CP violation from the standard model to strings

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A review of *CP* violation from the standard model to strings is given which includes a broad landscape of particle physics models, encompassing the nonsupersymmetric four-dimensional (4D) extensions of the standard model, and models based on supersymmetry, on extra dimensions, on strings, and on branes. The supersymmetric models discussed include complex minimal supergravity unified model and its extensions, while the models based on extra dimensions include five-dimensional models including models based on warped geometry. *CP* violation beyond the standard model is central to achieving the desired amount of baryon asymmetry in the Universe via baryogenesis and leptogenesis. They also affect a variety of particle physics phenomena: electric dipole moments, $g-2$, relic density and detection rates for neutralino dark matter in supersymmetric theories, Yukawa unification in grand unified and string based models, and sparticle production cross sections, and their decay patterns and signatures at hadron colliders. Additionally *CP* violations can generate *CP* even–*CP* odd Higgs mixings, affect the neutral Higgs spectrum, and lead to phenomena detectable at colliders. Prominent among these are the *CP* violation effects in decays of neutral and charged Higgs bosons. Neutrino masses introduce new sources of *CP* violation which may be explored in neutrino factories in the future. Such phases can also enter in proton stability in unified models of particle interactions. The current experimental status of *CP* violation is discussed and possibilities for the future outlined.

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I. INTRODUCTION

We begin with a brief history of the considerations that led to question the validity of *CP* symmetry as an exact symmetry for elementary particles. The history is tied to the issue of electric dipole moments and we retrace the steps back to 1950 when it was generally accepted that the particle electric dipole moments vanished due to parity symmetry. However, in 1950 it was first observed by Purcell and Ramsey (1950) that there was no experimental evidence for the parity symmetry for nuclear forces and for elementary particles, and thus the possible existence of an electric dipole moment for these needed to be tested experimentally. They and their graduate student James Smith then carried out such a test by showing experimentally in 1951 that the magnitude of the electric dipole moment of the neutron was less than $3\times 10^{-20}\, e\,\mathrm{cm}$ where e is the charge of the proton.[1] After the violation of parity symmetry proposed by Lee and Yang (1956) was confirmed (Wu *et al.*, 1957), it was argued that the elementary electric dipole moments would vanish due to the combined charge conjugation and parity symmetry, i.e., *CP* symmetry (or equivalently under a time reversal symmetry under the assumption of *CPT* invariance). However, it was then pointed out by Ramsey (1958) and independently by Jackson and collaborators (Jackson *et al.*, 1957) that *T* invariance was also an assumption and needed to be checked experimentally [a brief review of early history can be found in Ramsey (1998)]. Since then the search for *CP* violations has been vigorously pursued. The *CP* violation was eventually discovered in the kaon system by Val Fitch, James Cronin, and collaborators in 1964 (Christenson *et al.*, 1964). Shortly thereafter it was pointed out by Andre Sakharov (1967) that *CP* violations play an important role in generating the baryon asymmetry in the Universe. However, it has recently been realized that sources of *CP* violation beyond what exist in the standard model are needed for this purpose. In this context over the past decade a significant body of work on *CP* violation beyond the standard model has appeared. It encompasses nonsupersymmetric models, supersymmetric models, models based on extra dimensions and warped dimensions, and string models. There is currently no review which encompasses these developments. The purpose of this review is to bridge this gap. Thus in this review we present a broad overview of *CP* violation starting from the standard model and ending with strings. *CP* violation is central to understanding the phenomena in particle physics as well as in cosmology. Thus *CP* violation enters in *K* and *B* physics, and, as mentioned above, *CP* violation beyond the standard model is deemed necessary to explain the desired baryon asymmetry in the Universe. Further, new sources of *CP* violation beyond the standard model could also show up in sparticle production at the Large Hadron Collider, and in the new generation of experiments underway on neutrino physics. In view of the importance of *CP* violation in particle physics and in cosmology it is also important to explore the possible origins of such violations. These topics are the focus of this review. We now give a brief outline of the contents of this review.

In Sec. II we discuss *CP* violation in the standard model and the strong *CP* problem. The electroweak sector of the standard model contains one phase which appears in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The CKM matrix satisfies unitarity constraints including the well known unitarity triangle constraint where the three angles α,β,γ defined in terms of ratios involving the products of CKM matrix elements and their complex conjugates sum to π. In addition, the quantum chromodynamic (QCD) sector of the standard model brings in another source of *CP* violation—the strong *CP* phase θ_{QCD}. The natural size of this phase is $O(1)$ which would produce a huge contribution to the electric dipole moment (EDM) of the neutron in violation of existing experimental bounds. A brief discussion of these issues is given in Sec. II. A review of the experimental evidence for *CP* violation and of the searches for evidence of other *CP* violation such as in the electric dipole moment of elementary particles and of atoms is given in Sec. III. Here we discuss the current experimental situation in the *K* and *B* system. In the kaon system two parameters, ϵ (indirect *CP* violation) and ϵ' (direct *CP* violation), have played an important role in the discussion of *CP* violation in this system. Specifically the measurement of ϵ'/ϵ rules out the so-called superweak theory of *CP* violation while the measurement is consistent with the standard model prediction. In this section we also analyze the experimental constraints on the angles α,β,γ of the unitarity triangle discussed in Sec. II. The current experimental EDM limits of the electron, of the neutron, and of $^{199}\mathrm{Hg}$ are also discussed.

In Sec. IV we discuss *CP* violation in nonsupersymmetric extensions of the standard model. These include

[1] The experimental results of Purcell, Ramsey, and Smith while completed in 1951 were not published until much later (Smith *et al.*, 1957). However, they were quoted in other publications (Smith, 1951; Lee and Yang, 1956; Ramsey, 1956).

the left-right (LR) extensions, the two Higgs doublet model, and extensions with more than two Higgs doublets. It is shown that such extensions contain more sources of CP violation. For example, the LR extensions with the gauge group $SU(2)_L \times SU(2)_R \times U(1)_Y$ and three generations contain seven CP phases instead of one phase that the standard model has. Similarly it is shown that the number of allowed CP phases increases with the number of Higgs doublets. Further, new sources of CP violation arise as one increases the number of allowed generations. CP violation in the context of supersymmetric extensions of the standard model are discussed in Sec. V. Here one finds that the minimal supersymmetric standard model (MSSM) has a large number (i.e., 46) of phases which, however, is reduced to two phases in the minimal supergravity unified model (mSUGRA). However, more phases are allowed if one considers supergravity unified models with nonunivesal soft breaking at the grand unified (GUT) scale consistent with flavor changing neutral current (FCNC) constraints. A discussion of CP violation in extra dimension models is given in Sec. VI. In this section we exhibit phenomena of spontaneous vs explicit CP violation. In this section we also discuss CP violation in the context of warped extra dimensions.

A discussion of CP violation in strings is given in Sec. VII. It is shown that soft breaking in string models is parametrized by vacuum expectation values (VEVs) of the dilaton (S) and of the moduli fields (T_i) which carry CP violating phases. Additionally CP phases can occur in the Yukawa couplings. Thus CP violation is quite generic in string models. We give specific illustration of this in a Calabi-Yau compactification of an $E_8 \times E_8$ heterotic string and in orbifold compactifications. Here we also discuss CP violation in D brane models. Finally in this section we discuss the possible connection of SUSY CP phases with the CKM phase.

A discussion of computing the EDM of an elementary Dirac fermion is given in Sec. VIII while that of a charged lepton in supersymmetric models is given in Sec. IX. In Sec. X we analyze of the EDM of quarks in supersymmetry. The supersymmetric contributions to the EDM of a quark involve three different pieces which include the electric dipole, the chromoelectric dipole, and the purely gluonic dimension-6 operators. The contributions of each of these are discussed in Sec. X. Typically for low lying sparticle masses the supersymmetrtic contribution to the EDM of the electron and of the neutron is generally in excess of current experimental bounds. This poses a serious difficulty for supersymmetric models. Some ways to overcome these are also discussed in Sec. X. Two prominent ways to accomplish this include either a heavy sparticle spectrum with sparticle masses lying in the TeV region or the cancellation mechanism where contributions arising from the electric dipole, the chromoelectric dipole, and the purely gluonic dimension-6 operators largely cancel.

If the large SUSY CP phases can be made consistent with the EDM constraints, then such large phases can affect a variety of supersymmetric phenomena. We discuss several such phenomena in Sec. XI. These include analyses of the effect of CP phases on $g_\mu - 2$, on CP even–CP odd Higgs mixing in the neutral Higgs sector, and on the b quark mass. Further, CP phases can affect significantly the neutral Higgs decays into $b\bar{b}$ and $\tau\bar{\tau}$ and the decays of the charged Higgs into $\bar{t}b, \bar{\nu}_\tau \tau$ and the decays $H^\pm \to \chi^\pm \chi^0$. These phenomena are also discussed in Sec. XI. Some other phenomena affected by CP phases include the relic density of neutralino dark matter, proton decay via dimension-6 operators, the decay $B_s^0 \to \mu^+\mu^-$, decays of the sfermions, and the decay $B \to \phi K$. These are all discussed in some detail in Sec. XI. Finally in this section we discuss the T and CP odd operators and their observability at colliders. An analysis of the interplay between CP violation and flavor is given in Sec. XII. Here we discuss the mechanisms which may allow the muon EDM to be much larger than the electron EDM, and accessible to a new proposed experiment on the muon EDM which may extend the sensitivity of this measurement by several orders of magnitude and thus make it potentially observable. In this section an analysis of the effect of CP phases on $B \to X_s \gamma$ is also given. This FCNC process is of importance as it constrains the parameter space of MSSM and also constrains the analyses of dark matter. Section XIII is devoted to a study of CP violation in neutrino physics. Here a discussion of CP violation and leptogenesis is given, as well as a discussion on the observability of Majorana phases.

Future prospects for improved measurement of CP violation in experiments are discussed in Sec. XIV. These include improved experiments for the measurements of the EDMs, B physics experiments at the LHCb which is dedicated to the study of B physics, the Super Belle proposal, as well as superbeams which include the study of possible CP violation in neutrino physics. Conclusions are given in Sec. XV. Some further mathematical details are given in the Appendixes.

II. *CP* VIOLATION IN THE STANDARD MODEL AND THE STRONG *CP* PROBLEM

The electroweak sector of the standard model with three generations of quarks and leptons has one CP violating phase which enters via the Cabbibo-Kobayashi-Maskawa (CKM) matrix V. Thus the electroweak interactions contain the CKM matrix in the charged current sector,

$$g_2 \bar{u}_i \gamma_\mu V_{ij} (1-\gamma_5) d_j W^\mu + \text{H.c.}, \tag{1}$$

where $u_i = u, c, t$ and $d_j = d, s, b$ quarks. The CKM matrix obeys the unitarity constraint $(VV^\dagger)_{ij} = \delta_{ij}$ and can be parametrized in terms of three mixing angles and one CP violating phase. For the case $i \neq j$ the unitarity constraint can be displayed as a unitarity triangle, and there are six such unitarity triangles. Thus the unitarity of the CKM matrix for the first and third columns gives

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0. \tag{2}$$

One can display this constraint as a unitarity triangle by defining the angles α,β,γ so that

$$\alpha=\arg(-V_{td}V_{tb}^{*}/V_{ud}V_{ub}^{*}),$$

$$\beta=\arg(-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*}),$$

$$\gamma=\arg(-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*}), \tag{3}$$

which satisfy the constraint $\alpha+\beta+\gamma=\pi$. One can parametrize *CP* violation in a way which is independent of the phase conventions. This is the so-called Jarlskog invariant (Jarlskog, 1985) J which can be defined in nine different ways, and one of which is given by

$$J=\mathrm{Im}(V_{us}V_{ub}^{*}V_{cb}V_{cs}^{*}). \tag{4}$$

An interesting observation is that the CKM is hierarchical and allows for expansion in $\lambda\simeq 0.226$ so one may write V as a perturbative expansion in λ which up $O(\lambda^3)$ is given by

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{5}$$

In this representation the Jarlskog invariant is given by $J\simeq A^2\lambda^6\eta$, and the *CP* violation enters via η.

The standard model has another source of *CP* violation in addition to the one that appears in the CKM matrix. This source of *CP* violation arises in the strong interaction sector of the theory from the term $\theta(\alpha_s/8\pi)G\tilde{G}$, which is of topological origin. It gives a large contribution to the EDM of the neutron and consistency with current experiment requires $\bar{\theta}=\theta+\mathrm{Arg\,Det}(M_uM_d)$ to be small $\bar{\theta}<O(10^{-10})$. One solution to the strong *CP* problem is the vanishing of the up quark mass. However, analyses based on chiral perturbation theory and on lattice gauge theory appear to indicate a nonvanishing mass for the up quark. Thus resolution to the strong *CP* problem appears to require beyond the standard model physics. For example, one proposed solution is the Peccei-Quinn mechanism (Peccei and Quinn, 1977) and its refinements (Kim, 1979; Zhitnitskii, 1980; Dine *et al.*, 1981) which leads to axions. But currently severe limits exist on the corridor in which axions can exist. There is much work in the literature regarding how one may suppress the strong *CP* violation effects [for a review, see Dine (2000)]. In addition to the use of axions or a massless up quark one also has the possibility of using a symmetry to suppress the strong *CP* effects (Barr, 1984; Nelson, 1984).

The solution to the strong *CP* in the framework of left-right symmetric models has been discussed by Mohapatra *et al.* (1997) and Babu *et al.* (2002). Specifically in the analysis of Babu *et al.* (2002) the strong *CP* parameter $\bar{\theta}$ is zero at the tree level, due to parity (P), but is induced due to P-violating effects below the unification scale. In the analysis of Hiller and Schmaltz (2001) a solution to the strong *CP* problem using supersymmetry was proposed. Here one envisions a solution to the strong *CP* problem based on supersymmetric nonrenormalization theorem. In this scenario *CP* is broken spontaneously and its breaking is communicated to the MSSM by radiative corrections. The strong *CP* phase is protected by a SUSY nonrenormalization theorem and remains exactly zero while the loops can generate a large CKM phase from wave function renormalization. Another idea advocates promoting the U(1) *CP* violating phases of the supersymmetric standard model to dynamical variables, and then allowing the vacuum to relax near a *CP* conserving point Dimopoulos and Thomas (1996). In the analysis of Demir and Ma (2000) an axionic solution of the strong *CP* problem with a Peccei-Quinn mechanism using the gluino rather than the quarks was given and the spontaneous breaking of the new U(1) global symmetry was connected to the supersymmetry breaking with a solution to the μ problem (Demir and Ma, 2000). Finally, in the analysis of Aldazabal *et al.* (2004) a solution based on gauging away the strong *CP* problem was proposed. The work of Aldazabal *et al.* (2004) proposed a solution that involves the existence of an unbroken gauged $\mathrm{U}(1)_X$ symmetry whose gauge boson gets a Stueckelberg mass term by combining with a pseudoscalar field $\eta(x)$ which has axionlike coupling to $G\tilde{G}$. Thus the θ parameter can be gauged away by a $\mathrm{U}(1)_X$ transformation. The additional $\mathrm{U}(1)_X$ generates mixed gauge anomalies which are canceled by the addition of an appropriate Wess-Zumino term. We assume from here on that the strong *CP* problem is solved by one or the other of the techniques outlined above.

III. REVIEW OF EXPERIMENTAL EVIDENCE ON *CP* VIOLATION AND SEARCHES FOR OTHER EVIDENCE

There are currently four pieces of experimental evidence for *CP* violation. These consist of (i) the observation of indirect *CP* violation (ϵ), (ii) of direct *CP* violation (ϵ'/ϵ) in the kaon system, (iii) the observation of *CP* violation in B physics, and (iv) indirect evidence for *CP* violation due to the existence of baryon asymmetry in the Universe. Thus far the experimental evidence indicates that the *CP* violation in the K and B physics can be understood within the framework of the standard model. However, an understanding of baryon asymmetry in the Universe requires a new source of *CP* violation We review these below.

A. *CP* violations in the kaon system[2]

Historically the first indication for *CP* violation came from the observation of the decay $K_L\to\pi^+\pi^-$. In order

[2]For a review of this topic, see Winstein and Wolfenstein (1993) and Bertolini *et al.* (2000).

to understand this phenomenon we begin with the states K^0 (with strangeness $S=+1$) and $\bar{K}^0$ (with strangeness $S=-1$). From the above one can construct *CP* even and *CP* odd eigenstates,

$$K_{1,2} = \frac{1}{\sqrt{2}}(K^0 \pm \bar{K}^0). \tag{6}$$

One can arrange $\bar{K}^0$ to be the *CP* conjugate of K^0, i.e., $CP|K^0\rangle = |\bar{K}^0\rangle$, and in that case K_1 is the *CP* even and K_2 is the *CP* odd state. The decay of neutral K's come in two varieties: K_S (K_L) with lifetimes $\tau_S = 0.89\times 10^{10} s$ ($\tau_L = 5.2\times 10^{-8}$) with dominant decays $K_S \to \pi^+\pi^-, \pi^0\pi^0$ ($K_L \to 3\pi, \pi l \nu$). If these were the only decays, one would identify K_S with K_1 and K_L with K_2. However, the decay of $K_L \to \pi^+\pi^-$ provided the first experimental evidence for the existence of *CP* violation (Christenson *et al.*, 1964). This experiment indicates that the K_S (K_L) are mixtures of *CP* even and *CP* odd states and one may write

$$K_S = \frac{K_1 + \bar{\epsilon} K_2}{(1+|\bar{\epsilon}|^2)^{1/2}}, \quad K_L = \frac{K_2 + \bar{\epsilon} K_1}{(1+|\bar{\epsilon}|^2)^{1/2}}. \tag{7}$$

Experimentally one attempts to measure two independent *CP* violating parameters ϵ and ϵ' which are defined by

$$\epsilon = \frac{\langle(\pi\pi)_{I=0}|\mathcal{L}_W|K_L\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{L}_W|K_S\rangle}, \tag{8}$$

where $\mathcal{L}_W$ is the Lagrangian for the weak $\Delta S=1$ interactions, and

$$\epsilon' = \frac{\langle(\pi\pi)_{I=2}|\mathcal{L}_W|K_L\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{L}_W|K_L\rangle} - \frac{\langle(\pi\pi)_{I=2}|\mathcal{L}_W|K_S\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{L}_W|K_S\rangle}. \tag{9}$$

The parameter ϵ' is often referred to as a measure of direct *CP* violation while ϵ is referred to as a measure of indirect *CP* violation in the kaon system. An accurate determination of ϵ has existed for many years so that

$$|\epsilon| = (2.266 \pm 0.017) \times 10^{-3}. \tag{10}$$

The determination of direct *CP* violation is more recent and here one has (Burkhardt *et al.*, 1988; Alavi-Harati *et al.*, 1999; Fanti *et al.*, 1999)

$$\epsilon'/\epsilon = (1.72 \pm 0.018) \times 10^{-3}. \tag{11}$$

The above result rules out the so called superweak theory of *CP* violation (Wolfenstein, 1964) but is consistent with the predictions of the standard model. A detailed discussion of direct *CP* violation can be found in Bertolini *et al.* (2000). There are other kaon processes where *CP* violation effects can, in principle, be discerned. The most prominent among these is the decay $K_L \to \pi^0 \nu\bar{\nu}$. This process is fairly clean in that it provides a direct determination of the quantity $V_{td}V_{ts}^*$. The standard model prediction for the branching ratio is (Buras *et al.*, 2004) $\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = (3.0\pm 0.6)\times 10^{-11}$ while the current experimental limit is (Anisimovsky *et al.*, 2004) $\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) < 1.7\times 10^{-9}$. Thus an improvement in experiment by a factor of around 10^2 is needed to test the standard model prediction. On the other hand, significantly larger contribution to this branching ratio can arise beyond the standard model physics (Grossman and Nir, 1997; Colangelo and Isidori, 1998; Buras *et al.*, 2004, 2005). A new experiment, 391a, is underway at KEK which would have a significantly improved sensitivity for the measurement of this branching ratio and its results could provide a window to testing new physics in this channel.

We turn now to B physics. There is considerable literature in this area to which the reader is directed for details [(Carter and Sanda, 1980; Bigi and Sanda, 1981, 1984; Dunietz and Rosner, 1986); for reviews, see Hitlin and Stone (1991); Nardulli (1993); Nakada (1994); Harrison and Quinn (1996); Barberio (1998); Quinn (1998); Peruzzi (2004); Sanda (2004); Stone (2006)]. *CP* violations can occur in charged B or neutral B decays such as $B_d = \bar{b}d$ and $B_s = \bar{b}s$. In the B^0-$\bar{B}^0$ system the mass eigenstates can be labeled as B_H and B_L with

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle,$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle, \tag{12}$$

where p (q) may be parametrized by

$$p = \frac{1+\epsilon_B}{\sqrt{2(1+|\epsilon_B|^2)}},$$

$$q = \frac{1-\epsilon_B}{\sqrt{2(1+|\epsilon_B|^2)}}. \tag{13}$$

A quantity of interest is the mass difference between these states, i.e., $\Delta m_s = m_{B_H} - m_{B_L}$. Next consider a state f which is accessible to both B^0 and $\bar{B}^0$. A quantity sensitive to *CP* violation is the asymmetry which is defined by

$$a_f(t) = \frac{\Gamma(B^0(t)\to f) - \Gamma(\bar{B}^0(t)\to f)}{\Gamma(B^0(t)\to f) + \Gamma(\bar{B}^0(t)\to f)}, \tag{14}$$

where $B^0(t)$ $[\bar{B}^0(t)]$ denotes the states which were initially B^0 $[\bar{B}^0]$. The analysis of the asymmetry becomes specially simple if the final state is an eigenstate of *CP*. $A_f(t)$ may be written in the form

$$A_f(t) = A_f^c \cos(\Delta m t) + A_f^s \sin(\Delta m t), \tag{15}$$

where

$$A_f^c = \frac{1-|\lambda|^2}{1+|\lambda|^2}, \quad A_f^s = \frac{-2\,\mathrm{Im}\,\lambda}{1+|\lambda|^2}. \tag{16}$$

Here $\lambda \equiv q\bar{A}_f/pA_f$, where $A_f = \langle f|H|B^0\rangle$ and $\bar{A}_f = \langle f|H|\bar{B}^0\rangle$. An interesting aspect of a_f is that it is free of hadronic uncertainties and for the standard model case it is determined in terms of the CKM parameters. This would be the case if only one amplitude contributes to the decay $B^0(\bar{B}^0)\to f$. More generally one has more than one dia-

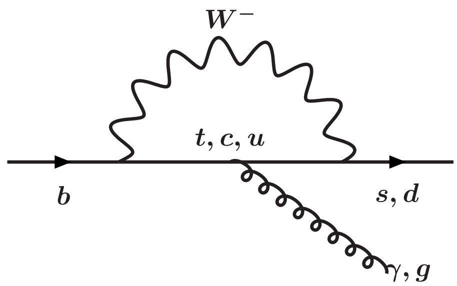

FIG. 1. The penguin diagram that contributes to B decays.

gram contributing with different CKM phase dependence which makes the extraction of CKM phases less transparent. Specifically $B^0(\bar{B}^0)$ decays may in general involve penguin diagrams which tend to contaminate the simple analysis outlined above. Gronau and London have proposed an isospin analysis which can disentangle the effect of the tree and penguin contributions when the final states in $B^0(\bar{B}^0)$ are $\pi^+\pi^-$ and $\pi^0\pi^0$ which is useful in the analysis of all the CKM angles (Gronau and London, 1990, 1991). The decay final states $J/\Psi K_S$ is interesting in that it is a *CP* eigenstate and it has a large branching ratio and to leading order is dominated by a single CKM phase. Specifically, the relation $\bar{A}_{J/\Psi K_S}/A_{J/\Psi K_S}=1$ holds to within a percent (Boos *et al.*, 2004), $A^s_{J/\Psi K_S}=\sin(2\beta)$ and $A^c_{J/\Psi K_S}=0$. Thus $B^0(\bar{B}^0)$ decay into this mode gives a rather clean measurement of $\sin 2\beta$. BaBar and Belle have both measured *CP* asymmetries utilizing the charm decays. Using the decays $B^0(\bar{B}^0)\to J/\Psi K_S$ and $B^0(\bar{B}^0)\to J/\Psi K_L$ BaBar and Belle have obtained a determination of the *CP* asymmetry $\sin(2\beta)$ and the world average for this is (Barberio *et al.*, 2006)

$$\sin(2\beta) = 0.685 \pm 0.032. \tag{17}$$

While the analysis of *CP* asymmetries in the $J/\Psi K_S$ system is the cleanest way to determine $\sin(2\beta)$ there are additional constraints on β that are indirect such as from Δm_d and Δm_s. These lead to a constraint on β with β lying in the range $(13°, 31°)$ at 95% C.L. (Charles *et al.*, 2005; Long, 2005).

The determination of α comes from the measurement of processes such as $B^0\to\pi^+\pi^-,\rho^+\rho^-$ since the combinations of phases that enter here are via $\sin[2(\beta+\gamma)] = -\sin(2\alpha)$. One problem arises due to the contribution of the penguin diagram, Fig. 1, which does not contain any weak phase. The penguin diagram can thus contaminate the otherwise neat weak phase dependence of this process. A possible solution comes from the fact that one can use the analysis of Grossman and Quinn (1998) to put an upper limit on the branching ratio for $B^0 \to \rho^0\rho^0$.

The current determination of α gives $\alpha = (96\pm 13\pm 11)°$ (Stone, 2006). The determination of γ comes from charged decays $B^\pm\to D^0K^\pm$. The current experimental values from BaBar and Belle are $\gamma = (67\pm 28\pm 13\pm 11)°$ and $\gamma=(67^{+14}_{-13}\pm 13\pm 11)°$ (Asner and Sun, 2006; Stone, 2006). A detailed analysis of global fits to the CKM matrix can be found in Charles (2006) and Charles *et al.* (2005).

We discuss now the $D^0-\overline{D^0}$ system. In analogy with the neutral B system we introduce the two neutral mass eigenstates D_1, D_2 defined by

$$|D_1\rangle = p|D^0\rangle + q|\overline{D^0}\rangle,$$

$$|D_2\rangle = p|D^0\rangle - q|\overline{D^0}\rangle. \tag{18}$$

The D mesons are produced as flavor eigenstates but they evolve as admixtures of mass eigenstates which govern their decays. The analysis of D^0 and $\overline{D^0}$ decays by BaBar (Aubert *et al.*, 2007) and by Belle (Staric *et al.*, 2007) finds no evidence of *CP* violation. For more details see Nir (2007b).

The fourth piece of experimental evidence for *CP* violation in nature is indirect. It arises from the existence of a baryon asymmetry in the Universe which is generally expressed by the ratio

$$n_B/n_\gamma = (6.1^{0.3}_{-0.2}) \times 10^{-10}. \tag{19}$$

An attractive picture for understanding baryon asymmetry is that the asymmetry was generated in the very early history of the Universe within the context of an inflationary universe starting with no initial baryon asymmetry [for a review on matter-antimatter asymmetry see Dine and Kusenko (2004)]. The basic mechanism of how this can come about was enunciated some time by Sakharov (1967). According to Sakharov there are three basic ingredients that govern the generation of baryon asymmetry. (i) One needs a source of baryon number violating interactions if one starts out with a universe which initially has no net baryon number. Such interactions arise quite naturally in grand unified models and in string models. (ii) One needs *CP* violating interactions since otherwise it would be a balance between processes producing particles vs processes producing antiparticles leading to a vanishing net baryon asymmetry. (iii) Finally, even with baryon number and *CP* violating interactions the production of a net baryon asymmetry would require a departure from thermal equilibrium. Thus one finds that one of the essential ingredients for the generation of the baryon asymmetry in the early Universe is the existence of *CP* violation. However, the *CP* violation in the standard model is not sufficient to generate the desired amount of baryon asymmetry and one needs a source of *CP* violation above and beyond what is present in the standard model. Such sources of *CP* violation are abundant in supersymmetric theories.

In addition to the baryon asymmetry in the Universe there are other avenues which may reveal the existence of new sources of *CP* violation beyond what exists in the standard model. The EDMs of elementary particles and atoms are prime candidates for these. The largest values of EDMs in the framework of the standard model (SM) are very small. The SM predicts for the case of the electron the value of $d_e \simeq 10^{-38} e$ cm and for the case of the neutron the value that ranges from 10^{-31} to $10^{-33}e$ cm (Gavela *et al.*, 1982; Khriplovich and Zhitnitsky, 1982; Shabalin, 1983; Bernreuther and Suzuki, 1991; Bigi and Uraltsev, 1991; Booth, 1993).

So far no electric dipole moment for the electron or neutron has been detected, and thus strong bounds on these quantities exist. For the electron the current experimental limit is (Regan *et al.*, 2002),

$$|d_e| < 1.6 \times 10^{-27} e \text{ cm} \quad (90\,\%\text{ C.L.}). \tag{20}$$

For the neutron the standard model gives $d_n \sim 10^{-32\pm1}e$ cm while the current experimental limit is (Baker *et al.*, 2006)

$$|d_n| < 2.9 \times 10^{-26} e \text{ cm} \quad (90\,\%\text{ C.L.}). \tag{21}$$

In each case one finds that the standard model prediction for the EDM is several orders of magnitude smaller than the current experimental limit and thus far beyond the reach of experiment even with improvement in sensitivity by one to two orders of magnitude. On the other hand, many models of new physics beyond the standard model generate much larger EDMs and such models are already being constrained by the EDM experiment. Indeed, improved sensitivities in future experiment may lead to a detection of such effects or put even more stringent constraints on the new physics models. The EDM of atoms also provides a sensitive test of *CP* violation. An example is Hg-199 for which the current limits are (Romalis *et al.*, 2001)

$$|d_{\text{Hg}}| < 2 \times 10^{-28} e \text{ cm}. \tag{22}$$

IV. *CP* VIOLATION IN SOME NON-SUSY EXTENSIONS OF THE STANDARD MODEL

While the standard model contains just one *CP* phase, more phases can appear in extensions of the standard model. In general, the violations of *CP* can be either explicit or spontaneous. The *CP* violation is called explicit if redefinitions of fields cannot make all couplings real in the interaction structure of the theory. The remaining phases provide an explicit source of *CP* violation. *CP* violation is called spontaneous if the model starts out with all couplings real but spontaneous breaking in the Higgs sector generates a nonremovable phase in one of the vacuum expectation values in the Higgs fields at the minimum of the potential. Returning to *CP* violation in the extension of the standard model, such extensions could be based on an extended gauge group, on an extended Higgs sector, or on an extended fermionic content [see, for example, Accomando *et al.* (2006)]. An example of a model with an extended gauge sector is the left-right (LR) symmetric model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ (Mohapatra and Pati, 1975). For n_g number of generations the number of phases is given by $N_L + N_R$ where $N_L = (n_g - 1)(n_g - 2)/2$ is exactly what one has in $SU(2)_L \times U(1)_Y$ model and $N_R = n_g(n_g+1)/2$ are additional sets of phases that arise in the LR model. For the case of three generations this leads to seven *CP* phases instead of just one *CP* phase that one has in the standard model. An analysis of EDM in LR models for the electron and neutron has been given by Frank (1999a, 1999b).

The simplest extension of the standard model with an extended Higgs sector is the so called two Higgs doublet model (2HDM) (Lee, 1973, 1974) which contains two SU(2) doublets which have exactly the same quantum numbers $\Phi_i = (\phi_i^+, \phi_i^0)$, $i=1,2$. One problem with the model is that it leads to flavor changing neutral currents (FCNCs) at the tree level if one allows couplings of both Φ_i to up and down quarks. The FCNCs can be suppressed by imposing a discrete $\mathcal{Z}_2$ symmetry (Glashow and Weinberg, 1977) such that under $\mathcal{Z}_2$ one has $\Phi_2 \to -\Phi_2$ and $u_{iR} \to -u_{iR}$ and the remaining fields are unaffected. Under the above symmetry the most general renormalizable scalar potential one can write is

$$\begin{aligned} V_0 = & -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\ & + [\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}]. \end{aligned} \tag{23}$$

However, with an exact $\mathcal{Z}_2$ discrete symmetry *CP* cannot be broken either explicitly or spontaneously in a 2HDM model (Branco, 1980a, 1980b; Mendez and Pomarol, 1991). Thus to have *CP* in the 2HDM model one must allow for violations of the discrete symmetry, but arrange for suppression of FCNCs. If the couplings allow for FCNCs at the tree level, then they must be suppressed either by heavy Higgs masses (Lahanas and Vayonakis, 1979; Branco *et al.*, 1985) or by adjustment of couplings or fine tunings so that FCNC are suppressed but *CP* violation is allowed (Liu and Wolfenstein, 1987).

However, hard breaking of the $\mathcal{Z}_2$ discrete symmetry is generally considered not acceptable. A more desirable possibility is violation of the discrete symmetry only via soft terms (Branco and Rebelo, 1985). Here the FCNCs are not allowed at the tree level but the inclusion of the soft terms allows for *CP* violation. Such a term is of the form

$$V_{\text{soft}} = -\mu_3^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \tag{24}$$

Soft breaking of the $\mathcal{Z}_2$ symmetry can allow both explicit and spontaneous *CP* violation. Thus explicit *CP* violation can occur in $V = V_0 + V_{\text{soft}}$ if one has (Grzadkowski *et al.*, 1999) $\text{Im}(\mu_3^{*4}\lambda_5) \neq 0$. For the case when $\text{Im}(\mu_3^{*4}\lambda_5) = 0$ a spontaneous violation of *CP* can arise. Specifically, in this case one can choose phases so that $\langle\Phi_1\rangle = v_1/\sqrt{2}$ $(v_1 > 0)$ and $\langle\Phi_2\rangle = e^{i\theta} v_2/\sqrt{2}$ $(v_2 > 0)$ with the normalization

$$\sqrt{v_1^2 + v_2^2} = 2m_W/g_2 = 246 \text{ GeV}. \tag{25}$$

The conditions for *CP* violation in a 2HDM model, both explicit and spontaneous, have more recently been studied using basis independent potentially complex invariants which are combinations of mass and coupling parameters. These invariants also are helpful in distinguishing between explicit and spontaneous *CP* violation in the Higgs sector. For further discussion, see the works of Lavoura and Silva (1994); Botella and Silva (1995); Branco *et al.* (2005); Davidson and Haber (2005); Ginzburg and Krawczyk (2005); Gunion and Haber (2005). While spontaneous breaking of *CP* discussed above involves SU(2) Higgs doublets which may enter in the spontaneous breaking of the electroweak symmetry, similar spontaneous violations of *CP* can occur in sectors not related to electroweak symmetry breaking.

In the absence of *CP* violation, the Higgs sector of the theory after spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry gives two *CP* even and one *CP* odd Higgs in the neutral sector. In the presence of *CP* violation, either explicit or spontaneous, the *CP* eigenstates mix and mass eigenstates are admixtures of *CP* even and *CP* odd states. The above leads to interesting phenomenology which has been discussed by Mendez and Pomarol (1991) and Grzadkowski *et al.* (1999). The number of independent *CP* phases increases very rapidly with increasing number of Higgs doublets. Thus suppose we consider an n_D number of Higgs doublets. In this case the number of independent *CP* phases that can appear in the unconstrained Higgs potential is (Branco *et al.*, 2005) $N_p = n_D^2(n_D^2-1)/4-(n_D-1)$. For $n_D=1,2,3$ one gets $N_p=0,2,16$, and thus the number of independent *CP* phases rises rather rapidly as the number of Higgs doublets increases. An analysis of the EDMs in the two Higgs model has been given by Hayashi *et al.* (1994) and Barger *et al.* (1997). Finally, one may consider extending the fermionic sector of theory with inclusion of additional generations. Such an extension brings in more possible sources of *CP* violation. Thus, for example, with four generation of quarks the extended CKM matrix will be 4×4. Such a matrix can be parametrized in terms of six angles and three phases (Barger *et al.*, 1981; Oakes, 1982). Thus generically extensions of the standard model will in general have more sources of *CP* violation than the standard model. We discuss *CP* violation in supersymmetric theories next. While the spontaneous breaking of *CP* discussed above involves SU(2) Higgs doublets which may enter in the spontaneous breaking of the electroweak symmetry, similar spontaneous violations of *CP* can occur in sectors not related to electroweak symmetry breaking.

V. *CP* VIOLATION IN SUPERSYMMETRIC THEORIES

Supersymmetric models are one of the leading candidates for new physics [for review see Nath *et al.* (1983a); Nilles (1984); Haber and Kane (1985); Martin (1997)] since they allow for a technically natural solution to the gauge hierarchy problem. However, supersymmtetry is not an exact symmetry of nature. Thus one must allow for breaking of supersymmetry in a way that does not violate the ultraviolet behavior of the theory and destabilize the hierarchy. This can be accomplished by the introduction of soft breaking. However, the soft breaking sector in the minimal supersymmetric standard model (MSSM) allows for a large number of arbitrary parameters (Dimopoulos and Georgi, 1981; Girardello and Grisaru, 1982). Indeed in softly broken supersymmetry the particle content of MSSM is additionally 21 masses, 36 mixing angles, and 40 phases (Dimopoulos and Sutter, 1995). which makes the model rather unpredictive.

The number of parameters is significantly reduced in the minimal supergravity unified models under the assumptions of a flat Kahler metric as explained below. The minimal supergravity model and supergravity model in general are constructed using techniques of applied $N=1$ supergravity, where one couples chiral matter multiplets and a vector multiplet belonging to the adjoint representation of a gauge group to each other and to supergravity. The supergravity couplings can then be described in terms of three arbitrary functions: the superpotential $W(z_i)$ which is a holomorphic function of the chiral fields z_i, the Kähler potential $K(z_i, z_i^\dagger)$, and the gauge kinetic energy function $f_{\alpha\beta}(z_i, z_i^\dagger)$ which transforms like the symmetric product of two adjoint representations. In supergravity models supersymmetry is broken in a so-called hidden sector and is communicated to the physical sector where quarks and lepton live via gravitational interactions. The size of the soft breaking mass, typically the gravitino mass $m_{3/2}$, is $\sim\kappa^2|\langle W_h\rangle|$, where W_h is the superpotential in the hidden sector where supersymmetry breaks and $\kappa=1/M_{\mathrm{Pl}}$, where M_{Pl} is the Planck mass. The simplest model where supersymmetry breaks in the hidden sector via a super Higgs effect is given by $W_h=m^2 z$, where z is the standard model singlet super-Higgs field. The breaking of supersymmetry by supergravity interactions in the hidden sector gives z a VEV of size $\sim\kappa^{-1}$, and thus with $m \sim 10^{10-11}$ GeV, the soft breaking mass is of size $\sim 10^3$ GeV.

In the minimal supergravity model one assumes that the Kähler potential has no generational dependence and is flat and further that the gauge kinetic energy function is diagonal and has no field dependence, i.e., one has effectively $f_{\alpha\beta}\sim\delta_{\alpha\beta}$. In this case one finds that the low energy theory obtained after integrating the GUT scale masses has the following soft breaking potential (Chamseddine *et al.*, 1982; Hall *et al.*, 1983; Nath *et al.*, 1983b):

$$\mathcal{V}_{SB} = m_{1/2}\bar{\lambda}^\alpha\lambda^\alpha + m_0^2 z_a z_a^\dagger + (A_0 W^{(3)} + B_0 W^{(2)} + \mathrm{H.c.}), \tag{26}$$

where $W^{(2)}$ is the quadratic and $W^{(3)}$ is cubic in the fields.

The physical sector of supergravity models consist of the MSSM fields, which include the three generations of quarks and leptons and their superpartners, and a pair of $SU(2)_L$ Higgs doublets H_1 and H_2 and their superpartners which are the corresponding Higgsino fields $\tilde{H}_1$ and $\tilde{H}_2$. For the case of MSSM one has

$$W^{(2)} = \mu_0 H_1 H_2,$$

$$W^{(3)} = \tilde{Q} Y_U H_2 \tilde{u}^c + \tilde{Q} Y_D H_1 \tilde{d}^c + \tilde{L} Y_E H_2 \tilde{e}^c. \tag{27}$$

Here H_1 is Higgs doublet that gives mass to the bottom quark and the lepton, and H_2 gives mass to the up quark. As is evident from Eqs. (26) and (27) the minimal supergravity theory is characterized by the parameters m_0, $m_{1/2}$, A_0, B_0, and μ_0. An interesting aspect of supergravity models is that they allow for spontaneous breaking of the $\mathrm{SU}(2)_L \times U(1)_Y$ electroweak symmetry (Chamseddine *et al.*, 1982). This can be accomplished in an efficient manner by radiative breaking using renormalization group effects (Ibanez and Ross, 1982, 2007; Inoue *et al.*, 1982; Alvarez-Gaume *et al.*, 1983; Ellis *et al.*, 1983; Ibanez and Lopez, 1984).

To exhibit spontaneous breaking one considers the scalar potential of the Higgs fields by evolving the potential to low energies by renormalization group effects such that

$$V = V_0 + \Delta V, \tag{28}$$

where V_0 is the tree level potential (Nath *et al.*, 1983a; Nilles, 1984; Haber and Kane, 1985)

$$\begin{aligned} V_0 = {} & m_1^2|H_1|^2 + m_2^2|H_2|^2 + (m_3^2 H_1 H_2 + \text{H.c.}) \\ & + \frac{g_2^2+g_1^2}{8}|H_1|^4 + \frac{g_2^2+g_1^2}{8}|H_2|^4 - \frac{g_2^2}{2}|H_1H_2|^2 \\ & + \frac{g_2^2-g_1^2}{8}|H_1|^2|H_2|^2 \end{aligned} \tag{29}$$

and ΔV is the one loop correction to the effective potential and is given by (Coleman and Weinberg, 1973; Weinberg, 1973; Arnowitt and Nath, 1992; Carena *et al.*, 2000)

$$\Delta V = \frac{1}{64\pi^2}\mathrm{Str}\left[M^4(H_1,H_2)\left(\ln \frac{M^2(H_1,H_2)}{Q^2} - \frac{3}{2}\right)\right]. \tag{30}$$

Here $\mathrm{Str} = \Sigma_i C_i (2J_i+1)(-1)^{2J_i}$, where the sum runs over all particles with spin J_i and $C_i(2J_i+1)$ counts the degrees of freedom of the particle i and Q is the running scale which is in the electroweak region. The gauge coupling constants and soft parameters are subject to the supergravity boundary conditions $\alpha_2(0)=\alpha_G=\frac{5}{3}\alpha_Y(0)$; $m_i^2(0)=m_0^2+\mu_0^2$, $i=1,2$; and $m_3^2(0)=B_0\mu_0$. As the potential evolves downwards from the GUT scale using renormalization group equations (Machacek and Vaughn, 1983, 1984, 1985; Jack *et al.*, 1994; Martin and Vaughn, 1994), a breaking of the electroweak symmetry occurs when the determinant of the Higgs mass^2 matrix turns negative so that (i) $m_1^2 m_2^2 - 2m_3^4 < 0$, and further for a stable minimum to exist one requires that the potential be bounded from below so that (ii) $m_1^2+m_2^2-2|m_3^2|>0$. Additionally one must impose the constraint that there be color and charge conservation. Defining $v_i = \langle H_i \rangle$ as the VEV of the neutral component of the Higgs H_i, the necessary conditions for the minimization of the potential, i.e., $\partial V/\partial v_i = 0$, gives two constraints. One of these can be used to determine the magnitude $|\mu_0|$ and the other can be used to replace B_0 by $\tan\beta \equiv \langle H_2\rangle/\langle H_1\rangle$. In this case the low energy supergravity model (mSUGRA) can be parametrized by m_0, $m_{1/2}$, A_0, $\tan\beta$, and $\mathrm{sgn}(\mu_0)$. It should be noted that fixing the value $|\mu|$ using radiative breaking does entail fine tuning but a measure of this is model dependent [see, for example, Chan (1998) and references therein]. The above discussion occurs when there are no *CP* violating phases in the theory. In the presence of *CP* phases $m_{1/2}$, A_0, μ_0 become complex and one may parametrize them so that

$$m_{1/2} = |m_{1/2}|e^{i\xi_{1/2}}, \quad A_0 = |A_0|e^{i\alpha_0}, \quad \mu_0 = |\mu_0|e^{i\theta_{\mu_0}}. \tag{31}$$

Now not all phases are independent. Indeed, in this case only two phase combinations are independent, and in the analysis of the EDMs one finds these to be $\xi_{1/2} + \theta_{\mu_0}$ and $\alpha_0 + \theta_{\mu_0}$. Often one rotates away the phase of the gauginos which is equivalent to setting $\xi_{1/2}=0$, and thus one typical choice of parameters for the complex mSUGRA (cmSUGRA) case is

$$\begin{aligned} & m_0, \quad |m_{1/2}|, \quad \tan\beta, \quad |A_0|; \\ & \alpha_0, \quad \theta_{\mu_0}\ (\text{cmSUGRA}). \end{aligned} \tag{32}$$

However, other choices are equally valid: thus, for example, the independent soft breaking parameters can be chosen to be m_0, $|m_{1/2}|$, $\tan\beta$, $|A_0|$, α_0, $\xi_{1/2}$. The mSUGRA model was derived using a super-Higgs effect which breaks supersymmetry in the hidden sector by VEV formation of a scalar super-Higgs field. Alternately one can view the breaking of supersymmetry as arising from gaugino condensation, in analogy with QCD, where one forms the condensate $q\bar{q}$ one has that the strong dynamics of an asymptotically free gauge theory in the hidden sector produces a gaugino condensate with $\langle \lambda\gamma^0\lambda\rangle = \Lambda^3$. The above can lead typically to supersymmetry breaking and a gaugino mass of size $m_{3/2} \sim \kappa^2\Lambda^3$. With $|\Lambda>| \sim (10^{12-13})$ GeV $m_{3/2}$ will again be in the electroweak region (Nilles, 1982; Ferrara *et al.*, 1983; Dine *et al.*, 1985; Taylor, 1990).

The assumption of a flat Kähler potential and flat kinetic energy function in supergravity unified models is essentially a simplification, and in general the nature of the physics at the Planck scale is largely unknown. For this reason one must also consider more general Kähler potentials (Soni and Weldon, 1983; Kaplunovsky and Louis, 1993) and allow for the nonuniversality of the gauge kinetic energy function. In this case the number of soft parameters grows, as also do the number of *CP* phases. Thus, for example, the gaugino masses will be complex and nonuniversal, and the trilinear parameter A_0, which is in general a matrix in the generation space, will be in general nondiagonal and complex. For simplicity to maintain the appropriate constraints on flavor changing neutral currents one can assume a diagonal form for A_0 at the GUT scale. Additionally, the Higgs masses for H_1 and H_2 at the GUT scale could also be nonuniversal. Thus in general for the nonuniversal supergravity unification a canonical set of soft parameters at the GUT scale will consist of (Matalliotakis and

Nilles, 1995; Olechowski and Pokorski, 1995; Polonsky and Pomarol, 1995; Nath and Arnowitt, 1997)

$$m_{H_i} = m_0(1+\delta_i), \quad i=1,2,$$

$$m_\alpha = |m_\alpha| e^{i\xi_\alpha}, \quad \alpha = 1,2,3,$$

$$A_a = |A_a| e^{i\alpha_a}, \quad a=1,2,3, \tag{33}$$

which contain several additional *CP* phases beyond the two phases in complex mSUGRA. However, not all phases are independent, as some phases can be eliminated by field redefinitions. Indeed in physical computations only a certain set of phases appear, as discussed by Ibrahim and Nath (2000c) (also see Appendix E). It should be kept in mind that for the case of nonuniversalities the renormalization group evolution gives an additional correction term at low energies (Martin and Vaughn, 1994).

As is apparent from the preceding discussion radiative breaking of the electroweak symmetry plays a central role in supergravity unified models. An interesting phenomena here is the existence of two branches of radiative breaking: one is the conventional branch known since the early 1980s [called the ellipsoidal branch (EB)] and the other was more recently discovered, i.e., called the hyperbolic branch (HB). The two branches can be understood by examining the condition of radiative breaking which is a constraint on the soft parameters m_0, $m'_{1/2}$, A_0 of the form (Chan *et al.*, 1998)

$$C_1 m_0^2 + C_3 m'^2_{1/2} + C'_2 A_0^2 + \Delta\mu^2_{\rm loop} = \frac{M_Z^2}{2} + \mu^2. \tag{34}$$

Here $\Delta\mu^2_{\rm loop}$ is the loop correction (Arnowitt and Nath, 1992; Carena *et al.*, 2000) and $m'_{1/2} = m_{1/2} + \frac{1}{2}A_0 C_4/C_3$, where C_i are determined purely in terms of the gauge and the Yukawa couplings but depend on the renormalization group scale Q. The behavior of radiative breaking is controlled in a significant way by the loop correction $\Delta\mu^2_{\rm loop}$ especially for moderate to large values of $\tan\beta$. For small values of $\tan\beta$ the loop correction $\Delta\mu^2$ is small around $Q \sim M_Z$, and C_i are positive and thus Eq. (34) is an ellipsoidal constraint on the soft parameters. For a given value of μ, Eq. (34) then puts an upper limit on the sparticle masses. However, for moderate to large values of $\tan\beta$, $\Delta\mu^2$ becomes sizable. Additionally C_i develop a significant Q dependence. It is then possible to choose a point $Q = Q_0$ where $\Delta\mu^2$ vanishes and quite interestingly here one finds that one of the C_i (specifically C_1) turns negative, drastically changing the nature of the symmetry breaking constraint Eq. (34) on the soft parameters. Thus in this case the soft parameters in Eq. (34) lie on the surface of a hyperboloid and thus for a fixed value of μ the soft parameters can get very large with m_0 getting as large as 10 TeV or larger. Direct observation of squarks and sleptons may be difficult on this branch, although charginos, neutralinos, and even gluino may be accessible. However, the HB does have other desirable features such as suppression of flavor changing neutral currents, and suppression of the SUSY EDM contributions. Further, HB still allows for satisfaction of relic density constraints with R parity conservation if the lightest neutralino is the lightest supersymmetric particle (LSP). We note in passing that the so called focus point region (Feng *et al.*, 2000) is included in the hyperbolic branch (Chan *et al.*, 1998; Lahanas *et al.*, 2003; Baer *et al.*, 2004).

There is a potential danger in supergravity theories in that the hierarchy could be destabilized by nonrenormalizable couplings in supergravity models since they can lead to power law divergences. This issue has been investigated by several: at one loop by Bagger and Poppitz (1993) and Gaillard (1995) and at two loops by Bagger *et al.* (1995). The analysis shows that at the one loop level the minimal supersymmetric standard model appears to be safe from divergences (Bagger and Poppitz, 1993). In addition to the breaking of supersymmetry by gravitational interactions, there are a variety of other scenarios for supersymmetry breaking. These include gauge mediated and anomaly mediated breaking for which reviews can be found in Giudice and Rattazzi (1999) and Luty (2005). Finally as is clear from the preceding discussion in supergravity models and in MSSM there is no *CP* violation at the tree level in the Higgs sector of the theory. However, this situation changes when one includes the loop correction to the Higgs potential. This leads to the generation of *CP* violating phase for one the Higgs VEVs and leads to mixings between the *CP* even and *CP* odd Higgs fields. This phenomenon is interesting from the experimental view point and will be discussed later.

While the standard model contribution to the EDMs of the electron and neutron is small and beyond observation of the current or future experiment, the situation in supersymmetric models is quite the opposite. Here the new sources of *CP* violation can generate large contributions to the EDMs even significantly above the current experimental limits. One needs special mechanisms to suppress the EDMs such as mass suppression (Nath, 1991; Kizukuri and Oshimo, 1992) or the cancellation mechanism to control the effect of large *CP* phases on the EDMs (Ibrahim and Nath, 1998a, 1998b, 1998c, 2000d; Chattopadhyay *et al.*, 2001; Ibrahim, 2001b). Specifically for the cancellation mechanism the phases can be large and thus affect a variety of *CP* phenomena which can be observed in low energy experiments and at accelerators. The literature on this topic is quite large.[3]

[3] A sample of these analyses can be found in Falk and Olive, 1998; Chattopadhyay *et al.*, 1999; Demir, 1999; Huang and Liao, 2000a, 2000b, 2002; Ibrahim and Nath, 2000a, 2000b, 2000c, 2001a, 2001b, 2002, 2003a, 2003b, 2003c, 2004, 2005; Akeroyd and Arhrib, 2001; Ibrahim *et al.*, 2001, 2004; Boz, 2002; Gomez *et al.*, 2004b, 2004c, 2005, 2006; Bartl *et al.*, 2006; Bartl, Hesselbach, *et al.* 2004e; Alan *et al.*, 2007.

VI. *CP* VIOLATION IN EXTRA DIMENSION MODELS

Recently there has been significant activity in the physics of extra dimensions (Antoniadis, 1990; Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998; Randall and Sundrum, 1999a, 1999b; Gogberashvili, 2002). One might speculate on the possibility of generating *CP* violation in a natural way from models derived from extra dimensions [for an early work see Thirring (1972)]. It turns out that it is indeed possible to do so (Khlebnikov and Shaposhnikov, 1988; Sakamura, 1999; Branco *et al.*, 2001; Chaichian and Kobakhidze, 2001; Chang and Mohapatra, 2001; Chang *et al.*, 2001; Dienes *et al.*, 2001; Huang *et al.*, 2002; Burdman, 2004; Grzadkowski and Wudka, 2004). The idea is to utilize properties of hidden compact dimensions in extra dimension models. Thus in extra dimension models after compactification the physical four-dimensional (4D) space is a slice of the higher dimensiional space and such a slice can be placed in different locations in extra dimensions. In the discussion below we label such a slice as a brane. We now present a simple argument which illustrates how *CP* violation in extra dimension models can arise (Chang and Mohapatra, 2001). Thus consider a U(1) gauge theory with left-handed fermions Ψ_i ($i=1-4$), where $i=1,2$ have charges +1 and $i=3,4$ have charges −1, and also consider a real scalar field Φ which is neutral. We assume that the fermion fields are in the bulk and the scalar field is confined to the $y=0$ brane. The fields Ψ_1, Ψ_2, and Φ are assumed to be even and Ψ_{3L}, Ψ_{4L} are assumed to be odd under $y\to -y$ transformation. Further, under *CP* symmetry define the fields to transform so that $\Psi_{1L}\to(\Psi_{3L})^c$, $\Psi_{2L}\to(\Psi_{4L})^c$, and $\Phi\to-\Phi$, where $(\Psi_L)^c$ has the meaning of a 4D charge conjugate of Ψ. One constructs a five-dimensional (5D) Lagrangian invariant under $y\to -y$ transformation of the form

$$M_5^{-1}\lambda_5\delta(y)\Phi[\Psi_{iL}^T C^{-1}\Psi_{2L} - (\Psi_{3L})^{cT}C^{-1}(\Psi_{4L})^c] + \mu[\Psi_{iL}^T C^{-1}\Psi_{2L} - (\Psi_{3L})^{cT}C^{-1}(\Psi_{4L})^c] + \text{H.c.} \tag{35}$$

On integration over the y coordinate the interaction terms in 4D arise from couplings on the $y=0$ brane and thus the zero modes of fields odd in y are absent, which means that the effective interaction at low energy in $(\lambda\Phi+\mu)\Psi_{1L}^{0T}\Psi_{2L}^{(0)}$ which violates *CP* provided $\mathrm{Im}(\lambda^*\mu)\neq 0$. Next we discuss a more detailed illustration of this *CP* violation arising from extra dimensions. This illustration is an explicit exhibition of how violations of *CP* invariance can occur in the compactification of a 5D QED (Grzadkowski and Wudka, 2004). Thus consider the Lagrangian in 5D of the form

$$\mathcal{L}_5 = -\tfrac{1}{4}V_{MN}^2 + \bar{\Psi}(i\gamma^M D_M - m_i)\Psi + \mathcal{L}_{gh}. \tag{36}$$

Here V_M is the vector potential in 5D space with coordinates z^M, where $M=0,1,2,3,5$ so that $z^M=(x^\mu,y)$, where $\mu=0,1,2,3$, and $D_M=\partial_M+ig_5qV_M$ is the gauge covariant derivative, with g_5 the U(1) gauge coupling constant and q the charge of fermion field. The theory is invariant under the following gauge transformations:

$$\psi(z)\to e^{-ig_5q\lambda}\psi(z),$$
$$V_M(z)\to V_M(z)+\partial_M\lambda(z), \tag{37}$$

and additionally under the *CP* transformations in five dimensions

$$z^m\to\eta^M z^M,\quad V^M\to\eta^M V^M,\quad \psi\to P\gamma^0\gamma^2\psi^*, \tag{38}$$

where $\eta^{1,2,3}=-1=-\eta^{0,5}$ and $P=1$. We compactify the theory in the fifth dimension on a circle with radius R assuming periodic boundary conditions for the gauge fields but assuming the twisted boundary condition for the fermion field,

$$\psi(x,y+R)=e^{i\alpha}\psi(x,y). \tag{39}$$

One can now carry out a mode expansion in four dimensions and recovers a massless zero mode $V_\mu(x)$ for the vector field (the photon). One also finds a massless field $\phi(x)$ which is the zero mode of the $V_5(x,y)$ expansion. This occurs because while V_5^n, $n\neq 0$ modes can be eliminated by an appropriate gauge choice, ϕ is a gauge singlet and remains in the spectrum. We note in passing that the presence of the zero mode is a consequence of the specific compactification chosen. Thus compactification on S^1/Z_2 rather than on the circle will remove the field ϕ. Now while ϕ is massless at the tree level, it can develop a mass when loop contributions are included. Thus an analysis of one loop effective potential gives (Grzadkowski and Wudka, 2004).

$$V_{\text{eff}}=\frac{1}{2\pi^4R^4}\sum_i[\beta_i^2 L_{i_3}(\gamma_i)+3\beta_i L_{i_4}(\gamma_i)+3L_{i_5}(\gamma_i)], \tag{40}$$

where $\beta_i=mR$, $\gamma_i=\exp(i\omega_iR-\beta_i)$, with $\omega_i=(\alpha_i+g_5q_iR\phi_0)$, $\phi_0=\langle\phi\rangle$, and L_{i_n} is the polylogarithm function.

Now it turns out that when one has a single fermion, there is no *CP* violation, but *CP* violation is possible when there are two fermions and one can assume the boundary conditions in this case so that $\psi_1(x,y+R)=\psi_1(x,y)$ and $\psi_2(x,y+R)=e^{i\alpha}\psi_2(x,y)$. In this situation the Yukawa couplings for the fermions violate *CP*. An interesting phenomenon here is that the above mechanism exhibits examples of both spontaneous *CP* violation and explicit *CP* violation (see Fig. 2). Thus for the case $\alpha=0,\pi$ one finds that the effective potential is symmetric in ϕ_0 and one has two degenerate minima away from $\phi_0=0$ and here one has spontaneous breaking of *CP*. For other choices of α, the effective potential is not symmetric in ϕ_0 and one has explicit violation of *CP*. The fact that *CP* is indeed violated in this example can be tested by an explicit computation of the fermion EDM which is nonvanishing and suppressed by the inverse size of the extra dimension.

We turn now to another mechanism for generation of *CP* violation in extra dimensional theories. This scenario is that of split fermions where the hierarchies of fermion masses and couplings are proposed to arise from a fermion location mechanism under a kink background

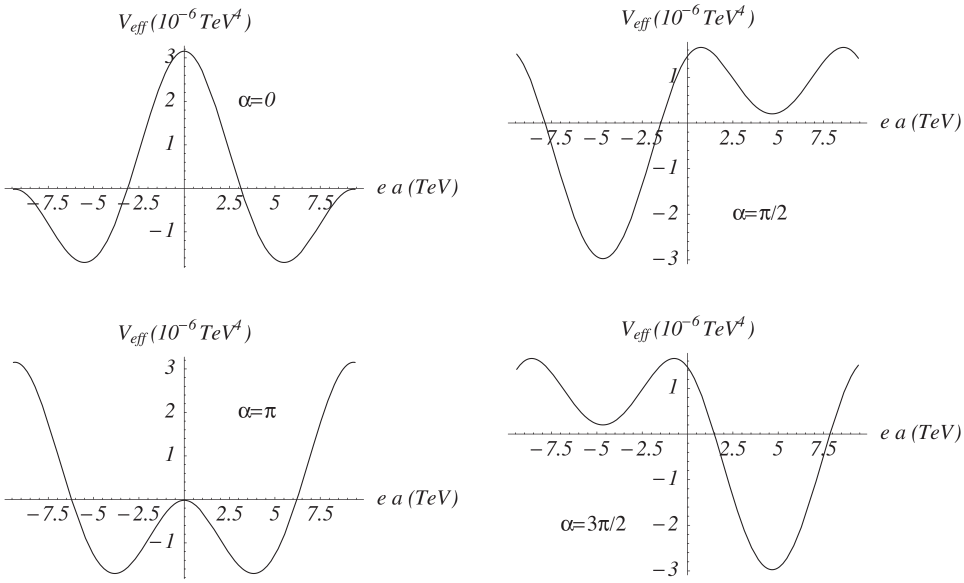

FIG. 2. The phenomena of spontaneous vs explicit breaking in a 5D compactification model (Grzadkowski and Wudka, 2004). The effective potential $V_{\rm eff}$ for four cases of twist angles with $\alpha=0$, $\pi/2$, π, $3\pi/2$. The cases $\alpha=0,\pi$ correspond to spontaneous breaking and $\alpha=\pi/2, 3\pi/2$ correspond to explicit breaking.

wherein the quark and leptons of different generations are confined to different points in a fat brane (Arkani-Hamed and Schmaltz, 2000; Kaplan and Tait, 2000, 2001; Mirabelli and Schmaltz, 2000). To illustrate the fat brane paradigm consider the (4+1)-dimensional action of two fermions:

$$S_5 = \int d^4x dy \{\bar{Q}[i\gamma_M \partial^M + \Phi_Q(y)]Q + \bar{U}[i\gamma_M\partial^M + \Phi_U(y)]U + \kappa H Q^c U\}. \tag{41}$$

The quantities $\Phi_{Q,U}$ are potentials which confine the quarks at different points in the extra dimension. As a model one may consider these as Gaussian functions centered around points l_q (i.e., functions of the form $\exp[-\mu^2(y-l_q)^2]$) and l_u where $1/2\sqrt{\mu}$ is the width of the Gaussian. After expanding the fields in their normal modes and integrating over the extra dimension, the Yukawa interaction in four dimensions including the generation index will take the form

$$\mathcal{L}_Y = \lambda^u_{ij} Q_i U_j H + \lambda^d_{ij} Q_i D_j H^*, \tag{42}$$

where λ^u_{ij} is defined by

$$\lambda^u_{ij} = \kappa_{ij} e^{-(1/2)\mu^2 (l_{q_i} - l_{u_i})}, \tag{43}$$

and λ^d_{ij} is similarly defined. The above structure indicates that the Yukawa textures are governed by the location of the quarks in the extra dimension. Detailed analyses, however, indicate that this scenario leads to an insufficient amount of *CP* violation to explain the value of ϵ_K in kaon decay. Thus the scenario above gives a value of the Jarskog invariant $J \leqslant 5\times 10^{-9}$ while one needs $J \sim 10^{-5}$ to get the proper value of ϵ_K. The above shortcoming can be corrected by extending the analysis to two extra dimensions (Branco *et al.*, 2001). In this case one finds the Jarlskog invariant $J \simeq 2.2\times 10^{-5}$ which is of desired strength to explain *CP* violation in kaon decay. An extension to include masses for the charged leptons and neutrinos has been carried out by Barenboim *et al.* (2001).

An analysis using the fermion localization mechanism for generating quark-lepton textures within a supersymmetric SU(5) GUT theory has been carried out by Kakizaki and Yamaguchi (2004) where the different SU(5) chiral multiplets are localized along different points in the extra dimension. The analysis allows one to generate a realistic pattern of quark masses and mixings and lepton masses. The *CP* violation is of sufficient strength here since $J \sim O(10^{-5})$. An additional feature of this model is that dimension 5 proton decay operators are also naturally suppressed due to the fact that these operators contain an overlap of wave functions of different chiral multiplets and are thus exponentially suppressed.

A similar analyses can be carried out in the framework of a nonfactorizable geometry (Chang *et al.*, 2000; Grossman and Neubert, 2000; Abe, Inagaki, and Muta, 2001; Huber and Shafi, 2001) based on the metric

$$ds^2 = e^{-2\sigma(y)}(dx)^2 - dy^2, \tag{44}$$

where $\sigma(y)=k|y|$. Under the Z_2 orbifold symmetry the 5D fermion transforms as $\Psi(-y)_\pm = \pm\gamma_5\Psi(y)_\pm$. $\Psi_\pm$ have the mode expansion

$$\Psi(x,y)_\pm = \frac{1}{\sqrt{2\pi r_c}} \sum_{n=0}^{\infty} \psi_{n\pm}(x) f^{(n)}_\pm(y). \tag{45}$$

The zero modes of $\Psi_\pm$ are the left handed and right handed Weyl spinors. Masses for these are generated by the 5D Higgs couplings of the form

$$\int d^4x dy \sqrt{-g}\lambda_{ij} H \bar{\Psi}_{i+}\Psi_{j-}. \tag{46}$$

For the zero mode they give rise to a Dirac mass term of the form (Huber and Shafi, 2001)

$$m_{ij} = (2\pi r_c)^{-1}\int_{-\pi r_c}^{\pi r_c} dy \lambda_{ij} H(y) f_{i+}^{(0)}(y) f_{i-}^{(0)}(y), \quad (47)$$

where

$$f^{(0)} = \left(\frac{e^{2\pi r_c(1/2-c)} - 1}{2\pi k r_c(1/2-c)}\right)^{-1/2} e^{(2-c)\sigma}, \quad (48)$$

with c a parameter that characterizes the location of the fermion in the extra dimension. For $c<1/2$ the fermion is localized near the $y=0$ brane while for $r=\pi r_c$ it is localized near $y=\pi r_c$ brane. With the appropriate choice of c's one may generate a realistic pattern of quark masses and mixings and a realistic CKM matrix. However, explicit determination of the Jarlskog invariant appears not to have been carried out. The texture models using extra dimensions do generally require a high level of fine-tuning in the selection of locations where fermions are placed. Thus models of this type do not appear natural. For related works on *CP* violation and extra dimensions see Sakamura (1999), Dooling *et al.* (2002), Huang *et al.* (2002), and Ichinose (2002).

VII. *CP* VIOLATION IN STRINGS

We discuss now the possible origins of *CP* violation in SUSY, string, and brane models [for review of string theory see Green *et al.* (1987a, 1987b) and Polchinski (1998a, 1998b)]. One possible origin is string compactification (Witten, 1985; Wu *et al.*, 1991; Kobayashi and Lim, 1995; Bailin *et al.*, 1998a, 1998b, 2000; Dent, 2001, 2002; Faraggi and Vives, 2002). One may call this hard *CP* violation since this type of *CP* violation can exist even without soft terms. Yukawa couplings which are now formed via string compactification will carry this type of *CP* violation and the CKM phase δ_{CKM} which arises from the Yukawa couplings is therefore a probe of *CP* violation arising from string compactification (assuming there is no *CP* violation arising from the Higgs sector). A second source of *CP* violation is via soft breaking. If SUSY contributions to K and B physics turn out to be small, then one has a plausible bifurcation, i.e., the *CP* violations in K and B physics are probe of string compactification, and baryogenesis and other *CP* phenomena that may be seen in sparticle decays, etc., become a probe of soft breaking.

Regarding soft breaking in string theory, such an analysis would entail specifying the Kähler potential, the superpotential, and the gauge kinetic energy function, on the one hand, and the mechanism of breaking, on the other hand. Each of these are model dependent. However, it is possible to parametrize the breaking as in gravity mediated breaking in supergravity. Thus one can write the general form of the soft terms in the form

$$V_{\mathrm{soft}} = m_\alpha^2 C_\alpha \bar{C}_{\bar{\alpha}} + A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma} C_\alpha C_\beta C_\gamma + \tfrac{1}{2}(B_{\alpha\beta}\mu_{\alpha\beta} C_\alpha C_\beta + \mathrm{H.c.}) + \cdots , \quad (49)$$

where the general expressions for the scalar masses m_α, trilinear couplings $A_{\alpha\beta\gamma}$, and the bilinear term B can be given. For the case $K_{\alpha\bar{\beta}}=\delta_{\alpha\bar{\beta}}K_\alpha$, one has (Kaplunovsky and Louis, 1993; Brignole *et al.*, 1994)

$$m_\alpha^2 = m_{3/2}^2 + V_0 - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \ln(K_\alpha),$$

$$A_{\alpha\beta\gamma} = cF^I[\partial_I K + \partial_I \ln(Y_{\alpha\beta\gamma}) - \partial_I \ln(K_\alpha K_\beta K_\gamma)],$$

$$B_{\alpha\beta} = cF^I[\partial_I K + \partial_I \ln(\mu_{\alpha\beta}) - \partial_I \ln(K_\alpha K_\beta)] + \cdots , \quad (50)$$

while the gaugino masses are given by

$$m_a = \frac{1}{2\,\mathrm{Re}(f_a)} F^I \partial_I f_a. \quad (51)$$

An efficient way to parametrize F^I is given by (Brignole *et al.*, 1994)

$$F^S = \sqrt{3} m_{3/2}(S+S^*)\sin\theta e^{-i\gamma_S},$$

$$F^i = \sqrt{3} m_{3/2}(T+T^*)\cos\theta \Theta_i e^{-i\gamma_i}, \quad (52)$$

where θ, Θ_i parametrize the Goldstino direction in the S, T_i field space and γ_S and γ_i are the F^S and F^i phases, and $\Theta_1^2+\Theta_2^2+\Theta_3^2=1$.

A. Complex Yukawa couplings in string compactifications

The Yukawa couplings arise at the point of string compactification, and it is interesting to ask how the Yukawa couplings develop *CP* phases. It is also interesting to determine if such phases are small or large. Consider, for example, the compactification of the $E_8\times E_8$ heterotic string on a six-dimensional Calabi-Yau (CY) manifold. In this case the massless families are either (1,1) or (2,1) harmonic forms. For the case when hodge number $h_{11}>h_{21}$, the massless mirror families are (1,1) forms while if $h_{21}>h_{11}$ the massless families are (2,1) forms. For the case when the families are (1,1) the cubic couplings among the families have been discussed by Strominger (1985). The analysis for the case when $h_{21}>h_{11}$ is more involved. One specific model of interest that can lead to complex Yukawas corresponds to compactification on the manifold K_0' (Schimmrigk, 1987; Gepner, 1988)

$$P^1 \equiv \sum_{i=0}^{3} z_i^3 + a_0(z_1 z_2 z_3) = 0,$$

$$P^2 \equiv \sum_{i=0}^{3} z_i x_i^3 = 0, \quad (53)$$

which is deformed from the manifold K_0 (corresponding to the case $a_0=0$) in the ambient space $CP^3\times CP^2$ by a single (2,1) form $(z_1z_2z_3)$. The K_0 has 35 h_{21} forms and 8 h_{11} forms, giving a Euler characteristic $\chi=2(h_{21}-h_{11})$ and the number of net massless families is $|\chi|/2$ (Sotkov and Stanishkov, 1988).

By modding out by two discrete groups Z_3 and Z_3' one gets a three generation model. The discrete symmetries are Z_3 and Z_3' where

$$Z_3\text{: } g\text{: } (z_0,z_1,z_2,z_3\text{:}x_1,x_2,x_3) \to (z_0,z_2,z_3,z_1;x_2,x_3,x_3,x1),$$

$$Z_3'\text{: } h\text{: } (z_0,z_1,z_2,z_3\text{:}x_1,x_2,x_3) \to (z_0,z_1,z_2,z_3;x_1,\alpha x_2,\alpha^2 x_3), \tag{54}$$

where $\alpha^3=1$, $\alpha\neq 1$. The group Z_3' is not freely acting and leaves three tori invariant. These invariant tori have to be blown up in order to obtain a smooth CY manifold. Such a blowing up procedure produces six additional (2,1) and (1,1) forms which, however, leave the net number of generations unchanged. One considers now the flux breaking of E_6 on this manifold. If one embeds a single factor, Z_3 or Z_3' in the E_6, then E_6 can break to $SU(3)^3$ or $SU(6)\times U(1)$ each of which leave the standard model gauge group unbroken. However, the case $SU(6)\times U(1)$ cannot be easily broken further since an adjoint representation does not arise in the massless spectrum. Thus typically one considers the $SU(3)^3$ possibility. In this case there are two possibilities: case A where Z_3 is embedded trivially and Z_3' is embedded nontrivially, and case B where Z_3' is embedded trivially and Z_3 is embedded nontrivially. Now for case A one may choose $U_g=(\mathrm{id})_C\times(\mathrm{id})_L\times(\mathrm{id})_R$, $U_h=(\mathrm{id})_C\times\alpha(\mathrm{id})_L\times\alpha(\mathrm{id})_R$, where U_g is defined so that $g\to U_g$ is a homomorphism of Z_3 into $E_6\ni U_g$ (Witten, 1985), and similarly for U_h, where (id) stands for an identity matrix, and C,L,R stand for color, left and right handed subgroups of $SU(3)^3$. The analysis of Yukawa couplings in this case has been carried out and the couplings can be made all real. Thus in this case there is no *CP* violation arising in the Yukawa sector at the compactification scale.

We consider next case B where essentially one has an interchange in the definitions of U_g and U_f so that

$$U_g=(id)_C\times\alpha(id)_L\times\alpha(id)_R,$$

$$U_h=(id)_C\times(id)_L\times(id)_R. \tag{55}$$

In this case the massless states that survive flux breaking of E_6 transform under Z_3 as follows:

$$Z_3L=L,\quad Z_3Q=\alpha Q,\quad Z_3Q^c=\alpha^2 Q^c,$$

$$Z_3\bar{L}=\bar{L},\quad Z_3\bar{Q}=\alpha^2\bar{Q},\quad Z_3\bar{Q}^c=\alpha\bar{Q}^c, \tag{56}$$

where the leptons transform as $L(1,3,\bar{3})$, quarks as $Q(3,\bar{3},1)$, and conjugate quarks as $Q^c(\bar{3},1,3)$. The barred quantities represent the mirrors, so that $\bar{L}(1,\bar{3},3)$, $\bar{Q}(\bar{3},3,1)$, and $\bar{Q}^c(3,1,\bar{3})$. In this model the number of generations and mirror generation are identical to that of the Tian-Yau model (Greene *et al.*, 1986, 1987) so that there are nine lepton generations and six mirror generations, seven quark generations and four mirror quark generations, seven conjugate quark generations and four mirror conjugate quark generations, providing us with three net families of quarks and leptons. The analysis of Yukawa couplings has been carried out on the manifold K_0 by many authors.

Our focus here is the $(27)^3$ couplings which are unaffected by the instantons (Distler and Greene, 1988) and one can use the techniques of Candelas (1988) to determine the couplings. An analysis for case B was carried out by Wu *et al.* (1991). The Yukawa couplings determined in this fashion have unknown normalizations for the kinetic energy. However, symmetries can be used to obtain constraints on the normalizations. Taking these normalization constraints into account it is found that Yukawas depend on α in a nontrivial manner, and thus *CP* is violated in an intrinsic manner. Further, the *CP* phase entering in the coupling is large. The *CP* violation on the K_0' manifold persists even when the modulus a_0 is real, so in this sense *CP* violation is intrinsic.

B. *CP* violation in orbifold models

Next we discuss the possibility of spontaneous *CP* violation in some heterotic string models. What we consider are field point limits of such models, and so we are essentially discussing supergravity models with the added constraint of modular invariance (T duality). The duality constraints have been utilized quite extensively in the analysis of gaugino condensation and SUSY breaking (Ferrara *et al.*, 1990; Font *et al.*, 1990; Nilles and Olechowski, 1990; Binetruy and Gaillard, 1991; Cvetic *et al.*, 1991; Gaillard and Nelson, 2007) and have also been utilized in the analysis of spontaneous breaking of *CP* (Acharya *et al.*, 1995; Bailin *et al.*, 1997; Dent, 2001, 2002; Giedt, 2002).

The scalar potential in supergravity and string theory is given by (Chamseddine *et al.*, 1982; Cremmer *et al.*, 1982)

$$V=e^K[(K^{-1})^i_j D_i W D_j^\dagger W^\dagger - 3WW^\dagger]+V_D, \tag{57}$$

where K is the Kähler potential, W is superpotential, and $D_iW=W_i+K_iW$, with the subscripts denoting derivatives with respect to the corresponding fields. As noted above we now use the added constraint of T-duality symmetry. Specifically we assume that the scalar potential in the effective four dimensional theory depends on the dilaton field S and on the (Kähler) moduli fields T_i ($i=1,2,3$), and it is invariant under modular transformations (to keep matters simple, we do not include here the dependence on the so called complex structure U moduli)

$$T_i\to T_i'=\frac{a_iT_i-ib_i}{ic_iT_i+d_i},\quad (a_id_i-b_ic_i)=1, \tag{58}$$

where $a_i,b_i,c_i,d_i\in Z$. Under the modular transformations, K and W undergo a Kähler transformation while the scalar potential V is invariant. For the Kähler potential we assume essentially a no scale form (Lahanas and Nanopoulos, 1987),

$$K=D(z)-\sum_i \ln(T_i+\bar{T}_i)+K_{IJ}Q_I^\dagger Q_J+H_{IJ}Q_IQ_J,$$

where $D(z)=-\ln(z)$, and for z one may consider

$$z = \left(S + \bar{S} + \frac{1}{4\pi^2} \sum_i^3 \delta_i^{\mathrm{GS}} \ln(T_i + \bar{T}_i) \right), \tag{59}$$

where δ_i^{GS} is the one loop correction to the Kähler potential from the Greene-Schwarz mechanism and Q are the matter fields consisting of the quarks, the leptons, and the Higgs. For the superpotential in the visible sector one may consider

$$W_v = \tilde{\mu}_{IJ} Q_I Q_J + \lambda_{IJK} Q_I Q_J Q_K. \tag{60}$$

Under T duality, Q's transform as

$$Q_I \to Q_I \Pi_i (ic_i T_i + d_i)^{n^i_{Q_I}}. \tag{61}$$

In general, K_{IJ}, H_{IJ}, μ_{IJ}, and λ_{IJK}, are functions of the moduli. The constraints on $n^i_{Q_I}$ are such that V is modular invariant. Analyses of soft SUSY breaking terms using modular invariance of the type above has been extensively discussed in the literature assuming moduli stabilization. In such analyses one generically finds that *CP* is indeed violated if one assumes that the moduli are in general complex.

However, minimization of the potential and stabilization of the dilaton VEV is a generic problem in such models and requires additional improvements. Often this is accomplished by nonperturbative corrections to the potential. Thus one might consider nonperturbative contributions to the superpotential so that

$$W_{\mathrm{np}} = \Omega(\sigma) \eta(T)^{-6}. \tag{62}$$

Here $\eta(T)$ is the Dedekind function, and we have assumed a single overall modulus T and $\sigma = S + 2\tilde{\delta}^{\mathrm{GS}} \ln \eta(T)$ and $\tilde{\delta}^{\mathrm{GS}} = -(3/4\pi)\delta^{\mathrm{GS}}$. Additionally one can assume nonperturbative corrections to the Kähler potential and treat $D(z)$ as a function determined by nonperturbative effects. The analysis shows that for a wide array of parameters minima typically occur at the self-dual points of the modular group, i.e., $T=1$ and $T = e^{i\pi/6}$. However, for some choices of the parameters T can take complex values away from the fixed point. Nonetheless *CP* phases arising from such points are small since in the soft parameters they come multiplied by the function $G(T,\bar{T}) = (T+\bar{T})^{-1} + 2d \ln[\eta(T)]/dT$ the imaginary part of which varies very rapidly as the real part changes. Thus large *CP* phases do not appear to arise using the moduli stabilization of the type above (Bailin *et al.*, 1997).

The situation changes significantly if W_{np} contains an additional factor $H(T)$ where

$$H(T) = \left(\frac{G_6(T)}{\eta(T)^{12}} \right)^m \left(\frac{G_4(T)}{\eta(T)^8} \right)^n P(j), \tag{63}$$

where $G_4(T)$ and $G_6(T)$ are Eisenstein functions of modular weight 4 and 6, m, n are positive integers, and $P(j)$ is a polynomial of $j(T)$ which is an absolute modular invariant. Alternately H can be expressed in the form

$$H(T) = (j - 1728)^{m/2} j^{n/2} P(j). \tag{64}$$

The form of $H(T)$ is dictated by the condition that no singularities appear in the fundamental domain. In this case to achieve dilaton stabilization with T modulus not only on the boundary of the fundamental domain but also inside the fundamental domain and thus T has a substantial imaginary part. In this case it is possible to get *CP* phases for the soft parameters which can lie in the range 10^{-4}–10^{-1} (Bailin *et al.*, 1997). Thus with the absolute modular invariant in the superpotential large *CP* phases can appear in the soft breaking in orbifold compactifications of the type discussed above.

In the analysis of Faraggi and Vives (2002) the issue of *CP* violation and FCNC in string models with an anomalous $\mathrm{U}(1)_A$-dilaton supersymmetry breaking mechanism was investigated. Here scalar masses arise dominantly from the $\mathrm{U}(1)_A$ contribution while the dilaton generates the main contribution to the gaugino masses. Further, the dilaton contributions to the trilinear terms and gaugino masses have the same phase. In this class of models the nonuniversal components of the trilinear soft SUSY breaking parameter are typically small and one has suppression of FCNC and *CP* in this class of models.

C. *CP* violation on *D* brane models

Considerable progress has occurred recently in the development of type I and type II string theory. Specifically D branes have provided a new and better understanding of type I string theory and connection with type IIB orientifolds. Further, the advent of D branes open up the possibility of a new class of model building [for recent reviews on D branes see Polchinski (1996) and Blumenhagen *et al.* (2005, 2006). Thus a stack of N D branes can produce generally an SU(N) gauge group or a subgroup of it, and open strings with both ends terminating on the same stack give rise to a vector multiplet corresponding to the gauge group of the stack. Further, open strings beginning on one end and ending on another transform like the bifundamental representations and can be chiral. Thus these are possible candidates for massless quarks, leptons, and Higgs fields. A simple possibility for model building occurs with compactification on $T^6/Z_2 \times Z_2$. In addition to the axion-dilaton field s the moduli space consists in this case of the Kähler (t_m) and the complex structure (u_m) moduli ($m=1,2,3$). For the moduli fields one has the Kähler potential of the form

$$K_0 = -\ln(s+\bar{s}) - \sum_{m=1}^{3} \ln(t_m + \bar{t}_m) - \sum_{m=1}^{3} \ln(u_m + \bar{u}_m). \tag{65}$$

Consider now complex scalars $C_i^{[99]}$ along the direction i with ends of the open string ending in each case on a $D9$ brane. In this case one can obtain the Kähler potential including the complex scalar field by the translation $t_m + \bar{t}_m \to t_m + \bar{t}_m - |c_m^{[99]}|^2$. For the case of strings with both ending on the same $D5_i$ brane one can show using either T duality (Ibanez *et al.*, 1999) or by use of Born-Infeld action (Kors and Nath, 2004; Kors, 2006) that the Kähler potential is modified by making the replacement $s+\bar{s} \to s+\bar{s}-|C_m^{[5_m,5_m]}|^2$. For the case when one has both $D9$ and $D5_m$ branes the modified Kähler potential reads

$$K^{[99+55]} = -\ln\left(s+\bar{s}-\sum_{m=1}^{3}|C_m^{[5_m5_m]}|^2\right) - \sum_{m=1}^{3}\ln\left(t_m+\bar{t}_{\bar{m}}-|C_m^{[99]}|^2 - \frac{1}{2}\sum_{n,p=1}^{3}\gamma_{mnp}|C_n^{[5_p5_p]}|^2\right). \tag{66}$$

To construct the Kähler potential for the case when one has open strings with one end on $D9$ branes and the other end on $D5_m$ branes, or for the case when open strings end on two different $D5$ branes, one can use the analogy to heterotic strings with $\mathbb{Z}_2$-twisted matter fields (Ibanez *et al.*, 1999; Kors and Nath, 2004). Alternately one can use string perturbation theory (Lust *et al.*, 2004, 2005; Bertolini *et al.* 2006). The result is

$$K^{[95]} = \frac{1}{2}\sum_{m,n,p=1}^{3}\gamma_{mnp}\frac{|C^{[95_m]}|^2}{(t_n+\bar{t}_{\bar{n}})^{1/2}(t_p+\bar{t}_{\bar{p}})^{1/2}} + \frac{1}{2}\sum_{m,n,p=1}^{3}\gamma_{mnp}\frac{|C^{[5_m5_n]}|^2}{(t_p+\bar{t}_{\bar{p}})^{1/2}(s+\bar{s})^{1/2}}. \tag{67}$$

Explicit formulas for the soft parameters using these results have been given in the literature. However, one needs to keep in mind that the configurations of the type discussed above are the so-called $\frac{1}{2}$BPS states, and in this case the spectrum of open states falls into $N=2$ multiplets, which implies that the spectrum is not chiral. Similar considerations apply to open strings which start and end on D_3 and D_7 branes, and results for these can be obtained by using T dualities.

For realistic model building one needs to work with intersecting D branes. Thus in Calabi-Yau orientifolds of type IIA one has $D6$ branes that intersect on the compactified six dimensional manifold. Sometimes it is convenient to work in the T-dual picture of type IIB strings where the geometrical picture of branes intersecting is replaced by internal world volume gauge field backgrounds, called fluxes on the $D9$ and $D5$ branes. The fluxes $\mathcal{F}_a^m$ where a labels the set of branes, are rational numbers, i.e., $\mathcal{F}_a^m = m_a^m/n_a^m$, in order to satisfy charge quantization constraints. The fluxes determine the number of chiral families. Further, the condition that $N=1$ supersymmetry be valid is a further constraint on the moduli and the fluxes and may be expressed in the form (Bachas, 1995; Berkooz *et al.*, 1996; Kors and Nath, 2004)

$$\sum_{m=1}^{3}\frac{s+\bar{s}}{t_m+\bar{t}_m}\mathcal{F}_a^m = \prod_{m=1}^{3}\mathcal{F}_a^m. \tag{68}$$

In the presence of fluxes the gauge kinetic energy function f_a is given by

$$f_a = \prod_{m=1}^{3}\mathbf{n}_a^{(m)}\left(s-\frac{1}{2}\sum_{m,n,p=1}^{3}\gamma_{mnp}\mathbf{F}_a^{(n)}\mathbf{F}_a^{(p)}t_m\right). \tag{69}$$

The computation of the Kähler metric for the case of an open string with both ending on some given stack a, $C_m^{[aa]}$, can be computed by dimensional reduction (Kors and Nath, 2004) or string perturbation theory (Lust *et al.*, 2004) and is given by

$$K^{[aa]} = \sum_{m=1}^{3}\frac{|C_m^{[aa]}|^2}{(s+\bar{s})(t_m+\bar{t}_{\bar{m}})(u_m+\bar{u}_{\bar{m}})}\frac{4\,\mathrm{Re}(f_a)}{1+\Delta_a^{(m)}},$$

$$\Delta_a^{(m)} = \frac{1}{2}\sum_{n,p=1}^{3}\gamma_{mnp}\frac{(t_n+\bar{t}_{\bar{n}})(t_p+\bar{t}_{\bar{p}})}{(s+\bar{s})(t_m+\bar{t}_{\bar{m}})}(\mathbf{F}_a^{(m)})^2. \tag{70}$$

Now the technique above using the heterotic dual or Born–Infeld works for $\frac{1}{2}$BPS brane configurations. However, for the bifundamental fields $C^{[ab]}$ that connect the different stacks of branes with different world volume gauge flux one needs an actual string perturbation calculation and here the result for the Kähler potential is (Lust *et al.*, 2004)

$$K^{[ab]} = \frac{|C^{[ab]}|^2}{\prod_{m=1}^{3}(u_m+\bar{u}_{\bar{m}})^{\theta_{ab}^{(m)}}}\frac{\Gamma(\theta_{ab}^{(m)})^{1/2}}{\Gamma(1-\theta_{ab}^{(m)})^{1/2}},$$

$$\theta_{ab}^{(m)} = \arctan\left(\frac{\mathbf{F}_a^{(m)}}{\mathrm{Re}(t_m)}\right). \tag{71}$$

Using the above one can obtain explicit expressions for the soft parameters. These have been worked out in detail in several papers. One can count the number of *CP* phases that enter in the analysis. They are the phases arising from s, t_m, u_m ($m=1,2,3$). These can be reduced with extra restrictions such as, for example, dilation dominance which would imply only one *CP* phase γ_s.

D. SUSY *CP* phases and the CKM matrix

A natural question is if there is a connection between the soft SUSY *CP* phases and the CKM phase δ_{CKM}. *A priori* it would appear that there is no connection between these two since they arise from two very different sources. Thus δ_{CKM} arises from the Yukawa interactions (assuming there is no *CP* violation in the Higgs sector) which from the string view point originates at the point when the string compactifies from ten dimensions to four dimensions. This is the point where we begin to identify various species of quarks and leptons and their couplings to the Higgs bosons. On the other hand, soft SUSY phases arise from the spontaneous breaking of supersymmetry and enter only in the dimension $\leqslant 3$ operators. Thus it would appear that they are disconnected. While this conclusion is largely true it is not entirely so. The reason for this is that in SUGRA models the trilinear soft term $A_{\alpha\beta\gamma}$ contains a dependence on Yukawa couplings so that (Nath *et al.*, 1983b; Kaplunovsky and Louis, 1993)

$$A_{\alpha\beta\gamma} = F^i \partial_i Y_{\alpha\beta\gamma} + \cdots . \tag{72}$$

Thus the phase of the Yukawa couplings enters in the phase of the trilinear coupling. However, the phase relationship between A and Y is not rigid, since even for the case when there is no phase in the Yukawa couplings one can generate a phase of A, and conversely even for the case when $\delta_{\rm CKM}$ is maximal one may constrain A to have zero phase. Further, it is entirely possible that the Yukawa couplings are all real and $\delta_{\rm CKM}$ arises from *CP* violation in the Higgs sector as originally conjectured (Lee, 1973, 1974; Weinberg, 1976). A more recent analysis of this possibility has been given by Chen *et al.* (2007).

On a more theoretical level it was initially thought that *CP* violation could occur in string theory in either of the two ways: spontaneously or explicitly (Strominger and Witten, 1985). However, it was conjectured later that *CP* symmetry in string theory is a gauge theory and it is not violated explicitly (Dine *et al.*, 1992; Choi *et al.*, 1993). We do not address this issue further here.

VIII. THE EDM OF AN ELEMENTARY DIRAC FERMION

If the spin-1/2 particle has electric dipole moment EDM d_f, it would interact with the electromagnetic tensor $F_{\mu\nu}$ through

$$\mathcal{L} = -\frac{i}{2} d_f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}, \tag{73}$$

which in the nonrelativistic limit reads

$$\mathcal{L} = d_f \psi_A^\dagger \vec{\sigma} \cdot \vec{E} \psi_A, \tag{74}$$

where ψ_A is the large component of the Dirac field. The above Lagrangian is not renormalizable, so it does not exist at the tree level of a renormalizable quantum field theory. However, it could be induced at the loop level if this theory contains sources of *CP* violation at the tree level. Thus suppose we determine the EDM of a particle with the field ψ_f due to the exchange of two other heavy fields: a spinor ψ_i and a scalar ϕ_k (see Fig. 3). The interaction that contains *CP* violation is given by

$$\mathcal{L} = L_{ik} \overline{\psi_f} P_L \psi_i \phi_k + R_{ik} \overline{\psi_f} P_R \psi_i \phi_k + \text{H.c.} \tag{75}$$

Here $\mathcal{L}$ violates *CP* invariance if and only if $\text{Im}(L_{ik} R_{ik}^*) \neq 0$. A direct analysis shows that the fermion ψ_f acquires a one loop EDM d_f which is given by

$$d_f = \frac{m_i}{16\pi^2 m_k^2} \text{Im}(L_{ik} R_{ik}^*) \left[Q_i A\left(\frac{m_i^2}{m_k^2}\right) + Q_k B\left(\frac{m_i^2}{m_k^2}\right) \right], \tag{76}$$

where

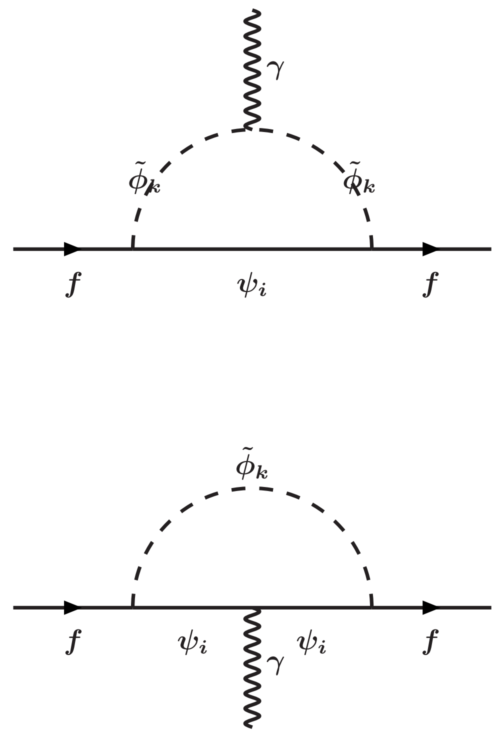

FIG. 3. Contributions to the electric dipole moment of a lepton or of a quark from the exchange of the charginos, the neutralinos, and the gluino. The internal dashed line in the loop is the scalar field ϕ_k, the solid line is the fermion field ψ_i, and the external wiggly line is the external photon line.

$$A(r) = \frac{1}{2(1-r)^2}\left(3 - r + \frac{2\ln r}{1-r}\right),$$

$$B(r) = \frac{1}{2(1-r)^2}\left(1 + r + \frac{2r\ln r}{1-r}\right). \tag{77}$$

We will utilize this result in EDM analyses in the following discussion.

IX. EDM OF A CHARGED LEPTON IN SUSY

We discuss now the EDM of a charged lepton in MSSM using the results of the previous section. As mentioned in Sec. IV, in softly broken supersymmetric models as many as 40 additional phases can appear. However, only certain combinations of phases appear in a given process and the number of such combinations depends on the process. We discuss now the details.

In these computations we use the Lagrangian of applied $N=1$ supergravity for the case of MSSM fields with inclusion of soft breaking (Nath *et al.*, 1983a; Nilles, 1984; Haber and Kane, 1985). The EDM of a charged lepton receives contributions from chargino, neutralino, and slepton exchanges. A discussion of the chargino and neutralino masses is given in Appendix A while a discussion of the slepton and squark masses is given in Appendix B. For the case of the charged lepton we find

$$d^E_{e\text{-chargino}}/e = \frac{\alpha_{\rm EM}}{4\pi \sin^2\theta_W} m^2_{\tilde{\nu}e} \sum_{i=1}^{2} \tilde{m}_{\chi_i^+} \,\mathrm{Im}(\Gamma_{ei}) A\left(\frac{\tilde{m}^2_{\chi_i^+}}{m^2_{\tilde{\nu}e}}\right), \tag{78}$$

where U and V are defined in Appendix A and $\Gamma_{ei} = \kappa_e U^*_{i2} V^*_{i1} = |\kappa_e| U^*_{R2i} U_{L1i}$. A direct inspection of Γ_{ei} shows that it depends on only one combination, i.e., $\xi_2+\theta_\mu+\theta_H$ where the phase θ_H comes from the Higgs sector and as discussed later is generated at the loop level.

The neutralino exchange contribution to the EDM of the fermion is as follows:

$$d^E_{f\text{-neutralino}}/e = \frac{\alpha_{\rm EM}}{4\pi \sin^2\theta_W} \sum_{k=1}^{2}\sum_{i=1}^{4} \mathrm{Im}(\eta_{fik}) \frac{\tilde{m}_{\chi_i^0}}{M^2_{\tilde{f}k}} \times Q_{\tilde{f}} B\left(\frac{\tilde{m}^2_{\chi_i^0}}{M^2_{\tilde{f}k}}\right), \tag{79}$$

where

$$\eta_{fik} = (a_0 X_{1i} D^*_{f1k} + b_0 X_{2i} D^*_{f1k} + \kappa_f X_{bi} D^*_{f2k}) \times (c_0 X_{1i} D_{f2k} - \kappa_f X_{bi} D_{f1k}), \tag{80}$$

with $a_0 = -\sqrt{2}\tan\theta_W (Q_f - T_{3f})$, $b_0 = -\sqrt{2} T_{3f}$, $c_0 = \sqrt{2}\tan\theta_W Q_f$, and in X_{bi}, $b=3$ (4) for $T_{3q} = -\frac{1}{2}$ ($\frac{1}{2}$). The following three combinations of phases appear in η_{fik}: $\xi_1+\theta_\mu+\theta_H$, $\xi_2+\theta_\mu+\theta_H$, and $\alpha_f+\theta_\mu+\theta_H$. We note in passing that the contribution from the neutrino Yukawa couplings to the lepton electric dipole moment has been computed by Farzan and Peskin (2004), and the charged Higgs contributions to the lepton EDM in a two-Higgs doublet model has been discussed by Kao and Xu (1992).

X. EDM OF QUARKS IN SUSY

The quarks receive contribution from the electric dipole operator (d^E_q), from the chromoelectric dipole operator (d^C_q), and from the purely gluonic dimension 6 operator of Weinberg (d^G_q). Thus

$$d_q = d^E_q + d^C_q + d^G_q. \tag{81}$$

We discuss these in further detail below.

A. The electric dipole moment operator contribution to EDM of quarks

The electric dipole moment operator receives contributions from the gluino, chargino, and neutralino exchanges. The gluino exchange contributes to the EDM of the quarks as follows:

$$d_{q\text{-gluino}}/e = -\frac{2\alpha_s}{3\pi} m_{\tilde{g}} Q_{\tilde{q}} \,\mathrm{Im}(\Gamma^{11}_q) \times \left[\frac{1}{M^2_{\tilde{q}1}} B\left(\frac{m^2_{\tilde{g}}}{M^2_{\tilde{q}1}}\right) - \frac{1}{M^2_{\tilde{q}2}} B\left(\frac{m^2_{\tilde{g}}}{M^2_{\tilde{q}2}}\right)\right], \tag{82}$$

where $\tilde{q}_1$ and $\tilde{q}_2$ are the mass eigenstates and $\Gamma^{1k}_q = e^{-i\xi_3} D_{q2k} D^*_{q1k}$, $\alpha_s = g_s^2/4\pi$, $m_{\tilde{g}}$ is the gluino mass, and $B(r)$ is as defined by Eq. (77). An explicit analysis gives $\Gamma^{12}_q = -\Gamma^{11}_q$, where

$$\mathrm{Im}(\Gamma^{11}_q) = \frac{m_q}{M^2_{\tilde{q}1} - M^2_{\tilde{q}2}} [m_0 |A_q| \sin(\alpha_q - \xi_3) + |\mu| \sin(\theta_\mu + \theta_H + \xi_3) |R_q|], \tag{83}$$

which holds for both signs of $M^2_{\tilde{q}1} - M^2_{\tilde{q}2}$. It is easy to see that combinations of phases that enter are $\alpha_q - \xi_3$ and $\xi_3+\theta_\mu+\theta_H$, or alternately one can choose them to be $\alpha_q+\theta_\mu+\theta_H$ and $\xi_3+\theta_\mu+\theta_H$.

The chargino contribution to the EDM for the up quark is as follows:

$$d_{u\text{-chargino}}/e = -\frac{\alpha_{\rm EM}}{4\pi\sin^2\theta_W} \sum_{k=1}^{2}\sum_{i=1}^{2} \mathrm{Im}(\Gamma_{uik}) \frac{\tilde{m}_{\chi_i^+}}{M^2_{\tilde{d}k}} \times \left[Q_{\tilde{d}} B\left(\frac{\tilde{m}^2_{\chi_i^+}}{M^2_{\tilde{d}k}}\right) + (Q_u - Q_{\tilde{d}}) A\left(\frac{\tilde{m}^2_{\chi_i^+}}{M^2_{\tilde{d}k}}\right)\right]. \tag{84}$$

Here $A(r)$ is as defined by Eq. (77) and

$$\Gamma_{uik} = \kappa_u V^*_{i2} D_{d1k} (U^*_{i1} D^*_{d1k} - \kappa_d U^*_{i2} D^*_{d2k}), \tag{85}$$

$$\kappa_u = \frac{m_u e^{-i\theta_H}}{\sqrt{2} m_W \sin\beta}, \quad \kappa_{d,e} = \frac{m_{d,e}}{\sqrt{2} m_W \cos\beta}, \tag{86}$$

and explicitly

$$\Gamma_{ui1(2)} = |\kappa_u| (\cos^2\theta_d/2) [U_{L2i} U^*_{R1i}] - (+) \tfrac{1}{2} |\kappa_u \kappa_d| (\sin\theta_d) [U_{L2i} U^*_{R2i}] e^{i\{\xi_2 - \phi_d\}}. \tag{87}$$

The EDM depends here only on two combinations of phases: $\alpha_d+\theta_\mu+\theta_H$ and $\xi_2+\theta_\mu+\theta_H$ with $\xi_2-\alpha_d$ a linear combination of the first two. A similar analysis hold for the chargino contributions to the down quark and one gets only two phase combinations which are identical to the case above with α_d replaced by α_u. The neutralino exchange contribution to the EDM of quarks is given by Eq. (79). The sum of the gluino, chargino, and neutralino exchanges discussed above gives the total contribution from the electric dipole operator to the quark EDM.

B. The chromoelectric dipole moment contribution to the EDM of quarks

For the case of quarks one has two more operators that contribute. These are the quark chromoelectric dipole moment ($\tilde{d}^C$) and the purely gluonic dimension 6 operator. For the operator $\tilde{d}^C$ we have the effective dimension 5 operator

$$\mathcal{L}_I = -\frac{i}{2}\tilde{d}^C\bar{q}\sigma_{\mu\nu}\gamma_5 T^a q G^{\mu\nu a}, \tag{88}$$

where T^a are the SU(3) generators. Contributions to $\tilde{d}^C$ of the quarks from the gluino, the chargino, and the neutralino exchange are given by

$$\tilde{d}^C_{q\text{-gluino}} = \frac{g_s\alpha_s}{4\pi}\sum_{k=1}^{2}\mathrm{Im}(\Gamma_q^{1k})\frac{m_{\tilde{g}}}{M^2_{\tilde{q}_k}}C\left(\frac{m^2_{\tilde{g}}}{M^2_{\tilde{q}_k}}\right), \tag{89}$$

$$\tilde{d}^C_{q\text{-chargino}} = -\frac{g^2 g_s}{16\pi^2}\sum_{k=1}^{2}\sum_{i=1}^{2}\mathrm{Im}(\Gamma_{qik})\frac{\tilde{m}_{\chi_i^+}}{M^2_{\tilde{q}k}}B\left(\frac{\tilde{m}^2_{\chi_i^+}}{M^2_{\tilde{q}k}}\right), \tag{90}$$

and

$$\tilde{d}^C_{q\text{-neutralino}} = \frac{g_s g^2}{16\pi^2}\sum_{k=1}^{2}\sum_{i=1}^{4}\mathrm{Im}(\eta_{qik})\frac{\tilde{m}_{\chi_i^0}}{M^2_{\tilde{q}k}}B\left(\frac{\tilde{m}^2_{\chi_i^0}}{M^2_{\tilde{q}k}}\right), \tag{91}$$

where $B(r)$ is defined by Eq. (77) and $C(r)$ is given by

$$C(r) = \frac{1}{6(r-1)^2}\left(10r - 26 + \frac{2r\ln r}{1-r} - \frac{18\ln r}{1-r}\right). \tag{92}$$

We note that all *CP* violating phases are contained in the factors $\mathrm{Im}(\Gamma_q^{1k})$, $\mathrm{Im}(\Gamma_{qik})$, and $\mathrm{Im}(\eta_{qik})$. But these are precisely the same factors that appear in the gluino, chargino, and neutralino contributions to the electric dipole operator.

C. The contribution of the purely gluonic operator to the EDM of quarks

The purely gluonic dimension 6 operator which contributes to the dipole moment is (Weinberg, 1989)

$$\mathcal{L}_I = -\tfrac{1}{6}\tilde{d}^G f_{\alpha\beta\gamma}G_{\alpha\mu\rho}G^{\rho}_{\beta\nu}G_{\gamma\lambda\sigma}\epsilon^{\mu\nu\lambda\sigma}, \tag{93}$$

where $G_{\alpha\mu\nu}$ is the gluon field strength tensor, $f_{\alpha\beta\gamma}$ are the Gell-Mann coefficients, and $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor with $\epsilon^{0123}=+1$. An analysis of d^G including the quark-squark-gluino exchange (see Fig. 4 where one of the loops contributing to this operator is shown) with gluino phase ξ_3 but with squark mass^2 matrix treated real has been given by Dai *et al.* (1990). Including the phases from A_t and μ in the squark mass^2 matrix the analysis of $\tilde{d}^G$ gives (Dai *et al.*, 1990; Ibrahim and Nath, 1998a)

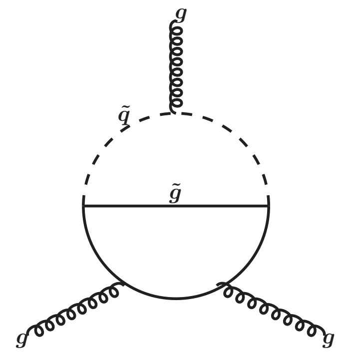

FIG. 4. The quark-squark-gluino exchange contribution to the purely gluonic dimension 6 operator. The dashed line in the upper semicircle in the loop is the squark $\tilde{q}$, the internal horizontal solid line is the gluino $\tilde{g}$, the solid line on the lower semicircle in the loop is the quark q, while the external wiggly lines are the gluons.

$$\tilde{d}^G = -3\alpha_s\left(\frac{g_s}{4\pi m_{\tilde{g}}}\right)^3[m_t(z_1^t - z_2^t)\mathrm{Im}(\Gamma_t^{12})H(z_1^t,z_2^t,z_t) + m_b(z_1^b - z_2^b)\mathrm{Im}(\Gamma_b^{12})H(z_1^b,z_2^b,z_b)]. \tag{94}$$

Here

$$\Gamma_q^{1k} = e^{-i\xi_3}D_{q2k}D^*_{q1k}, \quad z_\alpha^q = \left(\frac{M_{\tilde{q}\alpha}}{m_{\tilde{g}}}\right)^2, \quad z_q = \left(\frac{m_q}{m_{\tilde{g}}}\right)^2, \tag{95}$$

and $H(z_1,z_2,z_3)$ is defined by

$$H(z_1,z_2,z_3) = \frac{1}{2}\int_0^1 dx\int_0^1 du\int_0^1 dy\, x(1-x)u\frac{N_1N_2}{D^4}, \tag{96}$$

where

$$N_1 = u(1-x) + z_3x(1-x)(1-u) - 2ux[z_1y + z_2(1-y)],$$

$$N_2 = (1-x)^2(1-u)^2 + u^2 - \frac{1}{9}x^2(1-u)^2,$$

$$D = u(1-x) + z_3x(1-x)(1-u) + ux[z_1y + z_2(1-y)]. \tag{97}$$

For the case $m_{\tilde{q}},m_{\tilde{g}} \gg m_q$ one obtains the following expression for H:

$$H \simeq -\frac{m^2_{\tilde{g}}}{m^2_q}I(z_2^q), \tag{98}$$

where $I(z)$ is defined by

$$I(z) = \frac{1}{6(z-1)^2}[2(z-1)(11z-1) + (1-16z-9z^2)\ln z]. \tag{99}$$

The contribution of the last two operators to the EDM of quarks can be computed using dimensional analysis (Manohar and Georgi, 1984). This technique can be expressed using the "reduced" coupling constant rule. Thus, for example, for a coupling constant g appearing in an interaction of dimensionality (mass)^D and containing N field operators the reduced coupling is $(4\pi)^{2-N}M^{D-4}g$, where M is the chiral-symmetry breaking scale and has the value $M=1.19$ GeV. Thus the rule means that the reduced coupling of any term in the effective hadronic theory at energies below M is given by a product of the reduced coupling operators appearing in the effective Lagrangian at energies below M that produces this term. Using this rule for the chromoelectric and purely gluonic dimension 6 operators one finds there a contribution to the EDM of the quarks, given as follows:

$$d_q^E = d_q \eta^E, \quad d_q^C = \frac{e}{4\pi}\tilde{d}_q^C \eta^C, \quad d_q^G = \frac{eM}{4\pi}\tilde{d}_q^G \eta^G, \tag{100}$$

where η^E, η^C, and η^G are renormalization group evolution of d_q, $\tilde{d}_q^C$, and $\tilde{d}_q^G$ from the electroweak scale to the hadronic scale. A discussion of how these renormalization group factors are computed is discussed in Appendix C. Their numerical value is estimated to be $\eta^E \approx 0.61$ (Degrassi *et al.*, 2005), $\eta^C \approx \eta^G \sim 3.4$. The alternate technique to estimate contributions of the chromoelectric operator is to use the QCD sum rules (Khriplovich and Zyablyuk, 1996). To obtain the neutron EDM, we use the nonrelativistic SU(6) quark model which gives $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$.

D. The cancellation mechanism and other remedies for the *CP* problem in SUSY, strings, and branes

MSSM contains new sources of *CP* violation and these phases would induce EDMs of fermions in the theory. Taking the values of the model parameters at their phenomenologically favorable range ($m_{1/2} \sim m_0 \sim 100$ GeV, $\tan\beta \sim 10$, $\theta_\mu \sim \alpha_0 \sim 1$) one finds that the EDMs of the electron and neutron exceed the experimental bounds by several orders of magnitude. This problem is certainly a weakness of the low energy SUSY and needs to be corrected to make the theory viable. Various remedies have been suggested in the literature to overcome this problem. The first of these is the suggestion that the first generation of sleptons and the first two generations of squarks are very heavy (Nath, 1991) [see also Kizukuri and Oshimo (1992)]. This means the production and study of these particles at LHC will be difficult if not impossible. Another reason that this possibility is not attractive is that the annihilation rate of the lightest supersymmetric particle (LSP) may be too low in this range of masses and as a result the relic density of the LSP may be larger than the observed dark matter density. Another suggestion is that the phases are small $O(10^{-2})$ (Ellis *et al.*, 1982; Polchinski and Wise, 1983; Franco and Mangano, 1984; Dugan *et al.*, 1985; Weinberg, 1989; Arnowitt *et al.*, 1990, 1991; Braaten *et al.*, 1990a, 1990b; Dai *et al.*, 1990; Gunion and Wyler, 1990; Garisto and Wells, 1997). However, a small phase constitutes a fine tuning and there will not be any interesting display of *CP* violation in colliders. Moreover, electroweak baryogenesis cannot take place in this case (Kuzmin *et al.*, 1985). A third possibility proposed by Ibrahim and Nath (1998a, 1998b, 1998c) is that there are internal cancellations among the various contributions to the neutron and electron EDMs, leading to compatibility with experiment with large phases and a SUSY spectrum that is still within the reach of accelerators.

This is the most interesting solution because it leaves room for a host of nontrivial *CP* violating as well as *CP* conserving phenomena to be discovered at colliders and elsewhere. By *CP* violating properties, we mean those properties that vanish in the limit of *CP* conservation like the EDMs and the neutral Higgs bosons mixing. By *CP* conserving phenomena, we mean those properties that exist in the absence of *CP* violation but they differ if *CP* violation is included like $g_\mu - 2$. Following the work of Ibrahim and Nath (1998a, 1998b, 1998c) there has been much additional work on the cancellation mechanism in the literature (Falk and Olive, 1998; Ibrahim and Nath, 1998c, 2000d; Bartl *et al.*, 1999, 2001; Brhlik, Everett, Kane, and Lykken, 1999, 2000; Brhlik, Good, and Kane, 1999, 2001; Falk, Olive, *et al.*, 1999; Accomando *et al.*, 2000a, 2000b; Brhlik, Everett, Kane, King, *et al.*, 2000; Pokorski *et al.*, 2000; Barger *et al.*, 2001; Chattopadhyay *et al.*, 2001; Abel *et al.*, 2002).

As shown above, the quark and lepton EDMs in general depend on ten independent phases providing one with considerable freedom for the satisfaction of EDM constraints. Numerical analyses show the existence of significant regions of the parameter space where the cancellation mechanism holds. We describe here a straightforward technique for accomplishing the satisfaction of the electron and neutron EDM constraints. For the case of the electron one finds that the chargino component of the electron is independent of ξ_1 and the electron EDM as a whole is independent of ξ_3. Thus the algorithm to discover a point of simultaneous cancellation for the electron and neutron EDMs is a straightforward one. For a given set of parameters we vary ξ_1 until we reach the cancellation for the electron EDM since only one of its components (the neutralino) is affected by that parameter. Once the electric dipole moment constraint on the electron is satisfied we vary ξ_3 which affects only the neutron EDM keeping all other parameters fixed. By using this simple algorithm one can generate any number of simultaneous cancellations. The EDM of atoms also provides a sensitive test of *CP* violation. An example is the EDM of Hg-199 for which the current limits are given by Eq. (22). Among the phases that enter the EDM of Hg-199 is the phase α_s. We note that α_s enters only in $d_{\rm Hg}$ to one loop order, and thus it can be varied to achieve a simultaneous cancellation in $d_{\rm Hg}$ and a consistency with the experimental limits. Illustrative examples of points in the parameter space where cancellations occur and all EDM constraints are satisfied are given in Tables I and II in Appendix D. It needs to be emphasized that while cancellations among the various contributions to the EDMs are generic the suppres-

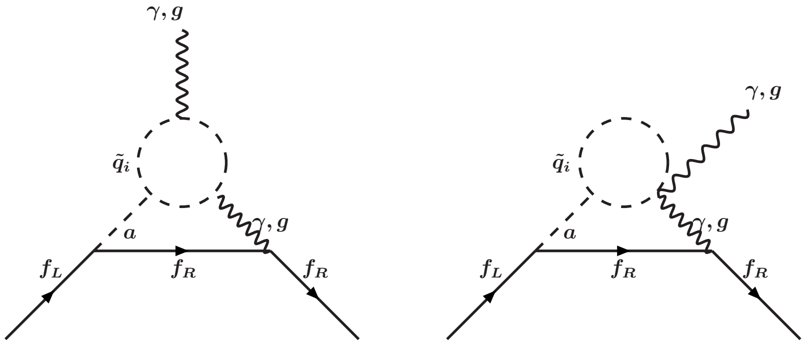

FIG. 5. Two loop Barr-Zee type diagrams that contribute to the EDMs in supersymmetry (Chang *et al.*, 1999).

sion of the EDMs for the electron and neutron do require fine tuning. On the positive side, the above, of course, leads to a narrowing of the parameter space of the theory.

In theories where the Higgs mixing parameter μ obeys the simple scaling behavior as the rest of the SUSY masses the EDMs exhibit a simple scaling behavior under the simultaneous scaling on m_0 and $m_{1/2}$. In the scaling region the knowledge of a single point in the MSSM parameter space where the cancellation in the EDMs occurs allows one to generate a trajectory in the m_0-$m_{1/2}$ plane where the cancellation mechanism holds and the EDMs are small. Thus under the transformation $m_0 \to \lambda m_0$, $m_{1/2} \to \lambda m_{1/2}$, μ itself obeys the same scaling, i.e., $\mu \to \lambda\mu$ in the large μ region. In this case d_e exhibits the scaling behavior

$$d_e \to \lambda^{-2} d_e. \tag{101}$$

The same scaling relation holds for the electric and chromoelectric operators of quarks,

$$d_q^E \to \lambda^{-2} d_q^E, \quad d_q^C \to \lambda^{-2} d_q^C. \tag{102}$$

For the gluonic dimension 6 operator we find the following scaling:

$$d_q^G \to \lambda^{-4} d_q^G. \tag{103}$$

Thus the scaling property of d_q will be more complicated. However, as λ increases the contribution of d_q^G will fall off faster than d_q^E and d_q^C and in this case one will have the scaling $d_q \to \lambda^{-2} d_q$ and so $d_n \to \lambda^{-2} d_n$. Thus scaling property of EDMs allows one to promote a single point in the SUSY parameter space where cancellation occurs to a trajectory in the parameter space. With the scaling property one can arrange the cancellation mechanism to work for the EDMs over a much larger region of the parameter space (Ibrahim and Nath, 2000d) than would otherwise be possible (Pospelov and Ritz, 2005). The scaling phenomenon also has implications for the satisfaction of the EDM constraints in string and *D*-brane models (Ibrahim and Nath, 2000d). As already stated in general only certain phase combinations appear in the analysis of a given physical quantity. Some examples of such combinations are given in Table III in Appendix E. For other solutions to the SUSY *CP* problem see Dimopoulos and Thomas (1996); Nir and Rattazzi (1996); Babu *et al.* (2000b); Abel *et al.* (2001).

E. Two loop contribution to EDMs

Two loop contributions to the EDMs can be quite significant. The analysis of Barr and Zee (1990) and Gunion and Wyler (1990) showed that significant contributions to the EDM of the electron and neutron can result if the Higgs boson exchange mediates *CP* violation. An analysis in the same spirit has been given by Chang *et al.* (1999) for the MSSM case. Here the *CP* phases arising from the Higgs boson couplings to the stop and the sbottom enter and these are not stringently constrained by data. Thus *CP* phases in the third generation could be quite substantial consistent with the EDM constraints. We discuss now the two loop analysis in further detail. We assume that the large CP phases arise only in the third generation trilinear soft parameters $A_{\tau,t,b}$ and the relevant two loop interactions arise via the *CP*-odd Higgs $a(x)$ (see Fig. 5) whose interactions with fermions and sfermions are given by

$$\mathcal{L}_a = \frac{g m_f}{2M_W} R_f i a \bar{f} \gamma_5 f + v \xi_f a(-\tilde{f}_1^* \tilde{f}_1 + \tilde{f}_2^* \tilde{f}_2), \tag{104}$$

where g is related to the W boson mass by $M_W = gv/2$, $R_f = \cot\beta\,(\tan\beta)$ for $T_3^f = \frac{1}{2}\,(-\frac{1}{2})$. The diagrams of Fig. 5 give the following contribution to the EDM of a fermion at the electroweak scale:

$$d_f/e = \frac{3\alpha_{em}}{64\pi^3} \frac{R_f m_f}{m_a^2} \sum_{q=t,b} \xi_q Q_f Q_q^2 [F(x_{1a}) - F(x_{2a})], \tag{105}$$

where $x_{ia} = (m_{\tilde{q}_i}/m_a)^2$ $(i=1,2)$, ξ_q $(q=t,b)$ are defined by

$$\xi_b = \frac{2 m_b \sin 2\theta_b \operatorname{Im}(A_b e^{i\delta_b})}{v^2 \sin 2\beta},$$

$$\xi_t = \frac{m_t \sin 2\theta_t \operatorname{Im}(\mu e^{i\delta_t})}{v^2 \sin^2\beta}, \tag{106}$$

and $\delta_q = \arg(A_q + R_q \mu^*)$. The function $F(x)$ is given by the loop integral

$$F(x) = \int_0^1 dy \frac{y(1-y)}{x - y(1-y)} \ln\left(\frac{y(1-y)}{x}\right). \tag{107}$$

Similarly the contribution to CEDM at the electroweak scale is given by

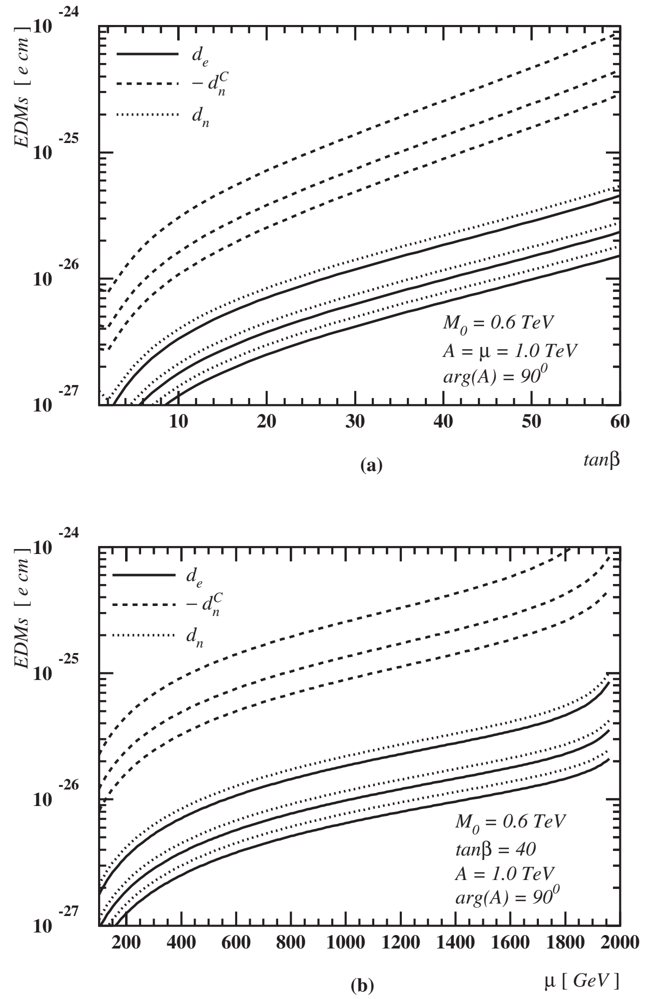

FIG. 6. Size estimate of the two loop contribution to the EDMs in supersymmetry with phases only in the third generation (Chang *et al.*, 1999).

$$d_f^C/e = \frac{\alpha_s}{128\pi^3}\frac{R_f m_f}{m_a^2}\sum_{q=t,b}\xi_q[F(x_{1a}) - F(x_{2a})]. \tag{108}$$

Numerical analysis of the EDM is given in Fig. 6 and indicates that one can satisfy the EDM constraints in certain ranges of the parameter space. However, it remains to be seen how one can naturally suppress phases in the first two generations while allowing them only in the third generation. The reader is also directed to several other works on two loop analyses of EDMs: Chang *et al.* (1990, 1991); Pilaftsis (2002); Degrassi *et al.* (2005); Feng *et al.* (2005, 2006). Specifically, a complete account of all dominant two-loop Barr-Zee type graphs in the *CP* violating MSSM is given in Pilaftsis (2002). The analyses of EDMs given in this section were based on the assumption of R parity conservation. For analyses of EDMs without R parity see Hall and Suzuki (1984); Keum and Kong (2001a, 2001b); Faessler *et al.* (2006).

XI. *CP* EFFECTS AND SUSY PHENOMENA

As noted earlier with the cancellation mechanism the phases can be large, and thus their effects could be visible in many supersymmetric phenomena.[4] Below we discuss several of these phenomena and refer to the literature above for others.

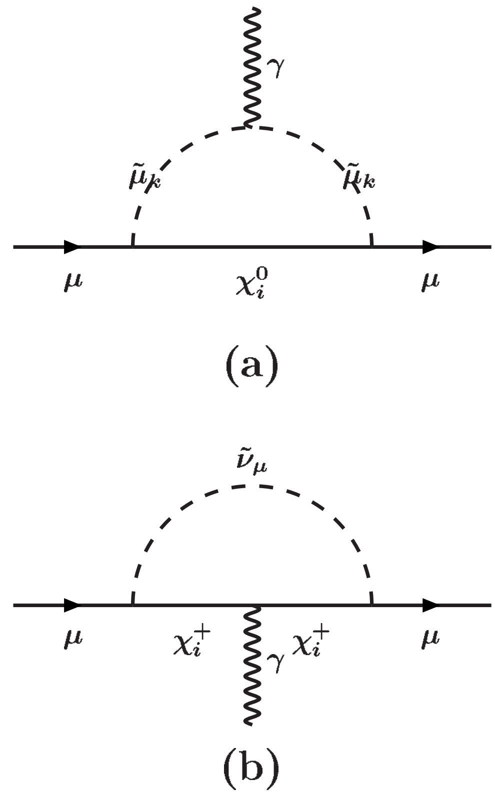

FIG. 7. Chargino and neutralino exchanges contributing to the muon $g-2$ which generate dependence of $g_\mu-2$ on phases.

A. SUSY phases and $g_\mu-2$

The effects of *CP* violating phases on the supersymmetric electroweak contributions to $g_\mu-2$ have been investigated (Ibrahim and Nath, 2000a, 2000c; Ibrahim *et al.*, 2001) (see Fig. 7). The parameter $a_\mu \equiv (g_\mu-2)/2$ is induced by loop corrections to the muon vertex with the photon field. In MSSM the muon interacts with other fermions ψ_i and scalars ϕ_k through

$$\mathcal{L} = L_{ik}\bar{\mu}P_L\psi_i\phi_k + R_{ik}\bar{\mu}P_R\psi_i\phi_k + \text{H.c.}, \tag{109}$$

where ψ_i stands for the neutralino (chargino) and ϕ_k stands for the smuon (scalar neutrino). The one loop contribution to a_μ is given by

[4]See Choi and Drees, 1998; Aoki *et al.*, 1999; Asatrian and Asatrian, 1999; Baek and Ko, 1999; Goto *et al.*, 1999; Ma *et al.*, 1999; Barr and Khalil, 2000; Choi, Guchait, *et al.*, 2000; Choi and Lee, 2000; Choi, Song, and Song, 2000; Dedes and Moretti, 2000a, 2000b; Huang and Liao, 2000a, 2000b, 2002; Kneur and Moultaka, 2000; Kribs, 2000; Mrenna *et al.*, 2000; Okada *et al.*, 2000.

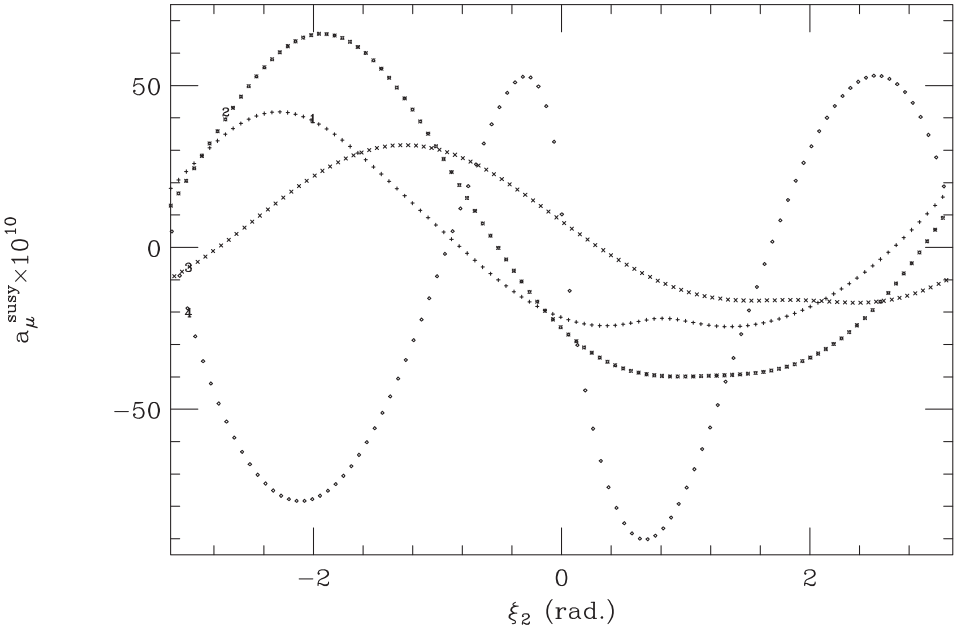

FIG. 8. The dependence of a_μ on a SUSY *CP* phase. The curves correspond to the four following cases (Ibrahim and Nath, 2000c): (1) $m_0=70$, $m_{1/2}=99$, $\tan\beta=3$, $|A_0|=5.6$, $\xi_1=-1$, $\xi_3=0.62$; $\theta_\mu=2.35$, $\alpha_{A_0}=0.4$; (2) $m_0=80$, $m_{1/2}=99$, $\tan\beta=5$, $|A_0|=5.5$, $\xi_1=-0.8$, $\xi_3=0.95$; $\theta_\mu=1.98$, $\alpha_{A_0}=0.4$; (3) $m_0=75$, $m_{1/2}=132$, $\tan\beta=4$, $|A_0|=6.6$, $\xi_1=-1$, $\xi_3=2.74$; $\theta_\mu=1.2$, $\alpha_{A_0}=-1.5$; (4) $m_0=70$, $m_{1/2}=99$, $\tan\beta=6$, $|A_0|=3.2$, $\xi_1=0.63$, $\xi_3=0.47$, $\theta_\mu=2.7$, $\alpha_{A_0}=-0.4$, where all masses are in GeV units and all phases are in rad.

$$a_\mu = a_\mu^1 + a_\mu^2. \tag{110}$$

Here a_μ^1 comes from the neutralino exchange contribution and a_μ^2 comes from the chargino exchange contribution so that

$$a_\mu^1 = \frac{m_\mu}{8\pi^2 m_i}\,\mathrm{Re}(L_{ik}R_{ik}^*) I_1\left(\frac{m_\mu^2}{m_i^2},\frac{m_k^2}{m_i^2}\right) + \frac{m_\mu^2}{16\pi^2 m_i^2}(|L_{ik}|^2+|R_{ik}|^2) I_2\left(\frac{m_\mu^2}{m_i^2},\frac{m_k^2}{m_i^2}\right) \tag{111}$$

and

$$a_\mu^2 = \frac{m_\mu}{8\pi^2 m_i}\,\mathrm{Re}(L_{ik}R_{ik}^*) I_3\left(\frac{m_\mu^2}{m_i^2},\frac{m_k^2}{m_i^2}\right) - \frac{m_\mu^2}{16\pi^2 m_i^2}(|L_{ik}|^2+|R_{ik}|^2) I_4\left(\frac{m_\mu^2}{m_i^2},\frac{m_k^2}{m_i^2}\right). \tag{112}$$

Here

$$I_1(\alpha,\beta) = -\int_0^1 dx \int_0^{1-x} dz \frac{z}{\alpha z^2 + (1-\alpha-\beta)z+\beta},$$

$$I_2(\alpha,\beta) = \int_0^1 dx \int_0^{1-x} dz \frac{z^2-z}{\alpha z^2 + (1-\alpha-\beta)z+\beta},$$

$$I_3(\alpha,\beta) = \int_0^1 dx \int_0^{1-x} dz \frac{1-z}{\alpha z^2 + (\beta-\alpha-1)z+1},$$

$$I_4(\alpha,\beta) = \int_0^1 dx \int_0^{1-x} dz \frac{z^2-z}{\alpha z^2 + (\beta-\alpha-1)z+1}. \tag{113}$$

In the supersymmetric limit the soft breaking terms vanish and a_μ should vanish as well (Ferrara and Remiddi, 1974; Barbieri and Giudice, 1993). A careful limit of Eqs. (111) and (112) shows that in the supersymmetric limit the sum of the W exchange contribution, in the standard model part, and of the chargino exchange contributions, in the supersymmetric counterpart, cancel. Thus

$$a_\mu^W + a_\mu^{\chi^+} = 0. \tag{114}$$

Similarly one can show that the Z boson exchange and contribution from the massive modes of the neutralino sector in the supersymmetric limit cancel,

$$a_\mu^Z + a_\mu^{\chi^0}(\text{massive}) = 0. \tag{115}$$

One can show that the massless part of the neutralino spectrum in the supersymmetric limit gives the value of $-\alpha_{\rm em}/2\pi$. Thus it gives the same magnitude but is opposite in sign to the famous photon exchange result.

The *CP* dependence of a_μ arises from the effect of the phases on the sparticle masses, and on their effects on L_{ik} and R_{ik} and significant variations can arise in a_μ as the phases are varied. This phenomenon is given in Fig. 8. Because of the significant dependence of a_μ on the phases it is possible to constrain the *CP* phases using the current data on a_μ; as done by Ibrahim *et al.* (2001). Further details on the analysis of this section are given in Appendix F.

B. SUSY *CP* phases and *CP* even–*CP* odd mixing in the neutral Higgs boson sector

Another important effect of *CP* violating phases is their role in determining the spectrum and *CP* properties of the neutral Higgs fields arising due to mixings of the *CP* even–*CP* odd Higgs (Pilaftsis, 1998a, 1998b; Pilaftsis and Wagner, 1999).

Such mixings between *CP* even and *CP* odd Higgs bosons cannot occur at the tree level, but are possible when loop corrections to the effective potential are included. To calculate such mixings we use the one loop effective potential as given by Eq. (28). We assume that the SU(2) Higgs doublets $H_{1,2}$ have nonvanishing vacuum expectation v_1 and v_2 so that we can write

$$(H_1) = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 + \phi_1 + i\psi_1 \\ H_1^- \end{pmatrix},$$

$$(H_2) = \frac{e^{i\theta_H}}{\sqrt{2}}\begin{pmatrix} H_2^+ \\ v_2 + \phi_2 + i\psi_2 \end{pmatrix}. \tag{116}$$

For the present case with the inclusion of CP violating effects, variations with respect to the fields $\phi_1, \phi_2, \psi_1, \psi_2$ give the following:

$$-\frac{1}{v_1}\left(\frac{\partial \Delta V}{\partial \phi_1}\right)_0 = m_1^2 + \frac{g_2^2 + g_1^2}{8}(v_1^2 - v_2^2) + m_3^2 \tan\beta \cos\theta_H, \tag{117}$$

$$-\frac{1}{v_2}\left(\frac{\partial \Delta V}{\partial \phi_2}\right)_0 = m_2^2 - \frac{g_2^2 + g_1^2}{8}(v_1^2 - v_2^2) + m_3^2 \cot\beta \cos\theta_H, \tag{118}$$

$$\frac{1}{v_1}\left(\frac{\partial \Delta V}{\partial \psi_2}\right)_0 = m_3^2 \sin\theta_H = \frac{1}{v_2}\left(\frac{\partial \Delta V}{\partial \psi_1}\right)_0, \tag{119}$$

where the subscript 0 means that the quantities are evaluated at the point $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$. As noted by Demir (1999) only one of the two equations in Eq. (119) is independent.

One can have sizable contributions to the potential corrections from top-stop, bottom-sbottom (Pilaftsis and Wagner, 1999; Choi, Drees, and Lee, 2000; Demir *et al.*, 2000b; Ibrahim and Nath, 2001a), $W-H^+-\chi^+$ sector (Ibrahim and Nath, 2001a), and from the $\chi^0-Z-h^0-H^0$ sector (Ibrahim and Nath, 2002; Ham *et al.*, 2003). The mass-squared matrix of the neutral Higgs bosons is defined by

$$M_{ab}^2 = \left(\frac{\partial^2 V}{\partial \Phi_a \Phi_b}\right)_0, \tag{120}$$

where $\Phi_a = (a = 1-4)$ are given by

$$\{\Phi_a\} = \{\phi_1, \phi_2, \psi_1, \psi_2\}, \tag{121}$$

and the subscript 0 means that we set $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$. The dominant contributions come from the stop, sbottom, and chargino. With the inclusion of the stop, sbottom, and chargino contributions one finds that θ_H is determined by

$$m_3^2 \sin\theta_H = \frac{1}{2}\beta_{h_t}|\mu||A_t|\sin\gamma_t f_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{1}{2}\beta_{h_b}|\mu||A_b|\sin\gamma_b f_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) - \frac{g_2^2}{16\pi^2}|\mu||\tilde{m}_2|\sin\gamma_2 f_1(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2), \tag{122}$$

where

$$\beta_{h_t} = \frac{3h_t^2}{16\pi^2}, \quad \beta_{h_b} = \frac{3h_b^2}{16\pi^2},$$

$$\gamma_t = \alpha_{A_t} + \theta_\mu, \quad \gamma_b = \alpha_{A_b} + \theta_\mu, \quad \gamma_2 = \xi_2 + \theta_\mu, \tag{123}$$

and $f_1(x,y)$ is defined by

$$f_1(x,y) = -2 + \ln\frac{xy}{Q^4} + \frac{y+x}{y-x}\ln\frac{y}{x}. \tag{124}$$

The inclusion of the stau and neutralino sectors in the analysis would contribute extra terms to Eq. (122) that are dependent on the phase $\gamma_\tau = \alpha_{A_\tau} + \theta_\mu$ and $\gamma_1 = \xi_1 + \theta_\mu$. The tree and loop contributions to M_{ab}^2 are given by

$$M_{ab}^2 = M_{ab}^{2(0)} + \Delta M_{ab}^2, \tag{125}$$

where $M_{ab}^{2(0)}$ are the contributions at the tree level and ΔM_{ab}^2 are the loop contributions with

$$\Delta M_{ab}^2 = \frac{1}{32\pi^2}\mathrm{Str}\left(\frac{\partial M^2}{\partial \Phi_a}\frac{\partial M^2}{\partial \Phi_b}\ln\frac{M^2}{Q^2} + M^2\frac{\partial^2 M^2}{\partial \Phi_a \partial \Phi_b}\ln\frac{M^2}{eQ^2}\right)_0, \tag{126}$$

and $e = 2.718$. Computation of the 4×4 Higgs mass^2 matrix in the basis of Eq. (121) gives

$$\begin{pmatrix} M_{11} + \Delta_{11} & -M_{12} + \Delta_{12} & \Delta_{13}s_\beta & \Delta_{13}C_\beta \\ -M_{12} + \Delta_{12} & M_{22} + \Delta_{22} & \Delta_{23}s_\beta & \Delta_{23}c_\beta \\ \Delta_{13}s_\beta & \Delta_{23}s_\beta & M_{33}s_\beta^2 & M_{33}s_\beta c_\beta \\ \Delta_{13}c_\beta & \Delta_{23}c_\beta & M_{33}s_\beta c_\beta & M_{33}c_\beta^2 \end{pmatrix}, \tag{127}$$

where $M_{11} = M_Z^2 c_\beta^2 + M_A^2 s_\beta^2$, $M_{12} = (M_Z^2 + M_A^2)s_\beta c_\beta$, $M_{22} = M_Z^2 s_\beta^2 + M_A^2 c_\beta^2$, $c_\beta, s_\beta = \cos\beta, \sin\beta$, and $M_{33} = M_A^2 + \Delta_{33}$, and $(c_\beta, s_\beta) = (\cos\beta, \sin\beta)$. Here the explicit Q dependence has been absorbed in m_A^2 which is given by

$$m_A^2 = (\sin\beta\cos\beta)^{-1}\Big(-m_3^2\cos\theta + \frac{1}{2}\beta_{h_t}|A_t||\mu|\cos\gamma_t f_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{1}{2}\beta_{h_b}|A_b||\mu|\cos\gamma_b f_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) + \frac{1}{2}\beta_{h_\tau}|A_\tau||\mu|\cos\gamma_\tau f_1(m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2) + \frac{g_2^2}{16\pi^2}|\tilde{m}_2||\mu|\cos\gamma_2 f_1(m_{\chi_1^+}^2, m_{\chi_2^+}^2)\Big) + \Delta_\chi. \tag{128}$$

Here Δ_χ is the contribution arising from the neutralino exchange and

$$\Delta_\chi = -\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{M_{\chi_j}^2}{D_j}\left[\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right)-1\right]$$
$$\times\{M_{\chi_j}^4(-g_2^2|\mu||\tilde{m}_2|\cos\gamma_2 - g_1^2|\mu||\tilde{m}_1|\cos\gamma_1)$$
$$+ M_{\chi_j}^2[g_2^2(|\tilde{m}_1|^2+|\mu|^2)|\mu||\tilde{m}_2|\cos\gamma_2$$
$$+ g_1^2(|\tilde{m}_2|^2+|\mu|^2)|\mu||\tilde{m}_1|\cos\gamma_1]$$
$$- g_2^2|\tilde{m}_1|^2|\mu|^3|\tilde{m}_2|\cos\gamma_2 - g_1^2|\tilde{m}_2|^2|\mu|^3|\tilde{m}_1|\cos\gamma_2\}, \tag{129}$$

where D_j is a polynomial of the neutralino mass M_{χ_j} and $\Delta\xi=\xi_1-\xi_2$. The first term in the second brace on the right hand side of Eqs. (128) is the tree term, while the second, third, and fourth terms come from the stop, sbottom, stau, and chargino exchange contributions. The remaining contributions in Eq. (128) arise from the neutralino sector. For Δ_{ij} one has

$$\Delta_{ij} = \Delta_{ij\tilde{t}} + \Delta_{ij\tilde{b}} + \Delta_{ij\tilde{\tau}} + \Delta_{ij\chi^+} + \Delta_{ij\chi^0}, \tag{130}$$

where $\Delta_{ij\tilde{t}}$ is the contribution from the stop exchange in the loops, $\Delta_{ij\tilde{b}}$ is the contribution from the sbottom exchange in the loops, $\Delta_{ij\tilde{\tau}}$ is the contribution from the stau loop, $\Delta_{ij\chi^+}$ is the contribution from the chargino sector, and $\Delta_{ij\chi^0}$ is the contribution from the neutralino sector. For illustration $\Delta_{ij\tilde{t}}$ are listed in Appendix G.

We note that the phases come to play a role here through the squark, slepton, chargino, and neutralino eigenvalues of their mass matrices. In the supersymmetric limit $M_{\chi_i^0}=(0,0,M_Z,M_Z)$, $(M_{h^0},M_{H^0})=(M_Z,0)$, $M_{\chi_i^+}=M_{H^+}=M_W$, and $M_{\tilde{q}_i}=m_q$. With this in mind one can see that all radiative corrections to the potential vanish in the supersymmetric limit. By introducing a new basis $\phi_1,\phi_2,\psi_{1D},\psi_{2D}$ where

$$\psi_{1D} = \sin\beta\psi_1 + \cos\beta\psi_2,$$
$$\psi_{2D} = -\cos\beta\psi_1 + \sin\beta\psi_2, \tag{131}$$

one finds that the field ψ_{2D} decouples from the other three fields and is a massless state (a Goldstone field). The Higgs mass^2 matrix $M^2_{\rm Higgs}$ of the remaining three fields is given by

$$\begin{pmatrix} M_{11}+\Delta_{11} & -M_{12}+\Delta_{12} & \Delta_{13} \\ -M_{12}+\Delta_{12} & M_{22}+\Delta_{22} & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & M_A^2+\Delta_{33}\end{pmatrix}. \tag{132}$$

We note that the basis fields $\{\phi_1,\phi_2,\psi_{1D}\}$ of the above matrix are the real parts of the neutral Higgs fields and a linear combination of their imaginary parts ψ_i. Thus these states are pure *CP* states where $\phi_{1,2}$ are *CP* even (scalars) and ψ_{1D} is *CP* odd (a pseudoscalar). What we are interested here is the mixing between the *CP* even and *CP* odd Higgs states in the eigenvectors of the above matrix and this mixing is governed by the off-diagonal elements Δ_{12} and Δ_{23} (see Fig. 9). These are found to be linear combinations of $\sin\gamma_t$, $\sin\gamma_b$, $\sin\gamma_1$, $\sin\gamma_2$, and $\sin\gamma_\tau$ where these phases are defined in Eq. (123). In the limit of vanishing *CP* phases the matrix elements Δ_{12} and Δ_{23} vanish and thus the Higgs mass^2 matrix factors into a 2×2 *CP* even Higgs matrix times a *CP* odd element. The effect of phases on *CP* even–*CP* odd Higgs boson mixings have been studied by Pilaftsis and Wagner (1999); Choi, Drees, and Lee (2000); Demir *et al.* (2000b); Ibrahim and Nath (2001a) and found to be significant. It has been shown that if a mixing effect among the *CP* even and the *CP* odd Higgs bosons is observed experimentally, then it is only the cancellation mechanism of EDMs that can survive (Ibrahim, 2001a). A more accurate determination of the VEV of the Higgs fields would require use of the two loop effective potential. An improved accuracy and scale dependence should be obtained with the full two loop effective potential (Martin, 2003).

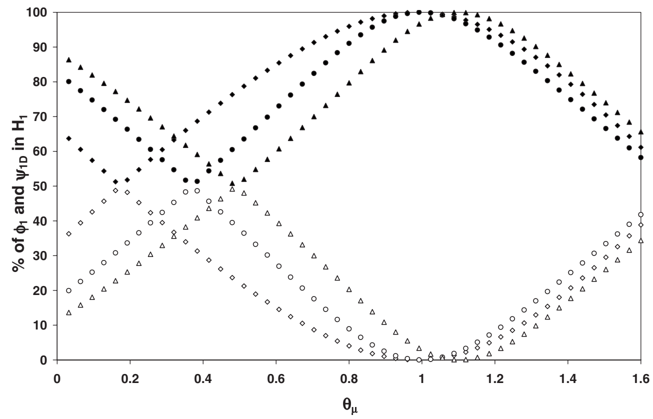

FIG. 9. The phenomenon of *CP* even–*CP* odd Higgs mixing via the SUSY *CP* phases. The *CP* even component ϕ_1 of H_1 (upper curves) and the *CP* odd component ψ_{1D} of H_1 (lower curves) including the stop, sbottom, stau, chargino, and neutralino sector contributions as a function of θ_μ. The common parameters are $m_A=300$, $Q=320$, $m_0=100$, $m_{1/2}=500$, $\xi_2=0.5$, $\alpha_0=0.3$, $|A_0|=1$. For curves with diamonds $\tan\beta=15$, $\xi_1=1.5$, for circles $\tan\beta=8$, $\xi_1=1.5$, and for triangles $\tan\beta=8$, $\xi_1=0.5$ where all masses are in GeV and all angles are in radians.

C. Effect of SUSY *CP* phases on the *b* quark mass

The running *b* quark mass is another object in MSSM where *CP* phases could have an impact. m_b can be written in the form

$$m_b(M_Z) = h_b(M_Z)\frac{v}{\sqrt{2}}\cos\beta(1+\Delta_b), \tag{133}$$

where $h_b(M_Z)$ is the Yukawa coupling for the *b* quark at the scale M_Z and Δ_b is the loop correction to m_b. The SUSY QCD and electroweak corrections are large in the large $\tan\beta$ region (Carena *et al.*, 1994; Hall *et al.*, 1994; Pierce *et al.*, 1997). At the tree level the *b* quark couples to the neutral component of the H_1 Higgs boson while the coupling to the H_2 Higgs boson is absent. Loop corrections produce a shift in the H_1^0 couplings and generate a nonvanishing effective coupling with H_2^0. Thus the effective Lagrangian can be written as (Babu *et al.*, 1999; Carena and Haber, 2003)

$$-\mathcal{L}_{\rm eff} = (h_b + \delta h_b)\bar{b}_R b_L H_1^0 + \Delta h_b \bar{b}_R b_L H_2^{0*} + \text{H.c.}, \tag{134}$$

where the star on H_2^0 is necessary in order to have a gauge invariant Lagrangian. The quantities δh_b and Δh_b receive SUSY QCD and SUSY electroweak contributions. The QCD contribution arises from corrections where gluinos and sbottoms are running in the loops. In the electrowak contributions, the sbottoms (stops) and neutralinos (charginos) are running in the loops. The basic integral that appears in the expressions of δh_b and Δh_b involving heavy scalars $\tilde{S}_1$, $\tilde{S}_2$ and a heavy fermion $\tilde{f}$ is,

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{m_{\tilde{f}} + \gamma_\mu k^\mu}{(k^2 - m_{\tilde{f}}^2)(k^2 - m_{\tilde{S}_1}^2)(k^2 - m_{\tilde{S}_2}^2)}. \tag{135}$$

In the approximation of the zero external momentum this integral could be written in the closed form,

$$I = \frac{m_{\tilde{f}}}{(4\pi)^2} f(m_{\tilde{f}}^2, m_{\tilde{S}_1}^2, m_{\tilde{S}_2}^2), \tag{136}$$

where the function $f(m^2, m_i^2, m_j^2)$ is given by

$$f(m^2,m_i^2,m_j^2) = [(m^2 - m_i^2)(m^2 - m_j^2)(m_j^2 - m_i^2)]^{-1} \times \left(m_j^2 m^2 \ln \frac{m_j^2}{m^2} + m^2 m_i^2 \ln \frac{m^2}{m_i^2} + m_i^2 m_i^2 \ln \frac{m_i^2}{m_j^2} \right) \tag{137}$$

for the case $i \neq j$ and

$$f(m^2,m_i^2,m_j^2) = \frac{1}{(m_i^2 - m^2)^2} \left(m^2 \ln \frac{m_i^2}{m^2} + (m^2 - m_i^2) \right)$$

for the case $i=j$. In the SUSY QCD the heavy fermion is the gluino and the heavy scalars are the sbottoms. In the chargino contribution, the chargino is the heavy fermion and the heavy scalars are the stops. In the neutralino part, the neutralino is the heavy fermion and the heavy scalars are the sbottoms.

The couplings δh_b and Δh_b are generally complex due to *CP* phases in the soft SUSY breaking terms. Electroweak symmetry is broken spontaneously by giving expectation values to H_1^0 and H_2^0. Thus one finds for the mass term

$$-\mathcal{L}_m = M_b \bar{b}_R b_L + \text{H.c.}, \tag{138}$$

where

$$M_b = \frac{h_b v \cos\beta}{\sqrt{2}} \left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b}{h_b} \tan\beta \right). \tag{139}$$

Here M_b is complex. By rotating the b quark field

$$b = e^{i/2\gamma_5\chi_b} b', \quad \tan\chi_b = \frac{\text{Im}\, M_b}{\text{Re}\, M_b} \tag{140}$$

one gets

$$-\mathcal{L}_m = m_b \bar{b}'_R b'_L + \text{H.c.}, \tag{141}$$

where m_b is real and positive and b' is the physical field,

$$m_b = \frac{h_b v \cos\beta}{\sqrt{2}} [(1 + \delta_R)^2 + \delta_I^2]^{1/2},$$

$$\delta_R = \text{Re}\left(\frac{\delta h_b}{h_b} \right) + \text{Re}\left(\frac{\Delta h_b}{h_b} \right) \tan\beta,$$

$$\delta_I = \text{Im}\left(\frac{\delta h_b}{h_b} \right) + \text{Im}\left(\frac{\Delta h_b}{h_b} \right) \tan\beta. \tag{142}$$

Thus one finds for the mass correction

$$\Delta_b \approx \text{Re}\, \frac{\Delta h_b}{h_b} \tan\beta + \text{Re}\, \frac{\delta h_b}{h_b}. \tag{143}$$

The SUSY *CP* violating phases in the SUSY QCD corrections are ξ_3, α_{A_b}, and θ_μ. These come from the vertices of $b\tilde{b}\tilde{g}$ and $\tilde{b}\tilde{b}H$. In the chargino part one finds the phases ξ_2, α_{A_t}, and θ_μ. In the case of neutralino we have ξ_2, ξ_1, α_{A_b}, and θ_μ. The corrections of the b quark mass are found to be dependent on θ_μ, ξ_3, and α_{A_0} as the values of these phases affect both the sign and magnitude of the correction. Thus the correction can vary from zero to as much as 30% in some regions of the parameter space and can also change its sign depending on the value of these phases. The effect of ξ_2 is less important and ξ_1 is found to be the least important phase (Ibrahim and Nath, 2003c). Similar results hold for the τ lepton and top quark masses. For the τ lepton the numerical size of the correction is as much as 5% and for the top quark is typically less than a percent.

D. SUSY *CP* phases and the decays $h \to b\bar{b}$, $h \to \tau\bar{\tau}$

As mentioned above, the spectrum of the neutral Higgs sector and its *CP* properties are sensitive to the *CP* violating phases through radiative corrections. The couplings of the quarks with the Higgs are also found to be dependent of these phases. Thus one can deduce the corrected effective interaction of the b quark with the lightest Higgs boson H_2 as

$$-\mathcal{L}_{\rm int} = \bar{b}(C_b^S + i\gamma_5 C_b^P) b H_2, \tag{144}$$

where

$$\begin{pmatrix} C_b^S \\ C_b^P \end{pmatrix} = \begin{pmatrix} \cos\chi_b & -\sin\chi_b \\ \sin\chi_b & \cos\chi_b \end{pmatrix} \begin{pmatrix} C_b^1 \\ C_b^2 \end{pmatrix}, \tag{145}$$

and

$$C_b^1 = \frac{1}{\sqrt{2}} [\text{Re}(h_b + \delta h_b) R_{21} + \{ -\text{Im}(h_b + \delta h_b) \sin\beta + \text{Im}(\Delta h_b) \cos\beta \} R_{23} + \text{Re}(\Delta h_b) R_{22}],$$

$$C_b^2 = -\frac{1}{\sqrt{2}}[-\mathrm{Im}(h_b+\delta h_b)R_{21} + \{-\mathrm{Re}(h_b+\delta h_b)\sin\beta + \mathrm{Re}(\Delta h_b)\cos\beta\}R_{23} - \mathrm{Im}(\Delta h_b)R_{22}].$$

The matrix R is the diagonalizing matrix of the Higgs mass^2 matrix,

$$RM^2_{\mathrm{Higgs}}R^T = \mathrm{diag}(m^2_{H_1}, m^2_{H_2}, m^2_{H_3}), \tag{146}$$

where we use the convention that in the limit of vanishing *CP* phases one has $H_1 \to H$, $H_2 \to h$, and $H_3 \to A$. These elements R_{ij} and the corrections δh_b and Δh_b are found to be sensitive functions of the *CP* violating phases and their values are all determined by SUSY radiative corrections of MSSM potential. The quantity $R_{b/\tau}$, defined as

$$R_{b/\tau} = \frac{\mathrm{BR}(h\to\bar{b}b)}{\mathrm{BR}(h\to\bar{\tau}\tau)}, \tag{147}$$

is found to be an important tool to discover supersymmetry. In the standard model, it is given by

$$R^{\mathrm{SM}}_{b/\tau} = 3\left(\frac{m_b^2}{m_\tau^2}\right)\left(\frac{m_h^2-4m_b^2}{m_h^2-4m_\tau^2}\right)^{3/2}(1+w), \tag{148}$$

where $1+w$ is the QCD enhancement factor (Gorishnii *et al.*, 1990),

$$1+w = 1+5.67\frac{\alpha_s}{\pi}+29.14\frac{\alpha_s^2}{\pi^2}. \tag{149}$$

By identifying m_h with m_{H_2}, the lightest Higgs boson in MSSM, we find a shift in $R_{b/\tau}$ value due to the supersymmetric effect including the effects due to *CP* phases as follows:

$$\Delta R_{b/\tau} = \frac{R_{b/\tau}-R^{\mathrm{SM}}_{b/\tau}}{R_{b/\tau}}. \tag{150}$$

The quantity $R_{b/\tau}$ in MSSM depends on the *CP* phase via C_b^S and C_b^P. Thus if a neutral Higgs is discovered and $R_{b/\tau}$ is measured and found to be different from what one expects in the standard model, then it would point to a nonstandard Higgs boson such as from MSSM (Babu and Kolda, 1999). The analysis of Ibrahim and Nath (2003b) showed that supersymmetric effects with *CP* phases can change the branching ratios by as much as 100% for the lightest Higgs boson decay into $\bar{b}b$ and $\bar{\tau}\tau$. Similar results are reported for the other heavier Higgs bosons. Thus the deviation from the standard model result for $R_{b/\tau}$ depends on the *CP* phases and it can be used as a possible signature for supersymmetry and *CP* effects. Similar analyses can also be given for the decay of the heavy Higgs, e.g., for $H^0 \to t\bar{t}$, $b\bar{b}$ and to $\chi^+\chi^-$ (Eberl *et al.*, 2004; Ibrahim, 2007) if allowed kinematically.

E. SUSY *CP* phases and charged Higgs decays $H^- \to \bar{t}b$, $H^- \to \bar{\nu}_\tau \tau$

In the neutral Higgs sector, the ratio $R^{h_0} = \mathrm{BR}(h^0 \to b\bar{b})/\mathrm{BR}(h^0 \to \tau\bar{\tau})$ is found to be sensitive to the supersymmetric loop corrections and to the *CP* phases. In an analogous fashion we may define the ratio $R^{H^-} = \mathrm{BR}(H^- \to b\bar{t})/\mathrm{BR}(H^- \to \tau\bar{\nu}_\tau)$ and it is also affected by SUSY loop corrections, and is sensitive to *CP* phases. Thus the tree level couplings of the third generation quarks to the Higgs bosons,

$$-\mathcal{L} = \epsilon_{ij}h_b\bar{b}_R H_1^i Q_L^j - \epsilon_{ij}h_t\bar{t}_R H_2^i Q_L^j + \mathrm{H.c.}, \tag{151}$$

receive SUSY QCD and the SUSY electroweak loop corrections which produce shifts in couplings similar to the case for neutral Higgs bosons. Thus the general effective interaction may be written as

$$-\mathcal{L}_{\mathrm{eff}} = \epsilon_{ij}(h_b+\delta h_b^i)\bar{b}_R H_1^i Q_L^j + \Delta h_b^i \bar{b}_R H_2^{i*} Q_L^i - \epsilon_{ij}(h_t+\delta h_t^i)\bar{t}_R H_2^i Q_L^j + \Delta h_t^i \bar{t}_R H_1^{i*} Q_L^i + \mathrm{H.c.} \tag{152}$$

We note that in the approximation

$$\delta h_f^1 = \delta h_f^2, \Delta h_f^1 = \Delta h_f^2 \tag{153}$$

one finds that the above Lagrangian preserves weak isospin. This is the approximation that is often used in the literature (Carena and Haber, 2003). However, in general, the above approximation will not hold and there will be violations of weak isospin. In the neutral Higgs interaction with the quarks and leptons of third generation, we examined $\delta h^1_{b,\tau}$, $\Delta h^2_{b,\tau}$, δh_t^2, and Δh_t^1. In the charged Higgs interaction with these particles we should similarly examine $\delta h^2_{b,\tau}$, $\Delta h^1_{b,\tau}$, δh_t^1, and Δh_t^2. The latter corrections have SUSY QCD contributions when gluinos, stops, and sbottoms are running in the loops and SUSY electroweak contributions when neutralinos and/or charginos, stops and/or sbottoms are running in the loops. The *CP* violating phases that enter δh_b^{2g} and Δh_b^{1g} are ξ_3, α_{A_t}, α_{A_b}, and θ_μ. The phases that appear in $\delta h^{2E}_{b,\tau}$ and $\Delta h^{1E}_{b,\tau}$ are ξ_1, ξ_2, α_{A_t}, α_{A_b}, α_{A_τ}, and θ_μ. The phases that enter the corrections δh_t^1 and Δh_t^2 are the same as in δh_b^2 and Δh_b^1. One can measure the size of the violation of weak isospin by defining r_b,

$$r_b = (|\Delta h_b^1|^2 + |\delta h_b^2|^2)^{1/2}(|\Delta h_b^2|^2 + |\delta h_b^1|^2)^{-1/2}. \tag{154}$$

Similar ratios could be defined for the top and tau, r_t and r_τ. The deviation of these quantities from unity is an indication of the violation of weak isospin in the Higgs couplings. It is found that such deviations from unity can be as much as 50% or more depending on the region of the parameter space one is in. It is also seen that these measures are sensitive functions of *CP* violating phases (Ibrahim and Nath, 2004). The interactions of the charged Higgs are thus governed by the Lagrangian

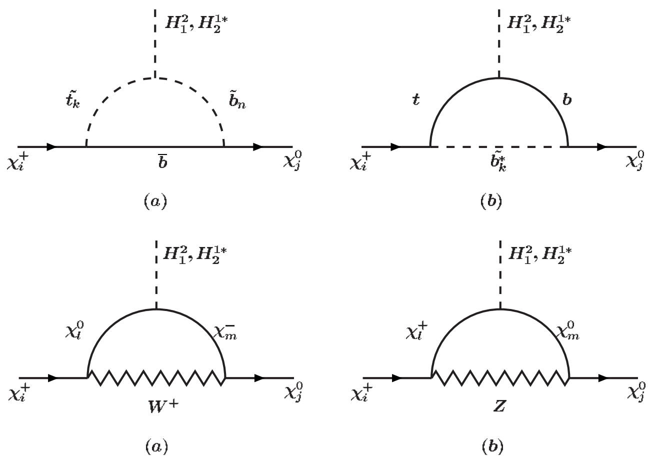

FIG. 10. Loop diagrams with *CP* dependent vertices that contribute to charged Higgs decays into charginos and neutralinos.

$$-\mathcal{L}=\bar{b}(B^s_{bt}+B^p_{bt}\gamma_5)tH^-+\bar{\tau}(B^s_{\nu\tau}+B^p_{\nu\tau}\gamma_5)\nu H^-+\text{H.c.},\tag{155}$$

where

$$\begin{aligned}B^s_{bt}&=-\tfrac{1}{2}(h_b+\delta h_b^2)e^{-i\theta_{bt}}\sin\beta+\tfrac{1}{2}\Delta h_b^1 e^{-i\theta_{bt}}\cos\beta\\&\quad-\tfrac{1}{2}(h_t+\delta h_t^{1*})e^{i\theta_{bt}}\cos\beta+\tfrac{1}{2}\Delta h_t^{2*}e^{i\theta_{bt}}\sin\beta,\\B^p_{bt}&=-\tfrac{1}{2}(h_t+\delta h_t^{1*})e^{i\theta_{bt}}\cos\beta+\tfrac{1}{2}\Delta h_t^{2*}e^{i\theta_{bt}}\sin\beta+\tfrac{1}{2}(h_b\\&\quad+\delta h_b^2)e^{-i\theta_{bt}}\sin\beta-\tfrac{1}{2}\Delta h_b^1 e^{-i\theta_{bt}}\cos\beta,\\B^s_{\nu\tau}&=-\tfrac{1}{2}(h_\tau+\delta h_\tau^2)e^{-i\chi_\tau/2}\sin\beta+\tfrac{1}{2}\Delta h_\tau^1 e^{-i\chi_\tau/2}\cos\beta,\end{aligned}\tag{156}$$

$B^p_{\nu\tau}=-B^s_{\nu\tau}$, and $\theta_{bt}=(\chi_b+\chi_t)/2$. The same holds for χ_τ with b replaced by τ. For $\tan\chi_t$ a similar expression holds with b replaced by t. The loop corrections to the charged Higgs couplings can be quite significant. Also the loop corrections can generate significant violations of the weak isospin in this sector.

F. SUSY *CP* phases and charged Higgs decays $H^\pm\to\chi^\pm\chi^0$

The decay $H^\pm\to\chi^\pm\chi^0$ is sensitive to *CP* violation phases even at the tree level. Inclusion of the loop corrections further enhance the effects of the *CP* phases (see Fig. 10). The tree level Lagrangian for $H^\pm\chi^\mp\chi^0$ is

$$\mathcal{L}=\xi_{ji}H_2^{1*}\bar{\chi}^0{}_jP_L\chi_i^++\xi'_{ji}H_1^2\bar{\chi}^0{}_jP_R\chi_i^++\text{H.c.},\tag{157}$$

where ξ_{ij} and ξ'_{ij} are given by

$$\begin{aligned}\xi_{ji}&=-gX_{4j}V^*_{i1}-\frac{g}{\sqrt{2}}X_{2j}V^*_{i2}-\frac{g}{\sqrt{2}}\tan\theta_W X_{1j}V^*_{i2},\\\xi'_{ji}&=-gX^*_{3j}U_{i1}+\frac{g}{\sqrt{2}}X^*_{2j}U_{i2}+\frac{g}{\sqrt{2}}\tan\theta_W X^*_{1j}U_{i2}.\end{aligned}\tag{158}$$

The phases that enter the couplings ξ_{ji} and ξ'_{ji} are ξ_1, ξ_2, and θ_μ. The loop corrections produce shifts in the couplings and the effective Lagrangian with loop corrected couplings is given by

$$\begin{aligned}\mathcal{L}_{\text{eff}}&=(\xi_{ji}+\delta\xi_{ji})H_2^{1*}\bar{\chi}^0{}_jP_L\chi_i^++\Delta\xi_{ji}H_1^2\bar{\chi}^0{}_jP_L\chi_i^+\\&\quad+(\xi'_{ji}+\delta\xi'_{ji})H_1^2\bar{\chi}^0{}_jP_R\chi_i^++\Delta\xi'_{ji}H_2^{1*}\bar{\chi}^0{}_jP_R\chi_i^++\text{H.c.}\end{aligned}\tag{159}$$

The phases that enter the corrections $\Delta\xi_{ij}$, $\delta\xi_{ij}$ are ξ_1, ξ_2, α_{A_t}, α_{A_b}, and θ_μ. This dependence arises from the shifts in the vertices of the charginos with top and sbottoms, charginos with bottoms and stops, neutralino with bottom and sbottoms, neutralino with tops and stops, W bosons with charginos and neutralinos, Z bosons with charginos and neutralinos, charged Higgs with neutralinos and charginos, and charged Higgs with stops and sbottoms. All these vertices enter in the loop corrections. Thus $\mathcal{L}_{\text{eff}}$ may be written in terms of the mass eigenstates as follows:

$$\mathcal{L}_{\text{eff}}=H^-\bar{\chi}^0{}_j(\alpha^S_{ji}+\gamma_5\alpha^P_{ji})\chi_j^++\text{H.c.},\tag{160}$$

where

$$\begin{aligned}\alpha^S_{ji}&=\tfrac{1}{2}(\xi'_{ji}+\delta\xi'_{ji})\sin\beta+\tfrac{1}{2}\Delta\xi'_{ji}\cos\beta+\tfrac{1}{2}(\xi_{ji}+\delta\xi_{ji})\cos\beta\\&\quad+\tfrac{1}{2}\Delta\xi_{ji}\sin\beta,\\\alpha^P_{ji}&=\tfrac{1}{2}(\xi'_{ji}+\delta\xi'_{ji})\sin\beta+\tfrac{1}{2}\Delta\xi'_{ji}\cos\beta-\tfrac{1}{2}(\xi_{ji}+\delta\xi_{ji})\cos\beta\\&\quad-\tfrac{1}{2}\Delta\xi_{ji}\sin\beta.\end{aligned}\tag{161}$$

From the above Lagrangian one can write down the decay rate of the charged Higgs into charginos and neutralinos,

$$\begin{aligned}\Gamma_{ji}(H^-\to\chi_j^0\chi_i^-)&=\frac{1}{4\pi M^3_{H^-}}[(m^2_{\chi_j^0}+m^2_{\chi_i^+}-M^2_{H^-})^2\\&\quad-4m^2_{\chi_i^+}m^2_{\chi_j^0}]^{1/2}[0.5(|\alpha^S_{ji}|^2+|\alpha^P_{ji}|^2)(M^2_{H^-}\\&\quad-m^2_{\chi_j^0}-m^2_{\chi_i^+})-0.5(|\alpha^S_{ji}|^2-|\alpha^P_{ji}|^2)\\&\quad\times(2m_{\chi_i^+}m_{\chi_j^0})].\end{aligned}\tag{162}$$

The charged Higgs decays are found to be more sensitive to the phases that enter both at the tree level as well as at the loop level such as θ_μ (Ibrahim *et al.*, 2004) relative to the phases such as α_A which enter only at the loop level.

G. Effect of *CP* phases on neutralino dark matter

If the lightest neutralino is the LSP then with R parity invariance it is a possible candidate for cold dark matter, and in this case the relic density (Falk *et al.*, 1995; Falk and Olive, 1998; Chattopadhyay *et al.*, 1999; Gomez *et al.*, 2005) as well as rates in experiments to detect neutralinos will be affected by the presence of *CP* phases (Chattopadhyay *et al.*, 1999; Falk, Ferstl, and Olive, 1999, 2000). We give a brief discussion of neutralino dark matter and highlight the effects of *CP* on neutralino dark matter analyses. A quantity of interest in experimental measurements is $\Omega_{\rm dm}h_0^2$, where $\Omega_{\rm dm} = \rho_{\rm dm}/\rho_c$, $\rho_{\rm dm}$ is the dark matter density, and ρ_c is the critical matter density needed to close the universe where

$$\rho_c = 3H_0^2/8\pi G_N \sim 1.88 \times h_0^2 \times 10^{-29}\ \text{g m/cm}^3. \quad (163)$$

Here H_0 is the Hubble constant, h_0 is its value in units of 100 km/sec Mpc, and G_N is the Newtonian constant. The current limit from WMAP3 on cold dark matter is (Spergel *et al.*, 2006)

$$\Omega_{\rm cdm}h^2 = 0.1045^{0.0072}_{-0.0095}. \quad (164)$$

In the Big Bang scenario the neutralinos will be produced at the time of the Big Bang and will be in thermal equilibrium with the background until the time of freezeout when they will go out of equilibrium. The procedure for computing the density of the relic neutralinos is well known using the Boltzmann equations. In general the analysis will involve co-annihilations and one will have processes of the type

$$\chi_i^0 + \chi_j^0 \to f\bar{f}, WW, ZZ, WH, \ldots . \quad (165)$$

Additionally co-annihilations with staus, charginos, and other sparticle species can also contribute. Thus the relic density of neutralinos $n = \Sigma_i n_i$ is governed by the Boltzmann equation (Lee and Weinberg, 1977; Gondolo and Gelmini, 1991; Griest and Seckel, 1991)

$$\frac{dn}{dt} = -3Hn - \sum_{ij} \langle \sigma_{ij} v \rangle (n_i n_j - n_i^{\rm eq} n_j^{\rm eq}). \quad (166)$$

Here σ_{ij} is the cross section for annihilation of particle types i, j, and $n_i^{\rm eq}$ is the number density of χ_i^0 in thermal equilibrium. Under the approximation $n_i/n = n_i^{\rm eq}/n^{\rm eq}$ one has the well known result

$$\frac{dn}{dt} = -3nH - \langle \sigma_{\rm eff} \rangle [n^2 - (n^{\rm eq})^2], \quad (167)$$

where $\sigma_{\rm eff} = \Sigma_{i,j}\sigma_{ij}\gamma_i\gamma_j$, and γ_i are the Boltzmann suppression factors $\gamma_i = n_i^{\rm eq}/n^{\rm eq}$. Explicitly one finds that the freezeout temperature is given by

$$x_f = \ln\left[x_f^{-1/2} \langle \sigma_{\rm eff} v \rangle_{x_f} m_1 \sqrt{\frac{45}{8\pi^6 N_f G_N}} \right], \quad (168)$$

where N_f is the number of degrees of freedom at freezeout and G_N is Newton's constant. The relic abundance of neutralinos at current temperatures is then given by

$$\Omega_{\chi^0} h_0^2 = \frac{1.07 \times 10^9\ \text{GeV}^{-1}}{N_f^{1/2} M_{\rm Pl}} \left[\int_{x_f}^{\infty} \langle \sigma_{\rm eff} v \rangle \frac{dx}{x^2} \right]^{-1}. \quad (169)$$

Here $x_f = m_1/T_f$, T_f is the freezeout temperature, $M_{\rm Pl} = 1.2 \times 10^{19}$ GeV, and $\langle \sigma v \rangle$ is the thermal average of σv so that

$$\langle \sigma v \rangle = \int_0^\infty dv v^2 (\sigma v) e^{v^2/4x} \Bigg/ \int_0^\infty dv v^2 e^{v^2/4x}. \quad (170)$$

The diagrams that contribute to $\langle \sigma v \rangle$ include the s channel Z, h, and A^0 poles, and the t and u channel squark and slepton exchanges. The Higgs boson and sparticle masses are affected by the *CP* phases of the soft parameters. Further, the vertices are also affected. Inclusion of the loop corrections to the vertices further enhances the dependence on phases (Gomez *et al.*, 2004c). Specifically the Yukawa couplings of bottom quark and neutral Higgs bosons are found to be sensitive to ξ_3 if one includes SUSY QCD corrections in the analysis. A detailed analysis to study the sensitivity of dark matter to the b quark mass and to the neutral Higgs boson mixings has been given by Gomez *et al.* (2004c). It is found that the relic density is sensitive to the mass of the b quark for large $\tan\beta$ and consequently also to the *CP* phases since the b quark mass is sensitive to the phases. In Fig. 11 we exhibit of the relic density and its sensitivity to phases. In the analysis presented in Fig. 11, the relic density was satisfied due to the annihilation through resonant Higgs poles, and one observes the sensitivity of the relic density to *CP* violating phases. The analysis of the relic density with inclusion of Yukawa unification constraint with inclusion of *CP* phases has been given by Gomez *et al.* (2005). An analysis of relic density in the presence of *CP* phases has also been given by Falk and Olive (1998); Argyrou *et al.* (2004); Nihei and Sasagawa (2004); Belanger, Boudjema, Kraml, *et al.* (2006).

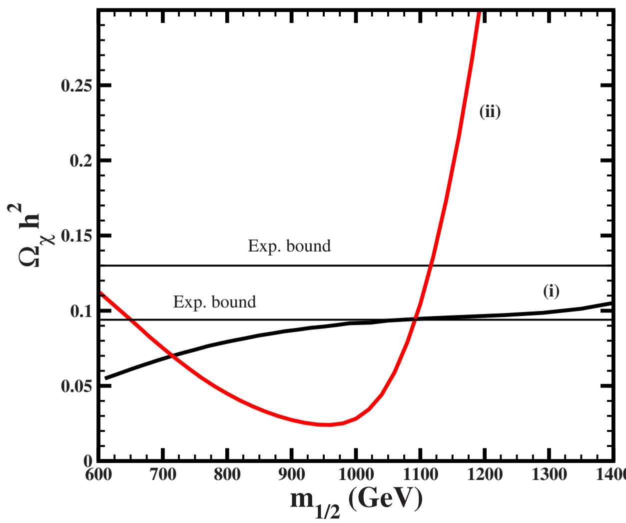

FIG. 11. (Color online) The relic density constrains with large phases from the analysis of Gomez *et al.* (2005). The curve labeled (i) is for the case m_0=1040, $|A_0|$=0, $\tan\beta$=40, θ_{mu}=2.9, α_A=0, ξ_1=1.0, ξ_2=0.15, ξ_3=0.5, while the curve labeled (ii) corresponds to m_0=1080, $|A_0|$=0, $\tan\beta$=40, θ_μ=0.6, α_A=0, ξ_1=0.5, ξ_2=−0.6, ξ_3=1.6. For case (i) EDM constraints are satisfied when $m_{1/2}$=1250 and for case (ii) they are satisfied when $m_{1/2}$=1100. All masses are in units of GeV and all angles in radians.

Typical dark matter experiments involve scattering of neutralinos of the Milky Way that reside in our vicinity with target nuclei. The basic Lagrangian that governs such scattering is the neutralino-quark scattering with neutralino and quarks in the initial and final states. The relative velocity of the LSP hitting the target is small, and so one can approximate the effective interaction governing the neutralino-quark scattering by an effective four-fermi interaction,

$$\mathcal{L}_{\rm eff} = \bar{\chi}\gamma_\mu\gamma_5\chi\bar{q}\gamma^\mu(AP_L + BP_R)q + C\bar{\chi}\chi m_q\bar{q}q + D\bar{\chi}\gamma_5\chi m_q\bar{q}\gamma_5 q + E\bar{\chi}i\gamma_5\chi m_q\bar{q}q + F\bar{\chi}\chi m_q\bar{q}i\gamma_5 q. \tag{171}$$

The deduction of Eq. (171) requires Fierz rearrangement which is discussed in Appendix H and further details are given in Appendix I. The first two terms A, B in Eq. (171) are spin-dependent interaction and arise from the Z boson and the sfermion exchanges. The effect of *CP* violating phases enter via the neutralino eigenvector components and the matrix $D_{\tilde{q}}$ that diagonalizes the squark mass matrix. Then the phases that play a role here are θ_μ, ξ_1, ξ_2, and α_{A_q}. The C term represents the scalar interaction which gives rise to coherent scattering. It receives contributions from the sfermion exchange and from the exchange of the neutral Higgs H_i mass eigenstates. The term D is nonvanishing in the limit when *CP* phases vanish. However, this term is mostly ignored in the literature as its contribution is suppressed because of the small velocity of relic neutralinos. In fact, the contributions of D, E, and F are expected to be relatively small and could be ignored. A significant body of work exists on the analysis of detection rates in the absence of *CP* phases (Nath and Arnowitt, 1995, 1997; Arnowitt and Nath, 1996), but much less so with inclusion of *CP* phases. Inclusion of the *CP* phases shows a significant effect of *CP* phases on the detection rates (Chattopadhyay *et al.*, 1999; Falk *et al.*, 2000 2000). The *CP* effects can be significant even with inclusion of the EDM constraints (Gomez *et al.*, 2004a; Nihei and Sasagawa, 2004).

H. Effect of *CP* phases on proton stability

CP violating phases can affect the nucleon stability in supersymmetric grand unified models with baryon and lepton number violating dimension 5 operators [Sakai and Yanagida (1982); Weinberg (1982); for a recent review see Nath and Perez (2007)]. Thus in a wide class of unified models including grand unified models, string, and brane models, baryon and lepton number violation arises via dimension $LLLL$ and $RRRR$ chiral operators of the form

$$\mathcal{L}_{5L} = \frac{1}{M}\epsilon_{abc}(Pf_1^u V)_{ij}(f_2^d)_{kl}(\tilde{u}_{Lbi}\tilde{d}_{Lcj})[\bar{e}^c_{Lk}(Vu_L)_{al} - \bar{\nu}^c_k d_{Lal}] + \cdots + \text{H.c.}, \tag{172}$$

$$\mathcal{L}_{5R} = -\frac{1}{M}\epsilon_{abc}(V^\dagger f^u)_{ij}(PVf^d)_{kl}(\bar{e}^c_{Ri}u_{Raj}\tilde{u}_{Rck}\tilde{d}_{Rbl}) + \cdots + \text{H.c.} \tag{173}$$

Here $\mathcal{L}_{5L}$ is the $LLLL$ and $\mathcal{L}_{5R}$ is the $RRRR$ lepton and baryon number violating dimension 5 operators, V is the CKM matrix, and f_i are related to quark masses, and P_i appearing in Eqs. (172) and (173) are the generational phases given by $P_i=(e^{i\gamma_i})$ with the constraint $\Sigma_i\gamma_i=0$ ($i=1,2,3$).

Using the above relations one generates the baryon and the lepton number violating dimension 6 operators by dressing the dimension 5 operators by the chargino, gluino, and neutralino exchanges. The dressing loops contain the *CP* phases both via the sparticle spectrum as well as via the vertices. This can be explicitly seen by elimination of the sfermion fields above via the relations

$$\tilde{u}_{iL} = 2\int[\Delta^L_{ui}L_{ui} + \Delta^{LR}_i R_{ui}],$$

$$\tilde{u}_{iR} = 2\int[\Delta^R_{ui}R_{ui} + \Delta^{RL}_i L_{ui}], \tag{174}$$

where $L_{ui}=\delta L_I/\delta\tilde{u}^\dagger_{iL}$, $R_{ui}=\delta L_I/\delta\tilde{u}^\dagger_{iR}$. Here L_I is the sum of fermion-sfermion-gluino, fermion-sfermion-chargino, and fermion-sfermion-neutralino and Δ's are the propagators. A detailed analysis of the specific mode $p\to\bar{\nu}K^+$ which is typically the dominant mode in supersymmetric decay modes of the proton is then given by the following with inclusion of *CP* phases:

$$\begin{aligned}\Gamma(p\to\bar{\nu}_iK^+) &= \frac{\beta_p^2 m_N}{M^2_{H_3}32\pi f^2_\pi}\left(1-\frac{m_K^2}{m_N^2}\right)^2|\mathcal{A}_{\nu_iK}|^2A_L^2(A_S^L)^2 \\ &\times\left|\left(1+\frac{m_N(D+3F)}{3m_B}\right)\left(1+\mathcal{Y}_i^{tk}+(e^{-i\xi_3}\mathcal{Y}_{\tilde{g}}+\mathcal{Y}_{\tilde{Z}})\delta_{i2}\right.\right. \\ &\left.+\frac{A_S^R}{A_S^L}\mathcal{Y}_1^R\delta_{i3}\right)+\frac{2}{3}\frac{m_N}{m_B}D\left(1+\mathcal{Y}_3^{tk}-(e^{-i\xi_3}\mathcal{Y}_{\tilde{g}}-\mathcal{Y}_{\tilde{Z}})\delta_{i2}\right. \\ &\left.\left.+\frac{A_S^R}{A_S^L}\mathcal{Y}_2^R\delta_{i3}\right)\right|^2,\end{aligned} \tag{175}$$

where

$$\mathcal{A}_{\nu_iK} = (\sin 2\beta M_W^2)^{-1}\alpha_2^2P_2m_cm_i^dV^\dagger_{i1}V_{21}V_{22} \times[\mathcal{F}(\tilde{c};\tilde{d}_i;\tilde{W})+\mathcal{F}(\tilde{c};\tilde{e}_i;\tilde{W})]. \tag{176}$$

In the above, A_L (A_S) are the long (short) suppression factors, D, F, f_π are the effective Lagrangian parameters, and β_p is defined by $\beta_pU_L^\gamma=\epsilon_{abc}\epsilon_{\alpha\beta}\langle 0|d^\alpha_{aL}u^\beta_{bL}u^\gamma_{cL}|p\rangle$, where U_L^γ is the proton wave function. Theoretical determinations of β_p lie in the range 0.003–0.03 GeV^3. Perhaps the more reliable estimate is from lattice gauge calculations which gives (Tsutsui *et al.*, 2004) $|\beta_p| = 0.0096(09)(^{+6}_{-20})$ GeV^3.

CP violating phases of the soft SUSY breaking sector enter in the proton decay amplitude. The *CP* phases enter the dressings in two ways, via the mass matrices of the charginos, the neutralinos and the sfermions, and via the interaction vertices. Taking account of this additional complexity, the analysis for computing the proton decay amplitudes follows the usual procedure. This effect is exhibited by considering R_τ,

$$R_\tau = \frac{\tau(p \to \bar{\nu} + K^+)}{\tau_0(p \to \bar{\nu} + K^+)}, \tag{177}$$

where $\tau(p \to \bar{\nu} + K^+)$ is the proton lifetime with *CP* violating phases and $\tau_0(p \to \bar{\nu} + K^+)$ is the lifetime without *CP* phases. This ratio is largely model independent. All model dependent features are contained mostly in the front factors which cancel out in the ratio. Since the dressing loop integrals enter in the proton decay lifetime in GUTs which contain the baryon and lepton number violating dimension 5 operators, the phenomena of *CP* violating effects on the proton lifetime should hold for a wide range of models of GUTs. The baryon and lepton number violating operators must be dressed by the chargino, gluino, and neutralino exchanges to generate effective baryon and lepton number violating dimension 6 operators at low energy. These dressing loops have vertices of quark-squark-chargino, quark-squark-neutralino, and quark-squark-gluino. From this structure one can read the phases that might enter the analysis. The chargino one has the phases θ_μ, α_{A_q}, and ξ_2. The neutralino vertex has beside the above set an extra phase ξ_1. The gluino vertex has the set θ_μ, α_{A_q}, ξ_3. Following the standard procedure (Weinberg, 1982; Nath *et al.*, 1985) one can obtain the effective dimension 6 operators for the baryon and lepton violating interaction arising from dressing of the dimension 5 operators. By doing so and estimating R_τ, one finds that this ratio is a sensitive function of *CP* phases (Ibrahim and Nath, 2000b). Modifications of the proton lifetime by as much as a factor of 2 due to the effects of the *CP* violating phases can occur. It has also been found that the *CP* phase effects could increase or decrease the proton decay rates and that the size of their effect depends highly on the region of the parameter space one is in.

I. SUSY *CP* phases and the decay $B_s^0 \to \mu^+\mu^-$

The branching ratio of the rare process $B_s^0 \to \mu^+\mu^-$ is another area where *CP* violating phase effects arise. It is known that the standard model value is rather small while in supersymmetric models it can get three orders of magnitude larger for large $\tan\beta$.[5]

Detecting such large values of B_s^0 would be a positive test for SUSY even before any sparticles are found. This decay is governed by the effective Hamiltonian

$$H_{\rm eff} = -\frac{G_F e^2}{4\sqrt{2}\pi^2} V_{tb} V^*_{td'} (C_S O_S + C_P O_P + C'_S O'_S + C'_P O'_P + C_{10} O_{10})_Q, \tag{178}$$

where C's are the coefficients of the Wilson operators O's defined by

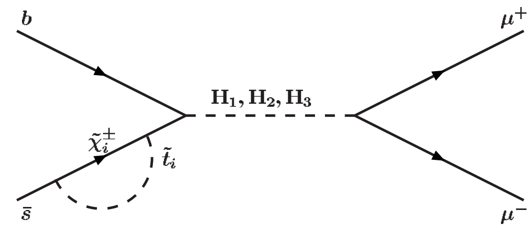

FIG. 12. The counterterm diagram which produces the leading term in amplitude proportional to $\tan^3\beta$ in the branching ratio $B_s^0 \to l^+ l^-$.

$$O_S = m_b(\bar{d}'_\alpha P_R b_\alpha)\bar{l}l, \quad O_P = m_b(\bar{d}'_\alpha P_R b_\alpha)\bar{l}\gamma_5 l,$$

$$O'_S = m_{d'}(\bar{d}'_\alpha P_L b_\alpha)\bar{l}l, \quad O'_P = m_{d'}(\bar{d}'_\alpha P_L b_\alpha)\bar{l}\gamma_5 l,$$

$$O_{10} = (\bar{d}'_\alpha \gamma^\mu P_L b_\alpha)\bar{l}\gamma_\mu\gamma_5 l, \tag{179}$$

and Q is the scale where the coefficients are evaluated. The branching ratio is a function of the coefficients $C_{S,P}$ and $C'_{S,P}$. In the counterterm diagram (see Fig. 12) which contributes to this ratio one can find vertices of $\bar{b}bH_i$, $\bar{s}\chi^-\tilde{t}$, and $\bar{\mu}\mu H_i$. The first two vertices are sensitive functions of the *CP* violating phases as explained in the different applications above. The phases that play a major role here are θ_μ, ξ_2, and α_{A_q}. Gluino and neutralino exchange diagrams also contribute which brings a dependence on additional phases ξ_1 and ξ_3. Inclusion of these (Ibrahim and Nath, 2003a) shows that the branching ratio can vary in some parts of the parameter space by up to one to two orders of magnitude due to the effect of *CP* phases. A demonstration of the strong effect of the phases on B decay branching ratio is given in Fig. 13. An analysis of this process using the so called resummed effective Lagrangian approach for Higgs mediated interactions in the *CP* violating MSSM has been given by Dedes and Pilaftsis (2003).

J. *CP* effects on squark decays

The interactions of $\bar{q}\tilde{q}'_i\chi^+_j$ and $\bar{q}\tilde{q}_i\chi^0_j$ do have *CP* violating phases at the tree level. These interactions are important for squark decays into fermions and such decays are expected to show up in the Large Hadron Collider when squarks become visible. The Lagrangian that governs the squark decays is given by

$$\mathcal{L} = g\bar{t}(R_{bij}P_R + L_{bij}P_L)\tilde{\chi}^+{}_j\tilde{b}_i + g\bar{b}(R_{tij}P_R + L_{tij}P_L)\tilde{\chi}^c{}_j\tilde{t}_i + g\bar{t}(K_{tij}P_R + M_{tij}P_L)\tilde{\chi}^0{}_j\tilde{t}_i + g\bar{b}(K_{bij}P_R + M_{bij}P_L)\tilde{\chi}^0{}_j\tilde{b}_i + \text{H.c.}, \tag{180}$$

where

[5] Choudhury and Gaur, 1999; Babu and Kolda, 2000; Bobeth *et al.*, 2001; Chankowski and Slawianowska, 2001; Huang *et al.*, 2001; Isidori and Retico, 2001; Arnowitt *et al.*, 2002; Buras *et al.*, 2002; Dedes *et al.*, 2002; Mizukoshi *et al.*, 2002; Xiong and Yang, 2002

$$\kappa_{t(b)} = \frac{m_{t(b)}}{\sqrt{2} m_W \sin\beta(\cos\beta)} \tag{181}$$

and

$$L_{bij} = \kappa_t V^*_{j2} D_{b1i},$$

$$R_{bij} = -(U_{j1} D_{b1i} - \kappa_b U_{j2} D_{b2i}),$$

$$K_{bij} = -\sqrt{2}[\beta_{bj} D_{b1i} + \alpha^*_{bj} D_{b2i}],$$

$$M_{bij} = -\sqrt{2}[\alpha_{bj} D_{b1i} - \gamma_{bj} D_{b2i}]. \tag{182}$$

The corresponding quantities with subscript t can be obtained by the substitution $b \to t$, $U \leftarrow \rightarrow V$.

The couplings R and L are functions of the phases θ_μ, ξ_2, and α_{A_q}. The set of phases that enter the couplings K and M is the same above set with an extra phase ξ_1. The loop corrections produce shifts in the couplings as follows:

$$\begin{aligned}\mathcal{L}_{\text{eff}} = {} & g\bar{t}(\tilde{R}_{bij} P_R + \tilde{L}_{bij} P_L)\tilde{\chi}^{+}{}_j \tilde{b}_i + g\bar{b}(\tilde{R}_{tij} P_R \\ & + \tilde{L}_{tij} P_L)\tilde{\chi}^{c}{}_j \tilde{t}_i + g\bar{t}(\tilde{K}_{tij} P_R + \tilde{M}_{tij} P_L)\tilde{\chi}^{0}{}_j \tilde{t}_i \\ & + g\bar{b}(\tilde{K}_{bij} P_R + \tilde{M}_{bij} P_L)\tilde{\chi}^{0}{}_j \tilde{b}_i + \text{H.c.},\end{aligned} \tag{183}$$

where $\tilde{R}_{bij} = R_{bij} + \Delta R_{bij}$ and ΔR_{bij} is the loop correction and other tildes are similarly defined. The loops that enter the analysis of Δ's have gluinos, charginos, neutralinos, neutral Higgs, charged Higgs, squarks, W, and Z boson exchanges. The masses of sparticles as well as the vertices where they enter are sensitive to the *CP* phases. The analysis using the loop corrected Lagrangian enhances the *CP* dependence of the masses and the vertices that already appear at the tree level (see Fig. 14). Recent analyses of stop and sbottom decays can be found in Bartl *et al.* (2003); Bartl, Hesselbach, *et al.* (2004); Ibrahim and Nath (2005).

K. $B \to \phi K$ and *CP* asymmetries

Like $B \to X_s + \gamma$, the decay $B \to \phi K_S$ has no tree level contribution and proceeds only via loop corrections. Thus the process presents a good testing ground for new physics since new physics also enters at the loop level. An interesting phenomenon concerns the fact that in the SM the *CP* asymmetry predicted for $B\phi K_S$ is the same as in $B \to J/\Psi K_s$ to $O(\lambda^2)$ (Grossman and Worah, 1997).

The current value of the $B \to J/\Psi K_S$ experimentally is

$$S_{J/\Psi K_s} = 0.734 \pm 0.055, \tag{184}$$

which is in excellent agreement with SM prediction of $\sin 2\beta = 0.715^{+0.055}_{-0.045}$. Although currently the experimental value for $S_{\phi K_S}$ (Aubert *et al.*, 2004),

$$S_{\phi K} = 0.50 \pm 0.25(\text{stat})^{+0.07}_{-0.04}(\text{syst}), \tag{185}$$

is consistent within 1σ of the SM prediction, its value has significantly in the past shown a 2.7σ deviation from the SM prediction, which triggered much theoretical activity to explain the large deviation.[6]

Although the discrepancy has largely disappeared it is still instructive to review briefly the possible processes that could make a large contribution to the $B \to \phi K_S$ process. It should be noted that the branching ratio $\text{BR}(B^0 \to \phi K_S) = (8.0 \pm 1.3) \times 10^{-6}$ is quite consistent with the SM result.

The time dependent asymmetries in $B \to \phi K_S$ are defined so that

$$\begin{aligned}\mathcal{A}_{\phi K}(t) &= \frac{\Gamma[\bar{B}(t) \to \phi K_S] - \Gamma[B(t) \to \phi K_S]}{\Gamma[\bar{B}(t) \to \phi K_S] + \Gamma[B(t) \to \phi K_S]} \\ &= -C_{\phi K}\cos(\Delta m_B t) + S_{\phi K}\sin(\Delta m_B t),\end{aligned} \tag{186}$$

where $S_{\phi K_S}$ and $C_{\phi K_S}$ are given by

$$C_{\phi K_S} = \frac{1 + |\lambda_{\phi K_S}|^2}{1 + |\lambda_{\phi K_S}|^2}, \quad S_{\phi K_S} = \frac{2\,\text{Im}\,\lambda_{\phi K_S}}{1 + |\lambda_{\phi K_S}|^2}, \tag{187}$$

and $\lambda_{\phi K_S}$ is defined by

$$\lambda_{\phi K_S} = -e^{-2i(\beta + \delta\beta)} \frac{\bar{A}(\bar{B}^0 \to \phi K_S)}{A(B^0 \to \phi K_S)}, \tag{188}$$

with β defined in the SM and $\delta\beta$ is any possible new physics contribution. Much of the work in trying to produce large effects within supersymmetric models has focused on generating corrections from flavor mixing in the quark sector using the mass insertion method (Hall *et al.*, 1986; Gabbiani *et al.*, 1996).

Thus, for example, an LL type mass insertion in the down quark sector will have the form

$$(\delta^d_{LL})_{ij} = [V^{d\dagger}_L (M_d)^2_{LL} V^d_L]_{ij}. \tag{189}$$

Here $(M^2_d)_{LL}$ is the LL down squark mass matrix, V^d_L is the rotation matrix that diagonalizes the down squark mass matrix, and $\tilde{m}$ is the average squark mass. Similarly one defines the mass insertions $(\delta^d_{RR})_{ij}$, $(\delta^d_{LR})_{ij}$, and $(\delta^d_{RL})_{ij}$. Among the supersymmetric contributions considered are the gluino mediated $b \to sq\bar{q}$ with $q = u, d, s, c, b$ and Higgs mediated $b \to ss\bar{s}$. Typically it is found that the LL and RR insertions give too small an effect but chirality flipping LR and RL insertions can generate sizable corrections to $B\phi K_S$. Thus, for example, $|(\delta^d_{LR})_{23}| \leq 10^{-2}$ can significantly affect $B \to \phi K_S$ while the constraints on $B \to X_s\gamma$ and ΔM_s are obeyed. The analysis in $B \to \phi K_S$ in supergravity grand unification with inclusion of *CP* phases was carried out by Arnowitt *et al.* (2003) and it was concluded that significant corrections to the asymmetries can arise with inclusion in the trilinear soft parameter A with mixings in the second and third generations either in the up sector or in

[6]See Ciuchini and Silvestrini, 2002; Datta, 2002; Hiller, 2002; Agashe and Carone, 2003; Arnowitt *et al.*, 2003; Baek, 2003; Chakraverty *et al.*, 2003; Chiang and Rosner, 2003; Dutta *et al.*, 2003; Kane *et al.*, 2003; Khalil and Kou, 2003; Kundu and Mitra, 2003; Cheng *et al.*, 2004

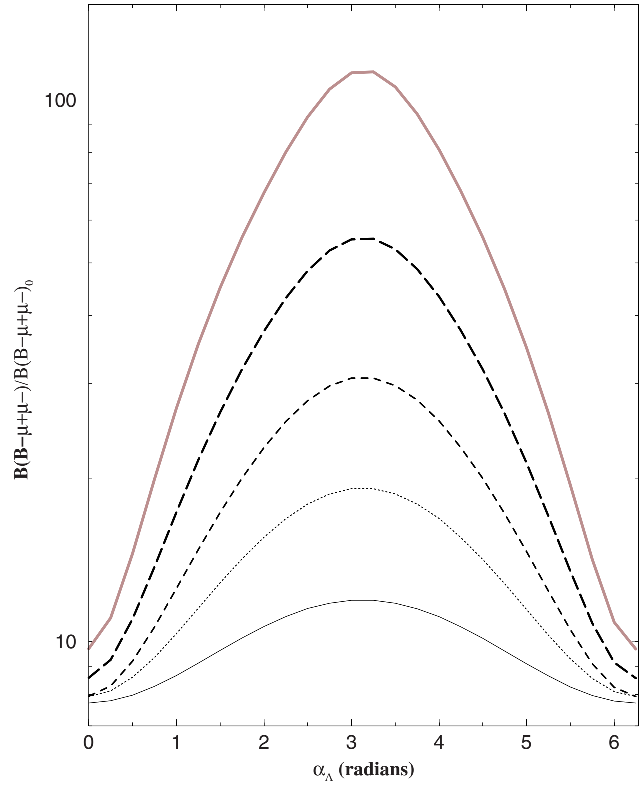

FIG. 13. (Color online) The strong dependence on α_A of the ratio of the branching ratios $B(B_s^0\to\mu^+\mu^-)/B(B_s^0\to\mu^+\mu^-)_0$, where $B(B_s^0\to\mu^+\mu^-)_0$ is the branching ratio when all phases are set to zero (Ibrahim and Nath, 2003a). The curves in ascending order are for values of $|A_0|$ of 1,2,3,4,5. The other parameters are m_0=200 GeV, $m_{1/2}$=200 GeV, $\tan\beta$=50, $\xi_1=\xi_2=\pi/4$, ξ_3=0, and θ_μ=2.

the down sector. A similar analysis of asymmetries in $B\to\eta' K$ has also been carried out by Gabrielli *et al.* (2005).

L. *T* and *CP* odd operators and their observability at colliders

In the previous sections we have discussed the effects of *CP* violation on several phenomena. The list of *CP* odd or *T* odd (assuming *CPT* invariance) is rather large [for a sample, see Bernreuther and Nachtmann (1991); De Rujula *et al.* (1991); Kane *et al.* (1992); Valencia (1994)]. We discuss briefly now the possibilities for the observation of *CP* in collider experiments. First we note that *CP* phases affect decays and scattering cross sections in two different ways. Thus in addition to generating a *CP* violating contribution to the amplitudes, they also affect the *CP* even part of the amplitudes which can affect the overall magnitude of decay widths and scattering cross sections. However, definite tests of *CP* violation can arise only via the observation of *T* odd or *CP* odd parts. As an example of the size of the effects induced by *CP* odd operators in supersymmetry on cross sections consider the process $e^+e^-\to t\bar{t}$. Here an analysis in MSSM including loop effects with *CP* phases gives (Christova and Fabbrichesi, 1993)

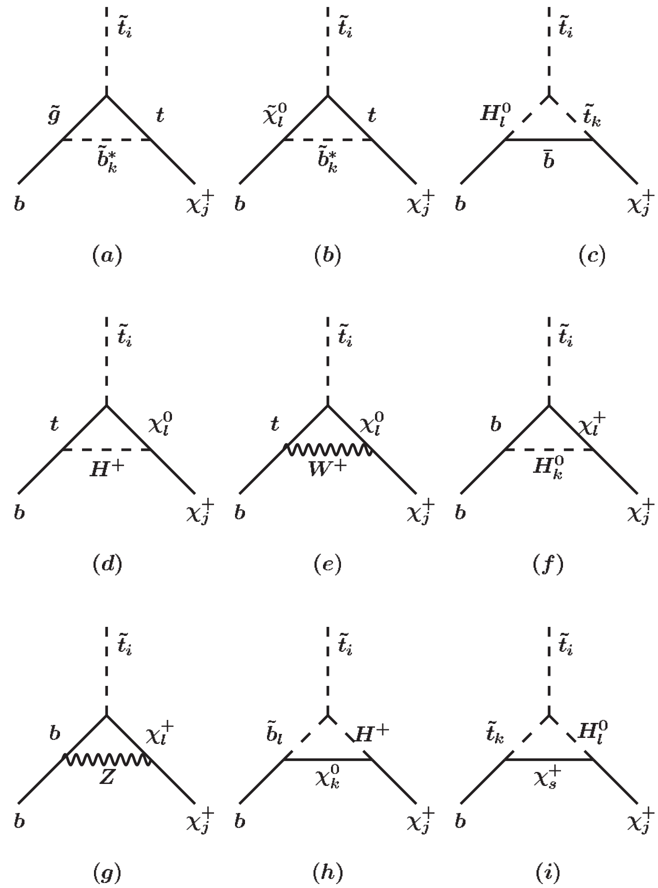

FIG. 14. A sample of one loop diagrams with *CP* phase dependent vertices that contribute to the decay of the stops.

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_0^{t\bar{t}}}{d\Omega}\left(1+c\frac{\alpha_s}{\pi}\sin(\alpha_{A_t}-\phi_{\tilde{g}})\frac{(\vec{J}\cdot\vec{p}\times\vec{k})}{|\vec{p}\times\vec{k}|}\right), \tag{190}$$

where $\vec{k}$ ($\vec{p}$) are the center of mass momentum of one of the initial (final) particles and $\vec{J}$ is the unit polarization vector of one the produced t quarks perpendicular to the production plane. c depends on the details of the sparticle spectrum and can vary significantly depending on the sparticle spectrum. The choice $c\sim 0.1$ gives the correction of the *T* odd observable to be of size $(10^{-1}\alpha_s/\pi)$ which is typically of the same size as the radiative corrections from the standard model. More generally with e^+e^- colliders in the process $e^+e^-\to X$ with momenta $\vec{p}_1,\vec{p}_2,\vec{p}$ a product of the type $(\vec{\xi}_i\times\vec{\xi}_j)\cdot\vec{\xi}_k$ where ξ_i is either a momentum or a polarization will give a *T*-odd observable. For example, one will have *T* odd operators of the type (Gavela *et al.*, 1989)

$$\mathcal{T}_1=(\vec{p}_1\times\vec{p}_2)\cdot\vec{S}_{e^-},$$

$$\mathcal{T}_2=\vec{p}\cdot(\vec{S}_{e^-}\times\vec{S}_{e^+}). \tag{191}$$

More generally with several particles ($i=1,\ldots,n,n>4$) one can form a *T* odd operator such as

$$\epsilon_{\alpha\beta\gamma\delta} p_i^\alpha p_j^\beta p_k^\gamma p_l^\delta. \quad (192)$$

An example of such an operator is the squark decay $\tilde{t} \to t + l^+ l^- + \chi_1^0$ which can also lead to an observable signal at the LHC (Langacker *et al.*, 2007). A study of the effects of *CP* violating phases of the MSSM on leptonic high-energy observables has been given by Choi, Drees, and Gaissmaier (2004). An efficient way to observe *CP* violation is via the use of polarized beams in e^+e^- colliders which is of interest in view of the proposed International Linear collider. A discussion on tests of supersymmetry at linear colliders can be found in Baer *et al.* (2004) and a detailed discussion of tests of *CP* asymmetries has been given by Moortgat-Pick *et al.* (2005).[7] An interesting issue concerns the possibility of expressing *CP* odd quantities in terms of basis independent quantities for the supersymmetric case similar to the Jarlskog invariant for the case of the standard model. Recent works in this direction can be found in and Lebedev (2003), Dreiner *et al.* (2007).

Finally we note that the computation of SUSY phenomena with *CP* phases is more difficult than computations without *CP* phases. In Appendix J we give a brief discussion of the tools necessary for the computation of SUSY phenomena with *CP* phases.

XII. FLAVOR AND *CP* PHASES

CP violation can influence flavor physics [for recent reviews see Fleischer (2006); Schopper (2006); Bigi (2007)] and thus such effects could be used as probes of the SUSY *CP* violation effects. This can happen in several ways. This could happen in *CP* violation effects in K and B physics, or if EDMs of leptons are measured and turn out to be in violation of scaling, and in possible future sparticle decays which may contain flavor dependent *CP* violating effects. We consider first *CP* violation in K and B physics. Essentially all phenomena seen here can be explained in terms of the *CP* violation with a standard model origin, i.e., arising from the phase δ_{CKM}. This means that unless some deviations from the standard model predictions are seen, the supersymmetric *CP* violation must be small. On the other hand, if significant deviations occur from the standard model predictions then one would need in addition to the large *CP* phases a new flavor structure. An example of this is flavor changing terms arising from the off diagonal component in the LR mass matrix $(\delta_{ij})_{LR}(d) = [m_{LR}^2(d)]_{ij}/\tilde{m}_q^2$ (Dine *et al.*, 1993, 2001; Khalil and Kobayashi, 1999; Masiero and Murayama, 1999; Demir *et al.*, 2000a, 2000b).

[7] A number of works related to the effects of *CP* on the Higgs and sparticle phenomena discussed in this section have been gives by Hollik *et al.*, 1998, 1999; Akeroyd and Arhrib, 2001; Boz, 2002, 2004a, 2004b; Bartl *et al.* 2006; Choi, Drees, Gaissmaier, and Song, 2004; Heinemeyer *et al.*, 2004; Ghosh *et al.*, 2005; Accomando *et al.*, 2006; Cheung *et al.*, 2006; Alan *et al.*, 2007

Further, if one adopts the viewpoint that the entire *CP* phenomena in the K and B system arise from the supersymmetric *CP* phases (Frere and Belen Gavela, 1983; Brhlik, Everett, Kane, King, and Lebedev, 2000) then one will need a new flavor structure. But such an assumption appears to be drastic since Yukawa couplings arising from string compactification will typically be complex. However, there are other ways in which *CP* violation can act as strong probes of flavor physics and vice versa. For instance, SUSY *CP* effects would be relevant in flavor changing neutral current processes such as $b \to s + \gamma$ and in $\mu \to e + \gamma$. Also if the EDM of the electron and the muon are eventually determined and a scaling violation is found, then such effects give us a connection between *CP* violation and flavor. Similarly the connection between *CP* and flavor can be obtained from collider data in the decays of sparticles. In the following we discuss two specific phenomena where *CP* and flavor affects can be significant. The issue of flavor and *CP* violation has been discussed in many papers (Masiero and Murayama, 1999; Demir *et al.*, 2000b; Chang *et al.*, 2003; Demir and Farzan, 2005; Ayazi and Farzan, 2007). Additional papers that discuss these issues are Ellis *et al.* (2006); Pospelov *et al.* (2006a, 2006b); Farzan (2007); Gronau (2007). *CP* and flavor violation in SO(10) is discussed in Harvey *et al.* (1980); Nath and Syed (2001); Babu *et al.* (2005); Chen and Mahanthappa (2005); Dutta *et al.* (2005).

A. d_μ vs d_e and possible scaling violations

The EDM of the muon and the electron are essentially scaled by their masses, so that

$$d_\mu / d_e \simeq m_\mu / m_e. \quad (193)$$

The current experimental limits on the muon EDM are much less stringent than on the electron EDM, and thus it is reasonable to ask if the EDM of the muon could be much larger than the EDM of the electron. If so, the improved experiments on the muon EDM may be able to detect it. Thus we explore the conditions under which significant violations of scaling may occur. Now we recall from our discussion on the EDM of the electron that large EDM for the electron generated by the chargino exchange may be canceled by contributions from the neutralino exchange. Thus one possibility in generating a large muon EDM is to upset this cancellation for the muon case. This appears possible by inclusion of flavor dependent nonuniversalities in the soft parameters. To make this idea concrete we consider that the chargino and neutralino exchange contributions to a lepton EDM are

$$d_l = \frac{e\alpha_{\rm EM}}{4\pi\sin^2\theta_W}\frac{\kappa_l}{m^2_{\tilde{\nu}_l}}\sum_{i=1}^{2}\tilde{m}_{\chi^+_i}\,{\rm Im}(U^*_{i2}V^*_{i1})A\left(\frac{\tilde{m}^2_{\chi^+_i}}{m^2_{\tilde{\nu}_l}}\right) + \frac{e\alpha_{\rm EM}}{4\pi\sin^2\theta_W}\sum_{k=1}^{2}\sum_{i=1}^{4}{\rm Im}(\eta^l_{ik})\frac{\tilde{m}_{\chi^0_i}}{M^2_{\tilde{l}_k}}Q_{\tilde{l}}B\left(\frac{\tilde{m}^2_{\chi^0_i}}{M^2_{\tilde{l}_k}}\right), \tag{194}$$

where A, B, and κ_l are defined earlier, and η^l_{ik} is given by

$$\eta^l_{ik} = [-\sqrt{2}\{\tan\theta_W(Q_l - T_{3l})X_{1i} + T_{3l}X_{2i}\}D^*_{l1k} - \kappa_l X_{3i}D^*_{l2k}](\sqrt{2}\tan\theta_W Q_l X_{1i}D_{l2k} - \kappa_l X_{3i}D_{l1k}). \tag{195}$$

Here X diagonalizes the neutralino matrix M_{χ^0} and D_l diagonalizes the slepton $(\text{mass})^2$ matrix. The chargino exchange contribution depends on the single phase combination $\xi_2+\theta_\mu$, while the neutralino exchange contribution depends additionally on the phase combinations $\theta_\mu+\xi_1$, and $\theta_\mu+\alpha_{A_{\tilde{l}}}$. Nonuniversalities can be introduced in two ways: via sneutrino masses which enter in the chargino exchange and via slepton masses that enter in the neutralino exchange diagram. One efficient way to introduce nonuniversalities in the slepton sector is via the trilinear coupling parameter A_l which can be chosen to be flavor dependent at the GUT scale. In this case the cancellation in the electron EDM sector would not imply the same exact cancellation in the muon sector and significant violations of the scaling relation can be obtained.

Since violations of scaling arise from the neutralino sector, we discuss this in further detail. Here the leading dependence of the lepton mass arises from n^l_{ik} while subleading dependence arises from the outside smuon mass factors in Eq. (194). Thus to understand the scaling phenomenon and its breakdown we focus on n^l_{ik} which can be expanded as follows using Eq. (195):

$$\eta^l_{ik} = a_0c_0X^2_{1i}D^*_{l1k}D_{l2k} + b_0c_0X_{1i}X_{2i}D^*_{l1k}D_{l2k} - \kappa_l a_0X_{1i}X_{3i}|D_{l1k}|^2 - \kappa_l b_0X_{2i}X_{3i}|D_{l1k}|^2 - \kappa_l c_0X_{1i}X_{3i}|D_{l2k}|^2 + \kappa^2_l X^2_{3i}D_{l1k}D^*_{l2k}, \tag{196}$$

where a_0, b_0, and c_0 are independent of the lepton mass. The first two terms on the right hand side of Eq. (196) are linear in lepton mass through the relation

$${\rm Im}(D^*_{l11}D_{l21}) = -{\rm Im}(D^*_{l12}D_{l22}) = \frac{m_l}{M^2_{\tilde{l}1} - M^2_{\tilde{l}2}}(m_0|A_l|\sin\alpha_f + |\mu|\sin\theta_\mu\tan\beta). \tag{197}$$

The third, fourth, and fifth terms on the right hand side of Eq. (196) have a leading linear dependence on the lepton mass through the parameter κ_l and have additional weaker dependence on the lepton mass through the diagonalizing matrix elements D_{ij}. The last term in Eq. (196) is cubic in the lepton mass. However, in most of the parameter space considered, the first term in Eq. (196) is the dominant one and controls the scaling behavior. Thus for the case when all soft SUSY breaking parameters including A are universal [i.e., $A_l=A$ in Eq. (197)], one finds that scaling results, i.e., $d_\mu/d_e \simeq m_\mu/m_e$. However, for the nonuniversal case, since the contribution from the A parameter is flavor dependent, we have a breakdown of scaling here. An analysis is given in Fig. 15. This breakdown can be seen by comparing d_μ for the nonuniversal cases (dashed line with triangles pointed down and dashed line with triangles pointed up) with d_e (solid line with squares) in Fig. 15.

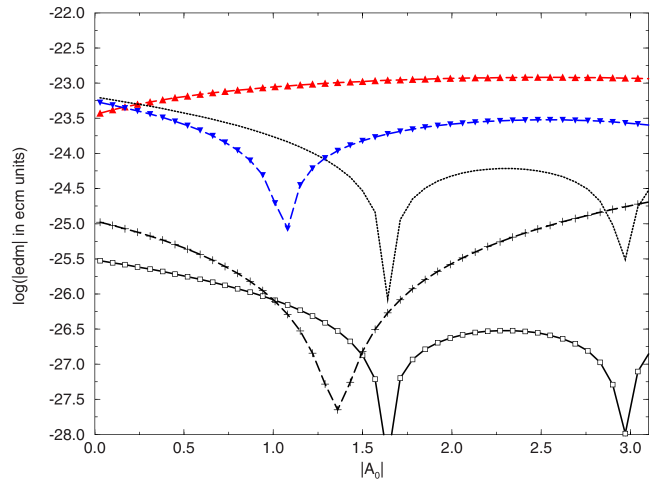

FIG. 15. (Color online) The strong flavor dependence via A nonuniversalities in enhancing the muon EDM relative to the electron EDM in the cancellation region from the analysis of Ibrahim and Nath (2001b). Plotted are the electron EDM d_e (solid line with squares), the neutron EDM d_n (dashed line with plus signs), and the muon EDM d_μ as a function of $|A_0|$ for $\tan\beta=20$, $m_0=200$, $m_{1/2}=246$, $\xi_1=0.28$, $\xi_2=-0.51$, $\xi_3=-0.11$, $\theta_\mu=0.4$, and $\alpha_{A_e}=1.02$ where all masses are in GeV. The curve with the dashed line with triangles pointed down is a plot of the muon EDM d_μ which have all the same parameters as for d_e and d_n except that $\alpha_{A_\mu}=0.0$ and the curve with the dashed line with triangles pointed up is a plot of the muon EDM d_μ which have all the same parameters as for d_e and d_n except that $|A_\mu|=6.0$ and $\alpha_{A_\mu}=-2.0$.

With the inclusion of nonuniversalities d_μ can be several orders of magnitude larger than d_e. Specifically values of d_μ could be as large as 10^{-24}–$10^{-23}e$ cm and within reach of proposed experiments which extend the search for the muon EDM to the range $10^{-24}e$ cm. An enhanced EDM for the muon relative to the electron EDM in excess of what scaling law allows can be generated with large neutrino mixings arising from the see-saw mechanism (Babu *et al.*, 2000a). Another analysis where lepton flavor violations are used to generate an enhancement of the muon EDM has been given by Feng *et al.* (2001).

B. SUSY *CP* phases and the FCNC process $B\to X_s\gamma$

There are other effects of the *CP* violating phases on the phenomenological constraint arises from the mea-

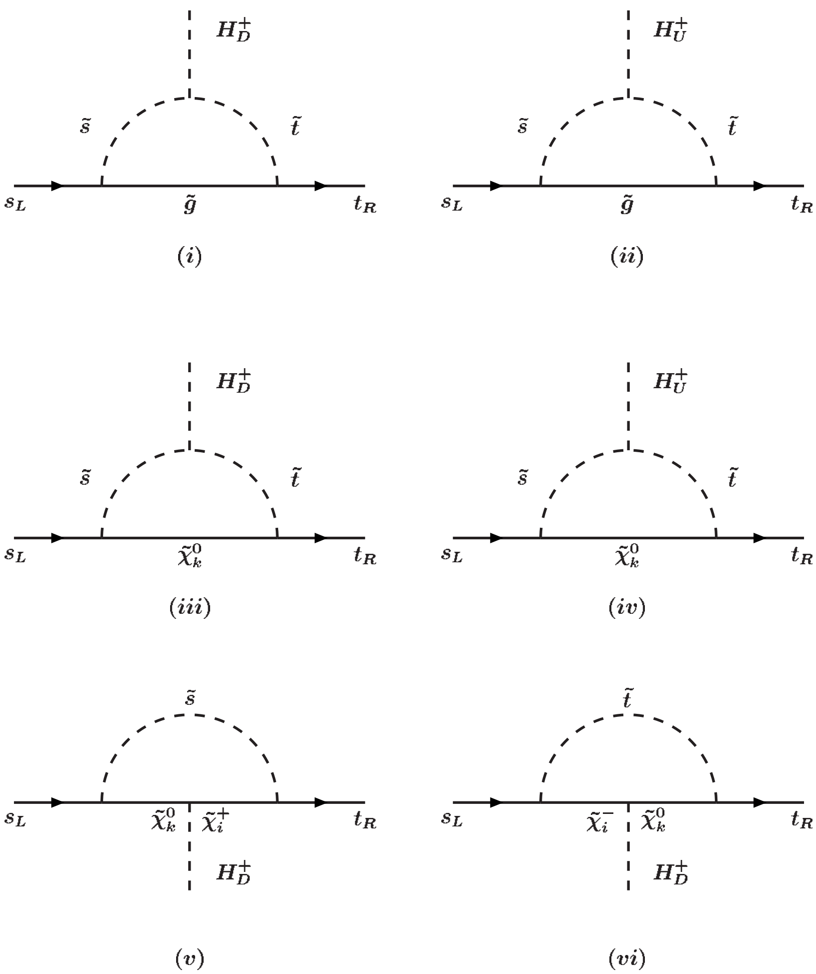

FIG. 16. A sample of diagrams with *CP* dependent vertices that contribute to the NLO corrections to the epsilons in $b\to s+\gamma$ decay. There are a total of 20 such diagrams.

surement of the rare decay $B\to X_s\gamma$. This decay only occurs at the one loop level in the standard model (Deshpande *et al.*, 1987; Altomari *et al.*, 1988; Dominguez *et al.*, 1988; Grinstein *et al.*, 1988; Casalbuoni *et al.*, 1993; Colangelo *et al.*, 1993; Falk *et al.*, 1994). The supersymmetric radiative corrections might be of the same order of magnitude as the standard model contribution (Bertolini *et al.*, 1991; Barbieri and Giudice, 1993; Barger *et al.*, 1993; Diaz, 1993; Garisto and Ng, 1993; Hewett, 1993; Lopez *et al.*, 1993; Nath and Arnowitt, 1994; Bertolini and Vissani, 1995; Goto and Okada, 1995; Baer *et al.*, 1998). It has recently been recognized that supersymmetric contributions can receive significant contributions from the next-to-leading order (NLO) corrections which are enhanced by large $\tan\beta$. These are typically parametrized by ϵ's. In addition to ϵ's there are two other loop (NLO) corrections which, however, are small and can be absorbed in a redefinition of the SUSY parameters (Degrassi *et al.*, 2000; Carena *et al.*, 2001). Currently the branching ratio of $B\to X_s\gamma$ is fairly accurately known experimentally (Barate *et al.*, 1998; Abe *et al.* (Belle), 2001; Chen *et al.*, 2001; Aubert *et al.*, 2002a, 2002b, 2005) and imposes significant constraints on model building. The current experimental value is

$$\mathrm{BR}(B\to X_s\gamma)=(355\pm 24^{+9}_{-10}\pm 3)\times 10^{-6} \tag{198}$$

as given by the Heavy Flavor Averaging group (Barberio *et al.*, 2006). The standard model result with QCD corrections (Chetyrkin *et al.*, 1997) including NLO gives (Gambino and Misiak, 2001) $\mathrm{BR}(B\to X_s\gamma)=(3.73\pm 0.30)\times 10^{-4}$. A similar robust prediction for supersymmetric models is needed. To analyze the NLO corrections for the supersymmetric case (see Fig. 16) one has to examine the effective Lagrangian describing the interaction of quarks with the charged Higgs fields $H^{\pm}$ and the charged Goldstones $G^{\pm}$ which we display below [see, e.g., Belanger *et al.* (2002); Demir and Olive (2002); Gomez *et al.* (2005, 2006)]:

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}}=&\frac{g}{\sqrt{2}M_W}G^{+}\left\{\sum_d m_t V_{td}\frac{1+\epsilon_t(d)\cot\beta}{1+\epsilon_{tt}\cot\beta}\bar{t}_R d_L\right.\\&\left.-\sum_u m_b V_{ub}\frac{1+\epsilon_b'(u)\tan\beta}{1+\epsilon_{bb}^{*}\tan\beta}\bar{u}_L b_R\right\}\\&+\frac{g}{\sqrt{2}M_W}H^{+}\left\{\sum_d m_t V_{td}\frac{1+\epsilon_t'(d)\tan\beta}{1+\epsilon_{tt}\cot\beta}\cot\beta\bar{t}_R d_L\right.\end{aligned}$$

$$+\sum_u m_b V_{ub}\frac{1+\epsilon_b(u)\cot\beta}{1+\epsilon^*_{bb}\tan\beta}\tan\beta\bar{u}_L b_R\Bigg\}+\text{H.c.},\tag{199}$$

where

$$\epsilon_t(b)=\frac{\Delta h_t^2}{h_t}+\tan\beta\frac{\delta h_t^1}{h_t},$$

$$\epsilon'_b(t)=\frac{\Delta h_b^{1*}}{h_b^*}+\cot\beta\frac{\delta h_b^{2*}}{h_b^*},$$

$$\epsilon'_t(b)=-\frac{\Delta h_t^2}{h_t}+\cot\beta\frac{\delta h_t^1}{h_t},$$

$$\epsilon_b(t)=-\frac{\Delta h_b^{1*}}{h_b^*}+\tan\beta\frac{\delta h_b^{2*}}{h_b^*},\tag{200}$$

and ϵ_{bb} and ϵ_{tt} are given by

$$\epsilon_{bb}=\frac{\Delta h_b^2}{h_b}+\cot\beta\frac{\delta h_b^1}{h_b},$$

$$\epsilon_{tt}=\frac{\Delta h_t^1}{h_t}+\tan\beta\frac{\delta h_t^2}{h_t}.\tag{201}$$

Using the above Lagrangian along with the interaction of quarks and W bosons one can write down the contributions to Wilson coefficients C_7 and C_8 in the effective Hamiltonian that governs the decay $b\to s\gamma$ [for further details see Kagan and Neubert (1998, 1999); Belanger *et al.* (2002); Belanger, Boudjema, Pukhov, and Semenov (2006)],

$$H_{\text{eff}}=-\frac{4G_F}{\sqrt{2}}V^*_{ts}V_{tb}\sum_{i=1}^{8}C_i(Q)O_i(Q),\tag{202}$$

where

$$O_2=(\bar{c}_L\gamma^\mu b_L)(\bar{s}_L\gamma_\mu c_L),$$

$$O_7=\frac{e}{16\pi^2}m_b(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu},$$

$$O_8=\frac{g_s}{16\pi^2}m_b(\bar{s}_L\sigma^{\mu\nu}T^a b_R)G^a_{\mu\nu}\tag{203}$$

as

$$C^W_{7,8}(Q_W)=F^{(1)}_{7,8}(x_t)+\frac{[\epsilon^*_{bb}-\epsilon'_b(t)]\tan\beta}{1+\epsilon^*_{bb}\tan\beta}F^{(2)}_{7,8}(x_t),\tag{204}$$

$$C^{H^\pm}_{7,8}(Q_W)=\frac{F^{(1)}_{7,8}(y_t)}{3\tan^2\beta}+\frac{1+\epsilon'_t(s)^*\tan\beta}{1+\epsilon^*_{bb}\tan\beta}F^{(2)}_{7,8}(y_t),\tag{205}$$

and x_t and y_t are defined by

$$x_t=\frac{m_t^2(Q_W)}{M_W^2},\quad y_t=\frac{m_t^2(Q_W)}{M_H^2},\tag{206}$$

and $F^{(1)}_{7,8}$ and $F^{(2)}_{7,8}$ are given by

$$F^{(1)}_7(x)=\frac{x(7-5x-8x^2)}{24(x-1)^3}+\frac{x^2(3x-2)}{4(x-1)^4}\ln x,$$

$$F^{(2)}_7(x)=\frac{x(3-5x)}{12(x-1)^3}+\frac{x(3x-2)}{6(x-1)^3}\ln x,$$

$$F^{(1)}_8(x)=\frac{x(2+5x-x^2)}{8(x-1)^3}-\frac{3x^2}{4(x-1)^4}\ln x,$$

$$F^{(2)}_8(x)=\frac{x(3-x)}{4(x-1)^3}-\frac{x}{2(x-1)^3}\ln x.\tag{207}$$

The C_7 and C_6 terms receive dominant exchange contributions from the W, charged Higgs, and charginos. The gluino and neutralino exchange terms can also contribute. The gluino exchange contributions have also been computed (Everett *et al.*, 2002). However, it turns out that in the minimal flavor violation scenario, the contributions from the gluino and neutralino exchanges are indeed relatively small. The analyses of $b\to s+\gamma$ beyond the MFV (minimal flavor violation) scenario where generational mixings are taken into account have been carried out by Foster *et al.* (2005a, 2005b) and Hahn *et al.* (2005). The most complete analyses of $B\to X_s\gamma$ in SUSY with the inclusion of NLO effects has been given by Buras *et al.* (2003); Degrassi *et al.* (2006); Gomez *et al.* (2006). Specifically in the analysis of Gomez *et al.* (2005, 2005) it was shown that ϵ's as well as the decay $B\to X_s\gamma$ are sensitive to the *CP* phases.

XIII. *CP* PHASES IN ν PHYSICS AND LEPTOGENESIS

Recent experiments discussed later in this section show that neutrinos are not massless. In general neutrinos could have either a Dirac mass, a Majorana mass, or perhaps a mixture of the two. For a neutrino to have a Dirac mass there must be a corresponding right handed neutrino to give a mass term of the type $m_D\bar{\nu}_L\nu_R+\text{H.c.}$ On the other hand, one can generate a Majorana mass term from just the left handed neutrinos, i.e., a mass term of the form $\nu_L^T C^{-1}m_L\nu_L+\text{H.c}$, where C is the charge conjugation matrix. For the case of three neutrino species the Majorana mass matrix is in general a symmetric mass matrix of the form (Mohapatra *et al.*, 2004, 2005; Nunokawa *et al.*, 2007)

$$\mathcal{M}_\nu=\begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau}\\ & m_{\mu\mu} & m_{\mu\tau}\\ & & m_{\tau\tau}\end{pmatrix}.\tag{208}$$

The Majorana neutrino mass matrix can be diagonalized by an orthogonal transformation so that

$$V^T \mathcal{M}_\nu V = \mathcal{M}_\nu^D, \tag{209}$$

where V can be written as $V=UK$ and the matrix U is similar to the CKM matrix and K is a diagonal matrix with two independent Majorana phases. For U one can use the parametrization

$$U = \begin{pmatrix} c_1 c_3 & c_3 s_1 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ -s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & c_1 c_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix}, \tag{210}$$

where $c_1=\cos\theta_{12}$, $c_2=\cos\theta_{23}$, $c_3=\cos\theta_{13}$ and similarly for s_1, s_2, and s_3, with θ_{ij} and δ constrained so that $0 \leqslant \theta_{ij} \leqslant \pi/2$ and $0 \leqslant \delta \leqslant 2\pi$. The matrix K is diagonal and can be taken to be

$$K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i\phi_2} \end{pmatrix}. \tag{211}$$

Thus we have three diagonal masses, three mixing angles, and three phases which together exhaust the full nine parameter set of the Majorana neutrino mass matrix. The Majorana *CP* phases do not enter in the neutrino oscillations, and only the Dirac phase δ does. Thus the oscillation probability from flavor ν_α to ν_β is given by

$$P(\nu_\alpha - \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i} \times \sin^2(\Delta m_{ij}^2 L/4E_\nu), \tag{212}$$

where $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$. From the solar neutrino and atmospheric neutrino data (Abdurashitov *et al.*, 1999; Hampel *et al.*, 1999; Altmann *et al.*, 2000; Fukuda *et al.*, 2000; Ambrosio *et al.*, 2001; Ahmad *et al.*, 2002a, 2002b) one finds that the neutrino mass^2 differences are given by

$$\Delta m^2_{\mathrm{sol}} = (5.4 - 9.5) \times 10^{-5}\ \mathrm{eV}^2,$$

$$\Delta m^2_{\mathrm{atm}} = (1.4 - 3.7) \times 10^{-3}\ \mathrm{eV}^2. \tag{213}$$

A fit to the solar and atmospheric data using the three neutrino generations gives constraints only on the neutrino mass differences and on the mixing angles. One has

$$\Delta m^2_{\mathrm{sol}} = ||m_2|^2 - |m_1|^2|,$$

$$\Delta m^2_{\mathrm{atm}} = ||m_3|^2 - |m_2|^2|,$$

$$\sin^2\theta_{12} = (0.23 - 0.39), \quad \sin^2\theta_{23} = (0.31 - 0.72),$$

$$\sin^2\theta_{13} < 0.054. \tag{214}$$

An interesting aspect of Eq. (214) is that the mixing angles θ_{12} and θ_{23} are large with θ_{23} being close to maximal while θ_{13} is small. This feature was rather unexpected and quite in contrast to the case of quarks where the mixings are small. An important point to note is that the neutrino oscillation experiments do not give us any information on the absolute value of the neutrino masses. Other experiments are necessary to provide information on the absolute values such as from cosmology and neutrinoless double beta decay. Thus from cosmology one has the following upper bound on each species of neutrino masses (Hannestad, 2003, 2004; Hannestad and Raffelt, 2004; Spergel *et al.*, 2006):

$$\sum_i |m_{\nu_i}| < 0.7 - 1\ \mathrm{eV}. \tag{215}$$

Similarly the neutrinoless double beta decay gives the following upper bound on the effective neutrino mass $|m_{ee}|$ (Klapdor-Kleingrothaus *et al.*, 2001; Bilenky, 2004):

$$|m_{ee}| < 0.2 - 0.5\ \mathrm{eV}, \tag{216}$$

where (Mohapatra *et al.*, 2005)

$$|m_\nu^{ee}| = |\cos^2\theta_{13}(|m_1|\cos^2\theta_{12} + |m_2|\sin^2\theta_{12} e^{2i\phi_1}) + \sin^2\theta_{13}|m_3|e^{2i\phi_2}|. \tag{217}$$

Several scenarios for the neutrino mass patterns have been discussed in order to explain the data. One possibility considered is that the third generation mass is much larger than the neutrino masses for the first two. Among these are the following: (i) $|m_{\nu_3}| \gg |m_{\nu_{1,\nu_2}}|$, (ii) $|m_{\nu_1}| \sim |m_{\nu_2}|$, $|m_{\nu_{1,\nu_2}}| \gg |m_{\nu_3}|$, (iii) $|m_{\nu_1}| \sim |m_{\nu_2}| \sim |m_{\nu_3}|$, $|m_{\nu_{1,\nu_2,\nu_3}}| \gg ||m_{\nu_i}| - |m_{\nu_j}||$. Neutrino oscillations are sensitive to δ but not to the Majorana phases (Barger *et al.*, 1980; Barger, Marfatia, and Whisnant, 2002). As is clear from Eq. (217), Majorana phases do enter in the neutrinoless double beta decay, but an actual determination of *CP* violation in $0\nu\beta\beta$ appears difficult (Barger, Glashow, *et al.*, 2002).

We discuss now the possible determination of δ in the next generation of neutrino experiments such as NOνA (Ayres *et al.*, 2002, 2004) and T2KK (Hagiwara *et al.*, 2007). We begin by noting that under the condition that *CPT* is conserved, the conservation of *CP* would require $P(\nu_\alpha \to \nu_\beta) - P(\bar\nu_\alpha \to \bar\nu_\beta) = 0$. In the presence of *CP* violation this difference is nonvanishing. Thus specifically one has (Berger *et al.*, 2007; Nunokawa *et al.*, 2007)

$$P(\nu_\mu \to \nu_e) - P(\bar\nu_\mu \to \bar\nu_e) = -16 J \sin\left(\frac{\Delta m_{12}^2 L}{4E}\right)\sin\left(\frac{\Delta m_{13}^2 L}{4E}\right)\sin\left(\frac{\Delta m_{23}^2 L}{4E}\right), \tag{218}$$

where E is the neutrino beam energy, L is the oscillation length, and J is the Jarlskog invariant for the neutrino mass matrix similar to the one for the quark mass matrix,

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin\delta. \tag{219}$$

We note that J depends on θ_{13} and δ both of which are currently unknown and thus one has only an upper limit for J so that $J \leqslant 0.04$. Thus the observation of a *CP* violation via Eq. (218) depends on other factors. For example, J vanishes if θ_{13} vanishes and thus the effect of *CP* violation via Eq. (218) would be unobservable. Simi-

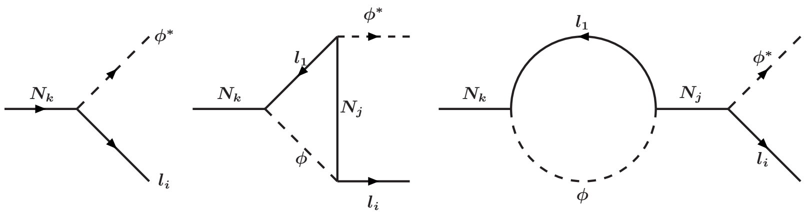

FIG. 17. Generation of lepton number asymmetry via decay of the right handed neutrino (N) by interference between the tree, vertex, and self-energy loop diagrams.

larly, if there was a degeneracy in the neutrino masses, for example, if $|m_{\nu_1}| \sim |m_{\nu_2}|$, then again the observation of *CP* violation via Eq. (218) would be difficult. However, aside from these extreme situations the process Eq. (218) holds the strong possibility that long baseline experiments should allow one to observe *CP* violation due to δ in the neutrino sector. Two experiments are ideally suited for this observation. One of these is NOνA (Ayres *et al.*, 2002, 2004) which will be a 25 kton liquid scintillator detector placed 810 km away from the NuMI neutrino beam in Fermilab (see Sec. XIV). The configuration will allow runs in the neutrino as well as in antineutrino mode. The second possibility is the T2KK detector (Hagiwara *et al.*, 2007) which is discussed in Sec. XIV.

A. *CP* violation and leptogenesis

As mentioned in Sec. I, achieving baryon asymmetry in the Universe requires three conditions: violation of baryon number, violation of C and of CP, and departure from thermal equilibrium. Quantitative analyses show that the standard model falls short of fulfilling these conditions. Specifically, the amount of *CP* violation is found not sufficient. Thus in the framework of the electroweak baryogenesis the effective *CP* suppression factor that enters is f_{CP} with (Shaposhnikov, 1986; Farrar and Shaposhnikov, 1993)

$$f_{CP} = T_C^{-12}(m_t^2 - m_c^2)(m_t^2 - m_c^2)(m_t^2 - m_u^2) \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) s_{12}s_{23}s_{31} \sin\delta, \tag{220}$$

where s_{ij}=sin θ_{ij} and θ_{ij} are the three mixing angles, δ is the CKM phase, and T_c is the temperature of the electroweak phase transition (EWPT). The EWPT is supposed to occur at values $T_c \sim 100$ GeV, which leads to $\delta_{CP} \sim 10^{-18}$–$10^{-20}$. A rough estimate of baryon asymmetry in EWPT is $B \simeq 10^{-8} f_{CP}$ and the standard model in this case leads to $B \simeq 10^{-26}$–10^{-28} which is far too small compared to the desired value of $B \sim 10^{-10}$. Additionally there are stringent constraints on the Higgs mass which are already in violation of the current limits. Analysis of baryogenesis in MSSM relieves some of the tension both because there are new sources of *CP* violation and also because the Higgs mass limits are significantly larger, e.g., $m_h \lesssim 120$ GeV. However, the analysis requires a significant fine tuning of parameters.

An attractive alternative to conventional baryogenesis [for reviews see Cohen *et al.* (1993); Riotto and Trodden (1999)] is baryogenesis via leptogenesis (Fukugita and Yanagida, 1986). For recent reviews see Buchmuller *et al.* (2005); Nardi *et al.* (2006); Chen (2007); Nir (2007a). The essential idea here is that if one can generate enough lepton asymmetry (see Fig. 17), then it can be converted into baryon asymmetry via sphleron interactions which violate $B+L$ but preserve $B-L$. Leptogenesis is a natural consequence of the see-saw mechanism (Minkowski, 1977; Gell-Mann and Slansky, 1979; Glashow, 1979; Yanagida, 1979; Mohapatra and Senjanovic, 1980) which is a popular mechanism for the generation of small neutrino masses [see also Schechter and Valle (1980, 1982) and Valle (2006) for early work on the see-saw phenomenology]. To generate a see-saw one needs heavy Majorana neutrinos and one can characterize the Lagrangian for the Majoranas by

$$L_N = M_i N_i N_i + \lambda_{i\alpha} N_i L_\alpha \phi, \tag{221}$$

where N_i are the Majorana fields and λ are in general complex and thus the λ terms violate *CP*. Further, L_N violates the lepton number and $B-L$. Thus the Lagrangian (221) has the general characteristics that might lead to the generation of baryon asymmetry via leptogenesis. The *CP* violation occurs in the decay of the Majoranas because of the overlap of the tree and loop.

One can define a *CP* asymmetry parameter so that

$$\epsilon_1 = \frac{\sum_\alpha [\Gamma(N_i \to l_\alpha \phi) - \Gamma(N_i \to \bar{l}_\alpha \phi^\dagger)]}{\sum_\alpha [\Gamma(N_i \to l_\alpha \phi) + \Gamma(N_i \to \bar{l}_\alpha \phi^\dagger)]}. \tag{222}$$

For the case of just two Majorana neutrinos the analysis of ϵ_1 gives

$$\epsilon_1 = C\left(\frac{M_2^2}{M_1^2}\right)\frac{\mathrm{Im}(\lambda\lambda^\dagger)_{12}^2}{(\lambda\lambda^\dagger)_{11}}, \tag{223}$$

where $C(z) = C_1(z) + C_2(z)$ and (Covi *et al.*, 1996)

$$C_1(z) = (8\pi)^{-1}\sqrt{z}\left[1 - (1+z)\ln\left(\frac{1+z}{z}\right)\right],$$

$$C_2(z) = (8\pi)^{-1}\frac{\sqrt{z}}{1-z}. \tag{224}$$

For the case of two singlets and $M_1 < M_2$ one has

$$\epsilon_1 = -\frac{3}{8\pi}\left(\frac{M_1}{M_2}\right)\frac{\mathrm{Im}(\lambda\lambda^\dagger)_{12}^2}{(\lambda\lambda^\dagger)_{11}}. \tag{225}$$

Next consider the case when the initial temperature T_i is larger than the mass of the lightest singlet neutralino N_1. In this case neglecting the decay effects of the heavier neutralinos, one can write the Boltzmann equations that govern the number densities n_{N_1} and n_{B-L} so that (Buchmuller and Plumacher, 2001; Buchmuller *et al.*, 2002, 2005)

$$\frac{dn_{N_1}}{dx} = -(D+S)(n_{N_1} - n_{N_1}^{\rm eq}),$$

$$\frac{dn_{B-L}}{dx} = -\epsilon_1 D(n_{N_1} - n_{N_1}^{\rm eq}) - W n_{B-L}. \tag{226}$$

Here $x \equiv M_1/T$ and $W = \Gamma_W/Hx$ is the washout term. The processes contributing to the Boltzmann equations are the decays, inverse decays, scattering processes with $\Delta L = 1$, and processes with $\Delta L = 2$, where $D = \Gamma_D/Hx$ includes decays and inverse decays and $S = \Gamma_S/Hx$ includes $\Delta L = 1$ scattering. Two parameters that enter prominently in the analysis are the effective mass $\tilde{m}_1$ which is defined by

$$\tilde{m}_1 = \frac{(\lambda\lambda^\dagger)_{11}|\langle\phi\rangle|^2}{M_1} \tag{227}$$

and the equilibrium neutrino mass $m*$ defined by

$$m* = \frac{16\pi^{5/2}\sqrt{g*}}{3\sqrt{5}}\frac{|\langle\phi\rangle|^2}{M_{\rm Pl}}, \tag{228}$$

where $g*$ is the total number of degrees of freedom ($g* = 106.75$ for SM). Numerically $m* \sim 10^{-3}$ eV.

The ratio $\tilde{m}_1/m*$ controls whether or not N_1 decays are out of equilibrium. When $\tilde{m}_1 < m*$ (the weak washout region), N_1 decay is slower than the Hubble expansion and leptogenesis can occur efficiently. For the case $\tilde{m}_1 > m*$ (the strong washout region) the back reactions that tend to washout are fast and leptogenesis is rather slow. However, even for $\tilde{m}_1/m* \gg 1$, a sufficient amount of lepton asymmetry can be generated. The solution to n_{B-L} can be obtained in the form

$$n_{B-L}(x) = n_{B-L}^f \exp\left(-\int_{x_1}^x dx' W(x')\right) - \frac{3}{4}\epsilon_1\kappa(x), \tag{229}$$

where $n_{B-L}^f = n_{B-L}(x=\infty)$ and $\kappa(x)$ is given by

$$\kappa(x) = -\frac{4}{3}\int_{x_1}^x \frac{D}{D+S}\frac{dn_{N_1}}{dx'}\exp\left(-\int_{x''}^x dx'' W(x'')\right). \tag{230}$$

The $B-L$ asymmetry is converted into baryon asymmetry by spheleron processes so that

$$\eta_B = \frac{a_{\rm sph}}{f}N_{B_L}^f - \frac{3}{4}\frac{a_{\rm sph}}{f}\epsilon_1\kappa_f, \tag{231}$$

where $a_{\rm sph}$ is the spheleron conversion factor ($a_{\rm sph} = 28/79$) and f is a dilution factor $f = n_\gamma^{\rm rec}/n_\gamma^*$ which depends on the photon production from the beginning of leptogenesis untill the point of recombination, and numerically $f = 2387/86$. One then has

$$\eta_B \simeq 10^{-2}\epsilon_1\kappa_f. \tag{232}$$

Now an upper limit on ϵ_1 can be obtained assuming that N_1 decay dominates the asymmetry as assumed above with a hierarchical pattern of heavy neutrino masses, and assuming that the decay of N_1 occurs for $T \geqslant 10^{12}$ GeV. In this case one can deduce, under the assumption $M_1/M_2 \ll 1$, the result (Davidson and Ibarra, 2002)

$$|\epsilon_1| \leqslant \frac{3}{16\pi}\frac{M_1(m_3 - m_2)}{|\langle\phi\rangle|^2}. \tag{233}$$

With $|m_3 - m_2| \leqslant \sqrt{\Delta m_{32}^2} \sim 0.05$ eV, one finds a lower bound on M_1 so that

$$M_1 \geqslant 2\times 10^9 \text{ GeV}. \tag{234}$$

This result implies a lower bound on the reheating temperature, and this bound appears to be in conflict with the upper bound on the reheating temperature to control the gravitino overproduction for the supersymmetric case. Consequently several variants of leptogenesis have been studied such as resonant leptogenesis (Pilaftsis, 1997; Pilaftsis and Underwood, 2004, 2005), soft leptogenesis (Grossman *et al.*, 2003, 2004; Boubekeur *et al.*, 2004), and nonthermal leptogenesis (Fujii *et al.*, 2002). The type of *CP* violation that occurs in leptogenesis involves neutrinos which are standard model singlets, and hence have no direct gauge interactions with the normal particles, and in addition are very heavy. Thus direct observation of *CP* violation that enters leptogenesis would be essentially impossible in laboratory experiments. However, in unified models *CP* phases could be interrelated across different sectors and thus indirect constraints on such phases could arise in such models.

B. Observability of Majorana phases

In the previous section we found that the leptogenesis does depend crucially on the Majorana phases [for a review of Majorana particles and their phases, see, e.g., Kayser (1984, 1985)]. However, these phases arise from heavy Majoranas and are not the same as the Majorana phases that arise in the light neutrino mass sector. It was noted in our discussion of the neutrino masses that Majorana phases do not enter in neutrino oscillations which depend only on the Dirac phase. The Majorana phases do enter in the neutrinoless double beta decay. However, they do so only in a *CP* even fashion and further their observation in the $0\nu\beta\beta$ appears difficult. The question one might ask is in what processes can the Majorana phases enter in a manifestly *CP* odd fashion? It is known that one such process is neutrino-antineutrino ($\nu \to \bar{\nu}$) oscillations (Schechter and Valle, 1981; Bernabeu and Pascual, 1983; de Gouvea *et al.*, 2003). The analysis of de Gouvea *et al.* (2003) sets out some simple criteria for their appearance in scattering phenomena. Thus consider the amplitude for the process X where

$$A_X = e^{i\xi_X}(A_1 + A_2 e^{i(\delta+\phi)}), \tag{235}$$

and we have pulled out a common phase factor $e^{i\xi_X}$ so A_1 has no phase dependent factor multiplying it, δ is a *CP* even phase, and ϕ is a *CP* odd phase. Then the mirror process $\bar{X}$ has the following amplitude:

$$A_{\bar{X}} = e^{i\xi_{\bar{X}}}(A_1 + A_2 e^{i(\delta-\phi)}), \tag{236}$$

where $A_{1,2}$ are assumed not to contain any *CP* violating effects and are the same in processes X and $\bar{X}$. The difference $\Delta\Gamma_{CP} = |A_{\bar{X}}|^2 - |A_X|^2$ is then given by

$$\Delta\Gamma_{CP} = 4A_1A_2 \sin(\delta)\sin(\phi). \tag{237}$$

The above simple analysis points to the following three conditions necessary for *CP* odd effects to arise in the process X vs its mirror process $\bar{X}$. These are (i) the existence of two distinct contributions to the amplitude, (ii) the two contributions must have a nonvanishing relative *CP* odd phase, and (iii) they must also have a nonvanishing relative *CP* even phase. The analysis of de Gouvea *et al.* (2003) considers the process

$$l_\alpha^+ W^- \to \nu \to l_\beta^- W^+ \tag{238}$$

for which one has the amplitude

$$A_X = \sum_i (\lambda_i U_{\alpha i} U_{\beta i}) \frac{m_i}{E} e^{-i(m_i^2 L/2E)} S, \tag{239}$$

where E is the energy of the intermediate state which propagates a microscopic distance L, U is the mixing matrix, and S depends on the initial and final states and on kinematical factors. For the *CP* conjugate process $l_\alpha^- W^+ \to l_\beta^+ W^-$ one has

$$A_{\bar{X}} = \sum_i (\lambda_i U_{\alpha i} U_{\beta i})^* \frac{m_i}{E} e^{-i(m_i^2 L/2E)} \bar{S}, \tag{240}$$

where the combination $(\lambda_i U_{\alpha i} U_{\beta i})$ is free of the phase convention (Bilenky *et al.*, 1984; Kayser, 1984; Nieves and Pal, 1987, 2003). Limiting the analysis to the case of two generations we can write

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}. \tag{241}$$

Under the approximation $\lambda_1 = 1 = \lambda_2$, $|\bar{S}| = |S|$, $\alpha = e$, and $\beta = \mu$ this leads to

$$\Delta\Gamma_{CP} = |A_{\bar{X}}|^2 - |A_X|^2 = \frac{m_1 m_2}{4E^2}|S|^2 \sin^2 2\theta \sin\left(\frac{(m_2^2 - m_1^2)L}{2E}\right) \sin\phi. \tag{242}$$

The above example satisfies all criterion set forth earlier for a *CP* odd effect to appear. *CP* odd effects can also appear in lepton number violating meson processes such as $K^\pm \to \pi^\mp \mu^\pm \mu^\pm$. Thus, for example, if we write

$$A_{K^+} = e^{i\xi_{K^+}}(A_{1K} + A_{2K} e^{i(\delta_K + \phi_K)}),$$

$$A_{K^-} = e^{i\xi_{K^-}}(A_{1K} + A_{2K} e^{i(\delta_K - \phi_K)}), \tag{243}$$

one will have $\Delta\Gamma_{CP}^K = |A_{K^-}|^2 - |A_{K^+}|^2$ given by

$$\Delta\Gamma_{CP}^K \propto 4A_{1K}A_{2K} \sin(\delta_K)\sin(\phi_K). \tag{244}$$

$\Delta L = 2$ contributions do arise with R parity violation in supersymmetry and contribute to $\Delta\Gamma_{CP}^K$. However, the effect turns out to be extremely small. Some possible cosmological effects of *CP* violation in neutrino oscillations have been considered by Khlopov and Petcov (1981).

XIV. FUTURE PROSPECTS

A. Improved EDM experiments

There are good prospects of improving the EDM bounds significantly. Future experiments may improve the sensitivity of EDM experiments by an order of magnitude or more (Dzuba *et al.*, 2002; Kawall *et al.*, 2004; Kozlov and Derevianko, 2006) and in some cases by a significantly larger factor (Lamoreaux, 2001; Semertzidis, 2004; Semertzidis *et al.*, 2004). A recent review on the current experimental situation and future prospects regarding the electron electric dipole moment has been given by Commins and DeMille (2006). Regarding the neutron EDM a sensitivity at the level of $1.7 \times 10^{-28} e$ cm could be achieved (Balashov *et al.*, 2007) and even a sensitivity of $10^{-29} e$ cm is possible (Harris, 2007). Regarding the EDM of ^{199}Hg improved measurements are in progress and a factor of three to four improvements over the next year or so is possible. Beyond that there are various projects aimed at improving the limit with diamagnetic atoms, using Xe-129, radioactive Ra or Rn. However, all are still in the development phase, so when one may expect better limits from these experiments is unclear. Regarding the muon EDM, one proposed experiment (Semertzidis, 2004; Semertzidis *et al.*, 2004) feasible at JPARC (Japan Proton Accelerator Reseach Complex) could extend the sensitivity to as much as $10^{-24} e$ cm. However, it appears that the earliest that muons may become available at JPARC is 2016. However, recently another proposal for muon EDM has been made where the existing muon beam μEI at PSI could be used. It is claimed that the muon EDM with a sensitivity of better than $d_\mu \sim 5 \times 10^{-23} e$ cm within one year of data taking is feasible (Adelmann and Kirch, 2006). Currently there is also an exploration underway regarding the possible determination of the deutron EDM using techniques similar to the ones used for muon EDM with the goal of reaching a sensitivity of $10^{-29} e$ cm (Semertzidis *et al.*, 2004; Semertzidis, 2007).

B. *B* physics at the LHCb

LHCb is one of the four detectors at the LHC, the other three being ATLAS, CMS, and ELLIS. Of the these ATLAS and CMS are the main particle physics detectors dedicated to the search for new physics such as supersymmetry or extra dimensions. While the ATLAS and CMS can also study B physics their capabilities in this respect are rather limited. On the other hand, LHCb is an experiment which is specifically dedicated to the study of B physics. Thus the B mesons produced in collisions at the LHC are likely to lie in angles close to the beam directions and a detector ideal for the study of B physics should be able to detect such particles. This is precisely what the LHCb is designed to do. Specifically the detection of charged particles will be accomplished by its ring-imaging Cherenkov (RICH) detector. The precise identification of the interaction region utilizes a vertex locator (VELO) which can be used for B tagging, and more generally for the separation of primary and secondary vertices. The number of B mesons produced at LHCb will be enormous. Even a luminosity of $10^{32}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ will lead to a number of $b\bar{b}$ events at the rate of $O(10^{11-12})$ per year. Thus the LHCb will have an unprecedented opportunity to study B physics in great depth.[8]

C. Super Belle proposal

The B factories are an ideal instrument for studying elements of the CKM matrix including the CP phase δ_{CKM}. The analyses provided by the B factories at SLAC (BaBar) and at KEK (Belle) have given a wealth of data and have improved measurements of the CKM elements. Specifically they have been able to measure time dependent CP asymmetries with good precision. Further improvements in the measurements of these elements will come only with significantly greater luminosity. The Super Belle proposal aims at achieving that by an upgrade of the KEKB collider to a luminosity of $10^{35-36}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. Such an improvement will also require an upgrade of the vertex detector for the Super Belle and specific proposals are under study (Kawasaki *et al.*, 2006)

D. Superbeams, ν physics, and *CP*

The answer to the question of whether or not CP phases appear in neutrino physics is of crucial relevance to our understanding of fundamental interactions. The observation of such phases in the light neutrino sector is possible using long baseline experiments and intense beams (Marciano, 2001; Diwan *et al.*, 2006; Marciano and Parsa, 2006) and its observation will give greater credence to the hypothesis that such phases also appear in the heavy neutrino sector which enter in leptogenesis. Thus the AIP 2004 study recommends "as a high priority, a comprehensive U.S. program to complete our understanding of neutrino mixing, to determine the character of the neutrino mass spectrum, and to search for CP violation among neutrinos" (Freedman and Kayser, 2004). Such high priority efforts could include improved $0\nu\beta\beta$ experiments, and super beams to study neutrino oscillations and detect CP phases. Specifically the study recommends "a proton driver in the megawatt class or above and neutrino superbeam with an appropriate very large detector capable of observing CP violation and measuring the neutrino mass-squared differences and mixing parameters with high precision." One such proposal is an upgraded Fermilab proton driver (FPD). Such an upgrade will improve the study of $\nu_\mu \to \nu_e$ oscillations by a significant factor (Geer, 2006). Thus the current Fermilab NuMI proton beam has 10^{13} protons at 120 GeV (a beam power of 0.2 MW). A secondary beam of charged pions is generated from the proton beam, and pions then decay producing a beam of tertiary ν_μ as they propagate along a long corridor to the target 735 km downstream. With 0.2 MW of proton beam power one generates only 10^{-5} interaction in a 1-kt detector at the far end. Thus an upgrade of the proton beam to deliver several megawatts of proton beam power coupled with an upgrade of the detector to 10 kt will significantly enhance the sensitivity of the detector to observe possible CP effects. A similar idea being discussed is T2KK where the far detector is put on the east coast of Korea along the Tokai to Kamioka (T2K) neutrino beam line (Hagiwara *et al.*, 2007).

XV. CONCLUSIONS

We have attempted here to give a broad overview of CP violation and the effect of CP phases arising from physics beyond the standard model. We know that CP violation beyond what is allowed in the standard model must exist in order that one generate the desired amount of baryon asymmetry in the Universe. We have examined the origin of such CP violation in some of the leading candidates for physics beyond the standard model. These include models based on extra dimensions, supersymmetric models with soft breaking, and string models. Specifically supersymmetric models and string models generate a plethora of new CP phases and one problem one encounters is that such phases lead to EDMs for the electron and neutron in excess of current limits. One way to limit to these is to fine tune the phases to be small which, however, is not satisfactory from the point of generation of baryon asymmetry. What one needs is a mechanism which allows at least some phases to be large while suppressing their contribution to the EDMs. One possibility is suppression of the EDMs by having a heavy sparticle spectrum. However, this possibility puts the sparticle masses at least for the first two generations in the several TeV range and thus outside the reach of the LHC. An alternative possibility of controlling the EDMs is the cancellation mechanism which allows for large phases consistent with the stringent limits on the EDMs from experiment. If the cancellation mechanism is valid, then the effect of CP phases will show up at colliders in a variety of supersymmetric phenomena. We have discussed some of these phenomena in this paper. One important such phenomenon is CP even $-CP$ odd Higgs

[8]See, e.g., http://www-pnp.physics.ox.ac.uk/lhcb/

mixing which would lead to discernible signals at hadron colliders and at a future International Linear Collider. Effects of *CP* could also be visible in $B_s^0 \to \mu^+\mu^-$, Higgs decays $h^0 \to b\bar{b}$, $\tau\bar{\tau}$, and sparticle decays. Dark matter analyses are also affected, specifically the detection cross section for neutralino-nucleon scattering.

The future proposed experiments will investigate *CP* phenomena with vastly increased data. Chief among these is the LHCb experiment which is dedicated to studying the *B* mesons. The proposed Super Belle will further add to these efforts. These will pin down the CKM matrix elements to a much greater precision than BaBar and Belle, and may shed light on the possibility whether or not new sources of *CP* violation are visible. However, if the sparticles are indeed observed, as one expects they will be, then a study of their branching ratios is likely to put significant limits on *CP* phases from sparticle decays.

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APPENDIX A: CHARGINO AND NEUTRALINO MASS MATRICES WITH PHASES

Here we present details on the diagonalization of the chargino and neutralino mass matrices which are in general complex. These appear in the analysis of Secs. IX and X. We consider the chargino mass matrix first. We have

$$M_C = \begin{pmatrix} \tilde{m}_2 & \sqrt{2}m_W \sin\beta \\ \sqrt{2}m_W \cos\beta & \mu \end{pmatrix}. \tag{A1}$$

The chargino matrix M_C is not Hermitian, is not symmetric, and is not real since μ and $\tilde{m}_2$ are complex. For simplicity we analyze its diagonalization for real $\tilde{m}_2$ and complex μ. Generalization for complex $\tilde{m}_2$ and μ is straightforward. M_C can be diagonalized by using the following biunitary transformation:

$$U'^* M_C V^{-1} = M_D. \tag{A2}$$

Here U' and V are Hermitian matrices and M_D is a diagonal matrix which, however, is not yet real. U' and V satisfy the relation

$$V(M_C^\dagger M_C)V^{-1} = \mathrm{diag}(|\tilde{m}_{\chi_1^+}|^2, |\tilde{m}_{\chi_2^+}|^2) = U'^*(M_C M_C^\dagger)(U'^*)^{-1}. \tag{A3}$$

We may parametrize U' so that

$$U' = \begin{pmatrix} \cos\dfrac{\theta_1}{2} & \sin\dfrac{\theta_1}{2}e^{i\phi_1} \\ -\sin\dfrac{\theta_1}{2}e^{-i\phi_1} & \cos\dfrac{\theta_1}{2} \end{pmatrix}, \tag{A4}$$

where

$$\tan\theta_1 = 2\sqrt{2}m_W(\tilde{m}_2^2 - |\mu|^2 - 2m_W^2\cos 2\beta)^{-1} \times (\tilde{m}_2^2\cos^2\beta + |\mu|^2\sin^2\beta + |\mu|\tilde{m}_2\sin 2\beta\cos\theta_\mu)^{1/2} \tag{A5}$$

and

$$\tan\phi_1 = |\mu|\sin\theta_\mu\sin\beta(\tilde{m}_2\cos\beta + |\mu|\cos\theta_\mu\sin\beta)^{-1}. \tag{A6}$$

Similarly we parametrize V so that

$$V = \begin{pmatrix} \cos\dfrac{\theta_2}{2} & \sin\dfrac{\theta_2}{2}e^{-i\phi_2} \\ -\sin\dfrac{\theta_2}{2}e^{i\phi_2} & \cos\dfrac{\theta_2}{2} \end{pmatrix}, \tag{A7}$$

where

$$\tan\theta_2 = 2\sqrt{2}m_W(\tilde{m}_2^2 - |\mu|^2 + 2m_W^2\cos 2\beta)^{-1} \times (\tilde{m}_2^2\sin^2\beta + |\mu|^2\cos^2\beta + |\mu|\tilde{m}_2\sin 2\beta\cos\theta_\mu)^{1/2} \tag{A8}$$

and

$$\tan\phi_2 = -|\mu|\sin\theta_\mu\cos\beta(\tilde{m}_2\sin\beta + |\mu|\cos\theta_\mu\cos\beta)^{-1}. \tag{A9}$$

We choose the phases of U' and V so that the elements of M_D will be positive. Thus we define $U = H\times U'$, where

$$H = (e^{i\gamma_1}, e^{i\gamma_2}), \tag{A10}$$

with γ_1, γ_2 the phases of the diagonal elements of M_D in Eq. (A2). With the above choice of phases one has

$$U^* M_C V^{-1} = \mathrm{diag}(\tilde{m}_{\chi_1^+}, \tilde{m}_{\chi_2^+}). \tag{A11}$$

Our choice of the signs and roots is such that

$$M^2_{(\tilde{m}_{\chi_1^+})(\tilde{m}_{\chi_2^+})} = \tfrac{1}{2}[\tilde{m}_2^2 + |\mu|^2 + 2m_W^2](+)(-) \times \tfrac{1}{2}[(\tilde{m}_2^2 - |\mu|^2)^2 + 4m_W^4\cos^2 2\beta + 4m_W^2 \times (\tilde{m}_2^2 + |\mu|^2 + 2\tilde{m}_2|\mu|\cos\theta_\mu\sin 2\beta)]^{1/2}, \tag{A12}$$

where the sign chosen is such that $\tilde{m}_{\chi_1^+} < \tilde{m}_{\chi_2^+}$ if

$$\tilde{m}_2^2 < |\mu|^2 + 2m_W^2 \cos 2\beta. \quad \text{(A13)}$$

For the neutralino mass matrix $M_{\tilde{\chi}^0}$ one has

$$\begin{pmatrix} \tilde{m}_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & \tilde{m}_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}. \quad \text{(A14)}$$

In the above $s_W = \sin\theta_W$, $s_\beta = \sin\beta$ where θ_W is the weak angle, and $c_\beta = \cos\beta$, and $s_\beta = \sin\beta$. The matrix M_{χ^0} is a complex non-Hermitian and symmetric matrix, which can be diagonalized by a unitary transformation such that

$$X^T M_{\chi^0} X = \mathrm{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}). \quad \text{(A15)}$$

APPENDIX B: SQUARK AND SLEPTON MASS^2 MATRICES WITH PHASES

In this appendix we give details on the diagonalization of the squark and slepton mass matrices that appear in Secs. IX and X. We consider the squark (mass)^2 matrix

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} M_{\tilde{q}11}^2 & M_{\tilde{q}12}^2 \\ M_{\tilde{q}21}^2 & M_{\tilde{q}22}^2 \end{pmatrix}. \quad \text{(B1)}$$

For the up squark case one has

$$M_{\tilde{u}11}^2 = M_{\tilde{Q}}^2 + m_u{}^2 + M_Z^2(\tfrac{1}{2} - Q_u s_W^2)\cos 2\beta,$$

$$M_{\tilde{u}12}^2 = m_u(A_u^* - \mu \cot\beta),$$

$$M_{\tilde{u}21}^2 = m_u(A_u - \mu^* \cot\beta),$$

$$M_{\tilde{u}22}^2 = m_{\tilde{U}}^2 + m_u{}^2 + M_Z^2 Q_u s_W^2 \cos 2\beta. \quad \text{(B2)}$$

Thus the squark mass^2 matrix is Hermitian and can be diagonalized by the unitary transformation

$$D_u^\dagger M_{\tilde{u}}^2 D_u = \mathrm{diag}(M_{\tilde{u}1}^2, M_{\tilde{u}2}^2), \quad \text{(B3)}$$

where one parametrizes D_u so that

$$D_u = \begin{pmatrix} \cos\dfrac{\theta_u}{2} & -\sin\dfrac{\theta_u}{2} e^{-i\phi_u} \\ \sin\dfrac{\theta_u}{2} e^{i\phi_u} & \cos\dfrac{\theta_u}{2} \end{pmatrix}. \quad \text{(B4)}$$

Here $M_{\tilde{u}21}^2 = |M_{\tilde{u}21}^2| e^{i\phi_u}$ and we choose the range of θ_u so that $-\pi/2 \leq \theta_u \leq \pi/2$, where $\tan\theta_u = 2|M_{\tilde{u}21}^2|/(M_{\tilde{u}11}^2 - M_{\tilde{u}22}^2)$. The eigenvalues $M_{\tilde{u}1}^2$ and $M_{\tilde{u}2}^2$ can be determined directly from Eq. (B1) so that

$$M_{\tilde{u}(1)(2)}^2 = \tfrac{1}{2}(M_{\tilde{u}11}^2 + M_{\tilde{u}22}^2)(+)(-)\tfrac{1}{2}[(M_{\tilde{u}11}^2 - M_{\tilde{u}22}^2)^2 + 4|M_{\tilde{u}21}^2|^2]^{1/2}. \quad \text{(B5)}$$

The $(+)$ in Eq. (A2) corresponds to the case so that for $M_{\tilde{u}11}^2 > M_{\tilde{u}22}^2$ one has $M_{\tilde{u}1}^2 > M_{\tilde{u}2}^2$ and vice versa. For our choice of the θ_u range one has

$$\tan\theta_u = \frac{2m_u|A_u m_0 - \mu^* \cot\beta|}{M_{\tilde{u}11}^2 - M_{\tilde{u}22}^2} \quad \text{(B6)}$$

and

$$\sin\phi_u = \frac{m_0|A_u|\sin\alpha_u + |\mu|\sin\theta_\mu R_u}{|m_0 A_u - \mu^* \cot\beta|}, \quad \text{(B7)}$$

where $R_u = \cot\beta$. The analysis for the down squark case proceeds in a similar fashion with the following changes:

$$M_{\tilde{d}11}^2 = M_{\tilde{Q}}^2 + m_d{}^2 - M_Z^2\left(\frac{1}{2} + Q_d s_W^2\right)\cos 2\beta,$$

$$M_{\tilde{d}12}^2 = m_d(A_d^* - \mu \tan\beta),$$

$$M_{\tilde{d}21}^2 = m_d(A_d - \mu^* \tan\beta),$$

$$M_{\tilde{d}22}^2 = m_{\tilde{D}}^2 + m_d{}^2 + M_Z^2 Q_d s_W^2 \cos 2\beta. \quad \text{(B8)}$$

The other changes are the modification of expressions for θ_d and ϕ_d. They read

$$\tan\theta_d = \frac{2m_d|A_u m_0 - \mu^* \tan\beta|}{M_{\tilde{d}11}^2 - M_{\tilde{d}22}^2} \quad \text{(B9)}$$

and

$$\sin\phi_d = \frac{m_0|A_d|\sin\alpha_d + |\mu|\sin\theta_\mu R_d}{|m_0 A_d - \mu^* \tan\beta|}, \quad \text{(B10)}$$

where $R_d = \tan\beta$. Finally for the case of the slectrons

$$M_{\tilde{e}11}^2 = M_{\tilde{L}}^2 + m_e^2 - M_Z^2(\tfrac{1}{2} - s_W^2)\cos 2\beta,$$

$$M_{\tilde{e}12}^2 = m_e(A_e^* - \mu \tan\beta),$$

$$M_{\tilde{e}12}^2 = m_e(A_e - \mu^* \tan\beta),$$

$$M_{\tilde{e}22}^2 = m_{\tilde{E}}^2 + m_e^2 - M_Z^2 s_W^2 \cos 2\beta. \quad \text{(B11)}$$

Expressions for θ_e and ϕ_e are identical to the case of the down quark with the replacement of d by e.

APPENDIX C: RG EVOLUTION OF ELECTRIC DIPOLE, COLOR DIPOLE, AND PURELY GLUONIC OPERATORS

In this appendix we discuss the renormatization group (RG) evolution of the EDMs discussed in Sec. X. As discussed, there are three competing operators that contribute to the EDM of the neutron. These are

$$\mathcal{O}_E = -\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu},$$

$$\mathcal{O}_{qC} = -\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 T^a q G^{\mu\nu a},$$

$$\mathcal{O}_G = -\tfrac{1}{6}f^{abc}G_a G_b \tilde{G}_c. \tag{C1}$$

The one loop RG evolution of the electric dipole and color dipole operators can be obtained using their anomalous dimensions since these operators are eigenstates under the renormalization group. Evolving these operators from a high scale $Q=M_Z$ to a low scale μ one finds

$$\mathcal{O}_i(\mu) = \Gamma^{-\gamma_i/\beta}\mathcal{O}_i(Q), \tag{C2}$$

where

$$\Gamma = \frac{g_s(\mu)}{g_s(Q)}, \quad \gamma_C = (29-2N_f)/3,$$

$$\gamma_E = 8/3, \quad \beta = (33-2N_f)/3, \tag{C3}$$

and N_f is the number of light quarks at the scale μ. Regarding the purely gluonic dimension 6 operator it obeys the following renormalization group equation (Weinberg, 1989; Boyd *et al.*, 1990; Braaten *et al.*, 1990a, 1990b; Dai *et al.*, 1990):

$$\mu\frac{\partial}{\partial\mu}\mathcal{O}_G = \frac{\alpha_s(\mu)}{4\pi}\left(\gamma_G\mathcal{O}_G - 6\sum_q m_q(\mu)\mathcal{O}_{qC}\right), \tag{C4}$$

where $\gamma_G=-3-2N_f$. The gauge coupling α_s and the running quark mass satisfy the RG equations

$$\mu\frac{\partial}{\partial\mu}g_s(\mu) = -\beta\frac{\alpha_s(\mu)}{4\pi}g_s(\mu) \tag{C5}$$

and

$$\mu\frac{\partial}{\partial\mu}m_q(\mu) = \gamma_m\frac{\alpha_s(\mu)}{4\pi}m_q(\mu), \tag{C6}$$

where $\gamma_m=-8$. The above operators contribute to the *CP* violating Lagrangian multiplied by coefficients which must cancel their μ dependence. This allows one to obtain for the coefficients the following relations:

$$d^{(E,C,G)}(\mu) \simeq \Gamma^{\gamma_{(E,C,G)}/\beta}d^{(E,C,G)}(Q), \tag{C7}$$

where Q is the high scale. In implementing the RG evolution one uses the matching conditions due to crossing the heavy thresholds for $q=b,c$. Thus, for example,

$$d^G(m_q^-) = d^G(m_q^+) + d^C(m_q)\frac{1}{8\pi}\frac{\alpha_s(m_q)}{m_q}. \tag{C8}$$

Using this technique one can evolve the EDMs from the electroweak scale $Q=M_Z$ down to the hadronic scale μ. A more up-to-date discussion of the RG evolution of operators including the mixings between the electric and the chromoelectric operators has been given by Degrassi *et al.* (2005).

TABLE I. Three parameter sets with A_0 in units of m_0.

Case	m_0, $m_{1/2}$, $\lvert A_0\rvert$	α_A, ξ_1, ξ_2, ξ_3
a	200, 200, 4	1, 0.5, 0.659, 0.633
b	370, 370, 4	2, 0.6, 0.653, 0.672
c	320, 320, 3	0.8, 0.4, 0.668, 0.6

APPENDIX D: SATISFACTION OF THE EDM CONSTRAINTS IN THE CANCELLATION MECHANISM

Here we give some examples of the parameter points where the cancellation mechanism discussed in Sec. X.D works to produce d_e, d_n, and d_{Hg} consistent with the current limits. Table I gives three sets of points a, b and c for which the corresponding EDMs d_e, d_n, and C_{Hg} are listed in Table II where C_{Hg} is related to the $\tilde{d}_d^C$, $\tilde{d}_u^C$, $\tilde{d}_s^C$ by . Using the experimental constraints on d_{Hg} one obtains the following constraint on C_{Hg}:

$$C_{Hg} < 3.0\times10^{-26}\ \text{cm}. \tag{D1}$$

The values of C_{Hg} listed in Table II are consistent with the above experimental constraint.

APPENDIX E: COMBINATION OF *CP* PHASES IN SUSY PROCESSES

The various phenomena discussed in Secs. IX and X involve several specific combinations of *CP* phases. We exhibit these combinations.

In Table III θ_1 is defined so that $\theta_1=\theta_\mu+\theta_H$ and the rest of phases are defined as in Eqs. (31) and (33).

APPENDIX F: DETAILS OF $g_\mu-2$ ANALYSIS IN SUSY WITH *CP* PHASES

Here we present further details on the analysis of a_μ discussed in Sec. XI.A but limiting ourselves to the case when the muon mass can be neglected relative to other masses. The chargino exchange contribution is given by

$$a_\mu^{\chi^-} = a_\mu^{21} + a_\mu^{22}, \tag{F1}$$

where for a_μ^{21} and a_μ^{22} we consider now the limit where $I_3(\alpha,\beta)$ and $I_4(\alpha,\beta)$ that appear in Eq. (113) have their first arguments set to zero. In this case one has

TABLE II. Electron, neutron, and H_g EDMs.

Case	d_e (e cm)	d_n (e cm)	C_{Hg} (cm)
a	1.45×10^{-27}	9.2×10^{-27}	7.2×10^{-27}
b	-1.14×10^{-27}	-7.9×10^{-27}	2.87×10^{-26}
c	-3.5×10^{-27}	7.1×10^{-27}	2.9×10^{-26}

TABLE III. Examples of *CP* phases in SUSY phenomena.

SUSY quantity	Combinations of *CP* phases
$p\to\overline{\nu_i}K^+$	$\xi_{1,2,3}+\theta_1,\ \alpha_{A_{t,b}}+\theta_1$
$b\to s+\gamma$	$\alpha_{A_{t,s,b}}+\theta_1,\ \xi_{1,2,3}+\theta_1$
H_i^0 mixing and spectrum	$\alpha_{A_{t,b,\tau}}+\theta_1,\ \xi_{1,2}+\theta_1$
$H^+\to\chi^0\chi^+$	$\alpha_{A_{t,b}}+\theta_1,\ \xi_{1,2}+\theta_1$
$g_\mu-2$	$\xi_{1,2}+\theta_1,\ \alpha_{A_\mu}+\theta_1$
$\tilde q\to q\chi$	$\alpha_{A_q}+\theta_1,\ \xi_{1,2,3}+\theta_1$
Dark matter	$\alpha_{A_q}+\theta_1,\ \xi_1+\theta_1$
$H^0\to\chi^+\chi^-$	$\xi_2+\theta_1,\ \alpha_{A_{b,t}}+\theta_1,\ \xi_1+\theta_1$
d_e (d_μ)	$\xi_{1,2}+\theta_1,\ \alpha_{A_e}+\theta_1(\alpha_{A_\mu}+\theta_1)$
d_n	$\xi_{1,2,3}+\theta_1,\ \alpha_{A_{ui}}+\theta_1,\alpha_{A_{di}}+\theta_1$

$$I_3(0,x)=-\tfrac{1}{2}F_3(x),\quad I_4(0,x)=-\tfrac{1}{6}F_4(x),\tag{F2}$$

where

$$F_3(x)=\frac{1}{(x-1)^3}(3x^2-4x+1-2x^2\ln x),$$

$$F_4(x)=\frac{1}{(x-1)^4}(2x^3+3x^2-6x+1-6x^2\ln x).\tag{F3}$$

In the limit considered above one has the following explicit expressions for the chargino contributions:

$$a_\mu^{21}=\frac{m_\mu\alpha_{\rm EM}}{4\pi\sin^2\theta_W}\sum_{i=1}^{2}\frac{1}{M_{\chi_i^+}}\,\mathrm{Re}(\kappa_\mu U_{i2}^*V_{i1}^*)F_3\left(\frac{M_{\tilde\nu}^2}{M_{\chi_i^+}^2}\right)\tag{F4}$$

and

$$a_\mu^{22}=\frac{m_\mu^2\alpha_{\rm EM}}{24\pi\sin^2\theta_W}\sum_{i=1}^{2}\frac{1}{M_{\chi_i^+}^2}(|\kappa_\mu U_{i2}^*|^2+|V_{i1}|^2)F_4\left(\frac{M_{\tilde\nu}^2}{M_{\chi_i^+}^2}\right),\tag{F5}$$

where

$$\kappa_\mu=\frac{m_\mu}{\sqrt{2}M_W\cos\beta}.\tag{F6}$$

Next we discuss the neutralino exchange contributions to a_μ. These are given by

$$a_\mu^{\chi^0}=a_\mu^{11}+a_\mu^{12},\tag{F7}$$

where

$$a_\mu^{11}=\frac{m_\mu\alpha_{\rm EM}}{2\pi\sin^2\theta_W}\sum_{j=1}^{4}\sum_{k=1}^{2}\frac{1}{M_{\chi_j^0}}\,\mathrm{Re}(\eta_{\mu j}^k)I_1\left(\frac{m_\mu^2}{M_{\chi_j^0}^2},\frac{M_{\tilde\mu_k}^2}{M_{\chi_j^0}^2}\right)\tag{F8}$$

and

$$a_\mu^{12}=\frac{m_\mu^2\alpha_{\rm EM}}{4\pi\sin^2\theta_W}\sum_{j=1}^{4}\sum_{k=1}^{2}\frac{1}{M_{\chi_j^0}^2}X_{\mu j}^kI_2\left(\frac{m_\mu^2}{M_{\chi_j^0}^2},\frac{M_{\tilde\mu_k}^2}{M_{\chi_j^0}^2}\right).\tag{F9}$$

Here $\eta_{\mu j}^k$ is defined by

$$\eta_{\mu j}^k=-\left(\frac{1}{\sqrt{2}}[\tan\theta_WX_{1j}+X_{2j}]D_{1k}^*-\kappa_\mu X_{3j}D_{2k}^*\right)\times(\sqrt{2}\tan\theta_WX_{1j}D_{2k}+\kappa_\mu X_{3j}D_{1k})\tag{F10}$$

and $X_{\mu j}^k$ is defined by

$$\begin{aligned}X_{\mu j}^k={}&\frac{m_\mu^2}{2M_W^2\cos^2\beta}|X_{3j}|^2+\frac{1}{2}\tan^2\theta_W|X_{1j}|^2(|D_{1k}|^2\\&+4|D_{2k}|^2)+\frac{1}{2}|X_{2j}|^2|D_{1k}|^2\\&+\tan\theta_W|D_{1k}|^2\,\mathrm{Re}(X_{1j}X_{2j}^*)\\&+\frac{m_\mu\tan\theta_W}{M_W\cos\beta}\,\mathrm{Re}(X_{3j}X_{1j}^*D_{1k}D_{2k}^*)\\&-\frac{m_\mu}{M_W\cos\beta}\,\mathrm{Re}(X_{3j}X_{2j}^*D_{1k}D_{2k}^*).\end{aligned}\tag{F11}$$

In the limit when the muon mass is neglected relative to other masses and the first argument in the double integral is taken to be zero one finds a simplification of the form factors so that

$$I_1(0,x)=\tfrac{1}{2}F_1(x),\quad I_2(0,x)=\tfrac{1}{6}F_2(x),\tag{F12}$$

where

$$F_1(x)=\frac{1}{(x-1)^3}(1-x^2+2x\ln x),\tag{F13}$$

and

$$F_2(x)=\frac{1}{(x-1)^4}(-x^3+6x^2-3x-2-6x\ln x).\tag{F14}$$

APPENDIX G: STOP EXCHANGE CONTRIBUTIONS TO HIGGS MASS² MATRIX

For completeness we present an analysis of the one loop contributions from the stop sector with inclusion of *CP* violating effects in the analysis of *CP* even–*CP* odd Higgs mixings discussed in Sec. XI.B. The contribution to the one loop effective potential from the stop and top exchanges is given by

$$\Delta V(\tilde t,t)=\frac{1}{64\pi^2}\left[\sum_{a=1,2}6M_{\tilde t_a}^4\left(\ln\frac{M_{\tilde t_a}^2}{Q^2}-\frac{3}{2}\right)-12m_t^4\left(\ln\frac{m_t^2}{Q^2}-\frac{3}{2}\right)\right].\tag{G1}$$

Using the above potential our analysis for $\Delta_{ij\tilde t}$ gives

$$\Delta_{11\tilde{t}} = -2\beta_{h_t} m_t^2 |\mu|^2 \frac{(|A_t|\cos\gamma_t - |\mu|\cot\beta)^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2), \tag{G2}$$

$$\begin{aligned}\Delta_{22\tilde{t}} = &-2\beta_{h_t} m_t^2 \frac{|A_t|^2[|A_t| - |\mu|\cot\beta\cos\gamma_t]^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 2\beta_{h_t} m_t^2 \ln\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right)\\ &+ 4\beta_{h_t} m_t^2 \frac{|A_t|[|A_t| - |\mu|\cot\beta\cos\gamma_t]}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right),\end{aligned} \tag{G3}$$

$$\begin{aligned}\Delta_{12\tilde{t}} = &-2\beta_{h_t} m_t^2 \frac{|\mu|[|A_t|\cos\gamma_t - |\mu|\cot\beta]}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right) + 2\beta_{h_t} m_t^2 f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\\ &\times \frac{|\mu|\,|A_t|[|A_t|\cos\gamma_t - |\mu|\cot\beta][|A_t| - |\mu|\cot\beta\cos\gamma_t]}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2},\end{aligned} \tag{G4}$$

$$\Delta_{13\tilde{t}} = -2\beta_{h_t} m_t^2 \frac{|\mu|^2|A_t|\sin\gamma_t[|\mu|\cot\beta - |A_t|\cos\gamma_t]}{\sin\beta(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2), \tag{G5}$$

$$\Delta_{23\tilde{t}} = -2\beta_{h_t} m_t^2 |\mu|\,|A_t|^2 \frac{\sin\gamma_t(|A_t| - |\mu|\cot\beta\cos\gamma_t)}{\sin\beta(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 2\beta_{h_t} \frac{m_t^2|\mu|\,|A_t|\sin\gamma_t}{\sin\beta(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right), \tag{G6}$$

and

$$\Delta_{33\tilde{t}} = -2\beta_{h_t} \frac{m_t^2|\mu|^2|A_t|^2\sin^2\gamma_t}{\sin^2\beta(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} f_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2). \tag{G7}$$

In the above analysis the D terms of the squark $(\text{mass})^2$ matrices are ignored to obtain approximate independence of the renormalization scale Q similar to the analysis of Demir (1999) and Carena *et al.* (2000).

APPENDIX H: FIERZ REARRANGEMENT RELATIONS INVOLVING MAJORANAS

Fierz rearrangements are known to be useful when manipulating interactions involving four fermions. Specifically such Fierz rearrangements are needed in the analysis of Sec. XI.G. Here we present these relations for the case when two of the fermions are Majoranas (such as neutralinos) and the other two are quarks. Thus any four fermi interactions with two Majoranas and two quarks can be written as involving the following combinations:

$$\begin{gathered}\bar{\chi}\chi\bar{q}q, \quad \bar{\chi}\gamma_5\chi\bar{q}\gamma_5 q, \quad \bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu q,\\ \bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu\gamma_5 q, \quad \bar{\chi}\gamma_5\chi\bar{q}q, \quad \bar{\chi}\chi\bar{q}\gamma_5 q.\end{gathered} \tag{H1}$$

For convenience define the 16 gamma matrices as follows:

$$\Gamma^A = \{1, \gamma^0, i\gamma^j, i\gamma^0\gamma_5, \gamma^j\gamma_5, \gamma_5, i\sigma^{0i}, \sigma^{ij}\}: \ i,j = 1-3 \tag{H2}$$

with the normalization

$$\mathrm{tr}(\Gamma^A\Gamma^B) = 4\delta^{AB}. \tag{H3}$$

The Fierz rearrangement formula with the above definitions and normalizations is

$$(\overline{u_1}\Gamma^A u_2)(\overline{u_3}\Gamma^B u_4) = \sum_{C,D} F_{CD}^{AB}(\overline{u_1}\Gamma^C u_4)(\overline{u_3}\Gamma^D u_2), \tag{H4}$$

where u_j are Dirac or Majorana spinors and

$$F_{CD}^{AB} = -(+)\tfrac{1}{16}\mathrm{tr}(\Gamma^C\Gamma^A\Gamma^D\Gamma^B) \tag{H5}$$

and where the plus sign is for commuting u spinors and the minus sign is for the anticommuting u fields. In our case we have to use the minus sign since we are dealing with quantum Majorana and Dirac fields in the Lagrangian. We give below the Fierz rearrangement for four combinations that appear commonly in neutralino-quark scattering. These are

$$\bar{\chi}q\bar{q}\chi = -\tfrac{1}{4}\bar{\chi}\chi\bar{q}q - \tfrac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}\gamma_5 q + \tfrac{1}{4}\bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu\gamma_5 q,$$

$$\bar{\chi}\gamma_5 q\bar{q}\chi = \tfrac{1}{4}\bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu q - \tfrac{1}{4}\bar{\chi}\chi\bar{q}\gamma_5 q - \tfrac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}q,$$

$$\bar{\chi}q\bar{q}\gamma_5\chi = -\tfrac{1}{4}\bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu q - \tfrac{1}{4}\bar{\chi}\chi\bar{q}\gamma_5 q - \tfrac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}q,$$

$$\bar{\chi}\gamma_5 q\bar{q}\gamma_5\chi = -\tfrac{1}{4}\bar{\chi}\chi\bar{q}q - \tfrac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}\gamma_5 q - \tfrac{1}{4}\bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu\gamma_5 q. \quad \text{(H6)}$$

The metric used above is $\eta_{\mu\nu}=(1,-1,-1,-1)$, and since χ's are Majoranas we have used the properties $\bar{\chi}\gamma_\mu\chi=0$ and $\bar{\chi}\sigma_{\mu\nu}\chi=0$.

APPENDIX I: EFFECTIVE FOUR-FERMI INTERACTION FOR DARK MATTER DETECTION WITH INCLUSION OF *CP* PHASES

In this appendix we present a derivation of the four-fermi neutralino-quark effective Lagrangian with *CP* violating phases given in Sec. XI.G. We begin by discussing the squark exchange contribution. From the fundamental supergravity Lagrangian of quark-squark-neutralino interactions

$$-\mathcal{L} = \bar{q}[C_{qL}P_L + C_{qR}P_R]\chi\tilde{q}1 + \bar{q}[C'_{qL}P_L + C'_{qR}P_R]\chi\tilde{q}2 + \text{H.c.} \quad \text{(I1)}$$

the effective Lagrangian for q-χ scattering via the exchange of squarks is given by (Chattopadhyay *et al.*, 1999; Falk, Ferstl, and Olive, 1999)

$$\mathcal{L}_{\text{eff}} = \frac{1}{M^2_{\tilde{q}1} - M^2_\chi}\bar{\chi}[C^*_{qL}P_R + C^*_{qR}P_L]q \times \bar{q}[C_{qL}P_L + C_{qR}P_R]\chi + \frac{1}{M^2_{\tilde{q}2} - M^2_\chi}\bar{\chi}[C^{*\prime}_{qL}P_R + C^{*\prime}_{qR}P_L]q\bar{q}[C'_{qL}P_L + C'_{qR}P_R]\chi, \quad \text{(I2)}$$

where

$$C_{qL} = \sqrt{2}(\alpha_{q0}D_{q11} - \gamma_{q0}D_{q21}),$$
$$C_{qR} = \sqrt{2}(\beta_{q0}D_{q11} - \delta_{q0}D_{q21}),$$
$$C'_{qL} = \sqrt{2}(\alpha_{q0}D_{q12} - \gamma_{q0}D_{q22}),$$
$$C'_{qR} = \sqrt{2}(\beta_{q0}D_{q12} - \delta_{q0}D_{q22}), \quad \text{(I3)}$$

and α, β, γ, and δ are given by

$$\alpha_{u(d)j} = \frac{gm_{u(d)}X_{4(3)j}}{2m_W\sin\beta(\cos\beta)},$$

$$\beta_{u(d)j} = eQ_{u(d)j}X'^*_{1j} + \frac{g}{\cos\theta_W}X'^*_{2j} \times (T_{3u(d)} - Q_{u(d)}\sin^2\theta_W),$$

$$\gamma_{u(d)j} = eQ_{u(d)j}X'_{1j} - \frac{gQ_{u(d)}\sin^2\theta_W}{\cos\theta_W}X'_{2j},$$

$$\delta_{u(d)j} = \frac{-gm_{u(d)}X^*_{4(3)j}}{2m_W\sin\beta(\cos\beta)}. \quad \text{(I4)}$$

Here g is the $SU(2)_L$ gauge coupling and

$$X'_{1j} = X_{1j}\cos\theta_W + X_{2j}\sin\theta_W,$$
$$X'_{2j} = -X_{1j}\sin\theta_W + X_{2j}\cos\theta_W. \quad \text{(I5)}$$

The effect of the *CP* violating phases enter via the neutralino eigenvector components X_{ij} and via the matrix D_{qij} that diagonalizes the squark mass^2 matrix.

Using the Fierz rearrangement one can now obtain the coefficients A, B, C, D, E, and F that appear in Eq. (171) in a straightforward fashion (Chattopadhyay *et al.*, 1999; Falk, Ferstl, and Olive, 1999). The first two terms (A,B) are spin-dependent interactions and arise from the Z boson and sfermion exchanges. For these one has

$$A = \frac{g^2}{4M_W^2}[|X_{30}|^2 - |X_{40}|^2][T_{3q} - e_q\sin^2\theta_W] - \frac{|C_{qR}|^2}{4(M^2_{\tilde{q}1} - M^2_\chi)} - \frac{|C'_{qR}|^2}{4(M^2_{\tilde{q}2} - M^2_\chi)}, \quad \text{(I6)}$$

$$B = -\frac{g^2}{4M_W^2}[|X_{30}|^2 - |X_{40}|^2]e_q\sin^2\theta_W + \frac{|C_{qL}|^2}{4(M^2_{\tilde{q}1} - M^2_\chi)} + \frac{|C'_{qL}|^2}{4(M^2_{\tilde{q}2} - M^2_\chi)}. \quad \text{(I7)}$$

The terms C, D, E, and F receive contributions from sfermions and from neutral Higgs and can be calculated using similar techniques.

APPENDIX J: COMPUTATIONAL TOOLS FOR SUSY PHENOMENA WITH *CP* PHASES

Numerical analysis of supersymmetric phenomena with *CP* phases is significantly more difficult than for the case when the phases are absent. First, most numerical integration codes for the renormalization group evolution, sparticle spectra, and for the analysis of sparticle decays and cross sections are not equipped to handle phases. Second, any physically meaningful set of parameters which include phases must necessarily satisfy the stringent EDM constraints which also require care. Significant progress has been in this direction by the so called CPsuperH (Lee *et al.*, 2004), which is a FORTRAN code that calculates the mass spectrum and decay widths of the neutral and charged Higgs bosons in MSSM with *CP* phases. There is significant room for further progress in this area.

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