Colloquium: Illuminating the Kapitza-Dirac effect with electron matter optics

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The observation of the Kapitza-Dirac effect raises conceptual, theoretical, and experimental questions. The Kapitza-Dirac effect is often described as diffraction of free electrons from a standing wave of light or stimulated Compton scattering. However, for the two-color Kapitza-Dirac effect these two interpretations appear to lead to paradoxical conclusions. The discussion of this paradox deepens our understanding of both of these versions of the Kapitza-Dirac effect.

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I. INTRODUCTION

de Broglie's prediction that matter, just as light, propagates as a wave was verified by Davisson and Germer (1927), when they observed electron diffraction from the periodic structure of a crystal lattice. After this observation it only took a few years for Kapitza and Dirac (1933) to propose that instead of using a material crystal one could also use the regular structure of a standing wave of light to diffract electrons (Fig. 1). (Kapitza and Dirac later both received the Nobel prize for unrelated work.)

The interaction between free electrons and photons is very weak. Kapitza and Dirac estimated that with the strongest readily available light source at the time, a mercury arc lamp, the relative strength of the deflected electrons relative to the undeflected electrons would be 10^{-14}. For this reason it is clear that attempts to measure the Kapitza-Dirac (KD) effect had to wait for the development of the laser.

After the development of the laser multiple attempts were made in the 1960s to observe the KD effect (Schwartz *et al.*, 1965; Bartell *et al.*, 1968; Takeda and Matsui, 1968; Pfeiffer, 1968). Reviews of these attempts including reasons why no effect was observed were given by Schwarz (1973) and Fedorov (1974, 1991). In the 1980s use was made of the fact that the intensity of light needed for near-resonant interaction with atoms is less by approximately nine orders of magnitude.

Pritchard's group at MIT showed that atoms could be diffracted with an off-resonant standing wave of light, which is often referred to as the KD effect (Gould *et al.*, 1986). Nowadays the KD effect is loosely defined as the "diffraction of a particle by a standing wave" (Photonics dictionary, 1996–2006). After Pritchard's observation in the diffractive regime, the atomic KD effect has been observed in the Bragg regime (Martin *et al.*, 1988), with slow atoms (Kunze *et al.*, 1997), and even with Bose-Einstein condensates (Ovchinnikov *et al.*, 1999). For the atomic KD effect the low requirements on the intensities of light open many possibilities: few-photon interactions (Domokos *et al.*, 1996), bichromatic light (Grimm *et al.*, 1994), and the use of evanescent waves (Hajnal and Opat, 1989) to name a few. In recent years the observation of molecular diffraction is one of the exciting new developments (Nairz *et al.*, 2001).

Bucksbaum *et al.* (1988) demonstrated a high-laser-intensity interaction with electrons, which appears to be the first observation of free electrons interacting with light. Diffraction was not observed, but the classical motion of electrons in the standing light wave was. Finally in 2001, more than 60 years after it was first proposed,

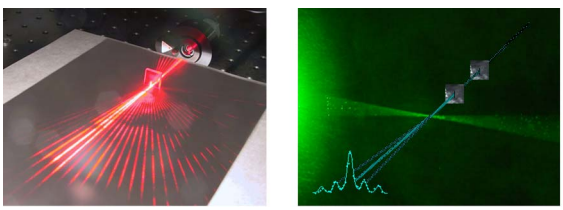

FIG. 1. (Color online) Matter-optics analogy. Diffraction of a light wave by a material grating is shown (left). A schematic of an electron matter wave diffracted by a laser beam, illustrating that the roles of the matter and light are reversed for the KD effect (right).

diffraction of electrons by a standing wave of light was observed (Freimund *et al.*, 2001, 2002).

The KD effect may serve as a testing ground for nonperturbative treatments in QED (Burnett *et al.*, 1993; Rosenberg, 1994; Guo, 1996; Li *et al.*, 2004). These nonperturbative approaches lead to new predictions (Radford, 2002). For an interesting review that considers the effect of increasing laser intensities, see Bucksbaum (1990). In extreme electromagnetic fields (Dietz and Probsting, 1998; Kao *et al.*, 2000; Tomaras *et al.*, 2001) even pair creation is predicted.

The KD effect is extensively used in atom optics. The effect serves as a coherent beam splitter. As such it has been used to construct matter interferometers for atoms (Giltner *et al.*, 1995; Rasel *et al.*, 1995). It is an open question whether or not an electron interferometer can be built using the KD effect (Batelaan, 2000). The advantage of using the KD effect for electrons to construct an interferometer is that there is no need for materials to be near the electrons, which would open the possibility of electron interferometry in the low-energy ranges typical for atomic physics (Forrey *et al.*, 1999).

In this paper some basic theoretical considerations and the experimental approach for the KD effect for electrons will be described. We discuss the KD effect from both the matter-wave and particle points of view and distinguish the Bragg from the diffractive regime. Fully quantum descriptions will be provided, but the discussion is first focused on the ponderomotive potential and on some intuitive qualitative pictures of the KD effect. We also discuss a modification of the KD effect that we hope deepens our understanding of the KD effect and indicates some new directions of work.

II. THEORETICAL DESCRIPTION OF THE KD EFFECT

A. The ponderomotive potential

To understand why an electron experiences a time-averaged potential in an oscillatory field it is sufficient to consider the classical motion of a charged particle placed in a standing wave of light. The electric and magnetic fields in a standing light wave are solutions to the Helmholtz equation and may be expressed by the vector potential $A_z = A_0 \cos kx \sin \omega t$ as follows:

$$E_z = -\frac{\partial}{\partial t} A_z = -A_0 \omega \cos kx \cos \omega t,$$
$$B_y = -\frac{\partial}{\partial x} A_z = A_0 k \sin kx \sin \omega t. \tag{1}$$

The resultant electric field oscillates $\pi/2$ out of phase with the magnetic field in both space and time.

The electric and magnetic fields are shown in Fig. 2. An electron that is placed in a standing wave of light, halfway between a maximum and a zero crossing in the electric field ($x = x_1$), will be accelerated along the z axis as shown in Fig. 2. The velocity acquired by the electron will lead to a Lorentz force $F_x = -ev_z B_y$ parallel to the light propagation direction. The oscillating electron velocity will lag behind by a phase of $\pi/2$ with respect to the electric field,

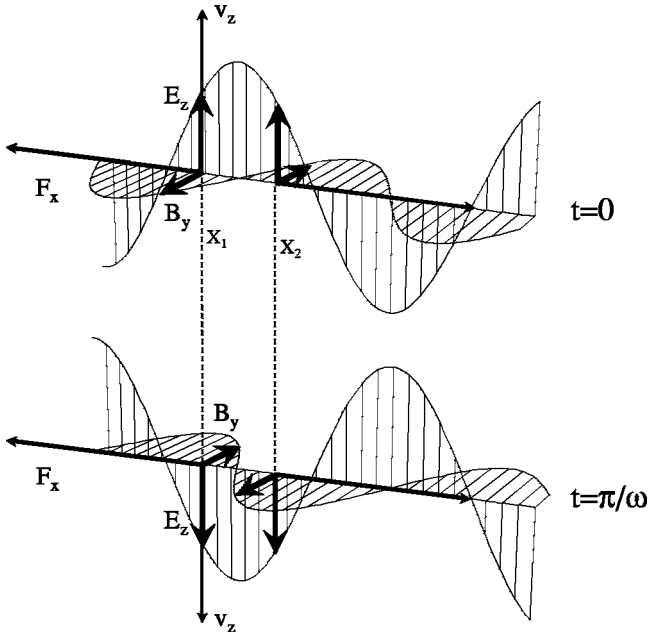

FIG. 2. Standing-wave fields. The electric (E_z) and magnetic (B_y) fields of a standing wave and the force (F_x) on an electron with velocity (v_z) at position x_1 and x_2 are given. An electron placed at x_1 feels a Lorentz force in the same direction at times separated by half a light period, even though the fields have switched direction. This gives rise to a time-averaged potential from which electrons can diffract.

$$v_z = \frac{eA_0}{m} \cos kx \sin \omega t. \tag{2}$$

Consequently, the electron's velocity oscillates in phase with the magnetic field and thus the Lorentz force in the x direction will not average to zero over an oscillation period because both the electron's velocity and the magnetic field change sign simultaneously. The resultant force will thus be directed along the x axis (Chan and Tsui, 1979).

To work our way towards a time-averaged potential for use in the Schrödinger equation we can first find the position dependence of the Lorentz force on the electron. An electron a quarter wave displaced from x_1 at $x = x_2$ experiences the same sign electric field as at position $x = x_1$, but the sign of the magnetic field is reversed. Thus the direction of the force at x_2 is reversed with respect to the force at x_1. From the spatial dependence of the force we can obtain an effective potential; the ponderomotive potential is

$$V_P = \frac{e^2 A_0^2}{4m} \cos^2 kx = \frac{e^2 I}{2m\varepsilon_0 c\omega^2} \cos^2 kx, \tag{3}$$

where the expression in terms of laser intensity is experimentally more useful. It is perhaps interesting that it was

only in 1957 that the presence of such ponderomotive[1] potentials was recognized (Boot and Harvie, 1957). An elegant more general derivation of potentials in rapidly oscillating fields based on classical perturbation theory is attributed to Kapitza in 1951 (Landau and Lifschitz, 1960). Classical electron motion in this potential leads to rainbow scattering, and its relation to quantum-mechanical motion has been discussed (Batelaan, 2000; Li *et al.*, 2004). Classical rainbow peaks are located at the maximum momentum transfer to the electrons. This occurs when electrons enter the potential at its highest gradient.

When considering only one-dimensional dynamics, quantum-mechanical motion is typically expected to become important when the de Broglie wavelength λ_{dB} is comparable to the typical length scale of the potential. This means that in our case we need to compare $\lambda_{dB,x}$ with the periodicity of the standing wave. The de Broglie wavelength $\lambda_{dB,x}$ is associated with the transverse electron motion along the standing wave. The periodicity of the standing-wave potential is $\lambda_{optical}/2$, where $\lambda_{optical}$ is the laser wavelength. When these two lengths are equal, λ_{dB} is on the order of $\lambda_{optical}/2$ and it is expected that the motion must be treated quantum mechanically.

When the particle's momentum, or its momentum transfer, along the standing wave is $p_x=\hbar k=\hbar/2\lambda_{optical}$ (see Fig. 3), the above relationship between the two wavelengths holds. This means that when electron motion on the order of a photon recoil is relevant, we expect that a quantum-mechanical treatment of motion is necessary. To be able to observe such a quantum-mechanical momentum transfer associated with a photon recoil, it is sufficient to select and detect the momentum of the electron beam better than the momentum transfer corresponding to the grating spacing. Consequently, in what follows, we focus our attention on the quantum-mechanical description of electron momenta along the standing-wave direction.

Before solving the Schrödinger equation it is useful at this point to inspect conceptual arguments corresponding to Heisenberg's uncertainty relations. To do so, we construct a schematized version of the sequence of events. First, an electron absorbs a photon from one laser beam. Subsequently the electron undergoes a stimulated Compton emission triggered by a photon of the counterpropagating laser beam. The total net momentum change is two photon recoils $2\hbar k$. This view affords a simple pictorial presentation of the KD effect (Fig. 3). For further discussion on the particle picture see Sec. II.C.

The boundary between Bragg and diffractive scattering is separated by the value of the uncertainty $\Delta\phi$ in the direction of interacting photons. This angular uncertainty gives an uncertainty of momentum transfer of $\hbar k\Delta\phi$. The $\Delta\phi$ and the laser-beam waist w are related through the position-momentum uncertainty relation. The uncertainty in $\Delta\phi$ allows for the conservation of momentum and energy in the scattering process. For a narrow laser-beam waist the value of $\Delta\phi_D$ is much larger than the diffraction angle $2\lambda_{dB,x}/\lambda_{optical}$ and the atom can be scattered into many orders. This is the diffractive regime, sometimes referred to as the Raman-Nath regime (Fig. 3, left). For a wide laser beam the value $\Delta\phi_B$ is much smaller than the diffraction angle and the atom cannot interact with two photons unless a special incident angle is chosen such that the momentum is conserved. This condition is satisfied at the Bragg angle θ_{Bragg} (Fig. 3, right).

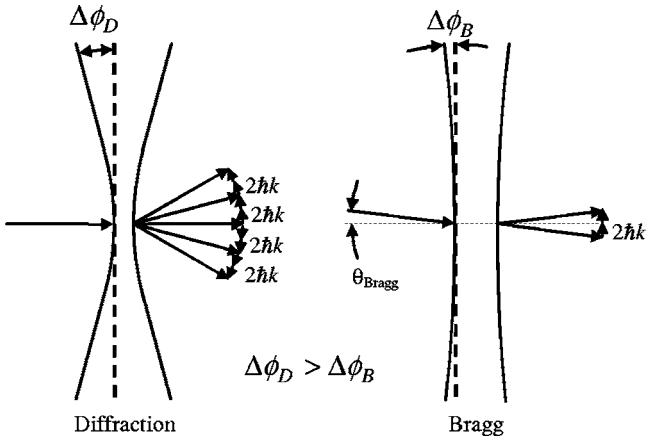

FIG. 3. The position-momentum uncertainty relation (Gould *et al.*, 1986; Martin *et al.*, 1988). Electrons moving from the left to the right through a laser focus are shown schematically. On the left, symmetric diffraction into many momentum states is shown; on the right, asymmetric Bragg scattering into two momentum states is shown. The diffracted electron beams are each separated by two photon momentum recoils $2\hbar k$. The diffractive regime can be tuned to the Bragg regime by reducing the laser-beam divergence from $\Delta\phi_D$ to $\Delta\phi_B$ with a less tight laser focus waist. The small uncertainty in the photon position for the diffractive case leads to a large uncertainty in the photon momentum (expressed in terms of $\Delta\phi_D$), which allows many diffraction orders to be reached. See text for a detailed explanation.

An energy-time argument can be given which is based on the energy-time uncertainty relation (Fig. 4). This view is also helpful in that it is closely related to the theory discussed below. During the interaction time $\Delta t=\Delta w/v$, an electron samples the frequency of a photon with an uncertainty of $1/\Delta t$. If the electron experiences a laser pulse of duration Δt, the interaction energy is uncertain by $\Delta E=\hbar/(2\Delta t)$. Energy conservation is satisfied by emitting and absorbing photons of different frequency from the light field. In the diffractive regime the time the electron spends in the narrow laser-beam waist is short. The ΔE associated with the short interaction time is much larger than the recoil shift of the electron δ_D $[=(2\hbar k)^2/2m]$, and a large number of diffraction orders can be reached (Fig. 4, right). In the Bragg regime

[1]The word "ponderomotive" stems from the Latin word "pondus" for weight. Cycle averaged potentials, which moving particles with mass experience in inhomogeneous fields, are typically referred to as "ponderomotive."

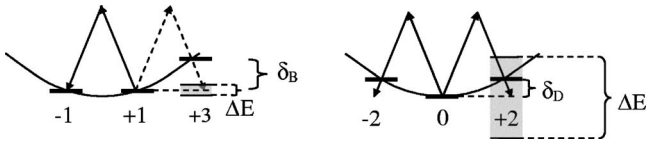

FIG. 4. The energy-time uncertainty relation. The momentum (in units of photon recoil) indicated along the horizontal axis versus kinetic energy of a free electron indicated along the vertical axis. In the Bragg regime (left) the interaction time is long and the associated energy uncertainty ΔE is much smaller than the recoil energy shift δ_B. This explains why only the two momentum states can be reached for states incident at the Bragg angle. In the diffractive regime (right) the argument is reversed ($\Delta E \gg \delta_D$).

the time the electron spends in the wide laser-beam waist is long. The ΔE associated with the long interaction time is much smaller than the recoil shift of the electron δ_B. The electron can then only be scattered when the electron enters the interaction region at the Bragg angle (Fig. 4, left).

We note that the use of the energy-time uncertainty relation is often discouraged in view of the absence of a time operator from which the uncertainty relation could be derived. This problem is discussed extensively in the literature. Since our system can be reduced to coupled two-level systems, an effective energy-time uncertainty relation can be defined (Hilgevoord, 1996, 1997).

The conceptual picture above is motivated by two theoretical treatments of the electron-light interaction. The wave picture is motivated by solving the Schrödinger equation using the ponderomotive potential obtained above. This wave treatment justifies the Heisenberg uncertainty relations, and the solution of the Schrödinger equation is sufficient to make predictions that are consistent with the experimental observations. The particle picture, however, is motivated by second quantization of the light field. We now proceed to present some basic ideas of both pictures that motivate our discussion.

B. The wave picture

Considering only motion along the standing wave, the photon recoils change the transverse velocity of electrons. For the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_0 \cos^2 kx, \tag{4}$$

the Schrödinger equation can be solved using a trial solution of the form

$$\psi = \sum_n c_n(t) e^{inkx}, \tag{5}$$

which describes the motion of the particle in terms of plane waves separated by one photon recoil each. With a minimum of manipulation, differential equations for the coefficients c_n can be found. These c_n coefficients are the amplitudes for finding the particle with a momentum of $n\hbar k$,

$$i\frac{dc_n}{dt} = \left(\varepsilon n^2 + \frac{V_0}{2\hbar}\right)c_n + \frac{V_0}{4\hbar}(c_{n-2} + c_{n+2}), \tag{6}$$

where $\varepsilon n^2 = \hbar k^2 n^2/2m$ is the kinetic energy (in units of $\hbar$) of each plane wave with momentum $n\hbar k$. The second term on the right-hand side of Eq. (6) shows that the momentum of the particle can only change by an even number of $\hbar k$ momentum recoils.

The solution of Eq. (6) can be found analytically, and it is interesting to consider it for two cases. When we ignore the kinetic energy term in Eq. (6) we are in the diffractive regime. This situation can occur when the particle does not have enough kinetic energy to move over the potential crests ($\varepsilon \ll V_0/\hbar$). The solution of Eq. (6) (which can be checked by direct substitution) is

$$d_m = i^m e^{-(i/\hbar)V_0 t} J_m(V_0 t/\hbar), \quad |d_m|^2 = J_m^2(V_0 t/\hbar). \tag{7}$$

where $d_m \equiv c_{n/2}$ and $m = 0, \pm 1, \pm 2, \ldots$. The Bessel function solutions represent a symmetric diffraction pattern that can be observed, where $|d_m|^2$ is the detection probability of finding the particle in the mth diffraction order.

In contrast, when we assume that the second term on the right-hand side of Eq. (6) can be ignored we are in the Bragg regime. This occurs when the particle can easily move over the potential crests ($\varepsilon \gg V_0/\hbar$). In this regime, as indicated in Fig. 4, only two diffraction orders couple,

$$\begin{aligned} c_1 &= e^{-i\varepsilon t}\cos(V_0 t/4\hbar) \\ c_{-1} &= -ie^{-i\varepsilon t}\sin(V_0 t/4\hbar) \end{aligned} \Rightarrow \begin{aligned} |c_1|^2 &= \cos^2(V_0 t/4\hbar), \\ |c_{-1}|^2 &= \sin^2(V_0 t/4\hbar), \end{aligned} \tag{8}$$

where the probability for finding a particle in both diffracted particle beams "pendulates" back and forth, as expressed by the so-called Pendellösung [Eq. (8)]. This oscillation can be observed as a function of both laser intensity or interaction time t_{int}. For the weaker potential used in this example for Bragg scattering the interaction has to be longer to reach a similar probability to diffract.

We are now in a position to make the connection to the conceptual time-energy picture discussed above. For diffraction to occur with a probability approaching unity, the argument of the Bessel function in Eq. (7) should be about unity, $V_0 t_{\text{int}}/\hbar \approx 1$, so that the ratio of the first- to zeroth-order diffraction amplitude is also about unity, $d_0/d_1 \sim 1$. Combining this with $\varepsilon \ll V_0/\hbar$ yields $\Delta E \gg \delta_D$, i.e., the same condition as in Fig. 4, right. A similar argument holds for the Bragg regime.

C. The particle picture

To arrive at Eq. (6) from Eqs. (4) and (5) we can write in Dirac's bra and ket notation the matrix element

$$\langle p_f|H|p_i\rangle = \langle p_f|p^2/2m + V_0\cos^2(kx)|p_i\rangle, \tag{9}$$

where the Hamiltonian couples the electron's initial and final momentum states. The term $p^2/2m$ gives rise to the kinetic energy, the first term on the right-hand side of Eq. (6). The second term in the Hamiltonian is the inter-

action term, which is due to the ponderomotive potential. The interaction term shifts the energy of the electron [the second term in Eq. (6)] and allows the electron to change its momentum by $\pm 2\hbar k$ [which are the third and fourth terms in Eq. (6)].

A similar matrix element can be written when the laser field is quantized. The usual second-quantization procedure involves replacing the complex amplitudes of the vector potential A by raising $\hat{a}^\dagger$ and lowering $\hat{a}$ operators (Milonni, 1994; Scully and Suhail Zubairy, 1997). The result is

$$\hat{A} = \{(\hat{a}_L + \hat{a}_L^\dagger)A_0 \cos(kx) + (\hat{a}_R + \hat{a}_R^\dagger)A_0 \cos(kx)\} \times \left(\frac{2\pi\hbar c^2}{\omega}\right)^{1/2}. \tag{10}$$

To distinguish the two counterpropagating laser beams, subscripts L and R are used. The relevant nonzero matrix element [Eq. (9)] after second quantization reads

$$A_{0\to 2\hbar k} = \langle p_i + 2\hbar k, (n+1)_L, (n-1)_R | \hbar\omega(\hat{a}_L^\dagger \hat{a}_R)\cos^2(kx) | (n)_R, (n)_L, p_i \rangle, \tag{11}$$

where the interaction term $\hbar\omega(\hat{a}_L^\dagger \hat{a}_R)\cos^2(kx)$ is a part of the Hamiltonian

$$\hat{H} = \frac{1}{2m}(\hat{p} - q\hat{A})^2. \tag{12}$$

We can evaluate the matrix element further using $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ and $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ to give

$$A_{0\to 2\hbar k} = \hbar\omega n. \tag{13}$$

Equation (13) is the basis for some of the earlier statements. We see that the stimulated emission [Eq. (11)] is enhanced by $\sqrt{n}$ as compared to spontaneous emission into an unoccupied mode. This explains why stimulated Compton scattering can dominate over the usual spontaneous Compton scattering. (Because this is a nonrelativistic argumentation, stimulated Thompson scattering might have been more appropriate.)

In Eq. (13) it is difficult to obtain a value for the number of photons n. Formally we would have to define the spatial modes of the laser fields and perform all spatial integrals to obtain an answer. Instead of attempting this difficult task we write the photon number as the product of the photon density ρ times the interaction volume V_{int}. The value of ρ is estimated from the laser intensity ($\rho = I/c$). We now ask what the interaction volume is that gives us agreement between the particle and wave picture. The answer is

$$V_{\text{int}} = r_0 \lambda^2, \tag{14}$$

where r_0 is the classical electron radius. Equation (14) gives a scattering amplitude that is (apart from factors of order one) equal to that of the wave picture:

$$\hbar\omega\rho r_0\lambda^2 = \frac{e^2 I}{m\varepsilon_0 c\omega^2} = 2V_0. \tag{15}$$

Consequently, the probability of scattering between two adjacent momentum states is given by

$$\left\{\frac{1}{\hbar}\left(\frac{e^2 I}{m\varepsilon_0 c\omega^2}\right)t_{\text{int}}\right\}^2. \tag{16}$$

For a low laser intensity Eq. (16) is also a good approximation of both the probability for diffractive [Eq. (7)] and Bragg [Eq. (8)] scattering.

It is not surprising that the classical electron radius appears in Eq. (14) given that the cross section for spontaneous Compton scattering is of the order of r_0^2. Qualitative arguments concerning different choices for the interaction volume can be given (Eberly, 1969). The conclusion is that to the photon the electron appears to be of the size of the classical electron radius (McGregor, 1992; Scully and Suhail Zubairy, 1997).

We hasten to emphasize that the above reasoning is far from rigorous. For a better treatment of second quantization and its effect on spontaneous and stimulated processes see Healy (1982). The effect of using number states and coherent states on atomic diffraction for fermions has been discussed recently (Meiser *et al.*, 2005). For an interesting first attempt to use coherent states within a QED approach to the Kapitza-Dirac effect see Clarke (2003). At present a finished full perturbative QED treatment of the KD effect including renormalization using coherent states is unknown. This means that the usual claim that the KD effect is due to stimulated Compton scattering is not rigorously justified. However, the lack of another explanation, the correct value of the momentum transfer $2\hbar k$, and nonrigorous arguments such as the one discussed above suggest this interpretation. The main purpose of this section is to point out that the full QED treatment is missing and to give a rough justification for the particle picture such as indicated in Figs. 5 and 6.

D. Momentum conservation

Although we can think of the KD effect as electron-wave diffraction, the KD effect highlights a different aspect of quantum mechanics than Davisson and Germer's (1927) or Tonomura's (1999, and references therein) famous experiments do. As mentioned above, Davisson and Germer were the first to show diffraction and thereby demonstrate the wave nature of the electron. Tonomura's experiments are the culmination of a series of experiments where the electron diffraction pattern is built up one particle at a time. It is one of the most dramatic examples of particle-wave duality. All three experiments concern electron diffraction, and it is fair to question if the KD experiment adds any new basic insights. There are some superficial differences; for instance, Tonomura's experiment uses a biprism for

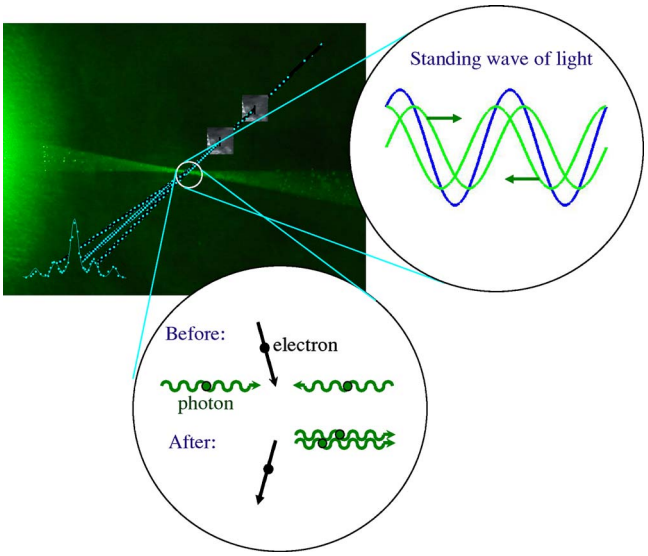

FIG. 5. (Color online) Wave and particle picture. The interaction between the electron beam and laser beam can be thought of as an electron wave experiencing a ponderomotive potential coinciding with a standing wave of light. The standing wave of light is the sum of two counterpropagating waves. Alternatively, we can think of the interaction as an electron undergoing a stimulated Compton scattering.

double-slit diffraction, while the Davisson-Germer and KD experiments concern crystal and "multislit" diffraction, respectively.

There is, however, a much more profound difference. Consider the following thought experiment. When a single electron hits the detection screen and its position is recorded, stop the experiment. Depending on one's point of view it may appear that there is not much to discuss about this experiment, since it is often stated that quantum mechanics makes statistical predictions about the behavior of ensembles of particles. Still, we can inspect conservation laws. In particular, we can ask, "Does momentum conservation hold for this thought experiment?" This would be very hard to prove experimentally for any experiment where the object that the electron interacts with is large and massive. Its recoil would be hard to measure not only because it is heavy, but also because it would be hard to isolate sufficiently from its environment. Moreover, it may be that only a part of the large object suffers the recoil imparted to it. Still, it is generally accepted that we expect momentum conservation to hold. This becomes clear by inspecting some of the literature concerning this topic. In one of Einstein's famous attempts to show that quantum mechanics is incomplete, he proposed to measure the recoil of a collimating slit to identify through which slit a particle went in a double-slit diffraction experiment, without disturbing the apparatus and thus the interference-diffraction pattern. Bohr refuted the argument using the Heisenberg uncertainty principle for the collimating slit (Bohr, 1949). Much later Wooters and Zurek's analysis made Bohr's argument quantitative (Wooters and Zurek, 1979). All of these authors assume momentum conservation to hold. In essence the diffracting particle and object are described after the interaction by an entangled wave function of the form $\psi_e(k)\psi_o(-k)+\psi_o(k)\psi_e(-k)$ (Wooters and Zurek, 1979). After measurement of the single electron that hits the detection screen, we are left with a single event that conserves momentum.

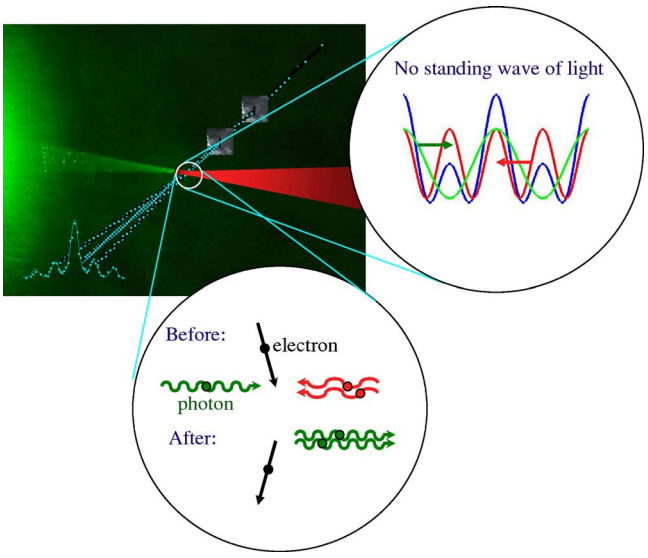

FIG. 6. (Color online) Wave and particle picture. The interaction between the electron beam and a counterpropagating beam may be considered to be conceptually difficult. In the wave picture the beating wave does not create a standing wave of light. Curiously, in the particle picture both energy and momentum can be conserved in an up-conversion process. It appears likely that this process can take place (for a discussion see Sec. II.D).

By which means is the momentum between the electron and slit exchanged? This is where the KD effect is interesting, because for the KD effect it appears that we can answer such a question. It is possible to identify the object with which the electron shares momentum: photons. And we have a candidate for the process by which momentum is exchanged: stimulated Compton scattering.

The particle picture is not used for diffraction of particles from a material grating. To predict a diffraction pattern that is consistent with experiment we do not need to consider that picture. Solving the Schrödinger equation with the appropriate potential will give good agreement with experiment. It is curious, however, that seemingly very similar physics experiments are described in different ways. For electron diffraction from a standing wave of light we have both a wave and a particle picture, but for electron diffraction from a grating we have only a wave picture. Because there is no particle picture for electron grating diffraction, it appears that the physical mechanism for momentum conservation is an unsolved problem.

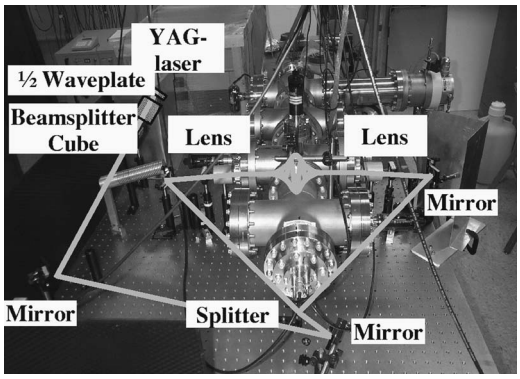

FIG. 7. The experimental setup used to observe the KD effect. A line is overlayed to schematically indicate the laser-beam paths.

III. THE KAPITZA-DIRAC EXPERIMENT

A. Experimental setup

An electron beam crosses two counterpropagating laser beams which form the standing-wave light grating (Fig. 7). To reach sufficiently high laser intensities, we used a Nd:YAG laser with 10-ns pulses and an energy of 0.2 J per pulse focused onto a 125-μm-diam beam waist. Each counterpropagating laser beam travels an equal distance not differing by more than 1 mm. This is well within the coherence length of the laser beam (5 mm) where the standing wave is formed. A 380-eV electron beam is collimated by two 10-μm-wide molybdenum slits separated by 24 cm. The electron beam runs from the top of the picture downward. The slits are held in the vacuum chamber by translation stages. A third slit cuts the height of the electron beam to the size of the laser-beam waist. Subsequently, the electron beam crosses the standing wave about 1 cm after the third slit. A fourth 10-μm slit, 24 cm downstream from the interaction region, is used to scan the electron beam profile.

The measured spatial width (full width at half maximum) of the electron beam is 25 μm. This is a considerably narrower width than the expected distance between the zeroth and first diffraction order, 56 μm $=2D\lambda_{dB}/\lambda_{opt}$, where D (=24 cm) is the distance from the grating to the detection plane and $\lambda_{opt}=532$ nm. We may thus expect the diffraction peaks to be resolved. For an energy of 380 eV the electron velocity is 1.1×10^7 m/s, while the de Broglie wavelength is 0.63 Å.

Electrons are detected as a function of time with an electron multiplier. Each laser pulse is used as a start signal, while the detection of electrons is used as the stop signal for a time to amplitude converter. A multichannel scaler records the pulses from the converter into coincidence time spectra. From the time spectra taken at various positions, the diffraction pattern is obtained directly.

B. Electron diffraction in the Raman-Nath regime

The diffraction pattern is shown in Fig. 8. The diffraction orders are clearly resolved and fall at their expected positions ($n\times56$ μm, $n=0,\pm1,\pm2,\ldots$). The heights of the diffraction peaks might be expected to be given by the analytic solution of the Schrödinger equation in the diffractive limit [Eq. (7)] (Freimund *et al.*, 2001). However, this is not exactly the case. Given that some electrons pass through less intense regions of the focused laser beam and some electrons pass through more intense regions, a numerical solution of the Schrödinger equation including averaging over the laser focus gives acceptable agreement with the experimental data (Fig. 8).

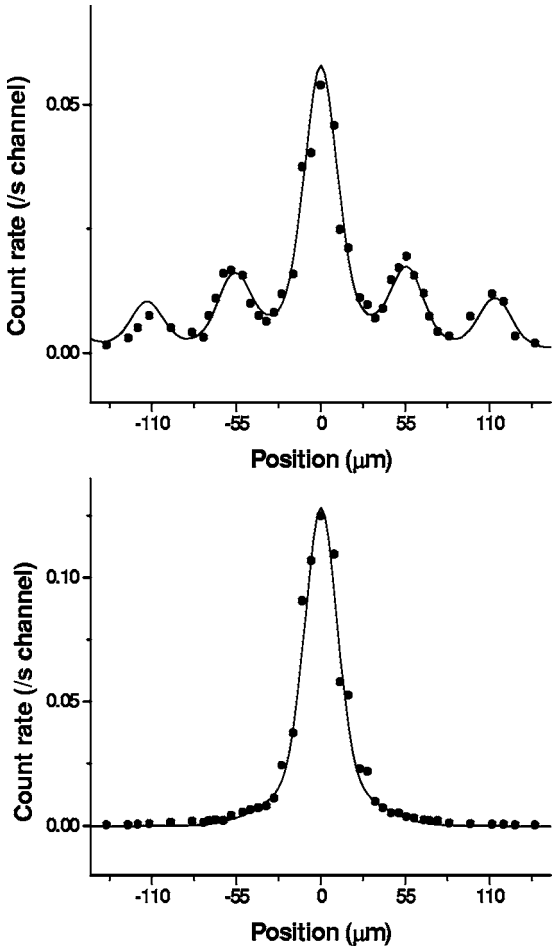

FIG. 8. Experimental data. The electron detection rate is presented as a function of detector position. Our data (solid dots) agree reasonably well with a numerical solution of the Schrödinger equation and clearly show the diffraction peaks that are the signature of the Kapitza-Dirac effect. The bottom figure shows the electron-beam profile with the laser beams turned off (bottom).

C. Electron diffraction in the Bragg regime

The main difference in the experimental setups used for observation of diffraction (Freimund *et al.*, 2001) and Bragg scattering (Freimund and Batelaan, 2002) involves increasing the beam width of the Nd:YAG laser at the region where it interacts with the electrons (see Figs. 3 and 4). This seemingly straightforward change of a pa-

rameter is a significant experimental obstacle. In the diffractive regime the electron beam can pass the laser beam under a wide angle of incidence. The Bragg angle needs to be found, a task usually achieved by rotating the mirror that reflects the laser to create the standing wave. Due to the high intensity of the laser and its short coherence length and the interaction of electron passing close by a mirror, the use of a mirror is problematic. Therefore, the standing wave is formed by counterpropagating two laser beams formed by a beam splitter.

We found that rotating the standing wave by optical means was difficult because the overlap of the laser beams and the quality of the standing wave were hard to maintain. Instead, we rotated the vacuum system containing the electron gun relative to the optics.

In the diffraction experiment, the laser is focused with a spherical lens to a diameter of 125 μm. For the Bragg case, the laser beam is focused with a cylindrical lens to a width of 8 mm and a height of about 200 μm. This effectively increases the cross-sectional area of the laser focus, lowers the intensity of the beam by two orders of magnitude, and increases the interaction time by two orders of magnitude. Because the interaction strength is dependent on the product of the intensity and interaction time [Eqs. (7) and (8)], Bragg scattering is expected to occur for these parameters. For the experimental parameters used the energy uncertainty $2/t_{\mathrm{int}}\hbar$ is less than the recoil shift ε of 0.8×10^{10} rad/s as expected. Unexpected is that the potential $V_0/\hbar$ ($\approx5\times10^{10}$ rad/s) is larger than the recoil shift. Also the interaction strength $V_0 t_{\mathrm{int}}/\hbar\approx30$ is stronger than necessary. This is attributed to an effective interaction length that is significantly less (see also below).

Figure 9, bottom, shows the expected asymmetric diffraction pattern. In Fig. 10 the angle of incidence is varied which allows the observation of the first- and second-order Bragg diffraction in the so-called rocking curve. This reveals another qualitative difference between the Bragg and diffraction regimes. In the Bragg regime, the profile appears as a peak centered on the Bragg angle. This is because there are no other angles that lead to conservation of energy and momentum (Fig. 4). In the diffraction regime, which has more laser beam divergence, the profile is approximately flat over many angles of incidence (Fig. 3) (Freimund and Batelaan, 2002). This excludes the possibility of observing asymmetric diffraction at a large misaligned angle of incidence. The mediocre quality of laser-beam collimation for the type of laser used prevented observation of sharper-peaked rocking curves.

The solid lines in Fig. 9, bottom, are numerical integrations of Eq. (6) at 0.2 J laser power using the initial distribution of Fig. 9, top. The theoretical calculation is in qualitative agreement with the experimental observation. However, the calculational parameter used for the laser width is 0.8 mm while experimentally it is 8 mm. This discrepancy can be attributed to the poor quality of our unfocused laser beam and to alignment difficulties with unfocused laser beams (Freimund and Batelaan, 2002). For a discussion on the effect of laser width on the KD effect see Fedorov (1974).

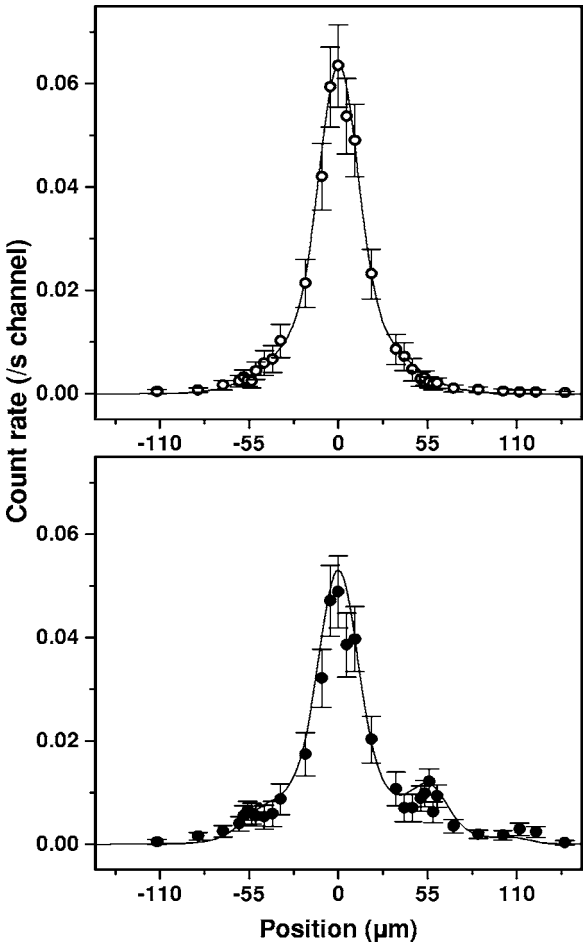

FIG. 9. Bragg regime. The detection rate is given as a function of detector position. With the laser turned off (top) a symmetric beam profile is found, while the laser light (bottom) produces Bragg scattering.

The data show the onset of the Bragg regime, but the experiment cannot probe deep into the Bragg regime. It should be expected that with a laser seeder (which was not available for this experiment) this problem can be overcome. These observations also reflect on the earlier attempts made to observe the KD effect. Unfocused beams were mostly used. Given the needed quality of the electron beam alignment and laser beam wave front the availability of current technology is a necessity.

IV. THE TWO-COLOR KAPITZA-DIRAC EFFECT: DIFFRACTION WITHOUT A GRATING?

The KD effect can be understood by thinking of the standing wave of light as a grating, as in the wave picture. In the particle picture we think of the KD effect as a collision that conserves energy and momentum (Fig. 5). Thus we have two alternative ways of thinking that both give the same result. However, when two laser beams with different wavelengths are used, the two ways of thinking appear to give conflicting answers. In the

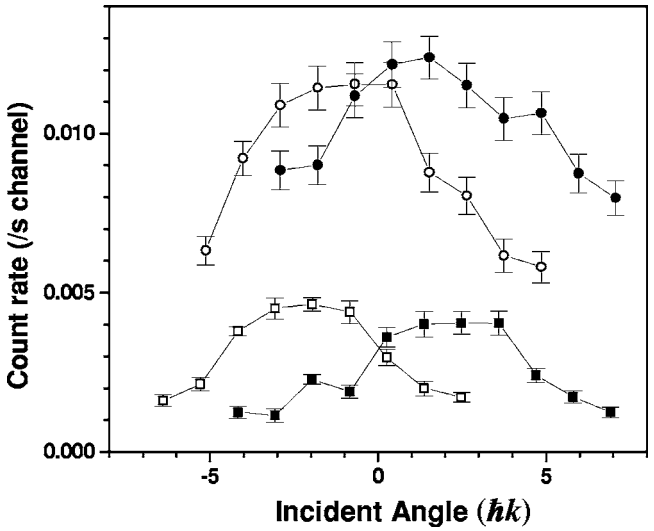

FIG. 10. Rocking curves. First-order (circles) and second-order (squares) Bragg scattering rates are shown as a function of the electron incident transverse momentum for the detector placed at the negative (open symbols) and positive (solid symbols) Bragg angles. The lines are a guide to the eye.

wave picture the two laser beams create a beat pattern and not a standing wave.

Such a beat pattern may not be expected to give rise to a time-averaged potential from which the matter wave could diffract. However, in terms of photon scattering, energy and momentum can be conserved and diffraction would be expected (Fig. 6). Hence the term "diffraction without a grating."

Instead of viewing this problem from the matter-optics point of view, it is also interesting to view this process from the nonlinear-optics point of view. The electron can be seen as the simplest nonlinear medium that supports the up- and down-conversion process (Smirnova *et al.*, 2004). In up-conversion n photons of frequency ω can be converted to one photon of frequency $n\omega$. Such a process and the related down-conversion processes will be labeled symbolically by ω-$n\omega$. Evidence that a single electron can be considered a nonlinear medium is that the process is nonlinear in the laser intensity. We first look at the classical electron motion following the approach for the KD effect that leads up Eq. (3). The vector potential for a laser field formed by one traveling beam of laser light of frequency ω counterpropagating with a laser beam of frequency 2ω is given by

$$\vec{A} = A_z\hat{z} = [A_1 \cos(kx - \omega t) + A_2 \cos(-2kx - 2\omega t)]\hat{z}, \tag{17}$$

where $A_i = \frac{1}{\omega_i}\sqrt{\frac{I}{\varepsilon_0 c}}$, so that the counterpropagating laser beams have equal intensity.

The equations of motion can now be numerically integrated to find the maximum electron deflection in the transverse direction parallel to the laser beam direction. In Fig. 11 the maximum transverse velocity of the electron is shown. The interaction time is the same as in the

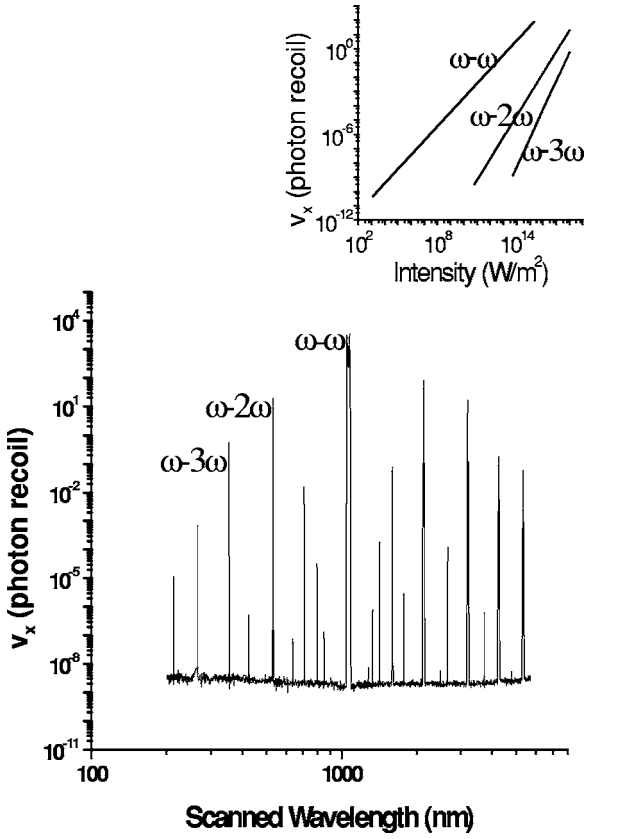

FIG. 11. Two-color electron interaction simulation. Two counterpropagating laser beams can provide a time-averaged potential to an electron when their wavelengths λ_1 and λ_2 satisfy $n\lambda_1 = m\lambda_2$, where n and m are integers. One wavelength is fixed at 1064 nm and the other is varied. Inset: The nonlinearity of the processes. The label ω-$n\omega$ indicates the frequency of photons involved in the process. For example, the two-color interaction can involve the conversion of two photons of frequency ω into one with frequency 2ω.

case of the KD effect, about 10^{-11} s. This means that we are looking at effects that survive many periods of the optical wave. Of the two counterpropagating waves one of the wavelengths is changed. When both are equal at 1064 nm we observe the largest effect. This is the effect of the ponderomotive potential associated with the KD effect. However, the peak at a wavelength of 532 nm (labeled "ω-2ω") is the focus of our discussion.

The calculation is performed at 10^{18} W/m^2 and shows a maximum electron velocity exceeding $10^{-5}c$. This effect can be cast into the shape of a time-averaged potential (Smirnova *et al.*, 2004)

$$V = \frac{7 v_z E_1^2 E_2}{16\omega^3 c^2} \sin 4kx. \tag{18}$$

Earlier we stated that we would not have expected to find a time-averaged potential based on the presence of a beat note in the field. However, when we consider the response of the electron to this field, the time-averaged potential is found. Quantum-mechanical perturbation theory gives the same result (Smirnova *et al.*, 2004).

It is interesting to inspect this potential. The periodicity of this grating corresponds to the momentum recoil given by the particle picture. One photon recoil of the field E_2 is combined with two photon recoils of the field E_1. Assuming that the electric field strengths of the two fields are identical, the potential and the maximum deflection are proportional to the third power of the electric field. The slope of the maximum electron velocity versus laser intensity obtained numerically (inset, Fig. 11) is 3/2 m^3/J. This shows that the process is nonlinear as expected for an up- or down-conversion process. The potential is also proportional to the electron's velocity. This is in contrast to the usual KD effect. In fact processes involving an even number of photons, such as the KD effect, do not require a nonzero electron velocity. Processes involving an odd number of photons, such as the two-color effect, do require a nonzero velocity (Freimund, 2003).

We now conclude that we do expect electron diffraction from this potential [Eq. (18)]. The particle picture suggested the correct answer. Why did the wave picture seemingly fail? And why did the wave picture work for the usual KD effect? The answer to the latter question is that the ponderomotive potential for the KD effect spatially coincides with the standing wave in the intensity of light. To identify a grating we should not look for a property associated with the light by itself, but ask what the light looks like to the electron. The presence of a periodic potential dictates the presence of diffraction. The conceptually striking idea is that the incoming wave participates in forming the grating and subsequently diffracts from that grating. In other words, the grating does not exist separately from the wave and cannot be found as such.

For the two-color KD effect the coincidence is not present. Mathematically the potential is proportional to $\int E_1^n E_2^m(t)dt$, where $n+m$ is the number of photons involved in the process. For the KD effect this integral is nonzero for $n+m=2$, which happens to coincide with the laser intensity, while for the two-color KD effect this integral is nonzero for $n+m=3$. This third moment of the electric field is not a physical parameter that is typically associated with the light itself and as such is not immediately recognized to give a time-averaged potential.

The above discussion describes the matter-optics point of view, which is the point of view of the electron. The main question is: "How is the electron affected by the presence of the light?" The nonlinear-optics point of view is obtained by answering the question, how is the light field affected by the presence of the electron? The electron (i.e., the medium) responds to E_1 by oscillating along the z axis with frequency ω. The Lorentz force modifies this to a figure-8 motion with the major axis along z, where the 2ω component is along the x axis. Thus, polarization $P_x^{(2)}(2\omega)=\chi_{xzz}^{(2)}(2\omega;\omega,\omega)E_{1,z}^2$ is created along the x axis. Simultaneously, the linear response to $\omega_2=2\omega$ induces polarization $P_x^{(1)}(2\omega)=\chi_{xz}^{(1)}(2\omega;2\omega)E_{2,z}$ at 2ω frequency. As in conventional nonlinear optics, mixing the two waves $P_x^{(2)}(2\omega)$ and $P_x^{(1)}(2\omega)$ induces a stationary polarization grating (Boyd, 1992).

Similar to conventional wave mixing, the matter wave of the electron diffracts from this stationary grating to generate the new wave; the phase matching is equivalent to momentum conservation. Conventionally, a nonlinear medium such as a beta barium borate (BBO) crystal is macroscopic and the momentum recoil cannot be observed. In our case, the nonlinear medium is a single electron and the momentum recoil is observed as the electron diffraction.

From the nonlinear-optics viewpoint, it would be surprising that the point electron can provide anything but spherical symmetry. Spherical symmetry does not support wave mixing. For χ^2 to be nonzero in the dipole approximation the medium cannot be reflection invariant in the z direction. In our case, the nonlinear response arises beyond the dipole approximation and, in general, requires no symmetry breaking. However, the second-harmonic component of the figure-8 motion is along the x axis. Therefore, the linear response $P_2(2\omega)=\chi_{xz}^{(1)}E_2$ is also needed along the x axis, orthogonal to E_2. The nonzero component $\chi_{xz}^{(1)}(2\omega;2\omega)$ of the linear susceptibility tensor originates exclusively from the Lorentz force and requires nonzero velocity in the z direction, $\chi_{xz}^{(1)}\propto v_z$, breaking the reflection symmetry. This is also why the potential is proportional to v_z.

Can the two-color experiment be performed? The intensity can easily be reached with femtosecond-pulsed lasers, and electron velocity changes of 10^3 m/s (about one photon recoil) can be observed (Freimund *et al.*, 2001; Freimund and Batelaan, 2002). There are two other potential problems involved with observing this effect. If during the duration of the laser pulse no electrons are present, no effect can be observed. For femtosecond pulses this problem requires the use of an on-demand electron pulse. This is possible with a picosecond source such as developed by Zewail (Lobastov *et al.*, 2005). A recent development may even push this into the femtosecond domain (Hommelhoff *et al.*, 2006). The other problem is that at these higher intensities spontaneous Compton scattering might occur. For harmonic motion the number of photons radiated by one electron during the interaction time t_{int} is given by

$$\frac{Pt_{\mathrm{int}}}{\hbar\omega}=\frac{e^4\omega^2A^2}{6\pi c^2\hbar\omega}\sqrt{\frac{\mu_0}{\varepsilon_0}}t_{\mathrm{int}}=\frac{e^4I}{3\pi\varepsilon_0c^3\hbar\omega}\sqrt{\frac{\mu_0}{\varepsilon_0}}t_{\mathrm{int}}\sim 10^{-5}. \tag{19}$$

Here the radiated power P is estimated from Larmor's expression for radiation combined with the amplitude of the harmonic motion of the electron [the amplitude can be obtained by integrating Eq. (2)]. The interaction time was chosen to be 30 fs. Alternatively, this rate is given by the billard-ball scattering rate times the interaction time:

$$Rt_{\mathrm{int}} = n\sigma c t_{\mathrm{int}} = \frac{I}{\hbar\omega c} r_0^2 c t_{\mathrm{int}}, \tag{20}$$

which gives the same answer, apart from factors of order unity. Yet another approach is to view the Compton scattering as scattering into a vacuum mode. This is a simple modification of Eqs. (15) and (16), obtained by choosing one of the fields to be unoccupied:

$$\left\{\frac{1}{\hbar}\left(\frac{e^2 I_{\mathrm{laser}}}{m\varepsilon_0 c\omega^2}\right)t_{\mathrm{int}}\right\}\left\{\frac{1}{\hbar}(\hbar\omega\rho_{\mathrm{vacuum}} r_0 \lambda^2)t_{\mathrm{int}}\right\}. \tag{21}$$

Effectively the intensity of one of the laser beams has been replaced by the vacuum photon density. In this case the vacuum photon density is given by

$$\rho = \int_{\omega}^{\omega+1/t_{\mathrm{int}}} \frac{\omega^2}{2\pi^2 c^3} d\omega \approx \frac{\omega^2}{c^3 t_{\mathrm{int}}}. \tag{22}$$

All three approaches [Eqs. (20)–(22)] give the same outcome. The result is that one-photon processes are negligible compared to the three-photon two-color effect. With amplified femtosecond lasers intensities can be reached where spontaneous emission is important. In this scenario one would use one laser beam to avoid the multiphoton processes discussed above. Other problems, such as the time it takes for electrons to be pushed out of the intense laser beam, then come into play (Park *et al.*, 2002).

V. SUMMARY AND CONCLUSIONS

The Kapitza-Dirac effect for electrons has been demonstrated in the diffractive regime and some features of the Bragg regime are emerging. The KD effect raises some conceptual, theoretical, and experimental questions. An example of a conceptual question is: "What is the origin of momentum conservation in the scattering process?" An example of a theoretical question is: "Can we perform a full quantum-electrodynamics calculation including renormalization of the KD effect?" An experimental question is: "Can we realize the two-color KD effect?"

A practical technological reason why the Kapitza-Dirac effect is interesting is that it provides a means by which to coherently split an electron beam without the presence of any material. This may be important for low-energy electron interferometry. No separate beam electron interferometers exist below about 200 eV. It is also clear that a device that relies on mechanical structure for producing coherence can also introduce decoherence (Hasselbach *et al.*, 2004; Gronniger *et al.*, 2005; Sonnentag and Hasselbach, 2005). It is interesting to investigate lower energies. Many collision phenomena become interesting at energies below several tens of eV, i.e., at energies comparable to atomic binding energies

The KD effect is part of the much broader field of electron matter optics. Within the field of electron matter optics, the physics underlying the electron microscope has been familiar for decades. This has provided a tool that is now widely used. The maturity of the field of electron microscopy would suggest that major new breakthroughs in the field of electron matter optics should not be expected. However, recent experiments show the opposite. Beautiful experiments on the Aharonov-Bohm effect involved the spatially coherent control of electron microscopy and interferometry (Tonomura, 1999). With ultrafast electron diffraction Zewail has added temporal control to coherent electron motion, which has started to resolve real-time molecular motion (Lobastov *et al.*, 2005). The KD effect shows that the coherent control of electron waves can now be done without material parts, which gives further possibilities to create matter-optics elements with spatial and temporal control. Hasselbach's demonstration of electron antibunching shows that not only first-order correlation effects but also second-order correlation effects are possible (Kiesel *et al.*, 2002). The work of Kasevich points in the direction of femtosecond control and attosecond dynamics (Hommelhoff *et al.*, 2006). We feel that these recent results are a sample of what may be the start of a new era of coherent electron control.

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