

Inclusive quasielastic electron-nucleus scattering

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This paper presents a review on the field of inclusive quasielastic electron-nucleus scattering. It discusses the approach used to measure the data and includes a compilation of data available in numerical form. The theoretical approaches used to interpret the data are presented. A number of results obtained from the comparison between experiment and calculation are then reviewed. The analogies to, and differences from, other fields of physics exploiting quasielastic scattering from composite systems are pointed out.

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I. INTRODUCTION

The energy spectrum of high-energy leptons (electrons in particular) scattered from a nuclear target displays a number of features. At low energy loss (ω),

peaks due to elastic scattering and inelastic excitation of discrete nuclear states appear; a measurement of the corresponding form factors as a function of momentum transfer $|\mathbf{q}|$ gives access to the Fourier transform of nuclear (transition) densities. At larger energy loss, a broad peak due to quasielastic electron-nucleon scattering appears; this peak—very wide due to nuclear Fermi motion—corresponds to processes by which the electron scatters from an individual, moving nucleon, which, after interaction with other nucleons, is ejected from the target. At even larger ω , peaks that correspond to excitation of the nucleon to distinct resonances are visible. At very large ω , a structureless continuum due to deep inelastic scattering (DIS) on quarks bound in nucleons appears. A schematic spectrum is shown in Fig. 1. At momentum transfers above approximately 500 MeV/c, the dominant feature of the spectrum is the quasielastic peak.

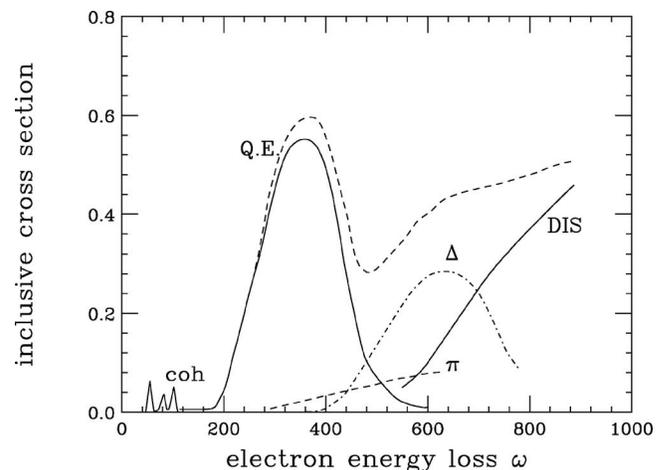


FIG. 1. Schematic representation of inclusive cross section as a function of energy loss.

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A number of questions have been investigated using quasielastic scattering:

- The quasielastic cross section integrated over electron energy loss is proportional to the sum of electron-nucleon cross sections. Historically, this has been exploited in order to measure the neutron charge and magnetic form factors using mainly light ($A < 4$) nuclear targets. Today the emphasis has shifted to exposing possible medium modifications of the nucleon form factors.
- Another integral property of the quasielastic peak, its width, provides a direct measure of the average momentum of nucleons in nuclei, and has been used to determine nuclear Fermi momenta; contrary to other observables such as densities, quasielastic scattering provides a *direct* determination via an observable sensitive to the momenta of nucleons.
- The *shape* of the quasielastic peak depends on the distribution in energy E and momentum \mathbf{k} of the initially bound nucleons. Precise measurements give indirect access to (integrated properties of) the nuclear spectral function $S(\mathbf{k}, E)$ describing this distribution. In particular, the tail of the quasielastic peak at large $|\mathbf{q}|$ and low ω is sensitive to the tail of the spectral function at large $|\mathbf{k}|$.
- Processes more complicated than one-nucleon knockout, in particular those related to non-nucleonic degrees of freedom and meson exchange currents (MEC), also play a role. They can be investigated by separating the quasielastic response into the longitudinal (charge) and transverse (magnetic plus convection) pieces, the latter being preferentially affected by MEC.
- *Scaling* is one of the unique features of quasielastic scattering. This refers to the fact that the inclusive cross section, which *a priori* is a function of two independent variables $|\mathbf{q}|$ and ω , depends on a single variable $y(q, \omega)$. This scaling property, a consequence of the kinematics of the underlying electron-nucleon elastic scattering process, provides a strong handle on the reaction mechanism. Further, the scaling violations that are observed reveal how the dynamics go beyond the impulse approximation (IA) picture of quasielastic scattering.

Inclusive quasielastic electron-nucleus scattering is not the only process of this type. This quasielastic process occurs in other areas of physics, and is being exploited to learn about the dynamics of the underlying composite system:

- (a) Quasielastic scattering of keV-energy photons from electrons bound in atoms provides information about the energy and momentum distribution of bound electrons.
- (b) Scattering of eV to keV neutrons from condensed-matter systems such as liquid helium provides a measurement of the He momentum distribution

and correlations, and has been exploited for a long time to isolate the effects of the Bose condensate in superfluid helium.

- (c) Quasielastic scattering of GeV-energy leptons on quarks bound in nucleons (DIS) has provided a wealth of information on the quark distribution functions; the observation of scaling violations in DIS has taught us much about the dynamics of strong interactions.

Inclusive electron scattering from nuclei is a subclass of quasielastic processes, the most obvious other representative being $(e, e'p)$. When compared to exclusive processes in which the knocked-out nucleon is detected, (e, e') corresponds to an integral over all final states of the nucleon, and consequently provides less specific information. On the other hand, (e, e') is more directly related to the dynamics of the initial hadronic (nuclear) ground state. The complications of the final hadronic continuum play a much smaller role. This is true particularly at large $|\mathbf{q}|$, as the electron is sensitive to the fate of the recoiling nucleon in a region of size $1/|\mathbf{q}|$; the final-state interaction (FSI) beyond that region affects only the more exclusive processes.

Quasielastic scattering from nuclei, compared to the other quasielastic processes mentioned above, has one distinct drawback one must deal with: the nucleon the electron scatters from is not elementary, but can be excited to various states. At large momentum transfer, the result is that only the low- ω side of the quasielastic peak can be exploited; the large- ω side is obscured by the overlap with Δ excitation.

During the past decade, remarkable progress in our understanding of quasielastic scattering has allowed us to define a number of features precisely. In particular, for light nuclei and nuclear matter, quasielastic scattering has provided accurate determinations of (integrated properties of) the nuclear spectral function. Detailed investigations of the reaction mechanism and the conditions necessary and/or sufficient for scaling have clearly shown that the dominant process is elastic scattering from individual nucleons. The different scale-breaking mechanisms have been identified. The effects beyond IA, mainly due to FSI, have come under much better control. The kinematic region in which more complicated processes, such as MEC, are important has also been identified, and the size of two-body current contributions has been understood.

In this review, we give a comprehensive discussion on the various aspects of quasielastic scattering. Quantitative understanding of the cross section starts from a description in terms of IA; effects beyond IA such as the role of FSI and MEC are addressed next. We then describe some experimental aspects important for the study of this reaction. We then give a compilation of the experiments that have been performed and point out where the cross sections (most often not published in numerical form) can be found. We also briefly describe experiments that provide the nucleon form factors needed to understand quasielastic scattering. We subse-

quently discuss scaling and the related superscaling. For light nuclei and nonrelativistic final states, exact calculations can be performed. For lower momentum transfers, an alternative approach, the use of the Euclidean response, is available and presented. We then study the results obtained after a longitudinal/transverse (L/T) separation of the cross section, and their impact on the Coulomb sum rule. A bothersome correction, namely, the effect of Coulomb distortion on the cross sections, is addressed as well. We also show how data for an important model system for nuclear theory, infinite nuclear matter, can be obtained. Last, we address other fields of quasielastic scattering and discuss their common aspects.

II. ELECTRON-NUCLEUS SCATTERING IN THE IMPULSE APPROXIMATION

A. Electron-nucleus cross section

The differential cross section of the process

$$e + A \rightarrow e' + X, \quad (1)$$

in which an electron of initial four-momentum $k_e \equiv (E_e, \mathbf{k}_e)$ scatters off a nuclear target to a state of four-momentum $k'_e \equiv (E_{e'}, \mathbf{k}_{e'})$, the target final state being undetected, can be written in the Born approximation as (Itzykson and Zuber, 1980)

$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2 E_{e'}}{Q^4 E_e} L_{\mu\nu} W^{\mu\nu}, \quad (2)$$

where $\alpha=1/137$ is the fine-structure constant, $d\Omega_{e'}$ is the differential solid angle in the direction specified by $\mathbf{k}_{e'}$, $Q^2=-q^2$, and $q=k_e-k_{e'} \equiv (\omega, \mathbf{q})$ is the four-momentum transfer.

The tensor $L_{\mu\nu}$, which can be written neglecting the lepton mass as

$$L_{\mu\nu} = 2[k_e^\mu k_{e'}^\nu + k_e^\nu k_{e'}^\mu - g^{\mu\nu}(k_e k_{e'})], \quad (3)$$

where $g^{\mu\nu} \equiv \text{diag}(1, -1, -1, -1)$ and $(k_e k_{e'}) = E_e E_{e'} - \mathbf{k}_e \cdot \mathbf{k}_{e'}$ is fully specified by the measured electron kinematic variables. All information on target structure is contained in the tensor $W^{\mu\nu}$, whose definition involves the initial and final nuclear states $|0\rangle$ and $|X\rangle$, carrying four-momenta p_0 and p_X , as well as the nuclear current operator J^μ ,

$$W^{\mu\nu} = \sum_X \langle 0 | J^\mu | X \rangle \langle X | J^\nu | 0 \rangle \delta^{(4)}(p_0 + q - p_X), \quad (4)$$

where the sum includes all hadronic final states.

The most general expression of the target tensor of Eq. (4), fulfilling the requirements of Lorentz covariance, conservation of parity, and gauge invariance, can be written in terms of two structure functions W_1 and W_2 as

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2}{M^2} \left(p_0^\mu - \frac{(p_0 q)}{q^2} q^\mu \right) \left(p_0^\nu - \frac{(p_0 q)}{q^2} q^\nu \right), \quad (5)$$

where M is the target mass and the structure functions depend on the two scalars Q^2 and $(p_0 q)$. In the target rest frame, $(p_0 q) = m\omega$ and W_1 and W_2 become functions of the measured momentum and energy transfer $|\mathbf{q}|$ and ω .

Substitution of Eq. (5) into Eq. (2) leads to

$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}} \right)_M \times \left[W_2(|\mathbf{q}|, \omega) + 2W_1(|\mathbf{q}|, \omega) \tan^2 \frac{\theta}{2} \right], \quad (6)$$

where θ and $(d\sigma/d\Omega_{e'})_M = \alpha^2 \cos^2(\theta/2)/4E_e \sin^4(\theta/2)$ denote the electron scattering angle and the Mott cross section, respectively.

The right-hand side of Eq. (6) can be rewritten singling out the contributions of scattering processes induced by longitudinally (L) and transversely (T) polarized virtual photons. The resulting expression is

$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}} \right)_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|, \omega) + \left(\frac{1}{2} \frac{Q^2}{|\mathbf{q}|^2} + \tan^2 \frac{\theta}{2} \right) R_T(|\mathbf{q}|, \omega) \right], \quad (7)$$

where the longitudinal and transverse structure functions are trivially related to W_1 and W_2 through

$$R_T(|\mathbf{q}|, \omega) = 2W_1(|\mathbf{q}|, \omega) \quad (8)$$

and

$$\frac{Q^2}{|\mathbf{q}|^2} R_L(|\mathbf{q}|, \omega) = W_2(|\mathbf{q}|, \omega) - \frac{Q^2}{|\mathbf{q}|^2} W_1(|\mathbf{q}|, \omega). \quad (9)$$

In principle, calculations of $W^{\mu\nu}$ of Eq. (4) at moderate momentum transfer ($|\mathbf{q}| < 0.5$ GeV/ c) can be carried out within nuclear many-body theory (NMBT), using nonrelativistic wave functions to describe the initial and final states and expanding the current operator in powers of $|\mathbf{q}|/m$ (Carlson and Schiavilla, 1998), where m is the nucleon mass. The available results for medium-heavy targets have been obtained mostly using the mean-field approach, supplemented by inclusion of model residual interactions to take into account long-range correlations (Dellafiore *et al.*, 1985).

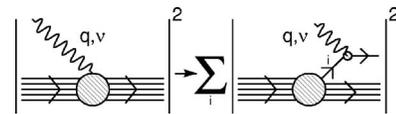


FIG. 2. Schematic representation of the IA regime, in which the nuclear cross section is replaced by the incoherent sum of cross sections describing scattering off individual nucleons, the recoiling $(A-1)$ -nucleon system acting as a spectator.

On the other hand, at higher values of $|\mathbf{q}|$, corresponding to beam energies larger than ~ 1 GeV, describing the final states $|X\rangle$ in terms of nonrelativistic nucleons is no longer possible. Due to the prohibitive difficulties involved in a fully consistent treatment of the relativistic nuclear many-body problem, calculations of $W^{\mu\nu}$ in this regime require a set of simplifying assumptions, allowing one to take into account the relativistic motion of final-state particles carrying momenta $\sim \mathbf{q}$, as well as inelastic processes leading to the production of hadrons other than protons and neutrons.

The IA scheme is based on the assumptions that (i) as the space resolution of the electron probe is $\sim 1/|\mathbf{q}|$, at large momentum transfer scattering off a nuclear target reduces to the incoherent sum of elementary scattering processes involving individual *bound* nucleons (see Fig. 2) and (ii) there is no FSI between hadrons produced in electron-nucleon scattering and the recoiling nucleus.¹ Under these assumptions, a relativistic particle in the final state is completely decoupled from the spectator system, and the description of its motion becomes a simple kinematic problem.

Within the IA picture, the nuclear current appearing in Eq. (4) is written as a sum of one-body currents

$$J^\mu \rightarrow \sum_i j_i^\mu, \quad (10)$$

while $|X\rangle$ reduces to the direct product of the hadronic state produced at the electromagnetic vertex, carrying momentum \mathbf{p}_x , and the state describing the $(A-1)$ -nucleon residual system, carrying momentum $\mathbf{p}_R = \mathbf{q} - \mathbf{p}_x$ (in order to simplify the notation, spin indices will be omitted),

$$|X\rangle \rightarrow |x, \mathbf{p}_x\rangle \otimes |\mathcal{R}, \mathbf{p}_R\rangle. \quad (11)$$

Using Eq. (11), we can replace

$$\begin{aligned} \sum_X |X\rangle\langle X| &\rightarrow \sum_x \int d^3p_x |x, \mathbf{p}_x\rangle\langle \mathbf{p}_x, x| \\ &\times \sum_{\mathcal{R}} d^3p_R |\mathcal{R}, \mathbf{p}_R\rangle\langle \mathbf{p}_R, \mathcal{R}|. \end{aligned} \quad (12)$$

Substitution of Eqs. (10)–(12) into Eq. (4) and insertion of a complete set of free nucleon states, satisfying

$$\int d^3k |N, \mathbf{k}\rangle\langle \mathbf{k}, N| = 1, \quad (13)$$

then leads to the factorization of the nuclear current matrix element according to

¹Coherent effects, not included in the impulse approximation picture, appear in DIS even at large $|\mathbf{q}|$ for values of the Bjorken scaling variable $x = Q^2/2m\omega \lesssim 0.2$, corresponding to very large electron energy loss.

$$\begin{aligned} \langle 0|J^\mu|X\rangle &= \left(\frac{m}{\sqrt{|\mathbf{p}_R|^2 + m^2}} \right)^{1/2} \langle 0|\mathcal{R}, \mathbf{p}_R; N, -\mathbf{p}_R\rangle \\ &\times \sum_i \langle -\mathbf{p}_R, N|j_i^\mu|x, \mathbf{p}_x\rangle, \end{aligned} \quad (14)$$

where the factor $(m/\sqrt{|\mathbf{p}_R|^2 + m^2})^{1/2}$ takes into account the implicit covariant normalization of $\langle -\mathbf{p}_R, N|$ in the matrix element of j_i^μ .

As a result, the incoherent contribution to Eq. (4) can be rewritten in the form

$$\begin{aligned} W^{\mu\nu} &= \sum_{x, \mathcal{R}} \int d^3p_R d^3p_x |\langle 0|\mathcal{R}, \mathbf{p}_R; N, -\mathbf{p}_R\rangle|^2 \\ &\times \frac{m}{E_R} \sum_i \langle -\mathbf{p}_R, N|j_i^\mu|x, \mathbf{p}_x\rangle\langle \mathbf{p}_x, x|j_i^\nu|N, -\mathbf{p}_R\rangle \\ &\times \delta^{(3)}(\mathbf{q} - \mathbf{p}_R - \mathbf{p}_x) \delta(\omega + E_0 - E_R - E_x), \end{aligned} \quad (15)$$

where E_0 is the target ground-state energy, $E_R = \sqrt{|\mathbf{p}_R|^2 + M_R^2}$, M_R being the mass of the recoiling system and E_x the energy of the final state X .

Finally, using the identity

$$\begin{aligned} \delta(\omega + E_0 - E_R - E_x) &= \int dE \delta(E - m + E_0 - E_R) \\ &\times \delta(\omega - E + m - E_x), \end{aligned} \quad (16)$$

and defining the nucleon spectral function as

$$S_N(\mathbf{k}, E) = \sum_{\mathcal{R}} |\langle 0|\mathcal{R}, -\mathbf{k}; N, \mathbf{k}\rangle|^2 \delta(E - m + E_0 - E_R), \quad (17)$$

where the index $N=p, n$ labels either a proton or a neutron, we can cast Eq. (4) in the form

$$\begin{aligned} W^{\mu\nu}(\mathbf{q}, \omega) &= \int d^3k dE \left(\frac{m}{E_{\mathbf{k}}} \right) [Z S_p(\mathbf{k}, E) w_p^{\mu\nu} \\ &+ (A - Z) S_n(\mathbf{k}, E) w_n^{\mu\nu}], \end{aligned} \quad (18)$$

A and Z being the target mass number and number of protons, respectively. In Eq. (18), $E_{\mathbf{k}} = \sqrt{|\mathbf{k}|^2 + m^2}$ and

$$\begin{aligned} w_N^{\mu\nu} &= \sum_x \langle \mathbf{k}, N|j_N^\mu|x, \mathbf{k} + \mathbf{q}\rangle\langle \mathbf{k} + \mathbf{q}, x|j_N^\nu|N, \mathbf{k}\rangle \\ &\times \delta(\tilde{\omega} + E_{\mathbf{k}} - E_x) \end{aligned} \quad (19)$$

with [see Eqs. (15) and (17)]

$$\tilde{\omega} = E_x - E_{\mathbf{k}} = E_0 + \omega - E_R - E_{\mathbf{k}} = \omega - E + m - E_{\mathbf{k}}. \quad (20)$$

The above equations show that within the IA scheme, the definition of the electron-nucleus cross section involves two important elements: (i) the tensor $w_N^{\mu\nu}$, defined by Eq. (19), describing the electromagnetic interactions of a bound nucleon carrying momentum \mathbf{k} , and (ii) the spectral function, defined by Eq. (17), yielding its momentum and removal energy distribution. These quantities will be further discussed in the following sections.

B. Electron scattering off a bound nucleon

While in electron-nucleon scattering in free space the struck particle is given the entire four-momentum transfer $q \equiv (\tilde{\omega}, \mathbf{q})$, in a scattering process involving a bound nucleon a fraction $\delta\omega$ of the energy loss goes into the spectator system. This mechanism emerges in a most natural fashion from the IA formalism.

Assuming that the current operators are not modified by the nuclear environment, the quantity defined by Eq. (19) can be identified with the tensor describing electron scattering off a *free* nucleon at four-momentum transfer $\tilde{q} \equiv (\tilde{\omega}, \mathbf{q})$. Hence, Eq. (19) shows that within IA, binding is taken into account through the replacement

$$q \equiv (\omega, \mathbf{q}) \rightarrow \tilde{q} \equiv (\tilde{\omega}, \mathbf{q}). \quad (21)$$

The interpretation of $\delta\omega = \omega - \tilde{\omega}$ as the amount of energy going into the recoiling spectator system becomes particularly transparent in the limit $|\mathbf{k}|/m \ll 1$, in which Eq. (20) yields $\delta\omega = E$.

In the case of quasielastic scattering $w_N^{\mu\nu}$ of Eq. (19) can be obtained from [compare to Eq. (5)]

$$w_N^{\mu\nu} = w_1^N \left(-g^{\mu\nu} + \frac{\tilde{q}^\mu \tilde{q}^\nu}{\tilde{q}^2} \right) + \frac{w_2^N}{m^2} \left(k^\mu - \frac{(k\tilde{q})}{\tilde{q}^2} \tilde{q}^\mu \right) \times \left(k^\nu - \frac{(k\tilde{q})}{\tilde{q}^2} \tilde{q}^\nu \right) \quad (22)$$

where $k \equiv (E_{\mathbf{k}}, \mathbf{k})$ and the two structure functions w_1 and w_2 are simply related to the measured electric and magnetic nucleon form factors G_{E_N} and G_{M_N} through

$$w_1^N = -\frac{\tilde{q}^2}{4m^2} \delta \left(\tilde{\omega} + \frac{\tilde{q}^2}{2m} \right) G_{M_N}^2, \quad (23)$$

$$w_2^N = \frac{1}{1 - \tilde{q}^2/4m^2} \delta \left(\tilde{\omega} + \frac{\tilde{q}^2}{2m} \right) \left(G_{E_N}^2 - \frac{\tilde{q}^2}{4m^2} G_{M_N}^2 \right). \quad (24)$$

While the replacement of ω with $\tilde{\omega}$ is reasonable on physics grounds and in fact quite natural in the context of the IA analysis presented above, it poses a considerable conceptual problem in that it leads to a violation of current conservation, which requires

$$q_\mu w_N^{\mu\nu} = 0. \quad (25)$$

A widely used prescription to overcome this difficulty was proposed in the early 1980s (de Forest, 1983). It amounts to using a tensor $\tilde{w}_N^{\mu\nu}$ whose time components are given by Eq. (19), while the longitudinal ones are obtained from the time components requiring that the continuity equation (25) be satisfied. Taking the z axis along the direction of \mathbf{q} , one obtains

$$\begin{aligned} \tilde{w}_N^{\mu\nu} &= w_N^{\mu\nu}(\tilde{q}) \quad \text{for } \mu \text{ and/or } \nu = 0, \\ \tilde{w}_N^{3\nu} &= \frac{\omega}{|\mathbf{q}|} w_N^{0\nu}(\tilde{q}), \end{aligned} \quad (26)$$

with \tilde{q} given by Eq. (21).

The above prescription is manifestly not unique. For example, one might have just as well chosen to use Eq. (19) to obtain the longitudinal components of $\tilde{w}^{\mu\nu}$ and the continuity equation to obtain the time components. However, differences between these two procedures to restore gauge invariance only affect the longitudinal response R_L . As a consequence, they are expected to become less and less important as the momentum transfer increases, electron scattering at large $|\mathbf{q}|$ being largely dominated by transverse contributions. Numerical calculations of the quasielastic cross section off ${}^3\text{He}$ have confirmed that these differences become indeed vanishingly small already in the pion production region (Meier-Hajduk *et al.*, 1989).

The uncertainty associated with the implementation of current conservation has been analyzed by Kim, who has compared the inclusive cross sections computed using the Coulomb gauge (equivalent to the deForest prescription) to those obtained from the Landau and Weyl gauges for a variety of targets and kinematic condition (Kim, 2004). The results corresponding to the different choices agree to better than $\sim 1\%$ for backward angle kinematics, thus confirming that the transverse response is unaffected by gauge ambiguities. For the longitudinal response, Kim finds more substantial ambiguities, as a consequence of the huge scalar and vector potentials present in the σ - ω model; these large potentials, however, only occur when trying to describe nuclei—strongly correlated systems—using a mean-field basis.

Other prescriptions based on the use of free nucleon currents (de Forest, 1983) were employed in the analysis of $(e, e'p)$ data in the region of large missing momentum and energy, where the effects of using different off-shell extrapolations of electron-nucleon scattering become sizable (Rohe, 2004). More fundamental approaches, involving the explicit calculation of the off-shell form factors, necessarily rely on oversimplified dynamical models (Naus and Koch, 1987; Naus *et al.*, 1990).

In conclusion, the violation of gauge invariance in the IA scheme, while in principle an intricate one, turns out to be only marginally relevant to inclusive electron scattering at large momentum transfer. The main effect of nuclear binding can be easily accounted for with the replacement $\omega \rightarrow \tilde{\omega}$.

C. The nuclear spectral function

The spectral function, defined by Eq. (17), gives the probability of removing a nucleon from the target nucleus leaving the residual system with energy $E_{\mathcal{R}} = E_0 - m + E$.

Within the shell-model picture, based on the assumption that nucleons in a nucleus behave as independent particles moving in a mean field, the spectral function reduces to

$$S_{\text{SM}}(\mathbf{k}, E) = \sum_{n \in \{F\}} |\phi_n(\mathbf{k})|^2 \delta(E - E_n), \quad (27)$$

where $\phi_n(\mathbf{k})$ is the momentum-space wave function associated with the single-particle shell-model (SM) state

n , E_n is the corresponding energy eigenvalue, and the sum is extended to all occupied states belonging to the Fermi sea $\{F\}$.

The results of electron- and hadron-induced nucleon knock-out experiments have provided overwhelming evidence of the inadequacy of the independent-particle model to describe the full complexity of nuclear dynamics. While the peaks corresponding to knock-out from shell-model orbits can be clearly identified in the measured energy spectra, the corresponding strengths turn out to be consistently and sizably lower than expected, independent of the nuclear mass number.

This discrepancy is mainly due to the effect of dynamical correlations induced by the nucleon-nucleon (NN) force, whose effect is not taken into account in the independent-particle model. Correlations give rise to scattering processes, leading to the virtual excitation of the participating nucleons to states of energy larger than the Fermi energy, thus depleting the shell-model states within the Fermi sea. As a result, the spectral function acquires tails extending to the region of large energy and momentum, where $S_{SM}(\mathbf{k}, E)$ of Eq. (27) vanishes.

The typical energy scale associated with NN correlations can be estimated considering a pair of correlated nucleons carrying momenta \mathbf{k}_1 and \mathbf{k}_2 much larger than the Fermi momentum (~ 250 MeV/ c). In the nucleus rest frame, where the remaining $A-2$ particles carry low momenta, $\mathbf{k}_1 \approx -\mathbf{k}_2 = \mathbf{k}$. Hence, knock-out of a nucleon of large momentum \mathbf{k} leaves the residual system with a particle in the continuum and requires an energy

$$E \approx E_{\text{thr}} + \mathbf{k}^2/2m, \quad (28)$$

much larger than the typical energies of shell-model states (~ 30 MeV). The above equation, where E_{thr} denotes the threshold for two-nucleon removal, shows that large removal energy and large nucleon momentum are strongly correlated.

Realistic theoretical calculations of the spectral function have been carried out within NMBT, according to which the nucleus consists of a collection of A nucleons whose dynamics are described by the nonrelativistic Hamiltonian

$$H = \sum_{i=1}^A \frac{\mathbf{k}_i^2}{2m} + \sum_{j>i=1}^A v_{ij} + \sum_{k>j>i=1}^A V_{ijk}. \quad (29)$$

In the above equation, \mathbf{k}_i is the momentum of the i th constituent and v_{ij} and V_{ijk} describe two- and three-nucleon interactions, respectively. The two-nucleon potential, which reduces to the Yukawa one-pion-exchange potential at large internucleon distance, is obtained from an accurate fit to the available data on the two-nucleon system, i.e., deuteron properties and ~ 4000 NN scattering data (Wiringa *et al.*, 1995). The additional three-body term V_{ijk} has to be included in order to account for the binding energies of the three-nucleon bound states (Pudliner *et al.*, 1995) and the empirical saturation properties of uniform nuclear matter (Akmal and Pandharipande, 1997); this term results from the fact that non-nucleonic constituents (such as Δ 's) have been excluded.

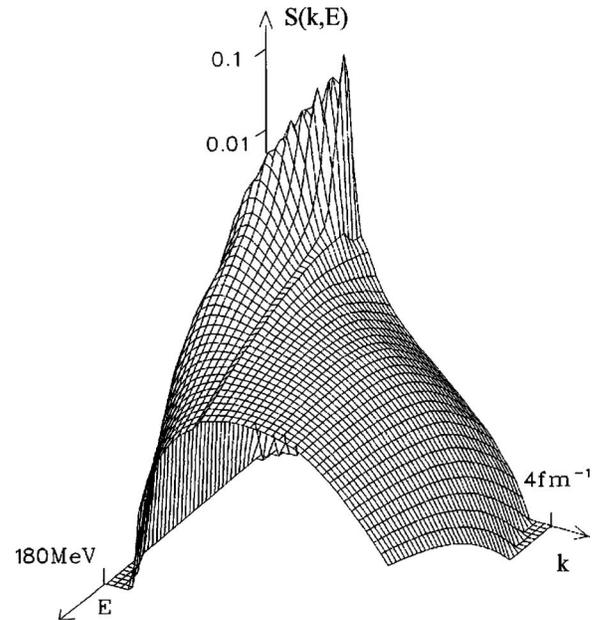


FIG. 3. Nuclear matter spectral function calculated using correlated basis function perturbation theory. From Benhar *et al.*, 1989.

The many-body Schrödinger equation associated with the Hamiltonian of Eq. (29) can be solved exactly, using stochastic methods, for nuclei with mass number $A \leq 12$. The resulting energies of the ground and low-lying excited states are in excellent agreement with the experimental data (Pieper and Wiringa, 2001). Accurate calculations can also be carried out for uniform nuclear matter, exploiting translational invariance and using either a variational approach based on cluster expansion and chain summation techniques (Akmal and Pandharipande, 1997) or G-matrix perturbation theory (Baldo *et al.*, 2000).

Nonrelativistic NMBT has been employed to obtain the spectral functions of the three-nucleon systems (Dieperink *et al.*, 1976; Ciofi degli Atti *et al.*, 1980; Meier-Hajduk *et al.*, 1983), oxygen (Geurts *et al.*, 1996; Polls *et al.*, 1997), and symmetric nuclear matter, having $A \rightarrow \infty$ and $Z=A/2$ (Benhar *et al.*, 1989; Ramos *et al.*, 1989). Calculations based on NMBT but involving some simplifying assumptions have also been carried out for ${}^4\text{He}$ (Ciofi degli Atti *et al.*, 1990; Morita and Suzuki, 1991; Benhar and Pandharipande, 1993).

As an example, Fig. 3 shows the results of a nuclear matter calculation² carried out using correlated basis function (CBF) perturbation theory (Benhar *et al.*, 1989). In addition to the peaks corresponding to single-particle states, i.e., to bound one-hole states of the $(A-1)$ -nucleon system, the resulting $S_N(\mathbf{k}, E)$ exhibits a broad background, extending up to $E \sim 200$ MeV and $|\mathbf{k}| \sim 800$ MeV/ c , associated with n -hole $(n-1)$ -particle $(A-1)$ -nucleon states in which at least one nucleon is

²As in symmetric nuclear matter $S_p(\mathbf{k}, E) = S_n(\mathbf{k}, E)$, the spectral function shown corresponds to an isoscalar nucleon.

excited to the continuum. The correlation ridge at $E \sim \mathbf{k}^2/2m$ [see Eq. (28)] is clearly visible. Note that, in the absence of interactions, the surface shown in Fig. 3 collapses to a collection of δ -function peaks distributed along the line $|E| = \mathbf{k}^2/2m$, with $|\mathbf{k}| < k_F \approx 250 \text{ MeV}/c$.

The proton spectral functions of nuclei with $A > 4$ have been modeled using the local density approximation (LDA) (Benhar *et al.*, 1994), in which the experimental information obtained from nucleon knock-out measurements is combined with theoretical calculations of the nuclear matter $S(\mathbf{k}, E)$ at different densities.

The kinematic region corresponding to low missing energy and momentum, where shell-model dynamics dominates, has been studied extensively by coincidence ($e, e'p$) experiments. The spectral function extracted from the data is usually written in the factorized form [compare to Eq. (27)]

$$S_{\text{MF}}(\mathbf{k}, E) = \sum_{n \in \{F\}} Z_n |\phi_n(\mathbf{k})|^2 F_n(E - E_n), \quad (30)$$

where the spectroscopic factor $Z_n < 1$ and the function $F_n(E - E_n)$, describing the energy width of the n th state, account for the effects of residual interactions not included in the mean-field picture. In the $Z_n \rightarrow 1$ and $F_n(E - E_n) \rightarrow \delta(E - E_n)$ limit, Eq. (30) reduces to Eq. (27).

The correlation contribution to the nuclear matter spectral function has been calculated using CBF perturbation theory for a wide range of density values (Benhar *et al.*, 1994). Within the LDA scheme, these results can be used to obtain the corresponding quantity for a finite nucleus of mass number A from

$$S_{\text{corr}}(\mathbf{k}, E) = \int d^3r \rho_A(\mathbf{r}) S_{\text{corr}}^{\text{NM}}(\mathbf{k}, E; \rho = \rho_A(\mathbf{r})), \quad (31)$$

where $\rho_A(\mathbf{r})$ is the nuclear density distribution and $S_{\text{corr}}^{\text{NM}}(\mathbf{k}, E; \rho)$ is the correlation part of the spectral function of uniform nuclear matter at density ρ . The correlation part of the nuclear matter spectral function can be easily singled out at zeroth order of CBF, being associated with two-hole-one-particle intermediate states. At higher orders, however, one-hole and two-hole-one-particle states are coupled, and the identification of the correlation contributions becomes more involved. A full account of the calculation of $S_{\text{corr}}^{\text{NM}}(\mathbf{k}, E)$ can be found in Benhar *et al.* (1994).

The full LDA spectral function is written in the form

$$S_{\text{LDA}}(\mathbf{k}, E) = S_{\text{MF}}(\mathbf{k}, E) + S_{\text{corr}}(\mathbf{k}, E), \quad (32)$$

the spectroscopic factors Z_n of Eq. (30) being constrained by the normalization requirement

$$\int d^3k dE S_{\text{LDA}}(\mathbf{k}, E) = 1. \quad (33)$$

A somewhat different implementation of LDA has also been proposed (Van Neck *et al.*, 1995). Within this approach, the nuclear matter spectral function is only used at $k > k_F(r)$, $k_F(r)$ being the local Fermi momentum, whereas the correlation background at $k < k_F(r)$ is

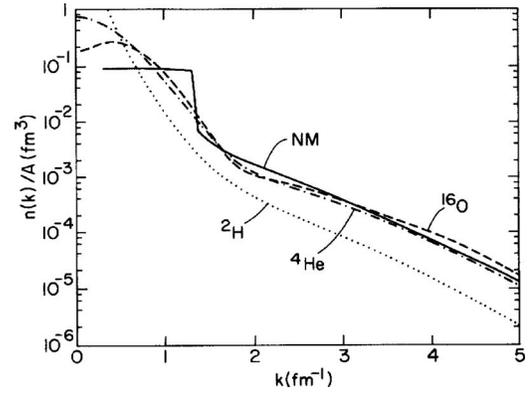


FIG. 4. Calculated momentum distribution per nucleon in ${}^2\text{H}$, ${}^4\text{He}$, ${}^{16}\text{O}$, and uniform nuclear matter (Schiavilla *et al.*, 1986; Benhar *et al.*, 1993).

incorporated in the generalized mean-field contribution. Comparison between the resulting oxygen momentum distribution and that obtained by Benhar *et al.* shows that they are in almost perfect agreement.

The LDA scheme is based on the premise that short-range nuclear dynamics are unaffected by surface and shell effects. The validity of this assumption is supported by the results of theoretical calculations of the nucleon momentum distribution

$$n(\mathbf{k}) = \int dE [Z S_p(\mathbf{k}, E) + (A - Z) S_n(\mathbf{k}, E)], \quad (34)$$

showing that for $A \geq 4$ the quantity $n(\mathbf{k})/A$ becomes nearly independent of A at large $|\mathbf{k}|$ ($\geq 300 \text{ MeV}/c$). This feature, illustrated in Fig. 4, suggests that the correlation part of the spectral function also scales with the target mass number, so that $S_{\text{corr}}^{\text{NM}}(\mathbf{k}, E)$ can be used to approximate $S_{\text{corr}}(k, E)$ at finite A .

A direct measurement of the correlation component of the spectral function of ${}^{12}\text{C}$, from the ($e, e'p$) cross section at missing momentum and energy up to $\sim 800 \text{ MeV}/c$ and $\sim 200 \text{ MeV}$, respectively, was carried out by the JLab E97-006 Collaboration (Rohe, 2004). The data from the preliminary analysis appear to be consistent with the theoretical predictions based on LDA.

D. Contribution of inelastic processes

The approach described in the previous sections is not limited to quasielastic processes. The tensor defined in Eqs. (18) and (19) describes electromagnetic transitions of the struck nucleon to any hadronic final state.

To take into account the possible production of hadrons other than protons and neutrons, one has to replace w_1^N and w_2^N given by Eqs. (23) and (24) with the inelastic nucleon structure functions extracted from the analysis of electron-proton and electron-deuteron scattering data (Bodek and Ritchie, 1981). The resulting IA cross sec-

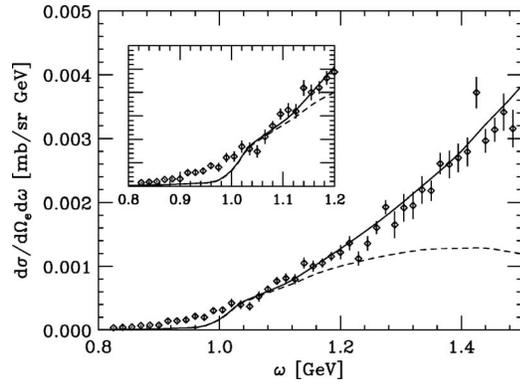


FIG. 5. Inclusive electron scattering cross section at $E_e = 3.595$ GeV and $\theta = 30^\circ$. The data points represent the extrapolated nuclear matter cross section (Day *et al.*, 1989) while the solid and dashed lines show the results of IA calculations carried out with and without inclusion of the inelastic contributions, respectively. From Benhar *et al.*, 1991.

tion can be written as in Eq. (6), the two nuclear structure functions W_1 and W_2 being given by (Benhar *et al.*, 1997)

$$W_1(|\mathbf{q}|, \omega) = \int d^3k dE \left\{ ZS_\rho(\mathbf{k}, E) \left(\frac{m}{E_k} \right) \times \left[w_1^p(|\mathbf{q}|, \tilde{\omega}) + \frac{1}{2} \frac{w_2^p(|\mathbf{q}|, \tilde{\omega}) |\mathbf{k} \times \mathbf{q}|^2}{m^2 |\mathbf{q}|^2} \right] + \dots \right\} \quad (35)$$

and

$$W_2(|\mathbf{q}|, \omega) = \int d^3k dE \left\{ ZS_\rho(\mathbf{k}, E) \left(\frac{m}{E_k} \right) \times \left\{ w_1^p(|\mathbf{q}|, \tilde{\omega}) \frac{q^2}{|\mathbf{q}|^2} \left(\frac{q^2}{\tilde{q}^2} - 1 \right) + \frac{w_2^p(|\mathbf{q}|, \tilde{\omega})}{m^2} \left[\frac{q^4}{|\mathbf{q}|^4} \left(E_k - \tilde{\omega} \frac{E_k \tilde{\omega} - \mathbf{k} \cdot \mathbf{q}}{\tilde{q}^2} \right)^2 - \frac{1}{2} \frac{q^2 |\mathbf{k} \times \mathbf{q}|^2}{|\mathbf{q}|^2} \right] \right\} + \dots \right\}, \quad (36)$$

where the dots denote the neutron contributions. Equations (35) and (36) are obtained using the prescription of Eq. (26) (de Forest, 1983) to preserve gauge invariance. Note that the standard expression (Atwood and West, 1973), widely used in studies of nuclear effects in deep inelastic scattering, can be recovered from the above equations replacing $\tilde{\omega} \rightarrow \omega$ and $E_k \rightarrow M_A - E_{\mathcal{R}}$.

As an example, Fig. 5 shows the quasielastic (dashed line) and total (solid line) inclusive cross sections of uniform nuclear matter, at beam energy $E_e = 3.595$ GeV and scattering angle $\theta = 30^\circ$, evaluated using a phenomenological fit of the nucleon structure functions w_1^N and w_2^N (Bodek and Ritchie, 1981) and the above-mentioned spectral function (Benhar *et al.*, 1989).

The data show that the transition from the quasielastic to the inelastic regime, including resonant and non-

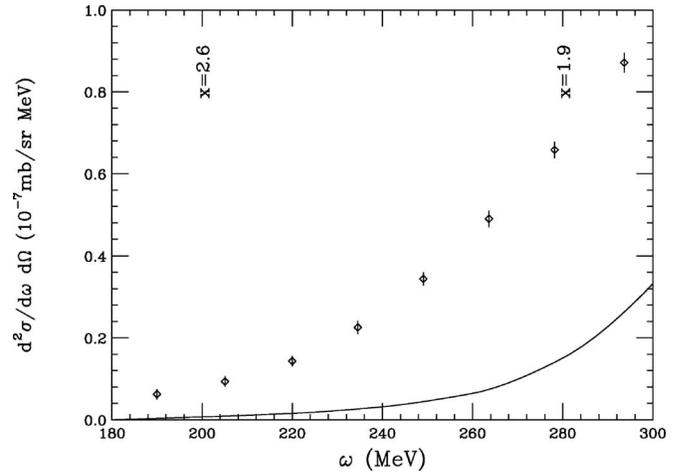


FIG. 6. Inclusive electron scattering cross section at $E_e = 7.26$ GeV and $\theta = 8^\circ$ for ${}^3\text{He}$. The data points are from Day *et al.* (1979); the solid line shows the IA calculation based on the ${}^3\text{He}$ spectral function (Meier-Hajduk *et al.*, 1983). Approximate values for the scaling variable x are indicated on top.

resonant pion production as well as deep inelastic processes, is a smooth one, thus suggesting the possibility of a unified representation.

The approach based on NMBT and the IA yields a good description of the measured cross section at energy loss $\omega \gtrsim 1$ GeV, corresponding to $x \lesssim 1.3$ (note that in the kinematics of Fig. 5, the top of the quasifree bump corresponds to $\omega = \omega_{\text{QE}} \sim 1.4$ GeV). On the other hand, the data at lower energy loss are largely underestimated.

The failure of IA calculations to explain the measured cross sections at $\omega \ll \omega_{\text{QE}}$ has long been recognized, and confirmed by a number of theoretical studies, carried out using highly realistic spectral functions (Meier-Hajduk *et al.*, 1983; Benhar *et al.*, 1989; Ciofi degli Atti *et al.*, 1992); see, e.g., Fig. 6. It has to be ascribed to FSI between the struck nucleon and the spectator particles, which move strength from the region of the quasifree bump to the low ω tail. This mechanism will be analyzed in the next section.

In conclusion, NMBT and the IA provide a consistent and computationally viable approach, yielding a quantitative description of the data in both the quasielastic and inelastic regimes, with the only exception being the region of very low-energy loss. Theoretical studies in which nuclear binding effects are included using realistic spectral functions also provide a quantitative account of the size and density dependence of the European Muon Collaboration (EMC) effect (Benhar *et al.*, 1997, 1999).

E. Different implementations of the IA scheme

In spite of the fact that the basic assumptions underlying IA can be unambiguously stated, in the literature one finds two different definitions of the IA inclusive cross section: the one discussed in Sec. II.A, involving the target spectral function, and another one, written in

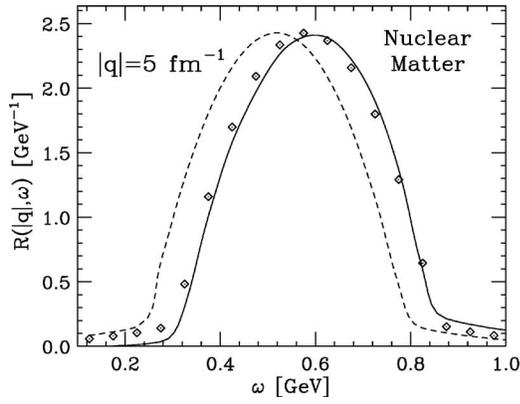


FIG. 7. Response of uniform nuclear matter at equilibrium density and $|\mathbf{q}| = 5 \text{ fm}^{-1}$. The solid and dashed lines have been obtained using the full spectral function (Benhar *et al.*, 1989) and the approximation of Eq. (37), respectively. The diamonds represent the results obtained when shifting the dashed line by $\bar{E} = 62 \text{ MeV}$ (Benhar *et al.*, 2001).

terms of the target momentum distribution of Eq. (34) (Rinat and Taragin, 1996).

It has been shown (Benhar *et al.*, 2001) that the definition in terms of the spectral function follows from minimal use of the assumptions involved in the IA scheme, and correctly takes into account the correlation between momentum and removal energy of the participating constituent. On the other hand, a more extended use of the same assumptions leads to a definition of the IA cross section in which the nucleon spectral function is written in the approximated form

$$S(\mathbf{k}, E) = \frac{1}{A} n(\mathbf{k}) \delta\left(E + \frac{|\mathbf{k}|^2}{2m}\right), \quad (37)$$

so that the information on the target removal energy distribution is lost.

Figure 7 shows a comparison between the response function of a nonrelativistic model of uniform nuclear matter³ obtained from the full spectral function (solid line) and the approximation of Eq. (37) (dashed line).

The two responses have similar shape, their width being dictated by the momentum distribution. However, they are shifted with respect to one another. The peak of the dashed curve is located at energy loss $\omega \sim |\mathbf{q}|^2/2m$, corresponding to elastic scattering off a free stationary nucleon, whereas the solid line, due to the removal energy distribution described by the spectral function, peaks at significantly larger energy. The shift is roughly given by the average nucleon removal energy⁴

³The response function shown in Fig. 7 is proportional to the inclusive cross section for the scattering of a scalar probe. The generalization to the electromagnetic target tensor of Eq. (4) is straightforward.

⁴Note that for spectral functions describing *correlated* nucleons, \bar{E} is much larger than the average of single-particle energies.

$$\bar{E} = \int d^3k dE E S(\mathbf{k}, E) = 62 \text{ MeV}. \quad (38)$$

This feature is illustrated by the diamonds of Fig. 7, which show the response obtained replacing $|\mathbf{k}|^2/2m \rightarrow (|\mathbf{k}|^2/2m - \bar{E})$ in the argument of the energy-conserving δ function of Eq. (37). The results of this calculation turn out to be much closer to those derived in the previous sections.

In addition to the shift in the position of the peak, the dashed and solid curves differ sizably at low energy transfer, where the response obtained using the momentum distribution is much larger. Obviously, to identify corrections to the response arising from mechanisms not included in the IA picture, one has to start from the definition involving the minimal set of approximations. The results of Fig. 7 show that a quantitative understanding of the effects of FSI, which are known to dominate the inclusive cross section at low ω , requires the use of the spectral function in the calculation of the IA cross section.

III. FINAL-STATE INTERACTIONS

The existence of strong FSI in quasielastic scattering has been experimentally long established. The results of a number of ($e, e'p$) measurements, covering the kinematic domain corresponding to $0.5 \leq Q^2 \leq 8.0 \text{ (GeV}/c)^2$ (Garino *et al.*, 1992; O'Neill *et al.*, 1995; Abbott *et al.*, 1998; Garrow *et al.*, 2002; Rohe *et al.*, 2005), show that the flux of outgoing protons is strongly suppressed with respect to the IA predictions. The observed attenuation ranges from 20–40 % in carbon to 50–70 % in gold.

Being only sensitive to rescattering processes taking place within a distance $\sim 1/|\mathbf{q}|$ of the electromagnetic vertex, the inclusive cross section at high momentum transfer in general is largely unaffected by FSI. However, the systematic discrepancies between data and the results of highly accurate IA calculations indicate that the effects of FSI can become appreciable, indeed dominant, in the low- ω region, where the cross sections produced by IA calculations become very small. As IA cross sections in this region are most sensitive to the high momentum and high removal energy tails of the nuclear spectral function,⁵ a quantitative understanding of FSI is required to unambiguously identify correlation effects.

In inclusive processes, FSI has two effects: (i) an energy shift of the cross section, due to the fact that the struck nucleon moves in the average potential generated by the spectator particles, and (ii) a redistribution of the strength, leading to the quenching of the quasielastic

⁵When ignoring the E distribution of $S(k, E)$, the cross section at low ω is related to the strength at large k . However, the spectral function exhibits a strong correlation between large $|\mathbf{k}|$ and large E . For example, in nuclear matter at equilibrium density, more than 50% of the strength at $|\mathbf{k}| = 1.5 \text{ fm}^{-1}$ (just above the Fermi surface) is located at $E > 80 \text{ MeV}$.

peak and an enhancement of the tails, as a consequence of NN scattering processes coupling the one-particle–one-hole final state to more complex n -particle– n -hole configurations.

Early attempts to include FSI effects were based on the Green's function formalism and multiple scattering theory, leading to a description of the dynamics in terms of a complex optical potential (Horikawa *et al.*, 1980). However, while providing a computationally practical scheme to account for the loss of flux in the one-nucleon removal channel, the optical potential model employed relies on the mean-field picture of the nucleus, and does not include the effect of dynamical NN correlations.

A similar approach has been adopted (Chinn *et al.*, 1989a, 1989b) to analyze the longitudinal and transverse responses of Eq. (7) and investigate the possible importance of relativistic effects using Dirac bound-state wave functions and optical potentials. Although the authors suggest that relativity may play an important role in suppressing the response functions, particularly R_L , the interpretation of their results is hindered by the large uncertainty associated with relativistic descriptions of nuclear dynamics and the appearance of the deep potentials, driven by the attempt to describe correlated systems using an independent-particle model. Systematic studies of relativistic effects carried out within NMBT show that they are indeed rather small. For example, when using the relativistic kinetic energy operator and when including boost corrections to the NN potential, a change of the binding energy of nuclear matter at equilibrium density of $\sim 10\%$ only has been found (Akmal *et al.*, 1998).

A different approach, based on NMBT and a generalization of Glauber theory of high-energy proton scattering (Glauber, 1959), was proposed in the early 1990s (Benhar *et al.*, 1991). This treatment of FSI, generally referred to as the correlated Glauber approximation (CGA), rests on the premises that (i) the struck nucleon moves along a straight trajectory with constant velocity (eikonal approximation), and (ii) the spectator nucleons are seen by the struck particle as a collection of fixed scattering centers (frozen approximation).

Under the assumptions given above, the expectation value of the propagator of the struck nucleon in the target ground state can be written in the factorized form (Petraki *et al.*, 2003)

$$U_{\mathbf{k}+\mathbf{q}}(t) = U_{\mathbf{k}+\mathbf{q}}^0(t) \bar{U}_{\mathbf{k}+\mathbf{q}}^{\text{FSI}}(t), \quad (39)$$

where $U_{\mathbf{k}+\mathbf{q}}^0(t)$ is the free-space propagator, while FSI effects are described by the quantity [$R \equiv (\mathbf{r}_1, \dots, \mathbf{r}_A)$]

$$\bar{U}_{\mathbf{k}+\mathbf{q}}^{\text{FSI}}(t) = \langle 0 | U_{\mathbf{k}+\mathbf{q}}^{\text{FSI}}(R; t) | 0 \rangle, \quad (40)$$

with

$$U_{\mathbf{k}+\mathbf{q}}^{\text{FSI}}(R; t) = \frac{1}{A} \sum_{i=1}^A e^{-i\Sigma_{j>i} t} \int_0^t dt' \Gamma_{\mathbf{k}+\mathbf{q}}(|\mathbf{r}_{ij} + \mathbf{v}t'|). \quad (41)$$

In Eq. (41), $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ and $\Gamma_{\mathbf{k}+\mathbf{q}}(|\mathbf{r}|)$ is the coordinate-space t matrix, related to the NN scattering amplitude at incident momentum $\mathbf{k} + \mathbf{q}$ and momentum transfer \mathbf{p} , $A_{\mathbf{k}+\mathbf{q}}(\mathbf{p})$, by

$$\Gamma_{\mathbf{k}+\mathbf{q}}(\mathbf{r}) = -\frac{2\pi}{m} \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} A_{\mathbf{k}+\mathbf{q}}(\mathbf{p}). \quad (42)$$

At large $|\mathbf{q}|$, $\mathbf{k} + \mathbf{q} \approx \mathbf{q}$ and the eikonal propagator of Eq. (41) becomes a function of t and the momentum transfer only.

The scattering amplitude extracted from the measured NN cross sections is generally parametrized in the form

$$A_{\mathbf{q}}(\mathbf{p}) = i \frac{|\mathbf{q}|}{4\pi} \sigma_{NN} (1 - i\alpha_{NN}) e^{-(\beta_{NN}|\mathbf{p}|)^2}, \quad (43)$$

where σ_{NN} and α_{NN} denote the total cross section and the ratio between the real and the imaginary part of the amplitude, respectively, while the slope parameter β_{NN} is related to the range of the interaction.

The quantity

$$P_{\mathbf{q}}(t) = \langle 0 | |U_{\mathbf{q}}^{\text{FSI}}(R; t)|^2 | 0 \rangle \quad (44)$$

measures the probability that the struck nucleon does not undergo rescattering processes during a time t after the electromagnetic interaction (Rohe *et al.*, 2005). In the absence of FSI, i.e., for vanishing $\Gamma_{\mathbf{q}}$, $P_{\mathbf{q}}(t) \equiv 1$.

Note that $P(t)$ is trivially related to the nuclear transparency $T_{\mathbf{q}}$, measured in coincidence ($e, e'p$) experiments (Garino *et al.*, 1992; O'Neill *et al.*, 1995; Abbott *et al.*, 1998; Garrow *et al.*, 2002; Rohe *et al.*, 2005), through

$$T_{\mathbf{q}} = \lim_{t \rightarrow \infty} P_{\mathbf{q}}(t). \quad (45)$$

It is important to realize that, as shown by Eqs. (40) and (41), the probability that a rescattering process occurs is not simply dictated by the nuclear density distribution $\rho_A(\mathbf{r}_j)$, the probability of finding a spectator at position \mathbf{r}_j . The rescattering probability depends upon the *joint* probability of finding the struck particle at position \mathbf{r}_i and a spectator at position \mathbf{r}_j , which can be written in the form

$$\rho^{(2)}(\mathbf{r}_i, \mathbf{r}_j) = \rho_A(\mathbf{r}_i) \rho_A(\mathbf{r}_j) g(\mathbf{r}_i, \mathbf{r}_j). \quad (46)$$

Due to the strongly repulsive nature of nuclear interactions at short range, $\rho^{(2)}(\mathbf{r}_i, \mathbf{r}_j)$ is dominated by NN correlations, whose behavior is described by the correlation function $g(\mathbf{r}_i, \mathbf{r}_j)$. The results of numerical calculations carried out within NMBT yield $g(\mathbf{r}_i, \mathbf{r}_j) \ll 1$ at $|\mathbf{r}_{ij}| < 1$ fm. This feature is illustrated in Fig. 8, showing the average $\bar{\rho}^{(2)}(\mathbf{r}_i, \mathbf{r}_j)$, defined as

$$\bar{\rho}^{(2)}(\mathbf{r}_{ij}) = \frac{1}{A} \int d^3R_{ij} \rho^{(2)}(\mathbf{r}_i, \mathbf{r}_j), \quad (47)$$

with $\mathbf{R}_{ij} = (\mathbf{r}_i + \mathbf{r}_j)/2$, evaluated for both NN and pp pairs with and without inclusion of dynamical correlation effects.

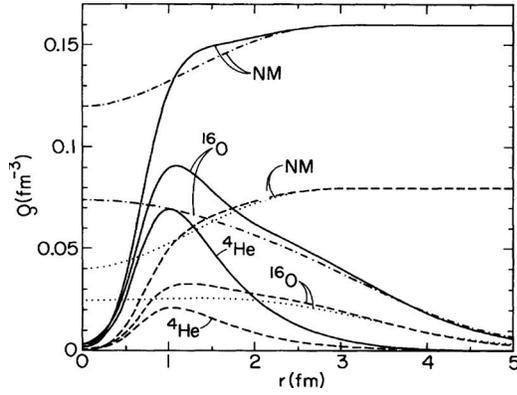


FIG. 8. Averaged distribution functions, defined by Eq. (47), of NN (solid lines) and pp (dashed lines) pairs in ${}^4\text{He}$, ${}^{16}\text{O}$, and uniform nuclear matter at equilibrium density (NM). The corresponding quantities obtained neglecting dynamical correlation effects are shown by the dot-dashed and dotted lines, respectively. From Benhar *et al.*, 1993.

From Eq. (41), it follows that within the CGA the energy shift and the redistribution of the inclusive strength are driven by the real and the imaginary part of the NN scattering amplitude, respectively. However, at large \mathbf{q} the imaginary part of $\Gamma_{\mathbf{q}}$, which gives rise to the real part of $\bar{U}_{\mathbf{q}}^{\text{FSI}}$, is dominant.

Neglecting the contribution of the real part of $A_{\mathbf{q}}$ altogether, the CGA inclusive cross section can be written as a convolution integral, involving the cross section evaluated within the IA, i.e., using Eqs. (2), (3), and (18), and a folding function embodying FSI effects,

$$\frac{d\sigma}{d\Omega_e d\omega} = \int d\omega' \left(\frac{d\sigma}{d\Omega_e d\omega'} \right)_{\text{IA}} f_{\mathbf{q}}(\omega - \omega'), \quad (48)$$

$f_{\mathbf{q}}(\omega)$ being defined as

$$\begin{aligned} f_{\mathbf{q}}(\omega) &= \delta(\omega) \sqrt{T_{\mathbf{q}}} + \int \frac{dt}{2\pi} e^{i\omega t} [\bar{U}_{\mathbf{q}}^{\text{FSI}}(t) - \sqrt{T_{\mathbf{q}}}] \\ &= \delta(\omega) \sqrt{T_{\mathbf{q}}} + F_{\mathbf{q}}(\omega), \end{aligned} \quad (49)$$

and normalized according to

$$\int_{-\infty}^{+\infty} d\omega f_{\mathbf{q}}(\omega) = 1. \quad (50)$$

The preceding equations show that the strength of FSI is governed by both $T_{\mathbf{q}}$ and the width of $F_{\mathbf{q}}(\omega)$. In the absence of FSI, $\bar{U}_{\mathbf{q}}^{\text{FSI}}(t) \equiv 1$, implying in turn $T_{\mathbf{q}} = 1$ and $f_{\mathbf{q}}(\omega) \rightarrow \delta(\omega)$.

In principle, the real part of the NN scattering amplitude can be explicitly included in Eq. (41) and treated on the same footing as the imaginary part. However, its effect turns out to be appreciable only at $t \sim 0$, when the attenuation produced by the imaginary part is weak. The results of numerical calculations show that an approximate treatment, based on the use of a time-independent optical potential, is adequate to describe the energy shift produced by the real part of $\Gamma_{\mathbf{q}}$ (Benhar *et al.*, 1994),

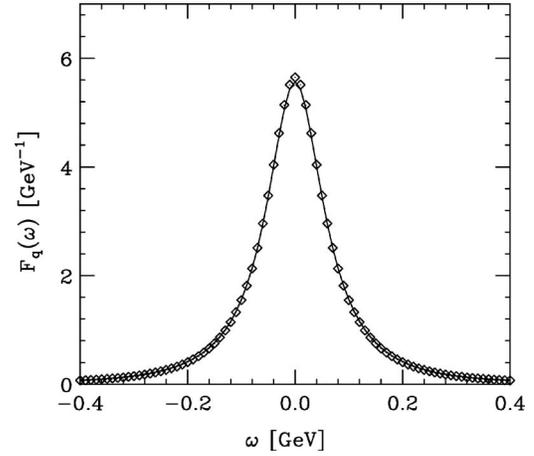


FIG. 9. Folding functions $F_{\mathbf{q}}$ of Eq. (49), calculated in uniform nuclear matter at equilibrium density. The solid line and diamonds correspond to momentum transfer $|\mathbf{q}| = 2.2$ and 3.4 GeV/ c , respectively (Benhar, 1999).

whose size of ~ 10 MeV is to be compared to a typical electron energy loss of a few hundred MeV.

The shape of the folding function is mainly determined by the total NN cross section σ_{NN} . In the energy region relevant to scattering of few-GeV electrons, σ_{NN} is dominated by the contribution of inelastic processes and nearly independent of energy. As a consequence, the scattering amplitude of Eq. (43) grows linearly with \mathbf{q} , and both $\bar{U}_{\mathbf{q}}^{\text{FSI}}(t)$ and the folding function $f_{\mathbf{q}}$ become independent of \mathbf{q} .

This feature is illustrated in Fig. 9, showing that the $F_{\mathbf{q}}$ calculated in uniform nuclear matter at equilibrium density at momentum transfers $|\mathbf{q}| = 2.2$ and 3.4 GeV/ c are nearly identical.

Dynamical NN correlations also affect the shape of the folding function. Inclusion of correlations through the distribution function $g(\mathbf{r}_i, \mathbf{r}_j)$ results in a strong quenching of the tails and an enhancement of the peak of $F_{\mathbf{q}}$, leading to a significant suppression of FSI effects.

The effect of FSI is illustrated in Fig. 10, showing the inclusive cross section of uniform nuclear matter at a beam energy $E_e = 3.595$ GeV and a scattering angle $\theta = 30^\circ$, corresponding to momentum transfer $|\mathbf{q}| \sim 2$ GeV/ c .

Comparison between theory and the data in Fig. 10 clearly shows that at $\omega < 1.1$ GeV, where quasielastic scattering dominates⁶ and which corresponds to $x > 1$, $x = Q^2/2m\omega$ being the Bjorken scaling variable, FSI effects are large and must be taken into account. The results obtained within the CGA are in good agreement with the data in the region $\omega > 800$ MeV, i.e., for $x \leq 1.8$, while at higher x the experimental cross section is largely overestimated. The dashed line has been obtained neglecting the effect of dynamical correlations on

⁶In the kinematics of Fig. 10, inelastic processes only contribute $\sim 5\%$ of the inclusive cross section at $\omega = 1.1$ GeV, and become negligibly small at lower ω .

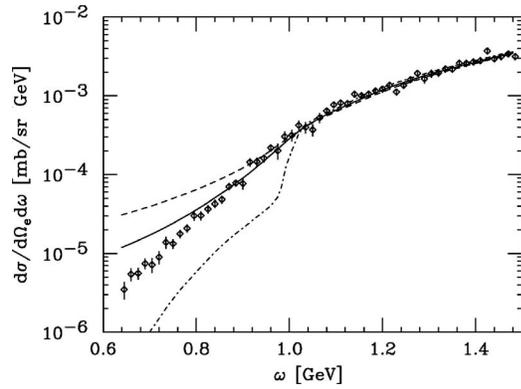


FIG. 10. Inclusive electron-scattering cross section at $E_e = 3.595$ GeV and $\theta = 30^\circ$. The data points represent the extrapolated nuclear matter cross section (Day *et al.*, 1989), while the solid and dashed lines show the results obtained in CGA including FSI effects, with and without taking into account correlation effects. For comparison, the IA cross section is also shown by the dot-dashed line (Benhar *et al.*, 1991).

the distribution function $g(\mathbf{r}_i, \mathbf{r}_j)$. Comparison between the solid and dashed lines provides a measure of the quenching of FSI due to NN correlations.

The ability of the CGA to provide a quantitative understanding of FSI in the region $x < 2$ is illustrated further in Fig. 11, showing the cross-section ratio

$$R = \frac{d\sigma(e + {}^{56}\text{Fe} \rightarrow e' + X) 2}{d\sigma(e + {}^2\text{H} \rightarrow e' + X) 56} \quad (51)$$

at $E_e = 3.595$ GeV and $\theta = 25^\circ$. Note that R of Eq. (51) is only defined up to $y \sim -700$ MeV/c, corresponding to $x = 2$, the kinematic limit for inclusive scattering off an $A = 2$ target (for the definition of y , see Sec. VII).

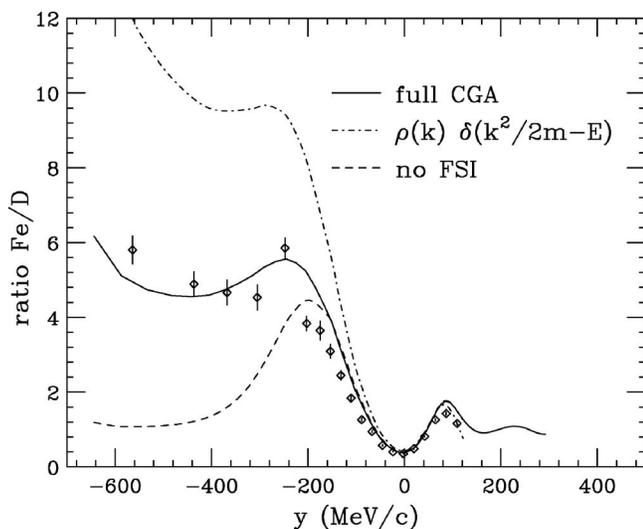


FIG. 11. Ratios of inclusive cross sections of iron and deuterium at $E_e = 3.595$ GeV and $\theta = 25^\circ$. Solid line, full calculation; dashed line, IA calculation, neglecting FSI in both iron and deuterium; dot-dashed line, calculation carried out using the approximate spectral function of Eq. (37) (Benhar *et al.*, 1995b).

The solid line in Fig. 11 corresponds to the full CGA calculation, providing a good description of the experiments over the whole range of y , whereas the IA results, represented by the dashed line, lie well below the data at $y < -200$ MeV/c ($x > 1.5$). For comparison, Fig. 11 also shows the results obtained using the approximate spectral function of Eq. (37), which turn out to largely overestimate the data at negative y .

The results of Figs. 11 and 6 clearly rule out the interpretation of the behavior of the cross-section ratio at $y \leq -200$ MeV/c in terms of scattering off strongly correlated nucleon pairs (Egiyan *et al.*, 2003). This interpretation in fact assumes the validity of the IA picture, which is known to fail at large negative y , and does not take into account the large effect of FSI.

Notwithstanding its success in describing the existing inclusive data at large negative y , the CGA appears consistently to overestimate FSI effects at larger $-y$. As the validity of the eikonal approximation is well established in the kinematic region apposite to scattering of few-GeV electrons, possible corrections to the CGA scheme are likely to be ascribable either to modifications of the NN scattering amplitude or to the inadequacy of the approximations leading to the convolution expression for the cross section.

It has been pointed out (Benhar *et al.*, 1991) that the use of the free-space amplitude to describe NN scattering in the nuclear medium may be questionable. Pauli blocking and dispersive corrections are known to be important at moderate energies (Pandharipande and Pieper, 1992). However, their effects on the calculated inclusive cross section have been found to be small in the kinematic region corresponding to $|\mathbf{q}| \geq 2$ GeV/c, and decrease as $|\mathbf{q}|$ increases (Benhar *et al.*, 1995a). Corrections to the amplitude associated with its extrapolation to off-shell energies are also expected to be small at $|\mathbf{q}| > 2$ GeV/c (Benhar and Liuti, 1996).

Modifications of the free-space NN cross section may also originate from the internal structure of the nucleon. It has been suggested (Brodsky, 1982; Mueller, 1982) that elastic scattering on a nucleon at high momentum transfer can only occur if the nucleon is found in the Fock state having the fewest number of constituents, so that the momentum transfer can be most effectively shared among them. This state is very compact, its size being proportional to $1/|\mathbf{q}|$, and therefore interacts weakly with the nuclear medium. Within this picture a nucleon, after absorbing a large momentum, travels through nuclear matter experiencing very little FSI, i.e., exhibits color transparency, before it evolves back to its standard configuration with a characteristic time scale.

Color transparency (CT) may be particularly relevant to the analysis of inclusive electron-nucleus scattering at $x > 1$, where elastic scattering is the dominant reaction mechanism, since it leads to a significant quenching of FSI. In fact, the influence of CT is expected to be much larger for (e, e') than for $(e, e'p)$; in (e, e') , FSI occur mainly very close to the electromagnetic vertex, at a distance of less than $\sim 1/|\mathbf{q}|$, where the compact configura-

tion has not yet evolved back to the ordinary proton. It has been suggested that the modification of the NN scattering amplitude due to onset of CT may explain the failure of CGA to reproduce the data in the region of very low ω (Benhar *et al.*, 1991, 1994; Benhar and Pandharipande, 1993).

However, recent measurements show no enhancement of the nuclear transparency up to $Q^2 \sim 8$ (GeV/c)² (Garrow *et al.*, 2002; Dutta *et al.*, 2003) and seem to rule out observable CT effects in $(e, e'p)$ at beam energies of few GeV. Therefore, the excellent agreement between the measured inclusive cross sections and the results of the theoretical calculation (Benhar *et al.*, 1991, 1994; Benhar and Pandharipande, 1993) carried out using CGA and the quantum diffusion model of CT (Farrar *et al.*, 1988) may be accidental.

An improved version of CGA has been recently proposed (Petraki *et al.*, 2003). Within this approach, the initial momentum of the struck nucleon, which is averaged over in CGA, is explicitly taken into account. As a result, one goes beyond the simple convolution form of the inclusive cross section and gets a generalized folding function, depending on both the momentum transfer \mathbf{q} and the initial momentum \mathbf{k} . Numerical calculations of the nonrelativistic response of uniform nuclear matter at $1 \leq |\mathbf{q}| \leq 2$ GeV/c show that the inclusion of this additional momentum dependence leads to a sizable quenching of the low-energy loss tail of the inclusive cross section, with respect to the predictions of CGA.

Even though they cover a limited kinematic range and have been obtained using a somewhat oversimplified model, the available results suggest that a better treatment of the momentum distribution of the struck nucleon may improve the agreement between theory and data in the region of $x > 2$, where CGA begins to fail. A systematic study of the dependence of the rescattering probability on the initial momentum, based on a relativistically consistent formalism, is presently being carried out (Benhar and Sick, 2007).

A different approach to describe FSI in inclusive electron-nucleus scattering, based on the relativistic generalization of the Gersch-Rodriguez-Smith (GRS) $1/|\mathbf{q}|$ expansion of the response of many-fermion systems (Gersch *et al.*, 1973), has been proposed (Gurvitz and Rinat, 2002). Numerical studies of the cross section of ${}^4\text{He}$ (Viviani *et al.*, 2003), carried out using realistic wave functions to compute the $\langle {}^4\text{He} | {}^3\text{He} \rangle$ and $\langle {}^4\text{He} | {}^3\text{He} \rangle$ transition matrix elements and a model spectral function (Ciofi degli Atti and Simula, 1994) to describe the many-body breakup channels, show that the generalized GRS expansion provides a fairly good overall description of the SLAC data (Day *et al.*, 1993) at a beam energy of 3.6 GeV.

IV. EXPERIMENTS

Studies of inclusive electron-nucleus scattering have been performed at a number of facilities. The bulk of the currently available data (at momentum transfers $|\mathbf{q}|$

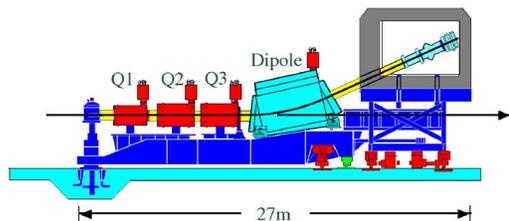


FIG. 12. (Color online) HMS spectrometer in Hall C at Jefferson Lab with four magnets arranged as QQD and the heavily shielded detector hut.

on the order of 500 MeV/c and above upon which we focus here) has been generated at Saclay (France), Bates-MIT (Boston), Jefferson Lab (Newport News), and SLAC (Stanford). Isolated data sets are available from other facilities (see the next section). Without exception, magnetic spectrometers have been used to momentum analyze scattered electrons. The magnet arrangements are varied, and at the four facilities mentioned above they were D(ipole), DD, Q(uadrupole)QQD, and QQDDQ systems, respectively. These spectrometers had a maximum bending momentum of 0.6, 0.9, 7, and 8 GeV/c, respectively and solid angles between 4 and 6.8 msr (Mo and Peck, 1965; Bertozzi *et al.*, 1979; Leconte *et al.*, 1980; Yan, 1997). As an example, we show in Figs. 12 and 13 the JLab High Momentum Spectrometer (HMS) of Hall C and its associated focal-plane detector setup. In most cases, the electron track in the focal plane was reconstructed using two to four planes of multiwire chambers, and the trigger involved the use of fast scintillator detectors, often two planes of segmented paddles. Typically the focal plane detector also included a Čerenkov detector needed to separate electrons from slower particles such as pions. At the high-energy facilities, a total absorption counter provided an additional, cruder, energy measurement to assist in the discrimination of electrons from other, often more numerous, charged particles.

The data acquisition systems of the earlier, lower-energy facilities were rather restricted because of computer speed, memory, and high data rates. Consequently, much of the data were preprocessed on-line and stored in spectra; this allowed little off-line analysis of the data. The data acquisition at the higher-energy facilities included event-mode acquisition where all coordinates,

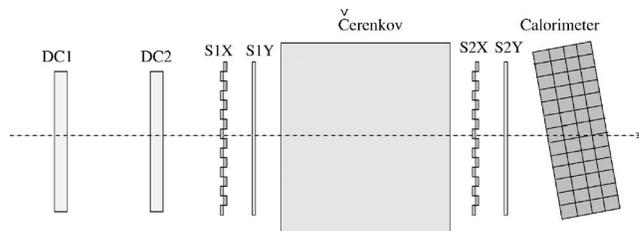


FIG. 13. HMS detector package, consisting of multiple planes of multiwire drift chambers, fast scintillators, a subatmospheric gas Čerenkov counter, and a lead glass total absorption counter.

times, and amplitudes are registered for further off-line analysis. Event-mode acquisition has important advantages in that during the course of the off-line analysis, shortcomings of the acquisition hardware can be discovered and (often) corrected. This typically leads to much more reliable cross sections.

In many cases, the product of effective solid angle and detector efficiency is not easily established. While the efficiency of the detector system can be determined by exploiting the redundancy of the detector elements, the effective solid angle of spectrometers (not using a solid-angle defining slit of well-known geometry) is difficult to establish. For that reason, many of the experiments, particularly the ones at the higher-energy facilities, use elastic scattering from a liquid hydrogen target as a check; the e - p cross sections are well known. The data on hydrogen are often also used to determine offsets in the scattered electron momenta and angles as reconstructed from positions and angles of electrons in the detectors.

The inclusive quasielastic spectrum extends over a momentum range that exceeds (in most cases) the momentum acceptance of the spectrometers. Consequently, data taking entails a sequence of runs at stepped values of the central spectrometer momentum. The resulting set of spectra, if taken with sufficient overlap, can then be used to determine both the dependence in $\delta p \equiv \Delta p/p_{\text{central}}$ due to detector acceptance and inefficiencies and the true ω dependence of the cross section (Crannell and Suelzle, 1966).

An experimental aspect peculiar to inclusive scattering is the need to accurately know the dispersion of the spectrometer. As the cross section is ultimately determined for bins in energy loss, the accuracy of the differential momentum dispersion enters directly in the accuracy of the cross sections. The dispersion can be determined using focal-plane scans with an elastic peak, provided that the relation between the integral over Bdl and the quantity measured—typically the B field at one point—is linear and has no offset at zero momentum. For spectrometers with high resolution, the dispersion can be determined using elastic scattering off nuclei with different mass. This determination of a precise dispersion often has not been given the appropriate attention.

After correcting the data for the various shortcomings of the spectrometers and detector packages (energy and angle offsets, detector inefficiencies, etc.), one major correction is the removal of radiative effects from external and internal bremsstrahlung. While the theory for radiative effects is well under control (Mo and Tsai, 1969), the specifics of quasielastic scattering lead to the need for extensive modeling. In quasielastic scattering, the energy loss can be very large as can be the energy loss due to radiative effects. Since radiation can be emitted before or after the quasielastic scattering process, the calculation of radiative effects requires knowledge of the quasielastic structure function for all incident electron beam energies down to $E_e - \omega_{\text{max}}$, where E_e is the incident energy and ω_{max} is the largest energy loss of interest. Collecting the required quasielastic data at the corresponding lower incident electron energies is often too

time consuming. As a consequence, only few such data or none at all are taken. In order to obtain the quasielastic cross section at all energies, extensive modeling of the data in the region covered by experiment is needed, such that a reliable extrapolation to the other regions is feasible. With the discovery of y scaling (see Sec. VII), this modeling has been greatly facilitated, as the scaling function $F(y)$ represents a quantity that is only very weakly dependent on energy and angle and can be interpolated over a large range. This unfortunately does not apply to the Δ region, which at large ω overlaps with the quasielastic peak. Modeling of the Δ contribution can be facilitated when using an approach based on superscaling (Amaro *et al.*, 2005).

Despite the difficulties introduced by radiative corrections, the impact on the overall accuracy is relatively small. For cross sections that do not depend greatly on ω , the effect of radiative corrections is small; the uncertainty introduced is also small if the model cross section is not grossly deficient.

There is another correction—not always considered—that is unique to cross-section measurements of continuum spectra at large energy losses. One must account for the contributions of electrons with momenta outside the momentum acceptance of the spectrometer, which interact with part of the spectrometer yoke or vacuum chamber, and are rescattered into the detector. Usually the trajectory of these electrons cannot be traced back through the spectrometer as their momentum is not known. This process can lead to background, which in general is in the % region, but can become problematic when, e.g., an L/T separation is performed. In this case, the effect is greatly enhanced in magnitude if the L or T contribution is small as compared to the other one, as happens at the larger momentum transfers where the longitudinal response gives a small contribution to the cross section. Such rescattering events can become especially important if the experiment uses long (liquid or gas) targets that extend beyond the acceptance of the spectrometer; in this case, electrons can enter the spectrometer through the solid-angle defining collimator and scatter from the vacuum chamber or pole pieces.

Experimentally, provisions against scattering from the yoke (but not the pole pieces) include the use of baffles on the high or low momentum wall of the vacuum chamber (Leconte *et al.*, 1980). In some cases, these backgrounds have also been subtracted by Monte Carlo simulations of electron paths through the spectrometer (Danel, 1990). They have been measured using a solid target displaced along the beam direction, and corrections up to 10% have been found (Danel, 1990). The most reliable method to remove these contributions has been accomplished via a secondary measurement of the energy of the electron in the focal plane, using some total-absorption counter (Jourdan, 1996). Once one knows with reasonable accuracy the energy of the detected particles, they can be traced back and one can find out whether they come directly from the target or not. For the QQDDQ 8 GeV spectrometer at SLAC, an acceptance-reducing collimator placed in between the

two dipoles allowed an experimental determination of the fractional contribution of rescattered events. At large energy loss, this fraction has been found to be quite substantial (Jourdan, 1996).

On the whole, this correction for rescattering has been given too little attention, particularly at the lower energies where their contribution is more important and where secondary energy measurements using total-absorption counters are not very effective.

One further correction that has to be applied to the data is the subtraction of electrons resulting from e^+e^- pair production in the target. The e^- from this charge-symmetric process gives a significant contribution to the inclusive cross section, particularly at large scattering angles and small energy loss ω where the quasielastic is very small. The effect of this contribution is usually handled by subtracting the spectrum, measured under identical kinematic conditions, of the pair produced e^+ .

V. DATA

The modern era of experiments dedicated to quasi-elastic electron-nucleus scattering began in 1969 with a series of data sets from Kharkov (Dementii *et al.*, 1969; Titov *et al.*, 1969, 1971, 1974; Titov and Stepula, 1972); a first systematic survey of quasielastic electron scattering from a range of nuclei (He to Pb) (Moniz *et al.*, 1971; Whitney *et al.*, 1974; McCarthy *et al.*, 1976) was made at Stanford. The first attempt to separate the response functions was performed at Stanford when data for ^{40}Ca and ^{48}Ca were collected at constant values of the three-momentum transfer (Zimmerman, 1969; Zimmerman and Yearian, 1976).

At about the same time, quasielastic electron scattering from ^{12}C was studied at the Harvard CEA at energies ranging from 1 to 4 GeV and angles from 8.5° to 18° using a half-quadrupole spectrometer (Stanfield *et al.*, 1971). At DESY, inclusive electron scattering data in the quasielastic region were studied (Heimlich, 1973; Zeller, 1973; Heimlich *et al.*, 1974) from ^6Li and ^{12}C at energies up to 2.7 GeV and angles as large as 15° .

The high-energy beam at SLAC was exploited in the mid-1970s to measure quasielastic cross sections at very high momentum transfer [≈ 4 (GeV/c) 2] on ^3He (Day *et al.*, 1979). A considerable body of quasielastic data was also measured at SLAC in the mid-1980s (Day *et al.*, 1987, 1993; Baran *et al.*, 1988; Baran, 1989; Potterveld, 1989; Sealock *et al.*, 1989; Chen, 1990; Mezziani *et al.*, 1992) using the high-intensity, low-energy (≤ 5 GeV) beam from the Nuclear Physics at SLAC (NPAS) injector.

Starting in the late 1970s, a series of experiments began to measure data for a wide range of nuclei (^2H to U), often as part of a program to separate the structure functions, both at Bates (Zimmerman *et al.*, 1978; Altemus *et al.*, 1980; Deady *et al.*, 1983, 1986; Hotta *et al.*, 1984; Blatchley *et al.*, 1986; Dow, 1987; O'Connell *et al.*, 1987; Dow *et al.*, 1988; Dytman *et al.*, 1988; Quinn *et al.*, 1988; VanReden *et al.*, 1990; Yates *et al.*, 1993) and at

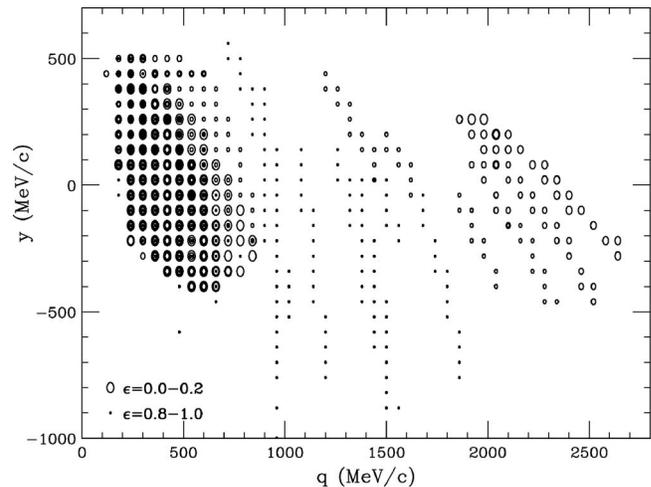


FIG. 14. Kinematic range covered by the (e, e') data for ^{12}C , plotted as a function of $|\mathbf{q}|$ and y (for definition, see Sec. VII). The symbols encode the range of ϵ covered by the data (bins of 0.2 in ϵ).

Saclay (Mougey *et al.*, 1978; Barreau *et al.*, 1983; Mezziani *et al.*, 1984, 1985; Marchand *et al.*, 1985; Zghiche *et al.*, 1994; Gueye *et al.*, 1999).

In 1996, an experiment at JLab, intended to extend the range in four-momentum transfer Q^2 and Bjorken scaling variable x , produced data in the quasielastic region out to $5(\text{GeV}/c)^2$ (Arrington, 1998; Arrington *et al.*, 1999). In the past year, a new experiment using the full energy of Jlab (6 GeV) extended these measurements even further using ^2H , ^3He , ^4He , Be, C, Cu, and Au targets.

As most of the data are not published in numerical form, we have prepared, as part of this review, a website, *Quasielastic Electron Nucleus Scattering Archive*, <http://faculty.virginia.edu/qes-archive>. In this archive, we have placed all radiatively unfolded cross sections we could locate in tabular form. The site also gives the references and some details on the experiments.

For some nuclei, quite a comprehensive set of cross sections is available. Figure 14 shows the range in $|\mathbf{q}|$ and y covered by the data for ^{12}C . The symbols incorporate information on $\epsilon = 1.0/[1.0 + (2\mathbf{q}^2/Q^2)\tan^2\theta/2]$, indicating that only for the lower momentum transfers is a large enough ϵ range is covered for reliable L/T separations.

VI. EXTRACTION AND USE OF NUCLEON FORM FACTORS

We began our discussion on the quasielastic scattering process by describing it as the incoherent scattering of high-energy electrons from the bound and quasifree nucleons. Any quantitative description of this process then depends on an accurate description of the elementary electron-nucleon (proton and neutron) cross section.

In this section, we briefly review the status of the form factors, with particular attention to the most recent developments. A comprehensive review is not called for

and the interested reader should consult [Gao \(2003\)](#) and [Hyde-Wright and de Jager \(2005\)](#) or the recent conference proceedings ([Day, 2005](#); [de Jager, 2005](#)).

The form factor data, until recently, have been obtained, in the case of the proton, through a Rosenbluth separation that exploits the linear dependence of the cross section on the polarization ϵ of the virtual photon. In terms of exchange of a single virtual photon, the cross section depends on the Sachs form factors G_E and G_M via

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{NS}} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2(\theta/2) \right], \quad (52)$$

with $\tau = Q^2/4M^2$ and $\sigma_{\text{NS}} = \sigma_{\text{Mott}} E'/E_0$. Rearranging this expression with $\epsilon^{-1} = 1 + 2(1 + \tau)\tan^2(\theta/2)$ yields

$$\sigma_R \equiv \frac{d\sigma}{d\Omega} \frac{\epsilon(1 + \tau)}{\sigma_{\text{NS}}} = \tau G_M^2(Q^2) + \epsilon G_E^2(Q^2). \quad (53)$$

By making measurements at a fixed Q^2 and variable $\epsilon(\theta, E_0)$, the reduced cross section σ_R can be fitted with a straight line with slope G_E^2 and intercept τG_M^2 . The Rosenbluth technique, because of the dominance of G_M^p over G_E^p at large Q^2 , demands strict control over the kinematics, acceptances, and radiative corrections over a large range of both incident and final electron energies and angles.

The analysis of quasielastic scattering has utilized widely accepted parametrizations or models of the nucleon form factors, based on data collected, starting in the 1950s through the 1990s. The data have been described by simple fits as well as sophisticated models based on the vector dominance model (VDM), dispersion relations and quark models; see [Gao \(2003\)](#), [Hyde-Wright and de Jager \(2004\)](#), and [Perdrisat et al. \(2007\)](#) for a review.

The lack of a free neutron target with which to study the neutron properties forces the experimental effort to investigate the neutron bound in a nucleus. The deuteron, weakly bound and with a ground state that, in principle, is calculable, has been the nucleus of choice for unpolarized studies. Until the early 1990s, the extraction of G_E^n was done most successfully through either small angle elastic electron scattering from the deuteron ([Drickey and Hand, 1962](#); [Benaksas et al., 1964](#); [Bumiller et al., 1970](#); [Galster et al., 1971](#); [Simon et al., 1981](#); [Platchkov et al., 1990](#)) or by quasielastic e -D scattering ([Hughes et al., 1965, 1966](#); [Bartel et al., 1973](#); [Lung et al., 1993](#)).

In the IA, the elastic electron-deuteron cross section is the sum of proton and neutron responses with deuteron wave-function weighting. The coherent nature of elastic scattering gives rise to an interference term between the neutron and proton response that allows the smaller G_E^n contribution to be extracted. Still, the large proton contribution must be removed. Experiments have been able to achieve small statistical errors but re-

main very sensitive to the deuteron wave-function model, leaving a significant residual dependence on the NN potential.

Quasielastic eD scattering provides a complementary approach to the extraction of G_E^n . In the IA model pioneered by [Durand and McGee \(Durand, 1961; McGee, 1967\)](#), the cross section is an incoherent sum of p and n cross sections. The extraction of G_E^n requires both a Rosenbluth separation and the subtraction of the sizeable proton contribution. It suffers, unfortunately, from unfavorable error propagation and a sensitivity to the deuteron structure.

G_M^n has been extracted from both inclusive and exclusive quasielastic scattering from the deuteron. G_M^n , when determined from the ratio ${}^2\text{H}(e, e'p)/{}^2\text{H}(e, e'n)$, always working near the top of the quasielastic peak, has the smallest uncertainties arising from FSI, MEC, and details of the ground-state wave function. For both the ratio technique and ${}^2\text{H}(e, e'n)p$, the absolute efficiency of the neutron detector must be determined. The most precise data are from MAMI ([Kubon et al., 2002](#)); recently G_M^n was measured out to $5 \text{ (GeV}/c)^2$ ([Brooks and Lachniet, 2005](#); [Lachniet, 2006](#)) at JLab.

It has been known for many years that the “small” nucleon electromagnetic form factors G_E^n and G_E^p at large Q^2 could be measured through spin-dependent elastic scattering from the nucleon, accomplished either through a measurement of the scattering asymmetry of polarized electrons from a polarized nucleon target ([Dombey, 1969](#); [Donnelly and Raskin, 1986](#); [Raskin and Donnelly, 1989](#)) or equivalently by measuring the polarization transferred to the nucleon ([Akhiezer and Rekalov, 1974](#); [Arnold et al., 1981](#)).

In the scattering of polarized electrons from a polarized target, an asymmetry appears in the elastic scattering cross section when the beam helicity is reversed due to the presence of a polarized piece Δ in addition to the unpolarized piece, Σ . The total cross section is

$$\sigma(h) = \Sigma + h\Delta, \quad h = \pm p_{\text{beam}}. \quad (54)$$

The asymmetry

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{\Delta}{\Sigma} \quad (55)$$

can be written schematically (a , b , c , and d are known kinematic factors) as

$$A = \frac{a \cos \Theta^* (G_M)^2 + b \sin \Theta^* \cos \Phi^* G_E G_M}{c (G_M)^2 + d (G_E)^2}, \quad (56)$$

where Θ^* and Φ^* fix the target polarization axis. With the target polarization axis in the scattering plane and perpendicular to \vec{q} ($\Theta^*, \Phi^* = 90^\circ, 0^\circ$), the asymmetry A_{TL} is proportional to $G_E G_M$. With the polarization axis in the scattering plane and parallel to \mathbf{q} ($\Theta^*, \Phi^* = 0^\circ, 0^\circ$), one measures the transverse asymmetry A_T , which in the case of a free nucleon would be insensitive to G_M^n (depending simply on the kinematic factors). In ${}^3\text{He}(\vec{e}, e')X$ scattering, the denominator contains contri-

butions arising from both protons and neutrons and at modest Q^2 is sensitive to G_M^n (Gao, 2003).

In elastic scattering of polarized electrons from a nucleon, the nucleon obtains (is transferred) a polarization whose components P_l (along the direction of the nucleon momentum) and P_t (perpendicular to the nucleon momentum) are proportional to G_M^2 and $G_E G_M$, respectively. Polarimeters are sensitive only to the perpendicular polarization components, so precession of the nucleon spin before the polarimeter in the magnetic field of the spectrometer (for the proton) or a dipole (inserted in the path of neutron) allows a measurement of the ratio P_t/P_l and the form factor ratio: $G_E/G_M = -(P_t/P_l)[(E_0 + E')/2M_N]\tan(\theta/2)$. The recoil polarization technique has allowed precision measurements of G_E^p to nearly $6 \text{ (GeV}/c)^2$ (Jones *et al.*, 2000; Gayou *et al.*, 2001, 2002)

These data on the proton have generated a great deal of activity as they have revealed a major discrepancy between the Rosenbluth data and the polarization transfer data for G_E^p at $Q^2 > 2 \text{ GeV}^2/c^2$. A detailed discussion of these topics is beyond the scope of this review; it has been discussed in the papers of Gao (2003), Hyde-Wright and de Jager (2004), and Perdrisat *et al.* (2007).

Extraction of the neutron form factors using polarization observables is complicated by the need to account for Fermi motion, MEC, and FSI, complications that are absent when scattering from a proton target. Fortunately it has been found for the deuteron that in kinematics that emphasize quasifree neutron knockout, both the transfer polarization P_t (Arenhoevel, 1987) and the beam-target asymmetry A_V^{eD} (Arenhoevel *et al.*, 1988) are especially sensitive to G_E^n and relatively insensitive to the NN potential describing the ground state of the deuteron and other reaction details; see Fig. 15. The utility of ^3He as a polarized neutron arises from the fact that in the ground state (dominantly a spatially symmetric S wave), the proton spins cancel and the ^3He spin is carried by the unpaired neutron. Calculations (Ishikawa *et al.*, 1998; Golak *et al.*, 2001) of the beam-target asymmetry from a polarized ^3He target have shown it to have only modest model dependence.

Extraction of the neutron form factors from beam-target asymmetry measurements in *inclusive* scattering is hindered by the dominant contribution of the proton, even with polarized ^3He where the protons are responsible for just 10% of the ^3He polarization. This is especially true in the case of the neutron charge form factor (Jones-Woodward *et al.*, 1991; Jones *et al.*, 1993), though methods to minimize the role of the proton have been proposed (Ciofi degli Atti *et al.*, 1995). G_M^n has been extracted from inclusive polarized electron-polarized ^3He scattering at Bates and Jefferson Lab out to $Q^2 = 0.6 \text{ (GeV}/c)^2$ (Gao *et al.*, 1994; Xu *et al.*, 2000, 2003). Inclusive asymmetry measurements on ^3He in the threshold region (Xiong *et al.*, 2001) have been used to successfully test nonrelativistic Faddeev calculations, which include MEC and FSI.

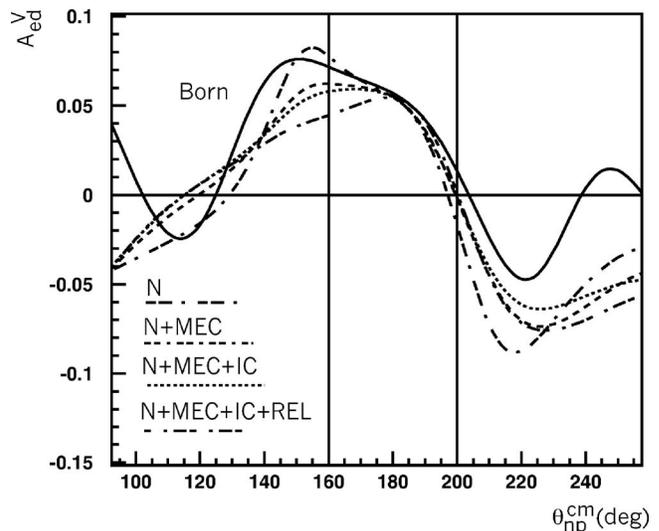


FIG. 15. A calculation (Arenhoevel *et al.*, 1988, 1989, 1992) of the electron-deuteron vector asymmetry A_V^{ed} at $Q^2=0.5 \text{ (GeV}/c)^2$, which demonstrates the insensitivity of A_V^{ed} to the reaction model and subnuclear degrees of freedom at angles near 180° corresponding to neutron along the momentum transfer.

Fortunately the development of polarized beams and targets has been able to leverage the utility of *exclusive* quasielastic scattering from bound neutrons in both ^3He and ^2H providing precision data on the electric form factors of the neutron. Coincidence measurements allow one to avoid completely the subtraction of the dominant proton.

Since the early work on the neutron at Bates (Jones *et al.*, 1993; Eden *et al.*, 1994) through $^3\text{He}(\vec{e}, e')X$ and $D(\vec{e}, e'\vec{n})p$, respectively, further development of high polarization beams and targets, together with high duty factor accelerators, has improved the data set (and outlook) for G_E^n (Meyerhoff *et al.*, 1994; Becker *et al.*, 1999; Herberg *et al.*, 1999; Ostrick *et al.*, 1999; Passchier *et al.*, 1999; Rohe *et al.*, 1999; Zhu *et al.*, 2001; Warren *et al.*, 2004) through either $\vec{D}(\vec{e}, e'n)p$, $^3\text{He}(\vec{e}, e'n)$, or $^2\text{H}(\vec{e}, e'\vec{n})p$ and G_M^n through $^3\text{He}(\vec{e}, e')X$ (Gao *et al.*, 1994; Xu *et al.*, 2000, 2003).

In order to understand the discrepancy between polarization transfer and Rosenbluth results for G_E^p , several investigators (Blunden *et al.*, 2003; Guichon and Vanderhaeghen, 2003; Chen *et al.*, 2004; Rekalov and Tomasi-Gustafsson, 2004) have explored the possibility of two-photon exchange corrections. While only incomplete calculations exist, the results of Blunden *et al.* (2003) and Chen *et al.* (2004) account for part of the difference.

The most recent work by Chen *et al.* (2004) describes the process in terms of hard scattering from a quark and uses generalized parton distributions to describe the quark emission and absorption. Chen *et al.* argued that when taking the recoil polarization form factors as input, the addition of the two-photon corrections reproduces

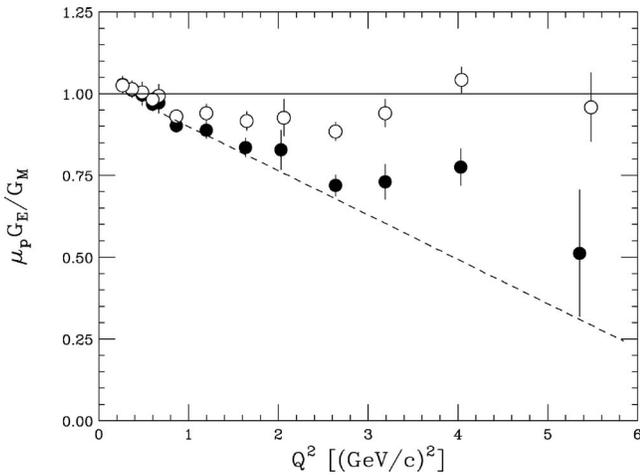


FIG. 16. Result from the global analysis of the L - T (Rosenbluth) data (Arrington *et al.*, 2007). Open (full) points are obtained before (after) two-photon exchange corrections. The dashed line is a fit to the polarization-transfer data.

the Rosenbluth data. However, Arrington (2005) has shown that when the corrections of Chen *et al.* are applied to the new Jefferson Lab Rosenbluth data, which have small errors (see below and Fig. 16), only one-half of the discrepancy is explained.

Direct tests for the existence of two-photon exchange include measurements of the ratio $\sigma(e^+p)/\sigma(e^-p)$, where the real part of the two-photon exchange amplitude leads to an enhancement, and in Rosenbluth data, where it can lead to nonlinearities in ϵ . There is no experimental evidence of nonlinearities in the Rosenbluth data, and the e^+/e^- ratio data (Mar *et al.*, 1968) are of only modest precision, making it difficult to absolutely confirm the presence of two-photon effects in these processes.

It is the imaginary part of the two-photon amplitude that can lead to single spin asymmetries, but again the existing data (Kirkman *et al.*, 1970; Powell *et al.*, 1970) are of insufficient precision to allow one to make a statement. There is, however, one observable that has provided unambiguous evidence for a two-photon effect in ep elastic scattering. Groups in both the United States (Wells *et al.*, 2001) and Europe (Maas *et al.*, 2005) have measured the transverse polarized beam asymmetry. These measurements are significant but have limited utility in solving the G_E^p discrepancy. The reader interested in more details about the existence of two-photon effects and their role in the form factor measurements should refer to Arrington (2005).

For quasielastic electron-nucleus scattering one therefore must ask the following question: Which form factors should one use in the interpretation of the data? What one needs is a parametrization of the electron-nucleon cross section for the various kinematics, and this parametrization is provided by the form factors G_E and G_M determined via Rosenbluth separation of the eN cross sections. Whether the G 's from Rosenbluth separations or polarization transfer, with or without two-

photon corrections, are the *true* (one-photon exchange) form factors is largely irrelevant.

In any case, the discrepancy is largely confined to the proton electric form factor G_E^p . At large momentum transfers [above $1(\text{GeV}/c)^2$], the quasielastic cross section is weakly dependent on G_E^p . For example, at forward angles at $1(\text{GeV}/c)^2$, the electric part of the proton cross section is approximately 30%, decreases to less than 20% at $2(\text{GeV}/c)^2$, and to less than 10% at $4(\text{GeV}/c)^2$. The electric proton contribution to quasielastic electron-nucleus scattering is almost a factor of 2 smaller due to the magnetic contribution of the neutron cross section.

We also note that another potential problem recently discussed (Bystritskiy *et al.*, 2007), namely, the difference between radiative corrections calculated by different authors, hardly influences the interpretation of quasielastic electron-nucleus scattering. For the time being, most of the data on both the nucleon and nuclei have been radiatively corrected using the approach of Mo and Tsai (including various improvements made over the years), and eventual shortcomings drop out when using nucleon form factors in predicting quasielastic cross sections.

VII. SCALING

In general, the inclusive cross section is a function of two independent variables, the momentum transfer \mathbf{q} and the energy transfer ω of the electron. (The L or T nature of the scattering might be considered as a third independent—discrete—variable.) *Scaling* refers to the dependence of the cross section on a single variable $y(\mathbf{q}, \omega)$, itself dependent on \mathbf{q} and ω . This scaling property is basically a consequence of momentum and energy conservation in the quasifree scattering process (West, 1975; Sick *et al.*, 1980).

Inclusive scattering by a “weakly” interacting probe such as the electron can often be interpreted in terms of IA (see Sec. II). Quantitative derivations of scaling in IA have been given in several places; see, e.g., Day *et al.* (1990). Here we first look at a *qualitative* consideration, which, however, contains much of the basic physics. Energy and momentum conservation for quasifree scattering off an initially bound nucleon with momentum \mathbf{k} yields

$$\omega = [(\mathbf{k} + \mathbf{q})^2 + m^2]^{1/2} - m + E + E_{\text{recoil}}, \quad (57)$$

with $E_{\text{recoil}} = \mathbf{k}^2/2m(A-1)$. Splitting \mathbf{k} into its components k_{\parallel} and k_{\perp} parallel and perpendicular to \mathbf{q} and assuming $|\mathbf{q}|, \omega \rightarrow \infty$, such that the k_{\perp}^2 and recoil- and removal-energy terms can be neglected, yields $(\omega + m)^2 = k_{\parallel}^2 + 2k_{\parallel}|\mathbf{q}| + |\mathbf{q}|^2 - m^2$. This equation reveals that $k_{\parallel} = y(\mathbf{q}, \omega)$; \mathbf{q} and ω are no longer independent variables. The same value of y ($\equiv k_{\parallel}$) can result from different combinations of $|\mathbf{q}|$ and ω . The cross section $\sigma(|\mathbf{q}|, \omega)$ divided by the electron-nucleon cross section $\sigma_{eN}(|\mathbf{q}|, \omega)$ and a kinematic factor gives a function $F(y)$ that only depends on y . This function F has an easy approximate interpretation: it represents the probability to find in the

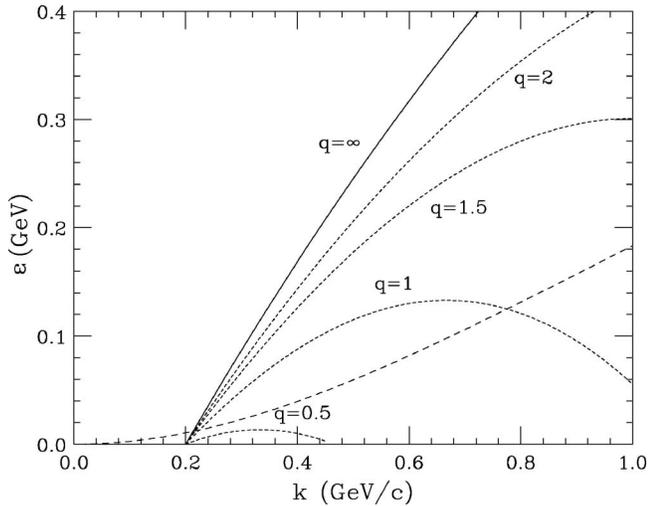


FIG. 17. Integration region of $S(\mathbf{k}, E)$ in Eq. (59) (below corresponding curves) contributing to $F(y)$ at $y = -0.2$ GeV/ c and given $|\mathbf{q}|$ (Day *et al.*, 1990). The long-dashed line shows the location of the “ridge” of $S(\mathbf{k}, E)$ where, at large $|\mathbf{k}|$, most of the strength is expected to occur.

nucleus a nucleon of momentum component y parallel to \mathbf{q} .

The quantitative derivation of y scaling is more involved (Pace and Salme, 1982; Day *et al.*, 1990). In the limit of very large momentum transfer, one also finds scaling; the main *quantitative* difference concerns the restriction of the region $|\mathbf{k}|, E$ which contributes to $F(y)$; Fig. 17 shows an example. In the limit of very large $|\mathbf{q}|$, the scaling function is given by

$$F(y) = 2\pi \int_y^\infty k dk \tilde{n}(y; |\mathbf{k}|), \quad (58)$$

with

$$\tilde{n}(y; |\mathbf{k}|) = \int_0^E dES(|\mathbf{k}|, E). \quad (59)$$

Allowing the upper integration limit to ∞ , \tilde{n} would then correspond to

$$n(k) = \int_0^\infty dES(|\mathbf{k}|, E) \quad (60)$$

and $F(y)$ would become the probability $n(k_{\parallel})$ to find a nucleon with momentum component k_{\parallel} in the nucleus. This approximation is quite reasonable; due to the rapid falloff of $S(\mathbf{k}, E)$ with increasing $|\mathbf{k}|$ or E , the integral is dominated by the region near $(|\mathbf{k}| = y, E = 0)$.

As an illustration of scaling in quasielastic electron scattering, we show in Fig. 18 some available inclusive scattering data for ${}^3\text{He}$. The data cover a large kinematic range and extend over many orders of magnitude in cross section, and the quasielastic peak shifts over a large range of ω with increasing $|\mathbf{q}|$. The same data, now shown in terms of the scaling function and plotted as a function of the scaling variable y (Fig. 19), show an im-

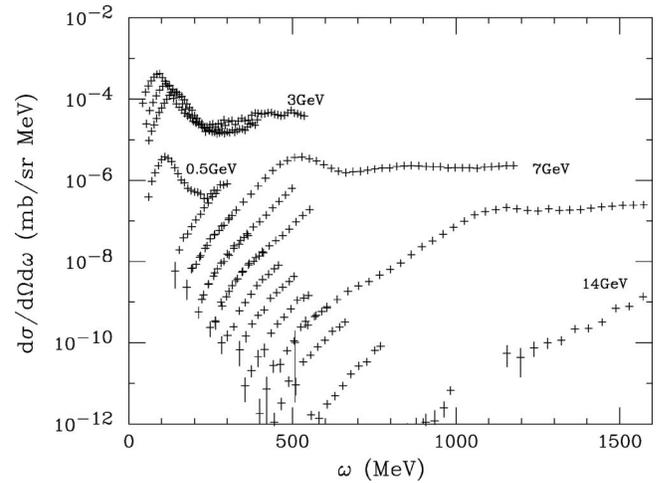


FIG. 18. Cross sections for ${}^3\text{He}(e, e')$ as a function of incident electron energy and energy loss, for different incident electron energies.

pressive scaling behavior for $y < 0$, i.e., for the low- ω side of the quasielastic peak. The cross section that differs by several orders of magnitude define the *same* function $F(y)$. For $y > 0$, the values strongly diverge, primarily due to the more involved kinematics and the different q dependence of the inelastic eN cross section (Δ excitation in particular), which contributes to the inclusive cross section at large ω and q .

In the IA, because of the distribution of strength in E of the spectral function, we expect that the asymptotic limit of the scaling function would be approached from below with increasing $|\mathbf{q}|$.

This scaling property allows the data to be exploited in several ways: The presence or absence of scaling tells us something about the reaction mechanism (we assumed IA without FSI to derive scaling), a residual $|\mathbf{q}|$ dependence of $F(y)$ can tell us something about the $|\mathbf{q}|$ dependence of the in-medium nucleon form factor and FSI, and the functional form of the experimental scaling function provides some insight into the nuclear spectral

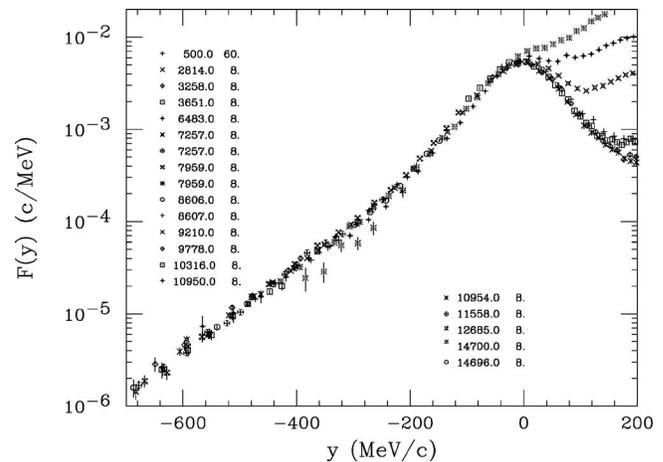


FIG. 19. Scaling function for ${}^3\text{He}(e, e')$. The various data sets are labeled by electron energy (MeV) and angle (deg).

function. We address some of these points in more detail below.

Reaction mechanism. The data show a marked deviation from scaling behavior for $y > 0$, indicating that in this region, processes other than quasifree scattering, such as MEC, pion production, Δ excitation, and DIS, contribute. This is confirmed by substituting MEC or Δ cross sections for the electron-nucleon cross section in our development of $F(y)$. In this instance, the data profoundly fail to scale for $y < 0$ as well. We thus may conclude in particular that, at places where the data approximately scale, contributions of reaction mechanisms other than quasifree scattering are smaller than the residual nonscaling of the data. Thus, scaling gives us direct information on the reaction mechanism, a question we must understand before we can use the data for quantitative interpretation.

Bound-nucleon form factor. The scaling function is secured by dividing the experimental cross sections by the electron-nucleon cross section. If the $|\mathbf{q}|$ dependence of the electron-nucleon cross section is not correct, the data will not scale, at least as long as the range of $|\mathbf{q}|$ covered is large enough to lead to a large variation of the elementary cross section. In order to exploit this idea quantitatively, one can compute the scaling function using a modified nucleon form factor describing the assumed revision of the bound-nucleon form factor, depending on one parameter, and then fit $F(y)$ using a flexible parametrization. The minimum of χ^2 of this fit gives the best value for the parameter modifying the nucleon form factor.

It has been found for iron, for example (Sick, 1986), that within the systematic error of 3% no change of the nucleon radius (more precisely the size parameter in the dipole formula) can be supported by the data. As the cross sections receive a $\sim 70\%$ contribution from magnetic scattering, this result mainly applies to the magnetic radius; the limit on any medium modification of the charge radius is twice as large. Given the number of models that predict a sizable influence of the nuclear medium on the nucleon form factors, the information provided by scaling behavior is quite constraining.

Constituent mass. In the calculation of the scaling variable y , the mass m_c of the constituent from which the electron scatters plays an important role, at least as long as the recoiling constituent is not ultrarelativistic (recoil energy > 3 GeV). The dependence of the scaling function on m_c can be exploited to learn about the nature of the constituent. This is important for small ω and large q , where it has been suggested that scattering from quark clusters or individual quarks plays a role (Pirner and Vary, 1981; Kumano and Moniz, 1988). For the kinematical region explored, the data are best explained by scattering from *nucleons*.

It is only possible to identify the participating constituent (its mass and form factor) if the data cover a large range of momentum transfer. If only a limited range is considered, accidental compensations can occur that then obscure an interpretation. For instance, the

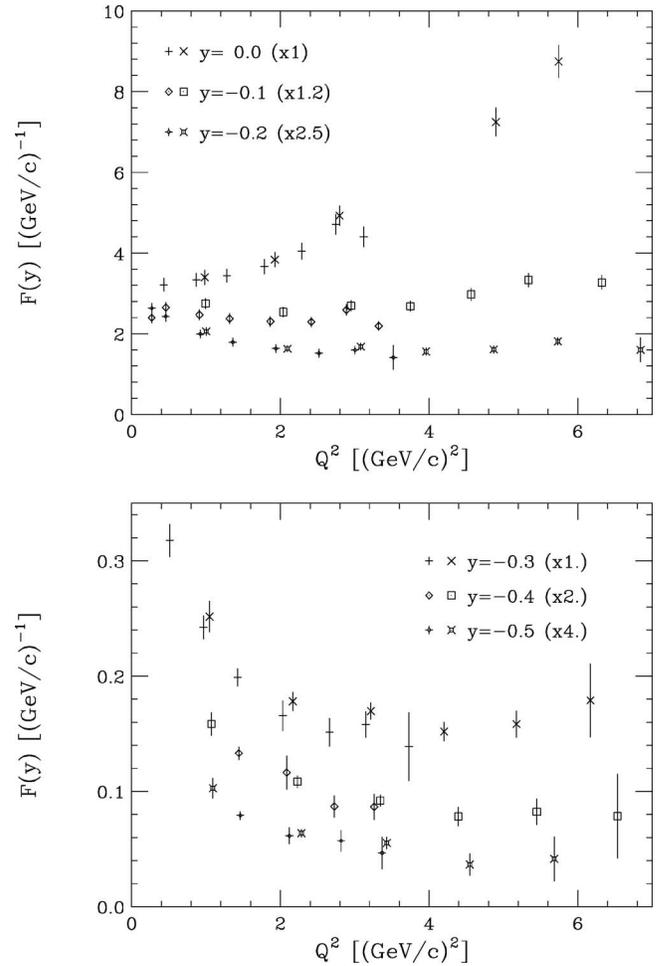


FIG. 20. Convergence of $F(y, Q^2)$ with increasing Q^2 , for different values of y (Arrington *et al.*, 1999). The rise of $F(y, Q^2)$ for $y=0$ and increasing Q^2 is due to the Δ contribution.

simultaneous observation of y scaling (nucleon mass and form factor) and ξ scaling (pointlike, massless constituents) was rather confusing until it could be shown to result from an accidental cancellation of q dependencies (Benhar and Liuti, 1995; Day and Sick, 2004).

Nucleon FSI. The final-state interaction of the knocked-out nucleon in general is of minor importance in inclusive scattering; the electron carries information only about the FSI that takes place within a distance of order $1/|\mathbf{q}|$ from the scattering vertex. Subsequent interactions of the recoil nucleon on its way out of the nucleus [which are much more important for, e.g., $(e, e'p)$] do not influence the scattered electron.

At low energy loss FSI does, however, play a role, as discussed in Sec. III. While the distribution of the spectral function $S(|\mathbf{k}|, E)$ in E leads to a convergence of $F(y, |\mathbf{q}|)$ from below with increasing $|\mathbf{q}|$, the FSI leads to a convergence from above. Figure 20, which presents the scaling function $F(y, Q^2)$ for iron for fixed values of y , shows this convergence from above, and, with the large momentum transfers recently made available at Jefferson Lab (Arrington *et al.*, 1999), that convergence of $F(y, Q^2)$ can be demonstrated for values of $-y$ as large as 0.5 GeV/ c .

Drawing conclusions on FSI starting from scaling should be done with caution. Scaling is usually derived assuming the IA, so the experimental observation of scaling would seemingly suggest that FSI are unimportant. This, however, is not correct. It has been shown (Benhar, 1999) how scaling in the presence of FSI can come about: for quasielastic scattering, the width of the quasielastic peak becomes constant at large $|\mathbf{q}|$ (due to relativistic kinematics). If at the same time the folding function that accounts for FSI becomes q independent—which is the case for NN scattering, where the total cross section is essentially independent of momentum—then the folded function also is independent of $|\mathbf{q}|$, and will scale. The same observation has been made for deep inelastic lepton-nucleon scattering (Paris and Pandharipande, 2002).

It has also been pointed out (Weinstein and Negele, 1982) that for a hard-core interaction the scaling function is not directly related to the momentum distribution, due to the effect of FSI. However, for less singular interactions, such as the Paris potential, it has been shown (Butler and Koonin, 1988) via a calculation in Brueckner-Goldstone theory that the full response does converge to the IA result at large $|\mathbf{q}|$.

Spectral function at large momentum. The properties of $S(\mathbf{k}, E)$ at large $|\mathbf{k}|$ are closely connected to the behavior of the inclusive cross section at large $|\mathbf{q}|$ (several GeV/c) and comparatively low ω (several hundred MeV). This is qualitatively obvious when considering the limit of $\omega=0$. It is only possible (working in the PWIA) to transfer a large momentum $|\mathbf{q}|$ to the nucleon with the result that the nucleon in the final state will have both small energy and hence small momentum $\mathbf{k} + \mathbf{q}$, if the initial nucleon had momentum $\mathbf{k} \sim -\mathbf{q}$ before the scattering.

The region of the low- ω tail of the quasielastic response is best studied by considering the ratio of the nuclear and deuteron response (the latter being well known experimentally and accurately calculable for any NN potential).

Figure 21 shows the ratio for one of the kinematics at large $|\mathbf{q}|$, small ω , where data are available. Data and theory are plotted as a function of the variable y_D , which is basically the component of the nucleon momentum \mathbf{k} parallel to \mathbf{q} , calculated for the deuteron. The dip at $y = 0$ results from the fact that the per nucleon momentum space density at low momenta is higher for the deuteron (see Fig. 3); the “plateau” in the region of $y < -300$ MeV/c results from the fact that the deuteron and nuclear momentum distributions have a similar falloff at large momenta and essentially differ by an overall factor.

Figure 21 shows that the particular nuclear matter spectral function used in this calculation (Benhar *et al.*, 1989) agrees well with the data. Tests carried out by renormalizing the spectral function at $|\mathbf{k}| > k_F$ have shown that the cross-section ratio at $y < -300$ MeV/c is essentially proportional to $S(|\mathbf{k}|, E)$ at $|\mathbf{k}| > k_F$. One needs to realize, however, that for a quantitative study

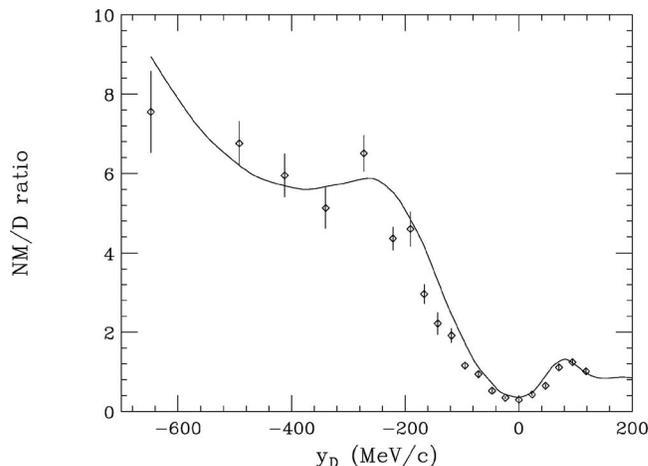


FIG. 21. Per nucleon cross section ratio of nuclear matter and the deuteron taken at 3.6 GeV and 30° , as a function of the scaling variable y . The result of the CBF calculation is shown as a solid line (Benhar *et al.*, 1994).

of $n(|\mathbf{k}|)$, it is important to include FSI, as has been done in Fig. 21, although it was suggested (Frankfurt *et al.*, 1993) that the FSI could cancel in this ratio (see also Fig. 11). Also, when dealing with nuclei with $A > 2$ it is important to use a spectral function $S(k, E)$ rather than just a momentum distribution (which ignores the E dependence of S); see Fig. 11.

In terms of the scaling variable x , the inclusive cross sections now reach up to $x \sim 3$. It is of course tempting to interpret the strength near $x=2$ ($x=3$) as originating from scattering off two (three) correlated nucleon systems having $2m$ ($3m$) mass. This type of interpretation (Egiyan *et al.*, 2003), however, ignores the fact that the data exhibit clear y scaling, hereby proving that the electron scatters from constituents with nucleonic mass and nucleonic form factor. The interpretation of cross-section ratios between nuclei as ratios of correlated strength is also hindered by the fact that x , unlike y , is not simply related to the momentum carried by the struck nucleon (Liuti, 1993). In addition, the strength at very negative y (large x) is strongly affected by A -dependent FSI; see Fig. 11.

Superscaling. Recently, the scaling idea has been pushed one step further (Donnelly and Sick, 1999). Motivated by the Fermi-gas model, in which all momentum distributions only differ by an overall scale factor (the Fermi momentum), Donnelly and Sick have investigated whether the scaling functions of different nuclei can also be related to each other by adjusting one overall scale factor. It turns out that this is possible for nuclei with $A \geq 12$; for lighter nuclei, deviations near the top of the quasielastic peak are visible. Figure 22 shows an example of the scaling function $f(\psi')$ plotted as a function of ψ' , which corresponds to the variable y scaled by a “Fermi momentum.”

This scaling as a function of nuclear mass number is actually better realized than the normal scaling, which is broken by the non-quasielastic contributions to the re-

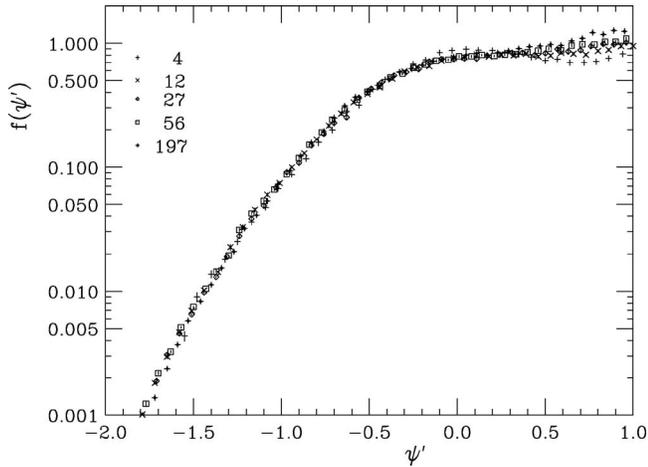


FIG. 22. Scaling function for nuclei $A=4-197$ and fixed kinematics ($|\mathbf{q}| \sim 1$ GeV/c) as a function of ψ' , which corresponds to the variable y scaled by a “Fermi momentum.”

sponse; these non-quasielastic contributions are not too dissimilar for different nuclei at the same kinematics. As the momentum used to scale y is a slow and smooth function of A , this superscaling property is particularly useful to interpolate data on $F(y)$ in order to predict $F(y)$ for nuclei not experimentally investigated.

For the *longitudinal* response, superscaling is particularly well realized, even at large energy loss where the transverse quasielastic response is obscured by inelastic eN scattering. Figure 23 shows the longitudinal response function determined via superscaling from the available separated data on C, Ca, and Fe. This function, incidentally, also displays a tail toward larger ω discussed in Sec. II.

This superscaling feature is particularly useful when one realizes that (e, e') and ν -induced quasielastic processes such as (ν, e) or (ν, μ) differ only by the elementary vertex, the underlying nuclear physics being the same. Cross sections for (e, e') together with superscal-

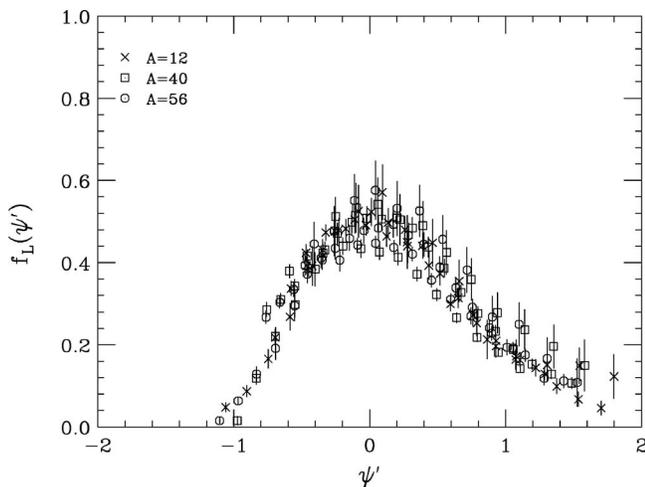


FIG. 23. Longitudinal superscaling function extracted (Donnelly and Sick, 1999) from the available data on C, Ca, and Fe, for momentum transfers between 300 and 570 MeV/c.

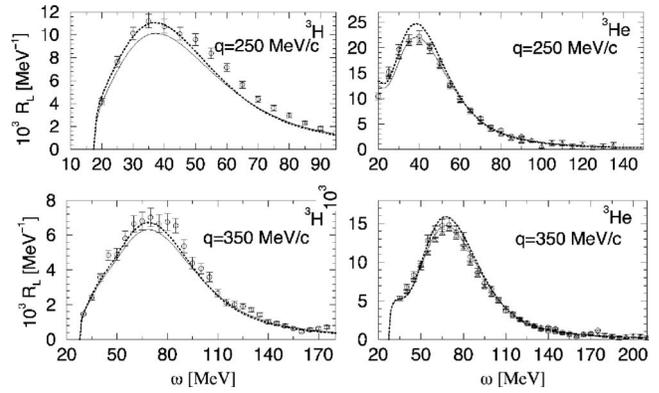


FIG. 24. Comparison of experimental and theoretical response (Efros *et al.*, 2004) for Argonne 18 potential (dotted) and Argonne V18+3BF (solid). Data are shown as open circles (Dow *et al.*, 1988) and squares (Marchand *et al.*, 1985).

ing then allow one to accurately predict the cross section for neutrino-induced reactions (Amaro *et al.*, 2005), currently an area of intense experimental activity.

VIII. LIGHT NUCLEI

Quasielastic scattering from light nuclei ($A \leq 4$) occupies a special place. For these systems, several methods to calculate the inclusive response have been used, often treating both the initial and the final state in less approximate ways than imposed by the complexities of the many-body system for heavier nuclei. For $A=3$, for example, the first calculation of a realistic spectral function was performed 30 years ago (Dieperink *et al.*, 1976). A variety of approaches to describe quasielastic scattering from light nuclei is available; we address some of their results below.

The longitudinal response for the $A=3$ nuclei has been calculated (Efros *et al.*, 2004) using the Lorentz integral transform technique (see also Sec. IX). They use ground-state wave functions expanded in terms of correlated sums of hyperspherical harmonics, calculated using different NN forces (e.g., Argonne 18, Bonn-A) and three-body force (3BF) models. As their calculation is essentially nonrelativistic, they restrict it to $|\mathbf{q}| < 500$ MeV/c, and consider the longitudinal response to avoid the difficulty of MEC. Figure 24 shows their results for ${}^3\text{He}$ and ${}^3\text{H}$ compared to data.

This calculation finds rather small differences between the responses calculated using different modern NN forces. The three-body force leads to a systematic reduction in the height of the quasielastic peak, presumably as a consequence of the tighter binding, which leads to a more compressed (extended) distribution in radial (momentum) space. While for ${}^3\text{He}$ the 3BF is helpful in explaining the data, this does not seem to be the case for the ${}^3\text{H}$ nucleus. In a recent paper (Efros *et al.*, 2005), the range of applicability of this nonrelativistic calculation has been extended.

The inclusive cross section has also been calculated separately for the two- and three-body breakup (Golak

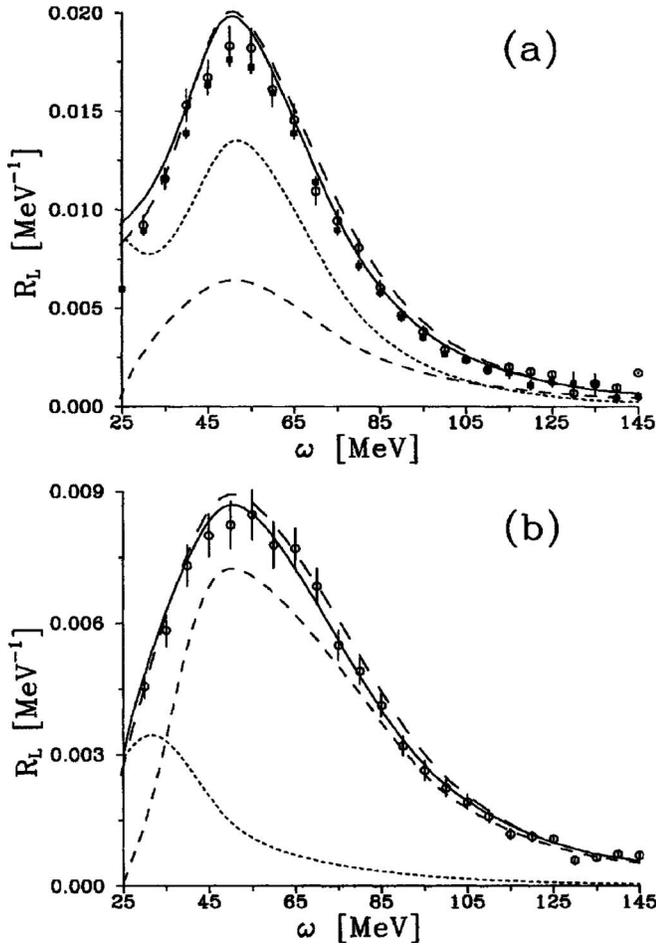


FIG. 25. Comparison of theoretical (Golak *et al.*, 1995) and experimental response at $|\mathbf{q}|=300$ MeV/c for (a) ${}^3\text{He}$ and (b) ${}^3\text{H}$. Dotted, two-body breakup; dashed, three-body breakup. Data are shown as open circles (Dow *et al.*, 1988) and squares (Marchand *et al.*, 1985).

et al., 1995). For the ground state, Golak *et al.* used a solution of the 34-channel Faddeev equation for the Bonn-B NN potential. For the final state, the authors separated the contribution from the (symmetrized) plane-wave approach and the one from rescattering processes, summed to all orders. This calculation again is nonrelativistic, and does not include MEC.

Figure 25 shows the results of Golak *et al.* for ${}^3\text{He}$ and ${}^3\text{H}$, with the contributions of two- and three-body breakup separately displayed.

The inclusive cross section has also been calculated using a realistic coupled-channel potential with single Δ -isobar excitation for the initial and final hadronic states, with the corresponding e.m. current with two-baryon contributions (Deltuva *et al.*, 2004). The potential is an extension of the CD-Bonn NN potential. The main Δ -isobar effects on observables result from the effective three-nucleon force and the corresponding effective two- and three-nucleon exchange currents. Both initial bound state and final continuum state are exact solutions of the three-particle scattering equations.

In particular, at low momentum transfer Deltuva *et al.*

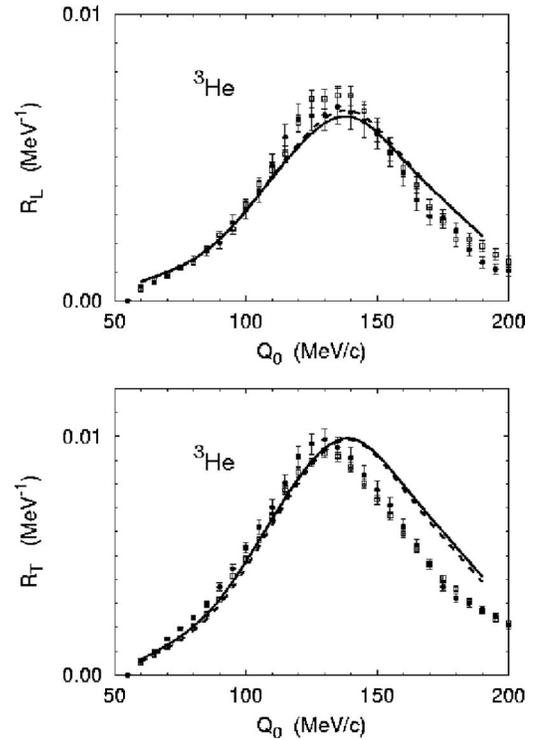


FIG. 26. Comparison of theoretical (Deltuva *et al.*, 2004) and experimental response at $|\mathbf{q}|=500$ MeV/c. Solid, coupled-channel nucleon- Δ ; dashed, purely nucleonic case. Data are shown as circles (Dow *et al.*, 1988) and squares (Marchand *et al.*, 1985).

found large effects due to the Δ in the threshold region of the transverse response. At the larger momentum transfers, e.g., at 500 MeV/c shown in Fig. 26 for ${}^3\text{He}$, the contribution of Δ degrees of freedom is smaller, in agreement with the observation made in Sec. IX. The small shift between data and calculation has been assigned to the use of nonrelativistic kinematics (necessary for consistency). Deltuva *et al.* have also calculated other e.m. observables such as elastic form factors and exclusive quasielastic cross sections, and find rather large changes when allowing for the coupled channel nucleon- Δ case.

As pointed out in Sec. VII, the scaling function $F(y)$ is closely related to the spectral function and momentum distribution. In the limit of very large momentum transfer, and when neglecting the effects due to finite E and FSI, the scaling function is given by

$$F(y) = 2\pi \int_{|y|}^{\infty} n(|\mathbf{k}|) d|\mathbf{k}|, \quad (61)$$

in which case the momentum distribution can be obtained from

$$n(|\mathbf{k}|) = \frac{1}{2\pi y} \frac{dF}{dy} \quad \text{with } k = |y|. \quad (62)$$

For the deuteron, neglecting the effects of finite E and FSI is not unreasonable as the data reach very high q . The $n(k)$ derived via scaling (Ciofi degli Atti *et al.*, 1987)

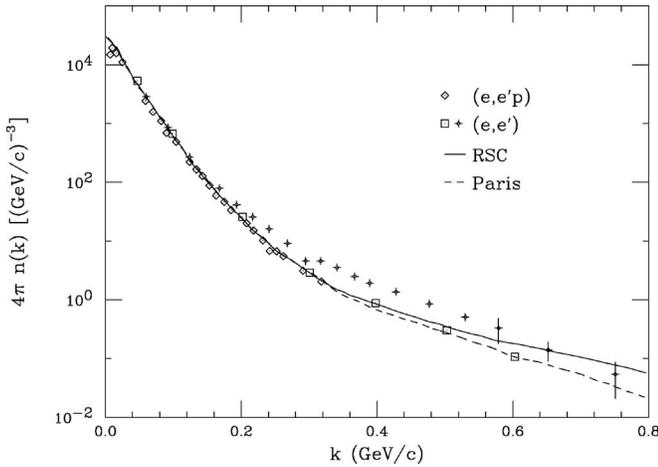


FIG. 27. Comparison of deuteron momentum distribution obtained from (e, e') (squares), $(e, e'p)$ (triangles), and theory (curves) (Ciofi degli Atti *et al.*, 1987).

is compared in Fig. 27 to the ones obtained from $(e, e'p)$ reactions and theory.

For heavier nuclei, both FSI and the effects of the distribution in E are no longer negligible, and can only be incorporated in predictions of $F(y, Q^2)$ starting from $S(\mathbf{k}, E)$.

IX. EUCLIDEAN RESPONSE

In previous sections, we discussed quasielastic scattering in terms of PWIA, with corrections for FSI. This type of theoretical description often is the only practical one, as a less approximate treatment of the final, relativistic continuum state is hard to come by.

For comparatively low momentum transfers, alternative and more reliable treatments are possible. For $A=3$ and $A=\infty$, the response in the nonrelativistic region can be calculated (Fabrocini and Fantoni, 1989; Golak *et al.*, 1995). For nuclei in between, integrals over the quasielastic response can be studied. These integrals can be expressed as expectation values of the ground-state wave function, without the need for an explicit treatment of the continuum state. This approach has been followed via both the Lorentz transform technique (Efros *et al.*, 1994; Leidemann *et al.*, 1997) and the use of the Laplace transform (Carlson and Schiavilla, 1992). The most extensive results, including one- and two-body currents, are available for the latter approach (Carlson *et al.*, 2002), which we discuss in more detail below.

The Euclidean response is defined as an integral over the quasielastic response

$$\tilde{E}_{T,L}(|\mathbf{q}|, \tau) = \int_{\omega_{\text{th}}}^{\infty} \exp[-\omega\tau] R_{T,L}(|\mathbf{q}|, \omega) d\omega, \quad (63)$$

with $\tau = Q^2/4m$. In the Lorentz transform technique, an additional Lorentzian factor that enhances the integrand at a given ω is used. The Lorentz transform technique has the advantage that the response as a function of ω can be reconstructed by the inverse transformation,

something that is not practical for the Euclidean response.

The longitudinal and transverse Euclidean response functions represent weighted sums of the corresponding $R_L(|\mathbf{q}|, \omega)$ and $R_T(|\mathbf{q}|, \omega)$: at $\tau=0$, they correspond to the Coulomb and transverse sum rules, respectively, while their derivatives with respect to τ evaluated at $\tau=0$ correspond to the energy-weighted sum rules. Larger values of τ correspond to integrals over progressively lower energy loss regions of the response.

In a nonrelativistic picture, $\tilde{E}_{T,L}$ can be obtained from

$$\begin{aligned} \tilde{E}_L(|\mathbf{q}|, \tau) &= \langle 0 | \rho^\dagger(\mathbf{q}) \exp[-(H - E_0)\tau] \rho(\mathbf{q}) | 0 \rangle \\ &\quad - \exp\left(-\frac{q^2\tau}{2Am}\right) |\langle 0(\mathbf{q}) | \rho(\mathbf{q}) | 0 \rangle|^2, \end{aligned} \quad (64)$$

and similarly for $\tilde{E}_T(|\mathbf{q}|, \tau)$, with the charge operator $\rho(\mathbf{q})$ replaced by the current operator $\mathbf{j}_T(\mathbf{q})$. The elastic contributions have been explicitly subtracted, and $|0(\mathbf{q})\rangle$ represents the ground state recoiling with momentum \mathbf{q} .

The study of the Euclidean response has the outstanding advantage that $\tilde{E}(|\mathbf{q}|, \tau)$ can be calculated from the ground-state properties alone; no explicit treatment of the final continuum state is required. For the $A=3, 4$ ground states, very precise wave functions are available, and the effects of MEC can be included using the two-body operators well established in elastic and inelastic electron scattering from light nuclei (for a review, see Sick, 2001).

The Euclidean response has the disadvantage that we lack an intuitive interpretation of this integrated quantity. Model studies (Carlson *et al.*, 2002) have shown the sensitivity of $\tilde{E}(|\mathbf{q}|, \tau)$ to properties of $R(|\mathbf{q}|, \omega)$; see Fig. 28.

Figure 28 (top) shows various modifications of the “normal” response, Fig. 28 (bottom) shows the effect on the ratios of the resulting Euclidean responses to the normal one. These studies show that, for the responses that can be extracted from the data, the region $0.01 \leq \tau \leq 0.05 \text{ MeV}^{-1}$ is the most relevant one for a comparison with theory. Below $\tau=0.01 \text{ MeV}^{-1}$, the contribution of the tail of the Δ in the experimental response is too important; above $\tau=0.05 \text{ MeV}^{-1}$, the response is totally dominated by the contribution of very small values of ω .

The calculations discussed here (Carlson *et al.*, 2002) have used the standard expressions for the one-body electromagnetic operators, obtained from a relativistic reduction of the covariant single-nucleon current. The two-body current operator consists of “model-independent” and “model-dependent” components, in the standard classification scheme (Riska, 1989). The model-independent terms are obtained (Schiavilla *et al.*, 1990) from the nucleon-nucleon interaction. For the model-dependent pieces, the calculation includes the isoscalar $\rho\pi\gamma$ and isovector $\omega\pi\gamma$ transition currents as well as the isovector current associated with excitation of intermediate Δ -isobar resonances. The two-body charge operators (Schiavilla *et al.*, 1990) include the π -,

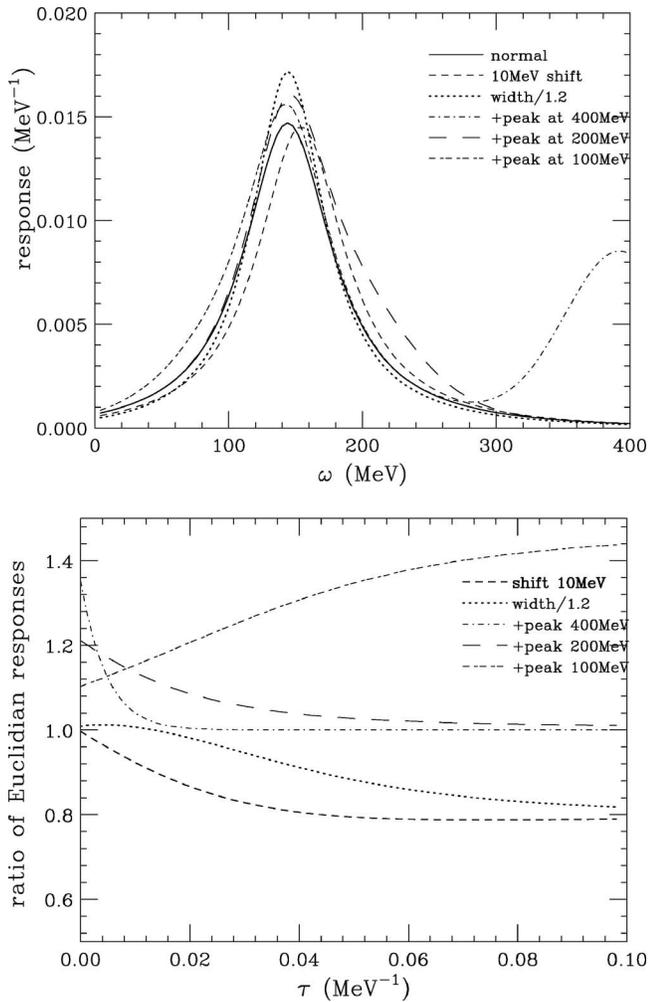


FIG. 28. Model responses derived from the “normal” response plus various modifications (top). The corresponding Euclidean response in terms of the ratio to the one from the normal one (bottom).

ρ -, and ω -meson exchange charge operators, the (isoscalar) $\rho\pi\gamma$ and (isovector) $\omega\pi\gamma$ couplings, and the single-nucleon Darwin-Foldy and spin-orbit relativistic corrections.

We show in Fig. 29 the results obtained for ${}^3\text{He}$ and ${}^4\text{He}$ at one value of $|\mathbf{q}|$. The ground-state wave functions used in this study were obtained with the variational Monte Carlo approach (Carlson *et al.*, 2002) and the Argonne v_8 NN interaction plus the UIX three-nucleon interaction.

The helium nuclei studied by Carlson *et al.* are of particular interest as for ${}^4\text{He}$ the excess in the transverse strength is maximal among all nuclei, and grows by a factor of 2 between $A=3$ and 4. This excess—presumably due to MEC—had not been understood in the past; many calculations of MEC for a multitude of nuclei gave results that were rather discordant, and always much too small. The data for the helium nuclei also show that this excess covers the entire quasielastic, and not only the “dip” region between the quasielastic and Δ peak.

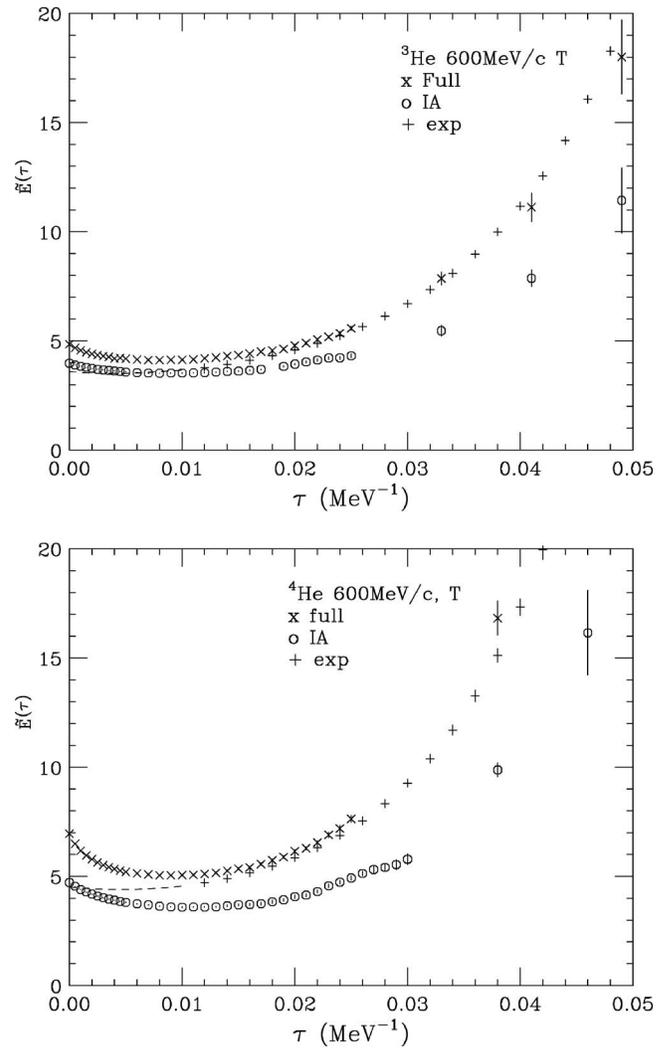


FIG. 29. Transverse Euclidean response divided by the proton magnetic form factor for ${}^3\text{He}$ and ${}^4\text{He}$ at 600 MeV/ c momentum transfer, scaled by $e^{\tau\omega_{qe}}$. Data (+), IA (○), and full calculation (×).

Figure 29 shows that the full calculation, which includes MEC, is in good agreement with the data (we pointed out above why the region $\tau < 0.01$ MeV $^{-1}$ should be ignored). The calculation predicts rather accurately the enhancement of the transverse strength due to MEC and the doubling between $A=3$ and 4, and it also does quite well in predicting the Q^2 dependence (not shown).

The good reproduction of the transverse strength at first glance comes as somewhat of a surprise, given the lack of success of previous MEC calculations (Donnelly *et al.*, 1978; Kohno and Ohtsuka, 1981; Van Orden and Donnelly, 1981 Alberico *et al.*, 1984; Blunden and Butler, 1989; Leidemann and Orlandini, 1990; Amaro and Lallena, 1992; Carlson and Schiavilla, 1994; Dekker *et al.*, 1994; Van der Sluys *et al.*, 1995; Anguiano *et al.*, 1996; Fabrocini, 1997a; Gadiyak and Dmitriev, 1998). Carlson *et al.* have therefore investigated in more detail the reason for the large MEC contribution. They have found that, in agreement with previous studies (Leidemann and Orlandini, 1990; Fabrocini, 1997b), MEC only pro-

duce large effects in combination with ground-state wave functions calculated *including* the short-range n - p correlations. As most previous calculations were based on independent-particle-type wave functions, the smallness of the resulting MEC contributions is thus understood. To verify this point further, Carlson *et al.* have repeated their calculation using the same operators, but with a Fermi-gas wave function. Instead of an enhancement factor of 1.47 coming from MEC at $|\mathbf{q}| = 600$ MeV/ c , they find a factor of 1.06 only, i.e., an eight times smaller MEC effect.

The results of Carlson *et al.* also show, somewhat surprisingly, that the MEC contribution is large at *low* momentum transfer. It decreases toward the larger Q^2 , in agreement with the expectation that at very large Q^2 it falls (Sargsian, 2001) like Q^{-4} relative to quasielastic scattering.

From the above discussion it becomes clear that the Euclidean response, despite inherent drawbacks, is a valuable quantity. Since the final continuum state does not have to be treated explicitly, calculations of much higher quality can be performed than for the response, and the role of two-body currents can be treated quantitatively. Comparison between data and calculation has shown in particular that for a successful prediction of MEC, *correlated* wave functions for the ground state are needed; such wave functions today are available up to $A \sim 12$ and for $A = \infty$. Unfortunately, the usage of the Euclidean response for the time being is restricted to a regime in which relativistic effects are not too large, such that they can be included as corrections.

X. L/T SEPARATION AND COULOMB SUM RULE

In the impulse approximation, and when neglecting the (small) contribution from nucleonic convection currents, the longitudinal and transverse response functions R_L and R_T contain the same information and have the same size. This has sometimes been called scaling of the zeroth kind (see Sec. VII). It was realized early on, however, that the transverse response receives significant contributions from meson exchange currents and Δ excitation (which are of a largely transverse nature). It is therefore clear that there is a high premium on separating the L and T responses, both because the L response is easier to interpret and because of the additional information contained in the T response.

The separation of the L and T responses is performed using the Rosenbluth technique, which is justified only in the single-photon exchange approximation. The cross section, divided by a number of kinematical factors

$$\frac{d\sigma}{d\Omega d\omega} \frac{\epsilon}{\sigma_{\text{Mott}}} \frac{|\mathbf{q}|^4}{Q^4} = \epsilon R_L(|\mathbf{q}|, \omega) + \frac{|\mathbf{q}|^2}{2Q^2} R_T(|\mathbf{q}|, \omega) = \Sigma, \quad (65)$$

is a linear function of the virtual photon polarization

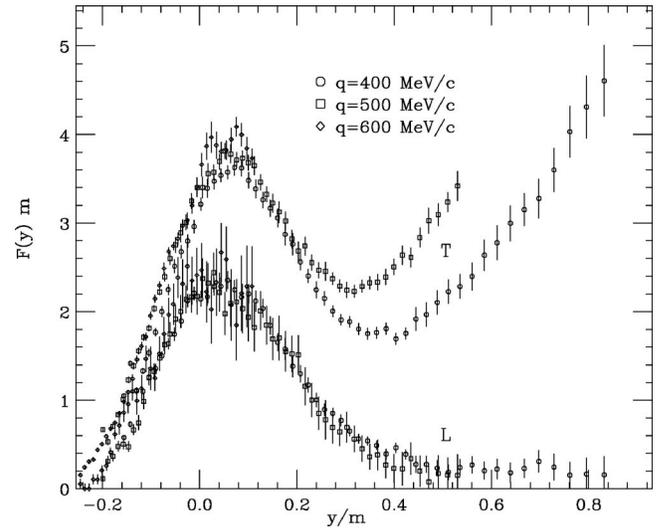


FIG. 30. Longitudinal (lower data set) and transverse responses of ^{12}C (Finn *et al.*, 1984), plotted in terms of the scaling function $F(y)$.

$$\epsilon = \left(1 + \frac{2|\mathbf{q}|^2}{Q^2} \tan^2 \frac{\theta}{2} \right)^{-1} \quad (66)$$

with \mathbf{q} (Q) being the 3- (4-) momentum transfer and ϵ varying from 0 to 1 for scattering angles θ between 180° and 0° . The slope of the linear function yields R_L and the intercept at $\epsilon=0$ yields R_T . Figure 30 shows an early example for an L/T separation, and demonstrates the excess observed for the transverse strength.

While conceptually very straightforward, this L/T separation is difficult in practice. It involves data taking at the same $|\mathbf{q}|$, but varying ϵ , i.e., varying beam energy. For an accurate separation of R_L and R_T , obviously the largest possible range in ϵ , hence beam energy, is required. As data are usually not taken at constant $|\mathbf{q}|$, but at a given beam energy and variable energy loss, obtaining the responses at constant $|\mathbf{q}|$ involves interpolations of the data. We show in Fig. 31 two examples for a Rosenbluth separation, performed on the low- and large- ω side of the quasielastic peak, which also illustrate the importance of the forward angle (high-energy) data for the determination of R_L , i.e., the slope of the fit.

The Rosenbluth technique is applicable in the plane-wave Born approximation, and fails once Coulomb distortion of the electron waves is present. Neglect of distortion is justified for the lightest nuclei alone, and only if R_T is not much bigger (or much smaller) than R_L . When one of the two contributions gets too small, even minor corrections due to Coulomb distortion can have large effects. At large $|\mathbf{q}|$, for instance, even the determination of the proton charge form factor via the Rosenbluth technique is significantly affected by Coulomb corrections (Arrington and Sick, 2004). In order to extract R_L and R_T in the presence of Coulomb distortion, the data must first be corrected for these effects; this is discussed in Sec. XI.

Here we concentrate on the discussion of the longitu-

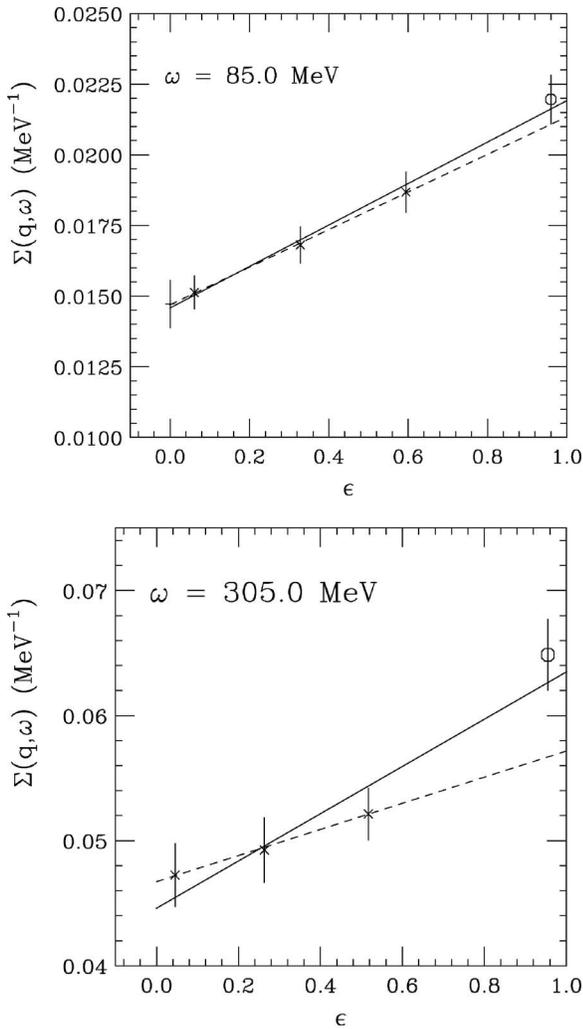


FIG. 31. Rosenbluth separation for ^{56}Fe and $|\mathbf{q}|=570$ MeV/c (Jourdan, 1996). The dashed lines are fits to the Saclay data alone; the solid lines are fits to the world data, which include the forward-angle SLAC data and emphasize the importance of a large range in ϵ .

dinal response. For practical reasons, the determination of the longitudinal response is possible only in a q range that is somewhat limited. In addition, at low $|\mathbf{q}|$, typically below twice the Fermi momentum, the response is affected by Pauli blocking, which in many of the approaches used to describe inclusive scattering is not properly treated. At large $|\mathbf{q}|$, typically above 0.8 GeV/c, the transverse response dominates the cross section, due to both the $|\mathbf{q}|^2/Q^2$ factor in Eq. (65) and the increasing Δ contribution (see Fig. 30), such that an accurate determination of R_L becomes very difficult. Energy-dependent experimental systematic errors must be handled with great care.

One particular use of the longitudinal response has received much attention: the determination of the Coulomb sum rule (CSR). In the nonrelativistic regime, when short-range correlations between nucleons and the effect of Pauli blocking is neglected, the CSR takes the simple form

$$S_L(|\mathbf{q}|) = \frac{1}{Z} \int_{\omega^+}^{\infty} \frac{R_L(|\mathbf{q}|, \omega)}{\tilde{G}_e^2} d\omega, \quad (67)$$

where $\tilde{G}_e^2 = (G_{ep}^2 + G_{en}^2 N/Z)$ and ω^+ is the threshold for particle emission. In the limit of large $|\mathbf{q}|$, S_L should be 1. In other words, when neglecting the small contribution from the neutron charge form factor G_{en} , the integral over the longitudinal response counts the number of protons times the square of the proton charge form factor G_{ep}^2 .

The history of the CSR is very checkered. Early work proposed the CSR as a tool to study short-range correlation SRC between nucleons (Czyz and Gottfried, 1963; Gottfried, 1963). These correlations move strength to large energy loss, and partly out of the physical region; at very large $|\mathbf{q}|$, these correlation contributions go to zero in the sum rule. The series of L/T separations performed at Bates (Altemus *et al.*, 1980) and Saclay (Meziani *et al.*, 1984) found effects in S_L that were much bigger than could be expected from SRC: in the region of $|\mathbf{q}|=350$ –550 MeV/c, S_L was up to 50% lower than expected, the deficit increasing with increasing $|\mathbf{q}|$ and increasing nuclear mass number A . These observations have been widely interpreted as a medium modification of the proton charge form factor (Noble, 1981; Celenza *et al.*, 1986; Mulders, 1986).

The experiments dealing with the CSR have received much of the attention. It was not generally known that some experiments (Altemus *et al.*, 1980) had suffered from rescattering of electrons on the “snout” connecting the scattering chamber and the spectrometer (Deady *et al.*, 1986) (see also Sec. IV). It was also a long time before a reanalysis (Jourdan, 1996) removed a number of deficiencies in the Saclay analysis of the data (see below).

L/T separations have been performed for a number of nuclei and momentum transfers (Zimmerman, 1969; Altemus *et al.*, 1980; Deady *et al.*, 1983, 1986; Barreau *et al.*, 1983; Finn *et al.*, 1984; Blatchley *et al.*, 1986; VanRenden *et al.*, 1990; Chen *et al.*, 1991; Meziani *et al.*, 1992; Yates *et al.*, 1993; Williamson *et al.*, 1997). Often (but not always, see, e.g., Williamson *et al.*, 1997) these separations were quite limited in the ϵ range as data from one facility only were included; from the determination of the proton form factors G_{ep} and G_{mp} , it is well known that more reliable results are obtained from an analysis of the world data spanning the largest possible range in ϵ (electron energy). In almost all cases, only approximate Coulomb corrections were included, using the effective momentum approximation version of the effective-momentum transfer approach (see Sec. XI). Particularly for data producing low S_L , the longitudinal response functions show an unphysical behavior at large ω : they dive steeply toward zero, and would, for responses without discontinuities, obviously be negative just beyond the range of ω shown (see, e.g., Jourdan, 1996). From microscopic calculations (Dieperink *et al.*, 1976; Dellafiore *et al.*, 1985; Fabrocini and Fantoni, 1989) using realistic nuclear spectral functions, we do

know, however, that the response at large ω should approach zero slowly, much more slowly than the response at low ω , as a consequence of the components of large removal energy E present in realistic spectral functions $S(\mathbf{k}, E)$ (see, e.g., Fig. 24).

The analysis of the data performed by Carlson *et al.* (2003) and Jourdan (1996) included the world data to employ the largest ϵ range and improved a number of aspects of the Saclay analysis. In particular, the integral in Eq. (67) was divided by the correct G_{ep} , the well-known relativistic corrections were included (de Forest, 1984), and Coulomb corrections in distorted wave Born approximation (DWBA) were made; in addition, the contribution of $R_L(\omega)$ above the upper integration limit—which experimentally is far from ∞ due to the limited range of data on $S_L(\omega)$ —was added. These L/T separations extend to $|\mathbf{q}| \sim 600$ MeV/ c , only for ${}^4\text{He}$, where the overlap of quasielastic and Δ strength is a lesser problem, the L strength is known up to 1 GeV/ c . The corrections to the 1984 analysis, which all happen to go in the same direction, increase the CSR by a substantial amount. For example, the CSR for the largest $|\mathbf{q}|$ and heaviest nucleus, ${}^{56}\text{Fe}$ and 570 MeV/ c analyzed by Jourdan (1996), amounts to 0.98 ± 0.15 . Figure 32 shows the corresponding L and T responses.

In a subsequent reanalysis of the (e, e') data (Morgenstern and Mezzani, 2001) resulting in a less extreme reduction in the CSR result, for iron the CSR was found to be 82% of the expected value. This result for the sum rule is still significantly smaller than that of Jourdan (1996), the difference being largely due to the use of the effective momentum approximation (EMA) for Coulomb corrections. This emphasizes that the Coulomb corrections are important for the determination of the CSR; in particular, for the largest q value of 570 MeV/ c and the large ω , where the difference from Jourdan (1997) and Morgenstern and Mezzani (2001) is largest, the backward angle data go down to scattered-electron energies as low as 130 MeV, where approximations such as EMA fail (Aste *et al.*, 2005).

The issue of medium modifications of proton electromagnetic structure has recently been revived by results of a polarization transfer ${}^4\text{He}(\vec{e}, e'\vec{p}){}^3\text{H}$ measurement carried out at Jefferson Lab (Strauch *et al.*, 2003). However, the interpretation of the experimental data in terms of medium-modified form factors is challenged by the results of a theoretical calculation, carried out using accurate three- and four-nucleon bound-state wave functions, a realistic model for the nuclear electromagnetic current operator, and a treatment of final-state interactions with an optical potential (Schiavilla *et al.*, 2005). Schiavilla *et al.* (2005) found no significant discrepancies between theory and experiment, both for the ratio of transverse to longitudinal polarization transfers and for the induced polarization, when free-nucleon electromagnetic form factors are used in the current operator.

To close the discussion of the Coulomb sum, we add as a caveat that even for the longitudinal response, the contribution of MEC is not entirely negligible. In the

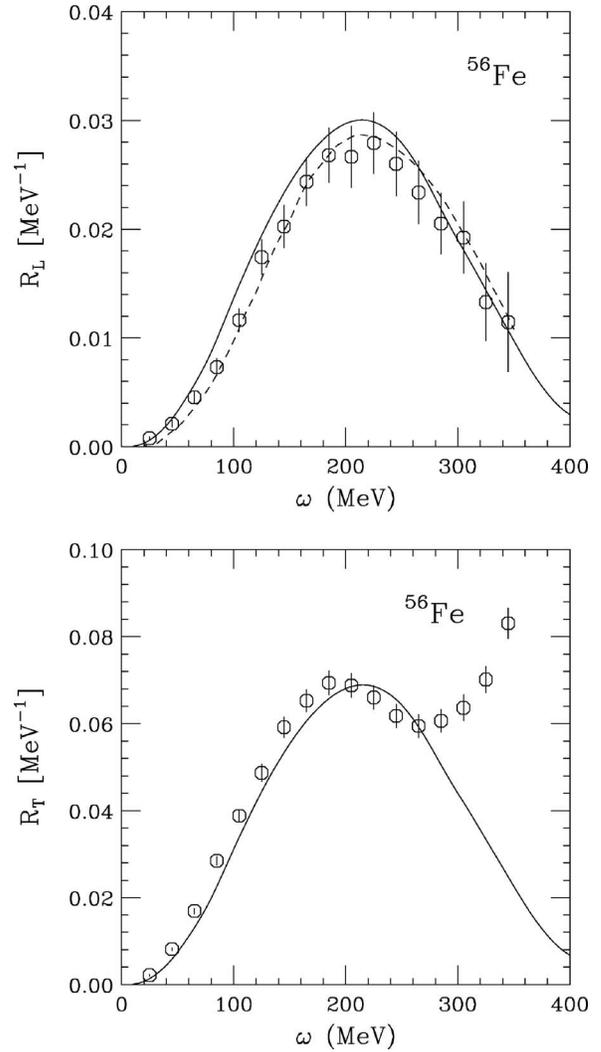


FIG. 32. Separated response functions for ${}^{56}\text{Fe}$ and $|\mathbf{q}| = 570$ MeV/ c . The solid curve corresponds to the CBF calculation (Fabrocini and Fantoni, 1989).

$|\mathbf{q}| \sim 1$ GeV/ c region, it has been shown (Carlson *et al.*, 2003) that for ${}^4\text{He}$ the contributions are of the order of 10%.

XI. COULOMB CORRECTIONS

The effects of the static Coulomb field of nuclei upon quasielastic scattering has posed a persistent problem. The presence of the Coulomb potential—for lead of order of 25–30 MeV in the nuclear interior—has a major effect on the quasielastic cross section measured in the several-hundred-MeV energy region. It invalidates the linear relation (65) used to separate the longitudinal and transverse responses with the Rosenbluth technique.

While the treatment of the Coulomb distortion on electron waves in the quasielastic region presents no conceptual problems, practical application has been difficult. Reliable calculations of Coulomb distortion have not been easily accessible to analyses of experimental data. As a consequence, most experiments have been

analyzed without considering Coulomb distortion effects, or by using relatively simple recipes.

The Coulomb distortion is more important for the lower-energy data and the heavier nuclei. Its effects are visible in the separations of longitudinal and transverse strength, where the small contribution—in general the longitudinal one—is most affected. It is likely that one of the main problems with the longitudinal strength—the diving to negative responses mentioned in Sec. X—is related to this aspect (Traini *et al.*, 1988).

The Coulomb distortion can be treated in the DWBA by using electron wave functions calculated as solutions of the Dirac equation for the known nuclear charge distribution; different programs have been employed (Cò and Heisenberg, 1987; Udias *et al.*, 1993; Kim *et al.*, 1996), in particular by the group of Onley and Wright (Zamani-Noor and Onley, 1986; Jin *et al.*, 1992). In these calculations, high-energy approximations (Lenz and Rosenfelder, 1971; Knoll, 1974; Rosenfelder, 1980) have been employed, as in general the electron energy is much higher than the Coulomb potential. Such calculations have been successfully carried out, and can serve as a benchmark. For a systematic analysis of experimental data, the numerical effort is usually too big and not practical. This is true in particular because often computer codes have been developed for $(e, e'p)$, in which case the (e, e') cross section has to be generated by summing over all possible initial and final states of the knocked-out nucleon.

Simplifications are possible when going to the eikonal approximation (Yennie *et al.*, 1965; Giusti and Pacati, 1987; Traini *et al.*, 1988). When using the lowest-order expansion in $Z\alpha$, one finds two dominating effects:

(i) As a consequence of the attractive electron-nucleus Coulomb interaction, the effective energy of the incident and scattered electrons at the moment of the scattering is increased by the Coulomb potential V . This has the consequence that the effective momentum transfer q_{eff} squared, on which the response functions R depend, is increased by a factor $(E_e + V)(E_{e'} + V)/E_e E_{e'}$.

(ii) Due to the attractive Coulomb interaction, the electron “plane” waves are focused onto the nucleus, hereby increasing the wave function at the location of the nucleus, with a corresponding increase in the scattering cross section.

In the calculation of the inclusive cross section, the overall effect of these corrections is accounted for by using the Mott cross section with the unmodified electron energy, but with q_{eff} as an argument of the response function. This approximation has been termed the effective momentum approximation. Versions in the literature differ by the choice of V . The results of EMA have been compared to the ones from DWBA calculations (Kim *et al.*, 1996; Jourdan, 1997).

Higher-order terms have been included (Traini *et al.*, 1988). These terms have been calculated using severe approximations in the expansion around $r=0$, and are not recommended (Traini, 2001). Due to the approximations, the second-order effects found are about as large

as the first-order effects, which is indicative of problems of the expansion.

The term EMA is often confused in the literature because of the fact that it is used for two different choices of the nuclear Coulomb potential V . This parameter V is often evaluated for the nuclear center $r=0$ assuming a homogeneous nuclear charge density, in which case $V = V_0 = 3Z\alpha/2R_{\text{eq}}$, with R_{eq} being the equivalent radius [a good approximation being $(1.1A^{1/3} + 0.86A^{-1/3})$ fm (Kim *et al.*, 1996)]. It has been recognized early on, however, that a better choice would be an appropriate *average* Coulomb potential (Rosenfelder, 1980). For this reason, many applications of EMA use for V the value at the nuclear surface, $V_s = (2/3)V_0$, where most nucleons are located.

In order to improve upon the quality of the Coulomb corrections without resorting to the full solution of the Dirac equation, one can employ the eikonal distorted wave Born approximation (EDWBA), where electron waves in the DWBA approach, including solutions of the Dirac equation, are calculated using the eikonal approximation (Aste *et al.*, 2004). In this case the electron current is modified by an additional eikonal phase and a change in amplitude. This type of calculation can be carried out for realistic shapes of the nuclear Coulomb potential, and EDWBA, contrary to the full solutions of the Dirac equation, can be extended more easily to the larger energies of interest for modern experiments.

While EDWBA is a fairly practical approach that can be employed on a routine basis, it still is much more involved than EMA. Aste and Jourdan have identified one problem of EMA, and have introduced an EMA-like approach called EMA' (Aste and Jourdan, 2004). In EMA, one tries to treat two distinct effects mentioned above: the increase of the electron momentum due to Coulomb interaction with the nucleus, and the focusing of the electron waves. To handle the increase of the electron momentum (and momentum transfer), it clearly makes sense to use a potential V that corresponds to the Coulomb potential averaged over all nucleons; here the use of the potential at the nuclear surface is a good approximation. For the focusing effect, on the other hand, the value for the nuclear center is a better approximation, as the focusing takes place all along the trajectory of the electron through the nucleus. As the electron approaches (leaves) the nuclear center, the focusing is smaller (larger). Hence the value at the center is a good compromise.

Subsequent studies using solutions of the Dirac equation in the Coulomb field of the nucleus (Aste *et al.*, 2005) indicate that, in addition to the enhanced focusing in the longitudinal direction accounted for by EMA', there is a reduction for noncentral electron trajectories. The two effects together give an overall focusing that is not far from the one obtained with EMA.

Recent studies by Tjon and Wallace (2006), performed using the eikonal expansion for the electron wave function, indicate that EMA somewhat overestimates the Coulomb effects. The authors give a recipe for correct-

ing the deficiencies, which, however, is not very practical for analyses of data.

The recent work of [Kim and Wright \(2005\)](#), who performed calculations of (e, e') both in full DWBA and also in an approximate version DW, which was found to agree well with DWBA, indicates that EMA is a good approximation for the transverse part, but that it works poorly for a longitudinal cross section. The authors, however, offer no explanation for the difference.

It would clearly be desirable to certify, via exact calculations, the validity of some EMA-type approach, as only such an approach could also be applied to the important region of the large- ω side of the quasielastic peak, where one has to deal with the overlap with a large Δ contribution, the Coulomb corrections for which also need to be dealt with properly.

The ideal way to experimentally check the Coulomb corrections is a comparison of electron and positron scattering. Unfortunately, positron beams are hard to come by, and experiments with the secondary positron beams are much more difficult than with electrons. One such experiment has been carried out ([Gueye et al., 1999](#)). The data, unfortunately, suffer from normalization problems ([Aste et al., 2004](#)); from the *position* of the quasielastic peak, one can, however, deduce that it is appropriate to use V_s for the calculation of q_{eff} .

XII. NUCLEAR MATTER

For nuclear matter, the Schrödinger equation for nucleons bound by the nucleon-nucleon interaction (deduced from NN scattering) can be solved with few approximations. The translationally invariant nature of the medium, where solutions can all be written in terms of plane waves, simplifies the calculation. As a consequence, the quality of nuclear matter wave functions is comparable to that for the $A=2,3,4$ nuclei. Due to the “exact” nature of the nuclear matter single-particle wave function, both the long-range and short-range properties are well under control. This is in contrast to finite nuclei, where calculations that are designed to do well in the long-range properties (mean-field calculations) usually do badly in the short-range aspects.

Unfortunately, only some integral properties such as density and binding energy are known experimentally; little is known on the short-range properties of nuclear matter. Quasielastic scattering here provides valuable information.

It is important to realize that inclusive electron scattering at large Q^2 is sensitive only to rather “local” properties of the medium. The spatial resolution of (e, e') is of order $1/|q|$, which at large transfer is small. In particular, the scattered electron is not sensitive to the interactions of the recoiling nucleon outside this range. This allows for an extrapolation from finite-nucleus data using the LDA.

The extrapolation procedure ([Day et al., 1989](#)) starts from consideration that the nuclear response is essentially the incoherent sum of contributions from indi-

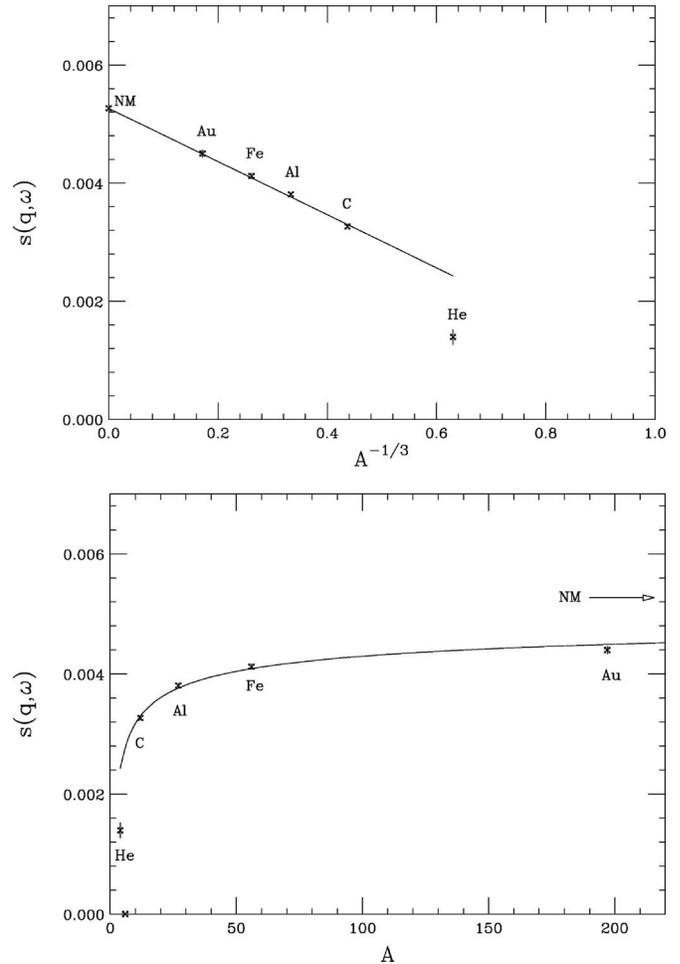


FIG. 33. Extrapolation of nuclear response per nucleon at fixed q and ω ($E=3.6$ GeV, $\theta=16^\circ$, and $\omega=180$ MeV) as a function of $A^{-1/3}$ (top) and as a function of A (bottom) where the extrapolated value of the nuclear matter response is indicated by the arrow.

vidual nucleons. As the average value of the density in the nuclear interior and the shape of the density distribution in the nuclear surface are approximately A independent, the response can be divided into a volume component, proportional to the mass number A , and a nuclear surface component proportional to $A^{2/3}$. It is the former one that is of interest when discussing nuclear matter. The ratio of the surface-to-volume contributions is thus proportional to $A^{-1/3}$. Extrapolation of the nuclear response per nucleon to $A^{-1/3}=0$ ($A \rightarrow \infty$) as a linear function of $A^{-1/3}$ yields the nuclear matter response.

In order to illustrate this approach, we reproduce in Fig. 33 one example for an extrapolation as a function of $A^{-1/3}$ ([Day et al., 1989](#)). Ignoring ^4He (for which the properties of the density cited above are not valid), the nuclear response $s(q, \omega)$ for $A=12-197$ is well fitted by a linear function of $A^{-1/3}$. The plot on the bottom of Fig. 33 gives the same extrapolation as a function of A . This figure reveals that the extrapolation as a function of A is unwieldy, though the curve better imparts the saturation

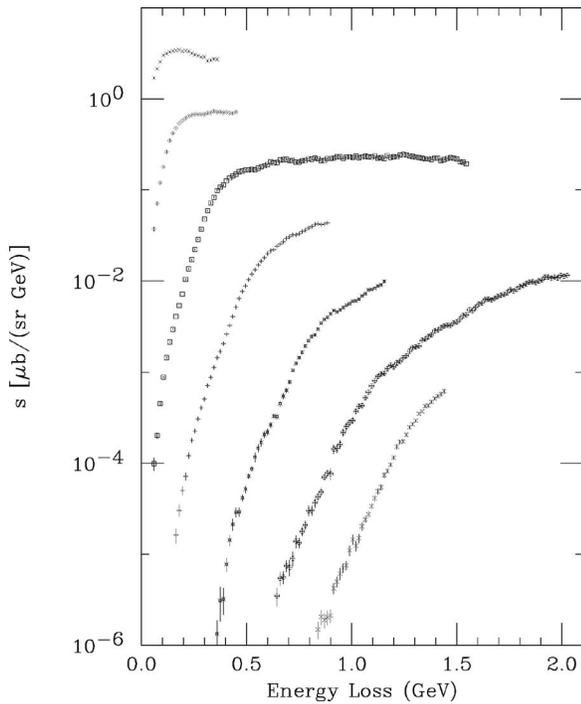


FIG. 34. Nuclear matter response at momentum transfers up to $3.5 \text{ (GeV}/c)^2$. From [Day *et al.*, 1989](#).

of the response. Even heavy nuclei significantly differ from nuclear matter due to the large fraction of surface nucleons.

In order to obtain the response function for symmetric nuclear matter, one makes use of additional knowledge. For quasielastic scattering, the relative contribution of protons and neutrons changes as a function of A . Although protons dominate due to the larger electron-proton cross section, the contribution of neutrons is not negligible. For the extrapolation, one assumes that the response functions for protons and neutrons are the same, and the trivial dependence on N, Z is removed by extrapolating the quantity

$$s(q, \omega) = \sigma(q, \omega) / (Z\sigma_{ep} + N\sigma_{en}). \quad (68)$$

The nuclear matter response has been extrapolated ([Day *et al.*, 1989](#)) from the nuclear response measured for finite nuclei over a large region of q and ω ; see Fig. 34. This was made possible by the availability of data for nuclei with $A=4, 12, 27, 56$, and 197 ([Day *et al.*, 1993](#)) taken at the same incident energies and angles. Recent data taken at Jefferson Lab with $A=3, 4, 9, 12, 63$, and 197 will allow this to be extended over an even larger range of q and ω .

XIII. RELATED AREAS

Inclusive scattering from composite systems is used as a tool in a number of areas ([Silver and Sokol, 1989](#)). The corresponding processes have many aspects that are closely related to quasielastic scattering from nuclei, but they also exhibit significant differences. We now address

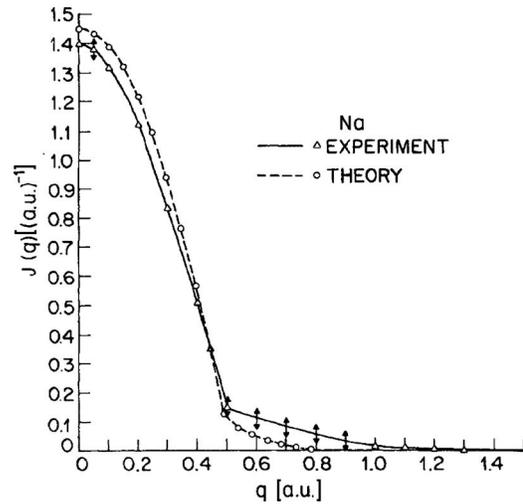


FIG. 35. Compton profile of Na as a function of electron momentum (in atomic units), with the contribution of core electrons removed and finite-resolution effects unfolded ([Eisenberger *et al.*, 1972](#)).

those areas in which inclusive scattering has been harnessed to study diverse composite systems.

These areas differ not only by the nature of the composite system investigated, but also by the probe used: photons, low-energy electrons, neutrons, high-energy electrons, muons, and neutrinos. The energies of the probes cover many orders of magnitude, from meV to GeV, scaling with the dimensions relevant for the composite systems.

Historically, the first area in which inclusive scattering became prominent was the measurement of the Compton profile, i.e., quasielastic scattering of photons or x rays from electrons bound in atoms ([Williams, 1977](#); [Cooper, 1985](#)). The Compton effect actually played a substantial role in the early validation of quantum ideas, and with the experimental work of [DuMond \(1947\)](#) it became a practical tool for the investigation of electron momentum distributions. Modern experiments involve energies in the region of ten to hundreds of keV, i.e., energies that in general are very large as compared to atomic Fermi energies (eV).

The observable in Compton scattering, the so-called Compton profile, is the longitudinal momentum distribution of the initially bound electrons, in direct correspondence with the scaling function $F(y)$ determined in quasielastic scattering from nuclei; see Sec. VII. The momentum distribution of the electrons exhibits a more complex structure, as not only the bound-electron $n(k)$ play a role, but also—and often more prominently—the conduction electrons for metallic targets.

As an example, we show in Fig. 35 the Compton profile of sodium. The contribution of the core electrons has been removed; the parabola-like part below a momentum of 0.5 a.u. is due to the free-electron Fermi gas and the tail at larger momenta is due to interelectron interactions ([Eisenberger *et al.*, 1972](#)).

When compared to quasielastic scattering on nuclei, Compton scattering possesses an additional feature: for

crystalline samples, the Compton profile can be measured for many different orientations of the probe. This allows one to derive the reciprocal form factor $B(\vec{r})$, from which one can extract the autocorrelation function in r space, a quantity that helps to deduce the spatial structure of the molecules in the crystal.

A variant of Compton scattering is electron Compton scattering, where the photon is replaced by an electron, typically in the 50 keV energy region. While the production of a good beam and the detection of the scattered particle is much easier, the requirement of very thin targets, unfortunately, partially offsets these advantages.

A second area in which inclusive scattering represents a popular tool is the scattering of low-energy neutrons from condensed-matter systems such as quantum liquids. Neutrons with energies ranging from 10 meV to 10 keV, from reactors or spallation neutron sources, are employed. The energies are again very high as compared to energy scales of the system, of order meV for liquid helium, for example.

Similar to quasielastic scattering from nuclei, the measured structure function $S(q, \omega)$ yields, in the impulse approximation, an integral over the momentum distribution. In the limit of large $|\mathbf{q}|$, it can be written as a function of one variable Y , the longitudinal momentum component.

In quasielastic neutron scattering, emphasis has been placed on the understanding of the role of FSI, for two reasons. First, the interaction between two atoms is fairly singular at short interatom distances r ; the typical Lennard-Jones potential rises very steeply (r^{-12}) at small r . This strong FSI has a pronounced effect on the observables, to the extent that the longitudinal momentum distribution can only be extracted after correction for FSI.

FSI was studied in great detail for a second reason. Much of the emphasis in the field was placed on the measurement of the fraction of Bose condensate in superfluid ^4He . This Bose condensate was expected to produce a δ -function-like spike in the momentum distribution $n(k)$ at $k=0$, which would lead to a spike in the response at $Y=0$. This feature was not seen, a fact that is now understood as a consequence of FSI.

The understanding of FSI in neutron scattering has many aspects that are parallel to the discussion given in Sec. III. In particular, it has been found that the main effect of FSI is a folding of the IA response. The folding function has a width governed by the atom-atom total cross section. It has also been found that for a quantitative understanding of FSI, it is imperative to include in the description of the initial state the atom-atom correlation function $g(|\mathbf{r}_i - \mathbf{r}_j|)$. The treatment of the atom-atom potential is comparatively difficult, yet is possible with hard-core perturbation theory (Silver, 1988).

As an example, we show in Fig. 36 the scaling function measured at a momentum transfer of 23 \AA on superfluid helium at 0.35 K (Sosnik *et al.*, 1990). At this temperature, a Bose condensate is predicted to occur. The $\delta(Y=0)$ function due to the condensate, smeared with the

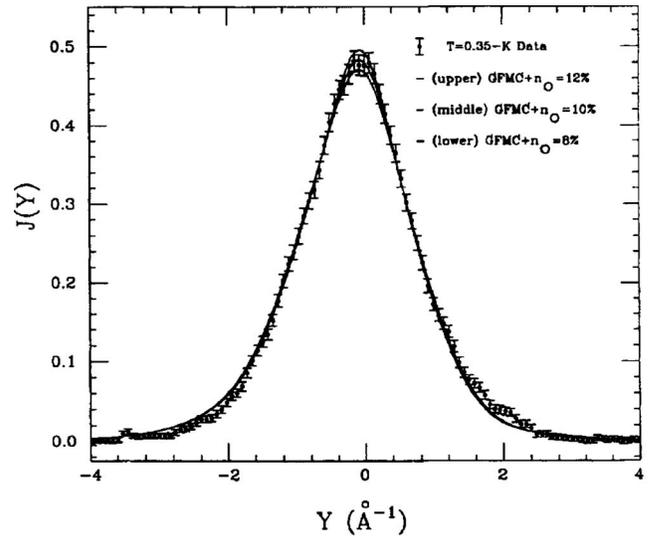


FIG. 36. Profile for neutron scattering from liquid helium at 0.35 K. The curves, calculated using GFMC and including FSI effects, are shown for different fractions of the Bose condensate, 10% giving the best fit. From Sosnik *et al.*, 1990.

FSI folding function, explains the data for a condensate fraction of 10%.

The last related area we address concerns deep inelastic scattering (DIS) of GeV electrons or muons from nucleons [for a review, see, e.g., Ellis *et al.* (1996)]. Here the energy spectrum of the inclusively scattered lepton is used to derive the momentum distribution of quarks bound in the nucleon. This process provided the first direct evidence for the existence of pointlike constituents of fractional charge, subject to asymptotic freedom (Gross and Wilczek, 1973; Politzer, 1973), and still is one of the main sources of information on nucleon structure.

DIS is generally analyzed in terms of the Bjorken scaling variable $x = Q^2/2m\omega$ (Bjorken and Paschos, 1969) appropriate for constituents of negligible rest mass. The variable y , which in the limit of small constituent mass equals the Nachtmann variable ξ (but for a trivial factor), is more appropriate for extending scaling to the lower momentum transfers (Benhar *et al.*, 2000). In terms of ξ , DIS and quasielastic scattering from nuclei have many things in common.

Apart from the parallel aspects (see Sec. II), DIS exhibits one special feature, namely the evolution of the scaling function with increasing Q^2 . This evolution is a consequence of the fact that the lepton, at larger and larger Q^2 , resolves more and more of the nucleon structure and the other nucleon constituents, the gluons. This evolution of the structure function has been studied in great detail, and is well understood in terms of quantum chromodynamics (QCD) (Gribov and Lipatov, 1972; Altarelli and Parisi, 1977). Figure 37 displays the evolution of the proton structure function with momentum transfer.

Unlike, e.g., the neutron scattering mentioned above, DIS is always analyzed in IA, neglecting the FSI of the recoiling quark; the structure functions are then inter-

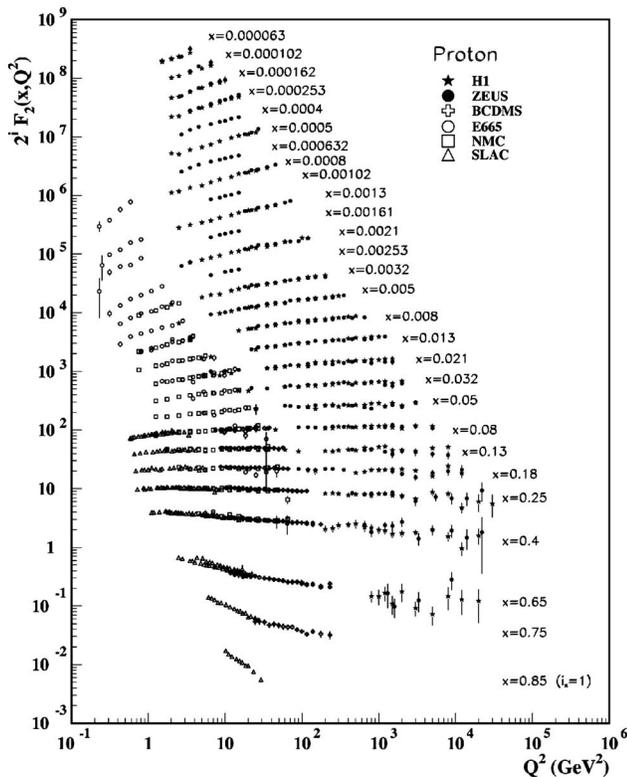


FIG. 37. Structure function F_2 as a function of the momentum transfer (shifted vertically by factors 2^i), for different values of the Bjorken scaling variable x . From Eidelman *et al.*, 2005.

interpreted directly as quark distribution functions. The effects of FSI, although known to be present even in the $Q^2 \rightarrow \infty$ limit (Brodsky *et al.*, 2002), are neglected despite the fact that model calculations indicate that they are of substantial size (Paris and Pandharipande, 2002).

Analogous experiments on DIS of neutrinos have also been performed; due to the small rates, the database in this area is much more restricted.

XIV. CONCLUSIONS

The field of inclusive quasielastic electron-nucleus scattering has seen important progress during the past decade, in terms of both experimental results and theoretical understanding.

Experiment has greatly benefited from the high-intensity GeV-energy facilities and the high-performance spectrometers and detectors that became available. This has allowed one to extend the data to extreme values of momentum transfer and energy loss. As a consequence, we now have, at least for selected nuclei, a fairly complete database (see the Web page <http://faculty.virginia.edu/qes-archive>, which gives a rather complete collection of the available cross sections). Not yet satisfactory is the situation for the longitudinal strength at large momentum transfer, where the database is very narrow. Improvements would also be desirable at low momentum transfer, where much of the database comes from experiments done in spectrum-acquisition mode.

Theory has also made considerable progress. Nuclear many-body theory today provides reliable spectral functions, which are at the basis of any quantitative understanding of quasielastic scattering, and especially important in the region of high momentum transfer and not too large energy loss. In particular, the nucleon-nucleon short-range correlations, which have long been known to play a major role, are now included in an adequate fashion.

The y -scaling analysis of the data clearly shows that elastic scattering off a single nucleon is the dominant reaction mechanism at $x > 1$. This is the region in which quantitative information on nuclear properties can be extracted.

For the treatment of the recoil-nucleon final-state interaction, various approaches have been developed, applicable in different regions of momentum transfer and energy loss. The results of calculations carried out within the scheme widely adopted at large momentum transfer (typically $|\mathbf{q}| > 1$ GeV), based on the eikonal approximation, indicate that final-state interaction effects are large, indeed dominant, in the low-energy-loss tail of the inclusive cross section, and their inclusion leads to a quantitative account of the existing data up to $x \sim 2$.

The role of meson exchange currents, which surprisingly have a large effect at rather low momentum transfer and across the entire quasielastic peak, has also been much better understood. Issues not yet satisfactorily resolved concern the final-state interaction at large momentum transfer and very low energy loss, corresponding to $x > 2$, and the role of non-nucleonic degrees of freedom at very large momentum transfer. Recent studies of FSI in $(e, e'p)$, carried out within the formalism described in Sec. III (Schiavilla *et al.*, 2005), show that the spin dependence of the NN scattering amplitude plays an important role. The possible relevance of these effects in inclusive processes requires further investigations. The treatment of knock-out processes accompanied by excitation of the nucleon also is not yet entirely satisfactory.

Recently, much consideration has been given to the connection between electron- and neutrino-nucleus scattering (Amaro *et al.*, 2005; Benhar, 2005). The generalization of the existing theoretical approaches and the exploitation of the measured (e, e') cross sections to predict the analogous neutrino-induced reactions, such as (ν, e) or (ν, μ) , will be of great importance to reduce the systematic uncertainty in the interpretation of neutrino oscillation experiments.

Only partially exploited is the relation to quasielastic scattering of hadronic probes (p, π, K, \dots) from which additional, and largely complementary, information could be learned.

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REFERENCES

- Abbott, D., *et al.*, 1998, Phys. Rev. Lett. **80**, 5072.
- Akhiezer, A. I., and M. P. Rekalo, 1974, Sov. J. Part. Nucl. **4**, 277.
- Akmal, A., and V. Pandharipande, 1997, Phys. Rev. C **56**, 2261.
- Akmal, A., V. Pandharipande, and D. Ravenhall, 1998, Phys. Rev. C **58**, 1804.
- Alberico, W., M. Ericson, and A. Molinari, 1984, Ann. Phys. (N.Y.) **154**, 356.
- Altarelli, G., and G. Parisi, 1977, Nucl. Phys. B **126**, 298.
- Altemus, R., *et al.*, 1980, Phys. Rev. Lett. **44**, 965.
- Amaro, J., and A. Lallena, 1992, Nucl. Phys. A **537**, 585.
- Amaro, J., *et al.*, 2005, Phys. Rev. C **71**, 065501.
- Anguiano, M., A. Lallena, and G. Cò, 1996, Phys. Rev. C **53**, 3155.
- Arenhoevel, H., 1987, Phys. Lett. B **199**, 13.
- Arenhoevel, H., W. Leidemann, and E. L. Tomusiak, 1988, Z. Phys. A **331**, 123.
- Arenhoevel, H., W. Leidemann, and E. L. Tomusiak, 1989, Z. Phys. A **334**, 363.
- Arenhoevel, H., W. Leidemann, and E. L. Tomusiak, 1992, Phys. Rev. C **46**, 455.
- Arnold, R. G., C. E. Carlson, and F. Gross, 1981, Phys. Rev. C **23**, 363.
- Arrington, J., 1998, Ph.D. thesis (CalTech).
- Arrington, J., 2005, Phys. Rev. C **71**, 015202.
- Arrington, J., C. D. Roberts, and J. M. Zanotti, 2007, J. Phys. G **34**, S23.
- Arrington, J., and I. Sick, 2004, Phys. Rev. C **70**, 028203.
- Arrington, J., *et al.*, 1999, Phys. Rev. Lett. **82**, 2056.
- Aste, A., and J. Jourdan, 2004, Europhys. Lett. **67**, 753.
- Aste, A., C. vonArx, and D. Trautmann, 2005, Eur. Phys. J. A **26**, 167.
- Aste, A., *et al.*, 2004, Nucl. Phys. A **743**, 259.
- Atwood, W., and G. West, 1973, Phys. Rev. D **7**, 773.
- Baldo, M., *et al.*, 2000, Phys. Lett. B **473**, 1.
- Baran, D., 1989, Ph.D. thesis (Northwestern University).
- Baran, D., *et al.*, 1988, Phys. Rev. Lett. **61**, 400.
- Barreau, P., *et al.*, 1983, Nucl. Phys. A **402**, 515.
- Bartel, W., *et al.*, 1973, Nucl. Phys. B **58**, 429.
- Becker, J., *et al.*, 1999, Eur. Phys. J. A **6**, 329.
- Benaksas, D., D. Drickey, and D. Frerejacque, 1964, Phys. Rev. Lett. **13**, 353.
- Benhar, O., 1999, Phys. Rev. Lett. **83**, 3130.
- Benhar, O., 2005, Nucl. Phys. B **139**, 15.
- Benhar, O., A. Fabrocini, and S. Fantoni, 1989, Nucl. Phys. A **505**, 267.
- Benhar, O., A. Fabrocini, and S. Fantoni, 2001, Phys. Rev. Lett. **87**, 052501.
- Benhar, O., and S. Liuti, 1995, Phys. Lett. B **358**, 173.
- Benhar, O., and S. Liuti, 1996, Phys. Lett. B **389**, 649.
- Benhar, O., and V. Pandharipande, 1993, Phys. Rev. C **47**, 2218.
- Benhar, O., V. Pandharipande, and S. Pieper, 1993, Rev. Mod. Phys. **65**, 817.
- Benhar, O., V. Pandharipande, and I. Sick, 1997, Phys. Lett. B **410**, 79.
- Benhar, O., V. Pandharipande, and I. Sick, 1999, Phys. Lett. B **469**, 19.
- Benhar, O., V. Pandharipande, and I. Sick, 2000, Phys. Lett. B **489**, 131.
- Benhar, O., and I. Sick, 2007, unpublished.
- Benhar, O., *et al.*, 1991, Phys. Rev. C **44**, 2328.
- Benhar, O., *et al.*, 1994, Nucl. Phys. A **579**, 493.
- Benhar, O., *et al.*, 1995a, Phys. Lett. B **359**, 8.
- Benhar, O., *et al.*, 1995b, Phys. Lett. B **343**, 47.
- Bertozzi, W., *et al.*, 1979, Nucl. Instrum. Methods **162**, 211.
- Bjorken, J., and E. Paschos, 1969, Phys. Rev. **185**, 1975.
- Blatchley, C., *et al.*, 1986, Phys. Rev. C **34**, 1243.
- Blunden, P., and M. Butler, 1989, Phys. Lett. B **219**, 151.
- Blunden, P. G., W. Melnitchouk, and J. A. Tjon, 2003, Phys. Rev. Lett. **91**, 142304.
- Bodek, A., and J. Ritchie, 1981, Phys. Rev. D **23**, 1070.
- Brodsky, S., 1982, *Proceedings of the 13th International Symposium on Multiparticle Dynamics* (World Scientific, Singapore), p. 963.
- Brodsky, S., *et al.*, 2002, Phys. Rev. D **65**, 114025.
- Brooks, W. K., and J. D. Lachniet, 2005, Nucl. Phys. A **755**, 261.
- Bumiller, F. A., *et al.*, 1970, Phys. Rev. Lett. **25**, 1774.
- Butler, M., and S. Koonin, 1988, Phys. Lett. B **205**, 123.
- Bystritskiy, Y., E. Kuraev, and E. Tomasi-Gustafson, 2007, Phys. Rev. C **75**, 015207.
- Carlson, J., and R. Schiavilla, 1992, Phys. Rev. Lett. **68**, 3682.
- Carlson, J., and R. Schiavilla, 1994, Phys. Rev. C **49**, 2880.
- Carlson, J., and R. Schiavilla, 1998, Rev. Mod. Phys. **70**, 743.
- Carlson, J., *et al.*, 2002, Phys. Rev. C **65**, 024002.
- Carlson, J., *et al.*, 2003, Phys. Lett. B **553**, 191.
- Celenza, L., A. Harindranath, and C. Shakin, 1986, Phys. Rev. C **33**, 1012.
- Chen, J., 1990, Ph.D. thesis (University of Virginia).
- Chen, J., *et al.*, 1991, Phys. Rev. Lett. **66**, 1283.
- Chen, Y. C., *et al.*, 2004, Phys. Rev. Lett. **93**, 122301.
- Chinn, C., A. Picklesimer, and J. van Orden, 1989a, Phys. Rev. C **40**, 790.
- Chinn, C., A. Picklesimer, and J. van Orden, 1989b, Phys. Rev. C **40**, 1159.
- Ciofi degli Atti, C., D. Day, and S. Liuti, 1992, Phys. Rev. C **46**, 1045.
- Ciofi degli Atti, C., S. Liuti, and S. Simula, 1990, Phys. Rev. C **41**, R2474.
- Ciofi degli Atti, C., E. Pace, and G. Salmè, 1980, Phys. Rev. C **21**, 805.
- Ciofi degli Atti, C., E. Pace, and G. Salmè, 1987, Phys. Rev. C **36**, 1208.
- Ciofi degli Atti, C., E. Pace, and G. Salmè, 1995, Phys. Rev. C **51**, 1108.
- Ciofi degli Atti, C., and S. Simula, 1994, Phys. Lett. B **325**, 276.
- Cò, G., and J. Heisenberg, 1987, Phys. Lett. B **197**, 489.
- Cooper, M., 1985, Rep. Prog. Phys. **48**, 415.
- Crannell, H., and L. Suelzle, 1966, Nucl. Instrum. Methods **44**, 133.
- Czyz, W., and K. Gottfried, 1963, Ann. Phys. (N.Y.) **21**, 47.
- Danel, J.-F., 1990, Ph.D. thesis (Université d'Orsay).
- Day, D., 2005, Nucl. Phys. A **755**, 151.
- Day, D., and I. Sick, 2004, Phys. Rev. C **69**, 028501.
- Day, D., *et al.*, 1979, Phys. Rev. Lett. **43**, 1143.
- Day, D., *et al.*, 1987, Phys. Rev. Lett. **59**, 427.
- Day, D., *et al.*, 1989, Phys. Rev. C **40**, 1011.
- Day, D., *et al.*, 1990, Annu. Rev. Nucl. Part. Sci. **40**, 357.
- Day, D., *et al.*, 1993, Phys. Rev. C **48**, 1849.
- Deady, M., *et al.*, 1983, Phys. Rev. C **28**, 631.
- Deady, M., *et al.*, 1986, Phys. Rev. C **33**, 1897.
- de Forest, T., 1983, Nucl. Phys. A **392**, 232.
- de Forest, T., 1984, Nucl. Phys. A **414**, 347.
- de Jager, K., 2005, in *Proceedings of SPIN 2004*, Trieste, Italy,

- edited by K. Aulenbacher *et al.* (World Scientific, Singapore).
- Dekker, M., P. Brussard, and J. Tjon, 1994, *Phys. Rev. C* **49**, 2650.
- Dellafore, A., F. Lenz, and F. Brieva, 1985, *Phys. Rev. C* **31**, 1088.
- Deltuva, A., *et al.*, 2004, *Phys. Rev. C* **70**, 034004.
- Dementii, N. G., *et al.*, 1969, *Sov. J. Nucl. Phys.* **9**, 142.
- Dieperink, A., *et al.*, 1976, *Phys. Lett.* **63B**, 261.
- Dombey, N., 1969, *Rev. Mod. Phys.* **41**, 236.
- Donnelly, T., and I. Sick, 1999, *Phys. Rev. C* **60**, 065502.
- Donnelly, T., *et al.*, 1978, *Phys. Lett.* **76B**, 393.
- Donnelly, T. W., and A. S. Raskin, 1986, *Ann. Phys. (N.Y.)* **169**, 247.
- Dow, K., 1987, Ph.D. thesis (MIT).
- Dow, K., *et al.*, 1988, *Phys. Rev. Lett.* **61**, 1706.
- Drickey, D. J., and L. N. Hand, 1962, *Phys. Rev. Lett.* **9**, 521.
- DuMond, J., 1947, *Rev. Sci. Instrum.* **18**, 626.
- Durand, L., 1961, *Phys. Rev. Lett.* **6**, 631.
- Dutta, D., *et al.*, 2003, *Phys. Rev. C* **68**, 064603.
- Dytman, S., *et al.*, 1988, *Phys. Rev. C* **38**, 800.
- Eden, T., *et al.*, 1994, *Phys. Rev. C* **50**, R1749.
- Efros, V., W. Leidemann, and G. Orlandini, 1994, *Phys. Lett. B* **338**, 130.
- Efros, V., *et al.*, 2004, *Phys. Rev. C* **69**, 044001.
- Efros, V., *et al.*, 2005, *Phys. Rev. C* **72**, 011002(R).
- Egiyan, K., *et al.*, 2003, *Phys. Rev. C* **68**, 014313.
- Eidelman, S., *et al.*, 2005, *Phys. Lett. B* **592**, 1.
- Eisenberger, P., *et al.*, 1972, *Phys. Rev. B* **6**, 3671.
- Ellis, R. K., W. J. Stirling, and B. R. Webber, 1996, *QCD and Collider Physics* (Cambridge University Press, Cambridge, UK).
- Fabrocini, A., 1997a, *Phys. Rev. C* **55**, 338.
- Fabrocini, A., 1997b, private communication.
- Fabrocini, A., and S. Fantoni, 1989, *Nucl. Phys. A* **503**, 375.
- Farrar, G., *et al.*, 1988, *Phys. Rev. Lett.* **61**, 686.
- Finn, J., R. W. Lourie, and B. H. Cottmann, 1984, *Phys. Rev. C* **29**, 2230.
- Frankfurt, L., *et al.*, 1993, *Phys. Rev. C* **48**, 2451.
- Gadiyak, V., and V. Dmitriev, 1998, *Nucl. Phys. A* **639**, 685.
- Galster, S., *et al.*, 1971, *Nucl. Phys. B* **32**, 221.
- Gao, H., 2003, *Int. J. Mod. Phys. E* **12**, 1.
- Gao, H., *et al.*, 1994, *Phys. Rev. C* **50**, R546.
- Garino, G., *et al.*, 1992, *Phys. Rev. C* **45**, 780.
- Garrow, K., *et al.*, 2002, *Phys. Rev. C* **66**, 044613.
- Gayou, O., *et al.*, 2001, *Phys. Rev. C* **64**, 038202.
- Gayou, O., *et al.*, 2002, *Phys. Rev. Lett.* **88**, 092301.
- Gersch, H., L. Rodriguez, and P. Smith, 1973, *Phys. Rev. A* **5**, 1547.
- Geurts, W., *et al.*, 1996, *Phys. Rev. C* **53**, 2207.
- Giusti, C., and F. Pacati, 1987, *Nucl. Phys. A* **473**, 717.
- Glauber, R., 1959, in *Lectures in Theoretical Physics*, edited by W. E. Brittin (Interscience, New York).
- Golak, J., *et al.*, 1995, *Phys. Rev. C* **52**, 1216.
- Golak, J., *et al.*, 2001, *Phys. Rev. C* **63**, 034006.
- Gottfried, K., 1963, *Ann. Phys. (N.Y.)* **21**, 29.
- Gribov, V., and L. Lipatov, 1972, *Sov. J. Nucl. Phys.* **15**, 438.
- Gross, D., and F. Wilczek, 1973, *Phys. Rev. Lett.* **30**, 1343.
- Gueye, P., *et al.*, 1999, *Phys. Rev. C* **60**, 044308.
- Guichon, P. A. M., and M. Vanderhaeghen, 2003, *Phys. Rev. Lett.* **91**, 142303.
- Gurvitz, S., and A. Rinat, 2002, *Phys. Rev. C* **65**, 024310.
- Heimlich, F., 1973, Report No. DESY-F23-73-1 (unpublished).
- Heimlich, F., *et al.*, 1974, *Nucl. Phys. A* **231**, 509.
- Herberg, C., *et al.*, 1999, *Eur. Phys. J. A* **5**, 131.
- Horikawa, Y., F. Lenz, and N. Mukhopadhyay, 1980, *Phys. Rev. C* **22**, 1680.
- Hotta, A., *et al.*, 1984, *Phys. Rev. C* **30**, 87.
- Hughes, E. B., *et al.*, 1965, *Phys. Rev.* **139**, B458.
- Hughes, E. B., *et al.*, 1966, *Phys. Rev.* **146**, 973.
- Hyde-Wright, C. E., and K. de Jager, 2004, *Annu. Rev. Nucl. Part. Sci.* **54**, 217.
- Ishikawa, S., *et al.*, 1998, *Phys. Rev. C* **57**, 39.
- Itzykson, C., and J. Zuber, 1980, *Quantum Field Theory* (McGraw-Hill, New York).
- Jin, Y., D. Onley, and L. Wright, 1992, *Phys. Rev. C* **45**, 1311.
- Jones, C. E., *et al.*, 1993, *Phys. Rev. C* **47**, 110.
- Jones, M. K., *et al.* (Jefferson Lab Hall A), 2000, *Phys. Rev. Lett.* **84**, 1398.
- Jones-Woodward, C. E., *et al.*, 1991, *Phys. Rev. C* **44**, R571.
- Jourdan, J., 1996, *Nucl. Phys. A* **603**, 117.
- Jourdan, J., 1997, in *Workshop on Electron-Nucleus Scattering*, edited by O. Benhar and A. Fabrocini (Edizioni ETS, Pisa), p. 13.
- Kim, K., 2004, *Phys. Rev. C* **70**, 027602.
- Kim, K., and L. Wright, 2005, *Phys. Rev. C* **72**, 064607.
- Kim, K., *et al.*, 1996, *Phys. Rev. C* **54**, 2515.
- Kirkman, H. C., *et al.*, 1970, *Phys. Lett.* **32B**, 519.
- Knoll, J., 1974, *Nucl. Phys. A* **223**, 462.
- Kohn, M., and N. Ohtsuka, 1981, *Phys. Lett.* **98**, 335.
- Kubon, G., *et al.*, 2002, *Phys. Lett. B* **524**, 26.
- Kumano, S., and E. Moniz, 1988, *Phys. Rev. C* **37**, 2088.
- Lachniet, J. D., 2006, Ph.D. thesis (Carnegie Mellon University).
- Lecante, P., *et al.*, 1980, *Nucl. Instrum. Methods* **169**, 401.
- Leidemann, W., V. Efros, and G. Orlandini, 1997, *Proceedings of the ELBA Workshop on Electron Nucleus Scattering*, edited by O. Benhar and A. Fabrocini (Edizioni ETS, Pisa), p. 220.
- Leidemann, W., and G. Orlandini, 1990, *Nucl. Phys. A* **506**, 447.
- Lenz, F., and R. Rosenfelder, 1971, *Nucl. Phys. A* **176**, 513.
- Liuti, S., 1993, *Phys. Rev. C* **47**, R1854.
- Lung, A., *et al.*, 1993, *Phys. Rev. Lett.* **70**, 718.
- Maas, F. E., *et al.*, 2005, *Phys. Rev. Lett.* **94**, 082001.
- Mar, J., *et al.*, 1968, *Phys. Rev. Lett.* **21**, 482.
- Marchand, C., *et al.*, 1985, *Phys. Lett.* **153B**, 29.
- McCarthy, J., *et al.*, 1976, *Phys. Rev. C* **13**, 712.
- McGee, I. J., 1967, *Phys. Rev.* **158**, 1500.
- Meier-Hajduk, H., U. Oelfke, and P. Sauer, 1989, *Nucl. Phys. A* **499**, 637.
- Meier-Hajduk, H., *et al.*, 1983, *Nucl. Phys. A* **395**, 332.
- Meyerhoff, M., *et al.*, 1994, *Phys. Lett. B* **327**, 201.
- Meziani, Z., *et al.*, 1984, *Phys. Rev. Lett.* **52**, 2130.
- Meziani, Z., *et al.*, 1985, *Phys. Rev. Lett.* **54**, 1233.
- Meziani, Z., *et al.*, 1992, *Phys. Rev. Lett.* **69**, 41.
- Mo, L., and C. Peck, 1965, Report No. SLAC TN-65-29 (unpublished).
- Mo, L., and Y. Tsai, 1969, *Rev. Mod. Phys.* **41**, 205.
- Moniz, E., *et al.*, 1971, *Phys. Rev. Lett.* **26**, 445.
- Morgenstern, J., and Z. Meziani, 2001, *Phys. Lett. B* **515**, 269.
- Morita, H., and T. Suzuki, 1991, *Prog. Theor. Phys.* **86**, 671.
- Mougey, J., *et al.*, 1978, *Phys. Rev. Lett.* **41**, 1645.
- Mueller, A., 1982, in *Proceedings of the 17th Rencontre Moriond*, edited by J. Than Thanh Van (Editions Frontier, Gif-sur-Yvette), p. 13.
- Mulders, P., 1986, *Nucl. Phys. A* **459**, 525.

- Naus, H., and J. Koch, 1987, Phys. Rev. C **36**, 2459.
- Naus, H., *et al.*, 1990, Nucl. Phys. A **509**, 717.
- Noble, J., 1981, Phys. Rev. Lett. **46**, 412.
- O'Connell, J., *et al.*, 1987, Phys. Rev. C **35**, 1063.
- O'Neill, T., *et al.*, 1995, Phys. Lett. B **351**, 87.
- Ostrick, M., *et al.*, 1999, Phys. Rev. Lett. **83**, 276.
- Pace, E., and G. Salme, 1982, Phys. Lett. **110B**, 411.
- Pandharipande, V., and S. Pieper, 1992, Phys. Rev. C **45**, 791.
- Paris, M., and V. Pandharipande, 2002, Phys. Rev. C **65**, 035203.
- Passchier, I., *et al.*, 1999, Phys. Rev. Lett. **82**, 4988.
- Perdrisat, C. F., V. Punjabi, and M. Vanderhaeghen, 2007, Prog. Part. Nucl. Phys. **59**, 694.
- Petraki, M., *et al.*, 2003, Phys. Rev. C **67**, 014605.
- Pieper, S., and R. Wiringa, 2001, Annu. Rev. Nucl. Part. Sci. **51**, 53.
- Pirner, H., and J. Vary, 1981, Phys. Rev. Lett. **46**, 1376.
- Platchkov, S., *et al.*, 1990, Nucl. Phys. A **510**, 740.
- Politzer, H., 1973, Phys. Rev. Lett. **30**, 1346.
- Polls, A., *et al.*, 1997, Phys. Rev. C **55**, 810.
- Potterveld, D., 1989, Ph.D. thesis (CalTech).
- Powell, T., *et al.*, 1970, Phys. Rev. Lett. **24**, 753.
- Pudliner, B., *et al.*, 1995, Phys. Rev. C **56**, 1720.
- Quinn, B., *et al.*, 1988, Phys. Rev. C **37**, 1609.
- Ramos, A., A. Polls, and W. Dickhoff, 1989, Nucl. Phys. A **503**, 1.
- Raskin, A. S., and T. W. Donnelly, 1989, Ann. Phys. (N.Y.) **191**, 78.
- Rekalo, M. P., and E. Tomasi-Gustafsson, 2004, Eur. Phys. J. A **22**, 331.
- Rinat, A., and M. Taragin, 1996, Nucl. Phys. A **598**, 349.
- Riska, D., 1989, Phys. Rep. **181**, 207.
- Rohe, D., 2004, Habilitationsschrift (Universität Basel).
- Rohe, D., *et al.*, 1999, Phys. Rev. Lett. **83**, 4257.
- Rohe, D., *et al.*, 2005, Phys. Rev. C **72**, 054602.
- Rosenfelder, R., 1980, Ann. Phys. (N.Y.) **128**, 188.
- Sargsian, M., 2001, Int. J. Mod. Phys. E **10**, 405.
- Schiavilla, R., V. Pandharipande, and D. Riska, 1990, Phys. Rev. C **41**, 309.
- Schiavilla, R., V. Pandharipande, and R. Wiringa, 1986, Nucl. Phys. A **449**, 219.
- Schiavilla, R., *et al.*, 2005, Phys. Rev. Lett. **94**, 072303.
- Sealock, R., *et al.*, 1989, Phys. Rev. Lett. **62**, 1350.
- Sick, I., 1986, in *Proceedings of the Conference on Weak and Electromagnetic Interactions in Nuclei*, edited by H. V. Klapdor (Springer, Berlin), p. 415.
- Sick, I., 2001, Prog. Part. Nucl. Phys. **47**, 245.
- Sick, I., D. Day, and J. McCarthy, 1980, Phys. Rev. Lett. **45**, 871.
- Silver, R., 1988, Phys. Rev. B **38**, 2283.
- Silver, R., and P. Sokol, 1989, *Momentum Distributions* (Plenum, New York).
- Simon, G. G., C. Schmitt, and V. H. Walther, 1981, Nucl. Phys. A **364**, 285.
- Sosnik, T., W. Snow, and P. Sokol, 1990, Phys. Rev. B **41**, 11185.
- Stanfield, K., *et al.*, 1971, Phys. Rev. C **3**, 1448.
- Strauch, S., *et al.*, 2003, Phys. Rev. Lett. **91**, 052301.
- Titov, Y., and E. Stepula, 1972, Sov. J. Nucl. Phys. **15**, 361.
- Titov, Y., *et al.*, 1969, Sov. J. Nucl. Phys. **9**, 772.
- Titov, Y., *et al.*, 1971, Sov. J. Nucl. Phys. **13**, 660.
- Titov, Y., *et al.*, 1974, Sov. J. Nucl. Phys. **19**, 240.
- Tjon, J., and S. Wallace, 2006, Phys. Rev. C **74**, 064602.
- Traini, M., 2001, Nucl. Phys. A **694**, 325.
- Traini, M., S. Turck-Chieze, and A. Zghiche, 1988, Phys. Rev. C **38**, 2799.
- Udias, J., *et al.*, 1993, Phys. Rev. C **48**, 2731.
- Van der Sluys, V., J. Ryckebusch, and M. Waroquier, 1995, Phys. Rev. C **51**, 2664.
- Van Neck, D., A. Dieperink, and E. Moya de Guerra, 1995, Phys. Rev. C **51**, 1800.
- Van Orden, J., and T. Donnelly, 1981, Ann. Phys. (N.Y.) **131**, 451.
- VanReden, K., *et al.*, 1990, Phys. Rev. C **41**, 1084.
- Viviani, M., A. Kievsky, and A. Rinat, 2003, Phys. Rev. C **67**, 034003.
- Warren, G., *et al.*, 2004, Phys. Rev. Lett. **92**, 042301.
- Weinstein, J., and J. Negele, 1982, Phys. Rev. Lett. **49**, 1016.
- Wells, S. P., *et al.*, 2001, Phys. Rev. C **63**, 064001.
- West, G., 1975, Phys. Rep., Phys. Lett. **18**, 263.
- Whitney, R., *et al.*, 1974, Phys. Rev. C **9**, 2230.
- Williams, B., 1977, *Compton Scattering* (McGraw-Hill, New York).
- Williamson, C., *et al.*, 1997, Phys. Rev. C **56**, 3152.
- Wiringa, R., V. Stoks, and R. Schiavilla, 1995, Phys. Rev. C **51**, 38.
- Xiong, F., *et al.*, 2001, Phys. Rev. Lett. **87**, 242501.
- Xu, W., *et al.*, 2000, Phys. Rev. Lett. **85**, 2900.
- Xu, W., *et al.*, 2003, Phys. Rev. C **67**, 012201.
- Yan, C., 1997, Prelim. Users Manual of HMS, JLab int. rep.
- Yates, T., *et al.*, 1993, Phys. Lett. B **312**, 382.
- Yennie, D., F. Boos, and D. Ravenhall, 1965, Phys. Rev. **137**, 882.
- Zamani-Noor, F., and D. Onley, 1986, Phys. Rev. C **33**, 1354.
- Zeller, D., 1973, Report No. DESY-F23-73-2 (unpublished).
- Zghiche, A., *et al.*, 1994, Nucl. Phys. A **572**, 513.
- Zhu, H., *et al.*, 2001, Phys. Rev. Lett. **87**, 081801.
- Zimmerman, P., 1969, Ph.D. thesis (Stanford University).
- Zimmerman, P., and M. Yearian, 1976, Z. Phys. A **278**, 291.
- Zimmerman, P., *et al.*, 1978, Phys. Lett. **80B**, 45.