The Problem of the Mechanism of Static Spark Discharge

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TABLE OF CONTENTS

I. INTRODUCTION

FOR the understanding of the mechanism of spark breakdown one must recall to mind that conduction currents in gases depend on the presence in the gap of electrons or ions created by some agent. If no electrons are produced there is no current. In the presence of photoelectrons, or those produced by radioactive processes, that is by outside agents, currents are observed that increase with the field, eventually tending towards a saturation value which in some cases is reached, in others not.[1] Application of still higher fields produces an ultimate increase in the current due to the multiplication of the electrons present by the *primary* process of ionization by collision in the gas (gas-intensified photoelectric or ionization current). This current of itself does not usually lead to a spark and is entirely dependent for its maintenance on the ionization imposed from outside. At some point in the increase of the field, the increase of the current may become very rapid. It leads to an unstable regime in which, through a transition which is sometimes explosive, the current readjusts itself to operate under a set of conditions where it is self-maintaining, i.e., it changes to an arc, glow discharge, corona or brush. In order to reach a self-maintaining condition the gas-intensified photoelectric or ionization current must develop a mechanism by which new electrons are generated in sufficient number by *secondary* processes in a sensitive portion of the gap to replace the externally imposed ionization. Thus to understand the problem of spark breakdown, one must list and discuss the various mechanisms of *secondary* ion liberation possible in a gap.

A. Processes in the gas itself

1. Ionization and excitation by *electron* impact.
2. Ionization by positive ion impact in the gas.
3. Photoelectric action in the gas.
4. Action of metastable atoms in the gas.

B. Processes at the cathode

1. Electron bombardment.
2. Positive ion bombardment.
3. Photoelectric effects.
4. Impact of metastable atoms.
5. Field currents.

[1] Loeb, *Kinetic Theory of Gases*, 2nd edition, p. 623 ff.; N. E. Bradbury, Phys. Rev. **40**, 980 (1932).

II. Possible Secondary Processes

A. 1. The ionization by electrons in the gas is essentially the *primary* process in all spark discharge.[2] In general outline it is determined by the rate of gain of energy of the electrons in the field and on their ability to produce ionization once they have sufficient energy.[3] The energy gain depends[3] on the free path of the electrons, which, aside from pressure and character of the gas, depends on the energy of the electrons.[4] The energy gain depends on the field strength existing and on the rate of loss of energy of electrons in impact with gas atoms or molecules. In monatomic gases the fractional loss of energy below the radiation or ionization potentials can be approximately calculated from classical conservation laws. It is quite low.[3, 5] In molecular gases which have vibrational energy levels the loss is materially greater.[6] Above the radiation potentials there is again additional loss of energy of electrons to radiation. As the probability of this loss is not too high, electrons can gain enough energy to ionize. Once having the ionizing energy, the probability of ionization or loss to radiation determines the ultimate destiny of the process. As free paths, rate of energy loss to elastic and inelastic impacts and the probabilities of ionization and radiation are all functions of the energy of the electron, and as the free paths at lower energies are not accurately known,[4] a quantitative theory of this process cannot be derived, largely owing to the mathematical complexity involved.[7] It must be added that while the number of excited and radiating molecules produced may not parallel ionization closely, the numbers, will be roughly proportional in any region of energy. At the lower electron energies and thus at lower gradients there will be a greater proportion of excitation acts relative to ionization.

Fortunately, while one cannot calculate the primary ionization by electrons in a field, one can measure it quite simply by a procedure developed by Townsend.[2] The quantity determined is α, the number of electrons created by ionization of molecules by electron impact per cm of advance in the field direction by a single electron. It turns out that $\alpha/p=f(X/p)$. Here p is the pressure in mm of Hg and X is the field strength in volts/cm. Only recently have the values of α/p been investigated in the whole region from $X/p=20$ (15,200 volts/cm $p=760$ mm) to $X/p=200$ where Townsend's accurate measurements begin.[8] It is the results of these investigations that make the present paper possible. The exact values of α/p for the inert gases are not at hand since Penning[9] has found that owing to the effect of metastable atoms in ionizing unbelievably small traces of impurity, previous investigations have not been conducted under proper conditions. In general, however, the curves of X/p against $\alpha/p=f(X/p)$ are essentially similar in form for all gases.

A. 2. Recent researches have proven conclusively that, except for certain special types of resonance ionization observed especially in the octette groups of the inert gases, the *positive ions do not ionize perceptibly, if at all, under 300 volts of energy*.[10] The neutral inert gas atoms among their own kind ionize at energies as low as 35 volts,[11] and alkali ions in inert gases in the most propitious cases as low as 60 volts.[10] Thus positive ions in a gas can hardly ionize gas molecules or atoms in fields where they gain less than about 50 volts per mean free path in the inert gases, or more nearly 200 volts per free path for other gases. *The ionization by positive*

[2] Townsend, *Electricity in Gases* (Oxford, 1914), chapter VIII.

[3] Compton and Langmuir, Rev. Mod. Phys. **2**, 204 (1930); L. B. Loeb, *Kinetic Theory of Gases*, 2nd edition (McGraw-Hill, 1934), p. 628 ff.; K. T. Compton, Phys. Rev. **7**, 489, 501, 509 (1916).

[4] R. B. Brode, Rev. Mod. Phys. **5**, 257 (1933).

[5] Cravath, Phys. Rev. **36**, 248 (1930); Morse, Allis and Lamar, ibid. **48**, 412 (1935).

[6] H. Baerwald, Physik. Zeits. **26**, 868 (1925); W. Harries, Zeits. f. Physik **42**, 26 (1927); H. Löhner, Ann. d. Physik **24**, 349 (1935).

[7] A recent attempt in this direction was made by M. J. Druyvesteyn, Physica **3**, 65 (1936), for Ne gas between values of X/p from 5 to 30. He assumed a linear variation of the excitation and ionizing probabilities in this region and electron impacts with *no* energy loss at all per impact.

[8] F. B. Sanders, Phys. Rev. **41**, 667 (1932); **44**, 1020 (1933); M. Paavola, Archiv f. Elektrotechnik **22**, 443, 450 (1929); K. Masch, ibid. **26**, 589 (1932); D. Posin, Phys. Rev. **47**, 258 (1935); **48**, 483 (1935).

[9] F. M. Penning, Phil. Mag. **11**, 961 (1931).

[10] O. Beeck, Physik. Zeits. **35**, 36 (1934); R. M. Sutton, Phys. Rev. **32**, 364 (1929); R. N. Varney, ibid. **47**, 483 (1935).

[11] O. Beeck, Ann. d. Physik **19**, 121, 129 (1934); Rostagni, Zeits. f. Physik **88**, 55 (1934); R. N. Varney, Phys. Rev. **49**, 204A (1936); **49**, 889A (1936); **50**, 159 (1936).

ions is thus definitely ruled out in all ordinary discharge phenomena.

A. 3. Ultraviolet radiations in gases are known to produce ionization if of sufficiently short wave-lengths.[12] Such radiations are relatively not as effective as ionizers in the body of the gas as they are in producing photoelectrons from an electrode surface owing to their relatively low absorption coefficients. Recent results of Cravath[13] have shown radiations from corona discharges to be effective as ionizers in air at atmospheric pressure. Cravath estimated that 1 such photoelectron was produced for every 10^4 ions generated in the discharge. The absorption coefficient of the radiations causing this effect was about 10 cm^{-1} in air at 760 mm pressure. Ions can *effectively* be produced in this fashion under the usual high pressure sparking conditions *only in heterogeneous gases*. In such cases the radiating potentials V_r for one gas must be greater than the ionization potential V_i for some other constituent. It is possible that at high electron energies some electrons belonging to inner shells can be excited to radiation at a potential $V_{r'}$ which is greater than V_i for outer electrons in the same molecular species. In this event a homogeneous gas might be ionized by a photoelectric process generated within itself. Such a mechanism is practically ruled out as an important factor owing to the rarity of its occurrence at low electron energies and the high probability of autoionization removing such excited states before they radiate. The only other mechanism by which a homogeneous gas may be ionized by a photoelectric process in the gas is by means of the *continuous recombination spectrum* of the ionized molecules or atoms which will give radiations capable of ionizing the gas. The emission of such spectra is feeble even in most low pressure discharge tubes. It will be feeble in the extreme in the dark discharge preceding spark breakdown.

Thus unless a gas is heterogeneous with V_r for one constituent greater than V_i for another, the photoelectric processes in the gas can be ruled out as an important mechanism in spark production. It is probable that in mixed gases like air at high pressures and with long gaps this mechanism may be of importance as Cravath has pointed out.

A. 4. Metastable atoms have not only been shown to be effective in ionizing a gas having admixtures of traces of gases of lower ionizing potential than the metastable state of the gas, but their presence has been shown materially to influence the sparking potential of the gas.[14] Their ability to diffuse against an electrical gradient makes them an important factor in discharge phenomena at lower pressures. Their action is in a large measure limited to the relatively pure inert gases as they are easily destroyed. In these gases, however, great care must be used to avoid complications produced by their action.

B. 1. Electron bombardment of surfaces, while highly efficient in liberating secondary electrons[15] cannot be considered here, as there is no mechanism by which electrons can be driven up to a cathode with sufficient energy to liberate electrons under spark discharge conditions.

B. 2. Positive ion bombardment of metal surfaces has long been considered an important method[16] for the secondary production of electrons in spark discharge. The data to date have been most discouragingly contradictory as regards the efficiency and even the possibility of such action at low positive ion energies.[17] The experimental difficulties encountered are largely responsible for the diversity of opinion. The difficulties lie in getting enough positive ions of low energies to strike different metal surfaces under conditions where photoelectric and other disturbing processes do not falsify the results. More or less definite results have been obtained above 10 to 20 volts energy of the impacting ions.[18, 19] These have shown that some of the

[12] R. C. Williamson, Phys. Rev. **21**, 107 (1923); Lawrence and Edlefsen, Phys. Rev. **34**, 233 (1929); O. Oldenburg, Zeits. f. Physik **38**, 370 (1926).

[13] A. M. Cravath, Phys. Rev. **47**, 254 (1935); Varney and Loeb, ibid. **48**, 822 (1935).

[14] F. M. Penning, Zeits. f. Physik **72**, 338 (1931); **78**, 454 (1932).

[15] F. M. Penning and Kruithoff, Physica **2**, 793 (1935).

[16] J. J. Thomson, *Conduction of Electricity Through Gases*, 3rd edition, Vol. 2 (Cambridge, 1933), p. 518. 2nd edition, p. 490 ff.

[17] Geiger and Scheel, *Handbuch der Physik*, 2nd edition, Vol. 22 (1933), part 2, pp. 127–133. Summarizes literature up to 1932.

[18] M. L. E. Oliphant, Proc. Roy. Soc. **A127**, 373 (1930); Oliphant and Moon, ibid. 388 (1930); F. M. Penning, Königl. Akad. Amsterdam **31**, 14 (1928); **33**, 841 (1930); Physica **8**, 13 (1928).

[19] W. J. Jackson, Phys. Rev. **28**, 524 (1926); **30**, 473 (1927).

diversity of opinion lay in (1) the really unknown conditions of the surfaces of different metals used as regards their work function due to gas films, and (2) a neglect in considering the possible effects of differences in the ionization potential V_i of the ion used in bombardment relative to the work function ϕ of the surface. The fact that metastable atoms having an excitation energy V_r greater than ϕ for a metal surface do liberate electrons from such a surface even when they strike the surface with their energy of thermal agitation only[20] leaves no doubt in one's mind that positive ions are capable of doing the same.[21] This action of positive ions has been asserted for many years by Holst and Oosterhuis[22] on the basis of image forces, by James Taylor[23] by some sort of photo-effect near the cathode, by A. von Hippel[24] on the basis of a local thermionic effect at the cathode. The process of liberation has been treated from the more modern point of view on the basis of the Sommerfeld electron theory of metals by Oliphant and Moon.[18] Experiments both by Oliphant and Moon and by Penning appear to have established the fact that positive ions of as low as 10 to 20 volts can liberate electrons from metals if V_i, their ionizing potential, is greater than ϕ. This applies then to ions of the inert gases and the common permanent gases with clean surfaces of Fe, Cu, Zn, Pt, W, etc. For many of the alkali ions on these metals this mechanism does not take place. Experiment at positive ion energies between 20 and 50 volts indicates that with the inert gases the number of electrons liberated per positive ion impact is of the order of 10 percent. At lower energies nothing is known of this quantity, although Penning's results if extrapolated indicate a material decrease in efficiency with energy. This is not indicated by theory, nor by Oilphant and Moon's experiments in inert gases.

All positive ions at energies above 100 volts liberate electrons from metal surfaces. The efficiencies increase with energy according to Oliphant and Moon and Penning in a nearly linear fashion. Extreme outgassing of metal surfaces seems to reduce[19] the number liberated per positive ion impact. Otherwise as with the photoelectric effect the gas layers on surfaces produce complicated effects. What the effect of gas at atmospheric pressures will be on this mechanism is not known. As the electrons may be ejected with high energies, one may expect some loss of such electrons back to the plate by diffusion processes at atmospheric pressure, as will be outlined below. It is thus clear that very little is known about this secondary mechanism precisely in the regime of positive ion energies which are involved in spark phenomena, i.e., from 0 to 10 volts. It can definitely be asserted that *there will be electrons liberated by this mechanism* and that *the efficiency of liberation can be expected to increase at least slowly with increasing electron energy.*

B. 3. Photoelectric yields at the cathode resulting from the longer wave-length ultraviolet radiations (3000–2000A) in discharge are not very effective owing to the small intensity of such radiations in the dark predischarge period, and their relatively smaller efficiency at the cathode. The photoelectric yields of the shorter wavelength radiations (formerly called entladungstrahlen), between 500 and 800A have recently been shown to be quite marked.[25] No careful study of such radiations as might be generated in the dark predischarge period have been made. The most significant investigation in this direction is that of Cravath[13] on corona discharge in air who found a radiation effective on Cu of absorption coefficient 2 cm^{-1} in air at atmospheric pressure. The estimated magnitude of the efficiency of this radiation was that it gave one photoelectron for every 10^5 primary electrons produced in the corona if all the radiation reached the electrode. *If this is generally true one must consider photoelectric effects at the cathode as one of the important secondary mechanisms in normal spark discharge.*

B. 4. Metastable atoms can produce electrons on impact with electrodes much as is the case with positive ions.[20] The ability of metastable

[20] M. L. E. Oliphant, Proc. Roy. Soc. **A124**, 228 (1929); Carl Kenty, Phys. Rev. **38**, 377 (1931); **43**, 181 (1935); S. Sonkin, ibid. **43**, 788 (1933); H. J. Couilliette, Proc. Roy. Soc. **A124**, 228 (1928).

[21] L. B. Loeb, *Nature of a Gas* (Wiley, 1931), page 120.

[22] Holst and Oosterhuis, Phil. Mag. **46**, 1117 (1923).

[23] James Taylor, Phil. Mag. **3**, 753 (1927); **4**, 505 (1927). Proc. Roy. Soc. **A114**, 73 (1927).

[24] A. von Hippel, Ann. d. Physik **81**, 1043 (1926); von Hippel and Blechschmidt, ibid. **86**, 1006 (1928).

[25] Carl Kenty, Phys. Rev. **44**, 891 (1933); Cashman and Huxford, ibid. **48**, 734 (1935).

atoms to move independently of fields is a most important property. They are easily destroyed and occur only in a limited group of gases. Their easy destruction[26] probably makes them more effective in the body of the gas than at the electrodes to which they must diffuse.

B. 5. Field currents are essentially caused by degenerate or by thermionic electrons whose escape over the potential barrier of a metal is facilitated by an externally applied field of sufficient magnitude. For most metals such fields must reach the magnitude of 10^7 volts/cm or more at local spots to cause a measurable emission.[27] At extremely low pressures, fields of this magnitude may occur in discharge and cold emission will play a role.[28] It is doubtful whether space charges ever could be maintained at any *appreciable gas pressure*, even over a free path from a liquid and possibly even solid cathode surface, which could cause gradients sufficient for field emission.[29] In ordinary sparking phenomena in gases, long before fields of such magnitude are reached, other secondary processes become so efficient that breakdown occurs and field emission is not necessary.

B. 6. It is necessary to draw attention to one feature of all secondary processes at a cathode which is often ignored. If secondary electrons escape into a gas from a surface giving a current i_0, and the electrons escape with a velocity C well above that of thermal agitation, the current i of such electrons measured at some distance from the cathode is given roughly by

$$i=i_0\frac{K_e(X/p)760(6\pi)^{\frac{1}{2}}}{C+K_e(X/p)760(6\pi)^{\frac{1}{2}}},$$

where K_e is the electron mobility and X/p has the usual significance.[1] It is seen that unless $760K_e(X/p)(6\pi)^{\frac{1}{2}}\sim C$ or greater, a considerable proportion of the initial electrons will return by diffusion to the cathode and i will be considerably smaller than i_0. This does not happen for electrons liberated in the body of the gas. It can happen for photoelectric and other processes at the cathode as C may be equivalent to volts of energy, while $760K_e(X/p)(6\pi)^{\frac{1}{2}}$ may well be less.

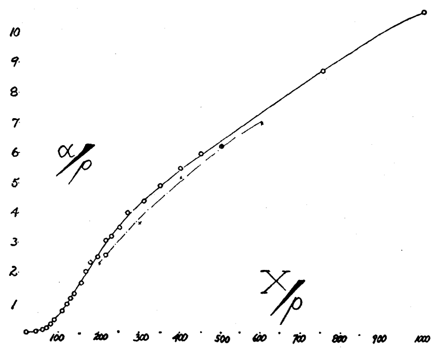

FIG. 1. Circled points are Posin's. Crosses represent Townsend's values. Values of Masch are close to Posin's as far as Masch went.

III. MECHANISMS OF CLASSICAL THEORY

Townsend early showed that if one had a photoelectric current in a gas in a field of strength X (volts per cm), as X increased beyond a certain value, the current began to increase faster than the approach to saturation called for. This was ascribed by him to multiplication of the initial and other electrons in the gas by ionization by collision. This increased current may be called a gas-intensified photoelectric current. It is studied by making X constant and varying the distance x between the electrodes. It transpires from such a study that $i=i_0e^{\alpha x}$ where α is the *number of new electrons formed per electron per cm path* in the field direction in the gas. Here i is the current at x cm from the cathode while the initial current from the cathode is i_0. Log i/i_0 plotted against x gives a straight line whose slope is α. Experiment shows that $\alpha/p=f(X/p)$ and curves of this function have been observed to be similar in general form for different gases. Such a curve for N_2 is shown in Fig. 1. Townsend[30] worked at low i_0, at low p (p expressed in mm of Hg) and small x and found the equation to hold fairly well. At a certain p and X/p, where x passed a certain

[26] E. W. Pike, Phys. Rev. **49**, 513 (1936).

[27] Millikan and Eyring, Phys. Rev. **27**, 51 (1926) (review of previous work); Fowler and Nordheim, Proc. Roy. Soc. **A119**, 173 (1928); W. V. Houston, Zeits. f. Physik **47**, 33 (1928); Phys. Rev. **33**, 361 (1929); R. T. K. Murray, ibid. **49**, 878A (1936).

[28] J. W. Flowers, Phys. Rev. **48**, 954 (1935).

[29] I. Langmuir, Science **58**, 290 (1923); Compton, Phys. Rev. **37**, 1077 (1931); L. Tonks, ibid. **48**, 562 (1935).

[30] Townsend, *Electricity in Gases* (Oxford, 1914), chapter IX.

value, the log i/i_0 *versus* x curves bent sharply upward. This he explained as being due to *ionization by positive ions by impact in the gas.* He assumed that each positive ion gave β new electrons per cm path in the field direction by ionization of neutral molecules. In this event he showed theoretically that

$$i/i_0=(\alpha-\beta)e^{(\alpha-\beta)x}/(\alpha-\beta e^{(\alpha-\beta)x}). \quad (1)$$

From his log i/i_0 curves at low pressure and small x he evaluated β (from the approximation for $\alpha \gg \beta$, $i/i_0=\alpha e^{\alpha x}/(\alpha-\beta e^{\alpha x})$), and the values of β so obtained fitted this theory quite well. He found that $\beta/p=F(X/p)$. The study of this phenomenon extended over a very narrow range of p, x and X/p and the results cannot be taken as proving the postulates. It had been assumed by J. J. Thomson[16] and also assumed as an alternative by Townsend,[31] that *positive ions could knock secondary electrons out of the cathode.* This process yields the equation

$$i=i_0e^{\alpha x}/[1-\gamma(e^{\alpha x}-1)]. \quad (2)$$

Here γ is the number of electrons emitted per positive ion impact. If one set $\gamma=\beta'/(\alpha-\beta')$ one gets

$$i=i_0(\alpha-\beta')e^{\alpha x}/(\alpha-\beta' e^{\alpha x}) \quad (3)$$

and it is seen that this yields an equation which is so like Eq. (1) used for calculating β in form that it is impossible to differentiate experimentally between them with the present degree of experimental accuracy. The equations above give $i=\infty$ at $x=\delta$ when $\alpha-\beta e^{(\alpha-\beta)\delta}=0$ (4)

or $$1-\gamma(e^{\alpha\delta}-1)=0. \quad (5)$$

That is at a certain value of X/p, when α and β, or α and γ, approach proper values one will have a self-sustained current when x reaches the value δ indicated above. Using his values of α and β at high values of X/p, Townsend was able to predict the values of sparking distance $x=\delta$ at the pressures studied. Hence the equation appeared proven and has until recent years been generally accepted.

IV. Other Mechanisms Proposed

Certain difficulties connected with the ability of positive ions to ionize by impact, initially pointed out by Holst and Oosterhuis,[22] have led to doubts being cast on the first of these mechanisms proposed by Townsend. As a result, numerous of the other secondary mechanisms have been proposed, to wit: ionization by positive ions at the cathode by Holst and Oosterhuis, Penning,[18] J. J. Thomson[16] and Rogowski,[32] by special photoelectric processes at the cathode for all sparks by Taylor,[23] in the low pressure corona by Werner[33] and Greiner[34] and in the gas and at the cathode by Cravath,[13] by metastables by Penning,[14] and by field currents at low pressure by Flowers.[28]

It is seen that the list of secondary mechanisms suggested above has been pretty well covered. While some of these processes have been invoked in special cases only (corona, etc.), it transpires that for the normal spark breakdown in gases at or near atmospheric pressure, nearly all of the mechanisms mentioned have been called upon. To what extent these are justified it is the purpose of this paper to determine. One may anticipate the answer by saying that as usual where there are so many possible solutions put forward with some degree of plausibility the solution is not the simple one to be hoped for. It is probable that each mechanism occurs to some extent in nearly all sparks, but depending on conditions, one mechanism predominates over the others. Hence the divergent results by different observers. One can say that while three of the processes are generally possible, two of them, ionization by positive ions in a gas and the field currents, occur under such conditions of X/p and p that other more likely phenomena supplant them under any but extraordinary conditions.

V. Modification of Townsend's Theory by Other Secondary Mechanisms

In 1928 R. B. Brode[35] and L. J. Neuman showed that on certain simplifying assumptions

[31] Townsend, reference 30, p. 330.

[32] W. Rogowski, Archiv f. Elektrotechnik **29**, 130 (1935).

[33] S. Werner, Zeits. f. Physik **90**, 384 (1934); **92**, 705 (1934).

[34] E. Greiner, Zeits. f. Physik **81**, 543 (1933).

[35] R. B. Brode and L. J. Neuman presented the equations before a Seminar class at the University of California in the spring of 1928. The results were never published before. One result as modified by Loeb is included here with Professor Brode's consent. It is important to note that Brode concluded from all these processes that the general form of the derived equation remains the same and it is thus impossible experimentally to distinguish one from the other.

one could for the cases of electron liberation by photons in the gas or at the cathode, or by impacts of metastable atoms in the gas or at the cathode arrive at equations closely similar to the two Townsend Eqs. (1) and (2) already cited. The assumptions made neglected processes by which metastable atoms or photons were removed from the gas without producing ionization and assumed the ion production as strictly proportional to the photons or metastable atoms produced. The similarities between the various equations are such that within the accuracy of present day measurements the resulting equations are practically interchangeable by a redefinition of the constants.

Later discussion will show that three of the cases included above, to wit: the action of metastable atoms (limited chiefly to inert gases), and the actions of *photons in the gas* (limited to long gaps and mixed gases at high pressures), are not of sufficient generality in the common sparking phenomena to warrant detailed discussion at this point. The action of *photons at the cathode does, however, merit serious consideration*. Accordingly the equation for this case will be deduced as a general illustration of the statement made with, however, certain amplifications which, as will be seen, do not alter the argument.

Let n_0 electrons be liberated per second at the cathode by an outside agent. These electrons plus the ηz electrons liberated at the cathode by the z photons that are created in the gas by electrons and reach the cathode constitute the total electron emission at the cathode n_0', i.e., $n_0'=n_0+\eta z$. Thus η is the fraction of the photons which produce electrons that succeed in leaving the cathode. dz is the number of photons available to the cathode produced in a distance dx of the gas in the field direction at a distance x from the cathode by electron impact. It is given by $dz=(n_0'+p)g\theta e^{-\mu x}dx$. Here n_0' is the number of electrons leaving the cathode and p is the number of new electrons created by collision in the distance from the cathode to x in the field direction. θ is the number of photons produced by an electron in advancing 1 cm in the gas in the field direction. The quantity μ is the absorption coefficient of the photons in the gas. These are heterogeneous in wave-length and μ therefore represents an average value. The factor g is a geometrical factor. It represents the fraction of the photons created in the gas that can reach the cathode. This is necessitated by the fact that photons are emitted in all directions so that only a fraction can reach the cathode. It is clear that g will be less than one-half. For electrodes whose linear dimensions are small compared to the plate distance it will vary with x and its average value will be less than $\frac{1}{2}$. For the sake of simplicity one can consider the case of infinite plane parallel electrodes in which case g will be 0.5 at the center, if the external ionizing agent acts uniformly over the cathode.

The generation of electrons by electron impact in the gas according to classical theory allows one to write $dp=(n_0'+p)\alpha dx$ in conformity with previous notation, α being the number of ions created per cm path in the field direction by electron impact. If $p=n_0'$ at $x=0$, integration between 0 and x yields $n_0'+p=n_0'e^{\alpha x}$, whence $p=n_0'(e^{\alpha x}-1)$, and $dz=n_0'e^{\alpha x}g\theta e^{-\mu x}dx$. Thus

$$z=(n_0'\theta/\alpha)ge^{(\alpha-\mu)x}+c \quad \text{and} \quad c=-n_0'\theta g/\alpha.$$

Accordingly

$$z=(n_0'\theta g/\alpha)[e^{(\alpha-\mu)x}-1]$$

and

$$n_0'=n_0+n_0'(\alpha\eta\theta g/\alpha)[e^{(\alpha-\mu)x}-1],$$

yielding

$$n=n_0e^{\alpha x}/[1-(\theta\eta g/\alpha)\{e^{(\alpha-\mu)x}-1\}].$$

Here n is the number of electrons reaching x at a steady state, i.e., $n=n_0'+p$. The equation for the number n of electrons set free by the n_0 electrons generated by an external agent at the cathode in a gap of length x is thus in this case

$$n=\alpha n_0e^{\alpha x}/(\alpha-\theta\eta g[e^{(\alpha-\mu)x}-1]). \tag{6}$$

Assuming for the present that $\alpha\gg\mu$ which is true below 10 cm pressure this equation has the form $n=n_0\alpha e^{\alpha x}/[\alpha-B(e^{\alpha x}-1)]$, with $B=\theta\eta g$. This is somewhat similar in form to Townsend's Eq. (1) for ionization by positive ions in the gas which is

$$n=n_0\alpha e^{\alpha x}/[\alpha-\beta(e^{\alpha x})],$$

when $\alpha\gg\beta$. It is closely similar to Eq. (2) for electron liberation at the cathode by positive ions which reads $n=n_0\alpha e^{\alpha x}/(\alpha-\alpha\gamma[e^{\alpha x}-1])$. In fact in regions where constants such as B, β and $\alpha\gamma$ can be determined $e^{\alpha x}\gg 1$, so that the equations are at present experimentally indistinguishable from each other. When x reaches some value δ a spark passes and Townsend assumes that this occurs approximately when the denominators of Eqs. (1), (2) and (6) approach 0. This gives the sparking equation under the three different secondary mechanisms as

$$\alpha/\beta = e^{\alpha\delta} \text{ (for ionization by positive ions in the gas)}, \quad (7)$$

$$1/\gamma = e^{\alpha\delta} \text{ (for electron liberation by positive ion impact at the cathode)}, \quad (8)$$

$$\alpha/B = \alpha/\theta\eta g = e^{(\alpha-\mu)\delta} \text{ (for electron liberation by photons at the cathode)}. \quad (9)$$

Aside from the factor μ which we shall neglect, it is seen that sparking is characterized in each case by a quantity α/β, $1/\gamma$ or $\alpha/\theta\eta g$ becoming equal to $e^{\alpha\delta}$.

Thus the problem breaks itself essentially into two parts, a study of the change of α with field strength and pressure, and the variation of the left-hand terms of (7), (8) and (9) with the same quantities. Now it is well known from Townsend's work that $\alpha/p = f(X/p)$ although the measurements extended over a limited range only. More recently $\alpha/p = f(X/p)$ has been experimentally determined over a long range of values for air and N_2.[8, 36] It probably varies similarly for all other gases at somewhat different ranges of X, p and X/p. Thus while Townsend's data covered a limited range of X/p, in which $f(X/p)$ had only one form, the conditions over the observed sparking range comprise three different regions in which $\alpha/p = f(X/p)$ has three distinct forms.

As regards the left-hand sides of the Eqs. (7), (8) and (9), some comments might be made. Townsend's original measurements enabled one to evaluate the quantity β in Eq. (1) and yielded the information that $\beta/p = F(X/p)$, where $F(X/p)$ for air from $X/p = 130$ to $X/p = 500$ varied possibly exponentially and somewhat faster than $\alpha/p = f(X/p)$. Recently Posin in this laboratory has evaluated β/p from $X/p = 100$ to $X/p = 1000$ in N_2 and finds β/p to be given by the curve of Fig. 2.[37]

[36] Varney, White, Posin and Loeb, Phys. Rev. **48**, 818 (1935); Jodelbauer, Zeits. f. Physik **92**, 116 (1934); D. Q. Posin, Phys. Rev. to be published.

[37] D. Q. Posin, Thesis, University of California 1935. Phys. Rev. to be published.

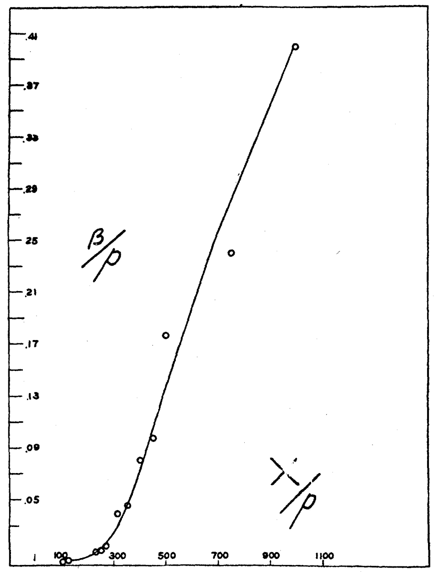

FIG. 2. β/p as $F(X/p)$ from Posin's data.

Since, however, it is known that *positive ions in this region of values of X/p cannot possibly ionize gas molecules by impact* one must attribute the β/p variation in the case of nitrogen as evaluated in Posin's experiments to possibly one or the other of the two alternative mechanisms cited. Thus one may interpret the β observed as approximately giving an $\alpha\gamma$ or a $\theta\eta g$ by writing $1/\gamma = \alpha/\beta$, or $\alpha/\theta\eta g = \alpha/\beta$. Posin has calculated the value of γ from the values of β and α and gives $\beta/\alpha = \gamma$ as a $\phi(X/p)$ as shown in Fig. 3. Neglecting absorption in the photon equation makes $\theta\eta g/p = \beta/p$. Hence experimental data give one a quantity in terms of which the factors γ or $\theta\eta g$ can be discussed.

There is one apparently striking difference between Eq. (8) for positive ion impact on the cathode and the Eq. (9) for photon impact on the cathode. The left-hand term of the equation for γ has no α in it while $\alpha/\theta\eta g$ involves an α. This is due to the fact that the *number of positive ions liberating γ electrons at the cathode is accurately equal to the electrons liberated in the gap. θ,* the number of photons produced per cm path of

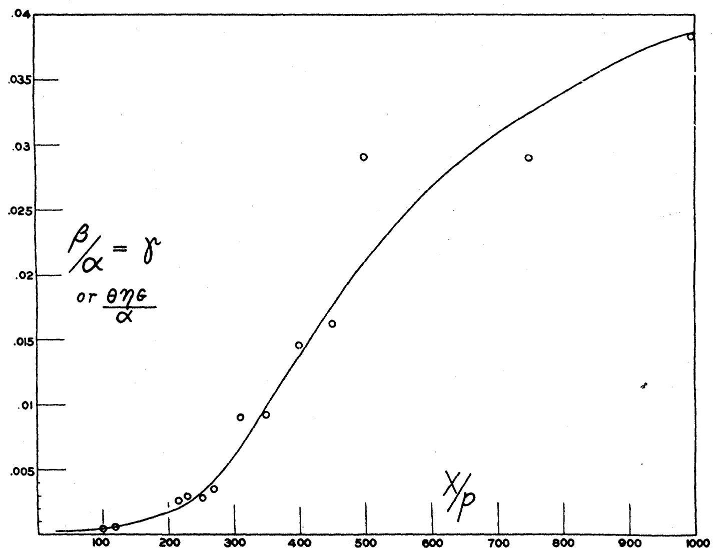

FIG. 3. Posin's β/α for N_2, representing either a γ for positive ion impact at the cathode or an $\eta\theta g/\alpha$ for photoelectric effect.

the electron in the field direction is, however, *not accurately proportional to* α. Cravath claims to have shown that in the range of values of $X/p=40$ to $X/p=100$ the number of photons liberated per electron formed by ionization by collision does not vary by more than a factor of two, while α varies by a factor of 100. Thus one might call $\theta/\alpha=\theta'$ where θ' is a factor of proportionality approximately constant. Then $\theta'\eta g$ would also be represented by the curve of Fig. 3.

Now it is seen that the experimentally evaluated γ or $\theta'\eta g$ increases at first rapidly and then more slowly as X/p increases and its over-all change in the region open to investigation is by a factor of 200. In the case of liberation of electrons by positive ions at the cathode this would mean that in a gas at values of $X/p=100$ to $X/p=1000$, where the positive ions strike the cathode on the average with from 0.5 to 10 volts energy the electron emission increases as observed. The effect of the loss of electrons by diffusion to the cathode at these values of X/p is negligible. These data are not *definitely* inconsistent with what little is so far known about the process of secondary emission at the cathode by positive ion bombardment. In the case of the photoelectric effect at the cathode the increase of $\theta\eta g/p$ with (X/p) at a faster rate than α/p is to be expected. The rough parallelism between α and θ merely means that electrons that are more efficient as ionizers are also in nearly the same proportion better at exciting radiation. At very high energies of the electrons, certainly beyond $X/p=500$ it is likely that the ionization probabilities will exceed the excitation probabilities and this could account for the flattening of the curve for $\theta'\eta g$ as X/p exceeds 500, for θ then increases less rapidly than α. That $\theta'\eta g$ increases at first from $X/p=100$ to $X/p=500$ may be in part due to the fact that θ increases faster than α at very low fields for θ appears at lower electron energies than α.* The initial increase of $\theta'\eta g$ is also possibly due to the fact that at higher electron energies higher states of excitation give harder photons. These will increase η, the liberation fraction at the cathode.

In the previous discussion the quantity μ in the exponent has been neglected. From corona

* Compare the excitation probabilities for electron impact in a gas with the ionization probabilities in the same gas when plotted as functions of the electron energy.

discharge in air Cravath has found μ to be about 2 cm^{-1} for the radiations acting to liberate electrons from metals, and 10 cm^{-1} for those active in the air at atmospheric pressure. Hence it is seen that for low values of α and large x, μ cannot be neglected. Aside from these experiments nothing is known about μ, particularly for the short wave-length radiations most active in liberating electrons. Little can then be concluded about the influence of the neglect of this factor except that in evaluating $\theta\eta g$ from the equation the neglect of the factor μ will make the values of $\theta\eta g$ appear smaller than they should be by the quantity $e^{-\mu x}$. Thus $\theta\eta g$ will appear to vary as the gap length x. Hence did this quantity play a role in Posin's experiments his value of $\beta/p = \theta\eta g/p$ would have been a function of x. This was not the case. In fact Posin found between distances x of 4 and 6 cm in N_2 that β was constant on a curve to within better than 2 percent. This means that μ must have been at most 0.05 or less. This was to be expected as β was determined at about 1 mm pressure such that μ as determined by Cravath would have been 2.6×10^{-3} so that its effect could not have been observed. It is seen then that at present one cannot on theory differentiate between the processes at work. Closely analogous discussion might be based on the equations derived from the action of metastable atoms and *photons in the gas*. However, physical considerations rule these out for the common static sparking phenomena for moderately long gaps 0.1–0.5 cm, and for all but the inert gases at or below atmospheric pressure. Thus in the cases of normal static spark breakdown one may with confidence consider either positive ion impact on the cathode or the photoelectric effect at the cathode the essential mechanism, with possibly a greater likelihood of the photoelectric effect predominating in most cases.

VI. The Effect of Variables on the Sparking Potential in Normal Static Breakdown

A. Paschen's law

With the data on hand for N_2 one may now turn to a study of the application of the equations discussed to the conditions for sparking. In view of the fact that the form of the equations yielding a spark using Townsend's original criterion, and assuming liberation of electrons at the cathode by either positive ion bombardment or photoelectric processes, are closely analogous; that is since one can write (neglecting μ) that a spark occurs at a gap length δ when $1/\gamma = e^{\alpha\delta}$ or when $1/\theta'\eta g = e^{\alpha\delta}$, one can for the circumstances of spark discharge usually studied set the conditions for a spark as $1/\gamma = e^{\alpha\delta}$ *without seriously committing oneself as to whether the quantity γ is caused by positive ions* (a true γ), *or by photoelectric effect*, $\gamma = \theta'\eta g$. In what follows then the equation in terms of γ will be used recognizing that it is *to be interpreted as applicable to either process*. Again it was seen that Posin found that γ as determined by measurement for N_2 varied as some function of X/p where $\phi(X/p)$ is given by the curve of Fig. 3. α/p is also a function of X/p, $\alpha/p = f(X/p)$, Fig. 1. Thus $1/\gamma = e^{\alpha\delta}$ can be written $\log_e 1/\phi(X/p) = p\delta f(X/p)$. Hence $\log 1/\phi(X\delta/p\delta) = p\delta f(X\delta/p\delta)$. Now for uniform fields $X\delta$ equals V_s, the sparking potential. Thus one can write

$$f(V_s/p\delta) = (1/p\delta)\log(1/\phi(V_s/p\delta)). \quad (10)$$

That is to say $V_s = F(p\delta)$. This is known as Paschen's law.[38] It can be shown to hold equally well where fields are nonuniform due to geometrical factors. Where the nonuniformity is caused by a space charge distortion Varney has found by calculations based on his equation[36] that it holds approximately although not accurately. The law has experimentally been verified within the limits of accuracy over a large range of working conditions.

B. The variation of sparking potential with pressure and gap length

Let it be assumed that the criterion for a spark which set $\alpha/\beta = 1/\gamma = e^{\alpha\delta}$ is approximately correct. Then Eq. (10) derived from it on the assumption of essentially linear potential gradients $f(V_s/p\delta) = (1/p\delta)\log(1/\phi(V_s/p\delta))$ should be open to verification once the form of f and ϕ are known. Townsend in his limited range from $X/p = 130$ to $X/p = 500$ for air and N_2 did not

[38] For a discussion of Paschen's law consult any standard work on spark discharge. Townsend, reference 30, p. 327; J. J. Thomson, 3rd edition, reference 16, vol. 2, p. 486; W. O. Schumann, *Durchbruchsfeldstärke von Gasen* (Springer, Berlin, 1923), pp. 51, 114.

derive expressions for f and ϕ. He used curves of his measured values of α/p and β/p plotted against X/p. Taking a given p and a given X he evaluated α and β, and from the expression $\alpha/\beta = e^{(\alpha-\beta)\delta}$ calculated the sparking distance δ. Setting his gap length at δ, at the value of p above chosen, he varied the potential across his plates until a spark occurred at V_s. The values of V_s were in close agreement with the product $X\delta$ computed from the values of X and δ chosen. This excellent confirmation of the theory in a limited region was not applied to the greater portion of the sparking potential curve. As a result of his extended observations of α/p over a range of X/p from about 1000 down to $X/p=20$ and of β/p from 1000 down to $X/p=100$ in N_2, Posin was in a position to test the theory still further. He has shown that from :

$X/p=20$ to $X/p=40$, (region I), $\alpha/p = A_1 e^{B_1 X/p}$ in agreement in form with a similar equation found by Sanders[6] for air. Here $A_1 = 5.76\times10^{-7}$, $B_1 = 0.245$.

$X/p=44$ to $X/p=176$ (region II), $\alpha/p = A_2(X/p - B_2)^2$. $A_2 = 1.17\times10^{-4}$, $B_2 = 32.2$, in analogy to an equation derived from Sanders'[8] curve for air by Jodelbauer.[36]

$X/p=176$ to $X/p=200$ (region III), the form of $f(X/p)$ changes from a relation $f(X/p)\sim(X/p)^2$ to $f(X/p)\sim(X/p)^{\frac{1}{2}}$ passing through a point of inflection with $f(X/p)=A_3(X/p)+B_3$. This region is such that it cannot conveniently be represented by any simple equation.

$X/p=200$ to $X/p=1000$ (region IV), $\alpha/p = (A_4X/p)^{\frac{1}{2}} - B_4$. $A_4 = 0.21$, $B_4 = 3.65$. The extent to which such equations hold is seen in Figs. 4, 5 and 6.

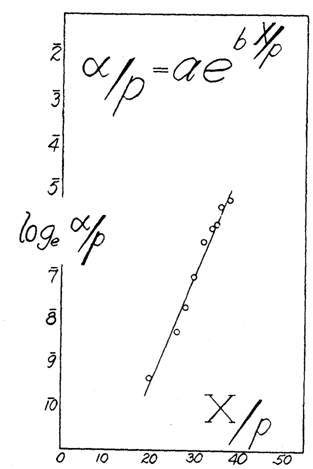

FIG. 4. $\alpha/p = A_1 e^{B_1 X/p}$ in region I at low X/p.

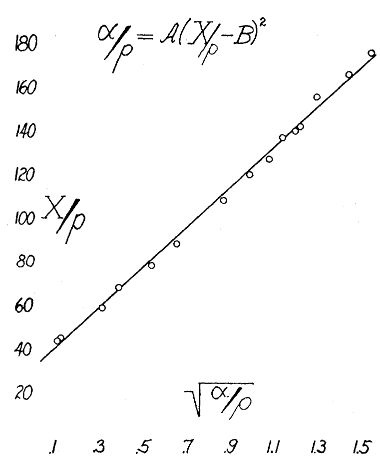

FIG. 5. $\alpha/p = A_2(X/p - B_2)^2$ in region II at intermediate X/p.

In each of these regions equations could be set up to give γ as a $\phi(X/p)$. Placing these equations with their constants into the functional relationship Eq. (10) would at once give equations yielding in their appropriate regimes relations between V_s and $p\delta$. Such relations it can readily be seen become too complex for evaluating V_s from $p\delta$ with any precision except by complicated graphical methods. Fortunately it turns out that except at lower values of $p\delta$ the equations are particularly insensitive to changes in γ. Hence for a survey of this type it is sufficient to set γ at some constant value in each of the regions I, II, and IV and putting in values of $p\delta$ to calculate the value for V_s as a function of p. The values of γ chosen by Posin are as follows, in region I, γ is not known but can be extrapolated for as about 10^{-5}. In region II a mean value of 10^{-3} was chosen for γ. In region IV γ was chosen as 0.02, the values ranging from 0.01 at $X/p=320$ to 0.04 at $X/p=1000$. At $X/p\sim200$ γ was set

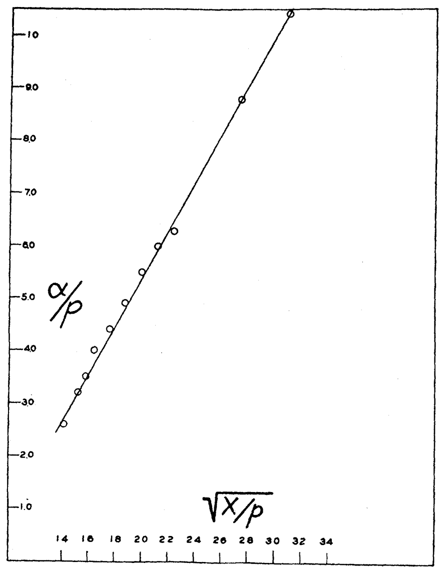

FIG. 6. $\alpha/p=(A_4X/p)^{\frac{1}{2}}-B_4$ in region IV at high X/p.

as 1.5×10^{-3}. Putting in constant values of γ gives the equations used in the following form.

Region I

$$V_s=\frac{p\delta}{B_1}\left[\log\left(\frac{1}{A_1p\delta}\log\frac{1}{\gamma}\right)\right],\quad \gamma=10^{-5}$$

$V_s=69p\delta-4.08p\delta\log p\delta.$ $p\delta$ from 800 to more than 10^4.

Region II

$$V_s=B_2p\delta+\left(\frac{1}{A_2}\log_e\frac{1}{\gamma}\right)^{\frac{1}{2}}(p\delta)^{\frac{1}{2}}.$$

$V_s=32.2p\delta+244(p\delta)^{\frac{1}{2}}$, $\gamma=10^{-3}$, $p\delta$ from 3 to 600.

Region IV

$$V_s=\frac{B_4^2}{A_4}p\delta+\frac{2B_4}{A_4}\log\frac{1}{\gamma}+\frac{\log^2 1/\gamma}{A_4}\frac{1}{p\delta},\quad \gamma=10^{-2}.$$

$V_s=63.5p\delta+73.5/p\delta+137,$ $p\delta$ from 0.4 to 1.2.

It is better to use $\gamma=1.5\times10^{-3}$ near $X/p=200$. This changes the equation to

$$V_s=63.4p\delta+203/p+226,\quad \gamma=1.5\times10^{-3},$$

$p\delta$ from 1.6 to 2.5.

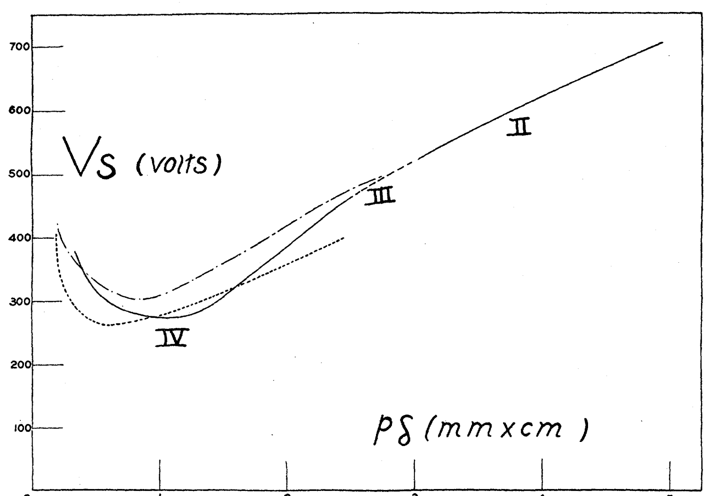

FIG. 7. Computed and observed sparking curves for N_2 in regions II, III, IV. Posin's calculated values—full curve; Strutt's observed values dashed curve; Hurst's observed values dot-dashed curve.

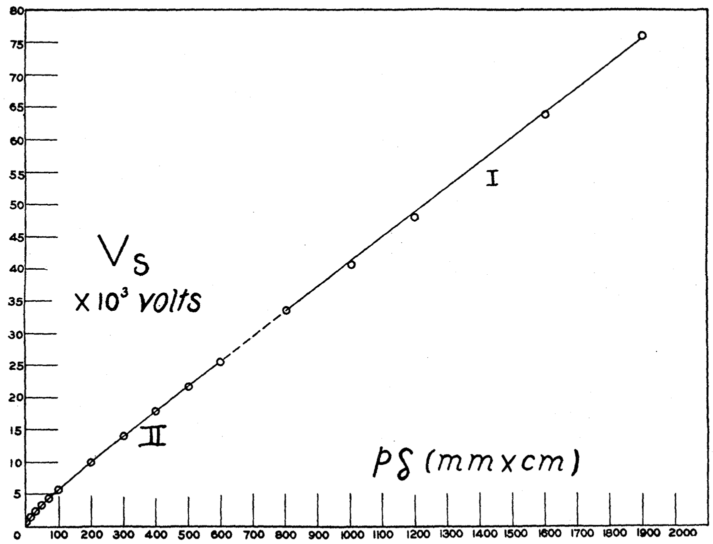

FIG. 8. Posin's computed sparking curve for N_2 in regions I and II. Circled points computed values.

The curves resulting from this test of the theory are shown in Figs. 7 and 8 as full lines. The upper dashed curve in Fig. 7 is Hurst's curve for N_2 and the lower dashed curve is that of Strutt.[39] There are no data for V_s in N_2 for higher values of X/p. It is seen that the computed curve falls fairly well between the two observed curves considering the approximation made. It may be well to point out one or two features of these equations having a bearing on spark discharge observations.

It is seen that in region I the curve of V_s against $p\delta$ is nearly a straight line. This is the relation so often used near atmospheric pressure in rule of thumb calculations. It results from the exponential character of $f(X/p)$ in this region. It is worth noting how relatively unimportant changes in γ are in this region. In region II the departure from linearity is still not very pronounced and V_s is still relatively insensitive to γ. In region III one might expect to find the minimum sparking potential at an X/p where $f(X/p)$ is linear. This is not the case. The relatively rapid change of γ with X/p in this region together with constants in the equation shift the minimum to lower values of $p\delta$. Physically the minimum is due to the fact that at high X/p, α/p increases more slowly than proportional to X/p, while the number of ions formed depends on $p\delta$. Thus larger changes in X/p are needed to increase the efficiency of the ionization process to compensate for decreases in $p\delta$, i.e., decreases in the number of molecules in the gap.

The minimum value of V_s computed from the equation for region IV is 274 volts. This is midway between the values observed by Strutt and Hurst. There is thus no doubt but that with more accurate values of β/p and a more careful solution of the equations one can get agreements between observed and computed values of V_s as good as required as long as there is no field distortion.

C. Effect of cathode material

If the quantity γ be varied by changing the work function of the cathode, the consequences can be viewed as follows.[40] Consider two cathodes

[39] H. E. Hurst, Phil. Mag. **11**, 534 (1900); R. I. Strutt, Phil. Trans. Roy. Soc. **A193**, 377 (1900).

[40] L. B. Loeb, Phys. Rev. **38**, 1891 (1931).

of different material such that for one $\gamma=\gamma_1$ and the other $\gamma=\gamma_2$, and assume that $\gamma_1=a\gamma_2$ where a lies between 1 and 10,

$$f(V_{s1}/p\delta)=\frac{1}{p\delta}\log\frac{1}{\gamma_1},\quad\text{and}\quad f(V_{s2}/p\delta)=\frac{1}{p\delta}\log\frac{1}{\gamma_2}.$$

Since in air and N_2 most measurements are made in region II where $f(X/p)=A_2(X/p-B_2)^2$,

$$V_{s1}=p\delta\left(B_2+\left(\frac{1}{A_2p\delta}\log\frac{1}{\gamma_1}\right)^{\frac{1}{2}}\right),$$

$$V_{s2}=p\delta\left(B_2+\left(\frac{1}{A_2p\delta}\log\frac{1}{\gamma_2}\right)^{\frac{1}{2}}\right).$$

Hence

$$\frac{V_{s1}-V_{s2}}{V_{s1}}=\frac{(1/A_2p\delta)^{\frac{1}{2}}[(\log_e 1/\gamma_1)^{\frac{1}{2}}-(\log_e 1/\gamma_2)^{\frac{1}{2}}]}{B_2+[(1/A_2p\delta)\log 1/\gamma_1]^{\frac{1}{2}}}.$$

At $X/p\sim 120$, $p\sim 1$ mm, $1/\gamma$ is ~ 2000 according to Posin,[37] $A_2\sim 10^{-4}$, $p\delta\sim 5$. In this region $[(1/A_2p\delta)\log 1/\gamma_1]^{\frac{1}{2}}\sim 300$, while $B_2\sim 30$. Thus B_2 can be neglected, giving $(V_{s_1}-V_{s_2})/V_{s_1}$ as 5 percent if $a=2$ and 14 percent if $a=5$. That is on theory one would expect a measurable change in sparking potential with change in γ at these pressures. In argon and inert gases this is found to be true according to Holst and Oosterhuis,[22] James Taylor[23] and L. J. Neuman.[41] Neuman found 20 percent change in A with Ni and Na electrodes at 0.3 mm, and 2 percent change at 10 mm pressure. At atmospheric pressures B_2 in N_2 is of the order of and perhaps ten times greater than $[(1/A_2p\delta)\log 1/\gamma_1]^{\frac{1}{2}}$. Under these circumstances the effect of the value of a becomes so small as to be negligible for most electrodes. Older data at atmospheric pressure[42] indicate complete independence of electrode material. No one has with modern techniques studied the effect over a pressure range in air or a molecular gas. Neuman[41] found in Ar that at 20 mm the difference in V_s for Ni and Na electrodes disappeared. A more careful study in Ar and N_2 for Na and Pt is being carried on in this laboratory. In the region where $f(X/p)=A_1e^{B_1(X/p)}$ the effect of electrode material will be still less while at high X/p where $f(X/p)\sim A_3(X/p)^n$ with $n<1$ the electrode material will be most significant.

[41] L. J. Neuman, Proc. Nat. Acad. Sci. **15**, 259 (1929).
[42] J. J. Thomson, *Conduction of Electricity in Gases*, 3rd edition, vol. 2 (Cambridge), p. 476. Reference 38, p. 20.

D. Influence of illumination or initial ionization

One may recall the equation

$$i=i_0e^{\alpha x}/(1-\gamma(e^{\alpha x}-1))$$

and assume $e^{\alpha x}\gg 1$. Previously the criterion for a spark was set by the condition that $i=.\infty$ as $1/\gamma=.e^{\alpha\delta}$ irrespective of i_0, where δ is the value of x at sparking. Now such a condition is indefinite and one must consider the effect of i_0 on sparking potential. This requires that one set the condition for a spark as a condition in which i reaches some value $\bar{\imath}$, where $\bar{\imath}$ is not defined, and probably cannot be defined in many cases, for sparks are not equilibrium phenomena. It is probable that under a given set of conditions a spark can materialize with a given value of $\bar{\imath}$. Without setting an exact value for $\bar{\imath}$ further conclusions can, however, be drawn. It is now possible to propose the following question. Given two sparks occurring, one with an initial current density i_0 and the other one with i_{00} due to gap illumination, what will be the change in sparking potential V_s in a gap exactly identical, except for the change in photoelectric current density? In other words, how does sparking potential change with gap illumination? If the current is $\bar{\imath}$ for a spark in this gap, then

$$\bar{\imath}=i_0e^{\alpha_0\delta}/(1-\gamma e^{\alpha_0\delta})=i_{00}e^{\alpha_{00}\delta}/(1-\gamma e^{\alpha_{00}\delta}),$$

and as $\bar{\imath}$ is constant for a spark in the gap

$$\frac{i_{00}}{i_0}=\left(\frac{1-\gamma e^{\alpha_{00}\delta}}{1-\gamma e^{\alpha_0\delta}}\right)\frac{e^{\alpha_0\delta}}{e^{\alpha_{00}\delta}},$$

$$\log\frac{i_{00}}{i_0}=\epsilon=\log\left[\frac{1-\gamma e^{\alpha_{00}\delta}}{1-\gamma e^{\alpha_0\delta}}\right]+(\alpha_0-\alpha_{00})\delta.$$

Hence

$$\frac{1}{\delta}\log\frac{i_{00}}{i_0}=\frac{\epsilon}{\delta}=\frac{1}{\delta}\log\frac{1-\gamma e^{\alpha_{00}\delta}}{1-\gamma e^{\alpha_0\delta}}+(\alpha_0-\alpha_{00}).$$

A priori one cannot say much in advance about the value of the term $(1/\delta)\log[(1-\gamma e^{\alpha_{00}\delta})/(1-\gamma e^{\alpha_0\delta})]$ since the values for $1-\gamma e^{\alpha_0\delta}$ are not known for a spark at an undefined $\bar{\imath}$. Let it be assumed for further study that $(1-\gamma e^{\alpha_{00}\delta})/$

$(1-\gamma e^{\alpha_0\delta})$ approximates unity. Then $\log 1=0$ and this equation takes the simple form $\epsilon/\delta=\alpha_0-\alpha_{00}$. If

$$\alpha/p=A_2(V_s/p\delta-B_2)^2,$$

$$\epsilon/A_2p\delta=(V_{s0}/p\delta-B_2)^2-(V_{s00}/p\delta-B_2)^2.$$

Neglecting first order terms in $V_s/p\delta$ one has

$$\epsilon/A_2p\delta=(V_{s0}/p\delta-V_{s00}/p\delta)(V_{s0}/p\delta+V_{s00}/p\delta).$$

Whence approximately

$$(V_{s0}-V_{s00})/V_{s0}=\epsilon p\delta/2A_2(V_{s0})^2.$$

In the past no great change of V_s with i_0 had been observed. Recently H. J. White[43] actually observed a change in V_s with illumination. Nearly simultaneously and independently Rogowski and Wallraff[44] observed a similar change. For this purpose a very high absolute value of illumination about 10^5 times the values commonly used was needed. This is *not indicated by the simple equation* above, for $\log(i_{00}/i_0)=\epsilon$ depends solely on the relative values. The absolute value of i_0 would, however, enter into the more correct equation which includes the $1-\gamma e^{\alpha_{00}\delta}$ terms. White observed that V_s plotted against i gave an hyperbola-like curve, V_s decreasing about 4 percent as i_0 increased by a factor of 10. More significant were the curves of ϵ plotted against $\alpha_0-\alpha_{00}$ taken from the values of V_{s_0} and $V_{s_{00}}$ by means of Sanders'[8] curve for air assuming a uniform gradient. White found that within the limits of accuracy $\alpha_0-\alpha_{00}$ was proportional to ϵ, but that the slope of the line was not quite proportional to $1/\delta$, being 1.7 instead of 2 for one gap and 2.7 instead of 3.0 for another. He ascribed this deviation to space charge formation on the basis of the simplified theory then proposed. While space charge formation could have been the cause, it is essential to note that he was omitting a quantity $(1/\delta)\log[(1-\gamma e^{\alpha_{00}\delta})/(1-\gamma e^{\alpha_0\delta})]$ which is not a constant but can be a very active function of α_0 and α_{00}, and may in part cause the difference. It is seen that unless ϵ/δ is commensurate in value with α_0, no great change can be expected. White observed $V_s\sim 16{,}000$ at $p\delta=380$, $A_2=\sim 10^{-4}$ and $\epsilon=2.3$ to give $(V_{s_0}-V_{s_{00}})/V_{s_0}=0.04$. From the simplified theory above the calculated value of $(V_{s_0}-V_{s_{00}})/V_{s_0}=0.017$.

The answer to the question as to why a lowering is not observed at low values of i_0 and i_{00} does not follow from the simple theory, but can be formulated as follows. Since $\bar{\imath}=i_0e^{\alpha_0\delta}/(1-\gamma e^{\alpha_0\delta})$ with $\bar{\imath}$ fixed, if i_0 is small one must decrease $1-\gamma e^{\alpha_0\delta}$ by increasing $\gamma e^{\alpha_0\delta}$ to give a spark. In regions where this quantity $1-\gamma e^{\alpha_0\delta}$ is very small, minute changes in $\alpha_0\delta$ (and hence of V_{s_0}) are needed to produce excessively large changes in $1-\gamma e^{\alpha_0\delta}$. Hence until i_0 reaches reasonable dimensions, and dimensions not too different from $\bar{\imath}$, so that $1-\gamma e^{\alpha_0\delta}$ can become larger and less sensitive, one will have $e^{\alpha_0\delta}/(1-\gamma e^{\alpha_0\delta})$ so sensitive to slight changes in $a_0\delta$ and hence in X/p as not notably to alter the sparking potential for small changes of i_0. That is changes in i_0 are so readily compensated by minute changes in α_0 and thus V_s as to be experimentally indistinguishable. It is obviously in the region of high i_0 where others had not worked, that White was working, otherwise the quantity $\log(1-\gamma e^{\alpha_{00}\delta})/(1-\gamma e^{\alpha_0\delta})$ could not have been neglected. The value of i_0 estimated by White at which the denominator became important was of the order of 5×10^{-5} amp. per cm^2, while previous workers rarely exceeded 5×10^{-12} amp. per cm^2.

In recent articles Rogowski and Fucks[45] and Fucks[46] have developed a theory of sparking based on positive ion impact on the cathode with a slowly changing γ. To this they add a space charge distortion and assume *contrary to all present knowledge* that the *ion velocity at sparking varies as* $X^{\frac{1}{2}}$ as does the electron velocity. Equations are deduced on these assumptions which make V_s vary as the $i_0^{\frac{1}{2}}$. This variation is observed according to Fucks for *part* of the region studied. The equations derived above for this mechanism yield variations of V_s with i_0 agreeing with observation fully as well as those of Fucks. Actually the data of Rogowski and Wallraff[44] are not complete nor absolute in value. More complete data than they give are needed for a crucial test of any theory.

[43] H. J. White, Phys. Rev. **48**, 113 (1935).
[44] Rogowski and Wallraff, Zeits. f. Physik **97**, 758 (1935).
[45] Rogowski and Fucks, Archiv f. Elektrotechnik **29**, 362 (1935).
[46] W. Fucks, Zeits. f. Physik **98**, 666 (1936).

E. Role of space charge formation

Another question arises for which the simple theory does not give the whole answer. This question involves the role of space charge formation in spark discharge. Actually Townsend's original investigations were conducted under conditions designed to avoid this complication. It is quite clear from the values of α found by Townsend at low p, small x and low intensities of photoelectric illumination, that space charges were not seriously active. Under different conditions the values of α observed by Sanders, Paavola, Masch and Posin[8] showed no indications of this either. Townsend also had estimated from the value of photoelectric current density he used that such currents should not lead to space charge formation. It is clear, however, that as α increases with higher X/p and large x, the multiplication of charges can lead to current densities i which might begin to yield space charges. Hence it was not entirely certain that the up-curving of the log i/i_0 *versus* x curves at appropriate X/p and x which Townsend ascribed to positive ion ionization might not be a space charge effect. The fact that these curves yielded a constant β along their length and that from the values of α and β at high X/p, and relatively low p and δ, Townsend could from the sparking potential V_s, assuming a uniform field, calculate a proper δ, however, confirmed his conviction of absence of space charge. The same applies to the conditions of Posin's[36, 37] measurements in N_2 above $X/p=100$, where special care was used to avoid space charges.

That, however, space charge formation might play a role in sparking at the higher pressures was first emphasized by Loeb and Rogowski[47] independently. These writers in 1928 concluded that if the ionization mechanism of Townsend by positive ions in the gas were to be maintained, it would be essential to assume the existence of space charges at the cathode of sufficient magnitude to cause positive ions to ionize. These space charges were assumed to be produced by positive ion accumulation and movement in the gap. *Since it is now known that positive ions cannot ionize in a gas, this original argument for the necessary existence of space charges is no longer valid.* Similar conclusions were later arrived at independently by Franck and von Hippel[48] and by Schumann.[49] Again certain evidence from the Kerr cell studies of spark breakdown in air in time intervals of 10^{-8} second indicated nonuniform gradients at the cathode in short gaps, and at the cathode, in midgap, and at the anode in longer gaps.[50] Finally, the sparking potentials observed in long sparks such as lightning discharge yielded on the assumption of uniform field strength gradients, fields of the order 3000 to 10,000 volts per cm, which correspond to an X/p far below any for which even an α appears in air at atmospheric pressure. Hence nonuniform fields in such breakdown must be postulated, and these in turn may be space charge conditioned.

Certain results obtained by Sanders, Paavola, and lately by Posin,[8] in which the log i/i_0 *versus* x curves showed upcurvings ascribed to a β, which, however, never yielded values of β which were constant over the upward bent portions of the curves, led Varney, White, Loeb and Posin[36] to set up and solve the complicated differential equation for the effect of space charges on the log i/i_0 *versus* x curves. The results of this study showed that: Depending on the electrode distance, pressure, potential, and intensity of illumination in a plane parallel electrode gap, the appearance of the upcurving of the log i/i_0 *versus* x curves and an ultimate spark breakdown could be caused by *space charge distortion due to primary electron ionization alone.* The possibility of such occurrences due to primary electron ionization alone is determined by the form of the $f(X/p)$ giving the value of α/p. If $f(X/p)$ is of the form $f(X/p)=(X/p)^n$ with $n>1$ the space charge distortion will occur. The higher the exponent n the more likely such a space charge distortion and a spark from this cause. Varney calculated that while with a photoelectric current density of 10^{-12} ampere per cm^2 notable upcurving due to space charges could not at about 2 mm pressure occur in Posin's plate distance of 10 cm, both marked upcurving and an ultimate spark at 8 cm distance could be expected at

[47] L. B. Loeb, J. Frank. Inst. **205**, 305 (1928); W. Rogowski, Archiv f. Elektrotechnik **20**, 99 (1928).

[48] Franck and von Hippel, Zeits. f. Physik **57**, 696 (1929).

[49] W. O. Schumann, Zeits. f. tech. Physik **11**, 58, 132, 195 (1930).

[50] F. G. Dunnington, Phys. Rev. **38**, 1535 (1931); H. J. White, ibid. **46**, 99 (1934).

$X/p=120$ when the current density was 10^{-11} amp. per cm^2. At 10^{-14} amp. per cm^2 there should be no perceptible space charge effect. In fact Posin,[37] using light from the positive column of a quartz Hg arc giving an *average* current density of 10^{-12} amp./cm^2 over an area of some cm^2 of cathode, found a marked distortion of the $\log i/i_0$ *versus* x curves yielding abnormal values of β and leading to a spark at about 6 cm. This is in as good agreement as possible with the theory in view of the impossibility of obtaining a true estimate of Posin's current density at any point. This density could easily have exceeded the *average* density by a factor of 10 or more. At values of the average current density of 10^{-14} amp./cm^2 Posin found that the upturned portions of the $\log i/i_0$ *versus* x curves yielded constant values of β along the curve with no sign of space charge distortion. Thus it appears that while *under some circumstances space charge distortion occurs, it did not play a role in Townsend's experimental domain.*

Further calculations by Varney[51] for $p=760$ and $X=30{,}000$ volts/cm with $X/p=40$ (i.e., about normal sparking conditions in N_2 at atmospheric pressure between plane parallel electrodes) yield the following data for sparking without secondary ionizing effect. If the density of photoelectric current is $i_0=5\times10^{-12}$ amp./cm^2 sparking occurs at $\delta=1$ cm. If $i_0=5\times10^{-15}$ amp./cm^2 sparking occurs at $\delta=1.7$ cm. If $i_0=5\times10^{-17}$ amp./cm^2 the spark occurs at around 2 cm. This means that even without secondary processes near the cathode the spark can occur by electron ionization alone due to the concentration of space charge near the cathode. As actually under the above conditions sparks always occur at 1 cm, and as no variation of sparking potential has been observed for values of i_0 between 10^{-17} amp./cm^2 and 10^{-12} amp./cm^2, it is clear that secondary and not primary processes must liberate electrons in all but the first case. One might even be permitted to speculate that the occurrence of a spark at 10^{-17} amp./cm^2 may be aided by the chance occurrence as a result of some secondary localized process, which in a limited micro area of cathode yields enough electrons to give i_0 a local value of the order of 10^{-12} amp./cm^2. Such a picture would explain the localized discharge streamers always observed. A current density of 10^{-12} amp./cm^2 amounts to 6×10^4 electrons per mm^2 per second or 6 electrons per mm^2 per 10^{-4} second. This is a long period in the time scale of spark breakdown. The spark cannot occur until enough photons or positive ions are liberated in the gap by the primary electron process (α) to make a localized secondary current of this order possible. All that increasing the intensity of illumination can do between 10^{-17} to 10^{-12} amp./cm^2 is to make the formation of the necessary density of ionization more probable and thus decrease the time for the formation to take place. Above 10^{-12} amp./cm^2 an increase in current density will lower the sparking potential by making the ionization effective in shorter gap lengths than 1 cm through space charge formation. This in fact was observed by White[43] and Rogowski and Wallraff[44] to be the case.

The theory makes it possible to calculate the amount of space charge distortion produced at the cathode by the primary electron process for the sparking value of the current density. This is surprisingly low at higher pressures, being but 14 percent in the case above while at $X/p=120$ the value of the field is 50 percent above that for a linear fall of potential taken as V_s/δ. This is interesting in that such distortion at the maximum point will not change X/p by a sufficient amount so as to change the form of the function $\alpha/p=f(X/p)$ from one where $f(X/p)=(X/p)^n$ with $n>1$ to one where $n<1$. The significance of such low space charge values for spark production illustrates the fact that it is not that the field must be materially increased to cause the spark but that *the field near the cathode* must be increased. It is the localization of the increase that is as effective as the increase itself. Whether under actual conditions sparking occurs solely due to the primary mechanism discussed above, becomes essentially an academic question. For it is clear that once such space charge accumulations begin, and probably well before they reach their maximum effectiveness secondary processes (positive ion bombardment at the cathode, photons at the cathode or in the

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[51] Further calculations made subsequent to the publication of reference 29 are here included through the kindness of Dr. Varney.

gas) will occur in sufficient magnitude so as to raise the current density to the sparking value. Thus the *space charge formation and distortion, while probably not the primary cause of breakdown, must and does under circumstances materially aid in preparing the conditions for more effective secondary processes, leading in some cases to lower sparking potentials.*

F. Time lag of sparking

If a sparking potential be applied to a given gap it will be observed that the spark discharge may occur at almost any interval of time after the gap has acquired the sparking potential V_s. The delay may be exceedingly short or very long (10^{-5} second to 30 minutes) provided inadequate ionization exists near the cathode. The time between the occurrence of the spark and the application of the potential is called the spark lag. In recent years this time lag has been the object of considerable study. By using a very short gap with a small volume of effective field strength the lag without artificially applied external ionization may be indefinitely long. Laue and Zuber[52] studied these lags down to about 10^{-3} second at normal breakdown voltage and as a function of the intensity of ionization. The lags were found to be purely statistical in time, that is, if the number of lags n out of n_0 that exceeded t seconds are plotted against t, then the curve of n/n_0 plotted against t is of the form $n=n_0e^{-\beta pt}$, where $\beta p=1/\tau$ and τ is the average time lag. From the plot of log (n_0/n) against t the straight line of slope $1/\tau=\beta p$ is obtained. Now as Laue points out, β represents the chance that an electron is liberated by the ionizing agent. In Zuber's case the *volume* of the gas was ionized by gamma-rays and β depended not only on the number of electrons liberated per unit volume per second but also on the chance that they were liberated in the effective volume of the gap. Thus here β depends on both the ion current and the volume. p is the chance that one electron will start the discharge. It depends on the overvoltage and on the mechanism active. In Zuber's work the range of the investigation was not very extensive. He established the law and showed that τ or βp was dependent on the intensity of illumination.

At the time no lower physical limit to τ was looked for. It was believed that the statistical lag was merely caused by the fact that a single electron had to be liberated at a propitious time and/or spot in the gap to start the discharge, and that $\beta p=1/\tau$ marked the chance that this would occur for a given set of conditions. It was, however, taken for granted that there must be a finite time τ which comprised the actual time to break down the gap (formative lag) once the electrons were produced with sufficient frequency in the gap to get rid of the statistical lag. This formative lag was believed to be very short for it was thought that it corresponded to the time taken for the charges to cross the electrode spacing in the high fields. Already relatively early P. O. Pedersen,[53] using Lichtenberg figures, estimated that this time could be of the order of 10^{-7} second.

When in 1928 Loeb and Rogowski[47] independently attempted to salvage the Townsend theory of secondary ionization by means of positive ions by assuming space charges in the gas, Loeb pointed out that the space charge formation created by the movement of positive ions alone (which was the most powerful one and easiest to invoke) required finite time intervals of the order of 10^{-5} second to develop in a gap of 0.1 cm at 760 mm in air. He stated that a finite limit for the statistical lags of Zuber and Laue should be found in this interval. Within a year the papers of Torok, Beams, Tamm, Rogowski[54] and others had appeared confirming Pedersen's short-time intervals in the spark mechanism. These and the later results of Strigel[55] indicated that these short time lags extended well down to 10^{-8} second but occurred only under conditions of rather high overvoltage (20 percent or more) and strong illumination, and were shorter the higher the overvoltage.

[52] K. Zuber, Ann. d. Physik **76**, 231 (1925); M. v. Laue, ibid. **76**, 261 (1925).

[53] P. O. Pederson, Ann. d. Physik **71**, 371 (1923).

[54] J. W. Beams, J. Frank. Inst. **206**, 809 (1928); J. J. Torok, Trans. A. I. E. E. **47**, 177 (1928); **48**, 46 (1930); Burraway, Archiv f. Elektrotechnik **16**, 14 (1926); R. Tamm, Archiv f. Elektrotechnik **20** (1928); W. Rogowski, Archiv f. Elektrotechnik **20**, 99 (1928); Lawrence and Dunnington, Phys. Rev. **35**, 396 (1930); F. G. Dunnington, ibid. **38**, 1535 (1931); Street and Beams, ibid. **38**, 416 (1931); L. v. Hamos, Ann. d. Physik **7**, 857 (1930).

[55] R. Strigel, Wissenschaft. Veröffent. aus dem Siemens-Konzern **11**, 53 (1932).

In consequence Loeb[56] modified his theory to take account of the spark discharge mechanism in high fields by which high space charges could be built up by single electron movements over short distances, due to the rapid increase in α with X/p. Shortly thereafter von Hippel and Franck[48] put forward their theory of the building up of space charges by *successive* electron avalanches which occur in intervals of 10^{-8} to 10^{-7} second. Similar calculations were made by W. O. Schumann,[49] Sämmer[57] and Kapzov.[58] Whichever picture of the exact mechanism of formation of this space charge caused by electron movement is taken, and doubtless under different conditions the different mechanisms each come into play, it is clear that these give rise to formative lags of a different order from those causing space charge by movement of positive ions, i.e., 10^{-8} in contrast to 10^{-5} second. Since the positive ion mechanism gives more powerful space charges and requires lower values of α one would expect that the formative lags taking 10^{-5} second would occur for low overvoltages. On the writer's suggestion Tilles[59] undertook the problem of studying the shorter lags. To this end he developed an ingenious device for measuring the time lags lying between 10^{-5} and 1 second. The method consisted in measuring with a ballistic galvanometer the constant output current given by a modified vacuum tube voltmeter between the time of application of the sparking potential and its fall during the spark. Two series were run, one with an impulse wave of short duration, another with an approach voltage of 96 percent of V_s and the sudden application of a voltage V_0 which was from 1 to 5 percent greater than V_s. Tilles obtained linear curves for log n_0/n plotted against t as had Zuber at low overvoltages and low intensity of illumination (see Fig. 9, curves a and b). The slopes of these lines varied in a linear fashion with overvoltage and with the logarithm of the intensity of ultraviolet illumination which was varied by a factor of 10^5. For these statistical lags Tilles found that for static breakdown with 96 percent approach voltage one may write for Cu spheres 0.952 cm radius and gap length 0.0683 cm at $V_s = 3820$ volts

$$\tau = 0.0037 e^{-0.39(\Delta V/V_s)100} I^{-0.76}.$$

Here $\Delta V/V_s \times 100$ is the percent overvoltage, ΔV being given by $V_0 - V_s = \Delta V$, V_0 being the applied potential and V_s that of normal sparking. This law holds from 1 to 10^{-4} second in his gap. I is the vacuum photoelectric current density measured in the gap in units of 10^{-12} ampere per cm^2 from a clean Ni plate. This does not actually correspond to the current per cm^2 from the Cu sphere because of the difference in photoemission from Cu and Ni. I is, however, *proportional* at constant pressure and approximately constant field strength to the actual number of photoelectrons liberated in the spark gap, and is proportional to the photoelectric current density.

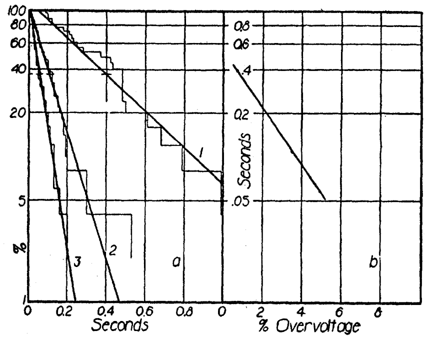

FIG. 9. Tilles' curves of statistical time lags. *a*, spark lag distributions, percent of lags greater than given by abscissa; *b*, average lag *vs.* percent overvoltage. Approach voltage = 96 percent; overvoltage, curve 1 = 0.7 percent; overvoltage curve 2 = 3.5 percent; overvoltage, curve 3 = 4.9 percent.

It is therefore possible to apply Tilles' measurements to the interpretation of the statistical time lags given by Laue. Thus one has

$$1/\tau = \beta p = (1/0.0037) e^{39(\Delta V/V)} I^{3/4}.$$

In these experiments β is directly proportional to I, the current density of photoelectrons. p contains the chance that one electron suffices and is modified by factors which increase this chance. That the overvoltage should affect p critically is clear and that the overvoltage should increase p very rapidly is more or less to be

[56] L. B. Loeb, Science **69**, 509 (1929); J. Frank. Inst. **210**, 15 (1930).
[57] J. J. Sämmer, Zeits. f. Physik **81**, 490 (1933).
[58] N. Kapzov, Physik. Zeits. Sowjetunion **6**, 82 (1934).
[59] A. Tilles, Phys. Rev. **46**, 1015 (1934).

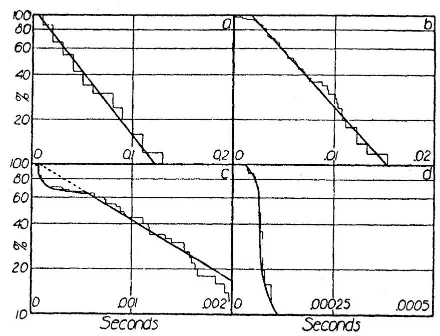

FIG. 10. Tilles' time lag curves. Spark lag distributions for different illuminations; *a*, random at $I=0.00183$; *b*, random at $I=0.0258$; *c*, transitional at $I=0.365$; *d*, peaked at $I=5.75$.

expected since changes in X/p strongly affect α/p and hence secondary electron emission, space charges, etc. The term $p\beta$ is proportional to I if one electron can suffice under proper conditions to start a spark. If two electrons simultaneously were required then $p\beta$ would be expected to vary with I^2, etc. As in theory β is proportional to I, it is clear that since βp is observed to be proportional to $I^{3/4}$, βp must contain in itself other conditions that act apparently *to reduce the effectiveness of increasing ultraviolet illumination.* Actually p would have to contain a factor $(I)^{1/4}$ in itself to give the observed change. Whether this result is spurious or whether the *formation of space charges during the approach voltage period and after* or other effects prolongs the time lag by making some of the electrons liberated less effective cannot be said. With surge impulse breakdown at lower illuminations the value of βp as a $f(I)$ changes to $\beta p \propto I$. In this case one can assume that each electron within the regime designated by the value of $e^{39(\Delta V/V_s)}$ will produce a spark. Under these conditions with weak illumination, it is possible that action of an inhibitory sort does not take place.

As the illumination intensity is increased the photo-current reaches a value such that the curves of log (i_0/i) against t alter their shape in a fashion indicating the existence of a minimum value of τ, in the shorter time intervals (see Fig. 10, *a*, *b*, *c* and *d*). At lower values of I in this regime only a small percentage of the breakdowns show the effect. At $I=5\times10^{-13}$ amp./cm^2, however, 30 percent of the lags are of a fixed length of about 10^{-4} second and 70 percent are statistical. Finally both at 3 and 5 percent overvoltage at about 2×10^{-12} amp./cm^2 the breakdowns are nearly all of one time τ, i.e., 10^{-4} second; that is, one goes from a statistical lag of decreasing τ to a formative lag of 10^{-4} second for the gap used, with sufficiently intense photoelectric emission, *overvoltage being constant* (see Fig. 11). In this region of time lag as overvoltages increase the formative lag at τ remains the lag, but decreases in value as with increasing V_s the electron avalanches produce the necessary ionization density much more effectively. Thus fewer electron avalanches are needed to produce the same gradient and τ decreases. Tilles' lags of 10^{-4} second are not determined by the time taken for positive ions to cross the gap. The time of crossing is much shorter than 10^{-4} second. These time lags must represent the time taken for a series of events possibly conditioned by the very short gaps used. White's[60] results in

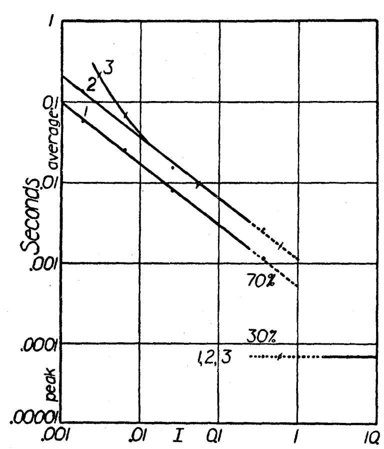

FIG. 11. Summary of Tilles' results. Master curve of average or peak values. Time lag of sparkover *vs.* illumination

Curve	Approach Voltage	Voltage
1	96%	105%
2	96%	103%
3	0%	105%

[60] H. J. White, Phys. Rev. **49**, 507 (1936).

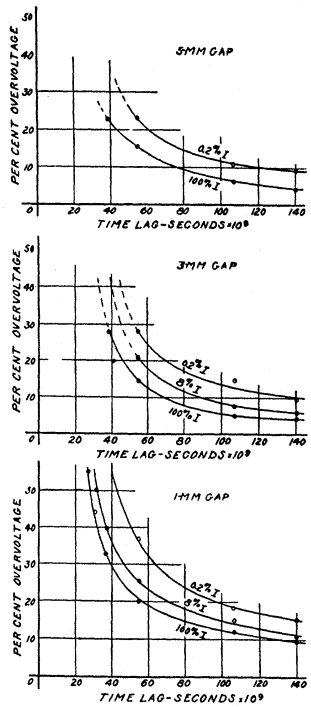

FIG. 12. White's time lag curves for air. Overvoltage against time lag for three gaps.

air in intervals of 6×10^{-8} sec. show that the lags at the same overvoltage are materially longer for a 1 mm gap than for a 5 mm gap, Fig. 12.

It is conceivable that with increased overvoltage the *electron movement* either in one, or in several successive avalanches following at short time intervals, will produce a space charge which is *not conditioned by ion movement,* and whose minimum value will depend on the time of electron crossing. One mode of such short-time interval space charge production has been calculated by Franck and von Hippel.[48, 49] It leads to lags of the order of several times 10^{-8} second. Quantitative measurements in this region were begun by H. J. White.[60] White used light from a spark to set off the gap to be studied and by means of the Kerr cell shutter observed the time between the flash of the initiating discharge and the breakdown of his experimental gap. Time lags of a nonstatistical sort were observed as a function of gap length, intensity of illumination and overvoltage. As White worked with very high illumination intensities he found that only small overvoltages of some 14 to 15 percent were needed to cause formative lags of the order of several times 10^{-8} to 10^{-7} second. The higher the intensity the lower the overvoltage required. Near 5×10^{-8} second the overvoltages rapidly increased (see Fig. 12). Calculations made by White indicated this time to be that of the order of an electron crossing the gap. White, however, pointed out that the same time intervals were of the order of time taken for the *rapid initial rise of intensity of light emission in the initiating spark source.* Hence the upward bend of overvoltage *versus* time lag curves at about 5×10^{-8} second had to be ascribed to a rapidly decreasing intensity of photoelectric illumination. Helium, unlike air, and CO_2 showed a very much higher set of overvoltages (about 30 percent) for the whole range. This is to be expected because of the low potentials at which helium yields appreciable values of α. At such potentials α is low and the ionization process is inefficient so that many impacts with atoms are needed to give much multiplication of ions. In short time intervals this requires greatly increased values of V_s in a given gap.

Very recently R. R. Wilson[61] has carried White's experiments further, using a steady, but only 10^{-5} times as strong a source of illumination (quartz Hg arc). Wilson used an approach voltage about equal to the normal sparking potential and suddenly applied an overvoltage. He raised the overvoltage applied in successive steps until a spark could be seen in his optical system, with a given time setting. He found that as overvoltage increased at first an occasional spark was observed when a given overvoltage was applied. He then increased the overvoltage by steps and recorded the percentage of sparks observed when the same overvoltage was repeatedly applied. He plotted this percentage of sparks as a function of overvoltage as seen in Fig. 13B. From these curves he chose

[61] R. R. Wilson, Phys. Rev. **49**, 210 (1936).

as his overvoltage for a spark in the time interval used, the point at which 50 percent of the overvoltage applications gave a spark. A series of such curves is shown in Fig. 13B and indicates why this criterion for the value of the overvoltage at a given time was resorted to. He found that corroded electrodes or a decrease in the intensity of illumination produced a decrease in the slope of the curves as shown in Fig. 13C and thus increased the overvoltage chosen for a given time lag. He found that materially higher overvoltages were required with his weak illumination than were observed by White for the same time lags. Two of his curves for air are given in Fig. 13A with weak and strong illumination. It is seen that Wilson's overvoltages decrease continually as the lag increases. At about 1×10^{-8} second his overvoltage curves rise relatively steeply. He found, however, *no lower limit to his lags even down to 10^{-9} second if sufficient overvoltage was applied*. For such short time measurements a vacuum switch was required for applying the potential. Wilson also studied breakdown, using a surge impulse with no approach voltage. He points out that with surge voltages and even with the application of high overvoltages above an approach voltage, the overvoltage values taken from the applied potential lose much significance. This follows since in the case of breakdowns occurring in 10^{-9} second it is possible that the actual voltage applied, owing to the reflection of the surge at the gap electrodes, may be materially higher than the static overvoltage applied. An indication that this occurs in the present work is seen in the actual observations of breakdowns which take place by the sudden application of surge potentials *under* the normal breakdown voltage of the gap for the time setting. Beams[62] observed similar difficulties in his work. Thus while the overvoltage observed by Wilson in surge impulse breakdown was only some 20 percent higher than that observed with the approach voltage, it may in reality have been much higher. Wilson also points out that the use of an approach voltage alters the gap

[62] J. W. Beams, J. Frank. Inst. **206**, 809 (1926).

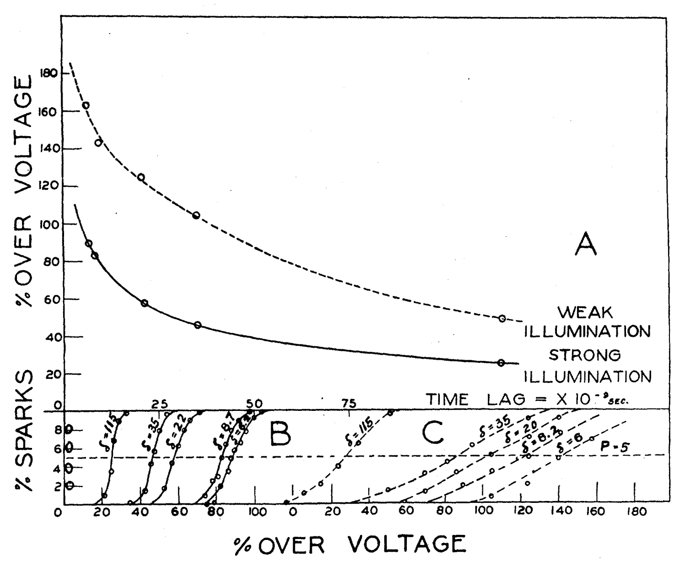

FIG. 13. *A*, Wilson's curves for time lag as a function of overvoltage for strong and weak illumination. *B*, *C*, curves of percentage of sparks against overvoltage for strong and weak illumination.

in that it builds up a strong gas-intensified photocurrent before the overvoltage is applied. The high electron densities produced by this are, however, all near the anode. With a photoelectric emission of 10^{-12} amp./cm^2 the currents achieved amount to some 10^{-8} amp./cm^2. In the area of 5 mm^2 where sparking occurs the current is 10^{-9} ampere, giving an equivalent of 10 electrons created in the gap in the 10^{-9} second of observed breakdown. What the influence on breakdown of such electron production in the gap preceding breakdown will be is undetermined. It is possible that such electron distributions in a gap before the breakdown may produce the anode streamers observed in Kerr cell studies.[50]

It must also be pointed out that in measurements of time lags by the procedures outlined the *definition of time lag varies*. In Wilson's work the *lag* is that between the application of the overvoltage and the appearance of a light emission in part of the spark gap. In White's work the lag is the time between the illumination of the gap by the auxiliary spark and the first appearance of light emission. In Tilles' work the lag was the time as measured by the linear charging of a condenser from the instant of application of the potential until the spark had so lowered the potential that the condenser ceased charging. It is essential that such differences in interpretation be kept in mind in the discussion of spark lag studies.

In the experiments of White and Wilson the lags are so short that *neither positive ions nor electrons could possibly have crossed the gap in the intervals used*. This is again borne out by a calculation of H. J. White.[60] White showed that if at his observed overvoltage an electron crossed the 5 mm gap used, the number of positive ions left behind by the one electron would be so great that the positive space charge would have produced field distortions preventing the electrons from ever reaching the anode. This condition in the time intervals investigated would lead exactly to the types of field distortion causing Dunnington's mid-gap streamers, which were also in evidence in White's and Wilson's gaps. Thus it is clear that there is no lower limit to the formative lags as regards the crossing time for electrons. This agrees with the mechanism postulated by Loeb.[56] At lower overvoltages it is probable that the electrons do cross the gaps giving other durations of time lags. There are indications in the longer formative lags due to the movement of the mid gap streamer with overvoltage that the successive electron avalanches of von Hippel and Franck[48] and Schumann[49] also occur. It is further possible that time lags can be conditioned on the movement of positive ions. Such lags have so far not been observed. Tilles' lags of 10^{-4} second cannot be interpreted in this fashion. More work is required in the region of longer lags extending from 10^{-7} to 10^{-4} second.

VII. The Occurrence of the Different Secondary Sparking Mechanisms

A. High vacuum sparks and field emission

According to classical sparking theory, when the product $p\delta$ becomes very small in a gap the sparking potential rises sharply. At pressures much below 10^{-5} mm with reasonably small gaps it should be virtually impossible to produce a spark breakdown owing to the absence of any molecules in the gap which can give ions. Actually such sparks have been well known for years. In the period 1923–25 several investigators arrived at an explanation of the phenomena on the basis of field current emission.[27] That is, electrons are emitted from the metals over the potential barrier in the presence of suitably high applied fields. The magnitude of these fields for most ordinary electrode materials as calculated from the contour of the cathode and the potential must be of the order of some 10^6 volts per cm, giving local fields at points or irregularities of 10^7 or more volts per cm. Under these conditions electrons can leak out over the potential barrier, and carry the current. The theoretical treatment of the problem by the Sommerfeld electron theory of metals has yielded equations in satisfactory agreement with experiment.[27] If the field current becomes great enough so that the minute point is heated, it vaporizes and a regular spark ensues through the medium of gases and metal vapors liberated from this point. Even at somewhat lower fields by increasing the temperature the thermionic threshold can be lowered by the field so that for heated electrodes the resulting currents can lead to sparks at appropriate fields, by vaporizing points.

While positive ion emission from metals has been observed at high temperatures, there is *no record to date of any field emission for positive ions* either of the regular type or one where thermionic emission is assisted by high fields.

Recent results of Flowers[28] on the sparking potentials in unsymmetrical gaps at relatively low pressures, which gaps are swept free of ions by an auxiliary field, have inclined him to interpret the sparks as due to a field emission. It is true that such an hypothesis would explain the peculiar independence of gas pressure and character observed by Flowers. On the other hand, the phenomenon is observed to occur whether the smaller electrode is positive or negative, and when the fields at the larger negative electrode could not under any circumstances be such as to give a field emission. It is clear that under such conditions other mechanisms should be scrutinized in seeking an explanation for the results before resorting to an explanation on the basis of a positive field emission. However, it must be realized that under any circumstances where the fields at local points can reach field emission values, *one may look for field emission at sufficiently low pressures.* However, there are little data at hand on the conditions of sparking at extremely low pressures, and the mechanisms marking the transition from field emission sparks to low pressure gaseous sparks have never been investigated.

B. Sparks at low pressures including corona discharge

1. Low pressure sparks. At low pressures and moderate gap lengths it is quite clear from the previous discussions that space charges play little role in the phenomenon. Unless X/p is such that the ions gain more than about 50 volts over a free path, in most gases ionization by positive ions in the gas cannot occur. Except in inert gases the metastable atoms cannot be seriously considered as giving the secondary mechanism. This leaves one with but two agents, to wit: positive ion impact on the cathode and photoelectric processes. With the absorption coefficient of 10 cm^{-1} found by Cravath for radiations photoelectrically active in air at atmospheric pressure, it is clear that the action of photons in a gas below 1 mm pressure can be neglected as a secondary agent. Thus there remain only the action of photons or positive ions at the cathode. This fact is clearly indicated by the effect of the work function of the cathode material on sparking potentials, as well as the peculiar polarization effects often observed as the result of discharges on electrodes at low pressures. Whether the photon or positive ion action at the cathode predominates cannot be definitely answered, although the discussion at the end of part 5 appears to favor the photons. In one case of low pressure discharge it is possible to gain a definite picture of the mechanism. This occurs in the case of the relatively low pressure corona discharge with dissimilar electrodes.

2. Low pressure corona. In the case of the low pressure corona discharge with the positive wire and negative cylinder the action of *photons* at the cathode has been shown to be the predominating action.[33, 34] The large cathode surface and small cathode field strength militate in favor of the photons relative to the positive ion impact as indicated by Penning.[9] The photons have been shown to be effective by Greiner.[34] Werner[33] has shown that only 1 out of 3 photoelectrons produced at the cathode in vacuum escape in the presence of gas of some 16 mm pressure. Space charges create less than 1 percent of the total field according to Werner, even in corona.[33] In the case of the negative wire and positive cylinder the whole picture changes. The fine wire has so little surface that the photons are mostly absorbed by the walls, or escape from the cylinder. The positive ions are drawn to the wire in a high field. The electrons are liberated by positive ions at the wire. The beaded appearance of the discharge indicates points of low work function at which the discharge concentrates. The negative corona is more disruptive, it extinguishes easily.[9] This indicates instability and that space charges *may* be active. Space charges will aid the secondary liberation of electrons as they aid in escape of electrons, give a greater field on the positive ions, and concentrate the events near the cathode.

In the corona breakdown studies the low

pressures and shorter gap lengths easily lead to the complications produced by the product $p\delta$ becoming too small, and this becomes particularly troublesome in the case of the inert gases where V_s is low and many collisions are required to give adequate ions.[63] Werner[33] estimates that $p\delta$ must allow about 70 collisions in H_2 and 500 in He. In the case of the corona with the positive wire, it appears that the value of ϕ, the work function of the cathode, makes little difference in V_s. This may in part be due to the enormous efficiency of the large cathode surface. It is also in part due to the relatively higher pressures in corona discharges of the type studied (16 mm) compared to those where electrode dependence with parallel plates was observed. Under these conditions, as was seen, the cathode material *appears* to exert relatively little influence. What the effect of the photoelectrically inactive metals Al and Mg as electrodes with a positive wire under carefully controlled conditions will be is not known.

C. The influence of metastable atoms

The influence of metastable atoms is most beautifully illustrated in the case of the low pressure corona.[9] If inert gases such as Ne or He are used *without any impurity* (i.e., Hg or Ar for which the excitation potential of the metastable state of the inert gas is greater than the ionizing potential of the impurity) the starting potentials V_s are high. The starting potential V_s for the corona discharge with positive wire is greater than V_s for the starting potential with a negative wire. Traces of the impurities (0.002 percent of Ar) immediately lower V_s for both cases but lower V_s for the positive wire much more. With just the correct amounts of impurity they make V_s with wire positive lower than V_s with wire negative. This neglected circumstance has falsified the results of many investigators as Penning has shown. The action depends on the long life of the metastable atoms which are created in the high field near the wire and *in general diffuse away from it*, carrying their ionizing power out into the gas. In the case of the positive wire the metastable atoms diffuse to the cathode cylinder or to the effective gas volume near the cathode in which electrons liberated can be of great value to the discharge mechanism. The action is most effective in the gas at a distance from the cathode because here the metastable atoms liberate electrons in the gas where the loss by diffusion to the cathode is small. In the case of the negative wire it is only the few metastable atoms that diffuse into the small volume near the wire from the regions further out in which they were created, that can materially aid in the discharge. For the intense field near the wire is the only region where electrons created by metastable atoms can be of value to the discharge. The number of metastable atoms effective in this region from the nature of the diffusion process and the geometry of the system are relatively few.

It is probable that most of the early studies of sparking conditions in inert gases both at low and high pressures should be repeated, using precautions such as indicated by Penning[9, 14] in order to eliminate the effects of metastable atoms on impurities of lower ionizing potential. The possibility of actions of this sort was not recognized at the time and inadequate precautions were taken to avoid impurities such as Ar in Ne and He or Hg in any of these gases.

D. Summary of the sparking mechanism under ordinary conditions

Granting the primary process of ionization by electron impact as presented, with $\alpha/p=f(X/p)$, and that to produce a breakdown enough electrons must be created near the cathode to make the discharge self-sustaining, one can conclude that a general equation of the *type* of Townsend's equation will lead to an evaluation of the sparking potential. This applies especially to conditions in which $X=V/\delta$, i.e., where no space charge distortion occurs. In it Paschen's law is strictly obeyed and $V_s=F(p\delta)$ depends on the form of $f(X/p)$ and $\phi(X/p)$ for electrons and secondary processes, although primarily on the former. As to the precise character of the secondary process mechanism the equation of the Townsend type cannot yield information by quantitative comparison with experiment, as the data will probably always be too inaccurate to discriminate between the various hypotheses. At pressures up to 100 mm and X/p greater than 40, the type of equation developed by

[63] J. S. Townsend, Phil. Mag. **28**, 83 (1914).

Townsend is found to hold experimentally for plane parallel electrodes. While the secondary process mechanism is not revealed by the equations definite information is at hand concerning what processes can and cannot occur under given circumstances. The following conclusions may be stated.

The mechanism for the primary ionization is paramount and ever present. The ionization *in the gas by positive ion impact cannot and does not enter into the sparking equations at any time.*[8] It is unlikely that field currents play a role except in extreme vacua.[28, 29] This leaves one with liberation of electrons at the cathode by positive ion impact, with photoelectric effects at the cathode, with photoelectric effects in the gas and with the effect of metastable atoms in the gas or at the cathode. There is in addition a chance that with sufficient initial ionization space-charge conditioned field distortion can lead through the primary process directly to a spark.[36] Which of these processes occurs is obviously determined by the particular conditions imposed by the experimental arrangements.

In the absence of inert gases and up to pressures of about 100 mm especially in pure gases, the action of metastable atoms and photoelectric processes *in* the gas are ruled out. Thus as at lower pressures the principal secondary mechanisms are positive ion impact at the cathode and photoelectric processes at the cathode. In this region, however, *space charges can begin* to play a role.

E. Sparking in long gaps and at high pressures

When the pressures approach atmospheric and the gap length increases to the order of several millimeters both the possibility of photoionization in the gas for any but the purest gases, and the influence of space charges must be considered. This is indicated by the following facts. At gap lengths of several mm at atmospheric pressure, and definitely at lower pressures with *adequate illumination*, the breakdown is preceded by a space charge distortion,[36] which as indicated, enhances materially the processes at the cathode and hence lowers the value of the sparking potential. The presence of distortion has been definitely verified by time lag studies,[59, 60, 24] by studies of the Townsend coefficients,[8, 37] and by theoretical considerations,[36, 48, 49, 57] as well as by observation of suppressed discharges[64] and discharges studied visually or photographically in short time intervals[50] including Lichtenberg figures[53] and recent cloud track pictures.[65, 66] How important space charge is under the conditions where it first appears is not known. That it is possible that at high pressures and long gap lengths space charge distortion can cause a spark on the basis of the primary mechanism appears to have been demonstrated mathematically.[36] Whether such a spark occurs or not is aside from the point, for accompanying any large space charge formation by a primary process is the presence of considerable ultraviolet radiation and positive ion production. Under these conditions the secondary process will be furnished by the photoelectric ionization of the gas or secondary emission from the cathode.

Since it is probable that the *photoelectric process in the gas* will occur at higher pressures with and probably before one reaches the field distortions needed to produce a spark on the basis of an electron ionization alone, it is obvious that the photoionization in the gas will account for certain discharges whose explanation has hitherto given trouble. It must be clear that in the static discharges of ultra-long gaps, such as lightning discharges, *photoionization or positive ion impact at the cathode cannot be invoked.* The same argument applies in the case of discharges from isolated positive conductors (needle points, wires and the high pressure corona[67]) for the fields at the cathode are exceedingly weak and the cathodes are too far distant to be involved. Hence *the regions in which the primary processes initiate must have the power of not only originating the high potential gradients locally needed for giving the discharge. They must also provide for the mechanism by which electron supplies are maintained and by which these gradients travel as they do in lightning strokes.* Such a mechanism in the absence of ionization by positive ions *can only occur with photoelectric processes in the gas.* That

[64] J. J. Torok, Trans. A. I. E. E. **13**, Feb. 1928.
[65] Nakaya and Yamasaki, Proc. Roy. Soc. **A148**, 466 (1935); H. Kraemer, Archiv f. Elektrotechnik **28**, 703 (1934); Bradley and Snoddy, Phys. Rev. **47**, 541 (1935).
[66] Flagler and Raether, Zeits. f. Physik **99**, 635 (1936).
[67] W. G. Hoover, Elec. Eng. **55**, 448 (1936).

such space charge distortions must occur is indicated by the fact that the *average* field strengths actually calculated in the case of positive point or wire corona discharges to planes at atmospheric pressure, and in the lightning discharges are exceedingly low (e.g., 3000 volts per cm for lightning discharges and 4,000 volts per cm for the positive point corona with a luminous path from point to plane at atmospheric pressure). These fields are 1/5 to 1/4 the fields which are required to give α a measurable value at atmospheric pressure. Hence unless space charge distortion occurs the mechanisms concerned cannot give rise to sparks. Furthermore in some cases these distortions must have the power of propagating themselves in order to bridge the gap. That the positive space charge dendrites emanating from the cathode can act as needle point electrodes in propagating the discharge has been emphasized by von Hippel.[68] How such channels propagate at the enormous speeds observed by Schonland and Collens[69] in lightning discharge and by Flagler and Raether[66] in sparks has been shown by Cravath and Loeb[70] on the basis of the ionization of the gas by ultraviolet radiation which produces ions in the distorted field ahead of the point.

[68] A. von Hippel, Naturwiss. **22**, 47 (1934).

[69] Schonland and Collens, Proc. Roy. Soc. **A143**, 654 (1934); **A152**, 595 (1935); Roy. Soc. South Africa, Meeting Oct. 16, 1935.

[70] Loeb and Cravath, Physics **6**, 125 (1935).

VIII. Conclusion

In conclusion one may say that while in detail there is a great deal to be learned about the individual processes of spark breakdown in many of the cases observed, in general the nature of the secondary mechanisms active in most sparks are pretty well understood. The essential conclusion to be drawn from this analysis is, however, that *there is no single definite secondary process which occurs universally in all discharge phenomena* as some would like to believe. There are at least five and, including very low pressures, six mechanisms by which the self-sustaining character of spark discharge can occur. Depending on the circumstances, any one or two in combination predominate to the exclusion of the others. In many cases the circumstances indicate at once the favored mechanism.

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