Colloquium: Field theoretic description of ultrarelativistic electron-positron plasmas

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Ultrarelativistic electron-positron plasmas can be produced in high-intensity laser fields and play a role in various astrophysical situations. Their properties can be calculated using QED at finite temperature. Here perturbative QED at finite temperature is used for calculating various important properties, such as the equation of state, dispersion relations of collective plasma modes of photons and electrons, Debye screening, damping rates, mean free paths, collision times, transport coefficients, and particle production rates, of ultrarelativistic electron-positron plasmas. In particular, the focus is on electron-positron plasmas produced with ultrastrong lasers.

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CONTENTS

I. INTRODUCTION

Ultrarelativistic plasmas, in which the thermal energy of the particles is much larger than their rest mass energy, were discussed first in the 1950s in the context of astrophysics. They occur in the early Universe as well as stellar and galactic high-energy processes (Zel'dovich and Novikov, 1971, 1983; Raffelt, 1996). For example, ultrarelativistic electron-positron plasmas (EPPs) can be created either by high temperatures in supernovae explosions [see, e.g., Hardy and Thoma (2001)] or by strong magnetic fields in so-called magnetars (Beskin *et al.*, 1993). The theoretical description of ultrarelativistic plasmas is based on transport theory (Silin, 1960) or thermal field theory (Tsytovich, 1961). The latter is an extension of quantum field theory, such as QED or QCD, in vacuum to finite temperature (or chemical potential) describing high-energy particle interactions in matter. For this purpose, two different approaches were developed: the imaginary time formalism (Matsubara, 1955; Kapusta, 1989) and the real time formalism (Landsmann and van Weert, 1987).

In the 1980s a new type of ultrarelativistic plasma became of interest: the quark-gluon plasma (QGP) (Müller, 1985). It corresponds to a new state of matter of deconfined quarks and gluons at extremely high temperatures above $k_B T = 150$ MeV. The early Universe should have been in this state for the first few microseconds after the Big Bang. Furthermore, the QGP is expected to be created in relativistic heavy-ion collisions, in particular within the accelerator experiments at the Relativistic Heavy Ion Collider (RHIC) (Gyulassy and McLerran, 2005). There, a hot and dense fireball ("little bang") of the size of an atomic nucleus is produced, which could be in the QGP phase for less than 10^{-22} s. Therefore the QGP in those experiments cannot be observed directly, its discovery relies on the comparison of theoretically predicted signatures with experimental data (circumstantial evidence). QCD at finite temperature has been applied for understanding the properties of the QGP and for calculating signatures for its formation. In addition to a nonperturbative method based on a numerical solution of the QCD equations (lattice QCD), perturbation theory at finite temperature has been used widely (Thoma, 1995a; Le Bellac, 1996). Lattice QCD allows the calculation of only static quantities, and hence not of most of the proposed signatures following from particle production. Perturbative QCD, on the other hand, has been used for computing both static and dynamic quantities. However, its predictions are limited by the fact that the QGP is a strongly coupled plasma at temperatures achievable in experiments. (At extremely high temperatures, far above the transition temperature, perturbative QCD should be reliable due to the asymptotic freedom of QCD.) Nonetheless, a wide variety of properties and quantities of the QGP have been considered in this way. Use of QCD started with the calculation of self-energies and dispersion relations in the high-temperature limit (Klimov, 1982; Weldon, 1982a, 1982b). In the high-temperature limit the results differ from the QED results only by trivial factors containing the QCD degrees of freedom (color and flavor). In addition, it can be shown that the gluon self-energy or polarization tensor, which is directly related to the dielectric function [see, e.g., Elze and Heinz (1989)], can also be derived within the transport (Vlasov) approach (Silin, 1960), since the high-temperature limit

corresponds to the classical limit. Other quantities, however, such as damping or production rates and transport coefficients, require a quantum field theoretic treatment. In particular, a resummation of certain diagrams, the hard thermal loops (HTLs), is needed for a consistent description, i.e., to obtain gauge-invariant, infrared-finite (or improved) results complete to leading order in the coupling constant (Braaten and Pisarski, 1990). Details about quantum field theoretic methods and their application to the physics of the QGP can be found in the review article by Thoma (1995a).

Recently, a third possibility for studying ultrarelativistic plasmas has been suggested: in extremely strong laser fields the creation of an ultrarelativistic EPP with temperature around 10 MeV could soon be realized (Liang *et al.*, 1998). For example, two opposite, circulary polarized laser pulses with a duration of 330 fs and an intensity of 7×10^{21} W/cm^2 can be made to impinge on a thin gold foil. In this way the target electrons can be heated up to about 10 MeV producing an ultrarelativistic EPP by pair creation. The positron density could reach about 5×10^{22} cm^{-3} (Shen and Meyer-ter-Vehn, 2001). This will allow the study of an ultrarelativistic EPP in the laboratory for the first time. Therefore predictions of the physical properties of such a system are needed.

The interaction of relativistic electrons and positrons is described by QED. In the presence of a thermal plasma background (heat bath) QED at finite temperature has to be considered. As discussed above, perturbative methods based on the imaginary (Matsubara) or real time formalism have been developed and applied to the physics of a QGP. Here we transfer the methods and results for properties of a QGP to the case of an EPP, where a perturbative treatment is more reliable than in the case of a strongly coupled QGP. Some applications of thermal field theory to astrophysical plasmas have been discussed [see, e.g., Altherr and Kraemmer (1992), and Thoma (2002), and references therein].

An important difference from nonrelativistic ion-electron plasmas (Lifshitz and Pitaevskii, 1981) is the relevant scales. In the nonrelativistic case, they are given by the masses of the plasma particles and the temperature T. For example, the electron plasma frequency reads

$$\omega_{pl} = \sqrt{\frac{4\pi e^2 \rho_e}{m_e}}, \tag{1}$$

and the Debye screening length due to the electrons in the plasma is

$$\lambda_D = \sqrt{\frac{k_B T_e}{4\pi e^2 \rho_e}}, \tag{2}$$

where ρ_e is the electron number density, T_e is the temperature of the electron component, and m_e is the electron mass.

In an ultrarelativistic plasma with $T \gg m$ the masses can be neglected, and the important scales are the temperature T, called the hard scale, and the soft scale eT, which determines the collective phenomena as we will

see below. Here we use natural units, i.e., $\hbar = c = k_B = 1$, as usual in quantum field theory. In these units $e = 0.3$, corresponding to a fine structure constant $\alpha = e^2/(4\pi) = 1/137$. For converting to Système International (SI) units we use 1 MeV $\triangleq 1.60 \times 10^{-13}$ J $\triangleq 5.08 \times 10^{12}$ m^{-1} $\triangleq 1.52 \times 10^{21}$ s^{-1}.

In the next section we discuss the equation of state of an equilibrated ultrarelativistic EPP. Then collective phenomena will be considered. Afterward, transport properties and particle production will be discussed. Finally, we describe properties of an EPP that is not in chemical equilibrium, as in the case of laser-induced plasmas. We will not consider here the formation process and equilibration of an EPP.

Many results presented here can be found in the literature for the case of a QGP (Thoma, 1995a) differing only by numerical factors, e.g., number of degrees of freedom. The purpose of this Colloquium is to summarize these results and to extend them to laser-induced EPPs as a reference for future experiments. As an example we consider a temperature of 10 MeV as it can be typically realized in laser-produced and supernovae EPPs.

Laser produced QED plasmas have also been discussed recently in two review articles (Marklund and Shukla, 2006; Mourou *et al.*, 2006) with emphasis on the production mechanism and nonlinear effects. Here, however, we want to focus on the properties of an equilibrated EPP as they can be calculated from perturbative QED. Such an EPP in thermal and chemical equilibrium might be the outcome of future laser experiments if the intensity can be increased further. As an example, we have chosen the predictions of Shen and Meyer-ter-Vehn (2001) based on a numerical simulation [particle in cell (PIC)], and cross sections for electron-positron production, in which two opposite laser beams are focused on a thin gold foil leading to a chemically nonequilibrated plasma (see Sec. VI). However, this proposal still awaits experimental confirmation. Other production mechanisms, based on the Schwinger pair production effect in strong fields (Schwinger, 1951), have shown that pair production can be efficient already far below the Schwinger critical field strength, requiring laser intensities of 5×10^{29} W/cm^2, in the case of time dependent and inhomogeneous fields, e.g., two oppositely directed pulsed laser beams in vacuum (see, e.g., Avetissian *et al.*, 2002; Di Piazza, 2004; Narozhny *et al.*, 2004; Dunne and Schubert, 2005; Gies and Klingmuller, 2005; Blaschke, Prozorkevich, Roberts, and Tarakanov, 2006; Blaschke, Prozorkevich, Smolyansky, *et al.*, 2006; Schützhold *et al.*, 2008). Furthermore, the pair production in an x-ray free electron laser (XFEL) has been discussed (Alkofer *et al.*, 2001; Ringwald, 2001; Roberts *et al.*, 2002). QED plasmas in strong magnetic fields have been considered by Danielsson and Grasso (1995) and more general in strong electromagnetic fields by Morozov *et al.* (2002a, 2002b). QED plasmas have also been studied by Melrose (2008).

II. EQUATION OF STATE

In this section we consider the equation of state of an EPP under the following assumptions: (1) ultrarelativistic EPP, i.e., $T \gg m$; (2) thermal and chemical equilibrium; (3) equal electron and positron density, i.e., vanishing chemical potential; (4) ideal gas, i.e., no interactions in the plasma; (5) infinitely extended, homogeneous, and isotropic EPP.

We relax some of these assumptions in the following sections. According to these assumptions the distribution function of the electrons and positrons is given by the Fermi-Dirac distribution

$$n_F(p) = \frac{1}{e^{p/T} + 1} \tag{3}$$

and of the photons by the Bose-Einstein distribution

$$n_F(p) = \frac{1}{e^{p/T} - 1}, \tag{4}$$

where the momentum p is identical to the energy E of the particles in the ultrarelativistic case. It should be noted that the photons are in equilibrium with electrons and positrons under the above assumptions, i.e., the system is actually an electron-positron-photon gas.

The particle and energy density can be calculated by integrating over the distribution functions. In the case of a QGP the results can be found in Müller (1985). The particle number density of the electrons and positrons follows from integration over the Fermi-Dirac distribution as

$$\rho_e^{eq} = g_F \int \frac{d^3p}{(2\pi)^3} n_F(p) = \frac{3}{\pi^2} \zeta(3) T^3 = 0.37 T^3, \tag{5}$$

where $g_F = 4$ is the number of degrees of freedom corresponding to the electrons and positrons in the two spin states. Assuming a temperature of $T = 10$ MeV, we find $\rho_e^{eq} = 370$ MeV$^3 = 4.9 \times 10^{40}$ m^{-3}.

The photon density follows accordingly by integrating over the Bose-Einstein distribution with $g_B = 2$ degrees of freedom corresponding to the two polarization states as $\rho_{ph}^{eq} = (2/\pi^2)\zeta(3) T^3 = 0.24 T^3$. The energy density of the electron-positron-photon gas is obtained from

$$\epsilon^{eq} = g_F \int \frac{d^3p}{(2\pi)^3} p n_F(p) + g_B \int \frac{d^3p}{(2\pi)^3} p n_B(p)$$
$$= \frac{11\pi^2}{60} T^4 = 1.81 T^4, \tag{6}$$

where the photons contribute 36% to the energy density. Here the Boltzmann law, $\epsilon^{eq} \sim T^4$, holds also for the fermions because we neglected their masses.

For $T = 10$ MeV we find $\epsilon^{eq} = 3.8 \times 10^{29}$ J m^{-3}. In a volume of 10^{12} m^3 (corresponding to the size of a neutron star) the total thermal energy of the EPP is 3.8×10^{41} J, which corresponds to about 10% of the entire energy

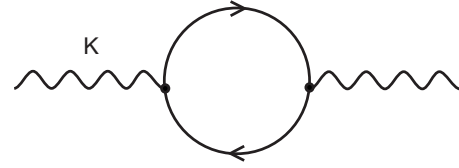FIG. 1. One-loop polarization tensor.

(without neutrinos) released in a supernova type II explosion. A volume of 1 μm^3 still contains an energy of 3.8×10^{11} J.

The Coulomb coupling parameter of the EPP, which is a measure for the nonideal behavior of a plasma (Ichimaru, 1982), is given by $\Lambda = e^2/dT$, where $d \simeq (\rho_e^{eq})^{-1/3} = 2.7 \times 10^{-14}$ m is the interparticle distance. For $T = 10$ MeV we find $\Lambda = 5.3 \times 10^{-3}$ which shows that the EPP is a weakly coupled plasma in contrast to the QGP where $\Lambda = O(1)$ (Thoma, 2005a, 2005b). Therefore the ideal gas results for the equation of state derived above are a good approximation. After all, interactions in the EPP play an important role, for example, in the collective behavior of the plasma, as discussed in the next section, and in equilibration of the plasma. Obviously, the interaction can be treated by perturbation theory.

III. COLLECTIVE PHENOMENA

Interactions between particles can be separated into two classes: individual collisions between the particles and long-range interactions of particles with the medium (Lifshitz and Pitaevskii, 1981). The latter lead to collective effects, which are characteristic of plasmas. The crucial quantity from which the collective phenomena are derived is the dielectric tensor relating the macroscopic electric field D_i in the medium to the external field E_i ($i = x, y, z$), i.e., in momentum space

$$D_i(\omega, \mathbf{k}) = \sum_j \epsilon_{ij}(\omega, \mathbf{k}) E_j(\omega, \mathbf{k}). \tag{7}$$

In the case of an isotropic medium it depends only on $k = |\mathbf{k}|$ and has two independent components,

$$\epsilon_{ij}(\omega, k) = \epsilon_T(\omega, k)\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) + \epsilon_L(\omega, k)\frac{k_i k_j}{k^2}. \tag{8}$$

The dielectric tensor is closely related to the polarization tensor or photon self-energy by [see, e.g., Elze and Heinz (1989)]

$$\epsilon_L(\omega, k) = 1 - \frac{\Pi_L(\omega, k)}{k^2},$$

$$\epsilon_T(\omega, k) = 1 - \frac{\Pi_T(\omega, k)}{\omega^2}, \tag{9}$$

where Π_L and Π_T are the longitudinal and transverse components of the polarization tensor, respectively.

The lowest-order diagram for the polarization tensor is shown in Fig. 1. Assuming the external momentum to be soft, i.e., ω and k to be much smaller than T, and the
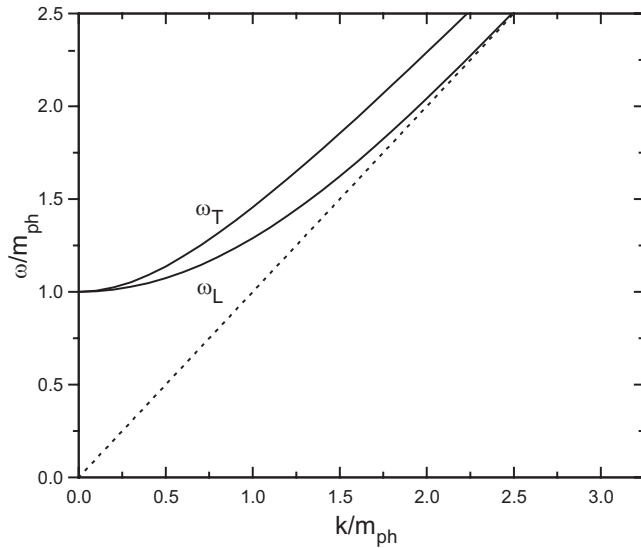
FIG. 2. Photon dispersion relation.

internal loop momenta to be hard (HTL approximation), an analytic result can be found using the real or imaginary time formalism (Klimov, 1982; Weldon, 1982a),

$$\Pi_L(\omega,k) = -3m_{ph}^2\left(1 - \frac{\omega}{2k}\ln\frac{\omega+k}{\omega-k}\right),$$

$$\Pi_T(\omega,k) = \frac{3}{2}m_{ph}^2\frac{\omega^2}{k^2}\left[1 - \left(1 - \frac{k^2}{\omega^2}\right)\frac{\omega}{2k}\ln\frac{\omega+k}{\omega-k}\right], \quad (10)$$

where $m_{ph}=eT/3$ is the effective photon mass. For $T=10$ MeV, we get $m_{ph}=1$ MeV.

The hard thermal loop approximation, which is identical to the high-temperature limit, consistently produces the lowest-order result in finite temperature QED (Braaten and Pisarski, 1990). The dielectric functions following from Eqs. (9) and (10) can also be derived from the classical Vlasov equation together with the Maxwell equations (Silin, 1960), since the high-temperature limit corresponds to the classical limit.

The dispersion relations of collective plasma modes, i.e., propagation of electromagnetic waves in the plasma, can be found using the Maxwell equation, leading to [see, e.g., Carrington *et al.* (2004)]

$$\epsilon_L(\omega,k) = 0,$$

$$\epsilon_T(\omega,k) = \frac{k^2}{\omega^2}. \quad (11)$$

Combining Eqs. (9)–(11) gives the dispersion relations $\omega_{L,T}(k)$ of the transverse and longitudinal plasma waves as shown in Fig. 2. The longitudinal branch, which does not exist in vacuum, is called a plasmon, as in the case of nonrelativistic plasmas. The transverse branch does not play a role in nonrelativistic plasmas but is equally important as the longitudinal one in relativistic plasmas. Both branches start at the plasma frequency $\omega_{pl}=\omega_{L,T}(k=0)=m_{ph}$ (Kajantie and Kapusta, 1985). Conse-

quently, the collective plasma waves have soft momenta of the order eT. At high momenta $k\gg m_{ph}$, the transverse mode approaches the free dispersion $\omega_T=k$, corresponding to a real photon in vacuum, whereas the longitudinal mode disappears, i.e., its spectral strength is exponentially suppressed (Pisarski, 1989). For $T=10$ MeV we find $\omega_{pl}=1.5\times10^{21}$ s^{-1}.

Electromagnetic plasma waves in an ultrarelativistic EPP have also been discussed by Medvedev (1999). For the plasma frequency the same dependence on the temperature and coupling constant was found [see Eq. (3) in Medvedev (1999)] if an equilibrium number density for the photons proportional to T^3 was used (see above) there. However, the numerical prefactor there is wrong and only one branch of plasma waves was discussed.

Another important quantity that can be derived from the polarization or dielectric tensor is the Debye screening length, entering the Yukawa potential of a heavy, nonrelativistic test charge in the EPP. The Debye screening length is given by the static limit of the longitudinal component of the polarization tensor $1/\Pi_L(\omega=0)$ (Kajantie and Kapusta, 1985), leading to $\lambda_D=1/\sqrt{3}m_{ph}$, which is 1.1×10^{-13} m for $T=10$ MeV.

Finally from Eq. (10) we see that the polarization tensor and the dielectric function become imaginary for $\omega^2<k^2$, i.e., below the light cone $\omega=k$, corresponding to Landau damping (Pisarski, 1988). We also observe that the plasma waves calculated at the lowest order of perturbation theory are undamped since they are located at $\omega>k$. This changes, however, at higher orders where the dispersion relation can intersect the light cone (Carrington *et al.*, 2004). It is interesting to note that these results are apart from some numerical color and flavor factors also valid for collective gluon modes in the QGP in the high-temperature limit (Thoma, 1995a).

A completely new phenomenon that does not appear in nonrelativistic plasmas is the existence of fermionic plasma waves, because all fermion masses are much too large in the nonrelativistic case. Their dispersion relations follow from the pole of the electron propagator containing the electron self-energy. Using again the hard thermal loop approximation, the electron self-energy reads [$P=(p_0,\mathbf{p})$, $p=|\mathbf{p}|$] (Klimov, 1982; Weldon, 1982b)

$$\Sigma(P) = -a(p_0,p)P^\mu\gamma_\mu - b(p_0,p)\gamma_0, \quad (12)$$

with

$$a(p_0,p) = \frac{1}{4p^2}[\text{tr}(P^\mu\gamma_\mu\Sigma) - p_0\text{tr}(\gamma_0\Sigma)],$$

$$b(p_0,p) = \frac{1}{4p^2}[P^2\text{tr}(\gamma_0\Sigma) - p_0\text{tr}(P^\mu\gamma_\mu\Sigma)], \quad (13)$$

where the traces over the γ matrices are given by
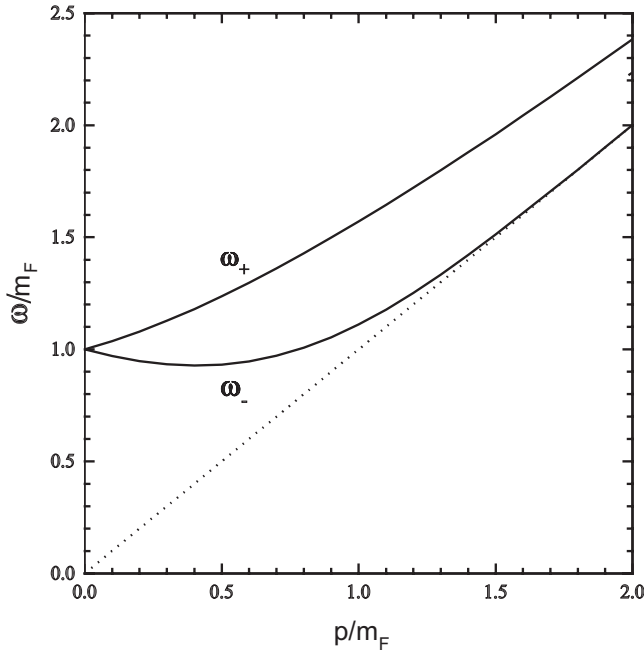
$$\text{tr}(P^\mu\gamma_\mu\Sigma) = 4m_F^2,$$

FIG. 3. Electron dispersion relation.

$$\mathrm{tr}(\gamma_0 \Sigma) = 2m_F^2 \frac{1}{p} \ln \frac{p_0 + p}{p_0 - p} \tag{14}$$

with the effective electron mass $m_F = eT/\sqrt{8}$ (Klimov, 1982; Weldon, 1982b), which is 1.1 MeV at $T = 10$ MeV.

The full electron propagator in the helicity representation is given by (Braaten *et al.*, 1990)

$$S^\star(P) = \frac{1}{2D_+(P)}(\gamma_0 - \hat{p} \cdot \gamma) + \frac{1}{2D_-(P)}(\gamma_0 + \hat{p} \cdot \gamma), \tag{15}$$

where

$$D_\pm(P) = -p_0 \pm p + \frac{1}{4p}[\pm \mathrm{tr}(P^\mu \gamma_\mu \Sigma) - (\pm p_0 - p)\mathrm{tr}(\gamma_0 \Sigma)]. \tag{16}$$

Again these results agree with those for quarks in the QGP apart from numerical pre-factors.

The dispersion relations following from the pole of this propagator are shown in Fig. 3. Two branches show up, one with a positive ratio of the helicity to the chirality (ω_+) following from $D_+ = 0$, and the other with a negative ratio (ω_-) following from $D_- = 0$; called a plasmino (Braaten *et al.*, 1990). The plasmino branch ω_-, which does not exist in vacuum, shows an interesting behavior, namely a minimum at $k = 0.41 m_F$. The same behavior of the quark dispersion in the QGP has been found. In this case sharp peaks in the dilepton production rate can appear due to the minimum in the plasmino branch, which leads to van Hove singularities (Braaten *et al.*, 1990; Peshier and Thoma, 2000). Whether something similar could be observed in the EPP, e.g., in the electron spectrum, is an interesting question which should be investigated in detail. It could open the exciting possibility of observing a new collective plasma wave, the plasmino, experimentally in a laser-induced EPP.
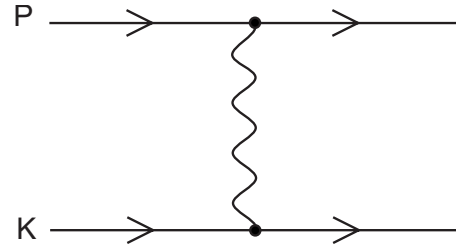

FIG. 4. Lowest order diagram for electron-electron scattering.

The collective quantities derived here perturbatively from the dielectric functions are linear phenomena. Nonlinear collective effects in electron-positron plasmas have been considered in, e.g., soliton formation (Lontano *et al.*, 2001), nonlinear Alfvén waves (Zhao *et al.*, 1994), nonlinear photon interactions (Tajima and Taniuti, 1990), plasma-enhanced photon splitting (Brodin *et al.*, 2007), and nonlinear self-modulation of radio pulses (Chian and Kennel, 1983).

IV. TRANSPORT PROPERTIES

Now we consider the interaction and properties of particles in the plasma with hard momenta, i.e., of the order of T or larger. In particular, we are interested in damping and transport rates, mean free paths, collision times, energy losses of these particles, and other transport properties such as the shear viscosity of the EPP. These quantities have been calculated perturbatively in high-temperature QCD (Thoma, 1995a). As in the case of the collective phenomena, the results can be taken almost directly from the QCD calculations.

It was shown by Braaten and Pisarski (1990) that a consistent treatment of gauge theories such as QED at finite temperature, i.e., for obtaining results that are gauge independent, infrared finite, and complete to leading order, require the use of an effective perturbation theory using resummed Green's functions based on the HTL approximation (the HTL resummation technique). The HTL method relies on the assumption that one can distinguish between soft and hard momenta, i.e., $eT \ll T$. This is approximately satisfied for QED, where $e = 0.3$, but not in QCD, where the corresponding coupling constant $g > 1$.

The damping rate of an electron or positron in the EPP is defined as the imaginary part of the dispersion relation $\omega_{L,T}(p)$. To lowest order it follows from the elastic scattering diagram of Fig. 4. In the case of a hard electron or positron with momentum of the order of T or higher, it exhibits a quadratic infrared (ir) divergence which can be reduced to a logarithmic one using a HTL resummed photon propagator. This logarithmic singularity is expected to be cut off by higher-order contributions, leading to (Thoma, 1995a)

$$\gamma_e = \frac{e^2 T}{4\pi} \ln \frac{1}{e} \tag{17}$$

within logarithmic accuracy, i.e., the constant under the logarithm is not determined. For $T = 10$ MeV we obtain

$\gamma_e = 86$ keV, which is much smaller than $\omega_{pl} = 1$ MeV, showing that the EPP is not overdamped.

Physically more important are the transport rates Γ, which are related to the mean free path and collision time of electrons and positrons in the EPP. They differ from the damping rate in cutting off the long-range interactions with small scattering angles θ by a factor $(1 - \cos \theta)$ under the integral defining the rate (Lifshitz and Pitaevskii, 1981). This leads to an improvement of the ir behavior (logarithmic instead of quadratic singularity in perturbation theory) and a finite result using the HTL method. Logarithmic divergent quantities can be treated consistently by splitting them into a soft part and a hard part, where the soft part is calculated using the HTL resummation technique (Braaten and Yuan, 1991). For the transport rate we find to logarithmic accuracy, by extending the QCD results of Thoma (1994) to QED,

$$\Gamma_e = \frac{e^4 T^3}{3\pi s} \ln \frac{1}{e}, \tag{18}$$

where the Mandelstam variable $s = (P + K)^2$ is the square of the sum of the four momenta of the incoming particles in the scattering diagram of Fig. 4. For deriving the above result we replaced $C_A g^2$ by e^2 and m_g by m_{ph} in Eq. (15) of Thoma (1994). To go beyond the logarithmic approximation, which is valid within about a factor of 2, one has to calculate explicitly the hard contribution from the QED scattering diagrams, as done by Thoma (1994) for the QCD case. For thermal particles we replace s by its thermal average $\langle s \rangle = 2\langle p \rangle_e \langle k \rangle_e \simeq 19.3 T^2$ (Thoma, 1994), where $\langle p \rangle_e = \langle k \rangle_e = \epsilon_e^{eq}/\rho_e^{eq} = 3.11 T$. Assuming again $T = 10$ MeV, we get $\Gamma_e = 0.54$ keV.

The mean free path λ_e^{mfp} and collision time τ_e of the plasma particles (electrons and positrons) are given by the inverse of the transport rate $1/\Gamma_e$, leading to $\lambda_e^{mfp} = 0.37$ nm and $\tau_e = 1.2 \times 10^{-18}$ s at $T = 10$ MeV.

In a nonrelativistic plasma the shear viscosity can be estimated from elementary kinetic theory as (Reif, 1965)

$$\eta_i = \frac{1}{3} \sum_i \rho_i \langle p_i \rangle \lambda_i^{mfp}, \tag{19}$$

where the sum is performed over the various components of the system. In a relativistic plasma the coefficient $1/3$ should be replaced by $4/15$ (de Groot *et al.*, 1980). Using the mean free path following from Eq. (18), the density of Eq. (5), and the thermal momentum $\langle p \rangle_e = 3.11 T$, the shear viscosity is given by (within logarithmic accuracy)

$$\eta_e = \frac{55.8 T^3}{e^4 \ln(1/e)}. \tag{20}$$

At $T = 10$ MeV the shear viscosity coefficient is $\eta_e = 7.9 \times 10^{10}$ Pa s.

Another quantity of interest in a plasma is its stopping power, or the energy loss of an energetic particle in the plasma. There are two contributions, namely the energy loss by collisions and the radiative loss by bremsstrahlung. In a relativistic plasma the latter becomes impor-
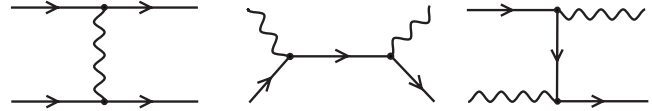

FIG. 5. Diagrams defining the collisional energy loss.

tant. The relevance of these contributions in the QGP has been the subject of controversy (Mustafa and Thoma, 2005). The collisional energy loss is given by the mean energy transfer divided by the mean free path leading to (Braaten and Thoma, 1991)

$$\frac{dE}{dx} = \frac{1}{v} \int d\gamma \omega, \tag{21}$$

where v is the particle velocity, γ, the damping or interaction rate proportional to the plasma density and the collision cross section, and ω, the energy transfer from the energetic particle to the plasma particle in the collision. Using for the collision cross section the lowest-order diagrams in Fig. 5, the collisional energy of a muon with mass M in an EPP has been calculated by Braaten and Thoma (1991), applying the HTL resummation technique,

$$\frac{dE}{dx} = \frac{e^4 T^2}{24\pi} \left(\frac{1}{v} - \frac{1 - v^2}{2v^2} \ln \frac{1 + v}{1 - v} \right) \left(\ln \frac{E}{M} + \ln \frac{1}{e} + A(v) \right), \tag{22}$$

where $A(v)$ is a slowly varying function of the muon velocity v between 1.3 and 1.5.

The collisional energy loss of an electron with energy $E \gg T$ is given by (Braaten and Thoma, 1991)

$$\frac{dE}{dx} = \frac{e^4 T^2}{48\pi} \ln \frac{15.3 E}{e^2 T}. \tag{23}$$

In Braaten and Thoma (1991) the factor 15.3 was replaced by 7.6 due to a numerical error, which was corrected by Thoma (1995a). This leads to an energy loss of 200 MeV/nm for an electron or positron with an energy of $E = 100$ MeV at $T = 10$ MeV, showing that such an electron is stopped (thermalized) within a fraction of a nanometer. (Note that the plasma density, proportional to T^3, on which the energy loss depends via the mean free path is hidden in the temperature dependence of the energy loss.) Recently it has been shown that this calculation can be improved by taking into account additional diagrams, changing this result slightly (Peigne and Peshier, 2008). So far no calculations of the radiative energy loss in an EPP have been performed to our knowledge.

The damping rate of a photon in an EPP follows from the diagram in Fig. 6, where a HTL resummed electron propagator has to be used in the case of soft momenta of the exchanged electron (positron). In contrast to the electron damping rate, the photon rate is found to be infrared finite using the HTL method, due to the presence of an electron propagator in Fig. 6 instead of the photon propagator in Fig. 4. Hence there is no need to
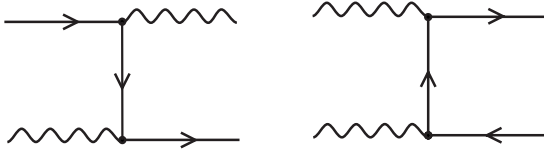
FIG. 6. Diagrams defining the photon damping rate.

cut off the long-range interaction and introduce a transport cross section. The result for a photon with energy $E=p$ reads (Thoma, 1995b)

$$\Gamma_{ph} = \frac{e^4 T^2}{64\pi E} \ln \frac{3.88E}{e^2 T}. \tag{24}$$

The mean free path and the collision time of photons in an EPP are given by $1/\Gamma_{ph}$. For a thermal photon with the mean momentum $\langle p \rangle_{ph} = \epsilon_{ph}^{eq}/\rho_{ph}^{eq} = 2.75T$ at $T=10$ MeV the mean free path $\lambda_{ph}^{mfp} = 0.28$ nm and the collision time $\tau_{ph} = 9.4 \times 10^{-19}$ s follow. Actually the damping rate given in Eq. (24) is a lower limit, as higher-order effects will enlarge it. As a matter of fact, the photon production rate in a QGP, which is the inverse process of the damping rate (Thoma, 1995b), was shown to be about a factor of 2 larger when bremsstrahlung is taken into account (Arnold *et al.*, 2001).

For the viscosity of the photon component, using the above mean free path, the photon density (see above), and the mean photon energy $\langle p \rangle_{ph} = 2.75T$, we find

$$\eta_{ph} = \frac{48.7T^3}{e^4 \ln(3.27/e)}, \tag{25}$$

corresponding to 3.5×10^{10} Pa s at $T=10$ MeV. Hence the viscosity of the EPP $\eta = \eta_e + \eta_{ph}$ has similar contributions from the electrons and photons.

A more advanced calculation of the total viscosity of the EPP based on the Kubo formula yields within logarithmic accuracy (Arnold *et al.*, 2000)

$$\eta = \frac{188T^3}{e^4 \ln(1/e)}. \tag{26}$$

This result is about a factor of 1.5 larger than the one presented here based on the elementary kinetic theory, which is typically valid within a factor of 2 (Reif, 1965).

V. PARTICLE PRODUCTION RATES

At high temperatures above 10 MeV other particle species will also be produced, e.g., muons with a mass of $m_\mu = 106$ MeV. Their rate follows to lowest order from the diagram in Fig. 7 (Born term). We assume that $m_e \ll T \ll m_\mu$ holds. The first inequality implies that the electron mass can be set to zero and the second inequality implies that muons are not equilibrated. Then the muon production rate to lowest order $(e^- e^+ \rightarrow \gamma^* \rightarrow \mu^- \mu^+)$ is given by
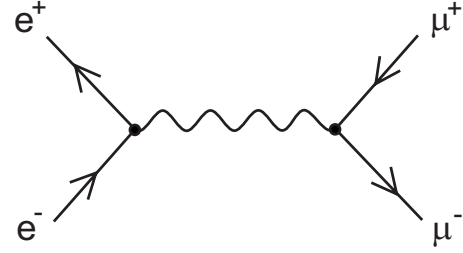

FIG. 7. Lowest order contribution of the muon production.

$$\frac{dN}{d^4x d^4p} = \frac{\alpha^2}{24\pi^4} \left(1 + \frac{2m_\mu^2}{M^2}\right) \left(1 - \frac{4m_\mu^2}{M^2}\right)^{1/2} \frac{T}{p}$$
$$\times \frac{1}{\exp(E/T)-1} \ln \frac{1+\exp[-(E+p)/(2T)]}{1+\exp[-(E-p)/(2T)]}, \tag{27}$$

where $M^2 = E^2 - p^2$ is the invariant mass of the virtual photon γ^*, E its energy, and $p = |\mathbf{p}|$ its momentum. This formula was derived by combining the cross section for the process in Berends *et al.* (1973) with the production rate in Cleymans *et al.* (1987), where the quark-antiquark annihilation process $q\bar{q} \rightarrow \mu^- \mu^+$ was considered. Here we assumed the chemical potential $\mu = 0$. The only difference from the process considered here come from the fractional charge of the quarks, i.e., we use here $\Sigma_i e_i^2 = 1$, and from the number of colors in the distribution functions in Eqs. (6a) and (6b) of Cleymans *et al.* (1987), i.e., the QGP rate is divided by a factor of 9. In addition, a factor $3/2$ appears in the electron-positron annihilation cross section—see Eq. (6) of Berends *et al.* (1973) after integration over the angle—compared to the quark-antiquark annihilation cross section in Eq. (5) of Cleymans *et al.* (1987), using the relative velocity of the ultrarelativistic particles $v_{q\bar{q}} = 1$. Because of $M^2 = E^2 - p^2 > 4m_\mu^2$ the rate is suppressed exponentially for temperatures below $2m_\mu$.

In order to obtain the spectrum from Eq. (27) one has to integrate over the space-time volume, taking into account the space-time evolution by using, for example, a hydrodynamical model. The total muon yield then follows from integration of the spectrum over the energy and momentum of the virtual photon.

At temperatures above 10 MeV hadron production also becomes important, in particular pion production (Kuznetsova *et al.*, 2008).

VI. NONEQUILIBRIUM AND FINITE CHEMICAL POTENTIAL

EPPs produced in strong laser fields are probably not in complete equilibrium. For example, it has been predicted by Shen and Meyer-ter-Vehn (2001) that a positron density of about 5×10^{28} m^{-3} at a temperature of 10 MeV can be reached. This density deviates from the equilibrium density (5) by 12 orders of magnitude. In the following we therefore assume that the EPP produced by lasers is in thermal but not in chemical equilibrium.

TABLE I. Properties of an EPP.

Quantity	Formula	Value at $T = 10$ MeV
Electron-positron density	$\rho_e^{eq} = 3/\pi^2 \zeta(3) T^3$	4.9×10^{40} m^{-3}
Photon density	$\rho_{ph}^{eq} = 2/\pi^2 \zeta(3) T^3$	3.2×10^{40} m^{-3}
Electron-positron energy density	$\epsilon_e^{eq} = 7\pi^2/60 T^4$	2.4×10^{29} J m^{-3}
Photon energy density	$\epsilon_{ph}^{eq} = \pi^2/15 T^4$	1.4×10^{29} J m^{-3}
Total energy density	$\epsilon^{eq} = 11\pi^2/60 T^4$	3.8×10^{29} J m^{-3}
Thermal electron momentum	$\langle p \rangle_e = \epsilon_e^{eq}/\rho_e^{eq} = 3.11 T$	31 MeV
Thermal photon momentum	$\langle p \rangle_{ph} = \epsilon_{ph}^{eq}/\rho_{ph}^{eq} = 2.75 T$	28 MeV
Interparticle distance	$d \simeq \rho_e^{eq-1/3}$	2.7×10^{-14} m
Coulomb coupling parameter	$\Lambda = e^2/(dT)$	5.3×10^{-3}
Effective photon mass	$m_{ph} = eT/3$	1 MeV
Plasma frequency	$\omega_{pl} = m_{ph}$	1.5×10^{21} s^{-1}
Debye screening length	$\lambda_D = 1/\sqrt{3} m_{ph}$	1.1×10^{-13} m
Effective electron mass	$m_F = eT/(2\sqrt{2})$	1.1 MeV
Electron damping rate	$\gamma_e = (e^2 T/4\pi) \ln(1/e)$	86 keV
Electron transport rate	$\Gamma_e = (e^4 T^3/3\pi s) \ln(1/e)$	0.54 keV for $s = 19.3 T^2$
Photon damping rate	$\Gamma_{ph} = (e^4 T^2/64\pi E) \ln(3.88 E/e^2 T)$	0.70 keV for $E = 2.75 T$
Electron mean free path	$\lambda_e^{mfp} = 1/\Gamma_e$	0.37 nm
Photon mean free path	$\lambda_{ph}^{mfp} = 1/\Gamma_{ph}$	0.28 nm
Electron collision time	$\tau_e = 1/\Gamma_e$	1.2×10^{-18} s
Photon collision time	$\tau_{ph} = 1/\Gamma_{ph}$	9.4×10^{-19} s
Electron viscosity	$\eta_e = 55.8 T^3/[e^4 \ln(1/e)]$	7.9×10^{10} Pa s
Photon viscosity	$\eta_{ph} = 48.7 T^3/[e^4 \ln(3.27/e)]$	3.5×10^{10} Pa s
Total viscosity	$\eta = \eta_e + \eta_{ph}$	$(1.1 - 1.6) \times 10^{11}$ Pa s
Electron energy loss	$dE/dx = (e^4 T^2/48\pi)\ln(15.3 E/e^2 T)$	200 MeV/nm for $E = 100$ MeV

Then we can replace the distribution functions for the electrons by Fermi-Dirac distributions multiplied by a fugacity factor λ describing the deviation from chemical equilibrium, $f_F(p) = \lambda n_F(p)$. This assumption has been used, for example, for describing the chemical equilibration of the QGP in ultrarelativistic heavy-ion collisions (Biró *et al.*, 1993). The fugacity is given by the ratio of the experimental to equilibrium particle density, since the experimental density follows from integrating over the nonequilibrium distribution, i.e.,

$$\rho_{expt} = g_F \int \frac{d^3 p}{(2\pi)^3} \lambda n_F(p) = \lambda \rho_{eq} \Rightarrow \lambda = 10^{-12}. \quad (28)$$

Using the real time formalism, QED perturbation theory and the HTL method can also be extended to nonequilibrium situations like the one discussed above (Carrington *et al.*, 1999). For example, the effective photon mass is given now by

$$m_{ph}^2 = \frac{4e^2}{3\pi^2} \int_0^\infty dp\, p f_F(p). \quad (29)$$

For $T = 10$ MeV we then find for the nonequilibrium photon mass $m_{ph}^{noneq} = \sqrt{\lambda} m_{ph} = 1$ eV and the plasma frequency $\omega_{pl}^{noneq} = 1.5 \times 10^{15}$ Hz. The Debye screening length in such an EPP is $\lambda_D = 0.1$ μm. In order to speak

of a plasma with collective behavior, its dimension L should be much larger than λ_D, i.e., at least of the order of 1 μm.

Furthermore, an anisotropic EPP can also be described by quantum field theoretic methods (Mrówczyński and Thoma, 2000). In this case instabilities can occur (Mrówczyński and Thoma, 2007).

Finally a possible difference between the positron and electron density can be treated by introducing a finite chemical potential μ, i.e., using the distribution

$$n_F(p) = \frac{1}{e^{(p \pm \mu)/T} + 1} \quad (30)$$

for the electrons (negative sign) and positrons (positive sign). This difference comes from the fact that the laser-produced EPP is embedded in the hot electron and cold ion background of the target. Therefore there will be an excess of electrons over positrons in the hot, relativistic EPP. The methods described above, such as the HTL resummation, can be generalized easily to this case (Vija and Thoma, 1995). For example, the energy density is given by

$$\epsilon^{eq} = \frac{11\pi^2}{60} T^4 + \frac{1}{2} T^2 \mu^2 + \frac{1}{4\pi^2} \mu^4 \quad (31)$$

or the effective photon energy by

$$m_{ph}^2 = \frac{e^2 T^2}{9}\left(1 + \frac{3\mu^2}{\pi^2 T^2}\right). \tag{32}$$

The muon production rate for finite chemical potential can be found by replacing the last three factors in Eq. (27) by Eq. (13) of Cleymans *et al.* (1987).

VII. CONCLUSIONS

The properties of ultrarelativistic EPPs can be treated by QED perturbation theory at finite temperature analogously to the treatment of the QGP by QCD, using the imaginary or real time formalism. A consistent treatment requires the use of the HTL resummation technique for many quantities under consideration. Many results can be more or less directly taken from the corresponding QGP calculation. The examples discussed here are the equation of state, the dielectric tensor, the dispersion relations of photons and electrons, the Debye screening length, damping and transport rates, mean free paths, collision times, shear viscosity, stopping power, and the muon production rate. In particular, collective phenomena, such as plasma waves, are of interest. In an EPP completely new phenomena appear, namely, collective fermion modes associated with the possibility of observation of van Hove singularities.

EPPs produced by strong lasers open the unique possibility to investigate the properties of these plasmas, which also exist in astrophysical systems, e.g., supernova explosions. However, current predictions indicate that laser-induced EPPs are not in chemical equilibrium. Their properties therefore require an extension of perturbative QED and the HTL resummation technique to nonequilibrium situations, which is possible within the real time formalism. As a first application, we discussed the Debye screening length in such an EPP, demonstrating that the EPP should have a spatial extension of at least 1 μm to exhibit characteristic plasma behavior.

Here we have considered a number of relevant properties of an ultrarelativistic, equilibrated EPP as a reference for future laboratory experiments and astrophysical EPPs, and we gave numbers for a plasma temperature of 10 MeV. The results are summarized in Table I.

New results, which have not been published so far, are the electron transport rate (18), the electron viscosity (22), and the photon viscosity (27). Also new is the application of the results, derived here or compiled from the literature, to relativistic laser plasmas as presented in the third row of Table I and in the discussion of nonequilibrium effects.

Further quantities, e.g., the radiative energy loss, or higher-order corrections beyond the leading logarithm approximation, to the quantities considered above could be evaluated in a way similar to that discussed above. An extension of the properties discussed in Secs. II–V to the case of a chemical nonequilibrated EPP would also be of interest. Finally, we did not consider here the formation of an EPP in a strong laser field, e.g., by the Schwinger mechanism, and its equilibration.

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