Alternatives to an elementary Higgs

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Strongly coupled and extra-dimensional models of electroweak symmetry breaking are reviewed. Models examined include warped extra dimensions, bulk Higgs, "little" Higgs, dilaton Higgs, composite Higgs, twin Higgs, quantum critical Higgs, and "fat" SUSY Higgs. Also discussed are current bounds and future LHC searches for this class of models.

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CONTENTS

I. INTRODUCTION

A key ingredient of the standard model (SM) of elementary particle physics is the electroweak sector that explains the relation between weak interactions and ordinary electromagnetism. This sector requires a mechanism that forces the vacuum to distinguish the weak gauge bosons, the W and Z, from the photon, thus breaking the symmetry of the electroweak gauge interactions. The W and Z bosons have large masses while the photon is exactly massless. This symmetry breaking goes by the name "Higgs mechanism." In addition to fields corresponding to the spin-1 gauge bosons (force carriers), the SM includes another boson field with zero spin, called the Higgs field, which is arranged to have a nonzero vacuum expectation value (VEV). In the absence of electroweak gauge couplings, three components of the Higgs field would have massless Goldstone excitations, that is there would be three exact Goldstone bosons (Goldstone, 1961). With the electroweak gauge interactions turned on, these excitations provide the missing longitudinal modes for the $W^{\pm}$ and Z. Using an elementary Higgs field (Englert and Brout, 1964; Guralnik, Hagen, and Kibble, 1964; Higgs, 1964a, 1964b; Weinberg, 1967) to arrange for the spontaneous breaking of electroweak gauge symmetry is certainly the simplest model that makes sense of the electroweak interactions, at least at tree level that is. At loop level one encounters large radiative corrections (aka quadratic divergences) that tend to drive the renormalized Higgs mass parameter, and hence the W and Z masses, up to the highest scale in the theory. In other words, in the absence of some incredible fine-tuning there should be some new

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physics beyond the SM near the TeV scale.[1] The existence of these quadratic divergences is a basic feature of elementary scalar fields, first noted by Weisskopf (1939), and is referred to as the hierarchy problem. In fact, Weisskopf argued that this was the explanation of why no one had ever discovered an elementary scalar field. Weisskopf's argument essentially still holds; the well-known loop holes are supersymmetry and compositeness. The quadratic divergences can be tamed with the introduction of supersymmetry, and the fine-tuning eliminated with superpartners below the TeV scale that cut off the divergence. On the other hand, if the scalar is composite rather than elementary, the new interactions that produce the composite can serve to cut off the divergence.

Now that the LHC has found a Higgs-like resonance (Aad *et al.*, 2012b; Chatrchyan *et al.*, 2012a) the next questions are: "Is there also supersymmetry?" and "Is the Higgs elementary?" Superpartners below the TeV scale can be uncovered by the LHC, but the question of whether the Higgs is elementary or not is much more subtle. There are a large variety of scenarios that cover a continuum of possibilities from elementary to composite [it was even suggested that a composite scalar acquires a small VEV that then induces the VEV of the elementary Higgs (Samuel, 1990; Chang *et al.*, 2015)]. There are even supersymmetry (SUSY) theories with a composite Higgs. In this review we will attempt to survey these possibilities, pointing out when their phenomenologies overlap and where there are unique signals.

Since the idea of a composite Higgs relies on having new strong interactions, we begin our review with a brief discussion of the prototypical model for breaking electroweak symmetry with strong interactions: technicolor. Although these models have been discarded, they set the stage for the more successful models that avoid the pitfalls of technicolor. An alternative approach to strong interactions is to use a weakly coupled "dual" description in terms of an extra dimension. Whether the extra dimension or the dual strong interactions are the fundamental description can only be answered at even higher energies, but the extra-dimensional description is certainly easier to calculate near the TeV scale. An alternative to canceling divergences with superpartners is to cancel them with partners of the same spin. Such "little" Higgs theories can in principle raise the compositeness or SUSY scale from 1 to 10 TeV. Extending the space-time symmetries to include conformal symmetry rather than supersymmetry is another possibility for protecting the Higgs mass from divergences. We also examine a more bottom-up approach that relies on constructing a general low-energy effective field theory for the Higgs. It will be useful to see how this effective theory can be matched on to some interesting composite models.

[1]It does not necessarily mean that TeV scale new physics stabilizing the Higgs mass will be easily accessible at the Large Hadron Collider (LHC); see, for instance the discussion of neutral naturalness models (Chacko, Goh, and Harnik, 2006; Craig *et al.*, 2015) in Sec. III.I. A recent proposal (Espinosa *et al.*, 2015; Graham, Kaplan, and Rajendran, 2015) relies on the cosmological evolution of the Universe to drive it near a critical point for electroweak symmetry breaking and thus alleviates the hierarchy problem without the need for TeV scale new physics at all.

A. An instructive failure: Technicolor

Technicolor[2] was the first alternative to an elementary SM Higgs; it is a beautiful idea that seems not to have lived up to its potential. In some ways technicolor is analogous to superconductivity, and somewhat ironically, before the Higgs mechanism was discovered by particle physicists, Anderson (1963) had emphasized how composite degrees of freedom lead to an effective photon mass in a Bardeen-Cooper-Schrieffer superconductor (Bardeen, Cooper, and Schrieffer, 1957), and Nambu had developed the field theory version of the composite Higgs mechanism in his work with Jona-Lasinio (Nambu and Jona-Lasinio, 1961a, 1961b). In a superconductor, electrons can attract each other (very weakly) by exchanging phonons (quanta of lattice vibrations) forming Cooper pairs. If the charged Cooper pairs undergo Bose condensation, then the lowest energy state of the system has an arbitrarily large charge (limited only by the number of electrons in the superconductor). Photons moving through this charged medium are effectively massive, as can be seen by the fact that magnetic fields cannot penetrate a superconductor (the Meissner effect). In a superconductor the analog of the Higgs is the Cooper pair, and the analog of the massive W and Z is the photon which has a penetration depth inversely proportional to its effective mass. However, a Cooper pair is so weakly bound that its physical size[3] (100s of nanometers) is much larger than its Compton wavelength, so there is no sense in which we can describe this system using an effective field theory with a Higgs-like field standing in for the Cooper pair. Thus the first known implementation of the Higgs mechanism does not have an elementary field but rather a loosely bound composite.

Weinberg (1976, 1979) and Susskind (1979) independently proposed that composite particles formed by a new strong interaction could replace the SM Higgs boson. The new interactions were supposed to be similar to those of quantum chromodynamics (QCD), and these theories were hence dubbed technicolor theories. Susskind showed that if the Higgs boson was absent from the standard model, QCD would provide electroweak symmetry breaking through quark composites (although it would give masses for the W and Z that are about a factor of 2600 too small). Technicolor theories thus harkened back to superconductivity where a gauge symmetry is broken by a composite of two fermions, a crucial difference being that the interactions responsible for superconductivity are quite weak, whereas the technicolor interactions must remain strong. Technicolor theories essentially resolve the fine-tuning problem by lowering the effective cutoff scale to 1 TeV. Remarkably, technicolor theories predicted the correct ratio for the W and Z masses. This is due to the global symmetry breaking pattern of QCD being $SU(2)\times SU(2)/SU(2)$ rather than simply the minimal breaking $SU(2)\times U(1)/U(1)$ required by electroweak gauge symmetry breaking. This enhanced symmetry is referred to as the custodial symmetry (Sikivie *et al.*, 1980), although many

[2]For a thorough review, see Hill and Simmons (2003).

[3]The electron pairs in high T_c superconductors are much smaller, so they look much more like an analog of the SM Higgs.

differ on whether they use this term to refer to the full $SU(2) \times SU(2)$ symmetry or the unbroken diagonal $SU(2)$ subgroup.

The Achilles' heel of technicolor has always been producing masses for the quarks and leptons. This requires several complicated extensions of the model [e.g., extended technicolor (Dimopoulos and Susskind, 1979; Eichten and Lane, 1980), and often extra pseudo-Goldstone bosons (PGB), and even then one finds problems with flavor changing neutral currents (FCNCs). The FCNC problem could be resolved for the first two generations (Holdom, 1981, 1985) by assuming approximately conformal behavior above 1 TeV [aka a "walking" rather than running coupling constant (Holdom, 1981; Appelquist, Karabali, and Wijewardhana, 1986; Yamawaki, Bando, and Matumoto, 1986; Appelquist and Wijewardhana, 1987)]. However, this is not enough to explain the top quark, and further model building is required as in Appelquist *et al.* (2004). A subtler problem with technicolor was revealed by the comparison with precision electroweak measurements (Holdom and Terning, 1990; Peskin and Takeuchi, 1990, 1992; Altarelli and Barbieri, 1991; Golden and Randall, 1991). Following the idea of scaling up QCD to obtain the correct W and Z masses, it was possible to scale up QCD data (essentially using QCD as an "analog computer") to predict the deviations of a technicolor theory from the SM. These deviations were not seen at the Stanford Linear Accelerator Center (SLAC) or the Large Electron Positron Collider (LEP) at CERN. It remained logically possible that there was some version of technicolor that does not behave like QCD, but in the absence of an explicit, workable model interest in technicolor waned during the 1990s.

For the remainder of this review we are interested in models that have a light composite scalar boson, unlike the QCD-like models just discussed.

B. Classifying the alternatives

While there have been many proposals for alternative models of electroweak symmetry breaking there are some basic concepts that can be applied to all the models that can impose some structure that allows us to easily compare and contrast them. One way to think about how to classify the range of models is to first consider the scaling dimension of the operator that breaks electroweak symmetry. For an elementary Higgs, the scaling dimension is obviously 1, up to perturbative corrections. In contrast, in an alternate universe where technicolor breaks electroweak symmetry, the operator that does the breaking is a fermion bilinear. At tree level this operator has scaling dimension 3, but at strong coupling it should be significantly smaller. In walking technicolor the scaling dimension was assumed to be 2. Thus the scaling dimension of the operator is an essential ingredient to understanding the ultraviolet (UV) completion of the Higgs sector. In theories with a light composite Higgs, it is especially important to know the scaling dimension of the square of the Higgs field, i.e., the mass term. If the scaling dimension of the mass term is less than 4, then the operator is relevant and can receive divergent corrections; if the scaling dimension of the mass term is greater than 4, then the operator is irrelevant and the hierarchy problem is solved. In general the scaling dimension of the mass term is not just twice the scaling dimension of the Higgs field, but if the composite Higgs is weakly coupled this should be a good approximation.

In the next section we will discuss extra-dimensional models in anti–de Sitter (AdS) space, and we will see that there is a direct connection between how the Higgs is localized in the extra dimension and the effective scaling dimension. In the original Randall-Sundrum (RS) model, where the Higgs is localized at one end of the extra dimension, the effective scaling dimension is infinite. When this model is generalized to allow the Higgs to extend into the extra dimension (aka the bulk Higgs model) one finds that the scaling dimension can vary from infinity all the way down to 1. For a fixed scaling dimension one is still free to vary how much of the W and Z masses come from the Higgs VEV and how much comes from mixing with higher resonances (i.e., from AdS curvature effects), so there is a two-dimensional parameter space of models.

An interesting way to keep a composite Higgs light is for it to be a pseudo-Goldstone boson corresponding to a global symmetry broken at a scale f that is much higher than the Higgs VEV v. This can occur with an elementary Higgs, which therefore has scaling dimension near 1 (as in little Higgs models), or via strong coupling or extra dimensions. In the explicit case of the minimal composite pseudo-Goldstone boson Higgs the scaling dimension turns out to be 2. In these types of models there are also additional mixing corrections to the W and Z masses which are parametrically of the order of v^2/f^2. The "fat" SUSY Higgs is another type of composite where the scaling dimension can range between 1 and 2. In its minimal form, the minimal composite supersymmetric standard model (MCSSM), we will see that the scaling dimension is fixed to be close to 1, with a small amount of mixing for the W and Z.

Several types of models use conformal invariance to try to keep the Higgs light. A dilaton Higgs is another example of a pseudo-Goldstone boson, this time from broken conformal invariance. Conformal technicolor (Luty and Okui, 2006) was an interesting attempt to find theories where the scaling dimension of the Higgs is close to 1, while the scaling dimension of the mass term is greater than 4. Generically if there is a parameter in the theory that can be adjusted to move from the phase where electroweak symmetry is broken to the symmetric phase, and the phase transition is continuous, we will find a light scalar in the broken phase that is a fluctuation of the order parameter. This is what happens in the SM, where the Higgs mass parameter in the Lagrangian has to be delicately adjusted to be close to the critical point with a light Higgs. In condensed matter parlance such transitions are called quantum critical points, since at zero temperature it is quantum fluctuations that dominate rather than thermal fluctuations. Experimentally we seem to be near such a critical point; this means that there is potentially a very long renormalization group (RG) flow, which usually results in approaching an infrared (IR) fixed point. This fixed point could be trivial (i.e., free) as in the SM, or nontrivial, that is an interacting conformal field theory (CFT). The quantum critical Higgs model assumes a quantum critical point where the light composites are weakly interacting; from the extra-dimensional

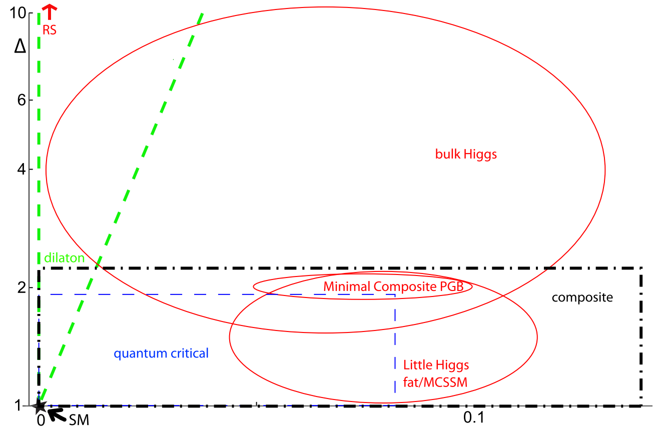

FIG. 1. Parameter space of alternative Higgs models, the vertical axis is the scaling dimension Δ of the operator that condenses and breaks the electroweak gauge symmetry, the horizontal axis is the fraction of the W and Z masses that arise from mixing. Precision electroweak measurements rule out models that lie too far to the right. The location of the SM, at the origin, is marked by a star. The dash-dotted lines denote the approximate range of composite Higgs models, while the thin blue dashes denote the approximate range of quantum critical Higgs models. The thick green dashed lines enclosing a wedge-shaped area denote the approximate region for dilaton models, with Δ denoting the dimension of the scale breaking condensate.

point of view it is a special case of the bulk Higgs where the scaling dimension is between 1 and 2. A rough sketch of the range of models we will consider is shown in Fig. 1.

II. EXTRA DIMENSIONS

In the late 1990s extra dimensions became a popular framework for extensions of the SM. Many early attempts (Antoniadis *et al.*, 1998; Arkani-Hamed, Dimopoulos, and Dvali, 1998; Appelquist, Cheng, and Dobrescu, 2001) did not really address the hierarchy problem until the advent of the warped extra-dimensional model of Randall and Sundrum (1999), generally referred to as RS. This type of model is a five-dimensional AdS space where the warped extra dimension is truncated at two 4D boundaries (aka 3-branes). The warping allows us to associate one end of the space with low energies, the IR, and the other end with high energies, the UV. Without the IR cutoff the theory resembles a CFT, a result that can be understood through the AdS/CFT correspondence.

A. The AdS/CFT correspondence

A major breakthrough of the late 1990s was the discovery of the AdS/CFT correspondence by Maldacena (1998), who conjectured (based on several independent consistency checks) that there is an exact equivalence between type IIB string theory on an $\mathrm{AdS}_5 \times S^5$ background and nongravitational 4D $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theories in the large N limit. In this correspondence operators of the CFT are associated with bulk fields in AdS_5, and the value of the field on the boundary of AdS_5 acts as a source for the CFT operator. While the initial excitement was mainly confined to the string community, soon it was realized that AdS/CFT has wide reaching consequences and applications in many branches of physics. For example, the presence of supersymmetry does not appear to be essential, and it was conjectured by Arkani-Hamed, Porrati, and Randall (2001), Perez-Victoria (2001), and Rattazzi and Zaffaroni (2001) that the proper interpretation of the original Randall-Sundrum models is in terms of a nonsupersymmetric version of the correspondence, whereby the bulk of an extra dimension with anti–de Sitter background corresponds to a large N limit of a nonsupersymmetric 4D CFT. It is actually not too hard to understand the underlying reason for this: the metric of 5D AdS space (in so-called conformal coordinates) is given by

$$ds^2 = \left(\frac{R}{z}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \tag{1}$$

which has an isometry $z \to e^\alpha z, x \to e^\alpha x$. The physical meaning is that a motion along the fifth dimension z is equivalent to a rescaling of the 4D coordinates, implying that movement along the fifth direction is actually a RG transformation. Since this leaves the metric invariant, one expects the corresponding 4D scaling to be a symmetry, hence a 4D CFT must be the underlying structure.

The RS model does not, however, have a full AdS space $0 \le z \le \infty$, but rather only coincides with a slice of it: the so-called UV brane (or Planck brane) at $z = R$ forming one of the boundaries, while the IR brane (TeV brane) at $z = R'$ forming the other boundary. The effect of the UV brane is to render the graviton zero mode normalizable (while gravity decouples in full AdS due to a non-normalizable graviton zero mode). Thus the presence of the UV brane will recouple gravity to the 4D CFT. Since gravity provides an explicit scale (the Planck scale), the correct interpretation of the UV brane is that it provides an explicit breaking at scale $1/z_{\mathrm{UV}}$ for the CFT.

The interpretation of the IR brane is more subtle: introducing the IR brane will provide a mass gap of order $1/z_{\mathrm{IR}}$ into the Kaluza-Klein (KK) expansion of all types of fields, calling for an interpretation different from that of the UV brane. The most natural interpretation is to assume that the presence of the IR brane signals a spontaneous breaking of the conformal symmetry: the theory was perturbed away from the exact fixed point, and as a result the coupling became strong and generated a condensate of scale $1/z_{\mathrm{IR}}$ resulting in the mass gap.

The next question is how to deal with global symmetries, which may or may not be weakly gauged. It is quite clear from the formulation of the correspondence that global symmetries in the CFT require bulk gauge fields in AdS: the conserved global current J_μ of the CFT must have a bulk vector field A_μ that it can couple to on the boundary. The question we need to answer next is what determines if this will be a global or a weakly gauged symmetry. Clearly this will again be set by the presence (or absence) of a 4D gauge field zero mode. Carefully examining the 5D Maxwell equation $\partial_\mu[\sqrt{g} g^{\mu\nu} F_{\nu\rho}] = 0$ in the background (1) shows that the profile $f(z)$ of a potential gauge zero mode $A_\mu(z, x) = f(z) A_\mu(x)$ is flat along the extra dimension. Imposing flat (Neumann) boundary conditions (BCs) for the gauge field profile

$$\partial_z f(z)|_{z=R,R'} = 0 \tag{2}$$

TABLE I. Summary of the AdS/CFT dictionary used in BSM model building from Csáki, Hubisz, and Meade (2005).

Bulk of AdS	$\leftrightarrow$	CFT
Inverse coordinate $(1/z)$ along AdS	$\leftrightarrow$	Energy scale in CFT
UV brane	$\leftrightarrow$	CFT has a cutoff
IR brane	$\leftrightarrow$	Conformal symmetry broken spontaneously by CFT
KK modes localized near IR brane	$\leftrightarrow$	Composites of CFT
Modes on the UV brane	$\leftrightarrow$	Elementary fields coupled to CFT
Gauge fields in bulk	$\leftrightarrow$	CFT has a global symmetry
Bulk gauge symmetry broken on UV brane	$\leftrightarrow$	Global symmetry not gauged
Bulk gauge symmetry unbroken on UV brane	$\leftrightarrow$	Global symmetry weakly gauged
Higgs on IR brane	$\leftrightarrow$	Strong CFT produces composite Higgs
BC breaking on IR brane	$\leftrightarrow$	CFT condensate breaks gauge symmetry
BC breaking on both branes	$\leftrightarrow$	Broken global symmetry with A_5 Goldstones

will allow the zero mode in the spectrum and thus corresponds to a weakly gauged symmetry. However imposing a Dirichlet BC $f(R) = 0$ on the UV brane will remove the gauge zero mode and hence the combination with a Dirichlet BC in the UV and Neumann BC in the IR corresponds to a global symmetry. Keeping a Neumann BC on the UV but imposing a Dirichlet BC on the IR will have the effect of raising the zero mode to a mass of order $1/z_{\mathrm{IR}}$: this is the case expected for a spontaneous breaking of the gauge symmetry. The CFT condensate that broke the CFT will also contribute to breaking the weakly gauged symmetry, similar to the technicolor models discussed in the previous section. Thus 5D models of technicolor can be built by imposing Dirichlet BCs for the appropriate combinations of bulk gauge field, yielding the so-called Higgsless models (Csáki, Grojean, Murayama *et al.*, 2004; Csáki, Grojean, Pilo, and Terning, 2004).

The final possibility of Dirichlet BCs on both UV and IR branes presents another important possibility with wide applications. A Dirichlet BC on the UV brane will render that symmetry global, and the additional breaking by a Dirichlet BC on the IR brane will produce a broken global symmetry, which should have the appropriate Goldstone bosons. Indeed it turns out that in this case the fifth component of the gauge field A_5 will have a zero mode, peaked on the IR brane. This is the mode often used for a holographic implementation of composite Higgs models where the Higgs arises as a Goldstone boson. A short summary of the AdS/CFT dictionary used by beyond the standard model (BSM) builders is presented Table I.

B. Realistic RS

There are many variations on RS models [for reviews, see Rattazzi (2003), Csáki, Hubisz, and Meade (2005), Sundrum (2005), Gherghetta (2006, 2011), Kribs (2006), Cheng (2010), and Ponton (2012)]. The first iterations had all the SM fields localized on the IR brane (aka the TeV brane). Through the AdS/CFT correspondence one sees that states localized on the IR brane are the analogs of strongly bound composites of the CFT, while states localized on the UV brane are external spectators of the strongly coupled CFT that have been added with a weak coupling. As such, the novel phenomenology of the early RS models is entirely due to the KK modes of 5D gravity, which includes spin-2 modes as well as a scalar radion (Davoudiasl, Hewett, and Rizzo, 2000b; Csáki, Graesser, and Kribs, 2001). The distance between the IR and UV branes is arbitrary, corresponding to the massless radion. In fact the size of the extra dimension is unstable to small perturbations (Csáki *et al.*, 1999) and must be stabilized (Goldberger and Wise, 1999), and this stabilization results in a mass for the radion (Csáki *et al.*, 2000; Goldberger and Wise, 2000; Geller, Bar-Shalom, and Soni, 2014). It can be shown that a light radion decays predominantly to gluons due to the trace anomaly (Giudice, Rattazzi, and Wells, 2001) as shown in Fig. 2.

The AdS/CFT correspondence suggests that the standard model states localized on the IR brane should have a plethora of higher dimension operators coupling them and that these operators should be suppressed by powers of the TeV scale. Since there is no experimental evidence of such operators, more realistic RS models were developed where the gauge bosons and light fermions live throughout the bulk of the extra dimension while the Higgs and the top quark are localized near the IR brane (Davoudiasl, Hewett, and Rizzo, 2000a, 2001; Gherghetta and Pomarol, 2000; Grossman and Neubert, 2000; Pomarol, 2000; Huber and Shafi, 2001; Csáki, Erlich, and Terning, 2002; Carena *et al.*, 2003). Since the Higgs is effectively a composite state with an inverse TeV size, there are no quadratic divergences in its mass. In these models the top needs to be localized near the IR brane so that it can have a large coupling to the Higgs.

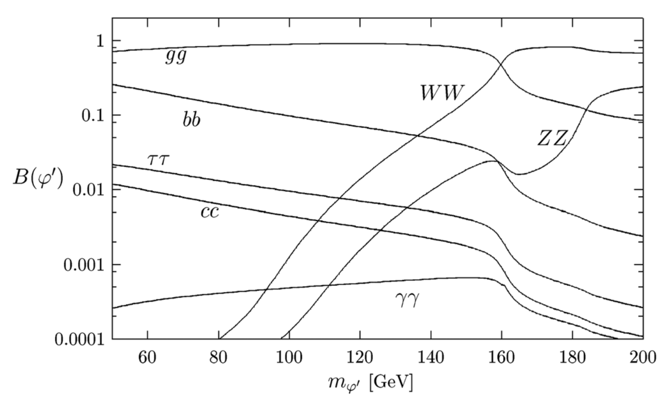

FIG. 2. Light radion decay fractions: the decay to gluons is enhanced while the decays to $b\bar{b}$ and $\gamma\gamma$ are suppressed. From Giudice, Rattazzi, and Wells, 2001.

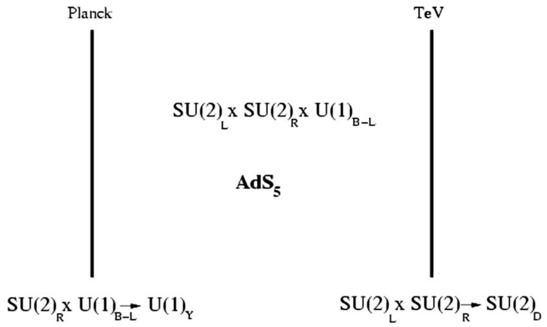

FIG. 3. Realistic RS models: the gauged $SU(2)_L \times SU(2)_R$ symmetry in the bulk corresponds to a custodial global symmetry in the hypothetical dual CFT. The gauge symmetry is broken to the SM gauge group on the Planck (UV) brane and is broken by the Higgs VEV down to the diagonal subgroup on the TeV (IR) brane.

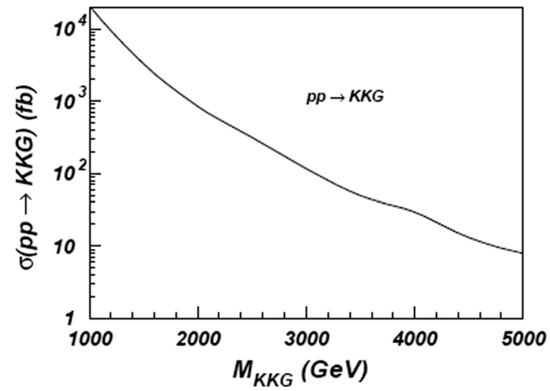

FIG. 5. KK gluons production cross section at the 14 TeV LHC. From Agashe et al., 2008.

We now focus on such realistic RS models. Using the rules of the AdS/CFT correspondence discussed in the previous section, we can relatively easily find the type of model that we are after. We want a theory that has an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ global symmetry, with the $SU(2)_L \times U(1)_Y$ subgroup weakly gauged and broken by a Higgs VEV on the IR brane. To have the full global symmetry, we need to have an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry in the bulk of AdS_5. To make sure that we do not get unwanted gauge fields at low energies, we need to break $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ on the UV brane, which we can do with another Higgs-like field, or with BCs. Finally, a Higgs localized on the TeV brane gets a VEV and breaks $SU(2)_L \times SU(2)_R$ to $SU(2)_D$. This setup implements the necessary custodial symmetry by means of a bulk $SU(2)_L \times SU(2)_R$ gauge symmetry (Agashe et al., 2003), as shown in Fig. 3. As in the SM and technicolor models, the custodial symmetry ensures the correct ratio of W and Z masses.

C. Realistic RS LHC searches

While deviations in Higgs couplings are expected in RS models their values are not uniquely predicted, so most of the search strategies focus on the extra associated particles. As mentioned previously the top quark must be localized near the TeV brane in order to get a large enough Yukawa coupling to the Higgs. This implies that the color $SU(3)$ gauge group is also in the bulk. All the bulk gauge fields will have a tower of KK modes. Since they are strongly interacting, KK gluons are a primary target for LHC searches (Lillie, Randall, and Wang, 2007). As seen in Fig. 4, KK gluons decay almost exclusively to $t\bar{t}$ pairs (Lillie, Shu, and Tait, 2007; Agashe et al., 2008). Figure 5 shows the production cross section for KK gluons at the 14 TeV LHC. A 3 TeV KK gluon has a 0.1 pb production cross section, so it is fairly easy to produce. KK gluons typically have a large width of the order of hundreds of GeV (Agashe et al., 2008) as seen in Fig. 6. To search for these broad resonances decaying to tops, effectively one needs to take advantage of the jet substructure (Seymour, 1994; Ellis et al., 2012; Larkoski, Salam, and Thaler, 2013) to develop a top tagger (Kaplan et al., 2008); see Plehn and Spannowsky (2012) and Shelton (2013) for reviews. A recent compact muon solenoid (CMS) analysis excludes KK gluons below 2.8 TeV (Chatrchyan et al., 2012b); see Fig. 7.

Other KK gauge boson states are much harder to find. For example, production of the KK excitations of the Z (aka the Z') is suppressed since it is localized on the IR brane while the light quarks are localized on the UV brane. The KK excitations can also decay to difficult final states (Agashe et al., 2007) as seen in Figs. 8–10. It would take about

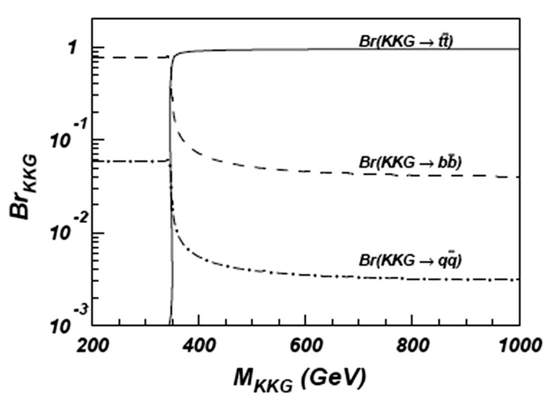

FIG. 4. KK gluons decay mainly to $t\bar{t}$ pairs. From Agashe et al., 2008.

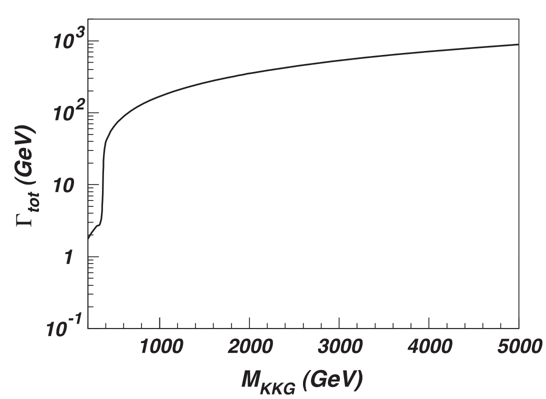

FIG. 6. KK gluons typically have a large width. From Agashe et al., 2008.

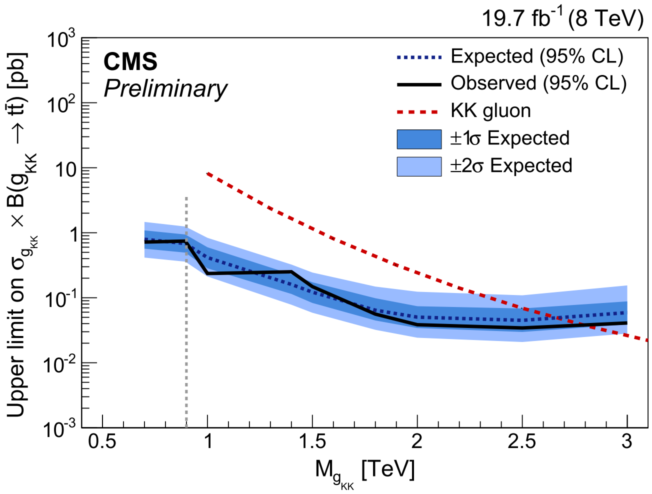

FIG. 7. CMS bound on KK gluon production. From Khachatryan *et al.*, 2015a.

100 fb^{-1} of integrated luminosity to uncover a 2 TeV Z', and 1 ab^{-1} for a 3 TeV Z' (Agashe *et al.*, 2007).

Currently LHC data put bounds around 1.3–2.7 TeV on KK gravitons for the original RS models with SM fields localized on the IR brane. These bounds arise from resonance searches in dileptons (see Fig. 11) and dijets (see Fig. 12). The diphoton channel currently gives KK graviton bounds around 1–2 TeV as shown in Fig. 13. These bounds are weakened for the case of the realistic RS when the SM fields are delocalized into the bulk, in which case the KK gravitons decay predominantly to W/Z (see Fig. 14).

Taking into account the trace anomaly, one sees (Csáki, Hubisz, and Lee, 2007) that loops on the brane generate significant couplings for the radion. In fact the discovery significance for the radion can be comparable to the Higgs (Csáki, Hubisz, and Lee, 2007; Grzadkowski, Gunion, and Toharia, 2012) as shown in Fig. 15. The Higgs searches at the LHC can also be used to set bounds on the RS radion; see, for example, Cho, Nomura, and Ohno (2013), Desai, Maitra, and Mukhopadhyaya (2013), and Bhattacharya *et al.* (2015). The bounds from decays to WW and ZZ (assuming a brane localized SM and as a function of its coupling and mass) are displayed in Fig. 16.

D. Bulk Higgs

Extra dimensions allow us to modify the realistic RS models by lifting the Higgs itself into the bulk as well (Chang and Ng, 2000; Davoudiasl, Lillie, and Rizzo, 2006; Cacciapaglia *et al.*, 2007; Dey, Mukhopadhyaya, and SenGupta, 2010; Cabrer, von Gersdorff, and Quiros, 2011; Das, Hundi, and SenGupta, 2011; Vecchi, 2011a; Archer, 2012; Frank, Pourtolami, and Toharia, 2013). In order to solve

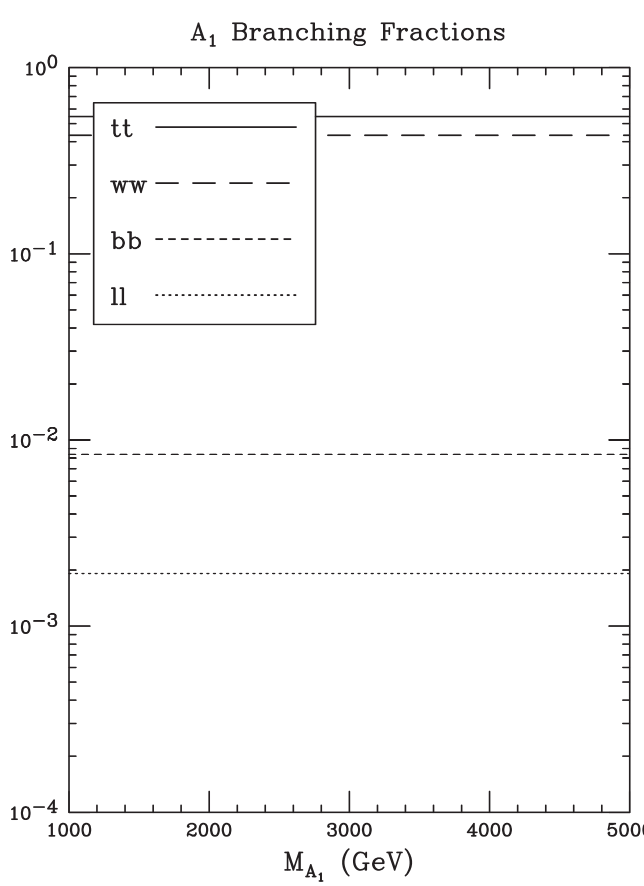

FIG. 8. Decay branching fractions for the first KK photon. From Agashe *et al.*, 2007.

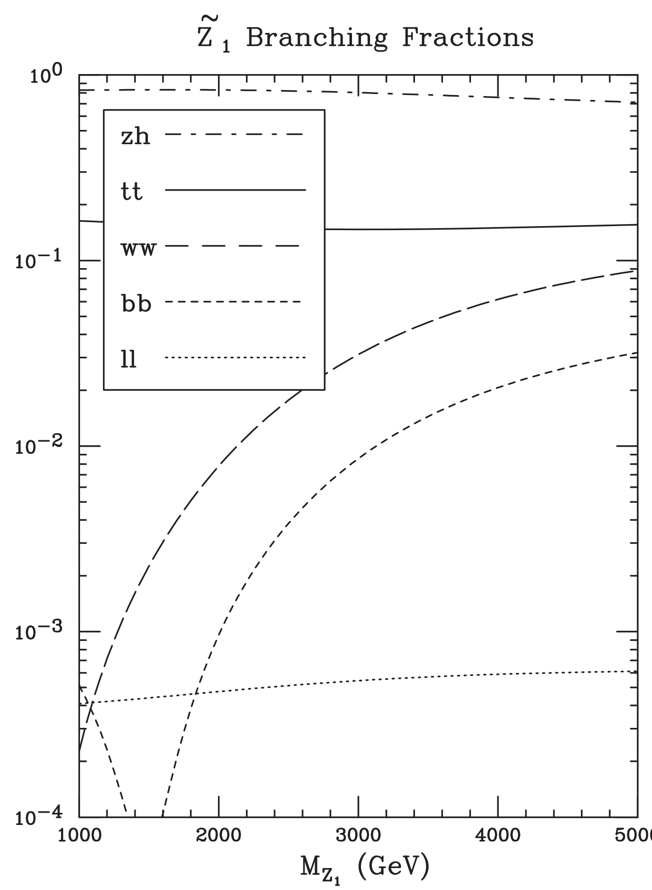

FIG. 9. Decay branching fractions for the first KK Z mode. From Agashe *et al.*, 2007.

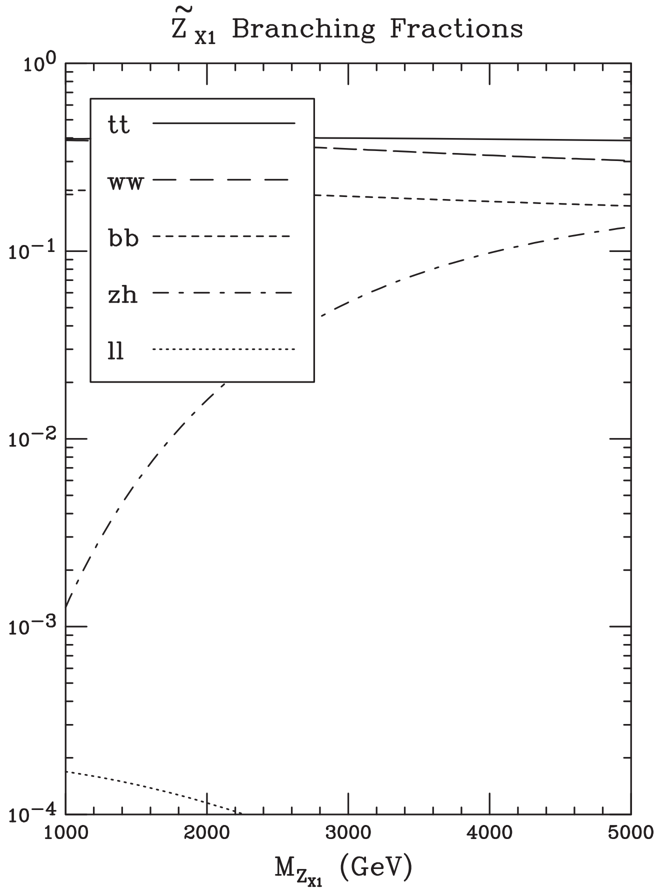

FIG. 10. Decay branching fractions for the KK Z_x mode. From Agashe *et al.*, 2007.

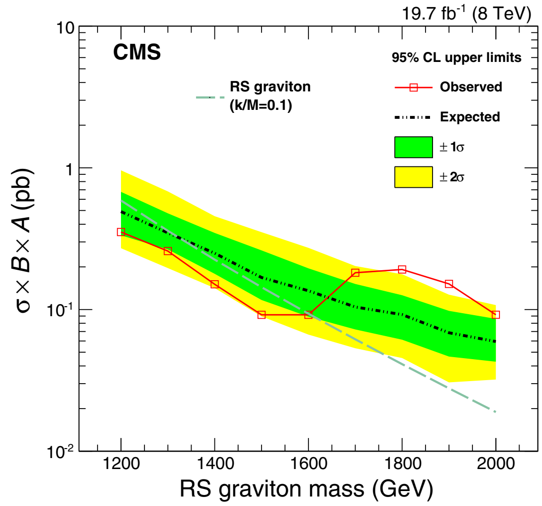

FIG. 12. Bound on the KK graviton from the dijet channel in the RS model. From Khachatryan *et al.*, 2015d.

the hierarchy problem, the profile should be peaked close to the IR brane, but does not necessarily have to be exactly localized (as we have assumed until now). In addition, the magnitude of the Higgs VEV does not have to exactly reproduce the value in the SM in order to obtain the correct W and Z masses. This can be understood in the following way. Raising the Higgs VEV will deform the wave functions of the gauge bosons, as a consequence even in the $v \to \infty$ limit one does not send the gauge boson masses to infinity. Instead one obtains Higgsless models (Csáki, Grojean, Murayama *et al.*, 2004), where the only role played by the Higgs is to enforce the IR brane boundary condition

$$g_{5L} A_\mu^{La} - g_{5R} A_\mu^{Ra} = 0, \tag{3}$$

where $g_{5L,R}$ are the bulk gauge couplings of the $SU(2)_{L,R}$ groups, while the orthogonal combination has a Neumann BC. In this case the masses of the W and Z bosons are set entirely by the size and geometry of the extra dimension, and one can think of such Higgsless models as extra-dimensional versions of technicolor, with the important significant difference that the Higgsless models could be weakly coupled. In this case one still expects perturbative unitarization of the WW, WZ scattering amplitudes; however, since there is no

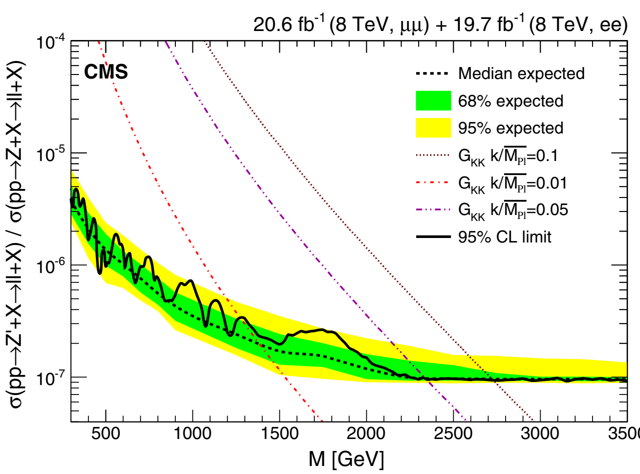

FIG. 11. Bound on the KK graviton from the dilepton channel in the RS model for different coupling strengths $k/\bar{M}_{\rm Pl}$. From Khachatryan *et al.*, 2015e.

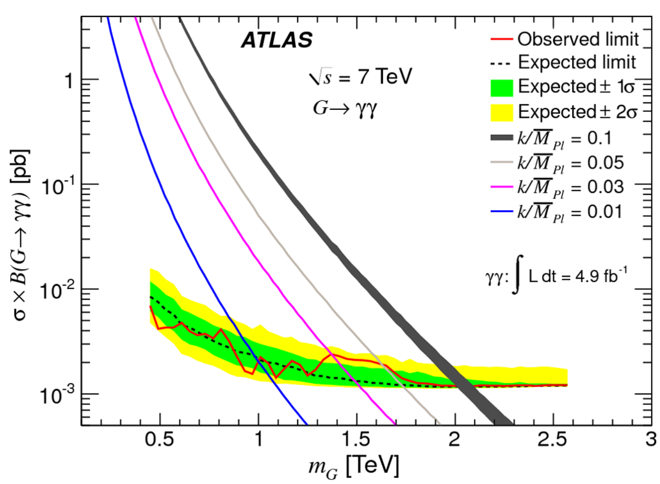

FIG. 13. Bound on the KK graviton from the diphoton channel in the RS model for different coupling strengths $k/\bar{M}_{\rm Pl}$. From Aad *et al.*, 2013b.

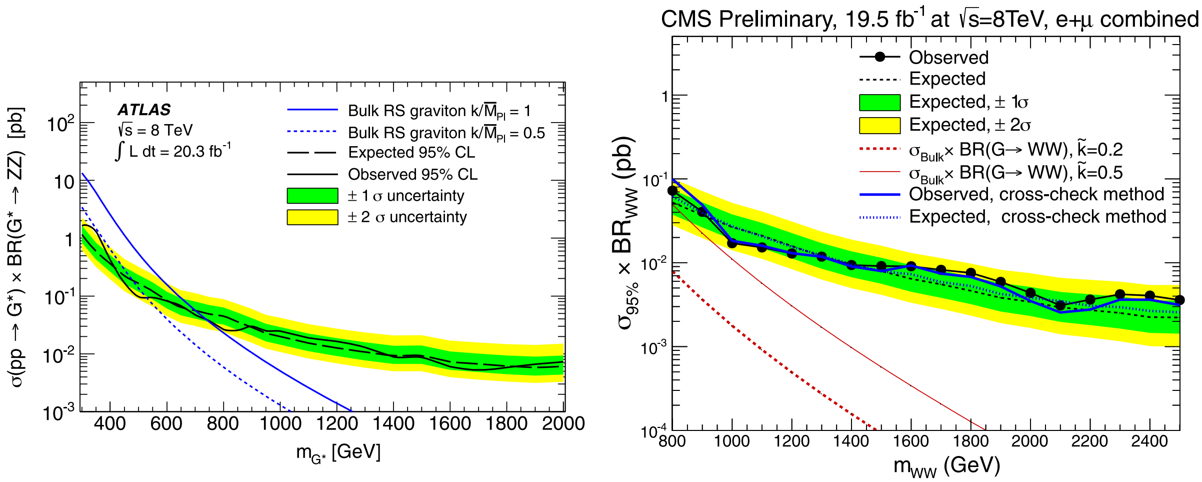

FIG. 14. Bound on the KK graviton from the $\ell\ell q\bar{q}\ell\nu q\bar{q}'$ and channel in the "bulk RS" model for different coupling strengths $k/\bar{M}_{\rm Pl}$. From Chatrchyan *et al.*, 2013, and Aad *et al.*, 2015a.

state analogous to the SM Higgs, the unitarization happens (Chivukula and He, 2002; Chivukula, Dicus, and He, 2002; Csáki, Grojean, Murayama *et al.*, 2004) via the exchange of the gauge boson KK modes $W', Z', W'', Z'', \ldots$. The generic large energy expansion of the WW scattering amplitude is given by

$$\mathcal{A} = A^{(4)}\frac{E^4}{M_W^4} + A^{(2)}\frac{E^2}{M_W^2} + A^{(0)} + \mathcal{O}\left(\frac{M_W^2}{E^2}\right). \tag{4}$$

The requirement of unitarity that the $A^{(4)}, A^{(2)}$ amplitudes must vanish imposes sum rules among the masses and couplings of the various KK modes of the W and Z given by (Csáki, Grojean, Murayama *et al.*, 2004)

$$\begin{aligned} g_{WWWW}^2 &= \sum_n g_{WWZ^{(n)}}^2 \\ g_{WWWW}^2 M_W^2 &= \frac{3}{4}\sum_n g_{WWZ^{(n)}}^2 M_{Z^{(n)}}^2 \end{aligned} \tag{5}$$

for the case of WW scattering, where $g_{WWZ^{(n)}}$ is the cubic coupling between the ordinary W and the various KK modes of the Z, and $M_{Z^{(n)}}$ is the mass of the KK Z's. A similar sum rule applies for WZ scattering.

Since a Higgs-like particle has been discovered, pure Higgsless models are excluded. Bulk Higgs models can, however, lead to situations where some of the features (e.g., the contribution of KK modes to unitarization) persists, albeit at a subleading level. By increasing the value of the Higgs VEV one merely fixes the size of the extra dimension by making sure the observed W and Z masses are still correctly reproduced. Such models have been referred to as gauge-phobic Higgs models (Cacciapaglia *et al.*, 2007) but we refer to them here as bulk Higgs models.

If we simply follow the AdS bulk gauge setup of the realistic RS models then we can take the Higgs field to be a bidoublet of $SU(2)_L \times SU(2)_R$ with $U(1)_{B-L}$ charge zero. The Lagrangian for the Higgs includes a bulk mass (μ^2 in units of the inverse curvature radius $k = R^{-1}$) and brane potentials. The brane potentials gives us the freedom to choose the Higgs BCs. To ensure that the Higgs has a VEV

$$\langle\mathcal{H}\rangle = \mathcal{H} = \begin{pmatrix} v(z) & 0 \\ 0 & v(z) \end{pmatrix}, \tag{6}$$

we can take

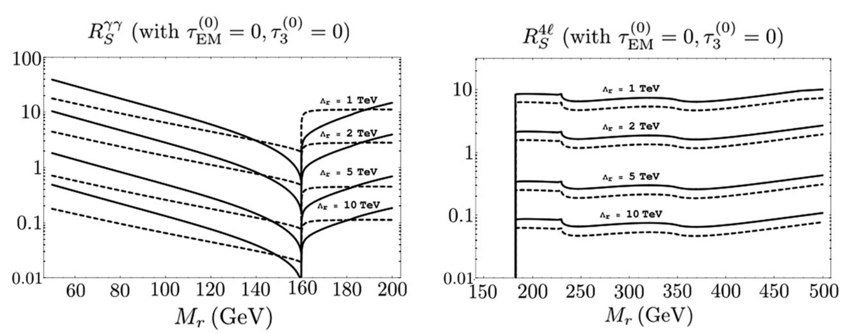

FIG. 15. Ratio of discovery significance for the radion in the realistic RS model (solid), and RS1 model (dashed) to that of a Higgs with the same mass. From Csáki, Hubisz, and Lee, 2007.

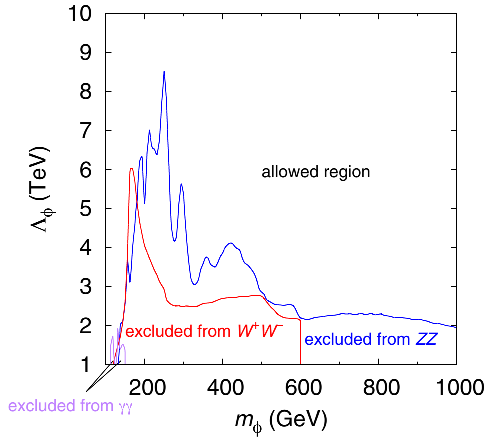

FIG. 16. Bound on the mass of the RS radion (m_ϕ) as a function of its coupling (Λ_Φ), where the coupling is given by $(\Phi/\Lambda_\Phi)T^\mu_\mu$. From Cho, Nomura, and Ohno, 2013.

$$V_{\mathrm{TeV}} = \left(\frac{R}{R'}\right)^4 \frac{\lambda R^2}{2}\left(\mathrm{Tr}|\mathcal{H}|^2 - \frac{v_{\mathrm{TeV}}^2}{2}\right)^2. \qquad (7)$$

The bulk equations of motion give us power law solutions for the Higgs VEV. A convenient way to parametrize the bulk profile of the Higgs VEV is to write

$$v(z) = a\left(\frac{z}{R}\right)^{2+\beta}, \qquad \beta = \sqrt{4+\mu^2}. \qquad (8)$$

The AdS/CFT correspondence suggests that in a CFT description the scaling dimension of the operator that breaks electroweak symmetry is $d = 2+\beta$. For a weakly coupled Higgs the dimension of its mass operator $|\mathcal{H}|^2$ is $2d = 4+2\beta$, so as long as $\beta > 0$ the mass operator is irrelevant, which is to say that it does not suffer from a quadratic divergence, and the hierarchy problem is solved. It is also possible to arrange for $\beta < 0$ (Klebanov and Witten, 1999; Cacciapaglia, Marandella, and Terning, 2009), in which case one can solve only the little hierarchy problem (cf. Sec. III.A).

A normalization V can be chosen (Cacciapaglia *et al.*, 2007) so that

$$v(z) = \sqrt{\frac{2(1+\beta)\log R'/R}{1-(R/R')^{2+2\beta}}}\,\frac{gV}{g_5}\frac{R'}{R}\left(\frac{z}{R'}\right)^{2+\beta}. \qquad (9)$$

This is useful for comparing with the SM limit where the elementary Higgs has scaling dimension $d = 1$ (and thus $\beta = -1$) and $V = 246$ GeV. As discussed in Sec. I.B, these models have a two-dimensional parameter space (V, β). We are already familiar with some of the limiting cases of the parameter space. The corner with large scaling dimension and small VEV is the realistic RS model; the large scaling dimension means that the Higgs is localized on the TeV brane. The corner with a large VEV and large scaling dimension corresponds to the Higgsless limit. The corner with $\beta = -1$ and $V = 246$ GeV is the SM. As mentioned, the fact that the SM has $\beta = -1$ is indicative of its hierarchy problem. In this section we restrict ourselves to $\beta \geq 0$ and defer discussion of $-1 < \beta < 0$ to Sec. IV.D. The relation of various models of electroweak symmetry breaking is shown in Table II.

TABLE II. Bulk Higgs parameters for benchmark points studied and limits for other models. The minimal pseudo-Goldstone boson Higgs model is discussed in Sec. III.E, and the quantum critical Higgs model is discussed in Sec. IV.D.

	V	β
Higgsless	∞	∞
RS1	246 GeV	∞
Bulk Higgs benchmark	300 GeV	2
Minimal PGB Higgs	246 GeV	0
Quantum critical benchmark	246 GeV	-0.3
Standard model	246 GeV	-1

Unitarization proceeds differently in the bulk Higgs model compared to the SM. The contributions to terms in the WW scattering amplitude that grow like energy squared have the following form:

$$\mathcal{A}^{(2)} \sim g^2_{WWWW} - \frac{3}{4}\sum_k \frac{M^2_{Z^k}}{M^2_W} g^2_{WWZ^k} - \frac{1}{4}\sum_k g^2_{WWH^k}, \qquad (10)$$

where the first term is the four W contact term, the first sum is over the KK modes of the Z, and the final sum is over the KK modes of the Higgs [cf. Bellazzini *et al.* (2012) and Falkowski, Rychkov, and Urbano (2012)]. In the SM the sums reduce to single terms. In Higgsless toy models the final sum vanishes but unitarization is maintained by the sum of Z KK modes, which is usually dominated by the lightest modes. In a bulk Higgs model there are contributions from all three terms, again usually dominated by the lightest modes. In every case $\mathcal{A}^{(2)} = 0$, but different modes are responsible for the unitarization.

It is useful to define the following parameter in order to quantify how "Higgsless" the model is:

$$\xi_{BH} \equiv \frac{\sum_k g^2_{WWH^k}}{g^2_{WWH}(SM)}. \qquad (11)$$

With the LHC data for the Higgs-like boson we are far away from the Higgsless limit $\xi_{BH} = 0$ and are roughly constrained to models with $\xi_{BH} > 0.9$. The production cross section times branching fractions of a bulk Higgs and an elementary Higgs are shown in Fig. 17. This suggests that the current data constrain us to $V < 300$ GeV.

As in any model with KK gauge bosons, care must be taken not to mess up the $Zb\bar{b}$ coupling. An interesting way of doing this was proposed by Agashe *et al.* (2006) who suggested that the top-bottom quark doublet should have a left-right interchange symmetry to protect the $Zb\bar{b}$ coupling from corrections. This has many implications (Carena *et al.*, 2006, 2007; Agashe *et al.*, 2008) including top-partner fermions with exotic electric charges (Contino and Servant, 2008; Dissertori *et al.*, 2010; Mrazek and Wulzer, 2010); cf. Sec. III.E.

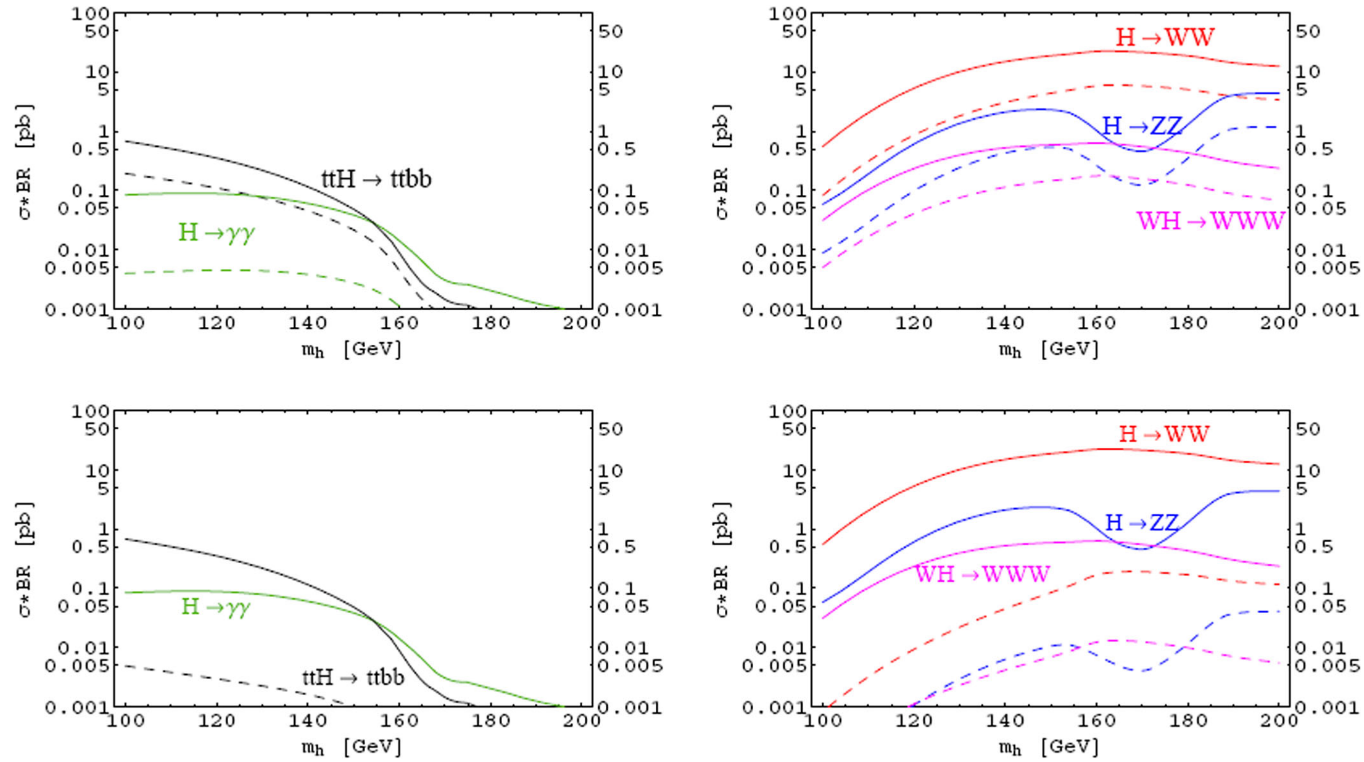

FIG. 17. Cross sections times branching ratios for various Higgs production and decay channels for the SM (solid lines) and bulk Higgs (dashed lines) for $\beta = 2$ with $V = 300$ (top) and $V = 500$ (bottom). From Cacciapaglia *et al.*, 2007.

E. Bulk Higgs LHC searches

The most promising channel for searching for these models is in WZ scattering (see Fig. 18) where one can observe the W' resonance (Birkedal, Matchev, and Perelstein, 2005; Agashe *et al.*, 2009) with 300 fb^{-1} of integrated luminosity. Further studies of the W' were performed by Davoudiasl, Hewett, and Rizzo (2001) and Hewett, Lillie, and Rizzo (2004). CMS has already performed a preliminary search in this channel (Khachatryan *et al.*, 2015c); the results are shown in Fig. 19. CMS finds a lower bound of 1500 GeV on the mass of a sequential W' decaying to WZ. Of course KK W's have suppressed couplings and the bound is weaker for smaller couplings. Model-independent analyses (Eboli, Gonzalez-Fraile, and Gonzalez-Garcia, 2012; Andreev, Osland, and Pankov, 2014) of ATLAS and CMS bounds on neutral spin-1 resonances [i.e., the Z' search (Agashe *et al.*, 2007; Langacker, 2009)] can also be applied to these models. Figure 20 shows the bounds in the mass-coupling plane for different Z' widths. A model-independent parametrizaton of WZ resonances has been advocated by Pappadopulo *et al.* (2014) and has been used in experimental analyses (Aad *et al.*, 2015c; Khachatryan *et al.*, 2015b).

A promising channel for a bulk Higgs search is $pp \to WH$ through an intermediate W' (Galloway, McElrath *et al.*, 2009). In contrast to the similar process in a little Higgs model (see

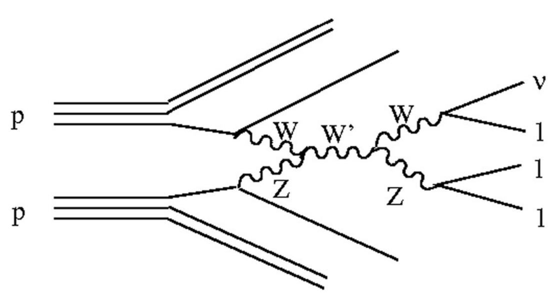

FIG. 18. WZ scattering in pp collisions.

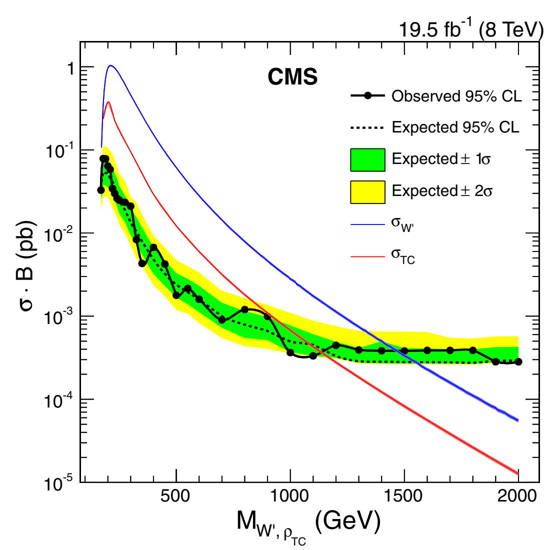

FIG. 19. CMS bound on the W' mass. From Khachatryan *et al.*, 2015c.

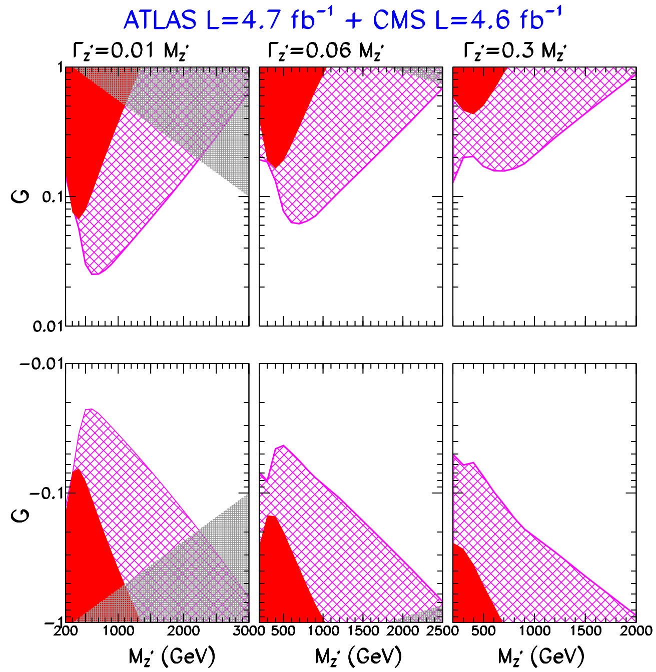

FIG. 20. Model-independent bounds on neutral spin-1 resonances in the mass-coupling plane for different Z' widths. From Gonzalez-Fraile, 2012.

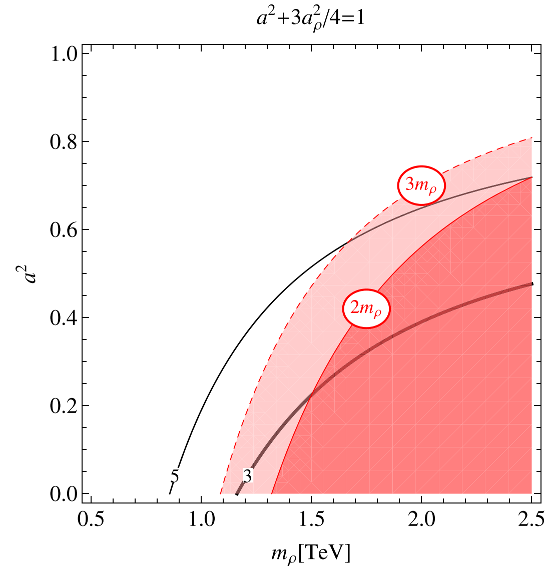

FIG. 22. Higgs and unitarity bounds combined to put an upper bound on the spin-1 resonance ρ for different cutoffs $\Lambda = 3$ TeV, 5 TeV, $2m_\rho$, and $3m_\rho$. Here a is the strength of the Higgs coupling to WW relative to the SM. From Bellazzini *et al.*, 2012.

Sec. III.F), the bulk Higgs has an enhanced coupling to WW'. As shown in Fig. 21, one finds an enhanced cross section relative to the SM for a broad range of V. The bulk Higgs signal also has very different kinematics from the SM background so it is relatively easy to introduce cuts that significantly reduce the background (Galloway, McElrath *et al.*, 2009). Another interesting way to get bounds on these models is to use bounds from the Higgs $\to WW$ search. Assuming that WW scattering is unitary, then if the Higgs coupling is not large enough, we need another resonance to unitarize the scattering, and this resonance cannot be too heavy or it cannot perform its job. With the observed Higgs mass this bound can be translated into an upper bound on the resonance mass as shown in Fig. 22.

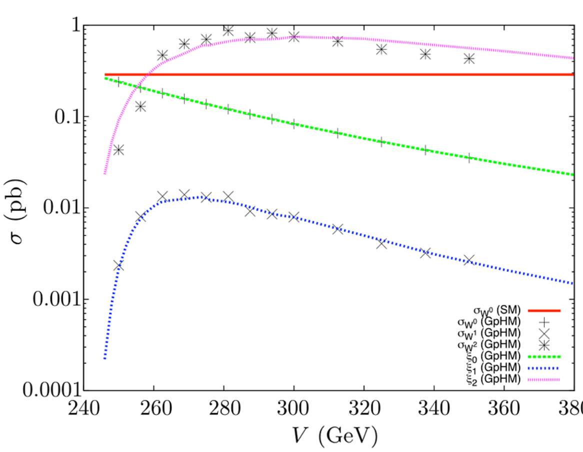

FIG. 21. Approximate contribution to $pp \to WH$ from an intermediate W'. From Galloway, McElrath *et al.*, 2009.

III. COMPOSITE PSEUDO-GOLDSTONE BOSON HIGGS

A. The "littlest" Higgs model

Solving the hierarchy problem resolves why the weak scale is small compared to the Planck scale (or some other very high scale). The little hierarchy problem refers to a smaller problem: why were there no new effects seen at LEP suppressed by just a few TeV? In other words, why is the weak scale small compared to 10 TeV? In order to address this we turn to an alternative where the Higgs is a pseudo-Goldstone boson. Models with pseudo-Goldstone boson Higgs bosons were proposed in the 1970s (Georgi and Pais, 1974, 1975) and thoroughly considered in the 1980s (Georgi and Kaplan, 1984; Georgi, Kaplan, and Galison, 1984; Kaplan and Georgi, 1984; Kaplan, Georgi, and Dimopoulos, 1984; Dugan, Georgi, and Kaplan, 1985). However, one-loop corrections to the Higgs mass meant that the cutoff scale of the effective theory could not be much above 1 TeV, which is no better than the SM. In the 21st century, the technique of dimensional deconstruction (Arkani-Hamed, Cohen, and Georgi, 2001a, 2001b; Hill, Pokorski, and Wang, 2001; Arkani-Hamed *et al.*, 2002a), which is essentially latticizing an extra dimension, led to the idea of collective breaking. If a global symmetry becomes exact when two different interactions go to zero separately, then the pseudo-Goldstone bosons that arise when this symmetry is spontaneously broken are doubly protected. If there are only loop corrections to the pseudo-Goldstone boson mass that raise it from being massless, then the leading contribution will come at two loop order, since is must be proportional to both of the couplings rather than just one as is the usual case. The way this actually comes about in specific models is that quadratic divergences of the SM are canceled by new "little partners" with the same spin as the SM particle, in contrast to SUSY where the superpartners have a different spin with the opposite statistics. The cancellation of the

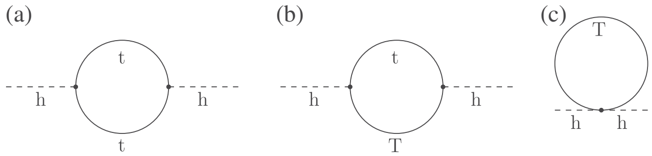

FIG. 23. Cancellation of the quadratic divergences in the top loop for little Higgs models. From Perelstein, 2007.

quadratic divergence is shown in Figs. 23 and 24. This extra suppression is enough to move the naturalness cutoff from 1 TeV, as it is in the SM, to 10 TeV. In this sense little Higgs theories solve the little hierarchy problem. What happens in such models at 10 TeV is usually left as an open question for a future UV completion. There are many different little Higgs models that have been proposed; for detailed reviews of little Higgs theories, see Schmaltz and Tucker-Smith (2005), Chen (2006), Cheng (2007), Perelstein (2007), and Bhattacharyya (2011). Here we limit ourselves mostly to the generic features of such models. The weak point of these models is that while the mass terms appear only through loops, the Higgs quartic coupling can be generated at tree level, which results in too large a quartic coupling and a physical Higgs mass that is generically above 125 GeV. Further tuning has to be introduced to get around this problem.

The so-called "littlest Higgs" model (Arkani-Hamed *et al.*, 2002b) is based on a nonlinear σ model describing the breaking of $SU(5)$ to $SO(5)$ (Arkani-Hamed *et al.*, 2002b). This can be arranged by giving a VEV to a symmetric tensor of the $SU(5)$ global symmetry Σ_0. The Goldstone bosons are the fluctuations around this VEV and can be parametrized by $\Pi = \pi^a X^a$, where the X^a are the broken generators of $SU(5)$. The nonlinear sigma model field is then

$$\Sigma(x) = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f}\Sigma_0, \tag{12}$$

where f is the analog of the pion decay constant that sets the scale of the symmetry breaking VEV. Gauging an $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ subgroup of $SU(5)$ completes the model. The model assumes that the quarks and leptons of the first two generations have their usual quantum numbers under $SU(2)_L \times U(1)_Y$ assigned under the first $SU(2)_1 \times U(1)_1$. The generators of $SU(2)_1$, called Q_1^a, correspond (in a convenient basis) to the upper-left 2×2 block of the $SU(5)$ generator, and similarly the generators of $SU(2)_2$ correspond to the lower-right 2×2 block. The uneaten Goldstone fields include a little Higgs doublet (h^0, h^+) and a complex triplet ϕ. One must ensure that ϕ does not get a

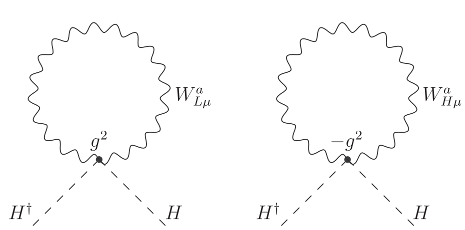

FIG. 24. Cancellation of the quadratic divergences in gauge loops for little Higgs models. From Perelstein, 2007.

large VEV; otherwise, it will give a large contribution to the isospin breaking T parameter.

As in the chiral Lagrangian of QCD, the kinetic energy term in the low-energy effective theory for the Goldstone field is

$$\frac{f^2}{8}\mathrm{Tr}D_\mu\Sigma(D^\mu\Sigma)^\dagger, \tag{13}$$

where the gauge covariant derivative is given by

$$D_\mu\Sigma = \partial_\mu\Sigma - i\sum_j[g_j W_j^a(Q_j^a\Sigma + \Sigma Q_j^{aT}) + g_j' B_j(Y_j\Sigma + \Sigma Y_j)]. \tag{14}$$

At the scale of symmetry breaking f (neglecting the Higgs VEV for the moment) the gauge group is broken to the diagonal subgroup. The gauge bosons of the four groups mix to form the light electroweak gauge bosons and heavy partners. In the (W_1^a, W_2^a) basis the mass matrix, which can be read off from Eq. (13), is

$$\frac{f^2}{4}\begin{pmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{pmatrix}. \tag{15}$$

Diagonalizing we find that the light (actually massless until we include the Higgs VEV) and heavy mass eigenstates are

$$W_L^a = sW_1^a + cW_2^a, \qquad W_H^a = -cW_1^a + sW_2^a, \tag{16}$$

where we have the usual result for the mixing angles

$$s = \frac{g_2}{\sqrt{g_1^2+g_2^2}}, \qquad c = \frac{g_1}{\sqrt{g_1^2+g_2^2}}. \tag{17}$$

There are analogous results for the $U(1)$ mass eigenstates. We can identify the light gauge bosons with the SM gauge bosons of $SU(2)_L \times U(1)_Y$, and quarks and leptons of the first two generations have their SM couplings. They also couple to the heavy gauge bosons (W_H^a, B_H) with strength $(-g_1 c, -g_1' c')$.

The kinetic term of the little Higgs field contains the coupling[4]

$$\mathcal{L}_{W^2h^2} = \frac{g_1 g_2}{4} W_{1\mu}^a W_2^{a\mu} h^2. \tag{18}$$

Expressing W_1 and W_2 in terms of the mass eigenstates we obtain a

$$\mathcal{L}_{W^2h^2} = cs(W_{\mu L}^a W_L^{a\mu} - W_H^{a\mu} W_{\mu H}^a)h^2 + (s^2 - c^2)W_{\mu L}^a W_H^{a\mu} h^2. \tag{19}$$

Now we can see exactly how the cancellation of the quadratic divergence comes about: the symmetry forces equal but opposite couplings to W_L^2 and W_H^2 so their one-loop contributions to the Higgs mass cancel. The mixed term does not

[4]For simplicity we work in a unitary gauge and keep track only of the $h \equiv \mathrm{Re}h^0$ component of the Higgs field.

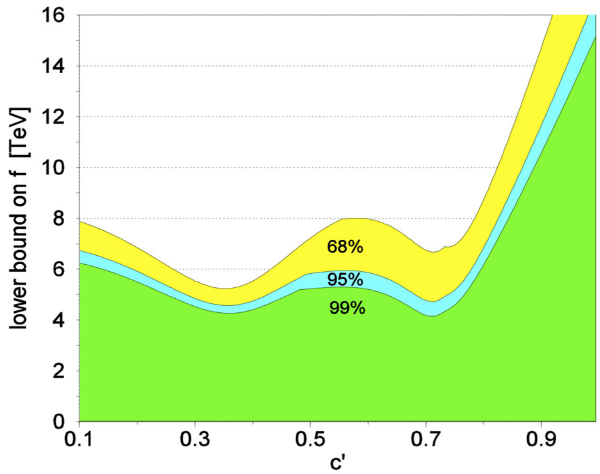

FIG. 25. Lower bound on the littlest Higgs scale f. From Csáki et al., 2003a.

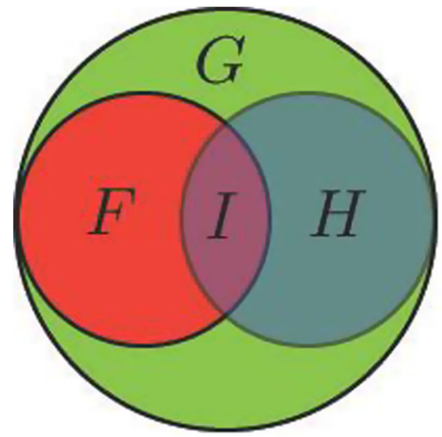

FIG. 26. Coset space for global symmetry group G spontaneously broken to H. If G has a weakly gauged subgroup F then I will be the unbroken gauge group. From Cheng, 2007.

contribute at one loop because we cannot close off the W_L propagator with W_H. A similar analysis shows that the top Yukawa coupling λ is related to the h^2 coupling to the top partner T:

$$\mathcal{L}_{\rm Yuk} = \lambda h t_R^\dagger t_L - \frac{\lambda}{2f} h^2 T_R^\dagger T_L + \text{H.c.} \tag{20}$$

Again the one-loop contributions cancel; see Fig. 23.

Integrating out W_H^a and B_H induces additional operators in the effective theory, which are quadratic in the light gauge fields and quartic in Higgs fields or quadratic in ϕ: These operators give corrections to the light gauge boson masses once the Higgs gets a VEV:

$$\langle h \rangle = \frac{v}{\sqrt{2}}, \qquad \langle \phi \rangle = v', \tag{21}$$

and including the effects of these higher dimension operators we find that the masses of the W and Z receive corrections of order v^2/f^2 and v'^2/v^2, which are potentially dangerous without some additional suppression mechanism.[5]

Exchanges of W_H^a and B_H also give corrections to the coupling of the $SU(2)_L \times U(1)_Y$ gauge bosons and additional four-fermion operators. So we see that little Higgs models can give big corrections to precision electroweak observables which results in a lower bound on the scale f. Figure 25 shows the bound in the case of the littlest Higgs model (Csáki et al., 2003a). To satisfy the bound the top partner must be heavy, and thus we find that there is still fine-tuning at the percent level in this model.

B. Variations on little Higgs

This type of scenario can be generalized to different breaking patterns. The general situation is shown in Fig. 26. Consider a global symmetry group G that is spontaneously broken to H, if G has a weakly gauged subgroup F then $I = F \cap H$ will be the unbroken gauge group. For a little Higgs model we want $I = SU(2)_L \times U(1)_Y$. The number of uneaten pseudo-Goldstone bosons is given by counting the number of broken generators $N(G) - N(H)$ and subtracting the number eaten by gauge symmetries $N(F) - N(I)$. A few examples of such models are as follows:

- minimal moose (Arkani-Hamed et al., 2002c): $G/H = SU(3)^2/SU(3)$, $F = [SU(2) \times U(1)]^2$;
- littlest Higgs (Arkani-Hamed et al., 2002b): $G/H = SU(5)/SO(5)$, $F = [SU(2) \times U(1)]^2$;
- simple group little Higgs (Kaplan and Schmaltz, 2003): $G/H = [SU(3)/SU(2)]^2$, $F = SU(3) \times U(1)$;
- "bestest little" Higgs (Schmaltz, Stolarski, and Thaler, 2010) $G/H = SO(6) \times SO(6)/SO(6)$, $F = SU(2)^4 \times U(1)$.

For a general approach to constructing little Higgs models using "moose" diagrams, see Arkani-Hamed et al. (2002c) and Gregoire and Wacker (2002). Other little Higgs models are discussed in Low, Skiba, and Tucker-Smith (2002), Chang (2003), Cheng and Low (2003), Contino, Nomura, and Pomarol (2003), Schmaltz (2003, 2004), Skiba and Terning (2003), Chang and Wacker (2004), Kaplan, Schmaltz, and Skiba (2004), Low (2004), Agashe, Contino, and Pomarol (2005), Batra and Kaplan (2005), Katz et al. (2005), Roy and Schmaltz (2006), Schmaltz and Thaler (2009), and Kearney, Pierce, and Thaler (2013). There have even been models that incorporate a little Higgs mechanism with SUSY (Birkedal, Chacko, and Gaillard, 2004; Csáki et al., 2006; Roy and Schmaltz, 2006).

Generically all of these little Higgs models give large tree-level corrections to precision electroweak constraints (Csáki et al., 2003a, 2003b; Hewett, Petriello, and Rizzo, 2003), which implies the existence of fine-tuning; see also Bazzocchi, Fabbrichesi, and Piai (2005), Casas, Espinosa, and Hidalgo (2005), and Grinstein and Trott (2008). Loop-level electroweak corrections (Gregoire, Tucker-Smith, and Wacker, 2004) have also been considered in the $SU(6)/Sp(6)$ little Higgs model, and in some regions of the parameter space the tuning can be weaker. Further studies of indirect constraints are given in Chivukula, Evans, and Simmons (2002), Huo and Zhu (2003),

[5]See the discussion of T parity in Sec. III.B.

Casalbuoni, Deandrea, and Oertel (2004), Choudhury *et al.* (2004), Kilic and Mahbubani (2004), Kilian and Reuter (2004), Lee (2004), Yue and Wang (2004), Buras, Poschenrieder, and Uhlig (2005), and Marandella, Schappacher, and Strumia (2005).

The most stringent constraints typically come from isospin breaking corrections, aka contributions to the T parameter, that show up in the differences between M_W and M_Z. Isospin violation can be suppressed in models that incorporate a custodial symmetry (Chang, 2003; Chang and Wacker, 2004). Further improvements in suppressing all precision electroweak corrections can be made in models that incorporate a new symmetry: T parity (Cheng and Low, 2003, 2004; Cheng, Low, and Wang, 2006). T parity is a Z_2 symmetry, reminiscent of R parity in the minimal supersymmetric standard model (MSSM), where ordinary particles are even and the new partners are odd. This implies that the new partners have to be pair produced and that the lightest T-parity odd particle (the LTP) is stable, a possible dark matter candidate, and a source of missing energy signatures. T parity significantly weakens the constraints from precision electroweak measurements (Han and Skiba, 2005; Hubisz *et al.*, 2006) since then T-odd particles can contribute only at loop level to precision electroweak observables. The requirement of T parity further increases the difficulty of finding a consistent UV completion however (Hill and Hill, 2007).

C. Effective theory of a pseudo-Goldstone boson Higgs

Without knowing the underlying theory one can always parametrize the effects of new physics with a low-energy, effective theory. At low energies the compositeness of the Higgs reveals itself in the deviations of the couplings in the effective theory (as compared to the SM couplings). Assuming that the Higgs boson is a CP-even weak doublet, one has (Giudice *et al.*, 2007; Contino *et al.*, 2013)

$$\begin{aligned}\Delta\mathcal{L}_{\rm eff} = {}& \mathcal{L}_{\rm SM} + \frac{\bar{c}_H}{2v^2}\partial^\mu(H^\dagger H)\partial_\mu(H^\dagger H) + \frac{\bar{c}_T}{2v^2}(H^\dagger \overleftrightarrow{D}^\mu H)(H^\dagger \overleftrightarrow{D}_\mu H) - \frac{\bar{c}_6\lambda}{v^2}(H^\dagger H)^3 \\ & + \left[\left(\frac{\bar{c}_u}{v^2}y_u H^\dagger H \bar{q}_L H^c u_R + \frac{\bar{c}_d}{v^2}y_d H^\dagger H \bar{q}_L H d_R + \frac{\bar{c}_l}{v^2}y_l H^\dagger H \bar{L}_L H l_R\right) + \text{H.c.}\right] \\ & + \frac{i\bar{c}_W g}{2m_W^2}(H^\dagger\sigma^i \overleftrightarrow{D}^\mu H)(D^\nu W_{\mu\nu})^i + \frac{i\bar{c}_B g'}{2m_W^2}(H^\dagger \overleftrightarrow{D}^\mu H)(\partial^\nu B_{\mu\nu}) + \frac{i\bar{c}_{HW} g}{m_W^2}(D^\mu H)^\dagger\sigma^i(D^\nu H)W^i_{\mu\nu} + \frac{i\bar{c}_{HB} g'}{m_W^2}(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu} \\ & + \frac{\bar{c}_\gamma g'^2}{m_W^2}H^\dagger H B_{\mu\nu}B^{\mu\nu} + \frac{\bar{c}_g g_S^2}{m_W^2}H^\dagger H G^a_{\mu\nu}G^{a\mu\nu}. \end{aligned} \tag{22}$$

The effects of the composite nature of the Higgs boson and resonances of the W and Z on precision electroweak measurements have been studied (Barbieri *et al.*, 2007; Orgogozo and Rychkov, 2012b; Ciuchini *et al.*, 2013; Pich, Rosell, and Sanz-Cillero, 2013), with the conclusion that the main constraint is that the Higgs WW coupling is close to the SM value $0.99 < 1 - \bar{c}_H/2 < 1.06$ [light fermion resonances can relax this constraint (Grojean, Matsedonskyi, and Panico, 2013; Pappadopulo, Thamm, and Torre, 2013)]. Deviations in the Higgs couplings of course mean that WW scattering is only partially unitarized by the Higgs (cf. Sec. II.D). A fairly general analysis of unitarization in this case has been given by Falkowski *et al.* (2011) and Bellazzini *et al.* (2012).

When the new physics sector is characterized by a single scale M and a coupling $g_* \equiv M/f$, and assuming that the classical action including the heavy fields involves at most two derivatives, the Wilson coefficients of the effective Lagrangian obey the following simple scaling:

$$\begin{aligned} &\bar{c}_H, \bar{c}_T, \bar{c}_6, \bar{c}_\psi \sim O\left(\frac{v^2}{f^2}\right), \qquad \bar{c}_W, \bar{c}_B \sim O\left(\frac{m_W^2}{M^2}\right), \\ &\bar{c}_{HW}, \bar{c}_{HB}, \bar{c}_\gamma, \bar{c}_g \sim O\left(\frac{m_W^2}{16\pi^2 f^2}\right). \end{aligned} \tag{23}$$

When the Higgs doublet is a composite pseudo-Goldstone boson of a spontaneously broken symmetry $\mathcal{G} \to \mathcal{H}$ of the strong dynamics (Contino, Nomura, and Pomarol, 2003; Agashe, Contino, and Pomarol, 2005; Giudice *et al.*, 2007), a further suppression of the contact operators to photon and gluons holds:

$$\bar{c}_\gamma, \bar{c}_g \sim O\left(\frac{m_W^2}{16\pi^2 f^2}\right) \times \frac{g_{\not{\mathcal{G}}}^2}{g_*^2}, \tag{24}$$

where $g_{\not{\mathcal{G}}}$ denotes any weak coupling that breaks the Goldstone symmetry (in minimal models the SM gauge couplings or Yukawa couplings). We stress that these estimates are valid at the UV scale M, at which the effective Lagrangian is matched onto explicit models. Renormalization effects between M and the electroweak scale mix operators with the same quantum numbers (Alonso *et al.*, 2014) and give in general subdominant corrections to the coefficients (Elias-Miró *et al.*, 2013, 2014; Grojean *et al.*, 2013; Cheung and Shen, 2015; Elias-Miró, Espinosa, and Pomarol, 2015). The estimates of $\bar{c}_{W,B}$ and $\bar{c}_T$ apply when these coefficients are generated at tree level. However, specific symmetry protections which might be at work in the UV theory, for example, R parity in SUSY theories, can force the leading corrections to arise at the one-loop level.

D. Pseudo-Goldstone boson Higgs

As mentioned earlier, the idea of the Higgs as a composite pseudo-Goldstone boson goes back to the 1980s (Georgi and Kaplan, 1984; Georgi, Kaplan, and Galison, 1984; Kaplan and

TABLE III. Cosets G/H from simple Lie groups with maximal subgroups. For each coset, the number of Goldstone bosons N_G and the Goldstone bosons representation under H and $SO(4) \simeq SU(2)_L \times SU(2)_R$ are given. From Mrazek *et al.*, 2011.

	G	H	N_G	$\mathrm{rep}[GB] = \mathrm{rep}[SU(2)\times SU(2)]$
Minimal	$SO(5)$	$SO(4)$	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$
Next to minimal	$SO(6)$	$SO(5)$	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
	$SO(6)$	$SO(4)\times SO(2)$	8	$\mathbf{4_2} + \bar{\mathbf{4}}_{\mathbf{-2}} = 2\cdot(\mathbf{2},\mathbf{2})$
	$SO(7)$	$SO(6)$	6	$\mathbf{6} = 2\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
	$SO(7)$	G_2	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
	$SO(7)$	$SO(5)\times SO(2)$	10	$\mathbf{10_0} = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
	$SO(7)$	$[SO(3)]^3$	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3\cdot(\mathbf{2},\mathbf{2})$
Composite two-Higgs	$Sp(6)$	$Sp(4)\times SU(2)$	8	$(\mathbf{4},\mathbf{2}) = 2\cdot(\mathbf{2},\mathbf{2})$ or $(\mathbf{2},\mathbf{2}) + 2\cdot(\mathbf{2},\mathbf{1})$
	$SU(5)$	$SU(4)\times U(1)$	8	$\mathbf{4_{-5}} + \bar{\mathbf{4}}_{\mathbf{5}} = 2\cdot(\mathbf{2},\mathbf{2})$
	$SU(5)$	$SO(5)$	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$

Georgi, 1984; Kaplan, Georgi, and Dimopoulos, 1984; Dugan, Georgi, and Kaplan, 1985). As in little Higgs models, the electroweak gauge symmetry is embedded in a larger global symmetry which is broken at a strong interaction scale $f \sim 1$ TeV, producing a Goldstone boson that is identified with the SM Higgs; for reviews, see Grojean (2009), Rychkov (2011), Contino (2011), Espinosa, Grojean, and Muhlleitner (2012), Bellazzini, Csáki, and Serra (2014), and Panico and Wulzer (2016). In an extra-dimensional setting[6] these models are also referred to as gauge-Higgs unification (Csáki, Grojean, and Murayama, 2003; Panico, Serone, and Wulzer, 2006; Medina, Shah, and Wagner, 2007; Serone, 2010) since the Goldstone boson shows up as the fifth component of a bulk gauge field. The effective theory has a cutoff $\Lambda \sim 4\pi f \sim 5$–10 TeV. The difference here is that there is no tree-level quartic allowed and the entire Higgs potential is loop generated. Top partners (as in little Higgs models, cf. Sec. III.A) and spin-1 partners are still needed to cancel the quadratic divergences. However, these models still must be tuned at the 1% level; as discussed later the origin of the tuning is the requirement of keeping v much smaller than f. Since the quartic is loop generated the Higgs is naturally light, around 100 GeV, in agreement with the observed value. Generically the deviations from SM Higgs couple at the 10%–20% level.

There are numerous ways to construct models where the Higgs can appear as a pseudo-Goldstone boson. We restrict ourselves to those with an unbroken custodial $SU(2) \times SU(2)$ symmetry, as usual. A list of examples is given in Table III [see also Bellazzini, Csáki, and Serra (2014)]. Some of these models have been investigated in the literature: the minimal model (Contino, Nomura, and Pomarol, 2003; Agashe, Contino, and Pomarol, 2005) (discussed in Sec. III.E), the next to minimal model (Gripaios *et al.*, 2009), and the composite two-Higgs doublet model (Mrazek *et al.*, 2011).

At lowest order in the weak gauge couplings, the effective Lagrangian is just a chiral Lagrangian with only derivative interactions. Thus the Higgs potential in these models is entirely generated by loop corrections à la Coleman-Weinberg (Coleman and Weinberg, 1973). Generically the potential takes the form

$$V(H) = \frac{g^2}{16\pi^2} f^4 \cos^n\left(\frac{H}{f}\right). \tag{25}$$

This means that there is no natural separation between the weak scale VEV v and the strong breaking scale f. Qualitatively the mass term and quartic term can be estimated as

$$m^2 \sim \frac{g^2\Lambda^2}{16\pi^2} \sim g^2 f^2, \qquad \lambda \sim \frac{g^2\Lambda^2}{16\pi^2 f^2} \sim g^2. \tag{26}$$

Consistency with precision electroweak measurements (especially the S parameter) (Carena *et al.*, 2007; Csáki, Falkowski, and Weiler, 2008) requires that approximately $v < 0.45f$. Detailed calculations show that generically achieving this suppression in these models requires a percent level tuning (Csáki, Falkowski, and Weiler, 2008), as shown in Fig. 27. This bound can be relaxed thanks to the contributions of the fermionic and bosonic resonances to the electroweak oblique parameters; see, for instance, Ciuchini *et al.* (2013) and Grojean, Matsedonskyi, and Panico (2013).

E. Minimal composite pseudo-Goldstone boson Higgs

The simplest implementation of the composite pseudo-Goldstone boson scenario is a 5D model (Contino, Nomura, and Pomarol, 2003; Agashe, Contino, and Pomarol, 2005; Agashe and Contino, 2006; Giudice *et al.*, 2007) known as the "minimal composite pseudo-Goldstone boson Higgs," usually shortened to just the "minimal composite Higgs." The model could be viewed as simply the low-energy effective theory for a particular pattern of symmetry breaking, without reference to a 5D theory. In the 5D description the Higgs is the fifth component of a gauge boson, and from its profile in the extra dimension, we can read off (via the AdS/CFT correspondence) that its scaling is 2. This model has a global $SO(5) \times U(1)_X$ symmetry in the bulk which is broken to $SU(2) \times U(1)_Y$ on the UV brane and $SO(4) \times U(1)_X$ on the IR (TeV) brane. The Goldstone boson of the $SO(5) \to SO(4)$ breaking is a vector of $SO(4)$ which is equivalent to a (2,2) of $SU(2)_L \times SU(2)_R$ which has the correct quantum numbers to be the Higgs. Gauging the SM $SU(2)_L \times U(1)_Y$ subgroup explicitly breaks the global symmetry and gives the Higgs a mass and quartic interactions. This can be thought of as an

[6] Often these models use more than 5 dimensions in order to get enough Higgs components.

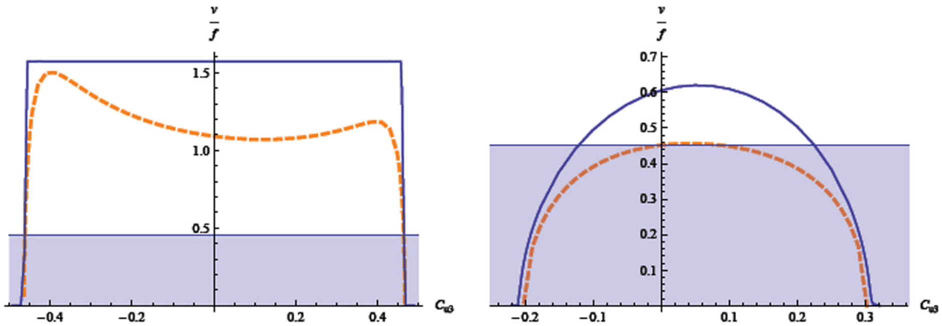

FIG. 27. The amount of tuning required to suppress v/f in pseudo-Goldstone boson Higgs models. Generically a tuning at the percent level is required to be in the acceptable shaded (blue) region, as shown on the left. By carefully choosing other input parameters the apparent fine-tuning in c_u is reduced, shown on the right, at the price of fine-tuning other parameters. From Csáki, Falkowski, and Weiler, 2008.

extra-dimensional version of a little Higgs model with custodial symmetry (Chang and Wacker, 2004).

As in a little Higgs model we construct the low-energy effective theory in terms of a field that provides a nonlinear realization of the symmetry. This field encodes the four Goldstone boson fluctuations around the VEV:

$$\Sigma(x) = \Sigma_0 e^{\Pi(x)/f}, \qquad \begin{array}{c} \Sigma_0 = (0,0,0,0,1), \\ \Pi(x) = -iX^a h^a(x)\sqrt{2}, \end{array} \tag{27}$$

where X^a are the broken $SO(5)$ generators. This is equivalent to

$$\Sigma = \frac{\sin(h/f)}{h}(h^1, h^2, h^3, h^4, h\cot(h/f)), \qquad h \equiv \sqrt{(h^a)^2}. \tag{28}$$

Following the classic paper of Callan, Coleman, Wess, and Zumino (Callan *et al.*, 1969; Coleman, Wess, and Zumino, 1969) we found the gauge field terms in the leading order Lagrangian (in momentum space) to be

$$\begin{aligned} \mathcal{L} = K^{\mu\nu}\Bigg\{ & \frac{1}{2}\left(\frac{f^2\sin^2(h/f)}{4}\right)(B_\mu B_\nu + W^3_\mu W^3_\nu - 2W^3_\mu B_\nu) \\ & + \left(\frac{f^2\sin^2(h/f)}{4}\right) W^+_\mu W^-_\nu \\ & - \frac{q^2}{2}\left[\frac{1}{g^2} W^{a_L}_\mu W^{a_L}_\nu + \frac{1}{g'^2} B_\mu B_\nu\right] + \cdots \Bigg\}, \end{aligned} \tag{29}$$

where the transverse factor $K^{\mu\nu}$ is just $K^{\mu\nu} = \eta^{\mu\nu} - q^\mu q^\nu$, W^a_μ are the $SU(2)_L$ gauge bosons, and B_μ is the $U(1)_Y$ gauge boson. Comparing with the gauge boson masses in the SM we can identify

$$v = f \sin\frac{\langle h\rangle}{f}. \tag{30}$$

It is useful to write expressions in terms of the ratios of the VEVs so we define

$$\xi \equiv \frac{v^2}{f^2} = \sin^2\frac{\langle h\rangle}{f}. \tag{31}$$

Expanding around the Higgs VEV, $h^a = (0,0,\langle h\rangle + h^0, 0)$, we obtain the following deviations of the couplings of the Higgs to the W and Z:

$$\begin{aligned} \kappa_W &\equiv \frac{g_{hWW}}{g^{\rm SM}_{hWW}} = 1 - \frac{\bar{c}_H}{2} = \sqrt{1-\xi}, \\ \kappa_Z &\equiv \frac{g_{hZZ}}{g^{\rm SM}_{hZZ}} = 1 - \frac{\bar{c}_H}{2} - 2\bar{c}_T = \sqrt{1-\xi}. \end{aligned} \tag{32}$$

These deviations in the Higgs coupling mean that unitarity breaks down at a scale

$$\Lambda \approx \frac{4\pi v}{\sqrt{\xi}}, \tag{33}$$

so below this scale additional resonances of the strongly coupled sector must start to contribute to WW scattering (cf. Sec. II.D).

As $\xi \to 0$ the Higgs couplings approach the SM values, so for sufficiently small ξ the model can pass most precision electroweak tests. A left-right symmetry to protect the $Zb\bar{b}$ coupling can also be implemented in this type of model (Agashe and Contino, 2006; Contino, Da Rold, and Pomarol, 2007), resulting in exotic top partners; cf. Sec. III.F.

F. Composite pseudo-Goldstone boson Higgs LHC searches

The signature of little Higgs models is a fermionic top partner that cancels the quadratic divergence in the top loop (Belyaev *et al.*, 2006; Han, Logan, and Wang, 2006; Meade and Reece, 2006; Matsumoto, Nojiri, and Nomura, 2007). For the model to address the little hierarchy problem this top partner has to be significantly below 1 TeV. The single lepton final state is often the optimal channel for finding such top partners (Aguilar-Saavedra, 2009). The gauge partners can be heavier since they have a smaller coupling to the Higgs, and this is easily arranged by having additional VEVs that contribute to the heavy gauge boson masses but not to the top-partner mass (Schmaltz, Stolarski, and Thaler, 2010). There have been many phenomenological studies (Burdman, Perelstein, and Pierce, 2003; Dib, Rosenfeld, and Zerwekh, 2003; Han *et al.*, 2003a, 2003b; Sullivan, 2003; Yue, Wang, and Yu, 2003; Birkedal-Hansen and Wacker, 2004; Park and Song, 2004; Perelstein,

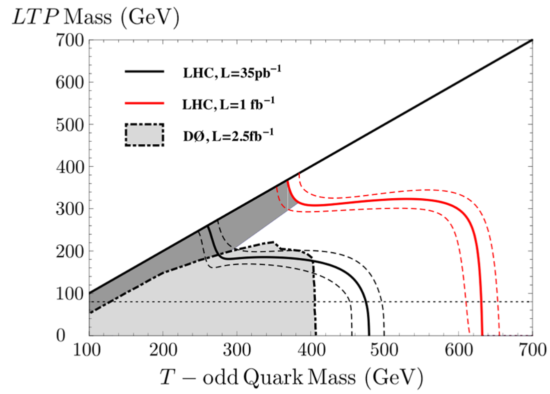

FIG. 28. Lower bound on the little Higgs scale f. From Perelstein and Shao, 2011.

Peskin, and Pierce, 2004; Azuelos *et al.*, 2005; Gonzalez-Sprinberg, Martinez, and Rodriguez, 2005; Hubisz and Meade, 2005; Kilian, Rainwater, and Reuter, 2005; Yue and Wang, 2005; Hektor *et al.*, 2007; Cacciapaglia, Deandrea, Ellis *et al.*, 2013; Reuter and Tonini, 2013; Barducci *et al.*, 2014; Reuter, Tonini, and de Vries, 2014; Yang, Liu, and Han, 2014). A model-independent (effective Lagrangian) approach was advocated by Buchkremer *et al.* (2013). As shown by Perelstein and Shao (2011), the SUSY search for jets and missing energy can be reinterpreted at a bound on the T-odd quark partner mass. The first 35 pb^{-1} of data (Khachatryan *et al.*, 2011) led to a bound around 450 GeV, while it was anticipated that a 1 fb^{-1} analysis should raise the bound to 600 GeV, as shown in Fig. 28.

CMS has performed a search (Chatrchyan *et al.*, 2014a) for the T-even top partner (the partner that cancels the divergence) which gives a bound at 475 GeV, assuming a branching fraction for $T \to t + Z$ of 100%, as shown in Figs. 29 and 30. In the little Higgs models the $T \to t + Z$ branching fraction is more like 25%, so the actual limit is around 375 GeV. One can also search for $T \to b + W$. Naturalness would suggest that the top-partner mass should be around 250 GeV in the absence of tuning, while a mass of 860 GeV would require a 10% tuning, and a mass of 3 TeV would require a 1% tuning. The combined LHC bound (Berger, Hubisz, and Perelstein, 2012) for 5 fb^{-1} assuming appropriate branching fractions is about 450 GeV, which implies at least a 20% fine-tuning. A CMS analysis (Chatrchyan *et al.*, 2014b) with 8 TeV data (allowing for decays into bW, tZ, and tH) found a lower bound of 687 GeV for vectorlike top partners using 19.6 fb^{-1} of data. More exotic top partners, for example, with an $SO(10)/SO(5)^2$ coset can have final states with b jets and a large number of electroweak gauge bosons (Kearney, Pierce, and Thaler, 2013). Model-independent parametrization of top partners has been advocated by De Simone *et al.* (2013) and Panico and Wulzer (2016) using an effective Lagrangian approach.

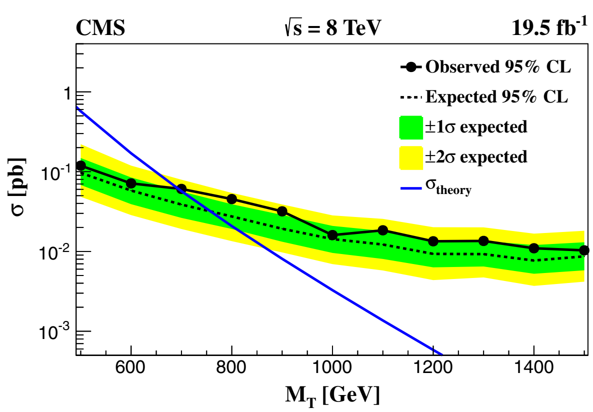

FIG. 29. CMS bound on the T-even top-partner production cross section for branching fractions into bW, tH, and tZ of 50%, 25%, and 25%, respectively. From Chatrchyan *et al.*, 2014a.

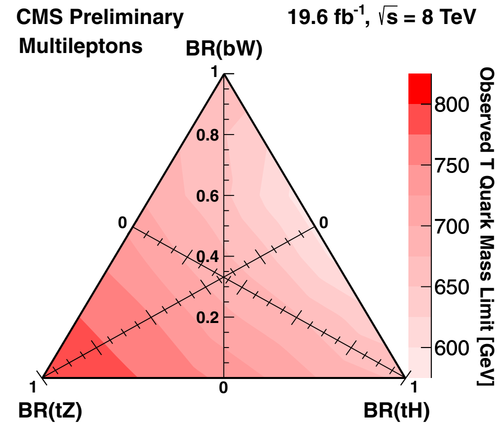

FIG. 30. CMS mass bound on the T-even top partner as a function of branching fractions into bW, tH, and tZ. From Chatrchyan *et al.*, 2014a.

Godfrey *et al.* (2012) studied the "bestest" model in some detail, focusing on top-partner pair production decaying to $b\bar{b}W^+W^-$ and $t\bar{t}ZZ$ and singly produced top partners from W exchange. With just 1.1 fb^{-1} of data there were already bounds on the top-partner mass of about 400 GeV.

The deviations of the Higgs couplings can provide interesting probes of composite pseudo-Goldstone boson models at the LHC (Duhrssen *et al.*, 2004; Espinosa, Grojean, and Muhlleitner, 2010; Azatov *et al.*, 2012; Azatov, Contino, and Galloway, 2012; Espinosa *et al.*, 2012; Cacciapaglia, Deandrea, La Rochelle, and Flament, 2013; Azatov and Galloway, 2013; Montull *et al.*, 2013). Typical deviations of the Higgs couplings in the minimal composite pseudo-Goldstone boson Higgs are shown in Fig. 31. One sees that generically the current LHC data are starting to set tight constraints on these models. A future stringent test will come from double Higgs production (Noble and Perelstein, 2008; Contino *et al.*, 2010; Gouzevitch *et al.*, 2013; Dolan, Englert, and Spannowsky, 2013) which can have contributions to the amplitude, shown in Fig. 32, that grow with energy squared. A search involving four W's (see Fig. 33) has been proposed (Contino *et al.*, 2010) which would take between 450 and 3500 fb^{-1} of luminosity for a 5σ signal for reasonable values of v/f. Composite Higgs models often have top-partner fermions with exotic electric charges (Contino and Servant, 2008; Dissertori *et al.*, 2010; Mrazek and Wulzer, 2010)

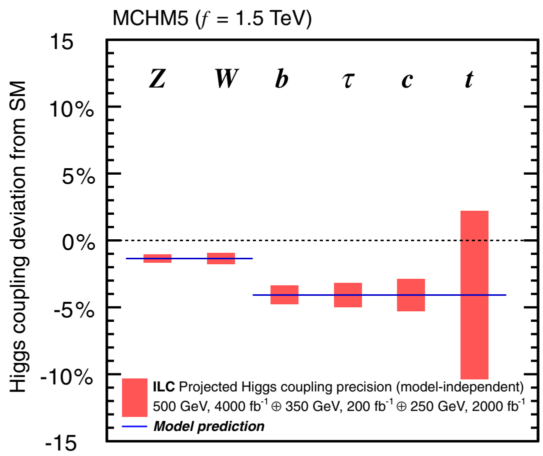

FIG. 31. Examples of deviations from the SM predictions of the Higgs couplings in the minimal composite models. The red bars denote the sensitivity of these couplings at the ILC. From Fujii *et al.*, 2015.

similar to the bulk Higgs models discussed in Sec. II.E. Top partners play a crucial role in the dynamics of electroweak symmetry breaking since the top sector is a dominant breaking of the global symmetry structure and contributes in an instrumental way to the generation of the pseudo-Goldstone boson Higgs potential [see Contino (2011) and Panico and Wulzer (2016) for a general discussion]. It has even been shown (Marzocca, Serone, and Shu, 2012; Pomarol and Riva, 2012) using general arguments based on the Weinberg sum rules that the mass of the Higgs around 125 GeV calls for top partners below 1 TeV. The best way to search for exotic top partners is in pair production $gg \to T_{5/3}\bar{T}_{5/3} \to t\bar{t}W^+W^-$ leading to same-sign dilepton final states. ATLAS and CMS started probing the interesting mass range with a lower bound on their mass close to 800 GeV (Aad *et al.*, 2013a; Chatrchyan *et al.*, 2014b) [nonminimal models could also feature exotic top partners with larger electric charges with striking decay channels, see, e.g., Pappadopulo, Thamm, and Torre (2013) and Matsedonskyi, Riva, and Vantalon (2014)]. These bounds can be further improved using single production channels (De Simone *et al.*, 2013). An idea of the potential reach of these LHC searches is shown in Table IV.

Unusual Higgs couplings may require unusual strategies to enhance particular signals (Fox, Tucker-Smith, and Weiner, 2011; Englert, Jaeckel *et al.*, 2012). Separating out the vector boson fusion production channel is especially useful since it probes the couplings at the heart of electroweak symmetry breaking (Chang *et al.*, 2012).

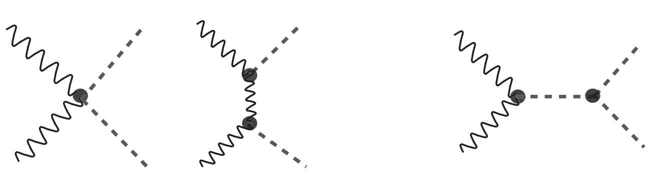

FIG. 32. Double Higgs production in composite pseudo-Goldstone boson Higgs models have contributions that grow with the square of the center-of-mass energy.

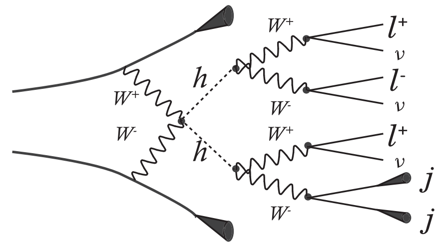

FIG. 33. Difficult 4 W search channel for a pseudo-Goldstone boson Higgs from strong double Higgs production by vector boson fusion. From Contino *et al.*, 2010.

G. Fat SUSY Higgs

Supersymmetric composite (aka fat) Higgs theories have an advantage over nonsupersymmetric theories in that many details of the composite sector are under theoretical control due to Seiberg duality (Seiberg, 1995). In the infrared limit a dual description often reduces to a weakly coupled gauge theory with a Yukawa coupling, and the size of the Yukawa coupling is set by ratios of strong interaction scales. This control allows for a much more detailed prediction in such theories. A further advantage over more conventional supersymmetric theories is that electroweak symmetry breaking can occur in the SUSY limit, thus avoiding having to tune SUSY breaking parameters against SUSY preserving parameters as happens in the MSSM.

Models with composite[7] Higgs fields (Luty, Terning, and Grant, 2001; Harnik *et al.*, 2004; Fukushima, Kitano, and Yamaguchi, 2011) need to address the problem of fitting the electroweak precision measurements. First the S parameter tends to grow with the size of the electroweak sector, but extra contributions to the S parameter from a composite Higgs are suppressed by the square of the VEV over the compositeness scale v^2/f^2, so these contributions are not necessarily troublesome here (Gripaios and West, 2006; Evans *et al.*, 2010; Pich, Rosell, and Sanz-Cillero, 2013). The precision electroweak fits of the SM prefer a small S, but, as is well known (Peskin and Wells, 2001), it is possible to have additional, correlated, contributions to S and T and still be consistent with precision electroweak fits.

An interesting class of fat SUSY Higgs models is where the particles that contribute in loops to the Higgs mass are at least partially composite (Delgado and Tait, 2005). In order for all the leading one-loop divergences to be determined by the strong dynamics, we need the Higgses, the electroweak gauge fields, and (some of) the SM fermions to be composite. The composite fields can be isolated from SUSY breaking, so that their superpartners are much lighter than those of the elementary fields. [This has also been explored in an extra-dimensional context where SUSY breaking is sequestered (Gherghetta and Pomarol, 2003; Goh, Luty, and Ng, 2005; Gherghetta, von Harling, and Setzer, 2011; Sundrum, 2011)].

[7] There have also been studies of models where the Higgs is mostly elementary but mixed with the composites of a strongly coupled SUSY sector (Heckman, Kumar, and Wecht, 2012).

TABLE IV. Potential reach of top (and bottom) partner searches for different charge states. From Agashe et al., 2013.

Collider	Luminosity	Pileup	3σ evidence	5σ discovery	95% C.L.
Top-partner pair production					
LHC 14 TeV	300 fb^{-1}	50	1340 GeV	1200 GeV	1450 GeV
LHC 14 TeV	3 ab^{-1}	140	1580 GeV	1450 GeV	1740 GeV
LHC 33 TeV	3 ab^{-1}	140	2750 GeV	2400 GeV	3200 GeV
Top-partner single production					
LHC 14 TeV	300 fb^{-1}	50	1275 GeV	1150 GeV	
LHC 14 TeV	3 ab^{-1}	140	1130 GeV	1000 GeV	
LHC 33 TeV	3 ab^{-1}	140	1350 GeV	1220 GeV	
LHC 100 TeV	3 ab^{-1}	50	1750 GeV	1600 GeV	
LHC 100 TeV	3 ab^{-1}	140	1750 GeV	1575 GeV	
Bottom-partner pair production					
LHC 14 TeV	300 fb^{-1}	50	1210 GeV	1080 GeV	1330 GeV
LHC 14 TeV	3 ab^{-1}	140	1490 GeV	1330 GeV	>1500 GeV
LHC 33 TeV	300 fb^{-1}	50	>1500 GeV	>1500 GeV	>1500 GeV
Charge 5/3 fermion pair production					
LHC 14 TeV	300 fb^{-1}	50	1.51 TeV	1.39 TeV	1.57 TeV
LHC 14 TeV	3 ab^{-1}	140	1.66 TeV	1.55 TeV	1.76 TeV
LHC 33 TeV	3 ab^{-1}	140	2.50 TeV	2.35 TeV	2.69 TeV

To get a realistic theory, the composites W and Z need to be mixed with elementary W and Z gauge bosons that couple to the elementary quarks and leptons. The electric theory of the simplest such model, the MCSSM model (Csáki, Shirman, and Terning, 2011; Csáki, Randall, and Terning, 2012), has a strongly coupled $SU(4)$ group and six flavors of quarks, $\mathcal{Q}$, $\bar{\mathcal{Q}}$. The model also allows small tree-level masses for the electric quarks $\mathcal{Q}$, $\bar{\mathcal{Q}}$.

The IR behavior of this strongly coupled theory is given by the Seiberg dual (Seiberg, 1995) with a dual gauge group $SU(2)_{\mathrm{mag}}$, six flavors of dual quarks q, $\bar{q}$, a gauge singlet meson M, and with the additional dynamical superpotential term that couples the meson to the dual quarks, with a Yukawa coupling y. The $SU(2)_{\mathrm{mag}} \times SU(2)_{\mathrm{el}}$ is eventually broken to the diagonal subgroup: the SM $SU(2)_L$ at a large scale $\mathcal{F}$.

The dual quarks contain the left-handed third generation quark doublet, two Higgses $H_{u,d}$, and two bifundamentals $\mathcal{H}, \bar{H}$ that will be responsible for breaking the $SU(2)_{\mathrm{mag}} \times SU(2)_{\mathrm{el}}$ to the diagonal and generating the partially composite W and Z. The embedding into the dual quarks is

$$q = Q_3, \mathcal{H}, H_d, \qquad \bar{q} = X, \bar{\mathcal{H}}, H_u, \tag{34}$$

where X is an exotic. From the q, $\bar{q}$ charge assignments it follows that the meson M contains the right-handed t, two singlets, two additional Higgses transforming under the elementary $SU(2)_{\mathrm{el}}$, and some exotics. All the exotic extra fields can be removed in an anomaly-free way through effective mass terms.

From the point of view of electroweak symmetry breaking and the light fermion masses this final model is basically identical at low energies to the usual fat Higgs models (Luty, Terning, and Grant, 2001; Harnik et al., 2004). The Higgs potential, for $\mathcal{F} \gg f$, involves an additional singlet S and is given by

$$\begin{aligned} V = {} & y^2|H_u H_d - f^2|^2 + y^2|S|^2(|H_u|^2 + |H_d|^2) + m_S^2|S|^2 \\ & + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + (ASH_uH_d + TS + \text{H.c.}) \\ & + \frac{g^2 + g'^2}{8}(|H_u|^2 - |H_d|^2)^2, \end{aligned} \tag{35}$$

where m_{S,H_u,H_d}^2, A, and T are soft supersymmetry breaking parameters, and the last term is the usual MSSM D term [an identical structure has been investigated in the nondynamical NMSSM context (Panagiotakopoulos and Pilaftsis, 2001)]. With the standard parametrization of the Higgs fields we defined the usual $\tan\beta$ as the ratio of the up- and down-type VEVs. The interaction with the singlet provides a sizable quartic, so $\tan\beta$ can be close to or less than 1. Minimizing the potential with respect to the scalar S we find that S develops a VEV.

Using holomorphic techniques one can map the effects of soft-SUSY breaking masses m_{UV}^2 in SUSY QCD over to the dual description (Cheng and Shadmi, 1998; Arkani-Hamed and Rattazzi, 1999; Luty and Rattazzi, 1999). At the edge of the conformal window, where the MCSSM model sits ($F = 4$, $N = 6$), one has a hierarchy of the soft-breaking terms, which takes the form

$$A, m_{\tilde{q},\tilde{g}} \sim \frac{m_{\mathrm{UV}}^2}{\Lambda} T \sim f^2 m_{\mathrm{UV}}. \tag{36}$$

This leads to parameters of order

$$\begin{aligned} & m_{\mathrm{UV}} \sim M_3 \sim \text{few} \times \text{TeV}, \\ & T \sim f^2 m_{\mathrm{UV}} \sim \text{few} \times 10^{-2}\ \text{TeV}^3, \\ & \Lambda \sim 5\text{–}10\ \text{TeV}, \qquad \mathcal{F} \sim \text{few} \times \text{TeV}, \\ & m_H \sim m_{\tilde{t}} \sim M_1 \sim M_2 \sim A \sim \text{few} \times 100\ \text{GeV}, \\ & \mu_{\mathrm{eff}} = y\langle S\rangle \sim A, \\ & f \sim 100\ \text{GeV}, \qquad \tan\beta \sim \mathcal{O}(1). \end{aligned}$$

In particular, the stop can remain light, even with a heavy gluino (Cleary and Terning, 2015).

H. Fat SUSY Higgs LHC searches

If the Higgs compositeness scale is at 10 TeV or above then it is very difficult to observe the small deviations in the Higgs couplings without some type of Higgs factory to do precision studies. As in the RS and little Higgs models we can look for the new associated particles that the composite model predicts. For SUSY composite models these are the usual superpartners, but they can have unusual spectra due to their composite nature. Some sample spectra have been generated (Csáki, Randall, and Terning, 2012); often the light stop $\tilde{t}_1$ is lighter than the lightest neutralino N_1. It is assumed that a low scale for supersymmetry breaking gives a lightest superpartner (LSP) gravitino with a mass of a few GeV or less. This type of spectrum corresponds to a stealthy stop scenario (Fan, Reece, and Ruderman, 2011), with the $\tilde{t}_1$ nearly degenerate with the t. The largest SUSY production process at the LHC in this scenario is $pp \to \tilde{t}_1\tilde{t}_1^*$. For a light gravitino, if the $\tilde{t}_1$ decays promptly to a top quark and a gravitino there is little missing energy and these events are very difficult to uncover, but novel search techniques using spin correlations are closing in on these processes (Aad *et al.*, 2015b).

The next largest SUSY production cross sections are the production of the lightest sbottom and the heavier stop $pp \to \tilde{b}_1\tilde{b}_1^*$ and $pp \to \tilde{t}_2\tilde{t}_2^*$. The $\tilde{b}_1$ decays to $\tilde{t}_1 W$, giving rise to $t\bar{t}WW$ final states and in principle, a small amount of missing energy. The $\tilde{t}_2$ decays mostly to $\tilde{t}_1 Z$ and again the $\tilde{t}_1$ will further decay to $t +$ gravitino. The final state for $pp \to \tilde{t}_2\tilde{t}_2^*$ thus mostly contains $t\bar{t}ZZ$ plus a small amount of missing energy. The $\tilde{b}_1$ production process would be an interesting channel, however all of these events will have very little missing transverse energy, so generic SUSY searches could miss it. Even though the light $\tilde{t}_1$ will be boosted, this will usually not bring the missing energy above the standard cuts. Other benchmark spectra have neutralino (N)LSPs, thus the usual missing energy signals of supersymmetry are expected. Because of the heaviness of the gluino and first two generations of squarks, the production rates are reduced from those of a vanilla MSSM. These spectra fall in the class of models considered by Papucci, Ruderman, and Weiler (2012).

I. Twin Higgs models and their UV completions

All models presented until now have the common feature that the particles responsible for canceling the quadratic divergences in the Higgs mass are charged under the SM gauge symmetries. In particular, the top partner carries color charge, implying a reasonably large minimal production cross section at the LHC. An alternative scenario (which is experimentally quite challenging) is the case referred to as "neutral naturalness" (Chacko, Goh, and Harnik, 2006; Burdman *et al.*, 2007, 2015; Chang, Hall, and Weiner, 2007; Craig and Howe, 2014; Cohen *et al.*, 2015; Craig *et al.*, 2015; Craig, Knapen, and Longhi, 2015a, 2015b), where the particles canceling the one-loop quadratic divergences are neutral under the SM. The canonical example for such theories is the twin Higgs model of Chacko, Goh, and Harnik (2006). This is an example of a pseudo-Goldstone boson Higgs theory, with an approximate global $SU(4)$ symmetry broken to $SU(3)$. This breaking can be parametrized by the 4-component field

$$H = e^{i\pi^a T^a/f}\begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix}, \tag{37}$$

where the T^a's are the seven broken generators of $SU(4)$ and the π^a's are the corresponding Goldstone bosons. The twin Higgs model is obtained by gauging the $SU(2)_A \times SU(2)_B$ subgroup of $SU(4)$, where $SU(2)_A$ is identified with the SM $SU(2)_L$, while $SU(2)_B$ is the twin $SU(2)$ group. Gauging this subgroup breaks the $SU(4)$ symmetry explicitly, but the forms of the corrections that are quadratically divergent are given by

$$\frac{\Lambda^2}{64\pi^2}(g_A^2|H_A|^2 + g_B^2|H_B|^2). \tag{38}$$

For $g_A = g_B$ this term is $SU(4)$ symmetric and does not contribute to the Goldstone masses. The $SU(4) \to SU(3)$ breaking will also result in the breaking of the twin $SU(2)_B$ group and as a result three of the seven Goldstone bosons will be eaten, leaving four Goldstone bosons corresponding to the SM Higgs doublet h. The leading expressions for H_A, H_B

$$H_A \sim h, \qquad H_B \sim \begin{pmatrix} 0 \\ f - \frac{h^2}{2f} \end{pmatrix} \tag{39}$$

nicely show the cancellation of the quadratic divergences of the h-dependent terms in Eq. (38). In fact imposing the Z_2 symmetry on the full model will ensure the cancellation of all one-loop quadratic divergences to the Higgs mass, since the two independent quadratic invariants $|H_A|^2$ and $|H_B|^2$ always combine into $|H|^2$ by the Z_2 symmetry. Logarithmically divergent terms can however arise, for example, from gauge loops and will be of the form $\kappa(|H_A|^4 + |H_B|^4)$, with

$$\kappa = \mathcal{O}\left(\frac{g^4}{16\pi^2}\log\frac{\Lambda}{gf}\right),$$

leading to a Higgs mass of the order of $g^2 f/4\pi$, which is of the order of the physical Higgs mass for $f \sim 1$ TeV. The quadratic divergences from the top sector can be eliminated if the Z_2 protecting the Higgs mass remains unbroken by the couplings that result in the top Yukawa coupling. This can be achieved by introducing top partners charged under a twin $SU(3)_c$ via the Lagrangian

$$\mathcal{L}_{\text{top}} = -y_t H_A \bar{t}_L^A t_R^A - y_t H_B \bar{t}_L^B t_R^B + \text{H.c.} \tag{40}$$

In this case the quadratic divergences are canceled by top partners that are neutral under the SM gauge symmetries.

One remaining question is how to obtain a Z_2 breaking VEV $\langle H_B \rangle = f \gg v = \langle H_A \rangle$. This can be achieved by adding a soft-breaking μ term for the Z_2 symmetry in the form $\mu |H_A|^2$ without the corresponding $|H_B|^2$ term.

Twin Higgs models are low-energy effective theories valid up to a cutoff scale of the order of $\Lambda \sim 4\pi f \sim 5$–$10$ TeV, beyond which a UV completion has to be specified. The simplest such possibility is to also make the Higgs composite, and UV complete the twin Higgs model via gauge and top partners at masses of the order of a few TeV. A concrete implementation is the holographic twin Higgs model (Geller and Telem, 2015), which also incorporates a custodial symmetry to protect the T parameter from large corrections. It is based on a warped extra-dimensional theory with a bulk $SO(8)$ gauge group, which incorporates the $SU(4)$ global symmetry discussed enlarged to contain the $SU(2)_L \times SU(2)_R$ custodial symmetry. In addition the bulk contains either a full $SU(7)$ group or an $SU(3) \times SU(3) \times U(1) \times U(1) \times Z_2$ subgroup of it to incorporate QCD, its twin, and hypercharge. The breaking on the UV brane is to the standard model and the twin standard model, while on the IR brane $SO(8) \to SO(7)$, giving rise to the seven Goldstone bosons, three of which will be again eaten by the twin W, Z. The main difference compared to ordinary composite Higgs models is that in composite twin Higgs models the cancellation of the one-loop quadratic divergences is achieved by the twin partners of order 700 GeV–TeV, which are uncharged under the SM gauge group. This allows the IR scale of the warped extra dimension to be raised to the multi-TeV range without reintroducing the hierarchy problem. The role of the composite partners is to UV complete the theory, rather than the cancellation of the one-loop quadratic divergences. For more details about the composite twin Higgs models, see Batra and Chacko (2009), Barbieri *et al.* (2015), Geller and Telem (2015), and Low, Tesi, and Wang (2015).

Another interesting variation of twin Higgs models is the so-called fraternal twin Higgs (Craig *et al.*, 2015), where only the third generation fermions are endowed with twin partners, those needed to cancel the quadratic divergence from the top loop and the states related to them by gauge symmetries. This may change the expected collider signals of the model significantly.

J. Twin Higgs LHC searches

In twin Higgs models the new particles have low production cross sections or are very heavy, so it is difficult to directly test them at the LHC. The only direct connection between the visible and the mirror sector is via the Higgs sector; thus one expects any direct signals to appear in Higgs physics. Because of the mixing with the twin Higgs, one expects in all scenario deviations of $\mathcal{O}(v^2/f^2) \lesssim 10\%$ in the Higgs couplings. Precision Higgs measurements should eventually be able to observe such deviations. More striking direct signals might also be expected depending on the details of the structure of the mirror sector, and, in particular, on the mirror QCD sector. The basic classes are models (Curtin and Verhaaren, 2015) with mirror QCD with long-lived glueballs (due to the absence of light mirror quarks), models with mirror QCD without long-lived glueballs (in the presence of light mirror quarks), and models without mirror QCD. For the case of mirror QCD with no light matter the glueballs of mirror QCD will be long lived and decay to the SM only via the mixing with the ordinary Higgs. Since the ordinary Higgs itself can decay to such glueballs one eventually ends up (Craig *et al.*, 2015; Curtin and Verhaaren, 2015) with displaced Higgs decays as depicted in Fig. 34. The expected reach from run II of the LHC for such displaced decays has recently been estimated by Csáki *et al.* (2015) and the main results are summarized in Fig. 35. The case of mirror QCD with light quark matter is expected to result in Higgs to invisible decays (where the invisibles are mirror jets). Finally in the case of no mirror QCD the only expected signal would be the deviation of the Higgs couplings from their SM values.

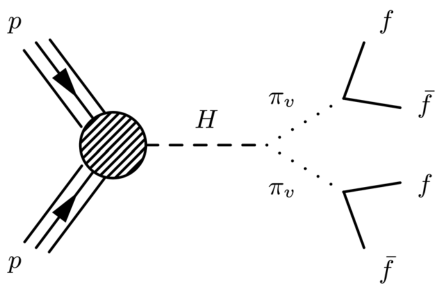

FIG. 34. Displaced decay of the SM Higgs due to decays to long-lived mirror glueballs which eventually decay to SM particles via mixing with the SM Higgs. From Csáki *et al.*, 2015.

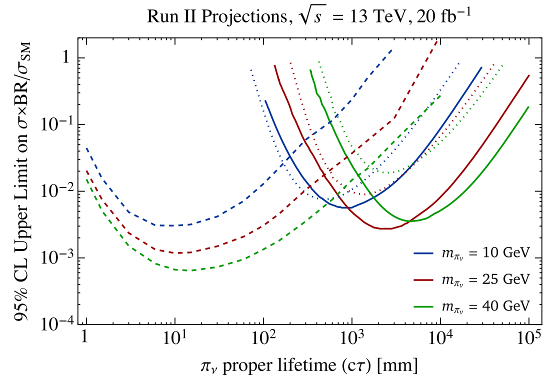

FIG. 35. Expected sensitivity of run II displaced searches for displaced Higgs decays. The vertical axis is the branching fraction of the Higgs into the displaced decays, while the horizontal is the lifetime of the metastable decay products, for three different values of the mass of these intermediate states. From Csáki *et al.*, 2015.

IV. BROKEN CONFORMAL SYMMETRY

A. Dilatons

Technicolor was one of the most appealing ideas for a natural electroweak symmetry breaking scenario. With the discovery of the Higgs-like particle pure technicolor models have been excluded. A natural question to ask is whether models of dynamical electroweak symmetry breaking could nevertheless produce a Higgs-like particle (Bando, Matumoto, and Yamawaki, 1986; Yamawaki, Bando, and Matumoto, 1986). Spontaneously broken conformal symmetry provides for this possibility since it produces a light scalar dilaton, the Goldstone boson of broken conformal symmetry. This is not too far fetched since the SM itself has a limit where the Higgs can be considered a dilaton: if the entire Higgs potential of the SM is turned off, the Higgs will be a classical flat direction, and the SM classically scale invariant. A Higgs VEV will break the scale invariance in addition to breaking the electroweak symmetry, and the physical Higgs boson can be identified with the dilaton of the spontaneously broken scale invariance. The couplings of the Higgs to other SM fields will be determined in this limit: the Higgs will couple to all sources of conformality breaking suppressed by the Higgs VEV, including couplings to masses and β functions in the case of massless gauge bosons. Thus the tree-level coupling of a dilaton to gauge bosons and fermions the same as the SM Higgs couplings [as guaranteed by low-energy theorems (Ellis, Gaillard, and Nanopoulos, 1976; Shifman *et al.*, 1979)] if the VEV that breaks conformal symmetry is the same as the VEV that breaks electroweak symmetry. The loop-level couplings to photons and gluons are model dependent. For example, if the gluons are composites of the conformal sector then the dilaton-gluon-gluon coupling is very different from the Higgs-gluon-gluon coupling (Low, Lykken, and Shaughnessy, 2012). If however the gluons merely weakly gauge a global symmetry of the conformal sector, then it is possible for this scenario to work (Goldberger, Grinstein, and Skiba, 2008; Foot, Kobakhidze, and McDonald, 2010; Ryskin and Shuvaev, 2010; Abe *et al.*, 2012; Bellazzini *et al.*, 2013; Chacko, Franceschini, and Mishra, 2013).

While QCD-like technicolor does not have a light dilaton (Holdom and Terning, 1988) there has been much work on trying to find technicolorlike models that do have a light dilaton (Bando, Matumoto, and Yamawaki, 1986; Yamawaki, Bando, and Matumoto, 1986; Dietrich, Sannino, and Tuominen, 2005; Sannino, 2009; Appelquist and Bai, 2010; Vecchi, 2010, 2011b; Elander and Piai, 2011, 2012, 2013; Hashimoto, 2011; Anguelova, Suranyi, and Wijewardhana, 2012; Fodor *et al.*, 2012; Matsuzaki and Yamawaki, 2012a, 2012b, 2012c; Kozlov, 2012; Lawrance and Piai, 2013). There have also been several other approaches to producing models with a Higgs-like dilaton (Jora, 2009; Foot, Kobakhidze, and McDonald, 2010; Ryskin and Shuvaev, 2010; Campbell, Ellis, and Olive, 2012; Coriano *et al.*, 2013); here we focus on the generic properties.

Scale transformations are part of the conformal group (Coleman, 1985). The scale transformation of an operator $\mathcal{O}$ is given by (for $x \to x' = e^{-\alpha} x$)

$$\mathcal{O}(x) \to \mathcal{O}'(x) = e^{\alpha\Delta}\mathcal{O}(e^{\alpha}x), \tag{41}$$

where Δ is the scaling dimension of $\mathcal{O}$. If all the operators in the action have scaling dimension 4, then the action is invariant. If scale invariance is broken spontaneously by the VEV of an operator $\langle\mathcal{O}\rangle = f^n$, then there must be a Goldstone boson that transforms as

$$\sigma(x) \to \sigma(e^{\alpha}x) + \alpha f. \tag{42}$$

The low-energy effective theory can simply be obtained by replacing the VEV with the nonlinear realization

$$f \to f\chi \equiv f e^{\sigma/f}. \tag{43}$$

In the spirit of Callan, Coleman, Wess, and Zumino (Callan *et al.*, 1969; Coleman, Wess, and Zumino, 1969) we can find the low-energy effective action by requiring that it is invariant under scale transformations:

$$\mathcal{L}_{\rm eff} = -a_{0,0}(4\pi)^2 f^4\chi^4 + \frac{f^2}{2}(\partial_\mu\chi)^2 + \frac{a_{2,4}}{(4\pi)^2}\frac{(\partial\chi)^4}{\chi^4} + \cdots. \tag{44}$$

It is quite unusual that there is a nonderivative term in the potential, but it is a simple consequence of the fact that we are dealing with a space-time symmetry and its transformation is canceled by the change in the volume measure. This effective theory is somewhat of an embarrassment since if $a_{0,0} \neq 0$ it does not describe spontaneous conformal breaking. If $a_{0,0} > 0$ then the vacuum will be at $f\chi = 0$ and there is no spontaneous breaking, if $a_{0,0} < 0$ then the vacuum will be at $f\chi \to \infty$ so there was no conformal theory to start with. Assuming that we actually can find theories that spontaneously break conformal symmetry (with a flat direction since $a_{0,0} = 0$) then we still need to be very careful that any perturbations do not reintroduce $a_{0,0} \neq 0$. If we add a perturbation $\lambda\mathcal{O}$ to the theory then the conditions for $a_{0,0}$ to remain small are that the scaling dimension of $\mathcal{O}$ is close to, but slightly below, 4 and the β function for λ is very small all the way from the UV down to the breaking scale $f\chi$ (Grinstein and Uttayarat, 2011; Bellazzini *et al.*, 2013; Chacko, Mishra, and Stolarski, 2013; Evans and Tuominen, 2013; Chacko and Mishra, 2013). If these conditions are satisfied $a_{0,0}$ will remain small and develop a weak dependence on λ through the running, which will pick out a unique minimum for χ and give the dilaton a small mass.

A spurion analysis is sufficient to determine the couplings of the dilaton in the low-energy theory (Bellazzini *et al.*, 2013). For SM particles the leading couplings are mass$/f$ for fermions and mass$^2/f$ for bosons, so they are very close to the SM Higgs couplings when $f = v$. The spurion analysis also gives the loop suppressed couplings to gluons and photons, but this can be seen even more simply through dilaton dependence of the breaking scale f. The IR gauge coupling is given by

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\mu_0)} - \frac{b_{\rm UV}}{8\pi^2}\ln\frac{\mu_0}{f} - \frac{b_{\rm IR}}{8\pi^2}\ln\frac{f}{\mu}, \tag{45}$$

where $b_{\rm IR}$ includes all the particles with masses below f and $b_{\rm UV}$ includes all the particles with masses below the UV cutoff μ_0. The coefficient of $F^{\mu\nu}F_{\mu\nu}$ in the low-energy theory is $-1/4g^2(\mu)$ and replacing f by $fe^{\sigma/f}$ we find a linear dilaton coupling

$$\frac{g^2}{32\pi^2}(b_{\rm IR} - b_{\rm UV})F^{\mu\nu}F_{\mu\nu}\frac{\sigma}{f}. \tag{46}$$

Thus if the SM particles are spectators of the conformal sector there contributions cancel out, and the low-energy coupling depends on the conformal sector contribution to $b_{\rm UV}$, which is completely model dependent. Of course the SM particles still contribute in the loops of the low-energy theory, and since they have SM-like couplings these loops will precisely mimic the SM Higgs couplings to photons and gluons. So, for example, if the conformal sector has no colored degrees of freedom, we expect the dilaton coupling to gg to be equal to the SM Higgs coupling to gg.

B. Dilaton Higgs LHC searches

LHC searches have been proposed for general dilatons (Barger, Ishida, and Keung, 2012a, 2012b; Coleppa, Gregoire, and Logan, 2012) but these studies tend to make model-dependent assumptions about the dilaton couplings that are incompatible with identifying it with the Higgs-like boson discovered at the LHC. The most direct way to constrain the dilaton Higgs models is to look for deviations in the couplings of the Higgs-like boson from the SM prediction; this is discussed further in Sec. III.C. A substantive difference between the SM Higgs and a dilaton Higgs is in the three- and four-point self-couplings. This can be addressed by double Higgs production searches (Goldberger, Grinstein, and Skiba, 2008; Noble and Perelstein, 2008; Coriano *et al.*, 2012; Gouzevitch *et al.*, 2013; Dolan, Englert, and Spannowsky, 2013; Cao *et al.*, 2014); cf. Sec. III.F.

C. Conformal technicolor

As discussed, technicolor theories solve the hierarchy problem by replacing the Higgs by an operator that has a dimension greater than 2. Naively a solution to the hierarchy problem would be that the analog of the quadratically divergent Higgs mass term (that leads to the hierarchy problem in the SM) is replaced by an operator with a dimension greater than 4, so that it cannot have a divergence. An alternative scenario known as "conformal technicolor" (Luty and Okui, 2006) posits a more subtle solution: what if the Higgs had a scaling dimension near 1, but the mass operator was still irrelevant. This certainly cannot happen at weak coupling, but if the Higgs is a composite of some strong dynamics this may be possible. This leads to a wealth of phenomenological possibilities (Luty, 2009; Evans *et al.*, 2010; Evans *et al.*, 2011; Fukushima, Kitano, and Yamaguchi, 2011; Gherghetta and Pomarol, 2011; Azatov, Galloway, and Luty, 2012; Morrissey, Plehn, and Tait, 2012; Orgogozo and Rychkov, 2012a). Valiant efforts have been made to extend the exact results on conformal theories to exclude this possibility with the result that the dimension of the Higgs must be larger than about 1.5 for its mass operator to be irrelevant (Rattazzi *et al.*, 2008; Rychkov and Vichi, 2009; Fitzpatrick and Shih, 2011; Poland and Simmons-Duffin, 2011; Green and Shih, 2012; Poland, Simmons-Duffin, and Vichi, 2012). Thus the Higgs would not look much like a free particle (cf. Sec. IV.D). An additional problem is why would such a theory have a Higgs that is light compared to the compositeness scale? The only known answer is that the Higgs-like scalar must be a pseudo-Goldstone boson and there must be some tuning involved to adjust its properties, in which case we have something very much like the minimal composite Higgs (see Sec. III.E).

D. Quantum critical points and the Higgs

A continuous phase transition that is tuned near the critical point has a light scalar degree of freedom that is the fluctuation of the order parameter. In the SM a small change in the Higgs mass parameter moves us from the broken phase to the symmetric phase, so it must be tuned to be very close to the critical point. Experimentally we have already seen a Higgs-like resonance, so if there is new physics beyond the standard model it also should be tuned to be close to a critical point. This type of transition at zero temperature is called a quantum critical point in order to emphasize that it is quantum fluctuations that dominate rather than thermal fluctuations. If we assume we are near a quantum critical point, then there is a very long RG flow which generically approaches either a trivial IR fixed point or a nontrivial fixed. The Higgs sector of the SM is an example of a theory that approaches a trivial fixed point. If we want to allow for the possibility that the correct extension of the SM involves a nontrivial fixed point, we need to study general CFTs with a variety of possible IR breakings of scale invariance. This is a formidable task, but we can greatly restrict the range of theories to be investigated since we know the Higgs resonance of such a theory must be weakly coupled since the observed Higgs is very similar to the SM Higgs. We see that the class of quantum critical Higgs (aka un-Higgs) models provides particular, concrete realizations of this scenario.

Georgi (2007b) introduced a useful technique for studying conformal sectors that couple to the standard model using two-point functions of operators[8] with scaling dimension between 1 and 2. Formally the phase space corresponding to the spectral density of this two-point function resembles the phase space for a fractional number of particles, hence the name "unparticles." For electroweak symmetry breaking these ideas have been applied in models where the Higgs couples to an approximately conformal sector and can mix with an unparticle (Delgado, Espinosa, and Quiros, 2007; Fox, Rajaraman, and Shirman, 2007; Delgado *et al.*, 2008a, 2008b; Lee, 2008a). For a nonelementary Higgs we need to study the case where the Higgs is a composite of an approximately conformal sector or in other words the Higgs field itself includes a continuum in its spectrum in addition to a pole [see Espinosa and Gunion (1999), Delgado, Espinosa, and Quiros (2007), van der Bij and Dilcher (2007), Delgado *et al.* (2008a, 2008b), Kikuchi and Okada (2008), and Lee (2008a, 2008b) for work on related ideas]. This is called the quantum critical Higgs scenario. It can be formulated in 4D (Stancato and Terning, 2009) or 5D (Falkowski and Perez-Victoria, 2009b). The 4D description is simpler to implement and will be described here, but the 5D description has broader implications for form factors (Bellazzini *et al.*, 2015) and is very useful for calculating precision electroweak observables (Falkowski and Perez-Victoria, 2009a).

[8]See also Georgi (2007a). The properties of higher n-point functions of unparticles are discussed by Georgi and Kats (2010).

Clearly a phenomenologically acceptable quantum critical Higgs cannot have a continuous spectrum that starts at zero energy; we must have a pole significantly below any continuum. Fortunately there is a simple way to parametrize the case with a finite threshold (Cacciapaglia, Marandella, and Terning, 2008; Fox, Rajaraman, and Shirman, 2007). Typically the introduction of a threshold introduces a single discrete state as well, which could play the role of the Higgs-like particle discovered at the LHC. An important part of seeing that the model is consistent is showing that a quantum critical Higgs still unitarizes WW scattering. This only happens due to the subtle behavior of the nonstandard Feynman vertices of unparticles coupled to gauge bosons. Such vertices are also crucial in the cancellation of anomalies for unfermions (Galloway, McRaven, and Terning, 2009). The quantum critical Higgs is taken to have a scaling dimension Δ between 1 and 2, and thus is a continuation of the bulk Higgs to smaller scaling dimensions. In the notation of Sec. II.D this is the range $-1 < \beta < 0$. For understanding the transition from $\Delta < 2$ to $\Delta > 2$ the 5D description (Falkowski and Perez-Victoria, 2009a) is essential.

For a quantum critical Higgs we crucially need couplings to the electroweak gauge bosons, so we need gauge covariant derivatives ($D_\mu = \partial_\mu - igA^aT^a$) in the kinetic term, and Yukawa couplings to the fermions, the largest being to the top quark. Thus, neglecting couplings to light quarks and leptons the minimal effective action (Cacciapaglia, Marandella, and Terning, 2008; Stancato and Terning, 2009) for a quantum critical Higgs is

$$S = \int d^4x - H^\dagger (D^2 + \mu^2)^{2-\Delta} H - V(H) - \lambda_t \bar{t}_R \frac{H^\dagger}{\Lambda^{\Delta-1}} \begin{pmatrix} t \\ b \end{pmatrix}_L + \text{H.c.} \tag{47}$$

Note that if the scaling dimension of the quantum critical Higgs Δ is larger than 1, then the Yukawa coupling has a dimension larger than 4 and is suppressed by a power of the cutoff. Calculating loop corrections in this theory generically lead to potential terms which allow for the possibility of a minimum at a nonzero VEV of the quantum critical Higgs field. To make further progress we need the couplings of the quantum critical Higgs to the gauge fields. The general result (Cacciapaglia, Marandella, and Terning, 2008) using Eq. (47) and the Mandelstam path-ordered exponential (Mandelstam, 1962; Terning, 1991) is

$$ig\Gamma^{a\alpha}(p,q) = -igT^a \frac{2p^\alpha + q^\alpha}{2p\cdot q + q^2} \times \{[\mu^2 - (p+q)^2]^{2-\Delta} - (\mu^2 - p^2)^{2-\Delta}\}. \tag{48}$$

As usual we expand the quantum critical Higgs field in small fluctuations around the VEV and the resulting Goldstone bosons will be eaten by the gauge bosons, leaving behind a physical Higgs mode. It is convenient to remove the mixing terms by including gauge fixing terms, and then the propagators for the gauge bosons are very different from the SM:

$$\Delta_W(q) = \frac{-i}{q^2 - M_W^2 + i\epsilon} \times \left[g_{\alpha\beta} + \left(\frac{\xi(q^2 - M_W^2)\mu^{2-2\Delta}}{K(q^2)[q^2 - \xi M_W^2/(2-\Delta)]} - \frac{1}{q^2}\right) q_\alpha q_\beta\right].$$

For practical calculations it seems best to use the Landau gauge $\xi = 0$, where the gauge boson propagators are the same as the SM Landau gauge propagators. The physical quantum critical Higgs propagator is

$$\Delta_h(q) = -\frac{i}{m^{4-2\Delta} - \mu^{4-2\Delta} + (\mu^2 - q^2 - i\epsilon)^{2-\Delta}}, \tag{49}$$

where we explicitly see the occurrence of a pole (to be identified with the resonance at 125 GeV) at

$$q^2 = \mu^2 - (\mu^{4-2\Delta} - m^{4-2\Delta})^{1/(2-\Delta)}, \tag{50}$$

and a continuum for $q > \mu$.

The effects of unparticles on unitarity have been studied for WW scattering (Greiner, 2007) and Higgs-Higgs scattering (He and Wen, 2008), assuming that the Higgs boson was an ordinary particle, and that the unparticle belonged to a non-SM sector. The nonstandard behavior of the quantum critical Higgs propagator (49) means that unitarization works differently in this model compared to the usual picture of heavy vector resonances discussed in Sec. II.D. Taking the SM Higgs exchange diagrams and replacing the Higgs propagator by a quantum critical Higgs propagator gives softer growth with energy that is not sufficient for unitarization. However, the full amplitude also has a new contribution from vertices with multiple gauge couplings that arise from expanding the path-ordered exponential. The requisite vertex is shown in Fig. 36. Once all the contributions are included there are no positive powers of energy in the scattering amplitude. Partial wave unitarity also puts mild constraints on the parameter values (Stancato and Terning, 2009). The unitarity of fermion scattering was discussed by Englert, Spannowsky *et al.* (2012), where special care was needed for $d > 1.5$.

From the usual top loop correction to the quadratic (quantum critical) Higgs term in the action we find

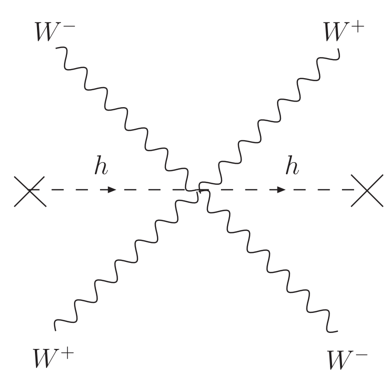

FIG. 36. The four gauge boson two (quantum critical) Higgs contribution to WW scattering. From Stancato and Terning, 2009.

$$\delta m_h^{4-2\Delta} = \frac{3|\lambda_t|^2}{8\pi^2}\Lambda^{4-2\Delta}. \tag{51}$$

So larger values of Δ lead to less sensitivity to the cutoff, smoothly matching onto the bulk Higgs (Sec. II.D) case ($\Delta > 2$) where the hierarchy problem is completely solved. Thus while we cannot expect to solve the full hierarchy we can resolve the little hierarchy problem. For $\Delta \sim 1.7$ the cutoff can be near 10 TeV. Beneke, Knechtges, and Muck (2011) provided an alternative interpretation of the little hierarchy in a theory with nonminimal gauge couplings.

E. Quantum critical Higgs LHC searches

The quantum critical Higgs reduces to the bulk Higgs model (Cacciapaglia *et al.*, 2007) when its scaling dimension is near 2 or larger (Cacciapaglia, Marandella, and Terning, 2009). We quantify the suppression of the gauge boson coupling with the definition

$$\xi^2 \equiv \frac{\sigma_{Unh}(e^+e^- \to HZ)}{\sigma_{\rm SM}(e^+e^- \to HZ)}. \tag{52}$$

Stancato and Terning (2009) showed that ξ^2 falls as Δ gets larger and is approximately zero, as expected, for $\Delta \to 2$. The suppression of gauge couplings was studied by Englert, Spannowsky *et al.* (2012) using pp collisions at the LHC, where again it was seen that for $\Delta \sim 1$ there is only a small suppression of the cross section compared to the SM, but a severe suppression already for values of Δ around 1.2. This sensitivity study was done before the Higgs discovery and focused on resonances above the ZZ threshold, so this type of analysis needs to be redone by experimentalists using the actual data with the resonance set to 125 GeV.

Another signature is that the fraction of top decays with a longitudinal W emitted is different than in the SM (Stancato and Terning, 2010). To compare with data, we must calculate the fraction of the top decays with a longitudinally produced W boson:

$$\mathcal{F}_0 \equiv \frac{\Gamma(t \to W_L^+ b)}{\Gamma(t \to W_L^+ b) + \Gamma(t \to W_T^+ b)}. \tag{53}$$

The current top quark data from ATLAS with 1.04 fb^{-1} yields the following value for $\mathcal{F}_0$ (Aad *et al.*, 2012a):

$$\mathcal{F}_0 = 0.67 \pm 0.07, \tag{54}$$

while CMS reports (Khachatryan *et al.*, 2015f) with 19.1 fb^{-1}

$$\mathcal{F}_0 = 0.720 \pm -0.039(\mathrm{stat}) \pm -0.037(\mathrm{syst}), \tag{55}$$

compared with the SM prediction $\mathcal{F}_0 = 0.699$. With more data the LHC promises to make a more accurate determination of this quantity; this process, however, can significantly constrain models only where the threshold μ is below the top mass.

The LHC constraints from the production cross section on a relatively heavy quantum critical Higgs using the $h \to ZZ \to 4\ell$ channel are straightforward (Englert, Netto *et al.*, 2012), but for masses below 200 GeV (as needed to identify the quantum critical Higgs pole with the Higgs-like boson discovered at the LHC), the analysis must be done by the experimental collaborations. Another search strategy is to look at the interference between the off-shell Higgs process $gg \to h \to ZZ \to 4\ell$ and the QCD process $gg \to ZZ \to 4\ell$ with just a top loop in the intermediate state. As shown in Fig. 37 this can uncover the continuum threshold, but it also provides access to the scaling dimension Δ (Bellazzini *et al.*, 2015). Double Higgs production can also yield complimentary information (Bellazzini *et al.*, 2015).

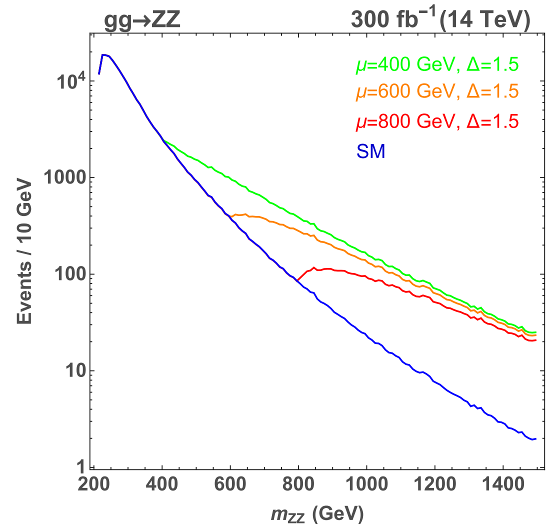

FIG. 37. The effects of a quantum critical Higgs two-point function in the production of on-shell Z-boson pairs, with different values of the threshold μ. From Bellazzini *et al.*, 2015.

V. CONCLUSIONS

All the models we considered should have some deviations in the Higgs coupling to SM particles, so the model-independent approach of constraining the couplings of an effective Higgs Lagrangian (Sec. III.C) is a robust method for attacking all models with nonelementary Higgs bosons. A good deal can be learned from the LHC, but some kind of Higgs factory is ultimately required. An especially interesting coupling is the Higgs to WW coupling, since deviations in this coupling imply TeV scale new physics in order to unitarize WW scattering.

In addition, different classes of models have additional signatures that are being searched for. The little Higgs models (Sec. III.F) need fermionic top partners in order to cancel the one-loop divergence in the Higgs mass. RS models (Sec. II.C) have KK gluons as well as KK W's, KK Z's (aka W' and Z'), and KK gravitons. The most promising LHC search is for the KK gluons, which can be enhanced using a top tagger. Bulk Higgs models (Sec. II.E) share these KK gauge bosons but the W and Z resonances need to be much lighter than in RS since they need to contribute to unitarization. However, a thorough W' search using WZ scattering would require around 300 fb^{-1} of luminosity. Some bulk Higgs models can share a feature of certain pseudo-Goldstone boson Higgs models (Sec. III.F) which is top-partner fermions with exotic electric charges (such as $+5/3$). The quantum critical Higgs scenario

(Sec. IV.E) is starting to be constrained by precision top decay polarization measurements. Fat SUSY Higgs models (Sec. III.H) have the usual superpartners but with an unusual mass spectrum.

The particular searches mentioned are well known and bounds have already been presented. In addition to these there are searches that should be done in the future. The bulk Higgs has an interesting signature in $pp \to WH$ through an intermediate W' which should be looked for. The quantum critical Higgs models can be tested using the $h \to ZZ \to 4\ell$ channel. Twin Higgs models (Sec. III.J) can be probed through invisible decays and displaced vertices. Finally pseudo-Goldstone boson Higgs and dilaton Higgs models can be probed through searching for an enhanced double Higgs production process.

More than ever Higgs physics is a heuristic compass in the quest for new physics beyond the standard model. The run II LHC program is set to reveal the first glimpses of this uncharted territory that will be fully explored later at future machines such as ILC, CLIC, CepC, SppC, and FCC (Gianoti, 2015).

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