# *Colloquium*: Time-reversal violation with quantum-entangled *B* mesons

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Symmetry transformations have been proven a bedrock tool for understanding the nature of particle interactions, formulating, and testing fundamental theories. Based on the up to now unbroken CPT symmetry, the violation of the CP symmetry between matter and antimatter by weak interactions, discovered in the decay of kaons in 1964 and observed more recently in 2001 in B mesons, strongly suggests that the behavior of these particles under weak interactions must also be asymmetric under time reversal T. However, until recent years there has not been a direct detection of the expected timereversal violation in the time evolution of any system. This Colloquium examines the field of timereversal symmetry breaking in the fundamental laws of physics. For transitions, its observation requires an asymmetry with exchange of initial and final states. A discussion is given of the conceptual basis for such an exchange with unstable particles, using the quantum properties of Einstein-Podolsky-Rosen entanglement available at B meson factories combined with the decay as a filtering measurement. The method allows a clear-cut separation of different transitions between flavor and CP eigenstates in the decay of neutral B mesons. These ideas have been implemented for the experiment by the BABAR Collaboration at SLAC's B factory. The results, presented in 2012, prove beyond any doubt the violation of time-reversal invariance in the time evolution between these two states of the neutral B meson.

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# I. INTRODUCTION

In particle physics not all processes are expected to run in the same way with time in one sense as they do in the opposite sense, a symmetry transformation known as time reversal *T*.

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Direct observation of this phenomenon in neutral B mesons was reported by the BABAR Collaboration at SLAC in November 2012 (Lees et al., 2012) and was echoed in other journals and magazines (The Economist, 2012; Bednaršek and Lézé, 2012; Johnston, 2012; Rao, 2012; Schwarzschild, 2012; Zeller, 2012). The so-called CPT theorem, applicable to phenomena described by a local quantum field theory with Lorentz invariance and Hermiticity, implies that the CP violation observed in 1964 with neutral kaons (Christenson et al., 1964, 1965) and in 2001 with neutral B mesons (Abe et al., 2001; Aubert et al., 2001) should also reveal independently a T violation for those systems. Why 48 years after the discovery of CP violation? The conceptual basis for solving the problem of how to probe time reversal with unstable systems was proposed (Bañuls and Bernabéu, 1999a, 2000; Wolfenstein, 1999a) in 1999 through the Einstein-Podolsky-Rosen (EPR) quantum entanglement of the two neutral B mesons produced at the so-called B factories, in addition to using their two decays as filtering measurements to project definite states of the neutral B meson. In this Colloquium we review the fundamentals of time-reversal symmetry, its implication for transitions between two quantum states of the neutral B meson, the implementation of genuine T asymmetries by means of specific decay channels, and the experimental analysis leading to the direct detection performed by BABAR at a significance of  $14\sigma$ .

The outline of this Colloquium is as follows. We first introduce the basics of time-reversal physics, presenting briefly the role of the time-reversal symmetry in the fundamental laws of classical and quantum mechanics, the different scenarios to search for T violation and to prove it experimentally, in particular, when considering unstable systems. In

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Sec. III we introduce the *B* factories and discuss how the quantum entanglement in decays of the  $\Upsilon(4S)$  resonance has been employed during the last decade to perform flavor tagging for exploring *CP* violation in neutral *B* mesons. Section IV discusses how the lack of definite states of the two mesons in the entangled system before their decay can lead to either flavor tagging or "*CP* tagging" for the preparation of neutral *B* meson states required to directly test time-reversal symmetry. Section V briefly presents the *BABAR* detector and data sample, describes how the time-reversal physics is extracted from the data, and summarizes the results and their interpretation. We conclude and discuss some perspectives in Sec. VI.

# **II. TIME-REVERSAL SYMMETRY IN PHYSICS**

Time enters at the most elementary level as a parameter in the description of physical phenomena, serving to identify the order of a sequence of events in the evolution of a physical system. Quantitatively it can be constructed in terms of a wellestablished and continuing sequence of repetitive events. If the period of repetition is constant, it may be used as a unit of time. An accurate definition of this unit is a prerequisite to reach a good precision in time measurements to observe the details of the evolution.

The symmetry transformation that changes a physical system with a given sense of the time evolution into another with the opposite sense is called time reversal *T*. It corresponds to changing the sign of the velocity vector **v** or the momentum **p**, without changing the position **r**. In the dynamical equations of motions, or their solutions, such a transformation corresponds formally to replacing *t* by -t. The *T* transformation changes the sign of other dynamical variables such as angular momentum. For fields, the magnetic field changes its sign under time reversal, whereas the electric field does not.

#### A. Classical and quantum mechanics

The time-reversal transformation in classical mechanics corresponds to substitute for each trajectory  $\mathbf{r}(t)$  the trajectory  $\mathbf{r}(-t)$ , i.e., to moving along the given trajectory with the opposite velocity at each point, as illustrated in Fig. 1. It is not obvious that the dynamics remains invariant under this *T* transformation. If the original trajectory is dynamically possible,  $d\mathbf{p}/dt = \mathbf{F}$  with a force  $\mathbf{F}$  depending on the sense (sign) of the velocity leads to a violation of *T* invariance.

In quantum mechanics, Wigner's time-reversal transformation (Wigner, 1932),

$$\psi(t) \to T\psi(t) \equiv \psi_T(t) = \psi^*(-t),$$
 (1)

keeps the Schrödinger equation  $i\hbar\partial\psi(t)/\partial t = \mathcal{H}\psi(t)$  invariant under a *T* transformation if the Hamiltonian  $\mathcal{H}$  is real. This has three fundamental consequences (Sachs, 1987; Lee, 1990; Branco, Lavoura, and Silva, 1999; Henley, 2013). First, the *T* operator is antiunitary. This property can be seen, for example, evaluating the scalar product of two states,

$$\langle \psi_T(t) | \phi_T(t) \rangle = \langle \psi(-t) | \phi(-t) \rangle^* = \langle \phi(-t) | \psi(-t) \rangle.$$
 (2)



FIG. 1 (color online). Trajectory of a stone falling from the leaning tower (left) and after time-reversal transformation (right). Leaning tower sketch from dragoart.com.

Thus, time reversal has to do with interchange of bra and ket states. Second, the complex conjugation implies that time reversal does not have observable and conserved eigenvalues, unless  $\psi(t)$  is purely real. Third, for a plane wave with momentum  $\mathbf{p}$ ,  $\psi(\mathbf{r},t) = \exp[i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar]$ , the time-reversed wave function is  $\psi^*(\mathbf{r}, -t) = \exp[i(-\mathbf{p}\cdot\mathbf{r}-Et)/\hbar]$ , i.e., the *T*-transformed function describes a particle with momentum  $-\mathbf{p}$  and energy *E*, thus it is not necessary to interpret the transformed function as a particle going backward in time. For this reason the *T* transformation is often referred to as "motion reversal" rather than "time reversal."

The T transformation is implemented in the space of states by the antiunitary operator  $U_T$  in such a way that, for spinless particles,

$$U_T \mathbf{r} U_T^{\dagger} = \mathbf{r}, \qquad U_T \mathbf{p} U_T^{\dagger} = -\mathbf{p},$$
(3)

and  $\psi_T(t) = U_T \psi(-t)$ . Equation (3) guarantees the invariance of the commutation rule between **r** and **p**, thus we might say that *T* transforms quantum mechanics into quantum mechanics. For a Hamiltonian  $\mathcal{H}$  invariant under time reversal  $[\mathcal{H}, U_T] = 0$ , the time-evolution operator  $\mathcal{U}(t, t_0)$ transforms as

$$U_T \mathcal{U}(t, t_0) U_T^{\dagger} = \mathcal{U}^{\dagger}(t, t_0).$$
(4)

The antiunitary character of  $U_T$  allows one to write  $U_T = UK$ , where U is unitary  $(U^{-1} = U^{\dagger})$  and K is an operator which complex conjugates all complex numbers. For the matrix elements of time-dependent transitions we have

$$\langle f | \mathcal{U}(t, t_0) | i \rangle = \langle f | U_T^{\dagger} U_T \mathcal{U}(t, t_0) U_T^{\dagger} U_T | i \rangle$$

$$= \langle U_T f | \mathcal{U}^{\dagger}(t, t_0) | U_T i \rangle^* = \langle U_T i | \mathcal{U}(t, t_0) | U_T f \rangle,$$
(5)

where time-reversal invariance is assumed when passing from the first to the second lines of Eq. (5). As a consequence, the comparison between  $i \rightarrow f$  and  $U_T f \rightarrow U_T i$  transitions is a genuine test of this invariance. It is because of these special properties that the role of time reversal is distinct from that of any other symmetry operation in physics and makes its experimental investigation significantly more difficult than other symmetries.

Therefore, time reversal in classical mechanics as well as in quantum mechanics is related to the following fundamental question (see Fig. 1): consider a point over a trajectory (a state in quantum mechanics), invert the velocities of all particles in that point, and let it evolve; shall we obtain the former initial point of the trajectory with all velocities reversed? Obviously, for a fair comparison the experiment should be repeated in the laboratory with exactly the same boundary conditions as in the *T* mirror, since the motion is not only determined by the equations of motion but also by the boundary conditions. From Newton's mechanics to electrodynamics, the dynamical laws of physics are symmetric under *T* transformation.

Motion-reversal symmetry implies, for a given configuration of energy-momenta and spin, the reciprocity relation (Sachs, 1987; Lee, 1990; Branco, Lavoura, and Silva, 1999; Henley, 2013): the probability of an initial state *i* being transformed into a final state *f* is the same as the probability that an initial state identical to *f*, but with momenta **p** and spins **s** reversed, transforms into the state *i* with momenta and spins reversed,

$$|\langle f|\mathcal{S}|i\rangle|^2 = |\langle U_T i|\mathcal{S}|U_T f\rangle|^2, \tag{6}$$

where S is the transition matrix determined by the Hamiltonian  $\mathcal{H}$ . Here  $|i\rangle \equiv |\mathbf{p}_i, \mathbf{s}_i\rangle$  and  $\langle f| \equiv \langle \mathbf{p}_f, \mathbf{s}_f|$  are the initial and final states,  $\langle U_T i|$  and  $|U_T f\rangle$  are the transformed states of  $|i\rangle$  and  $\langle f|$ , respectively,  $\langle U_T i| \equiv \langle -\mathbf{p}_i, -\mathbf{s}_i|$  and  $|U_T f\rangle \equiv |-\mathbf{p}_f, -\mathbf{s}_f\rangle$ . It should be noted that T invariance is a sufficient, but not necessary, condition for Eq. (6). Therefore, a breaking of reciprocity is an unambiguous signal for T violation. If S is Hermitian,  $|\langle f|S|i\rangle| = |\langle U_T i|S|U_T f\rangle| = |\langle U_T f|S|U_T i\rangle|$ ; in this case, T invariance implies T-odd invariance, and vice versa, where the T-odd transformation refers only to changing the sign of all odd variables under  $t \to -t$  in  $\mathcal{H}$ , without exchanging initial and final states. This occurs, for instance, to first order in weak interactions when final state interactions (FSI) can be neglected (Branco, Lavoura, and Silva, 1999).

The reciprocity relation establishes a connection between the differential cross sections for reactions  $a + b \rightarrow c + d$  and  $c + d \rightarrow a + b$  (detailed balance). It has been verified by experiment in nuclear reactions due to strong or weaker interactions, for example (Blanke *et al.*, 1983),

$$\alpha(0^+) + {}^{24}Mg(0^+) \rightarrow {}^{27}Al(5/2^+) + p(1/2^+).$$
 (7)

Here, if there is any T violation it cannot exceed one-half per mil.

## B. Complex systems and the arrow of time

When discussing T violation we should clearly distinguish t asymmetry of complex systems. For example, our daily experience shows us that when a vase falls and breaks into pieces it is not possible for the pieces of the group to fly back in reverse order, forming the vase. This macroscopic t asymmetry, also known as an "arrow of time," is in the nature of thermodynamics. As discussed by Eddington (1929), the

arrow of time is a property of entropy alone, a measurement of disorder: the arrow gives the direction of progressive increase of disorder in isolated systems. How is it then possible to generate thermodynamic irreversibility from fundamental laws that are *T* symmetric? The answer to this question is that the thermodynamic irreversibility is associated with the irreversibility of the initial conditions for systems with a large number of degrees of freedom, larger for more disordered states, making it very unlikely (although not forbidden) for a system to evolve from a disordered to a more ordered state. In the example of the falling vase, with  $\mathcal{O}(10^{24})$  of particles and collisions, it is not possible in practice to set up the initial conditions for the reversed process (positions and velocities).

There is no doubt that the Universe is expanding, even accelerating at the present time; thus we have a time-asymmetric behavior. But this Universe t asymmetry is perfectly compatible with *T*-symmetric laws of physics. If we think of the initial condition of our Universe, likely inflation (Guth, 1981; Guth and Pi, 1982; Ade *et al.*, 2014a, 2014b), as an improbable state (i.e., more ordered), the cosmological t asymmetry is probably connected with the arrow of time for complex systems. However, none of these time asymmetries is fundamental T violation.

In particle physics, as will be discussed further in detail in Sec. II.E, decays are an example of thermodynamically asymmetric processes: the number of variables or degrees of freedom describing the final state is larger than the number of variables needed to describe the initial state.

#### C. Discrete symmetries broken by weak interactions

It has been well known since 1957 that weak interactions have little respect for symmetries. That year space inversion (parity P) symmetry was discovered to be broken in beta decays (Lee and Yang, 1956; Garwin, Lederman, and Weinrich, 1957; Wu *et al.*, 1957). Then there was the hope that the combination of P with charge conjugation (CP) was a good symmetry. But just a few years later, in 1964, a small but unambiguous violation of the CP symmetry in K meson decays was discovered (Christenson *et al.*, 1964, 1965). More recently, in 2001, the B factory experiments *BABAR* and Belle observed that CP is violated in B mesons (Abe *et al.*, 2001; Aubert *et al.*, 2001).

The now well-established *CP* violation in the quark sector can be successfully accommodated within the standard model (SM) of particles and fields through the three-family Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing mechanism (Cabibbo, 1963; Kobayashi and Maskawa, 1973). It describes the coupling of the *W* boson to up and down quarks and conveys the fact that the quarks with definite properties under charged-current weak interactions are linear combinations of the quark mass eigenstates (Beringer *et al.*, 2012). For three families, the unitarity conditions of the quark-mixing matrix *V* are represented by triangles in the complex plane, as illustrated in Fig. 2, and lead to four fundamental parameters: three magnitudes and one single irreducible phase. In the Wolfenstein parametrization (Wolfenstein, 1983) we can write to  $O(\lambda_c^4)$  as



FIG. 2. The *bd* unitarity triangle representing the CKM unitarity conditions. The three sides are determined from semileptonic and nonleptonic *B* decays, including  $B^0-\bar{B}^0$  oscillations. Since they are of comparable length the angles are sizable and one expects large *CP* asymmetries in *B* decays in the SM. There are two other triangles which almost collapse to a line. This gives an intuitive understanding of why *CP* violation is small in the leading *K* decays (*ds* triangle) and in the leading  $B_s$  decays (*bs* triangle).

$$\begin{bmatrix} V_{uj} \\ V_{cj} \\ V_{tj} \end{bmatrix} = \begin{bmatrix} 1 - \lambda_c^2 & \lambda_c & A\lambda_c^3(\rho - i\eta) \\ -\lambda_c & 1 - \lambda_c^2/2 & A\lambda_c^2 \\ A\lambda_c^3(1 - \rho - i\eta) & -A\lambda_c^2 & 1 \end{bmatrix},$$

where the index *j* runs over *d*, *s*, and *b* quarks, and  $\lambda_c \approx 0.226$ ,  $A \approx 0.814$ ,  $\rho \approx 0.135$ , and  $\eta \approx 0.349$  (Beringer *et al.*, 2012). Extensive tests of the CKM mechanism using all experimental data show a high degree of consistency (Antonelli *et al.*, 2010). Historically, Kobayashi and Maskawa extended in 1973 the 2 × 2 Cabibbo mixing matrix to 3 × 3 to explain the *CP* violation discovered nine years before, thus anticipating the existence of the third family of quarks, quickly confirmed with the discovery of the  $\tau$  lepton in 1975 (Perl *et al.*, 1975) and of the fifth quark, the *b*, two years later (Herb *et al.*, 1977).

All local quantum field theories with Lorentz invariance and Hermiticity respect CPT symmetry (Pauli, Rosenfeld, and Weisskopf, 1955; Lüders, 1957, 2000), in accordance with all experimental evidence (Beringer et al., 2012); hence there is a straightforward theoretical connection between CP and Tviolation (matter-antimatter asymmetry defines a preferred sense of time evolution). Since the SM is based on a quantum field theory satisfying the CPT theorem, it follows that the source of *CP* violation also requires *T*-violating effects. With the complex Hermitian Lagrangian, genuine CP phases change sign for particles and antiparticles, so its experimental detection requires an interference experiment to observe the relative CP phase between the interfering complex amplitudes. Therefore, given the known CP violation in weak interactions in processes involving K and B mesons, T should also be broken in these systems. The question is whether the expected T asymmetry can be detected by an experiment that, considered by itself, clearly shows a motion-reversal asymmetry independent of CP asymmetries and CPT invariance.

#### **D.** Experiments probing *T* violation

There are two main types of experiments or observables that can be used to directly detect time-reversal noninvariance (Sachs, 1987; Wolfenstein, 1999a; Henley, 2013).



FIG. 3 (color online). A particle with spin is represented as a sphere with a spinning charge distribution. Its parity and time-reversed images are also shown, together with the corresponding magnetic  $\mu$  and electric *d* dipole moments.

A nonzero expectation value of a T-odd operator for a nondegenerate stationary state is the case for an electric dipole moment (EDM) of a particle with spin, which is also a P-odd, C-even quantity, as depicted in Fig. 3. A parity transformation about the midplane of the sphere flips the EDM d with respect to the magnetic dipole moment  $\mu$  (spin), which remains unchanged, whereas a time-reversal transformation flips  $\mu$ with respect to d, which is in this case unaffected. Thus, if either parity or time reversal are good symmetries, the particle cannot have an EDM since one can distinguish the mirror picture from the original one. A nonzero EDM can be generated by either strong T violation unless it is annulled by a Peccei-Quinn symmetry leaving the axion as a emnant (Peccei and Quinn, 1977) or by T violation in weak interactions. In the SM with the CKM mechanism, a nonvanishing EDM of the neutron appears only at the three-loop level. Hence, these experiments probe for physics beyond the SM. To date, no signals for an EDM have been found, although there are strong limits, as for the neutron and the electron,  $|d_n| < 2.9 \times 10^{-26} e \text{ cm}$  and  $|d_e| < 1.05 \times$ 10<sup>-27</sup> e cm (Beringer et al., 2012).

Experiments involving T-odd observables for weak decays (as well as electromagnetic transitions), constructed typically using odd products of the momentum and spin vectors of the decay products (initial and final states), are sometimes used. These observables are not genuine signals for T violation since initial and final states are not interchanged, and in general require detailed understanding of FSI effects since they can mimic T violation even with time-reversal symmetric dynamics (Sachs, 1987; Wolfenstein, 1999a). In some cases, T-odd CP asymmetries can be built by comparing particle and antiparticle decays, without knowing the CP-even FSI phases; see, for example, Gronau and Rosner (2011), and references therein.

We might also consider transition processes. As discussed before and illustrated in Fig. 4, the antiunitary character of the *T* operator demands the exchange of initial and final states to compare the probabilities  $|\langle f|\mathcal{U}(t,t_0)|i\rangle|^2$  and  $|\langle U_Ti|\mathcal{U}(t,t_0)|U_Tf\rangle|^2$ , once the initial conditions, namely, *i* in one case and  $U_Tf$  in the other, have been precisely realized. With stable particles one can consider neutrino  $\nu_e$  to  $\nu_{\mu}$  mixing (Cabibbo, 1978), but this requires high-luminosity and longbaseline neutrino facilities. Alternatively, neutral *K* and *B* mesons, the unique systems for which *CP* violation has been



FIG. 4 (color online). The antiunitary property of the *T* operator demands the exchange of initial and final states to set up an experiment directly probing time-reversal symmetry in transitions. Motion-reversal symmetry implies equal probabilities for transitions  $i \rightarrow f$  and  $i_T \leftarrow f_T$ , once the initial conditions, namely, *i* in one case and  $f_T$  in the other, have been precisely realized. Here  $i_T$  and  $f_T$  are the *T*-transformed states of *i* and *f*, respectively, with all momenta and spins reversed.

detected, are the best choice for an experiment probing T noninvariance. Nevertheless, the thermodynamic irreversibility of these systems makes this option difficult, as discussed next.

#### E. Unstable systems

A direct consequence of quantum dynamics is the negativeexponential time behavior of the decay of any unstable system into two or more particles, as given by the Fermi golden rule. The reversal of the exponential decay law reveals that the Ttransformation is not defined for a decaying state, thus it appears that the decay prevents proofs of motion reversal. This apparent imbalance in time has to do with initial conditions rather than with the dynamics under time reversal; we assumed that the initial system is the unstable particle, but it should be formed by some process before. Hence, to address the question of time reversal we have to choose an initial time earlier so that the production enters into the process (Sachs, 1987).

For example, we might consider the decay of a neutral *B* meson into the final state  $K^+\pi^-$ , i.e.,  $B^0 \to K^+\pi^-$ , with rate  $R_1$ . *CP* violation is known to be large in this decay (Aubert *et al.*, 2004c; Chao *et al.*, 2004), thus we have  $\bar{B}^0 \to K^-\pi^+$  with rate  $R_2 \neq R_1$ , as observed in Fig. 5. In the SM, this decay occurs through two different amplitudes (penguin and tree), with different weak phases and, in general, different strong phases. This is a general requirement (Bernabéu and Jarlskog, 1981) to generate a nonvanishing interference for particle and antiparticle decays.<sup>1</sup> This leads to a difference in the decay rates for *CP* conjugate processes, which we refer to as direct *CP* violation. The *B* meson production could be done in electron-positron annihilation reactions through the process



FIG. 5 (color online). Energy distributions of  $B^0 \to K^+\pi^-$  (solid circles and solid curve) and  $\bar{B}^0 \to K^-\pi^+$  (open circles and dashed curve) decays. Points with error bars are data and the curves are best fit projections. The difference between the two distributions is a signature of direct *CP* violation. From Aubert *et al.*, 2004c.

 $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\bar{B}^0$ . Here only one of the produced neutral *B* mesons is analyzed for its decay into  $K^+\pi^-$  or  $K^-\pi^+$ , whereas the other is not studied and might decay into any final state, say  $\bar{B}^0 \rightarrow \bar{X}$  or  $B^0 \rightarrow X$ . By *CPT* invariance the time-reversed processes,  $K^+\pi^- \rightarrow B^0$  and  $K^-\pi^+ \rightarrow \bar{B}^0$ , have expected rates  $R_1$  and  $R_2$ , respectively. However, the experiment probing motion reversal should form the two *B* mesons and the  $\Upsilon(4S)$ , through the chain reactions  $(K^+\pi^- \rightarrow B^0)(\bar{X} \rightarrow \bar{B}^0) \rightarrow \Upsilon(4S)$  and  $(K^-\pi^+ \rightarrow \bar{B}^0)(X \rightarrow B^0) \rightarrow \Upsilon(4S)$ . This is clearly a problem of thermodynamic irreversibility. Besides, even if we could build such a setup, strong-interaction processes in the kaon-pion annihilation would completely swamp the weak interaction process responsible for the decay.

We might consider motion reversal in the mixing, often also referred to as oscillation, of the pseudoscalar neutral K, B, and D mesons. In this case one compares the probability of a flavor eigenstate, say,  $K^0$  transforming into a  $\bar{K}^0$ , and vice versa. Since the states  $K^0$  and  $\bar{K}^0$  are particle and antiparticle, the two transitions are connected by both T and CP transformations. Even if CPT symmetry is broken, there exists no difference between CP and T in this case. Thus the two symmetry transformations are experimentally identical and lead to the same asymmetry. This flavor mixing or Kabir asymmetry (Kabir, 1970) is independent of time since the two processes have identical time dependence and is induced by the interference between the dispersive  $M_{12}$  and absorptive  $\Gamma_{12}$ contributions to the mixing of  $K^0$  and  $\bar{K}^0$  states. Here M and  $\Gamma$ are the  $2 \times 2$  mass and decay Hermitian matrices of the effective Hamiltonian describing neutral meson mixing  $\mathcal{H}_{eff}$  =  $M - i\Gamma/2$  (Branco, Lavoura, and Silva, 1999; Bigi and Sanda, 2000), and the index 1 (2) refers to the  $B^0$  ( $\bar{B}^0$ ) state.

Evidence at the  $4\sigma$  level for this asymmetry was found by the CPLEAR experiment at CERN in 1998 by studying the reaction  $p\bar{p} \rightarrow K^{\pm}\pi^{\mp}K^0(\bar{K}^0)$  in  $p\bar{p}$  collisions (Angelopoulos *et al.*, 1998). The strangeness (strange and antistrange flavor content) of the  $K^0$  and  $\bar{K}^0$  mesons at production time was determined by the charge of the accompanying charged kaon. Since weak interactions do not conserve strangeness, the  $K^0$  and  $\bar{K}^0$  mesons may subsequently transform into each other. The strangeness of the neutral kaon at decay time is determined through the semileptonic decays  $K^0 \rightarrow e^+\pi^-\nu_e$ 

<sup>&</sup>lt;sup>1</sup>The proof is as follows: if  $A_1 = a_1 \exp[i(\delta_1 + \phi_1)]$  and  $A_2 = a_2 \exp[i(\delta_2 + \phi_2)]$  are the two possible amplitudes for the process  $i \to f$ , with  $a_1$  and  $a_2$  real numbers, and  $\bar{i}$  and  $\bar{f}$  are the *CP*-conjugate states of *i* and *f*, respectively, then  $|A(i \to f)|^2 - |A(\bar{i} \to \bar{f})|^2 = -4a_1a_2\sin(\Delta\delta)\sin(\Delta\phi)$  do not vanish only if both  $\Delta\delta = \delta_1 - \delta_2$  and  $\Delta\phi = \phi_1 - \phi_2$  are nonvanishing.



FIG. 6 (color online). The CPLEAR asymmetry vs the neutralkaon decay time (in units of the  $K_s^0$  lifetime). The solid line represents the average. From Angelopoulos *et al.*, 1998.

and  $\bar{K}^0 \rightarrow e^- \pi^+ \bar{\nu}_e$ . The asymmetry, shown in Fig. 6, is effectively independent of time and reveals a net offset with respect to zero.

The interpretation of the asymmetry relies on two main aspects. First, in the framework of the Weisskopf-Wigner approach (Weisskopf and Wigner, 1930a, 1930b), the effect comes from the overlap (nonorthogonality) of the "stationary"  $K_{S}^{0}$  and  $K_{L}^{0}$  physical states. Second, the decay plays an essential role; indeed, within the SM the dispersive and absorptive contributions to  $K^0$ - $\bar{K}^0$  mixing (Fig. 7) are at leading order proportional to the mass and decay width differences between the  $K^0$  and  $\bar{K}^0$  mass eigenstates (Branco, Lavoura, and Silva, 1999; Beringer et al., 2012), respectively. The presence of the decay as an initial state interaction, essential to construct a nonvanishing interference for this observable, was argued by Wolfenstein to claim that this asymmetry "is not as direct a test of time-reversal violation as one might like" (Wolfenstein, 1999a, 1999b). In the neutral B system, where the decay width difference between the  $B^0$  and  $\bar{B}^0$  mass eigenstates is negligible, the measurement of this asymmetry has, in fact,



FIG. 7. The  $K^0 \rightarrow \bar{K}^0$  vs  $\bar{K}^0 \rightarrow K^0$  mixing asymmetry arises from the interference between the (a) dispersive and (b) absorptive contributions to  $K^0-\bar{K}^0$  mixing. (a) involves (c) short-distance box diagrams as well as (d) long-range interactions in which the intermediate states are off-shell mesons. The box diagram is matched by another where the quark triplet and the *W* bosons are interchanged. (b) is referred to as an "initial state interaction" since it involves decay to (d) intermediate on-shell states, and it is proportional to the decay width difference between the  $K^0$  and  $\bar{K}^0$ mass eigenstates.

brought negative results (Aubert *et al.*, 2006; Nakano *et al.*, 2006; Abazov *et al.*, 2012; Lees *et al.*, 2013). Others, however, have argued that its interpretation as a genuine signal for *T* violation is not affected by these arguments (Alvarez-Gaumé *et al.*, 1999; Ellis and Mavromatos, 1999; Gerber, 2004).

# III. B FACTORIES AND CP VIOLATION

At the asymmetric *B* factories, electron and positron beams collide with high luminosity at a center-of-mass (c.m.) energy of 10.58 GeV corresponding to the mass of the  $\Upsilon(4S)$ resonance, a vector particle with  $J^{PC} = 1^{--}$ . The  $\Upsilon(4S)$  is a bound state of a b and a  $\overline{b}$  quark that decays exclusively to a pair of B and  $\overline{B}$  mesons. Since the mass of the  $\Upsilon(4S)$  is only slightly higher than twice the mass of the B meson, the two B mesons have low momenta (about 330 MeV/c) and are produced almost at rest in the  $\Upsilon(4S)$  reference frame with no additional particles besides those associated with the Bdecays. The energy of the electron beam is adjusted to be between 2 and 3 times larger than that of the positrons, so that the c.m. frame has a Lorentz boost along the collision axis. Two B factory colliders, PEP-II at SLAC in California and KEKB at KEK in Japan, with their corresponding detectors, BABAR (Aubert et al., 2002, 2013) and Belle (Abashian et al., 2002), have been operating during the last decade, accumulating an integrated luminosity of data exceeding 500 fb<sup>-1</sup> and  $1 \text{ ab}^{-1}$ , respectively.

The electron- and positron-beam energies were designed to be 9.0 and 3.1 GeV at PEP-II, and 8.0 and 3.5 GeV at KEKB; thus the Lorentz boosts  $\beta\gamma$  are about 0.56 and 0.42, respectively. This allows a mean separation of the two *B* mesons along the collision axis in the laboratory frame of about 250 and 200  $\mu$ m. With a typical detector resolution of 100  $\mu$ m, it is possible to experimentally determine the distance  $\Delta z$  between the decay points of the two *B*s to obtain the decay time difference  $\Delta t \approx \Delta z / \beta \gamma c$ . This translates into a mean time separation of about 1.5 ps with a resolution ranging typically between 0.6 and 0.8 ps.

## A. Entangled neutral B mesons

The decay of the  $\Upsilon(4S)$  particle occurs through strong interactions, thus the system of the created pair of *B* and  $\bar{B}$  mesons inherits the  $\Upsilon(4S)$  quantum numbers. About 50% of the  $B\bar{B}$  pairs are  $B^0\bar{B}^0$ , when the hadronization process picks up a  $d\bar{d}$  quark pair, and 50%  $B^+B^-$  when it is a  $u\bar{u}$  pair.

Because *B* and  $\overline{B}$  are two pseudoscalar states of a unique (complex) field, Bose statistics and angular momentum conservation requires that the wave function of the  $B\overline{B}$  pair be in a *P*-wave, antisymmetric *CP*-even state (Lipkin, 1989),

$$|\Upsilon\rangle = \frac{1}{\sqrt{2}} [|B^{0}(t_{1})\rangle|\bar{B}^{0}(t_{2})\rangle - |\bar{B}^{0}(t_{1})\rangle|B^{0}(t_{2})\rangle].$$
(8)

The times  $t_1$  and  $t_2$  in Eq. (8) do not refer to time dependence but labels to characterize the states: state 1 (2) is labeled as the first (second) to decay, i.e.,  $t_1 < t_2$ . The antisymmetric entanglement is essential: even with  $B^0 - \bar{B}^0$  mixing between the production time at t = 0 and the first decay at  $t = t_1$ , the state  $|\Upsilon\rangle$  remains antisymmetric, with no trace of combinations  $|B^0\rangle|B^0\rangle$  or  $|\bar{B}^0\rangle|\bar{B}^0\rangle$ , and only  $|B^0\rangle|\bar{B}^0\rangle$  and  $|\bar{B}^0\rangle|B^0\rangle$ 



FIG. 8. A  $\overline{B}^0$  meson, containing a *b* quark, decays dominantly with lifetime ~1.5 ps through a  $b \rightarrow c$  transition. The virtual  $W^-$  gauge boson creates a negatively charged prompt lepton whose electric charge can be correlated with the *b* quark flavor. The charge of kaons produced through the  $b \rightarrow c \rightarrow s$  cascade also identifies the flavor of the *B* meson.

states appearing at any time. The state of the first *B* to decay at  $t_1$  dictates the state of the other *B*, without measuring (thus destroying) it, and then evolves in time and decays at  $t_2$ . The antisymmetric wave function defined by this EPR entanglement (Einstein, Podolsky, and Rosen, 1935; Reid *et al.*, 2009) is usually written in terms of the strong-interaction flavor eigenstates  $B^0$  and  $\bar{B}^0$ , as given by Eq. (8). Note that the basis choice of  $B^0$  and  $\bar{B}^0$  states is only a matter of convention. The individual state of each neutral *B* is not defined in the entangled state  $|\Upsilon\rangle$  before the first decay. The quantum collapse of the state at decay time  $t_1$  to a flavor-specific channel tags the second *B* as the orthogonal state.

Now consider that the first B decays semileptonically producing a negatively charged prompt lepton. As shown in Fig. 8, this decay proceeds through a  $b \rightarrow c$  transition, and the charge of the lepton is completely correlated with the flavor of the *b* quark, hence the parent *B* is a  $\overline{B}^0$  at the instant of its decay. The anticorrelation defined by the wave function determines that the other B at that time is a  $B^0$ , thereby it prepares (or tags) a  $B^0$  flavor state. In quantum mechanics language this means that the initial state of the Bmeson has been prepared or filtered as  $B^0$ . Neutral B mesons start to oscillate just after their production since their mixing rate  $\Delta m_d \approx 0.5 \text{ ps}^{-1}$  is comparable to their decay width  $\Gamma_d \approx 0.7 \text{ ps}^{-1}$ . Thus, the state prepared as  $B^0$  is a superposition of  $B^0$  and  $\overline{B}^0$  at a later time. As discussed further in Secs. III.B and IV, this quantum-mechanical superposition of states is of key importance for the experimental exploration of CP and, especially, time-reversal symmetries. Analogously, with a positively charged prompt lepton we have a  $\bar{B}^0$  tag. There are other signatures that can be used, like prompt charged kaons produced through a  $b \rightarrow c \rightarrow s$  cascade, as sketched in Fig. 8. In the following, we generically denote as  $\ell^{-}\bar{X}$  or  $\ell^{+}X$  any flavor-specific final state that can be used to identify the flavor of the decaying B. Note that for this procedure to work only right-sign decays should take place, i.e., the wrong-sign decays  $B^0 \to \ell^- \bar{X}$  and  $\bar{B}^0 \to \ell^+ X$  do not occur.

#### B. CP violation at B factories

Flavor tagging based on quantum entanglement has been the basis for a decade of CP violation physics at B factories. In these studies, the second B meson, the one that is not used for flavor tagging, is reconstructed into a CP-eigenstate final state, for example,  $J/\psi K_S^0$ , which has *CP*-odd parity, except for 0.1% due to *CP* violation in  $K^0$ - $\bar{K}^0$  mixing (Beringer *et al.*, 2012). The decay of the *B* meson into the flavor-eigenstate final state can occur after the decay into the *CP* eigenstate; to account for this situation the experiments define as convention a signed decay time difference,

$$\Delta t = t_{CP} - t_{\text{flavor}}.$$
(9)

*CP* violation means any difference between the decay rate  $B^0 \rightarrow J/\psi K_S^0$  and its *CP* conjugate  $\bar{B}^0 \rightarrow J/\psi K_S^0$ . The search for this asymmetry has been the "raison d'être" of the *B* factories. Experimentally everything is identical for the two processes, except the requirement of a  $\ell^- \bar{X}$  or a  $\ell^+ X$  decay of the *B* meson used for tagging. The large *CP* violation observed by the *B* factory experiments in 2001 (Abe *et al.*, 2001; Aubert *et al.*, 2001) follows from the comparison of the  $B^0$  and  $\bar{B}^0$  decay rates at an equal decay proper-time difference, as shown in Fig. 9(a). The difference between these two distributions signals *CP* violation. This is more evident in the time-dependent *CP* asymmetry, shown in Fig. 9(b), where the difference between the two distributions is normalized to their sum,

$$A_{CP,f} = \frac{\Gamma_{\bar{B}^0 \to f}(\Delta t) - \Gamma_{B^0 \to f}(\Delta t)}{\Gamma_{\bar{B}^0 \to f}(\Delta t) + \Gamma_{B^0 \to f}(\Delta t)}.$$
 (10)



FIG. 9 (color online). (a), (c) Flavor-tagged  $\Delta t$  distributions and (b), (d) *CP* asymmetries from the *BABAR* experiment, for (a), (b) *CP*-odd final states  $J/\psi K_S^0$ ,  $\psi(2S)K_S^0$ ,  $\chi_{c1}K_S^0$ , and  $\eta_c K_S^0$ , and the (c), (d) *CP*-even final states  $J/\psi K_L^0$ . The solid (dashed) curves in (a) and (c) represent the best fit projections in  $\Delta t$  for  $\bar{B}^0$  ( $B^0$ ) tags. The shaded regions represent the estimated background contributions to (a) and (c). The curves in (b) and (d) are the fit projections of the *CP* asymmetry between  $\bar{B}^0$  and  $B^0$  tagged events. From Aubert *et al.*, 2009.

It should be noted that the notion of a flavor tag adopted here as preparation of the initial state of the second B, using the measurement of the decay of the first B, is different from the one commonly used by the B factory experiments, where tagging refers to the flavor identification of the first B. The reason to adopt this convention, closer to the standard quantum mechanics language, will become more clear later.

One can also use final states with different *CP* parity, for example, the *CP*-even  $f = J/\psi K_L^0$ . Because of its opposite *CP* eigenvalue, the behaviors of the time distributions for  $B^0$ and  $\bar{B}^0$  are interchanged and the asymmetry is opposite, as observed in Figs. 9(c) and 9(d). Besides increasing the statistical power of the result, the use of final states with opposite *CP* parity provides a powerful cross-check of the *CP*-violating effect.

This *CP* asymmetry is induced by the interference of amplitudes involved in the two possible paths to reach the same final state f from a  $B^0$  (or its *CP* conjugate), either through decay without mixing  $B^0 \rightarrow f$  or through mixing followed by decay  $B^0 \rightarrow \overline{B}^0 \rightarrow f$ . Therefore, we usually refer to *CP* violation in the interference between decay amplitudes with and without mixing or simply indirect *CP* violation. This type of *CP* symmetry breaking can be compared to direct *CP* violation (due to the interference of different decay amplitudes) or *CP* violation in mixing (arising from the interference of the dispersive and absorptive contributions to mixing) (Branco, Lavoura, and Silva, 1999).

The time-dependent decay rates and the *CP* asymmetry can be parametrized in a model-independent way, assuming only quantum mechanics, as

$$\Gamma_{\bar{B}^0(B^0)\to f} \propto e^{-\Gamma_d |\Delta t|} \{1 + (-)[S_f \sin(\Delta m_d \Delta t) - C_f \cos(\Delta m_d \Delta t)]\},$$
(11)

$$A_{CP,f} = S_f \sin(\Delta m_d \Delta t) - C_f \cos(\Delta m_d \Delta t), \qquad (12)$$

where  $\Delta m_d$  is the mass difference between the physical states of the neutral *B* meson system, and  $\Gamma_d$  is the average total decay width. Equation (11) assumes a negligible difference between the decay rates of the mass eigenstates, i.e.,  $\Delta \Gamma_d = 0$ , and *CP* symmetry in mixing (independently of whether *CPT* and *T* are or are not violated). If *CP* symmetry in mixing holds, then the conditions |q/p| = 1 and z = 0 apply, where q, p, and z are three complex parameters introduced to define the eigenstates of well-defined mass and decay width (referred to as mass eigenstates) in terms of the strong-interaction (flavor) eigenstates (Branco, Lavoura, and Silva, 1999; Beringer *et al.*, 2012). Adopting an arbitrary sign convention,

$$|B_L\rangle \propto p\sqrt{1-z}|B^0\rangle - q\sqrt{1+z}|\bar{B}^0\rangle,$$
  

$$|B_H\rangle \propto p\sqrt{1+z}|K^0\rangle + q\sqrt{1-z}|\bar{B}^0\rangle.$$
(13)

Within the Weisskopf-Wigner approach, these parameters are related to the matrix elements of the 2 × 2 effective, non-Hermitian Hamiltonian  $\mathcal{H}_{eff}$  describing mixing. Note that the real and imaginary parts of the corresponding eigenvalues are the masses and decay widths, and their splittings are  $\Delta m_d$  and  $\Delta \Gamma_d$ , respectively. If either *CP* or *CPT* is a good symmetry in

mixing (independent of *T*), then z = 0, and if either *CP* or *T* is a symmetry of  $\mathcal{H}_{eff}$  (independent of *CPT*), |q/p| = 1. It then follows that *CP* with *CPT* violation in mixing is defined by  $z \neq 0$ , and *CP* with *T* violation is through  $|q/p| \neq 1$ . The curves shown in Fig. 9 represent the best fit projections in  $\Delta t$ using Eqs. (11) and (12).

The coefficients  $S_f$  and  $C_f$  are related to *CP* violation. Within the Weisskopf-Wigner approach, these are connected with the fundamental parameter describing indirect *CP* violation (Bigi and Sanda, 2000),

$$\lambda_f = \frac{q}{p} \frac{A_f}{A_f},\tag{14}$$

through

$$S_f = 2 \mathrm{Im} \lambda_f / (1 + |\lambda_f|^2),$$
  

$$C_f = (1 - |\lambda_f|^2) / (1 + |\lambda_f|^2),$$
(15)

where  $A_f = \langle f | \mathcal{D} | B^0 \rangle$  and  $\bar{A}_f = \langle f | \mathcal{D} | \bar{B}^0 \rangle$  are the  $B^0$  and  $\bar{B}^0$  decay amplitudes to the final state f, with  $\mathcal{D}$  the operator describing the B decay. For  $f = J/\psi K_S^0$  and  $f = J/\psi K_L^0$ , Eq. (14) becomes

$$\lambda_f = \eta_f \frac{q}{p} \frac{\bar{A}}{A} \frac{p_K}{q_K},\tag{16}$$

where  $A = \langle J/\psi K^0 | \mathcal{D} | B^0 \rangle$ ,  $\bar{A} = \langle J/\psi \bar{K}^0 | \mathcal{D} | \bar{B}^0 \rangle$ , and  $\eta_f = -1(+1)$  for  $f = J/\psi K_S^0 (J/\psi K_L^0)$  is associated with the *CP* parity of the final state. The factor  $p_K/q_K$  arises from  $K^0 \cdot \bar{K}^0$  mixing, essential for the interference because  $B^0$  and  $\bar{B}^0$  decay into  $J/\psi K^0$  and  $J/\psi \bar{K}^0$ , but not into  $J/\psi \bar{K}^0$  and  $J/\psi K^0$ , respectively.

Assuming that the amplitude  $A_f$  can be described by a single weak phase, the two contributions to CP violation in the parameter  $\lambda_f$ , CP with T violation and CP with CPT violation, can be identified easily by separating it into modulus and phase  $\lambda_f = |\lambda_f| \exp(i\phi_f)$  (Schubert *et al.*, 2014). CPT invariance in the decay requires  $|\bar{A}_f/A_f| = 1$ (Lee, Oehme, and Yang, 1957). For |q/p| = 1, experimentally well verified (Aubert et al., 2006; Nakano et al., 2006; Abazov et al., 2012; Lees et al., 2013), it follows that  $|\lambda_f| = 1$ . T invariance in the time evolution followed by decay requires  $\phi_f = 0$  or  $\pi$ , i.e.,  $\text{Im}\lambda_f = 0$  (Enz and Lewis, 1965). Instead, if  $A_f$  is the sum of two (or more) amplitudes with nonvanishing weak and strong phase differences between the two amplitudes, then we have  $|\bar{A}_f/A_f| \neq 1$ , even if  $\mathcal{D}$  is *CPT* symmetric. Thus, if  $|\bar{A}_f/A_f| = 1$  then we have either CPT symmetry in decay and a single amplitude, or an unlikely cancellation of T and CPT violation in decay amplitudes.

In the SM,  $B^0-\bar{B}^0$  mixing is dominated by the box diagrams shown in Fig. 10(a), leading to  $q/p \approx V_{tb}^* V_{td}/V_{tb} V_{td}^*$ (Branco, Lavoura, and Silva, 1999; Bigi and Sanda, 2000). For the final state  $f = J/\psi K_S^0$ , the *B* decay is dominated by the  $b \rightarrow c\bar{c}s$  tree amplitude in Fig. 10(b), followed by  $K^0-\bar{K}^0$ mixing (Fig. 7). Therefore, from Eq. (16) it follows



FIG. 10. (a) Box diagram corresponding to the SM shortdistance contributions to  $B^0 - \overline{B}^0$  mixing. This contribution is matched by a diagram where the quark triplet and the *W* bosons are interchanged. (b) Tree and (c) penguin SM diagrams for the  $B^0 \rightarrow c \overline{c} K^0$  decay. In the Wolfenstein parametrization the tree amplitude is  $\mathcal{O}(\lambda_c)$ , with  $\lambda_c \approx 0.226$ , whereas the penguin contribution is  $\mathcal{O}(\lambda_c^2)$  and has the same weak phase.

$$\lambda_f = \eta_f \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}.$$
 (17)

This leads to  $C_f = 0$  and  $S_f = \text{Im}\lambda_f = -\eta_f \sin 2\beta$ , where  $\beta \equiv$  $\arg[-(V_{cd}V_{cb}^*)/(V_{td}V_{tb}^*)]$  (Beringer *et al.*, 2012) is the angle between the t and c sides of the bd unitarity triangle, illustrated in Fig. 2. In the Wolfenstein parametrization,  $\beta \equiv -\arg V_{td}$ . Thus, the same magnitude is expected for the CP-even and CP-odd modes up to a small correction due to *CP* violation in  $K^0$ - $\bar{K}^0$  oscillations. Higher-order (penguin) amplitude contributions have either the same weak phase [Fig. 7(c)] or are CKM suppressed, so that these predictions apply within the SM up to  $\mathcal{O}(\lambda_c^4)$ , where  $\lambda_c \approx 0.226$  in the Wolfenstein parametrization (Wolfenstein, 1983; Beringer et al., 2012). The deviation due to the penguin contributions with different CKM phase has been estimated to be smaller than 1% (Boos, Mannel, and Reuter, 2004; Li and Mishima, 2007). Physics beyond the SM could modify the phase of q/p(thus  $S_f$ ) and  $C_f$ , although large effects are unlikely to be generated in the latter due to the dominance of the tree amplitude in decay (Chiang et al., 2010).

### **IV. TIME-REVERSAL EXPERIMENT CONCEPT**

We might be tempted to interpret these *CP* violation results as evidence for time-reversal noninvariance. The experimental study has been performed invoking  $\Delta \Gamma_d = 0$  and *CP* invariance in mixing (|q/p| = 1 and z = 0), although good agreement between the data points and curves observed in Fig. 9 hints that the effects of possible deviations from these assumptions are well below the statistical sensitivity of the current data. Indeed, dedicated studies have shown good agreement of the results with and without making these assumptions (Aubert *et al.*, 2004a, 2004b). Moreover, the results are consistent with  $C_f = 0$  and  $S_f \neq 0$ , and therefore, according to Eq. (14), with  $|\bar{A}_f/A_f| = 1$  for |q/p| = 1 and  $\mathrm{Im}\lambda_f \neq 0$ . Within the Weisskopf-Wigner approach these values are compatible with *CP* with *CPT* symmetry in decay, and *CP* with *T* violation in the interference of decay with and without mixing (Fidecaro, Gerber, and Ruf, 2013; Schubert *et al.*, 2014). This is not, however, the question of interest here, but to set up an experiment capable of demonstrating by itself motion-reversal noninvariance between states that are not *CP* conjugate to each other, as discussed previously.

# A. Entangled neutral B mesons revisited

The solution (Bañuls and Bernabéu, 1999a, 2000; Bernabéu, Martínez-Vidal, and Villanueva-Pérez, 2012; Wolfenstein, 1999a; Quinn, 2009) arises from the quantummechanical properties imposed by the EPR entanglement (Einstein, Podolsky, and Rosen, 1935; Reid *et al.*, 2009) between the two neutral *B* mesons produced in the  $\Upsilon(4S)$ resonance decay. Just as one *B* meson in the entangled pair is prepared in the  $\bar{B}^0$  or  $B^0$  state at the time when the other *B* is observed as a  $B^0$  or  $\bar{B}^0$  by a decay into  $\ell^+ X$  or  $\ell^- \bar{X}$ , respectively, the first decay of one *B* into the final states  $J/\psi K_S^0$  or  $J/\psi K_L^0$  prepares the other *B* into well-defined, orthogonal linear combinations of  $B^0$  and  $\bar{B}^0$  states. In fact, this idea offers the opportunity to separately explore time reversal, *CP*, and *CPT* symmetries, appropriately selecting different transitions defined by different decay channels.

For the entangled state of the two mesons produced by the  $\Upsilon(4S)$  decay, the individual state of each neutral *B* meson is not defined before its collapse as a filter imposed by the observation of the decay. Thus, the state  $|\Upsilon\rangle$  in Eq. (8) can be written in terms of any pair of orthogonal states of the individual *B* mesons, i.e., a linear combination of  $B^0$  and  $\bar{B}^0$ , that we denote  $B_+$ , and its orthogonal state,  $B_-$ ,

$$|\Upsilon\rangle = \frac{1}{\sqrt{2}} [|B_{+}(t_{1})\rangle|B_{-}(t_{2})\rangle - |B_{-}(t_{1})\rangle|B_{+}(t_{2})\rangle].$$
(18)

As in Eq. (8), the time evolution (including mixing) preserves only  $|B_+\rangle|B_-\rangle$  and  $|B_-\rangle|B_+\rangle$  terms.

Neglecting *CP* violation in  $K^{0}-\bar{K}^{0}$  mixing, which holds within  $\mathcal{O}(10^{-3})$  (Beringer *et al.*, 2012), and assuming that  $B^{0} \rightarrow J/\psi K^{0}$  and  $\bar{B}^{0} \rightarrow J/\psi \bar{K}^{0}$ , the normalized states

$$|B_{+}\rangle = \mathcal{N}\left(|B^{0}\rangle + \frac{A}{\bar{A}}|\bar{B}^{0}\rangle\right),$$

$$|B_{-}\rangle = \mathcal{N}\left(|B^{0}\rangle - \frac{A}{\bar{A}}|\bar{B}^{0}\rangle\right),$$
(19)

where  $\mathcal{N} = |\bar{A}|/\sqrt{|A|^2 + |\bar{A}|^2}$ , have the property that the former decays into  $J/\psi K_L^0$ , but not into  $J/\psi K_S^0$ , and the latter into  $J/\psi K_S^0$ , but not into  $J/\psi K_L^0$  (Lipkin, 1989; Bernabéu, Martínez-Vidal, and Villanueva-Pérez, 2012).

The proof is as follows. Adopting the same sign convention as in Eq. (13),<sup>2</sup> and assuming *CPT* invariance in kaon mixing, we have

$$|K_{S}^{0}\rangle = \mathcal{N}_{K}(p_{K}|K^{0}\rangle - q_{K}|\bar{K}^{0}\rangle),$$
  
$$|K_{L}^{0}\rangle = \mathcal{N}_{K}(p_{K}|K^{0}\rangle + q_{K}|\bar{K}^{0}\rangle),$$
  
(20)

<sup>&</sup>lt;sup>2</sup>The  $K_L^0$  state is experimentally found to be the heavier state.

with  $\mathcal{N}_K = 1/\sqrt{|p_K|^2 + |q_K|^2}$ . In the absence of wrongstrangeness *B* decays,  $\langle J/\psi \bar{K}^0 | \mathcal{D} | B^0 \rangle = \langle J/\psi K^0 | \mathcal{D} | \bar{B}^0 \rangle = 0$ , it is straightforward to show that

$$\langle J/\psi K_S^0(K_L^0) | \mathcal{D} | B_+ \rangle = \mathcal{N} \mathcal{N}_K[p_K A - (+)q_K A],$$

$$\langle J/\psi K_L^0(K_S^0) | \mathcal{D} | B_- \rangle = \mathcal{N} \mathcal{N}_K[p_K A - (+)q_K A].$$

$$(21)$$

Assuming now *CP* invariance in  $K^0-\bar{K}^0$  mixing, we have  $p_K = q_K$  and the above expressions yield

$$\langle J/\psi K_L^0 | \mathcal{D} | B_+ \rangle = \langle J/\psi K_S^0 | \mathcal{D} | B_- \rangle = 2p_K \mathcal{N} \mathcal{N}_K A,$$
  
 
$$\langle J/\psi K_S^0 | \mathcal{D} | B_+ \rangle = \langle J/\psi K_L^0 | \mathcal{D} | B_- \rangle = 0,$$
 (22)

hence  $B_+$  cannot decay into  $J/\psi K_S^0$  and  $B_-$  cannot decay into  $J/\psi K_L^0$ . Note that using Eq. (14) and inverting Eq. (20) it is straightforward to obtain Eq. (16) and the relation  $\lambda_{J/\psi K_L^0} = -\lambda_{J/\psi K_S^0}$ , as introduced in Sec. III.B. The fact that the two amplitude ratios differ only by a minus sign is of key importance for the definition of the  $B_{\pm}$  states. According to the definition in Eq. (19), these states are well behaved under  $B^0$  and  $\bar{B}^0$  rephasing, although they are not *CP* eigenstates independent of the flavor of the decay channel due to arbitrary quark phases and possible deviations from  $|A/\bar{A}| = 1$ .

The observation of the decay to the  $J/\psi K_S^0 (J/\psi K_L^0)$  final state at time  $t_1$  generates an automatic transfer of information to the (still living) partner meson. Hence, the other *B* meson at that time is in a  $B_+$  ( $B_-$ ) state because this is the state that cannot decay into  $J/\psi K_S^0 (J/\psi K_L^0)$ . We call the quantum preparation of the initial state at  $t_1$ , using the filter imposed by the observation at  $t_1$ , a " $B_+$  ( $B_-$ ) tag." Sometimes it is referred to as a "*CP* tag" (Bañuls and Bernabéu, 1999b) because it is defined through decays into *CP*-eigenstate final states. The  $B_+$  or  $B_-$  states prepared by entanglement at  $t_1$  evolve in time until they are observed at some later time  $t_2$  in a decaying final state filtering another linear combination state. For convenience, we restrict to flavor-specific final states  $\ell^-\bar{X}$  and  $\ell^+X$ that can be used to filter, assuming no wrong-sign *B* decays, the state of the *B* meson at  $t_2$  as  $\bar{B}^0$  and  $B^0$ , respectively.

Initial states  $B^0$  and  $\bar{B}^0$  can be prepared similarly, as already discussed in Sec. III.A. The observation of the decay to  $\ell^-\bar{X}$  $(\ell^+X)$  at time  $t_1$  dictates that the other B meson at that time is in a  $B^0$  ( $\bar{B}^0$ ) state because this state cannot decay into  $\ell^-\bar{X}$  $(\ell^+X)$ . As before, this requires assuming the absence of wrong-sign B decays, i.e.,  $B^0 \to \ell^-\bar{X}$  and  $\bar{B}^0 \to \ell^+X$  do not occur. We refer to the quantum preparation of this initial state at  $t_1$  as the  $B^0$  ( $\bar{B}^0$ ) tag. The  $B^0$  or  $\bar{B}^0$  states prepared by entanglement at  $t_1$  evolve in time until they are observed at some later time  $t_2$  in a decaying final state filtering a given  $B^0$ and  $\bar{B}^0$  linear combination state, which we select as  $B_+$  or  $B_$ by observing the appropriate decay channels.

By virtue of the EPR correlation of Eq. (18),  $B_+$  and  $B_$ have been defined as states orthogonal to the states, denoted as  $B_-^{\perp}$  and  $B_+^{\perp}$ , defined through the filter imposed by the observation at  $t_1$  of the decay into  $J/\psi K_S^0$  and  $J/\psi K_L^0$ , respectively, i.e.,  $\langle B_+ | B_-^{\perp} \rangle = \langle B_- | B_+^{\perp} \rangle = 0$ . The exchange of initial and final states with identical boundary conditions



FIG. 11 (color online). Basic concept of the time-reversal experiment. Electron-positron collisions at the asymmetric *B* factory produce  $\Upsilon(4S)$  resonances, which decay via strong interaction to an entangled pair of *B* mesons. When one *B* meson decays at  $t_1$ , the identity of the other is "tagged" without measuring it specifically. In the top panel, the *B* meson observed to decay to the final state  $\ell^+ X$  at  $t_1$  transfers information to the other meson and dictates that it is in a  $\bar{B}^0$  state. This surviving meson tagged as  $\bar{B}^0$  is observed at  $t_2$  to decay into a final state  $J/\psi K_L^0$  that filters the *B* meson to be in a  $B_+$  state, a linear combination of  $B^0$  and  $\bar{B}^0$  states. This case corresponds to a transition  $\bar{B}^0 \to B_+$ . To study time reversal we have to compare the rate at which this transition occurs to the rate of the time-reversed transition  $B_+ \to \bar{B}^0$  (bottom panel). Adapted from Zeller, 2012.

required by motion reversal imposes that the basis of tagging states  $(B_+, B_-)$  must be identical to the basis of states filtered by decay  $(B_+^{\perp}, B_-^{\perp})$ . This condition is met when  $B_+$  and  $B_-$ (and thus  $B_+^{\perp}$  and  $B_-^{\perp}$ ) are orthogonal to each other. It is then straightforward to prove that  $\langle B_-|B_+\rangle = 0$  if  $|A/\bar{A}| = 1$ . For this to hold, the decay amplitude *A* should have only one weak phase, as it is expected to apply at the 1% level or better for  $B^0 \rightarrow J/\psi K^0$  and  $\bar{B}^0 \rightarrow J/\psi \bar{K}^0$ , and have *CPT* symmetry in the decay amplitude (see Sec. III.B).

To clarify the foundations of the time-reversal experiment, consider the case illustrated in Fig. 11. In the top panel, the first B decays into a  $\ell^+ X$  final state (for example, a semileptonic final state producing a positively charged prompt lepton or an hadronic final state with a positively charged kaon). This final state filters the quantum-mechanical state of the other B at time  $t_1$  as  $\bar{B}^0$ . The surviving meson tagged as  $\bar{B}^0$ is observed later at  $t_2$  to decay into a final state  $J/\psi K_L^0$  that filters the B meson to be in a  $B_+$  state. Therefore, the event class  $(\ell^+ X, J/\psi K_L^0)$ , encapsulating a time ordering, undergoes a transition  $\overline{B}^0 \to B_+$  in the elapsed time  $t = t_2 - t_1$ . This transition has, according to the convention in Eq. (9), a decay time difference  $\Delta t = t > 0$ . As shown in the bottom panel, T transformation demands to set up the conditions for the mirrored transition  $B_+ \rightarrow \bar{B}^0$ . The  $B_+$  tag requires the first B to decay into the  $J/\psi K_S^0$  final state, whereas the surviving meson tagged at  $t_1$  as  $B_+$  has to be observed at  $t_2$  through its decay into a  $\ell^{-}\bar{X}$  flavor-specific final state. The time-ordered event class is now  $(J/\psi K_s^0, \ell^- \bar{X})$ , which according to Eq. (9) yields a decay time difference  $\Delta t < 0$ . Thus and from now on,

 $\Delta t > 0$  ( $\Delta t < 0$ ) refers to the time ordering in an event class with a flavor- (*CP*-) eigenstate decay channel appearing first. Note that this nontrivial *T* transformation is not defined by the " $\Delta t$  reversal" obtained just by flipping the time ordering of the decay channels for a given event class. For this to be achieved, EPR entanglement together with the availability of both flavor- and *CP*-eigenstate decay modes to filter appropriate neutral *B* states, has been key for the quantum preparation of the *B*-meson initial state in the transition and its motionreversed version; the problem of particle instability has been thus avoided.

#### B. Transitions among B-meson states

For the four initial states  $B^0$ ,  $\overline{B}^0$ ,  $B_+$ , and  $B_-$  prepared by entanglement, and the four final states  $B^0$ ,  $\overline{B}^0$ ,  $B_+$ , and  $B_$ available through decay, it is possible to construct eight transitions and their corresponding time-ordered event classes  $(f_1, f_2)$  given in Table I (Bernabéu, Martínez-Vidal, and Villanueva-Pérez, 2012). In this notation, the final state  $f_1$  is observed at time  $t_1$  and the final state  $f_2$  is observed at time  $t_2 = t_1 + t$ , where t > 0 is the elapsed time. The matching between event classes and transitions applies under the assumption of the absence of wrong-strangeness and wrong-sign *B* decays, *CP* invariance in  $K^0-\overline{K}^0$  mixing, and  $|\overline{A}/A| = 1$ . Recently an extended discussion, including wrong-strangeness and wrong-sign *B* decays, was presented (Applebaum *et al.*, 2014).

The observation of an asymmetry between the probabilities for transitions on the right- and left-hand sides of Table I,

$$\begin{split} |\langle \bar{B}^{0} | \mathcal{U}(t) | B_{-} \rangle|^{2} - |\langle B_{-} | \mathcal{U}(t) | \bar{B}^{0} \rangle|^{2}, \\ |\langle B_{-} | \mathcal{U}(t) | B^{0} \rangle|^{2} - |\langle B^{0} | \mathcal{U}(t) | B_{-} \rangle|^{2}, \\ |\langle \bar{B}^{0} | \mathcal{U}(t) | B_{+} \rangle|^{2} - |\langle B_{+} | \mathcal{U}(t) | \bar{B}^{0} \rangle|^{2}, \\ |\langle B_{+} | \mathcal{U}(t) | B^{0} \rangle|^{2} - |\langle B^{0} | \mathcal{U}(t) | B_{+} \rangle|^{2}, \end{split}$$
(23)

will be an unambiguous demonstration of motion reversal in time evolution of states that are not *CP* conjugate to each other. Here  $\mathcal{U}(t)$  is the time-evolution operator determined by the effective Hamiltonian  $\mathcal{H}_{eff}$ . Note that the notation has been changed in comparison to Sec. II.A since *t* now denotes the elapsed time. We immediately realize that each of the four

TABLE I. Time-ordered event classes  $(f_1, f_2)$  and their corresponding transitions between *B*-meson states. The matching between event classes and transitions applies under the assumption of the absence of wrong-strangeness and wrong-sign *B* decays, a single weak amplitude, i.e.,  $|\bar{A}/A| = 1$ , and no *CP* violation in  $K^0-\bar{K}^0$  mixing. The event classes and transitions of the right-hand side (Time reversed) are the time-reversed versions of those on the left-hand side (Reference).

Reference		Time reversed		
Event class	Transition	Event class	Transition	
$(\ell^+ X, J/\psi K_L^0)$	$\bar{B}^0 \rightarrow B_+$	$(J/\psi K^0_S, \ell^- \bar{X})$	$B_+ \rightarrow \bar{B}^0$	
$(J/\psi K^0_S, \ell^+ X)$	$B_+ \rightarrow B^0$	$(\ell^- \bar{X}, J/\psi K_L^0)$	$B^0 \rightarrow B_+$	
$(\ell^+ X, J/\psi K_S^0)$	$\bar{B}^0 \rightarrow B$	$(J/\psi K_L^0, \ell^- \bar{X})$	$B \to \bar{B}^0$	
$(J/\psi K_L^0, \ell^+ X)$	$B \to B^0$	$(\ell^- \bar{X}, J/\psi K_S^0)$	$B^0 \rightarrow B$	



FIG. 12 (color online). Expected time-dependent probability distributions for the eight time-ordered event classes given in Table I, shown as a planar map. They are identified by the flavor-specific ( $\ell^-\bar{X}$ ,  $\ell^+X$ ) and *CP*-eigenstate ( $J/\psi K_S^0$ ,  $J/\psi K_L^0$ ) decay products and the time ordering. The dashed arrows in the left and right panels indicate the time-ordered event classes connected by the three symmetries *CP* (vertical), *T* (oblique), and *CPT* (horizontal), independently of each other. The two panels of *t*-reverse decay channels are unconnected by the symmetries.

independent time-reversal comparisons in Table I uses a pair of time-ordered event classes involving four different final states at different times,  $\ell^+ X$  and  $\ell^- \bar{X}$  at times  $t_1$  (or  $t_2$ ) and  $t_2$ (or  $t_1$ ), and  $J/\psi K_S^0$  and  $J/\psi K_L^0$  at times  $t_2$  (or  $t_1$ ) and  $t_1$  (or  $t_2$ ), respectively. Therefore, time-reversal tests require the comparison of probabilities for event classes with opposite time ordering (opposite  $\Delta t$  sign) and different observed flavor- and *CP*-eigenstate final states, as illustrated in Fig. 12 (Bernabéu, Martínez-Vidal, and Villanueva-Pérez, 2012).

With the same approximations, differences like

$$|\langle B^0 | \mathcal{U}(t) | B_- \rangle|^2 - |\langle B_- | \mathcal{U}(t) | \bar{B}^0 \rangle|^2$$
(24)

or

$$|\langle B_{-}|\mathcal{U}(t)|B^{0}\rangle|^{2} - |\langle B_{-}|\mathcal{U}(t)|\bar{B}^{0}\rangle|^{2}$$
(25)

probe *CPT* and *CP* symmetry, respectively, in the time evolution of  $B^{0}-\bar{B}^{0}$  transitions. For each of these cases there are also four independent asymmetries, although clearly not all comparisons involving time-reversal, *CP*, and *CPT* transformations are independent. Whereas *CP* transformation requires the comparison of pairs of time-ordered event classes with different flavor-specific final states, but common *CP*eigenstate final state and same  $\Delta t$  sign, *CPT* demands a common flavor-specific final state, different *CP*-eigenstate final states, and opposite  $\Delta t$  sign (see Fig. 12). Therefore, the time-reversal case is the most challenging.

#### V. BABAR TIME-REVERSAL ANALYSIS

The *BABAR* detector recorded an integrated luminosity of 518 fb<sup>-1</sup> of data, of which 424 fb<sup>-1</sup> were taken at a c.m. energy corresponding to the mass of the  $\Upsilon(4S)$  resonance, 28 and 13.6 fb<sup>-1</sup> around the  $\Upsilon(3S)$  and  $\Upsilon(2S)$  resonances (10.36



FIG. 13 (color online). A sketch of the *BABAR* detector. The five subsystems and the solenoid magnet providing the 1.5 T magnetic field are indicated. Source: *BABAR* Collaboration.

and 10.02 GeV), 4 fb<sup>-1</sup> above the  $\Upsilon(4S)$ , and 48 fb<sup>-1</sup> below the resonances, of which 44 fb<sup>-1</sup> at a c.m. energy 40 MeV below the  $\Upsilon(4S)$  (Aubert *et al.*, 2013). These luminosities correspond to about 470 × 10<sup>6</sup>  $B\bar{B}$ , 690 × 10<sup>6</sup>  $c\bar{c}$ , and 500 × 10<sup>6</sup>  $\tau^+\tau^-$  pairs, and 121 × 10<sup>6</sup>  $\Upsilon(3S)$  and 99 × 10<sup>6</sup>  $\Upsilon(2S)$ resonances. For the time-reversal analysis (Lees *et al.*, 2012) *BABAR* used all  $B\bar{B}$  and  $\Upsilon(4S)$  off-resonance data.

The BABAR detector, sketched in Fig. 13, was designed as a general purpose detector for  $e^+e^-$  annihilation physics (Aubert et al., 2002). Surrounding the interaction point is a five-layer, double-sided silicon vertex tracker (SVT), which measures the angles and impact parameters of charged particle tracks. A 40-layer drift chamber (DC) surrounds the SVT and provides measurements of the momenta for charged particles. Charged hadron identification is achieved through measurements of energy loss in the tracking system and the Cherenkov angle obtained from a detector of internally reflected Cherenkov light (DIRC). A CsI(Tl) electromagnetic calorimeter (EMC) provides photon detection, electron identification, and  $\pi^0$  reconstruction. These components are inserted inside a solenoid magnet, which provides a 1.5 T magnetic field. The flux return of the magnet (IFR) is instrumented with resistive plate chambers and limited streamer tubes in order to detect muons and  $K_L^0$ s (Aubert *et al.*, 2013). Figure 14 illustrates the transverse view of the computer reconstruction of a  $B\bar{B}$  event. The detector performance has been very stable over the nine years of operation, supporting a broad flavor physics program.

Although the *CP*-violation and time-reversal analysis concepts are fundamentally different, the experimental analyses are closely connected to each other. Reconstruction algorithms, event selection criteria, calibration techniques, and descriptions of the background composition are common (Aubert *et al.*, 2009). However, the selected signal events for the time-reversal measurement require different treatment.

#### A. Event reconstruction and selection

In addition to  $J/\psi K_S^0$ ,  $\psi(2S)K_S^0$  and  $\chi_{c1}K_S^0$  decay modes are also considered; all these three final states have *CP*-odd parity and are hereafter denoted generically as  $c\bar{c}K_S^0$ . The



FIG. 14 (color online). Transverse view of the computer reconstruction of a  $B\bar{B}$  event in the *BABAR* detector. The display shows the decay products of the two *B* mesons as curved tracks in the central region of the detector (SVT and DC). Some particles deposit energy in the EMC calorimeter. Particle identification is assisted by measuring the Cherenkov rings (DIRC). Source: *BABAR* Collaboration.

 $c\bar{c}$  particle states are reconstructed in the decay channels  $J/\psi, \psi(2S) \rightarrow e^+e^-, \mu^+\mu^-, \psi(2S) \rightarrow J/\psi\pi^+\pi^-$ , and  $\chi_{c1} \rightarrow J/\psi\gamma$ . Whereas  $K_S^0$  mesons decay near the collision point and are reconstructed through their decays into  $\pi^+\pi^-$  and  $\pi^0\pi^0$  (the latter only for  $J/\psi K_S^0$ ), the long-living  $K_L^0$  mesons pass through the tracking systems undetected and are reconstructed by their hadronic interactions in the EMC and/or the IFR and the  $J/\psi$  in the *B* decay. The background rejection relies on vetoes to specific *B* decay modes, and on angular and event shape variables to suppress continuum events arising from  $e^+e^- \rightarrow q\bar{q}, q = u, d, s$  reactions. These variables exploit the different topology of  $q\bar{q}$  and  $B\bar{B}$  events, jetlike for the former and spherical for the latter, consequence of the large mass difference between light and *b* quarks.

*B*-meson candidates decaying to  $J/\psi K_L^0$  are characterized by the energy difference  $\Delta E = E_B^* - E_{\text{beam}}^*$  between the *B* energy and the beam energy in the  $e^+e^-$  c.m. frame, while for the  $c\bar{c}K_S^0$  final states the beam-energy substituted mass  $m_{\text{ES}} = \sqrt{(E_{\text{beam}}^*)^2 - |\mathbf{p}_B^*|^2}}$  is used, where  $\mathbf{p}_B^*$  is the *B* momentum in the c.m. frame. These two kinematic variables are based on the fact that *B* mesons are produced almost at rest in the c.m. frame and the beam energies are precisely known. Figure 15 shows the  $m_{\text{ES}}$  and  $\Delta E$  data distributions for the final sample of about 7800  $c\bar{c}K_S^0$  signal events with purities ranging from 87% to 96%, and 5800  $J/\psi K_L^0$  signal events with purities around 60%.

Since the mass of the  $\Upsilon(4S)$  is only slightly higher than twice the mass of the *B* meson, no additional particles are produced. Thus, the flavor identity of the *B* meson not associated with the reconstructed  $c\bar{c}K_S^0$  or  $J/\psi K_L^0$  final state is determined from the remaining particles in the event, on the basis of the charges of prompt leptons and kaons, pions from



FIG. 15 (color online). (a)  $m_{\rm ES}$  and (b)  $\Delta E$  distributions for the final *BABAR* sample of neutral *B* decays reconstructed in the  $c\bar{c}K_S^0$  and  $J/\psi K_L^0$  final states, respectively. The shaded regions represent the estimated background contributions. From Aubert *et al.*, 2009.

 $D^{*+} \rightarrow D^0 \pi^+$  decays, and high-momentum charged particles. The notation  $\ell^- \bar{X}$  ( $\ell^+ X$ ) introduced previously denotes all these inclusive final states that identify the flavor of the *B* as  $\bar{B}^0$  ( $B^0$ ). In practice, these flavor identity signatures are combined using a neural network whose output is used to divide the events into six hierarchical, mutually exclusive flavor categories of increasing misidentification probability. The proper-time difference between the decay of the two *B* mesons  $\Delta t$  is measured by the separation of the two decay vertices along the  $e^+e^-$  collision axis and the known boost, as discussed in Sec. III.

Besides the *CP* eigenstate final states, other high-statistics, flavor-specific final states like  $B^0 \rightarrow D^{*-}\pi^+$  and  $B^0 \rightarrow J/\psi K^{*0}[\rightarrow K^+\pi^-]$  (and their *CP* conjugates) are reconstructed and are used for calibrating the  $\Delta t$  resolution and the flavor misidentification probability. Charged *B*-meson decays like  $B^{\pm} \rightarrow J/\psi K^{\pm}, \psi(2S)K^{\pm}, J/\psi K^{*\pm}$  are likewise detected and used for systematic checks.

#### B. Signal data treatment and results

Assuming  $\Delta \Gamma_d = 0$ , the time dependence of each of the eight transitions depicted in Fig. 12 can be parametrized in a model-independent way as

$$\frac{g_{\alpha,\beta}^{\pm}(t)}{e^{-\Gamma_d t}} \propto 1 + S_{\alpha,\beta}^{\pm} \sin(\Delta m_d t) + C_{\alpha,\beta}^{\pm} \cos(\Delta m_d t), \quad (26)$$

where the lower indices  $\alpha = \ell^+, \ell^-$  and  $\beta = K_S^0, K_L^0$  stand for the final reconstructed decay modes  $\ell^+ X, \ell^- \bar{X}$  and  $c\bar{c}K_S^0, J/\psi K_L^0$ , respectively, and the upper indices encapsulate the time ordering (+) for  $B^0$  or  $\bar{B}^0$  tagged states and (-) for  $B_+$ or  $B_-$  tagged states. This expression is analogous to Eq. (11) with the distinction that is associated with eight pairs  $(S_{\alpha,\beta}^{\pm}, C_{\alpha,\beta}^{\pm})$  instead of only one, and t > 0.

The pairs of parameters  $(S_{\alpha,\beta}^{\pm}, C_{\alpha,\beta}^{\pm})$  are determined by a maximum likelihood fit to the measured  $\Delta t$  distributions of the four signal samples in which one *B* meson is reconstructed in a  $c\bar{c}K_S^0$  or  $J/\psi K_L^0$  decay mode, and the flavor content of the other *B* is identified through a  $\ell^-\bar{X}$  or  $\ell^+X$  decay mode. Neglecting time resolution, the elapsed time between the first and second decay is  $t = \Delta t$  for flavor tags, and  $t = -\Delta t$  for

 $B_{\pm}$  tags. As illustrated in Fig. 16, time resolution mixes events with positive and negative true  $\Delta t$ , i.e., a true event class  $(\ell^+ X, J/\psi K_L^0)$ , corresponding to a  $\bar{B}^0 \rightarrow B_+$  transition, might appear reconstructed as  $(J/\psi K_L^0, \ell^+ X)$ , corresponding to a  $B_- \rightarrow B^0$  transition, and vice versa. To separately determine the coefficients for event classes with true positive  $\Delta t$  (flavor tag) or true negative  $\Delta t$  ( $B_{\pm}$  tag), it is necessary to unfold the time ordering and the  $\Delta t$  resolution. This is accomplished by using a signal probability-density function for the four distributions of the form

$$\mathcal{H}_{\alpha,\beta}(\Delta t) = g^{+}_{\alpha,\beta}(\Delta t_{\text{true}})H(\Delta t_{\text{true}}) \otimes \mathcal{R}(\delta t; \sigma_{\Delta t}) + g^{-}_{\alpha,\beta}(-\Delta t_{\text{true}})H(-\Delta t_{\text{true}}) \otimes \mathcal{R}(\delta t; \sigma_{\Delta t}), \quad (27)$$

where  $\Delta t_{\text{true}}$  is the signed difference of proper times between the two *B* decays in the limit of perfect  $\Delta t$  resolution, *H* is the Heaviside step function,  $\mathcal{R}(\delta t; \sigma_{\Delta t})$  is the resolution function, with  $\delta t = \Delta t - \Delta t_{\text{true}}$ , and  $\sigma_{\Delta t}$  is the estimate of the  $\Delta t$ uncertainty obtained by the reconstruction algorithms. This unfolding procedure, which requires good  $\Delta t$  resolution and excellent knowledge of the resolution function, especially in the low  $|\Delta t|$  region, is of critical importance to resolve the time ordering and represents in practice the main experimental challenge of this time-reversal analysis (Lees *et al.*, 2012) in comparison to the associated *CP* violation analysis (Aubert *et al.*, 2009).

From the eight pairs of signal coefficients, reported on the left-hand side of Table II, one might construct two sets  $(\pm)$  of three pairs each of independent asymmetry parameters,  $(\Delta S_T^{\pm}, \Delta C_T^{\pm}), (\Delta S_{CP}^{\pm}, \Delta C_{CP}^{\pm})$ , and  $(\Delta S_{CPT}^{\pm}, \Delta C_{CPT}^{\pm})$ , as illustrated in Fig. 12 and also shown on the right-hand side of Table II. There are multiple choices to define the asymmetry parameters but for convenience two sets (panels in the first and second columns, and in the third and fourth columns of



FIG. 16 (color online). Unfolding of the measured  $\Delta t$  distributions: the reconstructed  $\Delta t$  distribution (bottom left) is unfolded to disentangle the  $\Delta t$  resolution function and the true  $\Delta t$ distribution (top left) used to define the  $B_{\pm}$  (true  $\Delta t < 0$ , top middle) and flavor (true  $\Delta t > 0$ , top right) tags. The  $\Delta t$  resolution mixes true  $B_{\pm}$  or flavor tags with fake flavor or  $B_{\pm}$  tags (bottom middle and bottom right), respectively. The case shown here corresponds to a signal sample in which the two *B* mesons have been reconstructed in the decay modes  $J/\psi K_L^0$  and  $\ell^+ X$ . The event classes are  $(J/\psi K_L^0, \ell^+ X)$  and  $(\ell^+ X, J/\psi K_L^0)$ , corresponding to transitions  $B_- \to B^0$  and  $\bar{B}^0 \to B_+$ , respectively.

TABLE II. Measured *BABAR* values of the  $S_{\alpha,\beta}^{\pm}$  and  $C_{\alpha,\beta}^{\pm}$  coefficients, and of the asymmetry parameters, defined as differences among coefficients for symmetry-transformed transitions as depicted in Fig. 12. The first uncertainty is statistical and the second systematic. The lower indices  $\ell^-$ ,  $\ell^+$ ,  $K_S^0$ , and  $K_L^0$  stand for reconstructed decay modes that identify the *B*-meson state as  $\bar{B}^0$ ,  $B^0$  and  $B_-$ ,  $B_+$ , respectively, and the upper indices encapsulate the time ordering (+) when the decay to  $\ell^-$  and  $\ell^+$  occurs first and (-) otherwise. The asymmetry parameters  $\Delta S_T^{\pm}$ ,  $\Delta C_T^{\pm}$  and the differences  $\Delta S_{CPT}^{\pm}$ ,  $\Delta C_{CPT}^{\pm}$ ,  $\Delta C_{CPT}^{\pm}$  are all motion-reversal violating. The first column refers to the index labeling in Fig. 12. From Lees *et al.*, 2012.

Transition	Coefficient	Result	Asymmetry parameter	Result
(b) $B_+ \to B^0$	$S^{-}_{\ell^{+}\kappa^{0}}$	$-0.66 \pm 0.06 \pm 0.04$	Reference	
	$C^{-}_{\ell^{+} K^{0}}$	$-0.05\pm 0.06\pm 0.03$	Reference	
(e) $B^0 \to B_+$	$S^+_{\ell^-\kappa^0}$	$0.51 \pm 0.17 \pm 0.11$	$\Delta S_T^-$	$1.17 \pm 0.18 \pm 0.11$
	$C^{+}_{\ell^{-}K^{0}}$	$-0.01 \pm 0.13 \pm 0.08$	$\Delta C_T^-$	$0.04 \pm 0.14 \pm 0.08$
(a) $\bar{B}^0 \to B_+$	$S^{t}_{\ell^+ \kappa^0}$	$-0.69 \pm 0.11 \pm 0.04$	$\Delta S_{CPT}^{-}$	$-0.03 \pm 0.13 \pm 0.06$
	$C^{+}_{\scriptscriptstyle {\scriptstyle {\scriptstyle \!$	$-0.02\pm 0.11\pm 0.08$	$\Delta C_{CPT}^{-}$	$0.03 \pm 0.12 \pm 0.08$
(f) $B_+ \to \bar{B}^0$	$S^{\iota , \kappa_L}_{\ell^- \kappa^0}$	$0.67 \pm 0.10 \pm 0.08$	$\Delta S_{CP}^{-}$	$1.33 \pm 0.12 \pm 0.06$
	$C^{-}_{\ell^{-}K^{0}}$	$0.03 \pm 0.07 \pm 0.04$	$\Delta C_{CP}^{-}$	$0.08 \pm 0.10 \pm 0.04$
(c) $\bar{B}^0 \to B$	$S^+_{\ell^+ \kappa^0}$	$0.55 \pm 0.09 \pm 0.06$	Reference	
	$C^{\ell}_{\mathscr{A}^+ \mathscr{K}_S}$	$0.01 \pm 0.07 \pm 0.05$	Reference	
(h) $B \to \bar{B}^0$	$S^{\iota}_{\ell^{-}\kappa^{0}}$	$-0.83 \pm 0.11 \pm 0.06$	$\Delta S_T^+$	$-1.37 \pm 0.14 \pm 0.06$
	$C^{-}_{\ell^{-} \kappa^{0}}$	$0.11 \pm 0.12 \pm 0.08$	$\Delta C_T^+$	$0.10 \pm 0.14 \pm 0.08$
(d) $B \to B^0$	$S^{\iota,\kappa_L}_{\varphi+\varphi^0}$	$0.70 \pm 0.19 \pm 0.12$	$\Delta S^+_{CPT}$	$0.16 \pm 0.21 \pm 0.09$
	$C^{-}_{\mathcal{L}^+ \mathcal{V}^0}$	$0.16 \pm 0.13 \pm 0.06$	$\Delta C^+_{CPT}$	$0.14 \pm 0.15 \pm 0.07$
(g) $B^0 \to B$	$S^{\ell^+,\kappa_L}_{\ell^-,\kappa^0}$	$-0.76 \pm 0.06 \pm 0.04$	$\Delta S^+_{CP}$	$-1.30 \pm 0.11 \pm 0.07$
	$C^{+}_{\ell^{-},K^{0}_{S}}$	$0.08 \pm 0.06 \pm 0.06$	$\Delta C_{CP}^+$	$0.07 \pm 0.09 \pm 0.03$

Fig. 12) of asymmetries related to each of the three discrete symmetries are chosen. The reference transitions are taken by convention  $B_+ \rightarrow B^0$  and  $\bar{B}^0 \rightarrow B_-$ , corresponding to event classes  $(c\bar{c}K_S^0, \ell^+X)$  and  $(\ell^+X, c\bar{c}K_S^0)$ , respectively (see Fig. 12 and Table II). This choice has the advantage that the breaking of time-reversal symmetry would directly manifest itself through any nonzero value of  $\Delta S_T^{\pm}$  or any difference between  $\Delta S_{CP}^{\pm}$  and  $\Delta S_{CPT}^{\pm}$ .

Systematic uncertainties in the measurement of the coefficients and asymmetry parameters in Table II are dominated by the knowledge of the  $\Delta t$  resolution and background composition of event classes containing  $J/\psi K_L^0$  final states, and any possible deviation of the experimental procedure observed in detailed Monte Carlo simulations (Aubert *et al.*, 2013). Besides, the experiment has performed a number of cross-checks, based on both simulated and data control samples, to assess the robustness of the results. Of special relevance is the check performed using event classes where the neutral *B* meson reconstructed in the  $c\bar{c}K_S^0$  and  $J/\psi K_L^0$  final states is replaced by a charged *B* decaying to  $c\bar{c}K^{\pm}$  and  $J/\psi K^{*+}$ , respectively. It is found that all coefficients and parameters, shown in Table II, are consistent with zero.

# C. Interpretation of results and significance

The ability of this analysis to describe the data can be assessed by visualizing the rate differences between the transitions and their time-reversed conjugates in Table I. The inequalities among probabilities are better determined in the form of asymmetries along the lines of Eq. (10). For transition  $\bar{B}^0 \rightarrow B_-$  (first entry in Table I),

$$A_{T}(\Delta t) = \frac{\mathcal{H}_{\ell^{-},K_{L}^{0}}^{-}(\Delta t) - \mathcal{H}_{\ell^{+},K_{S}^{0}}^{+}(\Delta t)}{\mathcal{H}_{\ell^{-},K_{L}^{0}}^{-}(\Delta t) + \mathcal{H}_{\ell^{+},K_{S}^{0}}^{+}(\Delta t)},$$
(28)

where  $\mathcal{H}_{\alpha,\beta}^{\pm}(\Delta t) = \mathcal{H}_{\alpha,\beta}(\pm \Delta t)H(\Delta t)$ . With this construction,  $A_T(\Delta t)$  is defined only for positive  $\Delta t$  values. Neglecting reconstruction effects,

$$A_T(t) \approx \frac{\Delta S_T^+}{2} \sin(\Delta m_d t) + \frac{\Delta C_T^+}{2} \cos(\Delta m_d t).$$
(29)

The three other asymmetries, corresponding to the last three entries in Table I, are constructed analogously and have the same time dependence, with  $\Delta S_T^+$  replaced by  $\Delta S_T^-$ ,  $\Delta S_{CP}^-$ ,  $\Delta S_{CPT}^-$ , and  $\Delta S_{CP}^+ - \Delta S_{CPT}^+$ , respectively, and equally for  $\Delta C_T^+$ .

Figure 17 shows the four time-reversal asymmetries constructed in this way. The data are well described by the solid curves, which represent the projection of the best fit to the eight  $(S_{\alpha,\beta}^{\pm}, C_{\alpha,\beta}^{\pm})$  pairs, as reported in Table II. These curves deviate significantly from the dashed curves, which represent the fit projection for time-reversal invariance, i.e.,  $\Delta S_T^{\pm} = 0$ ,  $\Delta C_T^{\pm} = 0$ ,  $\Delta S_{CP}^{\pm} = \Delta S_{CPT}^{\pm}$ , and  $\Delta C_{CP}^{\pm} = \Delta C_{CPT}^{\pm}$ . The fact that the dashed curves are not identically zero is a consequence of experimental effects, in particular, the asymmetry with respect to  $\delta t = 0$  of the time resolution function.

All eight  $C^{\pm}_{\alpha,\beta}$  in Table II are compatible with zero; therefore the  $|\bar{A}/A| = 1$  condition discussed in Sec. IV.A is validated within errors and the association between event classes and transitions in Table I is confirmed. It follows that the observation  $\Delta S^{\pm}_T \neq 0$  and  $\Delta S^{\pm}_{CP} \neq \Delta S^{\pm}_{CPT}$  in Table II is an unambiguous, direct detection of time-reversal violation in the time evolution of neutral *B* mesons,



FIG. 17 (color online). The four independent asymmetries  $A_T$  measured by the *BABAR* experiment between transitions: (a)  $\bar{B}^0 \rightarrow B_-$ , (b)  $B_- \rightarrow B^0$ , (c)  $\bar{B}^0 \rightarrow B_+$ , (d)  $B_+ \rightarrow B^0$ , and their time-reversed versions. The points with error bars represent the data, the solid and dashed curves represent the projections of the best fit results with and without time-reversal violation, respectively. From Lees *et al.*, 2012.

obtained through motion reversal in transitions that are not CP conjugate to each other. The violation of time-reversal symmetry is also clearly seen through the large differences between the solid and dashed curves in all four asymmetries shown in Fig. 17.

The significance of the observed time-reversal violation is evaluated on the basis of changes in log-likelihood value  $\ln \mathcal{L}$ with respect to the maximum  $(-2\Delta \ln \mathcal{L})$ . The difference  $-2\Delta \ln \mathcal{L}$  between the fit without *T* violation and the best fit is 226, including systematic uncertainties. Assuming Gaussian statistics, this corresponds to a significance of  $14\sigma$ , evaluated from the upper integral at  $-2\Delta \ln \mathcal{L}$  of the  $\chi^2$  probability distribution for 8 degrees of freedom<sup>3</sup> (*p* value) (Beringer *et al.*, 2012). Figure 18 shows *p*-value contours calculated



FIG. 18 (color online). Central values (point and square) and two-dimensional *p*-value contours (for  $p = 0.317, 4.55 \times 10^{-2}$ ,  $2.7 \times 10^{-3}, 6.3 \times 10^{-5}, 5.7 \times 10^{-7}$ , and  $2.0 \times 10^{-9}$ ) for the *T*asymmetry parameters  $(\Delta S_T^+, \Delta C_T^+)$  (dashed curves) and  $(\Delta S_T^-, \Delta C_T^-)$  (solid curves), as reported by *BABAR*. The *T*invariance point is shown as a + sign. From Lees *et al.*, 2012.

from the change  $-2\Delta \ln \mathcal{L}$  in two dimensions for the *T*-asymmetry parameters  $(\Delta S_T^+, \Delta C_T^+)$  and  $(\Delta S_T^-, \Delta C_T^-)$ . In the two cases we observe that the *T* invariance point is excluded at the  $6\sigma$  level. The difference  $-2\Delta \ln \mathcal{L}$  for fits assuming *CPT* or *CP* symmetry are 5 and 307, corresponding to  $0.3\sigma$  and  $17\sigma$ , consistent with *CPT* invariance and *CP* violation, respectively. These values, combined with those in Table II, are compatible with *CP* violation as due to time-reversal violation and *CPT* invariance. The larger significance of *CP* violation is because the comparison of probabilities for event classes, e.g.,  $(\ell^- \bar{X}, c\bar{c}K_S^0)$  and  $(\ell^+ X, c\bar{c}K_S^0)$  has a higher statistical and systematic significance than the comparison of, e.g.,  $(\ell^- \bar{X}, c\bar{c}K_S^0)$  and  $(\ell^- \bar{X}, c\bar{c}K_I^0)$ .

Following the discussion in Sec. IV.A and assuming *CP* conservation in mixing (i.e., *CP* with *T* invariance, and *CP* with *CPT* symmetry in mixing), all eight pairs of coefficients  $S_{\alpha,\beta}^{\pm}$  and  $C_{\alpha,\beta}^{\pm}$  are related,

$$S = S_{\ell^{+},K_{S}^{0}}^{+} = -S_{\ell^{-},K_{S}^{0}}^{+} = -S_{\ell^{-},K_{S}^{0}}^{-} = S_{\ell^{-},K_{S}^{0}}^{-} = -S_{\ell^{+},K_{L}^{0}}^{+}$$

$$= S_{\ell^{-},K_{L}^{0}}^{+} = S_{\ell^{+},K_{L}^{0}}^{-} = -S_{\ell^{-},K_{L}^{0}}^{-},$$

$$C = C_{\ell^{+},K_{S}^{0}}^{+} = -C_{\ell^{-},K_{S}^{0}}^{+} = C_{\ell^{-},K_{S}^{0}}^{-} = -C_{\ell^{-},K_{L}^{0}}^{-},$$

$$(30)$$

$$= -C_{\ell^{-},K_{L}^{0}}^{+} = C_{\ell^{+},K_{L}^{0}}^{-} = -C_{\ell^{-},K_{L}^{0}}^{-},$$

In the SM, the eight  $S^{\pm}_{\alpha,\beta}$  coefficients are measurements of  $S = \sin 2\beta \approx 0.7$ . The results in Table II lead to a mean value  $S = 0.686 \pm 0.029$ , which is consistent with the value obtained from the *CP* violation investigation based on the same data (Aubert *et al.*, 2009). Analogously, the measurements of the eight  $C^{\pm}_{\alpha,\beta}$  coefficients result in a mean value of  $0.022 \pm 0.021$ , consistent with both the previous *CP* analysis and zero.

## VI. CONCLUSION

The arrow of time in systems with large numbers of degrees of freedom is a thermodynamic property of entropy associated

<sup>&</sup>lt;sup>3</sup>Eight is the difference in the number of fit constraints with and without T violation.

with the irreversibility of boundary conditions. However, time's arrow is not related to the question of time-reversal symmetry in the fundamental laws of physics. Only two physical systems in nature, the unstable K and B mesons, have a relatively large expected breaking of time-reversal symmetry; in these systems CP violation has been observed and no experimental data contradict the CPT theorem. Therefore, K and B mesons are the best choice for an experiment directly detecting time-reversal noninvariance.

The main principle for a direct detection of time-reversal violation in transitions is the exchange of initial and final states. A unique opportunity arises from the quantummechanical properties imposed by the EPR entanglement between the two neutral B mesons produced in the  $\Upsilon(4S)$ resonance decay at B factories. The observation of the first Bdecaying into the flavor eigenstates  $\ell^+ X$  or  $\ell^- \bar{X}$ , or the *CP* eigenstates  $c\bar{c}K_S^0$  or  $J/\psi K_L^0$ , prepares (tags) the initial state of the other B as  $\bar{B}^{0}$ ,  $B^{0}$ ,  $B_{+}$ , or  $B_{-}$ , respectively. The initial states tagged by entanglement are filtered at a later time through its decay into a CP- or a flavor-eigenstate decay mode. The four B-meson states appearing as initial and final states make it possible to build eight different transition probabilities for the time evolution of the neutral B meson. In appropriate combinations, time-reversal, CP, and CPT symmetries can be analyzed separately through four time-dependent asymmetries, which can be expressed in terms of certain asymmetry parameters.

The BABAR experiment has measured the four timereversal asymmetries and extracted the corresponding asymmetry parameters. The results show a highly significant departure from motion-reversal symmetry. A precise exchange of initial and final states is needed to interpret the results as direct detection of time-reversal noninvariance. This requires the absence of both wrong-strangeness  $(B^0 \not\rightarrow c\bar{c}\bar{K}^0)$  and  $\bar{B}^0 \not\rightarrow c\bar{c}K^0$ ) and wrong-sign  $(B^0 \not\rightarrow \ell^- \bar{X} \text{ and } \bar{B}^0 \not\rightarrow \ell^+ X) B$ decays, *CP* invariance in  $K^0 - \bar{K}^0$  mixing, and  $|\bar{A}/A| = 1$ (a single decay amplitude and CPT symmetry in the  $B^0 \rightarrow$  $c\bar{c}K^0$  decay amplitude). All these effects are small and have been either accounted for in the systematic uncertainties (the second), directly demonstrated in the experimental analysis and incorporated in the uncertainties (the fourth), or neglected since their impact is well below the statistical sensitivity  $[\mathcal{O}(10\%)]$  according to measurements [the first,  $\mathcal{O}(0.1\%)$ ] or SM expectations [the third,  $\mathcal{O}(1\%)$ ].

Time-reversal and *CP* symmetry breakings are seen in two separate observations (the states involved in the transitions are not *CP* conjugate to each other) and the asymmetries are time dependent with only a  $\sin(\Delta m_d \Delta t)$  term, of order  $\mathcal{O}(10^{-1})$ , and are induced by the interference of decay amplitudes with and without mixing. The corresponding measurement of the weak phase from time-reversal asymmetries match those from *CP* asymmetries; therefore the observed *T* and *CP* violations balance each other, supporting *CPT* invariance in the time evolution of *B* mesons. This is in contrast to the flavor-mixing asymmetry in  $K^0$ - $\bar{K}^0$  transitions measured by CPLEAR, where *CP* and *T* transformations are identical and the asymmetry is time independent, of order  $\mathcal{O}(10^{-3})$ , and is produced by the interference between the dispersive and absorptive contributions to  $K^0$ - $\bar{K}^0$  mixing. The concept of direct detection of time-reversal violation in transitions might be extended to include systematic tests using pairs of *B* and *D* mesons created in the decay of the  $\Upsilon(4S)$  and  $\psi(3770)$  resonances (Bevan, Inguglia, and Zoccali, 2013), as well as pairs of *K* mesons from the  $\phi(1020)$  (Bernabéu, Di Domenico, and Villanueva-Pérez, 2013). In the latter case, there are important differences triggered by a nonvanishing decay width difference, the nonorthogonality of the  $K_L^0$  and  $K_S^0$  states, and the small effects expected within the SM. This opens the possibility to embark upon a complete time-reversal (and *CPT*) violation program in weak interactions to probe possible new physics contributions in tree and loop decays at future high-luminosity flavor factories, Belle II at SuperKEKB (Abe *et al.*, 2010; Aushev *et al.*, 2010) and KLOE-2 at DA $\Phi$ NE (Amelino-Camelia *et al.*, 2010).

The main limitation of the method is associated with the identification of the appropriate decay channels used to filter the states of the time-reversed transition. Specifically, it is necessary to identify pairs of decay channels that project into meson states orthogonal to each other. This orthogonality condition is satisfied by conjugate flavor-eigenstate decay channels  $(\ell^- \bar{X}, \ell^+ X)$  and by *CP* eigenstates of opposite CP parity with the same flavor content  $(c\bar{c}K_{s}^{0})$  $J/\psi K_L^0$ ). The fact that the *B* decay amplitudes to  $c\bar{c}K_S^0$  and  $J/\psi K_L^0$  are given by the same diagram followed by  $\tilde{K}^0$ - $\bar{K}^0$ mixing (needed to make possible the interference) is essential for the definition of the states  $B_{-}$  and  $B_{+}$ . The precise implementation of these restrictions imposes the requirements summarized above. An alternative time-reversal asymmetry based on EPR entanglement open to any pair of decay channels was recently suggested (Bernabéu, Botella, and Nebot, 2014), although the connection between the experiment and the time-reversal observables requires some theoretical input.

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