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The Physical Properties of Solid and Liquid Helium

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HE production of liquid helium and solid helium with the near approach to the absolute zero has raised great interest in low temperature problems. The discovery that liquid helium can exist in two forms has aroused further interest both on the experimental and the theoretical sides.1 In particular the curious properties of helium liquid II, to wit, its almost perfect conductivity for heat and its almost complete absence of viscosity give rise to great curiosity as to its structure. Before theories can be proposed it is well to collect the facts and the following pages will give a short abstract-helped out by diagrams as much as possible-of the physical properties of solid helium and the two forms of liquid helium.

No mention will be made of the effect of the low temperatures associated with liquid helium on the properties of other bodies such as for example the phenomenon of electrical superconductivity.

Nearly all the information has been taken from the papers published by the Leiden workers in the Leiden Communications, in Physica, and in the Reports of the Conferences on Low Temperature (Buenos Aires, 1932, The Hague, 1936).

The temperature scale has been fully worked out at Leiden with helium gas and vapor pressure thermometers, the virial coefficients of the gas having been carefully evaluated. The helium thermometer can be used down to 0.9° K. Below that other methods must be used. The Leiden laboratory takes the absolute zero as -273.14° C, which is a little above the commonly accepted value of -273.16° C.²

The order of presentation will be as follows:

. (1) The general description of the pressure-temperature diagram of liquid and solid helium, with remarks on the triple point phenomena and the lambda-line.

(2) The density and coefficient of expansions.

(3) Helium calorimetry. The latent and specific heats.

(4) The entropy diagram.

(5) The thermal conductivity.

¹W. H. Keesom, Nature **118**, 58, 81 (1926), **122**, 847 (1928). See also L. C. Jackson, *Low Temperature Physics*, ²W. H. Keesom, *Van der Horst and Taconis*. Leiden Communication, No. 230d, also Physica **1**, 332 (1933-1934).

- (6) The surface tension.(7) The viscosity.
- (8) The compressibility.
- (9) The dielectric constant.

(10) The electrical resistance.

(11) The velocity of sound.

(12) The refractive index.

(1) THE PRESSURE-TEMPERATURE DIAGRAM, ETC.

The first indication that the properties of very cold liquid helium may be peculiar was obtained when it was found to be impossible to solidify helium by the rapid evaporation of liquid helium; and that therefore the pressure-temperature diagram of helium must be very different from the pressure-temperature diagram of ordinary simple bodies.¹

Fig. 1 shows the p-T diagram of an ordinary liquid in the neighborhood of the triple point. In this diagram E is the triple point, EA is the vapor pressure curve of the solid, EB that of the liquid and EC the melting point line of the solid. Along EA solid and vapor may exist together in equilibrium, along EB liquid and vapor, and along EC solid and liquid. All three states can exist together in equilibrium only under the pressure and temperature given by the point E, hence its name, the *triple* point. The three lines



FIG. 1. The ordinary pressure-temperature diagram.



FIG. 2. A supposed beginning of the formation of the helium pressure-temperature diagram.

EA, EB, EC, divide the p-T diagram into three regions; if the substance is in the solid state its p and T must be that of some point in the region between EA and EC, if liquid it must be in the region between EC and EB, and if vapor the pand T must correspond to some point below EA and EB. Hence the reason for writing the words solid, liquid and vapor in these three regions.

The above diagram, of course, refers to the state of the substance when it is alone, i.e., not subjected to the pressure of any other substance. Under different pressure conditions subsidiary triple points may exist.

The line EA may be expected to end at the absolute zero, the line EC may end in another



FIG. 3. The helium pressure-temperature diagram, not to scale.



FIG. 4. The lower portion of the helium pressure-temperature diagram.

triple point corresponding to the coexistence of other possible states, the line EB ends at the *critical point* for it is impossible to have liquid present if the temperature is above the critical temperature. Recently, however, the apparent sudden ending of EB at the critical point has provoked some physicists and chemists to try to find some way of continuing the line in some form or other beyond B, but without much success at present.

The slopes of the lines AE, EB are always positive, the slope of EC is positive if the melting point of the body is raised by increase of pressure, but negative if the melting point is lowered by increase of pressure e.g., as with the change ice-water. Fuller details of the different forms of the p-T diagram will be found in treatises on the Phase Rule.

In the case of helium, however, solid helium and vapor of helium cannot exist in equilibrium with each other. If we imagine as shown in Fig. 2, that a pointer is stuck into the diagram at the triple poine E, and moved upwards along EE', splitting the line EA of Fig. 1 into two lines EA' and E'A''then a new region A'EE'A'' is created with three equilibrium lines.

If we suppose the pointer comes to rest at E' of Fig. 3 we get now a representation (not to scale) of the pressure-temperature diagram of helium at low temperatures. The region between EB, EE' and E'C is now labeled liquid helium I,





FIG. 5. The upper portion of the pressure-temperature diagram for helium.

and the region between EA', EE', and E'A'' is labeled liquid helium II.

We have therefore, referring to Fig. 3, the following equilibrium systems;

> (1) along EB liquid I and vapor, (2) along E'C liquid I and solid, (3) along E'A' liquid II and vapor, (4) along E'A'', liquid II and solid, (5) along EE', liquid I and liquid II.

The line EE' is called the lambda-line or more briefly, the λ -line. The point E is called the λ -point (par excellence). The name lambda is chosen because λ is often used to represent latent heat, i.e., the heat of transformation from one form of a substance to another form, and the transformation from liquid II to liquid I which takes place under the p and T conditions given by points on EE' seems to have the peculiar property of requiring no heat of transformation (see below). It is this property-among otherswhich makes the study of the change from liquid II to liquid I so interesting.

The point E may also be called the triple point of liquid helium I, liquid helium II and helium vapor, and E' the triple point of liquid helium I, liquid helium II, and solid helium.

Fig. 4 gives a graph, to scale, of the vapor pressure curves A'E, EB of the two forms of liquid helium and of the lower end of the lambdaline EE'.8

The coordinates of E (the lambda-point) are $T = 2.19^{\circ}$ K p = 3.865 cm of mercury = 0.0508 atmospheres.

The normal boiling point of helium liquid I is 4.22°K (indicated by point F).

The critical temperature of helium liquid I is 5.1°K and the critical pressure 2.3 atmospheres. The critical point is off the diagram.

Temperatures below 4°K are often measured with helium vapor-pressure thermometers. For this reason the equations of the curves A'E, EBhave been carefully worked out.4 They are for A'E (i.e., below 2.19°K),

$$\log_{10} p_{\text{cm Hg}} = 2.035 - 3.859/\text{T} + 0.922 \log_{10} T$$

and for EB (i.e., above 2.19°K),

 $\log_{10} p_{\rm em \ Hg} = 1.217 - 3.024/T + 2.208 \log_{10} T$

and the two curves are continuous in slope at the point E. (Differentiation of these two equations gives slightly different values of dp/dT at 2.19°K





⁸ Keesom, Buenos Aires Conference on Low Tempera-ture, p. 36 (1932). ⁴ Keesom, Weber, Nørgaard and Schmidt, Leiden Comm. Nos. 202b and c, Buenos Aires Conference (1932).



FIG. 7. Isobars of liquid helium in a density-temperature diagram.

but this is probably due to a slight inaccuracy (4 figures only) of the data.)

Fig. 5 gives to scale the melting point curve A''E'C of the solid, the lambda-line EE', and at the very bottom of the diagram, a portion of the vapor-pressure line A'EB.⁵

The coordinates of E' have been accurately fixed as $T=1.753^{\circ}$ K, p=29.91 atmospheres.

It will be noticed that dp/dT along the lambdaline is always negative.

The curve of equilibrium between solid helium and liquid helium II, viz., E'A'' shows no sign of dropping down to zero pressure as the temperature is reduced.

A brief description of the ordinary obvious properties of liquid and solid helium may be given at this stage.

Liquid helium I may be kept in a good Dewar's flask under atmospheric pressure (point F of Fig. 4). Liquid helium II however, can only be kept under partial vacuum conditions such as are maintained by an exhaust pump. In appearance liquid helium is like water. On account of its small refractive index and low surface tension (see below) it is hard to see its surface and meniscus. The easiest way of locating the surface is to view it by reflected light, when it looks much like a thin mica disk. Kept in a Dewar tube at temperatures between its normal boiling point (F) and the lambda-point (E) liquid helium I is continually bubbling throughout its volume (just like liquid air) but once the exhaust pumps bring the pressure below that of the lambda-point, the

liquid becomes perfectly quiescent. It is now liquid helium II and apparently there is sufficient evaporation from its surface to supply the demands of the pump. The extremely low viscosity and high thermal conductivity of liquid helium II (see below) are responsible for this change of behavior. The change from a bubbling liquid to a perfectly quiescent liquid brought about by a very slight change of pressure is a very interesting phenomenon to watch.

From the diagrams it is now obvious why solid helium can only be produced from liquid helium by increasing the pressure.

Solid helium is a clear crystalline solid and just as difficult to see as liquid helium. Its density and refractive index differ little from those of liquid helium In the early days it was discovered by the observation that an internal electromagnetically operated stirrer in the liquid stuck fast when the pressure was sufficiently increased.

(2) The Density (ρ) and the Coefficient of Expansion ($\alpha = -(1/\rho)(d\rho/dT)$) of Liquid Helium

An immense amount of work has been done on the variation of density with pressure and temperature.

Fig. 6 shows in the lower graph *AEB* the variation of density of liquids helium I and II under the pressure of the saturated vapor alone.⁶ It is a remarkable curve. The density is a maximum at



FIG. 8. Isochores of liquid helium on a pressure-temperature diagram.

⁶ Keesom, Leiden Comm. Supplement 75 (1933).

⁵ Keesom and Clusius, Leiden Comm. No. 216 (1931).

the triple point, the value being 0.1462 g cm⁻³ under these conditions. The coefficient of expansion of liquid helium I is positive. Its p-T curve has been traced all the way down to 2.19°K and the tangent shown in Fig. 6 gives the limiting slope of the curve at this point. From this slope the coefficient of expansion of liquid I at 2.19°K is found to be $\alpha_1 = +0.022$ deg.⁻¹.

Experimentally it is not so easy to measure the density of liquid II at temperatures just below the λ -point. To get at 2.19°K the limiting value, α_{11} of the coefficient of expansion of liquid II, use is made of the thermodynamic equation

$(dp/dT)_{\lambda} = \Delta C_p/T.v.\Delta \alpha$,

where $(dp/dT)_{\lambda}$ = the value of that differential along the λ -line just above the λ -point, $\Delta C_p = C_{p_{II}}$ $-C_{p_I}$ = the drop in the specific heat along a constant pressure line just above the λ -point as we pass from the region liquid II to liquid I, v = the specific volume at the λ -point, and $\Delta \alpha = \alpha_{II} - \alpha_{I}$.

From the measured values of $(dp/dT)_{\lambda}$, ΔC_p , $T(T=2.19^{\circ}\text{K})$, and v(v=1/0.1462cc) it is found that $\alpha_{II} - \alpha_{I} = -0.0648$, whence $\alpha_{II} = -0.0426$. The limiting tangent to the curve can now be drawn through the lambda-point and the lower part of the curve A'E continued (dotted) up to E. The upper curves HM, MN of Fig. 6 show the density-temperature relation of liquid II and liquid I under a pressure of one atmosphere.

An abstract of some of the immense amount of work on the variation of density (or specific volume) of the two forms of liquid helium with pressure and temperature is shown in Figs. 7, 8, and $9.^{7}$

In Fig. 7 we see how the density of liquid helium varies under constant pressure conditions for pressures ranging from the vapor pressure up to 30 atmospheres. For any given pressure it is seen that the two forms of liquid have equal maximum densities at points on the λ -line. The *isobars* are much alike in shape but at the higher pressures the isobars in liquid II turn up slightly more sharply when they meet the λ -line than they do at lower temperatures, i.e., the phenomenon of maximum density is more accentuated at higher pressures.

⁷Keesom and Miss Keesom, Leiden Comm. 224, 224d, 224e (1933); 240b (1935); Physica 1, 128 (1933–1934); 3, 105 (1936).



FIG. 9. The density-temperature diagram for solid and liquid helium.

In Fig. 8 we have the same data replotted to show the isochores (curves of constant specific volume or curves of constant density, also called isopycnals). The density data are given on the right of the figure. The isochores of liquid helium are made up of two branches, one for liquid II and one for liquid I which meet on the λ -line. For a given density the minimum pressure occurs at the λ -line. This diagram shows even better than Fig. 7 that the phenomenon of maximum density is more accentuated at the higher pressures. All the functions $(dp/dT)_v$, $(dv/dT)_p$, $(dv/dp)_T$ for liquid helium are discontinuous along the λ -line. In particular $(dp/dT)_v$ is negative for liquid II and positive for liquid I, therefore an isentropic expansion of liquid II is accompanied by an increase of temperature (note $dT/dp_s = Tv\alpha/c_p$, where v = specific volume, $\alpha = \text{coeff.}$ of expansion, and $c_p = \text{specific heat}$ at constant pressure).

TABLE I. Density of helium.

Temperature °K	Density of solid helium just at the melting pt.	Density of liquid helium just at the solidification pt.
2.2 3.0 4.0	0.202 g/cm ⁻³ 0.216 '' 0.230 ''	0.188 g/cm ⁻³ 0.203 " 0.217 "



FIG. 10. The specific heat at constant volume of solid helium.

The density of solid helium in equilibrium with the liquid forms may be obtained from Fig. 9.

The density varies from about 0.17 g cm^{-3} at 1.0°K to about 0.22 g cm⁻³ at 4.0°K. A few corresponding densities of melting and solidifying helium under the same pressure and temperature conditions are given⁸ in Table I. For densities and temperatures at which the helium is partly solid and partly liquid the solidification curve is accurately followed with rising as well as with falling temperatures for the solid-liquid II part of the curve. This however was not the case for the solid-liquid I part of the curve. The prolongation of the isochore 0.1820 shown in the upper part of Fig. 8 shows one instance of a supercooling of liquid I. With rising temperatures equilibrium was established very slowly and sometimes no definite point was reached for the transition. Indeed it is not easy to trace the isochores in the solid region. Even at 1.18°K the isochore 0.1820 did not leave the solidification curve to enter the solid region. As mentioned above the prolongation shown in Fig. 8 referred to supercooled liquid I.*

(3) CALORIMETRY OF LIQUID AND SOLID HELIUM

Despite the difficulty of experiments carried out at such low temperatures the calorimetry performed at Leiden on solid and liquid helium is possibly the most accurate calorimetry yet per-

formed.9 The heat is generated in electric heaters and the temperatures measured by resistance thermometers as in the Nernst-Eucken method. The rate of change of temperature with time gives the data necessary for the determination of specific heats and an absorption of heat at steady temperature (or even at constant density) provides the corresponding data for the calculation of latent heat. Use is made of the Clausius-Clapeyron equation

$$d\phi/dT = \lambda/T(v_2 - v_1)$$

to link up latent heats, specific volumes across an equilibrium line, and the rate of change of temperature of the equilibrium state with pressure.

(a) The latent heats

The latent heats of fusion of solid helium⁸ at 2.5°K, 3.0°K, 3.5°K, are 0.835, 1.089, 1.365 calorie per g, respectively (see Section on Entropy later). Looking up the previous curves we see that the density of liquid helium solidifying at 3.0°K is 0.203 g cm⁻³ while that of solid helium melting at 3.0°K is 0.216 g cm⁻³. Substituting in Clapeyron's equation we find that at 3.0°K the change of melting point with pressure is of the order of 1° per 50 atmospheres which checks up well with the slope of the curve E'C of Fig. 5.

The latent heats of vaporization of liquid helium measured directly over the range of 1.5°K to 3.0°K also agree well with the figures calculated by an application of the Clapevron equation to the change of state from liquid to vapor.¹⁰ The latent heat of vaporization is about 5.5 calorie per g. Note how small this is in comparison with the value for other liquids, e.g., the value for hydrogen is 110 and for water 540 calorie per g. Hence the importance of very good insulation for liquid helium containers. Latent heats of vaporization of liquid helium I at different temperatures may be calculated from the entropy diagram (Fig. 13).

The latent heat of transformation from the helium liquid II to helium liquid I is very small. The measured value is less than 0.00188 calorie per g. In all probability it is zero. This opens up a discussion as to whether helium liquid II and

⁸ Keesom and Miss Keesom, Leiden Comm. 240b, Physica 3, 105 (1936).
* I can find no reference to the detached line in the upper part of Fig. 9.

⁹ Keesom and Miss Keesom, Leiden Comm. 235d, Physica 2, 557 (1935). ¹⁰ Dana and Kamerlingh Onnes, Leiden Comm. 179c.

helium liquid I can be regarded as distinct phases in the thermodynamical sense. The change is sometimes called a phase change of the second order.

(b) The specific heats

The results of experiments on the specific heat of solid helium are shown in Fig. 10 for three different densities (or specific volumes).8 The specific heat diminishes as the density is increased. The dotted curves of Fig. 10 show how the specific heat would vary with temperature if the Debye function, θ ,* were either 25° or 30° or 35°K, respectively. The curves $C_v - T$ rise faster than they would if the specific heats followed Debye's T^3 formula. Apparently there is no definite θ for solid helium. The measurements of the specific heats of liquid helium exhibit some interesting features. The results are plotted in in Fig. 11. Considering the values obtained with the liquid in equilibrium with its vapor (curve A) it is noted that starting with liquid helium II at about 1°K and raising the temperature the specific heat increases fairly fast up to about 2.15°K and then from 2.15°K to 2.19°K it shoots suddenly from a value of about 2 cal. per g degree up to that of about 6 cal. per g degree. A further

elevation of temperature brings about a sudden drop to 1.5 cal. per g degree as the liquid changes to liquid I. This change at the lambda-point occurs within a range of temperature as small as 0.002 deg or even as small as 0.0002 degree. The specific heat of liquid I further decreases as the temperature rises to 2.5°K and then gradually increases up to about 2.0°K. The variations along curves B and C are similar but naturally the sudden change of specific heat occurs at different transition temperatures.

(4) THE ENTROPY DIAGRAM OF SOLID AND LIQUID HELIUM

Change of entropy is defined by the equation

ds = dQ/T,

where s signifies entropy and dQ is a small quantity of heat added reversibly at absolute temperature T. The equation usually refers to one gram of substance and changes of entropy are usually only required so that the entropy may be taken as zero at any specified state.

By integrating
$$s_2 - s_1 = \int_{1}^{2} dQ/T.$$

If the state or phase does not change

$$s_2 - s_1 = \int_0^2 c dT / T$$

where c is the appropriate specific heat of the substance. If c is constant this becomes

$$s_2 - s_1 = c \log_{\epsilon} (T_2/T_1) = 2.303c \log_{10} T_2/T_1.$$

If change of state occurs at constant pressure and temperature T, the heat absorbed is the appropriate latent heat $\boldsymbol{\lambda}$ and

$s_{\text{upper state}} - s_{\text{lower state}} = \lambda/T.$

As an introduction to the consideration of the entropy diagram of helium let us consider the entropy diagram of ice-water steam (Fig. 12) where entropies, as abscissae, are plotted against temperatures, as ordinates, and the entropy of 1 g of water at 0° C is taken as zero (point C).

Along the curve marked "water" the heating is supposed to occur always at the saturation vapor pressure. The curve CD shows the change from 0° to 100°C. If the pressure is not allowed to exceed one atmosphere ebullition occurs at 100°C



* Debye's function θ is defined by saying that two sub-

bebye's function θ is defined by saying that two sub-stances having Debye functions θ_1 and θ_2 , respectively, have equal specific heats at constant volume (C_{ν}) at temperatures T_1 and T_2 , respectively, if $T_1/T_2 = \theta_1/\theta_2$. Textbooks on thermodynamics furnish tables giving C_{ν} in terms of T/θ . Debye also showed that at very low temperatures C_{ν} is proportional to T^3 .

3.5





FIG. 12. Entropy-temperature diagram of ice-water steam.

and the change of entropy from water to steam (=539/373) is shown by DE, and if heating is continued on to 300°C (still at constant pressure) the line of superheat at 100°C viz. EF is followed. Curve CGHK gives the entropy change if beginning at 0°C and 4.6 mm pressure heating takes place gradually under the corresponding saturation pressures until the temperature reaches 200°C and the pressure 11,647 mm. Keeping p now constant vaporization occurs at 200°C along GH and superheat to 300°C traces the curve along HK. If we begin with ice at 0°C and atmospheric pressure the starting point of the graph would be B(BC=79.4/273.1). If we begin with ice at -50° C and atmospheric pressure A is the starting point. If we begin at -50° C under a very large pressure we still begin very nearly at A. If the pressure is adjusted such that melting occurs at -20° C the curve ALM is traced and followed by MC as the supercooled water at -20°C warms up under gradually decreasing pressures to 0°C. As stated above the pressures at points on the curve CDG are always the saturation pressures for the corresponding temperatures. If heating takes place under different but constant pressures different values of the specific heats will be required and the up-curve will depart slightly from CDG (this is brought out in the helium entropy diagrams, see below).

The two lines—the water line on the left and the saturated steam line on the right—will, if continued to higher temperatures bend over and join up at the critical temperature (see the helium diagram Fig. 13). The dotted lines passing up through the central region of mixed water and



FIG. 13. Temperature-entropy diagram of liquid helium.

water vapor are lines of constant dryness. For example along the curve marked $\frac{1}{4}$, one-quarter of the total mass of the stuff is in the form of steam and three-quarters of the mass in the form of water. Fig. 13 is the entropy diagram of helium corresponding to Fig. 12 for water.11 The additional temperatures and pressures (in atmospheres) inserted on the diagram are those for which careful observations have been made. The liquid line and saturated vapor line have been extrapolated above the normal boiling point to meet at the critical temperature. Lines of constant dryness (isohygrals) are inserted but not lines of superheat. The existence of two forms of liquid helium complicates the lower portion of the liquid line. Note the angle made at E in the saturated liquid curve due to the sudden drop in specific heat as we pass from liquid helium II to liquid helium I. Fig. 14 amplifies the lower lefthand portion of Fig. 13 and shows in addition a number of isobars. Looking at these in the helium liquid II region it is obvious that if we wished to lower the temperature of liquid helium II by an isentropic change the pressure must be increased, i.e., the liquid compressed. Note also that the two liquid regions overlap in the triangle formed by the λ -line *EE*', the saturated liquid line and the curve of solidification. Fig. 15 gives the entropy diagram of solid and liquid helium and a number of isochores.8 The diagram is even more complicated than the last and will repay careful study. One is lost in admiration of the quality of the experimental work that must have been done before these curves could have been drawn.

¹¹ Keesom and Miss Keesom, Physica 1, 161 (1933-1934).

The entropy calculations

The general method is as stated above. The zero for entropy in the case of helium is taken at the normal boiling point of liquid helium I $(T=4.22^{\circ}\text{K}, p=760 \text{ mm})$. The zero is at the right hand side of Figs. 14 and 15.

The specific heats of liquid helium I and II under the saturated vapor pressures being known the entropy at lower temperatures than 4.22°K can be calculated step by step back from 4.22°K to about 1.2°K. Below that extrapolation is employed using the T^3 law. This leads to a value of the entropy of liquid helium II at the absolute zero of $s_0 = -0.848$ calorie per degree per g. This should also by theory be the value of s_0 of solid helium. To trace up the temperatureentropy curve of an isochore (as in Fig. 15) the proceeding is as follows:

Select any given density, e.g., $\rho = 0.201$ g cm⁻³, for which specific heat data have been obtained. Plot C_v/T against T. The area under the curve $= \int_{T_0}^{T_0} (C_v/T) dT$ and therefore it is possible to find $s_T - s_0 = \int_{T_0}^{T_0} (c_v/T) dT$ for any given upper limit T. Integrate up to the melting point T_m for this given isochore, and so get s_m , the entropy of 1 g of solid helium just at the point of melting. This fixes the point A (Fig. 15) on the curve of fusion and this being done also for the two other isochores shown, the curve of fusion can be drawn in.

During the melting process

$$s = s_m + \int_{T_m} f(c/T) dT,$$

where c=apparent specific heat during fusion. (The correct value of c for the constant volume





change followed had been determined experimentally.) If the upper limit is set equal to T_{\bullet} where melting ceases, s will be the entropy (s_s) for a point B on the curve of solidification. This was also carried out for the three other constant densities and therefore now the curve of solidification E'B can be filled in. The specific heat of liquid helium being known the three isochores can now be extended into the liquid region (e.g., BC for $\rho=0.201$). Other isochores were determined in much the same way extending through both the helium liquid I and helium liquid II regions (see Fig. 15).

The melting curve and solidification curves being now completed on the diagram the latent heat of fusion (λ_m) can be got by drawing a constant temperature line cutting these two curves, reading off the entropies of the points of intersection and using the relation

$$s_s - s_m = \lambda_m / T$$
.

The latent heats given in Section 3 were obtained in this way, and they check up very well with those obtained by the use of Clapeyron's equation.

(5) THE THERMAL CONDUCTIVITY OF LIQUID HELIUM

The thermal conductivity of liquid helium has been recently measured¹² by using two flat coils of wire which may be employed either as electric heaters or as thermometers. These coils may be placed in thin copper boxes placed parallel to



FIG. 15. Isochores on a temperature-entropy diagram of solid and liquid helium.

 12 Keesom and Miss Keesom, Physica 3, 359 (1936).



FIG. 16. The surface tension-temperature diagram of liquid helium.

each other in the liquid helium and the rate of rise of temperature measured in one coil when heat is electrically generated in the other.

The thermal conductivity of liquid helium I shows no peculiarities, its value at 3.3°K being 6×10⁻⁵ calories per sec. per cm² per degree C per cm, approximately the same as that of a gas and about one-twentieth that of water.

If one works with liquid helium II at temperatures between 1.4°K and 1.75°K it is found that as soon as the heat is turned on in one coil the other coil experiences a rise of temperature. The heat is almost instantly distributed through liquid helium II which shows that its thermal conductivity is of a high order. Measurements gave a value of 190 calories per sec. per cm² per degree C per cm. This is about 200 times as great as that of copper at room temperatures and 14 times as great as that of copper at liquid hydrogen temperatures; and 3,000,000 times as great as that of liquid helium I. In fact liquid helium II is the best thermal conductor known and it may even be called a "supra-conductor."

Solid helium conducts heat just a little better than liquid helium I.

(6) THE SURFACE TENSION OF LIQUID HELIUM

This has been measured by the capillary tube method.13 The results are shown in Fig. 16. For helium liquid I the graph between surface tension and temperature is nearly linear between 4.2°K and 2.4°K, the temperature coefficient being about -1 dyne per cm per degree. When helium liquid II temperatures are attained the graph bends over nearly horizontally, the surface tension approaching a constant value of 3.2 dynes per cm at 1.5°K.

¹³ Van Urk, Keesom and Kamerlingh Onnes, Leiden Comm. 179.

(7) The Viscosity of Liquid Helium

Observations on the flow of liquid helium through fine tubes revealed the fact that liquid helium II is much more fluid than liquid helium I. To keep liquid helium I below 4.2°K (its normal boiling point) it must be kept under reduced pressure. Under these conditions it gently boils away bubbling occurring through its volume. A capillary tube viscosimeter can therefore not be used for liquid helium I below 4.2°K. The viscosity has however been measured in Toronto14 by using an oscillating conical-ended cylinder supported as a torsion pendulum, the rate of decrease of amplitude being measured at different temperatures as the pressure is gradually decreased. The results are shown in Fig. 17. The coefficient of viscosity of liquid helium I is about 110 micropoises (1 micropoise = 10^{-6} c.g.s. unit) at 4.2°K, rises in the usual way as the liquid is cooled up to 270 micropoises at 2.3°K and then as the temperature is taken past the λ -point (2.19°K) with the liquid helium II region drops rapidly to 33 micropoises. For comparison it may be noted that the coefficient of viscosity of water at 20°C is 10,000 micropoises. Liquid helium II is therefore almost superfluidic.

(8) THE COMPRESSIBILITY OF LIQUID HELIUM

Measurements of the compressibility (defined as $= -1/v(dv/dp)_T$ at 2.7°K and at pressures between 5 and 10 atmospheres gave a value of 8×10⁻³ cm² per kg-wt. or 0.8×10⁻¹⁰ c.g.s. unit.¹⁵ This may be compared with the values 0.39×10^{-10} for water and 0.38×10^{-10} for mercury. It has been stated that helium liquid has the greatest compressibility of all liquids. This however can only be true under certain limitations of the conditions for near the critical point all liquids have high compressibility.

The compressibility of solid helium is about $1\frac{1}{2} \times 10^{-3}$ cm² per kg-wt. at 3.7°K and 115 atmospheres.

(9) THE DIELECTRIC CONSTANT OF HELIUM

This, as shown in Fig. 18 also shows a discontinuity at the transition point of liquid helium I

¹⁴ Burton, Nature **135**, 265 (1935); Wilhelm, Misener and Clark, Proc. Roy. Soc. **A151**, 342 (1935). ¹⁵ Keesom, Buenos Aires, Conference, p. 117 (1932).



FIG. 17. The viscosity-temperature changes of liquid helium.

and liquid helium II, the curves¹⁶ being very similar to the density curves of Fig. 6.

(10) THE ELECTRICAL RESISTANCE OF LIQUID HELIUM

This has been measured¹⁷ by connecting in series a condenser and a galvanometer to a battery of 250 volts and immersing the condenser in liquid helium. No measurable deflection was obtained although the galvanometer scale could be read to one-tenth of a millimeter. This deflection corresponds to a specific resistance for helium of the order of 10¹⁵ ohm-cm. This figure may therefore be taken as the minimum possible value of the specific resistance and no measureable change was found over the whole temperature range from 4.22°K to 1.28°K. This range includes the λ -point (2.19°K) so that although the thermal conductivities of helium liquid I and liquid II differ greatly the electrical conductivities of the two forms of liquid are nearly equal.

(11) THE VELOCITY OF SOUND IN HELIUM GAS AT LIQUID HELIUM TEMPERATURES

At about 4.2°K (i.e., the normal boiling point) the velocity of sound in helium gas18 is of the

 ¹⁶ Wolfke and Keesom, Leiden Comm. 192a.
 ¹⁷ Wolfke and Keesom, Physica 3, 823 (1936).
 ¹⁸ Keesom and Van Itterbeek, Leiden Comm. 213b.





FIG. 18. The relation between the dielectric constant and temperature of liquid helium.

order of 110 meters per second. This value is in accordance with a specific heat ratio of about 1.7.

(12) THE REFRACTIVE INDEX OF LIQUID HELIUM

This has been recently measured¹⁹ in Toronto by a Wollaston cell method (determination of critical angle) the thin glass cell containing helium gas and being immersed in liquid helium. The refractive index of helium liquid II (against gaseous helium) is 1.028 ± 0.006 at a temperature just below 2.1°K. The refractive index of helium liquid I at just above 2.2°K is slightly less than this.

In this article we have briefly noted the chief properties of solid helium and the two forms of liquid helium in the region 1°K to 5°K leaving the diagrams to tell a fuller story. The behavior of liquid helium in the neighborhood of the λ -curve is possibly not unique. There are certain other substances, e.g., liquid crystals, the ammonium halides, methane etc. which have properties similar to those of liquid helium II. It is considered that liquid II is a more "well-ordered" liquid than liquid I and has properties approaching those of the crystalline solid helium. Up to the present this has not been confirmed by x-ray photography.

¹⁹ Wilhelm and Cove, Sept. (1936)—as yet unpublished.