REVIEWS OF MODERN PHYSICS, VOLUME 80, JULY–SEPTEMBER 2008

Waiting for precise measurements of $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

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(Published 5 August 2008)

In view of future plans for accurate measurements of the theoretically clean branching ratios $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$, which should occur in the next decade, the relevant formulas for quantities of interest are collected and their theoretical and parametric uncertainties are analyzed. In addition to the angle β in the unitarity triangle (UT), the angle γ can also be determined from these decays with respectable precision and in this context the importance of the recent NNLO QCD calculation on the charm contribution to $K^+ \to \pi^+ \nu\bar{\nu}$ and of the improved estimate on the long-distance contribution by means of chiral perturbation theory are presented. In addition to known expressions, several new ones that should allow transparent tests of the standard model (SM) and of its extensions are presented. While the review is centered around the SM, models with minimal flavor violation and scenarios with new complex phases in decay amplitudes and meson mixing are also discussed. A review of existing results within specific extensions of the SM, in particular the littlest Higgs model with T-parity, Z' models, the MSSM, and a model with one universal extra dimension are given. A new "golden" relation between B and K systems is derived that involves (β, γ) and $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$, and the virtues of (R_t, β), (R_b, γ), (β, γ), and $(\bar{\eta}, \gamma)$ strategies for the UT in the context of $K \to \pi\nu\bar{\nu}$ decays with the goal of testing the SM and its extensions are investigated.

DOI: 10.1103/RevModPhys.80.965 PACS number(s): 13.20.Eb, 12.15.Hh, 12.15.Ji

CONTENTS

I. INTRODUCTION

The rare decays of K and B mesons play an important role in the search for the underlying flavor dynamics and in particular in the search for the origin of CP violation (Buchalla, Buras, and Lautenbacher, 1996; Buras, 1998, 2003, 2005a, 2005b; Nir, 2001; Fleischer, 2002, 2004; Ali, 2003; Buchalla, 2003; Hurth, 2003; Isidori *et al.*, 2005). Among the many K and B decays, the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are very special as their branching ratios can be computed to an exceptionally high degree of precision that is not matched by any other loop-induced decay of mesons. In particular, the theoretical uncertainties in the prominent decays like $B \to X_s\mu^+\mu^-$ and $B_s \to \mu^+\mu^-$ amount typically to $\pm 10\%$ or larger at the level of the branching ratio, although progress in the calculation of the branching ratio of $B \to X_s\gamma$ at the next-to-next leading order (NNLO) level shows that in this case an error below 10% is possible in principle (Becher and Neubert, 2007; Misiak *et al.*, 2007). On the other hand, the corresponding uncertainties in $K_L \to \pi^0\nu\bar{\nu}$ amount to 1–2 % (Buchalla and Buras, 1993a, 1993b, 1999; Misiak and Urban, 1999). In the case of $K^+ \to \pi^+\nu\bar{\nu}$, the presence of the internal charm contributions in the relevant Z^0 penguin and box diagrams contained the theoretical perturbative uncertainty of $\pm 7\%$ at the next leading order level (Buchalla and Buras, 1994a, 1999), but this uncertainty has been recently reduced down to $\pm 1-2$ % through a complete NNLO calculation (Buras, Gorbahn, Haisch, *et al.*, 2005, 2006).

The reason for the exceptional theoretical cleanness of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ (Littenberg, 1989) is the fact that the required hadronic matrix elements can be extracted, including isospin breaking corrections (Marciano and Parsa, 1996; Mescia and Smith, 2007), from the leading semileptonic decay $K^+ \to \pi^0 e^+\nu$. Moreover, extensive studies of other long-distance contributions (Ecker *et al.*, 1988; Hagelin and Littenberg, 1989; Rein and Sehgal, 1989; Lu and Wise, 1994; Geng *et al.*, 1996; Fajfer, 1997; Buchalla and Isidori, 1998; Falk *et al.*, 2001) and of higher-order electroweak effects (Buchalla and Buras, 1998) have shown that they can safely be neglected in $K_L \to \pi^0\nu\bar{\nu}$ and are small in $K^+ \to \pi^+\nu\bar{\nu}$. In particular, the most recent improved calculation of long-distance contributions to $K^+ \to \pi^+\nu\bar{\nu}$ results in an enhancement of the relevant branching ratio by $6 \pm 2\%$. Further progress in calculating these contributions is possible in principle with the help of lattice QCD (Isidori, Martinelli, and Turchetti, 2006). Some recent reviews on $K \to \pi\nu\bar{\nu}$ can be found in Buras (2003, 2005a, 2005b), Isidori (2003), and Isidori *et al.* (2005).

We are fortunate that, while the decay $K^+ \to \pi^+\nu\bar{\nu}$ is CP conserving and depends sensitively on the underlying flavor dynamics, its partner $K_L \to \pi^0\nu\bar{\nu}$ is purely CP violating within the standard model (SM) and most of its extensions, and consequently depends also on the mechanism of CP violation. Moreover, the combination of these two decays allows us to eliminate the parametric uncertainties due to the Cabibbo-Kobayash-Maskwa (CKM) element $|V_{cb}|$ and m_t in the determination of the angle β in the unitarity triangle (UT) or equivalently of the phase of the CKM element V_{td} (Buchalla and Buras, 1994b, 1996). The resulting theoretical uncertainty in $\sin 2\beta$ is comparable to the one present in the mixing-induced CP asymmetry $a_{\psi K_S}$, and with the measurements of both branching ratios at the $\pm 10\%$ and $\pm 5\%$ level, $\sin 2\beta$ could be determined with ± 0.08 and ± 0.04 precision, respectively. This independent determination of $\sin 2\beta$ with a very small theoretical error offers a powerful test of the SM and of its simplest extensions in which the flavor and CP violation are governed by the CKM matrix, the so-called MFV (minimal flavor violation) models (Buras *et al.*, 2001b; D'Ambrosio *et al.*, 2002; Buras, 2003, 2005a, 2005b). Indeed, in $K \to \pi\nu\bar{\nu}$ the phase β originates in Z^0 penguin diagrams ($\Delta S = 1$), whereas in the case of $a_{\psi K_S}$ in the B_d^0-$\bar{B}_d^0$ box diagrams ($\Delta B = 2$). Any "nonminimal" contributions to Z^0 penguin diagrams and/or box B_d^0-$\bar{B}_d^0$ diagrams would then be signaled by the violation of the MFV "golden" relation (Buchalla and Buras, 1994b)

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}. \tag{1.1}$$

Now, strictly speaking, according to the common classification of different types of CP violation (Nir, 2001; Fleischer, 2002, 2004; Ali, 2003; Buchalla, 2003; Buras, 2003, 2005a, 2005b; Hurth, 2003), both the asymmetry $a_{\psi K_S}$ and a nonvanishing rate for $K_L \to \pi^0\nu\bar{\nu}$ in the SM and in most of its extensions signal the CP violation in the interference of mixing and decay. However, as the CP violation in mixing (indirect CP violation) in K decays is governed by the small parameter ε_K, one can show (Littenberg, 1989; Buchalla and Buras, 1996; Grossman and Nir, 1997), that the observation of $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ at the level of 10^{-11} and higher is a manifestation of a large direct CP violation with the indirect one contributing less than $\sim 1\%$ to the branching ratio. The great potential of $K_L \to \pi^0\nu\bar{\nu}$ in testing the physics beyond the SM has been summarized by Bryman *et al.* (2006).

Additionally, this large direct CP violation can be directly measured without essentially any hadronic uncertainties, due to the presence of $\nu\bar{\nu}$ in the final state. This should be contrasted with the popular studies of direct CP violation in nonleptonic two-body B decays (Nir, 2001; Fleischer, 2002; Ali, 2003; Buchalla, 2003; Buras, 2003, 2005a, 2005b; Hurth, 2003; Fleischer, 2004), which are subject to significant hadronic uncertainties. In particular, the extraction of weak phases requires generally rather involved strategies using often certain assumptions about the strong dynamics (Harrison and Quinn, 1998; Ball *et al.*, 2000; Anikeev *et al.*, 2001). Only a handful of strategies, which we review in Sec. IX, allow direct determinations of weak phases from nonleptonic B decays without practically any hadronic uncertainties.

Returning to Eq. (1.1), an important consequence of this relation is the following (Buras and Fleischer, 2001): for a given $\sin 2\beta$ extracted from $a_{\psi K_S}$, the measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ determines up to a twofold ambiguity the value of $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$, independent of any new parameters present in the MFV models. Consequently, measuring $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ will either select one of the possible values or rule out all MFV models. Recent analyses of the MFV models indicate that one of these values is unlikely (Bobeth *et al.*, 2005; Haisch and Weiler, 2007a). A spectacular violation of the relation (1.1) is found in the context of new physics scenarios with enhanced Z^0 penguins carrying a new CP-violating phase (Colangelo and Isidori, 1998; Nir and Worah, 1998; Buras and Silvestrini, 1999; Buras *et al.*, 2000, 2004a, 2004b, 1998; Atwood and Hiller, 2003). An explicit realization of such a scenario is the littlest Higgs model with T parity (Blanke *et al.*, 2007b), which we discuss in Sec. VIII.

Another important virtue of $K^+ \to \pi^+ \nu\bar{\nu}$ is a theoretically clean determination of $|V_{td}|$ or equivalently of the length R_t in the unitarity triangle. This determination is only subject to theoretical uncertainties in the charm sector, which amount after the recent NNLO calculation to $\pm 1-2$ %. The remaining parametric uncertainties in the determination of $|V_{td}|$ related to $|V_{cb}|$ and m_t should be soon reduced to the 1–2 % level. Finally, the decay $K_L \to \pi^0 \nu\bar{\nu}$ offers the cleanest determination of the Jarlskog CP invariant J_{CP} (Buchalla and Buras, 1996) or equivalently of the area of the unrescaled unitarity triangle that cannot be matched by any B decay. With the improved precision on m_t and $|V_{cb}|$, precise measurement of the height $\bar{\eta}$ of the unitarity triangle also becomes possible.

Clean determinations of $\sin 2\beta$, $|V_{td}|$, R_t, J_{CP}, and the UT in general, as well as the test of the MFV relation (1.1) and generally of the physics beyond the SM, put these two decays in the class of "golden decays," essentially on the same level as the determination of $\sin 2\beta$ through the asymmetry $a_{\psi K_S}$ and certain clean strategies for the determination of the angle γ in the UT (Nir, 2001; Fleischer, 2002, 2004; Ali, 2003; Buchalla, 2003; Buras, 2003, 2005a, 2005b; Hurth, 2003), which will be available at LHC (Ball *et al.*, 2000). We discuss the latter in Sec. IX. Therefore, precise measurements of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ are of utmost importance and should be aimed for, even when realizing that the determination of the branching ratios in question with an accuracy of 5% is extremely challenging.

With the NNLO calculation (Buras, Gorbahn, Haisch, *et al.*, 2005) at hand, the branching ratios of $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ within the SM can be predicted as

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = (8.1 \pm 1.1) \times 10^{-11}, \tag{1.2}$$

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}} = (2.6 \pm 0.3) \times 10^{-11}. \tag{1.3}$$

This is an accuracy of $\pm 14\%$ and $\pm 12\%$, respectively. We demonstrate that further progress on the determination of the CKM parameters coming in the next few years dominantly from BaBar, Belle, Tevatron, and later from LHC, as well as the improved determination of m_c relevant for $K^+ \to \pi^+ \nu\bar{\nu}$, should allow predictions for $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ with the uncertainties of $\pm 5\%$ or better. This accuracy cannot be matched by any other rare decay branching ratio in the field of meson decays.

On the experimental side, the AGS E787 Collaboration at Brookhaven was the first to observe the decay $K^+ \to \pi^+ \nu\bar{\nu}$ (Adler *et al.*, 1997, 2000). The resulting branching ratio based on two events and published in 2002 was (Adler *et al.*, 2002, 2004)

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (15.7^{+17.5}_{-8.2}) \times 10^{-11} \quad (2002). \tag{1.4}$$

In 2004, a new $K^+ \to \pi^+ \nu\bar{\nu}$ experiment, AGS E949 (Anisimovsky *et al.*, 2004, 2007), released its first results that are based on the 2002 running. One additional event has been observed. Including the result of AGS E787, the present branching ratio reads

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (14.7^{+13.0}_{-8.9}) \times 10^{-11} \quad (2004). \tag{1.5}$$

It is not clear, at present, how this result will be improved in the coming years as the activities of AGS E949 and the efforts at Fermilab around the CKM experiment (CKM Experiment, 2004) have unfortunately been terminated. On the other hand, the corresponding efforts at CERN around the NA48 Collaboration (2004) and at JPARC in Japan (J-PARC, 2004) could provide an additional 50–100 events at the beginning of the next decade.

The situation is different for $K_L \to \pi^0 \nu\bar{\nu}$. The older upper bound on its branching ratio from KTeV (Blucher, 2005), $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) < 2.9 \times 10^{-7}$, has recently been improved to

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) < 2.1 \times 10^{-7} \tag{1.6}$$

by E391 Experiment at KEK (Ahn *et al.*, 2006). While this is about four orders of magnitude above the SM expectation, the prospects for an improved measurement of $K_L \to \pi^0 \nu\bar{\nu}$ appear almost better than for $K^+ \to \pi^+ \nu\bar{\nu}$ from the present perspective.

Indeed, a $K_L \to \pi^0 \nu\bar{\nu}$ experiment at KEK, E391a (E391 Experiment, 2004), should in its first stage improve the bound in Eq. (1.6) by three orders of magnitude. While this is insufficient to reach the SM level, a few events could be observed if $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ turned out to be larger by one order of magnitude due to new physics contributions.

While an interesting experiment at Brookhaven, KOPIO (Bryman, 2002; Littenberg, 2002), that was supposed to in due time provide 40–60 events of $K_L \to \pi^0 \nu\bar{\nu}$ at the SM level, has unfortunately not been approved to run at Brookhaven, the ideas presented in this proposal will hopefully be realized one day. Finally, the second stage of the E391 experiment could, using the high intensity 50 GeV/c proton beam from JPARC (J-PARC, 2004), provide roughly 1000 SM events of $K_L \to \pi^0 \nu\bar{\nu}$, which would be eventful. Perspectives of a search for $K_L \to \pi^0 \nu\bar{\nu}$ at a Φ factory have been discussed by Bossi *et al.* (1999). Further reviews on experimental prospects for $K \to \pi\nu\bar{\nu}$ can be found in Barker and Kettell (2000), Belyaev *et al.* (2001), and Diwan (2002).

Parallel to these efforts, during the coming years we will witness unprecedented tests of the CKM picture of flavor and CP violation in B decays that will be available at SLAC, KEK, Tevatron, and CERN. The most prominent of these tests will involve CP violation in the B_s^0-$\bar{B}_s^0$ mixing and a number of clean strategies for the determination of the angles γ and β in the UT that will involve $B^\pm$, B_d^0, and B_s^0 two-body nonleptonic decays.

These efforts will be accompanied by the studies of CP violation in decays like $B \to \pi\pi$, $B \to \pi K$, and $B \to KK$, that in spite of being less theoretically clean than the quantities considered in the present review, will certainly contribute to the tests of the CKM paradigm (Cabibbo, 1963; Kobayashi and Maskawa, 1973). In addition, rare decays such as $B \to X_s \gamma$, $B \to X_{s,d} \mu^+\mu^-$, $B_{s,d} \to \mu^+\mu^-$, $B \to X_{s,d} \nu\bar{\nu}$, $B \to \tau\bar{\nu}$, $K_L \to \pi^0 e^+ e^-$, and $K_L \to \pi^0 \mu^+\mu^-$ will play an important role.

In 1994, two detailed analyses of $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, B_s^0-$\bar{B}_s^0$ mixing, and of CP asymmetries in B decays have been presented in the anticipation of future precise measurements of several theoretically clean observables, which could be used for determination of the CKM matrix and of the unitarity triangle within the SM (Buras, 1994; Buras *et al.*, 1994). These analyses were speculative as in 1994 even the top quark mass was unknown; none of the observables listed above have been measured and the CKM elements $|V_{cb}|$ and $|V_{ub}|$ were rather poorly known.

During the last 13 years, impressive progress has taken place: the top quark mass, the angle β in the UT, and the B_s^0-$\bar{B}_s^0$ mixing mass difference ΔM_s have been precisely measured, and three events of $K^+ \to \pi^+ \nu\bar{\nu}$ have been observed. We are still waiting for the observation of $K_L \to \pi^0 \nu\bar{\nu}$ and a precise direct measurement of the angle γ in the UT from tree level decays, but now we are rather confident that this will be realized in the next decade.

This progress makes it possible to considerably improve the analyses of Buras (1994) and Buras *et al.* (1994) within the SM and to generalize them to its simplest extensions. This is one of our goals in this review. We see that the decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$, as in 1994, play an important role in these investigations.

In this context, we emphasize that new physics contributions in $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$, in essentially all extensions of the SM,[1] can be parametrized in a model-independent manner by two parameters (Buras *et al.*, 1998), the magnitude of the short distance function X (Buras, 2003, 2005a, 2005b) and its complex phase:

$$X = |X| e^{i\theta_X} \tag{1.7}$$

with $|X| = X(x_t)$ and $\theta_X = 0$ in the SM. The important virtues of the $K \to \pi\nu\bar{\nu}$ system are as follows:

- $|x|$ and θ_X can be extracted from $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ and $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ without any hadronic uncertainties.
- As in many extensions of the SM, the function X is governed by the Z^0 penguins with top quark and new particle exchanges,[2] the determination of the function X is the determination of the Z^0 penguins that enter other decays.
- The theoretical cleanness of this determination cannot be matched by any other decay. For instance, decays such as $B \to X_{s,d} \mu^+\mu^-$ and $B_{s,d} \to \mu^+\mu^-$, which can also be used for this purpose, are subject to theoretical uncertainties of ±10% or more.

Already at this stage we emphasize that the clean theoretical character of these decays remains valid in essentially all extensions of the SM, whereas this is generally not the case for nonleptonic two-body B decays used to determine the CKM parameters through CP asymmetries and/or other strategies. While several mixing-induced CP asymmetries in nonleptonic B decays within the SM are essentially free from hadronic uncertainties, as the latter cancel out due to the dominance of a single CKM amplitude, this is often not the case in extensions of the SM in which the amplitudes receive new contributions with different weak phases implying no cancellation of hadronic uncertainties in the relevant observables. A classic example of this situation, as stressed by Ciuchini and Silvestrini (2002), is the mixing-induced CP asymmetry in $B_d^0(\bar{B}_d^0) \to \phi K_S$ decays that within the SM measures the angle β in the UT with small hadronic uncertainties. As soon as the extensions of the SM are considered in which new operators and new weak phases are present, the mixing-induced asymmetry $a_{\phi K_S}$ suffers from potential hadronic uncertainties that make the determination of the relevant parameters problematic unless the hadronic matrix elements can be calculated with sufficient precision. This is evident from the many papers on the anomaly in $B_d^0(\bar{B}_d^0) \to \phi K_S$ decays of which the subset is given by Fleischer and Mannel (2001), Ciuchini and Silvestrini (2002), Datta (2002), Hiller (2002), Raidal (2002), Grossman *et al.* (2003), and Khalil and Kou (2003).

The goal of the present review is to collect the relevant formulas for the decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ and to investigate their theoretical and paramet-

[1]Exceptions will be discussed in Sec. VIII.

[2]Box diagrams seem to be relevant only in the SM and can be calculated with high accuracy.

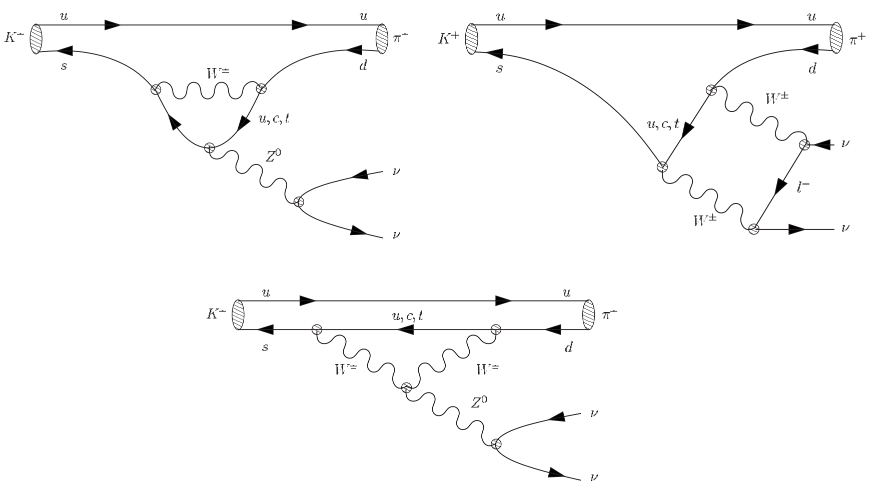

FIG. 1. The penguin and box diagrams contributing to $K^+ \to \pi^+ \nu\bar{\nu}$. For $K_L \to \pi^0 \nu\bar{\nu}$, only the spectator quark is changed from u to d.

ric uncertainties. In addition to known expressions, we derive new ones that should allow transparent tests of the SM and of its extensions. While our presentation is centered around the SM, we also discuss models with MFV and scenarios with new complex phases, in particular the littlest Higgs model with T parity, the MSSM, Z' models, and a model with one universal extra dimension. We also give a review of other models. Moreover, we investigate the interplay between the $K \to \pi\nu\bar{\nu}$ complex, the $B^0_{d,s}$-$\bar{B}^0_{d,s}$ mass differences $\Delta M_{d,s}$, and the angles β and γ in the unitarity triangle that will be measured precisely in two-body B decays one day.

Our review is organized as follows. Sections II and III can be considered as a compendium of formulas for the decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ within the SM. We also give formulas for the CKM factors and the UT that are relevant for us. In particular, in Sec. III we investigate the interplay between $K \to \pi\nu\bar{\nu}$, the mass differences $\Delta M_{d,s}$, and the angles β and γ. In Sec. IV, a detailed numerical analysis of the formulas of Secs. II and III is presented. Section V is a short guide to subsequent sections in which we review $K \to \pi\nu\bar{\nu}$ in various extensions of the SM. In Sec. VI we indicate how the discussion of previous sections is generalized to the class of the MFV models. In Sec. VII our discussion is further generalized to three scenarios involving new complex phases: a scenario with new physics entering only Z^0 penguins, a scenario with new physics entering only B^0_d-$\bar{B}^0_d$ mixing, and a hybrid scenario in which both Z^0 penguins and B^0_d-$\bar{B}^0_d$ mixing are affected by new physics. Here we derive a number of expressions that were not presented in the literature so far and illustrate how the new phases, and other new physics parameters, can be determined by means of the (R_b, γ) strategy (Buras, Parodi, and Stocchi, 2003) and the related reference unitarity triangle (Goto *et al.*, 1996; Cohen *et al.*, 1997; Grossman *et al.*, 1997; Barenboim *et al.*, 1999). While the discussion of Sec. VII is practically model independent within three scenarios considered, we give in Sec. VIII a review of existing results for both decays within specific extensions of the SM, such as little Higgs, Z' and supersymmetric models, models with extra dimensions, models with lepton-flavor mixing, and other selected models considered in the literature. In Sec. IX we compare the $K \to \pi\nu\bar{\nu}$ decays with other K and B decays used for the determination of the CKM phases and of the UT with respect to the theoretical cleanness. In Sec. X we describe the long-distance contributions that are taken into account in the numerical analyses. Finally, in Sec. XI we summarize our results and give an outlook for the future.

II. BASIC FORMULAS

A. Preliminaries

In this section, we collect the formulas for the branching ratios for the decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ that constitute the basis for this review. We also give values of the relevant parameters as well as recall formulas related to the CKM matrix and the unitarity triangle that are relevant for this review. Clearly, many formulas listed below have been presented previously in the literature, in particular in Buchalla and Buras (1996, 1999), Buchalla, Buras, and Lautenberger (1996), Buras (1998, 2003, 2005a, 2005b), Battaglia *et al.* (2003), and Buras, Parodi, and Stocchi (2003). Still the collection of them at one place and the addition of new ones should be useful for future investigations.

The effective Hamiltonian relevant for $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ decays can be written in the SM as follows (Buchalla and Buras, 1994a, 1999):

$$\mathcal{H}^{\mathrm{SM}}_{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_{\mathrm{w}}} \sum_{l=e,\mu,\tau} [V^*_{cs} V_{cd} X^l_{\mathrm{NL}} + V^*_{ts} V_{td} X(x_t)] \times (\bar{s}d)_{V\text{-}A} (\bar{\nu}_l \nu_l)_{V\text{-}A}, \tag{2.1}$$

with all symbols defined below. It is obtained from the relevant Z^0 penguin and box diagrams with the up, charm, and top quark exchanges shown in Fig. 1 and includes QCD corrections at the NLO level (Buchalla and Buras, 1993a, 1993b, 1994a, 1999; Misiak and Urban, 1999) and the NNLO calculated recently (Buras, Gorbahn, Haisch, *et al.*, 2005, 2006). The presence of up quark contributions is only needed for the GIM mechanism to work, otherwise only the internal charm and top contributions matter. The relevance of these contributions in each decay is spelled out below.

The index $l=e$, μ, τ denotes the lepton flavor. The dependence on the charged lepton mass resulting from box diagrams is negligible for the top contribution. In the charm sector, this is the case only for the electron and the muon but not for the τ lepton. In what follows, we give the branching ratios that follow from Eq. (2.1).

B. $K^+ \to \pi^+ \nu\bar{\nu}$

The branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ in the SM is dominated by Z^0 penguin diagrams with a significant contribution from the box diagrams. Summing over three neutrino flavors, it can be written as follows (Buchalla and Buras, 1999; Mescia and Smith, 2007):

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ (1+\Delta_{\mathrm{EM}}) \left[\left(\frac{\mathrm{Im}\,\lambda_t}{\lambda^5} X(x_t) \right)^2 + \left(\frac{\mathrm{Re}\,\lambda_c}{\lambda} P_c(X) + \frac{\mathrm{Re}\,\lambda_t}{\lambda^5} X(x_t) \right)^2 \right], \tag{2.2}$$

$$\kappa_+ = (5.173 \pm 0.025) \times 10^{-11} \left[\frac{\lambda}{0.225} \right]^8 . \tag{2.3}$$

An explicit derivation of Eq. (2.2) can be found in Buras (1998). Here $x_t = m_t^2/M_W^2$, $\lambda_i = V_{is}^* V_{id}$ are the CKM factors discussed below and κ_+ summarizes all remaining factors following from Eq. (2.1), in particular the relevant hadronic matrix elements that can be extracted from leading semileptonic decays of K^+, K_L, and K_S mesons. The original calculation of these matrix elements (Marciano and Parsa, 1996) has recently been significantly improved by Mescia and Smith (Mescia and Smith, 2007), where details can be found; in particular, Δ_{EM} amounts to -0.3%, which we neglect in what follows. In obtaining the numerical value in Eq. (2.3) (Mescia and Smith, 2007), the values (Yao *et al.*, 2006)

$$\sin^2\theta_w = 0.231, \qquad \alpha = \frac{1}{127.9}, \tag{2.4}$$

given in the minimal subtraction ($\overline{\mathrm{MS}}$) scheme have been used. Their errors are below 0.1% and can be neglected. There is an issue related to $\sin^2\theta_w$ that, although well measured in a given renormalization scheme, is a scheme-dependent quantity with the scheme dependence only removed by considering higher-order electroweak effects in $K \to \pi\nu\bar{\nu}$. An analysis of such effects in the large m_t limit (Buchalla and Buras, 1998) shows that in principle they could introduce a $\pm 5\%$ correction in the $K \to \pi\nu\bar{\nu}$ branching ratios, but with the $\overline{\mathrm{MS}}$ definition of $\sin^2\theta_w$ these higher-order electroweak corrections are found below 2% and can also be safely neglected. Similar comments apply to α. This pattern of higher-order electroweak corrections is also found in the $B^0_{d,s}$-$\bar{B}^0_{d,s}$ mixing (Gambino *et al.*, 1999). Yet, in the future, a complete analysis of two-loop electroweak contributions to $K \to \pi\bar{\nu}\nu$ would certainly be of interest.

The apparent large sensitivity of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ to λ is spurious as $P_c(X) \sim \lambda^{-4}$ and the dependence on λ in Eq. (2.3) cancels the one in Eq. (2.2) to a large extent. However, basically for aesthetic reasons it is useful to first write these formulas as given above. In doing this, it is essential to keep track of the λ dependence as it is hidden in $P_c(X)$ [see Eq. (2.13)], and changing λ while keeping $P_c(X)$ fixed would give wrong results. For later purposes, we also introduce

$$\bar{\kappa}_+ = \frac{\kappa_+}{\lambda^8} = (7.87 \pm 0.04) \times 10^{-6}. \tag{2.5}$$

The function $X(x_t)$ relevant for the top part is given by

$$X(x_t) = X_0(x_t) + \frac{\alpha_s(m_t)}{4\pi} X_1(x_t) = \eta_X X_0(x_t), \qquad \eta_X = 0.995, \tag{2.6}$$

where

$$X_0(x_t) = \frac{x_t}{8}\left[-\frac{2+x_t}{1-x_t} + \frac{3x_t-6}{(1-x_t)^2}\ln x_t \right] \tag{2.7}$$

describes the contribution of Z^0 penguin diagrams and box diagrams without the QCD corrections (Inami and Lim, 1981; Buchalla *et al.*, 1991) and the second term represents the QCD correction (Buchalla and Buras, 1993a, 1993b, 1999; Misiak and Urban, 1999) with

$$\begin{aligned} X_1(x_t) = {} & -\frac{29x_t - x_t^2 - 4x_t^3}{3(1-x_t)^2} - \frac{x_t + 9x_t^2 - x_t^3 - x_t^4}{(1-x_t)^3}\ln x_t \\ & + \frac{8x_t + 4x_t^2 + x_t^3 - x_t^4}{2(1-x_t)^3}\ln^2 x_t - \frac{4x_t - x_t^3}{(1-x_t)^2} \\ & \times L_2(1-x_t) + 8x\frac{\partial X_0(x_t)}{\partial x_t}\ln x_\mu, \end{aligned} \tag{2.8}$$

where $x_\mu = \mu_t^2/M_W^2$, $\mu_t = \mathcal{O}(m_t)$ and

$$L_2(1-x_t) = \int_1^{x_t} dt \frac{\ln t}{1-t}. \tag{2.9}$$

The μ_t dependence in the last term in Eq. (2.8) cancels to the order considered the μ_t dependence of the leading term $X_0[x_t(\mu_t)]$ in Eq. (2.6). The leftover μ_t dependence in $X(x_t)$ is below 1%. The factor η_X summarizes the NLO corrections represented by the second term in Eq. (2.6). With $m_t \equiv m_t(m_t)$, the QCD factor η_X is independent of m_t and $\alpha_s(M_Z)$ and is close to unity. Varying $m_t(m_t)$ from 150 to 180 GeV changes η_X by at most 0.1%.

The uncertainty in $X(x_t)$ is then fully dominated by the experimental error in m_t. The $\overline{\mathrm{MS}}$ top-quark mass,[3]

[3]We thank M. Jamin for discussions on this subject.

TABLE I. The parameter $P_c(X)$ in NNLO approximation for various values of $\alpha_s(M_Z)$ and $m_c(m_c)$ (Buras, Gorbahn, Haisch, *et al.*, 2006). The numerical values for $P_c(X)$ correspond to $\lambda=0.2248$, $\mu_W=80.0$ GeV, $\mu_b=5.0$ GeV, and $\mu_c=1.50$ GeV.

	$P_c(X)$						
$\alpha_s(M_Z)\backslash m_c(m_c)$ (GeV)	1.15	1.20	1.25	1.30	1.35	1.40	1.45
0.115	0.307	0.336	0.366	0.397	0.430	0.463	0.497
0.116	0.303	0.332	0.362	0.394	0.426	0.459	0.493
0.117	0.300	0.329	0.359	0.390	0.422	0.455	0.489
0.118	0.296	0.325	0.355	0.386	0.417	0.450	0.484
0.119	0.292	0.321	0.350	0.381	0.413	0.446	0.480
0.120	0.288	0.316	0.346	0.377	0.409	0.441	0.475
0.121	0.283	0.312	0.342	0.372	0.404	0.437	0.470
0.122	0.279	0.307	0.337	0.368	0.399	0.432	0.465
0.123	0.274	0.303	0.332	0.363	0.394	0.426	0.460

including one-, two-, and three-loop contributions (Melnikov and Ritbergen, 2000) and corresponding to the most recent $m_t^{\rm pole}=170.9\pm1.1\pm1.5$ GeV (Brubaker *et al.*, 2006), is given by

$$m_{\rm t}(m_{\rm t}) = 161.0 \pm 1.7 \text{ GeV}. \tag{2.10}$$

One finds then

$$X(x_t) = 1.443 \pm 0.017. \tag{2.11}$$

$X(x_t)$ increases with m_t roughly as $m_t^{1.15}$. After the LHC era the error on m_t should decrease below ±1 GeV, implying the error of ±0.01 in $X(x_t)$ that can be neglected for all practical purposes.

The parameter $P_c(X)$ summarizes the charm contribution and is defined through

$$P_c(X) = P_c^{\rm SD}(X) + \delta P_{c,u}, \tag{2.12}$$

with the long-distance contributions $\delta P_{c,u}=0.04\pm0.02$ (Isidori *et al.*, 2005). The short-distance part is given by

$$P_c^{\rm SD}(X) = \frac{1}{\lambda^4}\left[\frac{2}{3}X_{\rm NNL}^e + \frac{1}{3}X_{\rm NNL}^\tau\right], \tag{2.13}$$

where the functions $X_{\rm NNL}^l$ result from the NLO calculation (Buchalla and Buras, 1994a, 1999) and NNLO (Buras, Gorbahn, Haisch, *et al.*, 2005, 2006). The index l distinguishes between the charged lepton flavors in the box diagrams. This distinction is irrelevant in the top contribution due to $m_t \gg m_l$ but is relevant in the charm contribution as $m_\tau > m_c$. The inclusion of NLO corrections reduced considerably the large μ_c dependence [with $\mu_c=\mathcal{O}(m_c)$] present in the leading-order expressions for the charm contribution (Vainshtein *et al.*, 1977; Ellis and Hagelin, 1983; Dib *et al.*, 1991). Varying μ_c in the range $1\leqslant\mu_c\leqslant3$ GeV changes $X_{\rm NNL}^l$ by roughly 24% at NLO to be compared to 56% in the leading order. At NNLO, the μ_c dependence is further decreased, as discussed below.

The net effect of QCD corrections is to suppress the charm contribution by roughly 30%. For our purposes, we need only $P_c(X)$. In Table I, we give its values for different $\alpha_s(M_Z)$ and $m_c\equiv m_c(m_c)$. The chosen range for $m_c(m_c)$ is close to most recent estimates. For instance, $m_c(m_c)=1.286(13)$, 1.29(7)(13), and 1.29(7) (all in GeV) have been found from $R^{e^+e^-}(s)$ (Kuhn *et al.*, 2007), quenched combined with dynamical lattice QCD (Dougall *et al.*, 2006), and charmonium sum rules (Hoang and Jamin, 2004), respectively. Further references can be found in these papers and in Battaglia *et al.* (2003).

Finally, in Table II we show the dependence of $P_c(X)$ on $\alpha_s(M_Z)$ and μ_c at fixed $m_c(m_c)=1.30$ GeV.

Restricting the three parameters involved to the ranges

$$1.15 \leqslant m_{\rm c}(m_c) \leqslant 1.45 \text{ GeV}, \quad 1.0 \leqslant \mu_c \leqslant 3.0 \text{ GeV}, \tag{2.14}$$

$$0.115 \leqslant \alpha_s(M_Z) \leqslant 0.123, \tag{2.15}$$

one arrives at (Buras, Gorbahn, Haisch, *et al.*, 2005)

$$P_c(X)^{\rm SD} = (0.375 \pm 0.031_{m_c} \pm 0.009_{\mu_c} \pm 0.009_{\alpha_s}) \times\left(\frac{0.2248}{\lambda}\right)^4, \tag{2.16}$$

where the errors correspond to $m_c(m_c)$, μ_c, and $\alpha_s(M_Z)$, respectively. The uncertainty due to m_c is significant. On the other hand, the uncertainty due to α_s is small. In principle, one could add the errors in Eq. (2.16) linearly, which would result in an error of ±0.049. We think that this estimate would be too conservative. Adding the errors in quadrature gives ±0.033. This could be too optimistic, since the uncertainties are not statistically distributed. Therefore, as the final result for $P_c(X)$ we quote

$$P_c(X) = 0.41 \pm 0.05, \tag{2.17}$$

which we use in this review.

We expect that the reduction of the error in $\alpha_s(M_Z)$ to ±0.001 will decrease the corresponding error to 0.005, making it negligible. Concerning the error due to

TABLE II. The parameter $P_c(X)$ in NNLO approximation for various values of $\alpha_s(M_Z)$ and μ_c (Buras, Gorbahn, Haisch, *et al.*, 2006). The numerical values for $P_c(X)$ correspond to $\lambda=0.2248$, $m_c(m_c)=1.30$ GeV, $\mu_W=80.0$ GeV, and $\mu_b=5.0$ GeV.

	$P_c(X)$				
$\alpha_s(M_Z)\backslash\mu_c$ (GeV)	1.0	1.5	2.0	2.5	3.0
0.115	0.393	0.397	0.395	0.392	0.388
0.116	0.389	0.394	0.391	0.388	0.383
0.117	0.384	0.390	0.387	0.383	0.379
0.118	0.380	0.386	0.383	0.379	0.374
0.119	0.375	0.381	0.379	0.374	0.369
0.120	0.370	0.377	0.374	0.369	0.364
0.121	0.365	0.372	0.369	0.364	0.359
0.122	0.359	0.368	0.364	0.359	0.354
0.123	0.353	0.363	0.359	0.354	0.348

$m_c(m_c)$, it should be noted that increasing the error in $m_c(m_c)$ to ±70 MeV would increase the first error in Eq. (2.16) to 0.047, whereas its decrease to ±30 MeV would decrease it to 0.020. More generally, we have to a good approximation

$$\sigma[P_c(X)]_{m_c}=\left[\frac{0.67}{\mathrm{GeV}}\right]\sigma(m_c(m_c)). \tag{2.18}$$

From the present perspective, unless important advances in the determination of $m_c(m_c)$ are made, it will be difficult to decrease the error on $P_c(X)$ below ±0.03, although ±0.02 cannot be fully excluded. We use this information in our numerical analysis in Sec. IV.

C. $K_L\to\pi^0\nu\bar{\nu}$

The neutrino pair produced by $\mathcal{H}_{\mathrm{eff}}^{\mathrm{SM}}$ in Eq. (2.1) is a *CP* eigenstate with positive eigenvalue. Consequently, within the approximation of keeping only operators of dimension 6, as done in Eq. (2.1), the decay $K_L\to\pi^0\nu\bar{\nu}$ proceeds entirely through *CP* violation (Littenberg, 1989). However, as pointed out by Buchalla and Isidori (1998), even in the SM there are *CP*-conserving contributions to $K_L\to\pi^0\nu\bar{\nu}$, which are generated only by local operators of $d\geqslant 8$ or by long-distance effects. Fortunately, these effects are smaller by a factor of 10^5 than the leading *CP*-violating contribution and can be safely neglected (Buchalla and Isidori, 1998). As we discuss in Sec. VIII, the situation can be in principle different beyond the SM.

The branching ratio for $K_L\to\pi^0\nu\bar{\nu}$ in the SM is then fully dominated by diagrams with internal top exchanges with the charm contribution well below 1%. The branching ratio can be written then as follows (Buchalla and Buras, 1996; Buchalla, Buras, and Lautenberger, 1996; Buras, 1998):

$$\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})=\kappa_L\left(\frac{\mathrm{Im}\,\lambda_t}{\lambda^5}X(x_t)\right)^2, \tag{2.19}$$

$$\kappa_L=(2.231\pm0.013)\times10^{-10}\left[\frac{\lambda}{0.225}\right]^8, \tag{2.20}$$

where we have summed over three neutrino flavors. An explicit derivation of Eq. (2.19) can be found in Buras (1998). Here κ_L is the factor corresponding to κ_+ in Eq. (2.2). The original calculation of κ_L (Marciano and Parsa, 1996) has been recently significantly improved by Mescia and Smith (2007), where details can be found. Due to the absence of $P_c(X)$ in Eq. (2.19), $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ has essentially no theoretical uncertainties and is only affected by parametric uncertainties from m_t, $\mathrm{Im}\,\lambda_t$, and κ_L. These uncertainties should be decreased significantly in the coming years so that a precise prediction for $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ should be available in this decade. On the other hand, as discussed below, once this branching ratio has been measured, $\mathrm{Im}\,\lambda_t$ can be in principle determined with exceptional precision not matched by any other decay (Buchalla and Buras, 1996).

D. $K_S\to\pi^0\nu\bar{\nu}$

Next, mainly for completeness, we give the expression for $\mathrm{Br}(K_S\to\pi^0\nu\bar{\nu})$, which, due to $\tau(K_S)\ll\tau(K_L)$, is suppressed by roughly two orders of magnitude relative to $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$. We have (Bossi *et al.*, 1999)

$$\mathrm{Br}(K_S\to\pi^0\nu\bar{\nu})=\kappa_S\left(\frac{\mathrm{Re}\,\lambda_c}{\lambda}P_c(X)+\frac{\mathrm{Re}\,\lambda_t}{\lambda^5}X(x_t)\right)^2, \tag{2.21}$$

$$\kappa_S=\kappa_L\frac{\tau(K_S)}{\tau(K_L)}=(3.91\pm0.02)\times10^{-13}\left[\frac{\lambda}{0.2248}\right]^8. \tag{2.22}$$

Introducing the "reduced" branching ratio

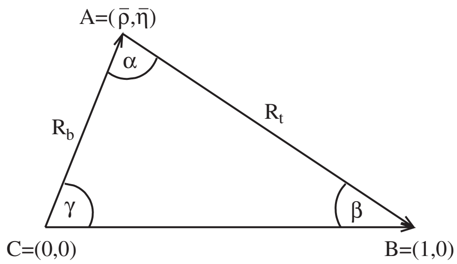

FIG. 2. Unitarity triangle.

$$B_3 = \frac{\mathrm{Br}(K_S \to \pi^0 \nu\bar{\nu})}{\kappa_S} \tag{2.23}$$

and analogous ratios B_1 and B_2 for $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ given in Eq. (3.24), we find a simple relation between the three $K \to \pi\nu\bar{\nu}$ decays,

$$B_1 = B_2 + B_3. \tag{2.24}$$

We emphasize that, while $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ being only sensitive to $\mathrm{Im}\,\lambda_t$ provides a direct determination of $\bar{\eta}$, $\mathrm{Br}(K_S \to \pi^0 \nu\bar{\nu})$ being only sensitive to $\mathrm{Re}\lambda_t$ provides a direct determination of $\bar{\varrho}$. The latter determination is not as clean as the one of $\bar{\eta}$ from $K_L \to \pi^0 \nu\bar{\nu}$ due to the presence of the charm contribution in Eq. (2.21). However, it is much cleaner than the corresponding determination of $\bar{\varrho}$ from $K_L \to \mu^+\mu^-$. Unfortunately, the small branching ratio $\mathrm{Br}(K_S \to \pi^0 \nu\bar{\nu}) \approx 5 \times 10^{-13}$ will not allow this determination in the foreseeable future. Therefore, we will not consider $K_S \to \pi^0 \nu\bar{\nu}$ in this review. Still one should not forget that the presence of another theoretically clean observable would be useful in testing extensions of the SM. Discussions of the complex $K_L \to \pi^0 \nu\bar{\nu}$ and $K_S \to \pi^0 \nu\bar{\nu}$ and its analogies to the studies of ε'/ε can be found in D'Ambrosio *et al.* (1994) and Bossi *et al.* (1999).

E. CKM parameters

1. Unitarity triangle, Im λ_t and Reλ_t

Concerning the CKM parameters, we use in our numerical analysis the Wolfenstein parametrization (Wolfenstein, 1983), generalized to include higher orders in $\lambda \equiv |V_{us}|$ (Buras *et al.*, 1994). This turns out to be useful in making the structure of various formulas transparent and gives results close to the ones obtained by means of the exact standard parametrization (Chau and Keung, 1984; Hagiwara *et al.*, 2002). The basic parameters are

$$\lambda, \quad A = \frac{|V_{cb}|}{\lambda^2}, \quad \bar{\varrho} = \varrho\left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right) \tag{2.25}$$

with ϱ and η the usual Wolfenstein parameters (Wolfenstein 1983). The parameters $\bar{\varrho}$ and $\bar{\eta}$, introduced by Buras *et al.* (1994), are useful as they describe the apex of the standard UT as shown in Fig. 2. More details on the unitarity triangle and the generalized Wolfenstein parametrization can be found in Buras *et al.* (1994), Battaglia *et al.* (2003), and Buras (2003, 2005a, and 2005b). Below, we only recall certain expressions that we need in the course of our discussion.

Parallel to the use of the parameters in Eq. (2.25) it will turn out to be useful to express the CKM elements V_{td} and V_{ts} as follows (Buras *et al.*, 2004a):

$$V_{td} = A R_t \lambda^3 e^{-i\beta}, \quad V_{ts} = -|V_{ts}| e^{-i\beta_s}, \tag{2.26}$$

with $\tan \beta_s \approx -\lambda^2 \bar{\eta}$. The smallness of β_s follows from the CKM phase conventions and the unitarity of the CKM matrix. Consequently, it is valid beyond the SM if three generation unitarity is assumed. R_t and β are defined in Fig. 2.

We have then

$$\lambda_t \equiv V_{ts}^* V_{td} = -\tilde{r}\lambda |V_{cb}|^2 R_t e^{-i\beta} e^{i\beta_s}$$
$$\text{with } \tilde{r} = \left|\frac{V_{ts}}{V_{cb}}\right| = \sqrt{1 + \lambda^2(2\bar{\varrho} - 1)} \approx 0.985, \tag{2.27}$$

where in order to avoid high powers of λ, we expressed the parameter A through $|V_{cb}|$. Consequently,

$$\mathrm{Im}\,\lambda_t = \tilde{r}\lambda |V_{cb}|^2 R_t \sin(\beta_{\mathrm{eff}}),$$
$$\mathrm{Re}\,\lambda_t = -\tilde{r}\lambda |V_{cb}|^2 R_t \cos(\beta_{\mathrm{eff}}), \tag{2.28}$$

with $\beta_{\mathrm{eff}} = \beta - \beta_s$.

Alternatively, using the parameters in Eq. (2.25), one has (Buras *et al.*, 1994)

$$\mathrm{Im}\,\lambda_t = \eta\lambda |V_{cb}|^2, \quad \mathrm{Re}\,\lambda_t = -\left(1 - \frac{\lambda^2}{2}\right)\lambda |V_{cb}|^2 (1 - \bar{\varrho}), \tag{2.29}$$

$$\mathrm{Re}\,\lambda_c = -\lambda\left(1 - \frac{\lambda^2}{2}\right). \tag{2.30}$$

The expressions for $\mathrm{Im}\,\lambda_t$ and $\mathrm{Re}\lambda_c$ represent to an accuracy of 0.2% the exact formulas obtained using the standard parametrization. The expression for $\mathrm{Re}\lambda_t$ in Eq. (2.29) deviates by at most 0.5% from the exact formula in the full range of parameters considered. After inserting the expressions (2.29) and (2.30) into the exact formulas for quantities of interest, further expansion in λ should not be made.

2. Leading strategies for $(\bar{\varrho}, \bar{\eta})$

Next, we have the following useful relations, which correspond to the best strategies for the determination of $(\bar{\varrho}, \bar{\eta})$ considered by Buras, Parodi, and Stocchi (2003).

(R_t, β) strategy:

$$\bar{\varrho} = 1 - R_t \cos\beta, \quad \bar{\eta} = R_t \sin\beta, \tag{2.31}$$

with R_t determined through Eq. (2.45) and β through $a_{\psi K_S}$. In this strategy, R_b and γ are given by

$$R_b = \sqrt{1 + R_t^2 - 2R_t \cos\beta}, \quad \cot\gamma = \frac{1 - R_t \cos\beta}{R_t \sin\beta}. \tag{2.32}$$

(R_b, γ) strategy:

$$\bar{\varrho} = R_b \cos\gamma, \quad \bar{\eta} = R_b \sin\gamma, \tag{2.33}$$

with γ (see Fig. 2) determined through clean strategies in tree dominated B decays (Ball *et al.*, 2000; Anikeev *et al.*, 2001; Nir, 2001; Fleischer, 2002, 2004; Ali, 2003; Buchalla, 2003; Buras, 2003, 2005a, 2005b; Hurth, 2003). In this strategy, R_t and β are given by

$$R_t = \sqrt{1 + R_b^2 - 2R_b \cos\gamma}, \quad \cot\beta = \frac{1 - R_b \cos\gamma}{R_b \sin\gamma}. \tag{2.34}$$

(β, γ) strategy:
Formulas in Eq. (2.31) and

$$R_t = \frac{\sin\gamma}{\sin(\beta+\gamma)}, \tag{2.35}$$

with β and γ determined through $a_{\psi K_S}$ and clean strategies for γ as in Eq. (2.33). In this strategy, the length R_b and $|V_{ub}/V_{cb}|$ can be determined through

$$R_b = \frac{\sin\beta}{\sin(\beta+\gamma)}, \quad \left|\frac{V_{ub}}{V_{cb}}\right| = \left(\frac{\lambda}{1-\lambda^2/2}\right) R_b. \tag{2.36}$$

$(\bar{\eta}, \gamma)$ strategy:

$$\bar{\varrho} = \frac{\bar{\eta}}{\tan\gamma} \tag{2.37}$$

with $\bar{\eta}$ determined, for instance, through $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ as discussed in Sec. III and γ as in the two strategies above.

As demonstrated by Buras, Parodi, and Stocchi (2003), the (R_t, β) strategy is useful now that the B_s^0-$\bar{B}_s^0$ mixing mass difference ΔM_s has been measured. However, the remaining three strategies turn out to be more efficient in determining $(\bar{\varrho}, \bar{\eta})$. The strategies (β, γ) and $(\bar{\eta}, \gamma)$ are theoretically cleanest as β and γ will be measured precisely in two-body B decays one day and $\bar{\eta}$ can be extracted from $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ subject only to the uncertainty in $|V_{cb}|$. Combining these two strategies offers a precise determination of the CKM matrix including $|V_{cb}|$ and $|V_{ub}|$ (Buras, 1994). On the other hand, these two strategies are subject to uncertainties coming from new physics that can enter through β and $\bar{\eta}$. The angle γ, the phase of V_{ub}, can be determined in principle without these uncertainties.

The strategy (R_b, γ), on the other hand, while subject to hadronic uncertainties in the determination of R_b, is not affected by new physics contributions as, in addition to γ, also R_b can be determined from tree level decays. This strategy results in the so-called reference unitarity triangle (RUT) as discussed by Goto *et al.* (1996), Cohen *et al.* (1997), Grossman *et al.* (1997), and Barenboim *et al.* (1999). We return to these strategies in the course of our presentation.

3. Constraints from the standard analysis of the UT

Other useful expressions that represent the constraints from the *CP*-violating parameter ε_K and $\Delta M_{s,d}$, which parametrize the size of $B_{s,d}^0$-$\bar{B}_{s,d}^0$ mixings, are as follows.

First, we have

$$\varepsilon_K = -C_\varepsilon \hat{B}_K \,\mathrm{Im}\,\lambda_t \{\lambda^4 \,\mathrm{Re}\,\lambda_c P_c(\varepsilon) + \mathrm{Re}\,\lambda_t \eta_2^{\mathrm{QCD}} S_0(x_t)\} e^{i\pi/4}, \tag{2.38}$$

where $S_0(x_t) = 2.27 \pm 0.04$ results from $\Delta S = 2$ box diagrams and the numerical constant C_ε is given by ($M_W = 80.4$ GeV)

$$C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} = 3.837 \times 10^4. \tag{2.39}$$

Next (Herrlich and Nierste, 1994, 1995, 1996; Jamin and Nierste, 2004),

$$P_c(\varepsilon) = \frac{\bar{P}_c(\varepsilon)}{\lambda^4} = (0.29 \pm 0.07)\left[\frac{0.2248}{\lambda}\right]^4,$$

$$\bar{P}_c(\varepsilon) = (7.3 \pm 1.7) \times 10^{-4}, \tag{2.40}$$

$\eta_2^{\mathrm{QCD}} = 0.574 \pm 0.003$ (Buras *et al.*, 1990; Buchalla, Buras, and Lautenbacher, 1996; Buras, 1998), and $\hat{B}_K$ is a nonperturbative parameter. In obtaining Eq. (2.38), a small term amounting to at most 5% correction to ε_K has been neglected. This is justified in view of other uncertainties, in particular those connected with $\hat{B}_K$, but in the future it should be taken into account (Andriyash *et al.*, 2004).

Comparing Eq. (2.38) with the experimental value for ε_K (Hagiwara *et al.*, 2002),

$$(\varepsilon_K)_{\mathrm{exp}} = (2.280 \pm 0.013) \times 10^{-3} \exp(i\pi/4), \tag{2.41}$$

one obtains a constraint on the UT that with the help of Eqs. (2.29) and (2.30) can be cast into

$$\bar{\eta}[(1-\bar{\varrho})|V_{cb}|^2 \eta_2^{\mathrm{QCD}} S_0(x_t) + \bar{P}_c(\varepsilon)]|V_{cb}|^2 \hat{B}_K = 1.184 \times 10^{-6}\left[\frac{0.2248}{\lambda}\right]^2. \tag{2.42}$$

Next, the constraint from ΔM_d implies

$$R_t = \frac{1}{\lambda}\frac{|V_{td}|}{|V_{cb}|} = 0.834\left[\frac{|V_{td}|}{7.75 \times 10^{-3}}\right]\left[\frac{0.0415}{|V_{cb}|}\right] \times \left[\frac{0.2248}{\lambda}\right], \tag{2.43}$$

$$|V_{td}| = 7.75 \times 10^{-3}\left[\frac{230\ \text{MeV}}{\sqrt{\hat{B}_{B_d}}F_{B_d}}\right]\sqrt{\frac{\Delta M_d}{0.50/\text{ps}}}$$
$$\times\sqrt{\frac{0.55}{\eta_B^{\text{QCD}}}}\sqrt{\frac{2.40}{S_0(x_t)}}. \qquad (2.44)$$

Here $\sqrt{\hat{B}_{B_d}}F_{B_d}$ is a nonperturbative parameter and $\eta_B^{\text{QCD}}=0.551\pm0.003$ the QCD correction (Buras *et al.*, 1990; Urban *et al.*, 1998).

Finally, the simultaneous use of ΔM_d and ΔM_s gives

$$R_t = 0.935\tilde{r}\left[\frac{\xi}{1.24}\right]\left[\frac{0.2248}{\lambda}\right]\sqrt{\frac{17.8/\text{ps}}{\Delta M_s}}\sqrt{\frac{\Delta M_d}{0.50/\text{ps}}},$$

$$\xi = \frac{\sqrt{\hat{B}_{B_s}}F_{B_s}}{\sqrt{\hat{B}_{B_d}}F_{B_d}} \qquad (2.45)$$

with $\tilde{r}$ defined in Eq. (2.27) and ξ standing for a nonperturbative parameter that is subject to smaller theoretical uncertainties than the individual $\sqrt{\hat{B}_{B_d}}F_{B_d}$ and $\sqrt{\hat{B}_{B_s}}F_{B_s}$.

The main uncertainties in these constraints originate in the theoretical uncertainties in $\hat{B}_K$, $\sqrt{\hat{B}_d}F_{B_d}$, $\sqrt{\hat{B}_s}F_{B_s}$, and to a lesser extent in ξ (Hashimoto, 2005; Dawson *et al.*, 2006),

$$\hat{B}_K = 0.79 \pm 0.04 \pm 0.08, \quad \sqrt{\hat{B}_d}F_{B_d} = 214 \pm 38\ \text{MeV},$$

$$\sqrt{\hat{B}_s}F_{B_s} = 262 \pm 35\ \text{MeV}, \quad \xi = 1.23 \pm 0.06. \qquad (2.46)$$

The QCD sum-rule results for the parameters in question are similar and can be found in Battaglia *et al.* (2003). Finally (Battaglia *et al.*, 2003; Abulencia *et al.*, 2006),

$$\Delta M_d = (0.507 \pm 0.005)/\text{ps}, \quad \Delta M_s = (17.77 \pm 0.12)/\text{ps}. \qquad (2.47)$$

Extensive discussion of the formulas (2.38), (2.42), (2.44), and (2.45) can be found in Battaglia *et al.* (2003). For our numerical analysis, we use (Bona *et al.*, 2005)

$$\lambda = 0.2258 \pm 0.0014, \quad A = 0.816 \pm 0.016,$$

$$|V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}, \qquad (2.48)$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.088 \pm 0.005, \quad R_b = 0.38 \pm 0.01, \qquad (2.49)$$

$$\beta = (22.2 \pm 0.9)^\circ, \quad \beta_s = -1^\circ \qquad (2.50)$$

with the value of β following from the UT fit and slightly higher than the one determined from measurements of the time-dependent *CP* asymmetry $a_{\psi K_S}(t)$ that give (Abe *et al.*, 2002; Aubert *et al.*, 2002a; Browder, 2004; Barberio *et al.*, 2007)

$$(\sin 2\beta)_{\psi K_S} = 0.675 \pm 0.026, \quad \beta = 21.2 \pm 1.0. \qquad (2.51)$$

III. PHENOMENOLOGICAL APPLICATIONS IN THE SM

A. Preliminaries

During the past ten years, several analyses of $K \to \pi\nu\bar{\nu}$ decays within the SM were presented, in particular in Buchalla and Buras (1999), D'Ambrosio and Isidori (2002), Buras (2003, 2005a, 2005b), Kettell *et al.* (2004), Charles *et al.* (2005), Haisch (2005), Bona *et al.* (2006a), and Mescia and Smith (2007). Moreover, correlations with other decays have been pointed out (Buras and Silvestrini, 1999; Bergmann and Perez, 2000, 2001; Buras *et al.*, 2000). In this section, we collect and update many of these formulas and derive a number of new useful expressions. In the next section, a detailed numerical analysis of these formulas will be presented. Unless explicity stated, all formulas are given for $\lambda = 0.2248$. The dependence on λ can easily be found from the formulas of the preceding section. When the dependence is introduced, it is often useful to replace $\lambda^2 A$ by $|V_{cb}|$ to avoid high powers of λ. On the whole, the issue of the error in λ in $K \to \pi\nu\bar{\nu}$ decays is really not an issue if changes are made consistently in all places, as emphasized earlier.

B. Unitarity triangle and $K^+ \to \pi^+\nu\bar{\nu}$

1. Basic formulas

Using Eqs. (2.28) in (2.2), we obtain (Buras *et al.*, 2004a)

$$\text{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+[\tilde{r}^2 A^4 R_t^2 X^2(x_t) + 2\tilde{r}\bar{P}_c(X)A^2 R_t X(x_t) \times \cos\beta_{\text{eff}} + \bar{P}_c(X)^2], \qquad (3.1)$$

with $\beta_{\text{eff}} = \beta - \beta_s$, $\tilde{r}$ given in Eq. (2.27) and

$$\bar{P}_c(X) = \left(1 - \frac{\lambda^2}{2}\right)P_c(X). \qquad (3.2)$$

In the context of the unitarity triangle, the expression following from Eqs. (2.2) and (2.29) is also useful (Buras *et al.*, 1994),

$$\text{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \bar{\kappa}_+|V_{cb}|^4 X^2(x_t)\frac{1}{\sigma}[(\sigma\bar{\eta})^2 + (\varrho_c - \bar{\varrho})^2], \qquad (3.3)$$

where

$$\sigma = \left(\frac{1}{1 - (\lambda^2/2)}\right)^2. \qquad (3.4)$$

The measured value of $\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$ then determines an ellipse in the $(\bar{\varrho}, \bar{\eta})$ plane centered at $(\varrho_c, 0)$ (see Fig. 3) with

$$\varrho_c = 1 + \frac{\lambda^4 P_c(X)}{|V_{cb}|^2 X(x_t)} \qquad (3.5)$$

and having the squared axes

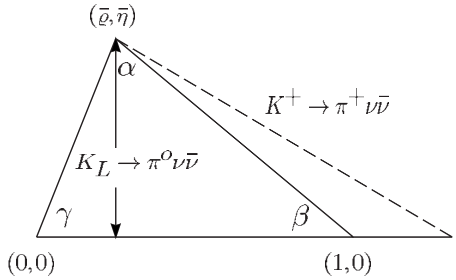

FIG. 3. Unitarity triangle from $K \to \pi\nu\bar{\nu}$.

$$\bar{\varrho}_1^2 = r_0^2, \quad \bar{\eta}_1^2 = \left(\frac{r_0}{\sigma}\right)^2, \tag{3.6}$$

where

$$r_0^2 = \left[\frac{\sigma \mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})}{\bar{\kappa}_+ |V_{cb}|^4 X^2(x_t)}\right]. \tag{3.7}$$

Note that r_0 depends only on the top contribution. The departure of ϱ_c from unity measures the relative importance of the internal charm contributions, $\varrho_c \approx 1.37$.

Then imposing the constraint from $|V_{ub}/V_{cb}|$ allows us to determine $\bar{\varrho}$ and $\bar{\eta}$ with

$$\bar{\varrho} = \frac{1}{1-\sigma^2}\left[\varrho_c - \sqrt{\sigma^2\varrho_c^2 + (1-\sigma^2)(r_0^2 - \sigma^2 R_b^2)}\right],$$

$$\bar{\eta} = \sqrt{R_b^2 - \bar{\varrho}^2}, \tag{3.8}$$

where $\bar{\eta}$ is assumed to be positive. Consequently,

$$R_t^2 = 1 + R_b^2 - 2\bar{\varrho}, \quad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}),$$

$$|V_{td}| = A\lambda^3 R_t. \tag{3.9}$$

The determination of $|V_{td}|$ and of the unitarity triangle in this way requires knowledge of $|V_{cb}|$ (or A) and of $|V_{ub}/V_{cb}|$. Both values are subject to theoretical uncertainties present in the existing analyses of tree level decays (Battaglia *et al.*, 2003). Whereas the dependence on $|V_{ub}/V_{cb}|$ is rather weak, the strong dependence of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ on A or $|V_{cb}|$, as seen in Eqs. (3.1) and (3.3), made in the past a precise prediction for this branching ratio and the construction of the UT difficult. With the more accurate value of $|V_{cb}|$ obtained recently (Battaglia *et al.*, 2003) and given in Eq. (2.48), the situation improved significantly. We return to this in Sec. IV. The dependence of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ on m_t is also strong. However, m_t is known already within $\pm 1\%$ and consequently the related uncertainty in $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ is substantially smaller than the corresponding uncertainty due to $|V_{cb}|$.

As $|V_{ub}/V_{cb}|$ is subject to theoretical uncertainties, a cleaner strategy is to use $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ in conjunction with β determined through the mixing-induced *CP* asymmetry $a_{\psi K_S}$. We investigate this strategy in the next section.

2. $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, β, $\Delta M_d/\Delta M_s$, or γ

Buchalla and Buras (1999) derived an upper bound on $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ within the SM. This bound depends only on $|V_{cb}|$, X, ξ, and $\Delta M_d/\Delta M_s$. With the precise value for the angle β now available, this bound can be turned into a useful formula for $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ (D'Ambrosio and Isidori, 2002) that expresses this branching ratio in terms of theoretically clean observables. In the SM and any MFV model, this reads

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \bar{\kappa}_+ |V_{cb}|^4 X^2 \left[\sigma R_t^2 \sin^2\beta + \frac{1}{\sigma}\left(R_t \cos\beta + \frac{\lambda^4 P_c(X)}{|V_{cb}|^2 X}\right)^2\right], \tag{3.10}$$

with σ defined in Eq. (3.4) and $\bar{\kappa}_+$ given in Eq. (2.5). It can be considered as the fundamental formula for a correlation between $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, β, and any observable used to determine R_t. This formula is theoretically very clean with the uncertainties residing only in $|V_{cb}|$, $P_c(X)$, and $\bar{\kappa}_+$. However, when one relates R_t to some observable new uncertainties could enter. Buchalla and Buras (1999) and D'Ambrosio and Isidori (2002) proposed to express R_t through $\Delta M_d/\Delta M_s$ by means of Eq. (2.45). This implies an additional uncertainty due to the value of ξ in Eq. (2.46).

Here we point out that if the strategy (β, γ) is used to determine R_t by means of Eq. (2.35), the resulting formula that relates $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, β, and γ is even cleaner than the one that relates $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, β, and $\Delta M_d/\Delta M_s$. We have then

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \bar{\kappa}_+ |V_{cb}|^4 X^2 \left[\sigma T_1^2 + \frac{1}{\sigma}\left(T_2 + \frac{\lambda^4 P_c(X)}{|V_{cb}|^2 X}\right)^2\right], \tag{3.11}$$

where

$$T_1 = \frac{\sin\beta \sin\gamma}{\sin(\beta+\gamma)}, \quad T_2 = \frac{\cos\beta \sin\gamma}{\sin(\beta+\gamma)}. \tag{3.12}$$

Similarly, the following formulas for R_t could be used in conjunction with Eq. (3.10):

$$R_t = \frac{\tilde{r}}{\lambda}\sqrt{\frac{\mathrm{Br}(B \to X_d\nu\bar{\nu})}{\mathrm{Br}(B \to X_s\nu\bar{\nu})}}, \tag{3.13}$$

$$R_t = \frac{\tilde{r}}{\lambda}\sqrt{\frac{\tau(B_s)}{\tau(B_d)}\frac{m_{B_s}}{m_{B_d}}}\left[\frac{F_{B_s}}{F_{B_d}}\right]\sqrt{\frac{\mathrm{Br}(B_d \to \mu^+\mu^-)}{\mathrm{Br}(B_s \to \mu^+\mu^-)}}, \tag{3.14}$$

with $\tilde{r}$ given in Eq. (2.26). In particular, Eq. (3.13) is essentially free of hadronic uncertainties (Buchalla and Isidori, 1998), and Eq. (3.14), not involving $\hat{B}_{B_s}/\hat{B}_{B_d}$, is a bit cleaner than Eq. (2.45).

C. $K_L \to \pi^0 \nu\bar{\nu}$, $\bar{\eta}$, Im λ_t, and the (β,γ) strategy

1. $\bar{\eta}$ and Im λ_t

Using Eqs. (2.19) and (2.28), we find

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \bar{r}^2 A^4 R_t^2 X^2(x_t) \sin^2 \beta_{\mathrm{eff}}. \tag{3.15}$$

In the context of the unitarity triangle, the expression following from Eqs. (2.19) and (2.29) is useful,

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) = \bar{\kappa}_L \eta^2 |V_{cb}|^4 X^2(x_t),$$

$$\bar{\kappa}_L = \frac{\kappa_L}{\lambda^8} = (3.39 \pm 0.03) \times 10^{-5} \tag{3.16}$$

from which $\bar{\eta} = \eta(1-\lambda^2/2)$ can be determined,

$$\bar{\eta} = 0.351 \sqrt{\frac{3.34 \times 10^{-5}}{\bar{\kappa}_L}\left[\frac{1.53}{X(x_t)}\right]} \times \left[\frac{0.0415}{|V_{cb}|}\right]^2 \sqrt{\frac{\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})}{3 \times 10^{-11}}}. \tag{3.17}$$

The determination of $\bar{\eta}$ in this manner requires knowledge of $|V_{cb}|$ and m_t. With an improved determination of these two parameters, useful determination of $\bar{\eta}$ should be possible.

On the other hand, the uncertainty due to $|V_{cb}|$ is not present in the determination of Im λ_t as (Buchalla and Buras, 1996)

$$\mathrm{Im}\,\lambda_t = 1.39 \times 10^{-4}\left[\frac{\lambda}{0.2248}\right]\sqrt{\frac{3.34 \times 10^{-5}}{\bar{\kappa}_L}\left[\frac{1.53}{X(x_t)}\right]} \times \sqrt{\frac{\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})}{3 \times 10^{-11}}}. \tag{3.18}$$

This formula offers the cleanest method to measure Im λ_t in the SM and all MFV models in which the function X takes generally different values than $X(x_t)$. This determination is even better than the one with the help of the *CP* asymmetries in B decays that require knowledge of $|V_{cb}|$ to determine Im λ_t. Measuring Br($K_L \to \pi^0 \nu\bar{\nu}$) with 10% accuracy allows us to determine Im λ_t with an error of 5% (Buchalla and Buras, 1996; Buchalla, Buras, and Leutenberger, 1996; Buras, 1998).

The importance of the precise measurement of Im λ_t is clear: the areas A_Δ of all unitarity triangles are equal and related to the measure of *CP* violation J_{CP} (Jarlskog, 1985a, 1985b),

$$|J_{CP}| = 2A_\Delta = \lambda\left(1 - \frac{\lambda^2}{2}\right)|\mathrm{Im}\,\lambda_t|. \tag{3.19}$$

2. A new "golden relation"

Next, in the spirit of the analysis by Buras (1994), we use the clean *CP* asymmetries in B decays and determine $\bar{\eta}$ through the (β,γ) strategy. Using Eqs. (2.31) and (2.35) in Eq. (3.17), we obtain a new "golden relation"

$$\frac{\sin\beta \sin\gamma}{\sin(\beta+\gamma)} = 0.351 \sqrt{\frac{3.34 \times 10^{-5}}{\bar{\kappa}_L}\left[\frac{1.53}{X(x_t)}\right]} \times \left[\frac{0.0415}{|V_{cb}|}\right]^2 \sqrt{\frac{\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})}{3 \times 10^{-11}}}. \tag{3.20}$$

This relation between β, γ, and Br($K_L \to \pi^0 \nu\bar{\nu}$) is clean and offers a test of the SM and of its extensions. Similarly to the golden relation in Eq. (1.1), it connects the observables in B decays with those in K decays. Moreover, it has the following two important virtues:

- It allows us to determine $|X|$,

$$|X| = F_1(\beta,\gamma,|V_{cb}|,\mathrm{Br}(K_L)) \tag{3.21}$$

  with $\mathrm{Br}(K_L) = \mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$. The analytic expression for the function F_1 can easily be extracted from Eq. (3.20).

- As $X(x_t)$ should be known with high precision once the error on m_t has been decreased, the relation (3.20) allows us to determine $|V_{cb}|$ with a remarkable precision (Buras, 1994),

$$|V_{cb}| = F_2(\beta,\gamma,X,\mathrm{Br}(K_L)). \tag{3.22}$$

The analytic formula for F_2 can easily be obtained from Eq. (3.20).

At first glance one could question the usefulness of determing $|V_{cb}|$ in this manner, since it is usually determined from tree level B decays. On the other hand, one should realize that one determines here the parameter A in the Wolfenstein parametrization that enters the elements V_{ub}, V_{cb}, V_{ts}, and V_{td} of the CKM matrix. Moreover, this determination of A benefits from the weak dependence on Br($K_L \to \pi^0 \nu\bar{\nu}$), which is only with a power of 0.25. The weak point of determining $|V_{cb}|$ in this way is contributions from new physics that could enter through the function X, whereas the standard determination of $|V_{cb}|$ through tree level B decays is free from this dependence. Still, a determination of $|V_{cb}|$, that in precision can almost compete with the usual tree diagram determinations and is theoretically cleaner, is clearly of interest within the SM.

D. Unitarity triangle from $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

The measurement of Br($K^+ \to \pi^+ \nu\bar{\nu}$) and Br($K_L \to \pi^0 \nu\bar{\nu}$) can determine the unitarity triangle completely (see Fig. 3), provided m_t and $|V_{cb}|$ are known (Buchalla and Buras, 1994b). Using these two branching ratios simultaneously allows us to eliminate $|V_{ub}/V_{cb}|$ from the analysis, which removes a considerable uncertainty in the determination of the UT, even if it is less important for $|V_{td}|$. Indeed, it is evident from Eqs. (2.2) and (2.19) that, given Br($K^+ \to \pi^+ \nu\bar{\nu}$) and Br($K_L \to \pi^0 \nu\bar{\nu}$), one can extract both Im λ_t and Re λ_t. One finds (Buchalla and Buras, 1994b; Buchalla, Buras, and Lautenbacher, 1996; Buras, 1998)

$$\mathrm{Im}\,\lambda_t = \lambda^5 \frac{\sqrt{B_2}}{X(x_t)},$$

$$\mathrm{Re}\,\lambda_t = -\lambda^5 \frac{\frac{\mathrm{Re}\,\lambda_c}{\lambda} P_c(X) + \sqrt{B_1 - B_2}}{X(x_t)}, \tag{3.23}$$

where we have defined the "reduced" branching ratios

$$B_1 = \frac{\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})}{\kappa_+}, \quad B_2 = \frac{\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})}{\kappa_L}. \tag{3.24}$$

Next using the expressions for $\mathrm{Im}\,\lambda_t$, $\mathrm{Re}\,\lambda_t$, and $\mathrm{Re}\,\lambda_c$ given in Eqs. (2.29) and (2.30), one finds

$$\bar{\varrho} = 1 + \frac{P_c(X) - \sqrt{\sigma(B_1 - B_2)}}{A^2 X(x_t)}, \quad \bar{\eta} = \frac{\sqrt{B_2}}{\sqrt{\sigma} A^2 X(x_t)} \tag{3.25}$$

with σ defined in Eq. (3.4). An exact treatment of the CKM matrix shows that the formulas (3.25), in particular the one for $\bar{\eta}$, are rather precise [Buchalla and Buras (1994b)].

E. sin 2β from $K \to \pi\nu\bar{\nu}$

Using Eq. (3.25), one finds subsequently (Buchalla and Buras, 1994b)

$$\sin 2\beta = \frac{2 r_s}{1 + r_s^2},$$

$$r_s = \sqrt{\sigma} \frac{\sqrt{\sigma(B_1 - B_2)} - P_c(X)}{\sqrt{B_2}} = \cot \beta. \tag{3.26}$$

Thus, within the approximation of Eq. (3.25), $\sin 2\beta$ is independent of V_{cb} (or A) and m_t, and as we see in Sec. IV, these dependences are fully negligible.

It should be stressed that $\sin 2\beta$ determined this way depends only on two measurable branching ratios and on the parameter $P_c(X)$, which is dominantly calculable in perturbation theory, as discussed in the preceding section. $P_c(X)$ contains a small nonperturbative contribution $\delta P_{c,u}$. Consequently, this determination is almost free from any hadronic uncertainties, and its accuracy can be estimated with a high degree of confidence. The recent calculation of NNLO QCD corrections to $P_c(X)$ improved significantly the accuracy of determing $\sin 2\beta$ from the $K \to \pi\nu\bar{\nu}$ complex.

Alternatively, combining Eqs. (3.1) and (3.15), one finds (Buras *et al.*, 2004a)

$$\sin 2\beta_{\mathrm{eff}} = \frac{2\bar{r}_s}{1 + \bar{r}_s^2}, \quad \bar{r}_s = \frac{\sqrt{B_1 - B_2} - \bar{P}_c(X)}{\sqrt{B_2}} = \cot \beta_{\mathrm{eff}}, \tag{3.27}$$

where $\beta_{\mathrm{eff}} = \beta - \beta_s$. As $\beta_s = \mathcal{O}(\lambda^2)$, we have

$$\cot \beta = \sigma \cot \beta_{\mathrm{eff}} + \mathcal{O}(\lambda^2) \tag{3.28}$$

and consequently one can verify that Eq. (3.27), while being slightly more accurate, is numerically close to Eq. (3.26). This formula turns out to be more useful than Eq. (3.26) when SM extensions with new complex phases in X are considered. We return to this in Sec. VII.

Finally, as in the SM and more generally in all MFV models there are no phases beyond the CKM phase, the MFV relation (1.1) should be satisfied. The confirmation of this relation would be an important test for the MFV idea. Indeed, in $K \to \pi\nu\bar{\nu}$ the phase β originates in the Z^0 penguin diagram, whereas in the case of $a_{\psi K_S}$ it originates in the B_d^0-$\bar{B}_d^0$ box diagram. We discuss the violation of this relation in particular new physics scenarios in Secs. VII and VIII.

F. The angle γ from $K \to \pi\nu\bar{\nu}$

We have seen that a precise value of β can be obtained both from the *CP* asymmetry $a_{\psi K_S}$ and from the $K \to \pi\nu\bar{\nu}$ complex in a theoretically clean manner. The determination of the angle γ is much harder. As discussed in Sec. IX and in Nir (2001), Fleischer (2002, 2004), Ali (2003), Buchalla (2003), and Hurth (2003), there are several strategies for γ in B decays but only few of them can be considered as theoretically clean. They all are experimentally challenging and a determination of γ with a precision of better than $\pm 5^\circ$ from these strategies alone will only be possible at LHCB and after a few years of running (Ball *et al.*, 2000; Anikeev *et al.*, 2001). A determination of γ with precision of $\pm(1-2)^\circ$ should be possible at Super-B (Super-B, 2007).

Here we point out that the $K \to \pi\nu\bar{\nu}$ decays offer a clean determination of γ that in accuracy can compete with the strategies in B decays, provided the uncertainties present in $|V_{cb}|$, in m_t, and in particular in m_c present in P_c, can be further reduced and the two branching ratios measured with an accuracy of 5%.

The relevant formula, which has not been presented in the literature so far, can be directly obtained from Eq. (3.25). It reads

$$\cot \gamma = \sqrt{\frac{\sigma}{B_2}} [A^2 X(x_t) - \sqrt{\sigma(B_1 - B_2)} + P_c(X)]. \tag{3.29}$$

We investigate it numerically in Sec. IV.

G. A second route to UT from $K \to \pi\nu\bar{\nu}$

Instead of using the formulas for $\mathrm{Im}\,\lambda_t$ and $\mathrm{Re}\,\lambda_t$ in Eq. (3.23), it is instructive to construct the UT by using Eq. (3.27) to find β and subsequently determine R_t from Eq. (3.1) with the result

TABLE III. Input for the determination of the branching ratios $Br(K^+\to\pi^+\nu\bar\nu)$ and $Br(K_L\to\pi^0\nu\bar\nu)$ in three scenarios. The corresponding $(\bar\varrho,\bar\eta)$ are also given.

	Scenario A	Scenario B
β	$(22.2\pm0.9)°$	$(22.2\pm0.5)°$
γ	$(64.6\pm4.2)°$	$(64.6\pm2.0)°$
$\lvert V_{cb}\rvert/10^{-3}$	41.6 ± 0.6	41.6 ± 0.3
R_b	0.381 ± 0.014	0.381 ± 0.007
m_t (GeV)	161 ± 1.7	161 ± 1.0
$P_c(X)$	0.41 ± 0.05	0.41 ± 0.02
$\bar\eta$	0.344 ± 0.016	0.344 ± 0.008
$\bar\varrho$	0.163 ± 0.028	0.163 ± 0.014

$$R_t=\frac{\sqrt{B_1-\bar P_c^2\sin^2\beta_{\rm eff}}-\bar P_c\cos\beta_{\rm eff}}{\tilde r A^2 X(x_t)}. \tag{3.30}$$

This (R_t,β) strategy by means of $K\to\pi\nu\bar\nu$ decays gives then $(\bar\varrho,\bar\eta)$ as given in Eq. (2.31) and in particular

$$\cot\gamma=\frac{1-R_t\cos\beta}{R_t\sin\beta}. \tag{3.31}$$

IV. NUMERICAL ANALYSIS IN THE SM

A. Introducing scenarios

In our numerical analysis, we consider various scenarios for the CKM elements and the values of the branching ratios $Br(K^+\to\pi^+\nu\bar\nu)$ and $Br(K_L\to\pi^0\nu\bar\nu)$ that should be measured in the future. In choosing the values of these branching ratios, we are guided in this section by their values predicted in the SM. We consider then the following:

- Scenario A for the present elements of the CKM matrix and a future scenario B with improved elements of the CKM matrix and the improved value of P_c through the reduction in the error of m_c and α_s. They are summarized in Table III. The accuracy of β in Table III corresponds to the error in $\sin 2\beta$ of ±0.023 for scenario A and ±0.013 for scenario B. It should be achieved, respectively, at B factories and LHCB. As discussed by Boos *et al.* (2004), even at this level of experimental precision, theoretical uncertainties in the determination of β through $a_{\psi K_S}$ can be neglected. The accuracy of γ given in Table III in scenarios A and B can presumably be achieved through the clean tree diagram strategies in B decays that will only become effective at LHC and Super-B. We discuss them in Sec. IX.
- Scenarios I and II for the measurements of $Br(K^+\to\pi^+\nu\bar\nu)$ and $Br(K_L\to\pi^0\nu\bar\nu)$ that together with future values of $|V_{cb}|$, m_t, and P_c should allow the determination of the UT, that is, of the angles β and γ and of the sides R_b and R_t, from $K\to\pi\nu\bar\nu$ alone. These scenarios are summarized in Table IV. Scenario I corresponds to the first half of the next decade, while scenario II is more futuristic.

TABLE IV. Input for the determination of CKM parameters from $K\to\pi\nu\bar\nu$ in two scenarios.

	Scenario I	Scenario II
$Br(K^+\to\pi^+\nu\bar\nu)/10^{-11}$	8.0 ± 0.8	8.0 ± 0.4
$Br(K_L\to\pi^0\nu\bar\nu)/10^{-11}$	3.0 ± 0.3	3.0 ± 0.2
m_t (GeV)	161 ± 1.7	161 ± 1.0
$P_c(X)$	0.41 ± 0.05	0.41 ± 0.02
$\lvert V_{cb}\rvert/10^{-3}$	41.6 ± 0.6	41.6 ± 0.3

In the remainder of the review, we frequently refer to Tables III and IV indicating which observables listed there are used at a given time in our numerical calculations.

B. Branching ratios in the SM

With the CKM parameters of scenario A given in Table III, we find using Eqs. (2.2) and (2.19)

$$\begin{aligned}Br(K^+\to\pi^+\nu\bar\nu)_{\rm SM}&=(8.1\pm0.6_{P_c}\pm0.5)\times10^{-11}\\&=(8.1\pm1.1)\times10^{-11},\end{aligned} \tag{4.1}$$

$$Br(K_L\to\pi^0\nu\bar\nu)_{\rm SM}=(2.6\pm0.3)\times10^{-11}. \tag{4.2}$$

The parametric errors come from the CKM parameters and the value of m_t and have been added in quadrature. In the case of $Br(K_L\to\pi^0\nu\bar\nu)$, only parametric uncertainties matter. For $Br(K^+\to\pi^+\nu\bar\nu)$ in the SM (4.1), we additionally have the error due to $P_c(X)$, which was added linearly.

The central value of $Br(K^+\to\pi^+\nu\bar\nu)$ in Eq. (4.1) is below the central experimental value in Eq. (1.5), but within theoretical, parametric, and experimental uncertainties, the SM result is fully consistent with the data. We also observe that the error in $P_c(X)$ constitutes still a significant portion of the full error.

One of the main origins of the parametric uncertainties in both branching ratios is the value of $|V_{cb}|$. As pointed out by Kettell *et al.* (2004), with the help of ε_K the dependence on $|V_{cb}|$ can be eliminated. Indeed, from the expression for ε_K in Eq. (2.38) and

$$\frac{{\rm Im}\,\lambda_t}{{\rm Re}\lambda_t}=-\tan\beta_{\rm eff},\quad \beta_{\rm eff}=\beta-\beta_s, \tag{4.3}$$

which follows from Eq. (2.28), ${\rm Im}\,\lambda_t$ and ${\rm Re}\,\lambda_t$ can be determined subject mainly to the uncertainty in $\hat B_K$ that should be decreased through lattice simulations in the future. Note that β will soon be determined with high precision from the $a_{\psi K_S}$ asymmetry.

We next investigate what kind of predictions one will get in a few years when β and γ will be measured with high precision through theoretically clean strategies at

TABLE V. Values of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ in the SM in units of 10^{-11} obtained through various strategies.

Strategy	$\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ (10^{-11})	$\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ (10^{-11})
Scenario A	8.10 ± 1.11	2.64 ± 0.30
	$8.10\pm0.62_{P_c}\pm0.49$	
Scenario B	8.10 ± 0.52	2.64 ± 0.15
	$8.10\pm0.25_{P_c}\pm0.27$	

LHCB (Ball *et al.*, 2000; Anikeev *et al.*, 2001). As pointed out by Buras, Parodi, and Stocchi (2003), the use of β and γ is the most powerful strategy to get $(\bar{\varrho},\bar{\eta})$. With the input of scenario B of Table III, we find

$$\mathrm{Im}\,\lambda_t=(1.38\pm0.04)\times10^{-4},$$

$$\mathrm{Re}\,\lambda_t=-(3.19\pm0.07)\times10^{-4}\quad(\text{scenario B}). \tag{4.4}$$

The results for the branching ratios in this scenario are given in Table V, where we have separated the error due to P_c from the parametric uncertainties.

In Table VI, we present the anatomy of parametric uncertainties given in Table V. Adding these uncertainties in quadrature gives the values in Table V. We observe that $|V_{cb}|$ plays a prominent role in these uncertainties.

Finally in Fig. 4 we show $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ as a function of γ for different values of β and $|V_{cb}|$. We observe that the dependence on β is rather weak, while the dependence on γ is very strong. Also the dependence on $|V_{cb}|$ is significant. This implies that a precise measurement of γ one day will also have a large impact on the prediction for $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$.

C. Impact of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ on the UT

1. Preliminaries

We then reverse the analysis and investigate the impact of present and future measurements of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ on $|V_{td}|$ and on the UT. To this end, one can take as additional inputs the values of $|V_{cb}|$ and β. One finds immediately that a precise value of $|V_{cb}|$ is now required in order to obtain a satisfactory result for $(\bar{\varrho},\bar{\eta})$. Indeed, $K\to\pi\nu\bar{\nu}$ decays are an excellent means to determine $\mathrm{Im}\,\lambda_t$ and $\mathrm{Re}\,\lambda_t$ or equivalently the *sd* unitarity triangle and in this respect have no competition from any B decay, but in order to construct the standard *bd* triangle of Fig. 2 from these decays, $|V_{cb}|$ is required. Here the *CP* asymmetries in B decays measuring directly angles of the UT are superior as the value of $|V_{cb}|$ is not required. Consequently, the precise value of $|V_{cb}|$ is of utmost importance if we make useful comparisons between various observables in K and B decays. On the other hand, in some relations such as Eq. (1.1), the $|V_{cb}|$ dependence is absent to an excellent accuracy.

2. $|V_{td}|$ from $K^+\to\pi^+\nu\bar{\nu}$

Taking the present experimental value of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ in Eq. (1.5), we determine first the UT side R_t and next the CKM element $|V_{td}|$. Using then the accurate expression for $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ in Eq. (3.10) and the values of $|V_{cb}|$ and β in the present scenario A of Table III, we find

$$R_t=1.35\pm0.70,\quad |V_{td}|=(12.6\pm6.6)\times10^{-3}, \tag{4.5}$$

where the dominant error arises due to the error in the branching ratio. The central values obtained here are large compared to the SM ones, but in view of the large errors one cannot say anything conclusive yet.

We consider then scenarios I and II of Table IV but do not take yet the values for $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ into account. As an additional variable, we take β or R_b in the scenario B of Table III. In Table VII, we give the values of R_t and $|V_{td}|$ resulting from this exercise. The precise value of β or R_b does not matter much in the determination of R_t and $|V_{td}|$, which is evident from the inspection of the $(\bar{\varrho},\bar{\eta})$ plot. This is also the reason why with the assumed errors on β and R_b, the two exercises in Table VII give essentially the same results.

In order to judge the precision achievable in the future, it is instructive to show the separate contributions of the uncertainties involved. In general, $|V_{td}|$ is subject to various uncertainties of which the dominant ones are given below,

$$\frac{\sigma(|V_{td}|)}{|V_{td}|}=\pm0.39\frac{\sigma(P_c)}{P_c}\pm0.70\frac{\sigma(\mathrm{Br}(K^+))}{\mathrm{Br}(K^+)}\pm\frac{\sigma(|V_{cb}|)}{|V_{cb}|}. \tag{4.6}$$

We find then

$$\frac{\sigma(|V_{td}|)}{|V_{td}|}=\pm5.0\%_{P_c}\pm7.0\%_{\mathrm{Br}(K^+)}\pm1.4\%_{|V_{cb}|}\quad(\text{scenario I}) \tag{4.7}$$

and

TABLE VI. The anatomy of parametric uncertainties in $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ corresponding to the results of Table V.

Strategy	$\sigma\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ (10^{-11})	$\sigma\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ (10^{-11})
Scenario A	$\pm0.33_{\bar{\varrho}}\pm0.06_{\bar{\eta}}\pm0.33_{\vert V_{cb}\vert}\pm0.13_{m_t}$	$\pm0.25_{\bar{\eta}}\pm0.16_{\vert V_{cb}\vert}\pm0.06_{m_t}$
Scenario B	$\pm0.17_{\bar{\varrho}}\pm0.03_{\bar{\eta}}\pm0.16_{\vert V_{cb}\vert}\pm0.08_{m_t}$	$\pm0.12_{\bar{\eta}}\pm0.08_{\vert V_{cb}\vert}\pm0.04_{m_t}$

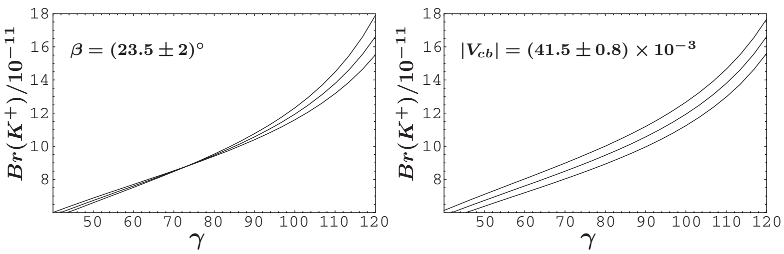

FIG. 4. $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ as a function of γ for different values of β and $|V_{cb}|$.

$$\frac{\sigma(|V_{td}|)}{|V_{td}|}=\pm2.0\%_{P_c}\pm3.5\%_{\mathrm{Br}(K^+)}\pm1.0\%_{|V_{cb}|}\quad(\text{scenario II}). \tag{4.8}$$

Adding the errors in quadrature, we find that $|V_{td}|$ can be determined with an accuracy of ±8.7% and ±4.2%, respectively. These numbers are increased to ±9.2% and ±4.3% once the uncertainties due to m_t, α_s, and β (or $|V_{ub}/V_{cb}|$) are taken into account. As a measurement of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ with a precision of 5% is challenging, the determination of $|V_{td}|$ with an accuracy better than ±5% from $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ seems difficult from the present perspective.

3. Impact on UT

The impact of $K^+\to\pi^+\nu\bar{\nu}$ on the UT is illustrated in Fig. 5, where we show lines corresponding to several selected values of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$. The construction of the UT from both decays shown there is described below.

D. Impact of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ on the UT

1. $\bar{\eta}$ and Im λ_t

We consider next the impact of a future measurement of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ on the UT. As discussed in the previous section, this measurement will offer theoretically clean determinations of $\bar{\eta}$ and in particular of Im λ_t. The relevant formulas are given in Eqs. (3.17) and (3.18), respectively. Using scenarios I and II of Table IV, we find

$$\bar{\eta}=0.367\pm0.019,$$

$$\mathrm{Im}\,\lambda_t=(1.47\pm0.07)\times10^{-4}\quad(\text{scenario I}), \tag{4.9}$$

$$\bar{\eta}=0.367\pm0.013,$$

$$\mathrm{Im}\,\lambda_t=(1.47\pm0.05)\times10^{-4}\quad(\text{scenario II}). \tag{4.10}$$

The obtained precision in the case of scenario II is truly impressive. We stress the clean character of these determinations.

2. Completing the determination of the UT

In order to construct the UT, we need still another input. It could be β, γ, R_b, or R_t. It turns out that the most effective in this determination is γ, as in the classification of Buras, Parodi, and Stocchi (2003) the $(\bar{\eta},\gamma)$ strategy belongs to the top class together with the (β,γ) pair. The angle γ should be known with high precision in five years. Still it is of interest to see what one finds when β is used instead of γ. R_b is not useful here as it generally gives two solutions for the UT.

In analogy to Table VII, we show in Table VIII the values of $\bar{\varrho}$ and $|V_{td}|$ resulting from scenarios I and II without using $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$. As an additional variable, we use β or γ. We observe that, with the assumed errors on β and γ, the use of γ is more effective than the use of β. Moreover, while going from scenario I to II for $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ has a significant impact when β is used, the impact is rather small when γ is used instead. Both features are consistent with the observations made by Buras, Parodi, and Stocchi (2003) in the context of $(\beta,\bar{\eta})$ and $(\gamma,\bar{\eta})$ strategies. In particular, the last feature is directly related to the fact that γ is larger by a factor of 3 than β.

The main message from Table VIII is that, using a rather precise value of γ, a precise determination of $|V_{td}|$ becomes possible, where the branching fraction of $K_L\to\pi^0\nu\bar{\nu}$ needs to be known only to about 10% accuracy.

3. A clean and accurate determination of $|V_{cb}|$ and $|V_{td}|$

Next, combining β and γ with the values of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ and m_t, clean determination of $|V_{cb}|$ by means of Eq. (3.22) is possible. In turn also $|V_{td}|$ can be determined. In Table IX, we show the values of $|V_{cb}|$ and $|V_{td}|$

TABLE VII. The values for R_t and $|V_{td}|/10^{-3}$ (in parentheses) from $K^+\to\pi^+\nu\bar{\nu}$ for various cases.

	Scenario I	Scenario II
Scenario B (β)	0.897±0.086 (8.42±0.80)	0.897±0.056 (8.42±0.51)
Scenario B (R_b)	0.897±0.086 (8.42±0.80)	0.897±0.056 (8.42±0.51)

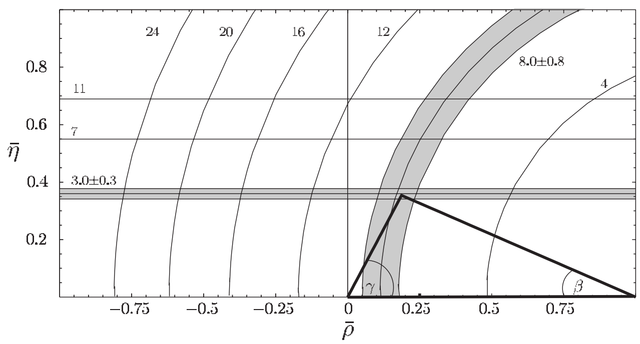

FIG. 5. The UT from $K \to \pi\nu\bar{\nu}$ in scenario I of Table IV. Lines corresponding to several values of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ (in units of 10^{-11}) are also shown.

obtained using scenarios I and II for $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ in Table IV with β and γ in scenario B of Table III.

We observe that the errors on $|V_{cb}|$ are larger than presently obtained from semileptonic B decays. But one should emphasize that this determination is essentially without any theoretical uncertainties. The high precision on $|V_{td}|$ is a result of a precise measurement of R_t by means of the (β, γ) strategy and a rather accurate value of $|V_{cb}|$ obtained with the help of $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$. Again also in this case the determination is theoretically clean.

E. Impact of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ on UT

Buchalla and Buras (1996) discussed the determination of the UT from both decays in explicit terms. The relevant formulas have been given in Sec. III. Here we confine our discussion to the determination of $\mathrm{Im}\,\lambda_t$, $\sin 2\beta$, and γ. We consider again two scenarios for which the input parameters are collected in Table IV. This time no other parameters beside those given in this table are required for the construction of the UT and the determination of these three quantities in question.

F. $\mathrm{Im}\,\lambda_t$ from $K_L \to \pi^0\nu\bar{\nu}$

As oposed to $\sin 2\beta$ and γ, only $K_L \to \pi^0\nu\bar{\nu}$ is relevant here. Using Eq. (3.18), we find that the error from m_t is roughly 1% and will soon be decreased even below that. Neglecting it, we find

$$\frac{\sigma(\mathrm{Im}\,\lambda_t)}{\mathrm{Im}\,\lambda_t} = \pm 0.5\frac{\sigma(\mathrm{Br}(K_L))}{\mathrm{Br}(K_L)} = \begin{cases} 5.0\% & \text{scenario I} \\ 3.3\% & \text{scenario II}, \end{cases} \tag{4.11}$$

which in the case of scenario II is already an impressive accuracy.

TABLE IX. The values for $|V_{cb}|$ and $|V_{td}|$ (in parentheses) in units of 10^{-3} from $K_L \to \pi^0\nu\bar{\nu}$, β and γ for various cases.

	Scenario I	Scenario II
Scenario B	43.1 ± 1.2 (8.23 ± 0.76)	43.1 ± 0.9 (8.23 ± 0.48)

G. The angle β from $K \to \pi\nu\bar{\nu}$

We next investigate the separate uncertainties in the determination of $\sin 2\beta$ coming from P_c, $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) \equiv \mathrm{Br}(K^+)$, and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) \equiv \mathrm{Br}(K_L)$. We find first

$$\frac{\sigma(\sin 2\beta)}{\sin 2\beta} = \pm 0.31\frac{\sigma(P_c)}{P_c} \pm 0.55\frac{\sigma(\mathrm{Br}(K^+))}{\mathrm{Br}(K^+)} \pm 0.39\frac{\sigma(\mathrm{Br}(K_L))}{\mathrm{Br}(K_L)}. \tag{4.12}$$

This leads to

$$\begin{aligned} \sigma(\sin 2\beta) &= 0.030_{P_c} + 0.041_{\mathrm{Br}(K^+)} + 0.029_{\mathrm{Br}(K_L)} \\ &= 0.080 \quad (\text{scenario I}) \end{aligned} \tag{4.13}$$

and

$$\begin{aligned} \sigma(\sin 2\beta) &= 0.011_{P_c} + 0.020_{\mathrm{Br}(K^+)} + 0.018_{\mathrm{Br}(K_L)} \\ &= 0.038 \quad (\text{scenario II}), \end{aligned} \tag{4.14}$$

where the errors have been added in quadrature apart from the one in P_c, which has been added linearly. The uncertainties due to $|V_{cb}|$ and m_t are fully negligible.

We observe the following:

- The uncertainty in $\sin 2\beta$ due to P_c alone amounted to 0.04 at NLO, implying that a NNLO calculation of P_c was desirable. On the other hand, now, at NNLO, the pure perturbative uncertainty in $\sin 2\beta$ amounts to ±0.006% (Buras, Gorbahn, Haisch, *et al.*, 2006) to be compared with ±0.025% at NLO
- The accuracy of the determination of $\sin 2\beta$, after the NNLO result became available, depends dominantly on the accuracy with which both branching ratios will be measured. In order to decrease $\sigma(\sin 2\beta)$ down to 0.02, they have to be measured with an accuracy better than 5%. Also, the reduction of the error in m_c relevant for P_c would be desirable.

TABLE VIII. The values for $\bar{\varrho}$ and $|V_{td}|/10^{-3}$ (in parentheses) from $K_L \to \pi^0\nu\bar{\nu}$ for various cases.

	Scenario I	Scenario II
Scenario B (β)	0.101 ± 0.052 (9.12 ± 0.51)	0.101 ± 0.040 (9.12 ± 0.37)
Scenario B (γ)	0.174 ± 0.018 (8.49 ± 0.16)	0.174 ± 0.017 (8.49 ± 0.16)

H. The angle γ from $K\to\pi\nu\bar{\nu}$

We next investigate, in analogy to Eq. (4.12), the separate uncertainties in the determination of γ coming from P_c, $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$, $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$, and $|V_{cb}|$. The relevant expression for γ in terms of these quantities is given in Eq. (3.29). We find then

$$\frac{\sigma(\gamma)}{\gamma} = \pm 0.75\frac{\sigma(P_c)}{P_c} \pm 1.32\frac{\sigma(\mathrm{Br}(K^+))}{\mathrm{Br}(K^+)} \pm 0.07\frac{\sigma(\mathrm{Br}(K_L))}{\mathrm{Br}(K_L)} \pm 4.11\frac{\sigma(|V_{cb}|)}{|V_{cb}|} \pm 2.34\frac{\sigma(m_t)}{m_t}. \tag{4.15}$$

This gives

$$\sigma(\gamma) = 5.7^\circ_{P_c} + 8.2^\circ_{\mathrm{Br}(K^+)} + 0.4^\circ_{\mathrm{Br}(K_L)} + 3.7^\circ_{|V_{cb}|} + 1.5^\circ_{m_t} = 19.6^\circ \tag{4.16}$$

and

$$\sigma(\gamma) = 2.3^\circ_{P_c} + 4.1^\circ_{\mathrm{Br}(K^+)} + 0.3^\circ_{\mathrm{Br}(K_L)} + 1.9^\circ_{|V_{cb}|} + 0.9^\circ_{m_t} = 9.4^\circ \tag{4.17}$$

for scenarios I and II, respectively, where the errors have been added in quadrature.

We observe the following:

- The uncertainty in γ due to P_c alone amounted to 8.6° at the NLO level, implying that a NNLO calculation of P_c was desirable. The pure perturbative uncertainty in γ amounts to ±1.2% at NNLO, compared to ±4.9% at NLO. Again, the reduction of the error in m_c relevant for P_c would be desirable.
- The dominant uncertainty in the determination of γ in scenarios I and II besides the one of P_c resides in $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$. In order to lower $\sigma(\gamma)$ below 5°, a measurement of this branching ratio with an accuracy of better than 5% is required. The measurement of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ has only a small impact on this determination.

I. Summary

In this section, we have presented a detailed numerical analysis of the formulas in Sec. III. First working in two scenarios, A and B, for the input parameters that should be measured precisely through B physics observables in this decade, we have shown how the accuracy of the predictions on the branching ratios will improve with time.

In the case of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$, there are essentially no theoretical uncertainties and the future accuracy of the prediction on this branching ratio within the SM depends fully on the accuracy with which $\mathrm{Im}\,\lambda_t$ and m_t can be determined from other processes. We learn from Table V that the present error of roughly 12% will be decreased to 6% when scenario B is be realized. As seen in Table VI, the progress on the error on $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ will depend on the progress on $|V_{cb}|$.

The case of $K^+\to\pi^+\nu\bar{\nu}$ is a bit different as now also the uncertainty in P_c enters. As discussed in Sec. II, this uncertainty comes, on the one hand, from the scale uncertainty and, on the other hand, from the error in m_c. The scale uncertainty dominated at NLO while the error on m_c is mainly responsible for the present error in P_c after NNLO has been completed. Formula (2.18) quantifies this explicitly. The anatomy of parametric uncertainties in $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ is presented in Table VI. As in the case of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$, here the reduction of the error in $|V_{cb}|$ will also be important.

As seen in Table V, the present error in $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ due to P_c amounts roughly to ±8%, which is smaller roughly by a factor of 1.5 than before the NNLO results for P_c where available. It is also seen in this table that in order to benefit from the improved values of the CKM parameters and of m_t, the uncertainty in P_c also has to be reduced through the improvement of m_c. It appears to us that the present error of 8% due to P_c could be decreased to 3% one day with the present total error of 14% reduced to 7%.

In the main part of this section, we have investigated the impact of the future measurements of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ on the determination of the CKM matrix. The results are self-explanatory and demonstrate that the $K\to\pi\nu\bar{\nu}$ decays offer powerful means in the determination of the UT and of the CKM matrix.

Clearly, future determination of various observables by means of $K\to\pi\nu\bar{\nu}$ will depend crucially on the accuracy with which $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ can be measured. Our discussion shows that it is certainly desirable to measure both branching ratios with an accuracy of at least 5%.

On the other hand, the uncertainties due to P_c, $|V_{cb}|$, and to a lesser extent m_t are also important ingredients of these investigations.

V. A GUIDE TO SECS. VI–VIII

Until now our discussion was confined to the SM. In the next three sections, we discuss the decays $K\to\pi\nu\bar{\nu}$ in various extensions of the SM.

In the case of most K and B meson decays, the effective Hamiltonian in the extensions of the SM becomes generally much more complicated than in the SM in that new operators, new complex phases, and new one-loop short-distance functions and generally new flavor violating couplings can be present. A classification of various possible extensions of the SM from the point of view of an effective Hamiltonian and valid for all decays can be found in Buras (2005a).

As emphasized at the beginning of this review in the case of $K\to\pi\nu\bar{\nu}$, the effective Hamiltonian in essentially all extensions of the SM is found from $\mathcal{H}_{\mathrm{eff}}^{\mathrm{SM}}$ in Eq. (2.1) by replacing $X(x_t)$ as follows [Buras *et al.* (1998)]:

$$X(x_t) \to X = |X| e^{i\theta_X}. \tag{5.1}$$

Thus, the only effect of new physics is to modify the magnitude of the SM function $X(x_t)$ and/or introduce a new complex phase θ_X that vanishes in the SM.

Clearly, the simplest class of extensions are models with minimal flavor violation in which $\theta_X = 0$, π and $|X|$ is only modified by loop diagrams with new particle exchanges but the driving mechanism of flavor and CP violation remains to be the CKM matrix. As in this class of models the basic structure of effective Hamiltonians in other decays is unchanged relative to the SM and only modifications in the one-loop functions, analogous to X, are allowed, the correlations between $K \to \pi\nu\bar{\nu}$ and other K and, in particular, B decays, valid in the SM remain true. A review of these correlations has been given by Buras (2003).

In the following section, we summarize the present status of $K \to \pi\nu\bar{\nu}$ in the models with MFV. As we show, the recently improved bounds on rare B decays, combined with the correlations in question, do not allow for a large departure of $K \to \pi\nu\bar{\nu}$ from the SM within this simplest class of new physics.

Much more spectacular effects in $K \to \pi\nu\bar{\nu}$ are still possible in models in which the phase θ_X is large. We discuss this in Sec. VII in a model-independent manner. We also discuss situations in which simultaneously to $\theta_X \neq 0$, also new complex phases in B_d^0-$\bar{B}_d^0$ mixing are present, and illustrate how these new phases, including θ_X, could be extracted from future data.

While Secs. VI and VII have a more model-independent character and analyze implications of the replacement (5.1) with arbitrary $|X|$ and θ_X, Sec. VIII can be considered as a guide to the literature on the new physics effects in $K \to \pi\nu\bar{\nu}$. In particular, we discuss the littlest Higgs model with T parity, Z' models, the MSSM with MFV, general supersymmetric models, models with universal extra dimensions, and models with lepton flavor mixing. Finally, we comment on essentially all new physics analyses done until the summer of 2007.

VI. $K \to \pi\nu\bar{\nu}$ and MFV

A. Preliminaries

A general discussion of the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ in the framework of minimal flavor violation (MFV) has been presented by Buras and Fleischer (2001). Earlier papers in specific MFV scenarios like two Higgs doublet can be found in Belanger *et al.* (1992) and Cho (1998), where additional references are given. We recall that in almost all extensions of the SM, the effective Hamiltonian for $K \to \pi\nu\bar{\nu}$ decays involves only the $(V\text{-}A) \otimes (V\text{-}A)$ operator of Eq. (2.1), and consequently for these decays there is no distinction between the constrained MFV (CMFV) (Buras *et al.*, 2001b; Blanke *et al.*, 2006) and more general formulation of MFV (D'Ambrosio *et al.*, 2002) in which additional non-SM operators are present in certain decays. Consequently in MFV or CMFV, all formulas of Secs. II and III for $K \to \pi\nu\bar{\nu}$ remain valid except for the following:

- The function $X(x_t)$ is replaced by the real valued master function (Buras, 2003, 2005a, 2005b) $X(v)$ with v denoting collectively the parameters of a given MFV model.
- If the function $X(v)$ is allowed to also take negative values, the following replacements should effectively be made in all formulas of Secs. II and III (Buras and Fleischer, 2001):

$$X \to |X|, \quad P_c(X) \to \mathrm{sgn}(X) P_c(X). \tag{6.1}$$

Here we assume also that the B^0-$\bar{B}^0$ function $S(v) > 0$, as in the SM. In fact, as found recently by Altmannshofer *et al.* (2007) and Blanke and Buras (2007), in all models with CMFV, $S(v) > S(v)_{\mathrm{SM}}$. On the other hand, we allow first for negative values of the function $X(v)$. The values of $X(v)$ and $S(v)$ can be calculated in any MFV model.

B. $K^+ \to \pi^+\nu\bar{\nu}$ versus $K_L \to \pi^0\nu\bar{\nu}$

An important consequence of Eqs. (3.26) and (1.1) is the following MFV relation (Buras and Fleischer, 2001):

$$B_1 = B_2 + \left(\frac{\cot\beta\sqrt{B_2} + \mathrm{sgn}(X)\sqrt{\sigma}P_c(X)}{\sigma} \right)^2, \tag{6.2}$$

which, for a given $\sin 2\beta$ extracted from $a_{\psi K_S}$ and $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, allows us to predict $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$. We observe that in the full class of MFV models, independent of any new parameters present in these models, only two values for $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$, corresponding to two signs of X, are possible. Consequently, measuring $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ will either select one of these two possible values or rule out all MFV models. In fact, the recent analysis of Haisch and Weiler (2007) showed that $X < 0$ is basically ruled out and as $X > 0$ gives larger branching ratios for the same $|X|$, we will therefore not consider $X < 0$ any further.

Buras and Fleischer (2001) presented a detailed numerical analysis of the relation (6.2). In view of the improved data on $\sin 2\beta$ and $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, we updated and extended this analysis. This is shown in Fig. 6, where we show $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ as a function of $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ for several values of $a_{\psi K_S}$. These plots are universal for all MFV models.

We also observe, as in Buras and Fleischer (2001), that the upper bound on $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ following from the data on $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\sin 2\beta \leqslant 0.719$ is substantially stronger than the model-independent bound following from isospin symmetry (Grossman and Nir, 1997),

$$\mathrm{Br}(K_{\mathrm{L}} \to \pi^0\nu\bar{\nu}) < 4.4 \mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}). \tag{6.3}$$

With the data in Eq. (1.5), which imply

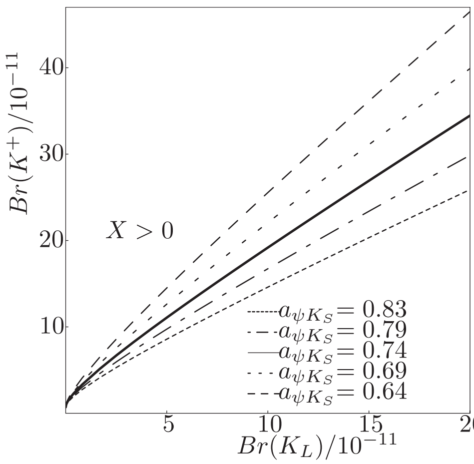

FIG. 6. $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ as a function of $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ for several values of $a_{\psi K_S}$ in the case of $\mathrm{sgn}(X)=1$.

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) < 3.8 \times 10^{-10} \quad (90\ \%\ \mathrm{C.L.}), \qquad (6.4)$$

one finds from Eq. (6.3)

$$\mathrm{Br}(K_\mathrm{L} \to \pi^0 \nu\bar{\nu}) < 1.7 \times 10^{-9} \quad (90\ \%\ \mathrm{C.L.}), \qquad (6.5)$$

which is still two orders of magnitude lower than the upper bound from the KTeV experiment at Fermilab (Blucher, 2005), yielding $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) < 2.9 \times 10^{-7}$ and the bound from KEK, $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) < 2.1 \times 10^{-7}$ (Ahn *et al.*, 2006).

On the other hand, taking the experimental bound $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ in Eq. (1.5) and $a_{\psi K_S} \leqslant 0.719$, we find from Eq. (6.2)

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{MFV}} \leqslant 2.0 \times 10^{-10}, \quad \mathrm{sgn}(X) = +1. \qquad (6.6)$$

Bobeth *et al.* (2005) performed a detailed analysis of several branching ratios for rare K and B decays in MFV models. Using the presently available information on the UUT, summarized by Bona *et al.* (2006b), and from the measurements of $\mathrm{Br}(B \to X_s \gamma)$, $\mathrm{Br}(B \to X_s l^+ l^-)$, and $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$, the upper bounds on various branching ratios within the CMFV scenario have been found. Recently this analysis was updated and generalized to include constraints from the observables in $Z \to b\bar{b}$ decay (Haisch and Weiler, 2007). The results of this analysis are collected in Table X together with the results within the SM.

Finally, anticipating that the leading role in constraining this kind of physics will eventually be taken over by $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, and $B_{s,d} \to \mu^+ \mu^-$, which are dominated by the function $C(v)$, Bobeth *et al.* (2005) and Haisch and Weiler (2007) provided plots for several branching ratios as functions of $C(v)$.

The main results from Bobeth *et al.* (2005) and Haisch and Weiler (2007) are the following:

The existing constraints coming from $K^+ \to \pi^+ \nu\bar{\nu}$, $B \to X_s \gamma$, $B \to X_s l^+ l^-$, and $Z \to b\bar{b}$ do not allow within the CMFV scenario of Buras *et al.* (2001b) for substantial departures of the branching ratios for all rare K and B decays from the SM estimates. This is evident from Table X.

This could be at first glance a rather pessimistic message. On the other hand, it implies that finding practically any branching ratio enhanced by more than a factor of 2 with respect to the SM will automatically signal either the presence of new *CP*-violating phases or new operators, strongly suppressed within the SM, at work. In particular, recalling that in most extensions of the SM the decays $K \to \pi \nu\bar{\nu}$ are governed by the single $(V\text{-}A) \otimes (V\text{-}A)$ operator, the violation of the upper bounds on at least one of the $K \to \pi \nu\bar{\nu}$ branching ratios will either signal the presence of new complex weak phases at work or new contributions that violate the correlations between the B decays and K decays.

As $a_{\psi K_S}$ in MFV models determines the true value of β and the true value of γ will be determined in tree level strategies in B decays one day, the true value of $\bar{\eta}$ can also be determined in a clean manner. Consequently, using Eq. (3.21) offers probably the cleanest measurement of $|X|$ in the field of weak decays.

VII. SCENARIOS WITH NEW COMPLEX PHASES IN $\Delta F = 1$ AND 2 TRANSITIONS

A. Preliminaries

In this section, we consider three simple scenarios beyond the framework of MFV, in which X becomes a

TABLE X. Bounds for various rare decays in CMFV models at 95% probability, the corresponding values in the SM at 68% and 95% CL, and the available experimental information (Haisch and Weiler, 2007b).

Observable	CMFV (95% CL)	SM (68% CL)	SM (95% CL)	Experiment
$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) \times 10^{11}$	[4.29, 10.72]	7.15 ± 1.28	[5.40, 9.11]	$(14.7^{+13.0}_{-8.9})$ (Anisimovsky *et al.*, 2004, 2007)
$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) \times 10^{11}$	[1.55, 4.38]	2.79 ± 0.31	[2.21, 3.45]	$< 2.1 \times 10^4$ (90% CL) (Ahn *et al.*, 2006)
$\mathrm{Br}(K_L \to \mu^+ \mu^-)_{\mathrm{SD}} \times 10^9$	[0.30, 1.22]	0.70 ± 0.11	[0.54, 0.88]	
$\mathrm{Br}(\bar{B} \to X_d \nu\bar{\nu}) \times 10^6$	[0.77, 2.00]	1.34 ± 0.05	[1.24, 1.45]	
$\mathrm{Br}(\bar{B} \to X_s \nu\bar{\nu}) \times 10^5$	[1.88, 4.86]	3.27 ± 0.11	[3.06, 3.48]	< 64 (90% CL) (Barate *et al.*, 2001)
$\mathrm{Br}(B_d \to \mu^+ \mu^-) \times 10^{10}$	[0.36, 2.03]	1.06 ± 0.16	[0.87, 1.27]	$< 3.0 \times 10^2$ (95% CL) (Bernhard, 2006)
$\mathrm{Br}(B_s \to \mu^+ \mu^-) \times 10^9$	[1.17, 6.67]	3.51 ± 0.50	[2.92, 4.13]	$< 5.8 \times 10^1$ (95% CL) (Maciel, 2007)

complex quantity as given in Eq. (1.7), and the universal box function $S(v)$ entering ε_K and $\Delta M_{d,s}$ not only becomes complex but generally becomes nonuniversal with

$$S_K(v) = |S_K(v)| e^{i2\varphi_K}, \quad S_d(v) = |S_d(v)| e^{i2\varphi_d},$$
$$S_s(v) = |S_s(v)| e^{i2\varphi_s} \tag{7.1}$$

for K^0-$\bar{K}^0$, B_d^0-$\bar{B}_d^0$, and B_s^0-$\bar{B}_s^0$ mixing, respectively. If these three functions are different from each other, some universal properties found in the SM and MFV models, which have been reviewed by Buras (2003, 2005a, 2005b), are lost. In addition, the mixing-induced CP asymmetries in B decays do not measure the angles of the UT but only sums of these angles and of φ_i. In particular,

$$S_{\psi K_S} = \sin(2\beta + 2\varphi_{B_d}). \tag{7.2}$$

Equally importantly, the rare K and B decays, governed in models with MFV by the real universal functions X, Y, and Z, are described now by nine complex functions ($i = K, d, s$) (Blanke *et al.*, 2007b)

$$X_i = |X_i| e^{i\theta_K^i}, \quad Y_i = |Y_i| e^{i\theta_Y^i}, \quad Z_i = |Z_i| e^{i\theta_Z^i} \tag{7.3}$$

that result from the SM box and penguin diagrams and analogous diagrams with new particle exchanges. In the SM and CMFV models, the independence of the functions in Eq. (7.3) of i implies strong correleations between various branching ratios in K, B_d, and B_s system and consequently strong upper bounds as shown in Table X. In models with new complex phases, this universality is generally broken and consequently, as we show in the next section, the bounds in Table X can be strongly violated.

As in the $K \to \pi\nu\bar{\nu}$ system, only one function is present. We drop the index i and denote it by

$$X = |X| e^{i\theta_X}. \tag{7.4}$$

In order to simplify the presentation, we assume here that $S_s = S_0(x_t)$ as in the SM but we take $S_d(v)$ to be complex with $S_d(v) \neq S_0(x_t)$. This will allow us to change the relation between R_t and $\Delta M_d / \Delta M_s$ in Eq. (2.45). We leave open whether $S_K(v)$ receives new physics contributions. We relax these assumptions in concrete models in the next section.

An example of general scenarios with new complex phases is the scenario in which new physics enters dominantly through enhanced Z^0 penguins involving a new CP-violating weak phase. It was first considered by Buras *et al.* (1998, 2000), Colangelo and Isidori (1998), Buras and Silvestrini (1999), and in the context of rare K decays and the ratio ε'/ε measuring direct CP violation in the neutral kaon system, and was generalized to rare B decays by Buchalla *et al.* (2001) and Atwood and Hiller (2003). Subsequently this particular extension of the SM has been revived by Buras *et al.* (2004a, 2004b), where it was pointed out that the anomalous behavior in $B \to \pi K$ decays observed by CLEO, BABAR, and Belle (Aubert *et al.* 2002b, 2003, 2004; Bornheim *et al.*, 2003; Chao *et al.*, 2004) could be due to the presence of enhanced Z^0 penguins carrying a large new CP-violating phase around $-90°$.

The possibility of important electroweak penguin contributions behind the anomalous behavior of the $B \to \pi K$ data has already been pointed out by Buras and Fleischer (2000), but only in 2005 has this behavior been independently observed by the three collaborations in question. Recent discussions related to electroweak penguins can be also found in Beneke and Neubert (2003) and Yoshikawa (2003). Other conjectures in connection with these data can be found in Gronau and Rosner (2003a, 2003b) and Chiang *et al.* (2004).

The implications of the large CP-violating phase in electroweak penguins for rare K and B decays and $B \to X_s l^+ l^-$ have been analyzed in detail by Buras *et al.* (2004a, 2004b) and subsequently the analyses of $B \to X_s l^+ l^-$ and $K_L \to \pi^0 l^+ l^-$ have been extended by Isidori *et al.* (2004) and Rai Choudhury *et al.* (2004), respectively. It turns out that in this scenario, several predictions differ significantly from the SM expectations with most spectacular effects found precisely in the $K \to \pi\nu\bar{\nu}$ system.

Meanwhile, the data on $B \to \pi K$ decays have changed considerably and the case for large electroweak penguin contributions in these decays is much less convincing (Gronau and Rosner, 2006; Baek and London, 2007; Fleischer, 2007; Fleischer *et al.*, 2007; Jain *et al.*, 2007; Silvestrini, 2007). Still the general formalism developed for the $K \to \pi\nu\bar{\nu}$ system in the presence of new complex phases (Buras, 1998; Buras *et al.*, 2004a, 2004b) remains valid and we discuss it below. Moreover, in the next section we discuss three explicit models: the Littlest Higgs model with T-parity (LHT), a Z' model, and the MSSM in which the function X becomes a complex quantity and departures of $K \to \pi\nu\bar{\nu}$ rates from the SM ones can be spectacular.

The scenarios with complex phases in B_d^0-$\bar{B}_d^0$ mixing have been considered by many Bertolini *et al.* (1987), Nir and Silverman (1990a, 1990b), Bergmann and Perez (2000, 2001), D'Ambrosio and Isidori (2002), Laplace (2002); Laplace *et al.* (2002), Fleischer *et al.* (2003), and Bona *et al.* (2006b).

Recently this scenario has been revived through the possible inconsistencies between UUT and the RUT signaled by the discrepancy between the value of $\sin 2\beta$ from $S_{\psi K_S}$ and its value obtained from tree-level measurements. We return to this issue below.

In what follows, we first review the formulas for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays obtained by Buras *et al.* (2004a, 2004b) for the case of a complex X. Subsequently, we discuss implications of this general scenario for the relevant branching ratios.

Next we consider scenarios with new physics present only in B_d^0-$\bar{B}_d^0$ mixing and the function X as in the SM. Here the impact on $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ comes only through modified values of the CKM parameters but, as we will show below, this impact is rather interesting.

Finally we consider a hybrid scenario with new physics entering both $K \to \pi\nu\bar{\nu}$ decays and B_d^0-$\bar{B}_d^0$ mixing. In this discussion, the (R_b, γ) strategy (RUT) for the determination of the UT will play an important role.

B. A large new *CP*-violating phase θ_X

In this general scenario, the function X becomes a complex quantity (Buras *et al.*, 1998), as given in Eq. (1.7), with θ_X a new complex phase that originates from new physics contributions to the relevant Feynman diagrams. Explicit realizations of such an extension of the SM will be discussed in Sec. VIII. In what follows, it is useful to define the following combination of weak phases:

$$\beta_X \equiv \beta - \beta_s - \theta_X = \beta_{\text{eff}} - \theta_X. \tag{7.5}$$

Following Buras *et al.* (2004a), the branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are given as follows:

$$\text{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+[\tilde{r}^2 A^4 R_t^2 |X|^2 + 2\tilde{r}\bar{P}_c(X)A^2 R_t |X| \times \cos\beta_X + \bar{P}_c(X)^2], \tag{7.6}$$

$$\text{Br}(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L \tilde{r}^2 A^4 R_t^2 |X|^2 \sin^2\beta_X, \tag{7.7}$$

with κ_+ given in Eq. (2.3), κ_L given in Eq. (2.20), $\bar{P}_c(X)$ defined in Eq. (3.2), β_X in Eq. (7.5), and $\tilde{r}$ in Eq. (2.27).

Once $\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\text{Br}(K_L \to \pi^0\nu\bar{\nu})$ have been measured, the parameters $|X|$ and β_X can be determined, subject to ambiguities that can be resolved by considering other processes, such as the nonleptonic B decays and rare decays discussed by Buras *et al.* (2004a). Combining Eqs. (7.6) and (7.7), the generalization of Eq. (3.27) to the scenario considered can be found (Buras *et al.*, 1998, 2004a),

$$\sin 2\beta_X = \frac{2\bar{r}_s}{1+\bar{r}_s^2}, \quad \bar{r}_s = \frac{\varepsilon_1\sqrt{B_1 - B_2} - \bar{P}_c(X)}{\varepsilon_2\sqrt{B_2}} = \cot\beta_X, \tag{7.8}$$

where $\varepsilon_i = \pm 1$. Moreover,

$$|X| = \frac{\varepsilon_2\sqrt{B_2}}{\tilde{r}A^2 R_t \sin\beta_X}, \quad \varepsilon_2 \sin\beta_X > 0. \tag{7.9}$$

The "reduced" branching ratios B_i are given in Eq. (3.24).

These formulas are valid for arbitrary $\beta_X \neq 0°$. For $\theta_X = 0°$ and $\varepsilon_1 = \varepsilon_2 = 1$, one obtains from Eq. (3.27) the SM result in Eq. (3.27). On the other hand, for $99° \leqslant \beta_X \leqslant 125°$ one has $\varepsilon_1 = -1$ and $\varepsilon_2 = 1$.

As in this scenario it is assumed that there are no significant contributions to $B_{s,d}^0$-$\bar{B}_{s,d}^0$ mixings and ε_K, in particular no complex phases, the determination of the CKM parameters through the standard analysis of the unitarity triangle proceeds as in the SM with the input parameters given in Sec. II.E. Consequently, β and β_s are already known from the usual analysis of the UT and the measurement of $\bar{r}_s$ in $K \to \pi\nu\bar{\nu}$ decays will provide a theoretically clean determination of θ_X and β_X. Similarly, a clean determination of $|X|$ can be obtained from Eq. (7.9), with R_t determined by means of Eq. (2.35).

It has been pointed out by Buras *et al.* (2004b) that in the case of $\beta_X \approx 90°$, in spite of the enhanced value of $|X|$, $\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$ does not significantly differ from the SM estimate because the enhancement of the first term in Eq. (7.6) can be compensated to a large extent by suppression of the second term $[\cos\beta_X \ll \cos(\beta - \beta_s)]$. Consequently, $\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$ in this case is strongly dominated by the "top" contribution given by the function X and charm-top interference is either small or even destructive.

On the other hand, $\beta_X \approx 90°$ implies a spectacular enhancement of $\text{Br}(K_L \to \pi^0\nu\bar{\nu})$ by one order of magnitude. Consequently, while $\text{Br}(K_L \to \pi^0\nu\bar{\nu}) \approx (1/3)\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$ in the SM, it is substantially larger than $\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$ in such a scenario. The large enhancement of $\text{Br}(K_L \to \pi^0\nu\bar{\nu})$ seen here is mainly due to the large weak phase β_X, as

$$\frac{\text{Br}(K_L \to \pi^0\nu\bar{\nu})}{\text{Br}(K_L \to \pi^0\nu\bar{\nu})_{\text{SM}}} = \left|\frac{X}{X_{\text{SM}}}\right|^2 \left[\frac{\sin\beta_X}{\sin(\beta - \beta_s)}\right]^2 \tag{7.10}$$

and to a lesser extent due to the enhanced value of $|X|$, which generally could be bounded by other processes.

Inspecting Eqs. (7.6) and (7.7), one observes (Buras *et al.*, 2004a) that the very strong dominance of the top contribution in these expressions implies a simple approximate expression,

$$\frac{\text{Br}(K_L \to \pi^0\nu\bar{\nu})}{\text{Br}(K^+ \to \pi^+\nu\bar{\nu})} \approx 4.4 \times (\sin\beta_X)^2 \approx 4.2 \pm 0.2. \tag{7.11}$$

We note that $\text{Br}(K_L \to \pi^0\nu\bar{\nu})$ is then close to its model-independent upper bound (Grossman and Nir, 1997) given in Eq. (6.3). It is evident from Eq. (7.8) that this bound is reached when the reduced branching ratios B_1 and B_2 in Eq. (3.24) are equal to each other.

A spectacular implication of such a scenario is a strong violation of the MFV relation (Buchalla and Buras, 1994b) in Eq. (1.1). Indeed, with $\beta_X \approx \pm 90°$,

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = \sin 2\beta_X \neq (\sin 2\beta)_{\psi K_S} = 0.675 \pm 0.026. \tag{7.12}$$

In the next section, we investigate this violation in two specific models. In Fig. 7, we show—in the spirit of the plot in Fig. 6—$\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$ as a function of $\text{Br}(K_L \to \pi^0\nu\bar{\nu})$ for fixed values of β_X that was presented by Buras *et al.* (2004a). As this plot is independent of $|X|$, it offers direct measurement of the phase β_X. The first line on the left represents the MFV models with $\beta_X = \beta_{\text{eff}} = \beta - \beta_s$, already discussed in Sec. VI, whereas the first line on the right corresponds to the model-independent Grossman-Nir bound (Grossman and Nir, 1997) given in Eq. (6.3). Note that the value of β_X corresponding to this

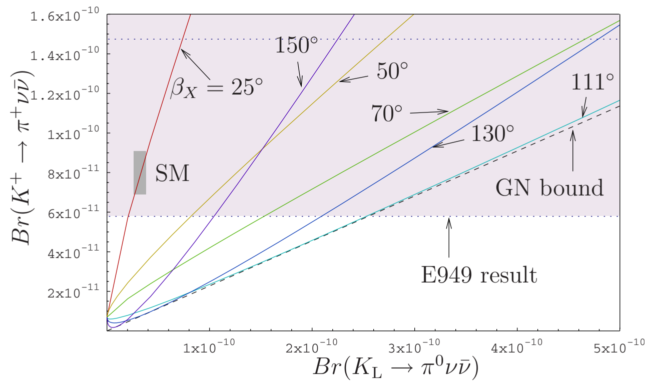

FIG. 7. (Color online) $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ as a function of $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ for various values of β_X (Buras *et al.*, 2004a). Dotted horizontal lines indicate the lower part of the experimental range (1.4) and the gray area the SM prediction. Also shown is the bound in Eq. (6.3).

bound depends on the actual value of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ since at this bound ($B_1 = B_2$) we have (Buras *et al.*, 2004a)

$$(\cot \beta_X)_{\mathrm{bound}} = -\frac{\bar{P}_c(X)}{\varepsilon_2\sqrt{B_2}}. \tag{7.13}$$

For the central values of $\bar{P}_c(X)$ and B_2 found in the latter paper, the bound corresponds to $\beta_X = 107.3°$. As only $\cot \beta_X$ and not β_X is directly determined by the values of the branching ratios in question, the angle β_X is determined only up to discrete ambiguities, seen already in Fig. 7. These ambiguities can be resolved by considering simultaneously other quantities discussed by Buras *et al.* (2004a).

C. General discussion of θ_X and $|X|$

In Fig. 8, we show the ratio of the two branching ratios in question as a function of β_X for three values of $|X| = 1.25$, 1.5, and 2.0. We observe that for β_X in the vicinity of 110°, this ratio is close to the bound in Eq. (6.3). However, even for $\beta_X = 50°$ the ratio is close to unity and by a factor of 3 higher than in the SM.

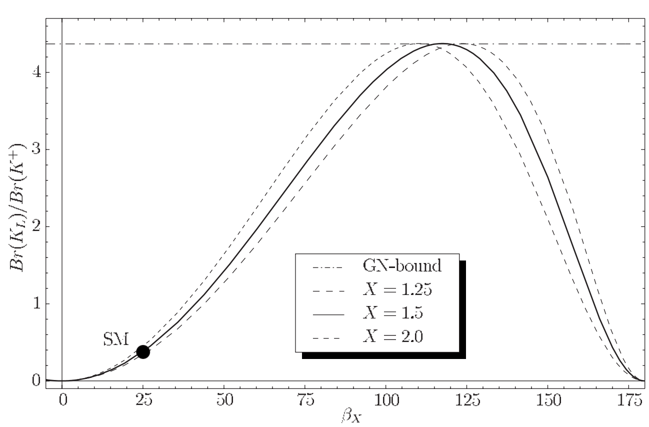

FIG. 8. The ratio of the $K \to \pi\nu\bar{\nu}$ branching ratios as a function of β_X for $|X| = 1.25$, 1.5, and 2.0. The horizontal line is the bound in Eq. (6.3).

TABLE XI. Values of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and of $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ (in parentheses) in units of 10^{-11} for different values of θ_X and $|X|$ with $\beta = 22.2°$ and $|V_{cb}| = 41.6 \times 10^{-3}$.

$\theta_X/|X|$	1.25	1.50	1.75	2.00	2.25
−90°	2.3	3.3	4.5	6.0	7.6
	(10.1)	(14.5)	(19.8)	(25.8)	(32.7)
−60°	3.8	5.0	6.5	8.3	10.2
	(12.1)	(17.4)	(23.6)	(30.9)	(39.1)
−30°	5.1	6.7	8.4	10.4	12.6
	(8.1)	(11.6)	(15.8)	(20.7)	(26.1)
0°	6.0	7.8	9.7	11.9	14.3
	(2.1)	(3.0)	(4.1)	(5.4)	(6.8)
30°	6.3	8.0	10.0	12.3	14.7
	(0.11)	(0.16)	(0.22)	(0.29)	(0.36)
60°	5.8	7.4	9.3	11.5	13.8
	(4.1)	(5.9)	(8.0)	(10.5)	(13.3)
90°	4.6	6.1	7.8	9.7	11.8
	(10.1)	(14.5)	(19.8)	(25.8)	(32.7)

Finally, in Table XI, we give the values of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ for different values of $|X|$ and θ_X, $\beta = 22.2°$, and $|V_{cb}| = 41.6 \times 10^{-3}$. In this context, we refer to scaling laws for FCNC processes pointed out by Buras and Harlander (1992), from which it follows that the dependence of $K \to \pi\nu\bar{\nu}$ branching ratios on $|V_{cb}|$ and $|X|$ is encoded in a single variable

$$Z = A^2|X|. \tag{7.14}$$

This observation allows us to make the following replacement in Table XI:

$$|X| \to |X|_{\mathrm{eff}} = \left[\frac{|V_{cb}|}{41.5 \times 10^{-3}}\right]^2 |X|, \tag{7.15}$$

so that for $|V_{cb}| \neq 41.6 \times 10^{-3}$ the results correspond to different values of $|X|$ obtained by rescaling the values for $|X|$ there by means of Eq. (7.15).

As beyond the SM the uncertainties in the value of $|X|$ are substantially larger than the ones in $|V_{cb}|$, the error in $|V_{cb}|$ can be absorbed into the one of $|X|_{\mathrm{eff}}$.

D. New complex phases in the B_d^0-$\bar{B}_d^0$ mixing

We next move to the scenario in which $X = X_{\mathrm{SM}}$, but there are new contributions to B_d^0-$\bar{B}_d^0$ mixing. This scenario has been considered in detail in many papers by (Bertolini *et al.*, 1987; Nir and Silverman, 1990a, 1990b; Bergmann and Perez, 2000, 2001; D'Ambrosio and Isidori, 2002; Laplace, 2002; Laplace *et al.*, 2002; Fleischer *et al.*, 2003). As summarized by Nir and Silverman, this scenario can be realized in supersymmetric models with (a) a heavy scale for the soft-breaking terms; (b) new sources of flavor symmetry breaking only in the soft-breaking terms, which do not involve the Higgs fields; and (c) Yukawa interactions similar to the SM case. However, as emphasized by Fleischer *et al.* (2003) and discussed in Sec. VIII, this scenario is not representative for all supersymmetric scenarios, in particular those with important mass insertions of the left-right type and Higgs mediated FCNC amplitudes with large $\tan\beta$. Nonsupersymmetric examples like littlest Higgs with T parity and Z' models can also provide new phase effects in B_d^0-$\bar{B}_d^0$ mixing, but generally such effects are simultaneously present in B_s^0-$\bar{B}_s^0$ mixing and $K\to\pi\nu\bar{\nu}$.

We recall that, in the presence of a complex function S_d, the off-diagonal term M_{12}^d in the neutral B_d^0 meson mass matrix has the phase structure

$$M_{12}^d = \frac{\langle B_d^0|H_{\rm eff}^{\Delta B=2}|\bar{B}_d^0\rangle}{2m_{B_d}} \propto e^{i2\beta}e^{i2\varphi_d}|S_d|, \tag{7.16}$$

with $|S_d|$ generally differing from $S_0(x_t)$. If S_s remains unchanged, then

- the asymmetry $a_{\psi K_S}$ does not measure β but $\beta+\varphi_d$ and
- the expression for R_t in Eq. (2.45) becomes

$$r_d R_t = 0.920\tilde{r}\left[\frac{\xi}{1.24}\right]\left[\frac{0.2248}{\lambda}\right]\sqrt{\frac{18.4/ps}{\Delta M_s}}\sqrt{\frac{\Delta M_d}{0.50/ps}},$$

$$r_d^2 \equiv \left|\frac{S_d}{S_0(x_t)}\right|. \tag{7.17}$$

As a consequence of these changes, the true angle β differs from the one extracted from $a_{\psi K_S}$ and also R_t and $|V_{td}|$ will be modified if $r_d\neq 1$.

As X is not modified with respect to the SM, the impact on $K\to\pi\nu\bar{\nu}$ amounts exclusively to the change of the true $\beta_{\rm eff}$ and R_t in the formulas (3.1) and (3.15). A particular pattern of a possible impact on $K\to\pi\nu\bar{\nu}$ in the scenario in question has been presented by Fleischer *et al.* (2003).

In the meantime, the data on the CP asymmetry $S_{\psi K_S}$ and the observables in $B_{s,d}^0$-$\bar{B}_{s,d}^0$ systems have improved so much that the allowed values for r_d and φ_{B_d} are strongly constrained. Also, there is now a slight tension between the values of $|V_{ub}|$ and $\sin 2\beta$ as inputed into the fits, potentially hinting toward some nonvanishing (negative) phase φ_{B_d} (Blanke *et al.*, 2006; Bona *et al.*, 2006c). However, since there are some open questions concerning the value of $|V_{ub}|$, it remains to be seen how this situation develops further. The implication of this for the $K\to\pi\nu\bar{\nu}$ decays is that, due to the higher value of $\bar{\eta}$ obtained from the RUT fit, the values for both branching ratios are larger than those found using CKM values from an overall fit of the unitarity triangle.

E. A hybrid scenario

The situation is more involved if new physics effects enter both X and S. Similarly to the previous two scenarios, the golden relation in Eq. (1.1) is violated, but now the structure of a possible violation is more involved,

$$[\sin 2(\beta-\theta_X)]_{\pi\nu\bar{\nu}} \neq [\sin 2(\beta+\varphi_d)]_{\psi K_S}. \tag{7.18}$$

Since θ_X originates in new contributions to the decay amplitude $K\to\pi\nu\bar{\nu}$ and θ_d in new contributions to the B_d^0-$\bar{B}_d^0$ mixing, it is likely that $\theta_X\neq\varphi_d$.

The most straightforward strategy to disentangle new physics contributions in $K\to\pi\nu\bar{\nu}$ and the B_d^0-$\bar{B}_d^0$ mixing in this scenario is to use the reference unitarity triangle that results from the (R_b,γ) strategy. Having the true CKM parameters at hand, one can determine θ_X and $|X|$ from $K\to\pi\nu\bar{\nu}$ and φ_d and $|S_d|$ from the B_d^0-$\bar{B}_d^0$ mixing and $a_{\psi K_S}$.

In order to illustrate these ideas in explicit terms, we investigate, in the remainder of this section, how the presence of new contributions in $K\to\pi\nu\bar{\nu}$ and the B_d^0-$\bar{B}_d^0$ mixing could be signaled in the $(\bar{\varrho},\bar{\eta})$ plane.

Beginning with $K\to\pi\nu\bar{\nu}$, we write

$$X = r_X X_{\rm SM} e^{i\theta_X}. \tag{7.19}$$

Then formulas (7.6) and (7.7) apply with

$$|X| \to X_{\rm SM}, \quad R_t \to r_X R_t. \tag{7.20}$$

We proceed then as follows:

- From the measured ${\rm Br}(K^+\to\pi^+\nu\bar{\nu})$ and ${\rm Br}(K_L\to\pi^0\nu\bar{\nu})$, we determine the "fake" angle β in the unitarity triangle with the help of Eq. (7.8). We denote this angle by β_X, which we defined in Eq. (7.5). In what follows we neglect β_s, but it can be taken straightforwardly into account if necessary.
- The height of the fake UT from $K\to\pi\nu\bar{\nu}$ is then given by

$$\bar{\eta}_{\pi\nu\bar{\nu}} = r_X R_t \sin\beta_X = \frac{\sqrt{B_2}}{\tilde{r}A^2 X_{\rm SM}}, \tag{7.21}$$

where we set $\varepsilon_2=+1$ in order to be concrete. As seen, this height can be found from ${\rm Br}(K_L\to\pi^0\nu\bar{\nu})$ and $X_{\rm SM}$.

Now we go to the B_d^0-$\bar{B}_d^0$ mixing, where we introduced the parameter r_d defined in Eq. (7.17). We proceed then as follows:

- The asymmetry $a_{\psi K_S}$ determines the fake angle β, which we denote by $\beta_d=\beta+\theta_d$.
- The fake side R_t, to be denoted by $(R_t)_d$, is given as follows:

$$(R_t)_d = r_d R_t. \tag{7.22}$$

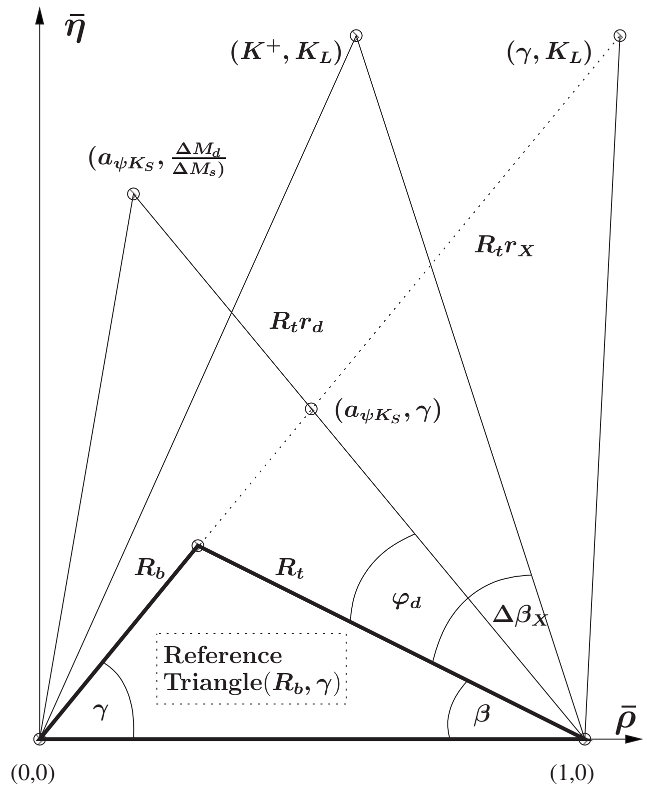

FIG. 9. Fake unitarity triangles compared to the reference triangle, $\Delta\beta_X = -\theta_X$.

It can be calculated from Eq. (7.17) subject to uncertainties in ξ.

Clearly, generally the fake UT's resulting from $K \to \pi\nu\bar{\nu}$ and the $(\Delta M_d/\Delta M_s, \beta)$ strategy, discussed above, will differ from each other, from the true reference triangle, and also from the UT obtained from the (γ, β) and $(\bar{\eta}, \gamma)$ strategies, if the determinations of $\bar{\eta}$ and β are affected by new physics.

We show these five different triangles in Fig. 9. Comparing the fake triangles with the reference triangle, all new physics parameters in $K \to \pi\nu\bar{\nu}$ and B_d^0-$\bar{B}_d^0$ mixing can be easily extracted. Figure 9 has only illustrative character. We know already from the recent analyses of the UT (Blanke *et al.*, 2006; Bona *et al.*, 2006c) that the phase φ_{B_d} is constrained to be much smaller than depicted in this figure. Moreover, a negative value seems to be favored.

F. Correlation between $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ and $\mathrm{Br}(B \to X_{s,d}\nu\bar{\nu})$

The branching ratios for the inclusive rare decays $B \to X_{s,d}\nu\bar{\nu}$ can be written in the models with a new complex phase in X as follows (Buras *et al.*, 2004a) $(q=d,s)$:

$$\mathrm{Br}(B \to X_q\nu\bar{\nu}) = 1.58 \times 10^{-5}\left[\frac{\mathrm{Br}(B \to X_c e\bar{\nu})}{0.104}\right] \times \left|\frac{V_{tq}}{V_{cb}}\right|^2 \left[\frac{0.54}{f(z)}\right] |X|^2, \tag{7.23}$$

where $f(z) = 0.54 \pm 0.04$ is the phase-space factor for $B \to X_c e\bar{\nu}$, with $z = m_c^2/m_b^2$, and $\mathrm{Br}(B \to X_c e\bar{\nu}) = 0.104 \pm 0.004$.

Formulas (7.7) and (7.23) imply interesting relations between the decays $K_L \to \pi^0\nu\bar{\nu}$ and $B \to X_{s,d}\nu\bar{\nu}$ that are generalizations of similar relations within the MFV models (Bergmann and Perez, 2000, 2001; Buras and Fleischer, 2001) to the scenario considered here,

$$\frac{\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})}{\mathrm{Br}(B \to X_s\nu\bar{\nu})} = \frac{\kappa_L}{1.58 \times 10^{-5}}\left[\frac{0.104}{\mathrm{Br}(B \to X_c e\bar{\nu})}\right] \times \left[\frac{f(z)}{0.54}\right] A^4 R_t^2 \sin^2\beta_X, \tag{7.24}$$

$$\frac{\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})}{\mathrm{Br}(B \to X_d\nu\bar{\nu})} = \frac{\kappa_L}{1.58 \times 10^{-5}}\left[\frac{0.104}{\mathrm{Br}(B \to X_c e\bar{\nu})}\right] \times \left[\frac{f(z)}{0.54}\right] \frac{A^4\bar{r}^2}{\lambda^2} \sin^2\beta_X. \tag{7.25}$$

The experimental upper bound on $\mathrm{Br}(B \to X_s\nu\bar{\nu})$ reads (Barate *et al.*, 2001)

$$\mathrm{Br}(B \to X_s\nu\bar{\nu}) < 6.4 \times 10^{-4} \quad (90\ \%\ \mathrm{C.L.}). \tag{7.26}$$

Using this bound and setting $R_t = 0.95$, $f(z) = 0.58$, and $\mathrm{Br}(B \to X_c e\bar{\nu}) = 0.10$, we find from Eq. (7.24) the upper bound

$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) \leq 4.4 \times 10^{-9}(\sin\beta_X)^2 = \begin{cases} 6.3 \times 10^{-10}, & \beta_X = 22.2° \\ 3.9 \times 10^{-9}, & \beta_X = 111° \end{cases} \tag{7.27}$$

at 90% C.L. for the MFV models and a scenario with a large new phase, respectively. In the case of the MFV models, this bound is weaker than the bound in Eq. (6.6) but, as the bound in Eq. (7.26) should be improved in the B-factory era, the situation could change in the coming years. Concerning the scenario with a complex phase θ_X of Sec. VII.B, no useful bound on $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ from Eq. (7.26) results at present as the bound in Eq. (7.27) is weaker than the model-independent bound in Eq. (6.5).

VIII. $K \to \pi\nu\bar{\nu}$ IN SELECTED NEW PHYSICS SCENARIOS

A. Preliminaries

In this section, we review the results for decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ in selected new physics scenarios. Our goal is to indicate the size of new physics contributions in the branching ratios in question. Due to several free parameters present in some of these extensions, actual predictions for the branching ratios are not precise and often depend sensitively on some parameters involved. The latter could then be determined or bounded efficiently once precise data on $K \to \pi\nu\bar{\nu}$ and other rare decays will be available. While we only present the results for $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$, most analyses discussed below used all available constraints from other observables known at the time of a given analysis. A detailed analysis of these constraints is clearly beyond the scope of this review. A general discussion of $K \to \pi\nu\bar{\nu}$ beyond the SM can be found in Grossman and Nir (1997). In writing this section, we also benefited from D'Ambrosio and Isidori (2002), Isidori (2003), and Bryman *et al.* (2006).

B. Littlest Higgs models

One of the most attractive solutions to the so-called little hierarchy problem that affects the SM is provided by little Higgs models. They are perturbatively computable up to ~10 TeV and have a rather small number of parameters, although their predictivity can be weakened by a certain sensitivity to the unknown ultraviolet (UV) completion of the theory. In these models, in contrast to supersymmetry, the problematic quadratic divergences to the Higgs mass are canceled by loop contributions of new particles with the same spin statistics of the SM ones and with masses around 1 TeV.

The basic idea of little Higgs models (Arkani-Hamed *et al.*, 2001) is that the Higgs model is naturally light as it is identified with a Nambu-Goldstone boson of a spontaneously broken global symmetry.

The most economical, in matter content, is the littlest Higgs (LH) model (Arkani-Hamed *et al.*, 2002), where the global group SU(5) is spontaneously broken into SO(5) at the scale $f \approx \mathcal{O}(1\ \text{TeV})$ and the electroweak sector of the SM is embedded in an SU(5)/SO(5) nonlinear sigma model. Gauge and Yukawa Higgs interactions are introduced by gauging the subgroup of SU(5): $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$. In the LH model, the new particles appearing at the TeV scales are the heavy gauge bosons $(W_H^{\pm}, Z_H, A_H)$, the heavy top (T), and the scalar triplet Φ.

In the original LH model (Arkani-Hamed *et al.*, 2002), the custodial SU(2) symmetry, of fundamental importance for electroweak precision studies, is unfortunately broken already at tree level, implying that the relevant scale of new physics f must be at least 2–3 TeV in order to be consistent with electroweak precision data (Csaki *et al.*, 2003; Han *et al.*, 2003a, 2003b; Hewett *et al.*, 2003; Chen and Dawson, 2004a, 2004b; Kilian and Reuter, 2004; Yue and Wang, 2004). As a consequence, contributions of new particles to FCNC processes turn out to be at most 10–20 % (Huo and Zhu, 2003; Buras *et al.*, 2005a, 2005b; Buras, Poschenrieder, Uhlig, *et al.*, 2006), which will not be easy to distinguish from the SM due to experimental and theoretical uncertainties. In particular, a detailed analysis of particle-antiparticle mixing in the LH model has been given by Buras *et al.* (2005b) and the corresponding analysis of rare K and B decays has recently been presented by Buras, Poschenrieder, Uhlig, *et al.* (2006).

More promising and more interesting from the point of view of FCNC processes is the littlest Higgs model with a discrete symmetry (T parity) (Cheng and Low, 2003, 2004) under which all new particles listed above, except T_+, are odd and do not contribute to processes with external SM quarks (T even) at tree level. As a consequence, the new physics scale f can be reduced to 1 TeV and even below it, without violating electroweak precision constraints (Hubisz, Meade, Noble, *et al.*, 2006).

A consistent and phenomenologically viable LHT requires introduction of three doublets of "mirror quarks" and three doublets of "mirror leptons" which are odd under T parity, transform vectorially under $SU(2)_L$, and can be given a large mass. Moreover, there is an additional heavy T_- quark that is odd under T parity (Low, 2004).

Mirror fermions are characterized by new flavor interactions with SM fermions and heavy gauge bosons, which involve in the quark sector two new unitary mixing matrices analogous to the CKM matrix (Chau and Keung, 1984; Hagiwara *et al.*, 2002). They are V_{Hd} and V_{Hu}, respectively, involved when the SM quark is of down- or up-type, and satisfying $V_{Hu}^{\dagger} V_{Hd} = V_{\text{CKM}}$ (Kobayashi and Maskawa, 1973). V_{Hd} contains three angles, like V_{CKM}, but three (non-Majorana) phases (Blanke, Buras, Duling, *et al.*, 2007), i.e., two additional phases relative to the SM matrices, that cannot be rotated away in this case.

Because of these new mixing matrices, the LHT model does not belong to the (MFV) class of models (Buras *et al.*, 2001b; D'Ambrosio *et al.*, 2002; Buras, 2003) and significant effects in flavor observables are

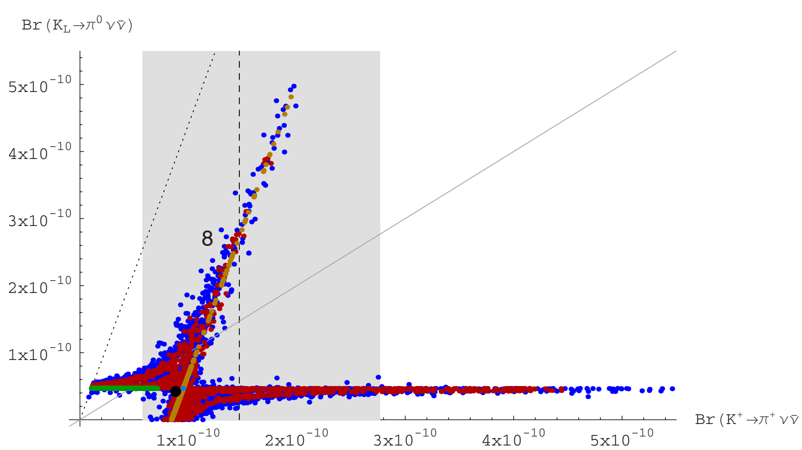

FIG. 10. (Color online) $\text{Br}(K_L \to \pi^0 \nu\bar{\nu})$ as a function of $\text{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ in the LHT model. The shaded area represents the experimental 1σ range for $\text{Br}(K^+ \to \pi^+ \nu\bar{\nu})$. The GN bound is displayed by the dotted line, while the solid line separates the two areas where $\text{Br}(K_L \to \pi^0 \nu\bar{\nu})$ is larger or smaller than $\text{Br}(K^+ \to \pi^+ \nu\bar{\nu})$.

possible, without adding new operators to the SM ones. Finally, it is important to recall that little Higgs models are low-energy nonlinear sigma models, whose unknown UV-completion introduces a theoretical uncertainty, as discussed by Buras, Poschenrieder, Uhlig, *et al.* (2006) and Blanke, Buras, Recksiegel, *et al.* (2007).

The flavor physics analysis in the LHT model can be found in the case of quark sector by Blanke *et al.* (2006, 2007b) and Hubisz, Lee, and Paz (2006) and in the lepton sector by Blanke *et al.* (2007a) and Choudhury *et al.* (2007). Here we summarize the results obtained for $K \to \pi\nu\bar{\nu}$ decays obtained by Blanke, Buras, Recksiegel, *et al.* (2007).

The presence of new flavor violating interactions between ordinary quarks and mirror quarks described by the matrix

$$V_{Hd} = \begin{pmatrix} c_{12}^d c_{13}^d & s_{12}^d c_{13}^d e^{-i\delta_{12}^d} & s_{13}^d e^{-i\delta_{13}^d} \\ -s_{12}^d c_{23}^d e^{i\delta_{12}^d} - c_{12}^d s_{23}^d s_{13}^d e^{i(\delta_{13}^d-\delta_{23}^d)} & c_{12}^d c_{23}^d - s_{12}^d s_{23}^d s_{13}^d e^{i(\delta_{13}^d-\delta_{12}^d-\delta_{23}^d)} & s_{23}^d c_{13}^d e^{-i\delta_{23}^d} \\ s_{12}^d s_{23}^d e^{i(\delta_{12}^d+\delta_{23}^d)} - c_{12}^d c_{23}^d s_{13}^d e^{i\delta_{13}^d} & -c_{12}^d s_{23}^d e^{i\delta_{23}^d} - s_{12}^d c_{23}^d s_{13}^d e^{i(\delta_{13}^d-\delta_{12}^d)} & c_{23}^d c_{13}^d \end{pmatrix}$$

introduces complex phases in the short-distance functions X_i, Y_i, and Z_i and breaks the universality and correlations between K, B_d, and B_s systems characteristic for the MFV models. Spectacular results are found in particular for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays. First one finds

$$0.7 \leq |X| \leq 4.7, \quad -130° \leq \theta_X \leq 55° \tag{8.1}$$

to be compared with $|X|=1.44$ and $\theta_X=0$ in the SM. As discussed in Sec. VII.B, a large phase θ_X can change the pattern of branching ratios in the $K \to \pi\nu\bar{\nu}$ system. This is seen in Fig. 10, where we show the correlation between $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ in the LHT model. The experimental 1σ range for $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ (Adler *et al.*, 2002; Anisimovsky *et al.*, 2004, 2007) and the model-independent Grossman-Nir (GN) bound (Grossman and Nir, 1997) are also shown. The different shaded areas in the figure correspond to different scenarios for the V_{Hd} matrix, whose discussion is beyond the scope of this review.

We observe that there are two branches of possible points. The first one is parallel to the GN bound and leads to possible huge enhancements in $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ so that values as high as 5×10^{-10} are possible, being at the same time consistent with the measured value for $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$. The second branch corresponds to values for $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ being rather close to its SM prediction, while $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ is allowed to vary in the range $[1\times10^{-11}, 5\times10^{-10}]$, however values above 4×10^{-10} are experimentally not favored. We note also that for certain parameter values of the model, $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ can be significantly suppressed.

In Fig. 11, we show the ratio $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})/\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ as a function of the phase β_X^K, displaying again the GN bound. We observe that the ratio can be significantly different from the SM prediction, with a possible enhancement of an order of magnitude.

The most interesting implications of this analysis are as follows:

- If $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ is found sufficiently above the SM prediction but below 2.3×10^{-10}, basically only two values for $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ are possible within the LHT model. One of these values is close to the SM value in Eq. (1.2) and the second much larger.
- If $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ is found above 2.3×10^{-10}, then only $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ with a value close to the SM one in Eq. (1.3) is possible.
- The violation of the MFV relation (1.1). We show this in Fig. 12, where the ratio of $\sin 2\beta_X^K$ over $\sin(2\beta+2\varphi_{B_d})$ is plotted versus δ_{13}^d. As φ_{B_d} is constrained by the measured $S_{\psi K_S}$ asymmetry to be at most a few degrees (Blanke *et al.*, 2006; Bona *et al.*, 2006c), large violations of the relation in question can only follow from the $K \to \pi\nu\bar{\nu}$ decays. As seen in Fig. 12, they can be spectacular.

Finally, in Fig. 13 we show $\mathrm{Br}(K_L \to \pi^0 e^+e^-)$ and $\mathrm{Br}(K_L \to \pi^0\mu^+\mu^-)$ versus $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$. We observe a

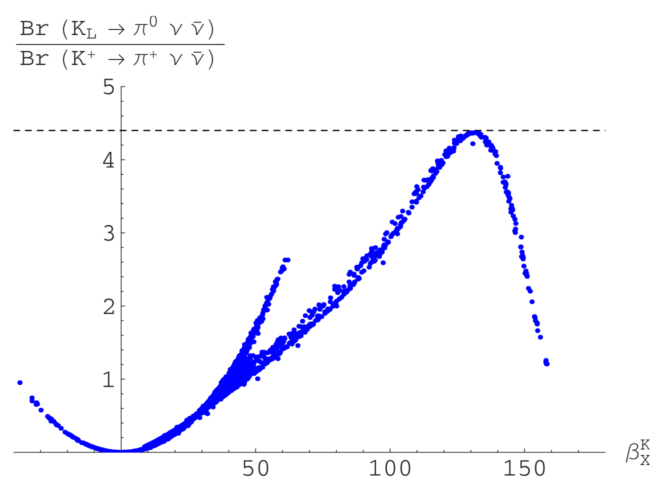

FIG. 11. (Color online) $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})/\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ in the LHT model as a function of β_X^K. The dashed line represents the GN bound.

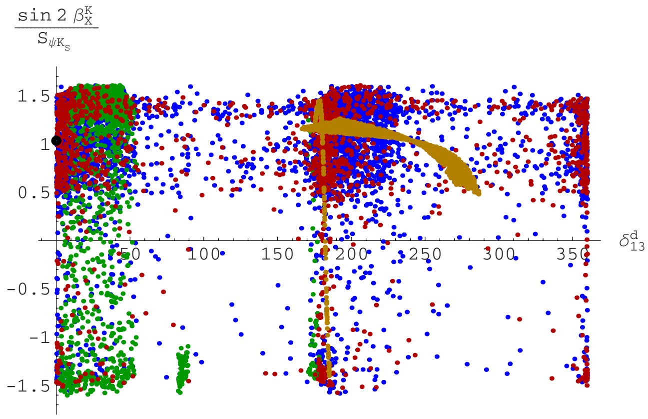

FIG. 12. (Color online) $\sin 2\beta_X^K/\sin(2\beta+2\varphi_{B_d})$ as a function of δ_{13}^d in the LHT model.

strong correlation between $K_L\to\pi^0\ell^+\ell^-$ and $K_L\to\pi^0\nu\bar{\nu}$ decays that we expect to be valid beyond the LHT model, at least in models with the same operators present as in the SM. We note that a large enhancement of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ automatically implies significant enhancements of $\mathrm{Br}(K_L\to\pi^0\ell^+\ell^-)$ and that different models and their parameter sets can then be distinguished by the position on the correlation curve. Moreover, measuring $\mathrm{Br}(K_L\to\pi^0\ell^+\ell^-)$ should allow a rather precise prediction of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ at least in models with the same operators as the SM. This should distinguish the LHT model from models with more complicated operator structure in $K_L\to\pi^0 l^+l^-$ (Mescia *et al.*, 2006), and consequently different correlations between $K_L\to\pi^0\nu\bar{\nu}$ and $K_L\to\pi^0 l^+l^-$.

As emphasized by Buras *et al.* (2000) and Buras and Silvestrini (1999), there exist correlations between $K\to\pi\nu\bar{\nu}$ decays, $K_L\to\mu^+\mu^-$, and ε'/ε, that could bound the size of the enhancement of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$. Unfortunately, the hadronic uncertainties in $K_L\to\mu^+\mu^-$ and in particular in ε'/ε lower the usefulness of these correlations at present. More promising, in the context of supersymmetric models and also generally, appear the correlations between $K\to\pi\nu\bar{\nu}$ and rare FCNC semileptonic decays like $B\to X_{s,d}l^+l^-$, $B_{s,d}\to l^+l^-$, and in particular $B\to X_{s,d}\nu\bar{\nu}$, because in these decays the main deviations from the SM can also be encoded in an effective $Z\bar{b}q$ $(q=s,d)$ vertex (Buchalla *et al.*, 2001; Atwood and Hiller, 2003). We discussed the correlation with $B\to X_{s,d}\nu\bar{\nu}$ in the preceding section.

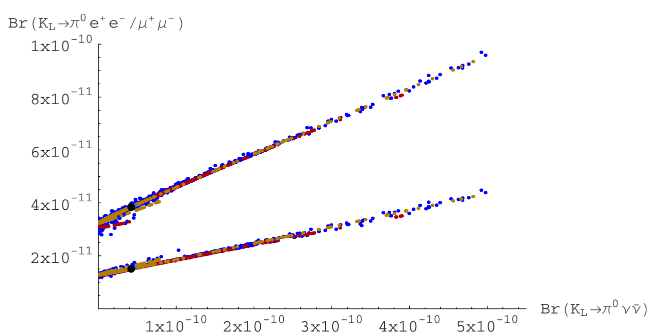

FIG. 13. (Color online) $\mathrm{Br}(K_L\to\pi^0 e^+e^-)$ (upper curve) and $\mathrm{Br}(K_L\to\pi^0\mu^+\mu^-)$ (lower curve) as functions of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ in the LHT model. The corresponding SM predictions are represented by dark points.

Recently, the correlation between ϵ'/ϵ and the decays $K\to\pi\nu\bar{\nu}$ has been investigated in the context of the LHT model for specific values of the relevant hadronic matrix elements entering ϵ'/ϵ (Blanke, Buras, Recksiegel, *et al.*, 2007). The resulting correlation between $K_L\to\pi^0\nu\bar{\nu}$ and ϵ'/ϵ is strong but less pronounced in the case of $K^+\to\pi^+\nu\bar{\nu}$. With the hadronic matrix elements evaluated in the large-N limit, $(\epsilon'/\epsilon)_{\mathrm{SM}}$ turns out close to the experimental data and significant departures of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0 l^+l^-)$ from the SM expectations are unlikely, while $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ can be enhanced by a factor of 5. On the other hand, modest departures of the relevant hadronic matrix elements from their large-N values allow for a consistent description of ϵ'/ϵ within the LHT model accompanied by large enhancements of $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0 l^+l^-)$, but only modest enhancements of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$. This analysis demonstrates that without a significant progress in the evaluation of the hadronic parameters in ϵ'/ϵ, the role of this ratio in constraining physics beyond the SM will remain limited.

C. Z' models

An additional neutral gauge boson can appear in several extensions of the standard model, such as left-right symmetric models, supersymmetric models with an additional U(1) factor, often arising in the breaking process of several grand unified theory models, such as the breaking chain $\mathrm{SO}(10)\to\mathrm{SU}(5)\times\mathrm{U}(1)$ or $E_6\to\mathrm{SO}(10)\times\mathrm{U}(1)$, or in 331 models, where the $\mathrm{SU}(2)_L$ of the SM is extended to an $\mathrm{SU}(3)_L$. In general, direct collider searches have already placed lower bounds on a general Z' mass, but FCNC processes can also provide valuable information on these particles, since additional contributions appear at tree level, if the Z' transmits flavor changes. General, model-independent analyses of B decays as well as the mass differences ΔM_s can be found in Grossman *et al.* (1999), Langacker and Plumacher (2000), and Barger *et al.* (2004a, 2004b). Additional interest in these contributions with respect to the B meson system has arisen in the context of the CP asymmetries in $B_d^0\to\phi K_S$. In general, one finds that sizeable contributions are still possible but are rather unpredicable in this model-independent context. On the other hand, the predictive power increases if the analysis is performed in a specific model.

As an example of this situation, we discuss the recent analysis (Promberger *et al.*, 2007) performed in the minimal 331 model (Frampton, 1992; Pisano and Pleitez, 1992). Here one has an $\mathrm{SU}(3)_c\times\mathrm{SU}(3)_L\times\mathrm{U}(1)$ that is broken down to the electromagnetic U(1) in two steps. In this process, the additional Z' boson appears, along

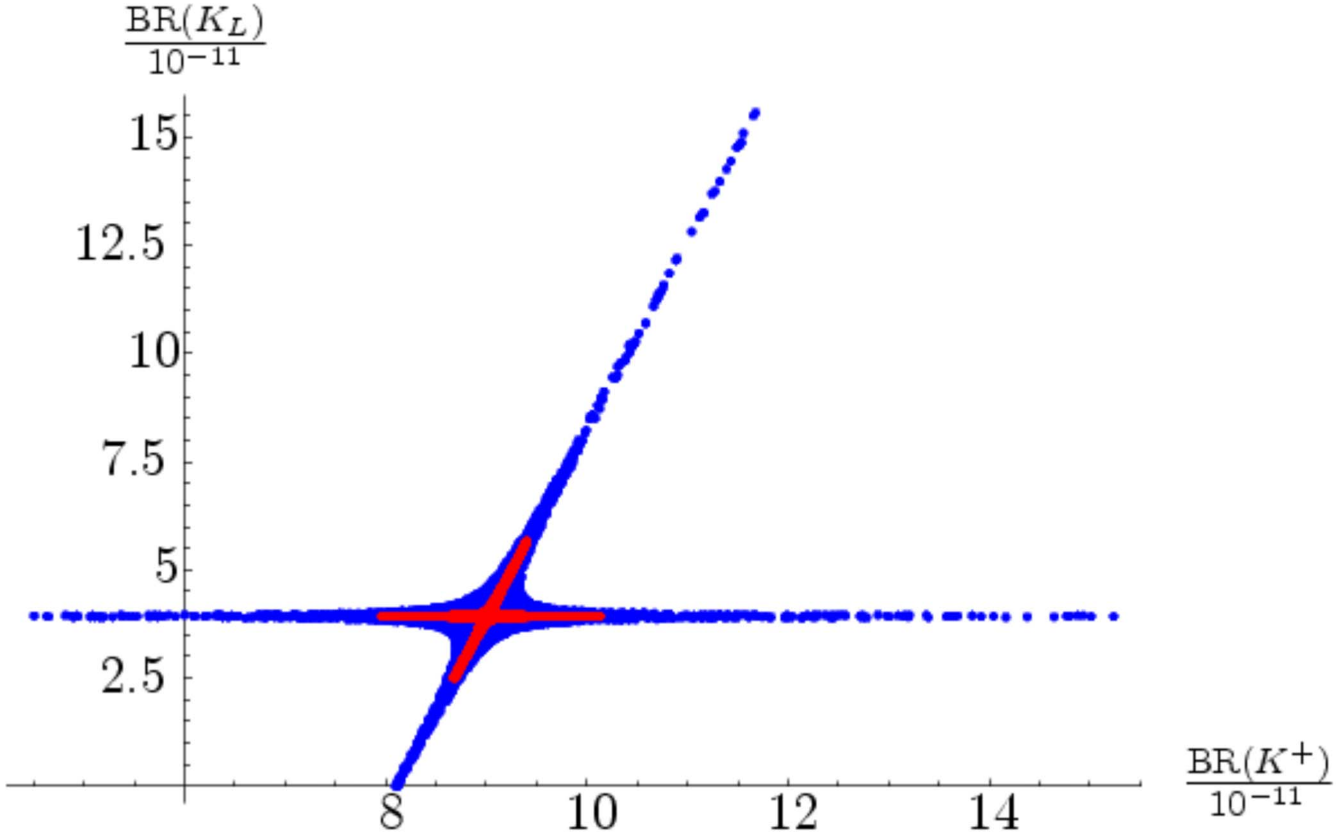

FIG. 14. (Color online) A projection onto the $K_L \to \pi^0 \nu\bar{\nu}$–$K^+ \to \pi^+ \nu\bar{\nu}$ plane including the upper bounds from ΔM_K and ϵ_K for $M_{Z'}=5$ TeV and 1 TeV.

with several charged gauge bosons that play no role in low-energy processes involving quarks, since they always couple to the additional heavy quarks that fill the left-handed triplet. Finally, the third generation of quarks is treated differently from the first two, transforming as an antitriplet. In this setup, taking into account also the asymptotic freedom of QCD, one finds that anomalies are canceled precisely in the case of the three generations, thereby explaining this feature of the SM, where the number of generations is fixed from observation. Apart from FCNC processes, constraints on the Z' mass come also from electroweak precision observables, but new contributions appear at the one-loop level, so that constraints from FCNCs are actually more interesting. Also, the model develops a Landau pole at a scale of several TeV, which constrains the new energy scale from above, thereby complementing the bounds from direct searches and FCNC observables.

The flavor nonuniversality reflected in the different transformation property of the third generation leads to the flavor changing Z' vertices. The FCNC processes under investigation then depend on the Z' mass as well as the weak mixing matrix required to diagonalize the Yukawa coupling of the down quark sector (as long as one is studying B or K meson processes). Also, one finds that the different processes decouple from each other, i.e., that sd, bd, and bs transitions are constrained independently, so that only constraints from ΔM_K and ε_K are used to constrain the branching fractions of $K_L \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$. This leads to an allowed region in the $K_L \to \pi^0 \nu\bar{\nu}$-$K^+ \to \pi^+ \nu\bar{\nu}$ plane shown in Fig. 14. We show the corresponding areas for $M_{Z'}=1$ TeV as well as $M_{Z'}=5$ TeV, where one observes that the allowed region shrinks with increasing $M_{Z'}$. The pattern is similar to the one shown in the LHT model in that there exist two possible branches, where $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ is close to the SM on one of them, while $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ is on the other. This is due to the different strength of the ε_K and ΔM_K bounds, respectively, and large modifications arise in those areas where the phase of the new contribution is such that it does not strongly modify ε_K. Therefore, a similar structure should appear whenever the $K \to \pi^0 \nu\bar{\nu}$ decays are constrained mainly by these two quantities. On the other hand, the minimal 331 model has a somewhat leptophobic nature, so that the effects are not expected to be as large as, for example, in the LHT model, but the current experimental central value can be reached, in particular, for $M_{Z'} < 2$ TeV.

Additionally, a measurement of both branching fractions fixes both the absolute value and phase of the new contributions (this is true in all Z' models) and allows predictions for the observables ΔM_K and ε_K (this is of course only true if the model is explicitly fixed). Another interesting feature of this model is that there are significant differences between the vector and axial vector coupling, which cancel each other out in the $V-A$ difference, to which $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ is sensitive, so that, in comparison, one finds stronger modifications in the $K_L \to\pi^0 l^+ l^-$ branching fraction than in $K_L\to\pi^0\nu\bar{\nu}$ (Promberger *et al.*, 2007). Finally, significant modifications can also be found in the angle $\beta|_{K\pi\nu\nu}$, which may be as large as 45° for small values of $M_{Z'}$.

On the other hand, recently (He and Valencia, 2004, 2006) the decays $K\to\pi\nu\bar{\nu}$ were analyzed in models that are variations of left-right symmetric models in which right-handed interactions, involving in particular a heavy Z' boson, single out the third generation (He and Valencia, 2002, 2003). Contributions of these new nonuniversal FCNC interactions appear at both the tree and one-loop levels. The tree level contributions involving Z' of the type $(\bar{s}d)_{V+A}(\bar{\nu}_\tau\nu_\tau)_{V+A}$ can be severely constrained by other rare decays, ε_K, and in particular B_s^0-$\bar{B}_s^0$ mixing. Before the measurement of ΔM_s, these could enhance $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ to the central experimental value in Eq. (1.5) and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ could be as high as 1.4×10^{-10}. These enhancements were accompanied by an enhancement of ΔM_s, and finding ΔM_s in the vicinity of SM expectations has significantly limited these possibilities (He and Valencia, 2006). On the other hand, new one-loop contributions involving a Z' boson may be important because of the particularly large τ neutrino coupling. They are not constrained by B_s^0-$\bar{B}_s^0$ mixing and can give significant enhancements of both branching ratios even if $\Delta M_s\approx(\Delta M_s)_{\mathrm{SM}}$. Unfortunately, the presence of many free parameters in these new one-loop contributions does not allow us to make definite predictions, but an enhancement by a factor of 2 still seems possible (He and Valencia, 2006).

Finally, FCNC processes at the tree level arise also if there is an additional vectorlike quark generation, or if there is only one additional isosinglet down-type or up-type quark, as one can encounter in certain E_6 GUT theories, or some models with extra dimensions. In this case, the SM Z boson itself can transmit flavor changes, since the mixing matrix of the respective quark sector is no longer unitary and therefore does not cancel out in the neutral Z current, causing FCNCs in the respective sector where the additional quark appears. The most recent analysis of the $K\to\pi\nu\bar{\nu}$ decays in this model was presented by Deshpande *et al.* (2004), while a complete analysis of FCNC processes in this type of scenario can be found in Barenboim *et al.* (2001). Here the authors obtain constraints on the matrix element U_{sd} (here $U=V^\dagger V$, with V the mixing matrix that diagonalizes the down-quark sector) from $K^+\to\pi^+\nu\bar{\nu}$, ε_K, $\varepsilon'/\varepsilon_K$. Additionally, they emphasized that the $K\to\pi\nu\bar{\nu}$ decays can be valuable for constraining this element further, if the decays are precisely measured. In fact, one finds there a figure somewhat similar in spirit to the one shown in Fig. 14, which shows an analogous interplay of constraints in the K physics sector. We have included this figure as Fig. 15.

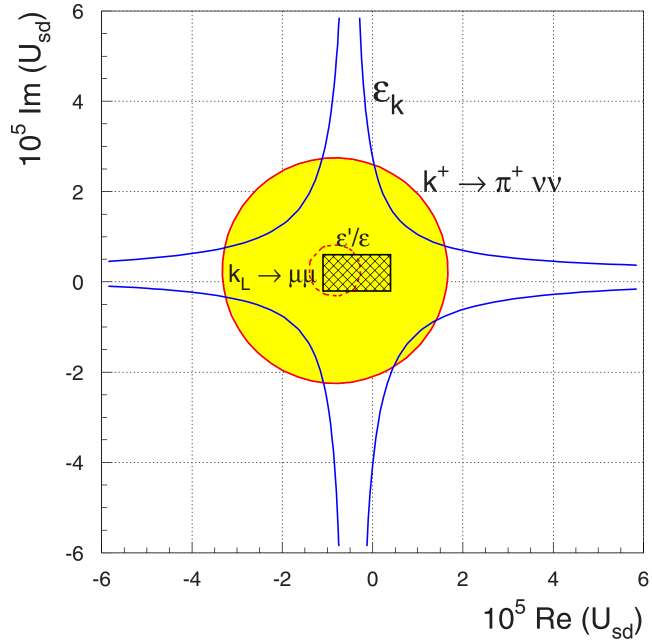

FIG. 15. (Color online) Effect of the constraints from ε_K, $K^+\to\pi^+\nu\bar{\nu}$, $K_L\to\mu^+\mu^-$, and ε'/ε on the U_{sd} FCNC coupling in the case of an extra isosinglet down quark (Barenboim *et al.*, 2001).

D. MSSM with MFV

There are many new contributions in MSSM, such as charged Higgs, chargino, neutralino, and gluino contributions. However, in the case of $K\to\pi\nu\bar{\nu}$ and MFV, it is a good approximation to keep only charged Higgs and chargino contributions.

To our knowledge, the first analyses of $K\to\pi\nu\bar{\nu}$ in this scenario can be found in Bertolini and Masiero (1986), Giudice (1987), Mukhopadhyaya and Raychaudhuri (1987), and Bigi and Gabbiani (1991), and subsequently in Couture and Konig (1995), Goto *et al.* (1998), and Buras *et al.* (2001a). In the latter analysis, constraints on the supersymmetric parameters from ε_K, $\Delta M_{d,s}$, $B\to X_s\gamma$, $\Delta\varrho$ in the electroweak precision studies and from the lower bound on the neutral Higgs mass have been imposed. Supersymmetric contributions affect both the loop functions such as $X(v)$ present in the SM and the values of the extracted CKM parameters such as $|V_{td}|$ and $\mathrm{Im}\,\lambda_t$. As the supersymmetric contributions to the function $S(v)$ relevant for the analysis of the UT are always positive [see also Altmannshofer *et al.* (2007)], the extracted values of $|V_{td}|$ and $\mathrm{Im}\,\lambda_t$ are always smaller than in the SM. Consequently, $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$, which are sensitive to $|V_{td}|$ and $\mathrm{Im}\,\lambda_t$, respectively, are generally suppressed relative to SM expectations. The supersymmetric contributions to the loop function $X(v)$ can compensate for the suppression of $|V_{td}|$ and $\mathrm{Im}\,\lambda_t$ only for special values of supersymmetric parameters, so that in these cases the results are close to SM expectations.

Setting λ, $|V_{ub}|$, and $|V_{cb}|$, all unaffected by SUSY contributions, at their central values one finds (Buras *et al.*, 2001a)

$$0.65 \leq \frac{\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})}{\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}}} \leq 1.02,$$

$$0.41 \leq \frac{\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})}{\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}}} \leq 1.03. \quad (8.2)$$

We observe that significant suppressions of the branching ratios relative to SM expectations are still possible. More importantly, finding experimentally at least one of these branching ratios above the SM value would exclude this scenario, indicating new flavor violating sources beyond the CKM matrix. Similarly, in the MSSM based on supergravity, a reduction of both $K \to \pi\nu\bar{\nu}$ rates up to 10% is possible (Goto *et al.*, 1998). Buras *et al.* (2001a) provided a compendium of phenomenologically relevant formulas in the MSSM that should turn out to be useful once the relevant branching ratios have been accurately measured and the supersymmetric particles have been discovered at Tevatron, LHC, and the e^+e^- linear collider. The study of the unitarity triangle can be found in Ali and London (1999a, 1999b, 1999c, 2001). Inclusion of NLO QCD corrections to the processes discussed by Buras *et al.* (2001a) has been performed by Bobeth *et al.* (2002). These corrections reduce mainly the renormalization scale uncertainties present in the analysis of Buras *et al.* (2001a), without modifying the results in Eq. (8.2) significantly.

E. General supersymmetric models

In general supersymmetric models, the effects of supersymmetric contributions to rare branching ratios can be larger than discussed above. In these models, new *CP*-violating phases and new operators are present. Moreover, the structure of flavor violating interactions is much richer than in the MFV models.

The new flavor violating interactions are present because generally the sfermion mass matrices $\tilde{M}_q^2$ can be nondiagonal in the way in which all neutral quark-squark-gaugino vertices and quark and lepton mass matrices are flavor diagonal. Instead of diagonalizing sfermion mass matrices, it is convenient to consider their off-diagonal terms as new flavor violating interactions. This so-called mass-insertion approximation (Hall *et al.*, 1986) has been reviewed by Gabbiani *et al.* (1996) and Misiak *et al.* (1998), where further references can be found.

Within the MSSM with R-parity conservation, sizable nonstandard contributions to $K \to \pi\nu\bar{\nu}$ decays can be generated if the soft-breaking terms have a non-MFV structure. The leading amplitudes giving rise to large effects are induced by (i) chargino and up-squark loops (Buras *et al.*, 1998, 2000; Colangelo and Isidori, 1998; Nir and Worah 1998) and (ii) charged Higgs and top quark loops (Isidori and Paradisi, 2006). In the first case, large effects are generated if the left-right mixing (A term) of the up squarks has a non-MFV structure (D'Ambrosio *et al.*, 2002). In the second case, deviations from the SM are induced by non-MFV terms in the right-right down sector, provided the ratio of the two Higgs vacuum expectation values ($\tan\beta = v_u/v_d$) is large ($\tan\beta \sim 30$–50).

The effective Hamiltonian encoding SD contributions in the general MSSM has the following structure:

$$\mathcal{H}_{\mathrm{eff}}^{(\mathrm{SD})} \propto \sum_{l=e,\mu,\tau} V_{ts}^* V_{td} [X_L (\bar{s}_L \gamma^\mu d_L)(\bar{\nu}_{lL} \gamma_\mu \nu_{lL}) + X_R (\bar{s}_R \gamma^\mu d_R)(\bar{\nu}_{lL} \gamma_\mu \nu_{lL})], \quad (8.3)$$

where the SM case is recovered for $X_R = 0$ and $X_L = X_{\mathrm{SM}}$. In general, both X_R and X_L are nonvanishing, and the misalignment between quark and squark flavor structures implies that they are both complex quantities. Since the $K \to \pi$ matrix elements of $(\bar{s}_L \gamma^\mu d_L)$ and $(\bar{s}_R \gamma^\mu d_R)$ are equal, the combination $X_L + X_R$ allows us to describe all SD contributions to $K \to \pi\nu\bar{\nu}$ decays. More precisely, we use the SM expressions for the branching ratios with the following replacement:

$$X_{\mathrm{SM}} \to X_{\mathrm{SM}} + X_L^{\mathrm{SUSY}} + X_R^{\mathrm{SUSY}}, \quad (8.4)$$

with $X_{L,R}^{\mathrm{SUSY}}$ complex quantities. In the limit of almost degenerate superpartners, the leading chargino and up-squarks contribution is (Colangelo and Isidori, 1998)

$$X_L^{\chi^\pm} \approx \frac{1}{96} \left[\frac{(\delta_{LR}^u)_{23} (\delta_{RL}^u)_{31}}{\lambda_t} \right] = \frac{1}{96\lambda_t} \left[\frac{(\tilde{M}_u^2)_{2_L 3_R}}{(\tilde{M}_u^2)_{LL} (\tilde{M}_u^2)_{RR}} \right] \left[\frac{(\tilde{M}_u^2)_{3_R 1_L}}{(\tilde{M}_u^2)_{LL} (\tilde{M}_u^2)_{RR}} \right]. \quad (8.5)$$

Here $(\delta_{AB}^q)_{ij}$ result from a convenient parametrization (Gabbiani *et al.*, 1996; Misiak *et al.*, 1998) of the nondiagonal terms $(\tilde{M}_u^2)_{iAjB}$ in squark mass matrices with $A, B = L, R$ and $i, j = 1, 2, 3$ standing for quark generation indices. As pointed out by Colangelo and Isidori (1998), a remarkable feature of the above result is that no extra $\mathcal{O}(M_W/M_{\mathrm{SUSY}})$ suppression and no explicit CKM suppression is present (as it happens in the chargino and up-squark contributions to other processes). Furthermore, the (δ_{LR}^u)-type mass insertions are not constrained by other B and K observables. This implies that large departures from SM expectations in $K \to \pi\nu\bar{\nu}$ decays are allowed, as confirmed by the complete analyses in Buras, Ewerth, Jager, *et al.* (2005) and Isidori, Mescia, Paradisi, *et al.* (2006). In particular, Buras, Ewerth, Jager, *et al.* (2005) found that both branching ratios can be as large as a few times 10^{-10} with $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ often larger than $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ and close to the GN bound. One also finds (Isidori, Mescia, Paradisi, *et al.* 2006) that $K \to \pi\nu\bar{\nu}$ are the best observables to determine or constrain from experimental data the size of the off-diagonal (δ_{LR}^u) mass insertions or, equivalently, the up-type trilinear terms $A_{i3}[(\tilde{M}_u^2)_{i_L 3_R} \approx m_t A_{i3}]$. Their measurement is therefore extremely interesting also in the LHC era.

In the large $\tan\beta$ limit, the charged Higgs and top-quark exchange leads to (Isidori and Paradisi, 2006)

$$X_R^{H^\pm} \approx \Bigg[\left(\frac{m_s m_d t_\beta^2}{2M_W^2}\right) + \frac{(\delta_{RR}^d)_{31}(\delta_{RR}^d)_{32}}{\lambda_t}\left(\frac{m_b^2 t_\beta^2}{2M_W^2}\right)\frac{\epsilon_{RR}^2 t_\beta^2}{(1+\epsilon_i t_\beta)^4}\Bigg] f_H(y_{tH}), \tag{8.6}$$

where $y_{tH}=m_t^2/M_H^2$, $f_H(x)=x/4(1-x)+x\log x/4(x-1)^2$, and $\epsilon_{i,RR}t_\beta=\mathcal{O}(1)$ for $t_\beta=\tan\beta\sim 50$. The first term of Eq. (8.6) arises from MFV effects and its potential $\tan\beta$ enhancement is more than compensated by the smallness of $m_{d,s}$. The second term on the r.h.s. of Eq. (8.6), which would appear only at the three-loop level in a standard loop expansion, can be largely enhanced by the $\tan^4\beta$ factor and does not contain any suppression due to light quark masses. Similarly to the double mass-insertion mechanism of Eq. (8.5), in this case the potentially leading effect is also the one generated when two off-diagonal squark mixing terms replace the two CKM factors V_{ts} and V_{td}.

The coupling of the $(\bar{s}_R\gamma^\mu d_R)(\bar{\nu}_L\gamma_\mu\nu_L)$ effective FCNC operator, generated by charged-Higgs and top-quark loops, is phenomenologically relevant only at large $\tan\beta$ and with non-MFV right-right soft-breaking terms: a specific but well-motivated scenario within grand-unified theories [see, e.g., Moroi (2000) and Chang *et al.* (2003)]. These nonstandard effects do not vanish in the limit of heavy squarks and gauginos, and have a slow decoupling with respect to the charged-Higgs boson mass. As shown by Isidori and Paradisi (2006), the B-physics constraints still allow a large room of nonstandard effects in $K\to\pi\nu\bar{\nu}$ even for flavor-mixing terms of CKM size (see Fig. 16).

A systematic study of $K\to\pi\nu\bar{\nu}$ decays in flavor supersymmetric models was performed by Nir and Worah (1998) and Nir and Raz (2002). These particular models are designed to solve naturally the CP and flavor problems characteristic for supersymmetric theories.[4] They are more constrained than the general supersymmetric models just discussed, in which parameters are tuned to satisfy the experimental constraints.

Models with exact universality of squark masses at a high-energy scale with the A terms proportional to the corresponding Yukawa couplings, models with approximate CP, quark and squark alignment, approximate universality, and heavy squarks were analyzed by Nir and Worah (1998) and Nir and Raz (2002) in general terms. It has been concluded that in most of these models, the impact of new physics on $K\to\pi\nu\bar{\nu}$ is sufficiently small so that in these scenarios one can get information on the CKM matrix from these decays even in the presence of supersymmetry. On the other hand, supersymmetric contributions to B_d^0-$\bar{B}_d^0$ mixing in models with alignment, with approximate universality and heavy squarks, can significantly affect the asymmetry $a_{\psi K_S}$, so that in these models the golden relation (1.1) can be violated. However, such scenarios have been put under large pressure in view of the recent data on D^0-$\bar{D}^0$ mixing (Ciuchini *et al.*, 2007; Nir, 2007).

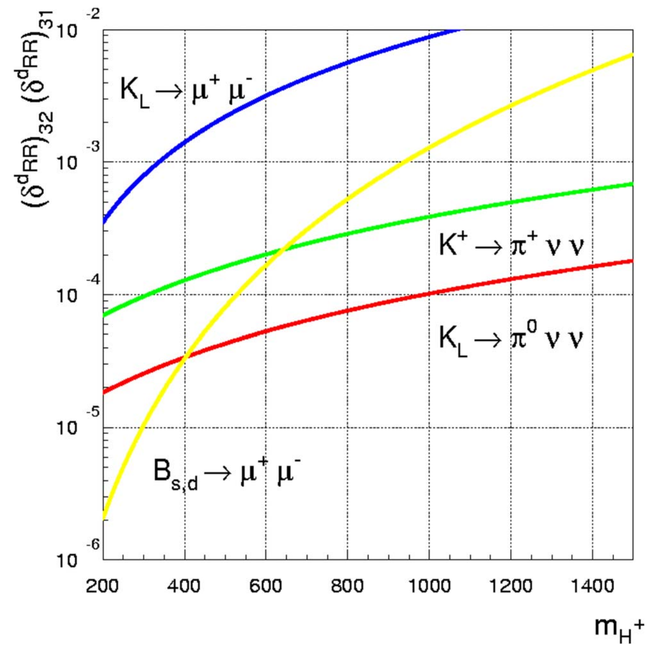

FIG. 16. (Color online) Supersymmetric contributions to $K\to\pi\nu\bar{\nu}$. Sensitivity to $(\delta_{RR}^d)_{23}(\delta_{RR}^d)_{31}$ of various rare K and B decays as a function of M_{H^+}, setting $\tan\beta=50$, $\mu<0$, and assuming almost degenerate superparteners [the bounds from the two $K\to\pi\nu\bar{\nu}$ modes are obtained assuming a 10% measurement of their branching ratios while, the $B_{s,d}\to\mu^+\mu^-$ bounds refer to the present experimental limits (Isidori and Paradisi, 2006)].

Finally, in supersymmetric models with nonuniversal A terms, enhancements of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ up to 1.5×10^{-10} and 2.5×10^{-10} are possible, respectively (Chen, 2002). Significant departures from SM expectations have also been found in supersymmetric models with R-parity breaking (Deandrea *et al.*, 2004), but all these analyses should be reconsidered in view of experimental constraints.

F. Models with universal extra dimensions

The decays $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ have been studied in the SM model with one extra universal dimension by Buras, Spranger, and Weiler (2003). In this model (ACD) (Appelquist *et al.*, 2001), all SM fields are allowed to propagate in all available dimensions and the relevant penguin and box diagrams receive additional contributions from Kaluza-Klein (KK) modes. This model belongs to the class of CMFV models and the only additional free parameter relative to the SM is the compactification scale $1/R$. Extensive analyses of the

[4]See the review by Grossman *et al.* (1998).

TABLE XII. Upper bound on $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ in units of 10^{-11} for different values of ξ, $1/R$, and $\Delta M_s=18/\mathrm{ps}$ ($21/\mathrm{ps}$). From Buras Spranger, and Weiler, 2003.

ξ	$1/R=300$ GeV	$1/R=400$ GeV	SM
1.30	12.0 (10.7)	11.3 (10.1)	(10.8) (9.3)
1.25	11.4 (10.2)	10.7 (9.6)	10.3 (8.8)
1.20	10.7 (9.6)	10.1 (9.1)	9.7 (8.4)
1.15	10.1 (9.0)	9.5 (8.5)	9.1 (7.9)

precision electroweak data, and analyses of the anomalous magnetic moment of both the muon, and $Z\to b\bar{b}$ vertex have shown the consistency of the ACD model with the data for $1/R\geq 300$ GeV. We refer the reader to Buras, Spranger, and Weiler (2003) and Buras, Poschenrider, Spranger, *et al.* (2004) for relevant references.

For $1/R=300$ and 400 GeV, the function X is found with $m_t=167$ GeV to be $X=1.67$ and 1.61, respectively. This should be compared with $X=1.53$ in the SM. In contrast to the analysis in the MSSM discussed by Buras *et al.* (2001a) and above, this 5–10 % enhancement of the function X is only insignificantly compensated by the change in the values of the CKM parameters. Consequently, the clear prediction of the model is the enhanced branching ratios $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$, albeit by at most 15% relative to the SM expectation. These enhancements allow us to distinguish this scenario from the MSSM with MFV.

The enhancement of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ in the ACD model is interesting in view of the experimental results in Eq. (1.5) with the central value higher by a factor of 1.8 than the central value in the SM. Even if the errors are substantial and this result is compatible with the SM, the ACD model with a low compactification scale is closer to the data. In Table XII, we show the upper bound on $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ in the ACD model obtained by Buras, Spranger, and Weiler (2003) with formula (3.10), and X replaced by its enhanced value in the model in question. To this end, $|V_{cb}|\leq 0.0422$, $P_c(X)<0.47$, $m_t(m_t)<172$ GeV, and $\sin 2\beta=0.734$ have been used. Table XII illustrates the dependence of the bound on the nonperturbative parameter ξ, $1/R$, and ΔM_s. We observe that for $1/R=300$ GeV and $\xi=1.30$, the maximal value for $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ in the ACD model is close to the central value in Eq. (1.5).

Clearly, in order to distinguish these results and the ACD model from the SM, other quantities that are more sensitive to $1/R$ should be considered simultaneously. In this respect, the sizable downward shift of the zero ($\hat{s}_0$) in the forward-backward asymmetry A_{FB} in $B\to X_s\mu^+\mu^-$ and the suppression of $\mathrm{Br}(B\to X_s\gamma)$ by roughly 20% at $1/R=300$ GeV appear to be most interesting results (Buras, Poschenrieder, Spranger, *et al.*, 2004).

As the most recent analysis of the $B\to X_s\gamma$ decay at the NNLO level results in its SM branching ratio being more than one σ below the experimental values, the model in question is put therefore under considerable pressure and the values of $1/R$ as low as 300 GeV appear rather improbable from the present perspective (Haisch and Weiler, 2007a). A decrease of the experimental error without a significant change of its central value and a better understanding of nonperturbative effects in the $B\to X_s\gamma$ decay could result in $1/R\approx\mathcal{O}(1\ \mathrm{TeV})$ and consequently small new physics effects in $K\to\pi\nu\bar{\nu}$ decays in this model.

G. Models with lepton-flavor mixing

In the presence of flavor mixing in the leptonic sector, the transition $K_L\to\pi^0\nu_i\bar{\nu}_j$, with $i\neq j$, could receive significant CP-conserving contributions (Grossman and Nir, 1997). Subsequently this issue was analyzed by Perez (1999, 2000) and Grossman *et al.* (2004). Here we summarize the main findings of these papers.

Perez (1999, 2000) analyzed the effect of light sterile right-handed neutrinos leading to scalar and tensor dimension-6 operators. As shown there, the effect of these operators is negligible if the right-handed neutrinos interact with the SM fields only through their Dirac mass terms.

Larger effects are expected from the operators

$$O_{sd}^{ij}=(\bar{s}\gamma_\mu d)(\bar{\nu}_L^i\gamma^\mu\nu_L^j), \tag{8.7}$$

which for ($i\neq j$) create a neutrino pair that is not a CP eigenstate. As shown by Grossman *et al.* (2004), the condition for a nonvanishing $K_L\to\pi^0\nu\bar{\nu}$ rate in this case is strong. One needs either CP violation in the quark sector or a new effective interaction that violates both quark and lepton universality. One finds then the following pattern of effects:

- If the source of universality breaking is confined to mass matrices, the effects of lepton-flavor mixing get washed out in the $K\to\pi\nu\bar{\nu}$ rates after the summing over the neutrino flavors. There are in principle detectable effects of lepton mixing only in cases in which there are two different lepton-flavor mixing matrices, although they cannot be large.
- In models in which simultaneous violation of quark and lepton universality proceeds entirely through Yukawa couplings, the CP-conserving effects in $K\to\pi\nu\bar{\nu}$ are suppressed by Yukawa couplings. As shown by Grossman *et al.* (2004), even in the MSSM with flavor violation and large $\tan\beta$, these types of effects are negligible.
- In exotic scenarios, such as R-parity violating supersymmetric models, lepton flavor mixing could generate sizable CP-conserving contributions to $K_L\to\pi^0\nu\bar{\nu}$ and generally in $K\to\pi\nu\bar{\nu}$ rates.

H. Other models

There exist other numerous analyses of $K\to\pi\nu\bar{\nu}$ decays within various extensions of the SM. For completeness, we describe them here.

Carlson *et al.* (1996) calculated the rate for $K_L \to \pi^0 \nu\bar{\nu}$ in several extensions of the SM Higgs sector, including the Liu-Wolfenstein two-doublet model of spontaneous CP violation and the Weinberg three-doublet model. They concluded that although in the usual two Higgs doublet model, with CP violation governed by the CKM matrix, some measurable effects could be seen, in models in which CP violation arises either entirely or predominantly from the Higgs sector, the decay rate is much smaller than in the SM.

The study of $K \to \pi\nu\bar{\nu}$ in models with four generations, extra vectorlike quarks, and isosinglet down quarks can be found in Hattori *et al.* (1998), Huang *et al.* (2001), Hawkins and Silverman (2002), Hung and Soddu (2002), Yanir (2002), and Aguilar-Saavedra (2003). In particular, in four generation models (Hattori *et al.*, 1998; Huang *et al.*, 2001; Yanir, 2002) due to three additional mixing angles and two additional complex phases, $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ can be enhanced by one to two orders of magnitude with respect to SM expectations and also $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ can be significantly enhanced. Unfortunately, due to many free parameters, the four generation models are not very predictive. A new analysis of $K \to \pi\nu\bar{\nu}$ in a model with an extra isosinglet down quark was given by Deshpande *et al.* (2004). Putting all available constraints on the parameters of this model, the authors concluded that $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ can still be enhanced up to the present experimental central value, while $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ can reach 1×10^{-10}.

The decays $K \to \pi\nu\bar{\nu}$ have also been investigated in a seesaw model for quark masses (Kiyo *et al.*, 1999). In this model, there are scalar operators $(\bar{s}d)(\bar{\nu}_\tau \nu_\tau)$, resulting from LR box diagrams, that make the rate for $K_L \to \pi^0 \nu\bar{\nu}$ nonvanishing even in the CP-conserving limit and in the absence of lepton-flavor mixing. But the enhancement of $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ due to these operators is at most of order 30% even for $M_{W_R} = 500$ GeV with a smaller effect in $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$.

The effects of electroweak symmetry breaking on rare K and B decays, including $K \to \pi\nu\bar{\nu}$, in the presence of new strong dynamics, have been worked out by Buchalla, Burdman, Hill, *et al.* (1996) and Burdman (1997). Deviations from the SM in $K \to \pi\nu\bar{\nu}$ have been shown to be correlated with the ones in B decays (Burdman, 1997).

The implications of a modified effective $Zb\bar{b}$ vertex on $K \to \pi\nu\bar{\nu}$, in connection with the small disagreement between the SM and the measured asymmetry A_{FB}^b at LEP, have been discussed by Chanowitz (1999, 2001). While the predictions are rather uncertain, an enhancement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ by a factor of 2, toward the central experimental value, is possible.

Enhancement of both $K \to \pi\nu\bar{\nu}$ branching ratios up to 50% has been found in a five-dimensional split fermions scenario (Chang and Ng, 2002) and the decay $K^+ \to \pi^+ \nu\bar{\nu}$ turns out to be the best for providing the constraints on the bulk SM in the Randall-Sundrum scenario (Burdman, 2002).

I. Summary

We have seen in this and the preceding section that many scenarios of new physics allow still for significant enhancements of both $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$: $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ can still be enhanced by factors of 2–3 and $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})$ could be larger by an order of magnitude than expected within the SM. While most models concentrate on possible enhancements of both branching ratios, their suppressions in several scenarios are still possible. This is the case in particular of the MSSM with MFV and in several models in which CP violation arises from the Higgs sector.

Because most models contain several free parameters, definite predictions for $K \to \pi\nu\bar{\nu}$ can only be achieved by considering simultaneously as many processes as possible so that these parameters are sufficiently constrained.

IX. COMPARISON WITH OTHER DECAYS

After this exposition of $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ decays in the SM and its most studied extensions, we compare the potential of these two clean rare decays in extracting the CKM parameters and in testing the SM and its extensions with other prominent K and B decays for which a rich literature exists. A subset of relevant references will be given below.

In the K system, the most investigated parameters in the past are ε_K and the ratio ε'/ε that describe, respectively, the indirect and direct CP violation in $K_L \to \pi\pi$ decays and the rare decays $K_L \to \mu^+\mu^-$ and $K_L \to \pi^0 e^+ e^-$. None of them can compete in the theoretical cleanness with the decays considered here, but some of them are still useful.

While $K_L \to \mu^+\mu^-$ and ε'/ε suffer from large hadronic uncertainties, the case considered for the decays $K_L \to \pi^0 \mu^+\mu^-$ and $K_L \to \pi^0 e^+ e^-$ is much more promising. They provide an interesting and complementary window to $|\Delta S| = 1$ SD transitions. While the latter is theoretically not as clean as the $K \to \pi\nu\bar{\nu}$ system, it is sensitive to different types of SD operators. The $K_L \to \pi^0 \ell^+ \ell^-$ decay amplitudes have three main ingredients: (i) a clean direct-CP-violating (CPV) component determined by SD dynamics, (ii) an indirect-CPV term due to K^0-$\overline{K}^0$ mixing, and (iii) a LD CP-conserving (CPC) component due to two-photon intermediate states. Although generated by different dynamics, these three components are of comparable size and can be computed (or indirectly determined) to good accuracy within the SM (Buchalla *et al.*, 2003; Isidori *et al.*, 2004). In the presence of nonvanishing NP contributions, the combined measurements of $K \to \pi\nu\bar{\nu}$ and $K_L \to \pi^0 \ell^+ \ell^-$ decays provide a unique tool to distinguish among different NP models.

Most advanced analyses of these decays within the SM can be found in Buchalla *et al.* (2003), Friot *et al.* (2004), and Isidori *et al.* (2004), where further references can be found. We also mention the recent analyses of these decays in the context of the MSSM (Isidori, Mescia, Paradisi, *et al.*, 2006) and other NP scenarios (Mescia *et al.*, 2006), in particular in the LHT model (Blanke, Buras, Recksiegel, *et al.*, 2007).

The situation with B decays is different. First, there are many more channels than in K decays, which allows us to eliminate or reduce many hadronic uncertainties by simultaneously considering several decays and using flavor symmetries. Also, the fact that now the b quark mass is involved in the effective theory allows us to calculate hadronic amplitudes in an expansion in the inverse power of the b quark mass and invoke related heavy quark effective theory, heavy quark expansions, QCD factorization for nonleptonic decays, perturbative QCD approach, and others. In recent years, considerable advances in this field have been made (Battaglia *et al.*, 2003). While in semileptonic tree level decays this progress allowed a decrease in the errors on the elements $|V_{ub}|$ and $|V_{cb}|$ (Battaglia *et al.*, 2003), in the case of prominent radiative decays like $B\to X_s\gamma$ and $B\to X_s l^+l^-$ these methods allowed for a better estimate of hadronic uncertainties. In addition, during the last decade theoretical uncertainties in these decays have been considerably reduced through the computations of NLO and in certain cases NNLO QCD corrections (Buchalla, Buras, and Lautenbacher, 1996; Buras, 1998; Nir, 2001; Fleischer, 2002; Ali, 2003; Buchalla, 2003; Hurth, 2003; 2004; Misiak *et al.*, 2007).

In the case of nonleptonic decays, various strategies for the determination of the unitarity triangle angles have been proposed. Reviews of these strategies have been presented by Fleischer (2002, 2004); Cavoto *et al.* (2007) and see also Nir (2001), Ali (2003), Buchalla (2003), Buras (2003, 2005a, 2005b), and Hurth (2003). These strategies generally use simultaneously several decays and are based on plausible dynamical assumptions that can be further tested by invoking other decays.

There is no doubt that these methods will give us considerable insight into flavor and QCD dynamics, but it is fair to say that most of them cannot match the $K\to\pi\nu\bar{\nu}$ decays with respect to the theoretical cleanness. On the other hand, there exist a number of strategies for the determination of the angles and also sides of the unitarity triangle that certainly can compete with the $K\to\pi\nu\bar{\nu}$ complex and in certain cases are even slightly superior to it, provided corresponding measurements can be made precisely.

Yet the present status of FCNC processes in the B_d system indicates that the new physics in this system enters only at a subleading level. While certain departures from the SM are still to be clarified, this will not be easy, particularly in the case of nonleptonic decays.

More promising from the point of view for the search of new physics is the B_s system. While the measurement of ΔM_s did not reveal large contributions from NP, the case of the CP asymmetry $S_{\psi\phi}$ and of the branching ratios $\mathrm{Br}(B_{d,s}\to\mu^+\mu^-)$ could be different as they all are strongly suppressed within the SM. The experiments at LHC will undoubtedly answer the important question of whether these observables signal NP beyond the SM. Even more detailed investigations will be available at a Super-B machine.

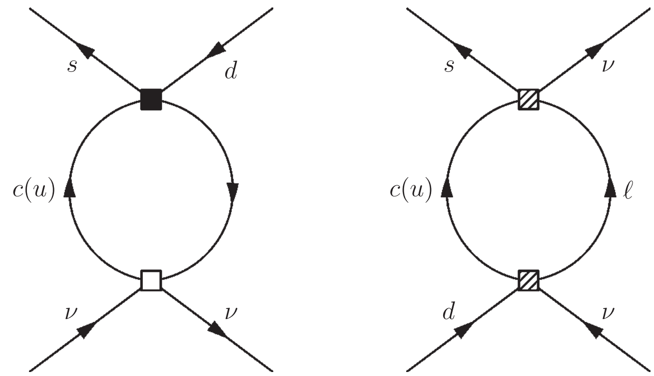

FIG. 17. One-loop diagrams with light quarks that generate higher-dimensional operators From Isidori *et al.*, 2005.

X. SUBLEADING CONTRIBUTIONS TO $K\to\pi\nu\bar{\nu}$

In this section, we discuss the subleading contributions to the decays $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ that we have discussed so far. More detailed discussions and explicit calculations have been presented by Ecker *et al.* (1988), Hagelin and Littenberg (1989), Rein and Sehgal (1989), Lu and Wise (1994), Geng *et al.* (1996), Fajfer (1997), Buchalla and Isidori (1998), and Isidori *et al.* (2005). These effects can be potentially interesting, especially when $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ are measured with an accuracy of 5%.

Accordingly, we begin with the discussion of $K^+\to\pi^+\nu\bar{\nu}$, where there can be, in principle, two additional following contributions to the branching ratio:

- Effects through soft u quarks in the penguin loop that induce an on-shell $K^+\to\pi^+Z^0\to\pi^+\nu\bar{\nu}$ transition as well as similar processes induced by W-W exchange. These are long-distance effects and addressed (Ecker *et al.*, 1988; Hagelin and Littenberg, 1989; Rein and Sehgal, 1989; Lu and Wise, 1994; Geng *et al.*, 1996; Fajfer, 1997) in chiral perturbation theory.
- Higher-dimensional operators contributing to the OPE in the charm sector (Falk *et al.*, 2001).

Most recently, both effects were investigated in detail by Isidori *et al.* (2005), paying particular attention to the cancellation of the renormalization scale dependence between both contributions.

Therefore, we follow Isidori *et al.* (2005) with a more elaborate discussion of both effects in detail: In particular, concerning the effects of higher-dimensional operators, the results of Falk *et al.* (2001) have been fully confirmed. These contributions have to be considered only in the charm sector, if one assumes a natural scaling of M_K^2/m_q^2 in the Wilson coefficients. The scaling of the Inami-Lim functions then leads to an overall scaling of

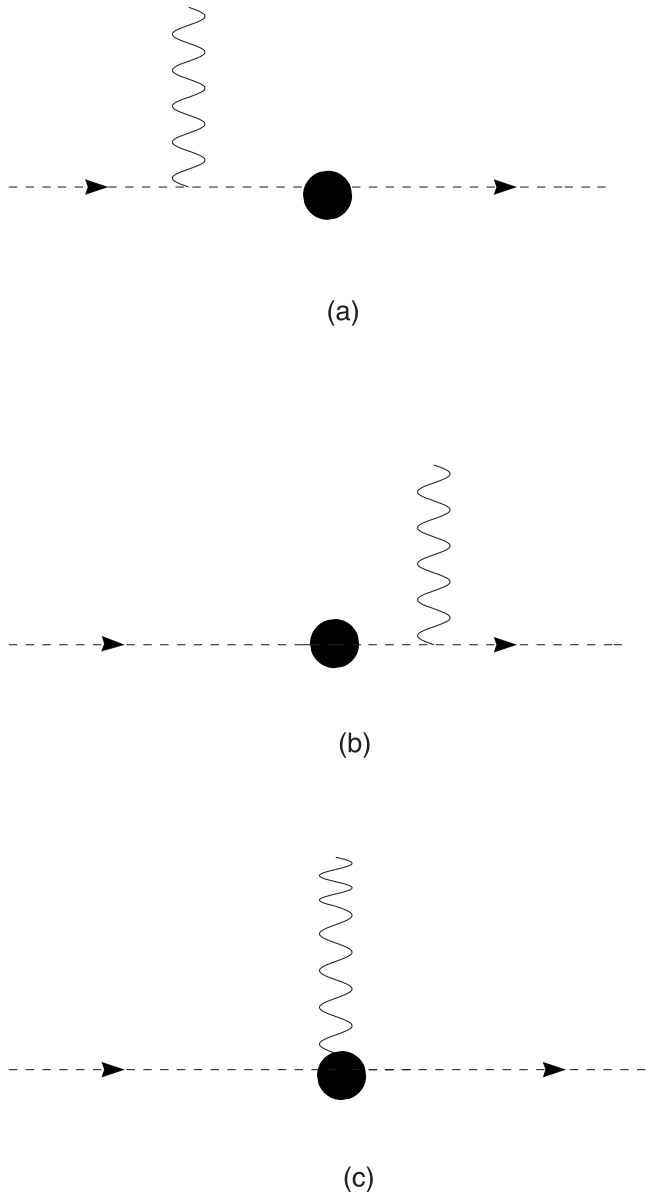

FIG. 18. Leading-order chiral perturbation theory diagrams contributing to a $K^+ \to \pi^+ Z^0$ vertex [from Lu and Wise (1994)]. Dashed lines denote the pion and kaon, while the wavy line denotes the Z^0, and the dot indicates the insertion of a flavor-changing effective vertex.

M_K^2/M_W^4, which is independent of the quark masses. The top contribution is then suppressed by CKM factors.

Going to dimension 8, one finds two operators that appear when expanding the penguin and box diagrams:

$$O_1^l = \bar{s}\gamma^\nu(1-\gamma^5)d(i\partial)^2[\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l],$$

$$\begin{aligned} O_2^l = {} & \bar{s}\gamma^\nu(1-\gamma^5)(iD)^2 d\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l \\ & + 2\bar{s}\gamma^\nu(1-\gamma^5)(iD^\mu)d\bar{\nu}_l\gamma_\nu(1-\gamma^5)(\partial_\mu)\nu_l \\ & + \bar{s}\gamma^\nu(1-\gamma^5)d\bar{\nu}_l\gamma_\nu(1-\gamma^5)(i\partial)^2\nu_l, \end{aligned} \tag{10.1}$$

where D^μ is the covariant derivative involving the gluon field. The coefficients of these operators are determined by matching the diagrams in Fig. 17 onto these operators, where one finds that the neutral coupling in the left diagram generates O_1^l while the charged coupling in the right diagram is responsible for O_2^l. These coefficients were given by Falk *et al.* (2001) and Isidori *et al.* (2005).

While the matrix element of O_1^l can be reliably estimated and gives a negligible contribution compared to the leading dimension-6 terms, the matrix element of O_2^l is harder to estimate, due to the gluon appearing in the covariant derivative. A numerical estimate is performed using the Lorentz structure and parametrizing the remaining hadronic effects by a bag factor, which is determined by matching onto the genuine long-distance contributions and demanding that the renormalization scale dependence should cancel. Further progress can be achieved through lattice calculations (Isidori, Martinelli, and Turchetti, 2006).

While the discussion so far is rather straightforward, the genuine long-distance effects from u quark loops have received much more attention (Hagelin and Littenberg, 1989; Rein and Sehgal, 1989; Lu and Wise, 1994; Geng *et al.*, 1996; Fajfer, 1997; Isidori *et al.*, 2005). Again, we follow Isidori *et al.* (2005), where the most recent and complete discussion has been given. In particular, it was shown that previous calculations missed several terms that are necessary to obtain the correct matching between short- and long-distance components in the amplitude.

In order to address these effects, one begins with the chiral effective $\Delta S=1$ Hamiltonan [see, e.g., D'Ambrosio and Isidori (1998), for a review]. From the chiral transformation properties, one finds that this Hamiltonian consists of pieces that transform as $(8_L,1_R)$ and $(27_L,1_R)$ under the chiral symmetry group $SU(3)_L \times SU(3)_R$. Experimentally, one finds that the octet piece is enhanced (this corresponds to the usual $\Delta I=1/2$ rule) so that $(27_L,1_R)$ can be neglected. To lowest order in the chiral expansion and using only the octet contribution, there is one operator that contributes,

$$\mathcal{L}^{(2)}_{|\Delta S|=1} = G_8 F^4 \langle \lambda_6 D^\mu U^\dagger D_\mu U\rangle, \tag{10.2}$$

where $G_8 \approx 9\times 10^{-6}\ \mathrm{GeV}^{-2}$, U is the conventional representation of the pseudoscalar meson fields, and $\langle\,\rangle$ implies a trace. Using the Hamiltonian thus obtained, one finds that the leading-order diagrams in CHPT (Fig. 18) cancel (Ecker *et al.*, 1988; Lu and Wise, 1994). However, to be consistent, there are additional operators to be included since the $SU(2)_L$ generators are broken and, for an effective chiral Lagrangian, also non-gauge-invariant operators with the correct representation must be added [this is not necessary for the $K\to\pi\gamma$ vertex (Ecker *et al.*, 1988)]. Including these operators also leads to the same parametric renormalization scale dependence as in the short-distance part of the amplitude. Then, the complete chiral Lagrangian is given by (Isidori *et al.*, 2005)

$$\mathcal{L}^{(2)}_{|\Delta S|=1} = G_8 F^4 \left\langle \lambda_6 \left[D^\mu U^\dagger D_\mu U - 2ig_Z Z_\mu U^\dagger D^\mu U \left(Q - \frac{a_1}{6} \right) \right] \right\rangle, \tag{10.3}$$

where a_1 is related to the coupling of the Z to the $U(1)_L$ charge (Lu and Wise, 1994). One finds then that the $O(p^2)$ terms do not cancel for the charged $(K^+ \to \pi^+ Z)$ amplitude,

$$\mathcal{A}(K^+ \to \pi^+ \nu\bar{\nu})_Z = \frac{G_F}{\sqrt{2}} G_8 F^2 [4p^\mu] \sum_l \bar{\nu}_l \gamma_\mu (1-\gamma_5)\nu_l. \tag{10.4}$$

Isidori *et al.* (2005) extended this calculation to $O(p^4)$, which involved several one-loop diagrams. The final contributions come from W-W exchange diagrams (Hagelin and Littenberg, 1989; Isidori *et al.*, 2005), which correspond, on the short-distance side, to the contributions from O_2^l and accordingly should cancel the respective renormalization scale dependence. We give the tree level result (Isidori *et al.*, 2005),

$$\mathcal{A}(K^+ \to \pi^+ \nu\bar{\nu})_{WW} = G_F^2 F^2 \lambda \sum_{l=e,\mu} 2p^\mu \bar{\nu}_l \gamma_\mu (1-\gamma_5)\nu_l. \tag{10.5}$$

Summing up all contributions, one can include all subleading effects discussed in this section by shifting the value of $P_c(X)$,

$$P_c^{(6)} \to P_c^{(6)} + \delta P_{c,u}, \quad \delta P_{c,u} = 0.04 \pm 0.02, \tag{10.6}$$

which implies a shift of roughly 6% in the branching ratio.

We now turn to $K_L \to \pi^0 \nu\bar{\nu}$. Long-distance contributions are equivalent to CP-conserving effects and have been comprehensively studied by Buchalla and Isidori (1998). As for $K^+ \to \pi^+ \nu\bar{\nu}$, there are effects from soft up quarks, which are treated in chiral perturbation theory, and higher-dimensional operators in the charm sector, which are actually short-distance effects. It is found that they are suppressed by several effects, reinforcing the theoretically clean character of this decay. We now briefly describe these effects.

The contributions from soft up quarks in the penguin loops have been studied by Geng *et al.* (1996) and Buchalla and Isidori (1998). As is the case for $K^+ \to \pi^+ \nu\bar{\nu}$, the leading diagrams appear at one-loop order. They have been calculated explicitly by Buchalla and Isidori (1998), who found, taking into account also phase-space suppression, that the CP-conserving long-distance contributions are suppressed by approximately a factor of 10^{-5} compared to the dominant top contribution.

The next contribution that can be important is higher-dimensional operators in the OPE. As Buchalla and Isidori (1998) studied only CP-conserving contributions, only one operator that is antisymmetric in neutrino momenta survives from the expansion of the box diagrams (contributions from Z^0 penguins also drop out for the same reason),

$$H_{\mathrm{CPC}} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\Theta_W} \lambda_c \ln \frac{m_c}{\mu} \frac{1}{M_W^2} \times T_{\alpha\mu} \bar{\nu} (\overleftarrow{\partial}^\alpha - \partial^\alpha) \gamma^\mu (1-\gamma_5) \nu, \tag{10.7}$$

$$T_{\alpha\mu} = \bar{s} \overleftarrow{D}_\alpha \gamma_\mu (1-\gamma_5) d - \bar{d} \gamma_\mu (1-\gamma_5) D_\alpha s. \tag{10.8}$$

There arise now several suppression factors: First, there is the naive suppression of the operator scaling, which is estimated to be $\mathcal{O}(\lambda_c M_K^2 / \mathrm{Im}\, \lambda_t M_W^2) \approx 10\%$ compared to the leading top contribution. Here the smallness of M_K/M_W is compensated by the ratio of CKM factors $\lambda_c / \mathrm{Im}\, \lambda_t$.

The suppression is more severe when the matrix elements are calculated, since the leading-order K_L-π^0 matrix element in chiral perturbation theory is

$$\langle \pi^0(p) | T_{\alpha\mu} | K_L(k) \rangle = -\frac{i}{2} \left[(k-p)_\alpha (k+p)_\mu + \frac{1}{4} m_K^2 g_{\alpha\mu} \right], \tag{10.9}$$

which vanishes when multiplied with the leptonic current in the operator due to the equations of motion and the negligible neutrino masses. The chiral suppression of the NLO (p^4) terms leads to an additional reduction of higher-dimensional operator contributions by $m_K^2 / 8\pi^2 f_\pi^2 \approx 20\%$. Finally, one also has to take into account phase-space effects, which further suppress these terms.

Estimating the $\mathcal{O}(p^4)$ matrix elements and performing the phase-space calculations, Buchalla and Isidori (1998) found that short-distance CP-conserving effects are suppressed by a factor of 10^{-5} compared to the dominant top contribution and conclude that they are "safely negligible, by a comfortably large margin."

It is then fair to say, from the present perspective, that long-distance effects are well under control especially in $K_L \to \pi^0 \nu\bar{\nu}$, but also in $K^+ \to \pi^+ \nu\bar{\nu}$, where the contributions and its uncertainty can be rather reliably quantified and included in numerical analyses. This is gratifying, since the NNLO calculation is available and of the same order of magnitude.

XI. CONCLUSIONS AND OUTLOOK

In the present review, we summarized the present status of the rare decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$, paying particular attention to theoretical and parametric uncertainties. Our analysis reinforced the importance of these decays in testing the SM and its extensions. We pointed out that the clean theoretical character of these decays remains valid in all extensions of the SM, whereas this is often not the case for nonleptonic two-body B decays used to determine the CKM parameters through CP asymmetries and/or other strategies. Here, in extensions of the SM in which new operators and new weak phases are present, the mixing-induced asymmetry $a_{\phi K_S}$ and other similar asymmetries can suffer from potential hadronic uncertainties that make the determination of the relevant parameters problematic unless the hadronic matrix element can be calculated with sufficient precision. In spite of advances in nonperturbative calculations of nonleptonic amplitudes for B decays (Beneke *et al.*, 1999; Keum *et al.*, 2001a, 2001b, 2002; Bauer, Fleming, Pirjol, *et al.*, 2002; Bauer *et al.*, 2002a, 2002b; Beneke and Feldmann, 2003; Beneke and Neubert, 2003; Stewart, 2003), we are still far away from precise calculations of nonleptonic amplitudes from first principles. On the other hand, the branching ratios for $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ can be parametrized in all extensions of the SM by a single complex function X (real in the case of MFV models) that can be calculated in perturbation theory in any given extension of the SM.

There exists, however, a handful of strategies in the B system that, similarly to $K\to\pi\nu\bar{\nu}$, are very clean. Moreover, in contrast to $K\to\pi\nu\bar{\nu}$, there exist strategies involving B decays that allow not only a theoretically clean determination of the UT but also one free from new physics pollution.

Our main findings are as follows:

- Our present predictions for the branching ratios read

$$\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})_{\mathrm{SM}} = (8.1\pm1.1)\times10^{-11}, \tag{11.1}$$

$$\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})_{\mathrm{SM}} = (2.6\pm0.3)\times10^{-11}. \tag{11.2}$$

  This is an accuracy of ±14% and ±12%, respectively.

- Our analysis of theoretical uncertainties in $K\to\pi\nu\bar{\nu}$, which come exclusively from the charm contribution to $K^+\to\pi^+\nu\bar{\nu}$, reinforced the importance of the recent NNLO calculation to this contribution (Buras, Gorbahn, Haisch, *et al.*, 2005, 2006). Indeed, the ±18% uncertainty in $P_c(X)$ coming dominantly from the scale uncertainties and the value of $m_c(m_c)$ translates into an uncertainty of ±7.0% in the determination of $|V_{td}|$, ±0.04 in the determination of $\sin 2\beta$, and ±10% in the prediction for $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$. The NNLO analysis reduced the uncertainty in P_c to 12%, and further progress on the determination of $m_c(m_c)$ could reduce the error in $P_c(X)$ down to ±5%, implying the reduced error in $|V_{td}|$ of ±2%, in $\sin 2\beta$ of ±0.011, and ±3% in $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$.

- Further progress on determining the CKM parameters, which in the next few years will come from BaBar, Belle, and Tevatron and later from LHC, should allow predictions for $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ with uncertainties of roughly ±5% or better. It should be emphasized that this accuracy cannot be matched by any other rare decay branching ratio in the field of meson decays.

- We analyzed the impact of precise measurements of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ on the unitarity triangle and other observables of interest, within the SM. In particular, we analyzed the accuracy with which $\sin 2\beta$ and the angle γ could be extracted from these decays. Provided both branching ratios can be measured with an accuracy of ±5%, an error on $\sin 2\beta$ of ±0.038 could be achieved. The determination of γ requires an accurate measurement of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and the reduction of errors in $P_c(X)$ and $|V_{cb}|$. With a measurement better than ±5% of $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and the reduction of errors in $P_c(X)$ and $|V_{cb}|$ anticipated, γ could be measured with an error of ±5°.

- We emphasized that the simultaneous investigation of the $K\to\pi\nu\bar{\nu}$ decay, the mass differences $\Delta M_{d,s}$, and the angles β and γ from clean strategies in two-body B decays should allow us to disentangle different new physics contributions to various observables and determine new parameters for extensions of the SM. The (R_t,β), (R_b,γ), (β,γ), and $(\bar{\eta},\gamma)$ strategies for UT when combined with $K\to\pi\nu\bar{\nu}$ decays are useful in this goal. This is the case in particular for the (R_b,γ) strategy that is related to the reference unitarity triangle (Goto *et al.*, 1996; Cohen *et al.*, 1997; Grossman *et al.*, 1997; Barenboim *et al.*, 1999). A graphical representation of these investigations is given in Fig. 9.

- We have presented a new "golden relation" between β, γ, and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$, given in Eq. (3.20), that with improved values of m_t and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ should allow a clean test of the SM one day. Another new relation is between β, γ, and $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$, which is given in Eq. (3.11). Although not as clean as the golden relation in Eq. (3.20) because of the presence of P_c, it should play a useful role in future investigations.

- We have presented results for both decays in models with minimal flavor violation and in several scenarios with new complex phases in Z^0 penguins and/or B_d^0-$\bar{B}_d^0$ mixing. We reviewed the results for $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ in a number of specific extensions of the SM. In particular, we discussed LHT, Z' and supersymmetry with MFV, more general supersymmetric models with new complex phases, models with universal extra dimensions, and models with lepton-flavor mixing. Each of these models has some characteristic predictions for the branching ratios in question, so that it should be possible to distinguish between various alternatives. Simultaneous investigations of other observables should be helpful in this respect. In some of these scenarios, departures from SM expectations are still allowed to be spectacular.

- Finally, we compared the usefulness of $K\to\pi\nu\bar{\nu}$ decays in testing various models with other decays. While in the K system $K\to\pi\nu\bar{\nu}$ decays have no competition, there are a handful of B decays and related strategies that are also theoretically clean. It is precisely the comparison between the results of these clean strategies in the B system with the ones obtained one day from $K\to\pi\nu\bar{\nu}$ decays that will be most interesting.

- In spite of an impressive agreement of the SM with the available data, large departures from SM expectations in B_s decays are still possible. However, even if future Tevatron and LHC data would not see any significant new physics effect in these decays, this will not imply necessarily that new physics is not visible in $K_L\to\pi^0\nu\bar{\nu}$, $K^+\to\pi^+\nu\bar{\nu}$, and $K_L\to\pi^0\ell^+\ell^-$. On the contrary, as seen in particular in the case of the LHT model (Blanke, Buras, Recksiegel, *et al.*, 2007), there are scenarios in which the effects in B physics are small, while large departures in these three decays will still be possible. It may then be that in the end, it will be K physics and not B physics that will offer the best information about the new phenomena at short distance scales, in accordance with the arguments in Bryman *et al.* (2006) and Grinstein *et al.* (2007).

We hope we convinced the reader that clean rare decays $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ deserve a prominent status in the field of flavor and CP violation and that precise measurements of their branching ratios are of utmost importance. We hope that our wait for these measurements will not be too long.

ACKNOWLEDGMENTS

We thank Jürgen Engelfried, Steve Kettell, and Laur Littenberg for information about the future prospects on $K\to\pi\nu\bar{\nu}$ experiments, and Gino Isidori and Steve Kettell for comments on the manuscript. Further, we thank Frederico Mescia and Christopher Smith for providing information on details of κ_+ and κ_L. F.S. thanks the IFAE, Barcelona, where final revisions and updates have been performed. The work presented here was supported in part by the German Bundesministerium für Bildung und Forschung under the Contracts No. 05HT4WOA/3, No. 05HT6WOA, and the DFG Project Bu. 706/1-2.

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