Leptonic *CP* violation

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Several topics on *CP* violation in the lepton sector are reviewed. A few theoretical aspects concerning neutrino masses, leptonic mixing, and *CP* violation will be covered, with special emphasis on seesaw models. A discussion is provided on observable effects which are manifest in the presence of *CP* violation, particularly, in neutrino oscillations and neutrinoless double beta decay processes, and their possible implications in collider experiments such as the LHC. The role that leptonic *CP* violation may have played in the generation of the baryon asymmetry of the Universe through the mechanism of leptogenesis is also discussed.

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I. INTRODUCTION

The violation of the product of the charge conjugation (*C*) and parity (*P*) symmetries, i.e., *CP* violation (CPV), is well established in the quark sector of the standard model (SM). At present, there is clear evidence that the Cabibbo-Kobayashi-Maskawa (CKM) matrix is complex, even if one allows for the presence of new physics in the B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ mixings. From a theoretical point of view, the complex phase in the CKM matrix may arise from complex Yukawa couplings and/or from a relative *CP*-violating phase in the vacuum expectation values (VEV) of Higgs fields. In either case, one expects an entirely analogous mechanism to arise in the lepton sector, leading to leptonic *CP* violation (LCPV).

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The discovery of neutrino oscillations provides evidence for nonvanishing neutrino masses and leptonic mixing. Therefore, it is imperative to look for possible manifestations of *CP* violation in leptonic interactions. The ideal playground for such a program relies on the phenomenon of neutrino oscillations. At present, several experiments are being planned to pursue such a task, including long-baseline facilities, superbeams, and neutrino factories. Hopefully, they will be able to measure the strength of *CP* violation and provide a knowledge of the leptonic mixing comparable to what is presently known about the quark sector. Yet, it is crucial to look for alternative manifestations of *CP* violation outside neutrino oscillations. In particular, the effects of Majorana-type phases may arise in neutrinoless double beta decay ($0\nu\beta\beta$) processes. The observation of such processes would establish the Majorana nature of neutrinos and, possibly, provide some information on the Majorana *CP* phases. In this review, we discuss the observable effects, which are manifest in the presence of leptonic *CP* violation. We present a short review of the neutrino oscillation formalism and summarize the prospects for the discovery of *CP* violation in the lepton sector. The possibility of extracting information about Majorana phases from $0\nu\beta\beta$ decay processes is also discussed.

The fact that neutrino masses are so small constitutes one of the most puzzling problems of modern particle physics. From a theoretical point of view, the smallness of neutrino masses can be elegantly explained through the seesaw mechanism, which can be realized in several ways depending on the nature of the heavy states added to the SM particle content. One of the most popular variants is the one in which the tree-level exchange of heavy neutrino singlets mediates the process of neutrino mass generation. The mechanism can be equally implemented considering, for instance, heavy scalar or fermion triplets. We review some of the realizations of the seesaw mechanism and discuss different parametrizations, which are useful when establishing a bridge between low-energy and high-energy *CP* violation in the lepton sector. This analysis will be relevant for the discussion of the connection between low-energy neutrino physics and leptogenesis, one of the most appealing scenarios for the generation of the baryon asymmetry of the Universe.

After the discovery of neutrino oscillations, several models have been put forward to offer an explanation for the pattern of neutrino masses and leptonic mixing. Future data from several kinds of experiments, ranging from kinematical searches to cosmology, will probably shed some light on the ultimate structure of the neutrino mass and mixing. In this regard, there are still fundamental questions to be answered: Are neutrinos Dirac or Majorana particles? What is the absolute neutrino mass scale? How are neutrino masses ordered? How large is the 1–3 leptonic mixing angle?

The explanation of the cosmological matter-antimatter asymmetry observed in nature constitutes one of the greatest challenges for modern particle physics and cosmology. We have entered a new era marked by outstanding advances in experimental cosmology and an unprecedented precision in measuring several cosmological parameters. In particular, the seven-year data recently collected from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite have placed

the observed baryon asymmetry in a rather narrow window. These measurements have also made it clear that the current state of the Universe is very close to a critical density and that the primordial density perturbations that seeded large-scale structure formation in the Universe are nearly scale invariant and Gaussian, which is consistent with the inflationary paradigm. Since any primordial asymmetry would have been exponentially diluted during inflation, a dynamical mechanism must have been operative after this period, in order to generate the baryon asymmetry that we observe today. The present review is not aimed at covering all the theoretical ideas on baryogenesis extensively developed over the last few years. Instead, we focus our discussion on the simplest leptogenesis scenarios, putting the emphasis on the role that leptonic *CP* violation may have played in the origin of matter. After briefly reviewing the simplest seesaw leptogenesis mechanisms, we analyze the possibility of establishing a bridge between leptonic *CP* violation at high and low energies. As it turns out, there is no model-independent relation between *CP* violation in leptogenesis and the observable phases of the low-energy leptonic mixing matrix. Such a link can only be established by restricting the number of free parameters in the leptonic flavor sector. From the model-building viewpoint, these restrictions are also necessary to fully reconstruct the neutrino mass matrix from low-energy data measured in feasible experiments.

In the analysis of lepton flavor models, a useful approach when addressing the question of *CP* violation is the construction of the *CP*-odd weak basis (WB) invariants. Independent of the basis choice and phase convention, any of these quantities should vanish if *CP* is an exact symmetry of the theory. Thus, in *CP*-violating theories which contain several phases, invariants constitute a powerful tool to investigate whether a particular model leads to leptonic *CP* violation at high and/or low energies. In our review, we briefly present such an invariant approach, in an attempt at relating leptogenesis with low-energy leptonic mixing phases. Finally, other interesting issues that we address here include the connection of leptogenesis with flavor symmetries and its viability under the hypothesis of minimal lepton flavor violation.

The layout of the review is as follows. In Sec. II, we review several topics related with fundamental aspects of neutrino masses, mixing, and *CP* violation in the lepton sector. First, in Sec. II.A, we study leptonic mixing and *CP* violation in the case when neutrino masses are generated by new physics which breaks the difference between baryon (B) and lepton (L) numbers, i.e., (B-L). Our analysis exclusively relies on the low-energy limit and not on any particular (B-L)-breaking mechanism to give masses to neutrinos. The construction of Dirac and Majorana unitarity triangles is presented in Sec. II.B, and the *CP* transformation properties in the lepton sector of the Lagrangian are discussed in Sec. II.C. The weak-basis invariants relevant for low-energy *CP* violation are then introduced in Sec II.D. In Sec. II.E, we recall the most popular versions of the seesaw mechanism for the neutrino mass generation, and in Sec. II.F we make a short digression on the origin of *CP* violation. We also briefly comment on the hypothesis of minimal lepton flavor violation in Sec. II.G. The present status of the neutrino mass and

mixing parameters and the basic aspects of neutrino oscillations in vacuum and matter are briefly reviewed in Secs. III.A and III.B. In the latter section, we focus on aspects related to CPV in neutrino oscillations and on the prospects of establishing CPV in future experiments. The possibility of probing CPV in $0\nu\beta\beta$ decays is discussed in Sec. III.C. In the framework of the type II seesaw mechanism, the *CP*-violating phases play a crucial role in the predictions for lepton flavor-violating charged-lepton decays, and also in the scalar triplet decays at accelerators, as discussed in Secs. III.D and III.E, respectively. Nonunitarity effects in the lepton sector are discussed in Sec. III.F. Section IV is devoted to the discussion of the possible role of leptonic *CP* violation in the origin of the matter-antimatter asymmetry in the context of leptogenesis. After reviewing the three main variants of this mechanism in Sec. IV.A, we discuss in Sec. IV.B how high-energy and low-energy *CP* violation can be related in some specific cases. We then briefly comment on the relevant *CP*-odd WB invariants for leptogenesis in Sec. IV.C. Finally, our conclusions and outlook are drawn in Sec. V.

II. NEUTRINO MASSES, MIXING, AND LEPTONIC *CP* VIOLATION

Neutrinos are strictly massless in the SM. No Dirac mass can be written since the right-handed neutrino field ν_R is not introduced, and no Majorana mass term can be generated, either in higher orders of perturbation theory or by nonperturbative effects, due to an exact (B-L) conservation. A Majorana mass term has the form $\nu_{L_i}^T C \nu_{L_j} m_{ij}$ and violates (B-L) by two units, so it is forbidden by the exact (B-L) symmetry. Because of the vanishing of neutrino masses, there is no leptonic mixing or leptonic *CP* violation in the SM. Any mixing generated in the diagonalization of the charged-lepton masses can be "rotated away" through a redefinition of the neutrino fields. Therefore, the experimental discovery of neutrino oscillations, pointing to nonvanishing neutrino masses, is a clear indication of physics beyond the SM.

A. The low-energy limit

We start by studying leptonic mixing and *CP* violation in an extension of the SM with neutrino masses generated by new physics which breaks (B-L). Our analysis follows an effective theory approach, without relying on any particular mechanism that breaks (B-L) and gives masses to neutrinos. Later on we present several realizations in which the (B-L)-breaking occurs due to the decoupling of heavy states.

1. Lepton mass terms

We assume that the gauge symmetry breaking has taken place and charged-lepton masses have been generated through the Yukawa couplings with the Higgs doublet, while Majorana neutrino masses arise from some unspecified (B-L)-breaking new physics. The Lagrangian mass terms are

$$\mathcal{L}_{\text{mass}} = -\bar{l}_L \mathbf{m}_l l_R - \tfrac{1}{2}\nu_L^T C \mathbf{m}_\nu \nu_L + \text{H.c.}, \tag{2.1}$$

where $l_{L,R} \equiv (e, \mu, \tau)_{L,R}^T$ stands for the SM charged-lepton fields, $\nu_L \equiv (\nu_e, \nu_\mu, \nu_\tau)_L^T$ are the left-handed neutrino fields,

and $\mathbf{m}_{l,\nu}$ are arbitrary complex matrices, being $\mathbf{m}_\nu$ symmetric.

There is clear evidence in the quark sector that the CKM mixing matrix is complex, even if one allows for the presence of new physics (Botella *et al.*, 2005). So, in analogy, we assume the existence of leptonic *CP* violation arising from complex lepton masses. The mass matrices of Eq. (2.1) are written in a weak basis, i.e., a basis for the lepton fields with real and flavor diagonal charged currents,

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \bar{l}_L \gamma_\mu \nu_L W^\mu + \text{H.c.} \tag{2.2}$$

The lepton mass matrices $\mathbf{m}_l$ and $\mathbf{m}_\nu$ encode all information about lepton masses and mixing. However, there is a redundancy of free parameters in these matrices so that not all of them are physical. This redundancy stems from the fact that one has the freedom to make a unitary WB transformation,

$$\nu_L = \mathbf{W}_L \nu_L', \qquad l_L = \mathbf{W}_L l_L', \qquad l_R = \mathbf{W}_R l_R', \tag{2.3}$$

under which all gauge currents remain real and diagonal, but the matrices $\mathbf{m}_l$ and $\mathbf{m}_\nu$ transform in the following way:

$$\mathbf{m}_l' = \mathbf{W}_L^\dagger \mathbf{m}_l \mathbf{W}_R, \qquad \mathbf{m}_\nu' = \mathbf{W}_L^T \mathbf{m}_\nu \mathbf{W}_L. \tag{2.4}$$

One may also use the freedom to make WB transformations to go to a basis where $\mathbf{m}_l = \mathbf{d}_l$ is real and diagonal. In this basis, one can still make the rephasing

$$l_{L,R}'' = \mathbf{K}_L l_{L,R}', \qquad \nu_L'' = \mathbf{K}_L \nu_L', \tag{2.5}$$

with $\mathbf{K}_L = \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3})$. Under this rephasing $\mathbf{d}_l$ remains invariant, but $\mathbf{m}_\nu$ transforms as

$$(\mathbf{m}_\nu'')_{ij} = e^{i(\varphi_i + \varphi_j)}(\mathbf{m}_\nu')_{ij}. \tag{2.6}$$

Since $\mathbf{m}_\nu'$ is an arbitrary complex symmetric matrix, it has $n(n + 1)/2$ phases, where n denotes the number of generations. One is still free to rephase Eq. (2.6) and further eliminate n phases. One is then left with

$$N_\phi = \tfrac{1}{2}n(n - 1) \tag{2.7}$$

physically meaningful phases.[1] It will be shown in the sequel that these phases in general violate *CP*. Note that the N_ϕ phases appear in a WB, prior to the diagonalization of both $\mathbf{m}_l$ and $\mathbf{m}_\nu$, and the generation of the leptonic mixing matrix. Note that N_ϕ coincides with the number of physical phases appearing in the leptonic mixing.

For three generations, $N_\phi = 3$, and one may use the rephasing of Eq. (2.6) in order to make, for example, all the diagonal elements of $\mathbf{m}_\nu$ real. For this choice, the three *CP*-violating phases can be identified with $\arg[(\mathbf{m}_\nu)_{12}]$, $\arg[(\mathbf{m}_\nu)_{13}]$, and $\arg[(\mathbf{m}_\nu)_{23}]$. It is clear that the individual phases of $(\mathbf{m}_\nu)_{ij}$ do not have any physical meaning, since they are not invariant under the rephasing given in Eq. (2.6). One may, however, construct polynomials of $(\mathbf{m}_\nu)_{ij}$, which are rephasing invariant (Farzan and Smirnov, 2007), such as

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[1]Alternatively, the parameter counting can be performed by analyzing the symmetries of the Lagrangian (Santamaria, 1993).

$$P_1 = (\mathbf{m}_\nu^*)_{11}(\mathbf{m}_\nu^*)_{22}(\mathbf{m}_\nu)_{12}^2,$$

$$P_2 = (\mathbf{m}_\nu^*)_{11}(\mathbf{m}_\nu^*)_{33}(\mathbf{m}_\nu)_{13}^2, \qquad (2.8)$$

$$P_3 = (\mathbf{m}_\nu^*)_{33}(\mathbf{m}_\nu^*)_{12}(\mathbf{m}_\nu)_{13}(\mathbf{m}_\nu)_{23}.$$

2. Leptonic mixing

The lepton mass matrices in Eq. (2.1) are diagonalized by the unitary transformations

$$\mathbf{U}_L^{l\dagger}\mathbf{m}_l\mathbf{U}_R^l = \mathbf{d}_l, \qquad \mathbf{U}^{\nu T}\mathbf{m}_\nu\mathbf{U}^\nu = \mathbf{d}_m, \qquad (2.9)$$

where $\mathbf{U}_{L,R}^l$ and $\mathbf{U}^\nu$ are unitary matrices; $\mathbf{d}_l$ and $\mathbf{d}_m$ are diagonal matrices. In terms of the lepton mass eigenstates, the charged current becomes

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}\bar{l}_L\gamma_\mu\mathbf{U}\nu_L W^\mu + \text{H.c.}, \qquad (2.10)$$

where $\mathbf{U} = \mathbf{U}_L^{l\dagger}\mathbf{U}^\nu$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix. The matrix $\mathbf{U}$ is unitary, so it

has n^2 parameters; $n(n-1)/2$ of these parameters can be used to define the $O(n)$ rotation, while n phases of $\mathbf{U}$ can be removed through the rephasing of n charged-lepton fields. Thus, one is left with $n(n-1)/2$ phases characterizing *CP* violation in $\mathbf{U}$. As mentioned, this number of phases coincides with the number of physical phases N_ϕ in the neutrino mass matrix, counted in a WB in which the charged-lepton mass matrix is diagonal and real.

For three generations, the 3×3 matrix $\mathbf{U}$ is conveniently parametrized by (Nakamura *et al.*, 2010)

$$\mathbf{U} = \mathbf{VK}, \qquad \mathbf{K} = \text{diag}(1, e^{i\alpha_1/2}, e^{i\alpha_2/2}), \qquad (2.11)$$

with $\alpha_{1,2}$ denoting the phases associated with the Majorana character of neutrinos (Bilenky, Hosek, and Petcov, 1980; Schechter and Valle, 1980; Doi *et al.*, 1981; Bernabeu and Pascual, 1983), and the unitary matrix $\mathbf{V}$ written, as in the case of the CKM quark mixing matrix, in terms of three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and one phase δ,

$$\mathbf{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \qquad (2.12)$$

Hereafter, $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ with the mixing angles chosen to lie in the first quadrant, and δ is a Dirac-type *CP*-violating phase. An alternative parametrization of the mixing matrix $\mathbf{U}$, which turns out to be more appropriate for the $0\nu\beta\beta$ analysis, is given by

$$\mathbf{U} = \mathbf{VK}', \qquad \mathbf{K}' = \mathbf{K}\,\text{diag}(1, 1, e^{i\delta}). \qquad (2.13)$$

In what follows, we also use the simplified notation

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{e1} & \mathbf{U}_{e2} & \mathbf{U}_{e3} \\ \mathbf{U}_{\mu 1} & \mathbf{U}_{\mu 2} & \mathbf{U}_{\mu 3} \\ \mathbf{U}_{\tau 1} & \mathbf{U}_{\tau 2} & \mathbf{U}_{\tau 3} \end{pmatrix} \qquad (2.14)$$

to denote the matrix elements of $\mathbf{U}$.

It is clear that the phase of a particular matrix element of $\mathbf{U}$ does not have any physical meaning. This reflects the fact that under a rephasing of the charged-lepton fields $l_{Lj} \rightarrow l'_{Lj} = e^{i\phi_j}l_{Lj}$ the matrix $\mathbf{U}$ transforms as

$$\mathbf{U}_{jk} \rightarrow \mathbf{U}'_{jk} = e^{i\phi_j}\mathbf{U}_{jk}. \qquad (2.15)$$

This is entirely analogous to what one encounters in the quark sector. The novel feature in leptonic mixing with Majorana neutrinos is that one cannot rephase Majorana neutrino phases since this would not leave invariant the neutrino mass terms. Note that we consider real neutrino masses, which satisfy Majorana conditions that do not contain phase factors. It should also be emphasized that rephasing invariance is a requirement for any physical quantity. In the quark sector, the simplest rephasing invariant functions of the CKM matrix elements $\mathbf{V}_{ij}$, apart from the trivial example of moduli, are the rephasing invariant quartets $\mathbf{V}_{ij}\mathbf{V}_{kj}^*\mathbf{V}_{kl}\mathbf{V}_{il}^*$. In the lepton sector with Majorana neutrinos, the simplest rephasing

invariant functions of the PMNS matrix elements $\mathbf{U}_{ij}$ are the bilinears of the type $\mathbf{U}_{ij}\mathbf{U}_{ik}^*$ (Nieves and Pal, 1987; Aguilar-Saavedra and Branco, 2000; Nieves and Pal, 2001), with $j \neq k$ and no summation over repeated indices. We then designate "Majorana-type" phases the following quantities:

$$\gamma_{jk} \equiv \arg(\mathbf{U}_{ij}\mathbf{U}_{ik}^*). \qquad (2.16)$$

From their definition, one can readily see that in the case of three generations there are six independent Majorana-type phases γ_{jk}. Using unitarity, one can then reconstruct the full matrix $\mathbf{U}$ from these six Majorana-type phases (Branco and Rebelo, 2009).

B. Dirac and Majorana unitarity triangles

In a SM-like theory with an arbitrary number of generations, quark mixing is defined through the CKM matrix which is unitary by construction. For three standard generations, unitarity leads to various relations among the moduli of the CKM matrix and rephasing invariant angles. These relations provide a crucial test of the SM and its mechanism of mixing and *CP* violation. We assume, for the moment, that the PMNS matrix $\mathbf{U}$ is unitary. Then one can construct six unitarity triangles from the orthogonality of the rows and columns of $\mathbf{U}$ (Aguilar-Saavedra and Branco, 2000). These triangles are analogous to the ones used in the quark sector to study various manifestations of *CP* violation. However, in the case of Majorana neutrinos there is an important difference. In the quark sector, the orientation of the unitarity triangles in the complex plane has no physical meaning, since under rephasing of the quark fields all triangles rotate. For example, one may choose in the quark sector, without loss of generality, any side of a given triangle to coincide with the real axis.
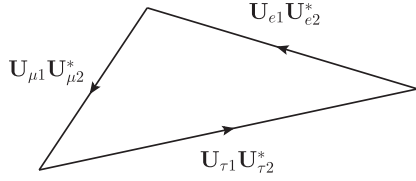
FIG. 1. Majorana unitarity triangle T_{12}. The arrows indicate the orientation of the triangle, which is determined by the Majorana phases and cannot be rotated in the complex plane.

In the lepton sector with Majorana neutrinos there are two types of unitarity triangles: Dirac triangles that correspond to the orthogonality of rows,

$$T_{e\mu}: \mathbf{U}_{e1}\mathbf{U}_{\mu 1}^* + \mathbf{U}_{e2}\mathbf{U}_{\mu 2}^* + \mathbf{U}_{e3}\mathbf{U}_{\mu 3}^* = 0,$$

$$T_{e\tau}: \mathbf{U}_{e1}\mathbf{U}_{\tau 1}^* + \mathbf{U}_{e2}\mathbf{U}_{\tau 2}^* + \mathbf{U}_{e3}\mathbf{U}_{\tau 3}^* = 0, \qquad (2.17)$$

$$T_{\mu\tau}: \mathbf{U}_{\mu 1}\mathbf{U}_{\tau 1}^* + \mathbf{U}_{\mu 2}\mathbf{U}_{\tau 2}^* + \mathbf{U}_{\mu 3}\mathbf{U}_{\tau 3}^* = 0,$$

and Majorana triangles that are defined by the orthogonality of columns,

$$T_{12}: \mathbf{U}_{e1}\mathbf{U}_{e2}^* + \mathbf{U}_{\mu 1}\mathbf{U}_{\mu 2}^* + \mathbf{U}_{\tau 1}\mathbf{U}_{\tau 2}^* = 0,$$

$$T_{13}: \mathbf{U}_{e1}\mathbf{U}_{e3}^* + \mathbf{U}_{\mu 1}\mathbf{U}_{\mu 3}^* + \mathbf{U}_{\tau 1}\mathbf{U}_{\tau 3}^* = 0, \qquad (2.18)$$

$$T_{23}: \mathbf{U}_{e2}\mathbf{U}_{e3}^* + \mathbf{U}_{\mu 2}\mathbf{U}_{\mu 3}^* + \mathbf{U}_{\tau 2}\mathbf{U}_{\tau 3}^* = 0.$$

It is clear from Eq. (2.17) that the orientation of Dirac triangles has no physical meaning since under the rephasing of the charged-lepton fields these triangles rotate in the complex plane, $\mathbf{U}_{ik}\mathbf{U}_{jk}^* \to e^{i(\phi_i - \phi_j)}\mathbf{U}_{ik}\mathbf{U}_{jk}^*$, in accordance with Eqs. (2.15). On the contrary, the orientation of Majorana triangles does have physical meaning since these triangles remain invariant under rephasing (cf. Fig. 1).

Leptonic *CP* violation with Majorana neutrinos has some novel features, when compared to *CP* violation in the quark sector. In the latter case, there is *CP* violation if and only if the imaginary part of a rephasing invariant quartet of the CKM matrix elements does not vanish. It is an important consequence of the unitarity of the CKM matrix that the imaginary part of all invariant quartets have the same modulus. The only meaningful phases in the quark sector are the arguments of rephasing invariant quartets. In the lepton sector, one may have an entirely analogous *CP* violation from the nonvanishing of the imaginary part of an invariant quartet of **U**. In the limit when **U** is unitary, again the imaginary part of all invariant quartets have the same modulus. Nevertheless, one may also have Majorana-type *CP* violation associated with the Majorana-type phases, identified as arguments of the rephasing invariant bilinears defined in Eq. (2.16).

In order to understand some of the special features of leptonic *CP* violation with Majorana neutrinos, it is instructive to study the limit of *CP* invariance. This case can be analyzed using the Majorana unitarity triangles of Eq. (2.18), which provide the necessary and sufficient conditions for *CP* conservation:

(i) Vanishing of their common area $\mathcal{A} = \frac{1}{2}|\mathrm{Im}Q|$, with $Q = \mathbf{U}_{ij}\mathbf{U}_{kj}^*\mathbf{U}_{kl}\mathbf{U}_{il}^*$ standing for any invariant quartet of **U** (no sum over repeated indices and $i \neq k, j \neq l$);

(ii) Orientation of all Majorana triangles along the direction of the real or imaginary axes.

The first requirement eliminates the possibility of Dirac-type *CP* violation while the second condition implies that Majorana phases do not violate *CP*. In order to understand requirement (ii), we assume that condition (i) is satisfied, i.e., all triangles collapse. If all Majorana triangles T_{jk} collapse along the real axis then $\gamma_{jk} = 0$ in Eq. (2.16). It is obvious that *CP* is conserved in this case and the leptonic mixing matrix **U** is real. If one of the triangles T_{jk} collapse along the imaginary axis, this means that the mass eigenstates ν_j and ν_k have opposite *CP* parities, but no *CP* violation is implied. One can make the triangle T_{jk}, which collapsed in the imaginary axis to collapse in the real axis instead, by multiplying the Majorana fields by $\pm i$ and rendering the corresponding mass eigenstate negative.

C. Majorana neutrinos and *CP* violation

In order to study *CP* violation in an extension of the SM with Majorana masses for left-handed neutrinos, it is convenient to consider the Lagrangian after the spontaneous gauge symmetry breaking. The relevant part of the Lagrangian reads

$$\mathcal{L} = -\bar{l}_L \mathbf{m}_l l_R - \frac{1}{2}\nu_L^T C \mathbf{m}_\nu \nu_L + \frac{g}{\sqrt{2}}\bar{l}_L \gamma_\mu \nu_L W^\mu + \mathrm{H.c.} \qquad (2.19)$$

The *CP* transformation properties of the various fields are dictated by the part of the Lagrangian which conserves *CP*, namely, the gauge interactions. One should keep in mind that gauge interactions in a WB do not distinguish the different generations of fermions and, consequently, the Lagrangian of Eq. (2.19) conserves *CP* if and only if there is a *CP* transformation defined by

$$CP\,l_L\,(CP)^\dagger = \mathbf{W}_L \gamma^0 C \bar{l}_L^T,$$

$$CP\,\nu_L\,(CP)^\dagger = \mathbf{W}_L \gamma^0 C \bar{\nu}_L^T, \qquad (2.20)$$

$$CP\,l_R\,(CP)^\dagger = \mathbf{W}_R \gamma^0 C \bar{l}_R^T,$$

where $\mathbf{W}_L$ and $\mathbf{W}_R$ are unitary matrices acting in generation space.

Often, in the literature, the transformations given in Eqs. (2.20) are referred to as generalized *CP* transformation. This is a misnomer, since the inclusion of the unitary matrices $\mathbf{W}_L$ and $\mathbf{W}_R$ is mandatory for a correct definition the *CP* transformation, in view of the flavor symmetry of gauge interactions. The lepton fields l_L and ν_L have to transform in the same way due to the presence of the left-handed charged-current interactions. Then, the Lagrangian of Eq. (2.19) conserves *CP* if and only if the lepton mass matrices $\mathbf{m}_\nu$ and $\mathbf{m}_l$ satisfy the following relations:

$$\mathbf{W}_L^T \mathbf{m}_\nu \mathbf{W}_L = -\mathbf{m}_\nu^*, \qquad \mathbf{W}_L^\dagger \mathbf{m}_l \mathbf{W}_R = \mathbf{m}_l^*. \qquad (2.21)$$

The above *CP* conditions are WB independent in the sense that if there exist matrices $\mathbf{W}_L$ and $\mathbf{W}_R$ that satisfy Eq. (2.21) when $\mathbf{m}_\nu$ and $\mathbf{m}_l$ are written in a particular WB, they will also

exist when the mass matrices are written in another WB. One can use this WB independence to study the *CP* restrictions in an appropriate WB. We perform this analysis in two different bases. We first consider the basis of real and diagonal charged-lepton mass matrix. At this stage, $\mathbf{m}_\nu$ is an arbitrary complex symmetric matrix. While keeping $\mathbf{m}_l$ diagonal, real, and positive, one can still make a WB transformation which renders the diagonal elements of $\mathbf{m}_\nu$ real. In this basis, Eq. (2.21) constrains $\mathbf{W}_L$ to be of the form

$$\mathbf{W}_L = \mathrm{diag}(\pm i, \pm i, \pm i). \qquad (2.22)$$

Substituting Eq. (2.22) into Eq. (2.21), one concludes that *CP* invariance constrains the elements of $\mathbf{m}_\nu$ to be either real or purely imaginary. Note, for instance, that the matrix

$$\mathbf{m}_\nu = \begin{pmatrix} |m_{11}| & |m_{12}| & i|m_{13}| \\ |m_{12}| & |m_{22}| & i|m_{23}| \\ i|m_{13}| & i|m_{23}| & |m_{33}| \end{pmatrix} \qquad (2.23)$$

does not lead to *CP* violation, since Eqs. (2.21) can be satisfied with $\mathbf{W}_L = \mathrm{diag}(i, i, -i)$. One could have also suspected that the matrix $\mathbf{m}_\nu$ defined in Eq. (2.23) would correspond to *CP* invariance since $\mathrm{Im}P_i = 0$, where P_i denotes the rephasing invariants given in Eqs. (2.8).

D. Weak-basis invariants and low-energy *CP* violation

We have seen that the existence of unitary matrices $\mathbf{W}_L$ and $\mathbf{W}_R$ satisfying Eqs. (2.21) is a necessary and sufficient condition for having *CP* invariance in the low-energy limit. We address now the question of finding *CP*-odd WB invariants which would detect *CP* violation in the lepton sector. Obviously, these WB invariants should be written in terms of $\mathbf{m}_\nu$ and $\mathbf{m}_l$. It is well known that, in the quark sector of the SM with three generations, there is only one *CP*-odd WB invariant which controls *CP* violation at low energies, namely, (Bernabeu, Branco, and Gronau, 1986; Gronau, Kfir, and Loewy, 1986),

$$\mathcal{J}_{\mathrm{quark}}^{CP} = \mathrm{Tr}[\mathbf{m}_u \mathbf{m}_u^\dagger, \mathbf{m}_d \mathbf{m}_d^\dagger]^3, \qquad (2.24)$$

where $\mathbf{m}_u$ and $\mathbf{m}_d$ denote the up and down quark mass matrices, respectively.[2]

In the case of three (Dirac or Majorana) neutrinos, one can write an entirely analogous *CP*-odd WB invariant which controls Dirac-type *CP* violation in the lepton sector

$$\mathcal{J}_{\mathrm{lepton}}^{CP} = \mathrm{Tr}[(\mathbf{m}_\nu \mathbf{m}_\nu^\dagger)^*, \mathbf{m}_l \mathbf{m}_l^\dagger]^3. \qquad (2.25)$$

This relation can be computed in any weak basis. The low-energy invariant (2.25) is sensitive to the Dirac-type phase δ and vanishes for $\delta = 0$. On the other hand, it does not depend on the Majorana phases α_1 and α_2 appearing in the leptonic mixing matrix $\mathbf{U}$. The quantity $\mathcal{J}_{\mathrm{lepton}}^{CP}$ can be fully written in terms of physical observables,

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[2]This invariant can also be written in the equivalent form $\mathcal{J}_{\mathrm{quark}}^{CP} = \mathrm{Im}\,\mathrm{Det}([\mathbf{m}_u \mathbf{m}_u^\dagger, \mathbf{m}_d \mathbf{m}_d^\dagger])$ (Jarlskog, 1985).

$$\mathcal{J}_{\mathrm{lepton}}^{CP} = -6i(m_\mu^2 - m_e^2)(m_\tau^2 - m_\mu^2)(m_\tau^2 - m_e^2)$$
$$\times \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2 \mathcal{J}_{e\mu}^{21}, \qquad (2.26)$$

where $\Delta m_{ji}^2 = m_j^2 - m_i^2$ are the light neutrino mass-squared differences. As shown in Sec. III.B, the quantity $\mathcal{J}_{e\mu}^{21}$ is the imaginary part of an invariant quartet appearing in the difference of the *CP*-conjugated neutrino oscillation probabilities $P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$. One can easily get

$$\mathcal{J}_{e\mu}^{21} \equiv \mathrm{Im}[\mathbf{U}_{11}\mathbf{U}_{22}\mathbf{U}_{12}^*\mathbf{U}_{21}^*]$$
$$= \tfrac{1}{8}\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\sin\delta, \qquad (2.27)$$

where θ_{ij} are the mixing angles in the standard parametrization of Eq. (2.12).

The requirement $\mathcal{J}_{\mathrm{lepton}}^{CP} \neq 0$ is a necessary and sufficient condition for having Dirac-type leptonic *CP* violation, independent of whether neutrinos are Majorana or Dirac particles. However, in the case of Majorana neutrinos there is also the possibility of Majorana-type *CP* violation. It is therefore interesting to find *CP*-odd invariants which could directly detect this type of *CP* violation, even in the limit when there is no Dirac-type *CP* violation. An example of such an invariant is (Branco, Lavoura, and Rebelo, 1986)

$$\mathcal{J}_{\mathrm{Maj}}^{CP} = \mathrm{Im}\,\mathrm{Tr}(\mathbf{m}_l\mathbf{m}_l^\dagger\mathbf{m}_\nu^*\mathbf{m}_\nu\mathbf{m}_\nu^*\mathbf{m}_l^T\mathbf{m}_l^*\mathbf{m}_\nu). \qquad (2.28)$$

The simplest way of verifying that $\mathcal{J}_{\mathrm{Maj}}^{CP}$ is sensitive to Majorana phases is by evaluating it for the particular case of two Majorana neutrinos. In this situation, there is only one Majorana-type phase and no Dirac-type phase. The leptonic mixing matrix can be parametrized by

$$\mathbf{U} = \begin{pmatrix} \cos\theta & -\sin\theta e^{i\gamma} \\ \sin\theta e^{-i\gamma} & \cos\theta \end{pmatrix}, \qquad (2.29)$$

where γ denotes the Majorana phase. An explicit evaluation of $\mathcal{J}_{\mathrm{Maj}}^{CP}$ gives

$$\mathcal{J}_{\mathrm{Maj}}^{CP} = \tfrac{1}{4}m_1 m_2 \Delta m_{21}^2 (m_\mu^2 - m_e^2)^2 \sin^2 2\theta \sin 2\gamma. \qquad (2.30)$$

It is worth pointing out that $\mathcal{J}_{\mathrm{Maj}}^{CP}$ shows explicitly some subtle points of Majorana-type *CP* violation. In particular, it shows that a phase $\gamma = \pi/2$ does not imply *CP* violation; it simply corresponds to *CP* invariance with the two neutrinos having opposite *CP* parities.

The invariants given in Eqs. (2.25) and (2.28) vanish if neutrinos are exactly degenerate in mass. In this limit, the parametrization of the mixing matrix $\mathbf{U}$ requires, in general, two angles and one *CP*-violating phase. This is to be contrasted to the case of Dirac neutrinos, in which there is no mixing or *CP* violation in the exact degeneracy limit. Therefore, leptonic *CP* violation may arise even when the three Majorana neutrinos have identical mass (Branco, Lavoura, and Rebelo, 1986). It is possible to construct a WB invariant which controls the strength of the *CP* violation in the latter case (Branco, Rebelo, and Silva-Marcos, 1999), namely,

$$\mathcal{J}_{\mathrm{deg}}^{CP} = \mathrm{Tr}[\mathbf{m}_\nu\mathbf{m}_l\mathbf{m}_l^\dagger\mathbf{m}_\nu^*, \mathbf{m}_l^*\mathbf{m}_l^T]^3. \qquad (2.31)$$

A necessary and sufficient condition for *CP* invariance is $\mathcal{J}_{\mathrm{deg}}^{CP} = 0$. The *CP*-odd invariant (2.31) can be expressed in

terms of lepton masses and mixing parameters by choosing the WB in which $\mathbf{m}_l\mathbf{m}_l^\dagger = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$. Parametrizing the mixing matrix $\mathbf{U}$ in the standard form of Eqs. (2.11) and (2.12), with vanishing θ_{13} and δ, and $\alpha_1 = 2\pi$, so that in the limit of *CP* invariance one of the Majorana neutrinos has relative *CP* parity opposite to the other two, one obtains

$$\mathcal{J}_{\text{deg}}^{CP} = -\frac{3i}{2}m^6(m_\tau^2 - m_\mu^2)^2(m_\tau^2 - m_e^2)^2(m_\mu^2 - m_e^2)^2$$
$$\times \cos(2\theta_{12})\sin^2(2\theta_{12})\sin^2(2\theta_{23})\sin\alpha_2, \quad (2.32)$$

where m denotes the common neutrino mass. The special feature of the WB invariant of Eq. (2.32) is the fact that, in general, it does not vanish, even in the limit of exact degeneracy of the three Majorana neutrino masses.

E. Seesaw mechanisms for neutrino mass generation

In the SM, quarks and charged fermions get masses through renormalizable Yukawa couplings with the Higgs doublet $\phi = (\phi^+, \phi^0)^T$, and their corresponding mass terms break the $SU(2)_L$ gauge symmetry as doublets. In contrast, a Majorana neutrino mass term, as the one given in Eq. (2.19), breaks $SU(2)_L$ as a triplet, and therefore it cannot be generated in the same way. This term is most likely to arise from higher dimensional operators, such as the lepton number violating ($\Delta L = 2$) dimension-five operator $\mathcal{O} = (\ell_\alpha\phi) \times (\ell_\beta\phi)/M$ (Weinberg, 1980), where $\ell = (l_L, \nu_L)$ is the SM lepton doublet. Once the Higgs field acquires a nonzero vacuum expectation value, $\langle\phi^0\rangle = v$, Majorana neutrino masses proportional to v^2/M are induced, in contrast to the quark and charged-lepton masses which are linear in v. Thus, if the mass scale M is much heavier than the electroweak breaking scale v, neutrinos could naturally get masses much smaller than all the other SM fermions.

The simplest and perhaps most attractive realization of the operator $\mathcal{O}$ in gauge theories is through the so-called seesaw mechanism. In this approach, the effective operator $\mathcal{O}$ is induced by the exchange of heavy particles with a mass scale M. Such heavy states are commonly present in grand unified theories (GUT). Several seesaw realizations are conceivable for neutrino mass generation (Mohapatra *et al.*, 2007; Nunokawa, Parke, and Valle, 2008). The following three types, schematically depicted in Fig. 2, are among the most popular ones:

- *Type I* (Minkowski, 1977; Gell-Mann, Ramond, and Slansky, 1979; Yanagida, 1979; Glashow *et al.*, 1980; Mohapatra and Senjanovic, 1980), mediated by heavy fermions, singlets under the $SU(3) \times SU(2) \times U(1)$ gauge symmetry;
- *Type II* (Konetschny and Kummer, 1977; Cheng and Li, 1980; Schechter and Valle, 1980; Lazarides, Shafi, and Wetterich, 1981; Mohapatra and Senjanovic, 1981), mediated by the exchange of $SU(2)$-triplet scalars;
- *Type III* (Foot *et al.*, 1989), mediated by the exchange of $SU(2)$-triplet fermions.

Below we briefly describe each of these realizations.

Adding two or three singlet fermions ν_R to the SM particle content is one of the simplest and rather natural possibilities
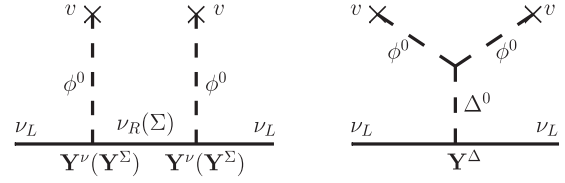

FIG. 2. Canonical seesaw mechanisms for neutrino mass generation. The left diagram corresponds to type I and type III seesaw masses, mediated by the tree-level exchange of singlet (ν_R) and triplet (Σ) fermions, respectively. The right diagram leads to type II seesaw neutrino masses via the exchange of a triplet scalar Δ.

to generate neutrino masses. Since the ν_R states are electroweak singlets, their masses are not protected by the electroweak symmetry and therefore can be very large. In the basis of diagonal charged-lepton Yukawa couplings, the relevant terms in the neutrino sector of the Lagrangian are

$$-\mathcal{L}^{\text{I}} = Y_{\alpha i}^{\nu*}\overline{\ell}_\alpha\tilde{\phi}\nu_{Ri} + \tfrac{1}{2}\overline{\nu_{Ri}}(\mathbf{m}_R)_{ij}\nu_{Rj}^c + \text{H.c.}, \quad (2.33)$$

where $\tilde{\phi} = i\sigma_2\phi^*$, $\mathbf{Y}^\nu$ is the Dirac-neutrino Yukawa coupling matrix, and $\mathbf{m}_R$ is the right-handed neutrino mass matrix. Notice that we have not included a Majorana mass term for left-handed neutrinos since this would require an enlargement of the scalar sector. For 3 generations and n_R heavy Majorana states, the type I seesaw Lagrangian of Eq. (2.33) contains altogether $7n_R - 3$ free parameters. The counting can be done as follows. In the mass basis of the singlet fermions, $N_i = \mathbf{U}_R^T\nu_R$, or, more precisely, in the basis where the $n_R \times n_R$ symmetric matrix $\mathbf{m}_R$ is diagonal, with positive and real eigenvalues M_i, i.e.,

$$\mathbf{U}_R^T\mathbf{m}_R\mathbf{U}_R = \mathbf{d}_M = \text{diag}(M_1, M_2, \ldots, M_{n_R}), \quad (2.34)$$

the Majorana mass term in Eq. (2.33) contains only n_R free parameters. In this basis, the Yukawa coupling matrix $\mathbf{Y}^\nu$ is an arbitrary $3 \times n_R$ complex matrix with $6n_R$ parameters. Of those, 3 phases can be removed by phase redefinitions of the charged-lepton fields l_L, thus remaining $3(2n_R - 1)$ physical parameters, to wit $3n_R$ moduli and $3(n_R - 1)$ phases.

After integrating out the heavy Majorana fields in the Lagrangian of Eq. (2.33), the effective mass matrix of the light neutrinos is given by the standard seesaw formula

$$\mathbf{m}_\nu = -v^2\mathbf{Y}^\nu\mathbf{m}_R^{-1}\mathbf{Y}^{\nu T}, \quad (2.35)$$

with the matrix $\mathbf{m}_\nu$ being diagonalized by the PMNS leptonic mixing matrix $\mathbf{U}$,

$$\mathbf{U}^T\mathbf{m}_\nu\mathbf{U} = \mathbf{d}_m = \text{diag}(m_1, m_2, m_3), \quad (2.36)$$

where m_i are the light neutrino masses.

The general type I seesaw framework introduces many more parameters than those required at low energies. Indeed, the effective neutrino mass matrix $\mathbf{m}_\nu$ can be written in terms of only nine physical parameters: the three light neutrino masses, and the three mixing angles and three phases that parametrize the mixing matrix $\mathbf{U}$.

We now consider the type II seesaw framework. In this case, the SM scalar sector is extended by introducing a scalar triplet Δ with hypercharge $+1$ (in the normalization of

hypercharge $-1/2$ for the lepton doublets) and mass M_Δ. In the SU(2) representation,

$$\Delta = \begin{pmatrix} \Delta^0 & -\Delta^+/\sqrt{2} \\ -\Delta^+/\sqrt{2} & \Delta^{++} \end{pmatrix}. \tag{2.37}$$

The relevant Lagrangian terms are, in this case,

$$\begin{aligned} -\mathcal{L}^{\mathrm{II}} = {}&(\mathbf{Y}^\Delta_{\alpha\beta}\ell^T_\alpha C\Delta\ell_\beta - \mu M_\Delta \tilde{\phi}^T\Delta\tilde{\phi} + \mathrm{H.c.}) \\ &+ M^2_\Delta \,\mathrm{Tr}(\Delta^\dagger\Delta), \end{aligned} \tag{2.38}$$

where $\mathbf{Y}^\Delta$ is a 3×3 symmetric complex coupling matrix and μ is a dimensionless coupling, which can be taken real without loss of generality. When compared to the type I seesaw Lagrangian of Eq. (2.33), the Lagrangian terms in Eq. (2.38) contain less free parameters. Indeed, only eleven parameters are required to fully determine the type II seesaw Lagrangian. Besides the two "unflavored" parameters μ and M_Δ, there are nine "flavored" parameters contained in the Yukawa matrix $\mathbf{Y}^\Delta$. In this sense, the type II seesaw is more economical, since the flavor structure of the neutrino mass matrix $\mathbf{m}_\nu$ is uniquely determined by the flavor structure of $\mathbf{Y}^\Delta$. The exchange of the heavy triplet leads to the effective neutrino mass matrix

$$\mathbf{m}_\nu = \frac{\mu v^2}{M_\Delta}\mathbf{Y}^\Delta. \tag{2.39}$$

Leptonic *CP* violation is thus encoded in the phases of the matrix $\mathbf{Y}^\Delta$.

Neutrino masses can also be generated by the tree-level exchange of two or three SU(2)-triplet fermions Σ_i with zero hypercharge,

$$\Sigma_i = \begin{pmatrix} \Sigma^0_i & \sqrt{2}\Sigma^+_i \\ \sqrt{2}\Sigma^-_i & -\Sigma^0_i \end{pmatrix}. \tag{2.40}$$

The Lagrangian that leads to the effective matrix $\mathbf{m}_\nu$ is similar to the type I seesaw Lagrangian of Eq. (2.33), but with different contractions of the SU(2) indices

$$-\mathcal{L}^{\mathrm{III}} = (\mathbf{Y}^\Sigma)^*_{\alpha i}\bar{\ell}_\alpha\tilde{\phi}\Sigma_i + \tfrac{1}{2}(\mathbf{m}_\Sigma)_{ij}\,\mathrm{Tr}(\bar{\Sigma}_i\Sigma^c_j) + \mathrm{H.c.} \tag{2.41}$$

The parameter counting is analogous to the type I case. In particular, eighteen (eleven) parameters are required to fully determine the high-energy neutrino sector in a model with three (two) triplet fermions. The effective light neutrino mass matrix exhibits the same seesaw structure of Eq. (2.35), with the obvious substitutions $\mathbf{Y}^\nu \to \mathbf{Y}^\Sigma$ and $\mathbf{m}_R \to \mathbf{m}_\Sigma$.

It is worth noting that, besides the three seesaw realizations discussed above, there are other types of unconventional seesaw schemes (Nunokawa, Parke, and Valle, 2008). For instance, in the so-called double-seesaw models (Mohapatra, R., 1986; Mohapatra and Valle, 1986), in addition to the conventional singlet fermions ν_R, one or more singlet fields S_i with lepton number $L = 1$ are added to the SM particle content. The relevant double-seesaw Lagrangian terms are

$$\begin{aligned} -\mathcal{L}^{\mathrm{IS}} = {}&\mathbf{Y}^{\nu*}_{\alpha i}\bar{\ell}_\alpha\tilde{\phi}\nu_{Ri} + \bar{S}_i(\mathbf{m}_{RS})_{ij}\nu_{Rj} \\ &+ \tfrac{1}{2}\bar{S}^c_i(\mathbf{m}_S)_{ij}S_j + \mathrm{H.c.}, \end{aligned} \tag{2.42}$$

where $\mathbf{m}_{RS}$ is an arbitrary complex matrix and $\mathbf{m}_S$ is a complex symmetric matrix. In this case, the effective mass matrix of the light neutrinos is given by

$$\mathbf{m}_\nu = -v^2\mathbf{Y}^\nu(\mathbf{m}^T_{RS})^{-1}\mathbf{m}_S\mathbf{m}^{-1}_{RS}\mathbf{Y}^{\nu T}. \tag{2.43}$$

The inverse seesaw is a variant of the double seesaw with a Majorana mass matrix $\mathbf{m}_S \ll v\mathbf{Y}^\nu \ll \mathbf{m}_{RS}$. Since in the limit $\mathbf{m}_S \to 0$ lepton number is conserved, this is a natural scenario in the 't Hooft sense ('t Hooft, 1980).

Finally, there are a variety of models of neutrino masses with the operator $\mathcal{O}$ being induced from physics at TeV or even lower energy scales (Chen and Huang, 2011). In such scenarios, loop and Yukawa coupling suppression factors typically guarantee the smallness of neutrino masses. Furthermore, $\Delta L = 2$ effective operators with dimension higher than five can give a dominant contribution to neutrino Majorana masses, if the leading effective operator $\mathcal{O}$ is forbidden due to a new symmetry or selection rule (Babu and Leung, 2001).

F. On the origin of *CP* violation

CP violation plays a central role in particle physics and has profound implications for cosmology. Yet the origin of *CP* violation is an entirely open question (Branco, Lavoura, and Silva, 1999). It is well known that if one allows for complex Yukawa couplings, *CP* violation arises in the SM with three or more fermion generations.

An alternative possibility is having *CP* as a good symmetry of the Lagrangian, only broken spontaneously by the vacuum. This is an attractive scenario which may be the only choice at a fundamental level, if one keeps in mind that pure gauge theories necessarily conserve *CP* (Grimus and Rebelo, 1997). The first model with spontaneous *CP* violation was suggested by T. D. Lee (1973) at a time when only two incomplete generations were known. Obviously, in the original Lee model with two generations, *CP* violation arises exclusively through the Higgs exchange. The Lee model has two Higgs doublets and no extra symmetry is introduced. As a result, fermions of a given charge receive contributions to their mass from the two Higgs fields. It can be readily verified that a nontrivial CKM mixing matrix is then generated by the relative phase between the two neutral Higgs VEV. However, since natural flavor conservation (NFC) is not implemented in the Higgs sector, there are dangerous Higgs-mediated flavor-changing neutral currents at tree level. One can implement NFC in the Higgs sector (Glashow and Weinberg, 1977; Paschos, 1977), but then three Higgs doublets are required in order to achieve spontaneous *CP* violation (Branco, 1980). The CKM matrix is, however, real in this model, which is in disagreement with the experimental evidence for a complex mixing matrix, even if one allows for the presence of new physics (Botella *et al.*, 2005).

One can envisage a simple model of spontaneous *CP* violation, which avoids the above difficulties while providing a possible common source for the various manifestations of *CP* violation (Branco, Parada, and Rebelo, 2003) in the quark and lepton sectors, as well as a solution to the strong *CP* problem. We outline below the main features of such a model (Branco, Parada, and Rebelo, 2003) in which all *CP*-breaking

effects share the same origin, namely, the VEV of a complex singlet scalar field. This minimal model consists of an extension of the SM with the following additional fields: three right-handed neutrinos ν_R, a neutral scalar singlet S, and a singlet vectorial quark D with charge $-1/3$. Furthermore, one imposes on the Lagrangian a Z_4 symmetry, under which the fields transform in the following manner:

$$\ell \to i\ell, \qquad l_R \to il_R, \qquad \nu_R \to i\nu_R,$$
$$D \to -D, \qquad S \to -S. \tag{2.44}$$

Under the above Z_4 symmetry, all other fields remain invariant. Furthermore, we impose *CP* invariance at the Lagrangian level. In the quark sector, the most general SU(3) × SU(2) × U(1) × Z_4 invariant Yukawa couplings can be written as

$$\mathcal{L}_{\text{quark}} = \bar{Q}_i \mathbf{Y}_{ij}^u \phi u_{Rj} + \bar{Q}_i \mathbf{Y}_{ij}^d \tilde{\phi} d_{Rj} + \tilde{M}\bar{D}_L D_R$$
$$+ \bar{D}_L(\mathbf{f}_{qi}S + \mathbf{f}'_{qi}S^*)d_{Ri} + \text{H.c.}, \tag{2.45}$$

while for the lepton sector they are

$$\mathcal{L}_{\text{lepton}} = \bar{\ell}_i \mathbf{Y}_{ij}^\ell \phi l_{Rj} + \bar{\ell}_i \mathbf{Y}_{ij}^\nu \tilde{\phi} \nu_{Rj}$$
$$+ \tfrac{1}{2}\nu_{Ri}^T C[(\mathbf{f}_\nu)_{ij}S + (\mathbf{f}'_\nu)_{ij}S^*]\nu_{Rj} + \text{H.c.} \tag{2.46}$$

Here Q, u_R, and d_R are the SM quark fields; $\mathbf{Y}^{u,d}$, $\mathbf{Y}^\ell$, $\mathbf{f}_{q,\nu}$, and $\mathbf{f}'_{q,\nu}$ are Yukawa coupling matrices. All couplings are assumed to be real so that the full Lagrangian is *CP* invariant. However, *CP* is spontaneously broken by the vacuum. Indeed, the Higgs potential contains terms of the form

$$V \propto (\mu^2 + \lambda_1 S^* S + \lambda_2 \phi^\dagger \phi)(S^2 + S^{*2}) + \lambda_3(S^4 + S^{*4}), \tag{2.47}$$

and, for an appropriate region of the parameter space, the scalar fields acquire VEV of the form $\langle \phi \rangle = v$ and $\langle S \rangle = Ve^{i\alpha}$.

It is possible to show that the phase α generates all *CP* violations, namely, nontrivial complex CKM and PMNS matrices, as well as the leptonic *CP* violation at high energies needed for leptogenesis. In order to verify that a nontrivial phase is generated in the CKM matrix $\mathbf{V}_{\text{CKM}}$, one has to recall that the mixing matrix connecting standard quarks is determined by

$$\mathbf{V}_{\text{CKM}}^{-1}\mathbf{h}_d\mathbf{V}_{\text{CKM}} = \mathbf{d}_d^2, \tag{2.48}$$

where

$$\mathbf{h}_d = \mathbf{m}_d\mathbf{m}_d^\dagger - \frac{\mathbf{m}_d\mathbf{M}_D^\dagger\mathbf{M}_D\mathbf{m}_d^\dagger}{\bar{M}^2}, \tag{2.49}$$

$\mathbf{d}_d^2 = \text{diag}(m_d^2, m_s^2, m_b^2)$, $\mathbf{m}_d = v\mathbf{Y}^d$, $\bar{M}^2 = \mathbf{M}_D\mathbf{M}_D^\dagger + \tilde{M}^2$, and $\mathbf{M}_D = V(\mathbf{f}_q^+ \cos\alpha + i\mathbf{f}_q^- \sin\alpha)$ with $\mathbf{f}_q^\pm \equiv \mathbf{f}_q \pm \mathbf{f}'_q$. Note that without loss of generality, we have chosen a weak basis with a diagonal and real up-quark mass matrix. The crucial point is then the following: the first term contributing to $\mathbf{h}_d$ in Eq. (2.49) is real since the matrix $\mathbf{m}_d$ is real due to the *CP* invariance of the Lagrangian; the second term in $\mathbf{h}_d$ is however complex, and of the same order of magnitude as the first one. As a result, $\mathbf{h}_d$ is a generic complex 3×3 Hermitian matrix, leading to a complex $\mathbf{V}_{\text{CKM}}$ matrix. For any specific model, one can explicitly check that *CP* violation *à la*

Kobayashi-Maskawa is generated by computing the *CP*-odd WB invariant given in Eq. (2.24). Having $\mathcal{J}_{\text{quark}}^{CP} \neq 0$ is a necessary and sufficient condition to have *CP* violation through the Kobayashi-Maskawa mechanism.

In the lepton sector, the neutrino mass matrix $\mathbf{m}_\nu$ is generated after the spontaneous symmetry breaking through the standard type I seesaw mechanism given in Eq. (2.35), with $\mathbf{m}_R = V(\mathbf{f}_\nu^+ \cos\alpha + i\mathbf{f}_\nu^- \sin\alpha)$ and $\mathbf{f}_\nu^\pm \equiv \mathbf{f}_\nu \pm \mathbf{f}'_\nu$. Although the Dirac-neutrino Yukawa coupling matrix $\mathbf{Y}^\nu$ is real, the matrix $\mathbf{m}_R$ is a generic complex symmetric matrix. As a result, the effective neutrino mass matrix $\mathbf{m}_\nu$ is a generic complex symmetric matrix, and the PMNS leptonic mixing matrix has, in general, three *CP*-violating phases. One can also check that the model has the *CP* violation necessary for leptogenesis to work.

An important constraint on models with spontaneous *CP* violation is related with the so-called domain-wall problem (Vilenkin, 1985). As pointed out in the seminal papers (Zeldovich, Kobzarev, and Okun, 1974; Kibble, 1976), the spontaneous breaking of a discrete global symmetry in the early Universe leads to the formation of domain walls with an energy density proportional to the inverse of the cosmological scale factor. Therefore, those objects could dominate over matter and radiation, overclosing the Universe. Although this represents a serious problem, several solutions have been put forward in order to solve it. One possible way to avoid the crippling effects of domain walls is to invoke an inflationary period that dilutes them away (Langacker, 1989). Note that this does not prevent the complex phase of $\langle S \rangle$ from generating a complex CKM matrix [see Eq. (2.49)]. An alternative way out relies on considering the existence of a (small) bare θ_{QCD} term (Krauss and Rey, 1992). In this case, it can be shown that the vacuum degeneracy connecting the two sides of the *CP* domain wall is lifted, resulting in a wall annihilation driven by the decay of a false vacuum. More interestingly, assuming that gravity breaks global discrete symmetries explicitly, then there is probably no domain-wall problem at all (Rai and Senjanovic, 1994; Dvali and Senjanovic, 1995). These few examples show that although this problem arises whenever *CP* is spontaneously broken, it is possible to overcome it independently of the dynamics behind the symmetry breaking. In particular, simple scenarios as the one outlined above could in principle generate complex CKM and PMNS matrices at low energies regardless of the solution chosen to the domain-wall problem.

G. The hypothesis of minimal lepton flavor violation

One of the proposals for the description of flavor-changing processes in the quark sector is the so-called hypothesis of minimal flavor violation (MFV) (Chivukula and Georgi, 1987; Buras *et al.*, 2001; D'Ambrosio *et al.*, 2002). It consists of assuming that even if there is new physics beyond the SM, Yukawa couplings are the only source flavor-changing processes. More precisely, the MFV hypothesis assumes that Yukawa couplings are the only source of the breaking of the large U(3)^5 global flavor symmetry present in the gauge sector of the SM with three generations.

If one assumes the presence of two Higgs doublets, the MFV principle can be implemented under the assumption of

NFC in the Higgs sector (Glashow and Weinberg, 1977; Paschos, 1977) or through the introduction of a discrete symmetry which leads to naturally suppressed flavor-changing neutral currents in the Higgs sector (Branco, Grimus, and Lavoura, 1996; Botella, Branco, and Rebelo, 2010). One of the interesting features of MFV in the quark sector is the prediction of the ratio of branching ratios of low-energy processes, which do not depend on the specific MFV model.

The MFV hypothesis has also been extended to the lepton sector (Cirigliano *et al.*, 2005) but, in contrast to the quark sector, this extension is not unique and requires additional input from physics at high energies. The reason is that the total lepton number may not be a symmetry of the theory since neutrinos can be Majorana particles. In order to extend MFV to the lepton sector, one has to make a choice between two possibilities

(i) Minimal field content: No new fields are introduced beyond the SM content and it is just assumed that some new physics at a high-energy scale generates an effective Majorana mass for the left-handed neutrinos.

(ii) Extended field content: Two or more right-handed neutrinos are introduced with gauge-invariant lepton number violating mass terms, which generate an effective seesaw neutrino mass matrix for light neutrinos.

CP violation was not considered at either low or high energies by Cirigliano *et al.* (2005). The inclusion of *CP* violation in a minimal lepton flavor violation (MLFV) scenario is crucial, for instance, in order to have a consistent framework to generate the baryon asymmetry through leptogenesis (Branco *et al.*, 2007). Subsequent suggestions (Davidson and Palorini, 2006; Cirigliano, Isidori, and Porretti, 2007; Cirigliano *et al.*, 2008; Gavela *et al.*, 2009; Alonso *et al.*, 2011) for MLFV did include *CP* violation in the lepton sector.

For definiteness, we analyze the MLFV hypothesis in the context of a minimal extension of the SM with three right-handed neutrinos ν_R. In this case, the relevant leptonic Yukawa coupling and right-handed Majorana mass terms are those given by Eq. (2.33) plus the usual charged-lepton Yukawa term $\bar{\ell}_i \phi \mathbf{Y}^\ell_{ij} l_{Rj}$. In the limit when these terms vanish, the Lagrangian of this extension of the SM has a large global flavor symmetry $SU(3)_\ell \times SU(3)_{l_R} \times SU(3)_{\nu_R} \times U(1)_\ell \times U(1)_{l_R} \times U(1)_{\nu_R}$. An interesting proposal for MLFV assumes that the physics leading to lepton-number violation through the generation of the mass matrix $\mathbf{m}_R$ is lepton blind, thus leading to an exact degenerate spectrum for the right-handed neutrinos at a high-energy scale. In this MLFV framework, the Majorana mass terms break $SU(3)_{\nu_R}$ into $O(3)_{\nu_R}$. Note that, even in the limit of exact degeneracy, $\mathbf{m}_R$ is not a WB invariant. Indeed, for a WB transformation under which $\nu_R \rightarrow \mathbf{V}_R \nu_R$, it transforms as $\mathbf{m}_R \rightarrow \mathbf{m}'_R = \mathbf{V}_R \mathbf{m}_R \mathbf{V}_R^T$. This transformation does not leave $\mathbf{m}_R$ invariant, even in the limit of exact degeneracy, since in general $\mathbf{V}_R \mathbf{V}_R^T \neq \mathbb{1}$.

It is worth emphasizing that MLFV in a framework with right-handed neutrinos is not as predictive as MFV in the quark sector (Branco *et al.*, 2007). A rich spectrum of possibilities is allowed for lepton flavor-violating (LFV)

processes and their correlation with low-energy neutrino physics and LHC physics [see, for example, Deppisch and Pilaftsis (2011)].

III. OBSERVABLE EFFECTS FROM LEPTONIC *CP* VIOLATION

Establishing the existence of LCPV is one of the main goals of the future neutrino physics program. The most promising way to search for CPV effects in the lepton sector is through the study of neutrino oscillations, which are sensitive to the Dirac *CP* phase δ entering the neutrino mixing matrix **U** of Eq. (2.11). The experimental sensitivity to LCPV depends strongly on the value of the reactor neutrino mixing angle θ_{13}, and also on the type of neutrino mass spectrum. In particular, if θ_{13} is not too small, then future experiments will be able to establish soon the existence (or not) of LCPV.

There are however other phenomena which, although being *CP* conserving, are also sensitive to the presence of *CP* phases in the lepton mixing matrix. For instance, predictions regarding neutrinoless beta decay rates change depending on the values of the Majorana phases $\alpha_{1,2}$. Other phenomena, which are triggered by the presence of new physics directly connected with neutrino masses and mixing may also be impacted from the fact that *CP* is violated in the lepton sector. A typical example is rare lepton flavor-violating decays like $l_j \rightarrow l_i \gamma$ ($j \neq i$), $l_j \rightarrow l_i l_k l_k$ ($i, k \neq j$) and μ-e conversion in nuclei (Raidal *et al.*, 2008). Ultimately, if the physics responsible for neutrino mass generation is close to the electroweak scale, then LCPV may also affect phenomena which could be observed at high-energy colliders like the LHC or a linear collider.

In this section, we present a general discussion about the possible direct and indirect effects of LCPV, with special emphasis on neutrino oscillations.

A. Neutrino oscillation parameters: Present status

The observation of a solar-neutrino deficit with respect to standard solar model predictions at the Homestake experiment (Cleveland *et al.*, 1998) provided the first hint in favor of neutrino oscillations. This observation has been confirmed by several other solar-neutrino experiments like SAGE (Abdurashitov *et al.*, 2002), Gallex (Hampel *et al.*, 1999), GNO (Altmann *et al.*, 2005), Kamiokande (Fukuda *et al.*, 2002), Super-Kamiokande (Smy *et al.*, 2004), and the Sudbury Neutrino Observatory (SNO) (Ahmad *et al.*, 2001). The data collected from these experiments led to the large mixing angle solution to the solar-neutrino problem, which was confirmed in 2002 by the KamLAND reactor neutrino experiment (Eguchi *et al.*, 2003).

A similar anomaly has also been observed in the atmospheric neutrino sector by the Irvine-Michigan-Brookhaven (IMB) (Becker-Szendy *et al.*, 1992), Kamiokande (Hirata *et al.*, 1992), MACRO (Ambrosio *et al.*, 2003), Soudan-2 (Sanchez *et al.*, 2003), and Super-Kamiokande (Fukuda *et al.*, 1998) experiments, which detected a ν_μ to ν_e-induced event ratio smaller than the expected. Atmospheric neutrino parameters are also constrained by the K2K (KEK to Kamioka) (Aliu *et al.*, 2005) and MINOS (Fermilab to
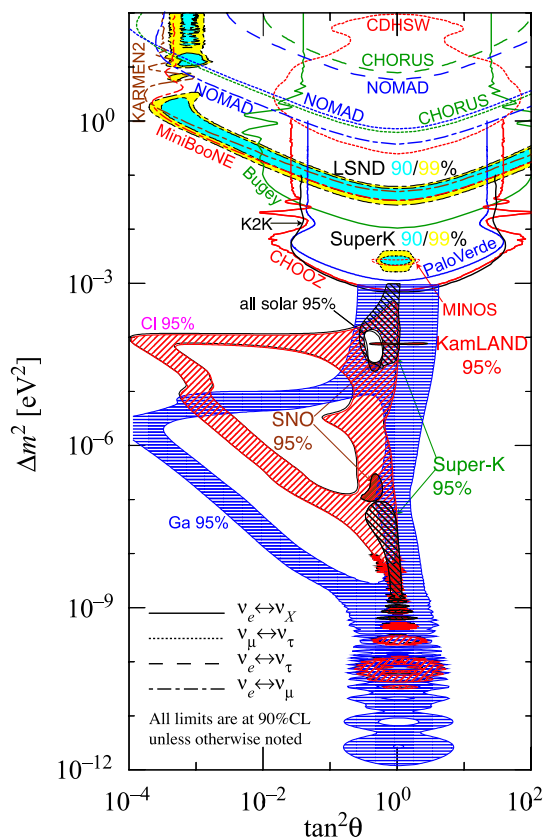
FIG. 3 (color online). Favored and excluded regions of neutrino mass-squared differences and mixing angles taken into account the data of several neutrino experiments. From Nakamura *et al.*, 2010.

three global analyses performed by Gonzalez-Garcia, Maltoni, and Salvado (GMS) (Gonzalez-Garcia, Maltoni, and Salvado, 2010), Schwetz, Tórtola, and Valle (STV) (Schwetz, Tortola, and Valle, 2011), and the Bari group (Fogli *et al.*, 2011).

In contrast with the quark sector, there are two large mixing angles in the lepton sector: θ_{12} and θ_{23}, sometimes referred as the *solar* and *atmospheric* neutrino mixing angles (see Table I). The current data indicate that, at their best-fit values, $\theta_{12} \simeq 34°$ and $\theta_{23} \simeq 45°$ (maximal atmospheric mixing), while the value of the remaining mixing angle θ_{13} are mainly constrained by reactor and accelerator neutrino experiments to be small. Recent data from the T2K (Abe *et al.*, 2011) and MINOS (Adamson *et al.*, 2011) experiments also indicate a relatively large value for θ_{13}. At 90% C.L., the T2K data are consistent with $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ for normal (inverted) hierarchy in the absence of Dirac *CP* violation. The MINOS Collaboration reports the best-fit values $2\sin^2(\theta_{23})\sin^2(2\theta_{13}) = 0.041^{+0.047}_{-0.031}(0.079^{+0.071}_{-0.053})$. These results have been taken into account in the global analyses performed by STV and the Bari group. As it is apparent from Table I, there is now an evidence for $\theta_{13} > 0$ at more than 3σ.

Neutrino oscillation experiments are not sensitive to the absolute neutrino mass scale since the oscillation frequency is controlled by the neutrino mass-squared differences Δm^2_{ji} and the neutrino energy. The current data are consistent with a three-neutrino scenario with $\Delta m^2_{21} \sim 7.6 \times 10^{-5}$ eV^2 and $|\Delta m^2_{31}| \sim 2.5 \times 10^{-3}$ eV^2, which implies a hierarchy among these two quantities such that

$$r \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \simeq \pm 0.03. \qquad (3.1)$$

The sign of Δm^2_{31} is not yet determined and therefore two types of neutrino mass spectrum are possible, namely,

$$\text{normally ordered (NO): } m_1 < m_2 < m_3,$$
$$\text{invertedly ordered (IO): } m_3 < m_1 < m_2. \qquad (3.2)$$

For each case, the neutrino masses can be expressed in terms of the lightest mass (m_1 and m_3 for the NO and IO cases, respectively), and the mass-squared differences Δm^2_{ji}

Soudan mine) (Michael *et al.*, 2006) accelerator long-baseline experiments. Both experiments observed that a fraction of the ν_μ neutrinos in the original beam disappear consistently with the hypothesis of neutrino oscillations.

Other experiments have provided useful data in constraining the neutrino parameter space. An illustrative way to present these data is given in Fig. 3, where the favored and excluded regions of neutrino mass-squared differences and mixing angles are shown, taking into account the results of several experiments. In Table I, we summarize the results of

TABLE I. Best-fit values with 1σ and 3σ errors for the three-flavor neutrino oscillation parameters, obtained by Gonzalez-Garcia, Maltoni, and Salvado (GMS) (Gonzalez-Garcia, Maltoni, and Salvado, 2010), Schwetz, Tórtola, and Valle (STV) (Schwetz, Tortola, and Valle, 2011), and the Bari group (Fogli *et al.*, 2011).

		GMS (Gonzalez-Garcia, Maltoni, and Salvado, 2010)	STV (Schwetz, Tortola, and Valle, 2011)	Bari (Fogli *et al.*, 2011)
$\Delta m^2_{21}[10^{-5}]$ eV^2		$7.59 \pm 0.20(^{+0.61}_{-0.69})$	$7.59^{+0.20}_{-0.18}(^{+0.60}_{-0.50})$	$7.58^{+0.22}_{-0.26}(^{+0.60}_{-0.59})$
$\Delta m^2_{31}[10^{-3}]$ eV^2	(NO)	$2.46 \pm 0.12(\pm 0.37)$	$2.50^{+0.09}_{-0.16}(^{+0.26}_{-0.36})$	$2.35^{+0.12}_{-0.09}(^{+0.32}_{-0.29})$
	(IO)	$-2.36 \pm 0.11(\pm 0.37)$	$-[2.40^{+0.08}_{-0.09}(\pm 0.27)]$	$-[2.35^{+0.12}_{-0.09}(^{+0.32}_{-0.29})]$
$\sin^2\theta_{12}$		$0.319 \pm 0.016(^{+0.053}_{-0.046})$	$0.312^{+0.017}_{-0.015}(^{+0.048}_{-0.042})$	$0.312^{+0.017}_{-0.016}(^{+0.052}_{-0.047})$
$\sin^2\theta_{23}$	(NO)	$0.46^{+0.08}_{-0.05}(^{+0.18}_{-0.12})$	$0.52^{+0.06}_{-0.07}(^{+0.12}_{-0.13})$	$0.42^{+0.08}_{-0.03}(^{+0.22}_{-0.08})$
	(IO)	$0.46^{+0.08}_{-0.05}(^{+0.18}_{-0.12})$	$0.52 \pm 0.06(^{+0.12}_{-0.13})$	$0.42^{+0.08}_{-0.03}(^{+0.22}_{-0.08})$
$\sin^2\theta_{13}$	(NO)	$0.0095^{+0.013}_{-0.007}(\leq 0.047)$	$0.013^{+0.007}_{-0.005}(^{+0.022}_{-0.012})$	$0.025 \pm 0.007(^{+0.025}_{-0.02})$
	(IO)	$0.0095^{+0.013}_{-0.007}(\leq 0.047)$	$0.016^{+0.008}_{-0.006}(^{+0.023}_{-0.015})$	$0.025 \pm 0.007(^{+0.025}_{-0.02})$

NO: $m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}$, $\quad m_3 = \sqrt{m_1^2 + |\Delta m_{31}^2|}$,

IO: $m_1 = \sqrt{m_3^2 + |\Delta m_{31}^2|}$, $\quad m_2 = \sqrt{m_3^2 + \Delta m_{21}^2 + |\Delta m_{31}^2|}$.

$$(3.3)$$

Depending on the value of the lightest neutrino mass, one can further classify the neutrino mass spectrum as being hierarchical (HI): $m_1 \ll m_2 < m_3$, inverted hierarchical (IH): $m_3 \ll m_1 < m_2$, or quasidegenerate (QD): $m_1 \simeq m_2 \simeq m_3 \simeq m_0 \gg |\Delta m_{31}^2|$, $m_0 \gtrsim 0.1$ eV. In the HI and IH limits, the neutrino masses are

$$m_2^{\text{HI}} \simeq \sqrt{\Delta m_{21}^2} \simeq 0.009 \text{ eV},$$
$$m_3^{\text{HI}} \simeq m_{1,2}^{\text{IH}} \simeq \sqrt{|\Delta m_{31}^2|} \simeq 0.05 \text{ eV}. \tag{3.4}$$

A direct kinematical bound is available for the effective electron-neutrino mass in β decay, $m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$. From the Mainz (Bonn *et al.*, 2002) and Troitzk (Lobashev *et al.*, 2001) experiments, $m_\beta < 2.3$ eV at 95% C.L., which implies $m_i < 2.3$ eV. In the future, the KATRIN experiment (Osipowicz *et al.*, 2001) is expected to reach a sensitivity of $m_\beta \simeq 0.2$ eV. The current 7-year WMAP data constrain the sum of neutrino masses to be less than 1.3 eV at 95% C.L. (Komatsu *et al.*, 2011) (within the standard cosmological model). Less conservative bounds can be obtained combining the data of several cosmological and astrophysical experiments (Abazajian *et al.*, 2011). The future Planck satellite data alone will allow one to set an upper bound on $\sum_i m_i$ of 0.6 eV at 95% C.L. (Hannestad, 2010). Concerning CPV in the lepton sector, the presently available neutrino data do not provide any information on the *CP* phases δ (Dirac) and $\alpha_{1,2}$ (Majorana). In the following, we discuss how LCPV can be probed in future experiments.

B. LCPV in neutrino oscillations

The existence of more than two neutrino flavors opens the possibility for the existence of *CP*-violating effects in the lepton sector, characterized by the *CP* phases of the neutrino mixing matrix **U**. Since neutrino oscillations depend directly on the way neutrinos mix among themselves and, consequently, on the existence of *CP* phases, it is not surprising that they represent the golden path for the search of LCPV. Yet, establishing CPV in the neutrino sector turns out to be a rather hard task. In the last years, several ideas have been brought together with the aim of overcoming these difficulties and finding the best strategy to detect CPV effects in neutrino oscillations. In particular, new experimental setups have been proposed in order to improve our knowledge of the neutrino parameters.

In this section, we review some basic aspects related to the formalism of LCPV and neutrino oscillations and discuss possible ways to search for CPV, pointing out the main difficulties inherent to this investigation. Moreover, we intend to draw a general picture of the prospects for the discovery of LCPV in future neutrino oscillation experiments. For more complete discussions about theoretical aspects of neutrino oscillations, we address the reader to other dedicated reviews

(Bilenky and Petcov, 1987; Akhmedov, 1999; Bilenky, Giunti, and Grimus, 1999; Mohapatra and Smirnov, 2006; Strumia and Vissani, 2006; Gonzalez-Garcia and Maltoni, 2008; Akhmedov, Smirnov, and Zatsepin, 2011) and textbooks (Fukugita and Yanagida, 2003; Giunti and Kim, 2007).

1. CPV in vacuum oscillations

If neutrinos are massive and mix, then a neutrino state produced via weak interactions (like nuclear beta and pion decays) is not a mass eigenstate. In this case, the weak eigenstates ν_α are a unitary linear combination of the mass eigenstates ν_k, in such a way that

$$|\nu_\alpha\rangle = \sum_{k=1}^{n} U_{\alpha k}^* |\nu_k\rangle, \tag{3.5}$$

where **U** is the lepton mixing matrix defined in Eq. (2.14). As first pointed out by Pontecorvo, the fact that mass and flavor eigenstates are different leads to the possibility of neutrino oscillations (Pontecorvo, 1968). The time evolution of a neutrino produced with a specific flavor is governed by

$$|\nu_\alpha(t)\rangle = \sum_{k=1}^{n} U_{\alpha k}^* e^{-iE_k t} |\nu_k\rangle, \tag{3.6}$$

where E_k is the energy of the neutrino mass eigenstate ν_k. For relativistic neutrinos, $E_k = \sqrt{p_k^2 + m_k^2} \simeq p_k + m_k^2/(2E_k)$. The $\nu_\alpha \rightarrow \nu_\beta$ transition amplitude is then given by

$$\mathcal{A}_{\alpha\beta}(t) = \sum_{k=1}^{n} U_{\beta k} e^{-iE_k t} U_{\alpha k}^*, \tag{3.7}$$

and the corresponding transition probability by $P_{\alpha\beta} = |\mathcal{A}_{\alpha\beta}|^2$. For $t = 0$ and $\alpha \neq \beta$, the above equation is equivalent to the definition of the Dirac unitarity triangles $T_{\alpha\beta}$ given in Eqs. (2.17). The time evolution of $\mathcal{A}_{\alpha\beta}$ can then be interpreted as a time-dependent rotation of the sides of these triangles.

Considering that for ultrarelativistic neutrinos $t \simeq L$ (where L is the distance traveled by neutrinos) and assuming equal momenta for all of the neutrino mass eigenstates ($p_k \equiv p \simeq E$ for any k), the $\nu_\alpha \rightarrow \nu_\beta$ oscillation probabilities can be further expressed as

$$P_{\alpha\beta}(L, E) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathcal{R}_{\alpha\beta}^{kj} \sin^2\frac{\Delta_{kj}}{2} + 2\sum_{k>j} \mathcal{J}_{\alpha\beta}^{kj} \sin\Delta_{kj},$$

$$(3.8)$$

where

$$\Delta_{kj} = \frac{\Delta m_{kj}^2 L}{2E}. \tag{3.9}$$

The quantities $\mathcal{R}_{\alpha\beta}^{kj}$ and $\mathcal{J}_{\alpha\beta}^{kj}$ are invariant combinations of the elements of **U** given by

$$\mathcal{R}_{\alpha\beta}^{kj} = \text{Re}[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*],$$
$$\mathcal{J}_{\alpha\beta}^{kj} = \text{Im}[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*]. \tag{3.10}$$

The above formulas show that the transition probabilities $\nu_\alpha \rightarrow \nu_\beta$ depend on the elements of the mixing matrix **U**,

$n - 1$ independent mass-squared differences, and the ratio L/E, which depends on the specific experimental setup. Within the simplest framework of two neutrinos, the oscillation probability is given by

$$P_{\alpha\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right), \qquad \alpha \neq \beta, \qquad (3.11)$$

being the survival probability $P_{\alpha\alpha} = 1 - P_{\alpha\beta}$. Therefore, to be sensitive to neutrino oscillations, experiments must be designed in such a way that $L \sim L_{\rm osc}$, with

$$L_{\rm osc} = \frac{4\pi E}{\Delta m^2} = 2.47 \frac{E[{\rm GeV}]}{\Delta m^2[{\rm eV}^2]} \text{ km.} \qquad (3.12)$$

The fact that *CP* violation in the lepton sector can be tested in neutrino oscillation experiments was first noted by Cabibbo (1978) and Barger, Whisnant, and Phillips (1980). Such tests require the comparison of transitions $\nu_\alpha \to \nu_\beta$ with the corresponding *CP*-conjugate channel $\bar{\nu}_\alpha \to \bar{\nu}_\beta$, or with $\nu_\beta \to \nu_\alpha$ if *CPT* invariance holds. For antineutrinos, the equivalent of Eqs. (3.5) and (3.6) reads

$$|\bar{\nu}_\alpha\rangle = \sum_{k=1}^n \mathbf{U}_{\alpha k}|\bar{\nu}_k\rangle, \qquad |\bar{\nu}_\alpha(t)\rangle = \sum_{k=1}^n \mathbf{U}_{\alpha k} e^{-iE_k t}|\bar{\nu}_k\rangle,$$

$$(3.13)$$

which lead to the following $\bar{\nu}_\alpha \to \bar{\nu}_\beta$ transition amplitudes and probabilities in vacuum:

$$\bar{\mathcal{A}}_{\alpha\beta}(t) = \sum_{k=1}^n \mathbf{U}^*_{\beta k} e^{-iE_k t}\mathbf{U}_{\alpha k}, \qquad \bar{P}_{\alpha\beta} = |\bar{\mathcal{A}}_{\alpha\beta}|^2,$$

$$(3.14)$$

respectively. It is straightforward to see that, due to *CPT* conservation, $\bar{P}_{\alpha\beta} = P_{\beta\alpha}$ (Cabibbo, 1978). The transformation properties of the neutrino flavor transitions under *CP*, *T*, and *CPT* are shown in Fig. 4.

Under *CP*, neutrinos transform into their antiparticles ($\nu_\alpha \leftrightarrow \bar{\nu}_\alpha$). Depending on whether we consider the case of Dirac or Majorana neutrinos, *CP* invariance in the lepton sector implies (Bilenky, Nedelcheva, and Petcov, 1984)

$$\mathbf{U}_{\alpha k} = \mathbf{U}^*_{\alpha k} \quad \text{(Dirac)}, \qquad (3.15)$$

$$\mathbf{U}_{\alpha k} = -i\rho_k \eta_k^{CP} \mathbf{U}^*_{\alpha k} \quad \text{(Majorana)}, \qquad (3.16)$$

where $\eta_k^{CP} = \pm i$ is the *CP* parity of the neutrino mass eigenstate with mass m_k and ρ_k is an arbitrary phase factor present in the Majorana condition $C\bar{\nu}_k^T = \rho_k \nu_k$. Therefore, *CP* invariance automatically leads to $\bar{P}_{\alpha\beta} = P_{\alpha\beta}$. Obviously,
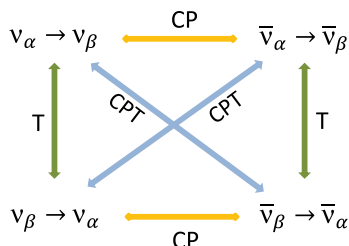

FIG. 4 (color online). Transformations of the different flavor-transition channels under *CP*, *T*, and *CPT*.

due to *CPT* conservation, *CP* invariance is equivalent to *T* invariance.

The most obvious way to measure *CP* violation in the neutrino sector is by looking at the differences $\Delta P_{\alpha\beta}^{CP} = P_{\alpha\beta} - \bar{P}_{\alpha\beta}$. Taking into account that $\bar{P}_{\alpha\beta}$ is obtained replacing $\mathbf{U}$ by $\mathbf{U}^*$ in Eq. (3.8), one has (Barger, Whisnant, and Phillips, 1980; Pakvasa, 1980)

$$\Delta P_{\alpha\beta}^{CP} = 4\sum_{k>j}\mathcal{J}_{\alpha\beta}^{kj}\sin\Delta_{kj}, \qquad (3.17)$$

which coincides with the *T*-violating probability differences $\Delta P_{\alpha\beta}^{\rm T} = P_{\alpha\beta} - P_{\beta\alpha}$. The above equation can be rewritten as

$$\Delta P_{\alpha\beta}^{CP} = -16\mathcal{J}_{\alpha\beta}^{21}\sin\frac{\Delta_{21}}{2}\sin\frac{\Delta_{13}}{2}\sin\frac{\Delta_{32}}{2}, \qquad (3.18)$$

with $\Delta P_{e\mu}^{CP} = \Delta P_{\mu\tau}^{CP} = \Delta P_{\tau e}^{CP} = \Delta P^{CP}$ and

$$\Delta P^{CP} = 4\mathcal{J}_{e\mu}^{21}(\sin\Delta_{21} + \sin\Delta_{32} + \sin\Delta_{13}). \qquad (3.19)$$

The invariant quantity $\mathcal{J}_{e\mu}^{21}$ has been defined in Eq. (2.27). From these results, it is clear that *CP* violation is absent in neutrino oscillations, if two (or more) neutrinos are degenerate in mass, or if one of the mixing angles is zero. Therefore, CPV in vacuum oscillations occurs as a pure three-flavor effect, and thus is suppressed by small mixing angles. Moreover, since $\bar{\nu}_\alpha \to \bar{\nu}_\alpha$ is the *CPT* transformed of $\nu_\alpha \to \nu_\alpha$, CPV cannot be observed in disappearance channels. Experimentally, the measurement of LCPV in neutrino oscillations requires sensitivity to the oscillatory behavior of the neutrino and antineutrino transition probabilities. In other words, L and E have to be such that at least one of the phases Δ_{kj} is of order 1. Indeed, if $\Delta_{kj} \ll 1$ for all k and j, then the transition probabilities are too small to be observed. On the other hand, in the limit $\Delta_{kj} \gg 1$, the averaged $\Delta P_{\alpha\beta}$ goes to zero. It is also important to note that, if the order-one phase corresponds to the largest Δm_{kj}^2, then $\Delta P_{\alpha\beta}^{CP} \simeq 0$ (Barger, Whisnant, and Phillips, 1980; Bilenky and Niedermayer, 1981). This can be readily understood considering the case $\Delta m_{32}^2 \simeq \Delta m_{31}^2 \gg \Delta m_{21}^2$. If $\Delta_{31} \simeq \Delta_{32} \simeq 1$ (short baseline) then $\Delta_{21} \ll 1$ and $\Delta P_{\alpha\beta}^{CP} \simeq 4(\mathcal{J}_{\alpha\beta}^{31} + \mathcal{J}_{\alpha\beta}^{32})\sin\Delta_{31} = 0$, due to the fact that $\mathcal{J}_{\alpha\beta}^{31} = -\mathcal{J}_{\alpha\beta}^{32}$ [see Eq. (3.10)]. Therefore, a measurement of the *CP*-odd asymmetry in neutrino oscillations can be performed only in long-baseline experiments (Arafune and Sato, 1997; Minakata and Nunokawa, 1997; Tanimoto, 1997; Bilenky, Giunti, and Grimus, 1998), as long as $|\mathcal{J}_{e\mu}^{21}|$ is not too small.

2. Matter-induced *CP* violation

The discussion presented in the previous section raises the question on whether a measurement of a nonzero ΔP^{CP} automatically implies that *CP* is violated in the lepton sector. Although this would be true in vacuum oscillations, matter effects in neutrino propagation (Wolfenstein, 1978; Barger *et al.*, 1980; Mikheev and Smirnov, 1985) can fake *CP* violation (Kuo and Pantaleone, 1987; Krastev and Petcov, 1988). Indeed, the presence of matter violates *C*, *CP*, and *CPT* due to the unequal number of particle and antiparticles (electrons and positrons) in the medium. In matter, the relevant effective Hamiltonian for neutrinos can be written as

$$\mathbf{H}' = \frac{1}{2E}[\mathbf{U}\,\mathbf{M}^2\mathbf{U}^\dagger + \mathbf{A}], \tag{3.20}$$

where $\mathbf{M}^2 = \mathrm{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)$ and $\mathbf{A} = \mathrm{diag}A(L), 0, 0$ with

$$A(L) \equiv 2\sqrt{2}EG_F N_e(L)$$

$$\simeq 2.3 \times 10^{-4} \text{ eV}^2 \, \frac{\rho(L)}{3 \text{ g cm}^{-3}} \, \frac{E}{\text{GeV}}. \tag{3.21}$$

Here $N_e(L)$ and $\rho(L)$ are the electron number and matter densities of the medium, respectively, as a function of the distance L. In the above estimate, the electron fraction number in matter has been considered to be $1/2$. Note that for an average density of 3 g cm^{-3} (which corresponds approximately to the Earth's lithosphere density), $AL/(2E) \simeq 0.6 \times 10^{-3}$ (L/km), meaning that matter effects are expected to be large for baselines $L \gtrsim 1000$ km.

For antineutrinos, the corresponding Hamiltonian $\bar{\mathbf{H}}'$ is obtained replacing $\mathbf{U}$ by $\mathbf{U}^*$ and A by $-A$ on the right-hand side of Eq. (3.20). Taking into account the neutrino evolution equation, one can show that the oscillation probabilities in matter do not depend on the Majorana phases $\alpha_{1,2}$ (Langacker *et al.*, 1987), just as in the vacuum oscillation regime.

The effective masses and mixing matrix for neutrinos and antineutrinos are obtained by diagonalizing $\mathbf{H}'$ and $\bar{\mathbf{H}}'$, respectively. The neutrino (antineutrino) transition probability in matter is then obtained replacing $\mathbf{U}$ by $\mathbf{U}'$ ($\bar{\mathbf{U}}'$) and Δm_{kj}^2 by $\Delta m_{kj}'^2$ ($\Delta \bar{m}_{kj}'^2$) in Eq. (3.8), where the primes refer to quantities in matter. As a result, one obtains for a constant matter-density profile

$$P'_{\alpha\beta}(L, E) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathcal{R}'^{lkj}_{\alpha\beta}\sin^2\frac{\Delta'_{kj}}{2} + 2\sum_{k>j} \mathcal{J}'^{lkj}_{\alpha\beta} \sin\Delta'_{kj}, \tag{3.22}$$

$$\bar{P}'_{\alpha\beta}(L, E) = \delta_{\alpha\beta} - 4\sum_{k>j} \bar{\mathcal{R}}'^{lkj}_{\alpha\beta}\sin^2\frac{\bar{\Delta}'_{kj}}{2} + 2\sum_{k>j} \bar{\mathcal{J}}'^{lkj}_{\alpha\beta} \sin\bar{\Delta}'_{kj}, \tag{3.23}$$

where $\mathcal{R}'^{lkj}_{\alpha\beta}$ and $\mathcal{J}'^{lkj}_{\alpha\beta}$ are now the invariants analogous to those defined in the vacuum regime [cf. Eqs. (3.10)],

$$\mathcal{R}'^{lkj}_{\alpha\beta} = \mathrm{Re}[\mathbf{U}'^*_{\alpha k}\mathbf{U}'_{\beta k}\mathbf{U}'_{\alpha j}\mathbf{U}'^*_{\beta j}],$$
$$\mathcal{J}'^{lkj}_{\alpha\beta} = \mathrm{Im}[\mathbf{U}'^*_{\alpha k}\mathbf{U}'_{\beta k}\mathbf{U}'_{\alpha j}\mathbf{U}'^*_{\beta j}], \tag{3.24}$$

and $\Delta'_{kj} = \Delta m_{kj}'^2 L/(2E)$. The corresponding quantities $\bar{\Delta}'_{kj}$, $\bar{\mathcal{R}}'^{lkj}_{\alpha\beta}$, and $\bar{\mathcal{J}}'^{lkj}_{\alpha\beta}$ are obtained replacing $\Delta m_{kj}'^2$ and $\mathbf{U}$ by $\Delta \bar{m}_{kj}'^2$ and $\bar{\mathbf{U}}$, respectively, in the previous expressions. It can be shown that the quantities $\mathcal{J}'^{lkj}_{\alpha\beta}$ and $\bar{\mathcal{J}}'^{lkj}_{\alpha\beta}$ are as good as $\mathcal{J}^{kj}_{\alpha\beta}$ for the proof of *CP* violation (Bilenky, Giunti, and Grimus, 1998; Harrison and Scott, 2000). However, the measurement of a *CP*-odd asymmetry in matter does not necessarily imply the existence of intrinsic CPV. From Eqs. (3.22) and (3.23), it is straightforward to show that $\Delta P'^{lCP}_{\alpha\beta} = P'_{\alpha\beta} - \bar{P}'_{\alpha\beta} \neq 0$ even if $\mathcal{J}'^{lkj}_{\alpha\beta} = \bar{\mathcal{J}}'^{lkj}_{\alpha\beta} = 0$, since the transition probabilities for neutrinos and antineutrinos are different in the

CP-conserving limit (Langacker *et al.*, 1987). *CP*-odd effects can also be observed in two-flavor neutrino oscillations due to the fact that the presence of matter may enhance, for instance, $\nu_e \leftrightarrow \nu_\mu$ oscillations and suppress the $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$, giving rise to a nonzero $\Delta P'^{lCP}_{e\mu}$. As for the survival probabilities, in general one has $P'_{\alpha\alpha} \neq \bar{P}'_{\alpha\alpha}$, contrarily to what happens in a vacuum. In conclusion, these fake CPV effects complicate the study of fundamental CPV in neutrino oscillations since *CP*-odd asymmetries can be observed even if $\delta = 0, \pi$.

Because of the *CPT*-violating character of the medium, *CP* and *T*-violation effects in matter are not directly connected.[3] Therefore, *T*-odd effects in matter can be analyzed independently of the *CP*-odd ones. The first simple observation is that there is no *T* violation in the two-flavor case. Taking the two flavors to be *e* and μ, unitarity implies $P'_{ee} + P'_{e\mu} = P'_{ee} + P'_{\mu e} = 1$, which in turn leads to the equality $P'_{e\mu} = P'_{\mu e}$. Thus, *T*-odd effects are present only for a number of neutrino flavors larger than 2. Moreover, in the presence of a symmetric matter-density profile, one can show that there are no matter-induced *T*-violating effects (Kuo and Pantaleone, 1987), since interchanging the final and initial neutrino flavors is equivalent to reversing the matter-density profile. In long-baseline neutrino oscillation experiments, matter effects due to the passage of neutrinos through the Earth are important. Since the Earth's matter density is not perfectly symmetric, the matter-induced *T* violation affects the *T*-odd asymmetries and therefore contaminates the determination of the fundamental *T* and *CP* asymmetries. Nevertheless, the asymmetries present in the Earth's density profile do not affect much the determination of the fundamental *CP*-violating phase δ (Akhmedov *et al.*, 2001).

It has been known for quite a long time that the most prominent oscillation channel for the study of three-flavor and matter effects in long-baseline experiments like neutrino factories is the so-called golden channel $\nu_e \rightarrow \nu_\mu$ (Barger, Whisnant, and Phillips, 1980; Minakata and Nunokawa, 1997; Tanimoto, 1997; De Rujula, Gavela, and Hernandez, 1999; Dick *et al.*, 1999; Cervera *et al.*, 2000; Donini *et al.*, 2000; Freund *et al.*, 2000). The exact formulas for the oscillation probabilities in matter are quite cumbersome due to the large number of parameters involved (Zaglauer and Schwarz, 1988; Ohlsson and Snellman, 2000). It is therefore convenient to consider expansions of $P_{\alpha\beta}$ and $\bar{P}_{\alpha\beta}$ in parameters which are known to be small. In the case of three-flavor neutrino oscillations, there are two rather obvious expansion parameters, namely, the mixing angle θ_{13} and the ratio r defined in Eq. (3.1). Approximate expressions for the oscillation probabilities in matter of constant density have been obtained for $\Delta m_{21}^2 \ll A$, Δm_{31}^2 (Cervera *et al.*, 2000; Freund, 2001; Asano and Minakata, 2011). Treating θ_{13} and r as small parameters, one has for the golden channel $\nu_e \rightarrow \nu_\mu$ (Cervera *et al.*, 2000)

$$P_{e\mu} \simeq T_1\sin^2 2\theta_{13} + r(T_2 + T_3)\sin 2\theta_{13} + r^2 T_4, \tag{3.25}$$

---

[3]Some interesting relations between *CP* and *T*-odd asymmetries can still be obtained for the matter-oscillation case (Minakata and Nunokawa, 1997; Koike and Sato, 2000; Akhmedov *et al.*, 2001).

at second order in $\sin 2\theta_{13}$ and r. The terms T_i in the above equation are (Huber *et al.*, 2006a)

$$T_1 \equiv s_{23}^2 f_\Delta^2 (1 - \hat{A}),$$

$$T_2 \equiv \sin\delta \sin\Delta \sin(2\theta_{12}) \sin(2\theta_{23}) f_\Delta(\hat{A}) f_\Delta(1 - \hat{A}),$$

$$T_3 \equiv \cos\delta \cos\Delta \sin(2\theta_{12}) \sin(2\theta_{23}) f_\Delta(\hat{A}) f_\Delta(1 - \hat{A}),$$

$$T_4 \equiv \cos^2(2\theta_{23}) \sin^2(2\theta_{12}) f_\Delta^2 (1 - \hat{A}), \tag{3.26}$$

where $f_\Delta(x) \equiv \sin(x\Delta)/x$ and

$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4E} \simeq 1.27 \frac{\Delta m_{31}^2}{\text{eV}^2} \frac{L}{\text{km}} \frac{\text{GeV}}{E}, \qquad \hat{A} \equiv \frac{A}{\Delta m_{31}^2}, \tag{3.27}$$

with A defined in Eq. (3.21). The corresponding antineutrino oscillation probability $\bar{P}_{e\mu}$ is obtained from $P_{e\mu}$, performing the replacements $(\delta \to -\delta, \hat{A} \to -\hat{A})$ in the coefficients T_i defined above. The sign of $\hat{A}$ is determined by the sign of Δm_{31}^2, and by whether one considers neutrino or antineutrino oscillations. The above approximate expressions are accurate as long as θ_{13} is not too large and $E \gtrsim 0.5$ GeV (Barger, Marfatia, and Whisnant, 2002). They are commonly used to illustrate some of the general features of the matter effects in the neutrino oscillation probabilities. In general, complete analyses are performed by integrating the evolution equations in matter and taking into account the Earth's matter-density profile provided by the preliminary reference Earth model (Dziewonski and Anderson, 1981).

3. Degeneracy problems

In the previous sections, we reviewed the basics of the neutrino oscillation formalism and how leptonic *CP* violation enters into the oscillation probabilities. The determination of the yet unknown neutrino parameters δ, θ_{13} and the sign of Δm_{31}^2, $\text{sgn}(\Delta m_{31}^2)$, from the knowledge of $P_{e\mu}$ and $\bar{P}_{e\mu}$ is usually plagued by degeneracies and correlations among the different parameters in the oscillation probabilities. Consequently, one cannot determine unambiguously the values of δ and θ_{13} (Burguet-Castell *et al.*, 2001; Minakata and Nunokawa, 2001) from a given measurement of the probabilities P and $\bar{P}$. The three twofold degeneracies related with the determination of the oscillation parameters in long-baseline neutrino experiments can be briefly summarized as follows.

a. CP degeneracy: (δ, θ_{13}) ambiguity

The *CP* degeneracy occurs as a consequence of the fact that two different sets (δ, θ_{13}) can lead to the same oscillation probabilities for fixed values of the remaining parameters (Burguet-Castell *et al.*, 2001; Koike, Ota, and Sato, 2002). For instance, there might be *CP*-conserving solutions which are degenerate with a *CP*-violating one. In the $(P, \bar{P})$ bi-probability space, the *CP* trajectories (for $\delta \neq n\pi/2$, with n integer) are ellipses (Minakata and Nunokawa, 2001) and therefore the degeneracy can be geometrically understood as the intersection of two ellipses with distinct values of θ_{13}. As a result, neutrino oscillation analysis relying on a monoenergetic beam at a fixed baseline L will necessarily lead to parameter ambiguities. If $\delta = n\pi$ or $(n - 1/2)\pi$, then the

ellipses collapse to a line and, in principle, θ_{13} can be determined. Nevertheless, a $(\delta, \pi - \delta)$ or $(\delta, 2\pi - \delta)$ ambiguity still remains (Barger, Marfatia, and Whisnant, 2002). Instead, if $\delta \simeq n\pi/2$, the ambiguous values of θ_{13} are very close to each other, being this case qualitatively similar to the previous ones.

b. Mass-hierarchy degeneracy: $\text{sgn}(\Delta m_{31}^2)$ ambiguity

In certain cases, the same values of P and $\bar{P}$ can be obtained for different pairs (θ_{13}, δ) and (θ_{13}', δ') when considering $\Delta m_{31}^2 > 0$ or $\Delta m_{31}^2 < 0$ (Minakata and Nunokawa, 2001). This is commonly known as the sign or mass-hierarchy degeneracy. As in the previous case, *CP*-conserving solutions with $\Delta m_{31}^2 > 0$ may be degenerate with *CP*-violating ones with $\Delta m_{31}^2 < 0$. The $\text{sgn}(\Delta m_{31}^2)$ ambiguity is only present for some values of δ and tends to disappear when matter effects become large, i.e., when L and θ_{13} are sufficiently large (Barger *et al.*, 2000; Lipari, 2000). Unlike the (δ, θ_{13}) ambiguity discussed above, where θ_{13} is resolved in the case $\delta = n\pi/2$, the $\text{sgn}(\Delta m_{31}^2)$ ambiguity can lead to different values of δ and θ_{13}, even if the condition $\delta = n\pi/2$ is verified. In total, this ambiguity can lead to a fourfold degeneracy since there may be four sets of (θ_{13}, δ) (two for $\Delta m_{31}^2 > 0$ and two for $\Delta m_{31}^2 < 0$) which give the same values of P and $\bar{P}$.

c. θ_{23} degeneracy: $(\theta_{23}, \pi/2 - \theta_{23})$ ambiguity

The extraction of δ and θ_{13} is affected by another ambiguity which is related with the atmospheric neutrino mixing angle θ_{23} (Fogli and Lisi, 1996; Barger, Marfatia, and Whisnant, 2002). Since only $\sin^2 2\theta_{23}$ enters in the ν_μ survival probabilities, it is straightforward to conclude that θ_{23} cannot be distinguished from $\pi/2 - \theta_{23}$. Obviously, for $\theta_{23} \simeq \pi/4$, which corresponds to the present best-fit value of this angle, the ambiguity is not present. Once again, *CP*-conserving and *CP*-violating solutions cannot be disentangled due to the θ_{23} ambiguity. Moreover, different values of θ_{13} can give the same P and $\bar{P}$, even if $\delta = n\pi/2$.

From the above discussion, one concludes that, in the worst case, there can be an eightfold degeneracy (Barger, Marfatia, and Whisnant, 2002) when determining δ and θ_{13} from the measurement of the probabilities P and $\bar{P}$, at a fixed baseline L and neutrino energy E. Moreover, for all the ambiguities, one may not be able to distinguish a *CP*-conserving solution from a *CP*-violating one. An example of the eightfold degeneracy is pictorially represented in Fig. 5, where the point corresponding to the true solution is degenerate with the clone ones (points II to VIII shown in the bottom panel) at the intersection of the corresponding ellipses in the bi-probability space. A complete analysis of the parameter degeneracy in neutrino oscillations can be found in Donini, Meloni, and Rigolin (2004); Minakata and Uchinami (2010), where the degeneracies are interpreted as being a result of the invariance of the oscillation probabilities under discrete mapping of the mixing parameters. Moreover, the analytical solution of all the clone solutions has been obtained as a function of the true one.

The existence of parameter ambiguities represents a major difficulty in the extraction of the neutrino parameters from the
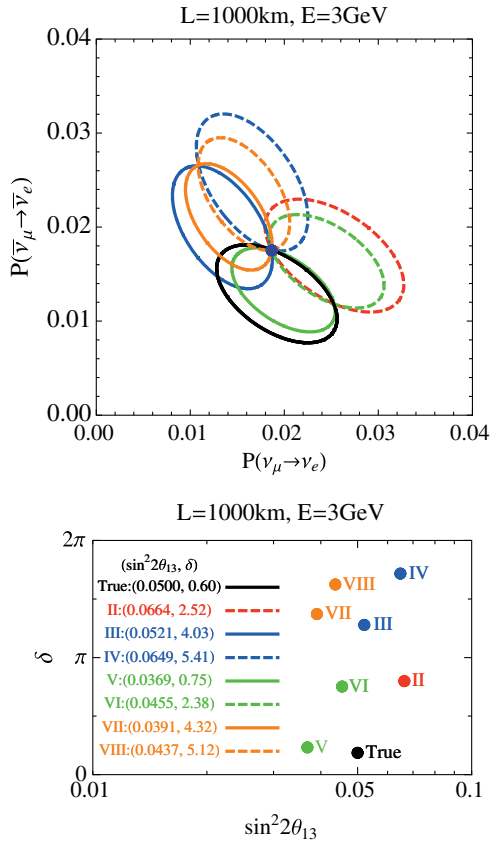
L=1000km, E=3GeV

L=1000km, E=3GeV

FIG. 5 (color online). Top panel: An illustrative example of the eightfold degeneracy in terms of the bi-probability plot in $P_{\mu e}$-$\bar{P}_{\mu e}$ space (Minakata and Nunokawa, 2001). Bottom panel: Values of $(\sin^2 2\theta_{13}, \delta)$ for the true solutions and the clone solutions II–VIII in $\sin^2 2\theta_{13}$-δ space. The correspondence between the ellipses (top panel) and the solution labels are made manifest by using the same color lines and/or symbols in both panels. From Minakata and Uchinami, 2010.

experimental measurements of oscillation probabilities. To overcome this limitation, a set of complementary measurements have to be performed for distinct oscillation channels, baselines, and energies (Burguet-Castell *et al.*, 2001; Ishitsuka *et al.*, 2005; Kajita *et al.*, 2007). It has also been shown that a good energy resolution is also important to resolve the degeneracies (Freund, Huber, and Lindner, 2001; Bueno *et al.*, 2002; Kajita, Minakata, and Nunokawa, 2002). A powerful method to reduce the impact of ambiguities is to perform measurements at the so-called "magic baseline" (Huber and Winter, 2003) which satisfies the condition $\sin(\hat{A}\Delta) = 0$. This choice leads to a simplified form of the oscillation probabilities since all terms in Eq. (3.25) will vanish, except the first one. This allows for a determination of $\sin^2 2\theta_{13}$ and $\mathrm{sgn}(\Delta m_{31}^2)$, which is free of correlations with the *CP* phase δ (Lipari, 2000; Barger, Marfatia, and Whisnant, 2002). It is straightforward to see that the first solution to the magic condition corresponds to $\sqrt{2}G_F n_e L = 2\pi$ which, for a constant matter-density profile, leads to

$$L_{\mathrm{magic}} \simeq 32726 \frac{1}{\rho[\mathrm{g/cm^3}]} \text{ km.} \qquad (3.28)$$

The magic baseline only depends on the matter density and taking an average $\rho \simeq 4.3 \text{ g/cm}^3$ one has $L_{\mathrm{magic}} \simeq 7630$ km.

The above baseline has the disadvantage that does not allow for the study of *CP* violation since the oscillation probabilities are independent from δ for $L \simeq L_{\mathrm{magic}}$. For this reason, the combination of the magic baseline with a shorter one (with better statistics) opens the possibility for the measurement of θ_{13}, $\mathrm{sgn}(\Delta m_{31}^2)$, and δ without much correlations. In particular, a detailed optimization study reveals that the combination of two baselines $L_1 = 4000$ km and $L_2 = 7500$ km is optimal for these studies (Kopp, Ota, and Winter, 2008).

The study of additional oscillation channels may also reduce the uncertainty in the determination of the neutrino oscillation parameters. For instance, it has been shown that the analysis of the "silver" channel $\nu_e \rightarrow \nu_\tau$ (Donini, Meloni, and Migliozzi, 2002) can be used to reduce the number of clone solutions and better determine θ_{13} and δ. In this case, the different behavior of the probability curves of different channels should reduce (or ideally eliminate) the impact of the degeneracies on the simultaneous fitting of the two sets of data. The combination of two superbeam facilities, one of them with a sufficiently long baseline and the other with a good θ_{13} sensitivity, could help to resolve the $\mathrm{sgn}(\Delta m_{31}^2)$ degeneracy (Minakata, Nunokawa, and Parke, 2003). One of these superbeam experiments could be combined with a reactor detector to determine the θ_{23} octant (Huber *et al.*, 2003; Minakata *et al.*, 2003). An upgraded version of the NOνA experiment (Ayres *et al.*, 2004) with a second detector off axis at a shorter baseline would also allow the determination of the neutrino mass-hierarchy free of degeneracies (Mena Requejo, Palomares-Ruiz, and Pascoli, 2005, 2006). Another possibility relies on combining long-baseline and atmospheric neutrino data to solve the θ_{23} and $\mathrm{sgn}(\Delta m_{31}^2)$ degeneracies (Huber, Maltoni, and Schwetz, 2005). These examples reveal the importance of working in the direction of establishing the optimum experimental facilities which reduce or even eliminate the impact of the ambiguities on the determination of the neutrino parameters in future neutrino oscillation experiments.

4. Future prospects for leptonic CPV in neutrino oscillation experiments

Even though neutrino physics has witnessed a series of successes in the last decade, there are still fundamental open questions about neutrinos. Among the ones for which neutrino oscillation experiments will seek an answer are

- How large is the θ_{13} mixing angle?
- Is there CPV in the lepton sector and, if so, what is the value of δ?
- How are neutrino masses ordered: is $\Delta m_{31}^2 > 0$ (NO) or $\Delta m_{31}^2 < 0$ (IO)?
- Is the atmospheric neutrino mixing angle θ_{23} exactly equal to $\pi/4$?
- Are there subdominant nonstandard interactions in the lepton sector?

From the theoretical perspective, a better knowledge of the oscillation parameters could give some hints about the origin of flavor in the lepton sector and, perhaps, on the neutrino mass generation mechanism. With this goal in mind, the major challenge for the upcoming neutrino oscillation experiments will be to probe for subleading effects in neutrino oscillations. In the last years, there has been an intense

activity towards finding the optimal experimental conditions and configurations that will allow one to answer the above questions.

It is beyond the scope of this review to give an exhaustive discussion of the physics reach of all future experiments. Instead, we aim at presenting a brief overview of the sensitivities and prospects in the measurement of θ_{13}, δ, and $\text{sgn}(\Delta m^2_{31})$, in future neutrino oscillation facilities. For further details, we refer the reader to other works exclusively dedicated to the subject (Apollonio *et al.*, 2002; Nunokawa, Parke, and Valle, 2008; Bandyopadhyay *et al.*, 2009; Bernabeu *et al.*, 2010; Mezzetto and Schwetz, 2010).

a. Upcoming reactor neutrino and superbeam experiments

Reactor neutrino experiments observe the disappearance of $\bar{\nu}_e$ antineutrinos produced in nuclear fission reactions in the core of a nuclear reactor. The neutrinos are detected through the inverse beta decay reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ with an energy threshold of approximately 1.8 MeV. Low-baseline reactor neutrino experiments like Gösgen (Zacek *et al.*, 1986), Bugey (Declais *et al.*, 1995), Palo Verde (Boehm *et al.*, 2001), and CHOOZ (Apollonio *et al.*, 2003) searched for $\bar{\nu}_e$ disappearance without success.[4] In the case that the detector is placed at a distance $L \sim 100$ km, the experiment becomes sensitive to the solar-neutrino oscillation parameters Δm^2_{21} and θ_{12}. The ongoing KamLAND experiment in Japan uses a 1 kton liquid-scintillator detector to measure the flux of $\bar{\nu}_e$ coming from a complex of 53 surrounding nuclear plants located at an average distance $L \sim 180$ km. The KamLAND data indicated a $\bar{\nu}_e$ disappearance, in agreement with the large mixing angle solution of the solar-neutrino data (Eguchi *et al.*, 2003).

Upcoming reactor neutrino experiments like Double CHOOZ in France (Ardellier *et al.*, 2006), Daya Bay in Japan (Guo *et al.*, 2007), and RENO in Korea (Ahn *et al.*, 2010) will have a typical baseline $L \sim 1$ km and therefore they will be looking for $\bar{\nu}_e$ disappearance driven by Δm^2_{31} and the small mixing angle θ_{13}. Consequently, the observation of a neutrino deficit in these experiments could be an indication for a nonzero θ_{13}. To increase the θ_{13} sensitivity, all these experiments will operate as multidetector setups. Double CHOOZ will be able to measure $\sin^2 2\theta_{13}$ down to 0.03, while Daya Bay and RENO aim at a sensitivity of $\sin^2 2\theta_{13} \sim 0.01$. Double CHOOZ has started to take data with one detector at the end of 2010 and it is expected to start operating with its two detectors by the middle of 2012. Daya Bay is currently under construction and full data taking is planned to start in 2012, while RENO has recently started its physics program.

In superbeam experiments, an intense proton beam is directed to a target, producing pions and kaons which subsequently decay into neutrinos. The resulting neutrino beam consists mainly of ν_μ with a small ν_e component. Because of the increased statistics, the precision of the leading atmospheric neutrino parameters is improved and the sensitivity to θ_{13} may become comparable (or slightly better) to that of

reactor neutrino experiments after a long running period. Moreover, under some circumstances, superbeam facilities may be able to provide some information regarding *CP* violation and the type of neutrino mass spectrum. The presence of ν_e in the original beam, which cannot be distinguished from the ones coming from the appearance process $\nu_\mu \rightarrow \nu_e$, is the main limitation of this kind of experiment. There are presently two superbeam experiments, namely, the "NuMI" (neutrinos at the main injector) off axis ν_e appearance experiment (NOνA) in the United States (Ayres *et al.*, 2004) which is still under construction, and the "Tokai to Kamioka" (T2K) experiment in Japan (Itow *et al.*, 2001). In NOνA, the neutrino beam is provided by the NuMI Fermilab facility and its far detector is planned to be located at a distance of 812 km. For T2K, the neutrino beam is produced at the Japan Research Complex (J-PARC), and the far detector (the Super-Kamiokande one) is located at a distance of 295 km. In order to reduce the systematic uncertainties, both experiments will have near detectors dedicated to study the unoscillated neutrinos.

The next round of reactor (Double CHOOZ, Daya Bay, and RENO) and accelerator (NOνA and T2K) neutrino experiments are mainly targeted to the measurement of the neutrino mixing angle θ_{13}, which, if large, could also be on the reach of MINOS and OPERA. However, it is also interesting to investigate how sensitive these experiments are to CPV and the neutrino mass hierarchy (NMH). This question was recently addressed in Huber *et al.* (2009), where the physics potential of the upcoming reactor and accelerator neutrino oscillation experiments has been analyzed.

In Fig. 6, the sensitivity limit and discovery potential of θ_{13} is given as a function of time for the reactor and superbeam experiments mentioned above. From the top panel of this figure, one concludes that the sensitivity will be dominated by the reactor neutrino experiments and, in particular, by Daya Bay as soon as it becomes operational. The same plot also shows that accelerator experiments are not competitive with the reactor ones. The discovery potential of θ_{13} is shown at the center and bottom of the same figure for the NH and IH spectrum, respectively. For the beam experiments, the dependence of the results on the *CP* phase δ is reflected by the corresponding shaded regions. Note that there is no dependence on δ for the reactor experiments since this phase does not appear in the P_{ee} disappearance probability. The comparison of the NH and IH results shows that the discovery potential of θ_{13} does not depend much on the type of neutrino mass hierarchy. In general grounds, one concludes that we can measure θ_{13} in the next generation of neutrino experiments, if $\theta_{13} \gtrsim 3°$.

The analysis of Huber *et al.* (2009) shows that NOνA is required for NMH discovery, due to its long baseline and significant matter effects. If $\sin^2 \theta_{13} \simeq 0.1$, the NMH can be established at 90% C.L. for about 40%–50% of all values of δ. Adding other experiments to NOνA slightly improves the situation in some cases. By themselves, NOνA and T2K do not have a significant CPV discovery potential. Yet, when combined, these two experiments can be sensitive to CPV for 30% of all values of δ, if $\Delta m^2_{31} < 0$. On the other hand, the same two experiments combined have no CPV discovery potential for the NH case (Huber *et al.*, 2009).

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[4]Recently, the improved predictions of the reactor antineutrino fluxes show that these experiments may have observed less neutrinos than expected (Mention *et al.*, 2011; Mueller *et al.*, 2011).
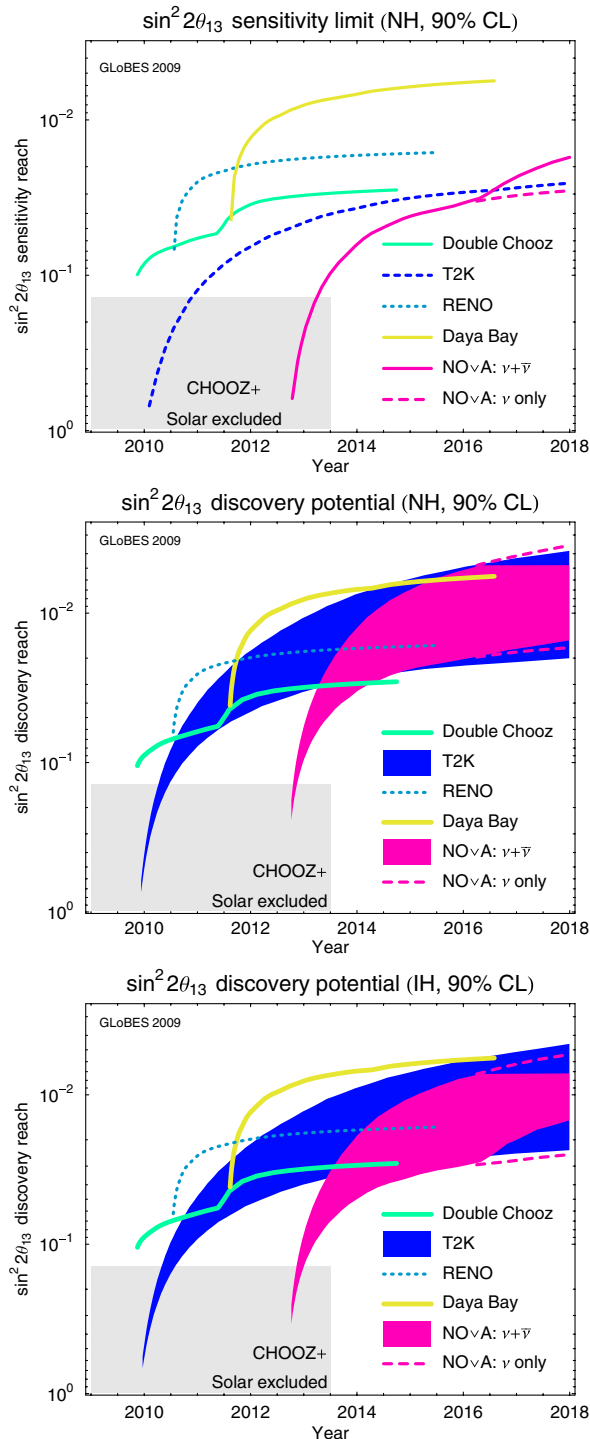
FIG. 6 (color online). Top: Time evolution of the sensitivity of θ_{13} at 90% C.L., defined as the limit which is obtained if the true value of θ_{13} is zero. Center and bottom: Discovery potential of θ_{13} at 3σ as a function of time (given as the smallest value of θ_{13}, which can be distinguished from zero), for a NH (center) and an IH (bottom) neutrino mass spectrum. From Huber *et al.*, 2009.

Nevertheless, the inclusion of reactor neutrino data significantly improves the situation to a point in which CPV can be established at 90% C.L. for about 20%–30% of all values of δ if $\sin^2\theta_{13} \gtrsim 0.04$. In conclusion, one can say that the CPV discovery potential in future reactor and superbeam experiments is rather marginal. If θ_{13} is close to its upper bound, the

sensitivity of these setups to CPV and the NMH can be greatly improved with upgraded versions of NOνA and T2K. In any case, although these experiments may give some indications about the value of θ_{13}, CPV, and the NMH, the confirmation of such hints will require a new generation of experiments like β beams or neutrino factories. One should also keep in mind that, even if θ_{13}, CPV, or sgn(Δm^2_{31}) are not measured, the upcoming beam experiments will increase the precision of the atmospheric neutrino parameters through the study of the $\nu_\mu \rightarrow \nu_\mu$ disappearance channel. In particular, deviations from maximal atmospheric mixing can be established at 3σ for $|\sin^2\theta_{23} - 0.5| \gtrsim 0.07$ (Huber *et al.*, 2009).

Recently, the T2K Collaboration reported the results of the first two physics runs (January to June 2010 and November to March 2011) (Abe *et al.*, 2011). The analysis of the events in the far detector with a single electronlike ring indicates electron-neutrino appearance from a muon-neutrino beam. T2K observed six of such events, which can hardly be explained if $\theta_{13} = 0$. Indeed, the probability to observe six or more events for vanishing θ_{13} is less than 1%. The 90% C.L. interval obtained from the T2K oscillation analysis is $0.03(0.04) < \sin^2(2\theta_{13}) < 0.28(0.34)$ with a best fit 0.11 (0.14), where the numbers in parenthesis correspond to the results in the case $\Delta m^2_{31} < 0$. Further data from T2K and reactor neutrino experiments will help to confirm these results and increase the precision on the determination of θ_{13}. Taking as a reference the best-fit value of the T2K analysis, then we can say that the prospects for determining the NMH and *CP* violation in the near future are very good.

Examples of second-generation superbeam experiments are the CERN superbeam project (Gomez-Cadenas *et al.*, 2001; Mezzetto, 2003a) based on a super proton linear particle accelerator (SPL), and the upgrade of T2K and T2HK (Itow *et al.*, 2001). In the former case, the MEMPHYS detector at Fréjus in France would detect the CERN SPL neutrinos located at a distance of 130 km. The T2HK beam would be produced at J-PARC in Tokai and sent to the Hyper-Kamiokande detector located at the Kamioka mine, 295 km far from the source. An alternative setup with a second detector placed in Korea (T2KK) at a distance of 1050 km has also been considered (Ishitsuka *et al.*, 2005). The discovery potential of θ_{13}, CPV, and NMH in those second-generation superbeam experiments has been investigated in Campagne *et al.* (2007).

The CPV discovery potential of T2HK and SPL is shown in Fig. 7, where the performance of the two experiments is compared. The results show that for maximal leptonic CPV, i.e., for $\delta = \pi/2$ or $3\pi/2$, CPV could be discovered at 3σ for $\sin^2 2\theta_{13} \gtrsim 10^{-3}$. Concerning the discovery potential of the mixing angle θ_{13}, the performance of T2HK and SPL is similar, and a measurement down to $\sin^2 2\theta_{13} \simeq 4 \times 10^{-3}$ is within their reach for all possible values of δ. Because of the short baseline of the upgraded superbeam experiments, the determination of the NMH at T2HK and SPL is rather limited. The combination of superbeam and β-beam experiments would also result in an increased θ_{13} sensitivity. For instance, the 5-year data set of SPL combined with a β-beam experiment would have a better sensitivity than a 10-year running of T2HK (Huber *et al.*, 2009). The SPL superbeam
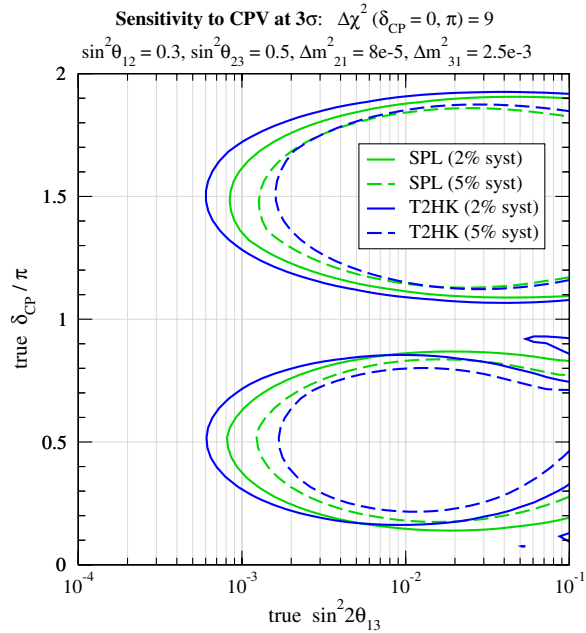
FIG. 7 (color online).   SPL and T2HK CPV discovery potential. Inside the regions delimited by the solid (dashed) lines, *CP* conservation can be excluded at the 3σ level, considering systematic errors of 2% (5%). From Campagne   *et al.*, 2007; Bandyopadhyay *et al.*, 2009.

combined with a neutrino factory could also help in solving the eightfold degeneracy (Burguet-Castell   *et al.*, 2002) described in Sec. III.B.3.

b. β-beam experiments

One of the main limitations of superbeam experiments is the ν_e contamination of the initial neutrino beam. A flavor-pure neutrino beam could be obtained using the β-beam concept (Zucchelli, 2002) in which highly boosted ν_es are obtained from the decay of accelerated unstable ions circulating in a storage ring. Pure electron-neutrino and antineutrino beams can be produced using ^{18}Ne and ^{6}He through the reactions ^{18}Ne $\rightarrow$ ^{18}Fe $+ e^{+} + \nu_e$ and ^{6}He $\rightarrow$ ^{6}Li $+ e^{-} + \bar{\nu}_e$ respectively. The neutrino energy can be accurately set by choosing the required Lorentz factor γ of the accelerated mother nuclei. β-beam experiments aim at studying the $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ appearance channels, which can be used to probe θ_{13} and *CP* violation. In principle, the $\nu_e \rightarrow \nu_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance can also be measured at a β-beam experiment, although in this case the performance is comparable with the one of reactor neutrino experiments. Although at present there are no concrete β-beam experiments planned, there has been a great effort to develop this kind of experimental setup (Lindroos and Mezzetto, 2010).

A standard low-energy experiment with sub-GeV neutrinos and a baseline of $L = 130$ km (distance from CERN to Fréjus) has been considered as a possible β-beam configuration (LE$\beta\beta$) (Mezzetto, 2003b; Bouchez, Lindroos, and Mezzetto, 2004). Possible candidate isotopes are ^{6}He and ^{18}Ne (Zucchelli, 2002) accelerated to a standard Lorentz factor $\gamma_{\text{He,Ne}} = 100$ at the CERN Super Proton Synchrotron (SPS) (Burguet-Castell   *et al.*, 2004; Mezzetto, 2006).
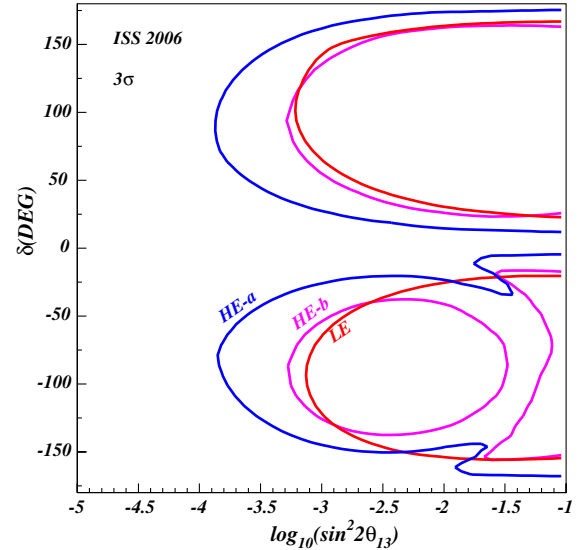

FIG. 8 (color online). 3σ sensitivity to CPV for the three β-beam configurations: LE$\beta\beta$, HE$\beta\beta$-a, and HE$\beta\beta$-b (see text for more details). From Bandyopadhyay   *et al.*, 2009.

High-energy β beams (HE$\beta\beta$) with $E = 1$–1.5 GeV and $L \simeq 700$ km   (CERN-Canfranc,   CERN-Gran   Sasso,   or Fermilab-Soudan) could also be an alternative. For such cases, the appropriate Lorentz factor $\gamma_{\text{He,Ne}} = 350$ is achievable at an upgraded SPS or the Tevatron (Burguet-Castell *et al.*, 2004). Alternatively, moderate values of $\gamma \sim 100$ could be appropriate if ions with higher endpoint kinetic energy like ^{8}Li or ^{8}B are used. Because of its larger baseline, the HE$\beta\beta$ setup would be sensitive to sgn(Δm^2_{31}) (Donini   *et al.*, 2005; Huber *et al.*, 2006b; Agarwalla, Choubey, and Raychaudhuri, 2007; Coloma   *et al.*, 2008; Meloni   *et al.*, 2008).

The 3σ sensitivity to CPV is shown in Fig. 8 for three β-beam configurations, namely, LE$\beta\beta$ with a 500 Mton water Čerenkov detector, HE$\beta\beta$ with a 500 Mton water Čerenkov detector (HE$\beta\beta$-a), and HE$\beta\beta$ with a liquid-scintillator detector (HE$\beta\beta$-b). From these results, one can see that the HE$\beta\beta$-a provides the best CPV sensitivity, with slightly worse results for negative values of δ due to the sgn(Δm^2_{31}) ambiguity. The potential of these β-beam setups to sgn(Δm^2_{31}) is limited to relatively high values of θ_{13}, namely, $\sin^2 \theta_{13} \gtrsim 0.03$. The extraction of θ_{13} and δ from the data is also more difficult for the LE$\beta\beta$ setup since the uncertainties are significantly larger and the eightfold degeneracy is present. The situation is improved for the HE$\beta\beta$-a case for which the intrinsic degeneracy is resolved.

The combination of superbeam and β-beam experiments has also been considered and, in particular, it has been shown that a 5-year run of SPL and β beam would result in a better sensitivity to θ_{13} than 10 years of T2HK (Huber *et al.*, 2009). Using distinct ions (Donini and Fernandez-Martinez, 2006) with a γ reachable at the CERN SPS could also help in resolving the degeneracies due to the different values of $L/\langle E \rangle$.

c. Electron-capture beams

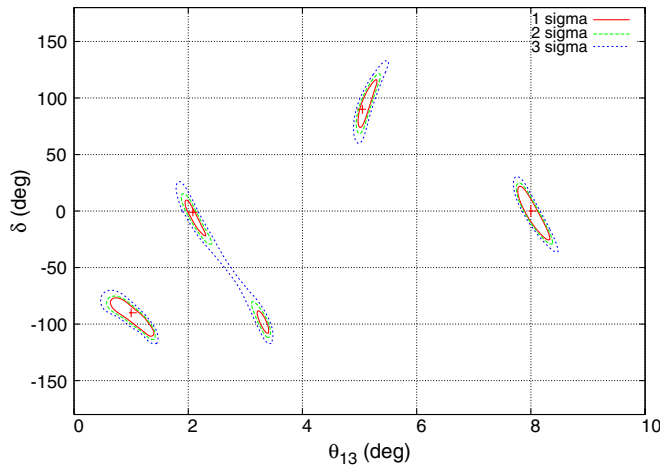In these experiments, neutrinos are obtained from electron-capture processes (Bernabeu   *et al.*, 2005; Sato, 2005;

FIG. 9 (color online). Fits for θ_{13} and δ in an electron-capture experiment with a baseline $L = 130$ km (CERN-Fréjus) and a 440 kton water Čerenkov detector. Two 5 yr-running periods with $\gamma = 195$ and $\gamma = 90$ have been considered. From Bernabeu *et al.*, 2005.

Bernabeu, Burguet-Castell, and Espinoza, 2006; Orme, 2010), in which an atomic electron is captured by a proton of the nucleus leading to a nuclear state of the same mass number A. The proton is replaced by a neutron, and an electron neutrino is emitted ($pe^- \rightarrow n\nu_e$) with fixed energy, since this is a two-body decay. Consequently, a flavor-pure and monochromatic neutrino beam can be obtained. The electron-capture beam concept is feasible if the ions decay fast enough. Recent discovery of nuclei far from the stability line having superallowed spin-isospin transitions to Gamow-Teller resonances turn out to be very good candidates. A particular choice is ^{150}Dy, with a neutrino energy at rest given by 1.4 MeV due to a unique nuclear transition from 100% electron capture in going to neutrinos. The oscillation channel to study is once more $\nu_e \rightarrow \nu_\mu$, being the prospects for the measurement of θ_{13} and *CP* violation quite impressive. Since only a neutrino beam is available, sensitivity to CPV is reached by performing runs at different values of γ. The attainable precision in such kinds of experiments (Bernabeu *et al.*, 2005) is illustrated in Fig. 9, where several values for θ_{13} and δ have been assumed. The contour lines correspond to the determination of the oscillation parameters at different confidence levels. It has also been shown that the combination of β and electron-capture beam experiments using boosted ytterbium could achieve remarkable results in what concerns the determination of the neutrino mass hierarchy, *CP*, violation and θ_{13} (Bernabeu *et al.*, 2009).

d. Neutrino factories

If θ_{13} happens to be very small, then its measurement will be only possible at a neutrino factory (NF). This idea was first discussed almost 15 years ago (Geer, 1998) and, since then, a great deal of effort has been made in order to plan and optimize the concept. In this type of experiment, muons are accelerated and stored in a storage ring. A boosted and collimated neutrino beam is obtained from the decays of the muons in the straight sections of the ring. Contrary to the β-beam and electron-capture experiments, at NFs the

neutrino beam contains both electron and muon (anti)neutrinos since $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$ (or $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$, if μ^+) are stored. The neutrino beam at a NF can be used to study the leading atmospheric neutrino parameters Δm_{31}^2 and θ_{23} through the study of the disappearance channels $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$. Nevertheless, the ultimate purpose of a NF is the measurement of subleading effects in the golden appearance channel $\nu_e \rightarrow \nu_\mu$ and its *CP* conjugated (Cervera *et al.*, 2000). The detection of golden channel events requires an effective charge separation of the muons produced in charged-current processes, due to the presence of wrong-sign muons originated from the disappearance channel. This could be achieved with a magnetized iron detector (MIND), which appears as the most straightforward solution for a high-fidelity muon charge measurement. Since the neutrino energy is typically very high (up to 25 or 50 GeV), the detector has to be placed at a distance of several thousand of kilometers in order for oscillations to occur. A very active research and development program is currently undergoing in the framework of the International Design Study for the Neutrino Factory (IDS-NF) (Bandyopadhyay *et al.*, 2009), to which the reader is referred for more details about the possible NF configurations and performance comparison. Here we limit ourselves to give a general idea about the θ_{13}, CPV, and NMH sensitivities at neutrino factories.

As mentioned, the determination of θ_{13} and δ at a NF suffers from several ambiguities. A possible solution to this problem is to combine golden measurements at different baselines or, if an efficient τ detector is available, to use the silver $\nu_e \rightarrow \nu_\tau$ oscillation channel (Donini, Meloni, and Migliozzi, 2002; Autiero *et al.*, 2004). The original IDS-NF setup considers a double-baseline NF with $L_1 \simeq$ 3000–5000 km, $L_2 \simeq L_{\text{magic}} \simeq 7500$ km, and a muon energy $E_\mu = 25$ GeV. Such a standard configuration is advantageous for several reasons: the sensitivity to very small values of θ_{13} and thus to several three-flavor effects (Huber *et al.*, 2006a), and the robustness against new physics effects like nonstandard interactions in the lepton sector (Kopp, Ota, and Winter, 2008) and systematic errors (Tang and Winter, 2009). An alternative setup with a lower muon energy $E_\mu = 5$ GeV, a totally active scintillator detector, and a baseline of $L \simeq$ 1300 km has also been considered as a possible low-energy neutrino factory configuration (Geer, Mena, and Pascoli, 2007; Bross *et al.*, 2008; Fernandez Martinez *et al.*, 2010; Tang and Winter, 2010). This kind of alternative is particularly suitable for large $\sin^2 2\theta_{13}$.

Since it is unlikely that the accelerator part of a NF will be specially built for this experiment, one has to assume that the neutrino beam will be produced at existing facilities. In such a case, the options are CERN, J-PARC, the Rutherford Appleton Laboratory, and the Fermi National Accelerator Laboratory (Apollonio *et al.*, 2009). As for the possible detector locations, a list of candidate sites in the United States (Cushman, 2006) and Europe (Rubbia, 2010) has been recently compiled. In Asia, possible detector sites are the Kamioka mine in Japan, the proposed Chinese underground laboratory at CPJL, YangYang in Korea, and the India-based Neutrino Observatory (INO) in India. The possibility of a green-field scenario in which neither the baseline nor the muon energy are constrained has also been considered
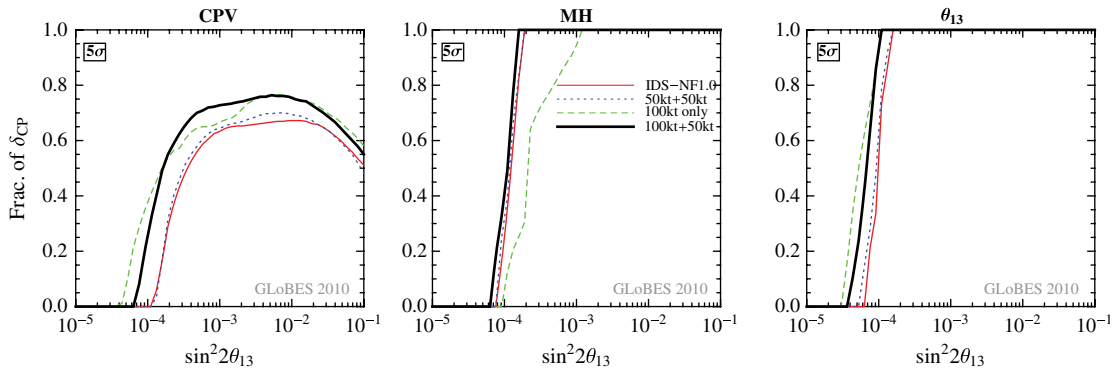
FIG. 10 (color online). 5σ discovery reach of CPV (left), NMH (center), and θ_{13} (right) for several NF setups: "50 kt + 50 kt" refers to a combination of two 50 kton MINDs at $L_1 = 4000$ km and $L_2 = 7500$ km, "100 kt only" to a 100 kton MIND at $L = 4000$ km, "100 kt + 50 kt" to a 100 kton MIND at $L_1 = 4000$ km and a 50 kton MIND at $L_2 = 7500$ km, and "IDS-NF 1.0" to the IDS-NF setup. From Agarwalla *et al.*, 2011.

in NF optimization studies (Bueno *et al.*, 2002; Huber *et al.*, 2006a; Agarwalla *et al.*, 2011).

As a representative analysis, we show in Fig. 10 (Agarwalla *et al.*, 2011) the CPV, θ_{13}, and NMH discovery potential for several NF setups. The results indicate that the θ_{13} sensitivity is comparable for all the cases considered, namely, $\sin^2 2\theta_{13}$ will be measurable at neutrino factories down to $\sim 10^{-4}$, corresponding to $\theta_{13} \sim 0.3°$. The "100 kt + 50 kt" setup is the one which performs better on the CPV discovery potential, while the NMH sensitivity is comparable to the one of the remaining two double-baseline options (see the figure caption for more details on the curve labels). The single-baseline configuration "100 kt only" has a rather worse NMH discovery reach than the other setups. In general, one can say that for $\sin^2 2\theta_{13} \gtrsim 10^{-2}$ a low-energy neutrino factory is quite effective. On the other hand, a double-baseline high-energy NF will be necessary for smaller values of θ_{13}. As mentioned, the next generation of reactor and superbeam experiments will be able to tell us if $\sin^2 2\theta_{13} \gtrsim 10^{-2}$, allowing for an optimization of a large θ_{13} scenario at neutrino factories.

To conclude, one can say that the θ_{13}, CPV, and NMH discovery potential of future experiments depends mainly on the true value of θ_{13}. If $\sin^2 2\theta_{13} \gtrsim 10^{-2}$, then the discovery potential of all the above considered experiments is comparable (although NFs will be able to perform more precise measurements). If the value of $\sin^2 2\theta_{13}$ is in the intermediate range $5 \times 10^{-4} \lesssim \sin^2 2\theta_{13} \lesssim 10^{-2}$, only β-beam experiments and neutrino factories will be able to probe on CPV and the NMH. In the worst case, in which $\sin^2 2\theta_{13} \lesssim 5 \times 10^{-4}$, neutrino factories seem to be the only hope to establish leptonic *CP* violation and identify the neutrino mass hierarchy. However, since the recent T2K and MINOS data indicate that $\sin^2 2\theta_{13}$ is not so small, most probably we will not have to wait for neutrino factories to discover LCPV and find out whether the neutrino mass spectrum is normal or inverted.

We conclude this section with a comment on the potential of measuring θ_{13} and NMH from supernova (SN) neutrinos. The time-dependent energy spectra of ν_e and $\bar{\nu}_e$ from a future SN can be valuable to obtain information on the neutrino mass and mixing pattern (Dighe and Smirnov, 2000). In fact, identifying the neutrino mass hierarchy is possible for θ_{13} as

small as 10^{-10} (Dasgupta, Dighe, and Mirizzi, 2008). For such small values of θ_{13}, the sensitivity of supernova neutrino oscillations to the mass hierarchy stems from collective neutrino oscillations that take place near the supernova core. Therefore, a future galactic SN may become extremely important for the understanding of neutrino mixing and SN astrophysics. Of course, the occurrence of a SN is a rare happening, and to take the most from SN neutrinos one must be prepared with the best detectors.

C. Neutrinoless double beta decay

An important process which may unveil crucial aspects about the fundamental nature of neutrinos is neutrinoless double beta decay ($0\nu\beta\beta$) (Tomoda, 1991; Vergados, 2002; Avignone, III, Elliott, and Engel, 2008), where even-even nuclei undergo the transition $(A, Z) \to (A, Z+2) + 2e^-$. This process obviously violates lepton number by two units and therefore the mechanism responsible for $0\nu\beta\beta$ can also induce Majorana neutrino masses. In short, the observation of $0\nu\beta\beta$ implies that neutrinos are Majorana particles (Schechter and Valle, 1982). Several scenarios beyond the SM predict the occurrence of $0\nu\beta\beta$ decay like, for instance, supersymmetric theories that violate lepton number and/or R parity (Mohapatra, R. N., 1986; Hirsch, Klapdor-Kleingrothaus, and Kovalenko, 1995; Hirsch, Klapdor-Kleingrothaus, and Kovalenko, 1998). The $0\nu\beta\beta$-decay width is usually factorized as $\Gamma_{0\nu\beta\beta} = G_{\text{kin}} |\mathcal{M}_{0\nu}|^2 F_{\text{part}}$, where G_{kin} is a known phase space factor, $\mathcal{M}_{0\nu}$ is the nuclear matrix element (NME), and F_{part} encodes the particle physics part of the process. In the simplest case, when $0\nu\beta\beta$ is driven by light Majorana neutrino exchange, $F_{\text{part}} \propto m_{ee}^2$, where m_{ee} is an effective electron-neutrino mass simply given by $m_{ee} = |(\mathbf{m}_\nu)_{11}|$ [see, e.g., Bilenky (2010) and Rodejohann (2011)].

Several experiments have been searching for $0\nu\beta\beta$ using different nuclei. Up to now, no indications in favor of this process have been obtained, although some members of the Heidelberg-Moscow collaboration claim to have observed $0\nu\beta\beta$ with a lifetime which corresponds to $m_{ee} \simeq 0.4$ eV (Klapdor-Kleingrothaus and Krivosheina, 2006). This result will be soon checked by an independent experiment. From the most precise $0\nu\beta\beta$ experiments, the upper bounds

$m_{ee} < 0.20\text{–}0.32 \text{ eV},$ Heidelberg-Moscow(^{76}Ge)

$< 0.30\text{–}0.71 \text{ eV},$ CUORICINO(^{130}Te)

$< 0.50\text{–}0.96 \text{ eV},$ NEMO(^{130}Mo)   (3.29)

obtained by the Heidelberg-Moscow (Baudis *et al.*, 1999), CUORICINO (Andreotti *et al.*, 2011), and NEMO (Arnold *et al.*, 2005) Collaborations have been inferred. In the future, $0\nu\beta\beta$ experiments like GERDA (Jochum, 2010), CUORE (Andreotti *et al.*, 2011), EXO (Gornea, 2010), MAJORANA (Gehman, 2008), SuperNEMO (Arnold *et al.*, 2010), SNO+ (Kraus and Peeters, 2010), KamLAND-ZEN (Terashima *et al.*, 2008), and others will be able to probe the value of m_{ee} down to a few 10^{-2} eV.

If the dominant contribution to $0\nu\beta\beta$ is due to the exchange of light active Majorana neutrinos, then m_{ee} depends exclusively on neutrino mass and mixing parameters which enter the definition of the neutrino mass matrix $\mathbf{m}_\nu$. Using the parametrization for the leptonic mixing matrix $\mathbf{U}$ given in Eq. (2.13), one has

$$m_{ee} = |c_{13}^2(m_1 c_{12}^2 + m_2 e^{-i\alpha_1} s_{12}^2) + m_3 e^{-i\alpha_2} s_{13}^2|.$$

(3.30)

This shows that the relation between the particle physics part of $0\nu\beta\beta$ decay and neutrino masses and mixing is direct in the sense that m_{ee} depends on parameters which define the neutrino mass matrix. Therefore, the observation of $0\nu\beta\beta$ decay can in principle provide valuable information about the type of neutrino mass spectrum (Bilenky, Pascoli, and Petcov, 2001; Pascoli and Petcov, 2002; Murayama and Pena-Garay, 2004), the absolute neutrino mass scale (Matsuda *et al.*, 2001; Pascoli, Petcov, and Wolfenstein, 2002; Joaquim, 2003; Choubey and Rodejohann, 2005), and the Majorana *CP*-violating phases (Barger and Whisnant, 1999; Czakon, Gluza, and Zralek, 2000; Branco, González Felipe, Joaquim, and Yanagida, 2003; Pascoli, Petcov, and Schwetz, 2006).

The presently available neutrino oscillation data already impose some constraints on the value of m_{ee}. In the case of a hierarchical neutrino mass spectrum ($m_1 \ll m_2 \simeq \sqrt{\Delta m_{21}^2} \ll m_3 \simeq \sqrt{\Delta m_{31}^2}$) one has

$$\frac{m_{ee}^{\text{HI}}}{\sqrt{\Delta m_{31}^2}} \simeq \left| s_{13}^4 + r c_{13}^4 s_{12}^4 + \frac{1}{2}\sqrt{r}s_{12}^2 \cos\alpha \sin^2(2\theta_{13}) \right|^{(1/2)},$$

(3.31)

where α is a Majorana-phase difference. If $\alpha = \pi$, cancellations in m_{ee}^{HI} may occur for

$$s_{13}^2 = \frac{r s_{12}^2}{1 + r s_{12}^2} \sim 0.01,$$

(3.32)

where in the numerical estimate we have used the STV best-fit values for the neutrino parameters given in Table I. Such values of s_{13}^2 are close to the best-fit points shown in Table I, and will be probed by future neutrino experiments as discussed in Sec. III.B.4.

In the case of an IH neutrino mass spectrum, the effective neutrino mass parameter is simply given by

$$m_{ee}^{\text{IH}} \simeq \sqrt{\Delta m_{31}^2}\sqrt{1 - \sin^2(2\theta_{12})\sin^2\frac{\alpha}{2}}.$$

(3.33)

It is straightforward to conclude that m_{ee}^{IH} is constrained to the range

$$\sqrt{|\Delta m_{31}^2|}(1 - 2s_{12}^2) \lesssim m_{ee}^{\text{IH}} \lesssim \sqrt{|\Delta m_{31}^2|},$$

(3.34)

which, taking into account the 3σ allowed ranges for the neutrino parameters given by the STV global analysis (Table I), leads to

$$0.013 \lesssim m_{ee}^{\text{IH}} \lesssim 0.05.$$

(3.35)

Therefore, near future $0\nu\beta\beta$ decay experiments will be able to test the IH neutrino mass spectrum when this process is dominated by neutrino exchange.

In Fig. 11, we show the dependence of m_{ee} on the lightest neutrino mass m_0 for both types of neutrino mass spectra, i.e., normal and inverted hierarchy (Vissani, 1999). The ranges of m_0 disfavored by kinematical neutrino mass searches (Mainz and Troitsk) and by cosmology are also shown (see the discussion at the end of Sec. III). The m_{ee} allowed region is shown for a normal (inverted) neutrino mass spectrum, taking the 3σ STV neutrino data of Table I, and varying the Majorana phases in the range $[0, 2\pi]$. These two regions overlap for $m_0 \gg \Delta m_{31}^2$, where neutrinos are quasidegenerate. The same regions would be delimited by the solid black
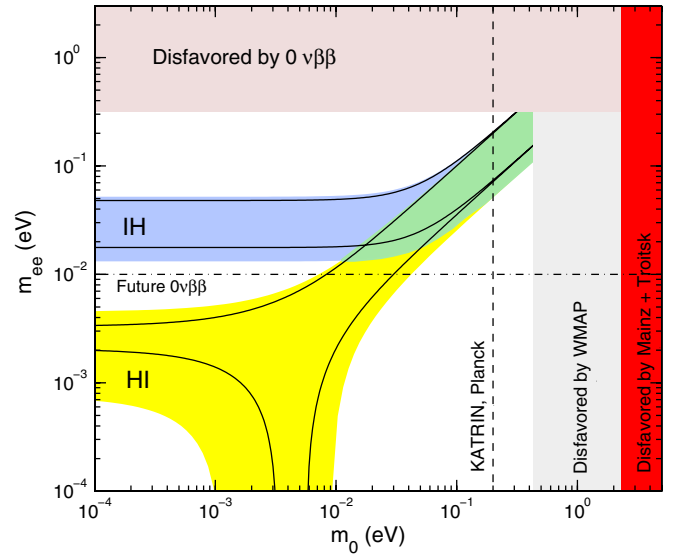

FIG. 11 (color online). Dependence of m_{ee} on the lightest neutrino mass m_0 for a normal (HI) and inverted (IH) neutrino mass spectrum. The yellow (light blue) region corresponds to 3σ intervals of the STV for a normal (inverted) neutrino mass spectrum. The Majorana phases $\alpha_{1,2}$ are varied in the interval $[0, 2\pi]$. The regions disfavored by kinematical searches and cosmology are delimited by the vertical shaded bands. The future Katrin and Planck satellite sensitivities are indicated by the vertical dashed line. The horizontal purple band refers to the m_{ee} region disfavored by the $0\nu\beta\beta$ Heidelberg-Moscow experiment. In turn, the dash-dotted horizontal line at $m_{ee} = 0.01$ eV illustrates the sensitivity of future $0\nu\beta\beta$ experiments. The m_{ee} allowed region delimited by the solid black lines is obtained when the best-fit values of the STV global neutrino data analysis are considered (see Table I).

lines if the best-fit values are considered. In this particular case, one can see that, even if the neutrino mixing angles are fixed, the Majorana phases have a strong impact on m_{ee}. It is also clear from this figure that the nonobservation of $0\nu\beta\beta$ in future experiments sensitive to m_{ee} down to 0.01 eV would exclude the IH and QD neutrino mass spectra. One should, however, keep in mind that the latter conclusion is valid under the assumption that the only contribution to $0\nu\beta\beta$ is the one mediated by the exchange of light active neutrinos.

If $0\nu\beta\beta$ decay is observed by future experiments, then one would *a priori* expect to learn something about Majorana-type *CP* violation in the lepton sector. In particular, a question which has been often addressed in the literature is whether one can extract the value of the phases α_1 and α_2 from a measurement of the $0\nu\beta\beta$ lifetime of a nucleus. Although this may seem an easy task from the mathematical point of view, the truth is that such a Majorana-phase determination is plagued by uncertainties in the determination of the NMEs $\mathcal{M}_{0\nu}$. Indeed, the computation of these quantities is a highly nontrivial many body problem (Menendez *et al.*, 2009). It has also been claimed that *CP* violation is not detectable via $0\nu\beta\beta$ (Barger *et al.*, 2002). The argument they present is based on the fact that if one considers x as being the sum of the uncertainty in the NME calculation and the experimental error, then the necessary condition for the discovery of *CP* violation requires that

$$\sin^2(2\theta_{12}) > 1 - \left(\frac{1-x}{1+x}\right)^2. \quad (3.36)$$

Taking the best-fit value of $\sin^2\theta_{12}$, one has $x < 0.46$. This is far beyond what seems reasonable to consider in view of the difficulties in calculating the NME, which presently suffers from an uncertainty factor of 2–3. More refined numerical studies confirmed the above general conclusion that, most probably, Majorana *CP* violation cannot be established in the near future $0\nu\beta\beta$ experiments. This could not be the case if the errors in the determination of m_{ee} and the sum of neutrino masses would not exceed 10%. In addition, the corresponding NME should be known within a factor of 1.5 (Pascoli, Petcov, and Schwetz, 2006), which seems to be a challenging target to reach.

Note that, although $0\nu\beta\beta$ decay depends on the Majorana phases $\alpha_{1,2}$, there is no distinction between the $0\nu\beta\beta$ rate of a nucleus and that of the corresponding antinucleus. In other words, $0\nu\beta\beta$ processes do not manifestly exhibit the violation of *CP*. Still, processes like neutrino $\leftrightarrow$ antineutrino oscillation and rare leptonic decays of K and B mesons (e.g., $K^\pm \to \pi^\mp l^\pm l^\pm$ and similar modes for the B meson) can actually be sensitive to Majorana-type CPV(de Gouvea, Kayser, and Mohapatra, 2003).

D. Lepton flavor violation and seesaw neutrino masses

In the quark sector, the only source of flavor and *CP* violation is the CKM mixing matrix. A large number of observables, mainly involving K and B-meson sectors, have been crucial to constrain the mixing angles and the *CP*-violating phase of this matrix, and to test the consistency of the CKM framework. In general, if there is new physics beyond the standard model (BSM), new sources of flavor and

CP violation are present. Their contributions to flavor and *CP*-violating processes may induce deviations from the SM predictions. The situation in the lepton sector is very different, since the only experimental evidence for flavor violation comes from neutrino oscillations, which require the existence of a nontrivial lepton mixing matrix **U**, which is the analog of the CKM matrix for leptons. This mixing matrix leads to LFV processes like, for instance, radiative charged-lepton decays $l_i \to l_j \gamma$ (Cheng and Li, 1977; Marciano and Sanda, 1977; Petcov, 1977). Moreover, if *CP* is violated in the lepton sector, charged-lepton electric dipole moments get also a nonzero contribution (Ng and Ng, 1996). However, due to the smallness of the neutrino masses, the corresponding observables are negligibly small and unaccessible to experiments.

The observation of any lepton flavor-violating process other than neutrino oscillations or the measurement of charged-lepton electric dipole moments would then be a direct signature of new physics. This is in clear contrast with what happens in the quark sector, in which new physics effects are subdominant to the SM ones. Up to now, none of these LFV processes have been observed and therefore only upper bounds on their rates are available. The present experimental limits for several charged-lepton LFV decays are shown in Table II. Several experiments aim at improving these bounds in the near future, namely, the MEG Collaboration plans to reach a sensitivity of $\mathrm{BR}(\mu \to e\gamma) \sim 10^{-13}$ (Cavoto, 2010) until the end of 2012, while a Super B factory would be able to probe LFV τ decays to a level of 10^{-9}. As for $\mu \to 3e$, the rather optimistic projected sensitivity is around 10^{-14} (Aysto *et al.*, 2001), while μ-e conversion in titanium could be tested at 10^{-18} by the J-PARC experiment PRISM/PRIME (Yoshimura, 2003).

If small neutrino masses are the only source of LFV, then the branching ratios (BR) for the radiative LFV charged-lepton decays are simply given by

$$\frac{\mathrm{BR}(l_i \to l_j \gamma)}{\mathrm{BR}(l_i \to l_j \bar{\nu}_j \nu_i)} = \frac{3\alpha}{32\pi} \left| \sum_{k=2,3} \mathbf{U}_{ik}^* \mathbf{U}_{jk} \frac{\Delta m_{k1}^2}{m_W^2} \right|^2$$

$$\leq \frac{3\alpha}{32\pi} \left| \frac{\Delta m_{31}^2}{m_W^2} \right|^2 \sim \mathcal{O}(10^{-53}), \quad (3.37)$$

where the unitarity of **U** and the present value for $|\Delta m_{31}^2|$ have been taken into account for the numerical estimate. The above result shows that, if neutrino masses are added to the

TABLE II. Present upper bounds for the branching ratios of flavor-violating charged-lepton decays $l_j \to l_i \gamma$ and $l_i \to l_j l_k l_k$ ($j, k \neq i$) and the μ-e conversion rate in titanium (Ti).

$\mu \to e\gamma$	2.4×10^{-12}	Adam *et al.* (2011)
$\tau \to \mu\gamma$	4.4×10^{-8}	Aubert *et al.* (2010)
$\tau \to e\gamma$	3.3×10^{-8}	Aubert *et al.* (2010)
$\mu^- \to e^+e^-e^-$	1.0×10^{-12}	Bellgardt *et al.* (1988)
$\tau^- \to \mu^+\mu^-\mu^-$	3.2×10^{-8}	Hayasaka *et al.* (2010)
$\tau^- \to e^+e^-e^-$	3.6×10^{-8}	Hayasaka *et al.* (2010)
$\tau^- \to e^+\mu^-\mu^-$	2.3×10^{-8}	Hayasaka *et al.* (2010)
$\tau^- \to e^-\mu^+\mu^-$	4.1×10^{-8}	Hayasaka *et al.* (2010)
$\tau^- \to \mu^+e^-e^-$	2.0×10^{-8}	Hayasaka *et al.* (2010)
$\tau^- \to \mu^-e^+e^-$	2.7×10^{-8}	Hayasaka *et al.* (2010)
$\mu \to e$ in Ti	4.3×10^{-12}	Hayasaka *et al.* (2010)

SM in order to explain the neutrino oscillation data, the rates of LFV processes turn out to be far beyond the sensitivity reach of future experiments. This is due to an extremely strong Glashow-Iliopoulos-Maiani (GIM) suppression mechanism (Glashow, Iliopoulos, and Maiani, 1970) in the lepton sector. Therefore, it is of extreme importance to explore BSM scenarios where this suppression is somehow alleviated.

Particularly interesting scenarios in which LFV is enhanced to observable levels are those when the new LFV sources are in some way related to those responsible for neutrino masses and mixing. For instance, if neutrino masses arise through the seesaw mechanism, then the seesaw mediators may induce LFV at either tree or one-loop level by participating directly in the decays. In such cases, the masses of these new states are required to be not too far from the electroweak scale. In the case of the type I seesaw (see Sec. II.E), the flavor dependence of the one-loop amplitudes of the processes $l_i \to l_j \gamma$ is roughly encoded in the coefficients $F_{ij} = (\mathbf{Y}^{\nu\dagger}\mathbf{d}_M^{-2}\mathbf{Y}^\nu)_{ij}$, where $\mathbf{Y}^\nu$ is the Dirac-neutrino Yukawa coupling matrix and $\mathbf{d}_M = \mathrm{diag}(M_1, M_2, M_3)$; M_i are the heavy Majorana neutrino masses. Instead, it follows from Eq. (2.35) that the effective neutrino mass matrix is proportional to the combination $\mathbf{Y}^\nu\mathbf{d}_M^{-1}\mathbf{Y}^{\nu T}$. From this simple (but effective) argument, one can see that there is no direct model-independent way of relating the neutrino data with LFV searches in this simple framework. This is mainly due to the fact that one cannot reconstruct the couplings $\mathbf{Y}^\nu$ and masses M_i, even if we know the effective neutrino mass matrix.

The situation is somehow different in the type II seesaw mechanism in which neutrino masses are generated by the tree-level exchange of scalar triplets. In this case, $l_i \to l_j \gamma$ is induced at one loop (Pich, Santamaria, and Bernabeu, 1984; Bilenky and Petcov, 1987; Mohapatra, 1992), while three-body charged-lepton LFV decays appear already at tree level (Barger *et al.*, 1982; Pal, 1983). The BRs for both cases are given by

$$\frac{\mathrm{BR}(l_i \to l_j \gamma)}{\mathrm{BR}(l_i \to l_j \bar{\nu}_j \nu_i)} = \frac{25\alpha}{768 G_F^2 \pi} \frac{|(\mathbf{Y}^{\Delta\dagger}\mathbf{Y}^\Delta)_{ij}|^2}{M_\Delta^4},$$
$$\frac{\mathrm{BR}(l_i^- \to l_j^+ l_k^- l_m^-)}{\mathrm{BR}(l_i \to l_j \bar{\nu}_i \nu_j)} = (1 + \delta_{km})\frac{|\mathbf{Y}_{ij}^\Delta|^2|\mathbf{Y}_{km}^\Delta|^2}{G_F^2 M_\Delta^4}. \quad (3.38)$$

Taking into account the bounds in Table II, one can use the above expressions to constrain combinations of the couplings $\mathbf{Y}^\Delta$, namely,

$$|(\mathbf{Y}^{\Delta\dagger}\mathbf{Y}^\Delta)_{ij}| \simeq 1.9 \times 10^3 \left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2 \sqrt{\frac{\mathrm{BR}(l_i \to l_j \gamma)}{\mathrm{BR}(l_i \to l_j \bar{\nu}_j \nu_i)}},$$

$$|\mathbf{Y}_{ij}^\Delta||\mathbf{Y}_{km}^\Delta| \simeq \frac{16.6}{\sqrt{1 + \delta_{km}}}\left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2 \sqrt{\frac{\mathrm{BR}(l_i^- \to l_j^+ l_k^- l_m^-)}{\mathrm{BR}(l_i \to l_j \bar{\nu}_i \nu_j)}}. \quad (3.39)$$

Note also that the flavor dependence of the BRs on the neutrino mass and mixing parameters is direct in the sense that $\mathbf{Y}^\Delta = M_\Delta \mathbf{m}_\nu/(\mu v^2)$ [see Eq. (2.39)]. Therefore the way in which the rates of LFV decays depend on the neutrino parameters is model independent. In order to eliminate the dependence on v, μ, and M_Δ, it is convenient to define ratios of BRs such as

$$R_{\tau j} \equiv \frac{\mathrm{BR}(\tau \to l_j \gamma)}{\mathrm{BR}(\mu \to e\gamma)} = \left|\frac{(\mathbf{m}_\nu^\dagger\mathbf{m}_\nu)_{\tau j}}{(\mathbf{m}_\nu^\dagger\mathbf{m}_\nu)_{\mu e}}\right|^2 \mathrm{BR}(\tau \to l_j \bar{\nu}_j \nu_\tau) \quad (3.40)$$

and

$$\hat{R}_{\tau jki} \equiv \frac{\mathrm{BR}(\tau^- \to l_j^+ l_k^- l_i^-)}{\mathrm{BR}(\mu \to 3e)}$$
$$= \frac{2}{1 + \delta_{ki}}\left|\frac{(\mathbf{m}_\nu)_{\tau j}(\mathbf{m}_\nu)_{ki}}{(\mathbf{m}_\nu)_{\mu e}(\mathbf{m}_\nu)_{ee}}\right|^2 \mathrm{BR}(l_i \to l_j \bar{\nu}_j \nu_i). \quad (3.41)$$

Using now the parametrization for $\mathbf{m}_\nu$ shown in Eq. (2.36), and taking into account the definitions (3.3), one can see that the quantities R_{ij} do not depend on the Majorana phases $\alpha_{1,2}$ and the lightest neutrino mass (Rossi, 2002; Joaquim and Rossi, 2007a). In contrast, the ratios $\hat{R}_{\tau jki}$ may depend on all neutrino parameters (Chun, Lee, and Park, 2003).

The dependence of $\mathrm{BR}(l_i \to l_j \gamma)$ on the neutrino parameters is (Joaquim, 2009; 2010)

$$\mathrm{BR}(\mu \to e\gamma) \propto c_{13}^2[r^2 c_{23}^2\sin^2(2\theta_{12}) + a^2 s_{13}^2 s_{23}^2$$
$$+ a|r|s_{13}\cos\delta\sin(2\theta_{12})\sin(2\theta_{23})],$$
$$\mathrm{BR}(\tau \to e\gamma) \propto c_{13}^2[r^2 s_{23}^2\sin^2(2\theta_{12}) + a^2 s_{13}^2 c_{23}^2$$
$$- a|r|s_{13}\cos\delta\sin(2\theta_{12})\sin(2\theta_{23})],$$
$$\mathrm{BR}(\tau \to \mu\gamma) \propto \{4|r|s_{13}\cos\delta\sin(2\theta_{12})\cos(2\theta_{23})$$
$$+ [2bc_{13}^2 - |r|(\cos(2\theta_{23}) - 3)$$
$$\times \cos(2\theta_{12})]\sin(2\theta_{23})\}^2$$
$$+ 16r^2 s_{13}^2\cos\delta\sin(2\theta_{12})\sin(2\theta_{23}). \quad (3.42)$$

These expressions are valid for both the NO and IO neutrino mass spectra with a and b defined as

$$\mathrm{NO}: a = 2(1 - |r|s_{12}^2) \simeq 2, \qquad b = -2 + |r| \simeq -2,$$
$$\mathrm{IO}: a = -2(1 + |r|s_{12}^2) \simeq -2, \qquad b = 2 + |r| \simeq 2, \quad (3.43)$$

and the parameter r given in Eq. (3.1).

From the above equations, one can immediately conclude that the ratios $R_{\tau j}$ depend on the lepton mixing angles, the Dirac *CP* phase δ, and the ratio r. Taking into account the present neutrino data summarized in Table I, one can study the dependence of $R_{\tau j}$ on θ_{13} and δ. This is shown in Fig. 12 where $R_{\tau\mu}$ (top panels) and $R_{\tau e}$ (bottom panels) are shown for the NO (left panels) and IO (right panels) neutrino mass spectra. From this figure, it is evident that the impact of δ on $R_{\tau j}$ can be significant for $s_{13} \sim 10^{-2}$. In particular, a flavor suppression may occur in the $\tau\mu$ and τe channels in the *CP*-conserving cases. This may be have profound impact on the LFV predictions of the type II seesaw (Joaquim and Rossi, 2007a; Joaquim, 2009).

As already mentioned, in the type II seesaw framework, the three-body LFV charged-lepton decay rates may also depend
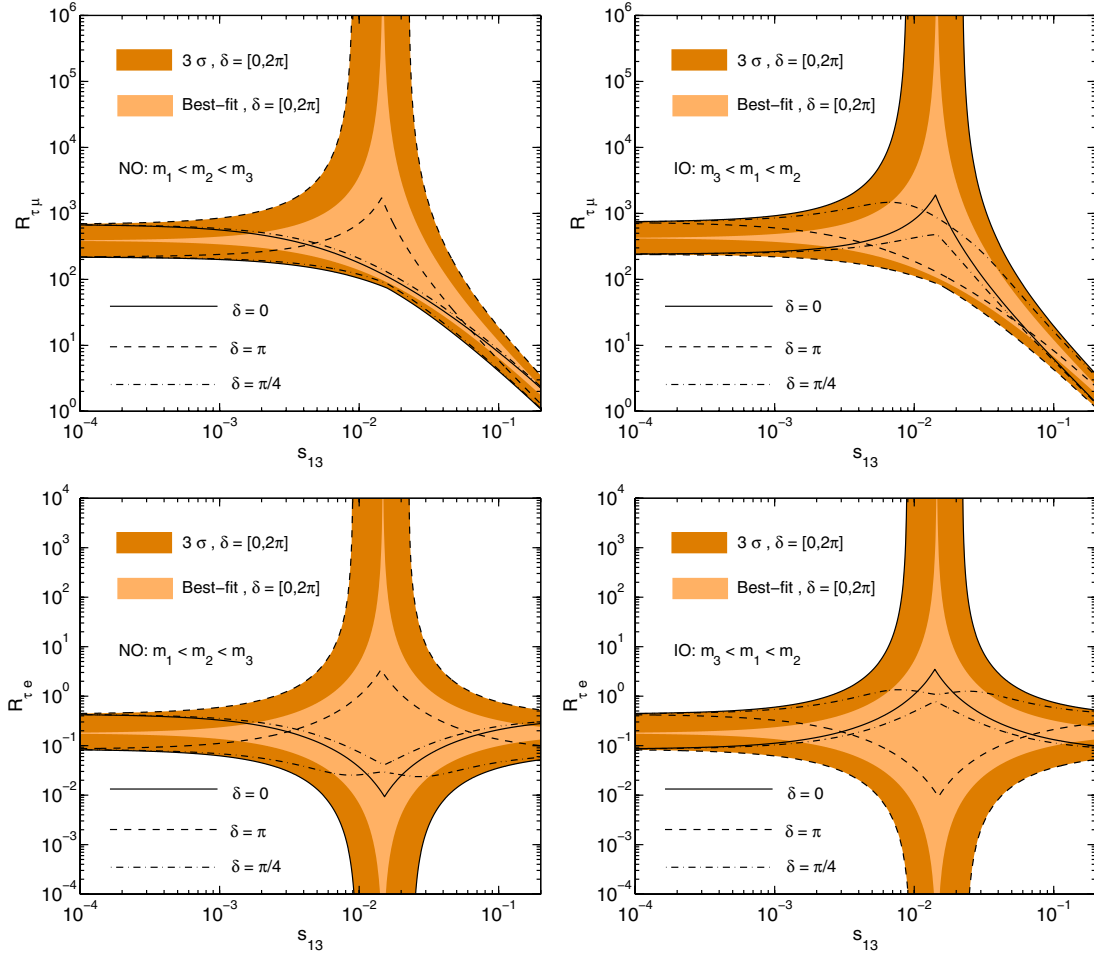
FIG. 12 (color online). Allowed regions for $R_{\tau\mu}$ (upper plots) and $R_{\tau e}$ (lower plots) defined in Eqs. (3.40) and (3.42) as a function of s_{13} and δ, for both the NO (left plots) and the IO (right plots) neutrino mass spectra. In dark (light) shading we show the 3σ (best-fit) allowed regions obtained by varying the *CP*-violating phase δ in the interval $[0, 2\pi]$ and using the neutrino data displayed in Table I. The solid, dashed, and dash-dotted lines delimits the 3σ region for $\delta = 0$, $\delta = \pi$, and $\delta = \pi/4$, respectively. Adapted from Joaquim, 2010.

on the neutrino mass scale and the Majorana *CP* phases. In some cases, this is not true though. For instance, for a HI neutrino mass spectrum and $\theta_{13} = \delta = 0$, the BRs of the decays $\mu^- \to e^+e^-e^-$ and $\tau^- \to e^+e^-e^-$ depend on the neutrino mixing parameters as

$$\mathrm{BR}(\mu^- \to e^+e^-e^-) \propto r^2 c_{12}^2 c_{23}^2 s_{12}^6,$$
$$\mathrm{BR}(\tau^- \to e^+e^-e^-) \propto r^2 c_{12}^2 s_{23}^2 s_{12}^6,$$
(3.44)

leading to $R_{\tau eee} \simeq \tan^2\theta_{23}\mathrm{BR}(\tau \to e\nu_\tau\bar\nu_e) \simeq 0.17$. Therefore, in this specific case, the observation of the $\tau^- \to e^+e^-e^-$ decay in the near future would exclude a scenario where these decays occur due to the exchange of the scalar triplet which gives rise to neutrino masses, for any value of the Majorana phases. This is not the case for the IH neutrino spectrum for which

$$\mathrm{BR}(\mu^- \to e^+e^-e^-) \propto c_{12}^2 s_{12}^2 c_{23}^2 \sin^2(\alpha_1/2)$$
$$\times [1 - \sin^2(2\theta_{12})\sin^2(\alpha_1/2)],$$
(3.45)

for $\theta_{13} = \delta = 0$ and at zero order in r. This expression exhibits a strong dependence on the (only) Majorana phase α_1. In Fig. 13, we show the dependence of the ratio $R_{\tau\mu\mu}$ on

the Majorana phases $\alpha_{1,2}$ for $s_{13} = 0.1$, $\delta = \pi/2$ (large Dirac *CP* violation), and a QD neutrino mass spectrum ($m_0 \simeq 0.1$ eV). The density plot of $\log_{10}(R_{\tau\mu\mu})$ shows that, depending on the values of α_1 and α_2, $R_{\tau\mu\mu}$ can change by several orders of magnitude.

If the neutrino mass mediators are very heavy, their direct effect on LFV processes becomes irrelevant. Still, they can participate indirectly on the generation of new LFV terms, as may happen in supersymmetric versions of the seesaw mechanism (Borzumati and Masiero, 1986; Rossi, 2002), when renormalizable Yukawa interactions involving the heavy and SM fields induce, through renormalization, LFV soft supersymmetry (SUSY) breaking. This scenario has been the subject of a large number of studies (Raidal *et al.*, 2008). In the SUSY type I seesaw, singlet neutrino superfields N_i with masses M_i are added to the minimal supersymmetric standard model (MSSM) superfield content, in such a way that the superpotential W is just $W = W_{\mathrm{MSSM}} + \mathbf{Y}^\nu NLH_2 + \frac{1}{2}M_iN_iN_i$, where L and H_2 are the lepton and Higgs superfields, respectively. Considering (flavor-blind) universal boundary conditions for the soft SUSY-breaking terms at a scale $\Lambda > M_i$, LFV terms may be generated at lower scales due to renormalization group effects induced by the presence of $\mathbf{Y}^\nu$ (Borzumati and
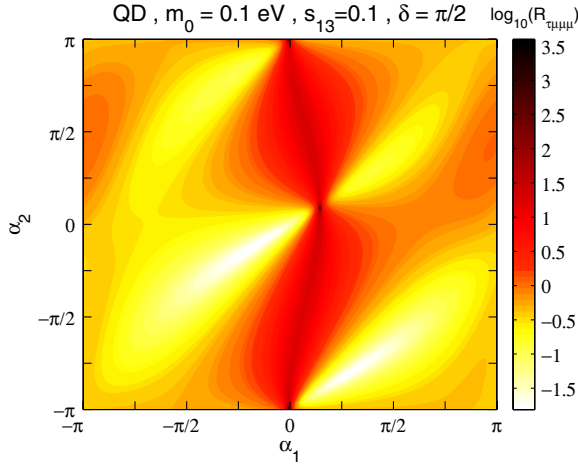
FIG. 13 (color online). Density plot of the ratio $R_{\tau\mu\mu}$ defined in Eq. (3.41) as a function of the Majorana phases $\alpha_{1,2}$ for a QD neutrino mass spectrum with $m_0 = 0.1$ eV, $s_{13} = 0.1$, and $\delta = \pi/2$.

Masiero, 1986). In particular, in the simplest case in which only the LFV effects induced in the left-handed scalar sector are relevant, the soft SUSY-breaking terms $\tilde{L}\mathbf{m}_{\tilde{L}}^2\tilde{L}$ are such that

$$(\mathbf{m}_{\tilde{L}}^2)_{ij} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2}(\mathbf{Y}^{\nu\dagger}\mathbf{Y}^\nu)_{ij}\ln\frac{\Lambda}{M} \qquad (i \neq j),$$

(3.46)

where m_0 and A_0 are the universal SUSY-breaking soft mass and trilinear parameters at the scale Λ. For simplicity, we have taken in the above expression a common mass M for all the heavy Majorana neutrinos. The existence of LFV entries in the slepton masses $(\mathbf{m}_{\tilde{L}}^2)_{ij}$ opens the window for the LFV processes discussed above at the loop level. For the specific case of radiative charged-lepton decays,

$$\mathrm{BR}(l_i \to l_j\gamma) \simeq \frac{48\pi^3\alpha}{G_F^2}|C_{ij}|^2\tan^2\beta\,\mathrm{BR}(l_i \to l_j\nu_i\bar{\nu}_j),$$

(3.47)

where the coefficients C_{ij} encode the LFV dependence of the rates. Taking a common mass m_S for the SUSY particles in the loops, one has

$$C_{ij} \sim \frac{g_2^2}{16\pi^2}\frac{(\mathbf{m}_{\tilde{L}}^2)_{ij}}{m_S^4} \qquad (i \neq j = e, \mu, \tau).$$

(3.48)

It is straightforward to see that the rates of the LFV processes depend on a combination of couplings, which is different from the one which appears in the neutrino mass matrix. Therefore, as in the case for the low-energy seesaw discussed above, a model-independent reconstruction of $(\mathbf{Y}^{\nu\dagger}\mathbf{Y}^\nu)_{ij}$ is not possible from low-energy data. For instance, it has been recently shown that the C_{ij} coefficients are not as sensitive to the unknown mixing angle θ_{13} as previously advocated (Casas *et al.*, 2011). In other words, the way that the SUSY LFV terms depend on the neutrino parameters in the SUSY type I seesaw mechanism is not model independent. Nevertheless, it can be shown that the phases entering in the neutrino mixing matrix may have a strong impact in LFV

processes (Petcov and Shindou, 2006) and the electric dipole moments of charged leptons (Ellis *et al.*, 2002; Masina, 2003; Farzan and Peskin, 2004; Joaquim, Masina, and Riotto, 2007).

In the case of the SUSY type II seesaw, the left-handed LFV soft scalar masses are given by (Rossi, 2002)

$$(\mathbf{m}_{\tilde{L}}^2)_{ij} - \frac{9m_0^2 + 3A_0^2}{8\pi^2}(\mathbf{Y}^{\Delta\dagger}\mathbf{Y}^\Delta)_{ij}\ln\frac{\Lambda}{M_\Delta},$$

(3.49)

where $\mathbf{Y}^\Delta$ are the couplings of the triplet with the lepton superfields and M_Δ the triplet mass. Note that, since the effective neutrino mass matrix $\mathbf{m}_\nu$ is again proportional to $\mathbf{Y}^\Delta$, the ratios of BRs defined in Eq. (3.40) are still valid in the SUSY case. In particular, the predictions shown in Fig. 12 also hold in the present case. The same is not true for three-body decays and μ-e conversion in nuclei which, in the MSSM, are induced at one loop due to the presence of LFV soft SUSY-breaking terms like $(\mathbf{m}_{\tilde{L}}^2)_{ij}$. Consequently, the rates for these processes will be also independent from the Majorana phases and the lightest neutrino mass (Joaquim and Rossi, 2007a). In general, this is valid in all cases with LFV in the soft SUSY-breaking sector induced by the couplings $\mathbf{Y}^\Delta$, as in the universal boundary condition limit (Rossi, 2002; Joaquim, 2009; Joaquim, 2010), or in the gauge-Yukawa SUSY-breaking mediation scenario (Joaquim and Rossi, 2006; 2007b). It has also been shown that, in a type II seesaw scenario with neutrino masses generated from Kähler effective terms, the same relation of LFV processes and neutrino data is obtained (Brignole, Joaquim, and Rossi, 2010; 2010).

In the previous examples, the *CP* phases affecting the LFV rates are those that can be potentially measured in neutrino experiments. However, it is well known that it is possible to probe on CPV in the leptonic sector by adopting an effective Lagrangian approach to extract some information on the *CP*-violating structure of the LFV effective operators (Treiman, Wilczek, and Zee, 1977; Zee, 1985; Okada, Okumura, and Shimizu, 2000; de Gouvea, Lola, and Tobe, 2001). For instance, this can be achieved by measuring the polarization of the final-state particles in $\mu \to e\gamma$ (Farzan, 2007; Ayazi and Farzan, 2009) and μ-e conversion in nuclei (Davidson, 2008). Similar conclusions can be drawn if one performs a spin measurement of the more energetic positron in the final state of $\mu^+ \to e^+e^-e^+$. Although such studies could shed some light on the *CP*-violating structure of the effective Lagrangian, the origin of such effects would be hardly identifiable, since their connection with *CP* violation in neutrino oscillations is difficult to establish without further theoretical assumptions. Still, it is undeniable that detecting such CPV effects in LFV processes could be a powerful tool for discriminating BSM scenarios in which the LFV effective operators arise.

E. Impact of LCPV at colliders

High-energy accelerators like the LHC may also provide valuable information about the neutrino mass generation mechanism. In particular, if the neutrino mass mechanism operates at scales not far from the electroweak scale, then new phenomena can manifest in colliders. Most of the research performed in this direction concerns the study of new

signals, which result from decays of the seesaw mediators (Han *et al.*, 2005; Bajc, Nemevsek, and Senjanovic, 2007; Akeroyd, Aoki, and Sugiyama, 2008; Kadastik, Raidal, and Rebane, 2008; del Aguila and Aguilar-Saavedra, 2009). Although these decays do not lead to explicit CPV effects, the presence of CPV phases affects the decay rates, since the couplings of the SM particles to the seesaw mediators depend on the phases $\alpha_{1,2}$ and δ of the lepton mixing matrix **U**.

The connection between LCPV, collider processes, and neutrino oscillation experiments is not straightforward to establish. In particular, in the case of the type I (III) seesaw, it is not possible to reconstruct in a model-independent way the couplings of the fermion singlets (triplets) with the Higgs and charged-lepton fields. However, the situation changes in the type II seesaw since, as mentioned, the couplings of the scalar triplet Δ with the lepton doublets have the same flavor structure as the effective neutrino mass matrix. In this framework, if the triplet mass is close to the electroweak scale, Δ may be produced in high-energy collisions. More specifically, the production of its doubly-charged Higgs component occurs via the Drell-Yan process $q\bar{q} \rightarrow \gamma^* Z^* \rightarrow \Delta^{++}\Delta^{--}$, and also (subdominantly) by photon-photon fusion $\gamma\gamma \rightarrow \Delta^{++}\Delta^{--}$. Provided the triplet VEV is small enough, the decays of $\Delta^{\pm\pm} \rightarrow l^\pm l^\pm$ are dominant over $\Delta^{++} \rightarrow \Delta^+\Delta^+$, $\Delta^{++} \rightarrow \Delta^+ W^+$, and $\Delta^{++} \rightarrow W^+ W^+$. In this case, the decay of the $\Delta^{\pm\pm}$ pair into four charged leptons gives a very clear signature, which is almost free of any SM background (Han *et al.*, 2005; del Aguila and Aguilar-Saavedra, 2009).

Assuming that neutrino masses are generated through the exchange of Δ, the decay rate of $\Delta^{\pm\pm} \rightarrow l_i^\pm l_j^\pm$ is proportional to $|(\mathbf{m}_\nu)_{ij}|^2$, which is sensitive to the LCPV phases. The branching ratios $\mathrm{BR}_{\Delta ij} \equiv \mathrm{BR}(\Delta^{\pm\pm} \rightarrow l_i^\pm l_j^\pm)$ are simply given by

$$\mathrm{BR}_{\Delta ij} = \frac{2}{1 + \delta_{ij}} \frac{\sum_k |m_k \mathbf{U}_{ik}\mathbf{U}_{jk}|^2}{\sum_n m_n^2}, \tag{3.50}$$

where δ_{ij} is the Kronecker symbol, introduced to account for the decays into charged leptons of the same flavor. The term in the denominator is $\sum_p m_p^2 = 3m_0^2 + \Delta m_{21}^2 + \Delta m_{31}^2$ for a NO neutrino mass spectrum, and $\sum_p m_p^2 = 3m_0^2 + \Delta m_{21}^2 + 2|\Delta m_{31}^2|$ for an IO one. The above BRs depend exclusively on the lepton mixing angles, CPV phases, and the neutrino masses. In some specific limits, very simple relations can be obtained. In particular, in the HI case (NO with $m_0 = 0$), and taking $\theta_{13} = 0$ one has

$$\mathrm{BR}_{\Delta ee}^{\mathrm{HI}} = \frac{rs_{12}^4}{1 + r}, \qquad \mathrm{BR}_{\Delta\mu e}^{\mathrm{HI}} = \frac{rc_{23}^2 \sin^2(2\theta_{12})}{2(1 + r)},$$

$$\mathrm{BR}_{\Delta\mu\mu}^{\mathrm{HI}} = \frac{rc_{12}^4 c_{23}^4 + s_{23}^4 + 2\sqrt{r}c_{12}^2 c_{23}^2 s_{23}^2 \cos\alpha_{21}}{1 + r}. \tag{3.51}$$

Note that, in this particular case, the $e^\pm e^\pm$ and $\mu^\pm e^\pm$ decays are suppressed by the parameter $r \ll 1$. Moreover, only the $\mu^\pm\mu^\pm$ channel is sensitive to leptonic CPV effects associated with the Majorana-phase difference $\alpha_{21} = \alpha_2 - \alpha_1$ (the decays into $\mu^\pm\tau^\pm$ are also sensitive to α_{21}). In the IH limit (IO with $m_0 = 0$) the above BRs are instead approximately given by
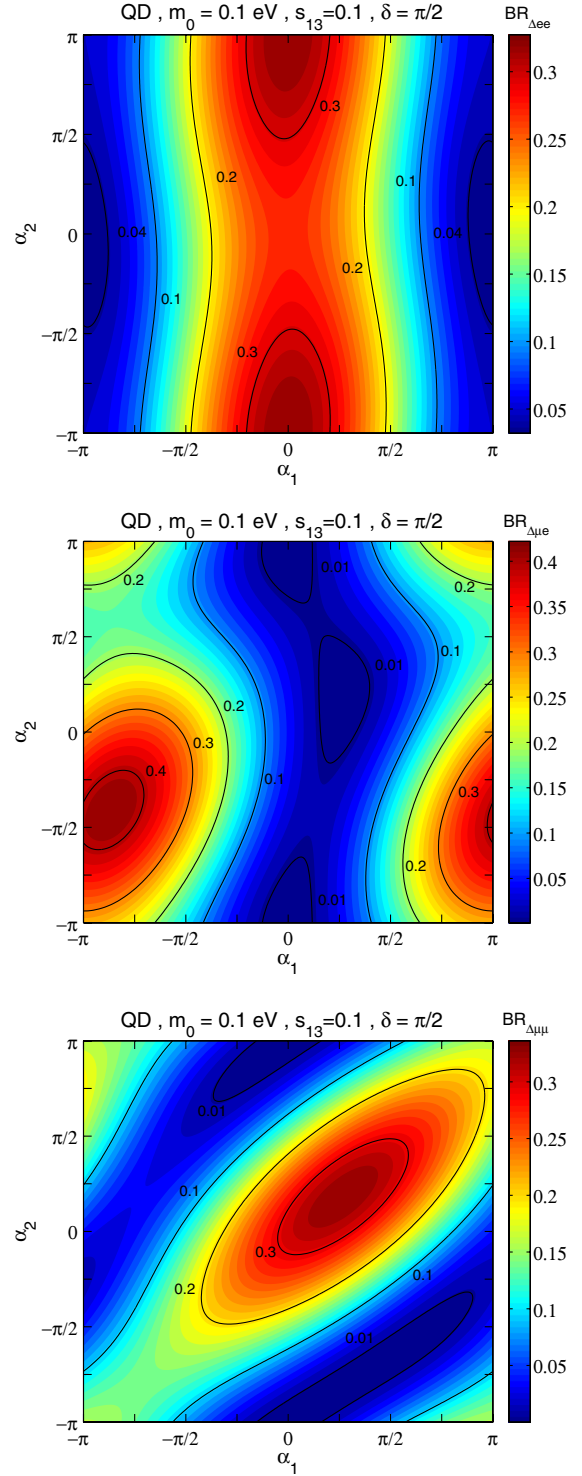

FIG. 14 (color online). Variation of $\mathrm{BR}_{\Delta ee}$ (top), $\mathrm{BR}_{\Delta\mu e}$ (center), and $\mathrm{BR}_{\Delta\mu\mu}$ (bottom) in the α_1–α_2 parameter space for a quaside-generate neutrino mass spectrum with $m_0 = 0.1$ eV, $s_{13} = 0.1$, and $\delta = \pi/2$. The remaining neutrino parameters are taken at the best-fit values of the STV analysis (see Table I).

$$\mathrm{BR}_{\Delta ee}^{\mathrm{IH}} \simeq \frac{1}{2}(s_{12}^4 + c_{12}^4 + 2c_{12}^2 s_{12}^2 \cos\alpha_1),$$

$$\mathrm{BR}_{\Delta\mu e}^{\mathrm{IH}} = \sin^2(2\theta_{12})c_{23}^2 \sin^2\frac{\alpha_1}{2}, \tag{3.52}$$

$$\mathrm{BR}_{\Delta\mu\mu}^{\mathrm{IH}} \simeq c_{23}^4 \mathrm{BR}_{\Delta ee}^{\mathrm{IH}}.$$

As for the QD case ($m_0 \gg \Delta m_{31}^2$), the following relations hold:

$$\mathrm{BR}_{\Delta ee(\mu)}^{\mathrm{QD}} \simeq \tfrac{2}{3} \mathrm{BR}_{\Delta ee(\mu)}^{\mathrm{IH}},$$

$$12\mathrm{BR}_{\Delta\mu\mu}^{\mathrm{QD}} = c_{23}^4 [3 + \cos(4\theta_{12}) + 2\sin^2(2\theta_{12})\cos\alpha_1]$$
$$+ 4s_{23}^4 + 2(c_{12}^2 \cos\alpha_{21} + s_{12}^2 \cos\alpha_2)\sin^2(2\theta_{23}).$$
$$(3.53)$$

The above results hold in the simple limits of HI, IH, and QD neutrino masses with $\theta_{13} = 0$. A complete study including the dependence on the lightest neutrino mass and CPV phases can be found in Garayoa and Schwetz (2008). The possibility of extracting information on the Majorana phases from the doubly-charged Higgs decays into leptons has been addressed in Akeroyd, Aoki, and Sugiyama (2008) and the connection with neutrinoless double beta decay in Petcov, Sugiyama, and Takanishi (2009). In particular, it has been shown that it is possible to extract some information about m_0 and $\alpha_{1,2}$ from BR_{ee}, $\mathrm{BR}_{\mu\mu}$, and $\mathrm{BR}_{e\mu}$.

In Fig. 14, we show how $\mathrm{BR}_{\Delta ee}$ (top panel), $\mathrm{BR}_{\Delta\mu e}$ (center panel), and $\mathrm{BR}_{\Delta\mu\mu}$ (bottom panel) depend on the Majorana phases $\alpha_{1,2}$, for the specific case $\delta = \pi/2$, $s_{13} = 0.1$ and a QD neutrino mass spectrum with $m_0 = 0.1$ eV. The results show that the rates for the decays of the triplets into leptons are considerably affected by the Majorana phases $\alpha_{1,2}$. In particular, one can see from these plots that $\mathrm{BR}_{\Delta\mu e}$ tends to be suppressed when $\mathrm{BR}_{\Delta ee}$ and $\mathrm{BR}_{\Delta\mu\mu}$ are larger.

F. Nonunitarity effects in the lepton sector

Searches for deviations from unitary mixing are a sensitive probe of physics beyond the SM. In the quark sector, several studies have been carried out in the direction of finding possible deviations from the unitarity of the CKM matrix. Similarly, nonunitarity (NU) effects may occur in the lepton sector in the presence of BSM physics. This is the case if, for instance, new states with mass far above the electroweak scale are added to the SM particle content. Probably the best example of such a framework is the seesaw mechanism described in Sec. III.E. In the type I version, the mass matrix is extended to a $(3 + n_R) \times (3 + n_R)$ form, where n_R is the number of heavy right-handed neutrinos with typical mass $M \gg v$. In this case, the NU of the lepton mixing matrix stems from the fact that this matrix is now a sub-block of a larger unitary one, since the complete theory has to respect probability conservation (Schechter and Valle, 1980). After the decoupling of these states, an effective dimension-six operator of the type $(\bar{\ell}\, \tilde{\phi}) i \partial (\tilde{\phi}^\dagger \ell)/M^2$ is generated (Broncano, Gavela, and Jenkins, 2003) which induces a contribution to the neutrino kinetic energy, suppressed by v^2/M^2, upon electroweak symmetry breaking. Therefore, a field redefinition is demanded to bring back the kinetic term to its canonical form. This, in turn, introduces NU mixing in the charged and neutral current Lagrangian terms.

In the conventional type I seesaw, the NU effects are too small to be observed. Nevertheless, this may not be the case in alternative realizations like the inverse seesaw (Mohapatra

and Valle, 1986; Gonzalez-Garcia and Valle, 1989), in which the effect of the mass suppression can be alleviated without prejudice of the smallness of neutrino masses. In other words, in this scenario the effective dimension-five operator responsible for the suppression of neutrino masses can be somehow decoupled from the dimension-six one, allowing at the same time not too small NU effects so that interesting new phenomenology may appear (Deppisch and Valle, 2005; Deppisch, Kosmas, and Valle, 2006; Malinsky *et al.*, 2009; Malinsky, Ohlsson, and Zhang, 2009; Dev and Mohapatra, 2010). Similar effects arise in other models with large light-heavy neutrino mixing (Nardi, Roulet, and Tommasini, 1995; Tommasini *et al.*, 1995), and in scenarios with extra dimensions where the mixing of Kaluza-Klein modes with the light neutrinos may induce NU effects (De Gouvea *et al.*, 2002; Branco, Delepine, Nobre, and Santiago, 2003; Bhattacharya, Dey, and Mukhopadhyaya, 2009). Another possible source of nonunitarity arises from loop corrections to the charged lepton or neutrino self-energies (Bellazzini *et al.*, 2011) which modify the corresponding kinetic terms, thus inducing NU effects. There can also be direct corrections to the lepton mixing matrix **U**.

In studying NU effects in the lepton sector, a model-independent approach can be adopted such that the sources of NU are not specified. In particular, we focus here on a framework called minimal unitarity violation (MUV), in which NU sources are allowed only in neutrino Lagrangian terms and three light neutrinos are considered (Antusch *et al.*, 2006). Under these assumptions, the mass and flavor neutrino eigenstates are related by a nonunitary 3×3 matrix **N** such that $\nu_\alpha = \mathbf{N}_{\alpha k}\nu_k$. In the corresponding mass basis, the charged and neutral current Lagrangian terms become (Schechter and Valle, 1980)

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}}(W_\mu^+ \bar{l}_\alpha \gamma_\mu P_L \mathbf{N}_{\alpha k} \nu_k + \mathrm{H.c.}), \qquad (3.54)$$

$$\mathcal{L}_{\mathrm{NC}} = -\frac{g}{\cos\theta_W}[Z_\mu \bar{\nu}_k P_L (\mathbf{N}^\dagger \mathbf{N})_{kj} \nu_j + \mathrm{H.c.}]. \qquad (3.55)$$

These modifications give rise to new effects in several physical phenomena such as neutrino oscillations, universality tests, and electroweak decays, which can be used to test unitarity in lepton mixing. In this direction, detailed analysis have been performed in the literature with the goal of quantifying the deviations from unitarity of **N**, taking into account several physical processes. In the following, we briefly review the main conclusions of those studies.

1. Neutrino oscillations with NU

In the presence of NU, the neutrino flavor and mass eigenstates cannot be simultaneously orthogonal. As a consequence, the oscillation probabilities $\nu_\alpha \rightarrow \nu_\beta$, as a function of the distance L travelled by neutrinos, now read (Czakon, Gluza, and Zralek, 2001)

$$P_{\alpha\beta} = \frac{|\sum_k \mathbf{N}_{\beta k} e^{-iE_k L} \mathbf{N}_{\alpha k}^*|^2}{(\mathbf{N}\mathbf{N}^\dagger)_{\alpha\alpha}(\mathbf{N}\mathbf{N}^\dagger)_{\beta\beta}}, \qquad (3.56)$$

which reduces to Eq. (3.8) in the limit of a unitary **N**. An immediate consequence of the above result is that a flavor transition is possible at zero distance ($L = 0$) before

oscillations (Langacker and London, 1988a), with a transition probability

$$P_{\alpha\beta}(L=0) = \frac{|(\mathbf{N}\,\mathbf{N}^\dagger)_{\beta\alpha}|^2}{(\mathbf{N}\,\mathbf{N}^\dagger)_{\alpha\alpha}(\mathbf{N}\,\mathbf{N}^\dagger)_{\beta\beta}} \neq \delta_{\alpha\beta}. \qquad (3.57)$$

This result can be probed at neutrino oscillation experiments with near detectors. In particular, the data from NOMAD (Astier *et al.*, 2001), Bugey (Declais *et al.*, 1995), KARMEN (Declais *et al.*, 1995), and the MINOS (Adamson *et al.*, 2008) near detector impose the following constraints on $\mathbf{N}\,\mathbf{N}^\dagger$:

$$|(\mathbf{N}\,\mathbf{N}^\dagger)_{e\alpha}| \simeq (1.00 \pm 0.04, <0.05, <0.09),$$
$$|(\mathbf{N}\,\mathbf{N}^\dagger)_{\mu\alpha}| \simeq (<0.05, 1.00 \pm 0.04, <0.013), \qquad (3.58)$$
$$|(\mathbf{N}\,\mathbf{N}^\dagger)_{\tau\alpha}| \simeq (<0.09, <0.013, ?),$$

at 90% C.L. (Antusch *et al.*, 2006).

In vacuum, the disappearance oscillation probability is then given by

$$P_{\alpha\alpha} = \sum_{k=1}^{3} |\mathbf{N}_{\alpha k}|^4 + \sum_{k\neq j=1}^{3} |\mathbf{N}_{\alpha k}|^2 |\mathbf{N}_{\alpha j}|^2 \cos\frac{\Delta m_{kj}^2 L}{2E}. \qquad (3.59)$$

Instead, the oscillation probabilities in matter are modified with respect to the unitary case since the effective potential felt by neutrinos is no longer diagonal (Bekman *et al.*, 2002; del Aguila and Zralek, 2002; Fernandez-Martinez *et al.*, 2007; Holeczek *et al.*, 2007). In addition, the NC contribution to the matter potential contributes to the evolution equation once it cannot be interpreted as a global phase.

Depending on the range of L/E, the above equation can be simplified and used to constrain the elements of $\mathbf{N}$ (or combinations of them), considering the experimental neutrino oscillation data suitable for each case. The combined fit of the KamLAND (Araki *et al.*, 2005), CHOOZ (Apollonio *et al.*, 2003), SNO (Ahmad *et al.*, 2002), and K2K (Ahn *et al.*, 2003) data allow for the following determination of $|\mathbf{N}|$ at 90% C.L. (Antusch *et al.*, 2006)

$$|\mathbf{N}_{ej}| \simeq (0.75\text{–}0.89, 0.45\text{–}0.66, <0.34),$$
$$|\mathbf{N}_{\mu 1}|^2 + |\mathbf{N}_{\mu 2}|^2 = 0.57\text{–}0.86, \qquad |\mathbf{N}_{\mu 3}| \simeq 0.57\text{–}0.86, \qquad (3.60)$$

where $|\mathbf{N}_{e2}|$ and $|\mathbf{N}_{e1}|$ are determined by the SNO and KamLAND data (combined with the others), respectively, and $|\mathbf{N}_{e3}|$ is constrained by CHOOZ. On the other hand, atmospheric and accelerator experiments do not allow for a discrimination between $|\mathbf{N}_{\mu 1}|^2$ and $|\mathbf{N}_{\mu 2}|^2$. Nevertheless,

these two quantities can be disentangled taking into account the constraints shown in Eq. (3.58), leading to the final result

$$|\mathbf{N}_{ej}| \simeq (0.75\text{–}0.89, 0.45\text{–}0.66, <0.27),$$
$$|\mathbf{N}_{\mu j}| \simeq (0.00\text{–}0.69, 0.22\text{–}0.81, 0.57\text{–}0.85). \qquad (3.61)$$

The absence of constraints for the elements in the third row of $\mathbf{N}$ is due to the lack of ν_τ oscillation signals.

2. NU constraints from electroweak decays

It has been known for quite a long time that nonunitarity of the leptonic mixing matrix induced by light-heavy neutrino mixing can manifest itself in tree-level processes like π, W, and Z decays (Langacker and London, 1988b; Nardi, Roulet, and Tommasini, 1992; Korner, Pilaftsis, and Schilcher, 1993; Nardi, Roulet, and Tommasini, 1994), in rare charged-lepton decays $l_j \to l_i \gamma$, $l_j \to 3l_j$, $l_j \to l_i l_i l_k$, and μ-e conversion in nuclei (Langacker and London, 1988a; Ilakovac and Pilaftsis, 1995; Tommasini *et al.*, 1995). The interest on this subject has been recently revived in a series of works, where the constraints on NU effects in the lepton sector have been analyzed, considering the above electroweak processes in view of the most recent experimental data (Antusch *et al.*, 2006; Abada *et al.*, 2007; 2008; Antusch, Baumann, and Fernandez-Martinez, 2009).

In the MUV framework, $W \to l_\alpha \nu_\alpha$ and invisible Z decays lead to the conditions

$$\frac{(\mathbf{N}\,\mathbf{N}^\dagger)_{\alpha\alpha}}{\sqrt{(\mathbf{N}\,\mathbf{N}^\dagger)_{ee}(\mathbf{N}\,\mathbf{N}^\dagger)_{\mu\mu}}} = f_\alpha, \qquad (3.62)$$

$$\frac{\sum_{\alpha\beta} |(\mathbf{N}\,\mathbf{N}^\dagger)_{\alpha\beta}|^2}{\sqrt{(\mathbf{N}\,\mathbf{N}^\dagger)_{ee}(\mathbf{N}\,\mathbf{N}^\dagger)_{\mu\mu}}} = 2.984 \pm 0.009, \qquad (3.63)$$

respectively, with $f_{e,\mu,\tau} = (1.000 \pm 0.024, 0.986 \pm 0.028, 1.002 \pm 0.032)$. On the other hand, from charged-lepton decays $l_\alpha \to l_\beta \gamma$, one can write

$$\frac{|(\mathbf{N}\,\mathbf{N}^\dagger)_{\alpha\beta}|^2}{\sqrt{(\mathbf{N}\,\mathbf{N}^\dagger)_{\alpha\alpha}(\mathbf{N}\,\mathbf{N}^\dagger)_{\beta\beta}}} = \frac{96\pi}{100\alpha_{\text{em}}} \frac{\text{BR}(l_\alpha \to l_\beta \gamma)}{\text{BR}(l_\alpha \to \nu_\alpha l_\beta \bar{\nu}_\alpha)}. \qquad (3.64)$$

The present experimental limits on the branching ratios entering the above expression are shown in Table II. The combination of constraints coming from electroweak decays leads then to the following limits[5] for $|\mathbf{N}\,\mathbf{N}^\dagger|$:

$$|\mathbf{N}\,\mathbf{N}^\dagger| \approx \begin{pmatrix} 1.002 \pm 0.005 & <7.2 \times 10^{-5} & <8.8 \times 10^{-3} \\ <7.2 \times 10^{-5} & 1.003 \pm 0.005 & <10^{-2} \\ <8.8 \times 10^{-3} & <10^{-2} & 1.003 \pm 0.005 \end{pmatrix}. \qquad (3.65)$$

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[5]We report here the result obtained in Antusch *et al.* (2006), improved by considering the most recent *BABAR* bounds on the radiative τ decays shown in Table II. In practice, this only affects the limits on $|(\mathbf{N}\,\mathbf{N}^\dagger)_{\tau\mu}|$ and $|(\mathbf{N}\,\mathbf{N}^\dagger)_{\tau e}|$ [see Eq. (3.64)].

In conclusion, data from weak decays provide strong constraints on the unitarity of the lepton mixing matrix, which is satisfied at the percent level. The improvement of the limits on the rare charged-lepton decays will further improve the bounds on leptonic NU effects. Moreover, future precision measurements performed in neutrino oscillation facilities will certainly play a crucial role in testing unitarity in the lepton mixing. It is also worth emphasizing that the above conclusions were drawn taking MUV as a reference framework in the analysis of lepton NU. If one goes beyond this simple scenario and considers particular cases with NU effects due to new physics, then other constraints may arise. For instance, if fermion triplets are added to the SM particle content, as in the type III seesaw mechanism, decay processes like $l_j \rightarrow l_i l_k l_k$ (cf. Table II) or μ-e conversion in nuclei are possible at tree level. Consequently, the constraints imposed on the NU of the lepton mixing matrix become stronger in this case when compared with the MUV ones. In particular, from the present bound on the μ-e conversion rate, one obtains $|(\mathbf{N}\mathbf{N}^\dagger)_{e\mu}| < 1.7 \times 10^{-7}$. Furthermore, the $|(\mathbf{N}\mathbf{N}^\dagger)_{e\tau}|$ and $|(\mathbf{N}\mathbf{N}^\dagger)_{\mu\tau}|$ bounds are also improved down to the level of $\sim 10^{-3}$ when considering the experimental bounds on the $\tau \rightarrow 3l$ rates (Abada *et al.*, 2008).

3. Nonunitarity and leptonic CPV

In analogy with the quark sector, the observation of LCPV would automatically raise the question on whether this signal can be explained within a minimal framework in which the only source of CPV in neutrino oscillations is the Dirac phase δ. This could not be the case if lepton mixing is nonunitary. For instance, in the previously discussed MUV framework, three extra phases in the leptonic mixing matrix $\mathbf{N}$ act as new sources of LCPV. At present, these phases are not bounded by the available neutrino oscillation and electroweak data. Although the MUV is a representative scenario of NU in the lepton sector, it has been shown that there is room for considerable new CPV effects even in such a limited framework (Fernandez-Martinez *et al.*, 2007; Altarelli and Meloni, 2009).

Following the notation of Fernandez-Martinez *et al.* (2007), one can parametrize deviations from unitarity by writing $\mathbf{N} = (\mathbb{1} + \boldsymbol{\eta})\mathbf{U}$, where $\boldsymbol{\eta}$ is a Hermitian matrix containing nine new parameters (six moduli and three phases). The bounds on $\eta_{\alpha\beta}$ can be easily obtained from the ones on $\mathbf{N}\mathbf{N}^\dagger$ considering that $(\mathbf{N}\mathbf{N}^\dagger)_{\alpha\beta} \simeq \delta_{\alpha\beta} + 2\eta_{\alpha\beta}$ (Fernandez-Martinez *et al.*, 2007). The main question is then how much room do these possible deviations from unitarity leave for the observation of nonstandard *CP* violation in neutrino oscillations. In order to understand this, one has to write the transition probabilities $P_{\alpha\beta}$ and *CP* asymmetries $A_{\alpha\beta} \equiv (P_{\alpha\beta} - \bar{P}_{\alpha\beta})/(P_{\alpha\beta} + \bar{P}_{\alpha\beta})$ in the MUV framework (Fernandez-Martinez *et al.*, 2007; Goswami and Ota, 2008; Altarelli and Meloni, 2009), which will receive new contributions from $\boldsymbol{\eta}_{\alpha\beta} \equiv \eta_{\alpha\beta} e^{i\theta_{\alpha\beta}}$, where $\theta_{\alpha\beta}$ are the new *CP*-violating phases.

In the MUV framework, the golden channel asymmetries $A_{e\mu}$ do not deviate significantly from the standard unitary case due to the strong bounds on $\eta_{e\mu}$. Since the new physics effects are already constrained to be small in this case, the above channel is probably the most appropriate for a clean determination of lepton mixing parameters. On the other hand, the transition probabilities and their corresponding asymmetries for the remaining oscillation channels may be considerably affected by new physics effects. For instance, for $\mu\tau$ oscillations (Fernandez-Martinez *et al.*, 2007; Altarelli and Meloni, 2009)

$$A_{\mu\tau} \simeq A_{\mu\tau}^{\text{SM}} - 4\eta_{\mu\tau} \cot\Delta_{31} \sin\delta_{\mu\tau}, \qquad (3.66)$$

where $A_{\mu\tau}^{\text{SM}}$ is the *CP* asymmetry in the standard unitary scenario, which is typically $\mathcal{O}(10^{-3})$, while the new contribution proportional to $\eta_{\tau\mu}$ can be as large as $\sim 10^{-1}$. In Fig. 15, we show the behavior of the *CP* asymmetries $A_{\alpha\beta}$ as a function of the Dirac *CP* phase δ [panels (a) to (c)] for $s_{13} = 0.1$ and several experimental setups (see the figure caption for more details). The parameters $\eta_{\alpha\beta}$ are varied in their allowed intervals and the phases $\delta_{\alpha\beta}$ are kept free. From Fig. 15(a) it is apparent the small impact of the new physics effects on $A_{e\mu}$ in the MUV framework. One should, however, keep in mind that in a more general picture with other new physics effects, the deviations with respect to the standard unitary scenario could be more significant. As for the $\mu\tau$ and $e\tau$ asymmetries, the NU effects can be quite dramatic, as illustrated in Figs. 15(b) and 15(c), where the solid lines indicate the result in the unitary case for which $\eta_{\alpha\beta} = 0$. This analysis shows that the new physics effects are more pronounced for the facilities with the smallest L/E, which makes neutrino factories with small baselines and large E more appropriate for the detection of new physics effects in $\nu_\mu \rightarrow \nu_\tau$ (Goswami and Ota, 2008).

The standard unitary picture for LCPV would be automatically disproved in case one or more asymmetries are not compatible with their bounds. If indeed the neutrino mixing and LCPV patterns are described by a unitary matrix, then the trajectory spanned by a pair of asymmetries is a well-defined line which is obtained by varying the value of δ. Therefore, in the standard unitary scenarios, any pair of measured asymmetries should fall in the corresponding line. Once one considers the MUV framework, the allowed space is enlarged outside these lines. This is shown in Fig. 15(d), where $A_{e\tau}$ is plotted against $A_{e\mu}$ (the least affected asymmetry), varying the MUV parameters in their allowed ranges. From this plot, one clearly distinguishes the closed line which corresponds to the case in which $\eta_{\alpha\beta} = 0$. Moreover, it is clear that the deviations to the standard unitary limit allowed by the present bounds on the MUV parameters are quite significant. One should also keep in mind that these results have been obtained in the MUV scenario, in which the new physics effects are pretty much constrained. Larger deviations to the standard unitary case could be observed in other frameworks with a wider allowed range for $A_{e\mu}$. Moreover, one should also take into account the experimental accuracy in the determination of the asymmetries, and the impact of the degeneracies discussed in Sec. III.B.3, which can make the task of testing the standard LCPV framework more difficult (Fernandez-Martinez *et al.*, 2007; Goswami and Ota, 2008; Altarelli and Meloni, 2009). In particular, it has been shown that deviations from the standard picture of LCPV could be established with a modest precision, when considering the
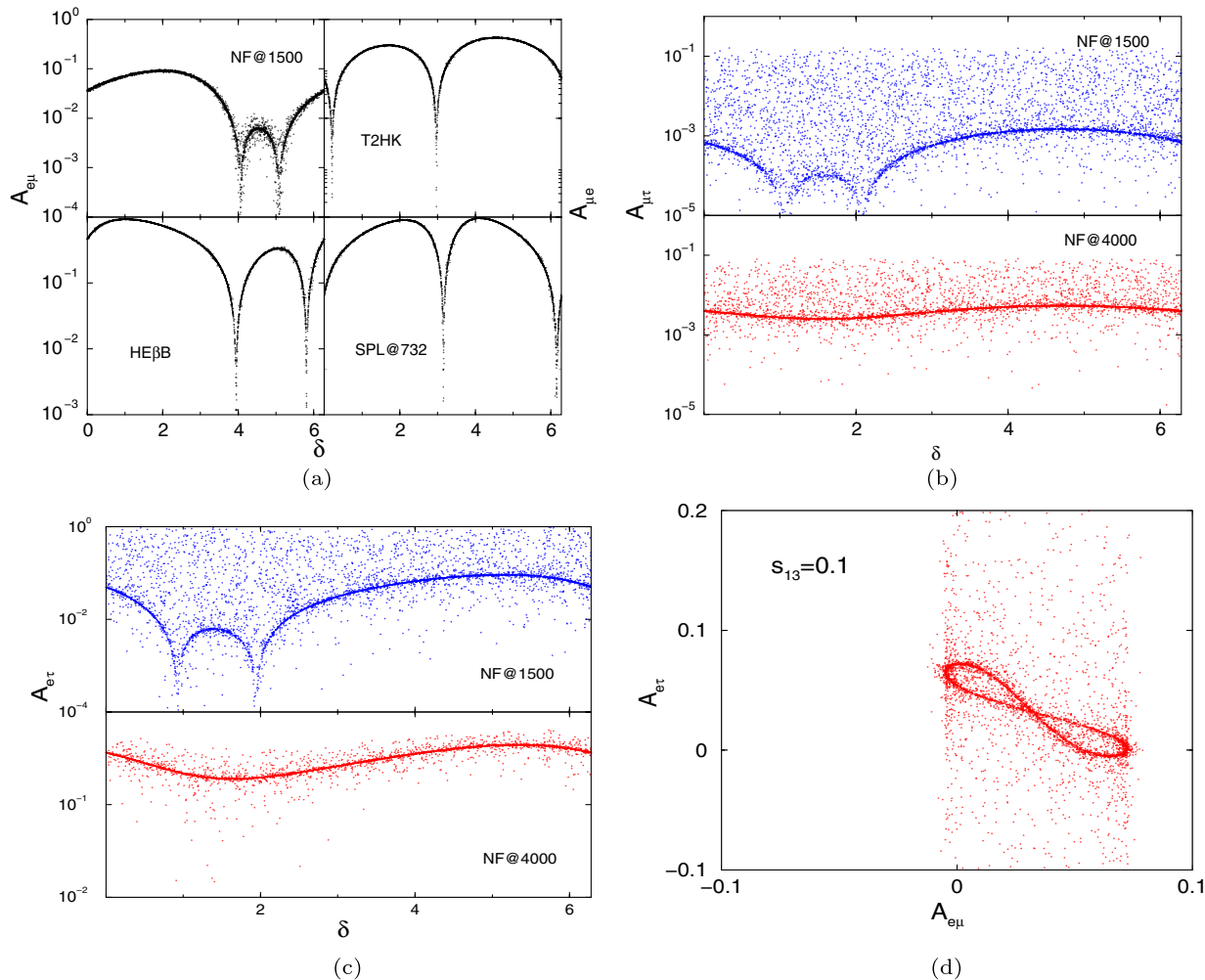
FIG. 15 (color online). (a) to (c) Scatter plots for $|A_{e\mu}|$, $|A_{\mu\tau}|$ and $|A_{e\tau}|$, respectively, as a function of the Dirac phase δ. The neutrino parameters are fixed at $s_{13} = 0.1$, $s_{12}^2 = 1/3$, $\theta_{23} = \pi/4$, $\Delta m_{21}^2 = 8 \times 10^{-5}$ eV^2, and $\Delta m_{31}^2 = 2.4 \times 10^{-3}$ eV^2. The results are presented considering several experimental setups, namely, HEβB (high-energy beta beam with $E = 1$ GeV and $L = 732$ km), the upgraded T2K, T2HK ($E = 0.75$ GeV and $L = 295$ km), the CERN superbeam project SPL ($E = 0.3$ GeV and $L = 130$ or 732 km), and neutrino factories (NF@L) with $E = 35$ and 30 GeV in panels (b) and (c), respectively. (d) $A_{e\tau}$ as a function of $A_{e\mu}$ considering a baseline $L = 1500$ km and $E = 30$ GeV. The neutrino parameters are the same as in the previous panels. In all cases, the MUV parameters are varied in their allowed ranges and the solid lines correspond to the standard unitary limit. From Altarelli and Meloni, 2009.

uncertainties on the $A_{\alpha\beta}$ asymmetries. This has been confirmed for a particular NF setup with detectors at $L = 1500$ km (Altarelli and Meloni, 2009) and $E = 50$ GeV.

IV. LEPTONIC *CP* VIOLATION AND THE ORIGIN OF MATTER

If we take for granted that inflation (Linde, 2008) took place in the early Universe, any primordial cosmological charge asymmetry would have been exponentially wiped out during the inflationary period. Thus, rather than being an initial accidental state, the observed dominance of matter over antimatter should be dynamically generated. In 1967, more than a decade before inflation was put forward and just three years after the discovery of *CP* violation in the $K_L \rightarrow 2\pi$ decays, Sakharov realized the need for generating the baryon asymmetry through a dynamical mechanism. Three necessary ingredients to create a baryon asymmetry from an

initial state with a baryon number equal to zero were formulated in his work (Sakharov, 1967)[6]: (i) baryon number violation, (ii) *C* and *CP* violation, and (iii) departure from thermal equilibrium.

The need for *B* violation is somehow obvious. If *B* is conserved by the interactions, and our Universe is initially symmetric ($B = 0$), then no baryon production may take place. Indeed, since the baryon number commutes with the Hamiltonian $\mathcal{H}$, i.e., $[B, \mathcal{H}] = 0$, at any time one has $B(t) = \int_0^t [B, \mathcal{H}] dt' = 0$. Thus, if *B* is conserved, the present asymmetry can only reflect asymmetric initial conditions. In grand unified theories, quarks and leptons are unified in the same

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[6]Sakharov did not enunciate these conditions as clearly as they are traditionally presented. The three key assumptions in his seminal paper "Violation of *CP*-invariance, *C* asymmetry, and baryon asymmetry of the Universe" are now known as the Sakharov conditions.

multiplets, thus baryon number violation mediated by gauge bosons and scalars is natural. In the SM, however, the baryon number and the lepton flavor numbers ($L_{e,\mu,\tau}$) are accidentally conserved, and it is not possible to violate these symmetries at any perturbative level. Nevertheless, due to the chiral anomaly, nonperturbative instanton effects may give rise to processes that violate (B + L) while conserving (B-L) ('t Hooft, 1976a; 1976b). Although exponentially suppressed at zero temperature, such configurations, often referred to as sphalerons (Klinkhamer and Manton, 1984), are frequent in the early Universe, at temperatures above the electroweak phase transition (Kuzmin, Rubakov, and Shaposhnikov, 1985).

The second Sakharov condition, namely, the violation of *C* and *CP* symmetries, is more subtle. The baryon number operator,

$$\hat{B} = \frac{1}{3}\sum_i \int d^3x \,{:}\psi_i^\dagger(\mathbf{x},t)\psi_i(\mathbf{x},t){:}, \tag{4.1}$$

where $\psi_i(\mathbf{x},t)$ denotes the quark field of flavor i and the colons represent the normal ordering, is *C* odd and *CP* odd. This can easily be seen by recalling how the *C*, *P*, and *T* operators act on the quark fields. Using the standard phase convention,

$$\begin{aligned} P\psi_i(\mathbf{x},t)P^{-1} &= \gamma^0\psi_i(-\mathbf{x},t), \\ P\psi_i^\dagger(\mathbf{x},t)P^{-1} &= \psi_i^\dagger(-\mathbf{x},t)\gamma^0, \\ C\psi_i(\mathbf{x},t)C^{-1} &= i\gamma^2\psi_i^\dagger(\mathbf{x},t), \\ C\psi_i^\dagger(\mathbf{x},t)C^{-1} &= i\psi_i(\mathbf{x},t)\gamma^2, \\ T\psi_i(\mathbf{x},t)T^{-1} &= -i\psi_i(\mathbf{x},-t)\gamma_5\gamma^0\gamma^2, \\ T\psi_i^\dagger(\mathbf{x},t)T^{-1} &= -i\gamma^2\gamma^0\gamma_5\psi_i^\dagger(\mathbf{x},-t). \end{aligned} \tag{4.2}$$

Thus,

$$\begin{aligned} P{:}\psi_i^\dagger(\mathbf{x},t)\psi_i(\mathbf{x},t){:}P^{-1} &={:} \psi_i^\dagger(-\mathbf{x},t)\psi_i(-\mathbf{x},t){:}, \\ C{:}\psi_i^\dagger(\mathbf{x},t)\psi_i(\mathbf{x},t){:}C^{-1} &= -{:}\psi_i^\dagger(\mathbf{x},t)\psi_i(\mathbf{x},t){:}, \\ T{:}\psi_i^\dagger(\mathbf{x},t)\psi_i(\mathbf{x},t){:}T^{-1}{:}\psi_i^\dagger(\mathbf{x},-t)\psi_i(\mathbf{x},-t){:}, \end{aligned} \tag{4.3}$$

and one obtains

$$\begin{aligned} C\hat{B}C^{-1} &= -\hat{B}, \qquad (CP)\hat{B}(CP)^{-1} = -\hat{B}, \\ (CPT)\hat{B}(CPT)^{-1} &= -\hat{B}. \end{aligned} \tag{4.4}$$

If *C* is conserved, then $[C,\mathcal{H}] = 0$ and from the time evolution of $\hat{B}$ and Eq. (4.4) one concludes

$$\begin{aligned} \langle\hat{B}(t)\rangle &= \langle e^{i\mathcal{H}t}\hat{B}(0)e^{-i\mathcal{H}t}\rangle \\ &= \langle C^{-1}e^{i\mathcal{H}t}C\hat{B}(0)C^{-1}e^{-i\mathcal{H}t}C\rangle \\ &= -\langle e^{i\mathcal{H}t}\hat{B}(0)e^{-i\mathcal{H}t}\rangle = -\langle\hat{B}(t)\rangle. \end{aligned} \tag{4.5}$$

Therefore, a nonzero expectation value $\langle\hat{B}\rangle$ requires that the Hamiltonian violates *C*. The same arguments apply to the *CP* symmetry.

Finally, the third Sakharov requirement can be understood as follows. In thermal equilibrium, thermal averages are described by the density operator $\rho = \exp(-\beta\mathcal{H})$, with $\beta = 1/T$. If the Hamiltonian is *CPT* invariant, using Eq. (4.4) it then follows

$$\begin{aligned} \langle\hat{B}\rangle_T &= \mathrm{Tr}(e^{-\beta\mathcal{H}}\hat{B}) = \mathrm{Tr}[(CPT)(CPT)^{-1}e^{-\beta\mathcal{H}}\hat{B}] \\ &= \mathrm{Tr}[e^{-\beta\mathcal{H}}(CPT)^{-1}\hat{B}(CPT)] = -\mathrm{Tr}(e^{-\beta H}\hat{B}) \\ &= -\langle\hat{B}\rangle_T, \end{aligned} \tag{4.6}$$

i.e., $\langle\hat{B}\rangle_T = 0$ in thermal equilibrium. In other words, in thermal equilibrium the rate for a given process that produces an excess of baryons is equal to the rate of its corresponding inverse process, so that no net asymmetry can be generated since the inverse process destroys the baryon excess as fast as the direct process creates it. Departure from thermal equilibrium is very common in the early Universe, when interaction rates cannot keep up with the expansion rate. A simple example is provided by the out-of-equilibrium decay of a heavy particle *X* with a mass $M_X > T$ at time of decay. In this case, the rate of the direct process is of order *T*, while the inverse decay rate is Boltzmann suppressed $\sim \exp(-M_X/T)$.

The present value of the baryon asymmetry of the Universe inferred from WMAP 7-year data combined with baryon acoustic oscillations is (Komatsu *et al.*, 2011)

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm 0.15) \times 10^{-10}, \tag{4.7}$$

where n_B, $n_{\bar{B}}$, and n_γ are the number densities of baryons, antibaryons, and photons at present time, respectively.[7] The explanation of such a small but nonzero number poses a challenge to both particle physics and cosmology. It is remarkable that the SM contains the three Sakharov ingredients. Yet not all of them are available in a sufficient amount. The baryon number is violated by the electroweak sphaleron processes, which are fast and unsuppressed in the early Universe. The *C* symmetry is maximally violated by the weak interactions, and *CP* is violated by the CKM phase. Nevertheless, if baryogenesis occurs at the electroweak phase transition scale $T_{\mathrm{ew}} \sim \mathcal{O}(100)$ GeV, the strength of *CP* violation, parametrized in the SM by the invariant $\mathcal{J}_{\mathrm{quark}}^{CP}$ of Eq. (2.24), seems insufficient to generate the required value of η_B. The naive estimate $\mathcal{J}_{\mathrm{quark}}^{CP}/T_{\mathrm{ew}}^{12} \sim 10^{-20}$ indicates that at such temperatures electroweak baryogenesis (Trodden, 1999) requires new sources of *CP* violation.[8] Finally, at the electroweak phase transition departure from thermal equilibrium takes place. However, a successful baryogenesis requires a strongly first order phase transition, which can occur if the Higgs mass is rather light, $m_{\mathrm{Higgs}} \lesssim 70$ GeV. This value is nevertheless well below the present experimental lower bound $m_H > 114.4$ GeV (Nakamura *et al.*, 2010).

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[7]An equivalent definition of the baryon asymmetry is the baryon-to-entropy ratio $Y_B = (n_B - n_{\bar{B}})/s$. The two measures are related as $Y_B \approx \eta_B/7.04$.

[8]In the cold electroweak baryogenesis scenarios, where baryogenesis takes place at temperatures well below T_{ew}, the strength of *CP* violation in the SM may be enough to account for the observed η_B (Garcia-Bellido *et al.*, 1999; Krauss and Trodden, 1999; Enqvist *et al.*, 2010; Tranberg *et al.*, 2010).

Thus, the explanation of the baryon asymmetry observed in our Universe requires new physics beyond the SM.

Among the several viable baryogenesis scenarios, leptogenesis (Fukugita and Yanagida, 1986) is undoubtedly one of the simplest, most attractive, and well-motivated mechanisms. Many aspects of leptogenesis have been widely discussed in the literature and there are excellent reviews on the subject [see, for instance, Buchmuller, Di Bari, and Plumacher (2005); Buchmuller, Peccei, and Yanagida (2005); and Davidson, Nardi, and Nir (2008)]. In its simplest realization, new heavy (bosonic or fermionic) particles are introduced in the theory in such a way that the interactions relevant for leptogenesis are simultaneously responsible for the nonvanishing and smallness of the neutrinos masses via the seesaw mechanism. The three Sakharov conditions are naturally fulfilled in this framework: the seesaw mechanism requires lepton-number violation and sphalerons partially convert the lepton asymmetry into a baryon asymmetry; neutrino complex Yukawa couplings provide the necessary source of *CP* violation; and last, departure from thermal equilibrium is guaranteed by the out-of-equilibrium decays of the new heavy particles. It is precisely on these simple thermal leptogenesis scenarios that this section of the review focuses. We do not aim at covering all the theoretical ideas on leptogenesis extensively developed over the last years. It is our goal, instead, to describe the role that leptonic *CP* violation may have played in the origin of matter.

A. Leptogenesis mechanisms

In this section, we briefly review the simplest nonsupersymmetric leptogenesis scenarios based on the seesaw mechanism for neutrino masses. As discussed in Sec. II.E, seesaw models are characterized by the properties of the exchanged heavy particles. In particular, in type I, type II, and type III seesaw mechanisms these particles are SU(3) × SU(2) × U(1)-singlet fermions, SU(2)-triplet scalars, and SU(2)-triplet fermions, respectively. As it turns out, thermal leptogenesis can be successfully implemented in each framework. Yet, in general, specific constraints must be satisfied in order to generate the required value of the baryon asymmetry.

The baryon asymmetry η_B produced by thermal leptogenesis can be obtained by taking into account the suppression factors given by the Sakharov conditions. The final asymmetry is the result of the rivalry between the processes that produce it and the washout processes that tend to erase it. Assuming that after inflation the Universe reheats to a thermal bath composed of particles with gauge interactions, the asymmetry can be estimated as the product of three factors: (the leptonic *CP* asymmetry ϵ produced in heavy particle decays) × (an efficiency factor η due to washout processes in scattering, decays, and inverse decays) × (a reduction factor due to chemical equilibrium, charge conservation, and the redistribution of the asymmetry among different particle species by fast processes). The computation of each of these factors is model dependent. In particular, the calculation of the efficiency factor $\eta (0 \leq \eta \leq 1)$ requires the solution of a full set of Boltzmann equations which describe the out-of-equilibrium dynamics of the processes involving the heavy particles responsible for leptogenesis. Simple analytical

estimates can also be obtained in some specific regimes (Giudice *et al.*, 2004; Buchmuller, Di Bari, and Plumacher, 2005; Abada *et al.*, 2006).

Departure from thermal equilibrium is provided by the expansion of the Universe, characterized by the Hubble expansion rate $H(T) \sim 1.66 g_*^{1/2} T^2/M_P$, where g_* is the number of relativistic degrees of freedom in the thermal bath ($g_* = 106.75$ within the SM) and $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass. Nonequilibrium takes place whenever a crucial interaction rate becomes smaller that H so that it is not fast enough to equilibrate particle distributions. Furthermore, flavor effects can play a significant role in this process. As first discussed in Barbieri *et al.* (2000); Endoh, Morozumi, and Xiong (2004) and more recently emphasized in Pilaftsis and Underwood (2005); Abada, Davidson, Josse-Michaux, Losada (2006); Abada *et al.* (2006); Nardi *et al.* (2006)) when the interactions mediated by the charged-lepton Yukawa couplings are in thermal equilibrium, the flavored leptonic asymmetries and the Boltzmann equations for individual flavor asymmetries must be properly taken into account. Since the time scale for leptogenesis is H^{-1} and the typical interaction rates for the charged-lepton Yukawa couplings y_α are $\Gamma_\alpha \simeq 10^{-2} y_\alpha^2 T$ (Cline, Kainulainen, and Olive, 1994), interactions involving the τ and μ Yukawa couplings are in equilibrium for $T \lesssim 10^{12}$ GeV and $T \lesssim 10^9$ GeV, respectively. Below these temperature scales the corresponding lepton doublets are distinguishable mass eigenstates and, as such, should be properly introduced into the Boltzmann equations.

Since the leptonic *CP* asymmetries are the relevant quantities in establishing a link between leptonic *CP* violation and the matter-antimatter asymmetry, in what follows we discuss these quantities in more detail within each seesaw framework.[9] Readers interested in a more complete understanding of the mechanism of leptogenesis are referred, e.g., to the recent pedagogical review (Davidson, Nardi, and Nir, 2008) and the extensive list of references quoted therein.

1. Type I seesaw leptogenesis

In the type I seesaw framework, at least two singlet fermions must be added to the SM particle content to correctly reproduce the observed neutrino mass square differences. The existence of more than one singlet fermion also turns out to be crucial for the mechanism of thermal leptogenesis. We consider the SM extended by three singlet fermions N_i ($i = 1, 2, 3$) with large Majorana masses M_i. In this case, the relevant Lagrangian interactions terms are given by Eq. (2.33). Working in the mass eigenbasis of the heavy neutrinos N_i and the charged leptons ℓ_α, the *CP* asymmetry ϵ_i^α in the lepton flavor α produced in the N_i decays is given by

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[9]The main conclusions of this section are expected to remain valid also in the minimal supersymmetric extension of each framework. Although new decay channels will enhance the generated *CP* asymmetry, these additional contributions tend to be compensated by the washout processes which are typically stronger than in the nonsupersymmetric case.

$$\epsilon_i^\alpha \equiv \frac{\Gamma(N_i \to \phi \ell_\alpha) - \Gamma(N_i \to \phi^\dagger \bar{\ell}_\alpha)}{\sum_\beta [\Gamma(N_i \to \phi \ell_\beta) + \Gamma(N_i \to \phi^\dagger \bar{\ell}_\beta)]}$$

$$= \frac{1}{8\pi} \frac{1}{\mathbf{H}_{ii}^\nu} \sum_{j \neq i} \{ \mathrm{Im}[\mathbf{Y}_{\alpha i}^{\nu*} \mathbf{H}_{ij}^\nu \mathbf{Y}_{\alpha j}^\nu][f(x_j) + g(x_j)]$$

$$+ \mathrm{Im}[\mathbf{Y}_{\alpha i}^{\nu*} \mathbf{H}_{ji}^\nu \mathbf{Y}_{\alpha j}^\nu] g'(x_j) \}, \tag{4.8}$$

where $\mathbf{H}^\nu \equiv \mathbf{Y}^{\nu\dagger} \mathbf{Y}^\nu$, $x_j = M_j^2/M_i^2$, and

$$f(x) = \sqrt{x}[1 - (1+x)\ln(1+x^{-1})],$$

$$g(x) = \sqrt{x} g'(x) = \frac{\sqrt{x}(1-x)}{(x-1)^2 + a_j^2}, \qquad a_j = \frac{\Gamma_{N_j}}{M_i} \tag{4.9}$$

are the vertex and self-energy one-loop functions, respectively. The quantity Γ_{N_j} denotes the N_j total tree-level decay rate,

$$\Gamma_{N_j} = \frac{\mathbf{H}_{jj}^\nu M_j}{8\pi}. \tag{4.10}$$

The *CP* asymmetry given in Eq. (4.8) arises from the interference of the tree-level and one-loop diagrams depicted in Fig. 16 (Covi, Roulet, and Vissani, 1996). The presence of complex phases in the Yukawa couplings involved as well as nonzero absorptive parts in the loop diagrams are necessary conditions to have a nonvanishing asymmetry. The last diagram in Fig. 16 corresponds to the wave-function corrections. The diagram with an internal ℓ_β is lepton flavor and lepton number violating. On the other hand, the diagram with an internal $\bar{\ell}_\beta$ is lepton flavor violating but lepton number conserving. Thus, it vanishes when summed over the lepton flavors (Covi, Roulet, and Vissani, 1996).

We note that in the self-energy loop functions g and g' of Eq. (4.9) the corrections due to the mixing of nearly degenerate heavy Majorana neutrinos have been included. They are parametrized here through the quantities a_j (Pilaftsis, 1997; Pilaftsis and Underwood, 2004; 2005). Anisimov, Broncano, and Plumacher, 2006 obtained a different regulator of the loop functions in the degenerate limit $M_i \sim M_j$. Instead of a_j^2, the term $(\sqrt{x}a_j - a_j)^2$ was found. Both results agree when $\mathbf{H}_{jj}^\nu \gg \mathbf{H}_{ii}^\nu$. The above corrections become relevant in the so-called resonant leptogenesis scenario (Pilaftsis and Underwood, 2004), i.e., in the limit when the mass splitting between N_i and N_j is comparable with their decay widths.

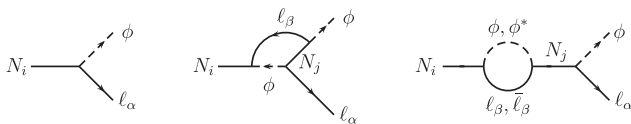Summing over the lepton flavors one recovers the standard result

$$\epsilon_i = \sum_\alpha \epsilon_i^\alpha = \frac{1}{8\pi} \frac{1}{\mathbf{H}_{ii}^\nu} \sum_{j \neq i} \mathrm{Im}[(\mathbf{H}_{ij}^\nu)^2][f(x_j) + g(x_j)]. \tag{4.11}$$

In the so-called N_1-dominated scenario with $M_1 \ll M_j$ ($j = 2, 3$), one has $x_j \gg 1$ and the one-loop functions are approximated by $f(x) \simeq -1/(2\sqrt{x})$, $g(x) \simeq -1/\sqrt{x}$, and $g'(x) \simeq -1/x$. In this case, the flavored asymmetry in Eq. (4.8) becomes

$$\epsilon_1^\alpha \simeq -\frac{3}{16\pi} \frac{1}{\mathbf{H}_{11}^\nu} \sum_{j \neq 1} \frac{M_1}{M_j} \mathrm{Im}[\mathbf{Y}_{\alpha 1}^{\nu*} \mathbf{H}_{1j}^\nu \mathbf{Y}_{\alpha j}^\nu], \tag{4.12}$$

while the unflavored asymmetry (4.11) reads

$$\epsilon_1 \simeq -\frac{3}{16\pi} \frac{1}{\mathbf{H}_{11}^\nu} \sum_{j \neq 1} \frac{M_1}{M_j} \mathrm{Im}[(\mathbf{H}_{1j}^\nu)^2]. \tag{4.13}$$

A remarkable feature of the unflavored asymmetry (4.13) is that it has the upper bound (Davidson and Ibarra, 2002; Hamaguchi, Murayama, and Yanagida, 2002)

$$|\epsilon_1| \lesssim \frac{3}{16\pi} \frac{M_1}{v^2} (m_{\max} - m_{\min})$$

$$\simeq 10^{-6} \left(\frac{M_1}{10^{10}\ \mathrm{GeV}} \right) \left(\frac{m_{\max} - m_{\min}}{m_{\mathrm{atm}}} \right), \tag{4.14}$$

where $v \approx 175$ GeV is the vacuum expectation value of the neutral component of the Higgs doublet; $m_{\max}$ and $m_{\min}$ are the largest and smallest light neutrino masses, respectively; m_{atm} is the atmospheric neutrino mass scale. Moreover, this bound gets more stringent for a quasidegenerate light neutrino spectrum ($m_{\max} \approx m_{\min}$). On the other hand, the asymmetry in a given flavor (4.12) is bounded by (Abada *et al.*, 2006)

$$|\epsilon_1^\alpha| \lesssim \frac{3}{16\pi} \frac{M_1 m_{\max}}{v^2} \sqrt{\frac{\mathbf{Y}_{\alpha 1}^{\nu*} \mathbf{Y}_{\alpha 1}^\nu}{\sum_\beta |\mathbf{Y}_{\beta 1}^\nu|^2}}, \tag{4.15}$$

which goes as the square root of the branching ratio to that flavor and is not suppressed for a degenerate light neutrino spectrum.

From the requirement that leptogenesis successfully reproduces the baryon asymmetry in Eq. (4.7), the bound in Eq. (4.14) leads to two important consequences (Buchmuller, Di Bari, and Plumacher, 2003; 2005; Giudice *et al.*, 2004):

(i) A lower bound on M_1 and the reheating temperature of the Universe, $M_1, T_{\mathrm{reh}} \gtrsim 2 \times 10^9$ GeV.

(ii) An upper bound on the light neutrino mass scale, $m \lesssim 0.15$ eV.

While the bound in (i) is not relaxed with the inclusion of flavor effects (Blanchet and Di Bari, 2007; Josse-Michaux and Abada, 2007), the arguments leading to the bound in (ii) do not apply in the flavored regime.[10] There is presently no

FIG. 16. Diagrams contributing to the *CP* asymmetry ϵ_i^α in type I seesaw leptogenesis. The last diagram corresponds to the wave function corrections: the one with an internal ℓ_β is lepton flavor and lepton-number violating, while the one with an internal $\bar{\ell}_\beta$ is lepton flavor violating but lepton-number conserving, thus giving no contribution to the unflavored *CP* asymmetry.

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[10]In the unflavored regime, the upper bound on the neutrino mass scale can be relaxed if, for instance, the expansion rate of the Universe is modified at the leptogenesis epoch due to brane cosmology (Bento, González Felipe, and Santos, 2006; Okada and Seto, 2006).

consensus on the precise upper bound on the light neutrino mass scale inferred from flavored leptogenesis. Analytical and numerical calculations (De Simone and Riotto, 2007; Josse-Michaux and Abada, 2007) suggest that one can easily saturate the cosmological bound and reach values of m up to 1 eV.

One may wonder whether the bound on M_1 (and T_{reh}) can be evaded without adding new particles or interactions. We recall that this bound applies only for hierarchical heavy neutrinos. For quasidegenerate N_i the leptonic *CP* asymmetries can be much larger than the upper value of Eq. (4.14). In particular, if $x_j - 1 = a_j$ (or, equivalently, $|M_j - M_i| \simeq \frac{1}{2}\Gamma_{N_j}$), the asymmetries ϵ_i^α are resonantly enhanced due to the self-energy contribution. In this case, the loop functions are approximately given by $g'(x) \simeq g(x) \simeq 4\pi/\mathbf{H}_{jj}^\nu$ so that at the resonance

$$\epsilon_{i,\text{res}}^\alpha \simeq -\frac{1}{2}\sum_{j\neq i}\left\{\frac{\text{Im}[\mathbf{Y}_{\alpha i}^{\nu*}\mathbf{H}_{ij}^\nu\mathbf{Y}_{\alpha j}^\nu]}{\mathbf{H}_{ii}^\nu\mathbf{H}_{jj}^\nu} + \frac{\text{Im}[\mathbf{Y}_{\alpha i}^{\nu*}\mathbf{H}_{ji}^\nu\mathbf{Y}_{\alpha j}^\nu]}{\mathbf{H}_{ii}^\nu\mathbf{H}_{jj}^\nu}\right\}$$

$$= -\sum_{j\neq i}\frac{\text{Re}[\mathbf{H}_{ij}^\nu]\,\text{Im}[\mathbf{Y}_{\alpha i}^{\nu*}\mathbf{Y}_{\alpha j}^\nu]}{\mathbf{H}_{ii}^\nu\mathbf{H}_{jj}^\nu}. \qquad (4.16)$$

After summing over the flavors one finds

$$\epsilon_{i,\text{res}} = -\frac{1}{2}\sum_{j\neq i}\frac{\text{Im}[(\mathbf{H}_{ij}^\nu)^2]}{\mathbf{H}_{ii}^\nu\mathbf{H}_{jj}^\nu}. \qquad (4.17)$$

Thus, one concludes that the resonantly enhanced *CP* asymmetry is not suppressed by the light neutrino masses or the heavy Majorana masses; it is just bounded by unitarity, $|\epsilon_i| \leq 1/2$. This in turn implies that leptogenesis can occur at a much lower energy scale.

Although theoretically challenging, it is possible to construct models in which the heavy Majorana neutrino mass splitting is naturally as small as the decay width at the leptogenesis scale. For instance, in the so-called radiative resonant leptogenesis scenario (González Felipe, Joaquim, and Nobre, 2004; Turzynski, 2004; Branco *et al.*, 2006), the required splitting can be generated by the renormalization group running from the GUT scale down to the leptogenesis scale, assuming that the heavy Majorana neutrinos are exactly degenerate at the GUT scale. The assumption of a completely degenerate right-handed neutrino spectrum at the GUT scale is compatible with the solar and atmospheric neutrino oscillation data (González Felipe and Joaquim, 2001). Such a degeneracy can be achieved, for instance, by imposing some discrete or Abelian symmetries (Branco *et al.*, 2006), or in models with minimal lepton flavor violation (Branco *et al.*, 2007; Cirigliano, Isidori, and Porretti, 2007; Cirigliano *et al.*, 2008) as described in Sec. II.G.

2. Type II seesaw leptogenesis

As shown in Sec. II.E, the type II seesaw is very economical in the sense that it has a single source of flavor structure, namely, the symmetric complex Yukawa coupling matrix $\mathbf{Y}^\Delta$ that couples the $SU(2)_L$ scalar triplet Δ to leptons. Furthermore, in its minimal realization, with only one scalar triplet, the flavor pattern of $\mathbf{Y}^\Delta$ uniquely determines the flavor structure of the low-energy effective neutrino mass matrix $\mathbf{m}_\nu$

of Eq. (2.39). There is, however, a drawback with leptogenesis in this minimal setup, namely, the leptonic *CP* asymmetry that is induced by the triplet decays is generated only at higher loops and is highly suppressed. Therefore, new sources for neutrino masses are required to implement thermal leptogenesis in a type II seesaw framework (Ma and Sarkar, 1998; Hambye, Ma, and Sarkar, 2001; D'Ambrosio *et al.*, 2004; Hambye *et al.*, 2004; Hambye and Senjanovic, 2004). These new sources could come, e.g., from other type I, type II, or type III contributions. For illustration, we describe a simple nonsupersymmetric leptogenesis scenario with only two scalar triplets, but other mixed seesaw leptogenesis scenarios are conceivable as well (Antusch and King, 2004; Hambye and Senjanovic, 2004; Hambye, Raidal, and Strumia, 2006). In particular, renormalizable left-right symmetric theories and grand unified models based on SO(10) provide a natural framework for the simultaneous presence of singlet fermions and Higgs triplets.

We consider the SM extended with two scalar triplets $\Delta_i (i = 1, 2)$ of hypercharge $+1$ (in the normalization with hypercharge $-1/2$ for the lepton doublets) and masses M_{Δ_i}. In the $SU(2)$ representation we write

$$\Delta_i = \begin{pmatrix} \Delta_i^0 & -\Delta_i^+/\sqrt{2} \\ -\Delta_i^+/\sqrt{2} & \Delta_i^{++} \end{pmatrix}. \qquad (4.18)$$

The relevant Lagrangian terms are given by Eq. (2.38), which include now the contributions from both scalar triplets,

$$\mathcal{L}_\Delta \ni \sum_i (-\mathbf{Y}_{\alpha\beta}^{\Delta_i}\ell_\alpha^T C\Delta_i\ell_\beta + \mu_i M_{\Delta_i}\tilde{\phi}^T\Delta_i\tilde{\phi} + \text{H.c.})$$

$$-\sum_i M_{\Delta_i}^2\,\text{Tr}(\Delta_i^\dagger\Delta_i), \qquad (4.19)$$

where $\mathbf{Y}^{\Delta_i}$ are symmetric 3×3 complex Yukawa coupling matrices and μ_i are dimensionless complex couplings.

In the presence of *CP*-violating interactions, the decay of Δ_i into two leptons generates a nonvanishing leptonic asymmetry for each triplet component $(\Delta_i^0, \Delta_i^+, \Delta_i^{++})$,

$$\epsilon_i^{\alpha\beta} = \Delta L\frac{\Gamma(\Delta_i^* \to \ell_\alpha + \ell_\beta) - \Gamma(\Delta_i \to \bar{\ell}_\alpha + \bar{\ell}_\beta)}{\Gamma_{\Delta_i} + \Gamma_{\Delta_i^*}}, \qquad (4.20)$$

where Γ_{Δ_i} denotes the total triplet decay width and the overall factor $\Delta L = 2$ arises because the triplet decay produces two leptons. It is useful to define

$$\mathcal{B}_i^\ell\Gamma_{\Delta_i} \equiv \sum_{\alpha,\beta}\Gamma(\Delta_i^* \to \ell_\alpha + \ell_\beta) = \frac{M_{\Delta_i}}{8\pi}\,\text{Tr}(\mathbf{Y}^{\Delta_i\dagger}\mathbf{Y}^{\Delta_i}),$$

$$\mathcal{B}_i^\phi\Gamma_{\Delta_i} \equiv \Gamma(\Delta_i^* \to \phi + \phi) = \frac{M_{\Delta_i}}{8\pi}|\mu_i|^2, \qquad (4.21)$$

where $\mathcal{B}_i^\ell \equiv \text{BR}(\Delta_i^* \to \ell + \ell)$ and $\mathcal{B}_i^\phi \equiv \text{BR}(\Delta_i^* \to \phi + \phi)$ are the tree-level branching ratios to leptons and Higgs doublets, respectively, $(\mathcal{B}_i^\ell + \mathcal{B}_i^\phi = 1)$. The total triplet decay width is then given by

$$\Gamma_{\Delta_i} = \frac{M_{\Delta_i}}{8\pi}[\text{Tr}(\mathbf{Y}^{\Delta_i\dagger}\mathbf{Y}^{\Delta_i}) + |\mu_i|^2]. \qquad (4.22)$$

When the triplet decays into leptons with given flavors ℓ_α and ℓ_β, a nonvanishing asymmetry $\epsilon_i^{\alpha\beta}$ is generated by the
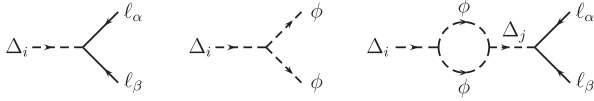
FIG. 17. Tree-level diagrams for the scalar triplet decays and one-loop diagram contributing to the *CP* asymmetry $\epsilon_i^{\alpha\beta}$ in type II seesaw leptogenesis.

interference of the tree-level decay process with the one-loop self-energy diagram shown in Fig. 17. One finds

$$\epsilon_i^{\alpha\beta} \simeq -\frac{g(x_j)}{2\pi} \frac{c_{\alpha\beta} \, \mathrm{Im}[\mu_i^* \mu_j \mathbf{Y}_{\alpha\beta}^{\Delta_i} \mathbf{Y}_{\alpha\beta}^{\Delta_j*}]}{\mathrm{Tr}(\mathbf{Y}^{\Delta_i\dagger}\mathbf{Y}^{\Delta_i}) + |\mu_i|^2} \qquad (j \neq i)$$

(4.23)

where $c_{\alpha\beta} = 2 - \delta_{\alpha\beta}$ for Δ_i^0 and Δ_i^{++}, $c_{\alpha\beta} = 1$ for Δ_i^+; $x_j = M_{\Delta_j}^2/M_{\Delta_i}^2$, and the loop function $g(x)$ is defined in Eq. (4.9), with the parameter a_j now given by $a_j = \Gamma_{\Delta_j}/M_{\Delta_j}$.

Recalling that in the type II seesaw framework under discussion the effective light neutrino mass matrix is

$$\mathbf{m}_\nu = \mathbf{m}_\nu^{(1)} + \mathbf{m}_\nu^{(2)}, \qquad \mathbf{m}_\nu^{(i)} = 2\mu_i^* \frac{v^2}{M_{\Delta_i}} \mathbf{Y}^{\Delta_i}, \quad (4.24)$$

and using

$$16\pi v^2 \Gamma_{\Delta_i}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2} = M_{\Delta_i}^2 [\mathrm{Tr}(\mathbf{m}_\nu^{(i)\dagger}\mathbf{m}_\nu^{(i)})]^{1/2}, \quad (4.25)$$

Eq. (4.23) can be recast in the more convenient form

$$\begin{aligned}
\epsilon_i^{\alpha\beta} &\simeq -\frac{g(x_j)}{4\pi} \frac{M_{\Delta_j}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2}}{v^2} \frac{c_{\alpha\beta} \, \mathrm{Im}[(\mathbf{m}_\nu^{(i)})_{\alpha\beta}(\mathbf{m}_\nu^{(j)})_{\alpha\beta}^*]}{[\mathrm{Tr}(\mathbf{m}_\nu^{(i)\dagger}\mathbf{m}_\nu^{(i)})]^{1/2}} \\
&= -\frac{g(x_j)}{4\pi} \frac{M_{\Delta_j}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2}}{v^2} \frac{c_{\alpha\beta} \, \mathrm{Im}[(\mathbf{m}_\nu^{(i)})_{\alpha\beta}(\mathbf{m}_\nu^*)_{\alpha\beta}]}{[\mathrm{Tr}(\mathbf{m}_\nu^{(i)\dagger}\mathbf{m}_\nu^{(i)})]^{1/2}}.
\end{aligned}$$

(4.26)

In the hierarchical limit $M_{\Delta_i} \ll M_{\Delta_j}$, Eq. (4.26) reduces to

$$\epsilon_i^{\alpha\beta} \simeq \frac{M_{\Delta_i}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2}}{4\pi v^2} \frac{c_{\alpha\beta} \mathrm{Im}[(\mathbf{m}_\nu^{(i)})_{\alpha\beta}(\mathbf{m}_\nu^*)_{\alpha\beta}]}{[\mathrm{Tr}(\mathbf{m}_\nu^{(i)\dagger}\mathbf{m}_\nu^{(i)})]^{1/2}}. \quad (4.27)$$

Summing over the final lepton flavors, Eq. (4.27) leads to the following expression for the unflavored asymmetry (Dorsner, Fileviez Perez, and González Felipe, 2006; Hambye, Raidal, and Strumia, 2006):

$$\epsilon_i = \sum_{\alpha,\beta} \epsilon_i^{\alpha\beta} = \frac{M_{\Delta_i}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2}}{4\pi v^2} \frac{\mathrm{Im}[\mathrm{Tr}(\mathbf{m}_\nu^{(i)}\mathbf{m}_\nu^\dagger)]}{[\mathrm{Tr}(\mathbf{m}_\nu^{(i)\dagger}\mathbf{m}_\nu^{(i)})]^{1/2}}.$$

(4.28)

It is then straightforward to show that the following upper bound holds (Hambye, Raidal, and Strumia, 2006):

$$\begin{aligned}
|\epsilon_i| &\leq \frac{M_{\Delta_i}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2}}{4\pi v^2} [\mathrm{Tr}(\mathbf{m}_\nu^\dagger \mathbf{m}_\nu)]^{1/2} \\
&= \frac{M_{\Delta_i}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2}}{4\pi v^2} \left(\sum_k m_k^2\right)^{1/2}.
\end{aligned}$$

(4.29)

Thus, unlike the type I seesaw case, the upper bound on the asymmetry increases as the light neutrino mass scale increases. For hierarchical light neutrinos one obtains

$$|\epsilon_i| \lesssim 10^{-6}(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2} \left(\frac{M_{\Delta_i}}{10^{10} \, \mathrm{GeV}}\right)\left(\frac{m_\mathrm{atm}}{0.05 \, \mathrm{eV}}\right). \quad (4.30)$$

We remark that, although the absolute maximum in Eqs. (4.29) and (4.30) is attained when $\mathcal{B}_i^\ell = \mathcal{B}_i^\phi = 1/2$, this situation does not necessarily correspond to a maximal baryon asymmetry. The efficiency of leptogenesis, dictated by the solution of the relevant Boltzmann equations, is not necessarily maximal in such a case. In fact, it turns out that the efficiency is minimal for $\mathcal{B}_i^\ell = \mathcal{B}_i^\phi = 1/2$ and maximal when either $\mathcal{B}_i^\ell \ll \mathcal{B}_i^\phi$ or $\mathcal{B}_i^\ell \gg \mathcal{B}_i^\phi$ (Hambye, Raidal, and Strumia, 2006). Consequently, in the limits when the efficiency is maximal the leptonic *CP* asymmetry is suppressed.

A major difference between type I and type II seesaw leptogenesis scenarios is that, unlike the singlet Majorana neutrinos, the scalar triplets couple to the SM gauge bosons. Since gauge interactions keep the triplets close to thermal equilibrium at temperatures $T \lesssim 10^{15}$ GeV, it may seem difficult to fulfill the third Sakharov condition. Nevertheless, estimates of the thermal leptogenesis efficiency (Hambye, Ma, and Sarkar, 2001; Hambye and Senjanovic, 2004) as well as a more precise calculation of it by solving the full set of Boltzmann equations (Hambye, Raidal, and Strumia, 2006) indicate that leptogenesis is efficient even at a much lower temperature. For hierarchical scalar triplets and in the absence of extra sources of *CP* violation, leptogenesis is efficient for $M_{\Delta_i} \gtrsim 10^9$ GeV.

If the scalar triplets are quasidegenerate in mass, the leptonic asymmetry can be resonantly enhanced provided that $|M_{\Delta_j} - M_{\Delta_i}| \sim 1/2\Gamma_{\Delta_j}$. In this case, from Eq. (4.26) one obtains

$$\epsilon_i^{\alpha\beta} \simeq \frac{(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2} c_{\alpha\beta} \, \mathrm{Im}[(\mathbf{m}_\nu^{(i)})_{\alpha\beta}(\mathbf{m}_\nu^{(j)})_{\alpha\beta}^*]}{[\mathrm{Tr}(\mathbf{m}_\nu^{(i)\dagger}\mathbf{m}_\nu^{(i)})]^{1/2}[\mathrm{Tr}(\mathbf{m}_\nu^{(j)\dagger}\mathbf{m}_\nu^{(j)})]^{1/2}}, \quad (4.31)$$

which, after summing over the lepton flavors, yields

$$\epsilon_{i,\mathrm{res}} \simeq \frac{(\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2} \, \mathrm{Im}[\mathrm{Tr}(\mathbf{m}_\nu^{(i)}\mathbf{m}_\nu^{(j)\dagger})]}{[\mathrm{Tr}(\mathbf{m}_\nu^{(i)\dagger}\mathbf{m}_\nu^{(i)})]^{1/2}[\mathrm{Tr}(\mathbf{m}_\nu^{(j)\dagger}\mathbf{m}_\nu^{(j)})]^{1/2}}. \quad (4.32)$$

This leads to the upper bound $|\epsilon_{i,\mathrm{res}}| \lesssim (\mathcal{B}_i^\ell \mathcal{B}_i^\phi)^{1/2}$, which is suppressed by neither the light neutrino masses nor the scalar triplet masses [it is just bounded by the unitarity constraint $|\epsilon_i| < 2\min(\mathcal{B}_i^\ell, \mathcal{B}_i^\phi)$]. This opens the possibility for type II seesaw leptogenesis scenarios at the TeV scale. We note, however, that in the latter case there is a dependence on M_{Δ_i} that strongly suppresses the leptogenesis efficiency when $M_{\Delta_i} \sim \mathcal{O}(\mathrm{TeV})$. Moreover, the final baryon asymmetry crucially depends on the triplet annihilation rate in the nonrelativistic limit, which is affected by nonperturbative corrections to the *s*-wave coefficient that reduce further the leptogenesis efficiency by about 30% (Strumia, 2009). Since after the electroweak symmetry breaking, at temperatures $T \lesssim m_\mathrm{Higgs}$, sphaleron interactions are suppressed and no longer can convert the lepton asymmetry into a baryon asymmetry, a stringent lower bound on the triplet mass is obtained. To successfully reproduce the observed baryon

asymmetry, a triplet mass $M_{\Delta_i} \gtrsim 1.6$ TeV is required (Strumia, 2009), which is too heavy to give detectable effects at the LHC (Nath *et al.*, 2010).

3. Type III seesaw leptogenesis

As explained in Sec. II.E, light neutrino masses can also be mediated by the tree-level exchange of $SU(2)$-triplet fermions with zero hypercharge. Such triplets naturally arise in theories based on grand unification, e.g., when the adjoint 24_F fermion representation is introduced in $SU(5)$, and their masses could be low enough to be accessible at the LHC (Bajc and Senjanović, 2007). Apart from the kinetic term, the type III seesaw Lagrangian has the same structure as in the type I seesaw case, but with different contractions of the $SU(2)$ indices in the Yukawa interaction terms [cf. Eq. (2.41)]. Thus, in what concerns neutrino masses, the type I and type III seesaw mechanisms share the same qualitative features. Yet, there are a few differences in the implementation of leptogenesis that are worth mentioning. First, in the *CP* asymmetry generated by the triplet fermion decay, the relative sign between the vertex and self-energy contributions is opposite to that of the type I seesaw case. Therefore, for a hierarchical triplet spectrum, the asymmetry turns out to be 3 times smaller than in the singlet fermion case. Nevertheless, this is compensated by the fact that the triplet has three components and, consequently, the final baryon asymmetry is 3 times bigger. Second, fermion triplets have gauge interactions which tend to keep them close to thermal equilibrium and reduce the efficiency of leptogenesis (Hambye *et al.*, 2004).

Since all of the conclusions previously drawn for type I seesaw leptogenesis essentially remain valid in the present case, we briefly comment next on the main differences. Considering the type III seesaw Lagrangian of Eq. (2.41) with three fermion triplets Σ_i ($i = 1, 2, 3$), the *CP* asymmetry generated in the decays of Σ_i into a lepton ℓ_α and the Higgs ϕ comes from the interference of the tree-level and one-loop graphs depicted in Fig. 18. It differs from its analogous of the type I case [cf. Eq. (4.8)] in the overall sign of the vertex contribution, and the substitutions $M_i \to M_{\Sigma_i}$, $\mathbf{Y}^\nu \to \mathbf{Y}^\Sigma$, $\mathbf{H}^\nu \to \mathbf{H}^\Sigma$ and $\Gamma_{N_i} \to \Gamma_{\Sigma_i}$. Thus, in a Σ_1-dominated scenario with hierarchical fermion triplets, $M_{\Sigma_1} \ll M_{\Sigma_j}$ ($j = 2, 3$), instead of the usual $f(x) + g(x) \simeq -3/(2\sqrt{x})$ factor, the factor $g(x) - f(x) \simeq -1/(2\sqrt{x})$ appears. This means, in particular, that the right-hand sides in Eqs. (4.12), (4.13), (4.14), and (4.15) get reduced by a factor of 3. On the other hand, the resonant asymmetries in Eqs. (4.16) and (4.17) remain
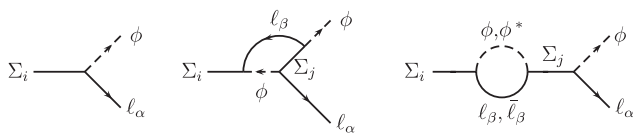
unaltered. As in the type II seesaw case, gauge interactions play a crucial role in the efficiency of the type III leptogenesis scenario. Assuming a hierarchical triplet mass spectrum and neglecting flavor effects, leptogenesis can succeed if $M_{\Sigma_1} \gtrsim 1.5 \times 10^{10}$ GeV and the neutrino mass scale is $m \lesssim 0.12$ eV (Hambye *et al.*, 2004). These bounds are slightly stronger than in type I leptogenesis. On the other hand, if leptogenesis occurs at the TeV scale, the correct amount of baryon asymmetry can only be generated for $M_{\Sigma_i} \gtrsim 1.6$ TeV (Strumia, 2009), which is too large to be within the energy reach of the LHC (Nath *et al.*, 2010). Accounting for flavor effects does not weaken this bound (Aristizabal Sierra, Kamenik, and Nemevsek, 2010).

4. Dirac leptogenesis

All the leptogenesis scenarios discussed in this section are based on the seesaw mechanism, which gives Majorana masses to the light neutrinos. Although well motivated from a theoretical and phenomenological viewpoint, this is not the only possibility to explain neutrino masses. Indeed, neutrinos could be Dirac particles and lepton number may not be violated at the perturbative level. It is therefore pertinent to ask whether leptogenesis can be implemented in such a framework. As it turns out, models with Dirac neutrinos and viable leptogenesis can be constructed as well (Akhmedov, Rubakov, and Smirnov, 1998; Dick *et al.*, 2000; Murayama and Pierce, 2002). The main idea behind the Dirac leptogenesis scenarios can be understood as follows. Suppose that the *CP*-violating decay of a heavy particle produces a nonzero lepton number $L < 0$ ($-L > 0$) for left-handed (right-handed) particles. Since the Yukawa interactions of the SM are fast enough, they rapidly equilibrate the left-handed and right-handed particles so that L goes to zero. However, this does not apply to Dirac neutrinos, which have Yukawa couplings exceedingly small, $y_\nu \lesssim \mathcal{O}(1 \text{ eV})/v \sim 10^{-11}$. For them, the equilibrium between the lepton number stored in each chirality occurs when $\Gamma_\nu/H \simeq y_\nu^2 M_P/T \gtrsim 1$, i.e., at a temperature which is far below the electroweak scale. Thus, by the time L equilibration takes place the left-handed lepton number has already been partially converted into a net baryon number by the sphalerons, leading to a universe with $B = L > 0$. Clearly, one of the consequences of Dirac leptogenesis is the absence of any signal in $0\nu\beta\beta$ decay searches.

B. Leptonic *CP* violation from high to low energies

One of the distinctive features of the leptogenesis mechanisms described in the previous section is the fact that the interactions relevant for leptogenesis can simultaneously be responsible for the nonvanishing and smallness of the neutrinos masses. This raises the question of whether there is a direct link between leptogenesis and low-energy leptonic observables. More specifically, if the strength of *CP* violation at low energies in neutrino oscillations is measured, what can one infer about the viability or nonviability of leptogenesis? From the sign of the baryon asymmetry, can one predict the sign of the *CP* asymmetries in neutrino oscillations, namely, the sign of the low-energy *CP* invariant $\mathcal{J}_{\text{lepton}}^{CP}$? Is there any connection between leptogenesis and the low-energy

FIG. 18. Diagrams contributing to the *CP* asymmetry ϵ_i^α in type III seesaw leptogenesis. As in the type I seesaw case, the last diagram involves two graphs, one which is lepton flavor and lepton-number violating and another which is lepton flavor violating but does not give contribution to the unflavored *CP* asymmetry.

Majorana phases measurable in $0\nu\beta\beta$ decay? The answers to these questions are, however, not straightforward.

In general, the seesaw framework contains many more (unconstrained) parameters than measurable quantities at low energies. We recall that, apart from the three charged-lepton masses, the lepton sector contains nine parameters: the three light neutrino masses plus the three mixing angles and three *CP*-violating phases contained in the PMNS leptonic mixing matrix $\mathbf{U}$. Only four of these nine parameters have been measured: the mass-squared differences (Δm_{21}^2, Δm_{31}^2) and two mixing angles (θ_{12}, θ_{23}). The lightest neutrino mass and the Dirac and Majorana phases in $\mathbf{U}$ are unknown. But even if these unknown parameters would be measured, and a partial correspondence with the leptonic sector at high energies could be established, there remain several high-energy free parameters which are not accessible to experiments. Some of the latter are relevant for leptogenesis. Consequently, any connection between leptogenesis and low-energy leptonic observables can only be found in a model-dependent way (Buchmuller and Plumacher, 1996; Branco *et al.*, 2001; Branco *et al.*, 2002). In particular, thermal leptogenesis can be unsuccessful despite the presence of low-energy leptonic *CP* violation. Conversely, leptogenesis can take place even without Dirac and/or Majorana phases at low energies (Branco *et al.*, 2002; Rebelo, 2003).

In this section we discuss some general aspects of the interplay between the leptonic *CP* violation responsible for leptogenesis at high energies and the one measurable at low energies, which originates from the leptonic mixing matrix $\mathbf{U}$. Our aim is to analyze some simple cases in which such a link can exist and manifest itself through the leptonic *CP* asymmetries. We restrict our discussion to the type I seesaw leptogenesis scenario. All the conclusions will be equally valid for the type III seesaw case (with some obvious changes in the notation). Other scenarios, in which the connection can be established taking into account not only the leptonic asymmetry but also the effects that affect the efficiency of leptogenesis (e.g., charged-lepton flavor effects), will be briefly commented on at the end of Sec. IV.B.2.

In order to address the above questions in a type I seesaw framework, one should keep in mind that, in the mass eigenbasis of the charged leptons and heavy Majorana neutrinos, all the information about the leptonic mixing and *CP* violation is contained in the Dirac-neutrino Yukawa coupling matrix $\mathbf{Y}^\nu$. It then becomes clear that any bridge between high-energy and low-energy *CP* violation can only be established for specific choices of this matrix. Below we describe a few possibilities.

1. Triangular parametrization

It can be easily shown that any arbitrary complex matrix can be written as the product of a unitary matrix $\mathbf{V}$ and a lower triangular matrix $\mathbf{Y}_\Delta$ (Morozumi *et al.*, 1997). In particular, the Dirac-neutrino Yukawa coupling matrix can be written as

$$\mathbf{Y}^\nu = \mathbf{V}\mathbf{Y}_\Delta, \qquad \mathbf{Y}_\Delta = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21}e^{i\beta_1} & y_{22} & 0 \\ y_{31}e^{i\beta_2} & y_{32}e^{i\beta_3} & y_{33} \end{pmatrix}, \quad (4.33)$$

where y_{ij} are real positive numbers. Since $\mathbf{V}$ is unitary, in general it contains six phases. However, three of these phases can be rephased away by a simultaneous phase transformation on the left-handed fields ℓ, which leaves the leptonic charged-current invariant. Furthermore, $\mathbf{Y}_\Delta$ defined in Eq. (4.33) can be rewritten in the form

$$\mathbf{Y}_\Delta = \mathbf{P}_\beta^\dagger \hat{\mathbf{Y}}_\Delta \mathbf{P}_\beta, \qquad (4.34)$$

where $\mathbf{P}_\beta = \mathrm{diag}(1, e^{-i\beta_1}, e^{-i\beta_2})$ and

$$\hat{\mathbf{Y}}_\Delta = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} & y_{22} & 0 \\ y_{31} & y_{32}e^{i\sigma} & y_{33} \end{pmatrix}, \qquad (4.35)$$

with $\sigma = \beta_3 - \beta_2 + \beta_1$. It follows then from Eqs. (4.33) and (4.34) that the matrix $\mathbf{Y}^\nu$ can be decomposed as

$$\mathbf{Y}^\nu = \mathbf{U}_\rho \mathbf{P}_\alpha \hat{\mathbf{Y}}_\Delta \mathbf{P}_\beta, \qquad (4.36)$$

where $\mathbf{P}_\alpha = \mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2})$ and $\mathbf{U}_\rho$ is a unitary matrix containing only one phase ρ. Therefore, in the mass eigenbasis of the charged leptons and heavy Majorana neutrinos, the phases ρ, α_1, α_2, σ, β_1, and β_2 are the only physical phases characterizing *CP* violation in the lepton sector.

The triangular parametrization given in Eq. (4.36) is in general not suitable to disentangle the phases appearing in the flavored leptogenesis asymmetries of Eq. (4.8), which depend on the quantities $\mathrm{Im}[\mathbf{Y}_{\alpha i}^{\nu*}\mathbf{H}_{ij}^\nu\mathbf{Y}_{\alpha j}^\nu]$. Nevertheless, for the unflavored leptogenesis asymmetry in Eq. (4.11), the relevant phases are only those contained in the matrix $\mathbf{H}^\nu = \mathbf{Y}^{\nu\dagger}\mathbf{Y}^\nu$. From Eqs. (4.33), (4.34), (4.35), and (4.36), we then conclude that these phases are σ, β_1, and β_2. Since the phases α_1, α_2, and ρ do not contribute to leptogenesis, and all six phases of $\mathbf{Y}^\nu$ are present in the leptonic mixing matrix $\mathbf{U}$, it is clear that a necessary condition for a direct link between the unflavored leptogenesis asymmetry and low-energy *CP* violation is the requirement that the matrix $\mathbf{V}$ in Eq. (4.33) contains no *CP*-violating phases. We note that, although the above condition was derived in a specific weak basis, and using the parametrization of Eq. (4.33), it can be applied to any model. A specific class of models which satisfy the above necessary condition in a trivial way are those for which $\mathbf{V} = \mathbb{1}$, leading to $\mathbf{Y}^\nu = \mathbf{Y}_\Delta$ (Branco *et al.*, 2003). This condition is necessary but not sufficient to allow for a prediction of the sign of the *CP* asymmetry in neutrino oscillations, given the observed sign of the baryon asymmetry and the low-energy neutrino data. A more restrictive class of matrices $\mathbf{Y}^\nu$ should be considered (Branco *et al.*, 2003). Below we illustrate the possibility of a direct link between leptogenesis and low-energy *CP* violation with a simple example.

We consider an N_1-dominated scenario with $M_1 \ll M_{2,3}$. Assuming that $y_{31} = 0$ and $\beta_3 = 0$, the matrix $\mathbf{Y}^\nu$ in Eq. (4.33) has the simple zero-texture structure[11]

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[11]Approximate texture zeros commonly arise in flavor model constructions based on the Froggatt-Nielsen mechanism (Froggatt and Nielsen, 1979).

$$\mathbf{Y}^\nu = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21}e^{i\beta_1} & y_{22} & 0 \\ 0 & y_{32} & y_{33} \end{pmatrix}, \tag{4.37}$$

so that $\mathrm{Im}[\mathbf{Y}^{\nu*}_{\mu 1}\mathbf{H}^\nu_{12}\mathbf{Y}^\nu_{\mu 2}]$ and $\mathrm{Im}[(\mathbf{H}^\nu_{12})^2]$ are the only nonvanishing quantities in the flavored and unflavored *CP* asymmetries of Eqs. (4.12) and (4.13), respectively. One obtains

$$\epsilon_1^\mu \simeq \frac{3}{16\pi}\frac{M_1}{M_2}\frac{y_{21}^2 y_{22}^2}{y_{11}^2 + y_{21}^2}\sin(2\beta_1), \qquad \epsilon_1^e = \epsilon_1^\tau = 0, \tag{4.38}$$

and summing over the flavors, $\epsilon_1 = \epsilon_1^\mu$. On the other hand, the strength of *CP* violation at low energies is controlled by the *CP* invariant $\mathcal{J}^{CP}_{\mathrm{lepton}}$ defined in Eq. (2.25), with the neutrino mass matrix given by the seesaw formula (2.35). In this case,

$$\begin{aligned}\mathcal{J}_{CP} &= -\frac{\mathrm{Im}[(\mathbf{m}_\nu\mathbf{m}_\nu^\dagger)_{12}(\mathbf{m}_\nu\mathbf{m}_\nu^\dagger)_{23}(\mathbf{m}_\nu\mathbf{m}_\nu^\dagger)_{31}]}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} \\ &= \frac{y_{11}^2 y_{21}^2 y_{32}^2 y_{22}^2 v^{12}}{M_1^3 M_2^3 \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}\sin(2\beta_1) \\ &\quad \times \left[y_{21}^2 y_{32}^2 + y_{11}^2 y_{22}^2 + y_{11}^2 y_{32}^2 + y_{33}^2(y_{11}^2 + y_{21}^2)\frac{M_2}{M_3} \right]. \end{aligned} \tag{4.39}$$

Thus, in this toy example not only the relative sign between the low-energy *CP* invariant $\mathcal{J}^{CP}_{\mathrm{lepton}}$ and the flavored (ϵ_1^μ) and unflavored (ϵ_1) asymmetries can be predicted (these quantities have the same sign), but also their dependence on the *CP*-violating phase β_1 is such that they are simultaneously maximized when $\beta_1 = \pi/4$. We also note that when $y_{33} = 0$ the texture of $\mathbf{Y}^\nu$ given in Eq. (4.37) corresponds to one of the textures considered in Frampton, Glashow, and Yanagida (2002). In this case, the heavy Majorana neutrino N_3 completely decouples, rendering this situation phenomenologically equivalent to the two right-handed neutrino cases discussed in Sec. IV.B.3.

2. Orthogonal parametrization

A particularly useful parametrization in the context of type I seesaw leptogenesis was proposed by Casas and Ibarra (2001). Using a complex orthogonal matrix $\mathbf{R}$, the Yukawa coupling matrix $\mathbf{Y}^\nu$ can be rewritten in the more convenient form for leptogenesis calculations,

$$\mathbf{Y}^\nu = v^{-1}\mathbf{U}^*\mathbf{d}_m^{1/2}\mathbf{R}\mathbf{d}_M^{1/2}, \tag{4.40}$$

where $\mathbf{d}_M$ and $\mathbf{d}_m$ are the diagonal mass matrices defined in Eqs. (2.34), (2.35), and (2.36), respectively. In this parametrization,

$$\mathbf{H}^\nu_{ij} = (\mathbf{Y}^{\nu\dagger}\mathbf{Y}^\nu)_{ij} = \frac{M_i^{1/2}M_j^{1/2}}{v^2}\sum_k m_k \mathbf{R}_{ki}^*\mathbf{R}_{kj}, \tag{4.41}$$

so that the flavored leptogenesis asymmetry given in Eq. (4.12) can be written in the form

$$\epsilon_1^\alpha \simeq \frac{3M_1}{16\pi v^2}\frac{\sum_{j,k}m_j^{1/2}m_k^{3/2}\,\mathrm{Im}[\mathbf{U}_{\alpha j}^*\mathbf{U}_{\alpha k}\mathbf{R}_{j1}\mathbf{R}_{k1}]}{\sum_k m_k|\mathbf{R}_{k1}|^2}, \tag{4.42}$$

while the unflavored asymmetry (4.13) reads

$$\epsilon_1 \simeq \frac{3M_1}{16\pi v^2}\frac{\sum_{j\neq 1}m_j^2\,\mathrm{Im}[\mathbf{R}_{j1}^2]}{\sum_k m_k|\mathbf{R}_{k1}|^2}. \tag{4.43}$$

It becomes evident that the unflavored asymmetry (4.43) or, more generally, the unflavored asymmetry defined in Eq. (4.11) does not depend on the low-energy *CP*-violating phases of $\mathbf{U}$, since the matrix $\mathbf{U}$ cancels out in the matrix $\mathbf{H}^\nu$, as can be seen from Eq. (4.41). It should be noted, however, that the above conclusion holds provided that the matrices $\mathbf{U}$ and $\mathbf{R}$ are independent from each other, i.e., if no constraints or specific *ansätze* are imposed on the matrix $\mathbf{Y}^\nu$. In particular, imposing some flavor symmetries or texture zeros on the matrix $\mathbf{Y}^\nu$ may lead to relations between the *CP*-violating phases in $\mathbf{U}$ and the *CP*-violating parameters in $\mathbf{R}$. In such cases, the parametrization in Eq. (4.40) may not be the most convenient for disentangling the *CP* violation responsible for leptogenesis from *CP* violation at low energies.

If the matrix $\mathbf{R}$ is real, i.e., if the only source of high-energy *CP* violation comes from the left-handed lepton sector, then the unflavored leptogenesis *CP* asymmetries ϵ_i vanish (Abada *et al.*, 2006; Nardi *et al.*, 2006). The fact that the matrix $\mathbf{R}$ is real when *CP* is an exact symmetry of the right-handed neutrino sector is easily understood once the matrix $\mathbf{Y}^\nu$ is written in its singular value decomposition, $\mathbf{Y}^\nu = \mathbf{V}_L^\dagger \mathbf{d}_\lambda \mathbf{V}_R$, where $\mathbf{V}_{L,R}$ are unitary matrices and $\mathbf{d}_\lambda = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ with λ_i the corresponding eigenvalues. The *CP* violation in the right-handed neutrino sector is thus encoded in the phases of $\mathbf{V}_R$. On the other hand, using the parametrization (4.40), one can also write $\mathbf{H}^\nu = \mathbf{d}_M^{1/2}\mathbf{R}^\dagger \mathbf{d}_m \mathbf{R}\mathbf{d}_M^{1/2}/v^2 = \mathbf{V}_R^\dagger \mathbf{d}_\lambda^2 \mathbf{V}_R$, which clearly shows that the orthogonal matrix $\mathbf{R}$ is real if and only if $\mathbf{V}_R$ is real.

The situation is, however, quite different when flavor effects are accounted for. We consider, for definiteness, the N_1-dominated scenario with $M_1 \ll M_{2,3}$ at temperatures $T \lesssim 10^{12}$ GeV. In this case, the flavored asymmetries are given by Eq. (4.42) and the relevant quantities are the combinations $\mathrm{Im}[\mathbf{U}_{\alpha j}^*\mathbf{U}_{\alpha k}\mathbf{R}_{j1}\mathbf{R}_{k1}]$, which explicitly depend on the PMNS matrix elements. Therefore, provided that $\mathbf{R} \neq \mathbb{1}$, the leptogenesis asymmetries ϵ_1^α do not vanish even if the matrix $\mathbf{R}$ is real. Furthermore, in the latter case the *CP*-violating effects responsible for leptogenesis are directly connected to the low-energy *CP*-violating phases in $\mathbf{U}$ (Branco, González Felipe, and Joaquim, 2007; Pascoli, Petcov, and Riotto, 2007). This becomes evident from the expression of the leptogenesis asymmetries,

$$\epsilon_1^\alpha = \frac{3M_1}{16\pi v^2}\frac{\sum_j \sum_{k>j}\sqrt{m_j m_k}(m_k - m_j)\mathbf{R}_{j1}\mathbf{R}_{k1}\mathcal{I}_{jk}^\alpha}{\sum_k m_k|\mathbf{R}_{k1}|^2}, \tag{4.44}$$

where

$$\mathcal{I}_{jk}^\alpha = \mathrm{Im}[\mathbf{U}_{\alpha j}^*\mathbf{U}_{\alpha k}] \tag{4.45}$$

are rephasing invariant quantities.

At this point, one may wonder whether a real matrix **R** can be naturally realized in some model. In general, once *CP* violation is allowed through the introduction of complex Yukawa couplings, it will arise in both the left-handed and right-handed sectors, leading to a complex PMNS matrix **U** as well as a complex orthogonal matrix **R**. The simplest way of restricting the number of *CP*-violating phases is through the assumption that *CP* is a good symmetry of the Lagrangian, only broken by the vacuum. A model with a complex leptonic mixing matrix **U** and real **R** can actually be constructed in a natural way. We consider the type I seesaw framework and impose *CP* invariance at the Lagrangian level. We also introduce three Higgs doublets, together with a Z_3 symmetry under which the left-handed fermion doublets ψ_{Lj} transform as $\psi_{Lj} \rightarrow e^{-i2\pi j/3}\psi_{Lj}$ and the Higgs doublets as $\phi_j \rightarrow e^{i2\pi j/3}\phi_j$, while all other fields transform trivially. One can show that there is a region of the parameter space where the vacuum violates *CP* through complex vacuum expectation values. Yet, due to the Z_3 restrictions on Yukawa couplings, the combination $\mathbf{Y}^{\nu\dagger}\mathbf{Y}^\nu$ turns out to be real, thus implying a real **R**, while a complex **U** is generated. The drawback of such a scheme is that leptogenesis must occur not far from the electroweak scale. However, one can envisage an alternative scenario where effective Yukawa couplings are generated by higher-order operators that involve singlet fields that acquire complex VEVs at very high energies. From a different viewpoint, the case of a real matrix **R** can also be realized within a class of models based on the so-called sequential dominance (King, 2007).

To illustrate the possibility of a direct link between leptogenesis and low-energy *CP* violation when the matrix **R** is real, we consider the following example. We assume a normal hierarchical light neutrino mass spectrum with $m_1 \simeq 0 \ll m_2 \simeq m_{\rm sol} \ll m_3 \simeq m_{\rm atm}$. In this case, Eq. (4.44) yields

$$\epsilon_1^\alpha \simeq \frac{3M_1}{16\pi v^2} \frac{m_{\rm atm}\sqrt{m_{\rm sol}m_{\rm atm}}\mathbf{R}_{21}\mathbf{R}_{31}I_{23}^\alpha}{m_{\rm sol}|\mathbf{R}_{21}|^2 + m_{\rm atm}|\mathbf{R}_{31}|^2}. \qquad (4.46)$$

We further assume that the *CP*-violating effects due to the low-energy Dirac-type phase δ are subdominant and can be neglected ($\delta \simeq 0$). Then, using the parametrization (2.11) and (2.12) of the mixing matrix **U**, one can show that

$$I_{23}^e \simeq -c_{13}s_{12}s_{13}\sin(\alpha_{12}/2),$$
$$I_{23}^\mu \simeq c_{13}s_{23}(-c_{12}c_{23} + s_{12}s_{13}s_{23})\sin(\alpha_{12}/2), \qquad (4.47)$$
$$I_{23}^\tau \simeq c_{13}c_{23}(c_{23}s_{12}s_{13} + c_{12}s_{23})\sin(\alpha_{12}/2),$$

with $\alpha_{12} = \alpha_1 - \alpha_2$. Therefore, in this simple example, the flavored leptogenesis asymmetries depend on the same Majorana-phase difference α_{12} that controls the effective Majorana mass parameter m_{ee} in $0\nu\beta\beta$ decay [cf. Eq. (3.30)]. We note, however, that the sign of ϵ_1^α cannot be uniquely predicted by the sign of $\sin(\alpha_{12}/2)$ since the product $\mathbf{R}_{21}\mathbf{R}_{31}$ can be positive or negative.

Before concluding this section, we briefly comment on the possibility of establishing a connection between leptogenesis and low-energy *CP* violation taking into account other effects (besides the leptonic *CP* asymmetries) that can affect the efficiency of leptogenesis. Assuming a particular prior on the parameter space (e.g., by restricting the orthogonal matrix **R** and the heavy and/or light neutrino mass spectra), it has been shown that flavored leptogenesis can work for any value of the PMNS phases and, therefore no direct connection can be established (Davidson *et al.*, 2007). On the other hand, for an inverted-hierarchical light neutrino mass spectrum, one can show that there exist regions in the leptogenesis parameter space where the purely high-energy contribution to the baryon asymmetry is highly suppressed and a successful leptogenesis can be achieved only if the necessary amount of *CP* violation is provided by the PMNS Majorana phases (Molinaro and Petcov, 2009a; 2009b).

3. Two right-handed neutrino case

Neutrino oscillation data do not demand the presence of three right-handed neutrinos in a type I seesaw framework. The solar and atmospheric neutrino mass scales could be associated to just two heavy Majorana neutrino masses. Such a two right-handed neutrino (2RHN) scenario has also the advantage of reducing the total number of free parameters so that the analysis of neutrino phenomenology and leptogenesis becomes much simpler (Frampton, Glashow, and Yanagida, 2002; Raidal and Strumia, 2003; Barger, *et al.*, 2004; González Felipe, Joaquim, and Nobre, 2004; Ibarra and Ross, 2004; Guo, Xing, and Zhou, 2007). To understand this, we recall that in the SM extended with three right-handed neutrinos the Lagrangian of the neutrino sector contains 18 parameters at high energies: 3 heavy Majorana masses plus 15 real parameters (9 moduli and 6 phases) needed to specify the Yukawa coupling matrix $\mathbf{Y}^\nu$. Of these, only 15 parameters are independent in what concerns the light neutrino mass matrix $\mathbf{m}_\nu$ obtained through the seesaw mechanism (the three Majorana masses M_i can be absorbed into $\mathbf{Y}^\nu$ by an appropriate rescaling of its elements). On the other hand, in the 2RHN case, there are altogether 11 parameters: 2 heavy Majorana masses together with 9 real parameters (6 moduli and 3 phases) that specify the 3×2 matrix $\mathbf{Y}^\nu$. Once again, performing the rescaling of the two heavy Majorana masses, the effective number is reduced to 9 parameters.

In the three right-handed neutrino case, the measurable quantities associated to the light neutrino mass matrix are 3 masses, 3 mixing angles, and 3 phases, while for two right-handed neutrinos this number is reduced by 2, since the lightest neutrino is massless and its associated Majorana phase vanishes. Thus, in the latter case there is no possibility of three quasidegenerate light neutrinos, and only two mass spectra are allowed: a normal hierarchy with $m_1 = 0$, $m_2 = m_{\rm sol}$, and $m_3 = m_{\rm atm}$ or an inverted hierarchy with $m_3 = 0$, $m_1 = m_{\rm atm}$, and $m_2 \approx m_{\rm atm} + m_{\rm sol}^2/(2m_{\rm atm})$.

The parameters in $\mathbf{Y}^\nu$ which are associated with the seesaw but are not determined by low-energy measurable quantities are most easily disentangled if this matrix is written in terms of the orthogonal parametrization of Eq. (4.40). The six (two) undetermined parameters of the 3RHN (2RHN) model would correspond precisely to those parameters that specify the complex orthogonal matrix **R**. The 2RHN model can then be thought of as the limiting case of the 3RHN model in which the heaviest right-handed neutrino N_3 decouples from the theory because it is very heavy or its Yukawa couplings are very small. From Eq. (4.40) one finds for the third column of the matrix **R**

$$\mathbf{R}_{i3} = \frac{v}{\sqrt{m_i M_3}}(\mathbf{U}^T \mathbf{Y}^\nu)_{i3}. \tag{4.48}$$

Thus, as $M_3 \to \infty$, $\mathbf{R}_{23}$, $\mathbf{R}_{33} \to 0$, while $\mathbf{R}_{13} \to 1$ due to orthogonality. Consequently, in the 2RHN model the orthogonal matrix $\mathbf{R}$ takes the simple 3×2 structure

$$\mathbf{R} = \begin{pmatrix} 0 & 0 \\ \cos z & -\sin z \\ \pm \sin z & \pm \cos z \end{pmatrix}, \tag{4.49}$$

where z is a complex angle and the $\pm$ signs account for a discrete indeterminacy in $\mathbf{R}$. Using this form, the elements of the Dirac-neutrino Yukawa coupling matrix read

$$\mathbf{Y}^\nu_{\alpha 1} = \sqrt{M_1}(\sqrt{m_2}\cos z U^*_{\alpha 2} \pm \sqrt{m_3}\sin z U^*_{\alpha 3})/v,$$
$$\mathbf{Y}^\nu_{\alpha 2} = \sqrt{M_2}(-\sqrt{m_2}\sin z U^*_{\alpha 2} \pm \sqrt{m_3}\cos z U^*_{\alpha 3})/v. \tag{4.50}$$

For an inverted hierarchy, the corresponding matrix $\mathbf{R}$ reads

$$\mathbf{R} = \begin{pmatrix} \cos z & -\sin z \\ \pm \sin z & \pm \cos z \\ 0 & 0 \end{pmatrix}, \tag{4.51}$$

and Eqs (4.50) become

$$\mathbf{Y}^\nu_{\alpha 1} = \sqrt{M_1}(\sqrt{m_1}\cos z U^*_{\alpha 1} \pm \sqrt{m_2}\sin z U^*_{\alpha 2})/v,$$
$$\mathbf{Y}^\nu_{\alpha 2} = \sqrt{M_2}(-\sqrt{m_1}\sin z U^*_{\alpha 1} \pm \sqrt{m_2}\cos z U^*_{\alpha 2})/v. \tag{4.52}$$

It is clear that without any assumption about the complex parameter z there is no direct link between the leptogenesis asymmetries and leptonic *CP* violation at low energies. Nevertheless, the fact that the number of unknown parameters at high energies is reduced with respect to the 3RHN case makes it possible to establish a connection between thermal leptogenesis and low-energy neutrino parameters with simple assumptions about the physics at high energies. For instance, assuming $M_1 \ll M_2$ and z real, the flavored leptogenesis asymmetries given in Eq. (4.46) for a normal hierarchical neutrino mass spectrum read

$$\epsilon^\alpha_1 \simeq \pm \frac{3M_1}{16\pi v^2} \frac{m_{atm}\sqrt{m_{sol}m_{atm}}}{m_{sol}\cos^2 z + m_{atm}\sin^2 z} \sin z \cos z I^\alpha_{23}, \tag{4.53}$$

with the rephasing invariant quantities I^α_{23} given by Eqs. (4.47) with the Majorana phase $\gamma_2 = 0$. On the other hand, the total (unflavored) asymmetry ϵ_1 would vanish in this case since $\sum_\alpha I^\alpha_{23} = 0$.

We note that the asymmetry (4.53) is maximal when

$$\sin z = \sqrt{\frac{m_{sol}}{m_{sol} + m_{atm}}} \approx \sqrt{\frac{m_{sol}}{m_{atm}}}, \tag{4.54}$$

which implies the upper bound

$$|\epsilon^\alpha_1| \leq \frac{3M_1 m_{atm}}{32\pi v^2} |I^\alpha_{23}|. \tag{4.55}$$

Nevertheless, we remark that a maximal *CP* asymmetry does not necessarily correspond to a maximal baryon asymmetry

since leptogenesis also crucially depends on the subsequent washout effects.

Yukawa coupling structures with texture zeros provide a well-motivated framework in which the number of high-energy parameters is reduced and relations among low-energy neutrino observable quantities may be implied. In the presence of a family symmetry, the charge assignment under the symmetry to particles may lead to one or several Yukawa couplings which are negligibly small compared to the others. It is clear that texture zeros are in general not WB invariant. This means that a given texture zero, which arises in a certain WB, may not be present or may appear in a different matrix entry in another WB. It is, however, important to distinguish among various types of texture zeros. Some of them have no physical meaning because they can be obtained through a WB transformation starting from arbitrary flavor matrices (Branco, Emmanuel-Costa, Felipe, and Serôdio, 2009). On the other hand, there are texture zeros that do have physical implications. Among the latter, one should distinguish between zeros that result from a flavor symmetry from those that just reflect an ad-hoc assumption on the flavor structure. It should be emphasized that even when texture zeros result from a family symmetry imposed on the Lagrangian, they are manifest only in a particular basis, namely, the basis where the symmetry is transparent. Furthermore, it has been shown (Branco, Rebelo, and Silva-Marcos, 2006) that a large class of sets of leptonic texture zeros imply the vanishing of certain *CP*-odd WB invariants. These invariants allow, for instance, one to recognize a flavor model, which is characterized by certain texture zeros in the matrix $\mathbf{Y}^\nu$ in the basis where the charged-lepton and right-handed neutrino mass matrices are diagonal, when the same model is written in an arbitrary WB where the zeros are not manifest.

The possibility of a texture zero in the (1, 1) position is quite interesting from the phenomenological point of view, since in the quark sector such a postulate, if applied to the up and down quark matrices, leads to the remarkably successful prediction for the Cabibbo angle $\theta_C = \theta_{12} = \sqrt{m_d/m_s}$ (Gatto, Sartori, and Tonin, 1968). Applying this rationale to the neutrino sector of the 2RHN model, i.e., imposing $\mathbf{Y}^\nu_{11} = 0$, would fix the value of the unknown parameter z in terms of low-energy neutrino data. From Eqs. (4.50) and (4.52), one finds

$$\tan z = \mp \sqrt{\frac{m_2}{m_3}}\frac{\mathbf{U}^*_{e2}}{\mathbf{U}^*_{e3}}, \qquad \tan z = \mp \sqrt{\frac{m_1}{m_2}}\frac{\mathbf{U}^*_{e1}}{\mathbf{U}^*_{e2}}, \tag{4.56}$$

for normal and inverted-hierarchical neutrino mass spectrum, respectively. Note also that imposing additional texture zeros in the neutrino Yukawa coupling matrix would yield relations among the mixing angles and neutrino masses. To see the implications for leptogenesis of a texture zero in the (1, 1) position, consider the unflavored asymmetry given in Eq. (4.43), rewritten as

$$\epsilon_1 \simeq \frac{3M_1}{16\pi v^2} \frac{(m_3^2 - m_2^2)\,\mathrm{Im}(\sin^2 z)}{m_2|\cos^2 z| + m_3|\sin^2 z|}. \tag{4.57}$$

Using the first relation in Eq. (4.56), ϵ_1 can then be expressed in terms of low-energy quantities as

$$\epsilon_1 \simeq \frac{3M_1(m_3^2 - m_2^2)}{16\pi v^2 m_{ee}} \frac{\text{Im}(U_{e2}^{*2}U_{e3}^2)}{|U_{e2}|^2 + |U_{e3}|^2}$$

$$\approx -\frac{3M_1 m_{\text{atm}}^2}{16\pi v^2 m_{ee}} \sin^2\theta_{13} \sin(2\delta + \alpha_1). \qquad (4.58)$$

Thus, in this simple example, there is a correlation between the sign of the baryon asymmetry and low-energy leptonic *CP* violation. Clearly, one texture zero is sufficient to establish such a link because the sign of ϵ_1 is determined by $\text{Im}(\tan^2 z)$, which in turn is fixed by Eq. (4.56). If we consider the flavored asymmetries ϵ_1^α given in Eq. (4.42), it would still be possible to write them in terms of low-energy observables. However, the direct connection between the sign of the baryon asymmetry and *CP* violation at low energies would be lost since the phase contributions to the individual asymmetries are more involved.

In the examples presented above, the heavy Majorana neutrinos have been assumed hierarchical in mass so that leptogenesis is dominated by the decays of N_1, the lightest of the heavy states. One can also envisage a situation when the heavy Majorana neutrino mass spectrum is exactly degenerate at energies above the leptogenesis scale. In this case, a small mass splitting among the heavy Majorana neutrino states can be generated, in a natural way, via the renormalization group running from the degeneracy scale down to the leptogenesis scale (González Felipe, Joaquim, and Nobre, 2004). To illustrate this, we consider again the minimal scenario with only two right-handed neutrinos, and assume that $M_1 = M_2 \equiv M$, at a scale $\Lambda > M$. The evolution of the right-handed neutrino mass matrix $\mathbf{m}_R$ as a function of the energy scale μ is governed by the renormalization group equation

$$\frac{d\mathbf{m}_R}{dt} = \mathbf{H}^{\nu T}\mathbf{m}_R + \mathbf{m}_R\mathbf{H}^\nu, \qquad t = \frac{1}{16\pi^2}\ln(\mu/\Lambda). \qquad (4.59)$$

Then, defining $\delta_N \equiv M_2/M_1 - 1$, which quantifies the degree of degeneracy between M_1 and M_2, the radiatively induced mass splitting at the decoupling scale M will be approximatively given by

$$\delta_N \simeq \frac{Mm_{\text{atm}}}{8\pi^2 v^2}(1 - \rho)\ln(\Lambda/M), \qquad (4.60)$$

where $\rho \equiv m_{\text{sol}}/m_{\text{atm}}$.

To analyze the implications for leptogenesis, we impose the particular texture zero $Y_{12}^\nu = 0$ (González Felipe, Joaquim, and Nobre, 2004). Then, in terms of the low-energy neutrino parameters, the unflavored *CP* asymmetries ϵ_i generated by the N_i decays read as

$$\epsilon_1 \simeq -\frac{3y_\tau^2}{64\pi} \frac{1 + \rho}{(1 - \rho)(\rho + x^2 - \Delta)} \tan\theta_{13} \sin(\alpha_1/2)$$

$$\times [\cot\theta_{12}\cos(\delta - \alpha_1/2) + \tan\theta_{13}\cos(\alpha_1/2)],$$

$$\epsilon_2 \simeq \frac{\rho + x^2 - \Delta}{1 + \rho x^2 + \Delta}\epsilon_1, \qquad (4.61)$$

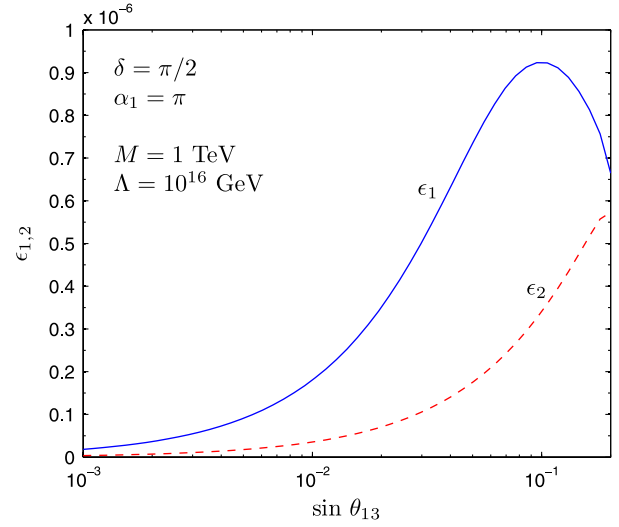where $x = \tan\theta_{13}/(\sqrt{\rho}\sin\theta_{12})$, y_τ is the τ Yukawa coupling, and

FIG. 19 (color online). The *CP* asymmetries ϵ_1 and ϵ_2 as functions of $\sin\theta_{13}$ generated in a minimal radiative leptogenesis scenario. The curves correspond to the approximate analytic expressions given in Eqs. (4.61).

$$\Delta = \tfrac{1}{2}(1 - \rho)[-1 + x^2 + \sqrt{1 + 2x^2\cos\alpha_1 + x^4}]. \qquad (4.62)$$

Taking, for instance, $\alpha_1 = \pi$ and $\delta = \pi/2$, the *CP* asymmetry ϵ_1 reaches its maximum value for $x = \sqrt{\rho}$. This corresponds to $\tan\theta_{13} = \rho\sin\theta_{12} \simeq 0.1$ and

$$|\epsilon_1^{\text{max}}| \simeq \frac{3y_\tau^2\cos\theta_{12}}{128\pi}\frac{1 + \rho}{1 - \rho} \simeq 10^{-6}. \qquad (4.63)$$

In Fig. 19, the *CP* asymmetries ϵ_i are plotted as functions of $\sin\theta_{13}$ taking $\Lambda = 10^{16}$ GeV, $M = 1$ TeV, $\delta = \pi/2$, $\alpha = \pi$, and assuming $y_\tau = 0.01$ in the analytical estimates. The curves correspond to the approximate expressions given in Eqs. (4.61). It is interesting to note that, in this case, the maximum of the leptogenesis asymmetry ϵ_1 is reached for $s_{13} \simeq 0.1$, which is the sensitivity range of future reactor and superbeam neutrino oscillation experiments.

4. Leptogenesis and flavor symmetries

Present neutrino data (see Table I) are in good agreement with the so-called tribimaximal (TB) leptonic mixing (Harrison, Perkins, and Scott, 2002),

$$\mathbf{U}_{\text{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \qquad (4.64)$$

corresponding to the mixing angles $\theta_{12} = \arcsin(1/\sqrt{3})$, $\theta_{23} = -\pi/4$, and $\theta_{13} = 0$ in the standard PDG parametrization (Nakamura *et al.*, 2010) given in Eq. (2.12). Since the above mixing matrix does not depend on any mass parameter, it is usually referred to as a mass-independent mixing scheme. If one assumes that the leptonic mixing is described at leading order by $\mathbf{U}_{\text{TB}}$, it is natural to consider that this special

structure arises due to a family symmetry. In particular, discrete symmetries are quite attractive, and the tetrahedral (alternating) group A_4, corresponding to even permutations of four objects, has been especially popular and featured in a large number of models of leptonic mixing (Altarelli and Feruglio, 2010).

From the phenomenological viewpoint, one of the attractive features of the mass-independent mixing schemes is that they lead to a predictive neutrino mass matrix structure which contains just a few parameters. The latter can then be directly related to neutrino observables such as the neutrino mass-squared differences, the absolute neutrino mass scale, and the effective mass parameter in $0\nu\beta\beta$ decays.

Besides restricting the number of relevant parameters, the imposition of certain flavor symmetries in the lepton sector of the theory may lead to constraints on the *CP* asymmetries in the framework of seesaw leptogenesis. In particular, it has been recently shown that type I and type III seesaw flavor models that lead to an exact mass-independent leptonic mixing have a vanishing leptogenesis *CP* asymmetry in leading order (Jenkins and Manohar, 2008; Bertuzzo *et al.*, 2009; Aristizabal Sierra *et al.*, 2010; González Felipe and Serôdio, 2010). To illustrate this fact, we consider the standard type I seesaw framework with three right-handed neutrinos ν_R. In this case, the relevant Lagrangian terms are given by Eq. (2.33), and the effective neutrino mass matrix $\mathbf{m}_\nu$ is obtained through the standard seesaw formula of Eq. (2.35).

We assume that the type I seesaw Lagrangian is invariant under the transformations of a given flavor symmetry group $\mathcal{G}$, so that left-handed and right-handed lepton fields transform as $\nu_L \rightarrow \mathcal{G}_L \nu_L$ and $\nu_R \rightarrow \mathcal{G}_R \nu_R$, respectively. Clearly, the generators $\mathcal{G}_L$ and $\mathcal{G}_R$ are unitary matrices built from the columns of the unitary matrices $\mathbf{U}$ and $\mathbf{U}_R$ that diagonalize the matrices $\mathbf{m}_\nu$ and $\mathbf{m}_R$, respectively. The Lagrangian invariance then implies that the Dirac-neutrino Yukawa coupling matrix $\mathbf{Y}^\nu$ should satisfy the symmetry relation $\mathcal{G}_L^T \mathbf{Y}^\nu \mathcal{G}_R^* = \mathbf{Y}^\nu$. To analyze the consequences of this relation for leptogenesis, we rewrite the symmetry equations in the basis in which the right-handed neutrino mass matrix is diagonal,

$$\mathcal{G}_R'^\dagger \mathbf{d}_M \mathcal{G}_R'^* = \mathbf{d}_M, \qquad \mathcal{G}_R'^T \mathbf{H}^\nu \mathcal{G}_R'^* = \mathbf{H}^\nu, \qquad (4.65)$$

with $\mathcal{G}_R' = \mathbf{U}_R^\dagger \mathcal{G}_R \mathbf{U}_R$. Assuming a nondegenerate heavy neutrino mass spectrum, the first relation in Eq. (4.65) requires the symmetry generators $\mathbf{G}_{R,i}'$ ($i = 1, 2, 3$) to be diagonal. Their explicit forms are thus given by $\mathbf{G}_{R,1}' = \mathrm{diag}(1, 1, -1)$, $\mathbf{G}_{R,2}' = \mathrm{diag}(1, -1, 1)$, and $\mathbf{G}_{R,3}' = \mathrm{diag}(-1, 1, 1)$. The action of any two of these matrices in the second relation of Eq. (4.65) would then enforce $\mathbf{H}^\nu$ to be diagonal, which in turn implies that the leptogenesis asymmetries (4.8) and (4.11) are equal to zero. The case of a degenerate heavy neutrino mass spectrum can be analyzed in a similar way. In the latter case, no leptogenesis *CP* asymmetry can be generated in leading order either (González Felipe and Serôdio, 2010). Note also that, due to the specific form of the matrix combination $\mathbf{H}^\nu$ that appears in the leptogenesis *CP* asymmetries, only the symmetry generators $\mathcal{G}_R$ are really needed in the above proof of vanishing leptogenesis.

Clearly, if the complete mass matrix symmetry is not imposed as the residual symmetry of the type I seesaw Lagrangian, the above conclusions do not necessarily remain valid. For instance, requiring the right-handed sector of the Lagrangian to be invariant just under the transformation $\nu_R \rightarrow \mathbf{G}_{R,1} \nu_R$ would lead to vanishing $\mathbf{H}_{13}^\nu$ and $\mathbf{H}_{23}^\nu$ off-diagonal elements. Yet a leptogenesis asymmetry could in principle be generated with a nonzero $\mathbf{H}_{12}^\nu$ matrix element.

In a type II seesaw framework, the interplay between flavor symmetries and the leptogenesis *CP* asymmetries is actually different (De Medeiros Varzielas, González Felipe, and Serôdio, 2011). In the latter case, we can see from Eq. (4.23) that the flavored leptonic *CP* asymmetries $\epsilon_i^{\alpha\beta}$ are proportional to the combination $\mathrm{Im}[\mu_i^* \mu_j \mathbf{Y}_{\alpha\beta}^{\Delta_i} \mathbf{Y}_{\alpha\beta}^{\Delta_j*}]$, while the unflavored asymmetry ϵ_i depends on $\mathrm{Im}[\mu_i^* \mu_j \mathrm{Tr}(\mathbf{Y}^{\Delta_i} \mathbf{Y}^{\Delta_j*})]$. To analyze the implications of discrete flavor symmetries for type II seesaw leptogenesis, it is convenient to rewrite the light neutrino mass matrix $\mathbf{m}_\nu = \mathbf{U}_{\mathrm{TB}} \mathbf{d}_m \mathbf{U}_{\mathrm{TB}}^T$ in terms of three contributions (De Medeiros Varzielas, González Felipe, and Serôdio, 2011),

$$\mathbf{m}_\nu = x\mathbf{C} + y\mathbf{P} + z\mathbf{D}, \qquad (4.66)$$

where x, y, and z are complex numbers;

$$\mathbf{C} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \qquad \mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad (4.67)$$

denote the well-known magic and μ-τ symmetric matrices, and $\mathbf{D}$ is the democratic matrix with all entries equal to $1/3$.

As it turns out, the type II seesaw leptonic asymmetry is in general nonvanishing. For leptogenesis to be viable at least two scalar SU(2) triplets are needed. Suppose, for instance, that both triplets are singlets under the family symmetry. Then, one of them can be associated to the $\mathbf{P}$ contribution and the other one to the $\mathbf{C}$ contribution in Eq. (4.66). If a third scalar triplet is available, it may be associated to the democratic component $\mathbf{D}$. In this minimal setup, unless a democratic contribution is present, the unflavored asymmetry ϵ_i is zero[12] because the product of the matrices $\mathbf{C}$ and $\mathbf{P}$ is traceless, which then implies $\mathrm{Tr}(\mathbf{Y}^{\Delta_i} \mathbf{Y}^{\Delta_j*}) = 0$. On the other hand, the flavored leptogenesis asymmetries do not necessarily vanish even when the democratic component is absent.

In addition to TB mixing, there are other mass-independent structures that can reproduce the observed leptonic mixing angles. Below we give some examples of such mass-independent schemes.

The transposed TB mixing has the mixing matrix (Fritzsch and Xing, 1996)

$$\mathbf{U}_{\mathrm{tTB}} = \begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 \\ -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix}, \qquad (4.68)$$

where the solar and atmospheric mixing angles are given by $\theta_{12} = \pi/4$ and $\theta_{23} = \arctan\sqrt{2}$, respectively. The

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[12]Note that, if each scalar triplet is simultaneously associated to the magic and μ-τ symmetric contributions, the unflavored asymmetry is, in general, nonvanishing.

well-known bimaximal structure has the mixing matrix (Barger *et al.*, 1998)

$$\mathbf{U}_B = \begin{pmatrix} \sqrt{\tfrac{1}{2}} & -\sqrt{\tfrac{1}{2}} & 0 \\ \tfrac{1}{2} & \tfrac{1}{2} & -\sqrt{\tfrac{1}{2}} \\ \tfrac{1}{2} & \tfrac{1}{2} & \sqrt{\tfrac{1}{2}} \end{pmatrix}, \tag{4.69}$$

and the corresponding mixing angles are in this case $\theta_{12} = \theta_{23} = \pi/4$. There are also two golden ratio proposals related to the quantity $\Phi = (1+\sqrt{5})/2$. The first matrix is (Kajiyama, Raidal, and Strumia, 2007)

$$\mathbf{U}_{GR} = \begin{pmatrix} \sqrt{\tfrac{1}{2}+\tfrac{1}{2\sqrt{5}}} & \sqrt{\tfrac{2}{5+\sqrt{5}}} & 0 \\ -\sqrt{\tfrac{1}{5+\sqrt{5}}} & \sqrt{\tfrac{1}{5-\sqrt{5}}} & \sqrt{\tfrac{1}{2}} \\ \sqrt{\tfrac{1}{5+\sqrt{5}}} & -\sqrt{\tfrac{1}{4}+\tfrac{1}{4\sqrt{5}}} & \sqrt{\tfrac{1}{2}} \end{pmatrix}, \tag{4.70}$$

with the associated angles $\theta_{12} = \arctan(1/\Phi)$ and $\theta_{23} = \pi/4$, while the second matrix reads (Rodejohann, 2009)

$$\mathbf{U}_{GR} = \begin{pmatrix} \tfrac{1+\sqrt{5}}{4} & \tfrac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}} & 0 \\ -\tfrac{\sqrt{5-\sqrt{5}}}{4} & \tfrac{1+\sqrt{5}}{4\sqrt{2}} & -\sqrt{\tfrac{1}{2}} \\ -\tfrac{\sqrt{5-\sqrt{5}}}{4} & \tfrac{1+\sqrt{5}}{4\sqrt{2}} & \sqrt{\tfrac{1}{2}} \end{pmatrix}, \tag{4.71}$$

with $\theta_{12} = \arccos(\Phi/2)$ and $\theta_{23} = -\pi/4$. Finally, the so-called hexagonal mixing (Giunti, 2003; Xing, 2003; Albright, Dueck, and Rodejohann, 2010) is described by the matrix

$$\mathbf{U}_H = \begin{pmatrix} \sqrt{\tfrac{3}{4}} & \tfrac{1}{2} & 0 \\ -\sqrt{\tfrac{1}{8}} & \sqrt{\tfrac{3}{8}} & -\sqrt{\tfrac{1}{2}} \\ -\sqrt{\tfrac{1}{8}} & \sqrt{\tfrac{3}{8}} & \sqrt{\tfrac{1}{2}} \end{pmatrix}, \tag{4.72}$$

which corresponds to the mixing angles $\theta_{12} = \pi/6$ and $\theta_{23} = -\pi/4$.

As in the TB case, the above mixing schemes predict the mixing angle $\theta_{13} = 0$ and therefore no Dirac-type *CP* violation. The conclusions for the leptogenesis asymmetries previously drawn are equally valid in all these cases.

C. *CP*-odd invariants for leptogenesis

Based on the most general *CP* transformations in the lepton sector that leave invariant the gauge interactions, we constructed WB invariants that need to vanish in order for *CP* invariance to hold at low energies (cf. Sec. II.D). *CP*-odd conditions derived from WB invariants are a powerful tool for model building, since they can be applied to any model without the need to go to a special basis. In this section, we are particularly interested in the construction of WB invariants which are sensitive to the *CP*-violating phases of leptogenesis.

In the case of unflavored leptogenesis, the *CP* asymmetry is only sensitive to phases appearing in the matrix $\mathbf{H}^\nu$ so that the relevant WB invariant conditions can be readily derived (Branco *et al.*, 2001)

$$I_1 \equiv \mathrm{Im}\,\mathrm{Tr}[\mathbf{H}^\nu(\mathbf{m}_R^\dagger\mathbf{m}_R)\mathbf{m}_R^*\mathbf{H}^{\nu*}\mathbf{m}_R] = 0,$$

$$I_2 \equiv \mathrm{Im}\,\mathrm{Tr}[\mathbf{H}^\nu(\mathbf{m}_R^\dagger\mathbf{m}_R)^2\mathbf{m}_R^*\mathbf{H}^{\nu*}\mathbf{m}_R] = 0,$$

$$I_3 \equiv \mathrm{Im}\,\mathrm{Tr}[\mathbf{H}^\nu(\mathbf{m}_R^\dagger\mathbf{m}_R)^2\mathbf{m}_R^*\mathbf{H}^{\nu*}\mathbf{m}_R(\mathbf{m}_R^\dagger\mathbf{m}_R)] = 0. \tag{4.73}$$

The choice of these invariant conditions is not unique. For instance, by replacing $\mathbf{m}_R$ by $\mathbf{m}_R^{*-1}$ in the invariants I_n, one can construct another set of invariants which, for hierarchical right-handed neutrinos, are more suitable for leptogenesis (Davidson and Kitano, 2004).

The quantities given in Eq. (4.73) can be evaluated in any convenient weak basis. In the WB in which the right-handed neutrino mass matrix $\mathbf{m}_R$ is diagonal and real, one obtains

$$I_1 = \sum_{i=1}^{3}\sum_{j>i}^{3} M_i M_j (M_j^2 - M_i^2)\mathrm{Im}[(\mathbf{H}_{ij}^\nu)^2] = 0,$$

$$I_2 = \sum_{i=1}^{3}\sum_{j>i}^{3} M_i M_j (M_j^4 - M_i^4)\mathrm{Im}[(\mathbf{H}_{ij}^\nu)^2] = 0, \tag{4.74}$$

$$I_3 = \sum_{i=1}^{3}\sum_{j>i}^{3} M_i^3 M_j^3 (M_j^2 - M_i^2)\mathrm{Im}[(\mathbf{H}_{ij}^\nu)^2] = 0.$$

The appearance of the quadratic combination $(\mathbf{H}_{ij}^\nu)^2$ in the above expressions simply reflects the well-known fact that phases of $\pi/2$ in $\mathbf{H}_{ij}^\nu$ do not imply *CP* violation. Note that Eqs. (4.74) constitute a set of linear equations in terms of the quantities $\mathrm{Im}[(\mathbf{H}_{ij}^\nu)^2]$, where the coefficients are functions of the right-handed neutrino masses M_i. The determinant of this system is equal to $M_1^2 M_2^2 M_3^2 (M_2^2 - M_1^2)^2 (M_3^2 - M_1^2)^2 (M_3^2 - M_2^2)^2$. It then follows that, if none of the M_i vanish and there is no degeneracy in the masses M_i, the simultaneous vanishing of I_1, I_2, and I_3 implies the vanishing of $\mathrm{Im}[(\mathbf{H}_{12}^\nu)^2]$, $\mathrm{Im}[(\mathbf{H}_{13}^\nu)^2]$, and $\mathrm{Im}[(\mathbf{H}_{23}^\nu)^2]$. This implies, in turn, that the unflavored type I leptogenesis asymmetries given in Eq. (4.11) are all equal to zero.

We note that the WB invariants I_i defined in Eq. (4.73) vanish if the heavy Majorana neutrinos are degenerate in mass. It is nevertheless possible to construct WB invariants which control the strength of *CP* violation in the latter case. For instance, the weak-basis invariant

$$\mathcal{J}_{\mathrm{deg}}^{CP} = M^{-6}\,\mathrm{Tr}[\mathbf{Y}^\nu\mathbf{Y}^{\nu T}\mathbf{Y}^\ell\mathbf{Y}^{\ell\dagger}\mathbf{Y}^{\nu*}\mathbf{Y}^{\nu\dagger}, \mathbf{Y}^{\ell*}\mathbf{Y}^{\ell T}]^3, \tag{4.75}$$

where M is the common heavy Majorana neutrino mass, does not vanish in the case of an exactly degenerate heavy Majorana neutrino mass spectrum. Thus, $\mathcal{J}_{\mathrm{deg}}^{CP} \neq 0$ would signal the violation of *CP* in this case.

For flavored leptogenesis, the phases appearing in $\mathbf{H}^\nu$ are still relevant. There is, however, the possibility of generating the required *CP* asymmetry even for $\mathbf{H}^\nu$ real. In this case, additional *CP*-odd WB invariant conditions are required, since I_i cease to be necessary and sufficient. A possible choice are the *CP*-odd WB invariant conditions obtained

from I_i through the substitution of $\mathbf{H}^\nu$ by $\hat{\mathbf{H}}^\nu = \mathbf{Y}^{\nu\dagger}\mathbf{Y}^\ell\mathbf{Y}^{\ell\dagger}\mathbf{Y}^\nu$, and which are sensitive to the additional phases appearing in flavored leptogenesis (Branco, González Felipe, Rebelo, and Serôdio, 2009).

V. CONCLUSIONS AND OUTLOOK

After almost 50 years since its discovery, *CP* violation is still at the core of particle physics and cosmology. In the quark sector, CPV has been established in both the kaon and *B* meson sectors, and the results obtained so far are compatible with the standard complex CKM mixing picture. With the discovery of neutrino masses, the natural expectation is that *CP* is also broken in the lepton sector. Indeed, in a unified description of fundamental particle physics, it is hard to imagine a scenario with CPV in the quark sector and not in the leptonic one.

The prospects for discovering LCPV in neutrino oscillation experiments mainly depend on the value of the reactor neutrino mixing angle θ_{13}. The smaller this angle is, the longer we will have to wait until experiments become sensitive to *CP* violating effects. In the best-case scenario, CPV could be discovered in the near future by combining the data of reactor neutrino and superbeam experiments (see Sec. III.B.4), if θ_{13} is not too small. If this is not the case, then we will probably have to wait for upgraded superbeams, β, electron-capture beams, or neutrino factories. The recent data from the T2K experiment in Japan indicate the appearance of ν_e from the original ν_μ neutrino beam with a number of observed *e*-like events, which exceed the expected ones. The probability of explaining the results with $\theta_{13} = 0$ is less than 1% and the obtained 90% C.L. interval for $\sin^2(\theta_{13})$ is [0.03(0.04), 0.28 (0.34)], with the numbers in parenthesis referring to the case in which $\Delta m_{31}^2 < 0$. Such an indication of a nonzero (and not very small) value of θ_{13} is a good omen for the prospects of discovering *CP* violation in the near future. With some luck, a hint for CPV could be provided by combining the data of superbeam (NOνA and T2K) and reactor neutrino experiments (Double Chooz, Daya Bay, and RENO). In any case, upgraded superbeams, β beams, or neutrino factories will be for sure necessary to confirm such a hint and measure the *CP*-violating phase δ.

It has been advocated that $0\nu\beta\beta$ decays could, in principle, provide some information about Majorana-type *CP* violation in the lepton sector. Although this is true in theory, the task of extracting information about the Majorana phases using $0\nu\beta\beta$ results is nontrivial. This holds even in the simplest scenario in which $0\nu\beta\beta$ is induced by the exchange of light Majorana neutrinos. As discussed in Sec. III.C, the main difficulty in the Majorana phase determination from $0\nu\beta\beta$ measurements resides on the uncertainties inherent to the nuclear matrix element determination. In particular, the precision required to make conclusive statements about Majorana *CP* violation seems to be far from what can be achieved. The observation of $0\nu\beta\beta$ would establish the Majorana nature of neutrinos, and therefore would favor some neutrino mass generation mechanisms over others. In the near future, the experimental sensitivity of $0\nu\beta\beta$ experiments will cover the region where $\Delta m_{31}^2 < 0$, covering the IH and QD neutrino spectrum cases. The combined study of

$0\nu\beta\beta$ and β decay, neutrino oscillations, and also cosmological data, will be crucial to improve the knowledge of neutrino fundamental parameters and test the minimal $0\nu\beta\beta$ mechanism.

If neutrino masses are generated at an energy scale not far from the electroweak scale, there is a hope to test the neutrino mass mechanism at high-energy colliders like the LHC. In such a case, it is straightforward to infer that the presence of *CP* violation in the neutrino sector would have an impact on the physical processes involving the neutrino mass mediators. In Secs. III.D and III.E, we illustrated how the leptonic *CP* phases affect the rates of several lepton decays in the context of the type II seesaw mechanism, in which neutrino masses are generated by the tree-level exchange of scalar triplets. The fact that the effective neutrino mass matrix is linear in the triplet-lepton-lepton couplings, allows one to write in a model-independent way the decay rates in terms of the low-energy neutrino parameters. In particular, we have seen that some decays are only sensitive to a particular set of *CP* phases. Therefore the detection of such decays complemented with neutrino data could provide extra information on leptonic CPV.

Another important question to be answered by future experiments is whether *CP* violation in the lepton sector follows the traditional CKM-like form with a unitary lepton mixing matrix. As discussed in Sec. III.F, deviations from unitarity in leptonic mixing appear in several extensions of the SM. Therefore, the detection of such effects would definitely point towards nonstandard physics. Nonunitarity effects are, in some cases, severely constrained by electroweak processes like radiative and three-body charged-lepton decays or leptonic *W* and *Z* decays. In Sec. III.F, we reviewed the present constraints on the unitarity of the leptonic mixing matrix in the context of the simple MUV hypothesis. In this framework, deviations from the standard *CP* violation scenario can be observed in future neutrino oscillation experiments like neutrino factories.

CP violation also plays a crucial role in cosmology, since the dynamical generation of the observed baryon asymmetry of the Universe requires that *CP* is violated. Once the SM is augmented with heavy states which can explain the smallness of neutrino masses, leptogenesis arises as the most natural and appealing mechanism to generate the excess of matter over antimatter. The *CP* violation present in the decays of the heavy Majorana neutrinos not only gives rise to a leptonic asymmetry but it is also present in the effective neutrino mass matrix determined by the seesaw mechanism. Thus, one would expect that a connection between *CP* violation at low energies and the one relevant for leptogenesis could be established. Unfortunately, establishing this connection in a model-independent way is not possible. In general, assumptions about the flavor structure of the neutrino couplings and/or masses have to be considered in order to make predictions. In Sec. IV, we showed a few examples in which a bridge between LCPV at low and high energies can be established. Obviously, the ultimate goal would be to test the leptogenesis mechanism at low energies, but this would be only possible if the lepton asymmetry is generated in the decays of particles that could be produced in accelerators. For certain, this will not be the case in a conventional leptogenesis framework in which the decaying seesaw mediators have masses much

larger than the electroweak scale. However, if the origin of lepton-number violation is related to physics within our reach, then there may be a hope to test the leptogenesis mechanism or, at least, get a hint for it.

The answers to many of the open questions discussed in this review depend on the capability of future experiments to explore the unknown. In the neutrino sector, the milestones achieved in recent years have already excluded many theoretical ideas. Still, there are important questions like the ones concerning leptonic *CP* violation which are waiting for answers. We hope to find them just around the corner.

ACKNOWLEDGMENTS

Note added in proof.—Recently, through the observation of electron-antineutrino disappearance, the Daya Bay Reactor Neutrino Experiment has measured the non-zero value (An, *et al.*, 2012) $\sin^2(2\theta_{13}) = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$ with a significance of 5.2σ. This confirms the T2K and MINOS experimental data presented in Sec. III.A and reinforces the prospects of a possible discovery of leptonic *CP* violation in the near future.

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