Erratum: *Colloquium*: Quantum fluctuation relations: Foundations and applications [Rev. Mod. Phys. 83, 771 (2011)]

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The first line of Eq. (51) contains some typos: it correctly reads

$$G[u;\lambda] = \mathrm{Tr}\mathcal{T} e^{iu[\mathcal{H}^H_\tau(\lambda_\tau) - \mathcal{H}(\lambda_0)]} e^{-\beta\mathcal{H}(\lambda_0)}/\mathcal{Z}(\lambda_0). \tag{51}$$

This compares with its classical analog, i.e., the second line of Eq. (27).

Quite surprisingly, notwithstanding the identity

$$\mathcal{H}^H_\tau(\lambda_\tau) - \mathcal{H}(\lambda_0) = \int_0^\tau dt \dot{\lambda}_t \frac{\partial \mathcal{H}^H_t(\lambda_t)}{\partial \lambda_t}, \tag{1}$$

one finds that generally

$$\mathcal{T} e^{iu[\mathcal{H}^H_\tau(\lambda_\tau) - \mathcal{H}(\lambda_0)]} \neq \mathcal{T} \exp\left[iu \int_0^\tau dt \dot{\lambda}_t \frac{\partial \mathcal{H}^H_t(\lambda_t)}{\partial \lambda_t}\right]. \tag{2}$$

As a consequence, it is not allowed to replace $\mathcal{H}^H_\tau(\lambda_\tau) - \mathcal{H}(\lambda_0)$, with $\int_0^\tau dt \dot{\lambda}_t \partial \mathcal{H}^H_t(\lambda_t)/\partial \lambda_t$ in Eq. (51). Thus, there is no quantum analog of the classical expression in the third line of Eq. (27). This is yet another indication that ''work is not an observable'' (Talkner, Lutz, and Hänggi, 2007)). This observation also corrects the second line of Eq. (4) of the original reference (Talkner, Lutz, and Hänggi, 2007).

The correct expression is obtained from the general formula

$$\mathcal{T} \exp[A(\tau) - A(0)] = \mathcal{T} \exp\left[\int_0^\tau dt \left(\frac{d}{dt} e^{A(t)}\right) e^{-A(t)}\right], \tag{3}$$

where $A(t)$ is any time dependent operator [in our case $A(t) = iu\mathcal{H}^H_t(\lambda_t)$]. Equation (3) can be proved by demonstrating that the operator expressions on either side of Eq. (3) obey the same differential equation with the identity operator as the initial condition. This can be accomplished by using the operator identity $de^{A(t)}/dt = \int_0^1 ds e^{sA(t)} \dot{A}(t) e^{(1-s)A(t)}$.

There are also a few minor misprints: (i) The symbol ds in the integral appearing in the first line of Eq. (55) should read dt. (ii) The correct year of the reference (Morikuni and Tasaki, 2010) is 2011 (not 2010).

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REFERENCES

Talkner P., E. Lutz, and P. Hänggi, 2007, Phys. Rev. E **75**, 050102(R).

