# How higher-spin gravity surpasses the spin-two barrier

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Aiming at nonexperts, the key mechanisms of higher-spin extensions of ordinary gravities in four dimensions and higher are explained. An overview of various no-go theorems for low-energy scattering of massless particles in flat spacetime is given. In doing so, a connection between the *S*-matrix and the Lagrangian approaches is made, exhibiting their relative advantages and weaknesses, after which potential loopholes for nontrivial massless dynamics are highlighted. Positive yes-go results for non-Abelian cubic higher-derivative vertices in constantly curved backgrounds are reviewed. Finally, how higher-spin symmetry can be reconciled with the equivalence principle in the presence of a cosmological constant leading to the Fradkin-Vasiliev vertices and Vasiliev's higher-spin gravity with its double perturbative expansion (in terms of numbers of fields and derivatives) is outlined.

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**CONTENTS**

I. Introduction 987
II. No-go Theorems in Flat Spacetime 990
A. Preamble: The gauge principle and the Fronsdal program 990
B. The Weinberg low-energy theorem 990
1. Charge conservation: The spin-one case 990
2. Equivalence principle: The spin-two case 991
3. Higher-order conservation laws: The higher-spin cases 991
C. Coleman-Mandula theorem and its avatar: No higher-spin conserved charges 991
D. Generalized Weinberg-Witten theorem 992
E. Velo-Zwanziger difficulties 992
III. Possible Ways Out 993
A. Masslessness 993
B. Asymptotic states and conserved charges 993
C. Lorentz minimal coupling 994
D. Flat background 994
E. Finite dimensionality of spacetime 994
IV. Various Yes-go Examples 994
A. Consistent cubic vertices in Minkowski spacetime 995
B. Cubic vertices in AdS spacetime 996
C. Main lessons 997
D. Higher-spin symmetry breakings 998
1. Higher-spin gauge symmetries are broken at the infrared scale 999
2. Dynamical symmetry breaking: Spin one versus higher spin 999
V. Fully Interacting Example: Vasiliev's Higher-Spin Gravity 999
A. Examples of non-Abelian gauge theories 999
B. The need for a complete theory 1000
C. Vasiliev's equations 1001
D. AdS/CFT correspondence: Vasiliev's theory from free conformal fields 1001
E. Emergence of extended objects 1003
VI. Conclusions and Outlook 1004
Appendix A: Weinberg Low-energy Theorem: *S* matrix and Lagrangian Dictionary 1004
1. Emission of a massless particle: Lorentz versus gauge invariances 1004
2. Cubic vertices 1005
Appendix B: Weinberg-Witten Theorem: A Lagrangian Reformulation 1005
1. Weinberg-Witten theorem 1005
2. Refinement of Weinberg-Witten theorem 1006

## I. INTRODUCTION

This review is an attempt at a nontechnical summary of how higher-spin gravity[1] manages to surpass the spin-two barrier: the stringent constraints on low-energy scattering in flat spacetime that seemingly forbid massless particles with spins greater than 2 to participate in the formation of any

[1]By the term "higher-spin gravity" we mean a theory where an extension of the spacetime isometry algebra by higher-spin generators is gauged.

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interacting quantum field theory.[2] While this may seem to call for radical measures, there exists a relatively conservative yet viable way out, namely, the dual usage of the cosmological constant as critical mass (infrared cutoff) and dimensionful coupling constant. This dual-purpose treatment of the cosmological constant leads to a successful exchange of what are leading and subleading terms in minimal coupling that lifts the spell of the no-go theorems, and, in particular, reconciles higher-spin gauge symmetry with the equivalence principle, leading up to the Fradkin-Vasiliev cubic action (Fradkin and Vasiliev, 1987b, 1987c; Vasiliev, 2001a, 2011; Alkalaev and Vasiliev, 2003) and Vasiliev's fully nonlinear equations of motion[3] (Vasiliev, 1990, 1992, 2003) [see, e.g., Vasiliev (2004a, 2004b) and Bekaert and *et al.* (2005) for some reviews].

Since our aim is to outline main ideas and results, we refrain from being technical and refer the interested reader to the already existing literature whenever necessary. Moreover, throughout the paper we mostly stick to the Fronsdal program (Fronsdal, 1978), i.e., the standard perturbative off-shell implementation of non-Abelian gauge deformations starting from the Fronsdal actions in constantly curved backgrounds. It is the gauge algebra (not necessarily an internal algebra) that we require to become non-Abelian similar to the diffeomorphism algebra in Einstein gravity. As for Vasiliev's higher-spin gravity, presently the most far-reaching construction of a full higher-spin gauge theory, we restrict ourselves[4] to a more brief address of how it presents a natural framework for a string-theory-like double perturbative expansion.

Now, why are higher-spin gauge fields interesting? Although massless fields of spin greater than 2 make perfect sense at the free level, their quantum interactions pose a main challenge to modern theoretical physics. In a nutshell, the problematics can be summarized as follows: consistent non-Abelian higher-spin gauge symmetries induce local higher-derivative generalizations of translations that seem to call for a nontrivial bosonic extension of spacetime itself, thus interfering with the basic assumptions of canonical second quantization that led up to the notion of free fields to begin with. Thus a satisfactory resolution seems certainly much more demanding than even that of quantizing ordinary general relativity (though the prolongation of the Einstein-Cartan reformulation of general relativity as a soldered Yang-Mills theory for the spacetime isometry algebra soon leads to infinite-dimensional algebras as well) which actually leaves room for a naive optimism: the quantization of higher-spin gauge theories could lead to a radically new view on quantum field theory altogether, and, in particular, on the formidable spin-two barrier set up by the requirement of power-counting renormalizability.

Indeed, at the classical level, there exist the aforementioned higher-spin gravities (Vasiliev, 1990, 1992, 1996, 2003; Sezgin and Sundell, 1998, 2001a,2001b, 2002a): these are special instances of interacting higher-spin gauge theories constituting what one may think of as the simplest possible higher-spin extensions of general relativity. Their minimal bosonic versions (in $D \geq 4$ ordinary spacetime dimensions) consist of a propagating scalar, metric, and tower of massless fields of even spins, $s = 4, 6, \ldots$ (these models can then be extended by various forms of "matter" and suitable higher-spin counterparts, in a supersymmetric setup in case fermions are included).

As mentioned, a key feature of higher-spin gravity is its double perturbative expansion: besides the expansion in numbers of fields, weighted by a dimensionless coupling $g$, there is a parallel albeit strongly coupled expansion in numbers of pairs of derivatives, weighted by a dimensionful parameter, the cosmological constant $\Lambda$, thus serving as both infrared and ultraviolet cutoff. Hence classical higher-spin gravity prefers a nonvanishing cosmological constant, unlike string theory in flat spacetime which also has a double perturbative expansion but with a strictly massless sector accessible at low energies in a weakly coupled derivative expansion.

Taking higher-spin gravity seriously as a model for quantum gravity, the key issue is thus whether its loop corrections[5], which are given in a weak-field expansion more reminiscent of the perturbative expansion of string theory than that of general relativity, may generate masses dynamically for the higher-spin fields? Remarkably, relying on arguments based on the anti–de Sitter/conformal field theory (AdS/CFT) correspondence (Girardello, Porrati, and Zaffaroni, 2003), the answer seems affirmative: the pattern of symmetry breaking is similar in spirit to that of ordinary quantum chromodynamics (QCD), with spin playing the role of color, the metric playing the role of an Abelian gauge field, and the Goldstone modes being two-particle states; in the leading order in perturbation theory, the spin-$s$ field acquires mass for $s > 2$ while the spin-$s-1$ Goldstone mode is the lightest bound state (in its parity sector) between the physical scalar and the massless spin-$s-2$ particle. The crucial missing ingredient is a "confinement mechanism" that causes $g$ to become large at low enough energies, thus creating a mass gap leading to a low-energy effective quantum gravity.

[2]These constraints on massless particle scattering appear only in spacetimes of dimension $D \geq 4$ to which we restrict our attention in this paper. Indeed, in dimension $D \leq 3$ massless fields of helicity $s \geq 2$ have no local propagating degrees of freedom. Pure massless higher-spin gravities in lower dimensions are of Chern-Simons type which do not share most of the exotic features of their higher-dimensional cousins discussed here.

[3]The precise link between, on the one hand, the Fradkin-Vasiliev cubic action and, on the other hand, the fully interacting Vasiliev equations, remains to be found.

[4]We thus leave out many other interesting features of the Vasiliev system, such as its unfolded, or Cartan integrable, formulation, and the link between its first-quantization, deformed Wigner oscillators, singletons, and compositeness of massless particles in anti–de Sitter spacetime.

[5]For related issues within the anti–de Sitter/conformal field theory (AdS/CFT) correspondence, see Klebanov and Polyakov (2002) and Sezgin and Sundell (2002b) and the recent advances due to Giombi and Yin (2009, 2010), which altogether point to the fact that four-dimensional higher-spin gravity should have a surprisingly simple ultraviolet behavior as a quantum field theory in anti–de Sitter spacetime, in the sense that its boundary dual is weakly coupled or even free, with a simple $1/N$ expansion.

Thus, the quantization of higher-spin gauge theories can lead to interesting models providing deepened insights into the interplay between quantum mechanics and geometry. These might be of relevance not only in the high-energy limit of quantum gravity and string theory, but also for providing new ideas in observational physics, such as, for example, in cosmology, where weakly coupled massless particles can serve as dark matter candidates. Finally, the development of the quantum theory of higher-spin fields may serve as a source of inspiration for seeking and testing new methods in quantum field theory, such as the application of deformation and geometric quantizations as well as topological models to dynamical systems with local degrees of freedom.

Having provided all of these motivations for quantizing higher-spin gauge fields, it is perhaps surprising to discover that there is a drastic gap between Vasiliev's on-shell approach to higher-spin gravity based on gauging a non-Abelian global-symmetry algebra and the Fronsdal program: the latter has so far been only partially completed, mainly at the cubic level [for a recent discussion on this issue, see, e.g., Bengtsson (2008) and references therein]. Hence a key question[6] is whether the Fronsdal program can be completed at the quartic level, even in the case of the aforementioned minimal bosonic model? This apparently straightforward problem will keep a number of interesting surprises in store, in particular, in view of the aforementioned properties of the AdS/CFT correspondence (Sezgin and Sundell, 2002b; Giombi and Yin, 2009, 2010) which have been derived using a rather different approach, as we return to in Sec. V and summarize in Sec. VI.

As far as more general interacting quantum field theories with higher-spin fields are concerned, open string field theory in flat spacetime provides a basic example thereof albeit with a massless sector restricted to spins less than or equal to 1. Recently, motivated by the similarities between open string theory and higher-spin gravities mainly at the level of free fields (Francia and Sagnotti, 2003, 2006), Sagnotti and Taronna (2011) deconstructed its first Regge trajectory and arrived at the germs of the non-Abelian interactions for massless totally symmetric tensors in flat spacetime (Boulanger and Leclercq, 2006; Boulanger, Leclercq, and Sundell, 2008) whose deformations into (A)dS spacetimes (Boulanger, Leclercq, and Sundell, 2008) lead to the Fradkin-Vasiliev cubic vertices. Moreover, Polyakov (2009) proposed to extend the open superstring in flat spacetime by sectors of states with novel world-sheet ghost numbers containing massless higher-spin particles in interaction. He also managed to show (Polyakov, 2010) that these higher-spin states interact with the closed-string graviton and that these interactions reproduce the aforementioned germs of Boulanger and Leclercq (2006) and Boulanger, Leclercq, and Sundell (2008).

As far as actual tensionless limits of strings are concerned, there is a vast literature which we cannot cover here. Of the various results that have been obtained, we simply point to the rather drastic difference between tensionless limits of, on the one hand, the open string in flat space and, on the other hand, the closed string in anti–de Sitter spacetime. A precise version of the former was taken by Bonelli (2003), Buchbinder *et al.* (2006), Fotopoulos *et al.* (2007), and Fotopoulos and Tsulaia (2007). It yields deformed Fronsdal actions albeit with Abelian $p$-form-like vertices that do not contain the non-Abelian interactions characteristic of the higher-spin gravities to be discussed in this review. Whether there exists a refined limit in the spirit of the aforementioned deconstruction by Sagnotti and Taronna (2011), leading to such couplings, remains to be seen.

As far as the closed AdS string is concerned, it exhibits a novel physical phenomenon that has no flat-space analog, whereby solitons, carrying quantum numbers of singletons, are formed at cusps (Engquist and Sundell, 2006); in the tensionless limit, their dynamics can be extracted by discretizing the Nambu-Goto action and degenerating spacetime to the Dirac hypercone leading to a direct connection between Vasiliev's higher-spin gravities and tensionless closed strings in which the graviton on both sides is identified (Engquist and Sundell, 2006). The resulting physical picture is also in accordance with the holographic proposals by Sundborg (2001) and Sezgin and Sundell (2002b) later dubbed "la grande bouffe" (Bianchi, Morales, and Samtleben, 2003).

Although these string-related theories are extremely interesting in their own right, here we are mainly concerned with non-Abelian interactions for strictly massless fields in flat spacetime and for their (A)dS analogs with their critical masses and the related higher-spin gravity.

In the case of strictly massless fields in flat spacetime, many $S$-matrix no-go theorems can be found in the literature (Weinberg, 1964; Coleman and Mandula, 1967; Haag, Lopuszanski, and Sohnius, 1975; Grisaru, Pendleton, and van Nieuwenhuizen, 1977; Benincasa and Cachazo, 2007; Porrati, 2008; Benincasa and Conde, 2011) that seemingly forbid interacting massless higher-spin particles. Since the relative strength of no-go theorems is measured by the weakness of their hypotheses, the $S$-matrix approach is usually advertised because it does not require assumptions about locality nor the Poincaré-covariant realization of the incoming quanta. At a closer inspection, however, it turns out that the $S$-matrix no-go results obtained so far concern only the spin-$s$ couplings involving $s$ derivatives such as, for example, two-derivative couplings between the graviton and other fields.

If one accepts that the spin-$s$ couplings contain more than $s$ derivatives, then these $S$-matrix arguments need to be reconsidered, and since the higher-spin interaction problem presents itself already at the classical level, it is anyway more satisfactory to pursue this analysis starting from purely Lagrangian arguments. And indeed, numerous cubic vertices, consistent at this order, have been found over the years in

[6]Here we want to stress that it is only by closing the quartic order that the cubic Lagrangian, including cubic curvature couplings known as cubic Born-Infeld terms, will be completely fixed (if it exists). Because of the double perturbative expansion, the Born-Infeld couplings dominate over the minimal couplings in physical amplitudes (assuming a deformed Fronsdal action with finite Born-Infeld "tail") and hence the quartic-closure problem must be addressed prior to any attempts to do physics with incomplete cubic actions. In other words, analyses based solely on current exchange may receive large corrections due to the exotic usage of the cosmological constant.

Minkowski and (A)dS spacetimes. They all exhibit higher-derivative couplings and will be reviewed here, as well as their relations with the Fradkin-Vasiliev vertices.

In summary, it may prove to be useful to confront the no-go theorems with the yes-go examples already in the classical Lagrangian framework, in order to emphasize the underlying assumptions of the no-go theorems, even if it may require an extra assumption about perturbative locality.

The paper is organized as follows: In Sec. II we begin by spelling out the gauge principle in perturbative quantum field theory and its "standard" implementation within the Fronsdal program for higher-spin gauge interactions. We then survey the problematics of nontrivial scattering of massless particles of spin greater than 2 in flat spacetime, and especially its direct conflict with the equivalence principle. In Sec. III we list possible ways to evade these negative results, both within and without the Fronsdal program. In Sec. IV we review results where consistent higher-spin interactions have been found, in both flat and (A)dS spacetimes. Because of the fact that consistent interacting higher-spin gravities indeed exist, at least for gauge algebras which are infinite-dimensional extensions of the (A)dS isometry algebra, an important question is related to the possible symmetry breaking mechanisms that give a mass to the higher-spin gauge fields. This is briefly discussed in Sec. IV.D. After reviewing why a classically complete theory is crucial in higher-spin gravity, we lay out in Sec. V the salient features of Vasiliev's approach to a class of potentially viable models of quantum gravity. We conclude in Sec. VI where we also summarize some interesting open problems. Finally we devote two Appendixes to the review of some $S$-matrix no-go theorems and to their reformulation in Lagrangian language. More precisely, Appendix A focuses on Weinberg's low-energy theorem, while Appendix B concentrates on the Weinberg-Witten theorem and its recent adaptation to gauge theories by Porrati.

## II. NO-GO THEOREMS IN FLAT SPACETIME

This section presents various theorems[7] that constrain interactions between massless particles in flat spacetime, potentially ruling out nontrivial quantum field theories with gauge fields with spin $s > 2$ and vanishing cosmological constant. The aim is to scrutinize some of their hypotheses in order to exhibit a number of conceivable loopholes that may lead to modified theories including massless higher spin, as summarized in Sec. III.

### A. Preamble: The gauge principle and the Fronsdal program

The key feature of the field-theoretic description of interacting massless particles is the gauge principle: a sensible perturbation theory requires compatibility between the interactions and some deformed version of the Abelian gauge symmetries of the free limit. The necessity of gauge invariance in perturbative quantum field theory stems from the fact that one and the same massless particle, thought of as a representation of the spacetime isometry group, in general admits (infinitely) many implementations in terms of quantum fields sitting in different Lorentz tensors obeying respective free equations of motion. For more information, see, e.g., Skvortsov (2008) and Boulanger, Iazeolla, and Sundell (2009).

Only a subset of these "carriers," namely, the primary curvature tensors and all of their derivatives, actually transform tensorially under isometry (implemented quantum mechanically via similarity transformations). The remaining carriers are different types of potentials obtained by integrating various curvature Bianchi identities (and which one may thus think of as representing different "dual pictures" of one and the same particle); such integrals in general transform under isometry with inhomogeneous pieces that one can identify as Abelian gauge transformations.

Thus, in the standard perturbative interaction picture one is led to the *Fronsdal program*: the construction of interaction Hamiltonians starting from Lorentz invariant and hence gauge-invariant nonlinear Lagrangians built from the aforementioned carriers.

We stress that the Fronsdal program is based on a working hypothesis: that standard canonical quantization of free fields in ordinary spacetime is actually compatible with the presence of higher-spin translations in higher-spin gauge theories. We proceed in this spirit in the bulk of this paper.

### B. The Weinberg low-energy theorem

The Weinberg low-energy theorem is essentially a by-product of dealing with the more general problem of emissions of soft massless particles. Consider a (nontrivial) scattering process involving $N$ external particles with (say, ingoing) momenta $p_i$ ($i = 1, 2, \ldots, N$) and spin $s_i$. The emission of an additional massless particle of integer spin $s$ with arbitrary soft momentum by the $i$th external particle is controlled by a cubic vertex of type $s$-$s_i$-$s_i$ (i.e., between a gauge boson of spin $s$ and two particles of spin $s_i$) with coupling constant $g_i^{(s)}$. The Weinberg low-energy theorem (Weinberg, 1964) states that Lorentz invariance of (or, equivalently, the absence of unphysical degrees of freedom from) the deformed amplitude imposes a conservation law of order $s-1$ on the $N$ external momenta[8]:

$$\sum_{i=1}^{N} g_i^{(s)} p_i^{\mu_1} \cdots p_i^{\mu_{s-1}} = 0. \tag{1}$$

#### 1. Charge conservation: The spin-one case

Lorentz invariance for the emission of a soft massless spin-one particle (such as a photon) leads to the conservation law $\sum_i g_i^{(1)} = 0$; thus it requires the conservation of the coupling constants (such as the electric charges) that characterize the interactions of these particles at low energies.

[7]The $S$-matrix no-go theorem (Benincasa and Cachazo, 2007) is not discussed here because it relies on slightly stronger assumptions than the others; see, e.g., the conclusion of Porrati (2008) for more comments.

[8]For pedagogical reviews, see, e.g., Weinberg (1995), Sec. 13.1 or Blagojevic (2002), Appendix G.

In order to prepare the ground for further discussion, we denote by "electromagnetic minimal coupling" the coupling of a charged particle to the electromagnetic field obtained by replacing the partial derivatives appearing in the Lagrangian describing the free, charged matter field in flat space, by the $u(1)$-covariant derivative, viz. $\partial_\mu \to \partial_\mu - ig_i^{(1)} A_\mu$.

### 2. Equivalence principle: The spin-two case

As argued by Weinberg (1964), the equivalence principle can be recovered as the spin-two case of his low-energy theorem. On one side, Lorentz invariance for the emission of a soft massless spin-two particle leads to the conservation law $\sum_i g_i^{(2)} p_i^\mu = 0$. On the other side, translation invariance implies momentum conservation $\sum_i p_i^\mu = 0$. Therefore, for generic momenta, Poincaré invariance requires all coupling constants to be equal: $g_i^{(2)} = g_j^{(2)} =: g^{(2)}$ $(\forall\ i, j)$. In other words, massless particles of spin two must couple in the same way to all particles at low energies.

This result has far-reaching consequences as it resonates with two deep properties of gravity, namely, its uniqueness and its universality. On the one hand, the local theory of a self-interacting massless spin-two particle is essentially[9] *unique*: in the low-energy regime (at most two derivatives in the Lagrangian) it must be described by the Einstein-Hilbert action. Therefore, the massless spin-two particle rightfully deserves the name "graviton."[10] On the other hand, the gravitational interaction is also *universal* (Weinberg, 1964): if there exists a single particle that couples minimally to the graviton, then all particles coupled to at least one of them must also couple minimally to the graviton. According to Weinberg himself, this theorem is the expression of the equivalence principle in quantum field theory, so, from now on, it will be referred to as the Weinberg equivalence principle. A proper understanding of this crucial theorem involves, however, some subtleties on the precise meaning of "minimal coupling."

Consider the quadratic Lagrangian $\mathcal{L}^{(0)}(\varphi_s, \partial\varphi_s)$ describing a free spin-$s$ matter field denoted by $\varphi_s$. In general relativity, the equivalence principle can be expressed by the Lorentz minimal coupling prescription, i.e., the assumption that the transformation rules of tensor fields under the Poincaré group extend naturally to the diffeomorphism group and the replacement of partial derivatives by Lorentz-covariant ones, viz. $\partial \to \nabla = \partial + g^{(2)}\Gamma_{\rm lin} + \cdots$, in the matter sector. It must be observed that this prescription does not apply to the spin-two field itself because the Einstein-Hilbert Lagrangian is not the covariantization of the Fierz-Pauli quadratic Lagrangian $\mathcal{L}^{(0)}(\varphi_2, \partial\varphi_2)$.

One focuses on cubic couplings $\mathcal{L}^{(1)}(h, \varphi_s, \partial\varphi_s)$ of the type 2-$s$-$s$, i.e., linear in the spin-two field $h_{\mu\nu}$ and quadratic in the spin-$s$ field $\varphi_s$. The symmetric tensor of rank two $\Theta^{\mu\nu} := \delta\mathcal{L}^{(1)}/\delta h_{\mu\nu}$ is bilinear in the spin-$s$ field. For consistency with the linearized diffeomorphisms $\delta_\xi h_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$, the cubic coupling $\mathcal{L}^{(1)}$ to a massless spin-two field $h_{\mu\nu}$ must arise through a bilinear conserved current of rank two, i.e., $\partial_\mu\Theta^{\mu\nu} \approx 0$, where the weak equality denotes the equality up to terms that vanish on the solutions of the free equations of motion for $\varphi_s$. For $s = 2$, the cubic self-coupling of type 2-2-2 coming in the Einstein-Hilbert Lagrangian gives rise to a conserved tensor $\Theta^{\mu\nu}$ which is equivalent to the Noether energy-momentum tensor $T^{\mu\nu}$ for the Fierz-Pauli Lagrangian. For $s \neq 2$, the cubic 2-$s$-$s$ coupling $\mathcal{L}^{(1)}$ comes from the Lorentz minimal coupling prescription applied to the free Lagrangian $\mathcal{L}^{(0)}$ if and only if $\Theta^{\mu\nu}$ is equal (possibly on shell and modulo an "improvement") to the Noether energy-momentum tensor $T^{\mu\nu}$ for $\mathcal{L}^{(0)}$. It is this precise condition on $\Theta^{\mu\nu}$ (for any spin) that should be understood as minimal coupling in the Weinberg equivalence principle.

### 3. Higher-order conservation laws: The higher-spin cases

Lorentz invariance for the emission of soft massless higher $(s \geq 3)$ spin particles leads to conservation laws of higher $(s - 1 \geq 2)$ order, i.e., for sums of products of momenta. For generic momenta, Eq. (1) has no solution when $s - 1 > 1$, therefore all coupling constants must be equal to zero: $g_i^{(s)} = 0$ for any $i$ when $s > 2$. In other words, as stressed by Weinberg in his book (Weinberg, 1995), p. 538: "massless higher-spin particles may exist, but they cannot have couplings that survive in the limit of low energy" (that is, they cannot mediate long-range interactions). Moreover, strictly speaking the Weinberg low-energy theorems concern only $s$-$s'$-$s'$ couplings.

Nevertheless, notice the existence of a simple solution for Eq. (1) corresponding to so-called trivial scattering, i.e., elastic scattering such that the outgoing particle states are permutations of the incoming ones, as in the case of free or possibly integrable field theories. For example, if we denote the ingoing momenta by $k_a$ $(a = 1, 2, \ldots, n)$ and the outgoing ones by $\ell_a$, then the higher-order conservation laws $\sum_a g_a^{(s)} k_a^{\mu_1} \cdots k_a^{\mu_{s-1}} = (-1)^{s-1} \sum_a g_a^{(s)} \ell_a^{\mu_1} \cdots \ell_a^{\mu_{s-1}}$ of order $s - 1 > 1$ imply that the outgoing momenta can only be permutations of the incoming ones, and that $g_a^{(s)} = g^{(s)}$ for all $a$ if $s$ is even, while $g_a^{(s)} = \epsilon_a g^{(s)}$ with $(\epsilon_a)^2 = 1$ for all $a$ if $s$ is odd.

## C. Coleman-Mandula theorem and its avatar: No higher-spin conserved charges

The Coleman-Mandula theorem (Coleman and Mandula, 1967) and its generalization to the case of supersymmetric theories with or without massless particles given by Haag, Lopuszanski, and Sohnius (1975) strongly restrict the symmetries of the $S$ matrix of an interacting relativistic field theory in four-dimensional Minkowski spacetime.[11] More precisely, (i) if the elastic two-body scattering amplitudes are generically nonvanishing (at almost all energies and angles), and (ii) if there is only a finite number of particle

[9]See, e.g., Boulanger *et al.* (2001) for a precise statement of the very general hypotheses and references therein for previous literature on this issue.

[10]A thorough discussion on the observability of the graviton is presented by Boughn and Rothman (2006) and Rothman and Boughn (2006).

[11]For an extended pedagogical review, see Weinberg (2000), Chapter 24.

species on and below any given mass shell, then the maximal possible extension of the Poincaré algebra is the (semi)direct sum of a superalgebra (a superconformal algebra in the massless case) and an internal symmetry algebra spanned by elements that commute with the generators of the Poincaré algebra.

In particular, this theorem rules out higher symmetry generators (equivalently, conserved charges) that could have come from higher-spin symmetries surviving at large distances. The argument goes as follows: the gauge symmetries associated with massless particles may survive at spatial infinity as nontrivial rigid symmetries. In turn, such symmetries should lead to the conservation of some asymptotic charges. Under the hypotheses of the generalized Coleman-Mandula theorem, nontrivial conserved charges associated with asymptotic higher-spin symmetries cannot exist.

This corollary of the generalized Coleman-Mandula theorem partially overlaps with the Weinberg low-energy theorem because the conservation law (1) precisely corresponds to the existence of a conserved charge $Q^{\mu_1\cdots\mu_{s-1}}$ which is a symmetric tensor of rank $s-1$ that commutes with the translations, but does not commute with the Lorentz generators.

#### D. Generalized Weinberg-Witten theorem

The Weinberg-Witten theorem (Weinberg and Witten, 1980) states that a massless particle of spin strictly greater than 1 cannot possess an energy-momentum tensor $T_{\mu\nu}$ which is both Lorentz covariant and gauge invariant.[12] Of course, this no-go theorem does not preclude gravitational interactions. In the spin-two case, it implies that there cannot exist any gauge-invariant energy-momentum tensor for the graviton. This proves that the energy of the gravitational field cannot be localized, but it obviously does not prevent the graviton from interacting with matter or with itself.

Recently, a refinement of the Weinberg-Witten theorem was presented (Porrati, 2008) that genuinely prevents massless particles of spin strictly greater than 2 from coupling minimally to the graviton in flat background. The minimality condition is stated according to the Weinberg equivalence principle, namely, it refers to Lorentz minimal spin-two coupling (see Sec. II.B.2). In the Lagrangian approach, the same result had already been obtained in various particular instances, where it was shown that the Lorentz minimal coupling prescription applied to free higher-spin gauge fields enters in conflict with their Abelian gauge symmetries (Aragone and Deser, 1979; Berends *et al.*, 1979; Aragone and La Roche, 1982; Boulanger and Leclercq, 2006). The complete no-go result ruling out the Lorentz minimal coupling of type 2-$s$-$s$ in the Lagrangian approach is given in Boulanger, Leclercq, and Sundell (2008).

In between the Lagrangian and the $S$-matrix approaches lies the light-cone approach where all local cubic vertices in dimensions from four to six have been classified [see, e.g., Metsaev (2006) and references therein] and where the same negative conclusions concerning the Lorentz minimal coupling of higher-spin gauge fields to gravity had already been reached and stated in complete generality.

[12]For a pedagogical essay, see, e.g., Loebbert (2008).

This being said, consistent cubic vertices between spin-two and higher-spin gauge fields do exist, even in Minkowski spacetime (Boulanger and Leclercq, 2006; Metsaev, 2006; Boulanger, Leclercq, and Sundell, 2008). Instead of describing Lorentz's minimal coupling, they contain more than two derivatives in total. As one can see, the generalized Weinberg-Witten theorem does not by itself forbid such type 2-$s$-$s$ interactions. The crux of the matter is to combine this theorem with the Weinberg equivalence principle.

Together, the Weinberg equivalence principle and the generalized Weinberg-Witten theorem prohibit the cross couplings of massless higher-spin particles with low-spin particles in flat spacetime (Porrati, 2008). The argument goes as follows: elementary particles with spin not greater than 2 are known to couple minimally to the graviton at low energy. Therefore (Weinberg's equivalence principle) all particles interacting with low-spin particles must also couple minimally to the graviton at low energy, but [generalized Weinberg-Witten theorem (Porrati, 2008) and identical results presented in Metsaev (2006) and Boulanger, Leclercq, and Sundell (2008)] massless higher-spin particles cannot couple minimally to gravity around the flat background. Consequently, at low energies massless higher-spin particles must completely decouple from low-spin ones. Hence, if the same Lagrangian can be used to describe both the low-energy phenomenology and the Planck-scale physics, then no higher-spin particles can couple to low-spin particles (including spin two) at all.

#### E. Velo-Zwanziger difficulties

In this section, we stress that, contrary to widespread prejudice, the Velo-Zwanziger difficulties do not constitute a serious obstruction to the general program of constructing consistent interactions involving higher-spin fields. The observed pathologies are nothing but symptoms of nonintegrability in the sense of Cartan of the differential equations under consideration. Thus, in order to avoid pathologies, it makes sense to follow a specific gauge principle,[13] which for high spins is nothing but a refined version (e.g., the Noether procedure) of the naive application of the minimal coupling prescription, as is the main topic of this review.

In particular, the electromagnetic interactions exhibit pathologies (such as seemingly superluminal propagation) in Minkowski spacetime already for massive spin-$\frac{3}{2}$ fields [see Velo and Zwanziger (1969) and Velo (1972) and a more recent analysis by Porrati and Rahman (2008, 2009) which contain a list of other relevant references on the issue] that are

[13]Weinberg emphasized a related point, while mentioning the Velo-Zwanziger paper and other related works (cf. references therein), in his book (Weinberg, 1995), p. 244: "The problems reported with higher spin have been encountered only for higher-spin particles that have been arbitrarily assumed to have only very simple interactions with external fields. No one has shown that the problems persist for arbitrary interactions. (...) There are good reasons to believe that the problems with higher spin disappear if the interaction with external fields is sufficiently complicated." One may reinterpret this by stating that consistency requires less simplistic interactions, namely, those governed by gauge invariance.

therefore not specific to higher spins and hence deserve a separate discussion. Indeed, the interactions between spin-$\frac{3}{2}$ and electromagnetic fields in gauged supergravities are well known to avoid the Velo-Zwanziger problems. In the case of spin-one self-interactions, a simple model to keep in mind is the Born-Infeld Lagrangian, whose expansion around a nontrivial electromagnetic background gives a linearized theory with causal structure governed by the Boillat metric whose light cone lies within that of the undeformed flat-space metric; see the discussion and references in Gibbons and Herdeiro (2001).

In order to think of a model containing spins greater than 1 and with higher-derivative corrections that have been added following a gauge principle, one can immediately go to string theory, where the Born-Infeld theory is subsumed into open string theory. Open strings propagating in electromagnetic backgrounds (Argyres and Nappi, 1990) contain massive spin-$s$ states with $s \geq \frac{3}{2}$ whose kinetic terms contain $2s - 2$ derivatives.

The actual physical problem is how to count degrees of freedom in the presence of extended spacetime gauge symmetries and the higher-derivative interactions that follow therefrom. In order to avoid nonintegrabilities in a systematic fashion, a natural resolution is to abandon the standard perturbative approach (formulating interactions in expansions around ordinary lower-spin backgrounds) in favor of the unfolded approach (Vasiliev, 1988, 1989, 1990, 1994) which allows a generalized perturbative formulation of field theory in the unbroken phase as well as in various generalized metric phases and/or tensorial spacetimes (Vasiliev, 2001b, 2002; Didenko and Vasiliev, 2004; Gelfond and Vasiliev, 2005).

To summarize this survey of no-go results, the genuine obstacles to massless higher-spin interactions are the Coleman-Mandula theorem, the low-energy Weinberg theorems, and the generalized Weinberg-Witten theorem.

## III. POSSIBLE WAYS OUT

In this section, we discuss the weaknesses of the various hypotheses underlying the no-go theorems for interacting massless higher-spin particles in flat spacetime. Correspondingly, we present conceivable ways to surpass the spin-two barrier. Of these openings, the principal escape route is the Fradkin-Vasiliev mechanism in which the cosmological constant plays a dual role as infrared and ultraviolet regulators. This leads to Vasiliev's fully nonlinear equations, which set a new paradigm for a realm of exotic higher-spin gravities that fit naturally into the contexts of weak-weak coupling holography and tensionless limits of extended objects. This "main route" will be discussed in more detail in Secs. V and VI.

### A. Masslessness

Implicitly, all of the aforementioned no-go theorems rely on the hypothesis of a *flat* spacetime background. Indeed, the notion of massless particles is unequivocal only in theories with Poincaré-invariant vacua. In constantly curved nonflat spacetimes, the mass operator (i.e., $\nabla^2$) is related to the eigenvalues of the second Casimir operators of the spacetime isometry algebra and of the Lorentz algebra. It is only in flat spacetime, however, that the eigenvalues of the mass operator are quantum numbers, which can be sent to zero leaving a strictly massless theory without any intrinsic mass scale.

Thus, as far as theories in Minkowski spacetime are concerned, one may consider interpreting massless higher-spin particles as limits of *massive* dittos. Such particles are consistent at low energies; on the experimental side, they are *de facto* observed in hadronic physics as unstable resonances albeit not as fundamental particles.[14] However, this high-energy limit has its own problems: it is singular in general as manifested by the van Dam–Veltman–Zakharov discontinuity in propagators of massive fields of spin greater than $\frac{3}{2}$. Indeed, on the theoretical side, this fact is related to the complicated nature of the tensionless limit of string theory in flat spacetime.

A clear physical picture of why the high-energy limit cannot be used to find massless higher-spin particles in flat spacetime is given by the example of higher-spin resonances in quantum chromodynamics. Dimensionless quantities depend on the ratio $E/m$, where $E$ and $m$ are the energy and the mass of the resonance, respectively. As $E$ goes to infinity with $m$ kept fixed is equivalent to $m$ tending to zero keeping $E$ constant, it follows that one must send $\Lambda_{\text{QCD}}$ to zero. In this limit, the size of a resonance grows indefinitely, however, and it becomes undetectable to an observer of fixed size, since the observer lives within the resonance's Compton wavelength.[15]

### B. Asymptotic states and conserved charges

The $S$-matrix theorems concern only particles that appear as asymptotic states. Moreover, within the perturbative approach, these asymptotic states are assumed to exist at all energy scales. Thus, an intriguing possibility is that there exists nonperturbatively defined higher-spin gauge theories in flat spacetime with mass gaps and confinement. We are not aware of any thorough investigations of such models and mechanisms so far, although Vasiliev's higher-spin gravities in four-dimensional anti–de Sitter spacetime have been conjectured to possess a perturbatively defined mass gap, resulting from dynamical symmetry breaking induced via radiative corrections (Girardello, Porrati, and Zaffaroni, 2003), as we comment on below.

As far as confinement is concerned,[16] one may ask whether the higher-spin charges of asymptotic states might all vanish, such as for color charges in QCD. Incidentally, Weinberg pointed out in his book (Weinberg, 2000), p. 13, that some subtleties arise in the application of the Coleman-Mandula theorem in the presence of infrared divergences, but that *there is no problem in non-Abelian gauge theories in which all massless particles are trapped—symmetries if unbroken would only govern S-matrix elements for gauge-neutral bound states.*

[14] Strictly speaking, one can arguably refer to the proton as stable while already the neutron is metastable while all other massive excitations are far more short lived.

[15] We thank one of the referees for this comment.

[16] This way out was briefly mentioned in the conclusions of Bekaert, Joung, and Mourad (2009).

### C. Lorentz minimal coupling

To reiterate slightly, the $S$-matrix no-go theorems[17] for higher-spin interactions are engineered for Poincaré-invariant relativistic quantum field theories aimed at describing physics at intermediate scales lying far in between the Planck and Hubble scales. In Lagrangian terms, the generalized Weinberg-Witten theorem can essentially be understood as resulting from demanding compatibility between linearized gauge symmetries and the Lorentz minimal coupling in the absence of a cosmological constant. This compatibility requires consistent cubic vertices with one and two derivatives for fermions and bosons, respectively. Vertices with these numbers of derivatives have the same dimension as the flat-space kinetic terms. If consistent, they therefore do not introduce any new mass parameter. Hence it is natural to extrapolate the Lorentz minimal coupling to all scales. In doing so, however, one needs to keep in mind not only the barrier for quantum fields in the ultraviolet but also in the infrared.

Pertinent to this statement is the generalized Weinberg-Witten theorem. The assumptions are that (i) the Lorentz minimal coupling term is always present, and (ii) the theory extends to all energies without encountering any infrared or ultraviolet catastrophe. To reiterate, the refined analysis relies crucially via assumption (i) on Weinberg's formulation of the equivalence principle,[18] which one may view as a low-energy constraint on the theory. The result is that massless higher-spin particles cannot couple with the universal graviton or anything that the latter couples to. In other words, if such massless higher-spin theories in flat background exist in the mathematical sense, they cannot be engineered to the low-energy physics that takes place in our Universe.

For instance, one can have a theory with two phases: A symmetric phase at high energy where higher-spin particles are massless and the Newton constant vanishes for all particles, and a broken phase, where higher-spin particles get a mass and the Newton constant is nonzero. This is an intriguing possibility; moreover it probably occurs in $AdS_4$ (Girardello, Porrati, and Zaffaroni, 2003); see the discussion in Sec. IV.D. Nothing forbids the existence of an *a priori* very warm universe where such exotic theories are relevant. After cooling and symmetry breakdown these may then yield an effective matter-coupled gravity theory in which the graviton is that field that couples to everything in always the same way, with a single coupling constant introduced, namely, Newton's constant.

The assumptions (i) and (ii) are indeed vulnerable to the possibility of phase transitions. This will be discussed in Sec. IV.D. Looking to the limits of the experimental as well as theoretical tests of the Lorentz minimal coupling, there is no reason *a priori* as to why the specific mechanism by which diffeomorphism invariance is implemented in Einstein's gravity should work at scales that are very small or very large. This suggests that the Lorentz minimal coupling can be rehabilitated within theories with infrared as well as ultraviolet cutoffs.

### D. Flat background

As already stressed, the strict definition of massless particle and $S$ matrix requires a flat spacetime. Passing to a slightly curved de Sitter or anti–de Sitter spacetime with cosmological constant $\Lambda$, one sometimes considers the existence of gauge symmetries as the criterion[19] of masslessness. Since there is no genuine $S$ matrix in AdS, a subtle and fruitful way out is that the $S$-matrix theorems do not apply any more when the cosmological constant $\Lambda$ is nonvanishing; instead one resorts to a holographic dual conformal field theory. This way out has been exploited successfully by the Lebedev school and has given rise to cubic vertices and full nonlinear equations of motion.

### E. Finite dimensionality of spacetime

Finally, in light of the recent progress made in amplitude calculations in ordinary relativistic quantum field theory (Bern *et al.*, 2007, 2009) as well as higher-spin gravity (Giombi and Yin, 2009, 2010), one may start raising criticism against the very assumptions behind the Fronsdal program: the higher-derivative nature of higher-spin interactions leads ultimately to a conceptual breakdown of the standard canonical approach to quantum field theory based on time slicing in ordinary spacetime. Although one can refer perturbatively to the canonical structure of the free fields (thought of as fluctuations around the spin-two background), the nonperturbative formulation of higher-spin symmetries leads toward an extension of spacetime by extra bosonic coordinates on which higher-spin translations act by linear differentiation. One may therefore think of a bosonic generalization of the superspace approach to supergravities, which is precisely what is provided by the unfolded dynamics program initiated by Vasiliev [for an illustration of the basic ideas in the context of higher-spin supergravity, see, for example, Engquist, Sezgin, and Sundell (2003)].

## IV. VARIOUS YES-GO EXAMPLES

In this section we give a review of the various positive results obtained over the years concerning consistent higher-spin cubic couplings in flat and AdS backgrounds. Section IV.A gathers together the results for cubic vertices in flat space, while Sec. IV.B essentially mentions the results obtained by Fradkin and Vasiliev in the late 1980s for cubic vertices in $(A)dS_4$. Section IV.C consists of a summary in the form of a general picture for non-Abelian higher-spin gauge theory, which seems to emerge from the known no-go theorems and yes-go examples. Of course, a word of caution should be added: the existence of consistent cubic couplings does not imply that a complete theory exists at all. However, the existence of full interacting equations (Vasiliev, 1990,

[17]Including the Coleman-Mandula theorem, since the conserved charges used in its arguments depend on the asymptotic behavior of interactions at large distances.

[18]See Eq. (B2) of Appendix B or Eq. (26) in Porrati (2008).

[19]This criterion is subtle, however, since for nonvanishing $\Lambda$, generic spins cannot have as many gauge symmetries as for vanishing $\Lambda$.

TABLE I. $s_1$-$s_2$-$s_2$ covariant vertices obtained by Berends, Burgers, and van Dam (1985).

| $\downarrow_{s_1}$ $\rightarrow^{s_2}$ | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 | $\frac{5}{2}$ | 3 |
|---|---|---|---|---|---|---|---|
| 0 | × | × | × | × | × | | |
| 1 | × | × | × | × | × | | |
| 2 | × | × | × | × | × | × | |
| 3 | × | × | × | × | × | × | × |
| $n$ | × | | | | | | |

1992, 2003) is a strong indication that a complete interacting Lagrangian[20] may exist, at least in (A)dS background. Actually, one of the open problems in higher-spin gravity is whether or not the Fronsdal program can be pursued beyond the cubic order in a standard fashion.

### A. Consistent cubic vertices in Minkowski spacetime

In the 1980s, the quest for high-spin interactions successfully started, taking flat spacetime as background. Using the light-cone gauge approach, higher-spin $s$-$s'$-$s''$ cubic vertices in four spacetime dimensions were found by Bengtsson, Bengtsson, and Brink (1983a, 1983b), Bengtsson, Bengtsson, and Linden (1987), and Fradkin and Metsaev (1991). These results, in the light-cone gauge approach, were considerably generalized later by Metsaev, 1993a, 1993c, 2006, 2007) and Fradkin and Metsaev (1995) with a complete classification of cubic (self- and cross)couplings for arbitrary massive and massless higher-spin fields, bosonic and fermionic, in dimensions four, five, and six. Mixed-symmetry fields were also considered therein. Moreover, Metsaev (1993b) obtained a wide class of cubic interactions for arbitrary fields in arbitrary dimension.

As far as manifestly Poincaré-invariant vertices in the Lagrangian approach are concerned, Berends, Burgers, and van Dam obtained a class of manifestly covariant, *non-Abelian* cubic couplings (Berends, Burgers, and van Dam, 1984, 1985). They used a systematization of the Noether procedure for introducing interactions, where the couplings are not necessarily of the form "gauge field times conserved current." Berends, Burgers, and van Dam (1985) obtained consistent and covariant cubic couplings of the kind $s_1$-$s_2$-$s_2$, for the values of $s_1$ and $s_2$ indicated in Table I. Of course, some of the vertices were already known before, such as, for example, in the cases 1-1-1, 2-2-2, and 2-$\frac{3}{2}$-$\frac{3}{2}$ corresponding to Yang-Mills, Einstein-Hilbert, and ordinary supergravity theories, respectively. There is a class of cross interactions $s_1$-$s_2$-$s_2$ for which the cubic vertices could easily been written. This class corresponds to the "Bell-Robinson" line $s_1 = 2s_2$ and below this line $s_1 > 2s_2$ (Berends, Burgers, and van Dam, 1986); see Deser and Yang (1990) in the $s_1 = 4 = 2s_2$ case and some more recent considerations by Manvelyan, Mkrtchyan, and Ruehl (2010c). In the aforementioned region $s_1 \geq 2s_2$, the gauge algebra remains *Abelian* at first order in a coupling constant although the gauge transformations for the spin-$s_2$ field are deformed. The reason is that the first-order deformation of the free spin-$s_2$ gauge transformations involves the spin-$s_2$ field only through its gauge-invariant Weinberg–de Wit–Freedman field strength (Weinberg, 1965; de Wit and Freedman, 1980).[21] Although they do not lead to non-Abelian gauge algebras, it is interesting that the cubic interactions on and below the Bell-Robinson line (i.e., for $s_1 \geq 2s_2$) have the form "spin-$s_1$ field times current" where the current is quadratic in (the derivatives of) the spin-$s_2$ field strength (Berends, Burgers, and van Dam, 1986; Deser and Yang, 1990) and is conserved on the spin-$s_2$ shell. Even more interesting, these currents can be obtained from some global invariances of the free theory by a Noether-like procedure, provided the constant parameters associated with these rigid symmetries be replaced by the gauge parameters of the spin-$s_1$ field (also internal color indices must be treated appropriately) (Berends, Burgers, and van Dam, 1986; Deser and Yang, 1990). The simplest class of cubic interactions below the Bell-Robinson line is provided by the couplings between scalar fields ($s_2 = 0$) and a collection of higher-spin tensor gauge fields through the Berends–Burgers–van Dam currents containing $s_1$ derivatives of the scalar fields (Berends, Burgers, and van Dam, 1986). Recently, they were reexamined by Bekaert (2006), Fotopoulos *et al.* (2007), and Bekaert, Joung, and Mourad (2009) as a toy model for higher-spin interactions. Note that these cubic interactions induce, at first order in the coupling constant, gauge transformations for the scalar field which are non-Abelian at second order and reproduce the group of unitary operators acting on free scalars on Minkowski spacetime (Bekaert, 2006; Bekaert, Joung, and Mourad, 2009).

As demonstrated by Boulanger, Leclercq, and Sundell (2008), in a flat background the non-Abelian 2-$s$-$s$ vertex is unique and involves a total number of $2s - 2$ derivatives. From $s = 3$ on, the non-Abelian 2-$s$-$s$ vertex in Minkowski spacetime is thus "nonminimal" and the full Lagrangian (if any) has no chance of being diffeomorphism invariant, a fact explicitly shown by Boulanger and Leclercq (2006) and Boulanger, Leclercq, and Sundell (2008). It was also shown by Boulanger, Leclercq, and Sundell (2008) that the unique and non-Abelian 2-$s$-$s$ vertex in Minkowski spacetime is nothing but the leading term in the flat limit of the corresponding AdS Fradkin-Vasiliev vertex that, among others, contains the Lorentz minimal coupling. That the minimal Lorentz coupling term in the Fradkin-Vasiliev vertex is *subleading* in the flat limit shows that the Weinberg equivalence principle is restored for higher spins in AdS spacetime but is lost in the flat limit. This supports the need to consider higher-spin interactions in the AdS background, at least if one wants to make a contact between higher-spin gauge fields and low-spin theories including Einstein-Hilbert gravity.

[20]As a matter of fact, a nonstandard action principle for Vasiliev's equations, which leads to a nontrivial quantization, was proposed by Boulanger and Sundell (2011).

[21]Note that one can trivially write down higher-derivative Born-Infeld–like consistent cubic interactions involving only gauge-invariant linearized field-strength tensors (Damour and Deser, 1987). However, these interactions deform neither the gauge algebra nor the gauge transformations at first order in some coupling constant. Nevertheless, they might be needed when pushing the non-Abelian cubic vertices to the next order in the coupling constants.

Recently, general results on the structure of cubic $s$-$s'$-$s''$ couplings ($s \leq s' \leq s''$) non-Abelian already at this order were given, showing, in particular, that the *maximum* number of derivatives involved in a non-Abelian coupling is $2s' - 1$ or $2s' - 2$, depending on the parity of the sum $s + s' + s''$ (Bekaert, Boulanger, and Leclercq, 2010). It was also shown that the cubic vertices saturating the upper derivative bound have a good chance of being extended to second order in the deformation parameter, as far as the Jacobi identity for the gauge algebra is concerned. Later on, the generic non-Abelian vertices were studied and explicitly built by Manvelyan, Mkrtchyan, and Ruehl (2010a, 2010b). Some classification results were also obtained about the structure of the Abelian cubic vertices. *A posteriori*, the approach (Manvelyan, Mkrtchyan, and Ruehl, 2010a, 2010b) to the writing of covariant non-Abelian vertices can be seen as the covariantization of the vertices already obtained in the light-cone approach by Bengtsson, Bengtsson, and Brink (1983a, 1983b) and Metsaev (2006, 2007) where, on top of the cubic coupling given by the light-cone gauge approach, terms are added which vanish in the spin-$s$ De Donder gauge.

With the advent of string field theory in the second half of the 1980s, the construction of higher-spin cubic vertices in flat space was carried out by Koh and Ouvry (1986), Bengtsson (1988), and Cappiello *et al.* (1989) in the so-called Becchi-Rouet-Stora-Tyutin (BRST) approach. This approach was indeed motivated by the BRST first quantization of the string and by the tensionless limit of open string field theory. More recently, this analysis was pursued by Bonelli (2003) and Buchbinder *et al.* (2006), Fotopoulos and Tsulaia (2007), Fotopoulos *et al.* (2007) [a review of the last three works plus other works can be found in Fotopoulos and Tsulaia (2009)]. The results obtained in this framework are encouraging, for instance, in the case of non-Abelian $s$-0-0 interactions (Fotopoulos *et al.*, 2007), although the higher-spin gauge field (self- and cross) interactions found in Fotopoulos and Tsulaia (2007) are Abelian, and therefore can hardly be related to the non-Abelian higher-spin theory of Vasiliev.

Before turning to the cubic interactions in AdS background, we continue with our review of positive results for higher-spin cubic vertices in flat space. Important results have recently been obtained by analyzing the tree-level amplitudes of the tensile (super)string. In what could be called a string and $S$-matrix approach, Polyakov (2009, 2010), Taronna (2010), and Sagnotti and Taronna (2011) obtained a plethora of vertices and recovered the vertices obtained in the previously cited approaches, thereby creating a direct link between open string theory and higher-spin gauge theory at the dynamical level. Moreover, in light of the uniqueness results of Boulanger, Leclercq, and Sundell (2008), one has a precise relation between the Fradkin-Vasiliev vertices and string theory.

Generically, the idea is that the non-Abelian flat-space cubic vertices obtained by Bekaert, Boulanger, and Cnockaert (2006) and Boulanger, Leclercq, and Sundell (2008) (which were shown to be related to the, appropriately taken, flat-space limit of the corresponding Fradkin-Vasiliev vertices) are also the seed for the construction of consistent massive higher-spin vertices in flat and AdS spacetimes. From these non-Abelian flat-space vertices, one can systematically construct massive and massless vertices in AdS and flat spaces by switching on mass terms à la Stückelberg and cosmological constant terms. This approach was used with success by Zinoviev (2009a, 2009b). See also the recent work by Zinoviev (2010) where the framelike formalism for higher-spin gauge fields is used.

### B. Cubic vertices in AdS spacetime

As mentioned in the previous section, at cubic level (i.e., at first order in perturbative deformation) Fradkin and Vasiliev found a solution to the higher-spin (gravitational, self- and cross) interaction problem by considering metric perturbations around $(A)dS_4$ background (Fradkin and Vasiliev, 1987b, 1987c). This was later extended to five dimensions (Vasiliev, 2001a), $\mathcal{N} = 1$ supersymmetry (Alkalaev and Vasiliev, 2003) and arbitrary dimensions (Vasiliev, 2011). For a recent analysis of the Fradkin-Vasiliev mechanism in arbitrary dimension $D$ and in the cases 2-$s$-$s$ and 1-$s$-$s$, see Boulanger, Leclercq, and Sundell (2008).

The Fradkin-Vasiliev construction was the starting point of dramatic progress leading recently to fully nonlinear field equations for higher-spin gauge fields in arbitrary dimension (Vasiliev, 2003). We will not detail their construction here but we simply comment that the use of a twistor variable and a Moyal-Weyl star product is central, although historically the usefulness of the star product was not immediately recognized. In a few words, the main problem with the higher-spin gravitational interaction was that, introducing the Lorentz minimal coupling terms in the action and gauge transformations, higher-spin gauge invariance could not be satisfied anymore. The solution provided by Fradkin and Vasiliev was to introduce a nonvanishing cosmological constant $\Lambda$ and expand the metric around an (A)dS background. The gauge variation of the cubic terms coming from the Lorentz minimal coupling around (A)dS are now canceled on the free shell, by the variation of a *finite* tail of additional nonminimal cubic vertices, each of them proportional to the linearized Riemann tensor around (A)dS and involving more and more (A)dS-covariant derivatives compensated by appropriate negative powers of the cosmological constant. In that gauge variation, the terms proportional to the free equations of motion are absorbed through appropriate corrections to the gauge transformations. This solution is the *Fradkin-Vasiliev mechanism*, and we call the gravitational cubic coupling they obtained the *quasiminimal coupling*, in the sense that the Lorentz minimal coupling is present and triggers a finite expansion of nonminimal terms.

A salient feature of the Fradkin-Vasiliev construction is that there are now two independent expansion parameters: the AdS mass parameter $\lambda \sim \sqrt{|\Lambda|}$ and the dimensionless deformation parameter $g := (\lambda \ell_p)^{(D-2)/2}$ that counts the order in the weak-field expansion, where the Planck length $\ell_p$ appears in front of the action through $1/\ell_p^{D-2}$ and where one works with dimensionless physical fields.

At the cubic level and for any given triplet of spins $\{s, s', s''\}$, there appears a finite expansion in *inverse* powers of $\lambda$, where the terms with the highest negative power of $\lambda$ bring the highest number of (A)dS-covariant derivatives acting on the weak fields. The highest power of $1/\lambda$ is proportional to $s''$, so that for unbounded spins the Fradkin-Vasiliev cubic Lagrangian is nonlocal. The massive parameter $\lambda$ simultaneously (i) sets the *infrared cutoff* via $|\Lambda| \sim \lambda^2$ and the critical masses $M^2 \sim \lambda^2$ for the dynamical fields, and (ii) dresses the derivatives in the interaction vertices thus enabling the Fradkin-Vasiliev mechanism. This dual role played by the cosmological constant is responsible for an exotic property of the Fradkin-Vasiliev cubic coupling.

*Exotic nonlocality of the Fradkin-Vasiliev Lagrangian.*— In the physically relevant cases where one has a separation of length scales, i.e., $\ell_p \ll \ell \ll \lambda^{-1}$, where $\ell \sim \|\varphi\|/\|\partial\varphi\|$ is some wavelength characterizing the physical system under consideration and where $\lambda^{-1}$ denotes here a generic infrared scale, not necessarily related to the cosmological constant, two situations can arise for perturbatively local (cf. Sec. IV.C) Lagrangians having vertices $V_n$ involving higher ($n \geq 3$) derivatives of the fields:

(a) Mild nonlocality: The theory is weakly coupled in the sense that $V_n \sim (\ell_p/\ell)^{n-2} \ll 1$. This situation arises for broken higher-spin symmetry, tensionful string sigma models, etc.

(b) Exotic nonlocality: The theory is strongly coupled in the sense that the vertices $V_n$ are proportional to $(\ell\lambda)^{-n+2} \gg 1$. This is the situation for the Fradkin-Vasiliev vertices. In the derivative expansion appearing within the Fradkin-Vasiliev mechanism, the terms involving the maximal number of derivatives are dominant since they contain the infrared cutoff instead of the ultraviolet one.

Finally, we make a comment related to the fully nonlinear Vasiliev equations in order to show that the same behavior appears order by order in the weak-field expansion. In this theory, the first-order corrections $T^{(1)}_{\mu\nu}$ to the stress tensor defined by $T_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R - \Lambda)$ arise in an expansion of the form $T^{(1)} = \sum_{n=0}^{\infty}\sum_{p+q=n}\lambda^{-n}\nabla^p\varphi_s\nabla^q\varphi_s$; see Kristiansson and Rajan (2003) for the scalar field contributions. One therefore sees the appearance of an *infinite derivative tail* in the standard field equations already at first order in the weak-field expansion (Sezgin and Sundell, 2002a). This leads to tree-level amplitudes depending on the following two dimensionless scales: (i) the weak-field expansion coupling $g = (\lambda\ell_p)^{(D-2)/2}$ that can always be taken to obey $g \ll 1$; and (ii) the derivative-expansion coupling $(\ell\lambda)^{-n+2}$ where $\ell$ is the characteristic wavelength. Thus the tails are strongly coupled around solutions that are close to the $\mathrm{AdS}_D$ solution since here $\ell\lambda \ll 1$.

### C. Main lessons

The first important lesson which one can draw from the previous discussions is that, contrary to widespread prejudices, many doors are left open for massless higher-spin particles. The second important lesson is that interactions for higher-spin gauge fields exist but are rather exotic. Some of their properties clash with standard lores inherited from low-spin physics, and indeed, there is no fundamental reason to expect that higher-spin fields must behave as their low-spin companions.

Some model-independent features of non-Abelian higher-spin gauge theories seem to emerge from all known no-go theorems and yes-go examples. It appears that most of the exotic properties of higher-spin fields can roughly be explained by mere dimensional arguments. As done in the previous section, we introduce a parameter $\ell$ with the dimension of a length and rescale all objects in order to work with dimensionless Lagrangian $\mathcal{L}$ and fields $\varphi$. The action takes the form $S = \ell^{-D}\int d^Dx\mathcal{L}(\varphi, \ell\partial\varphi, \ell^2\partial^2\varphi, \ldots)$ where each derivative is always multiplied by a factor of $\ell$. The Lagrangian counterpart of Feynman rules in $S$-matrix arguments is the weak-field expansion, i.e., the fields $\varphi$ are perturbations around some background for which the higher-spin Lagrangian $\mathcal{L}$ (if any) admits a usual perturbative power expansion in terms of these fields $\varphi$. Around a stable vacuum solution, this expansion starts with a quadratic kinetic term $\mathcal{L}^{(0)}$ with at most two derivatives and it goes on with vertices of various homogeneity degrees in $\varphi$: a cubic vertex $\mathcal{L}^{(1)}$, a quartic vertex $\mathcal{L}^{(2)}$, etc.

In the following we present four general facts (of which there is no proof in full generality but no counterexample has ever been found) that seem to capture universal properties of any massless higher-spin vertex.

*(i) Higher-spin vertices are local order by order in some length scale*

A function of the field and its derivatives (treated as independent variables) is said to be *local* if it depends only on a finite number of derivatives $\partial\varphi, \partial^2\varphi, \ldots, \partial^k\varphi$ (for some fixed integer $k$) and, moreover, if it depends only polynomially on these derivatives.

In the Lagrangian framework, the strong form of locality is the condition that the Lagrangian $\mathcal{L}$ must be a local function of the field $\varphi$, i.e., the total number of derivatives is bounded from above (so, in our conventions, the Lagrangian is a polynomial in the length parameter $\ell$). A weaker form of locality is the requirement that the Lagrangian $\mathcal{L}$ is *perturbatively local* in the sense that it admits a power series expansion in the fields and all their derivatives (so, in our conventions, each vertex must admit a power series expansion in the length scale $\ell$). Strictly speaking, this weak form of locality is rather a mild form of nonlocality because it is obviously not equivalent to the genuine requirement of locality. Nevertheless, it guarantees that somehow the nonlocality (if any) is under control: at each order in the length scale, the theory is local; the bound on the total number of partial derivatives is controlled by the power of $\ell$. Concretely, this means that there is no strong nonlocality (such as inverse powers of the Laplacian) and that, perturbatively, it can be treated as a local theory. Effective Lagrangians provide standard examples of perturbatively local theories.

We note that if at the cubic level one forgoes the assumption of perturbative locality, then the higher-spin gravitational minimal coupling around flat space would become automatically consistent. Remember that, in the early attempts to minimally couple higher-spin particles around flat space (Berends *et al.*, 1979; Aragone and Deser, 1980; Aragone and La Roche, 1982), the problem was that the higher-spin variation of the cubic Lagrangian creates terms $\delta_\varepsilon S^{\min} \sim \int\varepsilon\cdot(W\partial\varphi + \partial W\varphi)$ proportional to the spin-two linearized Weyl tensor $W$, where $\varepsilon$ is the higher-spin gauge parameter.

These terms cannot be compensated by an appropriate local gauge transformation for the spin-two field, since the linearized Weyl tensor (or its symmetrized and traceless derivative) does not vanish on shell. However, if one deals with wildly nonlocal operators and inserts the formal object "$\Box/\Box$" in front of the Weyl tensor, one can compensate the terms $\int \varepsilon \cdot (\frac{1}{\Box}\Box W \partial \varphi + \partial \frac{1}{\Box}\Box W \varphi)$ by appropriate nonlocal spin-two gauge transformations of the form $\delta h \sim \frac{1}{\Box}\partial^2(\varepsilon\partial\varphi + \partial\varepsilon\varphi)$, using the fact that, contrary to the Weyl tensor, the d'Alembertian of the Weyl tensor is proportional to the field equations for the spin-two field. Schematically, $\Box W \sim \partial C$ where $C$ denotes the (linearized) Cotton tensor which is itself a linear combination of the curl of the (linearized) Einstein tensor.

*(ii) Higher-spin vertices are higher derivative*

The higher-derivative property has been observed in all known examples of higher-spin cubic couplings. A summary of the general situation at the cubic level and in flat space is as follows.

*Cubic interactions* (Metsaev, 2006): In flat space, the total number $n$ of derivatives in any consistent local cubic vertex of type $s$-$s'$-$s''$ (with $s \le s' \le s''$) is bounded by

$$s' + s'' - s \le n \le s + s' + s''.$$

Therefore, the vertex contains at least $s''$ derivatives. In other words, the value of the highest spin involved ($s''$) gives the lowest number of derivatives that the cubic vertex must contain.

Note that this proposition applies to low and high spins. Examples of type 1-1-1 and 2-2-2 vertices are the cubic vertices in Yang-Mills and Einstein-Hilbert actions; they contain one and two derivatives, respectively. Examples of 2-$s$-$s$ vertices are, for low spins, the Lorentz minimal coupling ($s \le 3/2$) where the energy-momentum tensor involves two derivatives (also for $s = 2$) and, for high spins ($s > 2$) the higher-derivative nonminimal coupling mentioned before. The following two exotic properties of higher-spin particles are straightforward corollaries of results presented so far.

*Higher-derivative property*: In flat space, local cubic vertices including at least one massless particle of spin strictly higher than two contains three derivatives or more.

*Low-spin coupling*: In flat space, massless higher-spin particles couple nonminimally to low-spin particles. In (A)dS, they couple quasiminimally, thereby restoring Weinberg's equivalence principle (gravitational coupling) and the conventional definition of electric charge (electromagnetic coupling).

*(iii) Consistency requires an infinite tower of fields with unbounded spin*

A local cubic vertex is said to be perturbatively consistent at second order if it admits a local, possibly null, quartic continuation such that the resulting Lagrangian incorporating the cubic and associated quartic vertices (with appropriately modified gauge transformation laws) is consistent at second order in the perturbative coupling constant.

Note that the assumption of (perturbative) locality is crucial here. If this assumption is dropped, then consistency is automatic beyond the cubic level [see, e.g., the general theorem by Barnich and Henneaux (1993)] in the sense that any cubic vertex can be completed by nonlocal quartic vertices, etc. It is the assumption of (perturbative) locality that imposes very strong constraints on the set of possibilities.

In the local, non-Abelian deformation problem, a necessary requirement for the consistency of cubic vertices to extend to the quartic level is the closure of the algebra of gauge symmetries (at lowest order and possibly on shell). This imposes stringent constraints on the algebra in (A)dS spacetime (Fradkin and Vasiliev, 1987a): the presence of at least one higher-spin gauge field requires for consistency at quartic order an infinite tower of gauge fields with unbounded spin (more precisely the minimal spectrum seems to be a tower including all even spins). At the cubic level, the coupling constants of each cubic vertex are independent from each other. Another constraint coming from the consistency at the quartic level is that the coupling constants of the cubic vertices are expressed in terms of a single one. Surprisingly, similar results seem to apply in Minkowski spacetime (Metsaev, 1991).

When the spin is unbounded, higher-spin interactions are nonperturbatively nonlocal but perturbatively local, in the rough sense that the number of derivatives is controlled by the length scale. More precisely, at any finite order in the power expansion in $\ell$ the vertices are local, but if all terms are included, as usually required for consistency at the quartic level, then the number of derivatives is unbounded. Summarizing:

*Nonlocality*: The number of derivatives is unbounded in any perturbatively local vertex including an infinite spectrum of massless particles with unbounded spin.

The good news is that nonlocal theories do not automatically suffer from the higher-derivative problem. For nonlocal theories that are perturbatively local, the problem can be treated if the free theory is well behaved and if nonlocality is cured perturbatively [see Simon (1990) for a comprehensive review on this point].

*(iv) Massless higher-spin vertices are controlled by the infrared scale*

Concretely, in quantum field theory computations where massless particles are involved, one makes use of infrared and ultraviolet cutoffs where $\ell_{\rm IR}$ and $\ell_{\rm UV}$ denote the corresponding length scales ($\ell_{\rm UV} \ll \ell_{\rm IR}$). By definition of the cutoff prescription, the typical wavelength of physical excitations $\ell$ (roughly, the "size of the laboratory") must be such that $\ell_{\rm UV} < \ell < \ell_{\rm IR}$.

In low-spin physics, the ultraviolet scale is of the order of the Planck length $\ell_{\rm UV} \sim \ell_p$, interactions are controlled by that ultraviolet cutoff, and nonrenormalizable theories are weakly coupled in the low-energy regime $\ell \gg \ell_p$. In higher-spin gauge theory, the situation is turned upside down: interactions are controlled by the infrared cutoff $\ell_{\rm IR(higher\ spin)}$ (e.g., the AdS radius) and, since they are higher derivative, the theory is strongly coupled in the high-energy regime $\ell \ll \ell_{\rm IR(higher\ spin)}$.

#### D. Higher-spin symmetry breakings

While the transition from massless to massive higher-spin particles is well understood at the tree level via the Stückelberg mechanism, the higher-spin symmetry breaking remains unknown at the interacting level. The qualitative scenario is briefly discussed in Sec. IV.D.1 and, finally, a tentative summary of the possible scenarios is presented in Sec. IV.D.2.

#### 1. Higher-spin gauge symmetries are broken at the infrared scale

At energies of the order of the infrared cutoff for the higher-spin gauge theory, i.e., when $\ell \sim \ell_{\mathrm{IR(higher\ spin)}}$, higher-spin particles cannot be treated as "massless" any more. Instead, they get a mass of the order of $\ell^{-1}_{\mathrm{IR(higher\ spin)}}$ and, consequently, the higher-spin gauge symmetries are broken. Therefore, the no-go theorems do not apply any more. Hence, low-spin physics can be recovered at energy lower than the infrared cutoff of higher-spin gauge theory $\ell > \ell_{\mathrm{IR(higher\ spin)}}$.

*In Minkwoski spacetime*, a natural infrared scale of massless higher-spin particles is the ultraviolet scale of low-spin physics: $\ell_{\mathrm{IR(higher\ spin)}} \sim \ell_{\mathrm{UV(low\ spin)}} \sim \ell_p$. Then the corresponding massive higher-spin particles have masses not smaller than the Planck mass and the higher-spin interactions become "irrelevant" in the low-energy (sub-Planckian) regime. By naive dimensional analysis, in the high-energy (trans-Planckian) regime the scattering amplitudes should diverge since the theory is not (power-counting) renormalizable. However, for an infinite tower of higher-spin particles, the total scattering amplitudes may be extremely soft, or even finite. These possibilities are realized for tensile string theory around Minkowski spacetime where the ultraviolet scale is the string length $\ell_{\mathrm{UV(string)}} \sim \ell_s$, which is usually taken to be of the order of the ultraviolet scale for gravity $\ell_s \sim \ell_p$. The underlying symmetry principle behind such a phenomenon remains unknown, although the standard lore is that higher-spin symmetries play a key role in its understanding.

*In AdS spacetime*, the situation is drastically different because the natural infrared scale is the radius of curvature $\ell_{\mathrm{IR(higher\,spin)}} \sim R_{\mathrm{AdS}} \sim \lambda^{-1}$ and the ultraviolet scale may remain the Planck length $\ell_{\mathrm{UV(higher\ spin)}} \sim \ell_p$. The high-energy limit of higher-spin gauge theory is then equivalent to the flat limit $\ell \ll R_{\mathrm{AdS}}$. The Fradkin-Vasiliev cubic vertices and Vasiliev full nonlinear equations are precisely along these lines.

#### 2. Dynamical symmetry breaking: Spin one versus higher spin

The terminology "no-go theorem" assumes that the theorem (e.g., Coleman-Mandula's) is formulated negatively as the impossibility of realizing some idea (e.g., the mixing of internal and spacetime symmetries) under some conditions. If the idea proves to be possible then, retrospectively, the no-go theorem is read positively (by contraposition) as the necessity of some property (e.g., supersymmetry) for the idea to work. Similarly, one can speculate that maybe $S$-matrix no-go theorems (Weinberg, 1964; Coleman and Mandula, 1967; Porrati, 2008) on massless higher-spin particles should be read positively as providing a hint (if not a proof) that, at the infrared scale where these theorems are valid, an exotic mechanism, reminiscent of mass gap and confinement in QCD, must necessarily take place in any higher-spin gauge theory. At low energy, higher-spin particles must either decouple from low-spin ones or acquire a mass: in both cases, asymptotic massless higher-spin states are unobservable. Note that, usually, the elusive higher-spin symmetry breaking is presented as a "spontaneous" symmetry breaking such as the Brout-Englert-Higgs mechanism in the electroweak theory, but pursuing the analogy with QCD might be fruitful and one might think of a "dynamical" symmetry breaking where the Goldstone modes would be composite fields. From holographic arguments, Girardello, Porrati, and Zaffaroni (2003) indeed advocated for such a scenario whereby masses for all (even) higher-spin fields in Vasiliev's minimal theory in $\mathrm{AdS}_4$ are generated by quantum one-loop corrections while all low-spin gauge fields remain massless. We stress the direct similarity to the Schwinger mechanism in two-dimensional quantum electrodynamics (Schwinger, 1962) and the reminiscence to the saturation proposals for mass generation in three- and four-dimensional pure QCD; see, e.g., Aguilar and Papavassiliou (2008) and Aguilar, Binosi, and Papavassiliou (2010), and references therein.

A way to present a summary of the two phases of higher-spin gauge theory is by analogy with non-Abelian Yang-Mills theory (say quarkless QCD) whose main properties are listed as follows.

- High energy (unbroken symmetry): weak coupling ("asymptotic freedom")
- Low energy (broken symmetry): strong coupling ⇒ nonperturbative effects
  All asymptotic states must be massive ("mass gap") and singlet ("color confinement")

A plausible picture of non-Abelian higher-spin gauge theory is summarized as follows.

- High energy (unbroken symmetry): strong coupling
- Low energy (broken symmetry): decoupling of massless higher spins ⇐ no-go theorems
  All asymptotic higher-spin states must be massive and/or invariant under higher-spin symmetries

As one can see, perhaps the biggest difficulty with non-Abelian higher-spin gauge theory (with respect to its low-spin counterparts) is the absence of a phase with both unbroken symmetry and weak coupling (i.e., there is no analog of ultraviolet freedom for Yang-Mills theory, or infrared irrelevance for Einstein gravity), where the theory would be easier to study.

## V. FULLY INTERACTING EXAMPLE: VASILIEV'S HIGHER-SPIN GRAVITY

After repeating why a classically complete theory is key in higher-spin gravity, we lay out the salient features of Vasiliev's approach leading to a class of models that is not only the arguably most natural one but also a potentially viable brewing pot for actual semirealistic models of quantum gravity. We finally address the "state of the art" and what we believe to be some ways forward.

### A. Examples of non-Abelian gauge theories

It is not too much of an exaggeration to stress that the very existence of a fully interacting non-Abelian gauge field theory is a highly nontrivial fact, even at the classical level. Actually, looking to four spacetime dimensions, and focusing on bosonic gauge symmetries, notwithstanding the extreme importance that supersymmetry and matter couplings (which might be the same thing in higher-spin gravity) may play in order to have a phenomenologically viable model, one finds essentially three classes of models containing local degrees of freedom:

- Yang-Mills theories, i.e. the theory of self-interacting set of spin-one fields;
- general relativity, i.e. the theory of a self-interacting spin-two field; and
- higher-spin gravity, i.e., the theory of a self-interacting tower of critically massless even-spin fields.

Looking to their classical perturbation theories, one sees that higher-spin gravity distinguishes itself in the sense that it does not admit a strictly massless perturbative formulation on shell in terms of massless fields in flat spacetime. Instead it admits a generally covariant double perturbative expansion in powers of the following[22]:

- a dimensionless coupling constant $g$ counting the numbers of weak fields; and
- the inverse of a cosmological constant $\Lambda$ counting the numbers of pairs of derivatives.

Although higher-spin gravity still lacks a standard off-shell formulation, its on-shell properties nonetheless suggest a quantum theory in AdS spacetime in which localized higher-spin quanta interact in such a fashion that the resulting low-energy effective description be dominated by higher-derivative vertices such that the standard minimal spin-two couplings show up only as a subleading term. Thus one may think of higher-spin gravity as an effective flat-space quantum field theory with an *exotic cutoff*: a finite infrared cutoff, showing up as a cosmological constant in the gravitational perturbation theory, that at the same time plays the role of a massive parameter in higher-derivative interactions.

We mention again that the reason for this state of affairs can be explained directly in terms of the (mainly negative) results for higher-spin gauge theory in flat spacetime: if one removes $\Lambda$, i.e., attempts to formulate a strictly massless higher-spin gauge theory without any infrared cutoff, then one falls under the spell of various powerful (albeit restricted) no-go theorems concerning the couplings between massless fields with spin $s > 2$ and massless fields with spins $s \leq 2$ in flat spacetime.

As mentioned, the perhaps most striking constraint on gauge theories with vanishing cosmological constant $\Lambda = 0$ is the clear-cut clash between the equivalence principle, which essentially concerns the non-Abelian nature of spin-two gauge symmetries, and Abelian higher-spin gauge symmetry: on the one hand, all massless (as well as massive) fields must couple to a massless spin-two field via two-derivative vertices with the same universal coupling constant; on the other hand, such minimal couplings are actually incompatible with the free gauge transformations for spin-$s > 2$ fields as long as one assumes that these couplings play the dominant role at low energies.

[22]One can also define a Planck length $\ell_p = g\sqrt{|\Lambda|}$, but unlike general relativity, which contains only two derivatives, higher-spin gravity has no sensible expansion (in its unbroken phase) in powers of $\ell_p$. In this sense, the perturbation theory of higher-spin gravity is more similar in spirit to that of open string theory.

In other words, in flat spacetime there are severe no-go theorems forming a spin-two barrier that cannot be surpassed in the sense that massless particles of spins $s > 2$ cannot interact with massless particles of spins $s \leq 2$ provided the lower-spin sector contains finite minimal spin-two couplings. Thus, if one wishes to proceed in seeking strictly massless higher-spin gauge theories (with $\Lambda = 0$) then one is forced toward unnatural theories without any minimal spin-two couplings, whereas if one switches on a finite $\Lambda$ then one is naturally led into the realms of higher-spin gravity.

### B. The need for a complete theory

We now emphasize the need for a complete theory of higher-spin gravity already at the classical level, i.e., a consistent action principle, or alternatively, set of equations of motion, that contains a complete set of strongly coupled derivative corrections.

To this end, we return to the Fradkin-Vasiliev cancellation mechanism within the Fronsdal program: in the presence of a nonvanishing cosmological constant $\Lambda$, the Lorentz minimal cubic coupling (two derivatives) for a spin-$s$ field becomes embedded into the Fradkin-Vasiliev quasiminimal vertex terminating in the non-Abelian-type 2-$s$-$s$ vertex ($2s - 2$ derivatives) that remains consistent in the $\Lambda \to 0$ limit (Boulanger, Leclercq, and Sundell, 2008); this "top vertex" is thus the seed from which the subleading powers in $\Lambda$ are grown by imposing Abelian spin-$s$ gauge invariance. The crux of the matter, however, is that the cubic piece of a complete action (consistent to all orders) may in principle contain additional nonminimal interactions with more derivatives that are strongly coupled in the $\Lambda$ expansion.

Applying dimensional analysis one arrives at the following problem: for $\Lambda < 0$ the on-shell amplitude (Witten diagram) with three external massless gauge bosons need not vanish, and since $\Lambda$ now sets both the infrared cutoff (assuming the free theory to consist of standard tachyon-and-ghost free Fronsdal kinetic terms) and the mass scale for higher-derivative vertices, the contributions to the amplitude from vertices with $n$ derivatives grow similar to the $n$th power of a large dimensionless number. Thus, although the top (highest-derivative) vertex dominates the terms with fewer derivatives inside the quasiminimal coupling (including the Lorentz minimal coupling), it will in its turn be washed out by any genuinely nonminimal interaction, whose couplings (overall normalization in units of $\Lambda$) must hence be determined in order to estimate the three-particle amplitude.

Toward this end one may in principle work within a slightly refined Fronsdal program as follows: (i) fix a free Fronsdal action; (ii) parametrize all consistent cubic vertices including a *nonlocal Born-Infeld tail*, that is, a strongly coupled expansion in terms of Weyl tensors and their derivatives that cannot be replaced by a single effective Born-Infeld interaction with a finite coupling; and (iii) constrain the spectrum and cubic couplings by solving higher-order consistency conditions in the $g$ expansion (starting at quartic order).

However, without any guiding principle other than Lorentz and gauge invariance, this is an *a priori* intractable problem essentially due to the fact that the whole cubic tail must be fixed, which may require going to very high orders in the

$g$ expansion. Of course, in the simplest scenario, the complete cubic action could be fixed by quartic consistency, in which case there would be no interaction ambiguity at the cubic level. Thus, of all possible hypothetical outcomes the extreme cases are (i) quartic consistency suffices to completely fix the cubic action including its Born-Infeld tail; and (ii) quartic consistency rules out the cubic action altogether in which case the choice of free theory initiating the Fronsdal program would have to be revised.

In summary, to make the situation more tractable, one may resort to some additional guidance besides Lorentz and gauge invariance, or bias if one wishes to use that word, on what are suitable notions for "higher-spin multiplets," for selection of spectrum of fields, and "higher-spin tensor calculus," for construction of interactions.

How to proceed with this issue becomes most clear in higher-spin gravity: higher-spin gauge theories based on higher-spin algebras given by infinite-dimensional extensions of ordinary finite-dimensional spacetime isometry algebras. At this stage it is natural to rethink how unitary representations of the complete higher-spin algebra are mapped directly to fields living in infinite-dimensional geometries containing ordinary spacetime as a submanifold. Indeed one of the key instruments going into Vasiliev's formulation of fully nonlinear equations of motion for higher-spin gravities is *unfolded dynamics* (Vasiliev, 1988, 1989, 1990, 1994): a mathematically precise tool for manifestly diffeomorphism-invariant generalized spacetime reconstructions applying to finite-dimensional as well as infinite-dimensional cases.

### C. Vasiliev's equations

A working definition of higher-spin algebras developed by Fradkin, Konstein, and Vasiliev (Fradkin and Vasiliev, 1987a, 1988; Konshtein and Vasiliev, 1989, 1990) that has proven to be useful is that of Lie subalgebras of associative algebras obtained from the enveloping algebras of the spacetime isometry algebra by factoring out annihilators of their "fundamental," or ultrashort, unitary representations (singletons). In this setting, the higher-spin generators are monomials in the spacetime isometry generators, and higher-spin multiplets arise by tensoring together singletons (Flato and Fronsdal, 1978; Vasiliev, 2004c; Dolan, 2006) which introduce the germ of an extended object[23] as well as a precursor to AdS/CFT.

In order to construct higher-spin extensions of four-dimensional gravity, the simplest higher-spin algebras of this type can be realized in terms of elementary noncommutative twistor variables. As a result the full field content of a special class of higher-spin gravity theories, that we refer to as the minimal bosonic models and their matter-coupled and supersymmetrized extensions, is packed up into finite sets of "master" fields living on the product of a commutative spacetime and a noncommutative twistor space.

The feat of Vasiliev was then to realize that these master fields can be taken to obey remarkably simple-looking master equations built using exterior differential calculus on spacetime and twistor space, and star products on twistor space, reproducing the standard second-order equations in perturbation theory, in about the same way in which Einstein's equations arise inside a set of on-shell superspace constraints via constraints on the torsion and Riemann two-forms. As a result, Vasiliev's equations are diffeomorphic invariant, in the sense of unfolded dynamics, and perturbatively equivalent to a standard set of on-shell Fronsdal fields albeit with interactions given by a nonlocal double perturbative expansion resulting from the star products.

Looking at the twistor-space structure one sees that it services two purposes. In naive double perturbation theory, the expansion in the twistor variables combined with star products simply generates the higher-spin tensor calculus that one may take to define the minimal bosonic models after which one can naively strip off all the twistor variables by Taylor expansion and make contact with the standard tensorial equations of motion after having eliminated infinite towers of auxiliary fields.

A more careful look at these tensorial equations of motion reveals, however, Born-Infeld tails that are indeed strongly coupled, i.e., formally divergent for ordinary localized fluctuation fields and hence inequivalent to the canonical Born-Infeld interactions. Focusing on classical solutions in special sectors (boundary conditions) one then discovers that their resummation is tantamount to regularizations of star products that require one to perform the field-theoretic calculations inside the twistor space and not just by looking at Taylor expansions.

In other words, Vasiliev's complete higher-spin gravity is essentially nonlocal in spacetime but admits a quasilocal formulation in terms of star products on the direct product of commutative spacetime and noncommutative twistor space, where one can then proceed building classical observables and geometries for the theory.

This somewhat awkward albeit mathematically completely well-defined situation raises the issue of whether Vasiliev's equations should be viewed as natural representative for higher-spin gravity or not? Since there are no other known examples of classes of higher-spin gravities with local degrees of freedom, it is difficult to make any direct comparisons. However, lessons can be drawn by looking at the AdS/CFT correspondence.

### D. AdS/CFT correspondence: Vasiliev's theory from free conformal fields

In the previous sections we attempted to define a relation between the $S$ matrix and Lagrangian approaches in the case of vanishing cosmological constant. Switching on the cosmological constant the notion of the $S$ matrix becomes deformed into that of a holographic conformal field theory. Thus, one way of assessing to what extent a higher-spin gravity is "natural" is to ask oneself to what extent its dual conformal field theory is natural.

[23]The idea of treating algebras and their representations on a more equal footing, namely as various left-, right-, or two-sided modules arising inside the enveloping algebra and its tensor products, is in the spirit of modern algebra and deformation quantization. Indeed, further development of these thoughts lead to first-quantized systems linking higher-spin gravities to tensionless strings and branes (Engquist and Sundell, 2006).

Shortly after Maldacena's version of the AdS/CFT conjecture, which was derived within a stringy context involving strong- and weak-coupling dual descriptions of branes, the question came as to what the antiholographic dual of a weakly coupled CFT could be. Since a free CFT has infinitely many conserved currents of arbitrary spin, in addition to the stress-energy tensor, it was natural to expect the AdS dual to be a higher-spin gauge theory containing a graviton. With a noticeable precursor (Bergshoeff *et al.*, 1988), such ideas emerged progressively in a series of papers (Haggi-Mani and Sundborg, 2000; Konstein, Vasiliev, and Zaikin, 2000; Shaynkman and Vasiliev, 2001; Sundborg, 2001; Sezgin and Sundell, 2001a, 2002b, 2005; Witten, 2001; Klebanov and Polyakov, 2002; Mikhailov, 2002): the idea was born in the context of the type-IIB theory on $AdS_5 \times S^5$ (Haggi-Mani and Sundborg, 2000; Sezgin and Sundell, 2001a; Sundborg, 2001), and then pursued in a more general $D$-dimensional context, first at the level of kinematics (Konstein, Vasiliev, and Zaikin, 2000; Shaynkman and Vasiliev, 2001) and later at a dynamical level leading to the duality conjecture between a pure bosonic higher-spin gravity in any dimension and a theory of (a large number of) free conformal scalars in the vector representation of an internal symmetry group (Witten, 2001; Mikhailov, 2002; Sezgin and Sundell, 2002b), refined to include the strongly coupled fixed points of the three-dimensional $O(N)$ model and the Gross-Neveu model, in Klebanov and Polyakov (2002) and Sezgin and Sundell (2005), respectively. More precisely, the bilinear operators formed out of free fields couple to higher-spin sources identified as the boundary data of bulk higher-spin gauge fields. One should stress that although the boundary CFT is quadratic, it is nevertheless nontrivial since the bilinear operators actually couple to background sources; therefore the bulk dual theory is interacting. The concrete relation with Vasiliev's unfolded equations in four and five dimensions was elaborated on by Sezgin and Sundell (2001a, 2002b, 2005), and the fully nonlinear bosonic higher-spin gravity in any dimension was then found in Vasiliev (2003).

The agreement between Vasiliev's four-dimensional higher-spin gravity and the sector of bilinear operators formed out of free conformal scalars and spinors in three dimensions was verified at the level of scalar cubic couplings by Petkou (2003) and Sezgin and Sundell (2005), and, more recently, at the general cubic level by Giombi and Yin (2009, 2010) under certain prescriptions which still remain to be spelled out in their entirety. Thus the question of whether Vasiliev's higher-spin gravity is natural or not is equivalent to the question of whether free scalars (and spinors) are natural building blocks for three-dimensional conformal field theories with (unbroken or weakly broken) higher-spin currents. Or stated differently, thinking about Vasiliev's higher-spin gravity is about as natural as thinking about three-dimensional conformal field theories starting from free fields.

Intermediate developments are given by Das and Jevicki (2003), Leigh and Petkou (2003), Leonhardt, Meziane, and Ruehl (2003), Bonelli (2004), Leonhardt and Ruehl (2004), Hartnoll and Prem Kumar (2005), Ruehl (2005), Diaz and Dorn (2006), Elitzur *et al.* (2007), and Yonge (2007). More recently, the full checks of the conjecture for $AdS_4/CFT_3$ at the cubic level (Giombi and Yin, 2009, 2010) prompted a revived interest in the correspondence.[24] For instance, the conjecture was generalized in the presence of a Chern-Simons gauge field on the three-dimensional boundary (Aharony, Gur-Ari, and Yacoby, 2011; Giombi *et al.*, 2011). Another duality was proposed relating bosonic Vasiliev's theory on de Sitter bulk spacetime $dS_4$ and fermionic scalar fields Euclidean $CFT_3$ (Anninos, Hartman, and Strominger, 2011). The thermodynamic behavior of Vasiliev's higher-spin gravity was inferred from Conformal Field Theory computations (Shenker and Yin, 2011). Several attempts toward a constructive derivation of the bulk dual of a free CFT in the vector representation have been proposed, such as the bilocal field approach (Das and Jevicki, 2003; Jevicki, Jin, and Ye, 2011; Koch *et al.*, 2011) and the renormalization group (Douglas, Mazzucato, and Razamat, 2010).

Here we also stress that AdS/CFT is more to gauge field theory than what standard global-symmetry current algebra is to quantum field theory, essentially since the boundary currents are coupled to bulk gauge fields. Thinking of free conformal scalar fields, the case of two dimensions is very special, in that the stress tensor forms a closed operator algebra (the Virasoro algebra). Indeed, already in three dimensions one encounters the full higher-spin current algebra as one expands the operator product between two stress tensor generators (including a scalar current rather than a central term). Thus, in the case of four-dimensional theories of quantum gravity, it seems that the simplest, most natural procedure would be to start from Vasiliev-like higher-spin gravities and then seek symmetry breaking mechanisms that correspond to breaking the higher-spin currents, followed by taking limits in which these decouple from operator product expansions.

In fact, by putting more emphasis on the AdS/CFT correspondence, one provides further arguments (Girardello, Porrati, and Zaffaroni, 2003) as to why higher-spin gravity is a natural framework for seeking ultraviolet completions of general relativity. Ordinary general relativity together with various matter couplings (and without exotic vertices) may then appear at low energies as the result of the dynamical higher-spin symmetry breaking mechanism induced by radiative corrections proposed by Girardello, Porrati, and Zaffaroni (2003), provided that the induced noncritical mass gaps grow large at low energies. If so, higher-spin gravity may bridge general relativity and string theory, which might be needed ultimately in order to achieve nonperturbative unitarity.

[24]Note that recently, in the $AdS_3/CFT_2$ framework based on the bulk theories provided by Blencowe (1989) and Prokushkin and Vasiliev (1999), many interesting works appeared; see, e.g., Henneaux and Rey (2010), Campoleoni *et al.* (2010), Campoleoni, Fredenhagen, and Pfenninger (2011), Castro, Lepage-Jutier, and Maloney (2011), Chang and Yin (2011), Gaberdiel and Gopakumar (2011), Gaberdiel, Gopakumar, Hartman, and Raju (2011), Gaberdiel, Gopakumar, and Saha (2011), Gaberdiel and Hartman (2011), Gaberdiel and Vollenweider (2011), Kraus and Perlmutter (2011), and references therein.

### E. Emergence of extended objects

We now comment on the similarities and dissimilarities between higher-spin gravity, with its double perturbative expansion in terms of the dimensionless coupling $g$ and the cosmological constant $\Lambda$, and string theory, with its double perturbative expansion in terms of the string coupling $g_s$ and the string tension $T_s$. On the one hand, both of these theories are genuine higher-derivative theories which implies that at fixed orders in $g$ and $g_s$, respectively, there are vertices with fields of sufficiently high spins involving arbitrarily large inverse powers of their massive parameters $\Lambda$ and $T_s$, respectively. Thus, in order to understand their respective second quantizations ($g$ and $g_s$ expansions), one must first obtain a sufficiently sophisticated understanding of their first quantizations ($\Lambda$ and $T_s$ expansions). Now to its advantage string theory offers a massless window where its first-quantization is weakly coupled, whereas in dealing with unbroken higher-spin gravity one must face the whole packed-up content of its master fields.

A striking similarity between open string theory and higher-spin gravities occurs when one considers (Konstein, Vasiliev, and Zaikin, 2000) extensions of the higher-spin algebra by an internal, associative algebra [see also Vasiliev (2004b, 2006)]. In such cases, there exist colored, massless spin-two fields resembling the spin-two states of open strings. These states can be given Chan-Paton factors since their interactions are based on an associative algebra. This similarity was pointed out by Francia and Sagnotti (2003, 2006) to which we refer the interested reader for related discussions. We note that the existence of colored gravitons in extended higher-spin theories does not enter in contradiction with the results of Boulanger *et al.* (2001), since there it was assumed that the fields considered could have spin two at most and the background was taken to be flat.

At the classical level, the possibilities remain of having consistent truncations of closed-string theory down to higher-spin gravity, and of higher-spin gravity down to general relativity. For example, both of these types of truncations may turn out to be relevant in the case of the hypothetical tensionless type-IIB closed-string theory on $AdS_5 \times S^5$ that should be the antiholographic dual of free four-dimensional maximally supersymmetric Yang-Mills theory in its $1/N$ expansion (Sundborg, 2001; Sezgin and Sundell, 2002b). Here the hypothetical five-dimensional maximally supersymmetric higher-spin gravity [for the linearized theory, see Sezgin and Sundell (2001b)] can be identified as the Kaluza-Klein reduction of the "bent" first Regge trajectory of the flat-space string theory (Bianchi, Morales, and Samtleben, 2003; Sezgin and Sundell, 2005). The full tensionless string theory then involves a much larger higher-spin symmetry algebra bringing in mixed-symmetry fields with critical masses such that they fit into multipletons (Bianchi, Morales, and Samtleben, 2003; Sezgin and Sundell, 2005). As for consistent truncations of higher-spin gravity down to possibly matter-coupled (super)gravities, a look at the state of affairs in gauged supergravities arising from sphere reductions (de Wit and Nicolai, 1987; Nastase, Vaman, and van Nieuwenhuizen, 1999; Cvetic *et al.*, 2000) suggests that one should conjecture their existence in the case of maximal supersymmetry.

As far as the type-IIB superstring is concerned, its graviton in ten-dimensional flat spacetime admits a deformation into a graviton of five-dimensional anti–de Sitter spacetime. More generally, a key physical effect of having a negative cosmological constant is the formation of cusps on spiky closed strings (Gubser, Klebanov, and Polyakov, 2002; Kruczenski, 2005) [for generalizations to membranes, see Sezgin and Sundell (2002b)]. At the cusps, solitonic bound states arise, carrying the quantum numbers of singletons (Engquist and Sundell, 2006). In the case of folded long strings, the resulting two-singleton closed-string states are massless symmetric tensors with large spin realized by Flato and Fronsdal (1978). In the extrapolation of this spectrum to small spins, which is tantamount to taking a tensionless limit, resides the anti–de Sitter graviton. Engquist and Sundell (2006) argued that in order for the tensionless limit to lead to a closed-string field theory with nontrivial interactions, it should be combined with sending the cosmological constant to infinity in a discretized model with fixed mass parameter. This yields first-quantized $(0+1)$-dimensional models describing multisingleton states. These have continuum limits given by Wess-Zumino-Witten models with gauged $W$ algebras (rather than Virasoro algebras) that can be realized in terms of symplectic bosons (Engquist and Sundell, 2006; Engquist, Sundell, and Tamassia, 2007) and real fermions.

Engquist and Sundell (2006) furthermore argued that the coupling of these first-quantized models to higher-spin background fields requires their extension into Poisson sigma models in one higher dimension containing the original systems on their boundaries. In particular, in the case of a single singleton, that represents one string parton or membrane parton, these couplings are mediated via boundary and bulk vertex operators of a topological open string in the phase space of a singleton that is a particular example of the $C$ model of Cattaneo and Felder (2000); the consistency of this first-quantized system with disk topology then requires Vasiliev's equations.

The resulting physical picture provides a concrete realization for an extended object that is already present in the Flato-Fronsdal formula. This picture also matches well with the holographic framework: just as the weak-coupling stress tensor is deformed directly into the strong-coupling stress tensor on the CFT side, the graviton in higher-spin gravity is the continuation of that in closed-string theory. Moreover, the fact that topological $C$ models underlie general associative algebras directly explains why Vasiliev's equations are compatible with internal Chan-Paton factors.

One is thus led to contemplate a more profound underlying framework for quantum field theory in general, based on Poisson sigma models and topological summation and that would naturally incorporate the gauge principle as well as radiative corrections; in the case of the topological open string, the additional zero modes arising from cutting holes in the disk can then provide a first-quantized realization of the massive Goldstone modes of the Girardello-Porrati-Zaffaroni mechanism (Girardello, Porrati, and Zaffaroni, 2003).

## VI. CONCLUSIONS AND OUTLOOK

We discussed the key mechanism by which higher-spin gravity evades the no-go theorems and, in particular, how the equivalence principle is reconciled with higher-spin gauge symmetry.

Starting in flat spacetime, massless higher-spin particles cannot be reconciled with the equivalence principle. Nevertheless, the Weinberg-Witten theorem does not rule out higher-derivative energy-momentum tensors made out of higher-spin gauge fields. Hence massless higher-spin particles may couple nonminimally to a massless spin-two particle. However, in such a case the low-energy Weinberg theorem rules out the self-coupled Einstein-Hilbert action and minimally coupled matter, in particular, with low spins (i.e., $s = 0$, $1/2$, and 1), in contradiction with observations.

Going to AdS spacetime, the Lorentz minimal coupling reappears but only as a subleading term in a strongly coupled derivative expansion. In order to do weakly coupled calculations, even at the cubic level for higher-spin gravity, one thus needs a complete theory with the full derivative expansion under control. The simplest available candidate at the moment is Vasiliev's theory.

Remarkably, not only does it resolve all the difficulties reported in the no-go theorems, but actually it also seems to be the simplest unbroken higher-spin gravity in the sense that it corresponds, via AdS/CFT, to a free conformal field theory with only scalar and/or fermion fields, albeit in large number.

Two major open problems that need to be considered are as follows:

- Can the Fronsdal program be pursued until quartic vertices?
  It is not totally excluded that the answer be "no" under the requirement of perturbative locality. Moreover, scattering amplitudes in AdS can be defined without using an action principle, and the recent checks of the AdS/CFT correspondence in the context of higher-spin gravity at the cubic level were done by using the unfolded formalism in the bulk theory.
- Does the dimensionless coupling in higher-spin gravity become large at low energies in AdS?
  If the answer is "yes" then higher-spin gravity is a promising candidate for an effective quantum gravity theory. Drawing on our experience with QCD, since higher-spin gravity has been observed to be extremely soft at high energy, it is tempting to think that the coupling constant becomes weak in the ultraviolet and should grow in infrared, such that the dynamical higher-spin symmetry breaking, which is present already in the ultraviolet, gives rise to a finite mass gap allowing the identification of the low-energy and low-spin regime.

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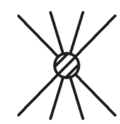
## APPENDIX A: WEINBERG LOW-ENERGY THEOREM: *S* MATRIX AND LAGRANGIAN DICTIONARY

Weinberg (1964) obtained stringent constraints on $S$-matrix elements by considering the effects tied to the emission of soft massless quanta.

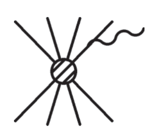
Consider an $S$-matrix element with $N$ external particles of momenta $p_i^\mu$ ($i = 1, 2, \ldots, N$) corresponding to the Feynman diagram

$$\mathcal{A}(p_1, \ldots, p_N) \quad = $$  , (A1)

where all external momenta $p_i$ are on their respective mass shells. For simplicity, all momenta are taken to be ingoing and the polarizations of these particles are left implicit in $\mathcal{A}$.

### 1. Emission of a massless particle: Lorentz versus gauge invariances

The amplitude for the further emission (or absorption) from any leg of a single massless spin-$s$ particle of momentum $q^\mu$ and polarization $\epsilon_{\mu_1\cdots\mu_s}(q)$ is denoted by $\mathcal{A}(p_1, \ldots, p_N; q, \epsilon)$:

$$\mathcal{A}(p_1, \ldots, p_N; q, \epsilon) \;=\; \epsilon_{\mu_1\ldots\mu_s}(q)\, \mathcal{A}^{\mu_1\cdots\mu_s}(p_1, \ldots, p_N; q) \;=$$  .

In general, the line of this extra particle can be attached to any other line, either internal or external.

In relativistic quantum field theory, the polarizations are not Lorentz-covariant objects: under Lorentz transformations, one has

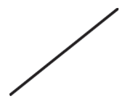
$$\epsilon_{\mu_1\cdots\mu_s}(q) \longrightarrow \epsilon_{\mu_1\cdots\mu_s}(q) + s q_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}(q)$$

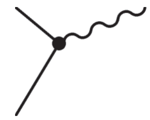
for some symmetric tensor $\xi$ where the round brackets denote complete symmetrization over the indices. This property is well known for massless particles and is the counterpart of gauge invariance in the Lagrangian approach. Lorentz invariance of the $S$ matrix and the decoupling of spurious degrees of freedom thus require the condition

$$q_{\mu_1}\mathcal{A}^{\mu_1\cdots\mu_s}(p_1,\ldots,p_N;q)=0,\quad \forall\ q. \tag{A2}$$

### 2. Cubic vertices

In the particular case where the Feynman diagram (A1) is a single straight line, i.e., it describes the free propagation of a single particle, then the modified Feynman diagram essentially is the tree-level process

$\mathcal{A}(p_1,p_2) \quad =$ 

$\mathcal{A}(p_1,p_2;q,\epsilon) \quad =$ 

so $\Gamma^{\mu_1\cdots\mu_s}(p_1,p_2;q):=\mathcal{A}^{\mu_1\cdots\mu_s}(p_1,p_2;q)$ is the part of the cubic vertex which corresponds to the Noether current in the Lagrangian approach. The conservation of the Noether current in the Lagrangian approach is equivalent to the Lorentz-invariance condition (A2) in the $S$-matrix approach.

We see this in more detail by considering a cubic vertex of type $s$-$s'$-$s'$ with $s\neq s'$. The massless particle of spin $s$ is of arbitrary momentum $q^\mu$ (so off shell) while the two particles of spins $s'$ are on shell with respective momenta $p_1$ and $p_2$. Writing explicitly the polarizations $\epsilon^{(1)}(p_1)$ and $\epsilon^{(2)}(p_2)$ of the two spin-$s'$ particles, the cubic vertex takes the form

$$\Gamma^{\mu_1\cdots\mu_s}(p_1,p_2;q)=\Gamma^{\mu_1\cdots\mu_s|\nu_1\cdots\nu_{s'}|\rho_1\cdots\rho_{s'}}(p_1,p_2;q)\times\epsilon^{(1)}_{\nu_1\cdots\nu_{s'}}(p_1)\epsilon^{(2)}_{\rho_1\cdots\rho_{s'}}(p_2).$$

In the Lagrangian language, the cubic interaction term corresponding to the cubic vertex is, without loss of generality, of the form

$$S^{(1)}[\varphi_s,\varphi_{s'}]:=\int d^Dx\mathcal{L}^{(1)},\qquad \mathcal{L}^{(1)}:=\varphi_{\mu_1\cdots\mu_s}\Theta^{\mu_1\cdots\mu_s}(\varphi_{s'},\varphi_{s'}),$$

where $\Theta^{\mu_1\cdots\mu_s}$ is bilinear in $\varphi_{s'}$. More precisely, we write the requirement of gauge invariance of the cubic action $S^{(1)}[\varphi_s,\varphi_{s'}]$ under linearized spin-$s$ gauge transformations $\delta^{(0)}_s\varphi_{\mu_1\cdots\mu_s}=s\partial_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$:

$$\delta^{(0)}_sS^{(1)}+\delta^{(1)}_sS^{(0)}=0,$$

where $S^{(0)}$ denotes the free part of the action, $\delta^{(0)}_s$ denotes the free spin-$s$ gauge transformations, and $\delta^{(1)}_s$ denotes the gauge transformations taken at linear order in the fields $\{\varphi_{s'},\varphi_s\}$ and linear in the spin-$s$ gauge parameter $\xi_{\mu_1\cdots\mu_{s-1}}$. The above equation implies that $\Theta^{\mu_1\cdots\mu_s}$ is a conserved current:

$$\partial_{\mu_1}\Theta^{\mu_1\cdots\mu_s}(\varphi_{s'},\varphi_{s'})\approx 0,$$

so that the Lorentz-invariance condition (A2) in the $S$-matrix approach is indeed equivalent to the conservation of the Noether current in the Lagrangian approach.

In momentum space,

$$S^{(1)}=\int d^Dqd^Dp_1d^Dp_2\delta(p_1+p_2+q)\Gamma^{\mu_1\cdots\mu_s|\nu_1\cdots\nu_{s'}|\rho_1\cdots\rho_{s'}}\times(p_1,p_2;q)\varphi_{\mu_1\cdots\mu_s}(q)\varphi_{\nu_1\cdots\nu_{s'}}(p_1)\varphi_{\rho_1\cdots\rho_{s'}}(p_2).$$

The cubic vertex with the lowest number of derivatives is of the form

$$\Gamma^{\mu_1\cdots\mu_s|\nu_1\cdots\nu_{s'}|\rho_1\cdots\rho_{s'}}(p_1,p_2;q)\propto\Gamma^{\mu_1\cdots\mu_s}(p_1,p_2;q)\eta^{\nu_1\rho_1}\cdots\eta^{\nu_{s'}\rho_{s'}},$$

where there is an implicit symmetrization over all $\nu$ indices and

$$\Gamma^{\mu_1\cdots\mu_s}(p_1,p_2;q)\propto(p_1-p_2)^{\mu_1}\cdots(p_1-p_2)^{\mu_s}$$

is the cubic vertex for a scalar particle coupled to a spin-$s$ massless particle. This coupling is called minimal in the sense that it contains the minimal amount of derivatives and also because it corresponds to a coupling with the Berends–Burgers–van Dam conserved currents associated with the rigid symmetries $\delta\varphi_{s'}(k)=i\xi^{\mu_1\cdots\mu_{s-1}}k^{\mu_1}\cdots k^{\mu_{s-1}}\varphi_{s'}(k)$ (Berends, Burgers, and van Dam, 1986) [see also Bekaert, Joung, and Mourad (2009) for more detail]. In the low-energy limit $q\rightarrow 0$, the only surviving cubic interaction is indeed the minimal coupling with $s$ derivatives.

The Lorentz-invariance condition (A2) on the amplitude $\mathcal{A}(p_1,\ldots,p_N;q,\epsilon)$ for the further emission (or absorption) of a soft massless spin-$s$ particle implies the conservation law of order $s-1$ on the $N$ external momenta (1) where each inserted minimal vertex $\Gamma^{\mu_1\cdots\mu_s}(p_i,-p_i-q;q)$ came up with a coupling constant $g^{(s)}_i$ [for more detail, see, e.g., Weinberg (1995), Sec. 13.1, or Blagojevic (2002), Appendix G]. Equivalently, these conservation laws can be obtained from the Noether charges associated with the above-mentioned rigid symmetries.

## APPENDIX B: WEINBERG-WITTEN THEOREM: A LAGRANGIAN REFORMULATION

### 1. Weinberg-Witten theorem

Weinberg and Witten (1980) designed their no-go theorem to eliminate "emergent gravity" theories where the graviton is a bound state of particles with spin one or lower. Its proof involves $S$-matrix manipulations which will be discussed in more detail in the next section on its refined version. If one assumes locality, then it becomes surprisingly easy to prove the Lagrangian version of the Weinberg-Witten theorem. Let $[s]$ denote the integer part of the spin $s$.

*Lemma*: Any local polynomial which is at least quadratic in a spin-$s$ massless field, nontrivial on shell and gauge invariant, must contain at least $2[s]$ derivatives.

*Proof:* Corollary 1 of Bekaert and Boulanger (2005) states that, on shell, any local polynomial which is gauge invariant may depend on the gauge fields only through the Weyl-like tensors. The latter tensors contain $[s]$ derivatives, thus the lemma follows. ■

A straightforward corollary of this lemma is a version of the Weinberg-Witten theorem.

Weinberg-Witten theorem (Lagrangian formulation):

(i) Any perturbatively local theory containing a charge current $J^\mu$ which is nontrivial, Lorentz covariant, and gauge invariant, forbids massless particles of spin $s > 1/2$.

(ii) Any perturbatively local theory containing a Lorentz-covariant and gauge-invariant energy-momentum tensor $T^{\mu\nu}$ forbids massless particles of spin $s > 3/2$.

*Proof*: In the free limit, any Noether current in a perturbatively local theory must be a quadratic local polynomial. For massless fields of spin $s > 1/2$, the lemma implies that this polynomial must contain at least two derivatives (or four derivatives if $s > 3/2$). However, the charge current contains one derivative and the energy-momentum tensor contains two derivatives. ■

The lower bound $s > 3/2$ of this version is slightly weaker than the lower bound $s > 1$ of the original Weinberg-Witten theorem (Weinberg and Witten, 1980). Anyway the case $s = 3/2$ is low spin and thereby is not a main concern of this paper.

#### 2. Refinement of Weinberg-Witten theorem

Porrati (2008) takes gauge invariance into account in order to still use Weinberg-Witten's argument but in a context where the stress-energy tensor need not be gauge invariant (or Lorentz covariant, which is the same in a second-quantized setting) any more.

In the original work (Weinberg and Witten, 1980) a particular matrix element was considered: elastic scattering of a spin-$s$ massless particle off a single soft graviton. The initial and final polarizations of the spin-$s$ particle are identical, say $+s$, its initial momentum is $p$ and its final momentum is $p + q$. The graviton is *off shell* with momentum $q$. The matrix element is

$$\langle +s, p+q|T_{\mu\nu}| + s, p\rangle. \tag{B1}$$

In the soft limit $q \to 0$ the matrix element is completely determined by the equivalence principle, as recalled above when reviewing Weinberg's low-energy theorem. Using the relativistic normalization for one-particle states $\langle p|p'\rangle = 2p_0(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}')$, we get

$$\lim_{q\to 0}\langle +s, p+q|T_{\mu\nu}| + s, p\rangle = p_\mu p_\nu. \tag{B2}$$

This is tantamount to saying that, at low energy, the only possible coupling between gravity and everything else is done via the minimal coupling procedure, bringing no more than two derivatives (or one if the spin is half-integer) in the interaction. More precisely, among all possible interaction terms there must always be that coming from minimal coupling $\partial \to \partial + \kappa\Gamma(h)$, with the nonvanishing coefficient $\kappa$ related to Newton's constant.

Since $q$ is spacelike (off-shell soft graviton), one goes in the frame in which $q^\mu = (0, -\mathbf{q})$, $p^\mu = (|\mathbf{q}|/2, \mathbf{q}/2)$, and $p^\mu + q^\mu = (|\mathbf{q}|/2, -\mathbf{q}/2)$ (the massless spin-$s$ particle is on shell), and deduces that a rotation $R(\theta)$ by an angle $\theta$ around the $\mathbf{q}$ direction acts on the one-particle states as $R(\theta)|p, +s\rangle = \exp(\pm i\theta s)|p, +s\rangle$, $R(\theta)|p+q, +s\rangle = \exp(\mp i\theta s)|p+q, +s\rangle$ since $R(\theta)$ is a rotation of $\theta$ around $\mathbf{p}$ but of $-\theta$ around $\mathbf{p}+\mathbf{q} = -\mathbf{p}$. Decomposing $T_{\mu\nu}$ under space rotations in terms of spherical tensors as the complex spin zero tensor $T_{0,0}$ plus the real components $\{T_{1,m}\}^1_{m=-1}$ and $\{T_{2,m}\}^2_{m=-2}$, one can write the following relation:

$$\begin{aligned} e^{\pm 2i\theta s}&\langle +s, p+q|T_{j,m}| + s, p\rangle \\ &= \langle +s, p+q|R^\dagger T_{j,m}R| + s, p\rangle \\ &= e^{i\theta m}\langle +s, p+q|T_{j,m}| + s, p\rangle \end{aligned} \tag{B3}$$

which admits, for $s > 1$, the only solution $\langle +s, p+q|T_{\mu\nu}| + s, p\rangle = 0$. If $T_{\mu\nu}$ is a tensor under Lorentz transformations then this implies that $\langle +s, p+q|T_{\mu\nu}| + s, p\rangle = 0$ in all frames, in contradiction with the equivalence principle (B2). This seems to kill gravity itself, but of course in that case as usually happens in gauge theories, $T_{\mu\nu}$ is not a Lorentz tensor (which is the same as saying that $T_{\mu\nu}$ is not gauge invariant).

One can define matrix elements for $T_{\mu\nu}$ that transform as Lorentz tensors only at the price of introducing nonphysical, pure-gauge states. This is what Porrati (2008) did in order to accommodate the Weinberg-Witten argument to gauge theories for spin-$s$ fields, $s > 1$ and prove that massless higher-spin particles cannot exist around a flat background if their tensor $T_{\mu\nu}$ appearing in $\langle +s, p+q|T_{\mu\nu}| + s, p\rangle$ complies with the equivalence principle (B2).

Denoting by $v$ all one-particle spin-$s$ states, whether or not spurious (pure gauge), the matrix element under consideration is denoted $\langle v', p+q|T_{\mu\nu}|v, p\rangle$. The method used by Porrati (2008) in order to derive the $S$ matrix is to perform the standard perturbative expansion of the effective action (where $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$)

$$\begin{aligned} A = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\tilde{h}^*_{\mu\nu}(q) \\ \times(\langle v', p+q|T^{\mu\nu}|v, p\rangle + \mathcal{T}^{\mu\nu}) + \mathcal{O}(h^2). \end{aligned} \tag{B4}$$

The linear interaction terms include the matrix element and another effective tensor $\mathcal{T}^{\mu\nu}$ which summarizes the effect of any other matter field but that we omit from now on without loss of generality. To linear order, Einstein's equations become

$$\begin{aligned} L_{\mu\nu}{}^{\rho\sigma}h_{\rho\sigma}(q) &= 16\pi G[\langle v', p+q|T^{\mu\nu}|v, p\rangle], \\ L_{\mu\nu}{}^{\rho\sigma} &= \delta^\rho_\mu\delta^\sigma_\nu q^2 - \eta_{\mu\nu}\eta^{\rho\sigma}q^2 - \delta^\rho_\mu q_\nu q^\rho \\ &\quad - \delta^\rho_\nu q_\mu q^\rho + \eta^{\rho\sigma}q_\mu q_\nu + \eta^{\mu\nu}q_\rho q_\sigma, \end{aligned} \tag{B5}$$

which is nothing but the Fourier transform of the symmetric differential operator $\vec{\mathcal{G}}^{\rho\sigma}_{\mu\nu}$ acting on the spin-two field $h_{\mu\nu}$ in the linearized (in $h_{\mu\nu}$) Einstein equations

$$\vec{\mathcal{G}}^{\rho\sigma}_{\mu\nu} h_{\rho\sigma} = \kappa T_{\mu\nu}(\varphi_s, \varphi_s) + \mathcal{O}(\kappa^2), \tag{B6}$$

where $T_{\mu\nu}(\varphi_s, \varphi_s)$ is the tensor bilinear in the spin-$s$ field $\varphi_s$ that gives the cubic 2-$s$-$s$ vertex in the action principle

$$S[h_{\mu\nu}, \varphi_s] = S^{\rm PF}[h_{\mu\nu}] + S^{\rm Fr}[\varphi_s] + \frac{\kappa}{2}\int d^D x h_{\mu\nu} T^{\mu\nu}(\varphi_s, \varphi_s) + \mathcal{O}(\kappa^2). \tag{B7}$$

To this same order in the metric fluctuation, a necessary condition is given by Porrati (2008) for the consistency of the gravitational interactions of high-spin massless particles:

$$\langle v, p+q | T^{\mu\nu} | v_s, p\rangle = L_{\mu\nu}{}^{\rho\sigma}\Delta_{\rho\sigma}(q) \tag{B8}$$

with $\Delta_{\rho\sigma}(q)$ analytic in a neighborhood of $q = 0$.

Equation (B4) provided Porrati the most general condition for the decoupling of the so-called spurious polarization $v_s$ (that we call sometimes "pure-gauge" states) from the $S$-matrix amplitudes. Decoupling occurs when one can reabsorb the change in the matrix element due to the substitution $v \to v + v_s$ with a local field redefinition of the graviton field.

In the Lagrangian language, this can be seen to originate from the requirement of gauge invariance of the cubic action $S^{(1)} := \frac{1}{2}\int d^D x h_{\mu\nu} T^{\mu\nu}(\varphi_s, \varphi_s)$ under linearized gauge transformations

$$\delta^{(0)} h_{\mu\nu} = 2\partial_{(\mu}\epsilon_{\nu)}, \tag{B9}$$

$$\delta^{(0)}\varphi_{\mu_1\cdots\mu_s} = s\,\partial_{(\mu_1}\epsilon_{\mu_2\cdots\mu_s)} \tag{B10}$$

up to terms that vanish on the surface of the free field equations:

$$\delta^{(0)} S^{(1)} + \delta^{(1)} S^{(0)} = 0, \tag{B11}$$

where $S^{(0)}$ denotes the free part of the action and $\delta^{(1)}$ denotes the gauge transformations taken at linear order in the field $\{h, \varphi\}$. The above equation can be rewritten as

$$\int d^D x\Big[\delta^{(0)} h_{\mu\nu}\frac{\delta S^{(1)}}{\delta h_{\mu\nu}} + \delta^{(0)}\varphi_{\mu_1\cdots\mu_s}\frac{\delta S^{(1)}}{\delta\varphi_{\mu_1\cdots\mu_s}} + \delta^{(1)} h^{\mu\nu}\vec{\mathcal{G}}^{\rho\sigma}_{\mu\nu} h_{\rho\sigma} + \delta^{(1)}\varphi_{\mu_1\cdots\mu_s}\frac{\delta S^{(0)}}{\delta\varphi_{\mu_1\cdots\mu_s}}\Big] = 0.$$

If, as assumed in the $S$-matrix approach, one takes the spin-$s$ particle on shell, then one sets $\delta S^{(0)}/\delta\varphi_{\mu_1\cdots\mu_s}$ to zero. If, in addition, one takes the Euler-Lagrange derivative of the result with respect to the gravitational field, noting that the only structure for $\delta^{(1)} h_{\mu\nu}$ which can contribute to Eq. (B11) with $S^{(1)} = \frac{1}{2}\int d^D x h_{\mu\nu} T^{\mu\nu}(\varphi_s, \varphi_s)$ is $\delta^{(1)} h_{\mu\nu} = R_{\mu\nu}(\varphi_s, \epsilon_s)$, one finds

$$T_{\alpha\beta}(\varphi_s, \delta^{(0)}\varphi_s) + \vec{\mathcal{G}}^{\mu\nu}_{\alpha\beta} R_{\mu\nu}(\varphi_s, \epsilon_s) = 0 \tag{B12}$$

which is (up to a convention of sign in front of the Fierz-Pauli action $S^{\rm FP} = \frac{1}{2}\int h_{\mu\nu}\vec{\mathcal{G}}^{\mu\nu}_{\alpha\beta} h^{\alpha\beta}$) the translation of Eq. (B8) in Lagrangian language.

Together with the principle of equivalence (B2), Eq. (B8) was the main assumption of the work (Porrati, 2008). We see that this condition (B8) is derived from the main equation (B11) in the Lagrangian formalism. Apart from the assumption of locality of $S^{(1)}$ which is relaxed in the $S$-matrix analysis (it would be interesting to see if this relaxation really gives new consistent solutions compared to the Lagrangian analysis), the Lagrangian analysis of Boulanger and Leclercq (2006) and Boulanger, Leclercq, and Sundell (2008) does not assume the equivalence principle and is based otherwise on a weaker form of Eq. (B8). That the spin-$s$ fields are put on shell in the $S$-matrix analysis can be viewed as an advantage (no *a priori* field-theoretical realization for the spin-$s$ fields).

Based on the sole two assumptions (B2) and (B8), Porrati is able to prove that no massless high-spin particle can minimally couple to gravity in flat space in complete accordance with the previous results of Aragone and Deser (1979), Berends *et al.* (1979), Aragone and La Roche (1982), Boulanger and Leclercq (2006), and Metsaev (2006) and with Boulanger, Leclercq, and Sundell (2008).

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