Axions and the strong *CP* problem

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Current upper bounds on the neutron electric dipole moment constrain the physically observable quantum chromodynamic (QCD) vacuum angle $|\bar{\theta}| \lesssim 10^{-11}$. Since QCD explains a great deal of experimental data from the 100 MeV to the TeV scale, it is desirable to explain this smallness of $|\bar{\theta}|$ in the QCD framework; this is the strong *CP* problem. There now exist two plausible solutions to this problem, one of which leads to the existence of a very light axion. The axion decay constant window, $10^9 \lesssim F_a \lesssim 10^{12}$ GeV for an $O(1)$ initial misalignment angle θ_1, has been obtained from astrophysical and cosmological data. For $F_a \gtrsim 10^{12}$ GeV with $\theta_1 < O(1)$, axions may constitute a significant fraction of the dark matter of the universe. The supersymmetrized axion solution of the strong *CP* problem introduces its superpartner the axino, which might have affected the evolution of the Universe significantly. The very light axion (theory, supersymmetrization, and models) using recent particle, astrophysical, and cosmological data, and present prospects for its discovery is reviewed here.

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I. OVERVIEW

Strong interaction phenomena have revealed that the discrete symmetries of charge conjugation C, parity P,

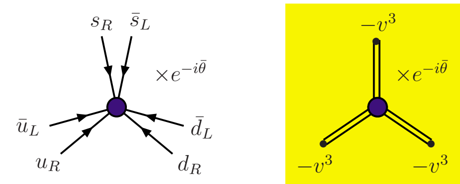

FIG. 1. (Color online) The determinental interaction of light quarks. Chiral symmetry breaking introduces the anomalous η' mass term from the quark condensations.

and time reversal T are separately good symmetries of nature. Therefore, quantum chromodynamics (QCD) based on the gauge group $SU(3)_c$ (Han and Nambu, 1965; Bardeen, Fritzsch, and Gell-Mann, 1972) must respect any combinations of these discrete symmetries C, P, and T to be accepted as the theory of strong interactions. Among these discrete symmetries, the CP symmetry is not necessarily respected in QCD due to the nonzero QCD vacuum angle θ, an issue known as the "strong CP problem." Since QCD is so successful phenomenologically, a possible solution to the strong CP problem is expected to be realized in nature. Currently the most attractive solution leads to the existence of a very light axion (Kim, 1979; Shifman, Vainstein, and Zakharov, 1980; Dine, Fischler, and Srednicki, 1981b; Zhitnitskii, 1981). Searches for QCD axions generated from the Sun (Andriamonje *et al.*, 2007; Inoue *et al.*, 2008) and remnant axions from the early Universe (Rosenberg, 2004; Carosi, 2007) are presently ongoing.

The story of axions started with the QCD U(1) problem (Weinberg, 1975) which is now understood, having been solved by the 't Hooft determinental interaction ('t Hooft, 1976, 1986). The determinental interaction is shown as the left diagram of Fig. 1 and the solution is shown as the shaded right diagram. The strong interaction causes the quark bilinears to condense with a vacuum expectation value (VEV) of order $v \simeq 260$ MeV. The phase of this interaction $\bar{\theta}$ originates from the QCD vacuum angle, which is known to be physical (Callan, Dashen, and Gross, 1976; Jackiw and Rebbi, 1976), and contributes to the neutron electric dipole moment (NEDM) with order $\bar{\theta}$ times the neutron size, a large value. Peccei and Quinn (PQ) observed that there exists a way to make $\bar{\theta}$ a phase by introducing a symmetry, now called $U(1)_{PQ}$; then physical amplitudes do not depend on $\bar{\theta}$, as in the massless quark case (Peccei and Quinn, 1977a, 1977b). In the standard model (SM), this phase is a pseudoscalar Goldstone boson called the "axion" among the multitude of Higgs fields as noted by Weinberg (1978) and Wilczek (1978). If the PQ idea was completed with Fig. 1, this axion would be exactly massless (but observable), and $\bar{\theta}$ would behave "unphysically" in having to choose the freedom an appropriate axion VEV, which was the original PQ idea. However, there exist subleading terms, proportional to one power of m_q, which close the quark lines with the current quark mass instead of a condensation. Then an axion potential develops, and the axion becomes a pseudo-Goldstone boson. The axion solution of the strong CP problem is cosmological in that the axion VEV chooses $\bar{\theta}=0$ at the minimum of this axion potential. The currently allowed axion is very light and long lived.

The properties of the axion (denoted as a) are mainly given by its decay constant F_a, which sets the scale of nonrenormalizable axion interactions through a/F_a. Initial axion searches placed F_a far above the electroweak scale and additional stringent bounds on F_a were obtained from studies of stellar evolution and cosmology (Kim, 1987). Axion astrophysics, started by Dicus, Kolb, Teplitz, and Wagoner (1978, 1980) using earlier ideas from Sato and Sato (1975) and Sato (1978) now gives a stringent lower bound on the decay constant, $F_a \geqslant 0.5 \times 10^9$ GeV, from the study of SN1987A (Raffelt, 1990a; Turner, 1990). With this large decay constant, the axion flux from the Sun is a small fraction of the solar neutrino flux, but may still be detectable by the CERN Axion Solar Telescope (CAST) experiment and by the Tokyo helioscope.

It is known that very light axions with F_a in the 10^{12} GeV region (axion mass in the μeV range) might compose some part of cold dark matter (CDM) in the Universe (Abbott and Sikivie, 1983; Dine and Fischler, 1983; Preskill, Wise, and Wilczek, 1983). The exact amount of axion CDM depends on the initial axion misalignment angle θ_1 at the time of axion creation when the universe temperature was around the axion decay constant, $T \sim F_a$. This observation puts the very light axion on the list of leading CDM candidate particles. If indeed these cosmic axions compose a significant fraction of CDM in the universe, they may be detectable by collection of axion-converted photons in cavity-type detectors (Sikivie, 1983) as tried by DePanfilis *et al.* (1987) and Hagmann *et al.* (1990) and now continuing at the Axion Dark Matter experiment (ADMX).

Cosmology including CDM was the leading candidate for the early Universe in the 1980s (Blumenthal, Faber, Primack, and Rees, 1984; Kolb and Turner, 1990; Weinberg, 2008). Since then this view has given way to the new cosmology with the discovery of dark energy (DE) in 1998 (Riess *et al.*, 1998; Perlmutter *et al.*, 1999). The current view of the dominant components of the Universe is $\Omega_{CDM} \simeq 0.23$ and $\Omega_\Lambda \simeq 0.73$ with only a few percent consisting of baryons (Spergel *et al.*, 2007). The most plausible dark matter candidates at present are the lightest supersymmetric (SUSY) particle (LSP), the axion, the axino, and the gravitino. Here we review the axion and its CDM-related possibilities.

The need for DM was suggested as early as the 1930s (Zwicky, 1933; Smith, 1936). Since then, evidence of nonluminous DM in the universe has been accumulating: examples include flat galactic rotation curves, Chandra satellite photos, and gravitational lensing effects. If the galactic bulge is the dominant mass in the galaxy, the rotational velocity v of a star located at r from the center should be $v \sim r^{-1/2}$. But the observed flat rotation curve [see, for example, McGaugh *et al.* (2007), and references therein] violates this expectation and implies an extended mass in the halo varying as $\rho(r) \sim 1/r^2$. The Chandra observation of x-ray and gravitational lensing images also implies this matter profile around the bullet cluster [Clowe *et al.* (2006)]. Circular gravitational lensing images [Jee *et al.* (2007)] also support the existence of DM. The DM density around the Solar system is $\rho_{\rm DM} \simeq 0.3$–0.45 GeV/cm^3.

Current CDM candidates are either incoherent particles or coherent oscillations of spin-0 fields. In this view bosonic collective motions such as the axion can be considered as CDM. The popular incoherent CDM particles are the weakly interacting massive particles (WIMPs) or decay products of WIMPs. A more frequently used independent distinction is between thermal and nonthermal relics, but there is no strict relation of correspondence between the incoherent and coherent particles and the thermal and nonthermal relics. WIMPs are massive particles with weak interaction cross sections, first discussed in terms of a heavy neutrino, corresponding to the right-hand side (RHS) crossing point of Fig. 2(a) (Lee and Weinberg, 1977b). The left-hand side (LHS) crossing point corresponds to a 10 eV neutrino (Cowsik and McClelland, 1972; Marx and Szalay, 1972). WIMPs, such as the LSP, are thermal relics when their number density is determined by the freezeout temperature and are nonthermal relics if their number density is determined by another mechanism such as the decay of heavier relics (Choi, Kim, Lee and Seto, 2008). In Fig. 2(b), we sketch the axion energy density in terms of the axion mass. The shape is flipped from that of Fig. 2(a), because in the axion case the low- and high-mass regions contribute Ω_a from different physics, one from the vacuum misalignment and the other from the hot thermal relics.

In addition to the heavy neutrino, SUSY with *R*-parity conservation allows the LSP to be just such a WIMP particle. The LSP interaction is "weak" since the interaction mediators (SUSY particles) are supposed to be in the 100 GeV range. For a WIMP to be a successful CDM candidate, usually the interaction cross section at the time of decoupling needs to be (Kolb and Turner, 1990; Spergel *et al.*, 2007)

$$\langle \sigma_{\rm int} v \rangle|_{\rm at\ decoupling} \approx 0.2 \times 10^{-26}\ {\rm cm^3\ s^{-1}}$$

$$\text{with } \Omega_m h^2 \simeq 0.113 \pm 0.009. \quad (1)$$

This is roughly the cross section for the LSP from low-energy SUSY, which is the reason why the DM community is so interested in the WIMP LSP. Some superweakly interacting particles such as gravitinos, axinos, and wimpzillas (Chung, Kolb, and Riotto, 1999) might be CDM candidates as well, but their cross sections do not fall in the range of Eq. (1). The CDM candidate particles are shown in the $\sigma_{\rm int}$ versus mass plane in Fig. 3 taken with minor modification from Roszkowski (2004). The incoherent fermions, such as the neutrino and the left ends of the bars of the axino and gravitino, correspond to the left crossing points of Fig. 2(a). The rest,

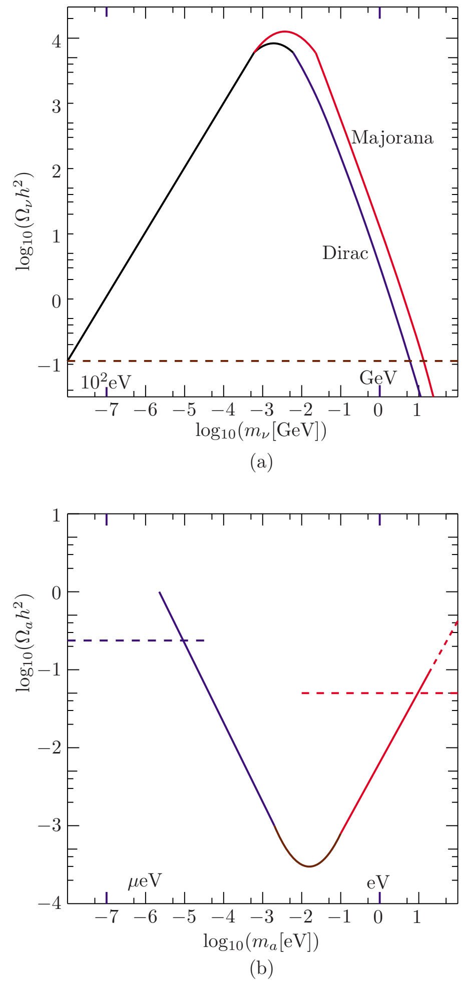

FIG. 2. (Color online) The Lee-Weinberg-type plots for (a) the neutrino $\Omega_\nu h^2$ (Kolb and Turner, 1990) and (b) the axion $\Omega_a h^2$, where h is the present Hubble constant in units of 100 km s^{-1} Mpc^{-1}. The dashed line in (a) is for $\Omega_\nu h^2 = 0.113$. In (b), it corresponds to the hadronic axion. The dashed lines correspond to the CDM and hot DM limits, respectively.

except for the axion, correspond more or less to the right crossing points of Fig. 2(a), with reheating after inflation considered if necessary. Currently, there are experimental efforts to discover the LSP as predicted by SUSY models. Direct cosmological searches are also ongoing (Jungman, Kamionkowski, and Griest, 1996; Bernabei *et al.*, 2003, 2008; Bertone, Hooper, and Silk, 2005; Lee *et al.*, 2007; Angle *et al.*, 2008; Behnke *et al.*, 2008; Ahmed *et al.*, 2009a). At the CERN Large Hadron Collider (LHC), the probable LSP mass ranges for LSPs produced by neutrolino decay will be looked for.

It is known that density perturbations must have begun growing much earlier than recombination time in order to become large enough to form galaxies in the young universe. For galaxy formation, therefore, DM is needed since proton density perturbations could not grow before the recombination time, but DM perturbations could. With DM, the equality point of radiation and matter energy densities can occur much earlier than the recombination time since DM is not prohibited from

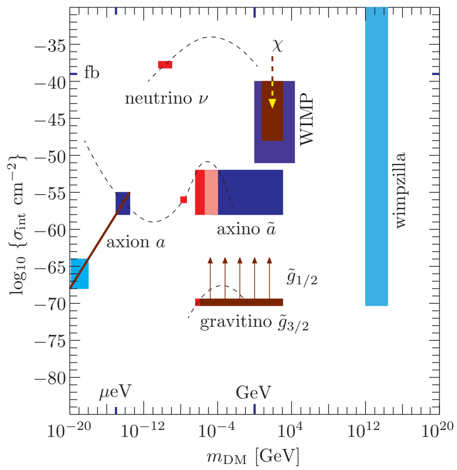

FIG. 3. (Color) Some proposed particles in the plane of the interaction cross section vs the corresponding particle mass m_i. The skeleton is taken from Roszkowski (2004). The dashed curves represent schematic shapes of Ω_i vs the corresponding particle mass m_i. The small red square box corresponds to the hot DM hadronic axion. Two small outside squares (cyan and blue) in the axion region are marked to show the plausible GUT and CDM axions, respectively. The abundances of the heavy axino, gravitino, and wimpzilla depend on how inflation ends.

collapsing by Silk damping (Silk, 1968). If the WIMP mass and interaction cross section fall in the region allowed by Eq. (1), the WIMP can be part of CDM. If the LSP were the only CDM component, then the LSP mass would give one number for the DM density, which may not be accurate. Thus, even if the LSP is contributing to the CDM density, we may need the axion to account for the correct amount of CDM around us. This is possible in the anthropic scenario of very light axions because it is equally probable for the initial axion misalignment angle θ_1 to take any value between 0 and π (Tegmark, Aguirre, Rees, and Wilczek, 2006).

Here we review the axion, which is probably the most interesting Nambu-Goldstone boson (Nambu, 1960; Goldstone, 1961; Nambu and Jona-Lasinio, 1961), as well as related issues. In Sec. II we discuss the strong *CP* problem and its plausible solutions. In Sec. III we review the most attractive solution giving the very light axion and present the axion theory in terms of possible axion couplings defined by c_1, c_2, and c_3 used throughout this review. In Sec. IV we present axion astrophysics and cosmology. Here we present a new number for the cosmic axion abundance in view of recent accurate data on light quark masses. In Sec. V we summarize the axion detection ideas and the ongoing axion detection experiments. In Sec. VI we summarize the proposed very light axion models, including superstring axions. Finally in Sec. VII we discuss cosmology with the axino, the axion's superpartner.

If the axion was observed, it would mark one of the most profound elementary particle discoveries because it would confirm experimentally the instanton-based arguments of QCD. In addition, if it were shown to be consistent with a cosmologically significant amount of axions, the CDM idea of bosonic collective motion would also be confirmed experimentally. If SUSY is correct and the axion is the solution to the strong *CP* problem, axino must have affected the evolution of the Universe as well.

II. THE STRONG *CP* PROBLEM AND SOLUTIONS

There are good reviews on the strong *CP* problem (Kim, 1987; Cheng, 1988; Peccei, 1989); here we outline a few key points. QCD with $SU(3)_c$ gluons is a confining gauge theory with three light quarks below 1 GeV and $\Lambda_{\rm QCD}=380\pm 60$ MeV (Groote, Körner, Schilcher, and Nasrallah, 1998). The classical gluon field equations have the instanton solution (Belavin, Polyakov, Schwartz, and Tyupkin, 1975),

$$G_\mu = if(r)g^{-1}(x)\partial_\mu g(x), \quad f(r)=\frac{r^2}{r^2+\rho^2}, \tag{2}$$

where the gauge coupling is absorbed in the gauge field, $g(x)$ is a pure gauge form with $G_{\mu\nu}\propto 1/r^4$ for a large r, and ρ is the instanton size. The (anti-)instanton solution satisfies the (anti-)self-duality condition $G_{\mu\nu}=\pm\tilde{G}_{\mu\nu}$ which carries the integer Pontryagin index

$$q=\frac{1}{16\pi^2}\int d^4x\,{\rm Tr}G\tilde{G}=\frac{1}{32\pi^2}\int d^4x\,G^a_{\mu\nu}\tilde{G}^{a\mu\nu}, \tag{3}$$

where $\tilde{G}^{a\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}$. The classical solution with $q=-\infty,\ldots,-1,0,+1,\ldots,+\infty$, introduces a new real number θ which parametrizes the $|\theta\rangle$ vacuum,

$$|\theta\rangle=\sum_{n=-\infty}^{\infty}e^{in\theta}|n\rangle. \tag{4}$$

Since the n's are integers, in view of Eq. (3), θ is a periodic variable with period 2π. It is known that θ is an observable parameter (Callan, Dashen, and Gross, 1976; Jackiw and Rebbi, 1976). In the θ vacuum, we must consider the *P*- and *T*- (or *CP*-) violating interaction parametrized by $\bar{\theta}=\theta_0+\theta_{\rm weak}$,[1]

$$\mathcal{L}=\bar{\theta}\{G\tilde{G}\}\equiv\frac{\bar{\theta}}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}, \tag{5}$$

where the curly bracket includes $1/32\pi^2$, θ_0 is the angle given above the electroweak scale, and $\theta_{\rm weak}$ is the value introduced by the electroweak *CP* violation. This observable $\bar{\theta}$ has led to the so-called strong *CP* problem from the upper bound on the NEDM. For QCD to become a correct theory, this *CP* violation by QCD must be sufficiently suppressed.

[1]With the canonical normalization of the gauge field, the RHS of Eq. (5) is multiplied by g_c^2.

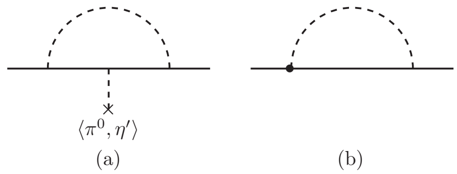

FIG. 4. Loop corrections for $\bar{n}n$-meson coupling. Insertion of the CP violation effect by VEVs of π^0 and η' in (a). They can be transferred to one vertex shown as a bullet in (b). With this bullet, CP violation is present because of a mismatch between the CP-conserving RHS vertex and CP-violating LHS vertex.

A. Neutron electric dipole moment

The interaction (5) is the anomaly term (Adler, 1969; Bell and Jackiw, 1969) which is the basis for solving ('t Hooft, 1986) the old U(1) problem of QCD (Weinberg, 1975). The important size of instantons for physics is near the scale where QCD becomes strong. 't Hooft (1976) showed that the determinental interaction of light quarks carries the same global symmetry as that of Eq. (5), and it is customary to use this light quark determinental interaction rather than treating the gluon interaction (5). The early estimates of the NEDM proportional to $\bar{\theta}$ from the determinental interaction are $2.7\times 10^{-16}\bar{\theta}e$ cm (Baluni, 1979) and $3.6\times 10^{-16}\bar{\theta}e$ cm (Crewther, Di Vecchia, Veneziano, and Witten, 1979). Other estimates from different methods are $11\times 10^{-16}\bar{\theta}e$ cm (Cea and Nardulli, 1984), $1.2\times 10^{-16}\bar{\theta}e$ cm (Schnitzer, 1984), $3\times 10^{-16}\bar{\theta}e$ cm (Musakhanov and Israilov, 1984), and $5.5\times 10^{-16}\bar{\theta}e$ cm (Kanaya and Kobayashi, 1981). Comprehensive reviews of the NEDM exist (Dar, 2000; Pospelov and Ritz, 2005). Recently, the NEDM has been estimated in the hard wall anti–de Sitter (AdS) QCD model with one extra dimension, $1.08\times 10^{-16}\bar{\theta}e$ cm (Hong, Kim, Siwach, and Yee, 2007).

The diagrams contributing to the NEDM are restricted. The neutron magnetic dipole moment arises at one loop in chiral perturbation theory. If we treat this neutron magnetic dipole moment operator $\mu_{\rm anom}\bar{n}\sigma^{\mu\nu}nF^{\rm em}_{\mu\nu}$ as a vertex, tree diagrams do not contribute to the NEDM, because the magnetic moment term has the same chiral transformation property as that of the mass term and hence by redefining an external neutron field one can remove the phases in the neutron mass and in the dipole moment operator together.

Let the U(1) chiral transformation of quarks in the broken phase be encoded in the neutron mass term as $m_n\bar{n}_L e^{i(\alpha_1'\eta'/f_{\eta'}-\alpha_8'\pi^0/f_\pi+\bar{\theta}/2)}n_R+$H.c. ($e^{i\alpha'\bar{\theta}}$ instead of $e^{3i\alpha'\bar{\theta}}$ because the baryon octet has spin $\frac{1}{2}$). The VEVs of π^0 and η' are calculated in Sec. III.B. The CP violation is present by a mismatch between the CP-conserving RHS vertex and the CP-violating LHS vertex as shown in Fig. 4(b). The mass term of Fig. 4(b) and the neutron magnetic dipole moment term of Fig. 5(b) have the same chiral transformation property and the phases appearing there can be simultaneously removed by redefining n_R,

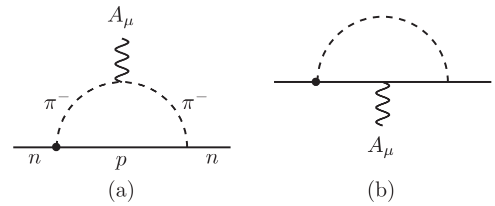

FIG. 5. Diagrams contributing to the NEDM with the bullet representing the CP violation effect. (a) is the physically observable contribution.

for example. However, the phase appearing in Fig. 5(a) cannot be removed by this phase redefinition and this contribution is physically observable. Since Fig. 5(a) is the physically observable NEDM, for the proton a similar argument leads to the same magnitude and opposite sign for the proton electric dipole moment, i.e., $d_n+d_p=0$. Now we estimate the NEDM as

$$\frac{d_n}{e}=\frac{g_{\pi NN}\overline{g_{\pi NN}}}{4\pi^2 m_N}\ln\left(\frac{m_N}{m_\pi}\right), \tag{6}$$

where the CP-violating scalar coupling $\overline{g_{\pi NN}}$ [the bullet of Fig. 5(a)] is estimated by Crewther, Di Vecchia, Veneziano, and Witten (1979) as

$$\overline{g_{\pi NN}}=-\bar{\theta}\frac{2(m_\Xi-m_\Sigma)m_u m_d}{f_\pi(m_u+m_d)(2m_s-m_u-m_d)}\approx -0.023\bar{\theta}, \tag{7}$$

where $Z=m_u/m_d\approx 0.48$, $m_d\approx 4.9$ MeV, and $m_s/m_d\simeq 20.1$. From Eq. (48) of Sec. III.B, we estimate the CP-violating scalar coupling as

$$\overline{g_{\pi NN}}=-\bar{\theta}\frac{Z}{(1+Z)}\simeq -\frac{\bar{\theta}}{3}. \tag{8}$$

Note that Eqs. (7) and (8) give a factor of ~ 10 difference. Existing calculations vary within a factor of 10. These old calculations depend on the various approximation methods used, but none of these estimated a VEV of π^0. For example, for Eq. (7), Eq. (11) of Crewther, Di Vecchia, Veneziano, and Witten (1979) uses the SU(3) symmetric baryon octet coupling due to the CP-violating interaction. On the other hand, for Eq. (8) the ground state vacuum of the mesonic fields has been used. After integrating out baryons, we look for the vacuum below the chiral symmetry scale. Then, the correct vacuum choice adds the value (8) to the value (7). But here we choose the one-order-larger value from the mesonic vacuum shift value (8) for an order of magnitude estimate, not concerning ourselves about the signs of the contributions. So we estimate the NEDM as $4.5\times 10^{-15}\bar{\theta}e$ cm from Eq. (8).

Since the recent upper bound on the NEDM is $|d_n|<2.9\times 10^{-26}e$ cm (Baker *et al.*, 2006), we must require

$$|\bar\theta| < 0.7\times 10^{-11}. \tag{9}$$

This extremely small upper bound on $\bar\theta$ has led to the so-called strong *CP* problem. $|\bar\theta|\lesssim 10^{-11}$ is perfectly allowed but its small value is not explained given that it could have chosen a value anywhere between 0 and $\sim\pi$. The strong *CP* problem is the quest to understand more satisfactorily why $\bar\theta$ is so unnaturally small.

B. Possible solutions

In the remainder of this paper, we simplify the notation replacing $\bar\theta$ by θ since there will not be much confusion. There are three explanations for the smallness of θ in the naturalness framework: case 1, calculable θ; case 2, massless up quark; case 3, axion. Here we discuss cases 1 and 2, and concentrate on case 3 in subsequent sections.

1. Calculable θ

The naturalness of a theory with a parameter β is defined by 't Hooft (1979): The theory is natural if the symmetry of the theory increases in the limit of vanishing β. A frequently quoted example is the Dirac fermion mass $m\bar\psi_L\psi_R$+H.c., where $m\to 0$ introduces a chiral symmetry $\psi\to e^{i\beta\gamma_5}\psi$ in the theory.

Regarding the strong *CP* problem, the appropriate symmetry is parity *P* or *CP* since the interaction (5) violates parity *P*, time reversal *T*, and *CP*, but conserves charge conjugation *C*. Requiring *CP* invariance in the Lagrangian is equivalent to setting θ_0 at zero. However, the observed weak interaction phenomena exhibit weak *CP* symmetry violations in the neutral *K* meson system and $B\to K^+\pi^-$ decay (Amsler *et al.*, 2008), and hence the needed introduction of *CP* violation in weak interactions with $\theta_0=0$ must be achieved spontaneously. In this process one necessarily introduces a $\theta_{\rm weak}$ part in θ which can be calculated and required to be sufficiently small within the bound given in Eq. (9). Along this line, many ideas have been proposed (Bèg and Tsao, 1978; Mohapatra and Senjanovic, 1978; Barr and Langacker, 1979; Segre and Weldon, 1979). This naturalness idea may be extended so as to effect only renormalizable couplings (Georgi, 1978). In any case, the introduction of weak *CP* violation by spontaneous mechanisms (Lee, 1973) or by soft scalar masses (Georgi, 1978) must be checked against various weak phenomena. The current weak *CP* violation data fit nicely with Kobayashi-Maskawa-type *CP* violation (Kobayashi and Maskawa, 1973), and these drastically different spontaneous weak *CP* violation ideas are probably difficult to fit to the data but are not considered ruled out yet (He, 2008), even though the spontaneous *CP* violation scheme (Branco, 1980) in the Weinberg model (Weinberg, 1976) is ruled out (Chang, He, and McKellar, 2001). It should be noted, though, that the models proposed above have difficulty in satisfying the bounds (9).

The Nelson-Barr-type weak *CP* violation however, mimics, the Kobayashi-Maskawa-type *CP* violation even though the fundamental reason for *CP* violation is spontaneous (Barr, 1984; Nelson, 1984). The scheme is designed such that the Yukawa couplings are real, i.e., $\theta_0=0$ from the *CP* invariance. Next, spontaneous *CP* violation is introduced through the singlet VEVs; this is the key difference from the previous calculable models. Thus, the spontaneous *CP* violation is required to occur much above the weak scale through the singlet VEVs, mediating it to light quarks through mixing with vectorlike heavy quarks. In modern terms, the heavy quarks can be considered as the mediation sector. Then, integrating out heavy fields we obtain the SM quarks with the Kobayashi-Maskawa-type weak *CP* violation. To ensure $\mathrm{Arg\,Det}\, M_q=0$ at tree level, specific forms for the Higgs couplings to the SM quarks and the superheavy vectorlike quarks are needed. Beyond the tree level, however, θ is generated at one loop, typically with the form (Goffin, Segrè, and Welson, 1980; Bento, Branco, and Parada, 1991),

$$\theta_{\rm weak}\approx\frac{1}{16\pi^2}\Delta f^2\sum(\text{loop integrals}), \tag{10}$$

where Δf^2 is the product of couplings and the Feynman loop integral is of $O(1)$. To satisfy the bound (9), the small coupling Δf^2 is needed. Some mechanism such as family symmetry may be needed to forbid $\theta_{\rm weak}$ at one loop (Nelson, 1984; Chang and Keung, 2004).

This kind of Nelson-Barr-type calculable $\theta_{\rm weak}$ can be mimicked in many extra-dimensional models including superstring theory. Recently, for example, $\theta_{\rm weak}$ was calculated to be $O(10^{-12})$ at a two-loop level in a sequestered flavor and *CP* model (Cheung, Fitzpatrick, and Randall, 2008).

Strictly speaking, the axion model also belongs to the class of calculable models but we separate it from the models with spontaneous *CP* violation because there it is not necessary to set $\theta_0=0$.

2. Massless up quark

Suppose that we chiral transform a quark as $q\to e^{i\gamma_5\alpha}q$. Then the QCD Lagrangian changes as

$$\int d^4x[-m_q\bar qq-\theta\{g_c^2G\tilde G\}]$$
$$\to\int d^4x[-m_q\bar qe^{2i\gamma_5\alpha}q-(\theta-2\alpha)\{g_c^2G\tilde G\}], \tag{11}$$

where $\{G\tilde G\}=(1/64\pi^2)\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$. If $m_q=0$, this is equivalent to changing $\theta\to\theta-2\alpha$. Thus, there exists a shift symmetry $\theta\to\theta-2\alpha$. It is known that the tunneling amplitude due to instanton solutions with a zero-mass quark vanishes ('t Hooft, 1976), which implies that the shift symmetry is an exact symmetry. In this case, θ is not physical, and hence there is no strong *CP* problem if the lightest quark (i.e., the up quark) is massless. The massless up quark solution must answer the question: Is the

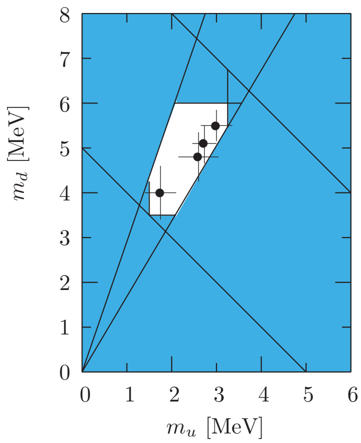

FIG. 6. (Color) The allowed m_u-m_d region (Manohar and Sachrajda, 2008). The two downward sloping lines are from the bound on $(m_u+m_d)/2$ and the two rising lines are from the bound on m_u/m_d, determined by the masses of the meson octet. The two vertical and horizontal boundaries are from the Particle Data Book bounds on $m_u=[1.5,3.3]$ MeV and $m_d=[3.5,6.0]$ MeV (Amsler *et al.*, 2008).

massless up quark phenomenologically viable? Weinberg's famous up-down quark mass ratio $Z=m_u/m_d$ gave $Z=5/9$ (Weinberg, 1977). It is very similar to the recent compilation of the light quark masses, $m_u=2.6^{+0.9}_{-1.1}$ MeV, $m_d=4.9^{+1.1}_{-1.4}$ Mev, and $Z=0.48^{+1.2}_{-1.3}$ shown in Fig. 6. This compilation is convincing enough to rule out the massless up quark possibility (Kaplan and Manohar, 1986). In this review, we use $Z=0.48$ when a number is needed though the appropriate bound may be $0.35<Z<0.60$ (Buckley and Murayama, 2007; Manohar and Sachrajda, 2008).

For some time the massless up quark possibility was taken seriously (Kaplan and Manohar, 1986). The reason is that, even if the Lagrangian mass for the up quark is zero, the 't Hooft determinental interaction may generate a useful up quark mass for chiral perturbation. There was confusion on this issue for some time (Leutwyler, 1990; Choi, 1992). Now, it is clear that the massless up quark possibility is ruled out, even without use of the lattice calculation of the ratio $m_u/m_d=0.410\pm0.036$ (Nelson, Fleming, and Kilcup, 2003).

III. AXIONS

The axion solution seems to be the most attractive one among three possible strong *CP* solutions, in particular at present when the massless up-quark possibility is excluded and calculable solutions need one-loop suppression.

Peccei and Quinn tried to mimic the symmetry $\theta\to\theta-2\alpha$ of the massless quark case of Eq. (11), by considering the full electroweak theory Lagrangian (Peccei and Quinn, 1977a, 1977b). They found such a symmetry if H_u is coupled only to up-type quarks and H_d couples only to down-type quarks,

$$\mathcal{L} = -\bar{q}_L u_R H_u - \bar{q}_L d_R H_d - V(H_u,H_d) + \text{H.c.} - \theta\{G\tilde{G}\}. \qquad (12)$$

Certainly, if we assign the same global charge under the γ_5 transformation to H_u and H_d, $q\to e^{i\gamma_5\alpha}q, H_u\to e^{i\beta}H_u, H_d\to e^{i\beta}H_d$, the flavor-independent part changes to

$$\mathcal{L}\to -\bar{q}_L e^{-i\gamma_5\alpha}u_R e^{i\beta}H_u - \bar{q}_L e^{-i\gamma_5\alpha}d_R e^{i\beta}H_d - V(e^{i\beta}H_u,e^{i\beta}H_d) + \text{H.c.} - (\theta-2\alpha)\{G\tilde{G}\}. \qquad (13)$$

Since the full Lagrangian must possess global symmetry, the potential V should not allow the H_uH_d and $(H_uH_d)^2$ terms. The choice of $\beta=\alpha$ achieves the same kind of θ shift as in the massless quark case, called PQ global symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$. Unlike an the massless up-quark case, here θ is physical. Even though the coefficient of $\{G\tilde{G}\}$ changes in the same way in Eqs. (11) and (13), these two cases differ in that the tunneling amplitude vanishes with a massless quark (a detailed discussion will be presented in Sec. III.B) but not without a massless quark. The reason is that the Higgs fields transform under $\mathrm{U}(1)_{\mathrm{PQ}}$, and one of the Higgs fields, called the axion a, has the shift symmetry $a\to a+\text{const}$ and corresponds to the Goldstone boson of the spontaneously broken $\mathrm{U}(1)_{\mathrm{PQ}}$ (Weinberg, 1978; Wilczek, 1978). As a result we call the resulting axion from Eq. (13) the Peccei-Quinn-Weinberg-Wilczek (PQWW) axion. If the consequence of the determinental interaction is only Fig. 1, then of the two bosons η' and a only η' obtains mass by the RHS diagram of Fig. 1 and a remains massless. If a remains massless, the strong *CP* problem is solved as envisioned by Peccei and Quinn (1977a) since for any θ we can choose the VEV $\langle a\rangle$ such that the final θ is zero. This was Peccei and Quinn's idea: that $\langle a\rangle$ has a shift symmetry mimicking that of the massless quark case. However, a has interactions and it can be produced in the stars and K meson decay, which differs from the massless quark case.

At the classical Lagrangian level, there seems to be no strong *CP* problem. But the axion coupling to $\{G\tilde{G}\}$ is generated at the one-loop level, which is the $\mathrm{U}(1)_{\mathrm{PQ}}$-QCD-QCD anomaly. The 't Hooft determinental interaction mentioned above is exactly this anomalous coupling. With this one-loop term, the Lagrangian is not invariant under the phase shift symmetry β or $a\to a+\text{const}$. Since it is explicitly broken at the one-loop level, the phase field β of the Higgs fields or axion a does not have a flat potential, i.e., Fig. 1 is not complete. Weinberg and Wilczek interpreted this phenomenon using the spontaneous symmetry breaking of the global symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$. It is said that θ is made dynamical where $\theta\equiv a/F_a$, but in the PQWW axion case the component was there from the beginning in the phases of the Higgs doublet fields. The free energy depending on $-\cos\theta$ is the potential for the axion. Since it is proportional to $-\cos\theta$, the minimum of the potential is at $\theta=0$ in *CP*-conserving theories (Vafa and Witten, 1984), and thus the vacuum chooses $\theta=0$. We discuss this effect below the chiral symmetry breaking scale in Sec. III.B. Thus, the axion solution of the strong CP problem is a kind of cosmological solution. Note, however, that the weak CP violation shifts θ a little bit, leading to $\theta \sim O(10^{-17})$ (Georgi and Randall, 1986).

The PQWW axion was ruled out quickly (Donnely *et al.*, 1978; Peccei, 1979), which was the reason for the popularity of calculable models in 1978 as discussed in Sec. II.B.1. Nowadays, cosmologically considered axions are very light, because of the phase of the SU(2) $\times$U(1) singlet scalar field σ. The simplest case is the Kim-Shifman-Vainstein-Zakharov (KSVZ) axion model (Kim, 1979; Shifman, Vainstein, and Zhakharov, 1980) which incorporates a heavy quark Q with the following coupling and the resulting chiral symmetry:

$$\mathcal{L} = -\bar{Q}_L Q_R \sigma + \text{H.c.} - V(|\sigma|^2) - \theta\{F\tilde{F}\}, \tag{14}$$

$$\mathcal{L} \to -\bar{Q}_L e^{i\gamma_5\alpha} Q_R e^{i\beta}\sigma + \text{H.c.} - V(|\sigma|^2) - (\theta - 2\alpha)\{G\tilde{G}\}. \tag{15}$$

Here Higgs doublets are neutral under $\mathrm{U(1)_{PQ}}$. By coupling σ to H_u and H_d, one can introduce a PQ symmetry also, not introducing heavy quarks necessarily, and the resulting axion is called the Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) axion (Zhitnitskii, 1980; Dine, Fischler, and Srednicki, 1981a, 1981b). In string models, most probably both heavy quarks and Higgs doublets contribute to the σ field couplings. The VEV of σ is much above the electroweak scale and the axion is a very light axion[2] The SU(2)$\times$U(1) singlet σ field may mix with the Higgs doublet component by a small amount, when in practice we can consider the axion as the phase of a singlet field $\sigma, \sigma=[(v+\rho)/\sqrt{2}]e^{ia/f_S}$ with $a \equiv a+2\pi N_{\mathrm{DW}}F_a$ and the axion period $2\pi N_{\mathrm{DW}}F_a$. Note that we use f_S for the VEV of σ or the value relevant in the field space and F_a defined from the coefficient of the anomaly term; namely, the coefficient of the anomaly $\{G\tilde{G}\}$ defines F_a as $\theta=a/F_a$ while the VEV(v) of $\sigma, \sigma \propto e^{ia/v}$, defines f_S. The periodicity 2π of θ implies that F_a cannot be larger than $v\equiv f_S$, and we have $F_a=f_S/N_{\mathrm{DW}}$. It has been shown that models with $N_{\mathrm{DW}}\neq 1$ have an energy crisis problem in the standard big bang cosmology (Sikivie, 1982). But models with $N_{\mathrm{DW}}=1$ do not have such a problem due to the mechanism of conversion of the two-dimensional axionic domain wall disks surrounded by axionic strings into radiation (Barr, Choi, and Kim, 1987).

A. Axion shift symmetry and reparametrization invariance

In the original PQWW axion model, the Lagrangian in the effective field theory language was extensively discussed (Donnelly *et al.*, 1978; Peccei, 1989). Here, due to the simplicity in the formulas, we present the variant-type axion models where the PQ charges are assigned only to the right-handed quark fields (Bardeen, Peccei, and Yanagida, 1987). This discussion will make it easier to introduce our general formulas below. The PQ current is (Bardeen, Peccei, and Yanagida, 1987)

$$J_\mu^{\mathrm{PQ}} = F_a \partial_\mu a + x\sum_{i=1}^{N_g} \bar{d}_{Ri}\gamma_\mu d_{Ri} + (1/x)\sum_{i=1}^{N} \bar{u}_{Ri}\gamma_\mu u_{Ri} + (-x)\sum_{i=N+1}^{N_g} \bar{u}_{Ri}\gamma_\mu u_{Ri}, \tag{16}$$

where N_g is the number of families, N is the number of up-type quarks coupled to H_u, and $x=\langle H_u\rangle/\langle H_d\rangle$. The color anomaly is nonvanishing, i.e., the divergence of J_μ^{PQ} is

$$\partial^\mu J_\mu^{\mathrm{PQ}} = \frac{1}{2}N\left(x+\frac{1}{x}\right)\frac{\alpha_c}{4\pi}G_{\mu\nu}^a\tilde{G}^{a\mu\nu} + m_u\bar{u}[i\gamma_5 e^{ia\gamma_5/F_a x}]u + m_d\bar{d}[i\gamma_5 e^{ia\gamma_5 x/F_a}]d, \tag{17}$$

where we considered the one-family model of u and d with $N=1$. If N is zero, there is no color anomaly. For a nonvanishing N, we have to pick up the component orthogonal to the longitudinal Z_μ. Since the axial-vector part of the Z_μ current is proportional to $J_{\mu 3}^5$, any axial U(1) current orthogonal to the longitudinal Z_μ is an $\mathrm{SU(2)_{flavor}}$ singlet current constructed in terms of right-handed quark fields. These include the currents corresponding to both η' and the PQ phase. Since η' is known to be heavy, we integrate out η' to obtain light fields below the chiral symmetry breaking scale. This corresponds to picking up an anomaly-free piece, orthogonal to the longitudinal Z_μ. It is

$$J_\mu^a = J_\mu^{\mathrm{PQ}} - \frac{1}{2}N\left(x+\frac{1}{x}\right)\frac{1}{1+Z}(\bar{u}\gamma_\mu\gamma_5 u + Z\bar{d}\gamma_\mu\gamma_5 d), \tag{18}$$

where $Z=m_u/m_d$. The divergence of Eq. (18) is proportional to $m_u m_d$, which must be the case for a particle orthogonal to η'.

Below we use the typical axion model (14) because it is simple to assign the PQ charges whenever an explicit example is needed. It has the following $\mathrm{U(1)_{PQ}}$ charges Γ,

Field	σ	Q_L	Q_R
Γ	1	$+\frac{1}{2}$	$-\frac{1}{2}$

In this example, the axial-vector current for $\mathrm{U(1)_{PQ}}$ is $J_\mu^5=\bar{Q}\gamma_\mu\gamma_5 Q+v\partial_\mu a$, where a is the phase field of $\sigma=(v/\sqrt{2})e^{ia/v}$. The current corresponds to the charge flow which satisfies the current conservation equation if the symmetry is exact. But the axial-vector current is in general violated at one loop by the anomaly (Adler, 1969;

[2]Once it was called an invisible axion (Wise, Georgi, and Glashow, 1981; Nilles and Raby, 1982) but it is better to call it a very light axion due to the possibility of its detection.

Bell and Jackiw, 1969) $\partial^\mu J^5_\mu=(N_Q g_c^2/32\pi^2)G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$, or $\partial^2 a=(N_Q g_c^2/32\pi^2 v)G^a_{\mu\nu}\tilde{G}^{a\mu\nu}+(m_Q/v)\bar{Q}i\gamma_5 Q$ with the Q number N_Q, which shows that the axion interaction with the SM fields is only the anomaly term (plus the anomalous coupling with the SM gauge fields). Here and in Eq. (17) we explicitly write the QCD coupling g_c^2, but in the remainder of the paper we absorb the gauge coupling in the gauge fields except in the experimental Sec. V. This axion is the one setting θ at zero; thus one needs the axion-gluon-gluon anomalous coupling for which the color anomaly of J^5_μ should exist. This kind of symmetry Γ is the PQ symmetry.

The axion is introduced as the Goldstone boson degree of a spontaneously, broken global $U(1)_{PQ}$ symmetry in renormalizable gauge models (Peccei and Quinn, 1977a; Kim, 1985) and/or as a pseudoscalar degree in a more fundamental theory where the axion interaction arises as a nonrenormalizable anomalous interaction in the effective low energy theory. The most compelling nonrenormalizable interaction was observed in the compactification of ten-dimensional (10D) superstring models (Witten, 1984). Below we treat the axion as being present as a dynamical degree at the electroweak scale, whether it arises from spontaneously broken PQ symmetry or from a more fundamental theory with a nonrenormalizable anomalous coupling, and focus on QCD interactions containing the axion degree, $a=\theta F_a$. Then we collectively write the most general form of its interactions: the c_1 term is the derivative coupling respecting the PQ shift symmetry, the c_2 term is the phase in the quark mass matrix, and the c_3 term is the anomalous coupling or the determinental interaction $\mathcal{L}_{\rm det}$,

$$\begin{aligned}\mathcal{L}_\theta &= \frac{1}{2}f_S^2\partial^\mu\theta\partial_\mu\theta-\frac{1}{4g_c^2}G^a_{\mu\nu}G^{a\mu\nu}+(\bar{q}_L i\not{D}_{q_L}+\bar{q}_R i\not{D}_{q_R})\\ &\quad + c_1(\partial_\mu\theta)\bar{q}\gamma^\mu\gamma_5 q-(\bar{q}_L m q_R e^{ic_2\theta}+\text{H.c.})\\ &\quad + c_3\frac{\theta}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}(\text{or }\mathcal{L}_{\rm det})+c_{\theta\gamma\gamma}\frac{\theta}{32\pi^2}F^i_{{\rm em}\,\mu\nu}\tilde{F}^{i\mu\nu}_{\rm em}\\ &\quad + \mathcal{L}_{{\rm leptons},\theta},\end{aligned} \tag{19}$$

where $\theta=a/f_S$ with the axion decay constant f_S up to the domain wall number ($f_S=N_{\rm DW}F_a$) and q is the fermion matrix composed of $SU(3)_c$ charge-carrying fields. When the singlet scalar fields are easier to discuss, we use f_S, and when the anomaly term is easier to discuss, we use F_a. $\mathcal{L}_{{\rm leptons},\theta}$ is the axion interaction with leptons. c_1, c_2, and c_3 are pregiven coupling constants below the axion scale f_S with the mass parameter m defined to be real and positive below the electroweak scale. Then, the determinental interaction can be used instead of the c_3 term,

$$\mathcal{L}_{\rm det}=-2^{-1}ic_3\theta(-1)^{N_f}\frac{e^{-ic_3\theta}}{K^{3N_f-4}}{\rm Det}(q_R\bar{q}_L)+\text{H.c.}, \tag{20}$$

where we multiplied the overall interaction by θ in the small-θ region and require the periodicity condition $c_3\theta=c_3\theta+2\pi$. The periodicity can be accommodated automatically if we replace $-2^{-1}ic_3\theta$ by 1, but then we must add a constant so that it vanishes at $\theta=0$. The sign is chosen such that the potential is a minimum at $\theta=0$ (Vafa and Witten, 1984). With the fixed phases, the c_3 term is given from the QCD vacuum structure (4), which does not have any dimensional coupling. But the instanton physics necessarily introduces the instanton sizes and hence a kind of QCD scale K for the interaction respecting the chiral transformation property for a flavor singlet operator $\mathcal{L}_{\rm det}$. We use either the anomaly term or $\mathcal{L}_{\rm det}$. The θ dependence of the form (20) is $-c_3\theta\sin(c_3\theta)$, which has the parity symmetry $\theta\to-\theta$. The Fourier expansion satisfying these constraints is

$$\begin{aligned}-2^{-1}c_3\theta\sin(c_3\theta)&=-2^{-1}[1-\cos(c_3\theta)]\\ &\quad+\sum_{n=2}a_n\cos(nc_3\theta),\end{aligned}$$

where the Fourier coefficients satisfy $\sum_{n=1}^{\infty}n^{2i}a_n=\delta_{i0}$. Neglecting the $n\geqslant 2$ terms, we use just the $\cos(c_3\theta)$ dependence.

In the defining phase Eq. (19), the PQWW axion is given by $c_1=0$, $c_2\neq 0$, and $c_3=0$, the KSVZ axion by $c_1=0$, $c_2=0$, and $c_3\neq 0$, the model-independent axion (Witten, 1984) in superstring models by $c_1=0$, $c_2=0$, and $c_3\neq 0$, and the DFSZ axion by $c_1=0$, $c_2\neq 0$, and $c_3=0$. In general, axion models from high energy will have $c_2\neq 0$ and $c_3\neq 0$, and the shift symmetry allows $c_1\neq 0$ in a different basis. For simplicity, we discuss Eq. (19) for one-flavor QCD first. For N_f flavors, both c_i and θ are defined from $N_f\times N_f$ matrices in addition to the anomalous coupling and hence the axion is included in Tr θ, which also contains the η' meson part of QCD. For N_f flavors, $c_i\theta$ must be replaced by Tr $c_i\theta$. For the following discussion, we refer to one-flavor QCD, but in Sec. III.B in the axion mass estimation we present the full N_f flavor QCD result with the chiral symmetry breaking taken into account.

For the case of the axion mass, the c_1, c_2, and c_3 terms may be relevant, but only the combination c_2+c_3 appears. This Lagrangian has a shift symmetry $a\to a+\text{const}$, which reparametrizes the couplings between c_1, c_2, and c_3. Explicitly, the axion-field-dependent changes of the quark fields $q_L\to e^{i\alpha a(x)}q_L$ and $q_R\to e^{-i\alpha a(x)}q_R$ give $c_1\to c_1-\alpha$, $c_2\to c_2-2\alpha$, $c_3\to c_3+2\alpha$, and it must give the same physics, i.e., (Georgi, Tomaras, and Pais, 1981; Kim, 1987),

$$\begin{aligned}&\Gamma_{1PI}[a(x),A^a_\mu(x);c_1,c_2,c_3,m,\Lambda_{\rm QCD}]\\ &\quad=\Gamma_{1PI}[a(x),A^a_\mu(x);c_1-\alpha,c_2-2\alpha,c_3\\ &\qquad+2\alpha,m,\Lambda_{\rm QCD}].\end{aligned} \tag{21}$$

The reparametrization symmetry dictates the nonderivative couplings satisfying $c_2+c_3=\text{const}$, which is one reason that we use $\theta=\theta_{\rm QFD}+\theta_{\rm QCD}=\theta_0+\theta_{\rm weak}$ as a physical parameter in axion models. Usually, transfer of all couplings of axions to the coefficient of $G\tilde{G}$, the axion decay constant F_a and θ are defined. Instead, if we use f_S (defined to be the VEV of the singlet Higgs field

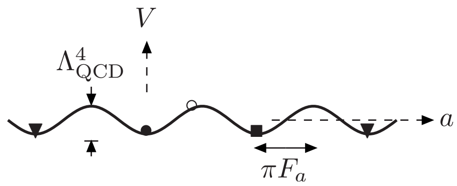

FIG. 7. The case with $N_{\rm DW}=3$ where three vacua are distinguished.

σ), there exists the coefficient c_3 defined in Eq. (19). The triangle diagrams may give an integer times θ and the instanton potential comes to the original value by a θ shift of $2\pi/(c_2+c_3)$, with $c_2+c_3=N_{\rm DW}$ not necessarily 1 in the pseudoscalar field space. Thus, this integer is called the domain wall number $N_{\rm DW}$ (Sikivie, 1982),

$$N_{DW}=|c_2+c_3|=\mathrm{Tr}\,\Gamma(f_{\rm colored})\ell(f_{\rm colored}), \tag{22}$$

where the trace is taken over all heavy and light quarks and ℓ is the index of the $\mathrm{SU}(3)_c$ representation of colored fermions and the PQ charge is given for the left-handed chiral representations. The height of the potential is $O(\Lambda^4_{\rm QCD})$ of the non-Abelian gauge interaction, which is shown in Fig. 7 with the domain wall number $N_{\rm DW}=3$: the bullet, the square, and the triangle denote different vacua. Two important properties of axions in *CP*-conserving theories are (i) the periodic potential with the period $2\pi F_a$ where F_a is defined in (19) with $F_a\equiv f_S/N_{\rm DW}$, and (ii) the minima at $a=0, 2\pi F_a, 4\pi F_a, \ldots$. This determines the cosine form of the potential. There exists the axion mixing with quark condensates as discussed in more detail later.

The derivative coupling, i.e., the c_1 term, can never contribute to the PQ symmetry breaking effect, especially to the axion mass. This axion gets its mass from the θ anomaly term which breaks the PQ symmetry. The global symmetry is not broken by the derivative term, which therefore cannot contribute to the axion mass. From the reparametrization invariance (21), the combination c_2+c_3 is the correct combination for the axion mass, as shown below. This derivation is included with a more complicated expression in the SUSY extension, but we show the c_2+c_3 dependence in this supergravity framework because it is the underlying symmetry in many axion models. Some of the following discussion is derived from Choi, Kim, and Nilles (2007).

1. Supersymmetrization

We now discuss the reparametrization invariance with the SUSY generalization. In the $\mathcal{N}=1$ SUSY models with chiral fields z, there are the superpotential $W(z)$ and the gauge kinetic function $f(z)$, both of which are holomorphic functions of z. The superpotential gives the c_2 term and the gauge kinetic function gives the anomaly term c_3. The PQ-invariant Lagrangian, the c_1 part, has shift symmetry under the shift of the axion supermultiplet: $A\to A+i\times\mathrm{const}$. This derivative coupling must appear from the D terms in SUSY models, i.e., through the Kähler potential. The real Kähler potential $K(z,z^*)$ must respect the PQ symmetry in the form of $A+\bar{A}$,

$$K=K_0[A+\bar{A}]+\{Z_q[A+\bar{A}]\bar{q}_1q_2+\mathrm{H.c.}\}, \tag{23}$$

where the ϑ^0 components of the fields are implied and the q's denote quark supermultiplets,

$$q=\varphi_q+i\vartheta\psi_q, \tag{24}$$

with the anticommuting variable ϑ. Here we used ϑ for the anticommuting Grassmann number since θ in this review is reserved for the axion $\theta=a/F_a$.

B. Axion mass

The axion mass arises from the anomaly coupling $\theta G\tilde{G}$. In this section, first we show that only the c_2 and c_3 couplings are relevant for the axion mass, and then we present the axion mass in the broken phase of the chiral symmetry. With SUSY, the discussion is a bit tricky, because the axion remains massless due to the *massless gluino* (as in the massless up-quark case with a spontaneously broken PQ symmetry). For the axion mass, therefore SUSY must be broken and here one has to look at how all supergravity terms contribute to the axion mass. Nevertheless, we have the master formula (21) for the axion, which must be valid even when SUSY is broken. In this regard, SUSY is not special for the axion mass; the chief constraint is only the anomaly consideration. Thus, the following discussion applies even without SUSY, but we discuss the axion mass in detail with the SUSY generalization to include the gluino effects and hence the c_1-type derivative couplings to matter (quarks) and gauginos (gluinos).

We have noted that there exists an anomaly coupling of the η' meson which is the mechanism solving the old U(1) problem of QCD. In addition to η', the axion a is introduced in the anomaly coupling and hence one must consider the mixing of η' and the axion (Bardeen and Tye, 1978; Baluni, 1979; Kim and Kim, 2006).

The c_3 term is the anomaly coupling of the axion, and we normalize the anomaly as the coefficient of $\epsilon^{\alpha\beta\gamma\delta}\varepsilon_{1\alpha}\varepsilon_{2\beta}k_{1\gamma}k_{2\beta}$. With this normalization, from $\epsilon_{\alpha\beta\gamma\delta}\partial^\alpha A^\beta\partial^\gamma A^\delta$ leading to $-\epsilon^{\alpha\beta\gamma\delta}k_{1\alpha}\varepsilon_{1\beta}k_{2\gamma}\varepsilon_{2\delta}$, the c_3 term anomaly is defined with $\mathcal{A}_3=1$.

It can be shown that, using the Kähler potential (23), the kinetic energy terms of fermions contain (Cremmer, Ferrara, Girardello, and van Pröyen, 1983; Nilles, 1984)

$$\sum_\psi Z_q(\bar{\psi}i\partial\!\!\!/\psi+\tfrac{1}{6}B_\mu\bar{\psi}\gamma^\mu\gamma_5\psi+\tfrac{1}{2}Y_{q,\mu}\bar{\psi}\gamma^\mu\gamma_5\psi)$$
$$+\sum_\lambda(\bar{\lambda}i\partial\!\!\!/\lambda-\tfrac{1}{2}B_\mu\bar{\lambda}\gamma^\mu\lambda), \tag{25}$$

where B_μ and $Y_{q,\mu}$ come from the auxiliary components of real K_0 and Z_q, respectively. In terms of the real parts $\mathcal{R}$ and $\mathcal{Y}$ of K_0 and $\mathcal{Z}$ [redefined from Z_q of Eq. (23)], we obtain

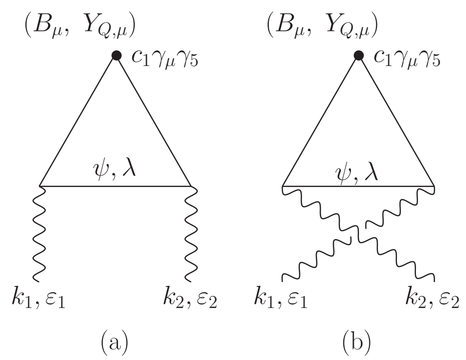

FIG. 8. The Feynman diagrams for generating anomalous $\theta G\tilde{G}$ couplings from c_1 for a fermion with mass m. For c_2, we replace $c_1\gamma_\mu\gamma_5$ by $c_2 m\gamma_5$.

$$B_\mu = \frac{i}{2}\left(\frac{\partial K_0}{\partial A}\partial_\mu A - \frac{\partial K_0}{\partial \bar{A}}\partial_\mu \bar{A}\right) = -\left(\frac{\partial \mathcal{R}}{\partial A}\partial_\mu a\right), \qquad (26)$$

$$Y_{q,\mu} = \frac{i}{2}\left(\frac{\partial K_0}{\partial A}\partial_\mu A - \frac{\partial K_0}{\partial \bar{A}}\partial_\mu \bar{A}\right) - i\left(\frac{\partial \ln Z_q}{\partial A}\partial_\mu A - \frac{\partial \ln Z_q}{\partial \bar{A}}\partial_\mu \bar{A}\right) = 2\left(\frac{\partial \ln \mathcal{Y}}{\partial A}\right)\partial_\mu a, \qquad (27)$$

where

$$G = -3\ln\left(\frac{-K}{3}\right) + \ln|W|^2, \quad K = -e^{-K_0/3}, \quad Z_q = e^{\mathcal{Z}},$$
$$K_0 = \mathcal{R} + iK_{0I} = \mathcal{R}, \quad Z_q = \mathcal{Y} + i\mathcal{I} = \mathcal{Y}. \qquad (28)$$

The c_3 term is an anomaly term. In addition to the c_3 term, the c_1 and c_2 couplings via loops of Fig. 8 will also generate anomaly terms. The derivative coupling, if it ever has to contribute to the axion mass, should do so via the anomaly through loops. In Fig. 8, the couplings for the triangle diagrams are represented in terms of c_1 and c_2. In supergravity models, we consider B_μ and $Y_{q,\mu}$ couplings, which are nothing but c_1. Consider a fermion with mass m. The derivative coupling through Fig. 8 contains the anomaly coupling through the coefficient of $\epsilon^{\alpha\beta\gamma\delta}\varepsilon_{1\alpha}\varepsilon_{2\beta}k_{1\gamma}k_{2\beta}$ [see, for example, Georgi, Tomaras, and Pais (1981)],

$$\mathcal{A}_1 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{-4f(x_1,x_2;q,k_1,k_2)}{m^2 - f(x_1,x_2;q,k_1,k_2)}, \qquad (29)$$

where

$$f = (x_1+x_2)(1-x_1-x_2)q^2 + 2x_1(1-x_1-x_2)q\cdot k_1 + x_1^2 k_1^2.$$

Also, the quark mass term of Fig. 8 gives

$$\mathcal{A}_2 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{2m^2}{m^2 - f(x_1,x_2;q,k_1,k_2)}. \qquad (30)$$

From Eqs. (29) and (30), we construct

$$\frac{1}{2}\mathcal{A}_1 + \mathcal{A}_2 = \int_0^1 dx_1 \int_0^{1-x_1} 2dx_2 = 1. \qquad (31)$$

When we calculate the axion mass in the real and positive quark mass basis as usual, the anomaly $(a/F_a)\{G\tilde{G}\}$ coupling (including the loop effect) is the sole source of the axion mass. In this basis, and also in any basis due to the reparametrization-invariant combination c_2+c_3, we do not have to discuss the contributions of the derivative couplings toward the axion mass. Even though the derivative coupling generates the anomaly, because it is derivative it does not contribute to the axion mass.

For one-flavor QCD, we can check the above statement explicitly using Eqs. (29)–(31). In the following two limiting cases, the integrals are easily computed as, using Eqs. (29) and (30): case (i), $m \ll \Lambda_{\rm QCD}$:

$$\Gamma_{1PI} = \frac{1}{16\pi^2}\epsilon^{\alpha\beta\gamma\delta}k_{1\gamma}k_{2\beta}\left[c_3 + 2c_1 + O\left(\frac{m^2}{k^2}\right)\right], \qquad (32)$$

case (ii), $m \gg \Lambda_{\rm QCD}$:

$$\Gamma_{1PI} = \frac{1}{16\pi^2}\epsilon^{\alpha\beta\gamma\delta}k_{1\gamma}k_{2\beta}\left[c_3 + c_2 + O\left(\frac{k^2}{m^2}\right)\right]. \qquad (33)$$

Consider the quark mass term and the one-flavor determinental interaction with the quark condensation, $\langle \bar{q}_L q_R\rangle \sim \Lambda_{\rm QCD}^3 e^{i\eta'/f}$. Then the potential takes the form

$$V = m\langle \bar{q}_L q_R\rangle e^{ic_2\theta} + \text{H.c.} + (c_3 + c_1\mathcal{A}_1 + c_2\mathcal{A}_2)\{G\tilde{G}\}. \qquad (34)$$

For the anomaly combination $c_3+c_1\mathcal{A}_1+c_2\mathcal{A}_2$, the reparametrization invariance Eq. (21) transforms $c_3+c_1\mathcal{A}_1+c_2\mathcal{A}_2$ to $c_3+2\alpha+(c_1-\alpha)\mathcal{A}_1+(c_2-2\alpha)\mathcal{A}_2=c_3+c_1\mathcal{A}_1+c_2\mathcal{A}_2$ where (31) is used, i.e., it is reparametrization invariant.

For case (i), we consider the light quark below the scale $\Lambda_{\rm QCD}^4$. Thus, we have

$$V = mv^3\cos\left(\frac{\eta'}{f} - c_2\theta\right) + \Lambda_{\rm QCD}^4\cos\left((c_3+2c_1)\theta + \frac{\eta'}{f}\right),$$

for which we choose $c_1=0$. (If we keep c_1, we must consider the kinetic mixing of a and η'.) Integrating out the heavy η' field as $\eta'/f = -c_3(a/f_S)$ from the $\Lambda_{\rm QCD}^4$ term, which is the larger one, we obtain

$$V = mv^3\cos\left((c_2+c_3)\frac{a}{f_S}\right),$$

from which

$$m_a \sim \sqrt{m\Lambda_{\rm QCD}^3}\frac{|c_2+c_3|}{f_S}. \qquad (35)$$

The quarks u, d, and s belong to this category.

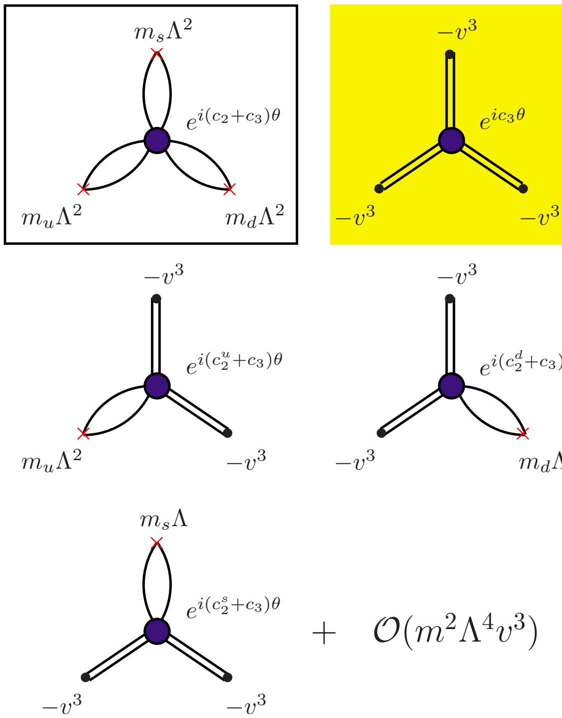

FIG. 9. (Color online) The 't Hooft determinental interaction. ● denotes the quark condensation and × denotes the insertion of the current quark mass. The diagram highlighted predominantly contributes to the η' mass, and $O(m_u m_d)$ is neglected.

For case (ii), the heavy quark does not condense, and integrating out the heavy quark gives

$$V = \Lambda_{\rm QCD}^4 \cos\left[(c_3+c_2)\frac{a}{f_S}\right]$$

from which the axion mass is given by

$$m_a \sim \Lambda_{\rm QCD}^2 \frac{|c_2+c_3|}{f_S}.$$

Again the axion mass depends only on the combination c_2+c_3. Heavy quarks above the chiral symmetry breaking scale c, b, and t give the c_2 term and vectorlike heavy quarks above the electroweak scale give the c_3 term when we write Eq. (19) just below the electroweak scale.

1. Axion mass with light quarks

In the real world, there exist three light quarks whose masses are much smaller than the QCD scale $\Lambda_{\rm QCD}$, and therefore the axion mass has the form anticipated in Eq. (35). Even though there are two light quarks the axion mass dependence has the form $\sqrt{m}$ as a result $F_a \gg f_\pi$. This is because of the way in which the leading term is picked up from the anomalous determinental interaction (Kim and Kim, 2006) as shown in Fig. 9.

In fact, this is obtained simply by noting that the instanton interaction is a U(1) singlet (Kim, 1987). Suppose we integrate out quark fields; then the quark mass parameters appear in the effective interaction as shown in the first diagram of Fig. 9. In this vacuum with a massless quark theory, the tunneling amplitude vanishes so that the strength of the first diagram must be proportional to m_q. With three quarks, we can generalize it as $1/(m_u^{-1}+m_d^{-1}+m_s^{-1})$. Suppose that there are only gluons and a very light axion a at low energy. Integrating out heavy fields, we are left with the flavor-independent coupling $aG\tilde{G}$. Here we are not considering η' even below the quark condensation scale. If quarks are added, the flavor singlet coupling $aG\tilde{G}$ can be split into quark mass terms with $\alpha_u \propto x/m_u$, $\alpha_d \propto x/m_d$, $\alpha_s \propto x/m_s$, etc., as if the quarks are not integrated over, $m_u \bar{u}_L u_R e^{i\alpha_u\theta} + m_d \bar{d}_L d_R e^{i\alpha_d\theta} + \cdots$, which shows that the flavor singlet coupling is of order $O(a/F_a)$. Then, even below the chiral symmetry breaking scale, we have the PQ charges proportional to $1/m_q$. With this definition of quark charges, the axion mass comes from integrating out $G\tilde{G}$, and is proportional to $\alpha_u+\alpha_d+\alpha_s$ which is $\sim m_u m_d m_s/[m_s(m_u+m_d)+m_u m_d]$ first shown for the PQWW axion (Baluni, 1979). This is true even in the heavy quark KSVZ-type axion models. Even if the light quarks do not have the same PQ charge as in some variant axion models (Krauss and Wilczek, 1986; Peccei, Wu, and Yanagida, 1986; Bardeen, Peccei, and Yanagida, 1987; Kim and Lee, 1989; Hindmarsh and Moulatsiotis, 1997), the axion mass has the same final form due to the reparametrization invariance, which will be shown below. As a result the axion mass formula we write below is quite general.

However, there was an assumption in this statement: η' was integrated out. So it is necessary to include η' to obtain a more accurate axion mass. The light mesons and axion interactions must appear from those of Fig. 9. In this framework, however, the flavor singlet condition must be invoked as a constraint. (This flavor singlet condition is the anomaly matching without η'.) Along the way, we would like to see how the $\sqrt{m}$ dependence arises below the chiral symmetry breaking scale in the KSVZ model.

In the presence of vectorlike heavy quarks, the heavy fields are integrated out; their sole effect is encoded in the low-energy effective theory as nonrenormalizable couplings suppressed by F_a, e.g., in the anomalous c_3-type couplings with the SM gauge bosons. It is assumed that the heavy quark does not condense, since the QCD coupling is very small due to the asymptotic freedom at the heavy-quark Compton wavelength scale and there does not exist a strong force to attract heavy quarks. Below the heavy quark scale, there are no massless mesons composed of heavy quarks. Therefore, the general form of the axion interaction, Eq. (19), is valid at low energy. First, the determinental interaction has the same chiral symmetry behavior as that of the anomaly term, and the anomaly term is removed in favor of the determinental interaction to include η' explicitly. Second, we choose the basis where the u and d quark masses are real. Since the strange quark mass is known to be below the QCD scale, we must include the strange quark with real and positive mass also in the instanton interaction. For simplicity, π^0 and η', arising from quark condensations $\bar{u}u$ and $\bar{d}d$ with decay constants f_π and $f_{\eta'}(\approx f_\pi)$ (Gell-Mann, Oakes, and Renner, 1968), are considered explicitly but with the η meson frozen. The effects of heavy quarks are included in the c_3 term. If we keep c_1, the kinetic mixing of mesons and axion is present, due to the PCAC relation $\langle 0|J^\mu_{5i}(x)|\text{meson}_j(k)\rangle = -ik^\mu f_i^2 e^{-ik\cdot x}\delta_{ij}$where $J^\mu_{5i}\sim\bar{q}\gamma^\mu\gamma_5 T_i q$. This would modify the axion mass, and hence it is easiest to calculate the axion mass by choosing the reparametrization parameter α such that $c_1=0$. In this basis, denoting π^0, η', and a in terms of dimensionless fields, $\theta_\pi=\pi^0/f_\pi$, $\theta_{\eta'}=\eta'/f_{\eta'}$, $\theta=a/F_a$, we obtain the following effective interaction below the chiral symmetry breaking scale:

$$\mathcal{L} = -m_u\langle\bar{u}_L u_R\rangle e^{i[(\theta_\pi+\theta_{\eta'})+c_2^u\theta]} - m_d\langle\bar{d}_L d_R\rangle e^{i[(-\theta_\pi+\theta_{\eta'})+c_2^d\theta]} + \text{H.c.} + \mathcal{L}_{\text{det}}, \tag{36}$$

where $\mathcal{L}_{\text{det}}$ is given in Eq. (20),

$$\mathcal{L}_{\text{det}} = (-1)^{N_f}K^{-5}(\langle\bar{u}_L u_R\rangle\langle\bar{d}_L d_R\rangle\langle\bar{s}_L s_R\rangle e^{i(2\theta_{\eta'}-c_3\theta)} + \cdots + \text{flavor singlet constraint}) + \text{H.c.}, \tag{37}$$

and K has the mass dimension arising from QCD instanton physics. The above form is consistent with the anomaly (32) with $c_1=0$. Note that the log det form in the effective Lagrangian was used by Veneziano (1979); Witten (1979, 1980); Di Vecchia and Veneziano (1980); and Di Vecchia *et al.* (1981) from the $1/N_c$ expansion consideration, but we use Eq. (37) because of its simplicity in the diagrammatic expansion. The sign of the first diagram inside the box in Fig. 9 is determined to be negative without the weak *CP* violation (Vafa and Witten, 1984). The QCD vacuum with the flavor independence of light quarks without the determinental interaction chooses $m_q\langle\bar{q}q\rangle=-|m_q|v^3$ and we choose the sign of all quark masses to be positive so that $\langle\bar{q}q\rangle=\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=-v^3$ (Dashen, 1971; Langacker and Pagels, 1973, 1979; Gasser and Leutwyler, 1982). Equation (37) is the instanton interaction of Fig. 9, which gives $\Lambda^4, m_u\Lambda^3, m_d\Lambda^3,\ldots$ by many ways of closing quark lines, shown in Fig. 9, but here one must invoke the *flavor singlet constraint*. The dominant term is the second diagram highlighted, which is flavor singlet and is the main source for the η' mass.

Now we restrict ourselves to the two-flavor case. For the axion, the key diagrams are those in the second line of Fig. 9. If there is more than one QCD axion, then the $O(m_u m_d)$ diagram will be important at the next-level axion mass. Integration over the instanton size includes large instantons, covering the chiral-symmetry-breaking range where mesons appear as dynamical degrees, where we invoke the flavor singlet constraint. The effective interaction Hamiltonian of θ_π, $\theta_{\eta'}$, and $\theta=a/f_S$ can be written, using the reparametrization invariance (21) with $N_f=3$ and η fixed, as [Huang (1993) and Kim and Kim (2006)]

$$\begin{aligned}-V &= m_u v^3\cos(\theta_\pi+\theta_{\eta'}) + m_d v^3\cos(-\theta_\pi+\theta_{\eta'}) \\ &+ \frac{v^9}{K^5}\cos[2\theta_{\eta'}-(c_2^u+c_2^d+c_3)\theta] \\ &+ m_u\frac{\Lambda_u^2 v^6}{K^5}\cos[-\theta_\pi+\theta_{\eta'}-(c_2^u+c_2^d+c_3)\theta] \\ &+ m_d\frac{\Lambda_d^2 v^6}{K^5}\cos[\theta_\pi+\theta_{\eta'}-(c_2^u+c_2^d+c_3)\theta],\end{aligned} \tag{38}$$

where Λ_u and Λ_d are parameters describing the result of the Feynman and instanton size integrations. The $(-1)^{N_f}$ term is canceled by the fermion loop or $(-v)$ factors. If $m_u=m_d$, Λ_u and Λ_d are equal. For $m_u\neq m_d$, Λ_u and Λ_d must be different. The instanton interaction is flavor independent, which should be respected in the interaction (38). The m_u and m_d linear terms from the determinental interaction should be flavor independent, i.e., $m_u\Lambda_u^2+m_d\Lambda_d^2$=flavor independent. Since it vanishes if one quark is massless, it must be a function of $m_u m_d$. Thus, the instanton size integration with current quark masses must give $m_u\Lambda_u^2+m_d\Lambda_d^2=2m_u m_d\tilde{L}^2/(m_u+m_d)$, which vanishes if any quark is massless. This is because the original gluon anomaly term $\{G\tilde{G}\}$ does not distinguish flavors, and the smallness of the current quark masses enables us to expand the 't Hooft determinental interaction in terms of powers of the current quark masses. Then, the 3×3 mass matrix M^2 of a, η', and π^0, taking into account the chiral symmetry breaking and the solution of the U(1) problem, is given as

$$M^2_{a,\eta',\pi^0} = \begin{pmatrix} c^2[\Lambda^4_{\eta'}+2\mu\Lambda^3_{\text{inst}}]/F^2 & -2c[\Lambda^4_{\eta'}+\mu\Lambda^3_{\text{inst}}]/f'F & 0 \\ -2c[\Lambda^4_{\eta'}+\mu\Lambda^3_{\text{inst}}]/f'F & [4\Lambda^4_{\eta'}+2\mu\Lambda^3_{\text{inst}}+m_+v^3]/f'^2 & -m_-v^3/ff' \\ 0 & -m_-v^3/ff' & (m_+v^3+2\mu\Lambda^3_{\text{inst}})/f^2 \end{pmatrix}, \tag{39}$$

where $c=c_2^u+c_2^d+c_3$, $F=f_S$, $f=f_\pi$, $f'=f_{\eta'}$, $\Lambda^4_{\eta'}=v^6/K'^2$, $\Lambda^3_{\text{inst}}=\tilde{L}^2v^3/K^2$, $m_+=m_u+m_d$, $m_-=m_d-m_u$, and

$$\mu = \frac{m_u m_d}{(m_u + m_d)}. \tag{40}$$

Certainly, Eq. (39) realizes the solution of the U(1) problem due to the $\Lambda_{\eta'}^4$ term in the (22) component. In the limit $f/F, f'/F \ll 1$, we obtain

$$m_{\pi^0}^2 \simeq \frac{m_+ v^3 + 2\mu\Lambda_{\rm inst}^3}{f_\pi^2}, \tag{41}$$

$$m_{\eta'}^2 \simeq \frac{4\Lambda_{\eta'}^4 + m_+ v^3 + 2\mu\Lambda_{\rm inst}^3}{f_{\eta'}^2}, \tag{42}$$

$$m_a^2 \simeq \frac{c^2}{F^2}\frac{Z}{(1+Z)^2} f_\pi^2 m_{\pi^0}^2 (1+\Delta), \tag{43}$$

where

$$\Delta = \frac{m_-^2}{m_+}\frac{\Lambda_{\rm inst}^3 (m_+ v^3 + \mu\Lambda_{\rm inst}^3)}{m_{\pi^0}^4 f_\pi^4}. \tag{44}$$

In this form, the π mass has the standard $m_+ v^3$ plus the instanton contribution to the light quark mass (Kaplan and Manohar, 1986; Choi, Kim, and Sze, 1988). From Eqs. (41) and (42), we estimate the parameter $\Lambda_{\eta'}^4$ which is the source of the solution of the U(1) problem: $\Lambda_{\eta'}^4 = (f_{\eta'}^2 m_{\eta'}^2 - f_\pi^2 m_\pi^2)/4 \approx (202\ \text{MeV})^4$ with $f_{\eta'} \simeq 86$ MeV and $f_\pi \simeq 93$ MeV. In any axion model, this form is valid with $|c| = N_{\rm DW}$. Using the standard definition on the axion decay constant $F_a = F/c$, we obtain

$$m_a^2 \simeq \frac{Z}{(1+Z)^2}\frac{f_\pi^2 m_\pi^2}{F_a^2}(1+\Delta). \tag{45}$$

Even though the instanton diagrams of Fig. 9 contain the summation of linear quark mass diagrams, the diagonalization process with mesons signals the predominant contribution of the lightest quark. The flavor singlet condition discussed before chooses the following linear quark mass dependence:

$$\mu = \left(\frac{1}{m_u} + \frac{1}{m_d} + \cdots\right)^{-1}. \tag{46}$$

Neglecting instanton contribution to the current quark masses, we obtain $m_a \approx 0.60$ eV $(10^7\ \text{GeV}/F_a)$, for the mass ratio $Z \simeq 0.48$ as summarized by Manohar and Sachrajda (2008). An earlier frequently cited Z is 5/9 (Weinberg, 1977; Gasser and Leutwyler, 1982). The correct axion mass has to include the current quark mass change due to instantons. However, the resulting estimate of Δ turns out to be small.

2. Comparison with old calculations

Now we comment on the old anomaly matching condition. If any quark mass is zero, there exists an exact symmetry $a \to a + \text{const}$, i.e., the axion is massless, above the chiral-symmetry-breaking scale. Below the chiral-symmetry-breaking scale, it is likely that this condition is satisfied. We denote the original current as $J_{\rm PQ}^\mu$. This current is anomalous above the chiral-symmetry-breaking scale, $\partial_\mu J_{\rm PQ}^\mu = (N_Q/32\pi^2) G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$, where N_Q is the number of heavy quarks with $\Gamma = 1/2$. Below the chiral-symmetry-breaking scale, we considered two pseudoscalar mesons which have anomalous couplings: η' and a. The global anomaly matching condition will work if there is no chiral symmetry breaking ('t Hooft, 1979). For chiral symmetry breaking, there are no massless fermions and we consider only color singlet mesons below the chiral symmetry breaking scale. Thus, the bosonic current must be anomaly-free after all heavy fields including η' are integrated out, i.e., we consider an anomaly-free current J_a^μ instead of $J_{\rm PQ}^\mu$ below the chiral-symmetry-breaking scale (Kim, 1987),

$$J_a^\mu = J_{\rm PQ}^\mu - \frac{N_Q}{2(1+Z)}(\bar{u}\gamma_\mu\gamma_5 u + Z\bar{d}\gamma_\mu\gamma_5 d), \tag{47}$$

where the divergence of the second current gives a singlet pseudoscalar density so that the axion does not mix with π^0. Equation (45) with Δ shows that the finite η' mass enters into the a-η' mixing.

3. Mesons without axions

Even if there is no axion, we can diagonalize the mass matrix. If $m_u = 0$, one starts with an exact up-quark chiral transformation, which leads to a Goldstone boson θ in the vacuum, $\langle \bar{u}u \rangle \neq 0$. This Goldstone boson couples to a neutron through $(c_1 \partial_\mu \theta)\bar{n}\gamma^\mu\gamma_5 n$. In reality, η' obtains mass by the anomaly, and the symmetry remains unbroken: it is the phase symmetry of $\langle \bar{u}u \rangle$. Therefore, any violation of the shift symmetry must be such that it goes away in the limit $\mu \to 0$; this is Dashen's theorem (Dashen, 1971). Thus, from Eq. (38) we obtain the VEVs of η' and π^0 for a small $\bar{\theta}$,

$$\frac{\langle \eta' \rangle}{f'} \simeq -\frac{\bar{\theta}}{2}(1+Z)\frac{\mu v^3}{\Lambda_{\eta'}^4},$$
$$\frac{\langle \pi^0 \rangle}{f} \simeq \bar{\theta}(1+Z)\frac{\mu}{m_+}. \tag{48}$$

The VEVs of η' and π^0 are vanishing if $\bar{\theta} = 0$ or any quark mass is zero. In addition, we can estimate the η' properties from the interaction $(v^9/K^5)\cos(2\eta'/f_{\eta'})$, where $f_{\eta'}$ is the η' decay constant and K has a mass dimension. This comes from the diagram of Fig. 9. Comparing $\pi^0 \to 2\gamma$ and $\eta' \to 2\gamma$ decay widths, 7.74 eV and 4.3 keV, respectively (Amsler *et al.*, 2008), we obtain $f_{\eta'}^2 = (4/3)(m_{\eta'}^3/m_\pi^3)[\Gamma(\pi^0 \to 2\gamma)/\Gamma(\eta' \to 2\gamma)] f_\pi^2$, or $f_{\eta'} \approx 86$ MeV. Fitting to the η' mass, we obtain $K = (v^9/f_{\eta'}^2 m_\eta'^2)^{1/5} = 240$ MeV.

4. The $\theta = 0$ vacuum with axions

We have shown above that the Lagrangian (38) chooses $\theta = 0$ in *CP*-conserving theories if $\theta_\pi = 0$ and

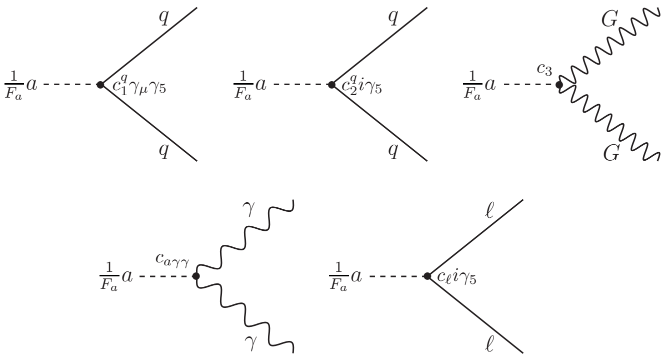

FIG. 10. The Feynman diagrams of axion couplings. G and γ are the gluon and photon, respectively. c_3 and $c_{a\gamma\gamma}$ couplings are anomalous.

$\theta_{\eta'}=0$, which is determined by QCD dynamics. However, if CP symmetry is broken, the vacuum value of θ is shifted from the $\theta=0$ value by the presence of any linear term of $\theta_\pi, \theta_{\eta'}$, or/and θ. The meson potential is invariant under CP symmetry with $CP(\pi)=CP(\eta')=CP(a)=-1$. Such linear terms are generated by considertion of CP-violating phases and chirality-flipping $(L\leftrightarrow R)$ insertions. As a result linear terms of a are generated by combining the 't Hooft determinental interaction and CP-violating weak interactions. Linear terms of π and η' can also be generated by considering the weak interactions alone without the determinental interaction, but the conditions of the flavor singlet, chirality-flipping $(L\leftrightarrow R)$, and CP-violating effects do not occur at the one-loop level. In the SM with the Kobayashi-Maskawa CP violation, Ellis and Gaillard (1979) showed that a finite correction occurs at the fourth order, $O(\alpha^2)$, leading to a small NEDM, but infinite corrections occur from $O(\alpha^7)$. These can give rise to a linear term of π. In the SM, the pioneering calculation with axions has been performed in chiral perturbation theory to obtain $\theta \lesssim 10^{-17}$ (Georgi, Kaplan, and Randall, 1986; Georgi and Randall, 1986). The estimated θ, however, is far below the current experimental limit of 10^{-11}.

C. Axion couplings

The axion interactions are given in Eq. (19) which are shown in Fig. 10 where we have not drawn the $aW\tilde{W}$ and $aZ\tilde{Z}$ diagrams which are orthogonal to the $a\gamma\tilde{\gamma}$. The diagrams of Fig. 10 are complete for the low-energy axion phenomenology, where the suppression factor $1/F_a$ by the axion decay constant is explicitly shown.

1. Axion-hadron coupling

When we discuss axion-hadron interactions, which are relevant low-energy laboratory experiments and physics at the core of supernovae, we must integrate out gluon fields. Technically, this is achieved using the reparametrization invariance to remove the $c_3\theta G\tilde{G}$ coupling. If we keep the c_3 coupling, we must consider the axion-gluon interactions also, which are hard to treat accurately at face value but must be the same as in the $c_3=0$ basis. In this way, the quark interactions are changed from the original values as follows:

$$c_1 \to \bar{c}_1 = c_1 + \frac{1}{2}c_3,$$
$$c_2 \to \bar{c}_2 = c_2 + c_3, \tag{49}$$
$$c_3 \to \bar{c}_3 = c_3 - c_3 = 0.$$

In the notation with overbars, there exist only $\bar{c}_1$ and $\bar{c}_2$.

We discuss one family without separating $c_{1,2}$ into $c_{1,2}^{u,d}$ first for an illustration, and then we discuss the cases with $c_{1,2}^{u,d}$ and write down formulas for three families. We define the initial parameters c_1, c_2, and c_3 together with the definition of the vacuum angle $\theta_0 \equiv \theta_{\rm QCD}$. In principle, the initial vacuum angle can be a free parameter. Here the vacuum angle $\theta_{\rm QCD}$ is defined such that $c_1=0$. Picking up the axion-dependent chiral rotation charge defined below the chiral symmetry breaking scale Eq. (47), the chiral quarks in the chiral perturbation theory are transformed as $q_L \to \exp(iQ_A\theta)q_L, q_R \to \exp(-iQ_A\theta)q_R$, where

$$Q_A = \frac{1}{2}\frac{M^{-1}}{\mathrm{Tr}\, M^{-1}}, \quad M^{-1} = \mathrm{diag}\left(\frac{1}{m_u}, \frac{1}{m_d}\right). \tag{50}$$

The derivative interactions of the axion are obtained in this way (Kaplan, 1985; Georgi, Kaplan, and Randall, 1986).

For the KSVZ axion, we have $c_1=c_2=0$ and $c_3=1$, and the coefficient of the gluon anomaly term is $a/F_a + \theta_{\rm QCD}$. Hence, redefining the axion as $a+F_a\theta_{\rm QCD}$, we obtain[3]

KSVZ axion $(c_1=0, c_2=0)$:

$$\bar{c}_1 = \tfrac{1}{2}c_3 = \tfrac{1}{2}, \tag{51}$$
$$\bar{c}_2 = c_2 + c_3 = 1.$$

Here $\bar{c}_2$ must be split according to the flavor singlet condition into $\bar{c}_2^u + \bar{c}_2^d$, Eq. (47), or (50).

For the DFSZ and PQWW axions, $c_1=0, c_2\neq 0$, and $c_3=0$. If a nonvanishing $\theta_{\rm QCD}$ is introduced here, we have, using the reparametrization invariance (21), $c_1' = -c_2/2, c_2'=0$, and $c_3'=c_2$. Then the coefficient of the gluon anomaly term is $c_2(a/f_S)+\theta_{\rm QCD}$, and hence, redefining the axion as $a+(f_S/c_2)\theta_{\rm QCD}$ and going back to the $\bar{c}_3=0$ basis, we obtain for one family,

DFSZ and PQWW axions:

$$\bar{c}_1 = \tfrac{1}{2}(-c_2 + \bar{c}_2), \tag{52}$$

[3]The sign convention is stated below.

$$\bar{c}_2 \neq 0, \quad \bar{c}_3 = 0.$$

Again, c_3' must be split according to the flavor singlet condition to $\bar{c}_2^u + \bar{c}_2^d$ according to the anomaly matching condition, Eq. (47).

When the heavy σ field and heavy quark fields are integrated out, the massless (at this level) degree $a = F_a\theta$ which appears from the phase of the singlet field $\sigma = (\langle\sigma\rangle + \rho/\sqrt{2})e^{i\theta}$ appears in the effective low-energy Lagrangian. If there are multiple SM singlets S_i carrying PQ charges and VEVs, then the axion component is

$$a = \frac{1}{V_a}\sum_i \Gamma_i V_i a(S_i), \quad V_a = \left(\sum_i \Gamma_i^2 V_i^2\right)^{1/2}, \tag{53}$$

where $a(S_i)$ is the phase field of S_i. The PQ charges are defined such that the smallest nonzero absolute value(s) of the PQ charges is 1 so that every scalar field returns to its original value after a 2π shift of its phase. We now discuss axion couplings after integrating out the heavy fields.

In the KSVZ model c_3 is calculated using the triangle diagram of heavy quarks for the global anomaly. The domain wall number $N_{\rm DW}$ is $N_{\rm DW} = {\rm Tr}\,\Gamma(Q_L)l(Q_L)$, with $F_a = V_a/N_{\rm DW}$ where $\Gamma(Q_L)$ (defined as $Q_L \to e^{i\Gamma(Q_L)\theta}Q_L$ under $a \to a + F_a\theta$) is the PQ charge and $l(Q_L)$ is the index of SU(3)$_c$ representation. Every field is represented in terms of left-handed fields, and the PQ charges are defined such that the SM singlet σ coupling to heavy quarks carries one unit of the PQ charge. If the light quarks also carry the PQ charge, then Eq. (22) gives $N_{\rm DW}$, which belongs to the generic very light axion model discussed below. The anomaly calculation gives the one-loop coupling $(N_{\rm DW}a/V_a)\{G\tilde{G}\}$, but since the vacuum angle θ or axion is given by the coefficient of $\{G\tilde{G}\}$, F_a is defined by dividing V_a of Eq. (53) by $N_{\rm DW}$ and hence $c_3 = \pm 1$ where the sign coincides with that of ${\rm Tr}\,\Gamma(Q_L)l(Q_L)$. As a convention, choose it to be $c_3 = +1$, which is choosing the effective PQ charges of heavy quarks to be positive. Transferring c_3 to c_2, we split $c_3 = c_2^u + c_2^d$ using the PQ charges of Eq. (50),

KSVZ axion:

$$\bar{c}_1^{u,d} = \tfrac{1}{2}\bar{c}_2^{u,d}, \tag{54}$$

$$\bar{c}_2^u = \frac{1}{1+Z}, \quad \bar{c}_2^d = \frac{Z}{1+Z},$$

In the DFSZ model, c_2^u and c_2^d are calculated by transferring the phase of σ to H_u and H_d with the PQ symmetry such that $\langle H_u^0\rangle = \sqrt{2}v_2 e^{i\Gamma_u a/V_\sigma}$ and $\langle H_d^0\rangle = \sqrt{2}v_1 e^{i\Gamma_d a/V_\sigma}$ if $H_u^* H_d^* \sigma^2$ defines the PQ charge of σ in terms of PQ charges Γ_u and Γ_d of H_u and H_d. Here $a = V_\sigma\theta$ is not the mass eigenstate and instead of V_σ the mass eigenstate $\tilde{a}$ uses the decay constant $F_a = [(\Gamma_u + \Gamma_d)^2 V_\sigma^2 + \Gamma_u^2 v_u^2 + \Gamma_d^2 v_d^2]^{1/2} \simeq (\Gamma_u + \Gamma_d)V_\sigma$ for $V_\sigma \gg v_u, v_d$, and the axion component $\tilde{a} = [(\Gamma_u + \Gamma_d)V_\sigma a + \Gamma_u v_{\rm EW} a(H_u) + \Gamma_d v_{\rm EW} a(H_d)]/F_a \simeq a$, and $F_a = V_\sigma/N_{\rm DW}$. In the DFSZ model they are given by Carena and Peccei (1989): $c_2^u = |v_d|^2/v_{\rm EW}^2$, $c_2^d = |v_u|^2/v_{\rm EW}^2$, and $c_1^{u,d} = c_3 = 0$. Using the reparametrization invariance Eq. (21), we can use $c_1'^u = -c_2^u/2$, $c_1'^u = -c_2^d/2$, $c_2'^u = c_2'^d = 0$, and $c_3' = c_2'^u + c_2'^d = 1$. Removing c_3' according to the flavor singlet condition, we obtain for one family,

DFSZ axion for one family:

$$\bar{c}_1^u = -\frac{|v_d|^2}{2v_{\rm EW}^2} + \frac{1}{2}\bar{c}_2^u, \quad \bar{c}_1^d = -\frac{|v_u|^2}{2v_{\rm EW}^2} + \frac{1}{2}\bar{c}_2^d,$$
$$\bar{c}_2^u = \frac{1}{1+Z}, \quad \bar{c}_2^d = \frac{Z}{1+Z}, \tag{55}$$

where $v_u = |\langle\sqrt{2}H_u^0\rangle|, v_d = |\langle\sqrt{2}H_d^0\rangle|, v_{\rm EW} = (v_u^2 + v_d^2)^{1/2}$. The PQ charges $c_2^u = |v_d|^2/v_{\rm EW}^2$ and $c_2^d = |v_u|^2/v_{\rm EW}^2$ of H_u and H_d are obtained by considering the orthogonal component to the longitudinal mode of the Z boson. Remember that the signs of $c_2^{u,d}$ are chosen from the convention that the PQ charges of $H_{u,d}$ are positive. This result is for one family.

If we have N_g families, we can calculate the couplings just below the electroweak scale where all quarks obtain masses. Thus, we obtain for three families $c_2^u = c_2^c = c_2^t = |v_d|^2/v_{\rm EW}^2$ and $c_2^d = c_2^s = c_2^b = |v_u|^2/v_{\rm EW}^2$. Using the reparametrization invariance, we can calculate c_1', c_2', and c_3', just above 1 GeV: $c_2' = 0$, $c_1^{i\prime} = -\frac{1}{2}c_2^i$, and $c_3' = N_g(\Sigma_i c_2^i) = N_g$. Then we integrate out the heavy quarks c, b, and t to obtain the effective couplings just above 1 GeV; this does not introduce any new c_2 terms. Now there are three light quarks u, d, and s for which we use the reparametrization invariance to remove the c_3' term such that the isosinglet condition is satisfied; $\partial^\mu J_\mu^a$ is anomaly-free where $J_\mu^a = J_\mu^{\rm PQ} - \alpha_u \bar{u}\gamma_\mu\gamma_5 u - \alpha_d \bar{d}\gamma_\mu\gamma_5 d - \alpha_s \bar{s}\gamma_\mu\gamma_5 s$ and $\partial^\mu J_\mu^{\rm PQ} = N_g\{G\tilde{G}\}$. Thus, $\alpha_u + \alpha_d + \alpha_s = N_g$ is satisfied and the SU(3)$_{\rm flavor}$ singlet condition of $\partial^\mu J_\mu^a$ determines

$$\alpha_u = \frac{m\alpha}{m_u}, \quad \alpha_d = \frac{m\alpha}{m_d}, \quad \alpha_s = \frac{m\alpha}{m_s}, \tag{56}$$

with

$$m\alpha = \frac{N_g m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} \simeq \frac{N_g(m_u + m_d)Z}{(1+Z)^2}.$$

Therefore, removing c_3' by the reparametrization invariance, we obtain

DFSZ axion for N_g families:

$$\bar{c}_2^u = \frac{1}{1+Z}N_g, \quad \bar{c}_2^d = \frac{Z}{1+Z}N_g, \tag{57}$$

$$\bar{c}_1^u = \frac{1}{2}\bar{c}_2^u - \frac{v_d^2}{v_{\rm EW}^2}, \quad \bar{c}_1^d = \frac{1}{2}\bar{c}_2^d - \frac{v_u^2}{v_{\rm EW}^2}. \tag{58}$$

If heavy quarks and also $H_{u,d}$ carry PQ charges, we must consider all these. If one SM singlet σ houses the axion, then we obtain

General very light axion:

$$\bar{c}_2^u = \frac{1}{1+Z}(1 \pm N_g), \tag{59}$$

$$\bar{c}_2^d = \frac{Z}{1+Z}(1 \pm N_g), \tag{60}$$

$$\bar{c}_1^u = \frac{1}{2(1+Z)}(1 \pm N_g) \mp \frac{|v_d|^2}{2v_{\rm EW}^2}\delta_{H_u}, \tag{61}$$

$$\bar{c}_1^d = \frac{Z}{2(1+Z)}(1 \pm N_g) \mp \frac{|v_u|^2}{2v_{\rm EW}^2}\delta_{H_d}, \tag{62}$$

where the PQ charges ($+$ or $-$) of H_u and H_d determine the sign ($-$ or $+$) in front of the DFSZ component and δ_H=1 or 0 if the corresponding Higgs doublets carry the PQ charges or not. For the MI axion from superstring theory, which is a hadronic axion, in principle there can exist an additional contribution to c_1 as pointed out in Sec. VI.F.1.

If there are no heavy degrees carrying the PQ charges above the electroweak scale, then c_2 in the so-called PQWW model is given by the PQ charges of H_u and H_d,

PQWW axion: Same as Eqs. (57) and (58). (63)

All models have $\bar{c}_1$ and $\bar{c}_2$. For the original c_2 term, different models give different values; for example, some variant axion models (Krauss and Wilczek, 1986; Bardeen, Peccei, and Yanagida, 1987; Kim and Lee, 1989; Hindmarsh, and Monlatshotis, 1997) have different c_2's from those of the PQWW axion. For astrophysical application, we must keep both $\bar{c}_1$ and $\bar{c}_2$. The $\bar{c}_1$ and $\bar{c}_2$ terms give the axial-vector and pseudoscalar couplings, respectively. The axion operator in the flavor SU(3) space can be written as

$$(\bar{c}_{1,2}^u - \bar{c}_{1,2}^d)F_3 + \frac{\bar{c}_{1,2}^u + \bar{c}_{1,2}^d}{\sqrt{3}}F_8 + \frac{\bar{c}_{1,2}^u + \bar{c}_{1,2}^d}{6}1, \tag{64}$$

where F_3 and $F_8/\sqrt{3}$ are the third component of the isospin and the hypercharge operators, respectively, and 1 is the identity operator.

The derivative couplings with nucleons and mesons below the chiral symmetry breaking are the defined as

$$\mathcal{L}_{\rm AV}^{\bar{c}_1} = \frac{\partial^\mu a}{F_a}\left[C_{\rm app}\bar{p}\gamma_\mu\gamma_5 p + C_{\rm ann}\bar{n}\gamma_\mu\gamma_5 n + iC_{a\pi NN}\left(\frac{\pi^+}{f_\pi}\bar{p}\gamma_\mu n - \frac{\pi^-}{f_\pi}\bar{n}\gamma_\mu p\right)\right], \tag{65}$$

$$\mathcal{L}_{a\pi\pi\pi}^{\bar{c}_1} = C_{a\pi\pi\pi}\frac{\partial^\mu a}{F_a f_\pi}(\pi^0\pi^+\partial_\mu\pi^- + \pi^0\pi^-\partial_\mu\pi^+ - 2\pi^+\pi^-\partial_\mu\pi^0), \tag{66}$$

where

$$C_{\rm app} = \bar{c}_1^u F + \frac{\bar{c}_1^u - 2\bar{c}_1^d}{3}D + \frac{\bar{c}_1^u + \bar{c}_1^d}{6}S, \tag{67}$$

$$C_{\rm ann} = \bar{c}_1^d F + \frac{\bar{c}_1^d - 2\bar{c}_1^u}{3}D + \frac{\bar{c}_1^u + \bar{c}_1^d}{6}S, \tag{67}$$

$$C_{a\pi NN} = \frac{\bar{c}_1^u - \bar{c}_1^d}{\sqrt{2}}, \quad C_{a\pi\pi\pi} = \frac{2(\bar{c}_1^u - \bar{c}_1^d)}{3}. \tag{68}$$

Here the axial-vector coupling parameters of the nucleon octet are given by F=0.47,D=0.81, and $S \simeq 0.13 \pm 0.2$ (Amsler *et al.*, 2008). For example, for the hadronic-axion couplings we obtain the results given by Kaplan (1985) and Chang and Choi (1993),

$$C_{\rm app} = \frac{1}{2(1+Z)}F + \frac{1-2Z}{6(1+Z)}D + \frac{1}{6}S,$$

$$C_{\rm ann} = \frac{Z}{2(1+Z)}F + \frac{Z-2}{6(1+Z)}D + \frac{1}{6}S, \tag{69}$$

$$C_{a\pi NN} = \frac{1-Z}{2\sqrt{2}(1+Z)}, \quad C_{a\pi\pi\pi} = \frac{1-Z}{3(1+Z)}.$$

For the DFSZ axion, there exist additional contributions from the extra terms in Eqs. (57) and (58).

Similar expressions might be attempted for the pseudoscalar couplings in terms of $\bar{c}_2^{u,d}$ and the pseudoscalar coefficients F', D', and S'. But for the axion current, corresponding to J_μ^a, there does not exist an anomaly as discussed in Eq. (47) and we do not write down the axion pseudoscalar couplings. The anomaly carried by axions above the chiral symmetry breaking scale is left over to η' below the chiral symmetry breaking scale and hence these pseudoscalar couplings are for the η' meson. The axial vector current of η' to the nucleon octet $N = q \otimes q \otimes q$ is

$$J_\mu^{\eta'} = f_{\eta'}\partial_\mu\eta' + g_N^5\bar{N}\gamma_\mu\gamma_5 T_0 N, \tag{70}$$

where T_0 is properly normalized, Tr $T_0^2 = \frac{1}{2}$ or $T_0 = 1/\sqrt{2N_f}$, and g_N^5 is determined by strong interaction dynamics. The original global symmetry breaking term (20) is transferred to $\bar{N}_L N_R e^{i\alpha_1'\eta'/f_{\eta'}}$ which is actually the nucleon mass term,

$$\Delta\mathcal{L} = -m_N\bar{N}_L N_R e^{i\alpha_1'\eta'/f_{\eta'}} + \text{H.c.} \tag{71}$$

For example, the SU(6) wave function of a spin-up neutron is

$$|n^\uparrow\rangle = \frac{1}{6\sqrt{2}}|4d^\uparrow u^\downarrow d^\uparrow - 2d^\downarrow u^\uparrow d^\uparrow - 2d^\uparrow u^\uparrow d^\downarrow - 2u^\uparrow d^\downarrow d^\uparrow + 4u^\downarrow d^\uparrow d^\uparrow - 2u^\uparrow d^\uparrow d^\downarrow - 2d^\uparrow d^\downarrow u^\uparrow - 2d^\downarrow d^\uparrow u^\uparrow + 4d^\uparrow d^\uparrow u^\downarrow\rangle, \tag{72}$$

where the quarks are now interpreted as constituent quarks below the chiral symmetry breaking. At low energy, this is the only relevant symmetry for consideration. The octet charge α_1' is determined by strong interaction dynamics. The ducaplet has a different U(1) charge α_1''. Two anomaly matching conditions, PQ-B-B and PQ-Q_{em}-Q_{em}, may be used but do not give useful information because of many form factors. So, the PQ charges of the current quarks being transferred to the constituent quarks in the octet with a multiplcation factor g_N^5, we obtain the PQ charge of the neutron as the PQ charge of one constituent quark. Thus, $\bar{N}_L N_R$ has the phase $\alpha' = 2g_N^5\sqrt{2N_f}/N_f$. If we guess that g_N^5 is similar to the octet form factor $g_A \simeq 0.75$ (Georgi, 1984, p. 100), α_1' is estimated as 1.22.

2. Axion-photon-photon coupling

As we calculated the c_3 coupling for the KSVZ axion, we can calculate the axion-photon-photon coupling by substituting the gluon lines by photon lines and the quark triangles by charged fermion triangles. Since we are interested in low-temperature experiments, we consider the energy scale below the electron mass. Therefore, considering $V_a = N_{\rm DW} F_a$, $c_{a\gamma\gamma}^0$ calculated from the PQ charges of charged fermions becomes

$$c_{a\gamma\gamma}^0 = \frac{\mathrm{Tr}\,\Gamma(Q_L) Q_{\rm em}^2}{N_{\rm DW}}. \tag{73}$$

Below the QCD chiral-symmetry-breaking scale, we chiral-transform light quarks to obtain

$$\mathcal{L}_{a\gamma\gamma} = c_{a\gamma\gamma}\frac{e^2}{32\pi^2 F_a} a F_{\mu\nu}^{\rm em}\tilde{F}_{\rm em}^{\mu\nu}, \tag{74}$$

where

$$c_{a\gamma\gamma} \simeq c_{a\gamma\gamma}^0 - c_{\chi \rm SB}, \tag{75}$$

where the chiral-symmetry-breaking effect, including the strange quark mass effect, is

$$c_{\chi \rm SB} = \frac{\frac{2}{3}(4 + 1.05Z)}{1 + 1.05Z} = [1.762, 2.260] \tag{76}$$

for a 20% allowance from the tree level chiral perturbation theory estimation (Kaplan and Manohar, 1986). For illustration, we take $c_{\chi \rm SB} \approx 1.98$ for $Z \simeq 0.5$ (Manohar and Sachrajda, 2008).

In the KSVZ model, $c_{a\gamma\gamma}^0$ is determined by the PQ charge-carrying heavy fermions. If there were only one neutral quark for this, then $c_{a\gamma\gamma}^0$ would be zero. If there is only one PQ charge-carrying heavy quark with the electromagnetic charge $Q_{\rm em}$, then $c_{a\gamma\gamma}^0 = Q_{\rm em}^2$. But, in realistic models from a fundamental theory it is more likely that there exist many PQ charge-carrying quarks, and the coupling given for one PQ charge-carrying heavy quark is presented just as an illustration.

In the DFSZ model, we consider only light quarks and leptons. The PQ charges of H_u and H_d determine the PQ charges of u and d quarks. For the PQ charge of e, we have two possibilities: H_d gives mass to e and the PQ charge of e is the same as that of d, or H_u gives mass to e and the PQ charge of e is opposite to that of u,

TABLE I. $c_{a\gamma\gamma}$ in several field theoretic models. The left block is for the KSVZ and the right block is for the DFSZ. (m,n) in the KSVZ block denotes m copies of $Q_{\rm em} = \frac{2}{3}$ and n copies of $Q_{\rm em} = -\frac{1}{3}$ heavy quarks with the same PQ charge. In the DHSZ block $x = \tan\beta = v_u/v_d$.

$Q_{\rm em}$	$c_{a\gamma\gamma}$	x	one Higgs couples to	$c_{a\gamma\gamma}$
0	–1.95	any x	(d^c, e)	0.72
$\pm\frac{1}{3}$	–1.28	any x	(u^c, e)	–1.28
$\pm\frac{2}{3}$	0.72			
± 1	4.05			
(m,m)	–0.28			

$$c_{a\gamma\gamma}^0 = -\frac{2v_d^2}{v_{\rm EW}^2}\left(\frac{2}{3}\right)^2 \times 3 - \frac{2v_u^2}{v_{\rm EW}^2}\left[\left(-\frac{1}{3}\right)^2 \times 3 + (-1)^2\right]$$
$$= -\frac{8}{3}, \quad \text{electron mass by } H_d, \tag{77}$$

$$c_{a\gamma\gamma}^0 = -\frac{2v_d^2}{v_{\rm EW}^2}\left[\left(\frac{2}{3}\right)^2 \times 3 - (-1)^2\right] - \frac{2v_u^2}{v_{\rm EW}^2}\left(-\frac{1}{3}\right)^2 \times 3$$
$$= -\frac{2}{3}, \quad \text{electron mass by } H_u^\dagger, \tag{78}$$

where the PQ charges of $H_{u,d}$ were chosen to be positive before. In applying Eq. (75), we must choose the PQ charges of light quarks to be positive and hence the signs of Eqs. (77) and (78) must be reversed. For the PQWW axion, the coupling is the same as those of Eqs. (77) and (78) with positive signs.

The KSVZ and DFSZ axion models arise in several different ways, for which the axion-photon-photon coupling has been tabulated by Kim (1998). In Table I, we list axion-photon-photon couplings for several very light axion models.

For a general light axion, the axion-photon-photon coupling depends on the ultraviolet completion of the theory. If the axion mass is lighter than $2m_e$, its lifetime is

$$\tau(a \to 2\gamma) = \frac{2^8\pi^3}{c_{a\gamma\gamma}^2 \alpha_{\rm em}^2}\frac{F_a^2}{m_a^3} \simeq \frac{3.65 \times 10^{24}}{c_{a\gamma\gamma}^2}\left(\frac{\rm eV}{m_a}\right)^5 \mathrm{s}$$
$$\simeq \frac{0.8 \times 10^7 t_U}{c_{a\gamma\gamma}^2}\left(\frac{\rm eV}{m_a}\right)^5, \tag{79}$$

where $Z \simeq 0.5$ and the age of the Universe $t_U \approx 4.35 \times 10^{17}$ s. For $c_{a\gamma\gamma} = O(1)$, the axion with 24 eV mass has the lifetime t_U (Moroi and Murayama, 1998; Hannestad, Mirizzi, Raffelt, and Wong, 2008).

3. Axion-lepton couplings

The tree level axion-lepton (l) coupling arises in the DFSZ and PQWW axions where the lepton mass term difines the c_l of Fig. 10 through the PQ charges of H_d or

H_u. The removal of the c_3 term does not change the coupling c_l, and hence we obtain the following tree-level couplings of the axion and lepton:

DFSZ axion:

$$\frac{m_l v_u^2}{N_{\rm DW} F_a v_{\rm EW}^2} \bar{l} i \gamma_5 l a, \quad \text{lepton mass by } H_d, \tag{80}$$

$$\frac{m_l v_d^2}{N_{\rm DW} F_a v_{\rm EW}^2} \bar{l} i \gamma_5 l a, \quad \text{lepton mass by } H_u^\dagger, \tag{81}$$

where the PQ charges of $H_{u,d}$ are chosen to be positive. For the PQWW axion, just F_a is replaced by $v_{\rm EW}$. For the KSVZ axion, the axion-lepton coupling occurs at higher order and is negligible in astrophysical applications. For the generic very light axion, the couplings given in Eqs. (80) and (81) are applicable.

Even though the tree level coupling of the axion with an electron is absent in the KSVZ model, the axion-electron coupling is present at one loop through the $c_{a\gamma\gamma}$ coupling (Srednicki, 1985)

$$2.2 \times 10^{-15} \left(\frac{m_a}{\rm eV}\right) \left[c^0_{a\gamma\gamma} \ln \frac{F_a}{m_e} - \frac{2}{3} \frac{4+Z}{1+Z} \ln \frac{\Lambda}{m_e} \right], \tag{82}$$

where Λ is the chiral symmetry breaking scale and $N_{\rm DW}^{-1}$ must be multiplied in models with $N_{\rm DW} \neq 1$. On the other hand, the DFSZ axion coupling to the electron is

$$1.4 \times 10^{-11} X_d \left(\frac{3}{N_g}\right) \left(\frac{m_a}{\rm eV}\right), \tag{83}$$

where $N_g = N_{\rm DW}/2$ is the number of families and $X_d = \sin^2 \beta = v_u^2 / v_{\rm EW}^2$ for the case of Eq. (80).

D. Old laboratory bounds on F_a

With the axion couplings discussed in Sec. III.C, one can estimate the axion production rates in various experiments. Null experimental results give the bounds on the relevant axion couplings. These have been discussed in earlier reviews (Kim, 1987; Cheng, 1988; Peccei, 1989). These old laboratory bounds, immediately studied after the proposal of the PQWW axion, basically rule out the PQWW axion, i.e., give an axion decay constant F_a greater than $O(10\text{ TeV})$,

$$F_a \gtrsim 10^4 \text{ GeV} \quad (\text{old laboratory bound}). \tag{84}$$

IV. AXIONS FROM OUTER SPACE

From Eq. (79), we note that the axion lifetime is longer than t_U for $m_a \sim 24$ eV, and this kind of axion is important in cosmology. For $m_a \lesssim 23$ keV with $c_{a\gamma\gamma} = 1$, the axion lifetime is longer than 10 min, allowing solar-generated axions below this mass to reach Earth. These examples illustrated the importance of studying low-mass axion effects in astrophysics and cosmology.

The window for F_a obtained from the astrophysical and cosmological constraints is given by

$$0.5 \times 10^9 \lesssim F_a \lesssim 2.5 \times 10^{12} \text{ GeV}, \tag{85}$$

where the upper bound is understood with an initial misalignment angle of order 1.

A. Axions from stars

In this section we present the key arguments leading to axion constraints from astrophysical sources. Axions have very small masses and therefore can be emitted without important threshold effects from stars, in analogy to neutrinos. The method to constrain axion models is basically the overall energy loss rate, whether using the individual stars (e.g., Sun and SN1987A) or the statistical properties of stellar populations (e.g., the stars in a globular cluster as a test population) (Kolb and Turner, 1990; Raffelt, 1996).

We may use the axion couplings to γ, p, n, and e to study the core evolution of a star. Simple bounds are obtained by comparing the energy loss rates by axion and by neutrino emission. Study of the evolutionary history of a star by axion emission may give a stronger bound than the one obtained from the energy loss rate but may not be as reliable. Since there are good reviews on axion astrophysics (Raffelt, 1990a, 2008a; Turner, 1990; Amsler *et al.*, 2008), here we briefly comment on axion physics in stars (Sun, low-mass red giants, supernovae) to cite reliable F_a bound.

With axion emission, the Sun consumes more fuel and needs an increased core temperature. From the Primakoff process $\gamma + Ze \to a + Ze$ in the hadronic axion models, Schlattl, Weiss, and Raffelt (1999) gave the axion emission rate $L_a \simeq 3.7 \times 10^{-2} L_\odot$ with a 20% increase of the ^{8}B flux with the increased core temperature. The ^{8}B neutrino flux gives the best bound on the solar axion emission rate. The measured ^{8}B neutrino flux $4.94 \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}$ (Aharmim *et al.*, 2005) is consistent with the axion emission if $L_a \leqslant 0.04 L_\odot$ (Bahcall, Serenelli, and Basu, 2005). This translates to an F_a bound of $F_a / c_{a\gamma\gamma} \geqslant 2.6 \times 10^6$ GeV for $L_a \leqslant 0.04 L_\odot$ (Schlattl, Weiss, and Raffelt, 1999).

For axion-electron coupling as in the DFSZ axion models, the axion emission from globular clusters gives a useful F_a bound (Raffelt and Dearborn, 1987). Stars in a globular cluster are assumed to have identical Y (helium fraction) and metallicity fraction. The helium core before ignition is degenerate and the bremsstrahlung emission is very effective, whereas the Primakoff emission is suppressed by the large plasma frequency and the helium ignition does not give a useful F_a bound for the KSVZ axion. However, after helium ignition the core degeneracy is lifted, the Primakoff effect becomes important, and the consumption of helium fuel is accelerated by the axion energy loss, shortening the helium-burning lifetimes. Horizontal branch stars in several globular clusters confirm the expected helium-burning lifetimes, which agrees with the standard prediction and the axion losses should not exceed $\varepsilon_a < 10\ \text{erg g}^{-1}\,\text{s}^{-1}$ in the cores of horizontal branch stars (Raffelt, 1990b;

Catelan, de Freista Pacheco, and Horvath, 1996), which leads to $F_a/c_{a\gamma\gamma} \geqslant 2\times 10^7$ GeV, a factor of 10 improvement over the solar bound. Note that this globular cluster bound is for models with an appreciable axion-electron coupling.

In the study of the axion emission in the small-mass red giants, the processes $\gamma+Z\to a+Z, e+Z\to a+e+Z$, and $\gamma+e\to a+e$ were considered. The early studies were simple comparisons of the axion and neutrino emission (Fukugita, Watamura, and Yoshimura, 1982a, 1982b; Krauss, Moody, and Wilczek, 1984). In the study of Dearborn, Schramm, and Steigman (1986), it is summarized as $F_a \geqslant 2.1\times 10^7 c_{a\gamma\gamma}$ GeV if the Primakoff process $\gamma+Z\to a+Z$ dominates and $F_a\geqslant 3.7\times 10^9 \sin^2\beta$ GeV if the Compton process dominates [for the DFSZ axion, viz., Eq. (80)]. The Primakoff process is present in any axion model, and hence the Primakoff process bound is almost model independent except in the region $m_a > 200$ keV where a is too heavy to be produced in the core of a star. But this threshold effect is irrelevant since the PQWW axion region is already excluded. Note, however, that there is no confirmed observation of neutrinos from the small-mass red giants, unlike from the Sun and SN1987A, and the possibility of dominant axion emission from red giants is not excluded by observation (Raffelt, 2008b). For the DFSZ axion, the region $m_a > 10^{-2}$ eV is excluded due to the large axion-electron coupling. For the hadronic axion, Raffelt and Dearborn (1987) argued that an axion mass greater than about $(2\text{ eV})/[(E/N-1.95)/0.72]$ would reduce the helium-burning time scale and is thus not allowed.

For supernovae explosion, the core temperature can go much higher than the temperature in the ignition phase of helium in the small-mass red giant cores. For supernovae, therefore, nuclear reactions are more important and the F_a bound can be very strong. As a result we use the axion couplings to nucleons discussed in Sec. III.C.1 to study the core evolution of supernovae. In the beginning, the bounds on the axion decay constant were obtained by comparing the nuclear burning rates of production of axions and neutrinos (Iwamoto, 1984; Pantziris and Kang, 1986). The discovery of SN1987A was important in that it propelled a much interest anew in the calculation of the axion production rate (Hatsuda and Yoshimura, 1988; Mayle, Ellis, Olive, Schramm, and Steigman, 1988; Raffelt and Seckel, 1988; Turner, 1988). In principle, the same kind of bound on F_a could be obtained from earlier supernovae studies. The studies after the discovery of SN1987A were performed with the derivative coupling and quartic terms of Sec. III.C.1 and obtained a bound $F_a \gtrsim 10^9$ GeV. But as pointed out by Carena and Peccei (1989), Choi, Kang, and Kim (1989), Turner, Kang, and Steigman (1989), Kang and Pantzisis (1991), and Sec. III.C.1, a proper treatment of nucleon states must be taken into account. For axion emission from supernovae, one must constrain the energy output to $\epsilon_a \leqslant 1\times 10^{19}\text{ erg g}^{-1}\text{ s}^{-1}$ (Raffelt, 1990a). The axion emission rate calculation of Raffelt (2008a) is

$$\epsilon_a = 3.0\times 10^{37}(\text{erg g}^{-1}\text{ s}^{-1}) C_N^2 F_{a\,\text{GeV}}^{-2} T_{a,30\text{ MeV}}^4 F, \quad (86)$$

where $F_{a,\text{GeV}} = F_a/\text{GeV}$, $T_{a,30\text{ MeV}} = T/(30\text{MeV})$, and $F = O(1)$. In a supernovae explosion the axion emission can be comparable to neutrino emission. Such remnant axions from all past supernovae explosions may be around us but will be difficult to detect because of the small $1/F_a$ (Raffelt, 2008b). For the smaller F_a region from supernovae explosions, axions can be trapped if the axion-nucleon interaction is strong enough. For the hadronic axion, this gives the bound on $m_a \geqslant 1$ eV (Raffelt, 1990a; Turner, 1990), and we have a hadronic axion window in the eV range.

For the KSVZ axion and the MI superstring axion, $\bar{c}_1$ terms are present. For example, we can simply take $\bar{c}_1^u = \frac{1}{3}$ and $\bar{c}_1^d = \frac{1}{6}$, corresponding to $Z=0.5$, and hence obtain $c_{\text{app}} = \frac{1}{3}F + \frac{1}{12}S \simeq 0.17$ for the KSVZ axion. Using c_{app} as C_N in Eq. (86), we obtain an F_a bound from supernovae,

$$F_a \geqslant 0.5\times 10^9\text{ GeV}. \quad (87)$$

The white dwarfs in the final evolutionary stage of low-mass stars ($M < 10\pm 2M_\odot$), with the theoretical model implemented in the DFSZ model, may give a stronger bound on F_a (Raffelt, 1986) for some region of the DFSZ parameter $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$. The recent study of the bremsstrahlung process gives the bound $F_a \geqslant 0.6\times 10^{10}\sin^2\beta$ GeV, and even fits the cooling diagram nicely with $F_a \simeq 1.2\times 10^{10}\times \sin^2\beta$ GeV for H_d giving mass to the electron (Isern, García-Berro, Torres, and Catalán, 2008). Note that $\tan\beta$ is known to be large ($\geqslant 30$) in SUSY grant unified theory (GUT) models, and the white dwarfs may give the strongest F_a bound for some DFSZ axion models.

The axion-nucleon coupling gets enhanced in a strong magnetic field. Magnetic fields as strong as $B > 10^{18}$ G in neutron stars have been assumed in the scalar virial theorem (Woltjer, 1964). With $B > 10^{20}$ G at the surface, the axion emission rate from neutron stars or white dwarfs will be enhanced by $O(1)$ compared to the $B=0$ case (Hong, 1998).

In summary, axions once produced in the hot plasma of a star most probably escape the core, taking out energy. This contributes to the energy loss mechanism of a star and is used to constrain axion models. From the nucleon-nucleon-a coupling, SN1987A gives the strongest astrophysical bound on the axion decay constant, $F_a > 0.5\times 10^9$ GeV (Raffelt, 1990a, 2008a; Turner, 1990).

B. Axions in the universe

Axions with $m_a \gtrsim 24$ eV have a lifetime shorter than the age of the Universe. In this case, axion decay might lead to photons that can be tested against the observed electromagnetic background of the Universe, as in some spontaneously broken flavor symmetric models, $\nu_i \to \nu_j a \to \nu_j\gamma\gamma$ (Berezhiani, Khlopov, and Khomeriki, 1990). However, in this case the needed decay constant, 10^6 GeV, is outside the current bound on F_a.

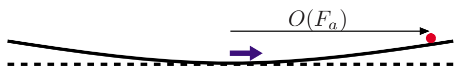

FIG. 11. (Color online) The almost flat axion potential. The misalignment angle is expected to be of order 1 but can also be very small as shown by the thick arrow.

The axion for $m_a \lesssim 24$ eV has a longer lifetime than the age of the Universe and can affect its evolution. The heavy thermal axions around the eV mass range of Fig. 2(b) become the hot DM in the Universe. For the 3–8 eV mass range, they accumulate in galaxy clusters where their slow decay produces a sharp line that, in principle, can be observed by telescope searches as suggested by Bershady, Ressell, and Turner (1991). In this case, the neutrino and axion hot DM must be considered together, which now constrains the axion mass to $m_a < 1.02$ eV (Hannestad, Mirizzi, Raffelt, and Wong, 2008a, 2008b), almost closing the hadronic axion window of 1–20 eV of Fig. 15.

But more attention is paid to axions behaving as the CDM candidate. The axion potential is almost flat as depicted in Fig. 11. Therefore, a chosen vacuum stays there for a long time, and starts to oscillate when the Hubble time H^{-1} is comparable to the oscillation period (the inverse axion mass), $3H \approx m_a$. This occurs when the temperature of the Universe is about 1 GeV (Abbott and Sikivie, 1983; Dine and Fischler, 1983; Preskill, Wise, and Wilczek, 1983). There exists the domain wall problem in the standard big bang cosmology (Sikivie 1982). The axion strings and domain wall problem have been summarized by Sikivie (2008). The axion cosmology is correlated to the reheating temperature $T_{\rm RH}$ in the inflationary models, where one must deal with both the inflaton and the axion. The density perturbations produced by perturbations of the inflaton field are adiabatic, $\delta\rho_{\rm matter}/\rho_{\rm matter} = (3/4)\delta\rho_{\rm rad}/\rho_{\rm rad}$. On the other hand, the perturbations produced by fluctuations of the axion field have isocurvature. If the reheating temperature $T_{\rm RH}$ is above the axion scale F_a, the limit on the isocurvature of less than 30% from the large-scale structure data can be used (Beltrán, García-Bellido, and Lesgourgues, 2007). This will be commented on more in Sec. IV.C on the anthropic argument.

In supersymmetric models, the reheating temperature is constrained to $T_{\rm RH} < 10^9$ or 10^7 GeV (if the gluino is lighter than the gravitino) from nucleosynthesis requirements in models with a heavy gravitino (Ellis, Kim, and Nanopoulos, 1984; Kawasaki, Kohri, and Moroi, 2005). So with SUSY the domain wall is not so problematic. In this case, the problem of string-radiated axions requiring axion mass $m_a > 10^{-3}$ eV (Davis, 1985; Harari and Sikivie, 1987; Dabholkar and Quashnock, 1990) is no longer problematic.

Axions are created at $T \simeq F_a$, but the axion vacuum $\langle a \rangle$ does not begin to roll until the Hubble parameter reaches the axion mass $3H = m_a$, which occurs at $T \simeq 1$ GeV. From then on, the classical field $\langle a \rangle$ starts to oscillate. For a small misalignment angle, the energy density behaves like that in the harmonic oscillator $m_a^2 F_a^2$, which is proportional to the axion mass times the number density. Thus, its behavior is like that of CDM, which is the reason that the axion DM is CDM even though its mass is very small and its interaction strength is much weaker than "weak." Even for a large misalignment angle, an adiabatic invariant I exists and one can estimate the current axion energy density. The axion field evolution with the adiabatic change of the axion mass has been considered before (Chang, Hagmann, and Sikivie, 1998, 1999).

The temperature-dependent axion mass (Gross, Pisarski, and Yaffe, 1981) enters in the determination of the cosmic temperature T_1 where $3H(T_1) \simeq m_a(T_1)$. The new estimate of T_1 for $F_a \leqslant 10^{16}$ GeV is a bit below 1 GeV, $T_1 \simeq 0.92$ GeV (Bae, Huh, and Kim, 2009). QCD has two phases: the quark-gluon phase and the chiral symmetry breaking hadronic phase. Near the critical temperature T_c, these two phases are separated above and below T_c. The critical temperature is estimated as 148^{+32}_{-31} (172^{+40}_{-34}) MeV for three (two) light quark flavors (Braun and Gies, 2007). So cosmology near T_c needs information on the temperature-dependent axion mass. This region is in the boundary of the weak and strong coupling regimes and it is very difficult to estimate the axion mass accurately. Early attempts in this direction are given in Steinhardt and Turner (1983); Seckel and Turner (1985); Turner (1986).

The 't Hooft determinental interaction is shown in Fig. 9. In the quark-gluon phase, we have the first diagram in the box, which is parametrized as $-K^{-5}(m_u m_d m_s/\bar\rho^6)\cos[(c_2+c_3)\theta]$ where $\bar\rho$ is the effective instanton size in the instanton size integration. [Gross, Pisarski, and Yaffe (1981); Eq. (6.15)] expressed the result as

$$n(\rho,0)\exp\{-\tfrac{1}{3}\lambda^2(2N+N_f) - 12A(\lambda)[1+\tfrac{1}{6}(N-N_f)]\}, \tag{88}$$

where $\lambda = \pi\rho T$, $A(\lambda) \simeq -\frac{1}{12}\ln(1+\lambda^2/3) + \alpha(1+\gamma\lambda^{-2/3})^{-8}$ with $\alpha = 0.012\,897\,64$ and $\gamma = 0.158\,58$, and the prefactor $n(\rho,0)$ is the zero-temperature density

$$n(\rho,0) = m_u m_d m_s C_N(\xi\rho)^3 \frac{1}{\rho^5}\left(\frac{4\pi^2}{g^2(1/\rho)}\right)^{2N} e^{-8\pi^2/g^2(1/\rho)}. \tag{89}$$

Here, the parameters are $\chi = 1.3391$ and $C_N = 0.160\,073$ for $N=3$ with the Pauli-Villars regularization (Gross, Pisarski, and Yaffe, 1981). Faleev and Silvestrov (1996) argue that the $\overline{\rm MS}$ scheme is suitable for the study, where $\xi = 1.3391$ and $C_N = 0.160\,073$ are presented for $N=3$. For the subsequent numerical illustration, we use the $\overline{\rm MS}$ scheme values. For the QCD coupling constant, we use the three-loop result (Amsler *et al.*, 2008; QCD by Hinchliffe, 2008),

$$\alpha_c(\mu) = \frac{g_c^2(\mu)}{4\pi}$$
$$\simeq \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\rm QCD}^2)}\Bigg[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln[\ln(\mu^2/\Lambda_{\rm QCD}^2)]}{\ln(\mu^2/\Lambda_{\rm QCD}^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda_{\rm QCD}^2)}\Bigg\{\bigg(\ln[\ln(\mu^2/\Lambda_{\rm QCD}^2)] - \frac{1}{2}\bigg)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\Bigg\}\Bigg], \quad (90)$$

where $\beta_0 = 11 - \frac{2}{3}N_f$, $\beta_1 = 51 - \frac{19}{3}N_f$, and $\beta_2 = 2857 - \frac{5033}{9}N_f + \frac{325}{27}N_f^2$. At $T = T_{\rm GeV}$ GeV (from 700 MeV to 1.3 GeV), we parametrize the instanton size integration of Eq. (88) as

$$V(\theta) = -C(T)\cos(\theta), \quad (91)$$

where $\theta = a/F_a$ and $C(T)$ is

$$C(T) = \alpha_{\rm inst}{\rm GeV}^4(T_{\rm GeV})^{-n}. \quad (92)$$

We obtain $\alpha_{\rm inst} = 4.715\times10^{-12}$ $(1.515\times10^{-11}, 1.185\times10^{-12})$, $n = 6.878$ $(6.789, 6.967)$ for $\Lambda_{\rm QCD} = 380$ $(440, 320)$ MeV (Bae, Huh, and Kim, 2008). Equating $3H(T)$ and $m(T) = \sqrt{C(T)/F_a^2}$, we obtain the following T_1 for $\Lambda_{\rm QCD} = 380$ MeV (Bae, Huh, and Kim, 2008):

$$T_{1,{\rm GeV}} = 0.931(F_{a,12})^{-0.184}. \quad (93)$$

For $F_a = 10^{12}$ GeV, we obtain $T_1 \simeq 0.93$ GeV. This number is smaller than those given in the 1980s because we used a smaller number for the product of current quark masses $m_u m_d m_s$ based on the recent compilation of light quark masses (Manohar and Sachrajda, 2008).

(1) *No sudden change in* $m_a(T)$: Since the potential varies much more slowly than the field itself, we can use the so-called adiabatic invariant theorem that if the potential is adiabatically changed, the area in the phase space swept by the periodic motion is unchanged in one axion oscillation (Landau and Lifshitz, 1976). In this case, for a small misalignment angle the adiabatic invariant is $\rho(t)/m(t)$, which can be interpreted as the conservation of the total axion number. For a large θ_1, the invariant is not the axion number density, but the CDM energy density, which can be related to the axion number density by a correction factor (Bae, Huh, and Kim, 2008). If we apply this until now, we obtain

$$\rho_a(T_\gamma = 2.73\ {\rm K}) = m_a(T_\gamma)n_a(T_\gamma)f_1(\theta_2) = \frac{\sqrt{Z}}{1+Z}m_\pi f_\pi \frac{3\times1.66 g_{*s}(T_\gamma)T_\gamma^3}{2\sqrt{g_*(T_1)}M_{\rm P}}\frac{F_a}{T_1} \times \frac{\theta_2^2 f_1(\theta_2)}{\gamma}\left(\frac{T_2}{T_1}\right)^{-3-n/2}, \quad (94)$$

where $f_1(\theta_2)$ is the anharmonic correction and we used $Z \equiv m_u/m_d \simeq 0.5$, $m_\pi = 135.5$ MeV, $f_\pi = 93$ MeV, and $g_{*s}(\text{present}) = 3.91$. γ is the entropy increase ratio from extra particles beyond the SM. This becomes roughly

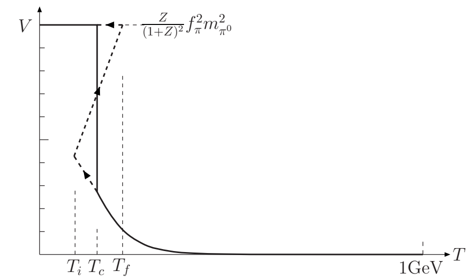

FIG. 12. Phase transition near the critical temperature $T_c \approx 150$ MeV.

$$1.449\times10^{-11}\frac{\theta_1^2}{\gamma}\left(\frac{F_{a,{\rm Gev}}}{10^{12}T_{1,{\rm GeV}}}\right)F(\theta_1, n){\rm eV}^4,$$

where θ_1 is the initial misalignment angle at T_1 and θ_2 is the angle at somewhat lower temperature T_2 where the adiabatic invariant I is calculated. The total correction factor $F(\theta_1, n)$ takes into account the anharmonic effect and the initial overshoot of the misalignment angle, presented by Bae, Huh, and Kim (2008). For the critical density $\rho_c = 3.9784\times10^{-11}(h/0.701)^2$ $({\rm eV})^4$ and $\Lambda_{\rm QCD} = 380 \mp 60$ MeV, the axion energy fraction, in terms of F_a only, is given by (Bae, Huh, and Kim, 2008)

$$\Omega_a = 0.3796ABC\left(\frac{\theta_1^2 F(\theta_1)}{\gamma}\right)\left(\frac{0.701}{h}\right)^2, \quad (95)$$

where $A = (m_u m_d m_s/3\times6\times103\ {\rm MeV}^3)^{-0.092}$, $B = (F_a/10^{12}\ {\rm GeV})^{1.184-0.010x}$ with $x = (\Lambda_{\rm QCD}/380\ {\rm MeV}) - 1$, and $C = (\Lambda_{\rm QCD}/380\ {\rm MeV})^{-0.733}$.

(2) *Sudden change in* $m_a(T)$: We now try to calculate the misalignment angle below the critical temperature of chiral symmetry breaking where a sudden phase change is experienced near the critical temperature T_c. The QCD interaction for light quarks below 1 GeV can be written as

$$\mathcal{L} = -(m_u\bar{u}_L u_R + m_d\bar{d}_L d_R + m_s\bar{s}_L s_R + {\rm H.c.}) - K^{-5}(\bar{u}_L u_R \bar{d}_L d_R \bar{s}_L s_R e^{-i\bar{c}_3\theta} + {\rm H.c.}), \quad (96)$$

where K has the mass dimension arising from QCD instanton physics. The 't Hooft determinental interaction ('t Hooft, 1976) written above is equivalent to the anomaly term and has the same chiral symmetry behavior. For $T_c \lesssim T \lesssim F_a$, quark bilinears are not developing VEVs, and the relevant determinental interaction for the axion is the first diagram inside the box of Fig. 9. Now, the importance of the determination of T_1 is how much the misalignment angle θ_1 can be shrunk at T_c.

In the hadronic phase below the critical temperature T_c, the axion potential is shown in Fig. 12. The value $\theta_1(T_c)$ is the boundary value at T_c we use in the effective Lagrangian below T_c. Below T_c, the quark bilinears develop VEVs and we must consider the possibilities of $\bar{q}_L q_R$ replaced with $\langle\bar{q}_L q_R\rangle$. The effective Lagrangian from the determinental interaction is shown in Fig. 9.

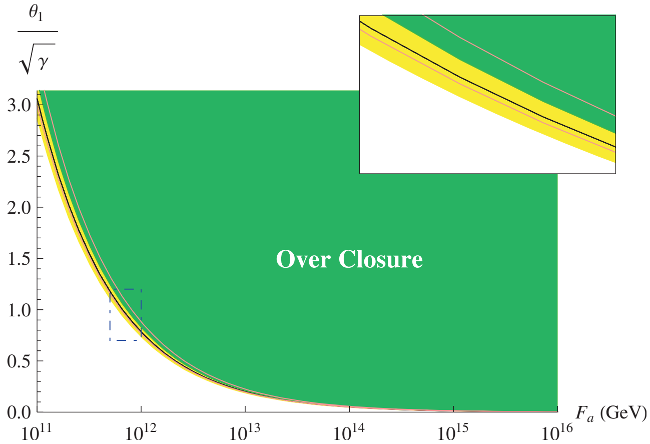

FIG. 13. (Color) F_a vs the misalignment angle $\theta_1/\sqrt{\gamma}$ as a function of Ω_a. The overclosure portion is from the precision measurement requiring $\Omega_a < 0.23$ (Komatsu *et al.*, 2009). The green region is the region excluded by the condition $\Omega_a \gtrsim 0.23$. The yellow band is the error bar region of $\Lambda_{\rm QCD}$ and the two red lines are the limits from the light quark mass bounds.

In the limit $F_a \gg f'$, the mass eigenstates in one-flavor QCD are

$$\eta'_{\rm mass} \simeq \begin{pmatrix} 1 \\ \dfrac{f'/F_a}{1+m/K'} \end{pmatrix}, \quad a_{\rm mass} \simeq \begin{pmatrix} \dfrac{-f'/F_a}{1+m/K'} \\ 1 \end{pmatrix}. \qquad (97)$$

Equation (38) with $v^3=0$ has minima at $\theta=2\pi n$ (n integer). For $v^3 \neq 0$, minima are at $\theta_{\eta'}=2\pi m$ (m integer) and $\theta=2\pi n$ (n integer). Therefore, the θ direction can be taken as the approximate axion direction even below T_c. The minimum point in the direction of the axion is not changed when one goes from $\theta_{\eta'} \neq 0$ to $\theta_{\eta'}=0$, i.e., above and below the critical temperature. [If the minimum of θ is shifted by π in going from $\theta_{\eta'}=0$ to $\theta_{\eta'}=\pi$, the shrunker $\theta_1(T_c)$ at T_c is near π, and we must start from $O(1)$ misalignment angle at T_c.] In most regions of the phase transition space, a time scale Δt is needed for the sound wave of quark bilinears to propagate to a large distance, which releases the latent heat to keep the temperature constant during the first-order phase transition (Mukhanov, 2005). Even if one considers supercooling toward a sudden phase transition, the parameter space for a sudden phase change is almost nil and the axion energy density presented in Eq. (95) is reliable (Bae, Huh, and Kim, 2008).

In Fig. 13, we present the exclusion plot for $m_u = 2.55$ MeV, $m_d=5.04$ MeV, and $m_s=104$ MeV (Manohar and Sachrajda, 2008) in the F_a vs $\theta_1/\sqrt{\gamma}$ space, including the anharmonic effect and the WMAP value (Dunkley *et al.*, 2009) of the CDM density combined with additional data (Komatsu *et al.*, 2009) $\Omega_{\rm DM}h^2 \simeq 0.1143 \pm 0.0034$. Note that F_a of order 10^{13} GeV is not very unnatural; it results from the new smaller masses for u and d (Manohar and Sachrajda, 2008).

If axions are the CDM component of the Universe, then they can be detected even though it may be very difficult. The feeble axion coupling can be compensated by the huge number of axions, since the number density is $\sim F_a^2$ and the cross section is $\sim 1/F_a^2$. So there is hope of detecting cosmic axions, which has been realized by Sikivie's cavity detector (Sikivie, 1983). But the Sikivie detector has technical limitations for the interesting large and median regions of the F_a window. For example, the F_a region $F_a > 10^{13}$ GeV advocated in anthropic arguments needs a too large cavity size and the supergravity mediation preferred region $F_a \sim 5 \times 10^{10}$ GeV requires $O(1.6\text{ mm})$ order cavities. For technically preferred axion masses in the region 10^{-6} eV, one needs a low-temperature cavity with dimension $O(>10^4\text{ cm}^3)$ and a magnetic field strength of $O(10\text{ T})$. The current status of cosmic axion search is shown in Fig. 14.

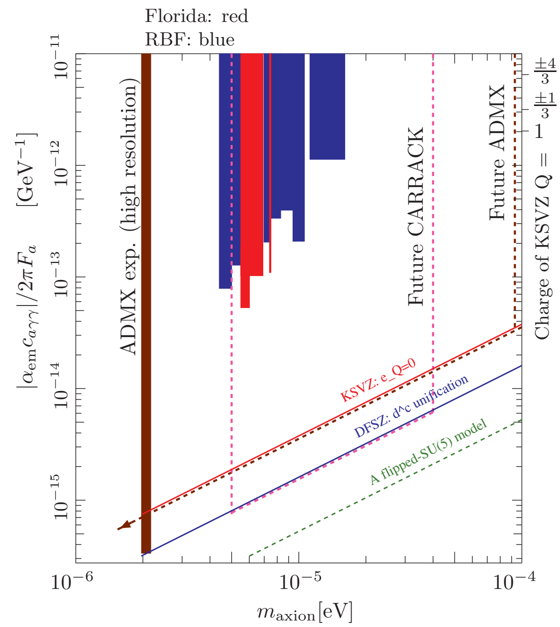

FIG. 14. (Color) The bounds on cosmic axion searches with some theoretical predictions. The coupling on the vertical axis is the coefficient of $\mathbf{E}\cdot\mathbf{B}$. The future CARRACK and ADMX experiments are from Tegmark, Aguirre, Rees, and Wilczek, 2006; Imai, 2008; and van Bibber, 2008.

C. Axion cosmology beyond the window

If $F_a \gg 10^{12}$ GeV, an $O(1)$ misalignment angle θ_1 is ruled out by the cosmic energy density argument. However, if $\theta_1 \ll 1$, the axion energy density can be within the closure density. Rather than fine tuning θ_1 to order 10^{-3} for a Planck scale F_a (Pi, 1984), the anthropic argument of Weinberg (Weinberg, 1987; Linde, 1988), that life forms can evolve in a universe with a sufficiently long lifetime, can be used for an allowable θ_1.

The homogeneous axion field value (with $a \to -a$ symmetry) right after inflation can take any value between 0 and πF_a or $\theta_1 = [0, \pi]$ because the height of the axion potential is negligible compared to the total energy density right after inflation. So in the axion context with

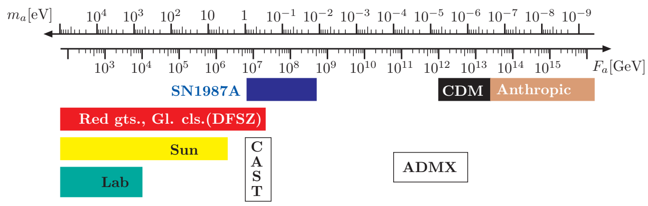

FIG. 15. (Color) A schematic for the F_a bounds.

only the misalignment production of axions, the CDM density is chosen as a random number by the spontaneous symmetry breaking of the $U(1)_{PQ}$. Even in the multicomponent CDM models including axions, the axion misalignment angle can act as the random number. This singles out axion physics, as stressed in Tegmark, Aguirre, Rees, and Wilczek (2006), from all other anthropic arguments without axions in that the selection of an axion vacuum is an unavoidable random process that fixes the key cosmological parameter. This also distinguishes axions from WIMPs, super-WIMPs, etc., where the abundance is fixed by particle physics parameters and not by a primordial random process. As a result Ω_a may be at the required value by an appropriate initial misalignment angle in models with axions with $F_a > 10^{12}$ GeV. Tegmark *et al.* studied the landscape scenario for 31 dimensionless parameters and some dimensionful parameters with which habitable planets are considered for the assumed nuclear physics parameters (Barr and Seckel, 1992). For example, Fig. 12 of Tegmark, Aguirre, Rees, and Wilczek (2006) presents the scalar fluctuation $Q \simeq \delta\rho/\rho$ versus the matter density per CMB photon ξ, in which the anthropically chosen point is shown as the star. In models with axions, this point results from a random number after inflation. If a WIMP is the sole candidate for CDM, one obtains just one number for $\delta\rho/\rho$ from particle physics parameters, which may not fit the observed point of that figure. Then we may need the CDM-favored WIMP and in addition the axion with $F_a > 10^{12}$ GeV, with the axion CDM fraction $R_a = \Omega_a/\Omega_{CDM}$. But this large F_a anthropic region has a potential conflict with the WMAP five-year data, as presented in the F_a vs E_I (=the inflation energy scale) plane of Fig. 2 of Hertzburg, Tegmark, and Wilczek (2008). For $R_a = 1$, for example, $F_a \geqslant 10^{14}$ GeV is inconsistent with the WMAP five-year data on the upper bound on the isocurvature fluctuation $\alpha_a < 0.072$ (Komatsu *et al.*, 2009).

From the study of outer space axions, we present a cartoon for the F_a bound in Fig. 15 where the future CERN Axion Solar Telescope (CAST) and ADMX experimental regions are also marked.

D. Quintessential axion

In light of SUSY breaking in supergravity, it is generally believed that at least a hidden confining force is needed at an intermediate scale. This hidden sector and the observable sector couple weakly in most phenomenological models. This scheme fits very well in the heterotic string framework and in heterotic *M* theory. In cosmology, on the other hand, we have had the important dark energy problem already for a decade (Riess *et al.*, 1998; Perlmutter *et al.*, 1999), which has led to much interest in quintessence models since the late 1980s (Wetterich, 1988). The quintessence related to axion physics is called the "quintessential axion" (QA) which was suggested in Kim and Nilles (2003, 2009). There have been attempts to identify one of the MD axions as the quintessential axion (Choi, 2000; Kim, 2000).

To explain the dark energy in terms of a QA, one requires the VEV of the QA not to roll down until recently. Of course, it is required for the current vacuum energy density of the classical QA to be of order $\lambda^4 \approx (0.003\text{eV})^4$. These two conditions restrict the QA decay constant f_q and the QA mass m_q. We can parametrize the QA (ϕ) potential as

$$V[\phi] = \lambda^4 U(\xi), \quad \xi = \frac{\phi}{f_q}. \tag{98}$$

For $\omega = p/\rho < -1 + \delta$, we require $f_q > \sqrt{(2-\delta)/6\delta} M_P |U'|$ where $U' = dU/d\xi$ (Kim and Nilles, 2003). Generically, one needs a Planckian scale quintessential axion decay constant f_q. So the QA mass is extremely small, $\lesssim 10^{-32}$ eV. As a result, there are two problems to be resolved to achieve the QA idea: a large decay constant and an extremely shallow QA potential.

It has long been believed that the MI axion has rather a robust model-independent prediction of its decay constant (Choi and Kim, 1985a; Svrcek and Witten, 2006). Recently, however, it was shown that the MI axion may not be model independent since the decay constant may depend on the compactification scheme in warped internal space, $ds^2 = h_w^2 \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n$ (Dasgupta, Firouzjahi, and Gwyn, 2008),

$$F_a = \sqrt{\frac{2}{\beta}\frac{m_s^2}{M_P}}, \tag{99}$$

where β depends on the warping in the compact space $y \in K$,

$$\beta = \frac{\int d^6y \sqrt{g_{(6)}} e^{-\phi} h_w^{-2}}{\int d^6y \sqrt{g_{(6)}} h_w^2}. \tag{100}$$

Thus, the MI axion with a small β can be a QA if the QCD axion decay constant can be in the intermediate scale. This possibility may be realizable in some composite axion models, as recently suggested in Kim and Nilles (2009).

V. AXION DETECTION EXPERIMENTS

There are currently a variety of experiments searching for axions, whether they are left over from the big bang or produced in stars or the laboratory. Though these experiments search for axions at a variety of mass and coupling scales they all rely on the Primakoff process, for which the following coupling, $c_{a\gamma\gamma}$ is given in Eq. (75):

$$\mathcal{L} = c_{a\gamma\gamma}\frac{a}{F_a}\{F_{\rm em}\tilde{F}_{\rm em}\}, \quad c_{a\gamma\gamma} \simeq \bar{c}_{a\gamma\gamma} - 1.98, \tag{101}$$

where $\bar{c}_{a\gamma\gamma}$=Tr $Q^2_{\rm em}|_{E \gg M_Z}$.

A. Solar axion search

1. Axion helioscopes

Axions produced in the nuclear core of the Sun will free-stream out and can possibly be detected on Earth via an axion helioscope, first described in 1983 (Sikivie, 1983, 1985) and developed into a practical laboratory detector in 1988 (van Bibber, McIntyre, Morris, and Raffelt, 1989). The technique relies on conversion of solar axions into low-energy x rays as they pass through a strong magnetic field. The flux of axions produced in the Sun is expected to follow a thermal distribution with a mean energy of $\langle E \rangle$=4.2 keV. The integrated flux at the Earth is expected to be $\Phi_a = g_{10}^2 3.67\times 10^{11}$ cm^{-2} s^{-1} with $g_{10} = (\alpha_{\rm em}/2\pi F_a)c_{a\gamma\gamma}10^{10}$ GeV (Zioutas *et al.*, 2005). The probability of a solar axion converting into a photon as it passes through a magnet with field strength **B** and length **L** is given as

$$P = \left(\frac{\alpha_{\rm em} c_{a\gamma\gamma} BL}{4\pi F_a}\right)^2 2L^2 \frac{1-\cos(qL)}{(qL)^2}. \tag{102}$$

Here $c_{a\gamma\gamma}$ is defined as the coupling of the axion to two photons as given in Eq. (101), while q is the momentum difference between the axion and the photon, defined as $q = m_a^2/2E$, where E is the photon energy. To maintain maximum conversion probability the axion and photon fields need to remain in phase over the length of the magnet, thus requiring $qL < \pi$ (van Bibber, McIntyre, Morris, and Raffelt, 1989). For low-mass axions $q \to 0$, leading to a maximum conversion probability. More massive axions will begin to move out of phase with the photon waves though this can be compensated for by the additon of a buffer gas to the magnet volume, thus imparting an effective mass to the conversion photon (van Bibber, McIntyre, Morris and Raffelt, 1989) and bringing the conversion probability back to the maximum. The gas pressure can be varied to tune to various axion masses.

An initial axion helioscope was built at Brookhaven in 1992 and used a 2.2 ton iron core dipole magnet oriented at the Sun with a proportional chamber for x-ray detection (Lazarus *et al.*, 1992). It was followed by a 4 T superconducting helioscope, developed by the University of Tokyo, which ran for 1 week with an evacuated bore in 1997 (Moriyama *et al.*, 1998; Ootani *et al.*, 1999) and for 1 month with a helium-filled bore in 2000 (Inoue *et al.*, 2002). Though both managed to set limits over a wide mass range their sensitivities were still well above even the most optimistic KSVZ axion couplings. Recently, though, the University of Tokyo group released data taken between December 2007 and April 2008, which were able to set a limit of $g_{a\gamma\gamma} < (5.6\text{–}13.4) \times 10^{-10}$ GeV^{-1} for the axion in the mass range $0.84 < m_a < 1.00$ eV (Inoue *et al.*, 2008).

In order to push into proposed axion model space, third-generation axion helioscopes have been developed at CERN (CAST) and at the University of Tokyo. Utilizing a prototype LHC magnet with L=9.3 m and B =9 T CAST began taking data in 2003. It utilizes a rail system to track the Sun for 90 min a day at sunrise and sunset, and its dual magnet bore allows it to employ up to four different x-ray detectors (one on each end of each magnet bore). Currently a time-projection chamber, a micromegas (micromesh gaseous structure) detector and an x-ray reflective telescope with a charge-coupled device detector are all used to detect converted x rays. Results from the combined 2003 and 2004 runs yield limits on axion-photon-photon couplings of $c_{a\gamma\gamma}/F_a < 7.6\times 10^{-8}$ GeV^{-1} (Andriamonje *et al.*, 2007). The experiment's second phase utilizing ^{4}He and ^{3}He buffer gases is currently under way with the latter gas allowing for axion searches in proposed model space up to mass ~1 eV.

2. Bragg diffraction scattering

An alternative to axion helioscopes was proposed in 1994: use of crystal detectors which meet the Bragg conditions to search for x rays generated by coherent axion-to-photon conversion (Paschos and Zioutas, 1994). Various dark matter WIMP search collaborations were able to look through their data sets and set limits on possible interactions from solar axions. These included germanium experiments such as COSME (Morales *et al.*, 2002) and SOLAX (Avignone *et al.*, 1998), CDMS (Ahmed *et al.*, 2009b), and the reactor germanium experiment TEXONO (Chang *et al.*, 2007), as well as the DAMA experiment (Bernabei *et al.*, 2001, 2003) which utilized NaI crystals. The limits from these searches can be seen in Fig. 16. One advantage of this technique is that its sensitivity is independent of axion mass, as long as one can neglect any nuclear recoils (Carosi and van Bibber, 2008).

3. Geomagnetic conversion

It has recently been pointed out (Davoudiasl and Huber, 2006) that solar axions might pass through the Earth and convert to x rays on the other side as they pass through the Earth's magnetic field. They could then be detected by x-ray telescopes and the solar x-ray background could be effectively shielded by the Earth.

B. Search for cosmic axions

Cosmic axions left over from the big bang may be detected utilizing microwave cavity haloscopes (Sikivie, 1983, 1985). The strategy relies on primordial axions drifting through a microwave cavity immersed in a strong static magnetic field in which they can resonantly

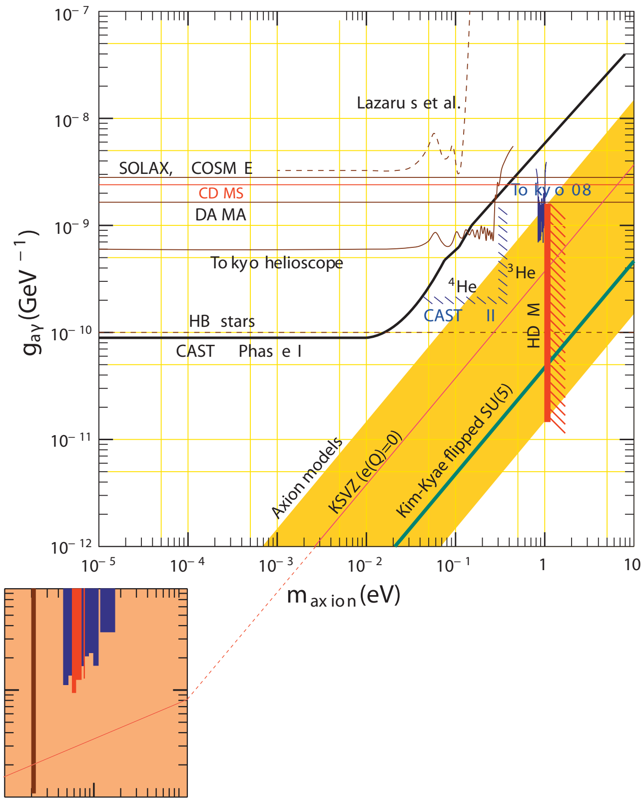

FIG. 16. (Color) Exclusion plot of axion-photon coupling vs axion mass (Carosi *et al.*, 2008). The black bold line limit is for phase 1 of the CAST experiment and results with inclusion of buffer gas are expected to increase the mass and reach plausible axion models. The field theoretic expectations are shown together with the string theory Z_{12-I} model of Choi, Kim, and Kim (2007). In the lower left apricot box, Fig. 14 is located.

convert to microwave photons, see Fig. 17. The cosmic axions' feeble interactions can be in part compensated by their large numbers; since the number density varies as $\sim F_a^2$ while their cross section varies as $\sim 1/F_a^2$. If the axion makes up the majority of CDM in the Universe, its local density is expected to be roughly 0.45 GeV/cm^3 (Gates, Gyuk, and Turner, 1995), which yields a number density of $\sim 10^{14}$ axions/cm^3 if one assumes a 4.5 μeV axion. The expected microwave signal will be a quasimonochromatic line beginning at the microwave frequency corresponding to the axion mass and slightly broadened upward due to the axion virial distribution, with expected velocities of order $10^{-3}c$, implying a spread in energies of $\delta E/E \sim 10^{-6}$.

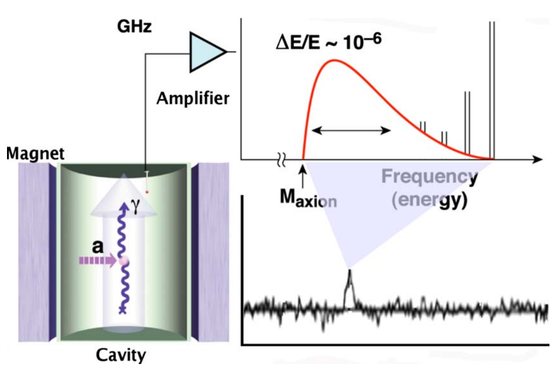

FIG. 17. (Color online) Outline of the general configuration of a resonant microwave cavity detector along with the associated singal expected from axion-photon conversions. This includes both the virial component and possible lines from coherent axions.

There could also be an additional signal from nonthermalized axions falling into the galaxy's gravitational well which would yield very sharp signals due to their low predicted velocity dispersion ($< 10^{-7}c$) (Sikivie, 2003).

1. General detector properties

Since the Lagrangian for axions coupling to a magnetic field goes as

$$\mathcal{L}_{a\gamma\gamma} = \left(\frac{\alpha_{\rm em} c_{a\gamma\gamma}}{2\pi F_a}\right) a\mathbf{E}\cdot\mathbf{B}, \tag{103}$$

the only resonant modes which can couple to axions are those that provide an axial electric field component (TM modes). The expected power generated from axion-to-photon conversions in the cavity is given by (Sikivie, 1985)

$$\begin{aligned} P_a &= \left(\frac{\alpha_{\rm em} c_{a\gamma\gamma}}{2\pi F_a}\right)^2 V B_0^2 \rho_a C_{lmn} \frac{1}{m_a} \min(Q_L, Q_a) \\ &= 0.5\times 10^{-26}\ \mathrm{W}\left(\frac{V}{500\ell}\right)\left(\frac{B_0}{7\mathrm{T}}\right)^2 C_{lmn}\left(\frac{c_{a\gamma\gamma}}{0.72}\right)^2 \\ &\quad \times \left(\frac{\rho_a}{0.5\times 10^{-24}\ \mathrm{g\ cm^{-3}}}\right) \\ &\quad \times \left(\frac{m_a}{2\pi(\mathrm{GHz})}\right)\min(Q_L, Q_a), \end{aligned} \tag{104}$$

where V is the cavity volume, B_0 is the magnetic field strength, ρ is the local axion mass density, m_a is the axion mass, C_{lmn} is a form factor which describes the overlap of the axial electric and magnetic fields of a particular TM$_{lmn}$ mode, Q_L is the microwave cavity's loaded quality factor (defined as center frequency over bandwidth), and Q_a is the axion quality factor defined as the axion mass over the axion's kinetic energy spread. The mode-dependent cavity form factor is defined as

$$C_{lmn} = \frac{\left|\int_V d^3x \vec{E}_\omega \cdot \vec{B}_0\right|^2}{B_0^2 V \int_V d^3x \epsilon |\vec{E}_\omega|^2} \tag{105}$$

where $\vec{E}_\omega(\vec{x})e^{i\omega t}$ is the oscillating electric field of the TM$_{lmn}$ mode, $\vec{B}_0(\vec{x})$ is the static magnetic field, and ϵ is the dielectric constant of the cavity space. For a cylindrical cavity with a homogeneous longitudinal $\vec{B}$ field the T_{010} mode yields the largest form factor with $C_{010} \approx 0.69$ (Bradley, 2003).

The mass range of cosmological axions is currently constrained between μeV and meV scales, corresponding to converted photon frequencies between several hundred MHz and several hundred GHz. Since larger microwave cavities correspond to lower resonant frequencies and lighter axions are more likely to contribute to the dark matter density, experiments have been designed to start searching at the low end of the frequency range. At these frequencies cavities can scan only a few kilohertz at a time in order to maintain the maximum quality factor. Axial metallic and dielectric tuning rods are utilized to tune the cavity's resonant frequency as it scans over the possible axion mass range. The scan rate is determined by the amount of time it takes for a possible axion signal to be detected over the microwave cavity's intrinsic noise, and is governed by the Dicke radiometer equation (Dicke, 1946),

$$\mathrm{SNR} = \frac{P_a}{\bar{P}_N}\sqrt{Bt} = \frac{P_a}{k_B T_S}\sqrt{\frac{t}{B}}. \tag{106}$$

Here P_a is the power generated by axion-photon conversions [Eq. (104)], $P_N = k_B B T_S$ is the cavity noise power, B is the signal bandwidth, t is the integration time, k_B is Boltzmann's constant and T_S is the system temperature (electronic plus physical temperature). The scan rate for a given signal-to-noise ratio is given by

$$\begin{aligned}\frac{df}{dt} &= \frac{12\ \mathrm{GHz}}{\mathrm{yr}}\left(\frac{4}{\mathrm{SNR}}\right)^2\left(\frac{V}{500\mathrm{l}}\right) \\ &\times\left(\frac{B_0}{7\mathrm{T}}\right)^4 C^2\left(\frac{c_{a\gamma\gamma}}{0.72}\right)^4\left(\frac{\rho_a}{0.45\ \mathrm{GeV/cm}^3}\right)^2 \\ &\times\left(\frac{3K}{T_S}\right)^2\left(\frac{f}{\mathrm{GHz}}\right)^2\frac{Q_L}{Q_a}.\end{aligned} \tag{107}$$

One can see from Eq. (106) that even a small expected signal power can be made detectable by increasing the signal power ($P_a \propto VB_0^2$), increasing the integration time t, or minimizing the system noise temperature T_S. Technology and costs limit the size and strength of the external magnets and cavities and integration times are usually $t \sim 100$ s in order to scan an appreciable bandwidth in a reasonable amount of time. As a result the majority of development has focused on lowering the intrinsic noise of the first-stage cyrogenic amplifiers.

2. Microwave receiver detectors

Initial experiments were undertaken at Brookhaven National Laboratory (DePanfilis *et al.*, 1987) and the University of Florida (Hagmann *et al.*, 1990), but their modest-sized cavities and magnet fields meant they were still factors of 10–100 times away from plausible axion model space. There are currenly two active second-generation experiments under way, the Axion Dark Matter Experiment (ADMX) at Lawrence Livermore National Laboratory (LLNL) and the Cosmic Axion Research with Rydberg Atoms in Cavities at Kyoto (CARRACK) experiment in Japan. Both experiments utilize large microwave cavities immersed in a strong static magnetic field to resonantly convert axions to photons but they go about detecting these photons in two different ways. ADMX uses ultrasensitive microwave receivers while CARRACK uses Rydberg atoms to detect single photons.

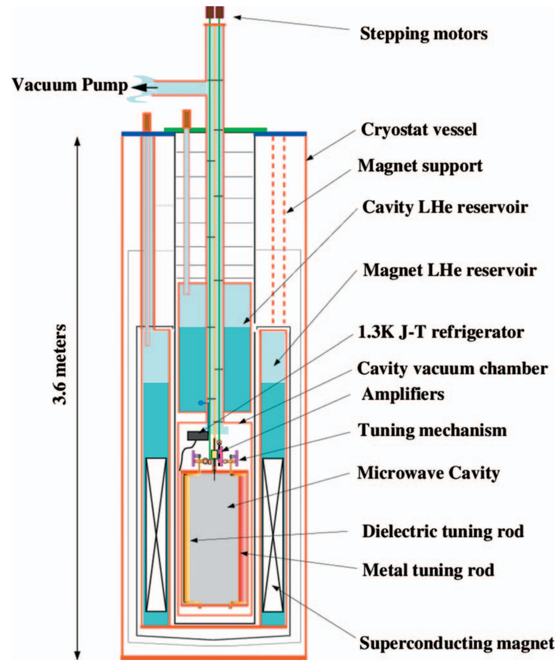

FIG. 18. (Color) Schematic of the ADMX experiment (Carosi and van Bibber, 2008).

The ADMX experiment is a collaboration of LLNL, MIT, the University of Florida, Lawrence Berkeley National Laboratory (LBNL), U.C. Berkeley, University of Chicago, and Fermilab, and has been operating in various modes since February 1996. A diagram of the experiment is shown in Fig. 18. ADMX consists of an 8.5 T superconducting magnet, 110 cm in length with a 60 cm clear bore. A 200 liter stainless steel microwave cavity plated in ultrapure copper is suspended below a cryogenic stage in the center of the B field. Power generated in the cavity is coupled to an adjustable antenna vertically input through the top cavity plate. Any signal is then boosted by extremely low-noise cryogenic amplifiers before being sent through a double-heterodyne mixing stage. Here the gigahertz range signal is mixed down to an intermediate 10.7 MHz, sent through a crystal bandpass filter, and then mixed down to audio frequencies at 35 kHz. This audio signal is then analyzed by fast-Fourier-transform electronics which measure over a 50 kHz bandwidth centered at 35 kHz. There is also a "high-resolution" channel in which the signal is mixed down to 5 kHz and sent through a 6-kHz-wide bandpass filter. Time traces of the voltage output, consisting of 2^{20} data points with a sampling frequency of 20 kHz, are then taken, resulting in a 52.4 s sample with 0.019 Hz resolution (Duffy *et al.*, 2006).

Since the system noise is dominated by the first stage of amplification, great care was taken in choosing the cryogenic amplifiers. The initial ADMX data runs utilized heterojunction field effect transistor (HFET) am-

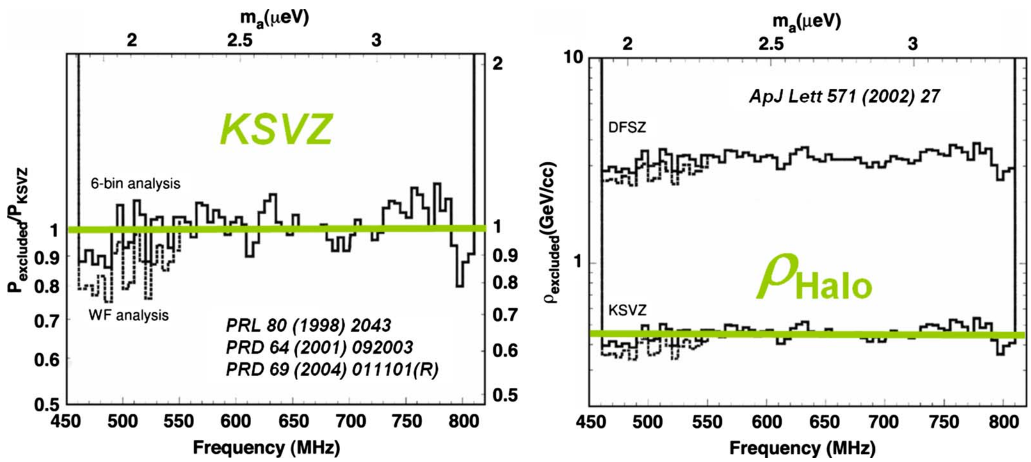

FIG. 19. (Color online) Medium-resolution limits from the ADMX experiment. The left plot shows limits to axion coupling assuming a dark matter halo density of $\rho=0.45\ \mathrm{GeV/cm^3}$ (upper region excluded). The right plot shows limits on the axion contribution to the dark matter halo density assuming the axion has either the KSVZ or DFSZ couplings (Bradley, 2003).

plifiers developed by the National Radio Astronomy Observatory (Daw and Bradley, 1997). Even though they had noise temperatures of only 2 K, the quantum noise limit at 1 GHz (defined as $T_q=h\nu/k_B$) is only 50 mK. As a result a much development went into replacing the HFETs with more sensitive superconducting quantum interference devices (SQUIDs) which had noise temperatures of only 15% of the quantum limit (Bradley, 2003). Currently data are being taken using the SQUIDs for the first stage of amplification 1920.

Results from the initial run using HFET amplifiers have already probed plausible axion model space in the axion mass range between 2.3 and 3.4 μeV (Bradley, 2003). Results from a high-resolution search have probed further into coupling space over a smaller mass range, 1.98–2.17 μeV (Duffy *et al.*, 2006). As of this review ADMX is scanning over the mass range corresponding to 800–900 MHz using SQUID amplifiers.

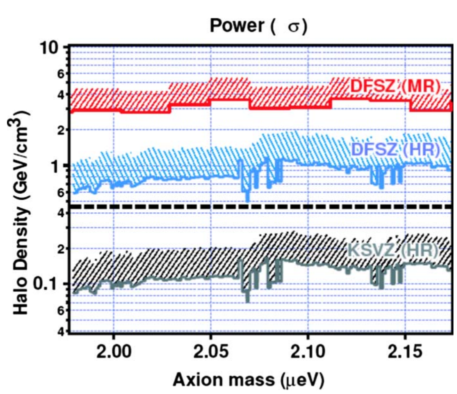

FIG. 20. (Color online) High-resolution limits from the ADMX experiment. Limits given in terms of axion contribution to the dark matter halo density assuming the axion has either the KSVZ or DFSZ coupling strength. The medium-resolution plot for that mass range for the DFSZ coupling is also given (Duffy *et al.*, 2006). The KSVZ axion with $e_Q=0$ shown above gives ρ_a at Earth less than $0.16\ \mathrm{GeV/cm^3}$, which corresponds to $\Omega_a \leqslant 0.36$. So the ADMX line of Fig. 14 using Eq. (95) crosses the $e_Q=0$ KSVZ line and goes down to the DFSZ line.

3. Rydberg atom detectors

The CARRACK experiment has published proof of concept papers for their detection technique using Rydberg atoms as opposed to low-noise amplifiers (Tada *et al.*, 2006). The experimental setup is shown in Fig. 21. In it rubidium atoms are excited into a Rydberg state ($|0\rangle \rightarrow |n\rangle$), and move through a detection cavity coupled to an axion conversion cavity. The spacing between energy levels is tuned to the appropriate frequency utilizing the Stark effect, and the Rydberg atoms' large dipole transition moment ensures efficient photon detection (one photon per atom, $|ns\rangle \rightarrow |np\rangle$). The atoms are then subjected to a selective field ionization allowing the atoms

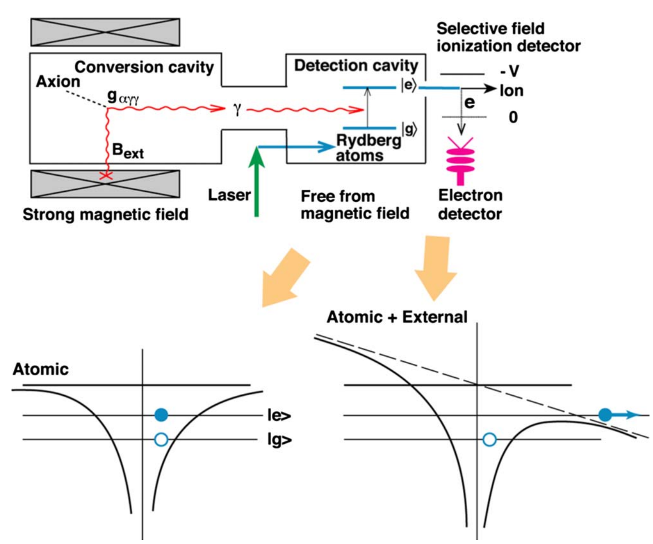

FIG. 21. (Color online) General schematic of CARRACK experiment utilizing Rydberg atoms to recover single photons generated in the microwave cavity (Tada *et al.*, 2006; Carosi and van Bibber, 2008).

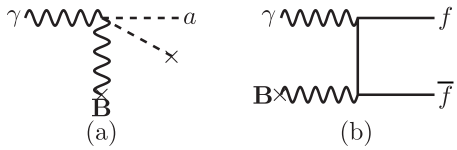

FIG. 22. Possible processes leading to a vacuum dichroism.

with the higher-energy state ($|np\rangle$) to be detected (Carosi and van Bibber, 2008). The advantage of this system is that Rydberg atoms act as single-photon detectors and thus do not suffer from quantum noise limitations.

Though still in the development phase CARRACK has already gone through two iterations, CARRACK 1 and CARRACK 2, and it has measured cavity emission at 2527 MHz down to a temperature of 67 mK, which is a factor of 2 below the quantum noise floor for that frequency. The eventual goal is 10 mK (Tada *et al.*, 2006). One disadvantage of this technique is that one cannot detect signals finer than the bandpass of the cavity, of order $\sim 10^{-5}$, which negates searches for late-infall coherent axions.

C. Laser searches

In addition to cosmological and solar axion searches there is also a class of laboratory axion searches that utilize laser photons ($\gamma_{\rm laser}$) traversing a magnetic field. Here the polarized laser photons can scatter off virtual photons (γ_v) provided by the magnetic field and convert into axions $\gamma_{\rm laser}+\gamma_v\to a$. Currently, laser axion searches fall into two general categories. The first technique looks for magneto-optical effects of the vacuum due to polarized laser photons disappearing from the beam as they are converted into axions. The second looks for photons converting into axions in the presence of a magnetic field, which are then transmitted through a wall and converted back into photons by a magnetic field on the other side, so-called light shining through walls experiments.

1. Polarization shift of laser beams

There can be axion-photon-photon anomalous coupling of the form $a\mathbf{E}\cdot\mathbf{B}$. A laser-induced axionlike particle search employing this coupling has been performed since the early 1990s by the Rochester-Brookhaven-Fermilab-Trieste (RBFT) group (Cameron *et al.*, 1991). A few years ago, the same type of experiment by the PVLAS Collaboration was performed with an initial positive signal with $F_a\sim 10^6$ GeV (Zavattini *et al.*, 2006) as discussed earlier. This has led to some exotic models where a vacuum dichroism is achieved by producing axionlike particles as shown in Fig. 22(a). Because of the nonrenormalizable interaction implied in Fig. 22(a), one may reconcile this model with the astrophysical bound (Mohapatra and Nasri, 2007), or if light millicharged particles are produced in a strong magnetic field as shown in Fig. 22(b), a vacuum dichroism is achieved as discussed in Gies, Jaeckel, and Ringwald (2006), Masso and Redondo (2006), and Kim (2007c). Here the polarization of the laser beam is looked for. With more data accumulation, there is no convincing evidence for an axionlike particle with $F_a\sim 10^6$ GeV at present, contrary to an earlier confusion (Dupays *et al.*, 2005; Chen *et al.*, 2007; Yoo, 2007; Zavattini *et al.*, 2007; Chou *et al.*, 2008). But this incident led to the current search for axionlike particles at DESY (Ringwald, 2008).

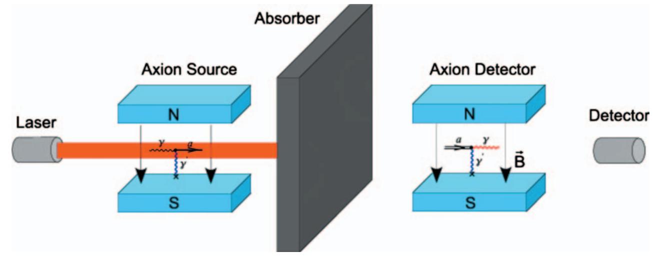

FIG. 23. (Color) Schemtic of "light shining through walls" experiment (Battesti *et al.*, 2008).

2. Light shining through walls

The "light shinging through walls" technique for searching for axions was first proposed in 1987 by van Bibber *et al.* (1987) and recently a model study has been presented (Adler, Gamboa, Mendéz, and López-Sarrión, 2008). The general experimental layout can be seen in Fig. 23 where polarized laser photons pass through the magnetic field with $\mathbf{E}\parallel\mathbf{B}$ and any converted axions (or other psuedoscalar particles) can continue through an absorber to be reconverted to photons on the other side.

The probability for a photon to convert into an axion as it traverses the "axion source" region is given by

$$P_{\gamma\to a}\propto\frac{1}{4}\left(\frac{\alpha_{\rm em}c_{a\gamma\gamma}}{2\pi F_a}BL\right)^2\frac{1-\cos(qL)}{(qL)^2}. \qquad (108)$$

This is the same probability for an axion to convert back into a detectable photon in the "axion detector" region on the other side of an absorber, which leaves the total probability for detecting a photon-axion-photon conversion as $P_{\gamma\to a\to\gamma}=P^2_{\gamma\to a}$ (ignoring photon detection efficiencies of course) (Battesti *et al.*, 2008). There is a maximum detectable axion mass for these laser experiments because the oscillation length becomes shorter than the magnetic field length, causing a degradation of the form factor $F(q)=1-\cos(qL)/(qL)^2$, but this can be compensated for using multiple discrete dipoles.

The first experiment using this technique was performed by the RBFT Collaboration in the early 1990s (Cameron *et al.*, 1993). Using two superconducting dipole magnets ($L=4.4$ m and $B=3.7$ T) and a laser ($\lambda=514$ nm and $P=3$ W) with an optical cavity providing ~ 200 reflections in the axion-generating region, they were able to set upper limits on axion couplings of $g_{a\gamma\gamma}<6.7\times 10^{-7}$ GeV (95% C.L.) for pseudoscalars with a maximum mass of $m_a<10^{-3}$ eV (Cameron *et al.*, 1993).

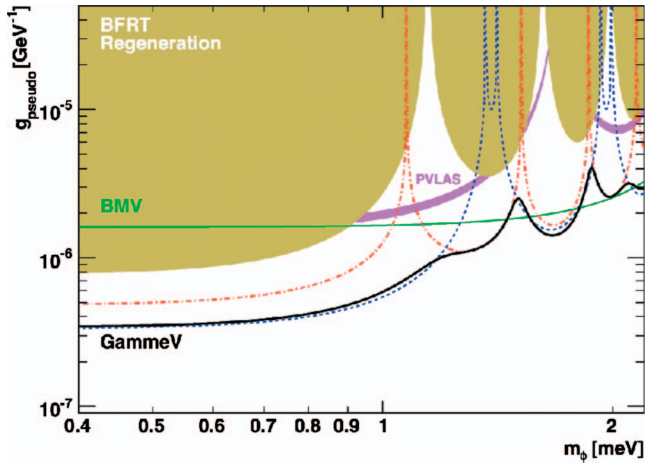

FIG. 24. (Color) Current limits on axion coupling from the GammeV Collaboration (Chou *et al.*, 2008; Yoo, 2008).

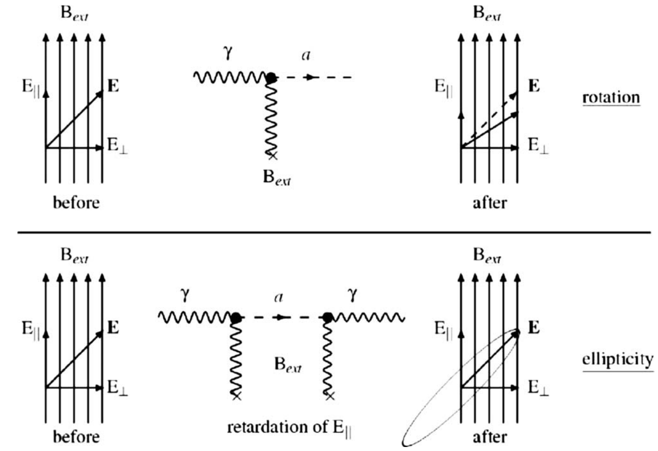

FIG. 25. The dichroism and birefringence effects. The upper plot shows the effect of dichroism as photons converting into axions cause a rotation of the linear beams polarization vector by an amount ϵ. The lower plot shows virtual axions inducing birefringence in which the linear beam acquires ellipticity Ψ (Battesti *et al.*, 2008).

Recent photon regeneration experiments include the BMV Collaboration at LULI (Robilliard *et al.*, 2007) which uses a short pulsed-field magnet and the GammeV Collaboration at Fermilab (Chou *et al.*, 2008) which uses a Tevatron dipole magnet (L=6 m and B=5 T) with an optical barrier in the middle. Both of these have ruled out the signal reported by the PVLAS (see next section). Figure 24 shows the current bounds from these latest regeneration experiments. Recently it has been shown that photon regeneration experiments can be resonantly enhanced by encompassing both the production and reconversion magnets in matched Fabry-Perot optical resonators (Sikivie, Tanner, and van Bibber, 2007).

3. Magneto-optical vacuum effects

An alternative to the shining light through walls technique is to look for the indirect effect of photons in polarized laser light converting into axions as the beam traverses a magnetic field. Figure 25 shows two different ways in which axion interactions can modify a polarized laser beam, induced dichroism and vacuum birefringence. Vacuum dichroism occurs when a polarized laser beam passes through a dipole magnet with the electric field component **E** at a nonzero angle ϕ relative to **B**. The photon component parallel to **B** will have a small probability to convert into axions, causing the polarization vector to rotate by an angle ϵ. Vacuum birefringence is due to the induced ellipticity of the beam (Ψ) as a result of virtual axions. It should be noted that higher-order QED diagrams, or "light-by-light scattering" diagrams, are expected to contribute to vacuum birefringence as well. Each of these effects can be estimated as

$$\Psi \approx N \frac{B^2 L^3 m_a^2}{384\omega}\left(\frac{c_{a\gamma\gamma}}{F_a}\right)^2 \sin(2\theta), \tag{109}$$

$$\epsilon \approx N \frac{B^2 L^2}{64}\left(\frac{c_{a\gamma\gamma}}{F_a}\right)^2 \sin(2\theta), \tag{110}$$

in the limit that $m_a^2 L/4\omega \ll 1$. Here L is the effective path length, N is the number of paths the light travels in the magnetic field, m_a is the axion mass, ω is the photon energy, and θ is the photon polarization relative to the magnetic field (Battesti *et al.*, 2008).

The initial experiment looking for magneto-optical vacuum effects was carried out by the RBFT Collaboration in the early 1990s (Semertzidis *et al.*, 1990). This experiment used a single-pass 8.8-m-long magnet with a magnetic field of $B \sim 2.1$ T and N=500. It set a limit on the polarization rotation of $\epsilon < 3.5 \times 10^{-10}$ which was still three orders of magnitude higher than that expected by light-by-light scattering and almost 15 orders of magnitude greater than an $m_a \sim 10^{-3}$ eV axion.

Recently the early PVLAS Collaboration reported the positive detection of vacuum dichroism. This experiment consists of a 1-m-long 5 T superconducting magnet with a angular frequency $\Omega_{\rm mag}$ of the magnet rotation and a 6.4-m-long Fabry-Pérot cavity giving the pass number $N = 2\Omega_{\rm mag}/\pi \sim 44\,000$. It registered a polarization shift of

$$\epsilon = (3.9 \pm 0.5) \times 10^{-12} \text{ rad pass}^{-1} \tag{111}$$

which translates to an allowed mass range of a neutral pseudoscalar boson of $1 \leq m_b \leq 1.5$ meV and a coupling strength of $1.5 \times 10^{-3} \leq c_{a\gamma\gamma}/F_a \leq 8.6 \times 10^{-3}$ GeV^{-1} (Zavattini *et al.*, 2006). Though the report of this positive signal has been retracted (Zavattini *et al.*, 2007, 2008), the interest it raised led to a number of more advanced experimental searches such as some of the new laser regeneration experiments mentioned previously.

TABLE II. Natural scales of F_a. For n large extra dimensions, the Planck mass is $M_P \simeq M_D(R/M_D)^{n/2}$.

Axions from	Order of F_a
String theory	String scale or Planck scale
M theory	String or the scale of the 11th dimension
Large extra (n) dimension	Combination of the fundamental mass M_D and extra dimension radius R
Composite models	Compositeness scale
Renormalizable theories	$\mathrm{U}(1)_{PQ}$-global-symmetry-breaking scale

VI. THEORIES FOR VERY LIGHT AXIONS

Axion couplings come in three types: the PQ-symmetry-preserving derivative coupling c_1 term, the PQ-symmetric c_2 term, and the anomalous c_3 term. The PQ symmetry gives a gluon anomaly and c_2+c_3 must be nonzero. Generally, we can therefore define a as a pseudoscalar field without potential terms except the one arising from the gluon anomaly under a particular basis (for example in the $c_2=0$ basis),

$$\frac{a}{F_a}\frac{1}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}. \tag{112}$$

Then we note that this kind of nonrenormalizable anomalous term can arise in several ways. The natural scales of F_a are shown in Table II. In any case, the essence of the axion solution (wherever it originates in Table II) is that the axion VEV $\langle a\rangle$ seeks $\theta=0$, whatever happened before. In this sense it is a cosmological solution. The potential arising from the anomaly term after integrating out the gluon field is the axion potential (with c_2+c_3) shown in Fig. 7.

A. SM singlets without SUSY

A complex SM singlet carrying the PQ charge can appear in many extensions of the SM: in grand unified theories (GUTs) (Wise, Georgi, and Glashow, 1981), in composite models (Choi and Kim, 1985c; Kim, 1985; Babu, Choi, Pati, and Zhang, 1994), and in models with extra dimensions (Di Lella, Pilaftsis, Raffelt, and Zioutas, 2000).

In the SU(5) GUT, the axion can be embedded in a complex $\mathbf{24}=\Sigma$ (Wise, Georgi, and Glashow, 1981), in which case the VEV $\langle\Sigma\rangle$ breaking SU(5) down to the SM and hence the axion decay constant is the GUT scale and is outside the axion window. On the other hand, a complex GUT singlet, whose VEV is not related to the GUT scale, can house the axion within the axion window. A SUSY generalization of the SU(5) GUT axion has been shown to be possible (Nilles and Raby, 1982). Recently, in view of the white dwarf evolution (Isern, García-Berro, Torres, and Catalán, 2008) with the two-dark-matter scenario (Huh, Kim, and Kyae, 2009) an electrophilic axion has been suggested in a SUSY flipped SU(5) (Bae, Huh, Kim, Kyae, and Viollier, 2009).

B. Composite axions

A SM singlet for the very light axion can arise as a composite meson with an extra confining force whose scale is much larger than the electroweak scale. This confining force can be the hidden sector gauge group in supergravity or just an extra gauge group. We call this extra confining gauge group "axicolor" SU(N). To create the QCD axion below the axicolor scale, there must be two classically conserved axial global symmetries (Choi and Kim, 1985c; Kim, 1985). With only one axial symmetry, a massless meson would not result, as in the case of one-flavor QCD there is no massless meson since the only meson η' becomes heavy by the instanton solution of the so-called U(1) problem ('t Hooft, 1986). For two axial symmetries, we can consider two kinds of axiquark, $Q^{A\alpha}$, $\bar{Q}_{A\alpha}$, q^A, and $\bar{q}_A$ where A is the SU(N) index and α is the $\mathrm{SU}(3)_c$ index. For these vectorlike representations, $(\mathbf{N},\mathbf{3})+(\bar{\mathbf{N}},\bar{\mathbf{3}})+(\mathbf{N},\mathbf{1})+(\bar{\mathbf{N}},\mathbf{1})$ under $\mathrm{SU}(N)\times\mathrm{SU}(3)_c$, mass terms are not introduced. The axicolor vacuum angle problem is solved basically by the massless axiquarks Q and q. Even though Q looks like a massless QCD quark, it cannot be considered as the massless quark solution of the QCD θ problem. After integrating out the axicolor degrees, we obtain an effective Lagrangian resulting from Q and q. The axibaryons are expected to be removed at the axicolor scale. Of two kinds of meson, one (the axicolor η') is removed at the axicolor scale and the other remains exactly massless. However, this massless axicolor meson couples to the QCD anomaly and becomes a QCD axion through the c_3 term, becoming the so-called hadronic axion. Of the two currents

$$\begin{aligned}\tilde{J}^5_\mu &= \bar{Q}\gamma_\mu\gamma_5 Q + \bar{q}\gamma_\mu\gamma_5 q,\\ J^5_\mu &= \bar{Q}\gamma_\mu\gamma_5 Q - 3\bar{q}\gamma_\mu\gamma_5 q,\end{aligned} \tag{113}$$

the divergence of J^5_μ corresponds to the massless meson a below the axicolor scale,

$$\partial^\mu J^5_\mu = \frac{2N}{32\pi^2}G^\alpha_{\mu\nu}\tilde{G}^{\alpha\mu\nu}, \tag{114}$$

and hence we obtain the effective interaction (112). In this minimal model, the domain wall number is N (Choi and Kim, 1985c, 1985d). In a supergravity model of preons, a similar mechanism was used to realize a composite axion (Babu, Choi, Pati, and Zhang, 1994), where the role of q-type matter is replaced by the metacolor gluino λ', metacolor is the binding force of preons. Even if the metacolor gluino obtains a mass of $O(100\text{ GeV})$, the QCD θ can be made to be within the experimental bound if F_a is greater than 10^{11} GeV.

The composite axion of Chun, Kim, and Nilles (1992b) is a composite made of hidden-color scalars whose bilinears develop VEVs and break the PQ symmetry. This idea has been made more concrete by Kim and Nilles (2009).

In the gauge-mediated SUSY-breaking scenario of Intriligator, Seiberg, and Shih (ISS) (2006), for example, an SU(N_c) confining group with N_f flavors satisfying $N_c+1\leqslant N_f<\frac{3}{2}N_c$ allows a SUSY-breaking local minimum. If $N_f-N_c\geqslant 3$ (for example, $N_c=7$ and $N_f=10$) with one type of $Q^{A\alpha}+\bar{Q}_{A\alpha}$ and N_f-3 flavors of the type $q^A+\bar{q}_A$, then there can exist a suitable local minimum where the composite axion envisioned in Eq. (113) can be realized. In this case, the SUSY-breaking scale and the composite axion scale are related, as first tried by Kim (1984).

C. Axions with extra dimensions

With large extra dimensions, the axion identification involves a few parameters: the fundamental scale mass M_F, the Kaluza-Klein (KK) radius R, and the number of extra dimensions n. In addition, there are several ways to allocate the field(s) containing the axion in the bulk and/or branes.

The possibility of large extra dimensions has been considered for the flat and warped extra dimensions. The TeV scale for M_F was the main motivation to look for the next level of the current experimental limit on millimeter-scale gravity (Antoniadis *et al.*, 1998; Arkani-Hamed, Dimopoulos, and Dvali, 1998). Because the axion scale is considered to be at the intermediate scale, a string theory at the intermediate scale M_F has also been considered (Burgess, Ibanez, and Quevedo, 1999). With the Randall-Sundrum-type warp factor (Randall and Sundrum, 1999a, 1999b), it is possible to introduce the intermediate scale with a Planck scale M_F via the Giddings-Kachru-Polchinski stabilization mechanism (Giddings, Kachru, and Polchinski, 2002).

Here we look only at the possibility of a TeV scale M_F. Since the Planck mass is given by $M_P\approx M_F(RM_F)^{n/2}$, we obtain $n\geqslant 2$ for $M_F\simeq 10$ TeV (Hannestad and Raffelt, 2002; Kanti, 2009). The Lagrangian in $4+n$ dimensions $[(4+n)\mathrm{D}]$ with a bulk field axion can be written as (Chang, Tazawa, and Yamaguchi, 2000),

$$\mathcal{L}_{\rm eff}=\int d^n y\left\{\frac{1}{2}M_F^n[\partial_\mu a\partial^\mu a+\partial_y a\partial^y a]+\frac{\xi\alpha_{\rm em}}{\pi}\frac{a}{\bar{v}_{\rm PQ}}F^{\rm em}_{\mu\nu}\tilde{F}^{{\rm em},\mu\nu}\right\} \tag{115}$$

where $\bar{v}_{\rm PQ}$ is the PQ-symmetry-breaking scale at the fundamental scale order M_F, and

$$a(x^\mu,\mathbf{y})=\sum_{\mathbf{n}=\mathbf{0}}^{\infty}a_{\mathbf{n}}(x^\mu)\cos\left(\frac{\mathbf{n}\cdot\mathbf{y}}{R}\right). \tag{116}$$

The four-dimensional (4D) PQ symmetry-breaking scale is $F_a\approx(M_P/M_F)^{A/n}\bar{v}_{\rm PQ}$ where $A=|\mathbf{n}|=\sqrt{n_1^2+\cdots+n_n^2}<n$ and F_a falls between $\bar{v}_{\rm PQ}$ and M_P. The very light axion is the $\mathbf{n}=\mathbf{0}$ component in Eq. (116), and the rest are the KK axions. The mass splitting of the KK axions is of order $1/R$, and the phenomenology of these KK axions for $a_{\rm KK}\to 2\gamma$ has been studied by Di Lella, Pilaftsis, Raffelt, and Zioutas (2000), from which we have $1/R\sim 1(10)$ eV for $n=2(3)$ for $M_F\approx 1$ TeV.

The possibility of a Z_2 odd 5D gauge field in a warped fifth dimension has been suggested for a QCD axion under the assumption that all unwanted PQ-symmetry-breaking effects are suppressed (Choi, 2004). One such constraint is that the bulk fields carry the vanishing PQ charge.

D. SUSY-breaking scale, axion and axino

The 4D supergravity interactions with the vanishing cosmological constant were obtained in 1983 (Cremmer, Ferrara, Girardello, and van Pröyen, 1983). The PQ symmetry can be embedded in the supergravity framework (Kim, 1984),

$$W_{\rm PQ}=(f_\delta A_1A_2-F_1^2)Z+(f_\epsilon A_1A_2-F_2^2)Z'+f_QA_1\bar{Q}_1Q_2, \tag{117}$$

where Z, Z', A_1, and A_2 are gauge singlet chiral fields, $\bar{Q}_1$ and Q_2 are chiral quark superfields, and f_δ, f_ϵ, F_1^2, and F_2^2 are parameters. The superpotential (117) leads to F-term SUSY breaking and PQ symmetry breaking at a common scale at order $O(F_1^2,F_2^2)$ if $f_\delta/f_\epsilon\neq F_1^2/F_2^2$. The f_Q term defines the PQ charge of the heavy quark and the resulting axion is of the KSVZ type. The PQ-symmetry-breaking scale is given by nonzero $\langle A_1A_2\rangle\simeq(F^2/\lambda)\cos(\beta-\alpha)$ and the SUSY-breaking scale is given by nonzero $Z_F=-F^2\sin\alpha\sin(\beta-\alpha)$ and $Z'_F=F^2\sin\alpha\sin(\beta-\alpha)$, where $\lambda=\sqrt{f_\delta^2+f_\epsilon^2}$, $F^2=\sqrt{F_1^4+F_2^4}$, $\tan\alpha=f_\epsilon/f_\delta$, and $\tan\beta=F_2^2/F_1^2$. The axino does not obtain mass at this level, but obtains a mass at order of the soft SUSY-breaking scale (Chun, Kim, and Nilles, 1992a; Chun and Lukas, 1995). Note that $\langle Z'/Z\rangle\simeq-\cot\alpha$ with $\langle Z,Z'\rangle=O(F^2/M)$. Early discussions of the axino can be found in Frère and Gérard (1983).

E. The μ problem

If Higgs doublets $H_{u,d}$ carry vanishing U(1) charges beyond the MSSM gauge charges, then the superpotential can contain a $W_\mu=-\mu H_uH_d$ term where μ can be of the order of the fundamental scale since it is a supersymmetric term. This is problematic for the TeV scale electroweak symmetry breaking; this is the so-called μ problem (Kim and Nilles, 1984). This μ term is a supersymmetric Higgsino mass term and can be forbidden in

W by introducing some symmetry, continuous or discrete. The widely discussed ones are the PQ and R symmetries. In the supergravity framework, if the Higgs doublets carry one unit of PQ charge then nonrenormalizable interactions of the form $S^2 H_u H_d / M_P$ can be present in W if S^2 and $H_u H_d$ carry opposite PQ charges. Then the resulting μ is of order F_a^2/M_P, which can be of order of the TeV scale (Kim and Nilles, 1984). With an intermediate hidden sector with hidden-sector squarks Q_1 and $\bar{Q}_2$, one may have a nonrenormalizable interaction of the form $Q_1 \bar{Q}_2 H_u H_d / M_P$. In this case the hidden-sector squark condensation at the intermediate mass scale can also generate a TeV scale μ (Chun, Kim, and Nilles, 1992b). [For the B_μ term, one may consider $(Q_1 Q_2^* / M_P^2) H_u H_d$ in the Kähler potential.] It is better for the superpotential to possess this kind of PQ symmetry and/or R symmetry (Hall and Randall, 1991; Dine and MacIntire, 1992; Casas and Muñoz, 1993; Kim and Nilles, 1994; Kim, 1999a). If so, even if the nonrenormalizable interactions are not considered, the gravity mediation scenario can generate a TeV scale μ via the Giudice-Masiero mechanism (Giudice and Masiero, 1988). In supergravity, the Higgsino mass term is present in the chiral fermion mass matrix given by Cremmer, Ferrara, Girardello, and van Pröyen (1983) and Nilles (1984):

$$e^{-G}[G^{ij} - G^i G^j - G^l (G^{-1})_l^k G_k^{ij}] \chi_{Li} \chi_{Lj}, \tag{118}$$

where $G = K(\phi, \phi^*) - \ln |W|^2$. The term $e^{-G} G^{ij}$ gives $\mu \sim m_{3/2}$ if K contains $H_u H_d$ (Giudice and Masiero, 1988) and $\mu \sim S^2/M_P$ if W contains $S^2 H_u H_d / M_P$ (Kim and Nilles, 1984).

In the next-to-MSSM (NMMSM) models with $W = S H_u H_d$, the μ term can be generated by the singlet VEV $\langle S \rangle$ at the electroweak scale (Cerdeño, Hugonie, López-Fogliani, Muñoz, and Teixeira, 2004; López-Fogliani and Muñoz, 2006). In a Z'-added MSSM (Z'MSSM), the μ term can also be successfully generated (Langacker, Paz, Wang, and Yavin, 2008).

Extending the MSSM gauge group which can be broken down to the MSSM at a high energy scale, one can generate a reasonable μ. For example, there exists an interesting solution to the problem of why there is only one pair of Higgsino doublets at low energy in the extended $SU(3)_W \times U(1)$ electroweak model (Lee and Weinberg, 1977a). This is dictated by the extended gauge symmetry. This one-pair problem is elegantly solved in the SUSY Lee-Weinberg-type model due to the antisymmetric [under $SU(3)_W$] Higgsino mass matrix (Kim, 2007b), reminiscent of the "color" introduction used to put low-lying baryons in the completely symmetric representation 56 in the old flavor-spin SU(6) (Han and Nambu, 1965).

Thus, explicit steps toward a successful μ in the gravity mediation scenario can be constructed in extra-singlet models, in SUSY-GUT models, through the superpotential, through the Kähler potential, and in composite models.

The loop effects are important sources of SUSY breaking in the gauge-mediated SUSY-breaking (GMSB) scenario (Dimopoulos and Raby, 1981; Dine, Fischler, and Srednicki, 1981a; Dine and Fischler, 1983; Dine and Nelson, 1993; Dine, Nelson, and Shirman, 1995), in anomaly mediation SUSY breaking (AMSB) (Giudice *et al.*, 1998; Randall and Sundrum, 1999a), and even in the mirage mediation scenario (Choi, Jeong, and Okumura, 2005; Loaiza-Brito, Martin, Nilles, and Ratz, 2005). GMSB has been suggested to solve the flavor problem (Gabbiani, Gabrielli, Masiero, and Silvestrini, 1996) present in the gravity mediation scenario. In this GMSB or any other loop-generated SUSY-breaking scenario, the soft terms generated by the supergravity effect are required to be subdominant compared to those arising from the loops, or at best comparable to them. If the loop terms are subdominant as in the GMSB or AMSB, then there are some problems.

First, the generation of μ is difficult because μ term generation via the Giudice-Masiero mechanism is subdominant at the TeV scale. One has to generate μ by employing the PQ and/or R symmetries; this method, however, does not belong to generating all TeV scale parameters dynamically. In this regard, another confining group around TeV scale has been proposed (Choi and Kim, 2000), and the model presented there is the type of composite $SU(2)_W$ axion discussed in Sec. VI.B, which was saved by introducing singlets and relevant couplings (Luty, Terning, and Grant, 2001). Then again it does not succeed in generating all TeV scale parameters dynamically.

Second, in the loop SUSY-breaking scenarios for generating all TeV scale electroweak parameters by loops there exists the B_μ/μ problem (Dvali, Giudice, and Pomarol, 1996). Since it occurs at loop orders, we consider $\int d^4\theta\, H_u H_d X^\dagger$ for μ and $\int d^4\theta\, H_u H_d X X^\dagger$ for B_μ where the auxiliary component of X develops a VEV. From this observation, one generically obtains $B_\mu \sim \mu\Lambda$ where $\Lambda \sim \mu/f^2$ can be greater than μ; this was remedied by making B_μ appear at two-loop order (Dvali, Giudice, and Pomarol, 1996). This B_μ/μ problem occurs essentially because of the difference of the engineering dimensions of the B_μ and μ terms. Both generically appear at one-loop order with the coefficient $g^2/16\pi^2$, and hence in describing the electroweak scale the B_μ term lacks one power of $g^2/16\pi^2$. Recently, a better solution employing a Kähler potential $H_u H_d (\ln X + \ln X^\dagger)$ has been suggested (Giudice, Kim, and Rattazzi, 2008), which can be compared to the original Giudice-Masiero Kähler potential $H_u H_d X^\dagger + \cdots$. There exist several more ideas about the B_μ/μ problem (Cohen, Roy, and Schmaltz, 2007; Cho, 2008; Murayama, Nomura, and Poland, 2008; Roy and Schmaltz, 2008).

Perhaps nonrenormalizable interactions are the easy solution of the μ and B_μ/μ problems even in the GMSB. Here, however, one introduces another scale. Without a detailed knowledge of the ultraviolet completion of the MSSM, the nonrenormalizable interactions are usually assumed to be suppressed by the Planck mass M_P. But,

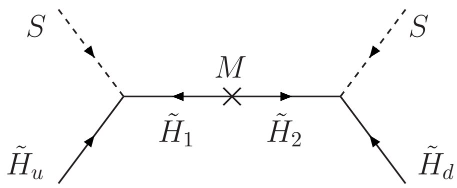

FIG. 26. The generation of the μ term by a seesaw mechanism.

there might be some heavy mass scale M, which can be somewhat smaller than the Planck mass M_P, for the seesaw mass of the required nonrenormalizable interactions. In string compactifications, it is known explicitly that M can be different from M_P (Choi and Kim, 2006; Kim, Kim, and Kyae, 2007; Choi and Kobayashi, 2008). A simple diagram giving an M dependence is shown in Fig. 26 where the SU(2) doublets H_1 and H_2 form a vectorlike superheavy pair. This is a kind of seesaw mechanism of Higgsino doublet pairs. For this scenario, a superpotential possessing the PQ symmetry can be constructed:

$$W=\tfrac{1}{2}mX^2+f_XX^3+\tfrac{1}{2}S^2T+XH_1H_2-f_1SH_1H_u -f_2SH_2H_d+\cdots, \tag{119}$$

where $\langle X\rangle=M$ and the H_uH_d term is forbidden by the PQ symmetry. Then the μ term for H_u and H_d (which give mass to up- and down-type quarks, respectively) is given by $\mu=\langle f_1f_2S^2/M\rangle$ (Kim and Nilles, 1984). If $\langle S\rangle$ is lowered to the hidden-sector confining scale of order $\sim 10^{10-12}$ GeV in the GMSB, the Higgsino mass can be made to be around the TeV scale by adjusting f_1f_2/M. One may construct models with appropriate F terms such that B_μ and $m^2_{\rm soft}$ are of the same order in the GMSB, e.g., through the PQ-symmetry-preserving term in the Kähler potential

$$\int d^4\vartheta\frac{f_1f_2T^*}{X}H_uH_d+\text{H.c.}, \tag{120}$$

which also gives a μ term.

In the so-called mixed mediation (M-mediation) scenario, with comparable moduli, anomaly, and gauge mediations, which includes in its parameter space the GMSB, the AMSB, the mirage mediation, and the deflected mirage mediation (Everett, Kim, Ouyang, and Zurek, 2008), the loop-generated μ term in general has a severe B_μ/μ problem. It seems that the model presented in an AMSB scenario (Pomarol and Rattazzi, 1999; Rattazzi, Strumia, and Wells, 2000) has the basic ingredient for the solution of B_μ/μ problem according to a PQ symmetry as stressed by Giudice, Kim, and Rattazzi (2008). This can also be gleaned from the axion shift symmetry in the mirage mediation scenario (Nakamura, Okumura, and Yamaguchi, 2008).

F. Axions from superstrings

The most interesting theory housing axions is superstring theory. Axions from strings are described by effective field theory below the compactification scale. If the axion arises from the spontaneous symmetry breaking of a tree-level global symmetry as discussed, the answer is simple: There is no such axion since string theory would not allow any global symmetry. If the compactification process leads to the SM, the renormalizable terms in this effective theory respect the gauge symmetry $SU(3)_c\times SU(2)\times U(1)_Y$ and the global symmetries of the baryon number $U(1)_B$ and the separate lepton numbers $U(1)_{Li}$. On the other hand, if the nonrenormalizable terms are allowed, one can write, for example, $q_Ll_Lu_Rd_R$, breaking both the baryon number and the lepton number symmetries. If the nonrenormalizable terms are included, the SM does not respect the baryon and lepton numbers symmetries. Similarly, there is no PQ global symmetry if we are allowed to write all nonrenormalizable terms. For the PQ symmetry, the situation is more severe. Suppose the singlet carrying the PQ charge is σ. Then $\sigma^*\sigma$ respects the PQ symmetry but σ^2 and σ^{*2} do not, which has led to a discussion of gravitational effects on the axion (Barr and Seckel, 1992; Ghigna, Lusignoli, and Roncadelli, 1992; Holman, Hsu, Kephart, Kolb, Watkins, and Widrow, 1992; Kamionkowski and March-Russell, 1992; Dobrescu, 1997). Therefore, the PQ symmetry cannot be discussed in general in terms of matter fields only, when we include gravity in the discussion as in string theory.

Thus, in string compactification one must consider the gravity multiplet also. Here the gauge singlet bosonic degrees in the gravity multiplet are the graviton g_{MN} $(M,N=0,1,\ldots,9)$, the antisymmetric tensor B_{MN}, and the dilaton Φ. In ten dimensions, g_{MN} and B_{MN} are gauge fields. A 4D action on Minkowski space x^μ $(\mu=0,1,2,3)$ is obtained by compactifying six internal spaces y^i $(i=4,\ldots,9)$ with the compact volume V_Z. Some bosonic degrees from the ten-dimensional (10D) antisymmetric tensor field behave like pseudoscalars in the 4D effective theory. Thus, the axion candidates, if they do not arise from the matter multiplets, must be in B_{MN}. The pseudoscalar fields in B_{MN} are like phase fields in axion models in field theory. Because there is no global symmetry in string theory, there must be no massless B_{MN}, otherwise the shift symmetry of B_{MN} would have worked as a global symmetry. From the tree-level equations of motion all pseudoscalar B_{MN} fields are not massless. For example, if a shift symmetry of B_{MN} is related to an anomaly as in the PQ current case, we consider that the shift symmetry is already broken. In other words, there is no shift symmetry of pseudoscalar B_{MN} unless it is anomalous.

One must deal with these bosonic degrees in string compactification to see whether these components lead to terms in the potential (or the superpotential in SUSY models), which is a technical and model-dependent procedure. Here we discuss axions from strings and comment on their phenomenological viability. Some relevant recent reviews describing details can be found in Conlon (2006) and Svrcek and Witten (2006). The M-theory discussion was presented in Choi (1997) and Svrcek and Witten (2006).

The pseudoscalar fields in B_{MN} come in two categories, one the tangential component $B_{\mu\nu}$ and the other B_{ij}. $B_{\mu\nu}$ can be discussed in any string compactification and hence is called "model independent" (MI) while B_{ij} depends on the compactification scheme as its internal coordinates i and j imply and is called "model dependent" (MD). After presenting the string formulas containing B_{MN}, we discuss the MI axion in Sec. VI.F.1 and then the MD axions present in much more speculative models in Sec. VI.F.2.

Now, there exists a standard formula for the string action [Polchinski (1988), Eq. (13.3.22)], which was lacking in the early days of string axions (Choi and Kim, 1985a; Witten, 1985a). The type-II dilaton $\phi_{\rm II}$ and coupling $g_{\rm II}=e^{\phi_{\rm II}}$ are related to the 10D gravitational coupling κ_{10} by $M_{10}^8=1/\kappa_{10}^2=4\pi/g_{\rm II}^2\ell_s^8$, where $\alpha'=\ell_s^2/(2\pi)^2$ (Polchinski, 1988). For the type-II string, there are NS-NS and R-R fluxes which can give anomalous couplings. These complicated systems housing pseudoscalars are reviewed in Conlon (2006) with the tentative that it is difficult to realize a QCD axion in string models with a workable moduli stabilization (Kachru, Kallosh, Linde, and Trivedi, 2003). In heterotic string models, there does not exist a reasonable moduli stabilization mechanism, even though an ambitious attempt has been proposed (Becker, Becker, Fu, Tseng, and Yau, 2006). However, we discuss heterotic string axions below because string axions were found first in heterotic string models and key couplings in axion phenomenology might be similarly discussed also for the type-II string. As in the type-II string, the heterotic string coupling is related to the dilaton Φ as $g_h=e^{\Phi}$. The kinetic energy terms of g_{MN}, B_{MN}, and A_M are [Polchinski (1988), Eqs. (12.1.39) and (12.3.36)],

$$\begin{aligned}\mathcal{L}_{\rm KE} &= \sqrt{-g_{10}}e^{-2\Phi}\left\{\frac{M_{10}^8}{2}R-\frac{M_{10}^8}{4}\left|dB_2-\frac{\omega_3}{M_{10}^2g_h^2}\right|^2 - \frac{M_{10}^8\alpha'}{8g_h^2}{\rm Tr}_v|F_2|^2\right\}\\ &= \sqrt{-g_{10}}\left\{\frac{2\pi}{g_h^2\ell_s^8}R-\frac{\pi}{g_h^2\ell_s^8}\left|dB_2-\frac{\omega_3}{M_{10}^2g_h^2}\right|^2 - \frac{1}{4(2\pi)g_h^2\ell_s^6}{\rm Tr}_v|F_2|^2\right\},\end{aligned} \tag{121}$$

where ${\rm Tr}_v$ is the trace over vector representation and the Chern-Simons three-form is

$$\omega_3={\rm Tr}_v(A_1\wedge dA_1+\tfrac{2}{3}A_1\wedge A_1\wedge A_1). \tag{122}$$

For $E_8\times E_8$, there is the adjoint representation and we use $\frac{1}{30}{\rm Tr}_a$ in place of ${\rm Tr}_v$. For the compact internal volume V_Z, the Planck mass is $M_P=4\pi V_Z/g_s^2\ell_s^8$ and the 4D gauge coupling constant is $g_{YM}^2=4\pi g_s^2\ell_s^6/V_Z$ or $\alpha_{YM}=g_s^2\ell_s^6/V_Z$. In most compactifications, the SM gauge fields arise from the level $k=1$ embedding and the coupling α_{YM} is the coupling strength at the compactification scale. If the SM gauge fields are embedded in the level k, the SM gauge coupling at the compactification scale will be smaller by the factor k. For interactions of B_{MN}, we consider the Bianchi identity, the gauge-invariant couplings of the gaugino χ (Derendinger, Ibanez, and Nilles, 1985; Dine, Rohm, Seiberg, and Witten, 1985), and the Green-Schwarz terms (Green and Schwarz, 1984),

$$dH=\frac{1}{16\pi^2}({\rm tr}R\wedge R-{\rm tr}F\wedge F),\quad H_{MNP}\bar{\chi}\Gamma^{MNP}\chi,$$
$$B\wedge{\rm tr}F\wedge F\wedge F\wedge F+\cdots, \tag{123}$$

where H_{MNP} is the field strength of B_{MN}, F is the field strength of the gauge field A, the gauge-invariant fermion coupling is the SUSY counterpart of the relevant terms of Eq. (121), and the ellipsis denotes more Green-Schwarz terms. It was argued that the H_{MNP} coupling to the gaugino must be a perfect square (Dine, Rohm, Seiberg, and Witten, 1985), which gives a vanishing cosmological constant even for a nonvanishing gaugino condensation with nonzero $\langle H_{MNP}\rangle$ (Derendinger, Ibanez, and Nilles, 1985; Dine, Rohm, Seiberg, and Witten, 1985).

1. Model-independent axion

$B_{\mu\nu}$ with μ and ν tangent to the 4D Minkowski spacetime is the MI axion present in all string compactifications (Witten, 1984). Because it is a 4D gauge boson, one cannot write potential terms in terms of $B_{\mu\nu}$ and it is massless if one neglects the anomaly term. The number of transverse degrees in $B_{\mu\nu}$ is 1, and it can be expressed as a pseudoscalar a by dualizing it, $H_{\mu\nu\rho}\propto F_a\epsilon_{\mu\nu\rho\sigma}\partial^\sigma a$. Even though it is massless at this level, the Bianchi identity of Eq. (123) gives an equation of motion of a as $\partial^2 a=(1/32\pi^2F_a)G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$, which hints that a might be an axion. For it to be really a QCD axion, c_2+c_3 should be nonzero as discussed in Sec. III.B. It is known that $c_3=1$ (Witten, 1985b) with c_3 defined in Eq. (19). The other possible couplings are given by the second term of Eq. (123),

$$\frac{F_a}{M_{10}^2}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma a_{\rm MI}\bar{\chi}\Gamma^{\mu\nu\rho}\chi=\frac{F_a}{M_{10}^2}\bar{\chi}\gamma_\sigma\gamma_5\chi\partial^\sigma a_{\rm MI}, \tag{124}$$

which is the c_1 term defined in Eq. (19). There is no c_2 term and $c_2+c_3=1$, and hence $H_{\mu\nu\rho}$ is really an axion and is model independent.[4] This is a hadronic axion. This MI hadronic axion can have a nonvanishing c_1 and hence its phenomenology might be different from that of the KSVZ hadronic axion. In Eqs. (61) and (62) for the MI hadronic axion, one has to add the relevant c_1 term from Eq. (124). The domain wall number of the MI axion has been shown to be $N_{\rm DW}=1$ by considering the coupling of the MI axion to a string $X^M(\sigma,\tau)$ on the world sheet

[4] Nevertheless, its properties may depend on models in warped space (Dasgupta, Firouzjahi, and Gwyn, 2008).

$\int d^2\sigma\,\epsilon^{\alpha\beta}B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$ (Witten, 1985a). F_a is about 10^{-3} times the Planck mass (Choi and Kim, 1985a), and the correct relation, obtained from Eq. (121), is $F_a/k = \alpha_c M_P/2^{3/2}\pi \sim 10^{16}$ GeV, where k is the level of the SM embedding and α_c is the QCD coupling constant (Svrcek and Witten, 2006). But the value $F_a \sim 10^{16}$ GeV most probably overcloses the universe.

An idea for lowering the MI axion decay constant may be the following. In some compactification schemes, an anomalous $U(1)_{an}$ gauge symmetry results, where the $U(1)_{an}$ gauge boson eats the MI axion so that the $U(1)_{an}$ gauge boson becomes heavy. This applies to the MI axion since the coupling $\partial^\mu a_{MI} A_\mu^{an}$ is present by the Green-Schwarz term (Witten, 1984; Chun, Kim, and Nilles, 1992b). In fact, even before considering this anomalous $U(1)_{an}$ gauge boson, the possibility was pointed out by Barr (1986); the theory became consistent after the anomalous $U(1)_{an}$ from string compactification was discovered (Atick, Dixon, and Sen, 1987; Dine, Seiberg, and Witten, 1987; Dine, Ichinose, and Seiberg, 1987). Then a global symmetry survives down the anomalous $U(1)_{an}$ gauge boson scale. A detailed scenario is the following. The anomalous $U(1)_{an}$ with the gauge transformation, $\theta_{an} \to \theta_{an} + \text{const}$ is obtained by calculating $U(1)_{an}$ charges of fermions. Thus, we have a nonvanishing c_2 in Eq. (19) as $m\bar{\psi}_L\psi_R \exp(ic_2\theta_{an})$ and c_3 of the MI axion as $c_3\theta_{MI}\{F\tilde{F}\}$. For all gauge group factors, the anomaly units are calculated and they are shown to be identical (Casas, Katehou, and Muñoz, 1988; Kim, 1988). For the MI axion to be part of a gauge boson, it must be a true Goldstone boson without an anomaly, i.e., it should be exactly massless; so we transform away the c_3 term by a phase redefinition of fermions such that $\bar{c}_2 = c_2 - c_3\langle\theta_{MI}\rangle/\theta_{an}$ and $\bar{c}_3 = 0$ can occur for all gauge fields, i.e., a_{MI} coupling to the anomalies vanishes for all gauge groups. Because the longitudinal gauge boson a_{MI} is removed, we are left with the $\bar{c}_2$ term only, $m\bar{\psi}_L\psi_R \exp(i\bar{c}_2\theta_{an}) + \text{H.c.}$, without the need to consider the gauge symmetry $U(1)_{an}$. At low energy, however, the term $m\bar{\psi}_L\psi_R \exp(i\bar{c}_2\theta_{an})$ has a global symmetry, $\theta_{an} \to \theta_{an} + \text{const}$, with θ_{an} not depending on x^μ. Thus, the interaction $m\bar{\psi}_L\psi_R \exp(i\bar{c}_2\theta_{an}) + \text{H.c.}$ explicitly shows a global U(1) axial symmetry or PQ symmetry below the $U(1)_{an}$ gauge boson mass scale: $\psi_L \to \psi_L e^{-i\theta_{an}/2}$ and $\psi_R \to \psi_R e^{i\theta_{an}/2}$. This global PQ symmetry can be broken in the axion window as in the field theoretic axion models. However, this idea about the decay constant does not work necessarily, because most fields, including those removed at the GUT scale, carry the $U(1)_{an}$ charge.

2. Model-dependent axion

In 4D, B_{MN} contains more pseudoscalars B_{ij} with i and j tangent to the compact space V_Z. If they are axions, these are MD axions. The number of massless B_{ij} modes at the KK mass level is the second Betti number of the compact space [Green, Schwarz, and Witten (1987), Eq. (14.3.10)], which was discussed in the early days in Witten (1984, 1985a) and Choi and Kim (1985b). The string propagation on $M_4 \times V_Z$ can be described by a suitable nonlinear σ model. In this σ model description, when a closed string topologically wraps V_Z nontrivially then there are world-sheet instantons due to the map $S_1 \to U(1)$. It is known that the world-sheet instantons are present precisely if the second Betti number is nonzero (Green, Schwarz, and Witten, 1987), and hence the MD axions are expected to receive non-negligible masses nonperturbatively (Dine, Seiberg, Wen, and Witten, 1986, 1987; Wen and Witten, 1986), but this may be a model-dependent statement (Polchinski, 2006). If a MD axion is known to have no potential term except the anomaly terms, then one should check the c_2 and c_3 couplings to confirm that it is really an axion. There has been no example presented yet in this way for a MD axion. If a MD axion is present, its decay constant is expected to be near the string scale as explicitly given by $F_{MD} = \alpha_C^{1/3} M_P / 2^{3/2}\pi k^{1/3} g_s^{2/3}$ from the anomaly term alone in Svrcek and Witten (2006). The Green-Schwarz term integrated over V_Z leads to this kind of decay constant for the MD axion (Choi and Kim, 1985b). However, as discussed, one has to calculate the corresponding c_2 term also to pinpoint the MD axion decay constant F_{MD}.

3. Toward a plausible QCD axion from string theory

A key problem in string axion models is to find a method obtaining a QCD axion at the axion window ($10^9 \leq F_a \leq 10^{12}$ GeV) but an attractive model in this direction is still lacking. Thus, the most pressing issue is the problem of introducing a detectable QCD axion from superstring theory. It includes the search for an approximate PQ symmetry and a detectable QCD axion.

The conditions for compactified manifolds in warped space needed to lower the MI axion decay constant have been discussed by Dasgupta, Firouzjahi, and Gwyn (2008), but its realization seems nontrivial.

The idea of localizing MD axions at fixed points in order to lower the decay constant has been proposed by I. W. Kim and J. E. Kim (2006). It uses the warp factor idea and one needs a so-called Giddings-Kachru-Polchinski throat (Giddings, Kachru and Polchinski, 2002) in the type-II string, but in the heterotic string a non-Kähler V_Z is needed (Becker, Becker, Fu, Tseng, and Yau, 2006). Indeed, a warp factor is obtained in this way, but it has power law behavior.

Intermediate scale string models can introduce the axion window as the ultraviolet completion scale (Burgess, Ibañez, and Quevedo, 1999). On the other hand, in this case the large radius used to generate the Planck mass is the scale needing explanation.

We note that a method of obtaining F_a in the axion window is through the composite axion from superstrings as discussed in Sec. VI.B. However, the composite axion has not been obtained so far from string construction.

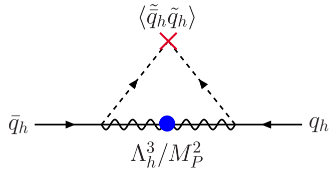

FIG. 27. (Color online) The hidden-sector squark condensation breaks chiral symmetry and generates hidden-sector quark masses.

Even if F_a is lowered, we must consider the hidden sector also in estimating the axion masses and decay constants as discussed below.

4. Hidden-sector confining forces, axion mixing, and approximate PQ symmetry

With the hidden-sector confining forces, we need at least two (QCD and one hidden-sector) θ's which have to be settled to zero, and hence we need at least two axions. For definiteness, consider only one more confining force at an intermediate scale, which may be the source of gravity mediation or GMSB. In this case, at least one MD axion is assumed to be present, and axion mixing must be considered. We assume that one decay constant is in the intermediate scale. Here there is an important (almost) theorem: the *cross theorem* on decay constants and condensation scales. Suppose that there are two axions a_1 with F_1 and a_2 with F_2 ($F_1 \ll F_2$) which couple to axion potentials with scales Λ_1 and Λ_2 ($\Lambda_1 \ll \Lambda_2$). The theorem states the following (Kim, 1999b, 2000; Kim and Kim, 2006): according to the diagonalization process in most cases with generic couplings, the larger potential scale Λ_2 chooses the smaller decay constant F_1, and the smaller potential scale Λ_1 chooses the larger decay constant F_2. So it is not enough just to obtain a small decay constant. The hidden sector may steal the smaller decay constant; the QCD axion is probably left with the larger decay constant. We can turn this around such that the hidden sector instanton potential is shallower than the QCD instanton potential since the instanton potential is proportional to the light quark mass as discussed in Sec. III.B. If the hidden-sector quark mass is extremely small, then the QCD axion can obtain the smaller decay constant, and the other axion is an extremely light axion which can be used to fit the observed dark energy (Riess *et al.*, 1998; Perlmutter *et al.*, 1999; Komatsu *et al.*, 2009). This is named the quintessential axion (Kim and Nilles, 2003, 2009). It can be easily realized if some hidden-sector squark condensations are very small, as Fig. 27 can generate hidden-sector quark masses (Kim and Kim, 2006).

Since it is difficult to obtain a reasonable light MD axion, attempts have been made to find an approximate PQ symmetry from string compactification. Only one reference exists using a realistic string compactification because of the difficulty of calculating all approximate PQ charges of quarks (Choi, Kim, and Kim, 2007). After all, the topologically attractive B_{ij} may not be the QCD axion we want. In this regard, we note that there already exists a field theoretic work regarding an approximate PQ symmetry, starting with a discrete Z_9 symmetry (Lazarides, Panagiotakopoulos, and Shafi, 1986). Later, gravitational nonperturbative effects such as wormholes and black holes were phenomenologically studied in view of any global symmetries (Giddings and Strominger, 1988; Lee, 1988; Gilbert, 1989). It is known that the PQ-symmetry-breaking operators in the superpotential must be forbidden up to dimension 8 (Barr and Seckel, 1992; Ghigna, Lusignoli, and Roncadelli, 1992; Holman, Hsu, Kephart, Kolb, Watkins, and Widrow, 1992; Kamionkowski and March-Russell, 1992; Dobrescu, 1997). If we introduce an *approximate* PQ symmetry, it is better to forbid the PQ-symmetry-breaking operators up to dimension 8 in the superpotential; possibly up to dimension 7 with reasonably small couplings somewhere.

In this spirit, it is worthwhile to check approximate PQ symmetries in string-derived models. The MSSMs presented in Kim (2007a, 2007b), Kim, Kim, and Kyae (2007), and Kim and Kyae (2007), satisfy most phenomenological constraints and one can check approximate global symmetries. But it is tedious work, and so far an approximate PQ symmetry has been checked out only for the flipped SU(5) model of Kim and Kyae (2007). In searching for an approximate global symmetry in a string-derived model, there are so many Yukawa couplings to be considered that a complete study up to all orders is almost impossible. For example (Choi, Kim, and Kim, 2007) presented $O(10^4)$ $d=7$ superpotential terms, and it is not a trivial task to find an approximate PQ symmetry direction, considering all these terms. Up to dimension-7 terms, there exists an approximate PQ symmetry which is spontaneously broken. The resulting axion coupling with photons has been calculated by Choi, Kim, and Kim (2007) and is shown in Fig. 16 together with the CAST and Tokyo axion search bounds (Andriamonje *et al.*, 2007; Inoue *et al.*, 2008). But the axion decay constant is not lowered. This is because the needed singlet VEVs, leading to the low energy MSSM, carry PQ charges. This is a generic problem for observable axions from superstrings. In comparison to the MI axion case with the anomalous $U(1)_{an}$, it may be easier to realize the observable axion with an approximate PQ symmetry.

VII. AXINO COSMOLOGY

Supersymmetrization of axion models includes the fermionic superpartner axino $\tilde{a}$ and the scalar superpartner saxion as discussed in Sec. VI.D. Both saxion and axino masses are split from the almost vanishing axion mass if SUSY is broken. The precise value of the axino mass depends on the model, specified by the SUSY-breaking sector and the mediation sector to the axion supermultiplet (Nilles, 1984). Most probably the saxion mass is around the soft mass scale M_{SUSY}. The axino mass should also be near this scale as well. But the axino mass can also be much smaller (Frere and Gerard, 1983;

Kim, Masiero, and Nanopoulos, 1984; Chun, Kim, and Nilles, 1992a) or much larger than $M_{\rm SUSY}$ (Chun and Lukas, 1995). Therefore, we take the axino mass as a free parameter here.

The decoupling temperature of the axino supermultiplet is of order (Rajagopal, Turner, and Wilczek, 1991)

$$T_{\tilde{a}\ \rm dcp} = (10^{11}\ {\rm GeV})\left(\frac{F_a}{10^{12}\ {\rm GeV}}\right)^2\left(\frac{0.1}{\alpha_c}\right)^3 \tag{125}$$

where α_c is the QCD coupling constant.

Saxion cosmology is a simple extension of the standard cosmology with saxion mass around the SUSY-breaking scale (Kim, 1991; Chang and Kim, 1996; Asaka and Yamaguchi, 1999), but its effect is not so dramatic as the effect of the axino. Therefore, here we focus on the axino cosmology (Rajagopal, Turner, and Wilczek, 1991; Covi, Kim, and Roszkowski, 1999; Covi, Kim, Kim, and Roszkowski, 2001; Choi, Kim, Lee, and Seto, 2008). In the moduli stabilization scenario of Kachru *et al.* (2003), the saxion VEV has been estimated by Choi and Jeong (2007).

The axino cosmology depends crucially on the nature of R parity. If R parity is conserved and the axino is lighter than the neutralino, then most probably the axino or gravitino (in the case of GMSB) is the LSP. If R parity is not conserved, the neutralino can decay to ordinary SM particles, as discussed by Allanach, Dedes, and Dreiner (2004).

Now we focus on R-parity conservation. The neutralino, if it is the LSP, is a natural candidate for dark matter. Due to TeV scale sparticle interactions, the thermal history of neutralinos allows them to be dark matter. But if a solution of the strong CP problem via the axion is imposed, the thermal history involves contributions from the axion sector, notably by the axino. Since axino cosmology depends on neutralino and gravitino number densities, we comment on the neutralino and gravitino cosmologies before discussing the effect of the axino. The neutralino cosmology depends on the neutralino freezeout temperature (Lee and Weinberg, 1977b; Drees and Nojiri, 1993) and the gravitino cosmology depends on the reheating temperature after inflation (Weinberg, 1982). Here we list several relevant temperatures in the axino cosmology,

$$\begin{aligned}
&T_{\tilde{a}\ \rm dcp}, \quad \text{axino decoupling temperature;}\\
&T_R, \quad \text{reheating temperature after inflation;}\\
&T_{\rm fr}, \text{ neutralino freezeout temperature;}\\
&T_{\tilde{a}\text{-rad}}, \text{ axino-radiation equality temperature;}\\
&T_D, \text{ radiation temperature right after } \tilde{a} \text{ decay.}
\end{aligned} \tag{126}$$

Here we are interested in the axino domination of the dark matter density. In the evolution history of cold axino dark matter, either a heavy axino has decayed already or it has not decayed yet. If the axino has not decayed yet, the current axino CDM can be estimated using $T_{\tilde{a}\ \rm dcp}$ or T_R. If it has decayed already, the past cold axino dark matter requires the existence of $T_R^{\rm min}$ at some earlier time,

$$\tfrac{4}{3} m_{\tilde{a}} Y_{\tilde{a}}(T_R^{\rm min}) = T_D \tag{127}$$

so that $Y_{\tilde{a}}(T_R) = n_{\tilde{a}}/s \geqslant Y_{\tilde{a}}(T_R^{\rm min})$ at the time of reheating after inflation, where $T_R^{\rm min}$ is the temperature above which axinos dominate the universe before they decay.

A. Neutralino and gravitino

The neutralino LSP seems the most attractive candidate for CDM simply because the TeV order SUSY-breaking scale introduces the LSP as a WIMP (Goldberg, 1983; Ellis, Hagelin, Nanopoulos, Olive, and Srednicki, 1984). The neutralino, which was in thermal equilibrium in the early universe, decouples and freezes out when the annihilation rate becomes smaller than the Hubble parameter. The freezeout temperature $T_{\rm fr}$ is normally given by $m_\chi/25$ (Lee and Weinberg, 1977b; Kolb and Turner, 1990), e.g., 4 GeV for a 100 GeV neutralino. Obviously, the neutralino relic density is not affected by the axino: $T_D > T_{\rm fr}$ since neutralinos were in thermal equilibrium after the axino decay. This is the standard neutralino dark matter. With the introduction of the axino, therefore, we study the case $T_D < T_{\rm fr}$.

Gravitinos in the universe are important if they dominate the dark matter fraction now or affected the result of nucleosynthesis. Thermal gravitinos produced at the Planckian time are important if $m_{3/2} \sim 1$ keV (Pagels and Primack, 1982). However, in the inflationary scenario these Planckian-time gravitinos are not important now. It was observed that heavy gravitino decay affects nucleosynthesis (Weinberg, 1982); this problem was suggested to be solved by inflation (Krauss, 1983; Khlopov and Linde, 1984). Then the gravitino number density is roughly estimated in terms of the reheating temperature after inflation, $n_{3/2} \propto T_R$. To estimate the cosmological bound on T_R rather accurately, a full supergravity interaction (Cremmer, Ferrara, Girardello, and van Pröyen, 1983) has been used and applied to the dissociation problem of rare light elements such as deuterium, etc., resulting in $T_R < 10^9$ GeV (Ellis, Kim, and Nanopoulos, 1984). A recent calculation of T_R has been performed using the nucleosynthesis code to look for ^{7}Li destruction and/or ^{6}Li overproduction (Kawasaki, Kohri, and Moroi, 2005; Kawasaki, Kohri, Moroi, and Yotsuyanagi, 2008), following the earlier work of Cyburt, Ellis, Fields, and Olive (2003), which led to a stronger bound, $T_R < 10^8$ GeV if the gravitino is lighter than the gluino and $T_R < 10^7$ GeV if the gravitino is heavier than the gluino. This gravitino problem is absent if the gravitino is the next LSP (NLSP), $m_{\tilde{a}} < m_{3/2} < m_\chi$, since a thermally produced gravitino would decay into an axino and an axion, which would not affect the BBN-produced light elements (Asaka and Yanagida, 2000).

If the gravitino is the LSP with the stau or neutralino as the NLSP, the gravitino can be the CDM even in the constrained MSSM (or mSUGRA) for some parameter space, avoiding the BBN and $b \to s\gamma$ constraints (Boehm, Djouadi, and Drees, 2000; Ellis, Olive, Santoso, and Spanos, 2004; Roszkowski, Ruiz de Austri, and Choi, 2005; Cerdeño, Choi, Jedamzik, Roszkowski, and Ruiz de Austri, 2006).

B. Axino

Thus, in SUSY theories we must consider a relatively small reheating temperature 10^{7-8} GeV. Axino cosmology must also be considered with this low reheating temperature.

In principle, the axion supermultiplet is independent of the observable sector, in which case we may take the axino mass as a free parameter from the keV scale to a value much larger than the gravitino mass (Chun, Kim, and Nilles, 1992a; Chun and Lukas, 1995). Light axinos ($m_{\tilde{a}} \lesssim 100$ GeV) can be a dark matter candidate and have been studied extensively as a warm dark matter candidate (Rajagopal, Turner, and Wilczek, 1991) with the reheating temperature given by Brandenburg and Steffen (2004), or a CDM candidate (Covi, Kim, and Roszkowski, 1999; Asaka and Yanagida, 2000; Covi, Kim, Kim, and Roszkowski, 2001; Roszkowski and Seto, 2007; Seto and Yamaguchi, 2007). Heavy axinos, however, cannot be the LSP; they can decay to the LSP plus light particles. This heavy axino decay to neutralinos has already been considered (Chun and Lukas, 1995). The heavy axino possibility was considered in studying cosmological effects of the saxion by Kawasaki and Nakayama (2008) and Kawasaki, Nakayama, and Senami (2008). A more complete cosmological analysis of the heavy axino has been discussed by Choi, Kim, Lee, and Seto (2008).

Since the CDM fraction of the universe is roughly 0.23 (Komatsu *et al.*, 2009), we focus on the possibility of the axino or axino-related neutralino being the CDM. For the axino to be the LSP, it must be lighter than the lightest neutralino and gravitino. In this case, we do not have T_D of Eq. (126). If the lightest neutralino is the NLSP, $m_{\tilde{a}} < m_\chi < m_{3/2}$, the thermal production (TP) mechanism gives the aforementioned bound on the reheating temperature after inflation. At a high reheating temperature, TP is dominant in axino production (Covi, Kim, and Roszkowski, 1999). If the reheating temperature is below the critical energy density line, there exists another axino CDM possibility from nonthermally produced (NTP) axinos which result from neutralino decay (Covi, Kim, Kim, and Roszkowski, 2001). This situation is shown in Fig. 28. We note that with R-parity conservation the double production of low-mass axinos is negligible in supernovae, and hence there is no useful exclusion region from SN1987A in the low-mass region.

Since the final axino energy fraction is reduced by the mass ratio $\Omega_{\tilde{a}} h^2 = (m_{\tilde{a}}/m_\chi)\Omega_\chi h^2$ for $m_{\tilde{a}} < m_\chi < m_{3/2}$, the stringent cosmologically constrained MSSM parameter space for m_χ can be expanded. As shown in Fig. 28, the NTP axinos can be CDM for a relatively low reheating

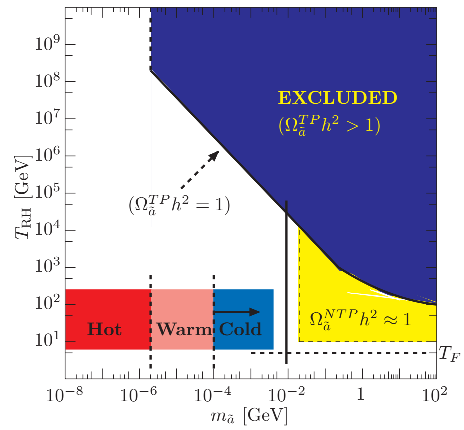

FIG. 28. (Color) Constraints of the reheating temperature as a function of the axino mass. The solid line is the upper bound from TP. The yellow region is the region where NTP can give cosmologically interesting results ($\Omega_{\tilde{a}}^{\rm NTP} h^2 \simeq 1$). The freezeout temperature is $T_{\rm fr} \approx m_\chi/20$.

temperature (<10 TeV) for 10 MeV $< m_{\tilde{a}} < m_\chi$. In Fig. 28 the thin dashed yellow corner on the RHS corresponds to MSSM models with $\Omega_\chi h^2 < 10^4$, and a small axino mass gives the possibility of the axino forming 23% of the closure density. If all SUSY mass parameters are below 1 TeV, then probably $\Omega_\chi h^2 < 100$ (the thick solid corner on the RHS) but a sufficient axino energy density requires $m_{\tilde{a}} > 1$ GeV. Thus, if the LHC does not detect the neutralino needed for closing of the universe, axino closing is a possibility (Covi, Roszkowski, and Small, 2002; Covi, Roszkowski, Ruiz de Austri, and Small 2004; Choi and Roszkowski, 2006; Choi, Roszkowski, and Ruiz de Austri, 2008). If the NLSP is a stau with axino or gravitino LSP, the previously forbidden stau LSP region is erased. In this case, the CDM axino is similar to the bino LSP case, but because of the charge on the stau it is easier to detect the stau signal at the LHC (Brandenburg, Covi, Hamaguchi, Roszkowski, and Steffen, 2005). However, it may be difficult to detect axinos (Kim and Kim, 2002).

In the GMSB scenario, the gravitino mass is generally smaller than the neutralino mass and possibly smaller than the axino mass. The cosmological effect for this case has been studied by Chun, Kim, and Kim (1994) and Kim and Kim (1995).

For a heavy axino decaying to a neutralino, we present a T_R vs $m_{\tilde{a}}$ plot for $F_a = 10^{11}$ GeV in Fig. 29. The region $T_R > T_{\tilde{a}\,\rm dcp}$ is above the dashed blue line. An axino lifetime greater than 0.1 s is denoted by the red shaded region on the LHS. The blue shaded region on the RHS is where the axino decays before the neutralino decouples ($T_D > T_{\rm fr}$). The magenta lines (horizontal) are the contours of the entropy increase due to the axino decay, $r \equiv S_f/S_0$. Above the $r=1$ line axinos dominate the universe before they decay. The green lines (vertical) denote $\langle \sigma_{\rm ann} v_{\rm rel} \rangle$, where $\sigma_{\rm ann}$ is the neutralino annihilation cross section, in units of GeV^{-2}, which are used to give

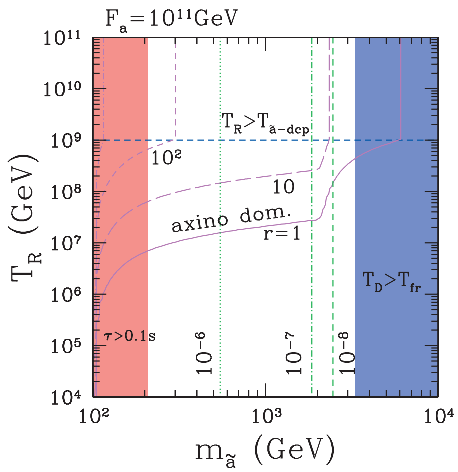

FIG. 29. (Color) The T_R vs $m_{\tilde{a}}$ plot for m_χ=100 GeV and F_a =10^{11} GeV.

the right amount of neutralino relic density. In Fig. 29 we use the neutralino and gluino masses m_χ=100 GeV and $m_{\tilde{g}}$=2 TeV, respectively. For a larger F_a and a heavier neutralino mass, the green lines move to the right (Choi, Kim, Lee, and Seto, 2008).

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REFERENCES

Abbott, L. F., and P. Sikivie, 1983, Phys. Lett. B **120**, 133.
Adler, S. C., *et al.*, 1996, Phys. Rev. Lett. **76**, 1421.
Adler, S. L., 1969, Phys. Rev. **177**, 2426.
Adler, S. L., J. Gamboa, F. Mendéz, and J. López-Sarrión, 2008, Ann. Phys. **323**, 2851.
Aharmin, B., *et al.* (SNO Collaboration), 2005, Phys. Rev. C **72**, 055502.
Ahmad, Q. R., *et al.* (SNO Collaboration), 2002, Phys. Rev. Lett. **89**, 011301.
Ahmed, Z., *et al.* (CDMS Collaboration), 2009a, Phys. Rev. Lett. **102**, 011301.
Ahmed, Z., *et al.* (CDMS Collaboration), 2009b, Phys. Rev. Lett. **103**, 141802.
Allanach, B. C., A. Dedes, and H. K. Dreiner, 2004, Phys. Rev. D **69**, 115002.
Amsler, C., *et al.* (Particle Data Group), 2008, Phys. Lett. B **667**, 1.
Andriamonje, S., *et al.* (CAST Collaboration), 2007, J. Cosmol. Astropart. Phys. **04**, 010.
Angle, J., *et al.* (Xenon Collaboration), 2008, Phys. Rev. Lett. **100**, 021303.
Antoniadis, I., N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, 1998, Phys. Lett. B **436**, 257.
Arkani-Hamed, N., S. Dimopoulos, and G. Dvali, 1998, Phys. Lett. B **429**, 263.
Asaka, T., and M. Yamaguchi, 1999, Phys. Rev. D **59**, 125003.
Asaka, T., and T. Yanagida, 2000, Phys. Lett. B **494**, 297.
Atick, J. J., L. J. Dixon, and A. Sen, 1987, Nucl. Phys. B **292**, 109.
Avignone, F. T., *et al.*, 1998, Phys. Rev. Lett. **81**, 5068.
Babu, K. S., K. Choi, J. C. Pati, and X. Zhang, 1994, Phys. Lett. B **333**, 364.
Bae, K.-J., J.-H. Huh, and J. E. Kim, 2008, J. Cosmol. Astropart. Phys. **09**, 005.
Bae, K. J., J.-H. Huh, J. E. Kim, B. Kyae, and R. D. Viollier, 2009, Nucl. Phys. B **817**, 58.
Bahcall, J. N., A. M. Serenelli, and S. Basu, 2005, Astrophys. J. **621**, L85.
Baker, C. A., *et al.*, 2006, Phys. Rev. Lett. **97**, 131801.
Baluni, V., 1979, Phys. Rev. D **19**, 2227.
Bardeen, W. A., H. Fritzsch, and M. Gell-Mann, 1972, in *Scale and Conformal Symmetry in Hadron Physics*, edited by R. Gatto (Wiley-Interscience, New York), p. 139.
Bardeen, W. A., R. D. Peccei, and T. Yanagida, 1987, Nucl. Phys. B **279**, 401.
Bardeen, W. A., and S.-H. H. Tye, 1978, Phys. Lett. B **76**, 580.
Barr, S. M., 1984, Phys. Rev. Lett. **53**, 329.
Barr, S. M., 1986, Phys. Lett. B **158**, 397.
Barr, S. M., K. Choi, and J. E. Kim, 1987, Nucl. Phys. B **283**, 591.
Barr, S. M., and P. Langacker, 1979, Phys. Rev. Lett. **42**, 1654.
Barr, S. M., and D. Seckel, 1992, Phys. Rev. D **46**, 539.
Battesti, R., *et al.*, 2008, in *Axions*, edited by M. Kuster *et al.*, Lecture Notes in Physics Vol. 741 (Springer, New York), p. 199.
Becker, K., M. Becker, J.-X. Fu, L.-S. Tseng, and S.-T. Yau, 2006, Nucl. Phys. B **751**, 108.
Bég, M. A. B., and H. S. Tsao, 1978, Phys. Rev. Lett. **41**, 278.
Behnke, E., *et al.* (COUPP Collaboration), 2008, Science **319**, 933.
Belavin, A. A., A. Polyakov, A. Schwartz, and Y. Tyupkin, 1975, Phys. Lett. B **59**, 85.
Bell, J. S., and R. Jackiw, 1969, Nuovo Cimento A **60**, 47.
Beltrán, M., J. García-Bellido, and J. Lesgourgues, 2007, Phys. Rev. D **75**, 103507.
Bento, L., G. C. Branco, and P. A. Parada, 1991, Phys. Lett. B **267**, 95.
Berezhiani, Z. G., M. Khlopov, and R. R. Khomeriki, 1990, Yad. Fiz. **52**, 104 [Sov. J. Nucl. Phys. **52**, 65 (1990)].
Bernabei, R., *et al.* (DAMA Collaboration), 2001, Phys. Lett. B **515**, 6.
Bernabei, R., *et al.* (DAMA Collaboration), 2003, Riv. Nuovo Cimento **26**, 1.
Bernabei, R., *et al.* (DAMA Collaboration), 2008, Eur. Phys. J. C **56**, 333.
Bershady, M. A., M. T. Ressell, and M. S. Turner, 1991, Phys. Rev. Lett. **66**, 1398.
Bertone, G., D. Hooper, and J. Silk, 2005, Phys. Rep. **405**, 279.
Blumenthal, G. R., S. M. Faber, J. R. Primack, and M. J. Rees, 1984, Nature (London) **311**, 517.
Boehm, C., A. Djouadi, and M. Drees, 2000, Phys. Rev. D **62**, 035012.
Bradley, R., 2003, Rev. Mod. Phys. **75**, 777.
Branco, G. C., 1980, Phys. Rev. Lett. **44**, 504.
Brandenburg, A., L. Covi, K. Hamaguchi, L. Roszkowski, and F. D. Steffen, 2005, Phys. Lett. B **617**, 99.
Brandenburg, A., and F. D. Steffen, 2004, J. Cosmol. Astro-

part. Phys. **08**, 008.
Braun, J., and H. Gies, 2007, Phys. Lett. B **645**, 53.
Buckley, M. R., and H. Murayama, 2007, e-print arXiv:0705.0542.
Burgess, C. P., L. E. Ibanez, and F. Quevedo, 1999, Phys. Lett. B **447**, 257.
Callan, C. G., R. F. Dashen, and D. J. Gross, 1976, Phys. Lett. B **63**, 334.
Cameron, R., *et al.* (RBFT Collaboration), 1991, Phys. Lett. A **157**, 125.
Cameron, R., *et al.* (RBFT Collaboration), 1993, Phys. Rev. D **47**, 3707.
Carena, M., and R. D. Peccei, 1989, Phys. Rev. D **40**, 652.
Carosi, G., 2007, in *6th International Conference of the Balkan Physical Union Strings*, edited by S. A. Cetin, and I. Hikmet, AIP Conf. Proc. No. 899 (AIP, Melville, NY), p. 30.
Carosi, G., and K. van Bibber, 2008, in *Axions*, edited by M. Kuster *et al.*, Lecture Notes in Physics Vol. 741 (Springer, New York), p. 135.
Carosi, G., K. van Bibber, and M. Pivovaroff, 2008, Contemp. Phys. **49**, 281.
Casas, A., E. K. Katehou, and C. Muñoz, 1988, Nucl. Phys. B **317**, 171.
Casas, A., and C. Muñoz, 1993, Phys. Lett. B **306**, 288.
Catelan, M., J. A. de Freista Pacheco, and J. E. Horvath, 1996, Astrophys. J. **461**, 231.
Cea, P., and G. Nardulli, 1984, Phys. Lett. B **144**, 115.
Cerdeño, D. G., K.-Y. Choi, K. Jedamzik, L. Roszkowski, and R. Ruiz de Austri, 2006, J. Cosmol. Astropart. Phys. **06**, 005.
Cerdeño, D. G., C. Hugonie, D. E. López-Fogliani, C. Muñoz, and A. M. Teixeira, 2004, J. High Energy Phys. **12**, 048.
Chang, D., X. G. He, and B. H. J. McKellar, 2001, Phys. Rev. D **63**, 096005.
Chang, D., and W.-Y. Keung, 2004, Phys. Rev. D **70**, 051901.
Chang, H. M., *et al.*, 2007, Phys. Rev. D **75**, 052004.
Chang, S., and K. Choi, 1993, Phys. Lett. B **316**, 51.
Chang, S., C. Hagmann, and P. Sikivie, 1998, e-print arXiv:hep-ph/9812321.
Chang, S., C. Hagmann, and P. Sikivie, 1999, Phys. Rev. D **59**, 023505.
Chang, S., and H. B. Kim, 1996, Phys. Rev. Lett. **77**, 591.
Chang, S., S. Tazawa, and M. Yamaguchi, 2000, Phys. Rev. D **61**, 084005.
Chen, S.-J., H.-H. Mei, and W.-T. Ni, 2007, Mod. Phys. Lett. A **22**, 2815.
Cheng, H.-Y., 1988, Phys. Rep. **158**, 1.
Cheung, C., A. L. Fitzpatrick, and L. Randall, 2008, J. High Energy Phys. **01**, 069.
Cho, H. Y., 2008, J. High Energy Phys. **07**, 069.
Choi, K., 1992, Nucl. Phys. B **383**, 58.
Choi, K., 1997, Phys. Rev. D **56**, 6588.
Choi, K., 2000, Phys. Rev. D **62**, 043509.
Choi, K., 2004, Phys. Rev. Lett. **92**, 101602.
Choi, K., and K. S. Jeong, 2007, J. High Energy Phys. **01**, 103.
Choi, K., K. S. Jeong, and K. Okumura, 2005, J. High Energy Phys. **0509**, 039.
Choi, K., K. Kang, and J. E. Kim, 1989, Phys. Rev. Lett. **62**, 849.
Choi, K., C. W. Kim, and W. K. Sze, 1988, Phys. Rev. Lett. **61**, 794.
Choi, K., and H. D. Kim, 2000, Phys. Rev. D **61**, 015010.
Choi, K., and J. E. Kim, 1985a, Phys. Lett. B **154**, 393.
Choi, K., and J. E. Kim, 1985b, Phys. Lett. B **165**, 71.
Choi, K., and J. E. Kim, 1985c, Phys. Rev. D **32**, 1828.
Choi, K., and J. E. Kim, 1985d, Phys. Rev. Lett. **55**, 2637.
Choi, K., J. E. Kim, and H. P. Nilles, 2007, private communication.
Choi, K.-S., I.-W. Kim, and J. E. Kim, 2007, J. High Energy Phys. **03**, 116.
Choi, K.-S., and J. E. Kim, 2006, *Quarks and Leptons from Orbifolded Superstring*, Lecture Notes in Physics Vol. 696 (Springer-Verlag, Berlin).
Choi, K.-S., and T. Kobayashi, 2008, Nucl. Phys. B **797**, 295.
Choi, K.-Y., J. E. Kim, H. M. Lee, and O. Seto, 2008, Phys. Rev. D **77**, 123501.
Choi, K. Y., and L. Roszkowski, 2006, in *Particles, Strings, and Cosmology*, edited by K. Choi, J. E. Kim, and D. Son, AIP Conf. Proc. No. 805 (AIP, Melville, NY), p. 30.
Choi, K.-Y., L. Roszkowski, and R. Ruiz de Austri, 2008, J. High Energy Phys. **0804**, 016.
Chou, A. S., *et al.* (GammeV Collaboration), 2008, Phys. Rev. Lett. **100**, 080402.
Chun, E. J., H. B. Kim, and J. E. Kim, 1994, Phys. Rev. Lett. **72**, 1956.
Chun, E. J., J. E. Kim, and H. P. Nilles, 1992a, Phys. Lett. B **287**, 123.
Chun, E. J., J. E. Kim, and H. P. Nilles, 1992b, Nucl. Phys. B **370**, 105.
Chun, E. J., and A. Lukas, 1995, Phys. Lett. B **357**, 43.
Chung, D. J. H., E. W. Kolb, and A. Riotto, 1999, Phys. Rev. D **60**, 063504.
Clowe, D., *et al.* (Chandra Collaboration), 2006, Astrophys. J. **648**, L109.
Cohen, A. G., T. S. Roy, and M. Schmaltz, 2007, J. High Energy Phys. **02**, 027.
Conlon, J. P., 2006, J. High Energy Phys. **05**, 078.
Covi, L., H. B. Kim, J. E. Kim, and L. Roszkowski, 2001, J. High Energy Phys. **05**, 033.
Covi, L., J. E. Kim, and L. Roszkowski, 1999, Phys. Rev. Lett. **82**, 4180.
Covi, L., L. Roszkowski, R. Ruiz de Austri, and M. Small, 2004, J. High Energy Phys. **06**, 003.
Covi, L., L. Roszkowski, and M. Small, 2002, J. High Energy Phys. **07**, 023.
Cowsik, R., and J. McClelland, 1972, Phys. Rev. Lett. **29**, 669.
Cremmer, E., S. Ferrara, L. Girardello, and A. van Pröyen, 1983, Nucl. Phys. B **212**, 413.
Crewther, R. J., P. Di Vecchia, G. Veneziano, and E. Witten, 1979, Phys. Lett. B **88**, 123; **91**, 487(E) (1980).
Cyburt, R. H., J. R. Ellis, B. D. Fields, and K. A. Olive, 2003, Phys. Rev. D **67**, 103521.
Dabholkar, A., and J. M. Quashnock, 1990, in *Cosmic Axions*, edited by C. Jones and A. C. Melissinos (World Scientific, Singapore), p. 286.
Dar, S., 2000, e-print arXiv:hep-ph/0008248.
Dasgupta, K., H. Firouzjahi, and R. Gwyn, 2008, J. High Energy Phys. **06**, 056.
Dashen, R. F., 1971, Phys. Rev. D **3**, 1879.
Davis, R., 1985, Phys. Rev. D **32**, 3172.
Davoudiasl, H., and P. Huber, 2006, Phys. Rev. Lett. **97**, 141302.
Daw, E., and R. F. Bradley, 1997, J. Appl. Phys. **82**, 1925.
Dearborn, D. S. P., D. N. Schramm, and G. Steigman, 1986, Phys. Rev. Lett. **56**, 26.
Degl'Innocenti, S., W. A. Dziembowski, G. Fiorentini, and B. Ricci, 1997, Astropart. Phys. **7**, 77.

DePanfilis, S., *et al.*, 1987, Phys. Rev. Lett. **59**, 839.
Derendinger, J.-P., L. Ibanez, and H. P. Nilles, 1985, Phys. Lett. **155B**, 65.
Dicke, R. H., 1946, Rev. Sci. Instrum. **17**, 268.
Dicus, D. A., E. W. Kolb, V. L. Teplitz, and R. W. Wagoner, 1978, Phys. Rev. D **18**, 1829.
Dicus, D. A., E. W. Kolb, V. L. Teplitz, and R. W. Wagoner, 1980, Phys. Rev. D **22**, 839.
Di Lella, L., A. Pilaftsis, G. Raffelt, and K. Zioutas, 2000, Phys. Rev. D **62**, 125011.
Dimopoulos, S., and S. Raby, 1981, Nucl. Phys. B **192**, 353.
Dine, M., and W. Fischler, 1983, Phys. Lett. B **120**, 137.
Dine, M., W. Fischler, and M. Srednicki, 1981a, Nucl. Phys. B **189**, 575.
Dine, M., W. Fischler, and M. Srednicki, 1981b, Phys. Lett. B **104**, 199.
Dine, M., I. Ichinose, and N. Seiberg, 1987, Nucl. Phys. B **293**, 253.
Dine, M., and D. A. MacIntire, 1992, Phys. Rev. D **46**, 2594.
Dine, M., and A. E. Nelson, 1993, Phys. Rev. D **48**, 1277.
Dine, M., A. E. Nelson, and Y. Shirman, 1995, Phys. Rev. D **51**, 1362.
Dine, M., R. Rohm, N. Seiberg, and E. Witten, 1985, Phys. Lett. B **156**, 55.
Dine, M., N. Seiberg, X. G. Wen, and E. Witten, 1986, Nucl. Phys. B **278**, 769.
Dine, M., N. Seiberg, X. G. Wen, and E. Witten, 1987, Nucl. Phys. B **289**, 319.
Dine, M., N. Seiberg, and E. Witten, 1987, Nucl. Phys. B **289**, 589.
Di Vecchia, P., F. Nocodemi, R. Pettorino, and G. Veneziano, 1981, Nucl. Phys. B **181**, 318.
Di Vecchia, P., and G. Veneziano, 1980, Nucl. Phys. B **171**, 253.
Dobrescu, B. A., 1997, Phys. Rev. D **55**, 5826.
Donnelly, T. W., S. J. Freedman, R. S. Lytel, R. D. Peccei, and M. Schwartz, 1978, Phys. Rev. D **18**, 1607.
Drees, M., and M. M. Nojiri, 1993, Phys. Rev. D **47**, 376.
Duffy, L., *et al.*, 2006, Phys. Rev. D **74**, 012006.
Dunkley, J., *et al.* (WMAP Collaboration), 2009, Astrophys. J., Suppl. Ser. **180**, 306.
Dupays, A., C. Rizzo, M. Roncadelli, and G. F. Bignami, 2005, Phys. Rev. Lett. **95**, 211302.
Dvali, G., G. F. Giudice, and A. Pomarol, 1996, Nucl. Phys. B **478**, 31.
Ellis, J., and M. K. Gaillard, 1979, Nucl. Phys. B **150**, 141.
Ellis, J., J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, 1984, Nucl. Phys. B **238**, 453.
Ellis, J., J. E. Kim, and D. V. Nanopoulos, 1984, Phys. Lett. B **145**, 181.
Ellis, J., K. A. Olive, Y. Santoso, and V. C. Spanos, 2004, Phys. Lett. B **588**, 7.
Everett, L. L., I.-W. Kim, P. Ouyang, and K. M. Zurek, 2008, Phys. Rev. Lett. **101**, 101803.
Faleev, S. V., and P. G. Silvestrov, 1996, Nucl. Phys. B **463**, 489.
Frère, J. M., and J. M. Gérard, 1983, Nuovo Cimento **37**, 135.
Fukugita, M., S. Watamura, and M. Yoshimura, 1982a, Phys. Rev. Lett. **48**, 1522.
Fukugita, M., S. Watamura, and M. Yoshimura, 1982b, Phys. Rev. D **26**, 1840.
Gabbiani, F., E. Gabrielli, A. Masiero, and L. Silvestrini, 1996, Nucl. Phys. B **477**, 321.
Gasser, J., and H. Leutwyler, 1982, Phys. Rep. **87**, 77.
Gates, E. I., G. Gyuk, and M. S. Turner, 1995, Astrophys. J. **449**, L123.
Gell-Mann, M., R. J. Oakes, and B. Renner, 1968, Phys. Rev. **175**, 2195.
Georgi, H., 1978, Hadronic J. **1**, 155.
Georgi, H., 1984, *Weak Interactions and Modern Particle Theory* (Benjaminn Cummings, Menlo Park, CA).
Georgi, H., D. B. Kaplan, and L. Randall, 1986, Phys. Lett. B **169**, 73.
Georgi, H., and L. Randall, 1986, Nucl. Phys. B **276**, 241.
Georgi, H., T. Tomaras, and A. Pais, 1981, Phys. Rev. D **23**, 469.
Ghigna, S., S. M. Lusignoli, and M. Roncadelli, 1992, Phys. Lett. B **283**, 278.
Giddings, S. B., S. Kachru, and J. Polchinski, 2002, Phys. Rev. D **66**, 106006.
Giddings, S. B., and A. Strominger, 1988, Nucl. Phys. B **307**, 854.
Gies, H., J. Jaeckel, and A. Ringwald, 2006, Phys. Rev. Lett. **97**, 140402.
Gilbert, G., 1989, Nucl. Phys. B **328**, 159.
Giudice, G. F., H. D. Kim, and R. Rattazzi, 2008, Phys. Lett. B **660**, 545.
Giudice, G. F., M. A. Luty, H. Murayama, and R. Rattazzi, 1998, J. High Energy Phys. **12**, 027.
Giudice, G., and A. Masiero, 1988, Phys. Lett. B **206**, 480.
Goffin, V., G. Segrè, and H. A. Welson, 1980, Phys. Rev. D **21**, 1410.
Goldberg, H., 1983, Phys. Rev. Lett. **50**, 1419; **103**, 099905(E) 2009.
Goldstone, J., 1961, Nuovo Cimento **19**, 154.
Green, M., and J. Schwarz, 1984, Phys. Lett. B **149**, 117.
Green, M., J. Schwarz, and E. Witten, 1987, *Superstring Theory* (Cambridge University Press, Cambridge).
Groote, S., J. G. Körner, K. Schilcher, and N. F. Nasrallah, 1998, Phys. Lett. B **440**, 375.
Gross, D., R. Pisarski, and L. Yaffe, 1981, Rev. Mod. Phys. **53**, 43.
Hagmann, C., *et al.*, 1990, Phys. Rev. D **42**, 1297.
Hall, L., and L. Randall, 1991, Nucl. Phys. B **352**, 289.
Han, M. Y., and Y. Nambu, 1965, Phys. Rev. **139**, B1006.
Hannestad, S., A. Mirizzi, G. G. Raffelt, and Y. Y. Y. Wong, 2007, J. Cosmol. Astropart. Phys. **08**, 015.
Hannestad, S., A. Mirizzi, G. G. Raffelt, and Y. Y. Y. Wong, 2008, J. Cosmol. Astropart. Phys. **04**, 019.
Hannestad, S., and G. G. Raffelt, 2002, Phys. Rev. Lett. **88**, 071301.
Harari, D., and P. Sikivie, 1987, Phys. Lett. B **195**, 361.
Hatsuda, T., and M. Yoshimura, 1988, Phys. Lett. B **203**, 469.
He, X.-G., 2008, Int. J. Mod. Phys. A **23**, 3282.
Hertzberg, M. P., M. Tegmark, and F. Wilczek, 2008, Phys. Rev. D **78**, 083507.
Hinchliffe, I., 2008, in *Particle Data Book* (Amsler *et al.*, 2008).
Hindmarsh, M., and P. Moulatsiotis, 1997, Phys. Rev. D **56**, 8074.
Holman, R., S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, 1992, Phys. Lett. B **282**, 132.
Hong, D. K., 1998, Phys. Lett. B **445**, 36.
Hong, D. K., H.-C. Kim, S. Siwach, and H. U. Yee, 2007, J. High Energy Phys. **11**, 036.
Huang, Z., 1993, Phys. Rev. D **48**, 270.
Huh, J.-H., J. E. Kim, and B. Kyae, 2009, Phys. Rev. D **79**, 063529.
Imai, K., 2008, Talk presented at PANIC 2008, Eilat, Israel,

Nov. 2008.
Inoue, Y., Y. Akimoto, R. Ohta, T. Mizumoto, A. Yamamoto, and M. Minowa, 2008, Phys. Lett. B **668**, 93.
Inoue, Y., T. Namba, S. Moriyama, M. Minowa, Y. Takasu, T. Horiuchi, and A. Yamamoto, 2002, Phys. Lett. B **536**, 18.
Intrilligator, K. A., N. Seiberg, and D. Shih, 2006, J. High Energy Phys. **04**, 021.
Isern, J., E. García-Berro, S. Torres, and S. Catalán, 2008, Astrophys. J. **682**, L109.
Iwamoto, N., 1984, Phys. Rev. Lett. **53**, 1198.
Jackiw, R., and C. Rebbi, 1976, Phys. Rev. Lett. **37**, 172.
Jee, M. J., *et al.*, 2007, Astrophys. J. **661**, 728.
Jungman, G., M. Kamionkowski, and K. Griest, 1996, Phys. Rep. **267**, 195.
Kachru, S., R. Kallosh, A. D. Linde, and S. P. Trivedi, 2003, Phys. Rev. D **68**, 046005.
Kamionkowski, M., and J. March-Russell, 1992, Phys. Lett. B **282**, 137.
Kanaya, K., and M. Kobayashi, 1981, Prog. Theor. Phys. **66**, 2173.
Kang, K., and A. Pantziris, 1991, Phys. Rev. D **43**, 241.
Kanti, P., 2009, in *Physics of Black Holes*, edited by E. Papantonopoulos, Lecture Notes in Physics Vol. 769 (Springer, Heidelberg), p. 387.
Kaplan, D. B., 1985, Nucl. Phys. B **260**, 215.
Kaplan, D. B., and A. V. Manohar, 1986, Phys. Rev. Lett. **56**, 2004.
Kawasaki, M., K. Kohri, and T. Moroi, 2005, Phys. Rev. D **71**, 083502.
Kawasaki, M., K. Kohri, T. Moroi, and A. Yotsuyanagi, 2008, Phys. Rev. D **78**, 065011.
Kawasaki, M., and K. Nakayama, 2008, Phys. Rev. D **77**, 123524.
Kawasaki, M., K. Nakayama, and M. Senami, 2008, J. Cosmol. Astropart. Phys. **03**, 009.
Khlopov, M., and A. D. Linde, 1984, Phys. Lett. **138**, 265.
Kim, H. B., and J. E. Kim, 1995, Nucl. Phys. B **433**, 421.
Kim, H. B., and J. E. Kim, 2002, Phys. Lett. B **527**, 18.
Kim, I.-W., and J. E. Kim, 2006, Phys. Lett. B **639**, 342.
Kim, J. E., 1979, Phys. Rev. Lett. **43**, 103.
Kim, J. E., 1984, Phys. Lett. B **136**, 378.
Kim, J. E., 1985, Phys. Rev. D **31**, 1733.
Kim, J. E., 1987, Phys. Rep. **150**, 1.
Kim, J. E., 1988, Phys. Lett. B **207**, 434.
Kim, J. E., 1991, Phys. Rev. Lett. **67**, 3465.
Kim, J. E., 1998, Phys. Rev. D **58**, 055006.
Kim, J. E., 1999a, Phys. Lett. B **452**, 255.
Kim, J. E., 1999b, J. High Energy Phys. **05**, 022.
Kim, J. E., 2000, J. High Energy Phys. **06**, 016.
Kim, J. E., 2007a, Phys. Lett. B **651**, 407.
Kim, J. E., 2007b, Phys. Lett. B **656**, 207.
Kim, J. E., 2007c, Phys. Rev. D **76**, 051701.
Kim, J. E., J.-H. Kim, and B. Kyae, 2007, J. High Energy Phys. **06**, 034.
Kim, J. E., and B. Kyae, 2007, Nucl. Phys. B **770**, 47.
Kim, J. E., and U. Lee, 1989, Phys. Lett. B **233**, 496.
Kim, J. E., A. Masiero, and D. V. Nanopoulos, 1984, Phys. Lett. B **139**, 346.
Kim, J. E., and H. P. Nilles, 1984, Phys. Lett. B **138**, 150.
Kim, J. E., and H. P. Nilles, 1994, Mod. Phys. Lett. A **9**, 3575.
Kim, J. E., and H. P. Nilles, 2003, Phys. Lett. B **553**, 1.
Kim, J. E., and H. P. Nilles, 2009, J. Cosmol. Astropart. Phys. **05**, 010.
Kobayashi, M., and T. Maskawa, 1973, Prog. Theor. Phys. **49**, 652.
Kolb, E. W., and M. S. Turner, 1990, *The Early Universe* (Addison-Wesley, Reading, MA).
Komatsu, E., *et al.* (WMAP Collaboration), 2009, Astrophys. J., Suppl. Ser. **180**, 330.
Krauss, L. M., 1983, Nucl. Phys. B **227**, 556.
Krauss, L. M., J. E. Moody, and F. Wilczek, 1984, Phys. Lett. B **144**, 391.
Krauss, L. M., and F. Wilczek, 1986, Phys. Lett. B **173**, 189.
Landau, L. D., and E. M. Lifshitz, 1976, *Mechanics* (Elsevier Butterworth-Heinemann, Amsterdam), Sec. 49.
Langacker, P., and H. Pagels, 1973, Phys. Rev. D **8**, 4595.
Langacker, P., and H. Pagels, 1979, Phys. Rev. D **19**, 2070.
Langacker, P., G. Paz, L.-T. Wang, and I. Yavin, 2008, Phys. Rev. D **77**, 085033.
Lazarides, G., C. Panagiotakopoulos, and Q. Shafi, 1986, Phys. Rev. Lett. **56**, 432.
Lazarus, D. M., G. C. Smith, R. Cameron, A. C. Melissinos, G. Ruoso, and Y. K. Semertzidis, 1992, Phys. Rev. Lett. **69**, 2333.
Lee, B. W., and S. Weinberg, 1977a, Phys. Rev. Lett. **38**, 1237.
Lee, B. W., and S. Weinberg, 1977b, Phys. Rev. Lett. **39**, 165.
Lee, H. S., *et al.* (KIMS Collaboration), 2007, Phys. Rev. Lett. **99**, 091301.
Lee, K., 1988, Phys. Rev. Lett. **61**, 263.
Lee, T. D., 1973, Phys. Rev. D **8**, 1226.
Leutwyler, H., 1990, Nucl. Phys. B **337**, 108.
Linde, A. D., 1988, Phys. Lett. B **201**, 437.
Loaiza-Brito, O., J. Martin, H. P. Nilles, and M. Ratz, 2005, in *Particles, Strings, and Cosmology*, edited by K. Choi, J. E. Kim, and D. Son, AIP Conf. Proc. No. 805 (AIP, Melville, NY), p. 198.
López-Fogliani, D. E., and C. Muñoz, 2006, Phys. Rev. Lett. **97**, 041801.
Luty, M. A., J. Terning, and A. K. Grant, 2001, Phys. Rev. D **63**, 075001.
Manohar, A. V., and C. T. Sachrajda, 2008, in *Particle Data Book* (Amsler *et al.*, 2008).
Marx, G. C., and A. S. Szalay, 1972, in *Proceedings of Neutrino-1972*, edited by Frankel and G. Marx (OMDK-TECHNIOFORM, Budapest), Vol. I, p. 191.
Masso, E., and J. Redondo, 2006, Phys. Rev. Lett. **97**, 151802.
Mayle, R., J. R. Ellis, K. A. Olive, D. N. Schramm, and G. Steigman, 1988, Phys. Lett. B **203**, 188.
McGaugh, S. S., *et al.*, 2007, Astrophys. J. **659**, 149.
Mohapatra, R. N., and S. Nasri, 2007, Phys. Rev. Lett. **98**, 050402.
Mohapatra, R. N., and G. Senjanovic, 1978, Phys. Lett. B **79**, 283.
Morales, A., *et al.*, 2002, Astropart. Phys. **16**, 325.
Moriyama, S., M. Minowa, T. Namba, Y. Inoue, Y. Takasu, and A. Yamamoto, 1998, Phys. Lett. B **434**, 147.
Moroi, T., and H. Murayama, 1998, Phys. Lett. B **440**, 69.
Mukhanov, V., 2005, *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge), p. 147.
Murayama, H., Y. Nomura, and D. Poland, 2008, Phys. Rev. D **77**, 015005.
Musakhanov, M. M., and Z. Z. Israilov, 1984, Phys. Lett. B **137**, 419.
Nakamura, S., K. Okumura, and M. Yamaguchi, 2008, Phys. Rev. D **77**, 115027.
Nambu, Y., 1960, Phys. Rev. **117**, 648.
Nambu, Y., and G. Jona-Lasinio, 1961, Phys. Rev. **122**, 345.

Nelson, A. E., 1984, Phys. Lett. B **136**, 387.
Nelson, D. R., G. T. Flemming, and G. W. Kilcup, 2003, Phys. Rev. Lett. **90**, 021601.
Nilles, H. P., 1984, Phys. Rep. **110**, 1.
Nilles, H. P., and S. Raby, 1982, Nucl. Phys. B **198**, 102.
Ootani, W., *et al.*, 1999, Phys. Lett. B **461**, 371.
Pagels, H., and J. R. Primack, 1982, Phys. Rev. Lett. **48**, 223.
Pantziris, A., and K. Kang, 1986, Phys. Rev. D **33**, 3509.
Paschos, E. A., and K. Zioutas, 1994, Phys. Lett. B **323**, 367.
Peccei, R. D., 1979, in *Proceedings of the 19th ICHEP*, edited by S. Homma *et al.* (Physical Society of Japan, Tokyo), p. 385.
Peccei, R. D., 1989, in *CP Violation*, edited by C. Jarlskog [Adv. Ser. Direct. High Energy Phys. (World Scientific Singapore), pp. 503–551].
Peccei, R. D., and H. R. Quinn, 1977a, Phys. Rev. Lett. **38**, 1440.
Peccei, R. D., and H. R. Quinn, 1977b, Phys. Rev. D **16**, 1791.
Peccei, R. D., T. T. Wu, and T. Yanagida, 1986, Phys. Lett. B **172**, 435.
Perlmutter, S., *et al.* (Supernova Cosmology Project), 1999, Astrophys. J. **517**, 565.
Pi, S.-Y., 1984, Phys. Rev. Lett. **52**, 1725.
Polchinski, J., 1988, *String Theory* (Cambridge University Press, Cambridge).
Polchinski, J., 2006, private communication.
Pomarol, A., and R. Rattazzi, 1999, J. High Energy Phys. **05**, 013.
Pospelov, M., and A. Ritz, 2005, Ann. Phys. **318**, 119.
Preskill, J., M. B. Wise, and F. Wilczek, 1983, Phys. Lett. B **120**, 127.
Raffelt, G. G., 1986, Phys. Lett. B **166**, 402.
Raffelt, G. G., 1990a, Phys. Rep. **198**, 1.
Raffelt, G. G., 1990b, Astrophys. J. **365**, 559.
Raffelt, G. G., 1996, *Stars for Laboratory for Fundamental Physics* (Cambridge University Press, Cambridge).
Raffelt, G. G., 2008a, in *Axions*, edited by M. Kuster *et al.*, Lecture Notes in Physics Vol. 741 (Springer, New York), p. 51.
Raffelt, G. G., 2008b, private communication.
Raffelt, G. G., and D. S. P. Dearborn, 1987, Phys. Rev. D **36**, 2211.
Raffelt, G., and D. Seckel, 1988, Phys. Rev. Lett. **60**, 1793.
Rajagopal, K., M. S. Turner, and F. Wilczek, 1991, Nucl. Phys. B **358**, 447.
Randall, L., and Sundrum, 1999a, Nucl. Phys. B **557**, 79.
Randall, L., and Sundrum, 1999b, Phys. Rev. Lett. **83**, 3370.
Rattazzi, R., A. Strumia, and J. D. Wells, 2000, Nucl. Phys. B **576**, 3.
Riess, A. G., *et al.*, 1998, Astron. J. **116**, 1009.
Ringwald, A., 2008, organized the 4th Petras Workshop, DESY, June 2008 (http/www.desy.de/ringwald).
Robilliard, C., *et al.* (BMV Collaboration), 2007, Phys. Rev. Lett. **99**, 190403.
Rosenberg, L. J., 2004, in *Intersections of Particle and Nuclear Physics*, edited by Z. Parsa, AIP Conf. Proc. No. 698 (AIP, Melville, NY), p. 332.
Roszkowski, L., 2004, in *Proceedings of PASCOS-2003*, edited by D. P. Roy and K. Sridha (Indian Academy of Sciences, Bangalore), p. 389.
Roszkowski, L., R. Ruiz de Austri, and K.-Y. Choi, 2005, J. High Energy Phys. **08**, 080.
Roszkowski, L., and O. Seto, 2007, Phys. Rev. Lett. **98**, 161304.
Roy, T. S., and M. Schmaltz, 2008, Phys. Rev. D **77**, 095008.
Sato, K., 1978, Prog. Theor. Phys. **60**, 1942.
Sato, K., and H. Sato, 1975, Prog. Theor. Phys. **54**, 1564.
Schlattl, H., A. Weiss, and G. Raffelt, 1999, Astropart. Phys. **B10**, 353.
Schnitzer, H. J., 1984, Phys. Lett. B **139**, 217.
Seckel, D., and M. S. Turner, 1985, Phys. Rev. D **32**, 3178.
Segrè, G., and A. Weldon, 1979, Phys. Rev. Lett. **42**, 1191.
Semertzidis, Y., *et al.* (RBFT Collaboration), 1990, Phys. Rev. Lett. **64**, 2988.
Seto, O., and M. Yamaguchi, 2007, Phys. Rev. D **75**, 123506.
Shifman, M. A., V. I. Vainstein, and V. I. Zakharov, 1980, Nucl. Phys. B **166**, 493.
Sikivie, P., 1982, Phys. Rev. Lett. **48**, 1156.
Sikivie, P., 1983, Phys. Rev. Lett. **51**, 1415.
Sikivie, P., 1985, Phys. Rev. D **32**, 2988.
Sikivie, P., 2003, Phys. Lett. B **567**, 1.
Sikivie, P., 2008, in *Axions*, edited by M. Kuster *et al.*, Lecture Notes in Physics Vol. 741 (Springer, New York), p. 19.
Sikivie, P., D. Tanner, and K. van Bibber, 2007, Phys. Rev. Lett. **98**, 172002.
Silk, J., 1968, Astrophys. J. **151**, 459.
Smith, S., 1936, Astrophys. J. **83**, 23.
Spergel, D. N., *et al.* (WMAP Collaboration), 2007, Astrophys. J., Suppl. Ser. **170**, 377.
Srednicki, M., 1985, Nucl. Phys. B **260**, 689.
Steinhardt, P., and M. S. Turner, 1983, Phys. Lett. B **129**, 51.
Svrcek, B., and E. Witten, 2006, J. High Energy Phys. **06**, 051.
Tada, M., *et al.* (CARRACK Collaboration), 2006, Phys. Lett. A **349**, 488.
Tegmark, M., A. Aguirre, M. Rees, and F. Wilczek, 2006, Phys. Rev. D **73**, 023505.
't Hooft, G., 1976, Phys. Rev. D **14**, 3432.
't Hooft, G., 1979, in *Recent Developments in Gauge Theories*, edited by G. 't Hooft *et al.* (Plenum, New York), p. 135.
't Hooft, G., 1986, Phys. Rep. **142**, 357.
Turner, M. S., 1986, Phys. Rev. D **33**, 889.
Turner, M. S., 1988, Phys. Rev. Lett. **60**, 1797.
Turner, M. S., 1990, Phys. Rep. **197**, 67.
Turner, M. S., H. Kang, and G. Steigman, 1989, Phys. Rev. D **40**, 299.
Vafa, V., and E. Witten, 1984, Phys. Rev. Lett. **53**, 535.
van Bibber, K., 2008, talk presented at IDM08, Stockholm, Sweden, Aug. 2008.
van Bibber, K., N. R. Dagedeviren, S. E. Koonin, A. K. Kerman, and H. N. Nelson, 1987, Phys. Rev. Lett. **59**, 759.
van Bibber, K., P. M. McIntyre, D. E. Morris, and G. G. Raffelt, 1989, Phys. Rev. D **39**, 2089.
Veneziano, G., 1979, Nucl. Phys. B **159**, 213b.
Weinberg, S., 1975, Phys. Rev. D **11**, 3583.
Weinberg, S., 1976, Phys. Rev. Lett. **37**, 657.
Weinberg, S., 1977, Festschrift for I. I. Rabi, Trans. of New York Academy of Sciences, 38, 185.
Weinberg, S., 1978, Phys. Rev. Lett. **40**, 223.
Weinberg, S., 1982, Phys. Rev. Lett. **48**, 1303.
Weinberg, S., 1987, Phys. Rev. Lett. **59**, 2607.
Weinberg, S., 2008, *Cosmology* (Oxford University Press, New York).
Wen, X. G., and E. Witten, 1986, Phys. Lett. B **166**, 397.
Wetterich, C., 1988, Nucl. Phys. B **302**, 645.
Wilczek, F., 1978, Phys. Rev. Lett. **40**, 279.
Wise, M. B., H. Georgi, and S. L. Glashow, 1981, Phys. Rev. Lett. **47**, 402.
Witten, F., 1979, Nucl. Phys. B **156**, 269.
Witten, E., 1980, Ann. Phys. **128**, 363.

Witten, E., 1984, Phys. Lett. B **149**, 351.
Witten, E., 1985a, Phys. Lett. B **153**, 243.
Witten, E., 1985b, Phys. Lett. B **155**, 151.
Woltjer, C., 1964, Astrophys. J. **140**, 1309.
Yoo, J. (GammeV Collaboration), 2008, in *Dark 2007*, edited by H. K. Klapdor-Kleingrothaus, and G. F. Lewis (World Scientific, Singapore), p. 536.
Zavattini, E., *et al.* (PVLAS Collaboration), 2006, Phys. Rev. Lett. **96**, 110406.
Zavattini, E., *et al.* (PVLAS Collaboration), 2007, Phys. Rev. Lett. **99**, 129901(E).
Zavattini, E., *et al.* (PVLAS Collaboration), 2008, Phys. Rev. D **77**, 032006.
Zhitnitskii, A. P., 1981, Sov. J. Nucl. Phys. **31**, 260.
Zioutas, K., *et al.* (CAST Collaboration), 2005, Phys. Rev. Lett. **94**, 121301.
Zwicky, F., 1933, Helv. Phys. Acta **6**, 110.