REVIEWS OF MODERN PHYSICS, VOLUME 82, APRIL–JUNE 2010

Colloquium: Transport in strongly correlated two dimensional electron fluids

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(Published 27 May 2010)

An overview of the measured transport properties of the two dimensional electron fluids in high mobility semiconductor devices with low electron densities is presented as well as some of the theories that have been proposed to account for them. Many features of the observations are not easily reconciled with a description based on the well understood physics of weakly interacting quasiparticles in a disordered medium. Rather, they reflect new physics associated with strong correlation effects, which warrant further study.

DOI: 10.1103/RevModPhys.82.1743 PACS number(s): 05.20.−y, 82.20.−w

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I. INTRODUCTION

As low-density two dimensional (2D) electronic systems with increasingly high mobility have become available, there has accumulated experimental evidence of a set of low-temperature phenomena which cannot be easily understood on the basis of traditional [Fermi-liquid-(FL) based] metal physics.[1] More precisely, these are systems with a large ratio between the typical potential and kinetic energies, $r_s \equiv 1/\sqrt{\pi n}(a_B)^2$, where n is the areal density of electrons and $a_B^* = \hbar^2 \epsilon / m^* e^2$ is the effective Bohr radius. A generic feature of electronic systems is a low-temperature evolution as a function of n from a conducting FL state at large n to an insulating "Wigner crystalline" (WC) state at low. To obtain as clear a perspective as possible on this physics, we focus on 2D electron (and hole) systems in which r_s is large, so interaction effects manifestly cannot be treated perturbatively, but in which the system remains a *fluid*, hence thermodynamically distinct from a WC.

It should be stressed that while the experimental data exhibit striking features which have been confirmed by multiple groups on various materials, there is disagreement concerning the correct theoretical perspective from which to view these experiments. Of the unsettled issues, the most vexed is the proper theoretical interpretation of the experimentally observed metal-insulator transition. There is also much that remains to be understood concerning the observed transport anomalies in samples which are far from the critical region. Here we summarize the salient experimental facts, stressing the anomalous character of the results obtained in conditions both close to and far from the metal-insulator transition. We also summarize some of the major theoretical approaches, including a discussion of their successes and shortcomings. The primary purpose of this Colloquium is to stimulate further experimental work on this subject, particularly work aimed at unraveling the noncritical behavior of the metallic and insulating states that occur in strongly correlated two dimensional electron fluids far from the metal-insulator transition.

II. EXPERIMENTAL SIGNATURES OF NON-FERMI-LIQUID BEHAVIOR

In this section, we summarize some of the experimental results on large r_s 2D electron and hole gases (2DEGs and 2DHGs) in highly conducting semiconductor heterostructures that we contend are incompatible with a Fermi-liquid-based theory and with the single-particle theory of weak localization. We focus on experiments on the following systems: n-Si metal-oxide-semiconductor field-effect transistors (MOSFETs),[2] p-GaAs heterojunctions and quantum wells,[3] n-GaAs heterojunctions,[4] p-SiGe quantum wells (Coleridge et al., 1997; Senz et al., 1999), AlAs quantum wells (Papadakis and Shayegan, 1998; De Poortere et al., 2002; Vakili et al., 2004), and n-SiGe structures (Okamoto et al., 2004; Lai et al., 2005, 2007; Lu et al., 2008). To orient the reader, the best current theoretical estimates (Tanatar and Ceperley, 1989; Attaccalite et al., 2002) of the critical value of r_s at which the energies of uniform FL and WC are equal are $r_s^\star \approx 38$, while for the devices we have in mind, $r_s \sim 5-20$ in the case of the Si-MOSFETs and $\sim 10-40$ for the p-GaAs devices.

One of the key points to note as we discuss the data below is the similarities in the structure of the data from the different devices. This implies that the observed anomalies represent robust "universal" behaviors of the 2DEG, which are largely independent of details. This is correct in spite of the fact that there are significant differences between the electronic structures of the various devices. In p-GaAs and n-GaAs heterostructures, the electrons or holes occupy a single band with an isotropic

[1]For experimental results on various 2D structures, see Zavaritskaya and Zavaritskaya (1987), D'Iorio et al. (1990), Shashkin et al. (1993, 2001, 2002, 2003, 2006), Kravchenko et al. (1994, 1995, 1996, 1998, 2000, 2002), Coleridge et al. (1997), Popović et al. (1997), Pudalov (1997), Pudalov et al. (1997, 1998, 2002, 2003), Simonian et al. (1997), Dultz et al. (1998), Hanein, Meirav, et al. (1998), Hanein, Shahar, et al. (1998), Papadakis and Shayegan (1998), Simmons et al. (1998, 2000), Feng et al. (1999, 2001), Hanein et al. (1999), Mertes et al. (1999), Mills et al. (1999, 2001), Okamoto et al. (1999, 2004), Sarachik and Kravchenko (1999), Senz et al. (1999), Yoon et al. (1999, 2000), Dultz and Jiang (2000), Ilani et al. (2000), Kravchenko and Klapwijk (2000), Abrahams et al. (2001), Fletcher et al. (2001), Vitkalov et al. (2001), De Poortere et al. (2002), Gao et al. (2002, 2003, 2005, 2006), Jaroszyński et al. (2002), Lilly et al. (2003), Noh et al. (2003), Pillarisetty et al. (2003), Prus et al. (2003), Zhu et al. (2003), Kravchenko and Sarachik (2004), Vakili et al. (2004), Das Sarma et al. (2005), Lai et al. (2005, 2007), Shashkin (2005), Tsui et al. (2005), Anissimova et al. (2007), Jaroszyński and Popović (2007), Lu et al. (2008), and McFarland et al. (2009).

[2]Transport and thermodynamic properties of dilute silicon MOSFETs have been extensively studied by Zavaritskaya and Zavaritskaya (1987), D'Iorio et al. (1990), Shashkin et al. (1993, 2001, 2002, 2003, 2006), Kravchenko et al. (1994, 1995, 1996, 1998, 2000, 2002), Popović et al. (1997), Pudalov (1997), Pudalov et al. (1997, 1998, 2002, 2003), Simonian et al. (1997), Feng et al. (1999, 2001), Mertes et al. (1999), Okamoto et al. (1999), Kravchenko and Klapwijk (2000), Fletcher et al. (2001), Vitkalov et al. (2001), Jaroszyński et al. (2002), Prus et al. (2003), Tsui et al. (2005), Anissimova et al. (2007), Jaroszyński and Popović (2007), and McFarland et al. (2009).

[3]For experiments performed on p-GaAs structures, see Dultz et al. (1998), Hanein, Meirav, et al. (1998), Simmons et al. (1998, 2000), Hanein et al. (1999), Mills et al. (1999, 2001), Yoon et al. (1999, 2000), Dultz and Jiang (2000), Ilani et al. (2000), Gao et al. (2002, 2003, 2005, 2006), Noh et al. (2003), and Pillarisetty et al. (2003).

[4]Dilute electron gases in n-GaAs heterojunctions have been studied by Hanein, Shahar, et al. (1998), Lilly et al. (2003), Zhu et al. (2003), and Das Sarma et al. (2005).

effective mass, while in n-Si-MOSFETs there are two degenerate valleys and correspondingly a nontrivial structure to the effective-mass tensor. In the heterostructures, the interactions between electrons at large separation are Coulombic, while in MOSFETs, the interaction between electrons is dipolar at distances large compared to the distance to the metal gate. In p-GaAs and p-SiGe, the spin-orbit coupling may be significant, while in n-GaAs and Si-MOSFETs it is clearly insignificant. In Si-MOSFETs the disorder potential is believed to have short-ranged correlations, while in most of the other 2D systems considered here, it is believed to be long-range correlated (i.e., due, primarily, to distant charged impurities).

A. Near critical samples with $\rho \sim h/e^2$

Both the 2DEG and the 2DHG exhibit a zero-temperature metal-insulator transition as a function of the electron or hole density n. This effect has been observed in all materials in which large r_s samples can be made with sufficiently high mobilities: p- and n-Si-MOSFETs, p- and n-GaAs quantum wells, p- and n-SiGe quantum wells, and n-AlAs quantum wells. Examples of the experimental data showing the resistivity $\rho(T)$ for different electron concentrations in various systems are presented in Fig. 1.

In all cases, at low temperatures T the resistivity $\rho(T)$ exhibits a "metallic" temperature dependence $[d\rho(T)/dT>0]$ for electron concentrations n in excess of a well-defined critical value n_c and dielectric behavior $[d\rho(T)/dT<0]$ for $n<n_c$. Moreover, typically at the lowest temperatures $\rho \ll h/e^2$ for $n>n_c$, while $\rho \gg h/e^2$ for $n<n_c$. (Note that, where the Drude formula is valid here, $\rho = h/e^2$ would correspond to a mean free path, ℓ, equal to the Fermi wavelength $k_F\ell = 2\pi$. The quantum of resistance h/e^2 is thus the upper limit for the possible regime of applicability of the Boltzmann transport theory.)

An issue that has been debated is whether there is an actual transition or just a rapid crossover. This issue is difficult to resolve unambiguously since a continuous metal-insulator transition is sharply defined only at zero temperature as the point at which the resistance changes nonanalytically from being finite (metallic) to infinite (insulating). (Note that under the heading "metallic" we include the possibility of a "perfectly metallic" phase in which ρ is nonzero at any nonzero T, but $\rho \to 0$ as $T \to 0$.) However, at nonzero temperature, even when there is a zero-temperature transition, at any $T>0$, such a transition would manifest itself as a rapid but analytic crossover. It is clear from Fig. 1 that the resistivity changes by many orders of magnitude as n varies over a modest range near n_c. The lower the temperature is, the more violent the resistivity changes. For comparison, in Fig. 1(f) we present data for the three dimensional (3D) metal-insulator transition in Si:P. No one doubts that these data reflect a zero-temperature metal-insulator transition, although from a purely empirical viewpoint,

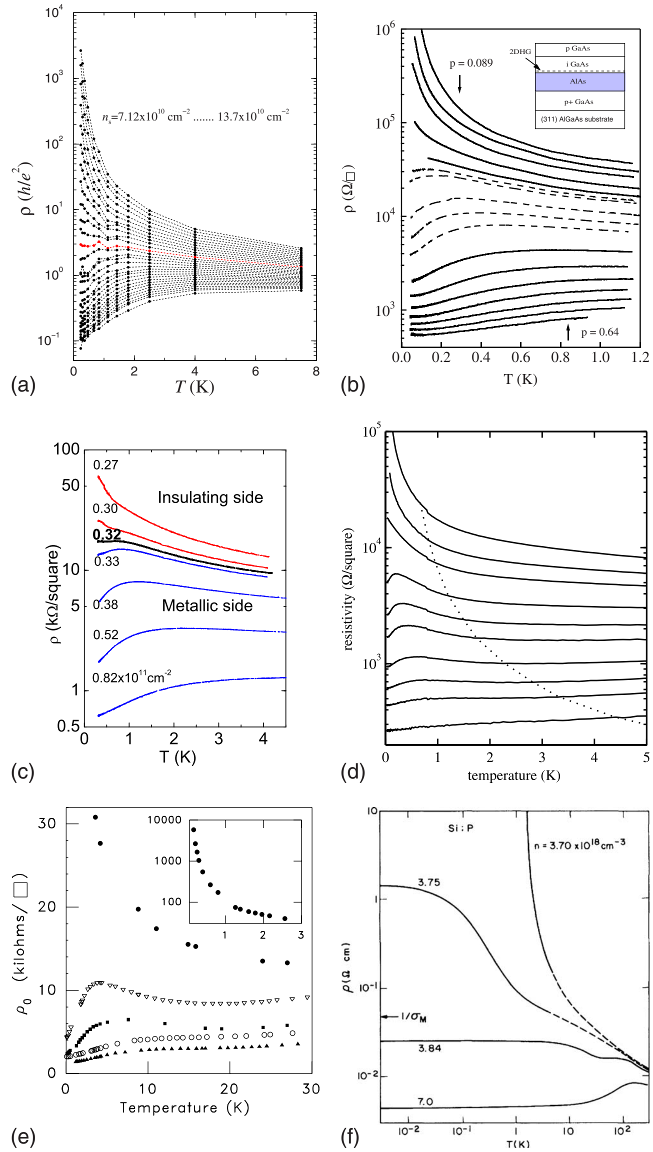

FIG. 1. (Color online) Critical behavior of the resistivity near the 2D metal-insulator transition in (a) Si-MOSFET, (b) p-GaAs/AlGaAs heterostructure, (c) n-SiGe heterostructure, (d) n-GaAs/AlGaAs heterostructure, and (e) p-SiGe quantum well. The corresponding ranges of r_s are (a) 14–19, (b) 12–32, (c) 27–47, (d) 5–14, and (e) 7–19. For comparison, we also show the critical behavior of the resistivity in Si:P, (f) a 3D system. Adapted from Kravchenko *et al.*, 1994, Hanein, Meirav, *et al.*, 1998, Lai *et al.*, 2005, Lilly *et al.*, 2003, Coleridge *et al.*, 1997, and Rosenbaum *et al.*, 1983, respectively.

the evidence is no better (and no worse) than in two dimensions.

We take the point of view that, at an empirical level, the experimental data presented in Fig. 1 represent a metal-insulator transition in two dimensions. It is, of course, possible that the nonanalytic behavior would ultimately be rounded out if it was possible to follow the physics to much lower temperatures than have yet been attained, but this would necessarily involve a crossover to new physics. However, the physics in the currently accessible range of temperatures is important to understand in its own right.

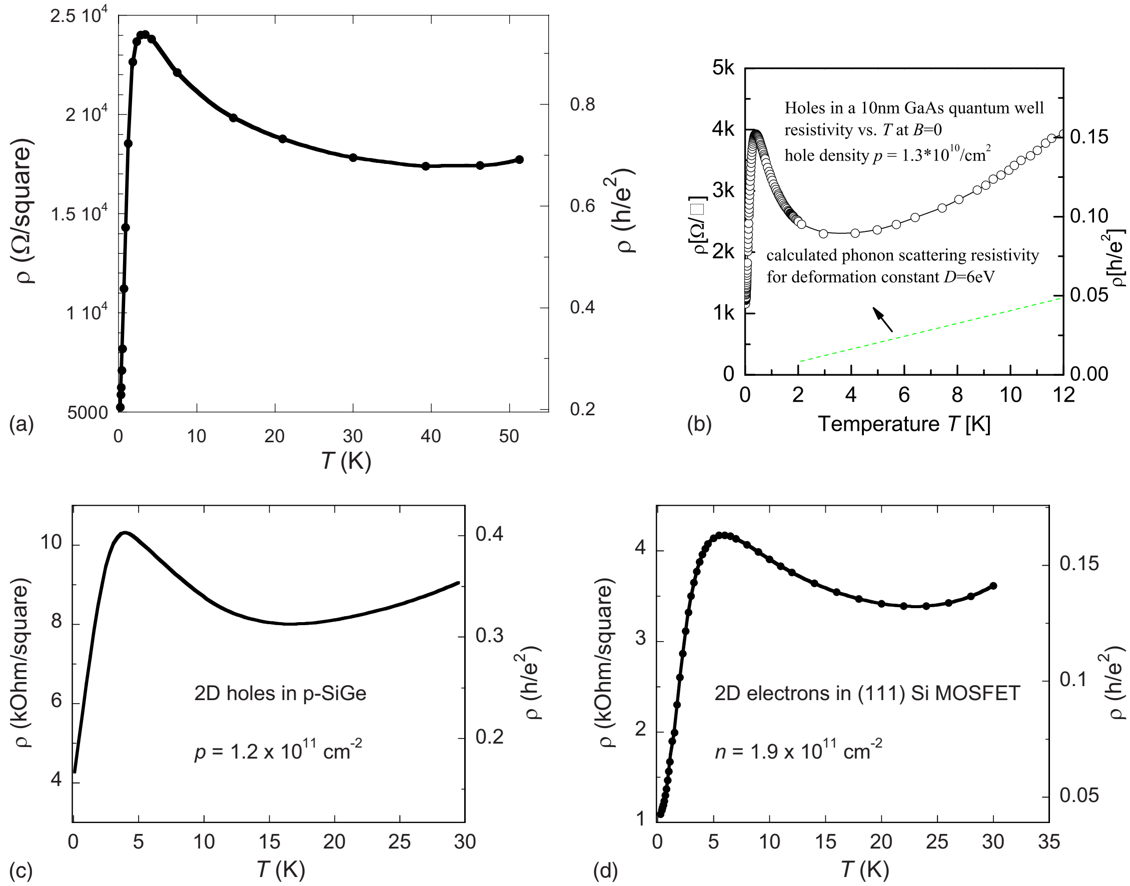

FIG. 2. (Color online) Nonmonotonic temperature dependence of the resistivity in (a) (100) Si-MOSFET, (b) p-GaAs quantum well, (c) p-SiGe quantum well, and (d) (111) Si-MOSFET deep in the metallic regime over an extended temperature range. The bare (nonrenormalized) Fermi temperatures are (a) 7.5, (b) 0.75, and (c) 7 K. Adapted from Gao *et al.*, 2005, Coleridge, 1997, and Mokashi and Kravchenko, 2009, respectively. Panel (d) courtesy of R. N. McFarland and B. E. Kane.

B. Strongly correlated highly metallic samples ($r_s \gg 1$ and $\rho \ll he^2$)

In this section, we focus on samples with $\rho \ll h/e^2$. In this limit, there is a small parameter, $1/k_F\ell \ll 1$, which permits a well-controlled perturbative expansion of physical quantities. Because the predictions of Fermi-liquid theory are sharp, discrepancies between experiment and theoretical expectations can be readily documented.

1. Temperature dependence of $\rho(T)$

As shown in Figs. 1(a)–1(e) and 2, in samples with large r_s and $\rho \ll h/e^2$ the resistance $\rho(T)$ is observed to increase with increasing temperature to a peak value $\rho(T_{\max})$. Depending on which type of device and the value of n, the ratio $\rho(T_{\max})/\rho(T_0)$ ranges from 2 to 10, where $T_0 \sim 25$ mK is the lowest temperature at which resistivity measurements are readily carried out.

This behavior has been observed in many 2D electronic systems: Si-MOSFETs, p- and n-GaAs quantum wells, p- and n-SiGe quantum wells, and n-AlAs quantum wells. Even at the maximum, the resistance $\rho_{\max} = \rho(T_{\max})$ is often smaller (and sometimes much smaller) than the quantum of resistance. For instance, the curves in Figs. 2(b)–2(d) and the corresponding curves in Figs. 1(b)–1(e) are deep in the metallic regime. Moreover, generally it is found that $k_B T_{\max} \sim E_F$, where E_F is the bare Fermi energy.

As a function of increasing temperature when $T > T_{\max}$, the resistance decreases, by as much as a factor of 2 (see Fig. 2), before ultimately starting to increase again at higher temperatures where, presumably, scattering from thermally excited phonons starts to be important. Still deeper in the metallic regime (i.e., at smaller r_s), the resistivity is a monotonically increasing function of the temperature [see the lowest curves in Figs. 1(a)–1(e)].

2. Parallel field magnetoresistance

As shown in Fig. 3, in Si-MOSFETs, p-GaAs heterojunctions, and SiGe quantum wells with large r_s and $\rho \ll h/e^2$, the resistance $\rho(T, B_\parallel)$ for temperatures $T < E_F$ exhibits a strongly *positive* magnetoresistance, increasing by as much as one order of magnitude as a function of $B_\parallel$ before saturating for $B_\parallel > B^* \sim E_F/g\mu_B$. Here μ_B is the effective Bohr magneton, g is the gyromagnetic ratio, and $B_\parallel$ is the magnetic field parallel to the film. In sufficiently thin samples, $B_\parallel$ has little effect on the or-

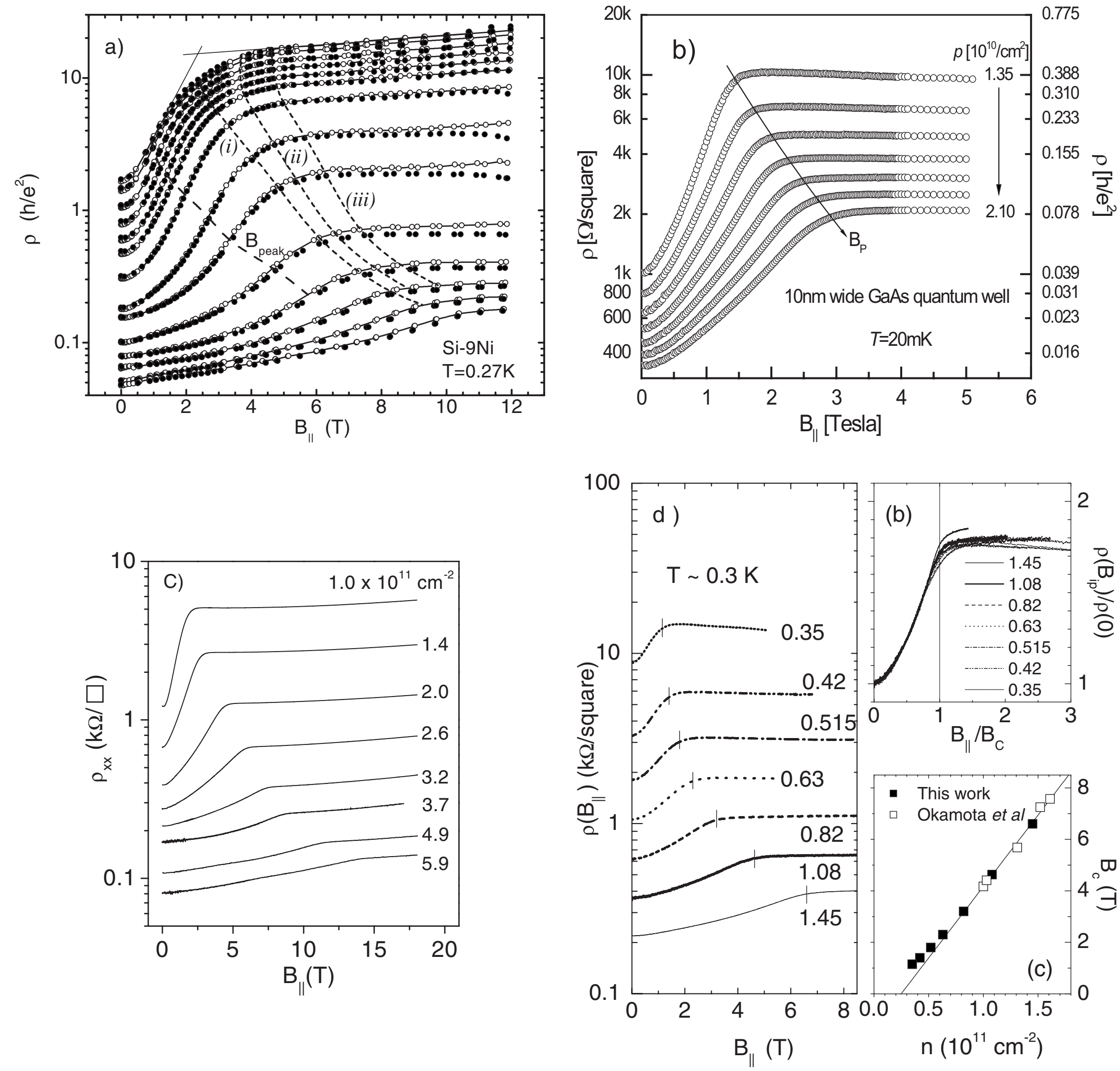

FIG. 3. Giant magnetoresistance in a parallel magnetic field in (a) a strongly metallic Si-MOSFET, (b) a 10-nm-wide p-GaAs quantum well, (c) a n-AlAs quantum well, and (d) a n-SiGe quantum well. Adapted from Pudalov *et al.*, 1998, De Poortere *et al.*, 2002, Lai *et al.*, 2005, and Gao *et al.*, 2006, respectively.

bital motion of the electrons, so it can be viewed as coupling only to the electron spin, and therefore the magnetoresistance is directly a function of the degree of spin polarization of the electron liquid.

In samples that are sufficiently close to the point of the metal-insulator transition, $B_{\|}$ can even induce a metal-insulator transition. This effect has been seen in Si-MOSFETs (see Fig. 4).

As shown in Figs. 5(a) and 5(b), the T dependence of ρ at low temperatures ($T < T_{\max}$) is largely eliminated when the electron spins are polarized. Specifically, for magnetic fields $B_{\|} > B^{*}$, the slope $d\rho(T, B_{\|})/dT$ is reduced from its $B_{\|}=0$ value—in some cases by as much as two orders of magnitude. This effect has been observed in Si-MOSFETs and p-GaAs quantum wells.

3. Magnetoresistance in a perpendicular magnetic field

The experimentally observed behavior of 2D strongly correlated electron liquids in a perpendicular magnetic field $B_{\perp}$ can be quite complex, as shown in Fig. 6. In part, this complexity reflects a combination of orbital and spin effects.

At small $B_{\perp}$ metallic samples often exhibit relatively small negative magnetoresistance. At larger $B_{\perp}$ the magnetoresistance is typically large and positive, while at still larger $B_{\perp}$ it becomes negative again as the system enters the quantum Hall regime.

The interplay between the quantum Hall states and the behavior of the system at large r_s and $B_{\perp} \to 0$ has only been partially explored between the quantum Hall and insulating regimes, which can be unambiguously identified in experiment, at least when the field is not too small. The phase boundary in the n-$B_{\perp}$ plane between the quantum Hall and insulating regimes can be identified in experiment in at least two ways: (a) Since the resistance is an increasing function of T in the quantum Hall phase (vanishing as $T \to 0$) and a decreasing function in the insulating phase, the phase boundary can

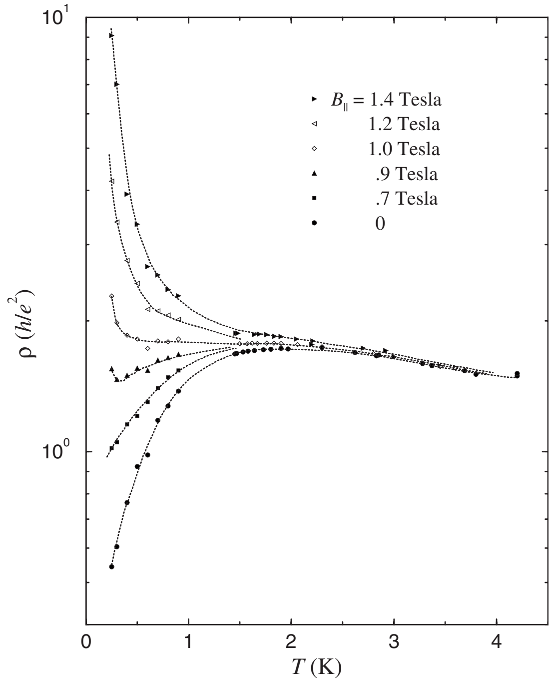

FIG. 4. Parallel magnetic-field-induced metal-insulator transition in a Si-MOSFET. Adapted from Simonian *et al.*, 1997.

be approximately identified as the points at which ρ_{xx} is T independent. (b) Since σ_{xx} vanishes as $T\rightarrow 0$ in both the insulating and quantum Hall phases but is nonzero in the critical regime, the phase boundary can be approximately identified as the points at which σ_{xx} has a local maximum at the lowest accessible T. At least when the field is not too small, these two methods produce essentially the same results, so the identification of the phase boundaries is unambiguous.

As shown on the left-hand side panel of Fig. 7, in a n-GaAs sample with relatively small r_s the phase boundary shifts up in energy as $B\rightarrow 0$, presumably reflecting the expected "floating" of the extended states (Khmelnitskii, 1984; Laughlin, 1984) (i.e., for noninteracting electrons, if all the states at $B_\perp=0$ are localized, then the lowest energy at which there is a localized state must diverge as $B_\perp\rightarrow 0$). This floating can be seen even better in Fig. 8, where the energy of the lowest extended state in a strongly disordered 2D hole system in a Ge/SiGe quantum well is observed to increase by one order of magnitude as $B_\perp\rightarrow 0$ (Hilke *et al.*, 2000).

In contrast, in high mobility p-GaAs samples with large r_s, as $B_\perp\rightarrow 0$ this phase boundary is observed to extrapolate to roughly the same value, n_c, which marks the zero-field metal-insulator transition in the same device (Dultz *et al.*, 1998) (see the right-hand side panel of Fig. 7). The analogous phase boundary, which also extrapolates to n_c, has been traced in Si-MOSFETs (Shashkin *et al.*, 1993), as shown in Fig. 9. While there are some notable differences in the way, within the quantum Hall regime, the various different integer quantum Hall phases terminate at low fields; in both p-GaAs and Si-MOSFETs the phase boundary between the quantum Hall and insulating phases clearly extrapolates to a finite zero-field critical value. This further corroborates the identification of n_c as a critical point.

At low fields, or elevated temperatures, and small r_s, one naturally observes the Shubnikov–de Haas oscillations of $\rho(B_\perp)$ in sufficiently clean 2D systems. Similar oscillations have been observed in both the Si-MOSFETs and p-GaAs quantum wells with large r_s (see Fig. 6). However, for the large r_s samples reported by Gao *et al.* (2003), these oscillations persist up to temperatures which are comparable with the bare Fermi energy. It is important to stress that FL theory not only predicts the existence of magnetic oscillations with period inversely proportional to the area enclosed by the Fermi surface but it also predicts that the amplitude of these oscillations should decrease in proportion to $\exp(-2\pi k_B T/\hbar\omega_c)$ where the cyclotron energy $\omega_c = eB/mc$ in turn must be less than E_F.

4. Spin magnetization of the electron gas at large r_s

The magnetization of the large r_s metal is widely observed to exhibit a strong dependence on n, which re-

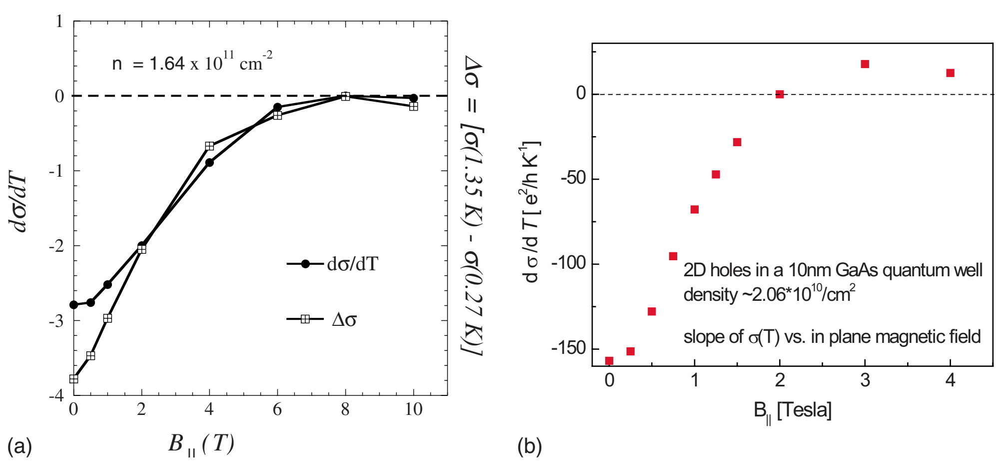

FIG. 5. (Color online) The slope $d\sigma/dT$ as a function of the parallel magnetic field (a) in a Si-MOSFET and (b) in a 10-nm-wide p-GaAs quantum well. Adapted from Tsui *et al.*, 2005 and Gao *et al.*, 2006, respectively.

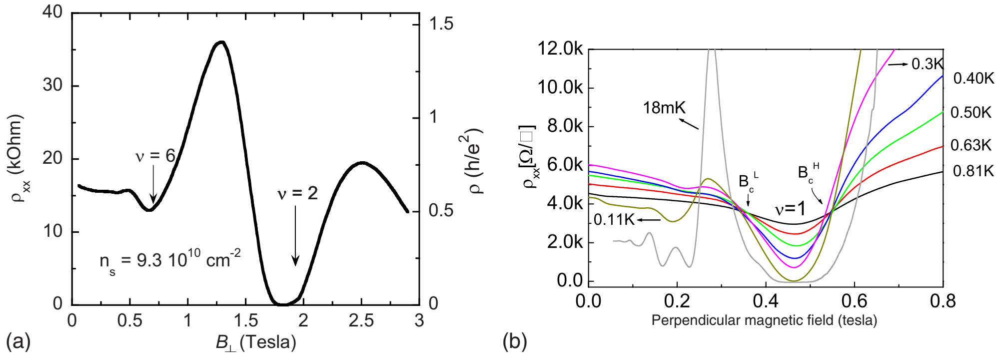

FIG. 6. (Color online) Magnetoresistance of the strongly correlated metallic 2D systems in a perpendicular magnetic field in (a) a Si-MOSFET and (b) a 10-nm-wide p-GaAs quantum well. Adapted from Kravchenko *et al.*, 2000 and Gao *et al.*, 2003, respectively.

flects the increasing importance of electron correlations. For instance, the in-plane magnetic field of complete spin polarization, taken from magnetization and magnetocapacitance measurements in clean Si-MOSFETs (Shashkin *et al.*, 2006), decreases significantly more strongly with decreasing n than does $E_F \propto n$, as shown in Fig. 10. Strong dependences of the magnetization on n have also been seen (Vitkalov *et al.*, 2001; Pudalov *et al.*, 2002; Zhu *et al.*, 2003; Vakili *et al.*, 2004; Lu *et al.*, 2008) in other types of devices with n near the critical density for the metal-insulator transition. However, there are subtle but important differences in the n dependences. For instance, in Si-MOSFETs, B^* appears to extrapolate to 0 at a positive value of $n=n^*$ (Shashkin *et al.*, 2001, 2006; Vitkalov *et al.*, 2001), while in the Si/SiGe quantum well devices studied by Lu *et al.* (2008), the n dependence of B^* over the accessible range of n was fitted by $B^* \sim n^{1.3}$, which vanishes only as $n \to 0$.

Complex evolution of the linear susceptibility χ with n has also been observed, but in contrast with the nonlinear response (as parametrized by B^*), the linear response can be strongly affected by rare regions of localized spins which make a large contribution to the low-T susceptibility (Prus *et al.*, 2003). Nevertheless, experiments (Shashkin *et al.*, 2006) showed that the n-dependent trends of χ are roughly in agreement with the results extracted from measurements of $B^*(n)$.

C. Strongly correlated highly resistive samples, $r_s \geqslant 1$ and $\rho > h/e^2$

The properties of the 2DEG at large r_s well on the insulating side of the metal-insulator transition have been less completely explored experimentally. In comparing results from different systems, one vexing issue is to what extent the large values of ρ reflect strong effects of disorder, as opposed to the intrinsic effects of correlations. We are primarily interested in data on the cleanest possible systems, where at the very least the correlation effects must have strongly enhanced the effects of the weak disorder.

Here we focus on experimental observations in strongly correlated highly resistive devices.

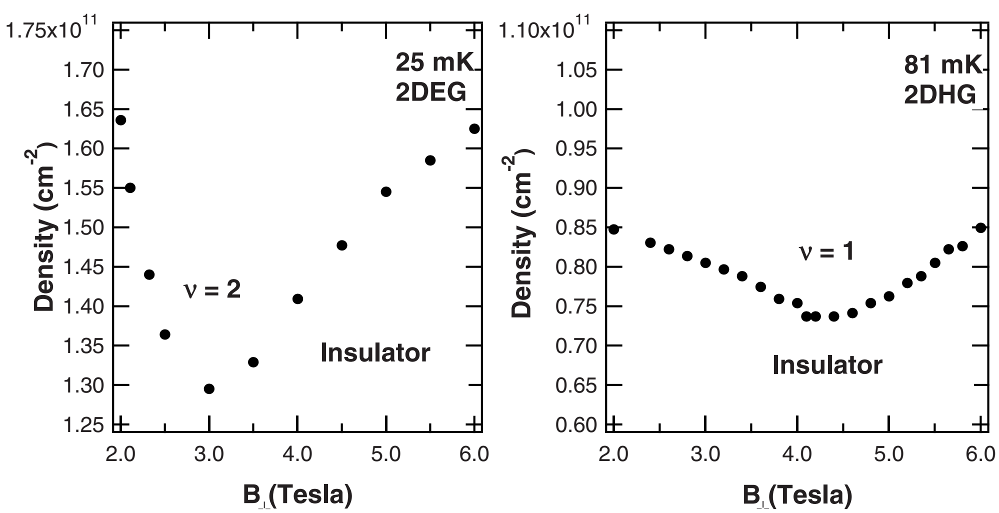

FIG. 7. Position of the lowest extended state on a n-$B_\perp$ diagram in a "weakly interacting" ($r_s \sim 1$) n-GaAs/AlGaAs heterostructure (left-hand side) and in a "strongly interacting" ($r_s \sim 10$) p-GaAs heterostructure (right-hand side). Adapted from Dultz *et al.*, 1998.

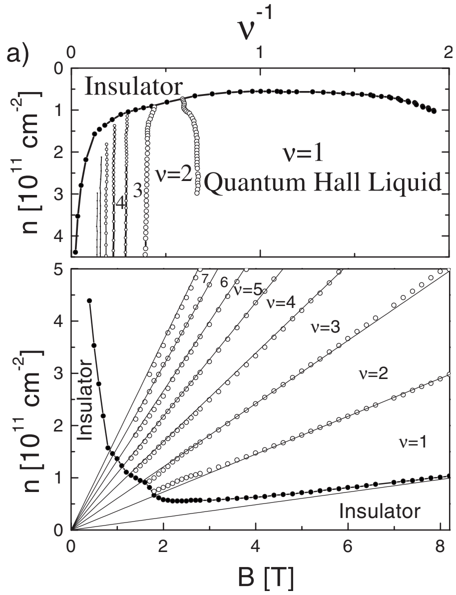

FIG. 8. A map of the extended states for a highly disordered 2D hole system in a Ge/SiGe quantum well. The open circles represent the positions of the extended state in the quantum Hall effect regime. The solid circles correspond to the position of the lowest extended state. Numbers show the value of $\rho_{xy}h/e^2$. Adapted from Hilke *et al.*, 2000.

1. Metallic T dependence in samples with resistivity $\rho > h/e^2$

The conventional theory of localization in the strong disorder limit $\rho \geqslant h/e^2$ predicts that the electronic transport should be due to hopping conductivity, and hence ρ should diverge strongly (exponentially) as $T \to 0$. On the other hand, in p-GaAs samples with large r_s, as shown in Fig. 11, the low-temperature resistivity for a range of n in which ρ is up to three times larger than h/e^2 sometimes exhibits a metallic temperature dependence, i.e., ρ *increases* with increasing temperature. Similar behavior has also been observed in Si-MOSFETs [see the two middle curves in Fig. 1(a)].

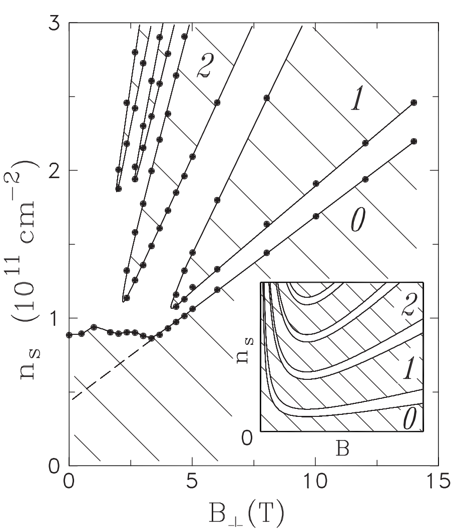

FIG. 9. Bands of the extended states (white areas) in a dilute Si-MOSFET. The shaded area corresponds to localized states. The inset shows the expected floating of the extended states (Khmelnitskii, 1984; Laughlin, 1984). Adapted from Shashkin *et al.*, 1993.

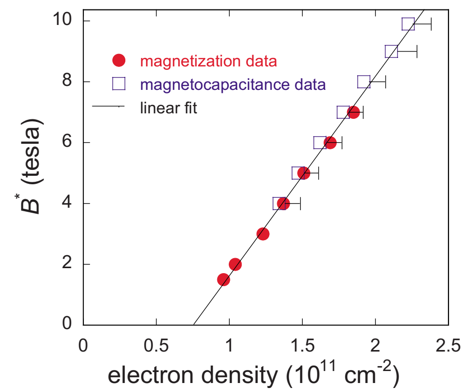

FIG. 10. (Color online) The critical magnetic field B^* needed for complete spin polarization in a Si-MOSFET as a function of the electron density. Adapted from Shashkin *et al.*, 2006.

2. Magnetoresistance for $r_s \geqslant 1$ and $\rho > h/e^2$

The magnetoresistance in a parallel magnetic field $B_\parallel$ on the insulating side of the metal-insulator transition is still large and positive, even in samples with ρ as high as 2 MΩ/$\square$. This can be seen in the data from Si-MOSFET devices shown in Fig. 12. At large enough $B_\parallel$ the magnetoresistance saturates in a way similar to that in metallic samples.

In a perpendicular magnetic field $B_\perp$ samples with $\rho(B=0) \geqslant h/e^2$ exhibit rather complex magnetoresistance. As shown in Figs. 13(a)–13(c), the resistance first increases strongly for small $B_\perp$, sometimes by orders of magnitude. At higher $B_\perp$, the magnetoresistance turns negative and eventually the system exhibits giant resis-

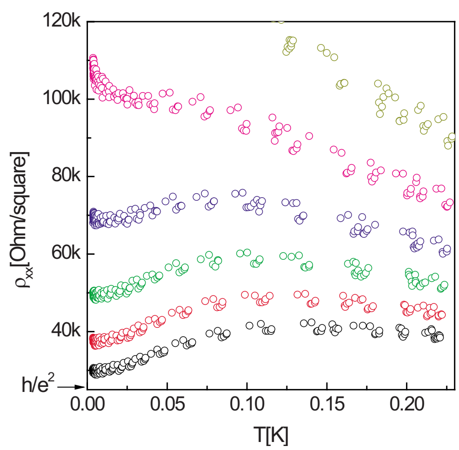

FIG. 11. (Color online) Metallic ($d\rho/dT > 0$) temperature dependence of the resistivity in a 30-nm-wide p-GaAs quantum well with $\rho > h/e^2$ (see four lower curves). 2D hole densities are 6.1×10^9, 5.9×10^9, 5.7×10^9, 5.5×10^9, 5.3×10^9, and 5.1×10^9 cm^{-2} from bottom to top curve. Adapted from Gao, 2003.

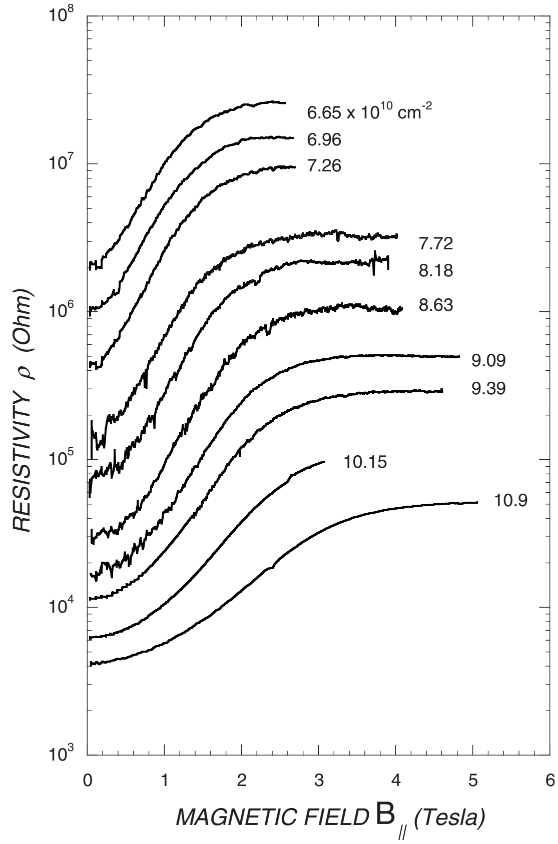

FIG. 12. Parallel field magnetoresistance in a Si-MOSFET at different electron densities across the metal-insulator transition. Adapted from Mertes *et al.*, 1999.

tance oscillations with deep minima at magnetic fields corresponding to Landau-level filling factors $\nu=1$ and 2. The same general features have been observed in p-GaAs samples (Gao, 2003), as shown in Fig. 13(d).

That the positive contribution to the magnetoresistance is largely a spin effect can be seen in Fig. 14, where, in the presence of a strong in-plane field, $B_{\parallel} \sim B^*$, the resistance at $B_{\perp}=0$ is much larger than for $B_{\parallel}=0$, but then, as a function of increasing $B_{\perp}$, the magnetoresistance is everywhere strongly negative all the way to the quantum Hall regime.

D. Drag experiments on double layers with $r_s \gg 1$ and $\rho \ll h/e^2$

Additional information concerning correlation effects can be obtained from measurements of the "drag" resistance in a system of two 2DEG layers which are electrically unconnected. Current I is passed through the lower (active) layer and the voltage V_D is measured on the upper (passive) layer. The drag resistance is defined to be the ratio $\rho_D = V_D/I$.

For relatively small r_s, measurements (Gramila *et al.*, 1993, 1994) of $\rho_D(T)$ in double layer 2DEGs are in qualitative agreement with Fermi-liquid theory (Price, 1983; Zheng and MacDonald, 1993). Specifically, the drag resistance is small, proportional to $(T/E_F)^2$ and to $(k_F d)^{-\alpha_d}$, where k_F is the Fermi momentum, d is the spacing between the two layers, and typically $\alpha_d=2$ or 4, depending on the ratio of d/ℓ.

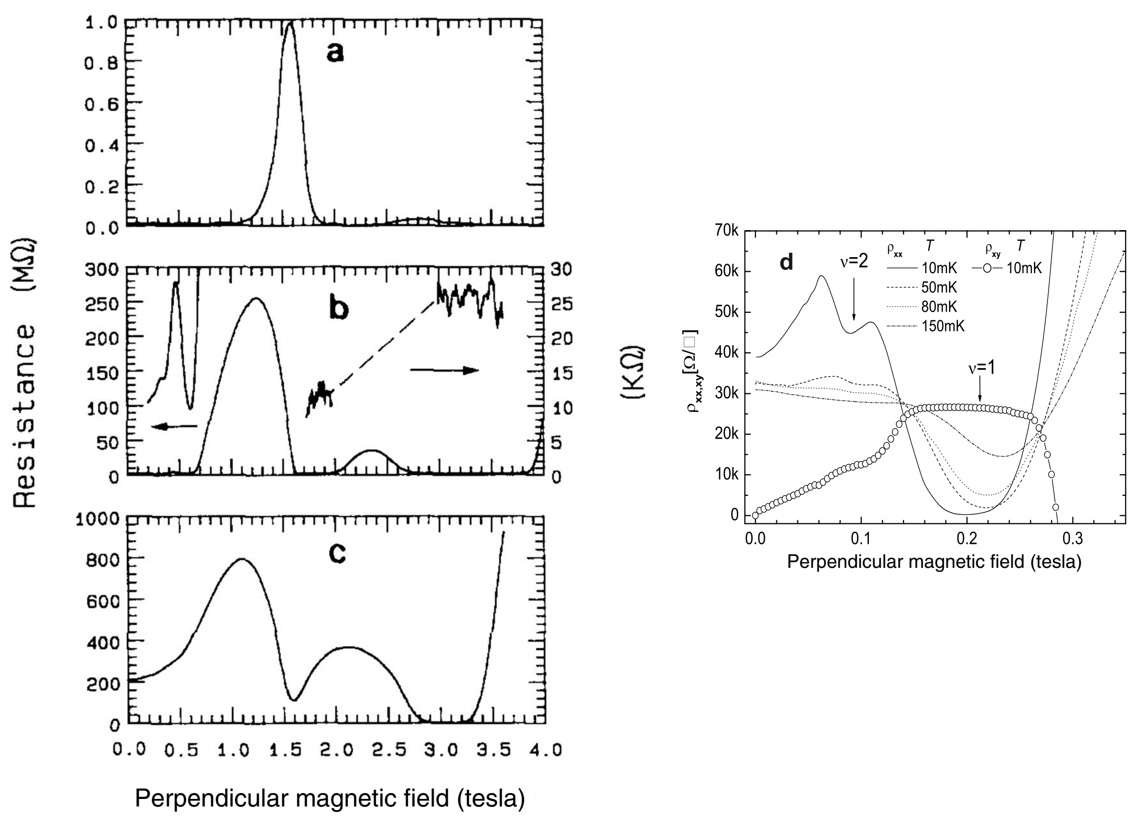

FIG. 13. Magnetoresistance of a highly resistive dilute 2D gas (a)–(c) in a perpendicular magnetic field in a Si-MOSFET and (d) in a 30-nm-wide p-GaAs quantum well. Adapted from D'Iorio *et al.*, 1990 and Gao, 2003, respectively.

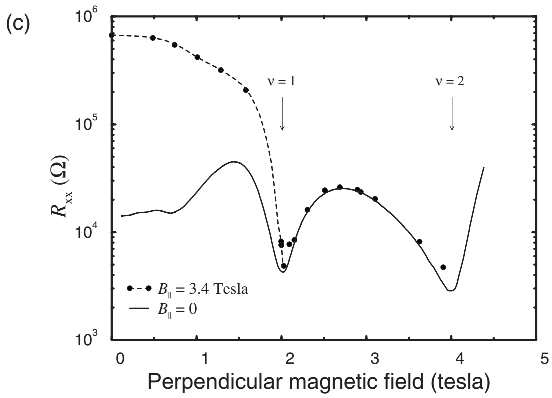

FIG. 14. $R_{xx}(B_\perp)$ in a silicon MOSFET in the presence of a parallel field $B_\parallel=3.4$ T used to suppress the metallic behavior (solid symbols). For comparison, the magnetoresistance in the absence of a parallel magnetic field is shown by the solid line. Adapted from Kravchenko *et al.*, 1998.

However, experiments (Pillarisetty *et al.*, 2003) on p-GaAs double layers with $r_s \sim 20-30$ yield results which differ significantly from liquid theory. These experiments were performed on samples with small resistances $\rho \sim (0.05-0.1)h/e^2$ in which quantum interference corrections to the Drude conductivity are presumably insignificant. The experimental data shown in Fig. 15 reveal the following features:

- The drag resistance in these samples is one to two orders of magnitude larger than expected on the basis of a simple extrapolation of the small r_s results.
- Whereas in a Fermi liquid $\rho_D(T) \sim T^2$, in large r_s devices $\rho_D \sim (T)^{\alpha_T}$ where the temperature exponent exhibits non-Fermi-liquid values $2 < \alpha_T < 3$.
- At low temperature, $\rho_D(T, B_\parallel)$ increases as a function of $B_\parallel$ by a factor of 10–20 and saturates when $B_\parallel > B^*$. A parallel magnetic field also appears to strongly suppress the temperature dependence of $\rho_D(T)$. In the presence of a nonzero $B_\parallel$, the value of $\alpha_T(B_\parallel)$ decreases with $B_\parallel$ and saturates for $B_\parallel > B^*$ at a value which is significantly smaller than the FL value $\alpha_T = 2$.
- The T and especially the $B_\parallel$ dependences of $\rho_D(T, B_\parallel)$ and the resistivities of the individual layers $\rho(B_\parallel, T)$ look qualitatively similar to one another, which suggests that both have a common origin (see Fig. 15). [In a Fermi liquid, $\rho(T, B_\parallel)$ is primarily determined by the electron-impurity scattering and $\rho_D(T, B_\parallel)$ by the interlayer electron-electron scattering, so there is no *a priori* reason for their T and $B_\parallel$ dependences to be similar.]

E. Comparison with small r_s devices

The anomalies discussed have been observed in samples with large r_s. In relatively smaller r_s high mobility devices (i.e., $r_s \sim 1$), the observed behavior is much

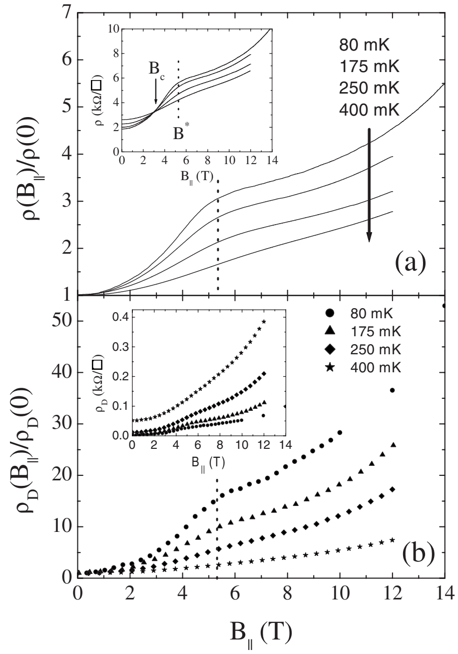

FIG. 15. In-plane magnetotransport data for $p_m = 2.15 \times 10^{10}$ cm^{-2} at $T = 80$, 175, 250, and 400 mK. (a) Inset: ρ vs $B_\parallel$. Main plot: Data from inset normalized by its $B_\parallel = 0$ value. (b) Inset: Corresponding data for ρ_D vs $B_\parallel$. Main plot: Data from inset normalized by its $B_\parallel = 0$ value. Adapted from Pillarisetty *et al.*, 2003.

more in line with the expectations of FL theory, modified by weak interference effects. For example, there is a smooth finite-temperature crossover as a function of disorder from a "weak localization" regime for $\rho < h/e^2$ to variable range hopping behavior for $\rho > h/e^2$; ρ is weakly (logarithmically) dependent on T for $\rho < h/e^2$ and $T \ll T_F$; the parallel field magnetoresistance is weak (logarithmic) and positive for $\rho < h/e^2$; the resistance diverges strongly (exponentially) with decreasing T whenever $\rho > h/e^2$; and the drag resistance is small compared to ρ [see, e.g., Ando *et al.* (1982) for a review]. Moreover, in the presence of a perpendicular magnetic field, small r_s devices exhibit "levitation of the delocalized states"; i.e., the phase boundary between the quantum Hall and the insulating phases tends toward ever higher values of the density as $B_\perp \to 0$, as shown on the left-hand side panel of Fig. 7 and in Fig. 8.

III. THEORETICAL CONSIDERATIONS

A. Good 2D "metals" with $r_s \ll 1$ and $k_F \ell \gg 1$

The properties of interacting electronic systems crucially depend on the dimensionless strength of the interactions r_s and the strength of the quenched disorder, which is parametrized by the dimensionless quantity $k_F\ell$, where k_F is the electron Fermi momentum and ℓ is the electron elastic mean free path. The theory of pure ($k_F\ell \gg 1$) and weakly interacting ($r_s \ll 1$) electron liquids is under good theoretical control. In this case, the Boltzmann theory yields a good first approximation to the transport properties, and any Fermi-liquid corrections to the bare electron mass and to the density of states are small. Moreover, the screening radius $\lambda_{sc} = a_B^*/4$, is much larger than the average distance between electrons, $\lambda_{sc} n^{1/2} = (4\sqrt{\pi})^{-1} r_s^{-1} \gg 1$.

In this limit, to a first approximation, the transport scattering rate can be calculated by perturbation theory. Specifically, ρ is proportional to the rate of transfer of momentum from the electrons to the lattice. At high temperatures the resistance of the 2D electron gas is determined by the electron-phonon scattering. At temperatures low compared to the effective Debye temperature, $T_{ph} \sim \hbar c k_F$, where c is the speed of sound, the electron-phonon scattering is no longer significant. [In some cases, a crossover temperature T_{ph}, below which electron-phonon scattering is unimportant, can be readily identified; for example, in Fig. 2, $T_{ph} > E_F$ is roughly the point at which $\rho(T)$ has a minimum.]

Since in all cases of interest here the Fermi momentum is much smaller than the reciprocal-lattice vector, electron-electron scattering conserves the total quasimomentum to very high order and therefore does not contribute directly to the resistance. [This is different from metals with large Fermi momenta where electron-electron umklapp processes determine the T dependence of the resistance at low T (Abrikosov, 1988).] Therefore the low-temperature resistance of the system is due to electron-impurity scattering.

Consider, for example, the case in which the dominant low-temperature scattering is from the Coulomb potential due to a random distribution of charged impurities. Naturally, the potential of an impurity is screened by the 2DEG itself. Two generic aspects of the screening are that it causes the potential to fall more rapidly when the distance from the impurity exceeds the screening length and it induces a rapidly oscillating component of the screened potential corresponding to the Friedel oscillations in the density. As is well known, scattering by an unscreened Coulomb potential produces singular forward scattering, leading to an infinite cross section. However, the transport scattering rate weights large-angle scattering more heavily. In three dimensions, the transport scattering cross section is still logarithmically divergent, so even when the screening length is long, it plays an essential role in cutting off this divergence. In two dimensions, however, the transport scattering rate is finite even in the limit $\lambda_{sc} \to \infty$, where in the Born approximation it is given by

$$\frac{\hbar}{\tau} = \frac{(2\pi)^2 e^4}{E_F} N_i \{1 + O([k_F \lambda_{sc}]^{-2})\}, \tag{1}$$

where N_i is the concentration of impurities. Moreover, to the extent that quantum inference between different scattering events can be neglected (Boltzmann transport), the conductivity is related to τ, the Fermi velocity v_F, and the density of states ν according to the Drude formula

$$\rho^{-1} = e^2 v_F^2 \tau \nu / 2. \tag{2}$$

Here ρ approaches a constant value as $T \to 0$. A parallel magnetic field, which we assume acts only on electron spins, changes the spin degeneracy at the Fermi level. In the limiting cases, $B_\parallel = 0$, there is a spin degeneracy $g_c = 2$, while for $B_\parallel \geq B^*$, $g_c = 1$. Since $E_F \propto v_F^2 \propto g_c^{-1}$ and $\nu \propto g_c$, we conclude that for $r_s \ll 1$,

$$\rho \propto g_c. \tag{3}$$

This means that the resistance *decreases* by a factor of 2 as $B_\parallel$ increases from 0 to B^*. By the same token the scale for the T dependence of $\rho(T)$ is set by $T_F = E_F / k_B$; since at temperatures large compared to T_F, $\tau \propto v_F^2 \propto T$, it follows that ρ decreases as T increases in proportion to T^{-2}. The only change in the T dependence of ρ in the presence of $B_\parallel > B^*$ is produced by the factor of 2 increase in T_F. Similar considerations (which we will not review explicitly) lead to the conclusion that the drag resistance in double layers is small in proportion to T^2/E_F and consequently ρ_D decreases as $B_\parallel$ increases.

This simple theoretical description in the limit $r_s \ll 1$ and $k_F \ell \gg 1$ gives a good account of the transport in clean high-density devices but is in drastic qualitative disagreement with all the experimental results on the transport properties of the large $r_s > 1$ devices presented in Sec. II.

B. Weak localization corrections and the theory of 2D localization

In two dimensions, the interference between multiple-scattering processes, i.e., the "weak localization corrections" to the conductivity, diverges at $T \to 0$ and $L \to \infty$ (Abrahams *et al.*, 1979; Gorkov *et al.*, 1979). Here L is the sample size [for a review, see Lee and Ramakrishnan (1985)]. This divergence, however, is only logarithmic, and therefore for $k_F \ell \gg 1$ and at accessible temperatures these corrections are small in comparison to the zeroth-order conductivity $G \equiv (h/e^2)\rho^{-1} = G_0 + \delta G$, where $G_0 \sim k_F \ell$ and

$$\delta G = -\ln[L/\ell]. \tag{4}$$

In infinite samples L in Eq. (4) should be reinterpreted as the phase-breaking length $L_\phi = \sqrt{D\tau_\phi}$, where $D = v_F \ell$ is the diffusion constant and $1/\tau_\phi \sim T^p$ is the phase-breaking rate, in which p depends on details of inelastic-scattering processes.

C. Interaction corrections

Impurities in a metal create the Friedel oscillations of the electron density. Due to the electron-electron interactions, the quasiparticles in the metal are scattered not only from the impurity but also from the modulations of the electron density. The interference between these two processes gives rise to corrections to the Drude resistance. These corrections are interesting because they are nonanalytic functions of T and $B_\parallel$, so at small enough temperatures they dominate the T and $B_\parallel$ dependences of the resistance.

In the diffusive limit ($L_T \equiv \sqrt{D/T} \gg \ell$) the interaction correction to the conductivity is logarithmically divergent as $L_T \to \infty$ (Altshuler *et al.*, 1980; Finkel'stein, 1983, 1984a, 1984b; Castellani *et al.*, 1984; Altshuler and Aronov, 1985; Finkel'stein, 1990),

$$\delta G = -\frac{1}{2\pi^2}\left\{1 + 3\left[1 - \frac{\ln(1+F_0)}{F_0}\right]\right\}\ln(L_T/\ell), \qquad (5)$$

where $F_0 < 0$ is an interaction constant in the triplet channel. Note that this same interaction parameter is also responsible for an enhancement of the spin susceptibility, $\chi = \chi_0/(1+F_0)$. For $r_s \ll 1$ (in which limit $|F_0| \ll 1$), Eq. (5) gives (up to a numerical factor) the same (negative) correction to the conductivity as Eq. (4). Note, however, that this correction has the opposite (metallic) sign when $\ln(1+F_0)/F_0 > 4/3$.

At higher temperatures, when $L_T \ll \ell$, the leading interaction effect involves the interference between a single electron scattering from an impurity and from the Friedel oscillations in the neighborhood of the same impurity. The interference corrections to the Drude formula in this so-called "ballistic regime" were considered by Zala *et al.* (2001a, 2001b).

The result is that in an intermediate interval of temperatures, $E_F \gg T \gg \hbar/\tau$, there is a T-dependent correction to the conductance which is linear in T. It consists of the sum of "singlet" and "triplet" contributions (Zala *et al.*, 2001a, 2001b),

$$G(T) - G(0) \propto G_0 \frac{T}{E_F}\left[1 + 3\frac{F_0}{1+F_0}\right]. \qquad (6)$$

The factor of 3 in the second (triplet) part of Eq. (6) reflects the existence of three channels in the triplet part on electron-electron interaction. [Equation (6) is written for the case in which there is no valley degeneracy, as in GaAs, but it is readily generalized to the valley degenerate case. For example, in the case of Si-MOSFETs, with a twofold valley degeneracy, this factor becomes 15.] For repulsive interactions F_0 is negative with a magnitude which, for small r_s, is proportional to r_s. Thus, at small r_s, $\rho(T)$ decreases linearly as T increases.

Of course, most experimental realizations of the 2DEG have $r_s \gtrsim 1$. At a phenomenological level, it is possible to imagine (Zala *et al.*, 2001a, 2001b) extrapolating to stronger interactions in the spirit of Fermi-liquid theory, in which case for $F_0 < -1/4$, a linearly increasing resistance would result. The range of validity of this sort of extrapolation, which is clearly sensible to some degree, is an unresolved theoretical issue in the field to which we will return.

D. The theory at $r_s \gg 1$

In an ideal MOSFET, where there is a metallic ground plane displaced by a distance d from the 2D electron gas, the electron potential energy changes its form as a function of d. For $r \ll d$ electrons interact via the Coulomb interaction and $V(r) \sim e^2/r$. For $r \gg d$ one has to take into account the interaction between electrons and their images in the ground plane and hence $V(r) \sim e^2 d^2/r^3$.

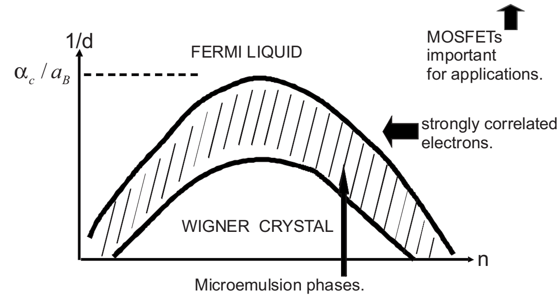

FIG. 16. Schematic phase diagram for 2D electrons in a MOSFET with no disorder.

The mean-field phase diagram for this problem is shown schematically in Fig. 16. The considerations that produce the reentrant character of transition line as a function of n are relatively simple. The kinetic energy of the 2D electron liquid scales with the electron density as $\sim n/m$. For Coulomb interactions, the typical interaction strength scales as $\sim e^2 n^{1/2}$, so the ratio of the potential to kinetic energy, $r_s \propto n^{-1/2}$, decreases with increasing density. Therefore, in the range of densities $n \gg d^{-2}$, increasing density always favors the FL. Conversely, for $n \ll d^{-2}$, the interactions between electrons are dipolar and hence effectively short ranged; i.e., the typical interact scales as $\sim e^2 d^2 n^{3/2}$. Thus, in this range of density, $r_s \propto n^{1/2}$ decreases with decreasing density, and hence decreasing density favors the FL, reflecting the same trends as does ^{3}He. One implication of this analysis is that there exists a critical value of the ratio $d/a_B = \alpha_c$ such that for $d < \alpha_c a_B$ there is no WC phase at any density. In this case there is a largest achievable value of $\max[r_s] \sim d/a_B^*$.

Fixed-node Monte Carlo simulations (Tanatar and Ceperley, 1989) performed under the *assumption* that there is a direct FL-WC transition yield the large critical value $r_s^{(c)} = 38 \gg 1$. By analogy, we think that it is likely that $\alpha_c \sim r_s^{(c)}$ is also large compared to 1, although as far as we know this issue has not been addressed using any quantitative methods.

The existence of a "highly correlated" 2D electron liquid in the range $1 < r_s < r_s^{(c)}$ derives from the large value of $r_s^{(c)}$. From this point of view, it is possible to consider the case $r_s < r_s^{(c)}$ and still treat r_s as a large parameter. If we do this, we see that the highly correlated 2D electron liquid has (at least) three characteristic energy scales: (1) E_F or more properly (Andreev, 1978, 1979) the renormalized Fermi energy, $E_F^* < E_F$, which contains a renormalized mass, (2) the interaction energy $V = r_s E_F$, and (3) the plasma frequency $\Omega_P \sim \sqrt{E_F V} = \sqrt{r_s} E_F$. For large r_s, these energies are quite distinct, with $V > \Omega_P > E_F^*$. As a consequence of the existence of this hierarchy of energy scales there are four distinct temperature intervals where the electron liquid behaves differently. We discuss these intervals below: $T<E_F$ where the system is in the Fermi-liquid state, $E_F<T<\Omega_P$ where the system is a nondegenerate strongly correlated and still highly quantum liquid, $\Omega_P<T<V$ where the system is a highly correlated classical liquid, and $T>V$ where the system is a classical electron gas.

E. The Fermi fluid to Wigner crystal transition in the absence of disorder

The classic studies of the Fermi fluid to WC transition were carried out under the assumption that, at $T=0$, there is a direct first-order transition where the ground-state energies cross. However, it has been shown (Lorenzana *et al.*, 2002; Spivak, 2003; Spivak and Kivelson, 2004, 2006; Jamei *et al.*, 2005; Biskup *et al.*, 2007; Giuliani *et al.*, 2007) that for the case of a relatively long-range potential,

$$V \sim 1/r^x, \quad 1 \geqslant x \geqslant 3, \tag{7}$$

a direct first-order liquid-crystal transition is forbidden in two dimensions. Instead, either the freezing transition is continuous [which is unlikely for well-known reasons (Brazovskii, 1975)] or between these two phases there must occur other phases, which we have called "microemulsion phases."

The largest theoretical uncertainty here is that no reliable estimates exist concerning the width in density of these novel phases. The part of the phase diagram where these phases should exist is indicated qualitatively by the shaded region in Fig. 16. A rich variety of phases, including bubble and stripe phases, is expected in this region. The existence of a stripe phase, for example, would be detectable macroscopically through a spontaneously generated resistance anisotropy.

Currently there is no direct experimental evidence of the existence of such phases. This is either due to the fact that, for some reason, the range of densities in which they exist is extremely small or because currently available samples are not pure enough to reflect the physics of the zero disorder limit. In any case, the existence of such phases, which are different from both the FL and the WC, significantly complicates the theory of strongly correlated disordered electronic systems.

F. Theoretical considerations concerning the metal-insulator transition in two dimensions with disorder

1. Anderson localization

It has been suggested (Abrahams *et al.*, 1979) that in absence of electron-electron interactions the logarithmic derivative

$$d \ln G / d \ln L = \beta(G) \tag{8}$$

is a function of G alone. According to Eq. (4), $\beta \sim -1/G$ as $G \to \infty$. The fundamental result of the single-particle theory of localization follows from this: even weak disorder leads to insulating behavior at $T=0$, and hence there can be no metal-insulator transition. On the face of it, this statement is inconsistent with the experiments summarized in Sec. II.

2. Transition in the presence of spin-orbit scattering

Since the existence of a metal-insulator transition is an important issue, it is worth considering the single-particle problem in the presence of spin-orbit coupling, where the theory predicts a zero-temperature transition between an "ideal metal" ($G=\infty$) and an insulator ($G=0$) (Hikami *et al.*, 1980; Evangelou and Ziman, 1987; Ando, 1989). Analytically, this follows again from a perturbative analysis of the interference corrections: $\delta G = +\frac{1}{4}\ln(L/\ell_{\mathrm{so}})$ for L large compared to the spin-orbit scattering length, $L \gg \ell_{\mathrm{so}} \geqslant \ell$. In other words, as long as G at length scale $\sim \ell_{\mathrm{so}}$ is large compared to 1, at low enough temperatures that $L_\phi > \ell_{\mathrm{so}}$, the conductivity is an *increasing* function of decreasing T "antilocalization." Under the same assumption of one-parameter scaling as in Eq. (8), this implies that $\beta \sim +1/G$ as $G \to \infty$, which in turn implies the stability of a perfectly metallic phase with $\rho=0$. On the other hand, the stability of an insulating phase for strong enough disorder is not debatable, i.e., $\beta<0$ for small enough G. Moreover, the validity of the one-parameter scaling in this problem and the existence of a metal-insulator transition have been tested in numerical studies (Asada *et al.*, 2004).

While this is the simplest model system in which a 2D metal-insulator transition exists, it is unlikely that it is relevant to the experiments discussed in Sec. II. Specifically, the spin-orbit interaction in n-silicon MOSFETs is quite weak and hence unlikely to be important in the accessible range of temperatures.

3. Scaling theories of a metal-insulator transition with strong interactions

Even in the absence of spin-orbit coupling, assuming the theory remains renormalizable, the β function is fundamentally modified by strong interactions. A key idea underlying most scaling theories of the metal-insulator transition (McMillan, 1981; Finkel'stein, 1990; Dobrosavljević *et al.*, 1997; Punnoose and Finkel'stein, 2001, 2005) is that the logarithmically diverging interaction correction to the conductivity in Eq. (5) can have the opposite sign and larger magnitude than the single-particle weak localization corrections in Eq. (4) when extrapolated to large values of r_s where $|F_0|$ is no longer small. Moreover, even when perturbation theory is still valid, i.e., where $G \gg 1$ and $r_s \ll 1$, the value of F_0 is subject to renormalization, which in the case of weak Coulomb interactions, $|F_0| \ll 1$, is of the form (Finkel'stein, 1990)

$$dF_0 / d\ln(L/\ell) \sim -1/k_F\ell. \tag{9}$$

Since F_0 is negative, this means that its magnitude grows with increasing L. It is therefore possible to imagine [in the spirit of the renormalization group (RG)] that even where the bare interactions are weak, at sufficiently long scales, F_0 grows until $|F_0| \lesssim 1$, at which point the β function would change sign.

If a metal-insulator transition occurs, it must occur at finite (dimensionless) interaction strength F^* and finite conductance G^*. (Here F may denote the strength of one or several interactions.) *A priori* the properties of such a fixed point cannot be computed perturbatively. Either the existence and character of such a fixed point must be inferred by extrapolating perturbative expressions for the β function to finite coupling strength (Castellani *et al.*, 1984; Finkel'stein, 1990; Chamon and Mucciolo, 2000; Punnoose and Finkel'stein, 2001, 2005; Nayak and Yang, 2003) or it must simply be conjectured (Dobrosavljević *et al.*, 1997) on phenomenological grounds. One problem with this scenario is that for $G \gg 1$, if the RG procedure is taken literally, the system evolves to a low-temperature state where $F_0 \to -1$, which in addition to changing the sign of the β function implies the existence of a magnetic instability, which surely must affect the physics. More generally, there exists no clear qualitative picture of what happens to the system when the parameter F_0 is significantly renormalized.

There are a number of experimentally relevant consequences of this scenario. Most importantly, it implies the existence of a true quantum phase transition between a (possibly ideal) metal and an insulator, with all the implied quantum critical phenomena. Second, valley degeneracy figures as an important factor for the character and possibly even the existence of a metal-insulator transition (Punnoose and Finkel'stein, 2001, 2005). Third, the theory predicts the existence of a peak in the temperature dependence of $\rho(T)$. For systems with $k_F\ell \gg 1$, $\rho(T)$ reaches a maximum as a function of T at the temperature (Finkel'stein, 1990; Punnoose and Finkel'stein, 2001, 2005)

$$T_{\max} \approx (v_F/\ell)\exp(-\alpha G), \tag{10}$$

corresponding to the length scale $L(T_{\max})$ at which β changes sign. Here α is a constant of order 1, which depends on the strength of the bare interactions. For $T < T_{\max}$, the system exhibits metallic T dependence. The theory also predicts an increase of the resistance at small values of $B_\parallel$. However, when the conductance of the system is large, $G \gg 1$, both these effects become small and manifest themselves only at exponentially low temperatures [see Eq. (10)].

G. 2DEG in the presence of a perpendicular magnetic field

In considering the phase diagram of the 2DEG in the n-$B_\perp$ plane with a fixed strength of disorder, there are a number of asymptotic statements (Kivelson *et al.*, 1992) that can be made with confidence on theoretical ground:

- First, for low enough density $n < n_{\min}$ and for any strength of magnetic field the system must be insulating; in the absence of disorder, this is a consequence of the Wigner crystallization, and in the presence of disorder, it can be traced to the strong tendency of dilute electrons to be strongly localized by disorder.
- For any fixed density and high enough magnetic field, $n\phi_0/B_\perp \ll 1$, it is similarly straightforward to show that the system must always be insulating. (Here $\phi_0 = hc/e$ is the quantum of flux.)
- Most interestingly, in the limit that both n and $B_\perp$ are large, but with $n\phi_0/B_\perp \sim 1$, there necessarily exist robust quantum Hall phases. To see this, note that in this limit $\hbar\omega_c$ is large compared to both the strength of the disorder and the electron-electron interactions, so inter-Landau-level scattering can be treated perturbatively. (Here $\omega_c = eB_\perp/mc$.) In this limit, the existence and stability of integer and fractional quantum Hall states are well established.

For present purposes, we neglect the complexity associated with the various distinct quantum Hall phases and simply discuss the considerations that determine the shape and topology of the curve $n^*(B_\perp)$ which encloses the regions of quantum Hall phases in the phase diagram (see Figs. 8 and 9). From the three observations above, it follows that

$$n^*(B_\perp) \sim B_\perp/(\phi_0) \quad \text{as } B_\perp \to \infty. \tag{11}$$

In the absence of electron-electron interactions and spin-orbit scattering, we know that all states at $B_\perp = 0$ are localized. This requires that $n^*(B_\perp) \to \infty$ as $B_\perp \to 0$. Within the single-particle theory, this constraint was accounted for by the notion of "levitation" of the delocalized states (Khmelnitskii, 1984; Laughlin, 1984). The idea here is that in the large field limit, when $\omega_c \gg 1/\tau$, there is a single delocalized energy level at the center of each Landau level, but that when at small field, $\omega_c \sim 1/\tau$, the delocalized states initially associated with each Landau level remain distinct but begin to levitate to higher energies. Thus, as $B_\perp \to 0$, the energies of the delocalized states diverge. Recent numerical studies (Sheng *et al.*, 2001) suggested that, even for noninteracting electrons, the fate of the delocalized states may be a more complex issue. However, as long as there is only a single insulating phase at $B_\perp = 0$, the divergence of $n^*(B_\perp)$ as $B_\perp \to 0$ is inescapable. Conversely, if $n^*(B_\perp)$ tends to a finite value, $n_c = n^*(0)$, as $B_\perp \to 0$, it probably implies the existence of a zero-field phase transition at this value of the density. Since n^* is unambiguously a critical density marking the point of a quantum phase transition, tracking it to the zero-field limit is a promising strategy for distinguishing a crossover from a true phase transition.

Comparing these theoretical expectations with the already discussed experiments, we see that in the large r_s p-GaAs and Si-MOSFETs (the right-hand side panel in Figs. 7 and 9), $n^*(B_\perp)$ extrapolates to a finite value as $B_\perp \to 0$. Conversely, in the small r_s n-GaAs device (left-hand side panel in Fig. 7 and Fig. 8), clear evidence of levitation is seen in the sense that $n^*(B_\perp)$ has a pronounced minimum at a value of $B_\perp$ which arguably corresponds to $\omega_c\tau\sim 1$ and then grows strongly as $B_\perp$ is further reduced.

H. Microemulsions of WC and FL in the presence of weak disorder

1. Effects of weak disorder at $T=0$

Starting from the clean limit, treating the interactions as strong and the disorder as a small perturbation leads to a rather different perspective on the metal-insulator transition. It is important to recognize, however, that even weak disorder is always a relevant perturbation in two dimensions. If we ignore the existence of microemulsion phases in the zero disorder limit, then in two dimensions disorder always rounds the first-order liquid to WC transition (Imry and Wortis, 1979; Aizenman and Wehr, 1989; Berker, 1993), resulting in a phase in which islands and continents of pinned WC coexist with seas and rivers of Fermi fluid. If, instead, we start with a spatially ordered pattern of alternating WC and Fermi fluid, characteristic of a microemulsion phase, then disorder destroys the long-range order, again resulting in a phase which consists of a disordered mixture of WC and Fermi fluid regions. Though the two scenarios differ greatly for weak disorder in the degree of local organization of the coexisting regions, at long distances they are difficult to distinguish.

Since at $T=0$ a WC does not conduct, this picture calls to mind a percolation-type metal-insulator transition. Of course, this picture does not take into account a variety of possibly important effects, including interference effects which could ultimately turn the percolative metal into an insulator. However, in the limit in which the characteristic sizes of the islands and continents are large, classical percolation could be a good description over a significant range of T.

2. Low-T thermal physics: The Pomeranchuk effect

The Fermi-liquid to crystal transition takes place at $r_s \gg 1$. In the Fermi-liquid state, the entropy density $S_{\mathrm{FL}} \approx nT/E_F$ is small for temperatures $T \ll E_F$. In contrast, for T large compared to J, the spin-exchange interaction between localized particles in the crystal state, the spin entropy of the crystal $S_C \approx n \ln 2$, is relatively large and temperature independent. Since for large r_s, J is exponentially smaller than E_F [see, e.g., Roger (1984)], there exists a broad range of temperatures, $E_F > T > J$, in which the crystal is the high-entropy state, $S_C > S_{\mathrm{FL}}$. In this range of temperatures and near the point of the crystal-liquid transition the system tends to freeze upon heating. Of course, at ultralow temperatures, $T \ll J$, or at high temperatures, $T > E_F$, the entropy of the crystal is smaller than that of the liquid, so the Pomeranchuk effect disappears.

The tendency of Fermi systems to freeze upon heating, known as the "Pomeranchuk effect," was originally discovered in the framework of the theory of ^{3}He (Pomeranchuk, 1950; Richardson, 1997), where it has been confirmed experimentally (see the phase diagram of ^{3}He in Fig. 17). In this regard, the Wigner crystal of electrons and the crystal of ^{3}He atoms are similar. Specifically, the exchange energy between spins in a Wigner crystal is exponentially small, $J \sim \exp(-\alpha\sqrt{r_s})$, where α is a number of order 1 (Roger, 1984; Chakravarty *et al.*, 1999; Voelker and Chakravarty, 2001). Thus at low temperatures the evolution of the random microemulsion phases as a function of T and $B_\parallel$ is dominated by the entropy changes associated with the spin degrees of freedom (Spivak, 2003; Spivak and Kivelson, 2004, 2006).

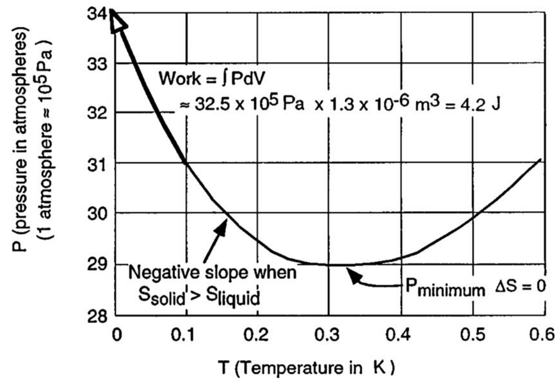

FIG. 17. The melting pressure of ^{3}He. Adapted from Richardson, 1997.

In the case of the two phase coexistence at low temperatures, the fraction of the system which is locally Wigner crystalline f_{WC} increases linearly with T. This behavior will dominate the T dependence of many important physical properties of the system at low T.

Similar considerations govern the $B_\parallel$ dependence of the phase diagram. Since the spin susceptibility $\chi_{\mathrm{WC}} \gg \chi_{\mathrm{FL}}$, the corresponding magnetization $M_{\mathrm{WC}} \gg M_{\mathrm{FL}}$ at small $B_\parallel$. Since the free energy of the system contains the term $-MB_\parallel$, there is a $B_\parallel$-induced increase of the WC fraction over a wide range of circumstances.

Since the resistances of the WC and FL are different, the Pomeranchuk effect is one of the most directly testable features of the microemulsion phases. Where the FL is the majority phase, it leads to the robust prediction that $\rho(T,B_\parallel)$ increases with T and $B_\parallel$ at low T, reflecting the purely thermodynamic fact that f_{WC} increases as T and $B_\parallel$ increase. However, for $B_\parallel > B^*$, the T and $B_\parallel$ dependences of $\rho(B_\parallel)$ are quenched since there is no spin entropy in the fully polarized system, so $f_{\mathrm{WC}}(T,B_\parallel)$ no longer depends strongly on these quantities. This effect can, in principle, produce arbitrarily large fractional changes in the low-T resistivity. Even on the insulating side of the transition, where at $T=0$ a majority of the system is Wigner crystalline, a temperature-induced increase in f_{WC} can result in an increase of ρ with increasing T in a limited range of temperatures even though $\rho > h/e^2$.

Many other more detailed aspects of the T and $B_\parallel$ dependences of ρ depend on the type of disorder and other "details." While some progress has been made in understanding the dynamical properties of microemulsion phases in the presence of weak disorder (Spivak and Kivelson, 2006), no satisfactory and/or quantitative theory of disordered microemulsions is currently available.

Of course, at high enough temperatures, where the electron liquid is nondegenerate, the WC always is the lower entropy phase. The Pomeranchuk effect reflects, more than anything, the low entropy of the degenerate Fermi liquid at $T<E_F$. The melting temperature of the WC is determined by the (possibly strongly renormalized) Fermi energy of the "competing" fluid. Above this temperature, the microemulsions give way to a uniform nondegenerate electron fluid.

3. Crossovers at higher temperatures ($T>T_F$) and large r_s

In the absence of disorder, umklapp scattering, and electron-phonon scattering, the long-wavelength properties of the electron fluid are governed by hydrodynamics rather than by the Boltzmann equation. In this limit, we need to talk about the viscosity of the fluid η rather than the conductivity, which is in any case infinite. For gentle enough disorder, we can think of the disorder as defining some form of effective medium through which the otherwise hydrodynamic fluid flows. This is a transport regime, which is not often recognized, in which the hydrodynamic healing length (e.g., the electron-electron mean free path $\ell_{e\text{-}e}$) is short compared to the distance between impurities or the length scale over which the disorder potential varies. In this case,

$$\rho \propto \eta, \tag{12}$$

where the proportionality constant is in general a complicated function of the strength and character of the disorder potential and η is the viscosity of the electron fluid in the absence of disorder. The conditions for the validity of this equation are generally violated at $T\to 0$, where $\ell_{e\text{-}e}\to\infty$, but for sufficiently strong interactions, it can be satisfied down to moderately low T.

Here we make a few remarks concerning the viscosity of the uniform fluid at $T>T_F$. As mentioned, for large r_s there is a broad range of temperatures in which $T_M \sim T_F<T<V$, where the electron fluid, although nondegenerate, is still highly correlated. (Here T_M is the melting temperature of the Wigner crystal.) It is generally observed that the viscosity of highly correlated fluids is a decreasing function of increasing temperature. This observation, combined with Eq. (12) and the low-temperature increase in resistivity produced by the Pomeranchuk effect, implies that for metallic samples with large r_s, there should generally be a maximum in the resistivity at $T\sim T_F$.

Looking at this problem more carefully, there are, as noted in Sec. III.D, two distinct ranges of T to be considered. Beyond this, our theoretical understanding of the viscosity of correlated fluids is crude at best. We thus rely on the following line of arguments to get a feeling for the expected T dependence of η. (i) For $V>T>\Omega_P$, T_F the electron fluid is a highly correlated classical fluid. There are many examples of such fluids—indeed, most classical fluids fall in this regime (Frenkel, 1946). The viscosity of classical fluids is widely observed (Frenkel, 1946; Ferrer and Kivelson, 1999) to be an exponentially increasing function of decreasing temperature. (ii) For $T_F \ll T \ll \Omega_P=\sqrt{E_F V}$ the fluid is still quantum but not degenerate and is still strongly correlated. It has been conjectured on theoretical grounds (Andreev, 1979) [see also Spivak and Kivelson (2006)] that in this regime

$$\eta(T) \propto 1/T. \tag{13}$$

IV. THEORETICAL INTERPRETATIONS OF EXPERIMENT

We now discuss some of the attempts that have been made to interpret the corpus of experimental observation summarized in Sec. II in terms of the various theoretical results outlined in Sec. III. As is clear from Sec. III, there is presently no well-controlled theory that treats non perturbatively both the strong correlation and disorder effects, so all such attempts involve an extrapolation of results from small r_s to large r_s from zero disorder to finite disorder or both. Thus, while we present arguments both in support of and against various proposed interpretations, none of our conclusions are irrefutable.

A. Explanations based on classical "Drude" formulas

A systematic attempt to explain a wide range of the experiments presented in Sec. II has been undertaken by Stern and Das Sarma (1985), Gold and Dolgopolov (1986), Das Sarma and Hwang (1999, 2000, 2003, 2004), and Dolgopolov and Gold (2000) employing the classical formulas for the resistivity which are valid for weak scattering and $r_s \ll 1$ and extrapolating them to the case $r_s \gg 1$. Manifestly, this approach involves extrapolating results from the weak-interaction regime into a regime in which they are strong. Since the electron-impurity scattering is treated perturbatively (Born approximation) and the electron screening is computed at the level of random phase approximation (RPA), there is no formal justification for the approach when $r_s>1$. However, the appeal of this approach is that it leads to explicit expressions for a wide range of physical quantities which can be directly compared with experiment. A number of striking quantitatively successful comparisons between this theory and experiment have been reported by Das Sarma and Hwang (1999, 2000, 2003, 2004).

There are, however, several aspects of this extrapolation that we find troubling. The extrapolation to large r_s does not simply involve quantitative shifts but qualitative changes. Whereas at small r_s, as discussed in Sec. III.A, both $d\rho/dT<0$ and $d\rho/dB_\parallel<0$ the classical formulas extrapolated to large r_s exhibit the opposite sign trends, $d\rho/dT>0$ and $d\rho/dB_\parallel>0$. The change of sign at $r_s \sim 1$ does admittedly bring the results into qualitative agreement with experiment on large r_s systems, but it cannot be said to be a "featureless" extrapolation. Moreover, the origin of the sign change can be traced to the fact, already discussed above, that at large r_s, the screening length λ_{sc} obtained in RPA is parametrically smaller than the spacing between electrons, $\lambda_{sc}\sqrt{\pi n} = (1/4)r_s^{-1} \ll 1$. Screening lengths less than the distance between electrons are clearly unphysical, and we worry that the same is true for other extrapolated results.

This same approach has been extended to explain the decrease of $\rho(T)$ as a function of increasing T at $T > T_{\max} \sim T_F$, where the electron gas is nondegenerate. Here the resistivity is computed using the kinetic theory of a weakly interacting electron-gas scattering from Coulomb impurities (Das Sarma and Hwang, 1999). In the framework of this approach the decrease of the resistance is associated with the increase of the velocity and decrease of the scattering rate as T increases. As a result $\rho \propto 1/T$. Again, semiquantitative success has been reported in the comparison of this theory with experiment. Since this theory neglects all correlations in the electron gas or between scattering events, it is well justified when $G \gg 1$ and $r_s \ll 1$ for all $T > T_F$ and even for $r_s \gg 1$ at temperatures $T \gg V$. However, the experiments in question deal with the temperature range $T_F < T < V \sim r_s T_F$ in samples with $r_s \gg 1$; in this regime, the electron liquid is still strongly correlated, although not quantum mechanically coherent.

Additionally, Drude theory does not incorporate an insulating state much less a metal-insulator transition. It seems to us that the anomalous behavior of the metallic state at large r_s is related to the appearance of a metal-insulator transition in the same devices.

B. Interaction corrections to the conductivity in the ballistic regime $L_T \ll l$

For $r_s \ll 1$ and at high enough temperatures that $L_T \ll \ell$ but low enough that $T \ll T_F$, the contribution (Zala *et al.*, 2001a, 2001b) to the resistivity of electrons scattering from the Friedel oscillations induced by impurities is given by Eq. (6). At small r_s, where $|F_0| \ll 1$, this expression gives the opposite sign of $d\rho/dT$ than is seen in experiments on samples with $r_s \gg 1$ and $G \gg 1$. However, when Eq. (6) is extrapolated to large enough r_s, where plausibly $F_0 < -1/4$, $d\rho/dT$ changes sign, and $\rho(T)$ becomes a linearly increasing function of T. [Where there is valley degeneracy, as in the case of Si-MOSFETs where there are two valleys, the same sign reversal occurs when $F_0 < -1/15$ (Zala *et al.*, 2001a, 2001b).] The same theory (Zala *et al.*, 2001a, 2001b) predicts a decrease in the magnitude of the triplet part of Eq. (6) by a factor of 3 when the electron gas is spin polarized by the application of $B_\parallel > B^*$. The reason is that the singlet and $L_z = 0$ triplet parts of the two-particle propagator are unaffected by $B_\parallel$, while the remaining two components of the triplet are suppressed by $B_\parallel$. While this approach also involves an extrapolation, involving a sign change at $r_s \sim 1$, there is certainly nothing unphysical about a Fermi-liquid parameter with a substantial magnitude: $F < -1/4$. However, there are other aspects of this explanation of the experiments presented in Sec. II that we find problematic.

First, this is a theory of *corrections* to the conductivity; even when the result is extrapolated to $r_s \sim 1$ and $T \sim T_F$, this approach only makes sense if the correction is small compared to the Drude conductivity itself G_0. Since the correction is a nonanalytic function of T, it can be the dominant contribution to $d\rho/dT$ for sufficiently small $T \ll E_F$. However, it is something of a stretch to interpret the large fractional changes in the conductivity seen in experiments on samples with $r_s \gg 1$ and $G \gg 1$ in these terms. Over a range of temperatures and fields, $T \sim E_F$ and/or $B_\parallel \sim B^*$, the analytic variations of the Drude conductivity (which were the focus of the theory discussed in Sec. IV.A) should generally be at least as large as these corrections. In more physical terms, it is hard to imagine that the scattering of the electrons from the induced Friedel oscillations of the electron density can make a larger contribution to the scattering cross section than the scattering from the impurity itself. Therefore, we think that this process (even when extrapolated to large r_s and taking the most optimistic viewpoint) cannot explain changes in the resistivity by more than a factor of 2. However, in experiment, ρ is observed to increase (admittedly more or less linearly with T) by a factor of 4 in large r_s p-GaAs heterostructures and by a factor of 10 in Si-MOSFETs.

Second, although this theory does predict a suppression of the T dependence of the resistivity by a parallel magnetic field, $d\rho(T)/dT|_{B_\parallel = B^*} / d\rho(T)/dT|_{B_\parallel = 0} < 1$, the measured ratio at low T is significantly smaller than that predicted by the theory (Zala *et al.*, 2001a, 2001b).

In short, we conclude that although the results of the perturbation theory (Stern and Das Sarma, 1985; Gold and Dolgopolov, 1986; Dolgopolov and Gold, 2000; Zala *et al.*, 2001a, 2001b) are likely relevant to experiments performed on samples with relatively small r_s and at relatively small values of $B_\parallel$ and T, they cannot explain results of experiments at $r_s \gg 1$ where almost all effects are of order one or larger.

C. Scaling theories of the metal-insulator transition in two dimensions

The scaling theories discussed in Sec. III.F.3 imply the existence of a metal-insulator transition at a sample-dependent critical density $n = n_c$. At criticality, the resistance approaches a finite limit ρ_c in the $T \to 0$ limit which is probably universal. Strong $n - n_c$ and T dependences of the resistivity near criticality are governed by universal critical exponents and scaling functions. The appeal of such an approach is the robustness implied by universality. However, our principle interest is with the behavior of samples over a broad range of r_s. Thus, other than to stress the importance of the existence of the transition itself, we have chosen not to focus particularly on experiments close to criticality. On the basis of the data in

Fig. 1, we identify an empirical value for the critical resistance $\rho_c \approx 1(h/e^2)$. Thus, we can identify samples with low-temperature resistances $\rho > \rho_c$ as "insulating" and with $\rho < \rho_c$ as metallic.

Any theory which is based on electron interference effects in practice predicts substantial fractional changes of $\rho(T, B_\parallel)$ over plausibly accessible ranges of T only in the near vicinity of the critical point. In particular, where $G \gg 1$, only fractionally small logarithmic variations are predicted. In contrast, in all systems described in this paper the transport anomalies take place in a wide range of the electron densities: up to five times the critical density [see, e.g., Altshuler *et al.* (2000)]. Therefore, most of the experimental data do not lie in the critical region. The experiments deep in the metallic region $\rho(T, B_\parallel)$ exhibit large changes in the resistance (sometimes by an order of magnitude) even in samples where the conductance is as large as $G \sim (10\text{–}20)e^2/h$ (see Figs. 1–4). In fact, the temperature dependence of the resistivity significantly increases in samples which are farther away for the critical region.

We therefore conclude that although this theory quantitatively describes experimental data on Si-MOSFETs in the close vicinity of the transition (Anissimova *et al.*, 2007), it cannot explain large effects far from the transition.

D. Interpretations based on electronic microemulsions and the Pomeranchuk effect

In an electron fluid with $r_s \gg 1$, the interaction energy is large compared to the kinetic energy, and so the short-range correlations must certainly be Wigner crystalline-like whatever the long-range emergent properties. No theoretically well-controlled treatment of this problem, capable of quantitative comparison with experiment, currently exists that treats on an equal footing the short-range crystalline correlations and the long-distance fluid character of the state. The nascent theory of electronic microemulsions is, however, an attempted first step in this direction. Here we sketch the ways that many of the most significant qualitative aspects of the experiments can be understood from this perspective.

1. Interpretation of the T and $B_\parallel$ dependences of the resistance for $T < T_F$

The T and $B_\parallel$ dependences of the resistance at $T < T_F$ can be qualitatively understood as consequences of the Pomeranchuk effect. As the fraction f_{WC} of Wigner crystal grows with increasing T and $B_\parallel$, this naturally produces an increasing ρ, i.e., a metallic T dependence and a positive magnetoresistance. Moreover, to the extent that the exchange energy $J \ll k_B T$, f_{WC} is a strongly nonanalytic function which, for $B_\parallel = 0$ and $T \ll T_F$, is linear in T and, for $T \ll B_\parallel$, is an initially linearly increasing function of $B_\parallel$ which saturates at $B_\parallel > B^*$. Moreover, the T dependence of f_{WC} is quenched by $B_\parallel$ in the range $T < B_\parallel$.

As discussed in Sec. II, the experimentally measured ρ in samples with $G \gg 1$ and $r_s \gg 1$ exhibits the same qualitative dependences as f_{WC}, including the dramatic one to two orders of magnitude decrease in $d\rho/dT$ produced by a field $B_\parallel > B^*$. The transport theory of microemulsions (Spivak and Kivelson, 2006) which relates ρ to f_{WC} is complex and incomplete. In the limit that the disorder potential is smooth and $f_{\mathrm{WC}} \ll 1$, the dominant contribution to the resistance comes from electrons scattering from rare islands of WC, which are themselves pinned at minima of the disorder potential. In this case, it is easy to see that $\rho \propto f_{\mathrm{WC}}$.

For other forms of disorder and in all cases where f_{WC} is not small, the dependence of ρ on f_{WC} is more complicated, although ρ is still a monotonically increasing function of f_{WC}. Thus, the Pomeranchuk effect gives a plausible explanation of the giant positive magnetoresistance in the insulating regime, $\rho > h/e^2$, as shown in Fig. 12. It also gives rise to the possibility that samples with $\rho > h/e^2$ can still have a metallic temperature dependence $d\rho/dT > 0$. Such behavior is occasionally seen, as shown in Fig. 11, although so far only relatively small effects in samples with ρ no larger than $\sim 3h/e^2$. However, in principle, the Pomeranchuk effect can produce arbitrary large effects. In particular, on a qualitative level it can explain the existence of a metal-insulator transition as a function of $B_\parallel$.

2. Interpretation of the magnetoresistance in a perpendicular magnetic field

Moving to the effect of a perpendicular magnetic field, the experimentally observed behavior can be quite complex, as shown in Figs. 6, 13, and 14. As is confirmed in tilted field experiments (also shown in the Fig. 14), this complexity reflects a combination of spin and orbital effects. The strong negative magnetoresistance seen at large fields, where the spins are fully polarized, generally reflects the existence of quantum Hall phases, as discussed in Sec. III.G. For intermediate values of $B_\perp$, where spin physics is important, the Pomeranchuk effect can readily account for the existence of a large positive magnetoresistance, as discussed above. [The small negative magnetoresistance, sometimes seen in metallic samples at small values of $B_\perp$ (see Fig. 6), is presumably due to weak localization effects.] This general analysis applies to samples in which the zero-field resistivity is either less than h/e^2 (Fig. 6) or greater than h/e^2 (Figs. 13 and 14). In the more resistive samples, a part of the negative magnetoresistance is presumably due to interference corrections to variable range hopping (Shklovskii and Spivak, 1991; Zhao *et al.*, 1991).

3. Interpretation of the T dependence of the resistance at $T > T_F$

The existence of a broad range of temperatures with $T_F r_s \sim V > T > T_F$ in which the electron fluid is still strongly correlated is one of the most clearly inescapables and at the same time widely neglected feature of the discussion of the 2DEG with large r_s. The best understanding we have of the transport theory is obtained by arguing in analogy with other strongly correlated fluids. As discussed in Sec. III.H.3, because there is no coherence, the Fermi statistics of the electrons is relatively unimportant, but so long as $T_F\sqrt{r_s}>T>T_F$, quantum effects could be crucial. Moreover, because the interactions are strong and the samples are "clean," it is plausible that $\rho\propto\eta$, where η is the viscosity of the electron fluid in the absence of disorder.

The best analogy we have to date is with the viscosity of liquids ^{3}He and ^{4}He in the same temperature range. The experimental fact that $\rho\sim 1/T$ in this temperature interval [see Fig. 2 and Mills *et al.* (1999)] is not inconsistent with the conjecture [Eq. (13)], that the viscosity of liquid He in the same "semiquantum" regime obeys $\eta\sim 1/T$. [Measured values (Andreev, 1979) of the viscosity in bulk ^{4}He are consistent with $\eta\sim 1/T$, but as far as we know no comparable data exist in bulk ^{3}He nor in ^{3}He or ^{4}He films.]

E. Interpretation of experiments based on percolation

Percolation is a classical concept whose relevance to quantum systems requires a separation of spatial scales (Efros and Shklovskii, 1984). Specifically, in order that different (randomly distributed) regions of the sample can be treated as sufficiently macroscopic to be characterized by a local value of the conductivity, the correlation length of the scattering potential must be large compared to the relevant microscopic lengths needed to define a local electronic phase. Percolation is a useful concept only if the disorder potential varies on distances large compared to the distance between electrons. In particular, since the quantum dephasing length L_ϕ diverges as $T\rightarrow 0$, percolation is never justified at low T because it neglects the localization, interference effects, etc., in the metallic state. As such percolation provides a valid description at intermediate temperatures. However, if the disorder has sufficiently long-range correlations, percolation effects can readily mimic the finite-T appearance of a metal-insulator transition.

Such a theory can be developed in a form which is applicable to samples with small r_s in the presence of a smooth scattering potential. (This situation can be realized, for example, in 2D samples where charged impurities are separated from the 2D electron liquid by a wide spacer.) In this case, to describe the transition one has to take into account nonlinear screening (Efros *et al.*, 1993; Fogler, 2004).

Das Sarma *et al.* (2005) and Manfra *et al.* (2007) applied the same sort of theory, extrapolated to large r_s, to explain experimental data for low-density electron liquids in the metallic state near the point of the metal-insulator transition. Again, despite the reported qualitative successes of this theory, we believe that there are significant problems with its straightforward application in the current context. For example, as $B_\parallel$ increases the Fermi energy of a weakly interacting electron liquid increases ultimately by a factor of 2. Generally, this would be expected to increase the fraction of the sample that is in the metallic phase. Therefore such a theory would result in a resistance which is a decreasing function of increasing $B_\parallel$, with even the possibility that it could induce an insulator to metal transition. Conversely, the experimental data presented in Fig. 4 show a field-driven metal to insulator transition.

Indeed, the assumption of very long-range correlations of the disorder is problematic in itself. Certainly, in Si-MOSFETs, it is generally accepted that the disorder has rather short-range correlations. More generally, the distance to the remote dopants is rarely significantly longer than the typical distance between electrons. Thus, the large separation of length scales assumed in a simple percolation analysis cannot be ubiquitous among the devices studied.

On the other hand, the metal-Wigner crystal transition (and associated intermediate microemulsion phases) in the presence of weak disorder yields a state which can naturally be described by percolation through the metallic fraction of the system. Even for short-range correlated disorder, the familiar disorder broadening of a first-order transition can lead to the "Imry-Ma" domains of large size (Imry and Wortis, 1979; Aizenman and Wehr, 1989; Baker, 1993). Although a percolation picture based on disorder induced phase coexistence has many features in common with the single-particle percolation picture (Efros *et al.*, 1993; Fogler, 2004; Das Sarma *et al.*, 2005; Manfra *et al.*, 2007) it has the important differences that (i) the large length scales that justify the percolation analysis are self-organized rather than inherited directly from the disorder potential and (ii) the resulting pattern of insulating and conducting phases is a strong function of any parameter, especially T and $B_\parallel$, which affects the thermodynamic balance between the two phases.

As discussed, this general type of theory is in qualitative agreement with much of the experimental data. For example, due to the Pomeranchuk effect, the fraction of the Wigner crystal increases with increasing $B_\parallel$, which in turn can lead to a field-induced transition from a metal to an insulator. Another advantage by considering of this regime is that it offers a simple explanation of why highly conducting samples can still have such strongly T- and $H_\parallel$-dependent conductivities. Unfortunately, however, no quantitative theory of the transport in this regime currently exists.

F. Experiments on double layers

We now turn to the anomalies in the measured drag resistance ρ_D (Pillarisetty *et al.*, 2003) in p-GaAs double layers with small resistances and large r_s.

A straightforward application of the Fermi-liquid-based theory valid at $r_s\ll 1$ yields results in qualitative contradiction with experimental results of Pillarisetty *et al.* (2003). For example, since the electron-scattering rate in the framework of the Fermi-liquid theory is inversely proportional to the Fermi energy, it is natural to expect that the drag resistance decreases as $B_\parallel$ increases. This is in contradiction with the experiment.

It was shown by Levchenko and Kamenev (2008) that interference corrections to ρ_D are increasingly important at low T and indeed do not vanish as $T \to 0$. This is in loose qualitative agreement with the fact that the experimental values of the drag resistance are much larger than the conventional theoretical values (Price, 1983; Zheng and MacDonald, 1993), which vanish in proportion to T^2 as $T \to 0$. Second, according to Levchenko and Kamenev (2008), $\rho_D \sim \rho^3$. This is also in qualitative agreement with experiment where both $\rho_D(B_\parallel)$ and $\rho^3(B_\parallel)$ increase with $B_\parallel$ and saturate at $B_\parallel > B^*$. On the other hand, as discussed, since this theory was derived in the limit $r_s \ll 1$ and $k_F l \gg 1$, it cannot answer the more basic question of why $\rho(T, B_\parallel)$ itself increases as a function of T and $B_\parallel$ nor why there is a maximum of $\rho(T)$.

A focused study based on an extrapolation of Fermi-liquid-based formulas to the region where $r_s \geqslant 1$ has been carried out by Das Sarma and Hwang (2005) in order to address the drag resistance data of Pillarisetty *et al.* (2003). According to Das Sarma and Hwang (2005), the large enhancement of the magnitude of the drag resistance arises from a combination of a number of separate (somewhat technical) features of the specific experimental system in question. Moreover, they found that both the resistance of individual layers and the drag magnetoresistance turn out to be increasing functions of $B_\parallel$. As mentioned in Sec. IV.A, our concern here is with the underlying assumptions in this approach: Despite its many successes, the formulas for the electron cross section used by Das Sarma and Hwang (2005) were derived under an assumption that the standard expressions for electron screening continue to apply when the screening length λ_{sc} is much smaller than the interelectron distance. Specifically, in this approach, the change of the sign of the drag magnetoresistance which occurs on extrapolating the small r_s results to large r_s that can be traced to the fact that $\lambda_{sc} \ll n^{-1/2}$, which is clearly unphysical.

An attempt to explain the experimental data (Pillarisetty *et al.*, 2003) based on the assumed existence of microemulsion phases was carried out by Spivak and Kivelson (2005). The main assumption of Spivak and Kivelson (2005) is that a phase consisting of *mobile* bubbles of Wigner crystal embedded in the Fermi liquid is responsible for the large drag resistance. Since the bubbles have different electron densities than the surrounding Fermi liquid, they produce a large electric potential which is seen by the electron liquid in the second layer, thus producing a large ρ_D. However, the existence of mobile bubbles at the relevant temperatures is an assumption that has not yet been tested since no reliable theoretical estimates have yet been made of the characteristic size of the bubbles nor the range of densities over which the bubbles are stable, and no direct experimental imaging of an electron microemulsion has yet been achieved. Once this assumption is accepted, all qualitative features of experiment (Pillarisetty *et al.*, 2003) can be explained as consequences of the Pomeranchuk effect.

G. n dependence of B^*

In discussing the n dependence of B^*, it is again useful to compare it to the analogous problem in ^{3}He. The problem of the density dependence of the saturation magnetic field and enhancement of the spin susceptibility in the strongly correlated Fermi-liquid ^{3}He near the crystallization transition has been discussed by Castaing and Nozieres (1979). There it was pointed out that two different scenarios can be imagined with different consequences for the evolution of the magnetic response:

(i) The system is nearly ferromagnetic, which means that the linear spin susceptibility is enhanced compared to its noninteracting value, but the saturation field B^* is not suppressed. This would mean that the system is close to a Stoner instability.

(ii) The system is nearly solid. In this case, both the linear and all nonlinear susceptibilities are enhanced, and B^* is suppressed compared to the noninteracting values.

We believe that the data in the large r_s 2DEG are more generally consistent with the second scenario.

V. CONCLUSION

We have summarized a large body of experimental data and theoretical arguments which strongly imply the existence of qualitatively new ("non-Fermi-liquid") physics of the 2DEG in the limit of large r_s and weak disorder. These phenomena include but are not limited to those associated with a metal-insulator transition. We have also critically discussed some of the attempts to make contact between theory and experiment concerning these behaviors.

The primary purposes of this paper are to bring into focus the problems of physics which are unresolved and to stimulate further study, especially experimental study of these problems. Because of the absence of a well-controlled theory of the metal-insulator transition in a strongly correlated 2DEG, there has been much controversy surrounding the interpretation of these experiments. However, it is important to note that there is no controversy concerning the experimental facts, especially given that similar phenomena are seen by many in a variety of device types.

We conclude our discussion with a list of some of the properties of the 2DEG which could readily be measured but which have not been seriously studied to date. More generally, the widely celebrated progress that is ongoing in the fabrication and broader distribution of increasingly high mobility Si- and (especially) GaAs-based devices will open the possibility of probing the intrinsic properties of clean 2DEGs at still lower densities and temperatures, where presumably all previously observed behaviors will have still larger amplitudes. Moreover, as high mobility devices made with other semiconductors become available, further tests of the universality of the phenomena, as well as interesting particulars associated with differences in band structure and the like, could add to our knowledge of this subject.

(i) There is surprisingly few data concerning the properties of strongly correlated electron liquids in the semiquantum regime $E_F < T < \sqrt{r_s} E_F \sim \sqrt{E_F V}$, where it is nondegenerate but still highly quantum mechanical, and in the classical regime $\sqrt{r_s} E_F < T < r_s E_F \sim V$, where it is still highly correlated but quantum effects are negligible. For example, experimental data on the magnetoresistance in a parallel magnetic field at $T > E_F$ have not been reported for any of the materials in question. (It is also worth noting that systematic studies of the viscosity of ^{3}He, especially ^{3}He films, in the semiquantum regime do not exist and would be useful for comparison.)

(ii) As the resistance is the most directly measured property of the 2DEG, most empirical information about its character comes from transport measurements. However, thermodynamic properties such as compressibility can be measured on 2DEGs (Eisenstein *et al.*, 1992). There have been some recent measurements of compressibility showing a drastic difference between the metallic and insulating phases (Dultz and Jiang, 2000; Ilani *et al.*, 2000; Allison *et al.*, 2006). It would be desirable, however, to perform more studies in samples with even higher mobility and r_s than those used by Dultz and Jiang (2000), Ilani *et al.* (2000), and Allison *et al.* (2006) to elucidate the difference between high r_s samples with strong metallic transport and the 2DEGs with lower r_s (Eisenstein *et al.*, 1992).

(iii) Thermoelectric and Nernst effects can reveal the strongly correlated nature of an electron system. However, we are aware of only a few papers (Moldovan *et al.*, 2000; Possanzini *et al.*, 2004; Faniel *et al.*, 2007) reporting thermoelectric measurements in strongly correlated 2DEGs, while the Nernst effect has never been measured in devices with large r_s.

(iv) Since drag effects are uniquely sensitive to charge-density fluctuations in the 2DEG, further experiments in double-layer systems, especially in the presence of perpendicular and more importantly parallel magnetic fields, could be illuminating. For example, the drag resistance in the semiquantum regime $T > T_F$ has never been measured.

(v) While isolated experiments exist on the quantum Hall effect in devices with moderately large r_s, systematic studies of the evolution from the quantum Hall states to the Wigner crystal state as a function of increasing r_s do not exist. Moreover, as discussed, there is much to be learned about the phases and phase transitions at $B_\perp = 0$ by following the evolution of the various quantum Hall phases to small $B_\perp$ in clean devices with large r_s.

(vi) It would be desirable to extend the few existing experiments (Lilly *et al.*, 2003; Zhu *et al.*, 2003) on low-density large r_s electron gases in ultraclean n-GaAs devices and in particular to measure effects of the magnetic field.

(vii) One promising avenue for obtaining more local information about the nature of the 2DEG at large r_s, more or less free of the complications due to quenched disorder, is to study their properties in mesoscopic devices such as “point contacts.” Up until now, however, they have primarily been studied only in three regimes: the regime where electron interactions are not important (van Wees *et al.*, 1988), the Coulomb blockade regime, and the Kondo regime (Goldhaber-Gordon *et al.*, 1998; Kouwenhoven and Glazman, 2001) in which there is a single localized electronic state through which tunneling between the two metallic reservoirs occurs. However, there is another interesting regime when there is a relatively large “depletion region” between two metallic reservoirs with high electron densities. In the depletion region, there is a strongly correlated electron liquid with low electron density whose density can be varied by changing a gate voltage. In this case as a function of gate voltage, aspects of the electronic microemulsion phases can be directly probed on a mesoscopic scale. In this context we mention that, as discussed by Spivak and Kivelson (2006), the Pomeranchuk effect can produce significant T and $B_\parallel$ dependences to the resistivity through the depletion region. It is interesting that it can produce dependences of $\rho(T, B_\parallel)$ which can mimic some of the behavior traditionally associated with the Kondo effect.

(viii) There are various new experimental methods (Tessmer *et al.*, 1998; Ilani *et al.*, 2001; Sciambi *et al.*, 2008) being developed which hold the promise of providing spatially resolved images of the evolving physics of the 2DEG at large r_s. Needless to say, such data could revolutionize our understanding of the strong correlation effects in the 2DEG.

(ix) Finally, although it may be a while until sufficiently well-ordered materials are available for these purposes, it is likely that advances in the study of double-layer graphene and related materials may open unprecedented opportunities to study the properties of the 2DEG at large r_s. Due its Dirac spectrum, r_s does not depend on n in a single layer of graphene, and more generally correlation effects are quite different than in problems with a quadratic dispersion. However, bilayers of graphene have a quadratic dispersion, and as a result $r_s \to \infty$ as $n \to 0$.

ACKNOWLEDGMENTS

We thank E. Abrahams, G. S. Boebinger, S. Chakravarty, S. Das Sarma, A. M. Finkel'stein, J. Folk, H.-W. Jiang, D. E. Khmelnitskii, P. A. Lee, L. Levitov, S. Maslov, V. M. Pudalov, M. P. Sarachik, D. Shahar, A. A. Shashkin, B. I. Shklovskii, D. C. Tsui, and S. A. Vitkalov for useful discussions. B.S. was supported by NSF Grant No. DMR-0704151. S.V.K. was supported by DOE Grant No. DE-FG02-84ER45153 and by BSF Grant No. 2006375. S.A.K. was supported by DOE Grant No. DE-AC02-76SF00515. X.P.A.G. was supported by CWRU startup fund and NSF Grant No. DMR-0906415.

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