABLE X. List of less precisely known input parameters needed for an update of the theory prediction of different mixing observables.

Parameter	Value	Reference
$f_{B_s}\sqrt{B}$	216(15) MeV	FLAG
$f_{B_d}\sqrt{B}$	175(12) MeV	FLAG
$\tilde{B}_S/B$	1.07(6)	Own average
$\tilde{B}_S/B(B_d)$	1.04(12)	Own average
$\bar{z}$	0.054 3964(229 532)	Own evaluation
m_t	173.34(76) GeV	arXiv:1403.4427
$\bar{m}_t(\bar{m}_t)$	165.696(73) GeV	Own evaluation
$\Lambda_{\mathrm{QCD}}^{(5)}$	233(8) MeV	Derived from NLO α_s
V_{us}	$0.22548^{+0.00068}_{-0.00034}$	CKMfitter
V_{cb}	$0.04117^{+0.00090}_{-0.00114}$	CKMfitter
V_{ub}/V_{cb}	0.0862278 ± 0.00442474	CKMfitter
γ	$1.17077^{+0.0169297}_{-0.0378736}$	CKMfitter
B_{R_0}/B	1 ± 0.3	HPQCD preliminary
B_{R_1}/B	1.71 ± 0.26	Own average
B_{R_2}	1 ± 0.5	VIA assumption
B_{R_3}	1 ± 0.5	VIA assumption
$B_{\tilde{R}_1}/B$	1.27 ± 0.16	Own average
$B_{\tilde{R}_3}$	1 ± 0.5	VIA assumption

$$R_1 \equiv \frac{m_s}{m_b} 4O_4, \qquad \tilde{R}_1 \equiv \frac{m_s}{m_b} 4O_5. \tag{A2}$$

Moreover one has to be aware of different normalization factors used in the definition of the matrix elements. Beneke, Buchalla, and Dunietz (1996) and Lenz and Nierste (2007) used

$$\langle R_1 \rangle = \frac{7}{3}\frac{m_s}{m_b} M_{B_s^0}^2 f_{B_s}^2 B_{R_1}, \tag{A3}$$

$$\langle \tilde{R}_1 \rangle = \frac{5}{3}\frac{m_s}{m_b} M_{B_s^0}^2 f_{B_s}^2 B_{\tilde{R}_1} \tag{A4}$$

This definition ensures that in vacuum insertion approximation the bag parameters B_{R_1} and $B_{\tilde{R}_1}$ have the value of 1. In the lattice literature different normalization factors, compared to $7/3$ and $5/3$, are used. Becirevic *et al.* (2002) and Carrasco *et al.* (2014) have

TABLE XI. List of additional and mostly preliminary determinations of lattice parameters needed for an update of the theory prediction of different mixing observables. Some of the values given here were simply read off plots provided by the different collaborations. The error of the RBC-UK evaluation cannot be estimated currently, because of missing $1/m_b$ corrections.

Parameter	Value	Collaboration
$f_{B_s}\sqrt{B}$	200(5–10) MeV	HPQCD
$\tilde{B}_S/B$	1.03	
B_{R_1}/B	1.98	
$B_{\tilde{R}_1}/B$	1.48	
$f_{B_s}\sqrt{B}$	211(8) MeV	ETMC
$\tilde{B}_S/B$	1.03	
B_{R_1}/B	1.46	
$B_{\tilde{R}_1}/B$	1.15	
$f_{B_s}\sqrt{B}$	227(7) MeV	Fermi-MILC
$\tilde{B}_S/B$	1.15	
B_{R_1}/B	1.60	
$B_{\tilde{R}_1}/B$	1.17	
$f_{B_s}\sqrt{B}$	262(?) MeV	RBC-UKQCD

$$\langle R_1 \rangle = 2\frac{m_s}{m_b} M_{B_s^0}^2 f_{B_s}^2 B_4', \tag{A5}$$

$$\langle \tilde{R}_1 \rangle = \frac{2}{3}\frac{m_s}{m_b} M_{B_s^0}^2 f_{B_s}^2 B_5', \tag{A6}$$

while Bouchard *et al.* (2011) and Dowdall *et al.* (2014) used

$$\langle R_1 \rangle = 2\frac{m_s}{m_b} M_{B_s^0}^2 f_{B_s}^2 B_4, \tag{A7}$$

$$\langle \tilde{R}_1 \rangle = \frac{2}{3}\frac{m_s}{m_b} M_{B_s^0}^2 f_{B_s}^2 B_5. \tag{A8}$$

For the top quark mass we did not take the PDG value, but a first combination of TeVatron and LHC results, presented by ATLAS, CDF, CMS, and D0 Collaborations (2014). $\Lambda_{\rm QCD}^{(5)}$ was derived from the NLO running of α_s using $\alpha_s(M_Z)$ and M_Z given as an input. The values of the CKM elements were taken from the web update of the CKMfitter group (Charles *et al.*, 2005); similar results are given by UTfit (Bona *et al.*, 2006b). Here the value of V_{ub} is taken from the fit and not from either an inclusive or an exclusive determination. Finally we also present in Table XI a list of additional lattice determinations for $f_{B_s}\sqrt{B}$ and $\tilde{B}_S/B$, given by HQPCD [LATTICE 2014 update by Dowdall *et al.* (2014)], ETMC (Carrasco *et al.*, 2014), the LATTICE 2015 update from the Fermilab-MILC Collaboration (Bouchard *et al.*, 2011), and the LATTICE 2015 update from RBC-UKQCD of Aoki *et al.* (2015).

APPENDIX B: ERROR BUDGET OF THE THEORY PREDICTIONS

In this Appendix we compare the error budget or our new standard model predictions with the ones given in 2011 by

TABLE XII. List of the individual contributions to the theoretical error of the mass difference ΔM_s within the standard model and comparison with the values obtained by Lenz and Nierste (2007, 2011).

$\Delta M_s^{\rm SM}$	This work	LN 2011	LN 2006
Central value	18.3 ps^{-1}	17.3 ps^{-1}	19.3 ps^{-1}
$\delta(f_{B_s}\sqrt{B})$	13.9%	13.5%	34.1%
$\delta(V_{cb})$	4.9%	3.4%	4.9%
$\delta(m_t)$	0.7%	1.1%	1.8%
$\delta(\alpha_s)$	0.1%	0.4%	2.0%
$\delta(\gamma)$	0.1%	0.3%	1.0%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.1%	0.2%	0.5%
$\delta(\bar{m}_b)$	<0.1%	0.1%	...
$\sum\delta$	14.8%	14.0%	34.6%

Lenz and Nierste (2011) and the ones given in 2006 by Lenz and Nierste (2007).

The error budget for the updated standard model prediction of $\Delta M_s^{\rm SM}$ is given in Table XII. For the mass difference we observed no improvement in accuracy compared to the 2011 prediction, because the by far dominant uncertainty (close to 14%) stems from $f_{B_s}\sqrt{B}$ and here the inputs are more or less unchanged. This will change as soon as new lattice values are available. The next important uncertainty is the accuracy of the CKM element V_{cb}, which contributes about 5% to the error budget. If one gives up the assumption of the unitarity of the 3×3 CKM matrix, the uncertainty can go up considerably. The uncertainties due to the remaining parameters play no important role. All in all we are left with an overall uncertainty of close to 15%, which has to be compared to the experimental uncertainty of about 1 per mille. This situation currently leaves some space for new physics contributions to the mass difference ΔM_s. With future improvements on the nonperturbative parameters a theoretical uncertainty in the range of 5%–10% is feasible.

Next we study the error budget of the decay rate difference $\Delta\Gamma_s$ in Table XIII. The uncertainty in the decay rate difference also did not change considerably compared to 2011. The dominant uncertainty is still the unknown bag parameter of the power suppressed operator R_2. This input did not improve since 2011. Here and in Lenz and Nierste (2007, 2011) we took the conservative assumption of $B_{R_{2,3}} = 1 \pm 0.5$. If in the future these parameters could be determined with an uncertainty of about ±10%, then an overall uncertainty of less than ±10% in $\Delta\Gamma_s$ would become feasible. First steps in the direction of a nonperturbative determination of B_{R_2} within the framework of QCD sum rules were done by Mannel, Pecjak, and Pivovarov (2007, 2011). There, however, only subleading contributions were determined. Thus a calculation of the leading (three-loop) contribution would be desirable. The second largest uncertainty stems from $f_{B_s}\sqrt{B}$, whose value also did not improve since 2011. There are, however, several new (mostly preliminary) results on the market—HQPCD [LATTICE 2014 update by Dowdall *et al.* (2014)], ETMC (Carrasco *et al.*, 2014), the LATTICE 2015 update from the Fermilab-MILC Collaboration (Bouchard *et al.*, 2011), and the LATTICE 2015 update from RBC-UKQCD of Aoki *et al.* (2015)—that seems to indicate that $f_{B_s}\sqrt{B}$ can be determined

TABLE XIII. List of the individual contributions to the theoretical error of the decay rate difference $\Delta\Gamma_s$ within the standard model and comparison with the values obtained by Lenz and Nierste (2007, 2011).

$\Delta\Gamma_s^{\rm SM}$	This work	LN 2011	LN 2006
Central value	0.088 ps^{-1}	0.087 ps^{-1}	0.096 ps^{-1}
$\delta(B_{\tilde{R}_2})$	14.8%	17.2%	15.7%
$\delta(f_{B_s}\sqrt{B})$	13.9%	13.5%	34.0%
$\delta(\mu)$	8.4%	7.8%	13.7%
$\delta(V_{cb})$	4.9%	3.4%	4.9%
$\delta(\tilde{B}_S)$	2.1%	4.8%	3.1%
$\delta(B_{R_0})$	2.1%	3.4%	3.0%
$\delta(\bar{z})$	1.1%	1.5%	1.9%
$\delta(m_b)$	0.8%	0.1%	1.0%
$\delta(B_{\tilde{R}_1})$	0.7%	1.9%	...
$\delta(B_{\tilde{R}_3})$	0.6%	0.5%	...
$\delta(B_{R_1})$	0.5%	0.8%	...
$\delta(B_{R_3})$	0.2%	0.2%	...
$\delta(m_s)$	0.1%	1.0%	1.0%
$\delta(\gamma)$	0.1%	0.3%	1.0%
$\delta(\alpha_s)$	0.1%	0.4%	0.1%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.1%	0.2%	0.5%
$\delta(\bar{m}_t(\bar{m}_t))$	0.0%	0.0%	0.0%
$\sum\delta$	22.8%	24.5%	40.5%

with an uncertainty as low as 5% in the near future. In most of these works not only the matrix element of Q, but also the full $\Delta B = 2$ operator basis is studied. This will provide improved values for the bag parameters B_S, $\tilde{B}_S$, B_{R_1}, $B_{\tilde{R}_1}$, and B_{R_0}, via Eq. (67). Number three in the error budget is the dependence on the renormalization scale, here a calculation of NNLO QCD corrections would be necessary to further reduce the error. First steps of such an endeavor were done by Asatrian, Hovhannisyan, and Yeghiazaryan (2012). The next important dependence is the CKM element V_{cb}, which leads currently to an uncertainty of about 5%. In the ratio $\Delta\Gamma_s^{\rm SM}/\Delta M_s^{\rm SM}$ one of the dominant uncertainties, the dependence on $f_{B_s}^2 B$ is canceling and we get for the error budget the values given in Table XIV. For $\Delta\Gamma_s/\Delta M_s$ we see a small improvement in the theoretical precision compared to 2011. The dominant uncertainty is given by the unknown matrix element of the dimension 7 operator R_2, followed by the uncertainty due to the renormalization scale dependence. The overall uncertainty is currently 17.3%, which is also the final theoretical uncertainty that can currently be achieved for $\Delta\Gamma_s$. Future investigations, i.e., nonperturbative determinations of the matrix element of R_2 and NNLO QCD corrections, might bring down this uncertainty to maybe 5%.

TABLE XIV. List of the individual contributions to the theoretical error of the ratio $\Delta\Gamma_s/\Delta M_s$ within the standard model and comparison with the values obtained by Lenz and Nierste (2007, 2011).

$\Delta\Gamma_s^{\rm SM}/\Delta M_s^{\rm SM}$	This work	LN 2011	LN 2006
Central value	48.1×10^{-4}	50.4×10^{-4}	49.7×10^{-4}
$\delta(B_{R_2})$	14.8%	17.2%	15.7%
$\delta(\mu)$	8.4%	7.8%	9.1%
$\delta(\tilde{B}_S)$	2.1%	4.8%	3.1%
$\delta(B_{R_0})$	2.1%	3.4%	3.0%
$\delta(\bar{z})$	1.1%	1.5%	1.9%
$\delta(m_b)$	0.8%	1.4%	1.0%
$\delta(m_t)$	0.7%	1.1%	1.8%
$\delta(B_{\tilde{R}_1})$	0.7%	1.9%	...
$\delta(B_{\tilde{R}_3})$	0.6%	0.5%	...
$\delta(B_{R_1})$	0.5%	0.8%	...
$\delta(B_{R_3})$	0.2%	0.2%	...
$\delta(\alpha_s)$	0.2%	0.8%	0.1%
$\delta(m_s)$	0.1%	1.0%	0.1%
$\delta(\gamma)$	0.0%	0.0%	0.1%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.0%	0.0%	0.1%
$\delta(V_{cb})$	0.0%	0.0%	0.0%
$\sum\delta$	17.3%	20.1%	18.9%

TABLE XV. List of the individual contributions to the theoretical error of the semileptonic CP asymmetries $a_{\rm sl}^s$ within the standard model and comparison with the values obtained by Lenz and Nierste (2007, 2011).

$a_{\rm fs}^{s,{\rm SM}}$	This work	LN 2011	LN 2006
Central value	2.22×10^{-5}	1.90×10^{-5}	2.06×10^{-5}
$\delta(\mu)$	9.5%	8.9%	12.7%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	5.0%	11.6%	19.5%
$\delta(\bar{z})$	4.6%	7.9%	9.3%
$\delta(B_{\tilde{R}_3})$	2.6%	2.8%	2.5%
$\delta(\gamma)$	1.3%	3.1%	11.3%
$\delta(B_{R_3})$	1.1%	1.2%	1.1%
$\delta(m_b)$	1.0%	2.0%	3.7%
$\delta(m_t)$	0.7%	1.1%	1.8%
$\delta(\alpha_s)$	0.5%	1.8%	0.7%
$\delta(B_{\tilde{R}_1})$	0.5%	0.2%	...
$\delta(\tilde{B}_S)$	0.3%	0.6%	0.4%
$\delta(B_{R_0})$	0.2%	0.3%	...
$\delta(B_{R_2})$	0.1%	0.1%	...
$\delta(m_s)$	0.1%	0.1%	0.1%
$\delta(B_{R_1})$	<0.1%	0.0%	...
$\delta(V_{cb})$	0.0%	0.0%	0.0%
$\sum\delta$	12.2%	17.3%	27.9%

TABLE XVI. List of the individual contributions to the theoretical error of the mixing quantities ΔM_d, $\Delta\Gamma_d$, and $a_{\rm sl}^{d,{\rm SM}}$ in the B^0 sector.

	$\Delta M_d^{\rm SM}$	$\Delta\Gamma_d^{\rm SM}$	$a_{\rm sl}^{d,{\rm SM}}$
Central value	0.528 ps^{-1}	2.61×10^{-3} ps^{-1}	-4.7×10^{-4}
$\delta(B_{\tilde{R}_2})$	...	14.4%	0.1%
$\delta(f_{B_d}\sqrt{B})$	13.7%	13.7%	...
$\delta(\mu)$	...	7.9%	9.4%
$\delta(V_{cb})$	4.9%	4.9%	0.0%
$\delta(\tilde{B}_S)$	...	4.0%	0.6%
$\delta(B_{R_0})$	...	2.5%	0.2%
$\delta(\bar{z})$	...	1.1%	4.9%
$\delta(m_b)$	0.0%	0.8%	1.3%
$\delta(B_{\tilde{R}_1})$	...	0%	...
$\delta(B_{\tilde{R}_3})$	...	0.5%	2.7%
$\delta(B_{R_1})$	...	0%	...
$\delta(B_{R_3})$	...	0.2%	1.2%
$\delta(m_s)$	...	...	...
$\delta(\gamma)$	0.2%	0.2%	1.1%
$\delta(\alpha_s)$	0.0%	0.1%	0.5%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.0%	0.1%	5.2%
$\delta(\bar{m}_t(\bar{m}_t))$	0.1%	0.0%	0.7%
$\sum\delta$	14.8%	22.7%	12.3%

The error budget for the semileptonic CP asymmetries is finally listed in Table XV. Here we witness some sizable reduction of the theory error. This quantity does not depend on $f_{B_s}\sqrt{B}$ and has only a weak dependence on R_2; thus the two least known parameters in the mixing sector do not affect the semileptonic asymmetries. The increase in precision stems mostly from better known CKM elements, in particular, of V_{ub}, in comparison to 2011. Currently the dominant uncertainty stems from the renormalization scale dependence followed by the dependence on V_{ub}. For a reduction of the overall theoretical uncertainty considerably below 10% a NNLO QCD calculation is mandatory.

Finally we present in Table XVI also the theory errors for the observables in the B^0 sector.

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