The spin structure of the proton

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This article reviews the present understanding of the QCD spin structure of the proton. The author first outlines the proton spin puzzle and its possible resolution in QCD. Then the review explores the present and next generation of experiments being undertaken to resolve the proton's spin-flavor structure, explaining the theoretical issues involved, the present status of experimental investigation, and the open questions and challenges for future investigation.

CONTENTS

I. INTRODUCTION

Understanding the spin structure of the proton is one of the most challenging problems facing subatomic physics: How is the spin of the proton built up from the intrinsic spin and orbital angular momentum of its quark and gluonic constituents? What happens to spin in the transition between current and constituent quarks in low-energy quantum chromodynamics (QCD)? Key issues include the role of polarized glue and gluon topology in building up the spin of the proton.

The story of the proton's spin dates from the discovery by Dennison (1927) that the proton is a fermion of spin $\frac{1}{2}$. Six years later Estermann and Stern (1933) measured the proton's anomalous magnetic moment, $\kappa_p = 1.79$ Bohr magnetons, revealing that the proton is not pointlike and has internal structure. The challenge of understanding the structure of the proton had begun!

We now understand the proton as a bound state of three confined valence quarks (spin-$\frac{1}{2}$ fermions) interacting through spin-1 gluons, with the gauge group being color SU(3) (Thomas and Weise, 2001). The proton is special because of confinement, dynamical chiral symmetry breaking, and the very strong color gauge fields at large distances.

Our present knowledge about the spin structure of the proton at the quark level comes from polarized deep-inelastic-scattering experiments (pDIS) which use high-energy polarized electrons or muons to probe the structure of a polarized proton and new experiments in semi-

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inclusive polarized deep-inelastic scattering, polarized proton-proton collisions, and polarized photoproduction experiments.

The present excitement and global program in high-energy spin physics was inspired by an intriguing discovery in polarized deep-inelastic scattering. Following pioneering experiments at SLAC (Alguard *et al.*, 1976, 1978; Baum *et al.*, 1983), recent experiments in fully inclusive polarized deep-inelastic scattering (Windmolders, 1999) have extended measurements of the nucleon's g_1 spin-dependent structure function (the inclusive form factor measured in these experiments) over a broad kinematic region where one is sensitive to scattering on the *valence* quarks plus the quark-antiquark *sea* fluctuations. These experiments have been interpreted to imply that quarks and antiquarks carry just a small fraction of the proton's spin (between about 15% and 35%)—less than half the prediction of relativistic constituent quark models (∼60%). This result has inspired vast experimental and theoretical activity in an effort to understand the spin structure of the proton. Before embarking on a detailed study of the spin structure of the proton it is essential to understand why the small value of this "quark spin content" measured in polarized deep-inelastic scattering caused such excitement and why it has challenged our understanding of the structure of the proton. We give a brief survey in Sec. I.A. An outline of the review is given in Sec. I.B.

Many elements of subatomic physics and quantum field theory are important in our understanding of the proton spin problem. These include the following:

- the dispersion relations for polarized photon-nucleon scattering;
- Regge theory and the high-energy behavior of scattering amplitudes;
- the renormalization of the operators which enter the light-cone operator product expansion description of high-energy polarized deep-inelastic scattering;
- perturbative QCD, the physics of large transverse momentum plus parton-model factorization;
- the nonperturbative and nonlocal topological properties of gluon gauge fields in QCD;
- the role of gluon dynamics in dynamical chiral symmetry breaking (the large mass of the η and η' mesons and the absence of a flavor-singlet pseudoscalar Goldstone boson in spontaneous chiral symmetry breaking).

The purpose of this article is to review our present understanding of the proton spin problem and the physics of the new and ongoing program aimed at resolving the spin-flavor structure of the nucleon.

A. Spin and the proton spin problem

Spin plays an essential role in particle interactions and the fundamental structure of matter, ranging from the subatomic world to large-scale macroscopic effects in condensed-matter physics (e.g., Bose-Einstein condensates, superfluidity, and exotic phases of low-temperature ^{3}He), and the structure of dense stars. Spin is essential for the stability of the known Universe. In applications, polarized neutron beams are used to probe the structure of condensed-matter and materials systems. Manipulating the spin of the electron may prove to be a key ingredient in designing and constructing a quantum computer—the new field of "spintronics" (Zutic *et al.*, 2004).

Spin is the characteristic property of a particle besides its mass and gauge charges. The two invariants of the Poincaré group are

$$\mathcal{P}_\mu \mathcal{P}^\mu = M^2,$$

$$\mathcal{W}_\mu \mathcal{W}^\mu = - M^2 s(s+1). \tag{1}$$

Here $\mathcal{P}$ and $\mathcal{W}$ denote the momentum and Pauli-Lubanski spin vectors, respectively, M is the particle mass, and s denotes its spin. The spin of a particle, whether elementary or composite, determines its equation of motion and its statistics properties. The discovery of spin and its properties are reviewed in Tomonaga (1997) and Martin (2002). Spin-$\frac{1}{2}$ particles are governed by the Dirac equation and Fermi-Dirac statistics whereas spin-0 and spin-1 particles are governed by the Klein-Gordon equation and Bose-Einstein statistics.

The proton's spin vector s_μ is measured through the forward matrix element of the axial-vector current

$$2Ms_\mu = \langle p,s|\bar{\psi}\gamma_\mu\gamma_5\psi|p,s\rangle, \tag{2}$$

where ψ denotes the proton field operator and M is the proton mass. The quark axial charges

$$2Ms_\mu \Delta q = \langle p,s|\bar{q}\gamma_\mu\gamma_5 q|p,s\rangle \tag{3}$$

then measure information about the quark spin content of the proton. Here q denotes the quark field operator. The flavor-dependent axial charges Δu, Δd, and Δs can be written as linear combinations of the isovector, SU(3) octet, and flavor-singlet axial charges,

$$g_A^{(3)} = \Delta u - \Delta d,$$

$$g_A^{(8)} = \Delta u + \Delta d - 2\Delta s,$$

$$g_A^{(0)} = \Delta u + \Delta d + \Delta s. \tag{4}$$

In semiclassical quark models Δq is interpreted as the amount of spin carried by quarks and antiquarks of flavor q.

In polarized deep-inelastic-scattering experiments one measures the nucleon's g_1 spin structure function as a function of the Bjorken variable x, the fraction of the proton's momentum which is carried by quark, antiquark, and gluon partons in incoherent photon-parton scattering with the proton boosted to an infinite-momentum frame. From the first moment of g_1, these experiments have been interpreted to imply a small value for the flavor-singlet axial charge:

$$g_A^{(0)}|_{\mathrm{pDIS}} = 0.15 - 0.35. \tag{5}$$

When combined with the octet axial charge measured in hyperon beta decays ($g_A^{(8)} = 0.58 \pm 0.03$) it corresponds to a negative strange-quark polarization,

$$\Delta s = -0.10 \pm 0.04 \tag{6}$$

—that is, polarized in the opposite direction to the spin of the proton.

The Goldberger-Treiman relation relates the isovector axial charge $g_A^{(3)}$ to the product of the pion decay constant f_π and the pion-nucleon coupling constant $g_{\pi NN}$, viz.,

$$2Mg_A^{(3)} = f_\pi g_{\pi NN} \tag{7}$$

through spontaneously broken chiral symmetry (Adler and Dashen, 1968). The Goldberger-Treiman relation leads immediately to the result that the spin structure of the proton is related to the dynamics of chiral symmetry breaking.

What happens to gluonic degrees of freedom? The axial anomaly, a fundamental property of quantum field theory, tells us that the axial-vector current which measures the quark spin content of the proton cannot be treated independently of the gluon fields that the quarks live in and that the quark spin content is linked to the physics of dynamical axial U(1) symmetry breaking in the flavor-singlet channel. For each flavor q the gauge-invariantly renormalized axial-vector current satisfies the anomalous divergence equation (Adler, 1969; Bell and Jackiw, 1969)

$$\partial^\mu(\bar{q}\gamma_\mu\gamma_5 q) = 2m\bar{q}i\gamma_5 q + \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}. \tag{8}$$

Here m denotes the quark mass and $(\alpha_s/4\pi)G_{\mu\nu}\tilde{G}^{\mu\nu}$ is the topological charge density. The anomaly is important in the flavor-singlet channel and intrinsic to $g_A^{(0)}$. It cancels in the nonsinglet axial-vector currents which define $g_A^{(3)}$ and $g_A^{(8)}$. In the QCD parton model the anomaly corresponds to physics at the maximum transverse momentum squared (Carlitz *et al.*, 1988). The anomaly contribution also involves nonlocal structure associated with gluon field topology—see Jaffe and Manohar (1990) and Bass (1998, 2003b). In dynamical axial U(1) symmetry breaking the anomaly and gluon topology are associated with the large masses of the η and η' mesons.

What values should we expect for the Δq? First, consider the static quark model. The simple SU(6) proton wave function

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}|u\uparrow(ud)_{S=0}\rangle + \frac{1}{\sqrt{18}}|u\uparrow(ud)_{S=1}\rangle - \frac{1}{3}|u\downarrow(ud)_{S=1}\rangle - \frac{1}{3}|d\uparrow(uu)_{S=1}\rangle + \frac{\sqrt{2}}{3}|d\downarrow(uu)_{S=1}\rangle \tag{9}$$

yields the values $g_A^{(3)} = \frac{5}{3}$ and $g_A^{(8)} = g_A^{(0)} = 1$.

In relativistic quark models one has to take into account the four-component Dirac spinor $\psi = (N/\sqrt{4\pi})\binom{f}{i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}g}$ where N is a normalization factor. The lower component of the Dirac spinor is p wave with intrinsic spin primarily pointing in the opposite direction to the spin of the nucleon. Relativistic effects renormalize the axial charges obtained from SU(6) by the factor $N^2\int drr^2(f^2 - \frac{1}{3}g^2)$ with a net transfer of angular momentum from intrinsic spin to orbital angular momentum—see, e.g., Jaffe and Manohar (1990).

Relativistic constituent quark models (which do not include gluonic effects associated with the axial anomaly) generally predict values of $g_A^{(3)} \simeq 1.25$ and $g_A^{(8)} \sim g_A^{(0)} \simeq 0.6$. For example, consider the MIT bag model. There, $N^2\int_0^R drr^2(f^2 - \frac{1}{3}g^2) = 0.65$, where R is the bag radius. This relativistic factor reduces $g_A^{(3)}$ from $\frac{5}{3}$ to 1.09 and $g_A^{(0)}$ to 0.65. Center-of-mass motion then increases the axial charges by about 20% bringing $g_A^{(3)}$ close to its physical value 1.26. Pion cloud effects are also important. In the SU(2) cloudy bag model one finds renormalization factors equal to 0.94 for the isovector axial charge and 0.8 for the isosinglet axial charges (Schreiber and Thomas, 1988) corresponding to a shift of total angular momentum from intrinsic spin into orbital angular momentum. The resultant predictions are $g_A^{(3)} \simeq 1.25$ (in agreement with experiment) and $g_A^{(0)} = g_A^{(8)} \simeq 0.6$. (Note that, at this level, relativistic quark-pion coupling models contain no explicit strange-quark or gluon degrees of freedom with the gluonic degrees of freedom understood to be integrated out into the scalar confinement potential.) The model prediction $g_A^{(8)} \simeq 0.6$ agrees with the value extracted from hyperon beta decays [$g_A^{(8)} = 0.58 \pm 0.03$ (Close and Roberts, 1993)] whereas the bag-model prediction for $g_A^{(0)}$ exceeds the measured value of $g_A^{(0)}|_{\mathrm{pDIS}}$ by a factor of 2–4.

The overall picture of the spin structure of the proton that has emerged from a combination of experiment and theoretical QCD studies can be summarized in the following key observations.

(1) Constituent quark-model predictions work remarkably well for the isovector part of the nucleon's g_1 spin structure function $(g_1^p - g_1^n)$: both for the first moment $\int_0^1 dx(g_1^p - g_1^n) \sim \frac{1}{6}g_A^{(3)}$, which is predicted by the Bjorken sum rule (Bjorken, 1966, 1970) and also over the whole presently measured range of Bjorken x (Bass, 1999). This includes the SLAC "small-x" region $(0.02 < x < 0.1)$—see Sec. IX.B—where one would, *a priori*, not necessarily expect quark-model results to apply. Constituent quark-model physics seems to be important in the spin structure of the proton probed at deep-inelastic Q^2! Furthermore, one finds the puzzling result that in the presently measured kinematics where accurate data exist the isovector part of g_1 considerably exceeds the isoscalar part of g_1 at small Bjorken x—the opposite to what is observed in unpolarized deep-inelastic scattering.

(2) In the singlet channel the first moment of the g_1 spin structure function for polarized photon-gluon fusion ($\gamma^* g \to q\bar{q}$) receives a negative contribution $-\alpha_s/2\pi$ from $k_t^2 \sim Q^2$, where k_t is the quark transverse momentum relative to the photon-gluon direction and Q^2 is the virtuality of the hard photon (Carlitz *et al.*, 1988). It also receives a positive contribution (proportional to the mass squared of the struck quark or antiquark) from low values of k_t, $k_t^2 \sim P^2$, m^2, where P^2 is the virtuality of the parent gluon and m is the mass of the struck quark. The contact interaction ($k_t \sim Q$) between the polarized photon and the gluon is flavor independent. It is associated with the QCD axial anomaly and measures the spin of the target gluon. The mass-dependent contribution is absorbed into the quark wave function of the nucleon.

(3) Gluon topology is associated with gluonic boundary conditions in the QCD vacuum and has the potential to induce a topological contribution to $g_A^{(0)}$ associated with Bjorken x equal to zero: topological $x=0$ polarization or, essentially, a spin "polarized condensate" inside a nucleon (Bass, 1998). This topology term is associated with a potential $J=1$ fixed pole in the real part of the spin-dependent part of the forward Compton amplitude and, if finite, is manifest as a "subtraction at infinity" in the dispersion relation for g_1 (Bass, 2003b). It is associated with dynamical axial U(1) symmetry breaking in the transition from constituent quarks to current quarks in QCD.

Summarizing these observations, QCD theoretical analysis leads to

$$g_A^{(0)} = \left(\sum_q \Delta q - 3\frac{\alpha_s}{2\pi}\Delta g \right)_{\text{partons}} + \mathcal{C}_\infty. \tag{10}$$

Here $\Delta g_{\text{partons}}$ is the amount of spin carried by polarized gluon partons in the polarized proton and $\Delta q_{\text{partons}}$ measures the spin carried by quarks and antiquarks carrying "soft" transverse momentum $k_t^2 \sim P^2$, m^2, where P is a typical gluon virtuality and m is the light-quark mass (Altarelli and Ross, 1988; Carlitz *et al.*, 1988; Efremov and Teryaev, 1988); $\mathcal{C}_\infty$ denotes the potential nonperturbative gluon topological contribution which has support only at Bjorken x equal to zero (Bass, 1998) so that it cannot be directly measured in polarized deep-inelastic scattering.

Since $\Delta g \sim 1/\alpha_s$ under QCD evolution, the polarized gluon term $[-(\alpha_s 2\pi)\Delta g]$ in Eq. (10) scales as $Q^2 \to \infty$ (Altarelli and Ross, 1988; Efremov and Teryaev, 1988). The polarized gluon contribution corresponds to two-quark-jet events carrying large transverse momentum $k_t \sim Q$ in the final state from photon-gluon fusion (Carlitz *et al.*, 1988).

The topological term $\mathcal{C}_\infty$ may be identified with a leading-twist subtraction at infinity in the dispersion relation for g_1 since $g_A^{(0)}|_{\text{pDIS}}$ is identified with $g_A^{(0)} - \mathcal{C}_\infty$ (Bass, 2003b). It probes the role of gluon topology in dynamical axial U(1) symmetry breaking in the transition from current to constituent quarks in low-energy QCD. The deep-inelastic measurement of $g_A^{(0)}$, Eq. (5), is not necessarily inconsistent with the constituent quark-model prediction 0.6 if a substantial fraction of the spin of the constituent quark is associated with gluon topology in the transition from constituent to current quarks (measured in polarized deep-inelastic scattering).

A direct measurement of the strange-quark axial charge, independent of the analysis of polarized deep-inelastic-scattering data and any possible subtraction at infinity correction, could be made using neutrino-proton elastic scattering through the axial coupling of the Z^0 gauge boson. Comparing the values of Δs extracted from high-energy polarized deep-inelastic scattering and low-energy νp elastic scattering will provide vital information about the QCD structure of the proton.

The vital role of quark transverse momentum in formula (10) means that it is essential to ensure that the theory and experimental acceptance are correctly matched when extracting information from semi-inclusive measurements aimed at disentangling the individual valence, sea, and gluonic contributions. For example, recent semi-inclusive measurements (Airapetian *et al.*, 2004, 2005a) using a forward detector and limited acceptance at large transverse momentum ($k_t \sim Q$) exhibit no evidence for the large negative polarized strangeness polarization extracted from inclusive data and may, perhaps, be more comparable with $\Delta q_{\text{partons}}$ than the inclusive measurement (6) which has the polarized gluon contribution included. Further semi-inclusive measurements with increased luminosity and a 4π detector would be valuable. On the theoretical side, when assessing models which attempt to explain the proton's spin structure it is important to look at the transverse-momentum dependence of the proposed dynamics plus the model predictions for the shape of the spin structure functions as a function of Bjorken x in addition to the first moment and the nucleon's axial charges $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}$.

New dedicated experiments are planned or underway to map out the spin-flavor structure of the proton and, especially, to measure the amount of spin carried by the valence and sea quarks and by polarized gluons in the polarized proton. These include semi-inclusive polarized deep-inelastic scattering (COMPASS at CERN and HERMES at DESY), polarized proton-proton collisions at the polarized proton-proton collider RHIC (Bunce *et al.*, 2004), and future polarized electron-proton collider studies (Bass and De Roeck, 2002). Experiments at Jefferson Laboratory are mapping out the spin distribution of quarks carrying a large fraction of the proton's momentum (Bjorken x) and promise to yield exciting new information on confinement-related dynamics.

Further interesting information about the structure of the proton will come from the study of "transversity" spin distributions (Barone *et al.*, 2002). Working in an infinite-momentum frame, these observables measure the distribution of spin polarized transverse to the momentum of the proton in a transversely polarized proton. Since rotations and Euclidean boosts commute and a series of boosts can convert a longitudinally polarized nucleon into a transversely polarized nucleon at infinite momentum, it follows that the difference between the transversity and helicity distributions reflects the relativistic character of quark motion in the nucleon. Furthermore, the transversity spin distribution of the nucleon is charge-parity odd ($C=-1$) and therefore valencelike (gluons decouple from its QCD evolution equation in contrast to the evolution equation for flavor-singlet quark distribution appearing in g_1) making a comparison of the different spin-dependent distributions most interesting. Studies of transversity-sensitive observables in lepton-nucleon and polarized proton-proton scattering are being performed by the HERMES (Airapetian *et al.*, 2005b), COMPASS, and RHIC (Adams *et al.*, 2004) experiments.

One would also like to measure the parton orbital angular momentum contributions to the proton's spin. Exclusive measurements of deeply virtual Compton scattering and single-meson production at large Q^2 offer a possible route to the quark and gluon angular momentum contributions through the physics and formalism of generalized parton distributions (Ji, 1998; Goeke *et al.*, 2001; Diehl, 2003). A vigorous program for studying these reactions is being designed and investigated at several major world laboratories.

B. Outline

This review is organized as follows. In the first part (Secs. II–VIII) we review the present status of the proton spin problem focusing on the present experimental situation for tests of polarized deep-inelastic spin sum rules and the theoretical understanding of $g_A^{(0)}$. In the second part (Secs. IX–XII) we give an overview of the present global program aimed at disentangling the spin-flavor structure of the proton and the exciting prospects for the new generation of experiments aimed at resolving the proton's internal spin structure. In Secs. II and III we give an overview of the spin sum rules for polarized photon-nucleon scattering, detailing the assumptions that are made at each step. Here we explain how these sum rules could be affected by potential subtraction constants (subtractions at infinity) in the dispersion relations for the spin-dependent part of the forward Compton amplitude. We next give a brief review of the partonic (Sec. IV) and possible fixed pole (Sec. V) contributions to deep-inelastic scattering. Fixed poles are well known to play a vital role in the Adler sum rule for W-boson nucleon scattering (Adler, 1966) and the Schwinger-term sum rule for the longitudinal structure function measured in unpolarized deep-inelastic ep scattering (Broadhurst *et al.*, 1973). We explain how fixed poles could, in principle, affect the sum rules for the first moments of the g_1 and g_2 spin structure functions. For example, a subtraction constant correction to the Ellis-Jaffe sum rule for the first moment of the nucleon's g_1 spin-dependent structure function would follow if there is a constant real term in the spin-dependent part of the deeply virtual forward Compton scattering amplitude. Section VI discusses the QCD axial anomaly and its possible role in understanding the first moment of g_1. The relationship between the spin structure of the proton and chiral symmetry is outlined in Sec. VII. This first part of the paper concludes with an overview in Sec. VIII of the different possible explanations of the small value of $g_A^{(0)}$ that have been proposed in the literature, how they relate to QCD, and possible future experimental tests which could help clarify the key issues. In the next part (Secs. IX–XIII) we focus on the new program to disentangle the proton's spin-flavor structure and the Bjorken-x dependence of the separate valence, sea, and gluonic contributions (Sec. IX), the theory and experimental investigation of transversity observables (Sec. X), quark orbital angular momentum and exclusive reactions (Sec. XI), and the g_1 spin structure function of the polarized photon (Sec. XII). A summary of key issues and challenging questions for the next generation of experiments is given in Sec. XIII.

Complementary review articles on the spin structure of the proton, each with a different emphasis, are given in Anselmino *et al.* (1995), Cheng (1996), Shore (1998), Lampe and Reya (2000), Fillipone and Ji (2001), Jaffe (2001), Barone *et al.* (2002), and Stoesslein (2002).

II. SCATTERING AMPLITUDES

In photon-nucleon scattering the spin-dependent structure functions g_1 and g_2 are defined through the imaginary part of the forward Compton scattering amplitude. Consider the amplitude for forward scattering of a photon carrying momentum q_μ ($q^2=-Q^2\leqslant 0$) from a polarized nucleon with momentum p_μ, mass M, and spin s_μ. Let $J_\mu(z)$ denote the electromagnetic current in QCD. The forward Compton amplitude

$$T_{\mu\nu}(q,p) = i\int d^4z e^{iq\cdot z}\langle p,s|T(J_\mu(z)J_\nu(0))|p,s\rangle \tag{11}$$

is given by the sum of spin-independent (symmetric in μ and ν) and spin-dependent (antisymmetric in μ and ν) contributions, viz.,

$$\begin{aligned} T^S_{\mu\nu} &= \frac{1}{2}(T_{\mu\nu}+T_{\nu\mu}) \\ &= -T_1\left(g_{\mu\nu}+\frac{q_\mu q_\nu}{Q^2}\right) \\ &\quad + \frac{1}{M^2}T_2\left(p_\mu+\frac{p\cdot q}{Q^2}q_\mu\right)\left(p_\nu+\frac{p\cdot q}{Q^2}q_\nu\right) \end{aligned} \tag{12}$$

and

$$T^A_{\mu\nu} = \frac{1}{2}(T_{\mu\nu} - T_{\nu\mu})$$
$$= \frac{i}{M^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left[s^\sigma\left(A_1 + \frac{\nu}{M}A_2\right) - \frac{1}{M^2}s\cdot qp^\sigma A_2\right]. \quad (13)$$

Here $\nu = p\cdot q/M$ and $\epsilon_{0123} = +1$; the proton spin vector is normalized to $s^2 = -1$. The form factors T_1, T_2, A_1, and A_2 are functions of ν and Q^2.

The hadron tensor for inclusive photon-nucleon scattering which contains the spin-dependent structure functions is obtained from the imaginary part of $T_{\mu\nu}$,

$$W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$$
$$= \frac{1}{2\pi}\int d^4z e^{iq\cdot z}\langle p,s|[J_\mu(z),J_\nu(0)]|p,s\rangle. \quad (14)$$

Here the connected matrix element is understood (indicating that the photon interacts with the target and not the vacuum). The spin-independent and spin-dependent components of $W_{\mu\nu}$ are

$$W^S_{\mu\nu} = -W_1\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) + \frac{1}{M^2}W_2\left(p_\mu + \frac{p\cdot q}{Q^2}q_\mu\right)$$
$$\times\left(p_\nu + \frac{p\cdot q}{Q^2}q_\nu\right) \quad (15)$$

and

$$W^A_{\mu\nu} = \frac{i}{M^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left[s^\sigma\left(G_1 + \frac{\nu}{M}G_2\right) - \frac{1}{M^2}s\cdot qp^\sigma G_2\right], \quad (16)$$

respectively. The structure functions contain all of the target-dependent information in the deep-inelastic process.

The cross sections for the absorption of a transversely polarized photon with spin polarized parallel $\sigma_{3/2}$ and antiparallel $\sigma_{1/2}$ to the spin of the (longitudinally polarized) target nucleon are

$$\sigma_{3/2} = \frac{4\pi^2\alpha}{\sqrt{\nu^2+Q^2}}\left[W_1 - \frac{\nu}{M^2}G_1 + \frac{Q^2}{M^3}G_2\right],$$

$$\sigma_{1/2} = \frac{4\pi^2\alpha}{\sqrt{\nu^2+Q^2}}\left[W_1 + \frac{\nu}{M^2}G_1 - \frac{Q^2}{M^3}G_2\right], \quad (17)$$

where we utilize usual conventions for the virtual-photon flux factor (Roberts, 1990). The spin-dependent and spin-independent parts of the inclusive photon-nucleon cross section are

$$\sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{\sqrt{\nu^2+Q^2}}\left[\frac{\nu}{M^2}G_1 - \frac{Q^2}{M^3}G_2\right] \quad (18)$$

and

$$\sigma_{1/2} + \sigma_{3/2} = \frac{8\pi^2\alpha}{\sqrt{\nu^2+Q^2}}W_1. \quad (19)$$

The G_2 spin structure function decouples from polarized photoproduction. For real photons ($Q^2=0$) one finds $\sigma_{1/2} - \sigma_{3/2} = (8\pi^2\alpha/M^2)G_1$. The cross section for the absorption of a longitudinally polarized photon is

$$\sigma_0 = \frac{4\pi^2\alpha}{\sqrt{\nu^2+Q^2}}W_L = \frac{4\pi^2\alpha}{\sqrt{\nu^2+Q^2}}\left[\left\{1 + \frac{\nu^2}{Q^2}\right\}W_2 - W_1\right]. \quad (20)$$

The W_2 structure function is measured in unpolarized lepton-nucleon scattering through the absorption of transversely and longitudinally polarized photons.

Our present knowledge about the high-energy spin structure of the nucleon comes from polarized deep-inelastic-scattering experiments. These experiments involve scattering a high-energy charged lepton beam from a nucleon target at large momentum transfer squared. One measures the inclusive cross section. The lepton beam (electrons at DESY, JLab, and SLAC and muons at CERN) is longitudinally polarized. The nucleon target may be either longitudinally or transversely polarized.

The relation between deep-inelastic lepton-nucleon cross sections and the virtual-photon-nucleon cross sections considered above is discussed and derived in various textbooks—see, e.g., Roberts (1990). Polarized deep-inelastic-scattering experiments have so far all been performed using a fixed target. Consider polarized ep scattering. We specialize to the target rest frame and let E denote the energy of the incident electron which is scattered through an angle θ to emerge in the final state with energy E'. Let $\uparrow\downarrow$ denote the longitudinal polarization of the electron beam. For a longitudinally polarized proton target (with spin denoted $\Uparrow\Downarrow$) the unpolarized and polarized differential cross sections are

$$\left(\frac{d^2\sigma\uparrow\Downarrow}{d\Omega dE'} + \frac{d^2\sigma\uparrow\Uparrow}{d\Omega dE'}\right)$$
$$= \frac{\alpha^2}{4E^2\sin^4\theta/2}\left[2\sin^2\frac{\theta}{2}W_1 + \cos^2\frac{\theta}{2}W_2\right] \quad (21)$$

and

$$\left(\frac{d^2\sigma\uparrow\Downarrow}{d\Omega dE'} - \frac{d^2\sigma\uparrow\Uparrow}{d\Omega dE'}\right)$$
$$= \frac{4\alpha^2}{M^3Q^2}\frac{E'}{E}[M(E + E'\cos\theta)G_1 - Q^2G_2]. \quad (22)$$

For a target polarized transverse to the electron beam the spin-dependent part of the differential cross section is

$$\left(\frac{d^2\sigma\uparrow\Rightarrow}{d\Omega dE'} - \frac{d^2\sigma\uparrow\Leftarrow}{d\Omega dE'}\right)$$
$$= \frac{4\alpha^2}{M^3Q^2}\frac{E'^2}{E}\sin\theta[MG_1 + 2EG_2]. \quad (23)$$

A. Scaling and polarized deep-inelastic scattering

In high Q^2 deep-inelastic scattering the structure functions exhibit approximate scaling. One finds

$$MW_1(\nu,Q^2) \to F_1(x,Q^2),$$

$$\nu W_2(\nu,Q^2) \to F_2(x,Q^2),$$

$$\frac{\nu}{M} G_1(\nu,Q^2) \to g_1(x,Q^2),$$

$$\frac{\nu^2}{M^2} G_2(\nu,Q^2) \to g_2(x,Q^2). \tag{24}$$

The structure functions F_1, F_2, g_1, and g_2 are to a very good approximation independent of Q^2 and depend only on x. (The small Q^2 dependence which is present in these structure functions is logarithmic and determined by perturbative QCD evolution.) Substituting Eq. (24) in the cross-section formula (22) for the longitudinally polarized target one finds that the g_2 contribution to the differential cross section and the longitudinal spin asymmetry is suppressed relative to the g_1 contribution by the kinematic factor $M/E \sim 0$, viz.,

$$\mathcal{A}_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{M\nu G_1 - Q^2 G_2}{M^3 W_1}$$

$$= \frac{g_1 - Q^2 g_2/\nu^2}{F_1} \to \frac{g_1}{F_1}. \tag{25}$$

For a transverse-polarized target this kinematic suppression factor for g_2 is missing meaning that transverse polarization is vital to measure g_2. We refer the reader to Roberts (1990) and Windmolders (2002) for the procedure of how the spin-dependent structure functions are extracted from the spin asymmetries measured in polarized deep-inelastic scattering.

In the (pre-QCD) parton model the deep-inelastic structure functions F_1 and F_2 are written as

$$F_1(x) = \frac{1}{2x} F_2(x) = \frac{1}{2} \sum_q e_q^2 \{q + \bar{q}\}(x) \tag{26}$$

and the polarized structure function g_1 is

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x). \tag{27}$$

Here e_q denotes the electric charge of the struck quark and

$$\{q + \bar{q}\}(x) = (q^\uparrow + \bar{q}^\uparrow)(x) + (q^\downarrow + \bar{q}^\downarrow)(x),$$

$$\Delta q(x) = (q^\uparrow + \bar{q}^\uparrow)(x) - (q^\downarrow + \bar{q}^\downarrow)(x) \tag{28}$$

denote the spin-independent (unpolarized) and spin-dependent quark parton distributions which measure the distribution of quark momentum and spin in the proton. For example, $\bar{q}^\uparrow(x)$ is interpreted as the probability of finding an antiquark of flavor q with plus component of momentum xp_+ (p_+ is the plus component of the target proton's momentum) and spin polarized in the same direction as the spin of the target proton. When we integrate out the momentum fraction x the quantity $\Delta q = \int_0^1 dx\, \Delta q(x)$ is interpreted as the fraction of the proton's spin which is carried by quarks (and antiquarks) of flavor q—hence the parton-model interpretation of $g_A^{(0)}$ as the total fraction of the proton's spin carried by up, down, and strange quarks. In QCD the flavor-singlet combination of these quark parton distributions mixes with the spin-independent and spin-dependent gluon distributions, respectively, under Q^2 evolution. The gluon parton distributions measure the momentum and spin dependence of glue in the proton. The second spin structure function g_2 has a nontrivial parton interpretation (Jaffe, 1990) and vanishes without the effect of quark transverse momentum—see, e.g., Roberts (1990).

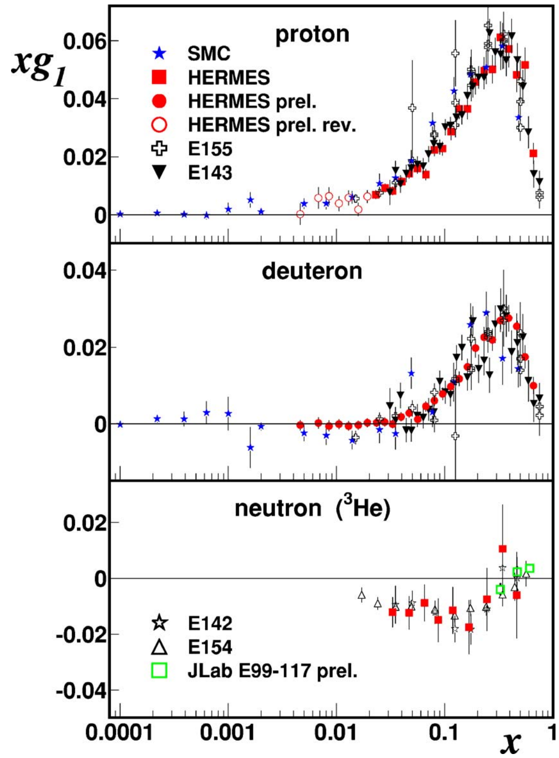

FIG. 1. (Color online) The world data on xg_1 with data points shown at the Q^2 at which they were measured. Figure courtesy of U. Stoesslein.

An overview of the world data on the nucleon's g_1 spin structure function is shown in Fig. 1 (which shows xg_1) and Fig. 2 (which shows g_1). There is a general consistency between all data sets. The largest range is provided by the SMC experiment (Adeva *et al.*, 1998a, 1999), namely, $0.000\,06 < x < 0.8$ and $0.02 < Q^2 < 100\ \text{GeV}^2$. This experiment used proton and deuteron targets with 100–200-GeV muon beams. The final results are given in the paper by Adeva *et al.* (1998a). The low-x data from SMC (Adeva *et al.*, 1999) are at a Q^2 well below 1 GeV^2, and the asymmetries are found to be compatible with zero. The most precise data come from the electron-scattering experiments at SLAC [E154 on the neutron (Abe *et al.*, 1997) and E155 on the proton

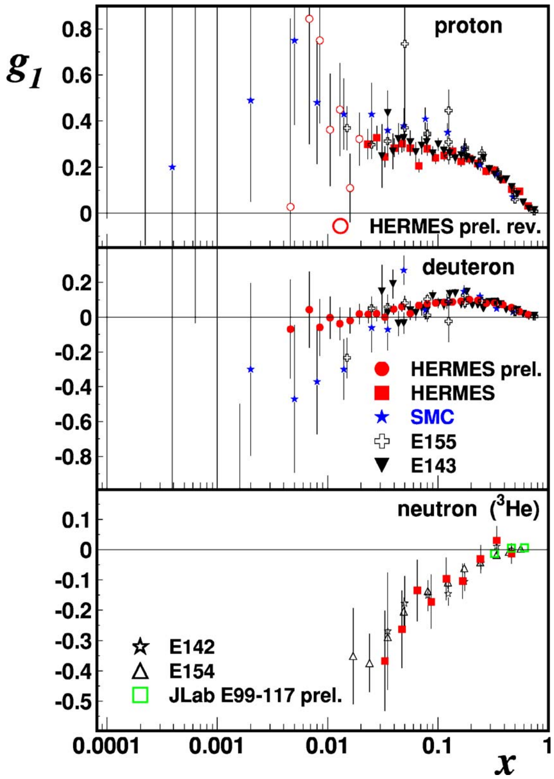

FIG. 2. (Color online) The world data on g_1 with data points shown at the Q^2 at which they were measured. Figure courtesy of U. Stoesslein.

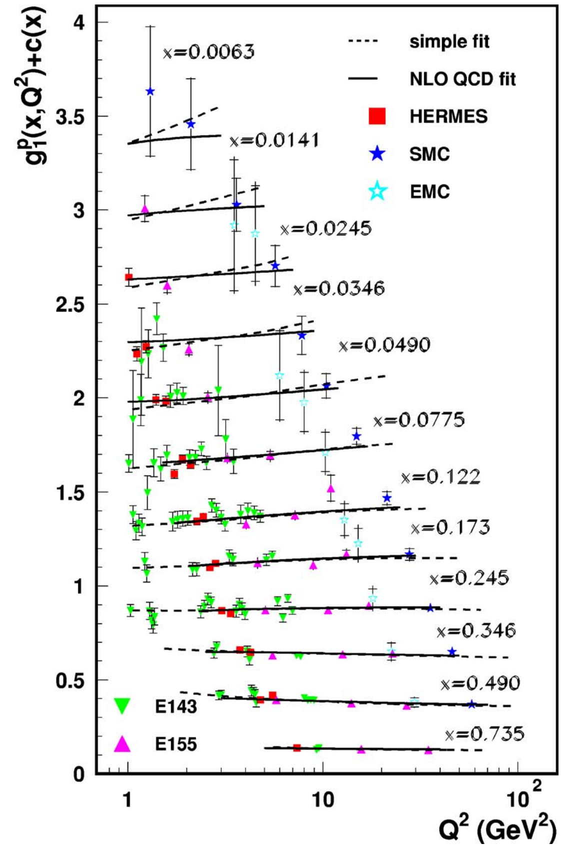

FIG. 3. (Color online) Q^2 dependence of g_1^p for $Q^2 > 1$ GeV^2 together with a simple fit according to Anthony *et al.* (2000) and a next-to-leading-order (NLO) perturbative QCD fit from Stoesslein (2002).

(Anthony *et al.*, 1999, 2000)], JLab (Zheng *et al.*, 2004a, 2004b, on the neutron), and HERMES at DESY (Ackerstaff *et al.*, 1997; Airapetian *et al.*, 1998, on the proton and neutron), with JLab focused on the large-x region. The methods for extracting the neutron's spin structure function from experiments using a deuteron or ^{3}He target are discussed in Piller and Weise (2000) and Thomas (2002).

Note the large isovector component in the data at small x (between 0.01 and 0.1) which considerably exceeds the isoscalar component in the measured kinematics. This result is in stark contrast to the situation in the unpolarized structure function F_2 where the small-x region is dominated by isoscalar pomeron exchange. Given the large experimental errors on the data little can presently be concluded about g_1 at the smallest x values (x less than about 0.006).

The structure-function data at different values of x (Figs. 1 and 2) are measured at different Q^2 values in the experiments, viz., $x_{\text{expt.}} = x(Q^2)$. For the ratios g_1/F_1 there is no experimental evidence of Q^2 dependence in any given x bin. The E155 Collaboration at SLAC found the following good phenomenological fit to their final data set with $Q^2 > 1$ GeV^2 and energy of the hadronic final state $W > 2$ GeV (Anthony *et al.*, 2000):

$$\frac{g_1^p}{F_1^p} = x^{0.700}(0.817 + 1.014x - 1.489x^2) \times \left(1 + \frac{c^p}{Q^2}\right),$$

$$\frac{g_1^n}{F_1^n} = x^{-0.335}(-0.013 - 0.330x + 0.761x^2) \times \left(1 + \frac{c^n}{Q^2}\right). \tag{29}$$

The coefficients $c^p = -0.04 \pm 0.06$ and $c^n = 0.13 \pm 0.45$ describing the Q^2 dependence are found to be small and consistent with zero. The Q^2 dependence of the g_1 spin structure function is shown in Fig. 3. It is useful to compare data at the same Q^2, e.g., for the comparison of experimental data with the predictions of deep-inelastic sum rules. To this end, the measured x points are shifted to the same Q^2 using either the (approximate) Q^2 independence of the asymmetry or performing next-to-leading-order QCD-motivated fits (Gehrmann and Stirling, 1996; Altarelli *et al.*, 1997; Adeva *et al.*, 1998b; Anthony *et al.*, 2000; Goto *et al.*, 2000; Glück *et al.*, 2001; Blümlein and Böttcher, 2002; Leader *et al.*, 2002; Hirai *et al.*, 2004) to the measured data and evolving the measured data points all to the same value of Q^2.

B. Regge theory and the small-x behavior of spin structure functions

The small-x or high-energy behavior of spin structure functions is an important issue both for the extrapolation of data needed to test spin sum rules for the first moment of g_1 and also in its own right.

Regge theory makes predictions for the high-energy asymptotic behavior of the structure functions:

$$W_1 \sim \nu^{\alpha},$$

$$W_2 \sim \nu^{\alpha-2},$$

$$G_1 \sim \nu^{\alpha-1},$$

$$G_2 \sim \nu^{\alpha-1}. \tag{30}$$

Here α denotes the (effective) intercept for the leading Regge exchange contributions. The Regge predictions for the leading exchanges include $\alpha=1.08$ for the Pomeron contributions to W_1 and W_2, and $\alpha \simeq 0.5$ for the ρ and ω exchange contributions to the spin-independent structure functions.

For G_1 the leading gluonic exchange behaves as $\{\ln \nu\}/\nu$ (Bass and Landshoff, 1994; Close and Roberts, 1994). In the isovector and isoscalar channels there are also isovector a_1 and isoscalar f_1 Regge exchanges plus contributions from the Pomeron a_1 and Pomeron f_1 cuts (Heimann, 1973). If one makes the usual assumption that the a_1 and f_1 Regge trajectories are straight lines parallel to the (ρ,ω) trajectories then one finds $\alpha_{a_1} \simeq \alpha_{f_1} \simeq -0.4$, within the phenomenological range $-0.5 \leqslant \alpha_{a_1} \leqslant 0$ discussed in Ellis and Karliner (1988). Taking the masses of the $a_1(1260)$ and $a_3(2070)$ states plus the $a_1(1640)$ and $a_3(2310)$ states from the Particle Data Group (Eidelman *et al.*, 2004) yields two parallel a_1 trajectories with slope $\sim 0.75\ \mathrm{GeV}^{-2}$ and a leading trajectory with slightly lower intercept: $\alpha_{a_1} \simeq -0.18$.

For this value of the a_1 intercept the effective intercepts corresponding to the soft Pomeron a_1 cut and the hard Pomeron a_1 cut are $\simeq -0.1$ and $\simeq +0.25$, respectively, if one takes the soft and hard Pomerons as two distinct exchanges (Cudell *et al.*, 1999).[1] In the framework of the Donnachie-Landshoff-Nachtmann model of soft Pomeron physics (Landshoff and Nachtmann, 1987; Donnachie and Landshoff, 1988), the logarithm in the $\ln \nu/\nu$ contribution comes from the region of internal momentum where two nonperturbative gluons are radiated collinear with the proton (Bass and Landshoff, 1994).

For G_2 one expects contributions from possible multi-Pomeron (three or more) cuts $[\sim (\ln \nu)^{-5}]$ and Regge Pomeron cuts $(\sim \nu^{\alpha_i(0)-1}/\ln \nu)$ with $\alpha_i(0)<1$ (since the Pomeron does not couple to A_1 or A_2 as a single gluonic exchange)—see Ioffe *et al.* (1984).

In terms of the scaling structure functions of deep-inelastic scattering the relations (30) become

$$F_1 \sim \frac{1}{x^{\alpha}},$$

$$F_2 \sim \frac{1}{x^{\alpha-1}},$$

$$g_1 \sim \frac{1}{x^{\alpha}},$$

$$g_2 \sim \frac{1}{x^{\alpha+1}}. \tag{31}$$

For deep-inelastic values of Q^2 there is some debate about the application of Regge arguments. In the conventional approach the effective intercepts for small-x or high-ν physics tend to increase with increasing Q^2 through perturbative QCD evolution which acts to shift the weight of the structure functions to smaller x. The polarized isovector combination $g_1^p - g_1^n$ is observed to rise in the small-x data from SLAC and SMC as $\sim x^{-0.5}$ although it should be noted that, in the measured x range, this exponent could be softened through multiplication by a $(1-x)^n$ factor—for example, associated with perturbative QCD counting rules at large x (x close to 1). For example, the exponent $x^{-0.5}$ could be modified to about $x^{-0.25}$ through multiplication by a factor $(1-x)^6$. In an alternative approach Cudell *et al.* (1999) have argued that the Regge intercepts should be independent of Q^2 and that the hard Pomeron revealed in unpolarized deep-inelastic scattering at HERA is a distinct exchange independent of the soft Pomeron which should also be present in low-Q^2 photoproduction data.

Detailed investigation of spin-dependent Regge theory and the low-x behavior of spin structure functions could be performed at SLAC or using a future polarized ep collider (e-RHIC) where measurements could be obtained through a broad range of Q^2 from photoproduction through the "transition region" to polarized deep-inelastic scattering. These measurements would provide a baseline for investigations of perturbative QCD-motivated small-x behavior in g_1. Open questions include: Does the rise in $g_1^p - g_1^n$ at small Bjorken x persist to small values of Q^2? How does this rise develop as a function of Q^2? Further possible exchange contributions in the flavor-singlet sector associated with polarized glue could also be looked for. For example, color coherence predicts that the ratio of polarized to unpolarized gluon distributions $\Delta g(x)/g(x) \propto x$ as $x \to 0$ (Brodsky *et al.* 1995) suggesting that, perhaps, there is a spin analog of the hard Pomeron with intercept about 0.45 corresponding to the polarized gluon distribution.

The s and t dependence of spin-dependent Regge theory is being investigated by the pp2pp experiment (Bültmann *et al.*, 2003) at RHIC which is studying polarized proton-proton elastic scattering at center-of-mass energies $50 < \sqrt{s} < 500$ GeV and four-momentum transfer $0.0004 < |t| < 1.3\ \mathrm{GeV}^2$.

[1] I thank P. V. Landshoff for valuable discussions on this issue.

III. DISPERSION RELATIONS AND SPIN SUM RULES

Sum rules for the (spin) structure functions measured in deep-inelastic scattering are derived using dispersion relations and the operator product expansion. For fixed Q^2 the forward Compton scattering amplitude $T_{\mu\nu}(\nu,Q^2)$ is analytic in the photon energy ν except for branch cuts along the positive real axis for $|\nu| \geq Q^2/2M$. Crossing symmetry implies that

$$A_1^*(Q^2,-\nu) = A_1(Q^2,\nu),$$

$$A_2^*(Q^2,-\nu) = -A_2(Q^2,\nu). \tag{32}$$

The spin structure functions in the imaginary parts of A_1 and A_2 satisfy the crossing relations

$$G_1(Q^2,-\nu) = -G_1(Q^2,\nu),$$

$$G_2(Q^2,-\nu) = +G_2(Q^2,\nu). \tag{33}$$

For g_1 and g_2 these relations become

$$g_1(x,Q^2) = +g_1(-x,Q^2),$$

$$g_2(x,Q^2) = +g_2(-x,Q^2). \tag{34}$$

We use Cauchy's integral theorem and the crossing relations to derive dispersion relations for A_1 and A_2. Assuming that the asymptotic behavior of the spin structure functions G_1 and G_2 yield convergent integrals we write the two unsubtracted dispersion relations:

$$A_1(Q^2,\nu) = \frac{2}{\pi}\int_{Q^2/2M}^{\infty} \frac{\nu' d\nu'}{\nu'^2 - \nu^2} \,\mathrm{Im}\, A_1(Q^2,\nu'),$$

$$A_2(Q^2,\nu) = \frac{2}{\pi}\nu\int_{Q^2/2M}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} \,\mathrm{Im}\, A_2(Q^2,\nu'). \tag{35}$$

These expressions can be rewritten as dispersion relations involving g_1 and g_2. We define

$$\alpha_1(\omega,Q^2) = \frac{\nu}{M}A_1,$$

$$\alpha_2(\omega,Q^2) = \frac{\nu^2}{M^2}A_2. \tag{36}$$

Then, the formulas in Eq. (35) become

$$\alpha_1(\omega,Q^2) = 2\omega\int_1^{\infty} \frac{d\omega'}{\omega'^2 - \omega^2} g_1(\omega',Q^2),$$

$$\alpha_2(\omega,Q^2) = 2\omega^3\int_1^{\infty} \frac{d\omega'}{\omega'^2(\omega'^2 - \omega^2)} g_2(\omega',Q^2), \tag{37}$$

where $\omega = 1/x = 2M\nu/Q^2$.

In general there are two alternatives to an unsubtracted dispersion relation:

(1) First, if the high-energy behavior of G_1 and/or G_2 (at some fixed Q^2) produced a divergent integral, then the dispersion relation would require a subtraction. Regge predictions for the high-energy behavior of G_1 and G_2—see Eq. (30)—each lead to convergent integrals so this scenario is not expected to occur, even after including the possible effects of QCD evolution.

(2) Second, even if the integral in the unsubtracted relation converges, there is still the potential for a subtraction at infinity. This scenario would occur if the real part of A_1 and/or A_2 does not vanish sufficiently fast enough when $\nu \to \infty$ so that we pick up a finite contribution from the contour (or "circle at infinity"). In the context of Regge theory such subtractions can arise from fixed poles [with $J=\alpha(t)=0$ in A_2 or $J=\alpha(t)=1$ in A_1 for all t] in the real part of the forward Compton amplitude. We shall discuss these fixed poles and potential subtractions in Sec. V.

In the presence of a potential subtraction at infinity the dispersion relations (35) are modified to

$$A_1(Q^2,\nu) = \mathcal{P}_1(\nu,Q^2) + \frac{2}{\pi}\int_{Q^2/2M}^{\infty} \frac{\nu' d\nu'}{\nu'^2 - \nu^2} \,\mathrm{Im}\, A_1(q^2,\nu'),$$

$$A_2(Q^2,\nu) = \mathcal{P}_2(\nu,Q^2) + \frac{2}{\pi}\nu\int_{Q^2/2M}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} \,\mathrm{Im}\, A_2(q^2,\nu'). \tag{38}$$

Here $\mathcal{P}_1(\nu,Q^2)$ and $\mathcal{P}_2(\nu,Q^2)$ denote the subtraction constants. Factoring out the ν dependence of these subtraction constants, we define two ν-independent quantities $\beta_1(Q^2)$ and $\beta_2(Q^2)$:

$$\mathcal{P}_1(\nu,Q^2) = \beta_1(Q^2),$$

$$\mathcal{P}_2(\nu,Q^2) = \beta_2(Q^2)\frac{M}{\nu}. \tag{39}$$

The crossing relations (32) for A_1 and A_2 are observed by the functions $\mathcal{P}_i$. Scaling requires that $\beta_1(Q^2)$ and $\beta_2(Q^2)$ (if finite) must be nonpolynomial in Q^2—see Sec. V. Equations (38) can be rewritten

$$\alpha_1(\omega,Q^2) = \frac{Q^2}{2M^2}\beta_1(Q^2)\omega + 2\omega\int_1^{\infty} \frac{d\omega'}{\omega'^2 - \omega^2} g_1(\omega',Q^2),$$

$$\alpha_2(\omega,Q^2) = \frac{Q^2}{2M^2}\beta_2(Q^2)\omega + 2\omega^3\int_1^{\infty} \frac{d\omega'}{\omega'^2(\omega'^2 - \omega^2)} g_2(\omega',Q^2). \tag{40}$$

Next, the fact that both α_1 and α_2 are analytic for $|\omega| \leq 1$ allows us to make the Taylor-series expansions (about $\omega = 0$)

$$\alpha_1(x,Q^2) = \frac{Q^2}{2M^2}\beta_1(Q^2)\frac{1}{x} + \frac{2}{x}\sum_{n=0,2,4,\ldots}\left(\frac{1}{x^n}\right)\int_0^1 dy y^n g_1(y,Q^2),$$

$$\alpha_2(x,Q^2) = \frac{Q^2}{2M^2}\beta_2(Q^2)\frac{1}{x} + \frac{2}{x^3}\sum_{n=0,2,4,\ldots}\left(\frac{1}{x^n}\right)\int_0^1 dy y^{n+2} g_2(y,Q^2), \tag{41}$$

with $x=1/\omega$.

These equations form the basis for the spin sum rules for polarized photon-nucleon scattering. We next outline the derivation of the Bjorken (1966, 1970) and Ellis-Jaffe (1974) sum rules for the isovector and flavor-singlet parts of g_1 in polarized deep-inelastic scattering, the Burkhardt-Cottingham sum rule for G_2 (Burkhardt and Cottingham, 1970), and the Gerasimov-Drell-Hearn sum rule for polarized photoproduction (Drell and Hearn, 1966; Gerasimov, 1966). Each of these spin sum rules assumes no subtraction at infinity.

A. Deep-inelastic spin sum rules

Sum rules for polarized deep-inelastic scattering are derived by combining the dispersion-relation expressions (41) with the light-cone operator production expansion. When $Q^2\to\infty$ the leading contribution to the spin-dependent part of the forward Compton amplitude comes from the nucleon matrix elements of a tower of gauge-invariant local operators multiplied by Wilson coefficients, viz.,

$$T^A_{\mu\nu} = i\epsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{n=0,2,4,\ldots}\left(-\frac{2}{q^2}\right)^{n+1} q^{\mu_1}q^{\mu_2}\cdots q^{\mu_n} \times \sum_{i=q,g}\Theta^{(i)}_{\sigma\{\mu_1\cdots\mu_n\}}E^i_n\left(\frac{Q^2}{\mu^2},\alpha_s\right), \tag{42}$$

where

$$\Theta^{(q)}_{\sigma\{\mu_1\cdots\mu_n\}} \equiv i^n\bar\psi\gamma_\sigma\gamma_5 D_{\{\mu_1}\ldots D_{\mu_n\}}\psi - \mathrm{tr} \tag{43}$$

and

$$\Theta^{(g)}_{\sigma\{\mu_1\cdots\mu_n\}} \equiv i^{n-1}\epsilon_{\alpha\beta\gamma\sigma}G^{\beta\gamma}D_{\{\mu_1}\ldots D_{\mu_{n-1}}G^{\alpha}_{\ \mu_n\}} - \mathrm{tr} \tag{44}$$

are local operators. Here $D_\mu=\partial_\mu+igA_\mu$ is the gauge-covariant derivative and the sum over even values of n in Eq. (42) reflects the crossing-symmetry properties of $T_{\mu\nu}$. The functions $E^q_n(Q^2/\mu^2,\alpha_s)$ and $E^g_n(Q^2/\mu^2,\alpha_s)$ are the respective Wilson coefficients. (Note that, for simplicity, in this discussion we consider the case of a single quark flavor with unit charge and zero quark mass. The results stated in Sec. III.B include the extra steps of using the full electromagnetic current in QCD.)

The operators in Eq. (42) may each be written as the sum of a totally symmetric operator and an operator with mixed symmetry,

$$\Theta_{\sigma\{\mu_1\cdots\mu_n\}} = \Theta_{\{\sigma\mu_1\cdots\mu_n\}} + \Theta_{[\sigma,\{\mu_1]\cdots\mu_n\}}. \tag{45}$$

These operators have the matrix elements

$$\langle p,s|\Theta_{\{\sigma\mu_1\cdots\mu_n\}}|p,s\rangle = \{s_\sigma p_{\mu_1}\cdots p_{\mu_n} + s_{\mu_1}p_\sigma p_{\mu_2}\cdots p_{\mu_n} + \cdots\}\frac{a_n}{n+1},$$

$$\langle p,s|\Theta_{[\{\sigma\mu_1]\cdots\mu_n\}}|p,s\rangle = \{(s_\sigma p_{\mu_1} - s_{\mu_1}p_\sigma)p_{\mu_2}\cdots p_{\mu_n} + (s_\sigma p_{\mu_2} - s_{\mu_2}p_\sigma)p_{\mu_1}\cdots p_{\mu_n} + \cdots\}\frac{d_n}{n+1}. \tag{46}$$

Now define $\tilde a_n = a^{(q)}_n E^q_{1n} + a^{(g)}_n E^g_{1n}$ and $\tilde d_n = d^{(q)}_n E^q_{2n} + d^{(g)}_n E^g_{2n}$, where E^i_{1n} and E^i_{2n} are the Wilson coefficients for a^i_n and d^i_n, respectively. Combining Eqs. (42) and (46) one obtains the following equations for α_1 and α_2:

$$\alpha_1(x,Q^2) + \alpha_2(x,Q^2) = \sum_{n=0,2,4,\ldots}\frac{\tilde a_n + n\tilde d_n}{n+1}\frac{1}{x^{n+1}}$$

$$\alpha_2(x,Q^2) = \sum_{n=2,4,\ldots}\frac{n(\tilde d_n - \tilde a_n)}{n+1}\frac{1}{x^{n+1}}. \tag{47}$$

These equations are compared with the Taylor-series expansions (41), from which we obtain the moment sum rules for g_1 and g_2:

$$\int_0^1 dx x^n g_1 = \frac{1}{2}\tilde a_n - \delta_{n0}\frac{1}{2}\frac{Q^2}{2M^2}\beta_1(Q^2) \tag{48}$$

for$=0,2,4,\ldots$ and

$$\int_0^1 dx x^n g_2 = \frac{1}{2}\frac{n}{n+1}(\tilde d_n - \tilde a_n) \tag{49}$$

for $n=2,4,6,\ldots$.

Note that

(1) The first moment of g_1 is given by the nucleon matrix element of the axial-vector current $\bar\psi\gamma_\sigma\gamma_5\psi$. There is no twist-2, spin-1, gauge-invariant, local gluon operator to contribute to the first moment of g_1 (Jaffe and Manohar, 1990).

(2) The potential subtraction term $(Q^2/2M^2)\beta_1(Q^2)$ in the dispersion relation in Eq. (41) multiplies a $1/x$ term in the series expansion on the left-hand side, and thus provides a potential correction factor to sum rules for the first moment of g_1. It follows that the first moment of g_1 measured in polarized deep-inelastic scattering measures the nucleon matrix element of the axial-vector current up to this potential subtraction at infinity term, which corresponds to the residue of any $J=1$ fixed pole with nonpolynomial residue contribution to the real part of A_1.

(3) There is no $1/x$ term in the operator product expansion formula (47) for $\alpha_2(x,Q^2)$. This is matched by the lack of any $1/x$ term in the unsubtracted version of the dispersion relation (41). The operator product expansion provides no information about the first moment of g_2 without additional assumptions concerning analytic continuation and the $x\sim 0$ behavior of g_2 (Jaffe, 1990). We shall return to this discussion in the context of the Burkhardt-Cottingham sum rule for g_2 in Sec. III.D.

If there are finite subtraction constant corrections to one (or more) spin sum rules, one can include the correction by reinterpreting the relevant structure function as a distribution with the subtraction constant included as twice the coefficient of a $\delta(x)$ term (Broadhurst *et al.*, 1973).

B. g_1 spin sum rules in polarized deep-inelastic scattering

The value of $g_A^{(0)}$ extracted from polarized deep-inelastic scattering is obtained as follows. One includes the sum over quark charges squared in $W_{\mu\nu}$ and assumes no twist-2 subtraction constant $[\beta_1(Q^2)=O(1/Q^4)]$. The first moment of the structure function g_1 is then related to the scale-invariant axial charges of the target nucleon by

$$\begin{aligned}\int_0^1 dx g_1^p(x,Q^2) &= \left(\frac{1}{12}g_A^{(3)} + \frac{1}{36}g_A^{(8)}\right)\left\{1+\sum_{\ell\geq 1} c_{\mathrm{NS}\ell}\alpha_s^\ell(Q)\right\} \\ &+ \frac{1}{9}\, g_A^{(0)}|_{\mathrm{inv}}\left\{1+\sum_{\ell\geq 1} c_{\mathrm{S}\ell}\alpha_s^\ell(Q)\right\} \\ &+ \mathcal{O}\left(\frac{1}{Q^2}\right) - \beta_1(Q^2)\frac{Q^2}{4M^2}. \end{aligned} \tag{50}$$

Here $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}|_{\mathrm{inv}}$ are the isotriplet, SU(3) octet, and scale-invariant flavor-singlet axial charges, respectively. The flavor-nonsinglet $c_{\mathrm{NS}\ell}$ and singlet $c_{\mathrm{S}\ell}$ Wilson coefficients are calculable in ℓ-loop perturbative QCD (Larin *et al.*, 1997). One then assumes no twist-2 subtraction constant $[\beta_1(Q^2)=O(1/Q^4)]$ so that the axial charge contributions saturate the first moment at leading twist.

The first moment of g_1 is constrained by low-energy weak interactions. For proton states $|p,s\rangle$ with momentum p_μ and spin s_μ,

$$\begin{aligned} 2Ms_\mu g_A^{(3)} &= \langle p,s|(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)|p,s\rangle, \\ 2Ms_\mu g_A^{(8)} &= \langle p,s|(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)|p,s\rangle. \end{aligned} \tag{51}$$

Here $g_A^{(3)}=1.2695\pm 0.0029$ is the isotriplet axial charge measured in neutron beta decay; $g_A^{(8)}=0.58\pm 0.03$ is the octet charge measured independently in hyperon beta decays [and SU(3)] (Close and Roberts, 1993). The assumption of good SU(3) here is supported by the recent KTeV measurement (Alavi-Harati *et al.*, 2001) of the Ξ^0 beta decay $\Xi^0\to\Sigma^+ e\bar{\nu}$. The nonsinglet axial charges are scale invariant.

The scale-invariant flavor-singlet axial charge $g_A^{(0)}|_{\mathrm{inv}}$ is defined by

$$2Ms_\mu\, g_A^{(0)}|_{\mathrm{inv}} = \langle p,s|E(\alpha_s)J_{\mu 5}^{GI}|p,s\rangle, \tag{52}$$

where

$$J_{\mu 5}^{GI} = (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)_{GI} \tag{53}$$

is the gauge-invariantly renormalized singlet axial-vector operator and

$$E(\alpha_s) = \exp\left(\int_0^{\alpha_s} d\tilde{\alpha}_s \gamma(\tilde{\alpha}_s)/\beta(\tilde{\alpha}_s)\right) \tag{54}$$

is a renormalization-group factor which corrects for the (two-loop) nonzero anomalous dimension $\gamma(\alpha_s)$ (Kodaira, 1980) of $J_{\mu 5}^{GI}$ and which goes to 1 in the limit $Q^2\to\infty$; $\beta(\alpha_s)$ is the QCD beta function. We are free to choose the QCD coupling $\alpha_s(\mu)$ at either a hard or a soft scale μ. The singlet axial charge $g_A^{(0)}|_{\mathrm{inv}}$ is independent of the renormalization scale μ and corresponds to the three flavor $g_A^{(0)}(Q^2)$ evaluated in the limit $Q^2\to\infty$. If we take $\alpha_s(\mu_0^2)\sim 0.6$ as typical of the infrared region of QCD, then the renormalization-group factor $E(\alpha_s)\simeq 1-0.13-0.03=0.84$ where -0.13 and -0.03 are the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ corrections, respectively.

In terms of the flavor-dependent axial charges

$$2Ms_\mu\Delta q = \langle p,s|\bar{q}\gamma_\mu\gamma_5 q|p,s\rangle \tag{55}$$

the isovector, octet, and singlet axial charges are

$$\begin{aligned} g_A^{(3)} &= \Delta u - \Delta d, \\ g_A^{(8)} &= \Delta u + \Delta d - 2\Delta s, \\ g_A^{(0)} &\equiv g_A^{(0)}|_{\mathrm{inv}}/E(\alpha_s) = \Delta u + \Delta d + \Delta s. \end{aligned} \tag{56}$$

The perturbative QCD coefficients in Eq. (50) have been calculated to $O(\alpha_s^3)$ precision (Larin *et al.*, 1997). For three flavors the coefficients are

$$\begin{aligned} \left\{1+\sum_{\ell\geq 1} c_{\mathrm{NS}\ell}\alpha_s^\ell(Q)\right\} &= \left[1-\left(\frac{\alpha_s}{\pi}\right) - 3.583\,33\left(\frac{\alpha_s}{\pi}\right)^2 - 20.215\,27\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right], \\ \left\{1+\sum_{\ell\geq 1} c_{\mathrm{S}\ell}\alpha_s^\ell(Q)\right\} &= \left[1-0.333\,33\left(\frac{\alpha_s}{\pi}\right) - 0.549\,59\left(\frac{\alpha_s}{\pi}\right)^2 - 4.447\,25\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right]. \end{aligned} \tag{57}$$

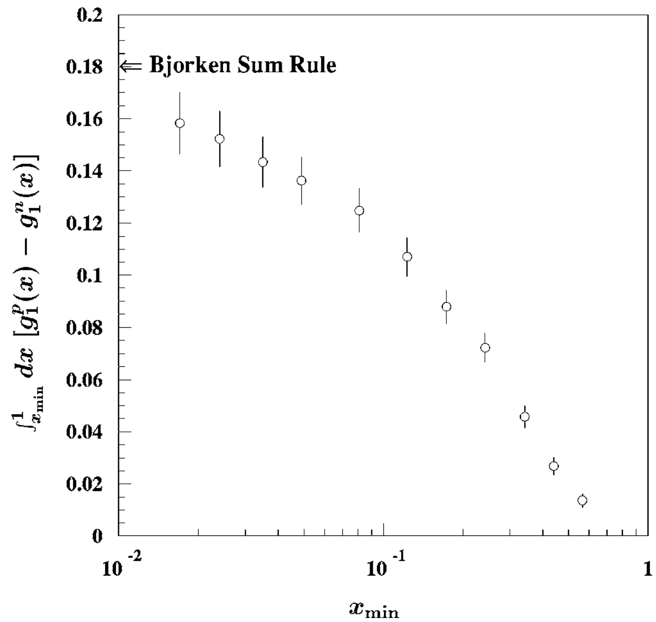

FIG. 4. Difference between the measured proton (SLAC E143) and neutron (SLAC E154) integrals calculated from a minimum x value, $x_{\min}$ up to x of 1. The value is compared to the theoretical prediction from the Bjorken sum rule which makes a prediction over the full x range. For the prediction, the Bjorken sum rule is evaluated up to third order in α_s (Larin *et al.*, 1997) and at $Q^2=5\ (\mathrm{GeV}/c)^2$. Error bars on the data are dominated by systematic uncertainties and are highly correlated point to point. Figure from Abe *et al.*, 1997.

In the isovector channel the Bjorken sum rule (1996, 1970)

$$I_{\mathrm{Bj}} = \int_0^1 dx(g_1^p - g_1^n) = \frac{g_A^{(3)}}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.583\left(\frac{\alpha_s}{\pi}\right)^2 - 20.215\left(\frac{\alpha_s}{\pi}\right)^3\right] \quad (58)$$

has been confirmed in polarized deep-inelastic-scattering experiments at the level of 10% [where the perturbative QCD coefficient expansion is truncated at $O(\alpha_s^3)$]. The E155 Collaboration at SLAC found $\int_0^1 dx(g_1^p - g_1^n) = 0.176 \pm 0.003 \pm 0.007$ using a next-to-leading-order QCD-motivated fit to evolve g_1 data from the E154 and E155 experiments to $Q^2=5\ \mathrm{GeV}^2$—in good agreement with the theoretical prediction 0.182 ± 0.005 from the Bjorken sum rule (Anthony *et al.*, 2000). Using a similar procedure the SMC experiment obtained $\int_0^1 dx(g_1^p - g_1^n) = 0.174^{+0.024}_{-0.012}$, also at $5\ \mathrm{GeV}^2$ (Adeva *et al.*, 1998b) and also in agreement with the theoretical prediction.

The evolution of the Bjorken integral (Abe *et al.*, 1997) $\int_{x_{\min}}^1 dx(g_1^p - g_1^n)$ as a function of $x_{\min}$ is shown for the SLAC data (E143 and E154) in Fig. 4. Note that about 50% of the sum rule comes from x values below about 0.12 and that about 10–20 % comes from values of x less than about 0.01.

Substituting the values of $g_A^{(3)}$ and $g_A^{(8)}$ from beta decays (and assuming no subtraction constant correction) in the first-moment equation (50) polarized deep-inelastic data implies

$$g_A^{(0)}|_{\mathrm{pDIS}} = 0.15 - 0.35 \quad (59)$$

for the flavor-singlet (Ellis-Jaffe) moment corresponding to the polarized strangeness $\Delta s = -0.10 \pm 0.04$ discussed in Sec. I. The measured value of $g_A^{(0)}|_{\mathrm{pDIS}}$ compares with the value 0.6 predicted by relativistic quark models and is less than 50% the value one would expect if strangeness were not important (viz., $g_A^0 = g_A^8$) and the value predicted by relativistic quark models without additional gluonic input.

The small-x extrapolation of g_1 data is the largest source of experimental error on measurements of the nucleon's axial charges from deep-inelastic scattering. The first polarized deep-inelastic experiments (Ashman *et al.*, 1988, 1989) used a simple Regge-motivated extrapolation ($g_1 \sim$ const) to evaluate the first-moment sum rules. More recent measurements quoted in the literature frequently use the technique of performing next-to-leading-order QCD-motivated fits to the g_1 data, evolving the data points all to the same value of Q^2 and then extrapolating these fits to $x=0$. Values extracted from these fits using the "modified minimal subtraction scheme" include $g_A^{(0)} = 0.23 \pm 0.04 \pm 0.06$ [SLAC experiment E155 at $Q^2=5\ \mathrm{GeV}^2$ (Anthony *et al.*, 2000)], $g_A^{(0)} = 0.19 \pm 0.05 \pm 0.04$ [SMC at $Q^2=1\ \mathrm{GeV}^2$ (Adeva *et al.*, 1998b)], and $g_A^{(0)} = 0.29 \pm 0.10$ [the mean value at $Q^2 = 4\ \mathrm{GeV}^2$ obtained by Blümlein and Böttcher (2002)].

Note that polarized deep-inelastic-scattering experiments measure g_1 between some small but finite value $x_{\min}$ and an upper value $x_{\max}$ which is close to 1. As we decrease $x_{\min} \to 0$ we measure the first moment

$$\Gamma \equiv \lim_{x_{\min}\to 0} \int_{x_{\min}}^1 dx g_1(x,Q^2). \quad (60)$$

Polarized deep-inelastic experiments cannot, even in principle, measure at $x=0$ with finite Q^2. They miss any possible $\delta(x)$ terms which might exist in g_1 at large Q^2. That is, they miss any potential (leading-twist) fixed-pole correction to the deep-inelastic spin sum rules.

Measurements of g_1 could be extended to smaller x with a future polarized ep collider. The low-x behavior of g_1 is itself an interesting topic. Small-x measurements, besides reducing the error on the first moment (and gluon polarization Δg in the proton—see Sec. IX.E), would provide valuable information about Regge and QCD dynamics at low x where the shape of g_1 is particularly sensitive to the different theoretical inputs discussed in the literature: e.g., $[\alpha_s \ln^2(1/x)]^k$ resummation and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution (Kwiecinski and Ziaja, 1999), possible Q^2-independent Regge intercepts (Cudell *et al.*, 1999), and the nonperturbative "confinement physics" to hard (perturbative QCD) scale transition. Does the color glass condensate of small-x physics (Iancu *et al.*,

2002) carry net spin polarization? We refer to Ziaja (2003) for a recent discussion of perturbative QCD predictions for the small-x behavior of g_1 in deep-inelastic scattering. In the conventional picture based on QCD evolution and no separate hard Pomeron trajectory, much larger changes in the effective intercepts, which describe the shape of the structure functions at small Bjorken x, are expected in g_1 than in the unpolarized structure function F_2 so far studied at HERA as one increases Q^2 through the transition region from photoproduction to deep-inelastic values of Q^2 (Bass and De Roeck, 2001). It will be fascinating to study this physics in future experiments, perhaps using a future polarized *ep* collider.

C. νp elastic scattering

Neutrino-proton elastic scattering measures the proton's weak axial charge $g_A^{(Z)}$ through elastic Z^0 exchange. Because of anomaly cancellation in the Standard Model the weak neutral current couples to the combination $u - d + c - s + t - b$, viz.,

$$J_{\mu 5}^{Z} = \frac{1}{2}\left\{ \sum_{q=u,c,t} - \sum_{q=d,s,b} \right\} \bar{q}\gamma_\mu \gamma_5 q. \tag{61}$$

It measures the combination

$$2g_A^{(Z)} = (\Delta u - \Delta d - \Delta s) + (\Delta c - \Delta b + \Delta t). \tag{62}$$

Heavy-quark renormalization-group arguments can be used to calculate the heavy t, b, and c quark contributions to $g_A^{(Z)}$ both at leading order (Collins *et al.*, 1978; Kaplan and Manohar, 1988; Chetyrkin and Kühn, 1993) and at next-to-leading order (Bass *et al.*, 2002). Working to next-to-leading order it is necessary to introduce "matching functions" (Bass *et al.*, 2003) to maintain renormalization-group invariance throughout. The result is

$$2g_A^{(Z)} = (\Delta u - \Delta d - \Delta s)_{\text{inv}} + \mathcal{H}(\Delta u + \Delta d + \Delta s)_{\text{inv}} + O(m_{t,b,c}^{-1}), \tag{63}$$

where $\mathcal{H}$ is a polynomial in the running couplings $\tilde{\alpha}_h$,

$$\mathcal{H} = \frac{6}{23\pi}(\tilde{\alpha}_b - \tilde{\alpha}_t)\left\{ 1 + \frac{125663}{82800\pi}\tilde{\alpha}_b + \frac{6167}{3312\pi}\tilde{\alpha}_t - \frac{22}{75\pi}\tilde{\alpha}_c \right\} - \frac{6}{27\pi}\tilde{\alpha}_c - \frac{181}{648\pi^2}\tilde{\alpha}_c^2 + O(\tilde{\alpha}_{t,b,c}^3). \tag{64}$$

Here $(\Delta q)_{\text{inv}}$ denotes the scale-invariant version of Δq which is obtained from linear combinations of $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}|_{\text{pDIS}}$ and $\tilde{\alpha}_h$ denotes Witten's renormalization-group-invariant running couplings for heavy-quark physics (Witten, 1976). Taking $\tilde{\alpha}_t = 0.1$, $\tilde{\alpha}_b = 0.2$, and $\tilde{\alpha}_c = 0.35$ in Eq. (64), one finds a small heavy-quark correction factor $\mathcal{H} = -0.02$, with leading-order terms dominant. The factor $\tilde{\alpha}_b - \tilde{\alpha}_t$ ensures that all contributions from b and t quarks cancel for $m_t = m_b$ (as they should).

Modulo the small heavy-quark corrections noted above, a precision measurement of $g_A^{(Z)}$, together with $g_A^{(3)}$ and $g_A^{(8)}$, would provide a weak-interaction determination of $(\Delta s)_{\text{inv}}$, complementary to the deep-inelastic measurement of Δs in Eq. (6). The singlet axial charge, in principle measurable in νp elastic scattering, is independent of any assumptions about the presence or absence of a subtraction at infinity correction to the Ellis-Jaffe deep-inelastic first moment of g_1, the $x \sim 0$ behavior of g_1, or SU(3) flavor breaking. Modulo any subtraction at infinity correction to the first moment of g_1, one obtains a rigorous sum rule relating deep-inelastic scattering in the Bjorken region of high-energy and high-momentum transfer to three independent, low-energy measurements in weak-interaction physics: the neutron and hyperon beta decays plus νp elastic scattering.

A precision measurement of the Z^0 axial coupling to the proton is therefore of very high priority. Ideas are being discussed for a dedicated experiment (Tayloe, 2002). Key issues are the ability to measure close to the elastic point and a very low duty factor ($\sim 10^{-5}$) neutrino beam to control backgrounds, e.g., from cosmic rays.

The experiment E734 at BNL made the first attempt to measure Δs in νp and $\bar{\nu} p$ elastic scattering (Ahrens *et al.*, 1987). This experiment extracted differential cross sections $d\sigma/dQ^2$ in the range $0.4 < Q^2 < 1.1$ GeV^2. Extrapolating the axial form factor $(1 - 2\Delta s|_{\text{inv}}/g_A^{(3)})/(1 + Q^2/M_A^2)^2$ to the elastic limit one obtains the value for Δs (Kaplan and Manohar, 1988): $\Delta s = -0.15 \pm 0.09$ taking the mass parameter in the dipole form factor to be $M_A = 1.032 \pm 0.036$ GeV. However, the data are also consistent with $\Delta s = 0$ if one takes the mass parameter to be $M_A = 1.06 \pm 0.05$ GeV, which is consistent with the world average and therefore equally valid as a solution. That is, there is a strong correlation between the value of Δs and the dipole mass parameter M_A used in the analysis which prevents an unambiguous extraction of Δs from the E734 data (Garvey *et al.*, 1993). A new dedicated precision experiment is required.

The neutral-current axial charge $g_A^{(Z)}$ could also be measured through parity violation in light atoms (Fortson and Lewis, 1984; Missimer and Simons, 1985; Campbell *et al.*, 1989; Khriplovich, 1991; Bruss *et al.*, 1998, 1999; Alberico *et al.*, 2002).

D. The Burkhardt-Cottingham sum rule

The Burkhardt-Cottingham sum rule (Burkhardt and Cottingham, 1970) reads

$$\int_{Q^2/2M}^{\infty} d\nu G_2(Q^2,\nu) = \frac{2M^3}{Q^2}\int_0^1 dx g_2 = 0, \quad \forall\, Q^2. \tag{65}$$

For deep-inelastic scattering, this sum rule is derived by assuming that the moment formula (49) can be analytically continued to $n = 0$. In general, the Burkhardt-Cottingham sum rule is derived by assuming no $\alpha \geq 0$ singularity in G_2 (or, equivalently, no $1/x$ or more singu-

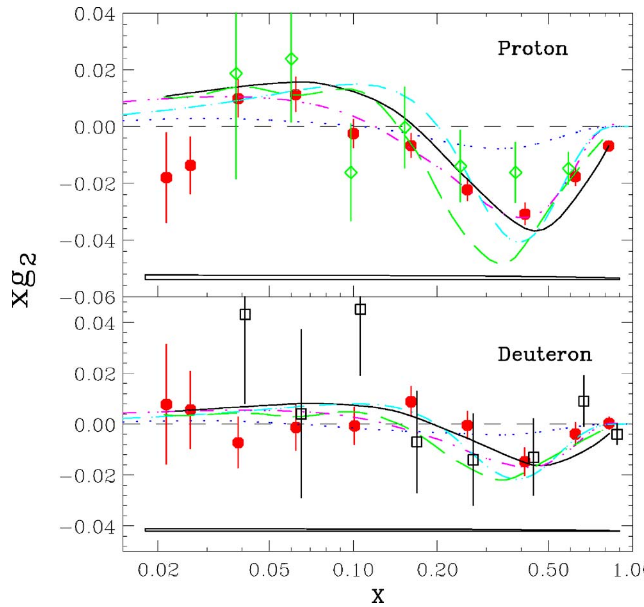

FIG. 5. (Color online) The Q^2 averaged measured of xg_2 (SLAC data) compared with the twist-2 Wandzura-Wilczek contribution g_2^{WW} term (solid line) and several quark-model calculations. Figure from Anthony *et al.*, 2003.

lar small behavior in g_2) and no subtraction at infinity (from an $\alpha = J = 0$ fixed pole in the real part of G_2) (Jaffe, 1990). The most precise measurements of g_2 to date in polarized deep-inelastic scattering come from the SLAC E155 and E143 experiments—see Fig. 5—which report $\int_{0.02}^{0.8} dx g_2^p = -0.042 \pm 0.008$ for the proton and $\int_{0.02}^{0.8} dx g_2^d = -0.006 \pm 0.011$ for the deuteron at $Q^2 = 5\ \mathrm{GeV}^2$ (Anthony *et al.*, 2003). New, even more accurate measurements of g_2 (for the neutron using a ^{3}He target) from Jefferson Laboratory (Amarian *et al.*, 2004) for Q^2 between 0.1 and 0.9 GeV^2 are consistent with the sum rule. Further measurements to test the Burkhardt-Cottingham sum rule would be most valuable, particularly given the SLAC proton result quoted above.

The formula (49) indicates that g_2 can be written as the sum

$$g_2 = g_2^{\mathrm{WW}}(x) + \bar{g}_2(x) \tag{66}$$

of a twist-2 term (Wandzura and Wilczek, 1977), denoted g_2^{WW},

$$g_2^{\mathrm{WW}} = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y), \tag{67}$$

and a second contribution $\bar{g}_2$, which is the sum of a higher-twist (twist-3) contribution $\xi(x,Q^2)$ and a "transversity" term $h_1(x,Q^2)$ which is suppressed by the ratio of the quark to target nucleon masses and therefore negligible for light u and d quarks (Cortes *et al.*, 1992),

$$\bar{g}_2(x,Q^2) = -\int_x^1 \frac{dy}{y} \frac{\partial}{\partial y}\left(\frac{m_q}{M} h_1(y,Q^2) + \xi(y,Q^2) \right). \tag{68}$$

The first moment of the twist-2 contribution g_2^{WW} vanishes through integrating the convolution formula (67). If one drops the transversity contribution from the formalism (being proportional to the light-quark mass), one obtains

$$\tilde{d}_2(Q^2) = 3\int_0^1 dx x^2 [g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2)] \tag{69}$$

for the leading twist-3 matrix element in Eq. (49). The values extracted from dedicated SLAC measurements are $d_2^p = 0.0032 \pm 0.0017$ for the proton and $d_2^n = 0.0079 \pm 0.0048$ for the neutron—that is, consistent with zero (no twist 3) at two standard deviations (Anthony *et al.*, 2003). These twist-3 matrix elements are related in part to the response of the collective color electric and magnetic fields to the spin of the nucleon. Recent analyses attempt to extract the twist-4 corrections to g_1. The results and the gluon field polarizabilities are small and consistent with zero (Deur *et al.*, 2004).

E. The Gerasimov-Drell-Hearn sum rule

The Gerasimov-Drell-Hearn (GDH) sum rule (Drell and Hearn, 1966; Gerasimov, 1966) for spin-dependent photoproduction relates the difference of the two cross sections for the absorption of a real photon with spin polarized antiparallel, $\sigma_{1/2}$, and parallel, $\sigma_{3/2}$, to the target spin to the square of the anomalous magnetic moment of the target. The GDH sum rule reads

$$\int_{\mathrm{threshold}}^{\infty} \frac{d\nu}{\nu}(\sigma_{1/2} - \sigma_{3/2}) = \frac{8\pi^2\alpha}{M^2}\int_{\mathrm{threshold}}^{\infty} \frac{d\nu}{\nu} G_1 = -\frac{2\pi^2\alpha}{M^2}\kappa^2, \tag{70}$$

where κ is the anomalous magnetic moment. The sum rule follows from the very general principles of causality, unitarity, Lorentz, and electromagnetic gauge invariance and the assumption that the g_1 spin structure function satisfies an unsubtracted dispersion relation. Modulo the no-subtraction hypothesis, the Gerasimov-Drell-Hearn sum rule is valid for a target of arbitrary spin S, whether elementary or composite (Brodsky and Primack, 1969)—for reviews see Bass (1997) and Drechsel and Tiator (2004).

The GDH sum rule is derived by setting $\nu = 0$ in the dispersion relation for A_1, Eq. (38). For small photon energy $\nu \to 0$,

$$A_1(0,\nu) = -\frac{1}{2}\kappa^2 + \tilde{\gamma}\nu^2 + O(\nu^4). \tag{71}$$

Here $\gamma_N = (\alpha/M^2)\tilde{\gamma}$ is the spin polarizability which measures the stiffness of the nucleon spin against electromagnetic-induced deformations relative to the axis defined by the nucleon's spin. This low-energy theorem follows from Lorentz invariance and electromagnetic gauge invariance (plus the existence of a finite mass gap between the ground state and continuum contributions to forward Compton scattering) (Gell-Mann

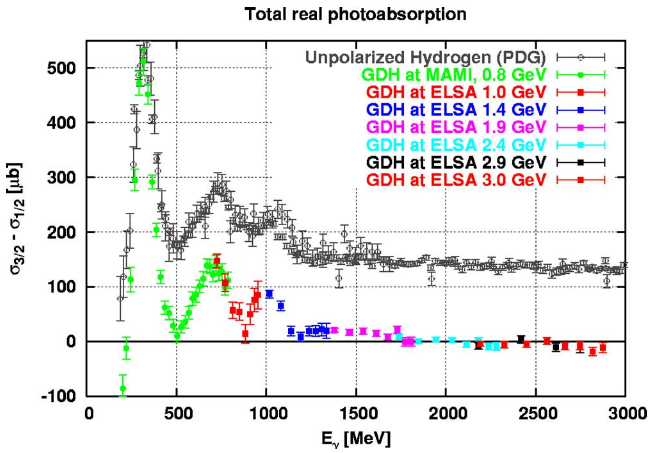

FIG. 6. (Color online) The spin-dependent photoproduction cross section for the proton target (ELSA and MAMI data). Figure courtesy of K. Helbing.

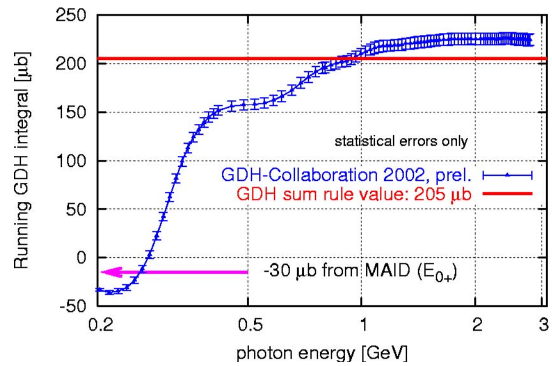

FIG. 7. (Color online) Running GDH integral for the proton (ELSA and MAMI). Figure courtesy of K. Helbing.

and Goldberger, 1954; Low, 1954; Brodsky and Primack, 1969).

The integral in Eq. (70) converges for each of the leading Regge contributions (discussed in Sec. II.B). If the sum rule were observed to fail (with a finite integral), the interpretation would be a subtraction at infinity induced by a $J=1$ fixed pole in the real part of the spin amplitude A_1 (Abarbanel and Goldberger, 1968).

Present experiments at ELSA and MAMI are aimed at measuring the GDH integrand through the range of incident photon energies $E_\gamma=0.14-0.8$ GeV (MAMI; Ahrens *et al.*, 2000, 2001, 2002) and 0.7–3.1 GeV (ELSA; Dutz *et al.*, 2003). The inclusive cross section for the proton target $\sigma_{3/2}-\sigma_{1/2}$ is shown in Fig. 6. This GDH integral on the proton is shown in Fig. 7 and is dominated by the Δ resonance contribution. [The contribution to the sum rule from the unmeasured region close to threshold is estimated from the MAID model (Drechsel *et al.*, 2003).] The combined data from the ELSA-MAMI experiments suggest that the contribution to the GDH integral for a proton target from energies $\nu<3$ GeV exceeds the total sum-rule prediction (-204.5 μb) by about 5–10 % (Helbing, 2002). Phenomenological estimates suggest that about $+25\pm10$ μb of the sum rule may reside at higher energies (Bass and Brisudova, 1999; Bianchi and Thomas, 1999) and that this high-energy contribution is predominantly in the isovector channel. (It should be noted, however, that any 10% fixed pole correction would be competitive with this high-energy contribution within the errors.) Further measurements, including at higher energy, would be valuable. Preliminary data on the neutron have just been released from MAMI and ELSA (Helbing, 2004). These data, if confirmed, suggest that the neutron GDH integal, if it indeed obeys the GDH sum rule, will require a large (mainly isovector) contribution (perhaps 45 μb) from photon energies E_γ greater than about 1800 MeV. With the warning that these data are still preliminary, it is interesting to note that, just like the measured g_1 at deep-inelastic Q^2, the high-energy part of the spin-dependent cross section $\sigma_{1/2}-\sigma_{3/2}$ at $Q^2=0$ seems to be largely isovector prompting the question whether there is some physics conspiracy to suppress the singlet term. It should be noted, however, that perturbative QCD-motivated fits to g_1 data with a positive polarized gluon distribution (and no node in it) predict that g_1 should develop a strong negative contribution at $x<0.0001$ at deep-inelastic Q^2—see, e.g., De Roeck *et al.* (1999), and references therein.

In addition to the GDH sum rule, one also finds a second sum rule for the nucleon's spin polarizability. This spin polarizability sum rule is derived by taking the second derivative of $A_1(Q^2,\nu)$ in the dispersion relation (38) and evaluating the resulting expression at $\nu=0$, viz., $(\partial^2/\partial\nu^2)A_1\,(Q^2,\nu)|_{\nu=0}$. One finds

$$\int_0^\infty \frac{d\nu'}{\nu'^3}(\sigma_{1/2}-\sigma_{3/2})(\nu') = 4\pi^2\gamma_N. \qquad (72)$$

In comparison with the GDH sum rule the relevant information is now concentrated more on the low-energy side because of the $1/\nu'^3$ weighting factor under the integral. Main contributions come from the $\Delta(1232)$ resonance and the low-energy pion photoproduction continuum described by the electric dipole amplitude E_{0+}. The value extracted from MAMI data (Drechsel *et al.*, 2003),

$$\gamma_p = (-1.01\pm0.13)\times10^{-4}\ \mathrm{fm}^4, \qquad (73)$$

is within the range of predictions of chiral perturbation theory.

Further experiments to test the GDH sum rule and to measure $\sigma_{1/2}-\sigma_{3/2}$ at and close to $Q^2=0$ are being carried out at Jefferson Laboratory, GRAAL at Grenoble, LEGS at BNL, and SPRING-8 in Japan.

We note two interesting properties of the GDH sum rule.

First, we write the anomalous magnetic moment κ as the sum of its isovector κ_V and isoscalar κ_S contributions, viz., $\kappa_N=\kappa_S+\tau_3\kappa_V$. One then obtains the isospin-dependent expressions:

$$(\text{GDH})_{I=0} = (\text{GDH})_{VV} + (\text{GDH})_{SS} = -\frac{2\pi^2\alpha}{m^2}(\kappa_V^2 + \kappa_S^2),$$

$$(\text{GDH})_{I=1} = (\text{GDH})_{VS} = -\frac{2\pi^2\alpha}{m^2} 2\kappa_V\kappa_S. \quad (74)$$

The physical values of the proton and nucleon anomalous magnetic moments $\kappa_p = 1.79$ and $\kappa_n = -1.91$ correspond to $\kappa_S = -0.06$ and $\kappa_V = +1.85$. Since $\kappa_S/\kappa_V \simeq -\frac{1}{30}$, it follows that $(\text{GDH})_{SS}$ is negligible compared to $(\text{GDH})_{VV}$. That is, to good approximation, the isoscalar sum rule $(\text{GDH})_{I=0}$ measures the isovector anomalous magnetic moment κ_V. Given this isoscalar measurement, the isovector sum rule $(\text{GDH})_{I=1}$ then measures the isoscalar anomalous magnetic moment κ_S.

Second, the anomalous magnetic moment is measured in the matrix element of the vector current. Furry's theorem tells us that the real-photon GDH integral for a gluon or a photon target vanishes. Indeed, this is the reason that the first moment of the g_1 spin structure function for a real polarized photon target vanishes to all orders and at every twist: $\int_0^1 dx g_1^\gamma(x,Q^2)$ independent of the virtuality Q^2 of the second photon that it is probed with (Bass *et al.*, 1998). Assuming correction to the GDH sum rule, this result implies that the two nonperturbative gluon exchange contribution to $\sigma_{1/2} - \sigma_{3/2}$, which behaves as $\ln \nu/\nu$ in the high-energy Regge limit, has a node at some value $\nu = \nu_0$ so that it does not contribute to the GDH integral. There is no axial anomaly contribution to the anomalous magnetic moment and hence no axial anomaly contribution to the GDH sum rule.

F. The transition region

Several experiments have explored the transition region between polarized photoproduction (the physics of the GDH sum rule) and polarized deep-inelastic scattering (the physics of the Bjorken sum rule and $g_A^{(0)}$ through the Ellis-Jaffe moment).

The Q^2-dependent quantity (Anselmino *et al.*, 1989)

$$\Gamma(Q^2) \equiv I(Q^2) = \int_{Q^2/2M}^{\infty} \frac{d\nu}{\nu} G_1(\nu, Q^2) = \frac{2M^2}{Q^2}\int_0^1 dx g_1(x,Q^2) \quad (75)$$

interpolates between the two limits with $I(0) = -\frac{1}{4}\kappa_N^2$ implied by the GDH sum rule. Measurements of $\int_0^1 dx g_1^p = (Q^2/2M^2) I(Q^2)$ are shown in Fig. 8. Note the negative slope predicted at $Q^2 = 0$ by the GDH sum rule and the sign change around $Q^2 \sim 0.3\ \text{GeV}^2$. The shape of the curve is driven predominantly by the role of the Δ resonance and the $1/Q^2$ pole in Eq. (75). Figure 8 also shows the predictions of various models (Soffer and Teryaev, 1993; Burkert and Ioffe, 1994) which try to describe the intermediate Q^2 range through a combination of resonance physics and vector-meson dominance at low Q^2 and scaling parton physics at deep inelastic (DIS) Q^2. Chiral perturbation theory (Bernard *et al.*, 2003) may describe the behavior of this "generalized GDH integral" close to threshold—see the shaded band in Fig. 8.

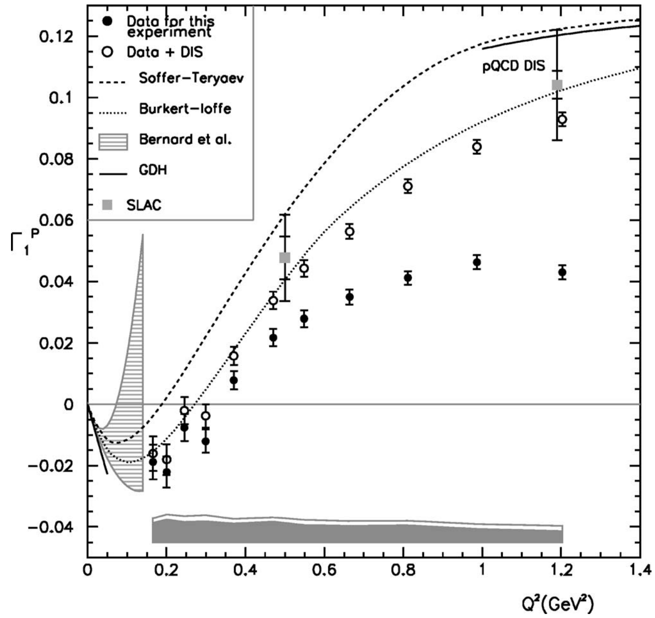

FIG. 8. Data from JLab (CLAS) and SLAC on the low Q^2 behavior of $\int_0^1 dx g_1^p$ compared to various theoretical models interpolating the scaling and photoproduction limits (Fatemi *et al.*, 2003).

In the model of Ioffe and collaborators (Anselmino *et al.*, 1989; Burkert and Ioffe, 1994) the integral at low to intermediate Q^2 for the inelastic part of $\sigma_A - \sigma_P$ is given as the sum of a contribution from resonance production, denoted $I^{\text{res}}(Q^2)$, which has a strong Q^2 dependence for small Q^2 and then drops rapidly with Q^2, and a nonresonant vector-meson dominance contribution which they took as the sum of a monopole and a dipole term, viz.,

$$I(Q^2) = I^{\text{res}}(Q^2) + 2M^2\Gamma^{\text{as}}\left(\frac{1}{Q^2+\mu^2} - \frac{C\mu^2}{(Q^2+\mu^2)^2}\right). \quad (76)$$

Here Γ^{as} is taken as

$$\Gamma^{\text{as}} = \int_0^1 dx g_1(x,\infty) \quad (77)$$

and

$$C = 1 + \frac{1}{2}\frac{\mu^2}{M^2}\frac{1}{\Gamma^{\text{as}}}\left(\frac{1}{4}\kappa^2 + I^{\text{res}}(0)\right). \quad (78)$$

The mass parameter μ is identified with rho-meson mass, $\mu^2 \simeq m_\rho^2$.

IV. PARTONS AND SPIN STRUCTURE FUNCTIONS

A. The QCD parton model

We now return to g_1 in the scaling regime of polarized deep-inelastic scattering. As noted in Sec. II.A, in the (pre-QCD) parton model g_1 is written as

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \Delta q(x), \tag{79}$$

where e_q denotes the quark charge and $\Delta q(x)$ is the polarized quark distribution.

In QCD we have to consider the effects of gluon radiation and (renormalization-group) mixing of the flavor-singlet quark distribution with the polarized gluon distribution of the proton. The parton-model description of polarized deep-inelastic scattering involves writing the deep-inelastic structure functions as the sum over the convolution of "soft" quark and gluon parton distributions with "hard" photon-parton scattering coefficients:

$$g_1(x) = \left\{ \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) \right\} \otimes C_{ns}^q + \frac{1}{9}\{(\Delta u + \Delta d + \Delta s) \otimes C_s^q + f \Delta g \otimes C^g\}. \tag{80}$$

Here $\Delta q(x)$ and $\Delta g(x)$ denote the polarized quark and gluon parton distributions, $C^q(z)$ and $C^g(z)$ denote the corresponding hard scattering coefficients, and f is the number of quark flavors liberated into the final state ($f = 3$ below the charm production threshold). The parton distributions contain all the target-dependent information and describe a flux of quark and gluon partons into the (target-independent) interaction between the hard photon and the parton which is described by the coefficients and which is calculable using perturbative QCD. The perturbative coefficients are independent of infrared mass singularities in the photon-parton collision which are absorbed into the soft parton distributions (and softened by confinement-related physics).

The separation of g_1 into hard and soft is not unique and depends on the choice of "factorization scheme." For example, one might use a kinematic cutoff on the partons' transverse momentum squared ($k_t^2 > \lambda^2$) to define the factorization scheme and thus separate the hard and soft parts of the phase space for the photon-parton collision. The cutoff λ^2 is called the factorization scale. The coefficients have the perturbative expansion $C^q = \delta(1-x) + (\alpha_s/2\pi) f^q(x, Q^2/\lambda^2)$ and $C^g = (\alpha_s/2\pi) f^g(x, Q^2/\lambda^2)$, where the strongest singularities in the functions f^q and f^g as $x \to 1$ are $\ln(1-x)/(1-x)_+$ and $\ln(1-x)$, respectively—see, e.g., Lampe and Reya (2000). The deep-inelastic structure functions are dependent on Q^2 and independent of the factorization scale λ^2 and the "scheme" used to separate the γ^*-parton cross section into hard and soft contributions. Examples of different schemes one might use include using modified minimal subtraction ($\overline{\text{MS}}$) ('t Hooft and Veltman, 1972; Bodwin and Qiu, 1990) to regulate the mass singularities which arise in scattering from massless partons, and cutoffs on other kinematic variables such as the invariant mass squared or the virtuality of the struck quark. Other schemes which have been widely used in the literature and analysis of polarized deep-inelastic scattering data are the "AB" (Ball *et al.*, 1996) and "CI" (chiral invariant; Cheng, 1996) or "JET" (Leader *et al.*, 1998) schemes. We illustrate factorization-scheme dependence and the use of these schemes in the analysis of g_1 data in Secs. VI.D and IX.C.

If the same scheme is applied consistently to all hard processes then the factorization theorem asserts that the parton distributions that one extracts from experiments should be process independent (Collins, 1993a). In other words, the same polarized quark and gluon distributions should be obtained from future experiments involving polarized hard QCD processes in polarized proton-proton collisions (e.g., at RHIC) and polarized deep-inelastic-scattering experiments. The factorization theorem for unpolarized hard processes has been successfully tested in a large number of experiments involving different reactions at various laboratories. Tests of the polarized version await future independent measurements of the polarized gluon and sea-quark distributions from a variety of different hard scattering processes with polarized beams.

B. Light-cone correlation functions

The spin-dependent parton distributions may also be defined via the operator product expansion. For g_1 this means that the odd moments of the polarized quark and gluon distributions project out the target matrix elements of the renormalized, spin-odd, composite operators which appear in the operator product expansion, viz.,

$$2Ms_+(p_+)^{2n}\int_0^1 dx x^{2n}\Delta q(x,\mu^2) = \langle p,s|[\bar{q}(0)\gamma_+\gamma_5(iD_+)^{2n}q(0)]_{\mu^2}|p,s\rangle, \tag{81}$$

$$2Ms_+(p_+)^{2n}\int_0^1 dx x^{2n}\Delta g(x,\mu^2) = \langle p,s|[\mathrm{Tr}\, G_{+\alpha}(0)(iD_+)^{2n-1}\tilde{G}_+^{\alpha}(0)]_{\mu^2}|p,s\rangle \quad (n \geqslant 1). \tag{82}$$

The association of $\Delta q(x,\mu^2)$ with quarks and $\Delta g(x,\mu^2)$ with gluons follows when we evaluate the target matrix elements in Eqs. (81) and (82) in the light-cone gauge, where $D_+ \to \partial_+$ and the explicit dependence of D_+ on the gluon field drops out. The operator product expansion involves writing the product of electromagnetic currents $J_\mu(z)J_\nu(0)$ in Eq. (11) as the expansion over gauge-invariantly renormalized, local, composite quark and gluonic operators at lightlike separation $z^2 \to 0$—the realm of deep-inelastic scattering (Muta, 1998). The subscript μ^2 on the operators in Eq. (82) emphasizes the dependence on the renormalization scale.[2]

Mathematically, the relation between the parton distributions and the operator product expansion is given in terms of light-cone correlation functions of point-split operator matrix elements along the light-cone. Define

$$\psi^{\pm} = P^{\pm}\psi, \tag{83}$$

where

$$P^{\pm} = \frac{1}{2}(1 \pm \alpha_3) = \frac{1}{2}\gamma^{\pm}\gamma^{\mp}. \tag{84}$$

The polarized quark and antiquark distributions are given by

$$\Delta\psi(x) = \frac{1}{2\sqrt{2}\pi}\int d\xi^- e^{-ixM\xi^-/\sqrt{2}}\langle p,s|(\psi^{+R})^\dagger(\xi^-)\psi^{+R}(0) - (\psi^{+L})^\dagger(\xi^-)\psi^{+L}(0)|p,s\rangle,$$

$$\Delta\bar{\psi}(x) = \frac{1}{2\sqrt{2}\pi}\int d\xi^- e^{-ixM\xi^-/\sqrt{2}}\langle p,s|\psi^{+L}(\xi^-)(\psi^{+L})^\dagger(0) - \psi^{+R}(\xi^-)(\psi^{+R})^\dagger(0)|p,s\rangle. \tag{85}$$

In this notation $\Delta q = \Delta\psi + \Delta\bar{\psi}$. The nonlocal operator in the correlation function is rendered gauge invariant through a path-ordered exponential which simplifies to unity in the light-cone gauge $A_+ = 0$. Taking the moments of these distributions reproduces the results of the operator product expansion in Eq. (48).[3] The light-cone correlation function for the polarized gluon distribution is

$$x\Delta g(x) = \frac{i}{2\sqrt{2}M\pi}\int d\xi^- e^{-ix\xi^- M/\sqrt{2}} \times \langle p,s|G_{+\nu}(\xi^-)\tilde{G}^{\nu}_{+}(0) - G_{+\nu}(0)\tilde{G}^{\nu}_{+}(\xi^-)|p,s\rangle. \tag{87}$$

In the light-cone gauge ($A_+ = 0$) one finds $G_a^{+\nu} = \partial^+ A_a^\nu - \partial^\nu A_a^+ = \partial^+ A_a^\nu$ so that

$$G^{+\nu}\tilde{G}^{+}_{\nu} = G^{+}_{R}G_{-L} - G^{+}_{L}G_{-R} = G^{+}_{R}G^{+R} - G^{+}_{L}G^{+L}. \tag{88}$$

Thus $\Delta g(x)$ measures the distribution of gluon polarization in the nucleon. One can evaluate the first moment of $\Delta g(x)$ from its light-cone correlation function. One first assumes that

$$\lim_{x\to 0^+} x\Delta g(x) = 0. \tag{89}$$

In $A_+ = 0$ gauge the first moment becomes

$$\int_0^1 dx\Delta g(x) = \frac{1}{\sqrt{2}M}[\langle A^\nu(\xi^-)\tilde{G}^{+}_{\nu}(0)\rangle|_{\xi^-\to\infty} - \langle A^\nu(0)\tilde{G}^{+}_{\nu}(0)\rangle] \tag{90}$$

—that is, the sum of the forward matrix element of the gluonic Chern-Simons current K_+ plus a surface term (Manohar, 1990) which may or may not vanish in QCD.

V. FIXED POLES

Fixed poles are exchanges in Regge phenomenology with no t dependence: the trajectories are described by $J = \alpha(t) = 0$ or 1 for all t (Abarbanel *et al.*, 1967; Brodsky *et al.*, 1972; Landshoff and Polkinghorne, 1972). For example, for fixed Q^2 a t-independent real constant term in the spin amplitude A_1 would correspond to a $J=1$ fixed pole. Fixed poles are excluded in hadron-hadron scattering by unitarity but are not excluded from Compton amplitudes (or parton distribution functions) because these are calculated only to lowest order in the current-hadron coupling. Indeed, there are two famous examples where fixed poles are required: in the Adler sum rule for W-boson nucleon scattering (by current algebra), and to reproduce the Schwinger-term sum rule for the longitudinal structure function measured in unpolarized deep-inelastic ep scattering. We review the derivation of these fixed pole contributions, and then discuss potential fixed pole corrections to the Burkhardt-Cottingham, g_1, and Gerasimov-Drell-Hearn sum rules.[4] Fixed poles in the real part of the forward Compton amplitude have the potential to induce subtraction at infinity corrections to sum rules for photon-nucleon (or lepton-nucleon) scattering. For example, a ν-independent term in the real part of A_1 would induce a subtraction constant correction to the spin sum rule for the first moment of g_1. Bjorken scaling at large Q^2 constrains the Q^2 dependence of the residue of any fixed pole in the real of the forward Compton amplitude [e.g., $\beta_1(Q^2)$ and $\beta_2(Q^2)$ in the dispersion relations (41)]. To be consistent with scaling these residues must decay as or faster than $1/Q^2$ as $Q^2\to\infty$. That is, they must be nonpolynomial in Q^2.

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[2] Note that the parton distributions defined through the operator product expansion include the effect of renormalization effects such as the axial anomaly (and the trace anomaly for the spin-independent distributions which appear in F_1 and F_2) in addition to absorbing the mass singularities in photon-parton scattering.

[3] Some care has to be taken regarding renormalization of the light-cone correlation functions. The bare correlation function from which we project out moments as local operators is ultraviolet divergent. Llewellyn Smith (1989) proposed a solution to this problem by defining the renormalized light-cone correlation function as a series expansion in the proton matrix elements of gauge-invariant local operators. For the polarized quark distribution this becomes

$$\langle\bar{\psi}(z_-)\gamma_+\gamma_5\psi(0)\rangle = \sum_n \frac{(-z_-)^n}{n!}\langle[\bar{\psi}\gamma_+\gamma_5(D_+)^n\psi](0)\rangle. \tag{86}$$

[4] We refer to Efremov and Schweitzer (2003) for a recent discussion of an "$x=0$" fixed-pole contribution to the twist-3, chiral-odd structure function $e(x)$.

A. The Adler sum rule

The first example we consider is the Adler sum rule for W-boson nucleon scattering (Adler, 1966):

$$\int_{Q^2/2M}^{+\infty} d\nu [W_2^{\bar{\nu}p}(\nu,Q^2) - W_2^{\nu p}(\nu,Q^2)] = \int_0^1 \frac{dx}{x}[F_2^{\bar{\nu}p}(x,Q^2) - F_2^{\nu p}(x,Q^2)] = \begin{cases} 4-2\cos^2\theta_C & \text{(BCT)} \\ 2 & \text{(ACT)}. \end{cases} \tag{91}$$

Here θ_C is the Cabibbo angle, and BCT and ACT refer to below and above the charm production threshold.

The Adler sum rule is derived from current algebra. The right-hand side of the sum rule is the coefficient of a $J=1$ fixed pole term

$$\frac{i}{\pi} f_{abc} F_c [(p_\mu q_\nu + q_\mu p_\nu) - M\nu g_{\mu\nu}]/Q^2 \tag{92}$$

in the imaginary part of the forward Compton amplitude for W-boson nucleon scattering (Heimann *et al.*, 1972). This fixed pole term is required by the commutation relations between the charge raising and lowering weak currents

$$q_\mu T^{\mu\nu}_{ab} = -\frac{1}{\pi}\int d^4x e^{iq\cdot x}\langle p,s|[J_a^0(x),J_b^\nu(0)]|p,s\rangle \delta(x^0) = -\frac{i}{\pi} f_{abc}\langle ps|J_c^\nu(0)|ps\rangle. \tag{93}$$

Here F_c is a generalized form factor at zero momentum transfer:

$$\langle p,s|J_c^\nu(0)|p,s\rangle \equiv p^\nu F_c. \tag{94}$$

The fixed pole term appears in lowest-order perturbation theory, and is not renormalized because it is a consequence of the charge algebra. The Adler sum rule is protected against radiative QCD corrections

B. The Schwinger-term sum rule

Our second example is the Schwinger-term sum rule (Broadhurst *et al.*, 1973) which relates the logarithmic integral in ω (or Bjorken x) of the longitudinal structure function $F_L(\omega,Q^2)$ $\left(F_L=\frac{1}{2}\omega F_2-F_1\right)$ measured in unpolarized deep-inelastic scattering to the target matrix element of the operator Schwinger term $\mathcal{S}$ defined through the equal-time commutator of the electromagnetic charge and current densities

$$\langle p,s|[J_0(\vec{y},0),J_i(0)]|p,s\rangle = i\partial_i \delta^3(\vec{y})\mathcal{S}. \tag{95}$$

The Schwinger-term sum rule reads

$$\mathcal{S} = \lim_{Q^2\to\infty}\left[4\int_1^\infty \frac{d\omega}{\omega}\tilde{F}_L(\omega,Q^2) - 4\sum_{\alpha>0}\gamma(\alpha,Q^2)/\alpha - C(q^2)\right]. \tag{96}$$

Here $C(Q^2)$ is the nonpolynomial residue of any $J=0$ fixed pole contribution in the real part of T_2 and

$$\tilde{F}_L(\omega,Q^2) = F_L(\omega,Q^2) - \sum_{\alpha\geqslant 0}\gamma(\alpha,Q^2)\omega^\alpha \tag{97}$$

represents F_L with the leading $(\alpha>0)$ Regge behavior subtracted. The integral in Eq. (96) is convergent because $\tilde{F}_L(\omega,Q^2)$ is defined with all Regge contributions with effective intercept greater than or equal to zero removed from $F_L(Q^2,\omega)$. The Schwinger term $\mathcal{S}$ vanishes in vector gauge theories like QCD.

Since $F_L(\omega,Q^2)$ is positive definite, it follows that QCD possesses the required nonvanishing $J=0$ fixed pole in the real part of T_2.

C. The Burkhardt-Cottingham sum rule

The third example, and the first in connection with spin, is the Burkhardt-Cottingham sum rule for the first moment of g_2 (Burkhardt and Cottingham, 1970):

$$\int_{Q^2/2M}^\infty d\nu G_2(Q^2,\nu) = \frac{2M^3}{Q^2}\int_0^1 dx g_2 = 0. \tag{98}$$

Suppose that future experiments find that the sum rule is violated and that the integral is finite. The conclusion (Jaffe, 1990) would be a $J=0$ fixed pole with nonpolynomial residue in the real part of A_2. To see this work at fixed Q^2 assume that all Regge-like singularities contributing to $A_2(\nu,Q^2)$ have intercept less than zero so that

$$A_2(\nu,Q^2) \sim \nu^{-1-\epsilon} \tag{99}$$

as $\nu\to\infty$ for some $\varepsilon>0$. Then the large-ν behavior of A_2 is obtained by taking $\nu\to\infty$ under the ν' integral giving

$$A_2(Q^2,\nu) \sim -\frac{2}{\pi\nu}\int_{Q^2/2M}^\infty d\nu' \,\mathrm{Im}\, A_2(Q^2,\nu'), \tag{100}$$

which contradicts the assumed behavior unless the integral vanishes; hence the sum rule. If there is an $\alpha(0)=0$ fixed pole in the real part of A_2, the fixed pole will not contribute to $\mathrm{Im}\, A_2$ and therefore not spoil the convergence of the integral.

One finds

$$\beta_2(Q^2) \sim -\frac{2}{\pi M}\int_{Q^2/2M}^\infty d\nu' \,\mathrm{Im}\, A_2(Q^2,\nu') \tag{101}$$

for the residue of any $J=0$ fixed pole coupling to $A_2(Q^2,\nu)$.

D. g_1 spin sum rules

Scaling requires that any fixed pole correction to the Ellis-Jaffe g_1 sum rule must have nonpolynomial residue. Through Eq. (41), the fixed pole coefficient $\beta_1(Q^2)$ must decay as or faster than $O(1/Q^2)$ as $Q^2 \to \infty$. The coefficient is further constrained by the requirement that G_1 contains no kinematic singularities (for example, at $Q^2=0$). In Sec. VI.C we will identify a potential leading-twist topological $x=0$ contribution to the first moment of g_1 through analysis of the axial anomaly contribution to $g_A^{(0)}$. This zero-mode topological contribution (if finite) generates a leading-twist fixed pole correction to the flavor-singlet part of $\int_0^1 dx g_1$. If present, this fixed pole will also violate the Gerasimov-Drell-Hearn sum rule (since the two sum rules are derived from A_1) unless the underlying dynamics suppresses the fixed pole's residue at $Q^2=0$. The possibility of a fixed pole correction to g_1 spin sum rules was raised in pre-QCD work as early as Abarbanel and Goldberger (1968) and Heimann (1973).

Note that any fixed pole correction to the Gerasimov-Drell-Hearn sum rule is most probably a nonperturbative effect. The sum rule (41) has been verified to $O(\alpha^2)$ for all $2 \to 2$ processes $\gamma a \to bc$ where a is either a real lepton, quark, gluon, or elementary Higgs target (Altarelli *et al.*, 1972; Brodsky and Schmidt, 1995), and for electrons in QED to $O(\alpha^3)$ (Dicus and Vega, 2001).

One could test for a fixed pole correction to the Ellis-Jaffe moment through a precision measurement of the flavor-singlet axial charge from an independent process where one is not sensitive to theoretical assumptions about the presence or absence of a $J=1$ fixed pole in A_1. Here the natural choice is elastic neutrino-proton scattering where the parity-violating part of the cross section includes a direct weak-interaction measurement of the scale-invariant flavor-singlet axial charge $g_A^{(0)}|_{\text{inv}}$.

A further test could come from a precision measurement of the Q^2 dependence of the polarized gluon distribution at next-to-next-to-leading-order accuracy where one becomes sensitive to any possible leading-twist subtraction constant—see the discussion below Eq. (128).

The subtraction constant fixed-pole-correction hypothesis could also, in principle, be tested through measurement of the real part of the spin-dependent part of the forward deeply virtual Compton amplitude. While this measurement may seem extremely difficult at the present time one should not forget that Bjorken believed when writing his original Bjorken sum-rule paper that the sum-rule would never be tested!

VI. THE AXIAL ANOMALY, GLUON TOPOLOGY, AND THE FLAVOR-SINGLET AXIAL CHARGE $g_A^{(0)}$

We next discuss the role of the axial anomaly in the interpretation of $g_A^{(0)}$.

A. The axial anomaly

In QCD one has to consider the effects of renormalization. The flavor-singlet axial-vector current $J_{\mu 5}^{GI}$ in Eq. (53) satisfies the anomalous divergence equation (Adler, 1969; Bell and Jackiw, 1969; Crewther, 1978)

$$\partial^\mu J_{\mu 5}^{GI} = 2f \partial^\mu K_\mu + \sum_{i=1}^{f} 2im_i \bar{q}_i \gamma_5 q_i, \tag{102}$$

where

$$K_\mu = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right] \tag{103}$$

is the gluonic Chern-Simons current and the number of light flavors f is 3. Here A_a^μ is the gluon field and $\partial^\mu K_\mu = (g^2/32\pi^2) G_{\mu\nu} \tilde{G}^{\mu\nu}$ is the topological charge density. Equation (102) allows us to define a partially conserved current

$$J_{\mu 5}^{GI} = J_{\mu 5}^{\text{con}} + 2f K_\mu, \tag{104}$$

viz., $\partial^\mu J_{\mu 5}^{\text{con}} = \Sigma_{i=1}^f 2im_i \bar{q}_i \gamma_5 q_i$.

When we make a gauge transformation U the gluon field transforms as

$$A_\mu \to U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} \tag{105}$$

and the operator K_μ transforms as

$$K_\mu \to K_\mu + i \frac{g}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\nu (U^\dagger \partial^\alpha U A^\beta) + \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} [(U^\dagger \partial^\nu U)(U^\dagger \partial^\alpha U)(U^\dagger \partial^\beta U)]. \tag{106}$$

Partially conserved currents are not renormalized. It follows that $J_{\mu 5}^{\text{con}}$ is renormalization scale invariant and the scale dependence of $J_{\mu 5}^{GI}$ associated with the factor $E(\alpha_s)$ is carried by K_μ. This is summarized in the equations

$$J_{\mu 5} = Z_5 \, J_{\mu 5}|_{\text{bare}},$$

$$K_\mu = K_\mu|_{\text{bare}} + \frac{1}{2f} (Z_5 - 1) \, J_{\mu 5}|_{\text{bare}},$$

$$J_{\mu 5}^{\text{con}} = J_{\mu 5}^{\text{con}}|_{\text{bare}}, \tag{107}$$

where Z_5 denotes the renormalization factor for $J_{\mu 5}$. Gauge transformations shuffle a scale-invariant operator quantity between the two operators $J_{\mu 5}^{\text{con}}$ and K_μ while keeping $J_{\mu 5}^{GI}$ invariant.

The nucleon matrix element of $J_{\mu 5}^{GI}$ is

$$\langle p,s | J_{\mu 5}^{GI} | p',s' \rangle = 2M [\tilde{s}_\mu G_A(l^2) + l_\mu l \cdot \tilde{s} G_P(l^2)], \tag{108}$$

where $l_\mu = (p' - p)_\mu$ and $\tilde{s}_\mu = \bar{u}_{(p,s)} \gamma_\mu \gamma_5 u_{(p',s')}/2M$. Since $J_{\mu 5}^{GI}$ does not couple to a massless Goldstone boson it follows that $G_A(l^2)$ and $G_P(l^2)$ contain no massless pole terms. The forward matrix element of $J_{\mu 5}^{GI}$ is well defined and

$$g_A^{(0)}|_{\text{inv}} = E(\alpha_s) G_A(0). \tag{109}$$

We would like to isolate the gluonic contribution to $G_A(0)$ associated with K_μ and thus write $g_A^{(0)}$ as the sum of (measurable) "quark" and "gluonic" contributions. Here one has to be careful because of the gauge dependence of the operator K_μ. To understand the gluonic contributions to $g_A^{(0)}$ it is helpful to go back to the deep-inelastic cross section in Sec. II.

B. The anomaly and the first moment of g_1

We specialize to the target rest frame and let E denote the energy of the incident charged lepton which is scattered through an angle θ to emerge in the final state with energy E'. Let $\uparrow\downarrow$ denote the longitudinal polarization of the beam and $\Uparrow\Downarrow$ denote a longitudinally polarized proton target. The spin-dependent part of the differential cross sections is

$$\frac{d^2\sigma\uparrow\Downarrow}{d\Omega dE'} - \frac{d^2\sigma\uparrow\Uparrow}{d\Omega dE'} = \frac{4\alpha^2 E'}{Q^2 E\nu}[(E + E'\cos\theta)g_1(x,Q^2) - 2xMg_2(x,Q^2)], \tag{110}$$

which is obtained from the product of the lepton and hadron tensors

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{Q^4}\frac{E'}{E}L^A_{\mu\nu}W^{\mu\nu}_A. \tag{111}$$

Here the lepton tensor

$$L^A_{\mu\nu} = 2i\epsilon_{\mu\nu\alpha\beta}k^\alpha q^\beta \tag{112}$$

describes the lepton-photon vertex and the hadronic tensor

$$\frac{1}{M}W^{\mu\nu}_A = i\epsilon^{\mu\nu\rho\sigma}q_\rho\left(s_\sigma\frac{1}{p\cdot q}g_1(x,Q^2) + [p\cdot q s_\sigma - s\cdot q p_\sigma]\frac{1}{M^2 p\cdot q}g_2(x,Q^2)\right) \tag{113}$$

describes the photon-nucleon interaction.

Deep-inelastic scattering involves the Bjorken limit $Q^2=-q^2$ and $p\cdot q = M\nu$ both $\to\infty$ with $x=Q^2/2M\nu$ held fixed. In terms of light-cone coordinates this corresponds to taking $q_-\to\infty$ with $q_+=-xp_+$ held finite. The leading term in $W^{\mu\nu}_A$ is obtained by taking the Lorentz index of s_σ as $\sigma=+$. (Other terms are suppressed by powers of $1/q_-$.)

If we wish to understand the first moment of g_1 in terms of the matrix elements of anomalous currents ($J^{\rm con}_{\mu5}$ and K_μ), then we have to understand the forward matrix element of K_+ and its contribution to $G_A(0)$.

Here we are fortunate in that the parton model is formulated in the light-cone gauge ($A_+=0$) where the forward matrix elements of K_+ are invariant. In the light-cone gauge the non-Abelian three-gluon part of K_+ vanishes. The forward matrix elements of K_+ are then invariant under all residual gauge degrees of freedom. Furthermore, in this gauge, K_+ measures the gluonic "spin" content of the polarized target (Manohar, 1990; Jaffe, 1996)—strictly speaking, up to the nonperturbative surface term we find from integrating the light-cone correlation function, Eq. (90). One finds

$$G_A^{(A_+=0)}(0) = \sum_q \Delta q_{\rm con} - f\frac{\alpha_s}{2\pi}\Delta g, \tag{114}$$

where $\Delta q_{\rm con}$ is measured by the partially conserved current $J^{\rm con}_{+5}$ and $-(\alpha_s/2\pi)\Delta g$ is measured by K_+. Positive gluon polarization tends to reduce the value of $g_A^{(0)}$ and offers a possible source for Okubo-Zweig-Iizuka (OZI) violation in $g_A^{(0)}|_{\rm inv}$. The connection between this more formal derivation and the QCD parton model will be explored in Sec. VI.D. In perturbative QCD $\Delta q_{\rm con}$ is identified with $\Delta q_{\rm partons}$ and Δg is identified with $\Delta g_{\rm partons}$—see Sec. VI.D and Altarelli and Ross (1983), Carlitz *et al.* (1988), Efremov and Teryaev (1988), and Bass *et al.* (1991).

C. Gluon topology, large gauge transformations, and connection to the axial U(1) problem

If we were to work only in the light-cone gauge we might think that we have a complete parton-model description of the first moment of g_1. However, one is free to work in any gauge including a covariant gauge where the forward matrix elements of K_+ are not necessarily invariant under the residual gauge degrees of freedom (Jaffe and Manohar, 1990). Understanding the interplay between spin and gauge invariance leads to rich and interesting physics possibilities.

We illustrate this by an example in covariant gauge. The matrix elements of K_μ need to be specified with respect to a specific gauge. In a covariant gauge we can write

$$\langle p,s|K_\mu|p',s'\rangle = 2M[\tilde{s}_\mu K_A(l^2) + l_\mu l\cdot\tilde{s}K_P(l^2)], \tag{115}$$

where K_P contains a massless Kogut-Susskind pole (Kogut and Susskind, 1974). This massless pole is an essential ingredient in the solution of the axial U(1) problem (Crewther, 1978) [the absence of any near massless Goldstone boson in the singlet channel associated with spontaneous axial U(1) symmetry breaking] and cancels with a corresponding massless pole term in G_P-K_P. The Kogut-Susskind pole is associated with the (unphysical) massless boson that one expects to couple to $J^{\rm con}_{\mu5}$ in the chiral limit and which is not seen in the physical spectrum.

We next define gauge-invariant form factors $\chi^g(l^2)$ for the topological charge density and $\chi^q(l^2)$ for the quark chiralities in the divergence of $J_{\mu5}$:

$$2Ml \cdot \tilde{s}\chi^g(l^2) = \langle p,s| \frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}|p',s'\rangle,$$

$$2Ml \cdot \tilde{s}\chi^q(l^2) = \langle p,s| \sum_{i=1}^{f} 2im_i\bar{q}_i\gamma_5 q_i|p',s'\rangle. \tag{116}$$

Working in a covariant gauge, we find

$$\chi^g(l^2) = K_A(l^2) + l^2 K_P(l^2) \tag{117}$$

by contracting Eq. (116) with l^μ. [Also, note the general gauge-invariant formula $g_A^{(0)} = \chi^q(0) + f\chi^g(0)$.]

When we make a gauge transformation any change $\delta_{\rm gt}$ in $K_A(0)$ is compensated by a corresponding change in the residue of the Kogut-Susskind pole in K_P, viz.,

$$\delta_{\rm gt}[K_A(0)] + \lim_{l^2 \to 0} \delta_{\rm gt}[l^2 K_P(l^2)] = 0. \tag{118}$$

As emphasized above, the Kogut-Susskind pole corresponds to the Goldstone boson associated with spontaneously broken $U_A(1)$ symmetry (Crewther, 1978). There is no Kogut-Susskind pole in perturbative QCD. It follows that the quantity that is shuffled between the $J_{+5}^{\rm con}$ and K_+ contributions to $g_A^{(0)}$ is strictly nonperturbative; it vanishes in perturbative QCD and is not present in the QCD parton model.

The QCD vacuum is understood to be a Bloch superposition of states characterized by different topological winding number (Callan *et al.*, 1976; Jackiw and Rebbi, 1976)

$$|\text{vac},\theta\rangle = \sum_n e^{in\theta}|n\rangle, \tag{119}$$

where the QCD θ angle is zero (experimentally less than 10^{-10})—see, e.g., Quinn (2004).

One can show (Jaffe and Manohar, 1990) that the forward matrix elements of K_μ are invariant under "small" gauge transformations (which are topologically deformable to the identity) but not invariant under "large" gauge transformations which change the topological winding number. Perturbative QCD involves only small gauge transformations; large gauge transformations involve strictly nonperturbative physics. The second term on the right-hand side of Eq. (106) is a total derivative; its matrix elements vanish in the forward direction. The third term on the right-hand side of Eq. (106) is associated with the gluon topology (Cronström and Mickelsson, 1983).

The topological winding number is determined by the gluonic boundary conditions at "infinity" (a large surface with boundary which is spacelike with respect to the positions z_k of any operators or fields in the physical problem; Crewther, 1978). It is insensitive to local deformations of the gluon field $A_\mu(z)$ or of the gauge transformation $U(z)$. When we take the Fourier transform to momentum space the topological structure induces a light-cone zero mode which can contribute to g_1 only at $x=0$. Hence we are led to consider the possibility that there may be a term in g_1 which is proportional to $\delta(x)$ (Bass, 1998).

It remains an open question whether the net nonperturbative quantity which is shuffled between $K_A(0)$ and $(G_A - K_A)(0)$ under large gauge transformations is finite or not. If it is finite and therefore physical, then when we choose $A_+ = 0$ this nonperturbative quantity must be contained in some combination of the $\Delta q_{\rm con}$ and Δg in Eq. (114).

In Secs. III and V we found that a $J=1$ fixed pole in the real part of A_1 in the forward Compton amplitude could also induce a "$\delta(x)$ correction" to the sum rule for the first moment of g_1 through a subtraction at infinity in the dispersion relation (40). Both the topological $x=0$ term and the subtraction constant $(Q^2/2M^2)\beta_1(Q^2)$ (if finite) give real coefficients of $1/x$ terms in Eq. (41). It seems reasonable therefore to conjecture that the physics of gluon topology may induce a $J=1$ fixed pole correction to the Ellis-Jaffe sum rule. Whether this correction is finite or not is an issue for future experiments.

Instantons provide an example of how to generate topological $x=0$ polarization (Bass, 1998). Quark-instanton interactions flip chirality, thus connecting left- and right-handed quarks. Whether instantons spontaneously or explicitly break axial U(1) symmetry depends on the role of zero modes in the quark-instanton interaction and how one should include nonlocal structure in the local anomalous Ward identity. Topological $x=0$ polarization is natural in theories of spontaneous axial U(1) symmetry breaking by instantons (Crewther, 1978) where any instanton-induced suppression of $g_A^{(0)}|_{\rm pDIS}$ is compensated by a shift of flavor-singlet axial charge from quarks carrying finite momentum to a zero mode $(x=0)$. It is not generated by mechanisms ('t Hooft, 1986) of explicit U(1) symmetry breaking by instantons. Experimental evidence for or against a subtraction at infinity correction to the Ellis-Jaffe sum rule would provide valuable information about gluon topology and vital clues to the nature of dynamical axial U(1) symmetry breaking in QCD.

D. Photon-gluon fusion

We next consider the role of the axial anomaly in the QCD parton model and its relation to semi-inclusive measurements of jets and high-k_t hadrons in polarized deep-inelastic scattering.

Consider the polarized photon-gluon fusion process $\gamma^* g \to q\bar{q}$. We evaluate the g_1 spin structure function for this process as a function of the transverse momentum squared of the struck quark k_t^2 with respect to the photon-gluon direction. We use q and p to denote the photon and gluon momenta and use the cutoff $k_t^2 \geq \lambda^2$ to separate the total phase space into hard $(k_t^2 \geq \lambda^2)$ and soft $(k_t^2 < \lambda^2)$ contributions. One finds (Bass *et al.*, 1998)

$$g_1^{(\gamma^* g)}|_{\text{hard}} = -\frac{\alpha_s}{2\pi}\frac{\sqrt{1-4(m^2+\lambda^2)/s}}{1-4x^2P^2/Q^2}\left\{(2x-1)\left(1-\frac{2xP^2}{Q^2}\right)\right.$$
$$\times\left[1-\frac{1}{\sqrt{1-4(m^2+\lambda^2)/s}\sqrt{1-4x^2P^2/Q^2}}\ln\left(\frac{1+\sqrt{1-4x^2P^2/Q^2}\sqrt{1-4(m^2+\lambda^2)/s}}{1-\sqrt{1-4x^2P^2/Q^2}\sqrt{1-4(m^2+\lambda^2)/s}}\right)\right]$$
$$\left.+\left(x-1+\frac{xP^2}{Q^2}\right)\frac{[2m^2(1-4x^2P^2/Q^2)-P^2x(2x-1)(1-2xP^2/Q^2)]}{(m^2+\lambda^2)(1-4x^2P^2/Q^2)-P^2x(x-1+xP^2/Q^2)}\right\} \tag{120}$$

for each flavor of quark liberated into the final state. Here m is the quark mass, $Q^2=-q^2$ is the virtuality of the hard photon, $P^2=-p^2$ is the virtuality of the gluon target, x is the Bjorken variable ($x=Q^2/2p\cdot q$), and s is the center-of-mass energy squared, $s=(p+q)^2=Q^2[(1-x)/x]-P^2$, for the photon-gluon collision.

When $Q^2\to\infty$ the expression for $g_1^{(\gamma^* g)}|_{\text{hard}}$ simplifies to the leading-twist (=2) contribution:

$$g_1^{(\gamma^* g)}|_{\text{hard}} = \frac{\alpha_s}{2\pi}\left\{(2x-1)\left[\ln\frac{1-x}{x}-1 + \ln\frac{Q^2}{x(1-x)P^2+(m^2+\lambda^2)}\right] + (1-x)\frac{2m^2-P^2x(2x-1)}{m^2+\lambda^2-P^2x(x-1)}\right\}. \tag{121}$$

Here we take λ to be independent of x. Note that for finite quark masses, phase space limits Bjorken x to $x_{\max}=Q^2/[Q^2+P^2+4(m^2+\lambda^2)]$ and protects $g_1^{(\gamma^* g)}|_{\text{hard}}$ from reaching the $\ln(1-x)$ singularity in Eq. (121). For this photon-gluon fusion process, the first moment of the hard contribution is

$$\int_0^1 dx g_1^{(\gamma^* g)}|_{\text{hard}} = -\frac{\alpha_s}{2\pi}\left[1+\frac{2m^2}{P^2}\frac{1}{\sqrt{1+4(m^2+\lambda^2)/P^2}} \times\ln\left(\frac{\sqrt{1+4(m^2+\lambda^2)/P^2}-1}{\sqrt{1+4(m^2+\lambda^2)/P^2}+1}\right)\right]. \tag{122}$$

The soft contribution to the first moment of g_1 is then obtained by subtracting Eq. (122) from the inclusive first moment (obtained by setting $\lambda=0$).

For fixed gluon virtuality P^2 the photon-gluon fusion process induces two distinct contributions to the first moment of g_1. Consider the leading-twist contribution, Eq. (122). The first term $-\alpha_s/2\pi$ in Eq. (122) is mass independent and comes from the region of phase space where the struck quark carries large transverse momentum squared $k_t^2\sim Q^2$. It measures a contact photon-gluon interaction and is associated (Carlitz *et al.*, 1988; Bass *et al.*, 1991) with the axial anomaly though the K_+ Chern-Simons current contribution to $J_{\mu 5}^{GI}$. The second mass-dependent term comes from the region of phase space where the struck quark carries transverse momentum $k_t^2\sim m^2,P^2$. This positive mass-dependent term is proportional to the mass squared of the struck quark. The mass-dependent term in Eq. (122) can safely be neglected for light-quark flavor (up and down) production. It is very important for strangeness and charm production (Bass *et al.*, 1999). For vanishing cutoff ($\lambda^2=0$) this term vanishes in the limit $m^2\ll P^2$ and tends to $+\alpha_s/2\pi$ when $m^2\gg P^2$ (so that the first moment of $g_1^{(\gamma^* g)}$ vanishes in this limit). The vanishing of $\int_0^1 dx g_1^{(\gamma^* g)}$ in the limit $m^2\ll P^2$ to leading order in $\alpha_s(Q^2)$ follows from an application (Bass *et al.*, 1998) of the fundamental GDH sum rule.

One can also analyze the photon-gluon fusion process using x-dependent cutoffs. Examples include the virtuality of the struck quark

$$m^2-k^2=P^2x+\frac{k_t^2+m^2}{(1-x)}>\lambda_0^2=\text{const}(x) \tag{123}$$

or the invariant mass squared of the quark-antiquark pair produced in the photon-gluon collision

$$\mathcal{M}_{q\bar{q}}^2=\frac{k_t^2+m^2}{x(1-x)}+P^2\geq\lambda_0^2=\text{const}(x). \tag{124}$$

These different choices of infrared cutoffs correspond to different jet definitions and different factorization schemes for photon-gluon fusion in the QCD parton model—see Bass *et al.* (1991), Mankiewicz (1991), Manohar (1991), and Bass *et al.* (1998). If we evaluate the first moment of $g_1^{(\gamma^* g)}$ using the cutoff on the quarks' virtuality, then we find "half of the anomaly" in the gluon coefficient through the mixing of transverse and longitudinal momentum components. The anomaly coefficient for the first moment is recovered with the invariant mass squared cutoff through a sensitive cancellation of large- and small-x contributions (Bass *et al.*, 1991).

We noted above that when one applies the operator product expansion, the first term in Eq. (122) corresponds to the gluon matrix element of the anomalous gluonic current K_+. This operator product expansion analysis can be generalized to the higher moments of $g_1^{(\gamma^* g)}$. The anomalous contribution to the higher moments is controlled by choosing the correct prescription for γ_5. One finds (Bass, 1992b; Cheng, 1996) that the axial anomaly contribution to the shape of g_1 at finite x is given by the convolution of the polarized gluon distribution $\Delta g(x,Q^2)$ with the hard coefficient

$$\tilde{C}^{(g)}|_{\mathrm{anom}} = -\frac{\alpha_s}{\pi}(1-x). \tag{125}$$

This anomaly contribution is a small-x effect in g_1; it is essentially negligible for x less than 0.05. The hard coefficient $\tilde{C}^{(g)}|_{\mathrm{anom}}$ is normally included as a term in the gluonic Wilson coefficient C^g—see Sec. IX.C. It is associated with two-quark-jet events carrying $k_t^2 \sim Q^2$ in the final state.

Equation (122) leads to the well-known formula stated in Sec. I:

$$g_A^{(0)} = \left(\sum_q \Delta q - 3\frac{\alpha_s}{2\pi}\Delta g\right)_{\mathrm{partons}} + \mathcal{C}_\infty. \tag{126}$$

Here Δg is the amount of spin carried by polarized gluon partons in the polarized proton and $\Delta q_{\mathrm{partons}}$ measures the spin carried by quarks and antiquarks carrying soft transverse momentum $k_t^2 \sim m^2, P^2$. Note that the mass-independent contact interaction in Eq. (122) is flavor independent. The mass-dependent term associated with low k_t breaks flavor SU(3) in the perturbative sea. The third term $\mathcal{C}_\infty = \frac{1}{2}\lim_{Q^2\to\infty}(Q^2/2M^2)\beta_1(Q^2)$ describes any fixed pole subtraction at infinity correction to $g_A^{(0)}$.

Equations (107) yield the renormalization-group equation

$$\left\{\frac{\alpha_s}{2\pi}\Delta g\right\}_{Q^2} = \left\{\frac{\alpha_s}{2\pi}\Delta g\right\}_\infty + \frac{1}{3}\{1/E(\alpha_s)-1\}g_A^{(0)}\Big|_{\mathrm{inv}}. \tag{127}$$

It follows that the polarized gluon term satisfies

$$\alpha_s \Delta g \sim \mathrm{const}, \quad Q^2 \to \infty. \tag{128}$$

This key result, first noted in the context of the QCD parton model by Altarelli and Ross (1988) and Efremov and Teryaev (1988), means that the polarized gluon contribution makes a scaling contribution to the first moment of g_1 at next-to-leading order. (In higher orders the Q^2 evolution of Δg depends on the value of $g_A^{(0)}|_{\mathrm{inv}}$ suggesting one method to search for any finite $\mathcal{C}_\infty$.)

The transverse-momentum dependence of the gluonic and sea-quark partonic contributions to $g_A^{(0)}$ suggests the interpretation of measurements of quark sea polarization will depend on the large-k_t acceptance of the apparatus. Let $g_1^{(\gamma^* g)}|_{\mathrm{soft}}(\lambda)$ denote the contribution to $g_1^{(\gamma^* g)}$ for photon-gluon fusion where the hard photon scatters on the struck quark or antiquark carrying transverse momentum $k_t^2 < \lambda^2$. Figure 9 shows the first moment of $g_1^{(\gamma^* g)}|_{\mathrm{soft}}$ for the strange and light (up and down) flavor production, respectively, as a function of the transverse-momentum cutoff λ^2. Here we set $Q^2 = 2.5\ \mathrm{GeV}^2$ (corresponding to the HERMES experiment) and $10\ \mathrm{GeV}^2$ (SMC). Following Carlitz *et al.* (1998), we take $P^2 \sim \Lambda_{\mathrm{QCD}}^2$ and set $P^2 = 0.1\ \mathrm{GeV}^2$. Observe the small value for the light-quark sea polarization at low transverse momentum and the positive value for the integrated strange sea polarization at low k_t^2: $k_t < 1.5$ GeV at the HERMES $Q^2 = 2.5\ \mathrm{GeV}^2$.

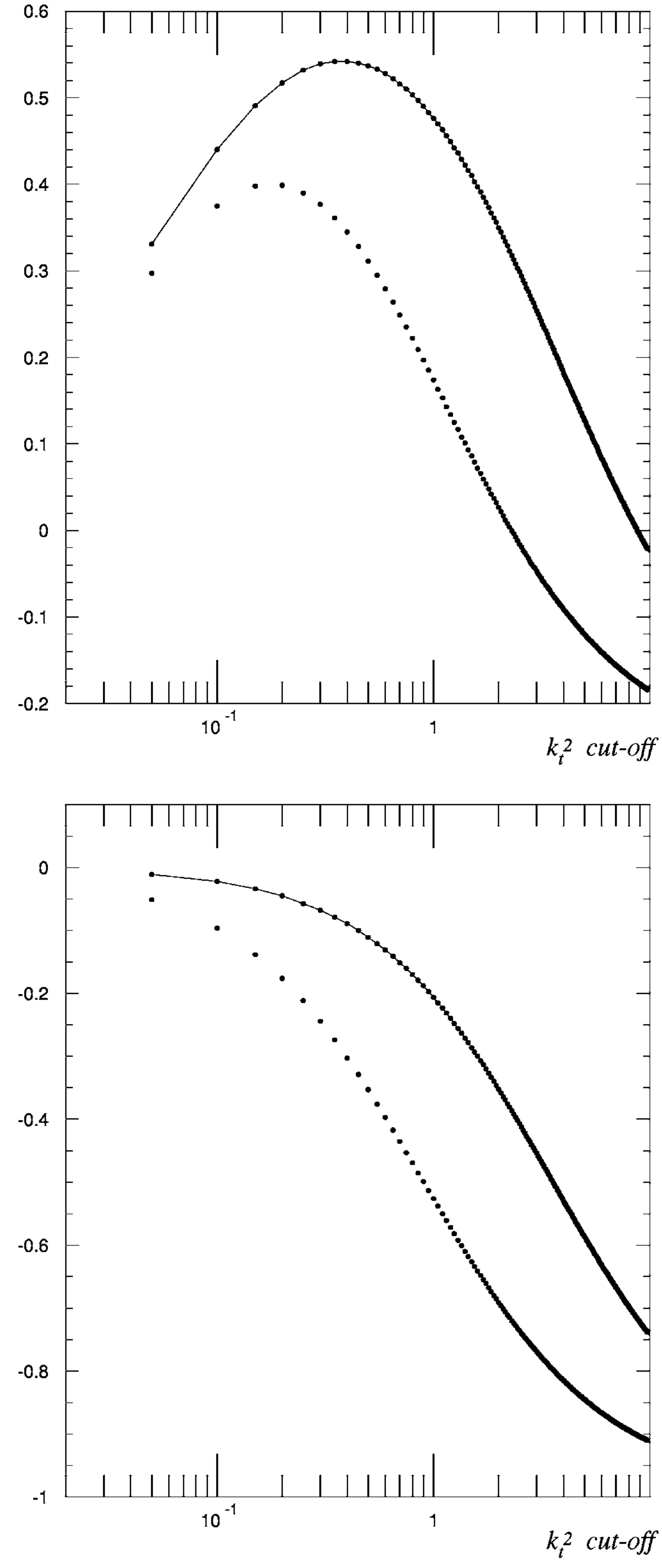

FIG. 9. $\int_0^1 dx\, g_1^{(\gamma^* g)}|_{\mathrm{soft}}$ for polarized strangeness production (top) and light-flavor (u or d) production (bottom) with $k_t^2 < \lambda^2$ in units of $\alpha_s/2\pi$ (Bass, 2003a). Here $Q^2 = 2.5\ \mathrm{GeV}^2$ (dotted line) and $10\ \mathrm{GeV}^2$ (solid line).

E. Choice of currents and "spin"

The axial anomaly presents us with three candidate currents we use to define the quark spin content: $J^{GI}_{\mu 5}$, the renormalization scale invariant $E(\alpha_s)J^{GI}_{\mu 5}$, and $J^{\rm con}_{\mu 5}$. One might also consider using the chiralities $\chi^{(q)}$ from Eq. (116). We next explain how each current yields gauge-invariant possible definitions.

First, note that if we try to define intrinsic spin operators

$$S_k = \int d^3x(\bar{q}\gamma_k\gamma_5 q), \quad k=1,2,3, \tag{129}$$

using the axial-vector current operators, then we find that the operators constructed using the gauge-invariantly renormalized current $J^{GI}_{\mu 5}$ cannot satisfy the (spin) commutation relations of SU(2) $[S_i,S_j](\mu^2) = i\epsilon_{ijk}S_k(\mu^2)$ at more than one scale μ^2 because of the anomalous dimension and the renormalization-group factor associated with $E(\alpha_s)$ and the axial anomaly (Bass and Thomas, 1993b). In order to satisfy SU(2) perhaps the most natural scale for normalizing the axial-vector current operators is $\mu\to\infty$—that is, using the scale-invariant current $\{E(\alpha_s)J_{\mu 5}\}$. Then we find $[S_i,S_j](\mu^2) = i\epsilon_{ijk}E(\alpha_s)S_k(\mu^2)$ if we use the gauge-invariant current renormalized at another scale. One might argue that gluon spin is renormalization scale dependent, Eq. (127), so one need not worry too much about this issue, but there are further points to consider.

Next choose the $A_0=0$ gauge and define two operator charges:

$$X(t) = \int d^3z J^{GI}_{05}(z),$$

$$Q_5 = \int d^3z J^{\rm con}_{05}(z). \tag{130}$$

Because partially conserved currents are not renormalized it follows that Q_5 is a time-independent operator. The charge $X(t)$ is manifestly gauge invariant whereas Q_5 is invariant only under small gauge transformations; the charge Q_5 transforms as

$$Q_5 \to Q_5 - 2fn, \tag{131}$$

where n is the winding number associated with the gauge transformation U. Although Q_5 is gauge dependent we can define a gauge-invariant chirality q_5 for a given operator $\mathcal{O}$ through the gauge-invariant eigenvalues of the equal-time commutator

$$[Q_5,\mathcal{O}]_- = -q_5\mathcal{O}. \tag{132}$$

The gauge invariance of q_5 follows since this commutator appears in gauge-invariant Ward identities (Crewther, 1978) despite the gauge dependence of Q_5. The time derivative of spatial components of the gluon field have zero chiralty q_5,

$$[Q_5,\partial_0 A_i]_- = 0 \tag{133}$$

but nonzero X charge

$$\lim_{t'\to t}[X(t'),\partial_0 A_i(\vec{x},t)]_- = \frac{ifg^2}{4\pi^2}\tilde{G}_{0i} + O(g^4 \ln|t'-t|). \tag{134}$$

The analogous situation in QED is discussed by Adler and Boulware (1969), Jackiw and Johnson (1969), and Adler (1970). Equation (133) follows from the nonrenormalization of the conserved current $J^{\rm con}_{\mu 5}$. Equation (134) follows from the implicit A_μ dependence of the (anomalous) gauge-invariant current $J^{GI}_{\mu 5}$. The higher-order terms $g^4\ln|t'-t|$ are caused by wave-function renormalization of $J_{\mu 5}$ (Crewther, 1978).

This formalism generalizes readily to the definition of baryon number in the presence of electroweak gauge fields. The vector baryon-number current is sensitive to the axial anomaly through the parity-violating electroweak interactions. If one requires that the baryon number is renormalization-group invariant and that the time derivative of the spatial components of the W-boson field have zero baryon number, then one is led to using the conserved vector-current analogy of q_5 to define the baryon number. Sphaleron-induced electroweak baryogenesis in the early Universe (Kuzmin *et al.*, 1985; Rubakov and Shaposhnikov, 1996) is then accompanied by the formation of a "topological condensate" (Bass, 2004) which (probably) survives in the Universe we live in today.

Last, we comment on the use of the chiralities χ^q and the quantity χ^g to define the "quark spin" and "gluon spin" content of the proton. This suggestion starts from the decomposition

$$g_A^{(0)} = \chi^q(0) + 3\chi^g(0) \tag{135}$$

but is less optimal because the separate quark and gluonic pieces are very much infrared sensitive and strongly dependent on the ratios of the light-quark masses m_u/m_d (Cheng and Li, 1989; Veneziano, 1989)—see also Gross *et al.* (1979) and Ioffe (1979). Indeed, for the polarized real-photon structure function g_1^γ the quantity $\chi^g_{\rm photon}\sim 30$ at realistic deep-inelastic values of Q^2 (Bass, 1992a)!

VII. CHIRAL SYMMETRY AND THE SPIN STRUCTURE OF THE PROTON

The Goldberger-Treiman relations relate the spin structure of the proton to spontaneous chiral symmetry breaking in QCD.

The isovector Goldberger-Treiman relation (Adler and Dashen, 1968)

$$2Mg_A^{(3)} = f_\pi g_{\pi NN} \tag{136}$$

relates $g_A^{(3)}$ and therefore $\Delta u - \Delta d$ to the product of the pion decay constant f_π and the pion-nucleon coupling constant $g_{\pi NN}$. This result is nontrivial. It means that the spin structure of the nucleon measured in high-energy, high-Q^2 polarized deep-inelastic scattering is intimately related to spontaneous chiral symmetry breaking and low-energy pion physics. The Bjorken sum rule can also be written $\int_0^1 dx(g_1^p - g_1^n) = \frac{1}{6}\{f_\pi g_{\pi NN}/2M\}\{1 + \Sigma_{\ell \geq 1} c_{\mathrm{NS}\ell} \alpha_s^\ell(Q)\}$ (modulo small chiral corrections ~5% coming from the finite light-quark and pion masses).

The flavor-singlet generalization of the Goldberger-Treiman relation was derived independently by Shore and Veneziano (1990, 1992) and Hatsuda (1990).

Isoscalar extensions of the Goldberger-Treiman relation are quite subtle because of the axial U(1) problem whereby gluonic degrees of freedom mix with the flavor-singlet Goldstone state to increase the masses of the η and η' mesons. The vacuum condensates $\langle \mathrm{vac}|\bar{q}q|\mathrm{vac}\rangle$ $(q=u,d,s)$ spontaneously break both chiral SU(3) and axial U(1) symmetry. One expects a nonet of would-be Goldstone bosons: the physical pions and kaons plus octet and singlet states. In the singlet channel the axial anomaly and nonperturbative gluon topology induce a substantial gluonic mass term for the singlet boson.

The Witten-Veneziano mass formula (Veneziano, 1979; Written, 1979) relates the gluonic mass term for the singlet boson to the topological susceptibility of pure Yang-Mills (glue with no quarks)

$$\tilde{m}_{\eta_0}^2 = -\frac{6}{f_\pi^2}\chi(0), \tag{137}$$

where $\chi(k^2) = \int d^4 z i e^{ik\cdot z} \langle \mathrm{vac}|TQ(z)Q(0)|\mathrm{vac}\rangle|_{\mathrm{YM}}$ and $Q(z)$ denotes the topological charge density. Without this singlet gluonic mass term the η meson would be approximately degenerate with the pion and the η' meson would have a mass $\sim\sqrt{2m_K^2 - m_\pi^2}$ after we take into account mixing between the octet and singlet bosons induced by the strange-quark mass.

In the chiral limit the flavor-singlet Goldberger-Treiman relation reads

$$2Mg_A^{(0)} = \sqrt{\chi'(0)}\, g_{\phi_0 NN}. \tag{138}$$

Here $\chi'(0)$ is the first derivative of the topological susceptibility and $g_{\phi_0 NN}$ denotes the one-particle irreducible coupling to the nucleon of the flavor-singlet Goldstone boson which would exist in a gedanken world where OZI is exact in the singlet axial U(1) channel. ϕ_0 is a theoretical object and not a physical state in the spectrum. The important features of Eq. (138) are first that $g_A^{(0)}$ factorizes into the product of the target-dependent coupling $g_{\phi_0 NN}$ and the target-independent gluonic term $\sqrt{\chi'(0)}$. The coupling $g_{\phi_0 NN}$ is renormalization scale invariant and the scale dependence of $g_A^{(0)}$ associated with the renormalization-group factor $E(\alpha_s)$ is carried by the gluonic term $\sqrt{\chi'(0)}$. Motivated by this observation, Narison, Shore, and Veneziano (1995) conjectured that any OZI violation in $g_A^{(0)}|_{\mathrm{inv}}$ might be carried by the target-independent factor $\sqrt{\chi'(0)}$ and suggested experiments to test this hypothesis by studying semi-inclusive polarized deep-inelastic scattering in the target fragmentation region (which allows one to vary the *de facto* hadron target—e.g., a proton or Δ resonance; Shore and Veneziano, 1998).

OZI violation associated with the gluonic topological charge density may also be important to a host of η and η' interactions in hadronic physics. We refer to a paper by Bass (2002b) for an overview of the phenomenology. Experiments underway at COSY-Jülich are measuring the isospin dependence of η and η' production close to threshold in proton-nucleon collisions (Moskal, 2004). These experiments are looking for signatures of possible OZI violation in the η' nucleon interaction. Anomalous glue may play a key role in the structure of the light-mass (about 1400–1600 MeV) exotic mesons with quantum numbers $J^{PC}=1^{-+}$ that have been observed in experiments at BNL and CERN. These states might be dynamically generated resonances in $\eta'\pi$ rescattering (Bass and Marco, 2002; Szczepaniak *et al.*, 2003) mediated by the OZI violating coupling of the η'. Planned experiments at the GSI in Darmstadt will measure the η mass in nuclei (Hayano *et al.*, 1999) and thus probe aspects of axial U(1) dynamics in the nuclear medium.

VIII. CONNECTING QCD AND QCD-INSPIRED MODELS OF THE PROTON SPIN PROBLEM

We now compare the various proposed explanations of the proton spin problem (the small value of $g_A^{(0)}$ extracted from polarized deep-inelastic scattering) in roughly the order that they enter the derivation of the g_1 spin sum rule:

(1) A subtraction at infinity in the dispersion relation for g_1 perhaps generated in the transition from current to constituent quarks and involving gluon topology and the mechanism of dynamical axial U(1) symmetry breaking. In the language of Regge phenomenology it is associated with a fixed pole in the real part of the spin-dependent part of the forward Compton amplitude.

 In this scenario the strange-quark polarization Δs extracted from inclusive polarized deep-inelastic scattering and neutrino-proton elastic scattering would be different. A precision measurement of νp elastic scattering would be very useful.

 Note that fixed poles play an essential role in the Adler and Schwinger-term sum rules—one should be on the lookout!

(2) SU(3) flavor breaking in the analysis of hyperon beta decays. Phenomenologically, SU(3) flavor symmetry seems to be well respected in the measured beta decays, including the recent KTeV measurement of the Ξ^0 decay (Alavi-Harati *et al.*, 2001). Leader and Stamenov (2003) have recently argued that even the most extreme SU(3)-breaking scenarios consistent with hyperon decays will still lead to a negative value of the strange-quark axial charge Δs extracted from polarized deep-inelastic data. Possible SU(3) breaking in the large N_c limit of

QCD has been investigated by Flores-Mendieta *et al.* (1998).

One source of SU(3) breaking that we have so far observed is in the polarized sea generated through photon-gluon fusion where the strange-quark mass term is important—see Eq. (122) and Fig. 9. The effect of including SU(3) breaking in the parton model for $\Delta q_{\rm partons}$ within various factorization schemes has been investigated by Glück *et al.* (2001).

(3) Topological charge screening and target independence of the spin effect generated by a small value of $\chi'(0)$ in the flavor-singlet Goldberger-Treiman relation. This scenario could be tested through semi-inclusive measurements where a pion or D meson is detected in the target fragmentation region, perhaps using a polarized ep collider with Roman pot detectors (Shore and Veneziano, 1998). These experiments could, in principle, be used to vary the target and measure g_1 for, e.g., Δ^{++} and Δ^{-} targets along the lines of the program that has been conducted in unpolarized scattering experiments (Holtmann *et al.*, 1994).

(4) Nonperturbative evolution associated with the renormalization-group factor $E(\alpha_s)$ between deep-inelastic scales and the low-energy scale where quark models might, perhaps, describe the twist-2 parton distributions (Jaffe, 1987). One feature of this scenario is that (in the four-flavor theory) the polarized charm and strange-quark contributions evolve at the same rate with changing Q^2 since $\Delta s - \Delta c$ is flavor nonsinglet (and therefore independent of the QCD axial anomaly) (Bass and Thomas, 1993a). Heavy-quark renormalization-group arguments suggest that Δc is small (Kaplan and Manohar, 1998; Bass *et al.*, 2002) up to $1/m_c$ corrections.

(5) Large gluon polarization $\Delta g \sim 1$ at the scale $\mu \sim 1$ GeV could restore consistency between the measured $g_A^{(0)}$ and quark-model predictions if the quark-model predictions are associated with $\Delta q_{\rm partons}$ (the low-k_t contribution to $g_A^{(0)}$) in Eq. (126). Δg can be measured through a variety of gluon-induced partonic production processes including charm production and two-quark-jet events in polarized deep-inelastic scattering, and prompt photon production and jet studies in polarized proton collisions at RHIC—see Sec. IX.E. First attempts to extract Δg from QCD-motivated fits to the Q^2 dependence of g_1 data yield values between 0 and 2 at $Q^2 \sim 1$ GeV^2—see Sec. IX.C.

How big should we expect Δg to be? Working in the framework of light-cone models one finds contributions from "intrinsic" and "extrinsic" gluons. Extrinsic contributions arise from gluon bremstrahlung $q_V \to q_V g$ of a valence quark and have a relatively hard virtuality. Intrinsic gluons are associated with the physics of the nucleon wave function (for example, gluons emitted by one valence quark and absorbed by another quark) and have a relatively soft spectrum (Bass *et al.*, 1999). Light-cone models including QCD color coherence at small Bjorken x and perturbative QCD counting rules at large x (Brodsky and Schmidt, 1990; Brodsky *et al.*, 1995) suggest values of $\Delta g \sim 0.6$ at low scales ~ 1 GeV^2—sufficient to account for about half of the "missing spin" or measured value of $g_A^{(0)}$.

Bag-model calculations give values $\Delta g \sim -0.4$ (note the negative sign) when one includes gluon exchange contributions and no "self-field" contribution where the gluon is emitted and absorbed by the same quark (Jaffe, 1996) and $\Delta g \sim 0.24$ (positive sign) when the self-field contribution is included (Barone *et al.*, 1998). A QCD sum-rule calculation (Saalfeld *et al.*, 1998) gives $\Delta g \sim 2 \pm 1$.

(6) Large negatively polarized strangeness in the quark sea (with small k_t). This scenario can be tested through semi-inclusive measurements of polarized deep-inelastic scattering provided that radiative corrections, fragmentation functions, and the experimental acceptance are under control.

Of course, the final answer may prove to be a cocktail solution of these possible explanations or include some new dynamics that has not yet been thought of.

In testing models of quark sea and gluon polarization it is important to understand the transverse momentum and Bjorken-x dependence of the different sea-quark dynamics. For example, sea-quark contributions to deep-inelastic structure functions are induced by perturbative photon-gluon fusion (Altarelli and Ross, 1988; Carlitz *et al.*, 1988; Efremov and Tervaev, 1988), pion and kaon cloud physics (Koepf *et al.*, 1992; Melnitchouk and Malheiro, 1999; Cao and Signal, 2003), instantons (Forte and Shuryak, 1991; Dolgov *et al.*, 1999; Nishikawa, 2004; Schafer and Zetocha, 2004), etc. In general, different mechanisms will produce sea with different x and k_t dependence.

Lattice calculations are also making progress in unraveling the spin structure of the proton (Mathur *et al.*, 2000; Negele *et al.*, 2004). Interesting new results (Negele *et al.*, 2004) suggest a value of $g_A^{(0)}$ about 0.7 in a heavy-pion world where the pion mass $m_\pi \sim 700-900$ MeV. Physically, in the heavy-pion world (away from the chiral limit) the quarks become less relativistic and it is reasonable to expect the nucleon spin to arise from the valence-quark spins. Sea-quark effects are expected to become more important as the quarks become lighter and sea production mechanisms become important. It will be interesting to investigate the behavior of $g_A^{(0)}$ in future lattice calculations as these calculations approach the chiral limit.

In an alternative approach to understanding low-energy QCD, Witten (1983a, 1983b) noticed that in the limit that the number of colors N_c is taken to infinity ($N_c \to \infty$ with $\alpha_s N_c$ held fixed) QCD behaves like a system of bosons and the baryons emerge as topological solitions called Skyrmions in the meson fields. In this model the spin of the large N_c "proton" is a topological quantum number. The "nucleon's" axial charges turn out to be sensitive to which meson fields are included in the model and the relative contribution of a quark source and pure mesons—we refer to the lectures of Aitchison (1988) for a more detailed discussion of the Skyrmion approach. Brodsky *et al.* (1988) found that $g_A^{(0)}$ vanishes in a particular version of the Skyrmion model with just pseudoscalar mesons. Nonvanishing values of g_A^0 are found using more general Skyrmion Lagrangians (Cohen and Banerjee, 1989; Ryzak, 1989), including with additional vector mesons (Johnson *et al.*, 1990).

IX. THE SPIN-FLAVOR STRUCTURE OF THE PROTON

A. The valence region and large *x*

The large-x region (x close to 1) is very interesting and particularly sensitive to the valence structure of the nucleon. Valence quarks dominate deep-inelastic structure functions for large and intermediate x (greater than about 0.2). Experiments at Jefferson Laboratory are making the first precision measurements of the proton's spin structure at large x—see Fig. 10

QCD-motivated predictions for the large-x region exist based on perturbative QCD counting rules and quark models of the proton's structure based on SU(6) [flavor SU(3) ⊗ spin SU(2)] and scalar diquark dominance. We give a brief explanation of these approaches.

(1) Perturbative QCD counting rules predict that the parton distributions should behave as a power-series expansion in $1-x$ when $x \to 1$ (Farrar and Jackson, 1975; Brodsky *et al.*, 1995). The fundamental principle behind these counting-rules results is that for the leading struck quark to carry helicity polarized in the same direction as the proton, the spectator pair carries spin zero and is bound through longitudinal gluon exchange. For the struck quark to be polarized opposite to the direction of the proton the spectator pair should be in a spin-1 state, and in this case one also has to consider the effect of transverse gluon exchange. Calculation shows that this is supressed by a factor of $(1-x)^2$. We use $q^{\uparrow}(x)$ and $q^{\downarrow}(x)$ to denote the parton distributions polarized parallel and antiparallel to the polarized proton. One finds (Brodsky *et al.*, 1995)

$$q^{\uparrow\downarrow}(x) \to (1-x)^{2n-1+2\Delta S_z}, \quad x \to 1. \tag{139}$$

Here n is the number of spectators and ΔS_z is the difference between the polarization of the struck quark and the polarization of the target nucleon. When $x \to 1$ the QCD counting rules predict that the structure functions should be dominated by valence quarks polarized parallel to the spin of the nucleon. The ratio of polarized to unpolarized structure functions should go to 1 when $x \to 1$. For the helicity parallel valence-quark distribution one predicts

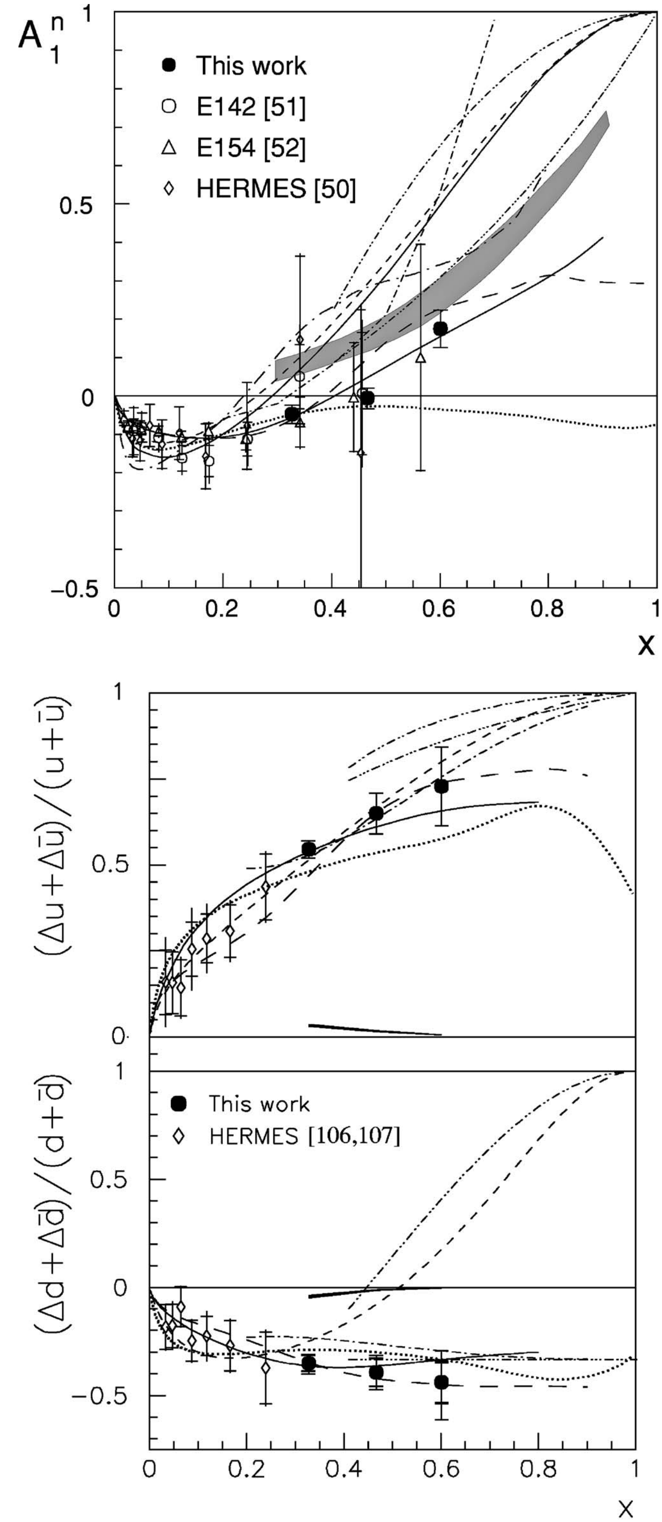

FIG. 10. Top: Recent data on $\mathcal{A}_1^n$ from the E99-117 experiment (Zheng *et al.*, 2004a, 2004b). Bottom: extracted polarization asymmetries for $u+\bar{u}$ and $d+\bar{d}$. For more details and references on the various model predictions, see Zheng *et al.* (2004a, 2004b).

$$q^{\uparrow}(x) \sim (1-x)^3, \quad x \to 1, \tag{140}$$

whereas for the helicity antiparallel distribution one obtains

TABLE I. QCD-motivated model predictions for the large-x limit of deep-inelastic spin asymmetries and parton distributions.

Model	$\Delta u/u$	$\Delta d/d$	$\mathcal{A}_1^p$	$\mathcal{A}_1^n$	d/u
SU(6)	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{9}$	0	$\frac{1}{2}$
Broken SU(6), scalar diquark	1	$-\frac{1}{3}$	1	1	0
QCD counting rules	1	1	1	1	$\frac{1}{5}$

$$q^{\downarrow}(x) \sim (1-x)^5, \quad x \to 1. \tag{141}$$

Sea distributions are suppressed and the leading term starts as $(1-x)^5$.

(2) Scalar diquark dominance is based on the observation that, within the context of the SU(6) wave function of the proton in Eq. (9), one-gluon exchange tends to make the mass of the scalar diquark pair lighter than the vector spin-1 diquark combination. One-gluon exchange offers an explanation of the nucleon-Δ mass splitting and has the practical consequence that in model calculations of deep-inelastic structure functions the scalar diquark term $(1/\sqrt{2})|u\uparrow(ud)_{S=0}\rangle$ in Eq. (9) dominates the physics at large Bjorken x (Close and Thomas, 1988).

In the large-x region (x close to 1) where sea quarks and gluons can be neglected the neutron and proton spin asymmetries are given by

$$\mathcal{A}_1^n = \frac{\Delta u + 4\Delta d}{u + 4d}, \quad \mathcal{A}_1^p = \frac{4\Delta u + \Delta d}{4u + d}. \tag{142}$$

Rearranging these expressions one obtains formulas for the separate up and down quark distributions in the proton:

$$\frac{\Delta u}{u} = \frac{4}{15}\mathcal{A}_1^p\left(4 + \frac{d}{u}\right) - \frac{1}{15}\mathcal{A}_1^n\left(1 + 4\frac{d}{u}\right),$$

$$\frac{\Delta d}{d} = \frac{4}{15}\mathcal{A}_1^n\left(4 + \frac{u}{d}\right) - \frac{1}{15}\mathcal{A}_1^p\left(1 + 4\frac{u}{d}\right). \tag{143}$$

The predictions of perturbative QCD counting rules and scalar diquark dominance models for the large-x limit of these asymmetries are given in Table I. On the basis of both perturbative QCD and SU(6), one expects the ratio of polarized to unpolarized structure functions, $\mathcal{A}_{1n}$, should approach 1 as $x \to 1$ (Melnitchouk and Thomas, 1996; Isgur, 1999). It is vital to test this prediction. If it fails, we understand nothing about the valence spin structure of the nucleon.

Interesting new data from the Jefferson Laboratory Hall A Collaboration on the neutron asymmetry $\mathcal{A}_1^n$ (Zheng *et al.*, 2004a) are shown in Fig. 10. These data show a clear trend for $\mathcal{A}_1^n$ to become positive at large x. The crossover point where $\mathcal{A}_1^n$ changes sign is particularly interesting because the value of x where this occurs in the neutron asymmetry is the result of a competition between the SU(6) valence structure (Close and Thomas, 1988) and chiral corrections (Schreiber and Thomas, 1988; Steffens, 1995). Figure 10 also shows the extracted valence polarization asymmetries. The data are consistent with constituent quark models with scalar diquark dominance which predict $\Delta d/d \to -1/3$ at large x, while perturbative QCD counting-rule predictions (which neglect quark orbital angular momentum) give $\Delta d/d \to 1$ and tend to deviate from the data, unless the convergence to 1 sets in very late.

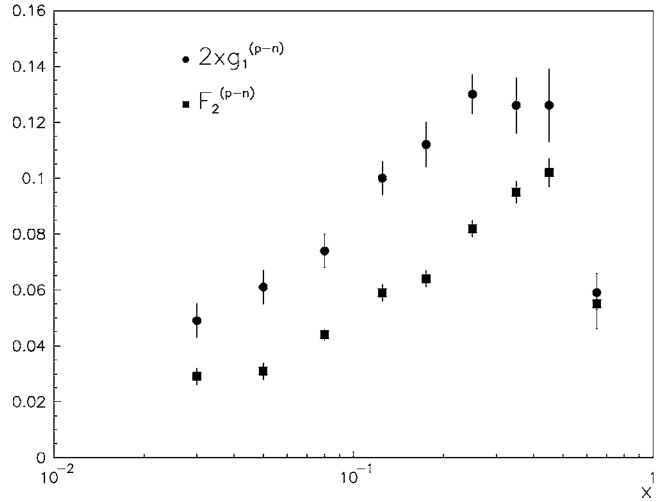

FIG. 11. The isovector structure functions $2xg_1^{(p-n)}$ (SLAC data) and $F_2^{(p-n)}$ (NMC) from Bass (1999).

A precision measurement of $\mathcal{A}_{1n}$ up to $x \sim 0.8$ will be possible following the 12-GeV upgrade of Jefferson Laboratory (Meziani, 2002).

B. The isovector part of g_1

Constituent quark-model predictions for g_1 are observed to work very well in the isovector channel. First, as highlighted in Sec. III.B, the Bjorken sum rule which relates the first moment of the isovector part of g_1, $g_1^p - g_1^n$, to the isovector axial charge $g_A^{(3)}$ has been confirmed in polarized deep-inelastic-scattering experiments at the level of 10% (Windmolders, 1999). Second, looking beyond the first moment, one finds the following intriguing observation about the shape of $g_1^p - g_1^n$. Figure 11 shows $2x(g_1^p - g_1^n)$ (SLAC data) together with the isovector structure function $F_2^p - F_2^n$ (NMC data). The ratio $R_{(3)} = 2x(g_1^p - g_1^n)/(F_2^p - F_2^n)$ is plotted in Fig. 12. It measures the ratio of polarized to unpolarized isovector quark distributions. In the QCD parton model[5]

[5]In a full description one should also include the perturbative QCD Wilson coefficients for the nonsinglet spin difference and spin-averaged cross sections. However, the effect of these coefficients makes a non-negligible contribution to the deep-inelastic structure functions only at $x < 0.05$ and is small in the kinematics where there is high-Q^2 spin data. There are no gluonic or singlet Pomeron contributions to the isovector structure functions $g_1^p - g_1^n$ and $F_2^p - F_2^n$.

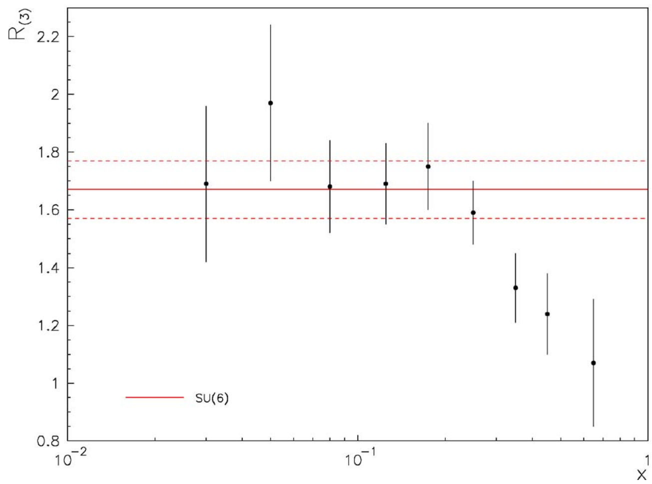

FIG. 12. (Color online) The ratio $R_{(3)}=2xg_1^{(p-n)}/F_2^{(p-n)}$ from Bass (1999).

$$2x(g_1^p - g_1^n) = \frac{1}{3}x[(u+\bar{u})^\uparrow - (u+\bar{u})^\downarrow - (d+\bar{d})^\uparrow + (d+\bar{d})^\downarrow] \tag{144}$$

and

$$F_2^p - F_2^n = \frac{1}{3}x[(u+\bar{u})^\uparrow + (u+\bar{u})^\downarrow - (d+\bar{d})^\uparrow - (d+\bar{d})^\downarrow]. \tag{145}$$

The data reveal a large isovector contribution in g_1 and the ratio $R_{(3)}$ is observed to be approximately constant [at the value $\sim 5/3$ predicted by SU(6) constituent quark models] for x between 0.03 and 0.2, and goes towards one when $x \to 1$ (consistent with the prediction of both QCD counting rules and scalar diquark dominance models). The small-x part of this data is very interesting. The area under $(F_2^p - F_2^n)/2x$ is determined by the Gottfried integral (Gottfried, 1967; Arneodo *et al.*, 1994) and is about 25% suppressed relative to the simple SU(6) prediction (by the pion cloud, Pauli blocking, etc.). The area under $g_1^p - g_1^n$ is fixed by the Bjorken sum rule [and is also about 25% suppressed relative to the SU(6) prediction—the suppression here being driven by relativistic effects in the nucleon and by perturbative QCD corrections to the Bjorken sum rule]. Given that perturbative QCD counting rules or scalar diquark models work and assuming that the ratio $R_{(3)}$ takes the constituent quark prediction at the canonical value of $x \sim \frac{1}{3}$, one finds (Bass, 1999) that the observed shape of $g_1^p - g_1^n$ is almost required to reproduce the area under the Bjorken sum rule (which is determined by the physical value of $g_A^{(3)}$—a nonperturbative constraint)! The constant ratio in the low- to medium-x range contrasts with the naive Regge prediction using a_1 exchange (and no hard Pomeron a_1 cut) that the ratio $R_{(3)}$ should fall and be roughly proportional to x as $x \to 0$. It would be very interesting to have precision measurements of g_1 at high energy and low Q^2 from a future polarized *ep* collider to test the various scenarios of how small-x dynamics might evolve through the transition region and the application of spin-dependent Regge theory.

C. QCD fits to g_1 data

In deep-inelastic-scattering experiments the different x data points on g_1 are each measured at different values of Q^2, viz., $x_{\text{expt.}}(Q^2)$. One has to evolve these experimental data points to the same value of Q^2 in order to test the Bjorken (Bjorken, 1966, 1970) and Ellis-Jaffe (Ellis and Jaffe, 1974) sum rules. Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution is frequently used in analyses of polarized deep-inelastic data to achieve this.

The λ^2 dependence of the parton distributions is given by the DGLAP equations (Altarelli and Parisi, 1977)

$$\frac{d}{dt}\Delta\Sigma(x,t) = \frac{\alpha_s(t)}{2\pi}\left[\int_x^1 \frac{dy}{y}\Delta P_{qq}\left(\frac{x}{y}\right)\Delta\Sigma(y,t) + 2f\int_x^1 \frac{dy}{y}\Delta P_{qg}\left(\frac{x}{y}\right)\Delta g(y,t)\right],$$

$$\frac{d}{dt}\Delta g(x,t) = \frac{\alpha_s(t)}{2\pi}\left[\int_x^1 \frac{dy}{y}\Delta P_{gq}\left(\frac{x}{y}\right)\Delta\Sigma(y,t) + \int_x^1 \frac{dy}{y}\Delta P_{gg}\left(\frac{x}{y}\right)\Delta g(y,t)\right], \tag{146}$$

where $\Sigma(x,t)=\Sigma_q\Delta q(x,t)$ and $t=\ln\lambda^2$. The splitting functions P_{ij} in Eq. (63) have been calculated at leading order by Altarelli and Parisi (1977) and at next-to-leading order by Mertig, Zijlstra, and van Neerven (Zijlstra and van Neerven, 1994; Mertig and van Neerven, 1996), and Vogelsang (Vogelsang, 1996).

Similar to the analysis that is carried out on unpolarized data, global next-to-leading-order perturbative QCD analyses have been performed on the polarized structure-function data sets. The aim is to extract the polarized quark and gluon parton distributions. These QCD fits are performed within a given factorization scheme, e.g., the "AB," chiral invariant (CI), or JET and modified minimal subtraction ($\overline{\text{MS}}$) schemes.

Let us briefly review these different factorization schemes. Different factorization schemes correspond to different procedures for separating the phase space for photon-gluon fusion into hard and soft contributions in the convolution formula (80). In the QCD parton-model analysis of photon-gluon fusion that we discussed in Sec. XI.D using the cutoff on the transverse momentum squared, the polarized gluon contribution to the first moment of g_1 is associated with two-quark-jet events carrying $k_t^2 \sim Q^2$. The gluon coefficient function is given by $C_{\text{PM}}^{(g)} = g_1^{(\gamma^* g)}|_{\text{hard}}$, where $g_1^{(\gamma^* g)}|_{\text{hard}}$ is taken from Eq. (121) with $Q^2 \gg \lambda^2$ and $\lambda^2 \gg P^2, m^2$. This transverse-momentum cutoff scheme is sometimes called the "chiral invariant" (CI) (Cheng, 1996) or JET (Leader *et al.*, 1998) scheme.

Different schemes can be defined relative to this k_t cutoff scheme by the transformation

$$C^{(g)}\left(x, \frac{Q^2}{\lambda^2}, \alpha_s(\lambda^2)\right) \to C^{(g)}\left(x, \frac{Q^2}{\lambda^2}, \alpha_s(\lambda^2)\right) - \tilde{C}^{(g)}_{\text{scheme}}(x, \alpha_s(\lambda^2)). \tag{147}$$

Here $\tilde{C}^{(g)}_{\text{scheme}}$ shall be α_s/π times a polynomial in x. The parton distributions transform as

$$\Delta\Sigma(x,\lambda^2)_{\text{scheme}} = \Delta\Sigma(x,\lambda^2)_{\text{PM}} + f\int_x^1 \frac{dz}{z}\Delta g\left(\frac{x}{z},\lambda^2\right)_{\text{PM}} \times \tilde{C}^{(g)}_{\text{scheme}}(z,\alpha_s(\lambda^2)),$$

$$\Delta g(x,\lambda^2)_{\text{scheme}} = \Delta g(x,\lambda^2)_{\text{PM}} \tag{148}$$

so that the physical structure function g_1 is left invariant under the change of scheme. The virtuality and invariant-mass cutoff versions of the parton model that we discussed in Sec. XI.D correspond to different choices of scheme.

The $\overline{\text{MS}}$ and AB schemes are defined as follows. In the $\overline{\text{MS}}$ scheme the gluonic hard scattering coefficient is calculated using the operator product expansion with $\overline{\text{MS}}$ renormalisation ('t Hooft and Veltman, 1972). One finds (Bass, 1992b; Cheng, 1996)

$$C^{(g)}_{\overline{\text{MS}}} = C^{(g)}_{\text{PM}} + \frac{\alpha_s}{\pi}(1-x). \tag{149}$$

In this scheme $\int_0^1 dx C^{(g)}_{\overline{\text{MS}}}=0$ so that $\int_0^1 dx \Delta g(x,\lambda^2)$ decouples from $\int_0^1 dx g_1$. This result corresponds to the fact that there is no gauge-invariant twist-2, spin-1, gluonic operator with $J^P=1^+$ to appear in the operator product expansion for the first moment of g_1. In the $\overline{\text{MS}}$ scheme the contribution of $\int_0^1 dx \Delta g$ to the first moment of g_1 is included in $\int_0^1 dx \Sigma_{\overline{\text{MS}}}(x,\lambda^2)$. The AB scheme (Ball *et al.*, 1996) is defined by the formal operation of adding the x-independent term $-\alpha_s/2\pi$ to the $\overline{\text{MS}}$ gluonic coefficient, viz.,

$$C^{(g)}_{\text{AB}}(x) = C^{(g)}_{\overline{\text{MS}}} - \frac{\alpha_s}{2\pi}. \tag{150}$$

In the $\overline{\text{MS}}$ scheme the polarized gluon distribution does not contribute explictly to the first moment of g_1. In the AB and JET schemes, on the other hand, the polarized gluon (axial anomaly contribution) $\alpha_s \Delta g$ does contribute explicitly to the first moment since $\int_0^1 dx C^{(g)} = -\alpha_s/2\pi$.

For the SMC data one finds for the $\overline{\text{MS}}$ (AB) scheme at a Q^2 of 1 GeV^2 (Adeva *et al.*, 1998b): $\Delta\Sigma = 0.19 \pm 0.05 (0.38 \pm 0.03)$ and $\Delta g = 0.25^{+0.29}_{-0.22}(1.0^{+1.2}_{-0.3})$, where $\Delta\Sigma = \Delta u + \Delta d + \Delta s$. The main source of error in the QCD fits comes from lack of knowledge about g_1 in the small-x region and (theoretically) the functional form chosen for the quark and gluon distributions in the fits. Note that these QCD fits in both the AB and $\overline{\text{MS}}$ schemes give values of $\Delta\Sigma$ which are smaller than the Ellis-Jaffe value of 0.6.

New fits are now being produced taking into account all the available data including new data from polarized semi-inclusive deep-inelastic scattering. Typical polarized distributions extracted from the fits are shown in Fig. 13. Given the uncertainties in the fits, values of Δg are extracted ranging from between about 0 and +2. In these pQCD analyses one ends up with a consistent picture of the proton spin: the low value of $\Delta\Sigma$ may be compensated by a large polarized gluon. The precision on Δg is, however, still rather modest. Moreover, it is vital to validate this model with direct measurements of Δg, as we discuss in Sec. IX.E. Also, the first moments depend on integrations from $x=0$ to 1. Perhaps there is an additional component at very small x?

D. Polarized quark distributions and semi-inclusive polarized deep-inelastic scattering

As noted above, there are several possible mechanisms for producing sea quarks in the nucleon: photon-gluon fusion, the meson cloud of the nucleon, instantons. In general the different dynamics will produce polarized sea with different x and transverse-momentum dependence.

Semi-inclusive measurements of fast pions and kaons in the current fragmentation region with final-state particle identification can be used to reconstruct the individual up, down, and strange quark contributions to the proton's spin (Close, 1978; Frankfurt *et al.*, 1989; Close and Milner, 1991). In contrast to inclusive polarized deep-inelastic scattering in which the g_1 structure function is deduced by detecting only the scattered lepton, the detected particles in the semi-inclusive experiments are high-energy (greater than 20% of the energy of the incident photon) charged pions and kaons in coincidence with the scattered lepton. For large energy fraction $z = E_h/E_\gamma \to 1$ the most probable occurrence is that the detected $\pi^\pm$ and $K^\pm$ contain the struck quark or antiquark in their valence Fock state. They therefore act as a tag of the flavor of the struck quark (Close, 1978).

In leading-order QCD the double-spin asymmetry for the production of hadrons h in semi-inclusive polarized γ^* polarized proton collisons is

$$A^h_{1p}(x,Q^2) \simeq \frac{\sum_{q,h} e_q^2 \Delta q(x,Q^2) \int_{z_{\min}}^1 D^h_q(z,Q^2)}{\sum_{q,h} e_q^2 q(x,Q^2) \int_{z_{\min}}^1 D^h_q(z,Q^2)}, \tag{151}$$

where $z_{\min} \sim 0.2$. Here

$$D^h_q(z,Q^2) = \int dk_t^2 D^h_q(z,k_t^2,Q^2) \tag{152}$$

is the fragmentation function for the struck quark or antiquark to produce a hadron h $(=\pi^\pm, K^\pm)$ carrying en-

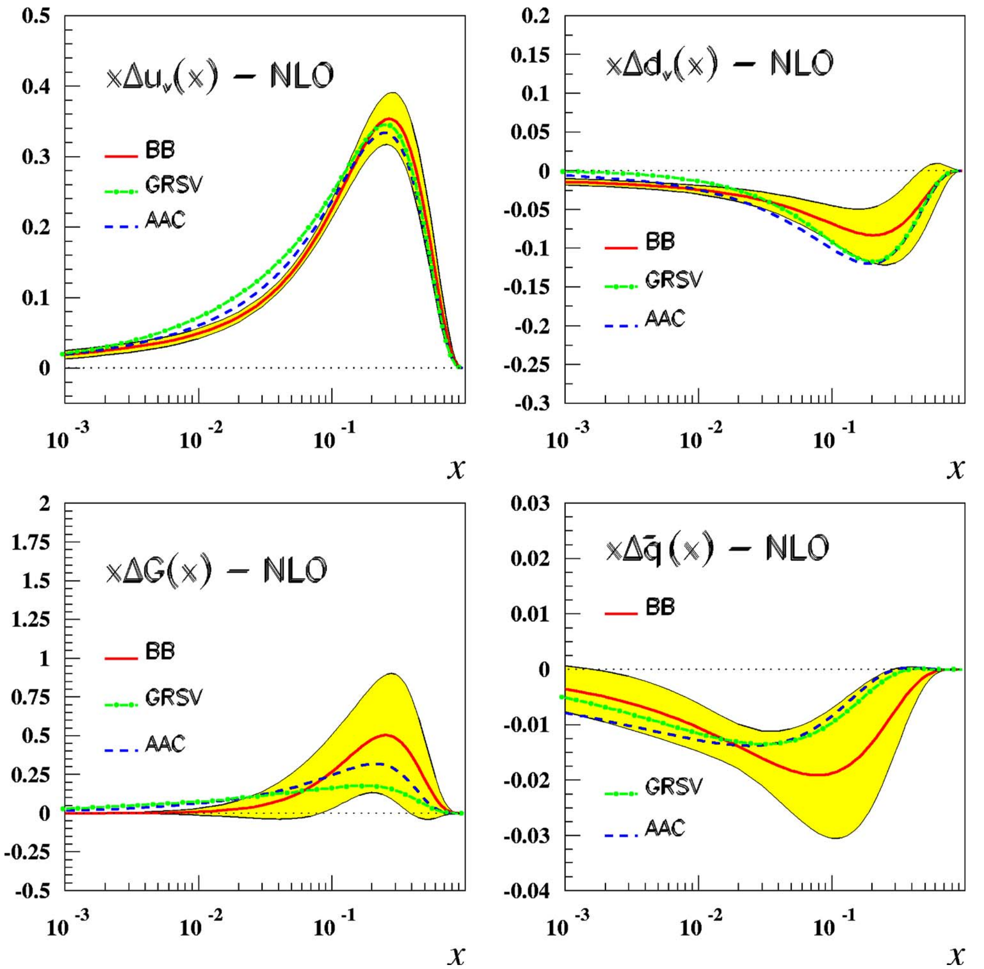

FIG. 13. (Color online) Polarized parton distribution functions from NLO pQCD [modified minimal subtraction ($\overline{\text{MS}}$)] fits at $Q^2=4$ GeV^2 using SU(3) flavor assumptions (Stoesslein, 2002).

ergy fraction $z=E_h/E_\gamma$ in the target rest frame; $\Delta q(x,Q^2)$ is the quark (or antiquark) polarized parton distribution and e_q is the quark charge. Note the integration over the transverse momentum k_t of the final-state hadrons (Close and Milner, 1991). (In practice this integration over k_t is determined by the acceptance of the experiment.) Since pions and kaons have spin zero, the fragmentation functions are the same for both polarized and unpolarized leptoproduction. Next-to-leading-order corrections to Eq. (151) are discussed by de Florian *et al.* (1998) and de Florian and Sassot (2000).

This program for polarized deep-inelastic scattering was pioneered by the SMC (Adeva *et al.*, 1998c) and HERMES (Ackerstaff *et al.*, 1999) Collaborations with new recent measurements from HERMES reported in Airapetian *et al.* (2004, 2005a). Figure 14 shows the latest results on the flavor separation from HERMES (Airapetian *et al.*, 2004), which were obtained using a leading-order (naive parton model) Monte Carlo code-based "purity" analysis. The polarizations of the up and down quarks are positive and negative, respectively, while the sea-polarization data are consistent with zero and not inconsistent with the negative sea polarization suggested by inclusive deep-inelastic data within the measured x range (Glück *et al.*, 2001; Blümlein and Böttcher, 2002). However, there is also no evidence from this semi-inclusive analysis for a large negative strange-quark polarization. For the region $0.023<x<0.3$ the extracted Δs integrates to the value $+0.03\pm0.03\pm0.01$ which contrasts with the negative value for the polarized strangeness (6) extracted from inclusive measurements of g_1. It will be interesting to see whether this effect persists in forthcoming semi-inclusive data from COMPASS. The HERMES data also favor an isospin symmetric sea $\Delta\bar{u}-\Delta\bar{d}$, but with large uncertainties.

An important issue for semi-inclusive measurements is the angular coverage of the detector (Bass, 2003a). The nonvalence spin-flavor structure of the proton extracted from semi-inclusive measurements of polarized deep-inelastic scattering may depend strongly on the transverse-momentum (and angular) acceptance of the detected final-state hadrons which are used to determine the individual polarized sea distributions. The present semi-inclusive experiments detect final-state hadrons produced only at small angles from the incident lepton beam (about 150 mrad angular coverage). The perturbative QCD "polarized gluon interpretation" (Altarelli and Ross, 1988; Efremov and Teryaev, 1988) of the inclusive measurement (6) involves physics at the maximum transverse momentum (Carlitz *et al.*, 1988; Bass, 2003a) and large angles—see Fig. 9. Observe the small value for the light-quark sea polarization at low transverse momentum and the positive value for the inte-

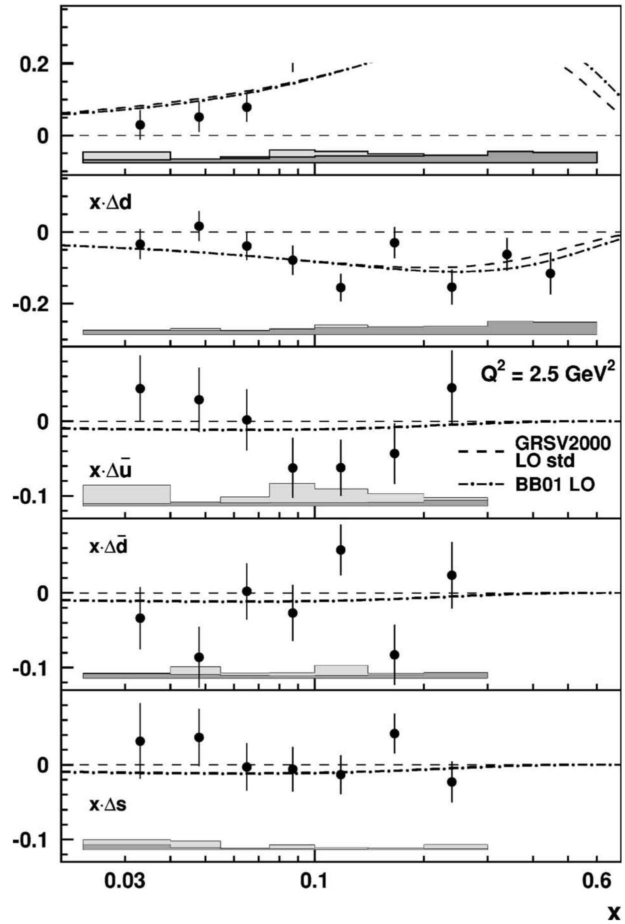

FIG. 14. Recent HERMES results (Airapetian *et al.*, 2004) for the quark and antiquark polarizations extracted from semi-inclusive DIS.

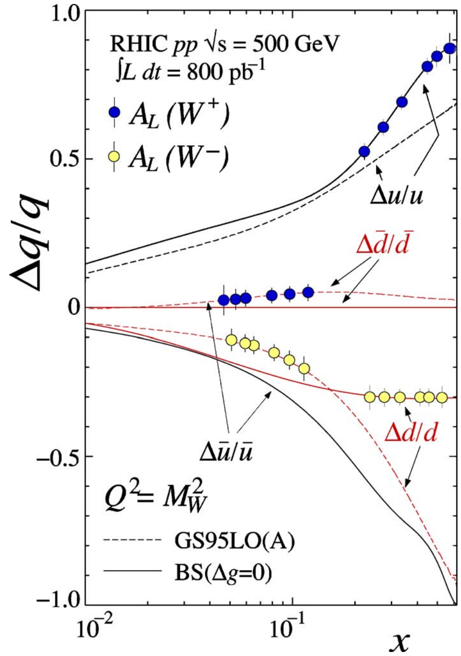

FIG. 15. (Color online) Expected sensitivity (Bunce *et al.*, 2000) for the flavor decomposition of quark and antiquark polarizations at RHIC. Reprinted, with permission, from the Annual Reviews of Nuclear and Particle Science, Volume 50 ©2000 by Annual Reviews www.annualreviews.org.

grated strange sea polarization at low k_t^2:$k_t<1.5$ GeV at the HERMES $Q^2=2.5\ \mathrm{GeV}^2$. When we relax the transverse-momentum cutoff, increasing the acceptance of the experiment, the measured strange sea polarization changes sign and becomes negative (the result implied by fully inclusive deep-inelastic measurements). For HERMES the average transverse momentum of the detected final-state fast hadrons is less than about 0.5 GeV whereas for SMC the k_t of the detected fast pions was less than about 1 GeV. Hence there is a question of whether the leading-order sea-quark polarizations extracted from semi-inclusive experiments with limited angular resolution fully include the effect of the axial anomaly or not.

Recent theoretical studies motivated by these data also include possible effects associated with spin-dependent fragmentation functions (Kretzer *et al.*, 2001), possible higher-twist effects in semi-inclusive deep-inelastic scattering, and possible improvements in the Monte Carlo calculations (Kotzinian, 2003).

The dependence on the details of the fragmentation process limits the accuracy of the method above. At RHIC (Bruce *et al.*, 2000) the polarization of the u, $\bar{u}$, d, and $\bar{d}$ quarks in the proton will be measured directly and precisely using W-boson production in $u\bar{d}\to W^+$ and $d\bar{u}\to W^-$. The charged weak boson is produced through a pure $V-A$ coupling and the chirality of the quark and antiquark in the reaction is fixed. A parity-violating asymmetry for W^+ production in pp collisions can be expressed as

$$A(W^+) = \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}. \qquad (153)$$

For W^- production u and d quarks should be exchanged. The expression converges to $\Delta u(x)/u(x)$ and $-\Delta\bar{d}(x)/\bar{d}(x)$ in the limits $x_1 \gg x_2$ and $x_2 \gg x_1$, respectively. The momentum fractions are calculated as $x_1 = (M_W/\sqrt{s})e^{y_W}$ and $x_2 = (M_W/\sqrt{s})e^{-y_W}$, with y_W the rapidity of the W. The experimental difficulty is that the W is observed through its leptonic decay $W\to l\nu$ and only the charged lepton is observed. With the assumed integrated luminosity of 800 pb^{-1} at $\sqrt{s}=500$ GeV, one can expect about 5000 events each for W^+ and W^-. The resulting measurement precision is shown in Fig. 15.

It has also been pointed out that neutrino factories would be an ideal tool for polarized quark-flavor decomposition studies. These would allow one to collect large data samples of charged-current events in the kinematic region (x,Q^2) of present fixed-target data (Forte *et al.*, 2001). A complete separation of all four flavors and antiflavors would become possible, including $\Delta s(x,Q^2)$.

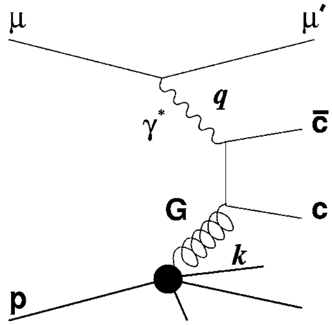

FIG. 16. $c\bar{c}$ production in photon-gluon fusion.

E. The polarized gluon distribution $\Delta g(x,Q^2)$

Motivated by the discovery of Altarelli and Ross (1988) and Efremov and Teryaev (1988) that polarized glue makes a scaling contribution to the first moment of $g_1, \alpha_s \Delta g \sim$ const, there has been a vigorous and ambitious program to measure Δg. The next-to-leading-order QCD-motivated fits to the inclusive g_1 data are suggestive that, perhaps, the net polarized glue might be positive but more direct measurements involving glue-sensitive observables are needed to really extract the magnitude of Δg and the shape of $\Delta g(x,Q^2)$ including any possible nodes in the distribution function. Possible channels include gluon-mediated processes in semi-inclusive polarized deep-inelastic scattering and hard QCD processes in high-energy polarized proton-proton collisions at RHIC.

COMPASS has been conceived to measure Δg via the study of the photon-gluon fusion process, as shown in Fig. 16. The cross section of this process is directly related to the gluon density at the Born level. The experimental technique consists of the reconstruction of charmed mesons in the final state. COMPASS will also use the same process with high-p_t particles instead of charm to access Δg. This may lead to samples with larger statistics, but these have larger background contributions, namely, from QCD Compton processes and fragmentation. The expected sensitivity on the measurement of $\Delta g/g$ from these experiments is estimated to be about $\delta(\Delta g/g)=0.1$ at $x_g \sim 0.1$.

HERMES was the first to attempt to measure Δg using high-p_t charged particles, as proposed for COMPASS above, and nearly real photons $\langle Q^2\rangle = 0.06\ \mathrm{GeV}^2$. The measurement is at the limit of where a perturbative treatment of the data can be expected to be valid, but the result is interesting: $\Delta g/g = 0.41 \pm 0.18 \pm 0.03$ at an average $\langle x_g \rangle = 0.17$ (Airapetian *et al.*, 2000). The SMC Collaboration have performed a similar analysis for their own data keeping $Q^2 > 1\ \mathrm{GeV}^2$. An average gluon polarization was extracted $\Delta g/g = -0.20 \pm 0.28 \pm 0.10$ at an average gluon momentum $x_g = 0.07$ (Adeva *et al.*, 2004).

The search for Δg is also one of the main physics drives for polarized RHIC. The key processes used here are high-p_t prompt photon production $pp \to \gamma X$, jet production $pp \to \mathrm{jets}+X$, and heavy-flavor production $pp \to c\bar{c}X, b\bar{b}X, J/\psi X$. Due to the first-stage detector capabilities most emphasis has so far been put on the prompt photon channel. Measurements of $\Delta g/g$ are expected in the gluon x range $0.03 < x_g < 0.3$.

These anticipated RHIC measurements of Δg have inspired new theoretical developments aimed at implementing higher-order calculations of partonic cross sections into global analyses of polarized parton distribution functions, which will benefit the analyses of future polarized pp data to measure Δg. Hard polarized reactions at RHIC and the polarized parton distributions that they probe are summarized in Table II.

In the first runs at RHIC the longitudinal double-spin asymmetry for production of a leading pion π^0 with large transverse momentum has been used as a surrogate jet to investigate possible gluon polarization in the proton. Next-to-leading-order perturbative QCD corrections to this process have been calculated by de Florian (2003) and Jäger *et al.* (2003). The data from PHENIX (Alder *et al.*, 2004; Fukao, 2005) are shown in Fig. 17 and are consistent with a significant (up to a few percent) negative asymmetry $\mathcal{A}_{LL}^{\pi}$ for pion transverse momentum $1 < p_t < 4$ GeV, in contrast with the predictions of leading-twist perturbative QCD calculations which do provide a good description of the unpolarized cross section in the same kinematics. It will be interesting to see whether this effect survives more precise data. The next-to-leading-order perturbative QCD analysis of Jäger *et al.* (2004) suggests that the leading power in p_t contribution to $\mathcal{A}_{LL}^{\pi}$ cannot be large and negative in the measured range of p_t within the framework of perturbative QCD independent of the sign of Δg. One would need to invoke power-suppressed contributions (though the leading power term seems to describe the corresponding unpolarized data) and/or nonperturbative effects. In-

TABLE II. Polarized partons from RHIC.

Reaction	LO subprocesses	Partons probed	x range
$pp \to \mathrm{jets}\ X$	$q\bar{q}, qq, qg, gg \to \mathrm{jet}\ X$	$\Delta q, \Delta g$	$x > 0.03$
$pp \to \pi X$	$q\bar{q}, qq, gg \to \pi X$	$\Delta q, \Delta g$	$x > 0.03$
$pp \to \gamma X$	$qg \to q\gamma, q\bar{q} \to g\gamma$	Δg	$x > 0.03$
$pp \to Q\bar{Q}X$	$gg \to Q\bar{Q}, q\bar{q} \to Q\bar{Q}$	Δg	$x > 0.01$
$pp \to W^{\pm}X$	$q\bar{q}' \to W^{\pm}$	$\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d}$	$x > 0.06$

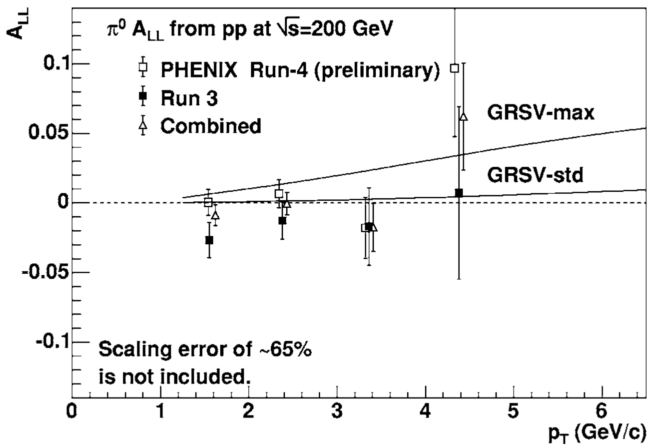

FIG. 17. The PHENIX (preliminary run-4) data (Fukao, 2005) for the spin asymmetry $\mathcal{A}_{LL}^{\pi}$ along with NLO perturbative QCD predictions for various Δg (Jäger *et al.*, 2004).

crease in precision and data at higher pion p_t (up to about 12 GeV) are expected from future runs.

Future polarized *ep* colliders could add information in two ways: by extending the kinematic range for measurements of g_1 or by direct measurements of Δg. A precise measurement of Δg is crucial for a full understanding of the proton spin problem. HERA has shown that large center-of-mass energy allows several processes to be used to extract the unpolarized gluon distribution. These include jet and high-p_t hadron production, charm production in both DIS and photoproduction, and correlations between multiplicities of the current and target hemisphere of the events in the Breit frame. The most promising process for a direct extraction of Δg is di-jet production (De Roeck *et al.*, 1999; Rädel and De Roeck, 2002). The underlying idea is to isolate boson-gluon fusion events where the gluon distribution enters at the Born level.

X. TRANSVERSITY

There are three species of twist-2 quark distributions in QCD. These are the spin-independent distributions $q(x)$ measured in the unpolarized structure functions F_1 and F_2, the spin-dependent distributions $\Delta q(x)$ measured in g_1, and the transversity distributions $\delta q(x)$.

The transversity distributions describe the density of transversely polarized quarks inside a transversely polarized proton (Barone *et al.*, 2002). Measuring transversity is an important experimental challenge in QCD spin physics. We briefly describe the physics of transversity and the program to measure it.

The twist-2 transversity distributions (Ralston and Soper, 1979; Artru and Mekhfi, 1990; Jaffe and Ji, 1992) can be interpreted in parton language as follows. Consider a nucleon moving with (infinite) momentum in the $\hat{e}_3$ direction, but polarized along one of the directions transverse to $\hat{e}_3$. Then $\delta q(x,Q^2)$ counts the quarks with flavor q, momentum fraction x, and their spin parallel to the spin of a nucleon minus the number antiparallel.

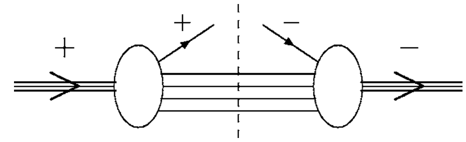

FIG. 18. Transversity in helicity basis.

That is, in analogy with Eq. (28), $\delta q(x)$ measures the distribution of partons with transverse polarization in a transversely polarized nucleon, viz.,

$$\delta q(x,Q^2) = q^{\uparrow}(x) - q^{\downarrow}(x). \tag{154}$$

In a helicity basis, transversity corresponds to the helicity-flip structure shown in Fig. 18 making transversity a probe of chiral symmetry breaking (Collins, 1993b). The first moment of the transversity distribution is proportional to the nucleon's C-odd tensor charge, viz., $\delta q = \int_0^1 dx\, \delta q(x)$ with

$$\langle p,s|\bar{q} i\sigma_{\mu\nu}\gamma_5 q|p,s\rangle = (1/M)(s_\mu p_\nu - s_\nu p_\mu)\delta q. \tag{155}$$

Transversity is C odd and chiral odd.

If quarks moved nonrelativistically in the nucleon, δq and Δq would be identical since rotations and Euclidean boosts commute and a series of boosts and rotations can convert a longitudinally polarized nucleon into a transversely polarized nucleon at infinite momentum. The difference between the transversity and helicity distributions reflects the relativistic character of quark motion in the nucleon. In the MIT bag model this effect is manifest as follows. The lower component of the Dirac spinor enters the relativistic spin depolarization factor with the opposite sign to Δq because of the extra factor of γ_μ in the tensor charge (Jaffe and Ji, 1992). That is, the relativistic depolarization factor $N^2\int_0^R drr^2\left(f^2 - \frac{1}{3}g^2\right)$ for Δq mentioned in Sec. I.A is replaced by $N^2\int_0^R drr^2\left(f^2 + \frac{1}{3}g^2\right)$ for δq, where

$$\psi = \frac{N}{\sqrt{4\pi}}\begin{pmatrix} f \\ i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} g \end{pmatrix}$$

is the Dirac spinor.

Little is presently known about the shape of the transversity distributions. However, some general properties can be deduced from QCD arguments. The spin distributions $\Delta q(x)$ and $\delta q(x)$ have opposite charge-conjugation properties: $\Delta q(x)$ is C even whereas $\delta q(x)$ is C odd. The spin-dependent quark and gluon helicity distributions (Δq and Δg) mix under Q^2 evolution. In contrast, there is no analog of gluon transversity in the nucleon so δq evolves without mixing, like a nonsinglet parton distribution function. Not coupling to glue or perturbative $\bar{q}q$ pairs, $\delta q(x)$ and the tensor charge promise to be more quark-model-like than the singlet axial charge (though they are both scale dependent) and should be an interesting contrast (Jaffe, 2001). Under QCD evolution the moments $\int_0^1 dx x^n \delta q(x,Q^2)$ decrease with increasing Q^2. In leading-order QCD the transversity distributions are bounded above by Soffer's inequality (Soffer, 1995)

$$|\delta q(x,Q^2)| \leqslant \frac{1}{2}[q(x,Q^2) + \Delta q(x,Q^2)]. \quad (156)$$

Experimental study of transversity distributions at leading twist requires observables which are the product of two objects with odd chirality—the transversity distribution and either a second transversity distribution or a chiral-odd fragmentation function. In proton-proton collisions the transverse double-spin asymmetry $\mathcal{A}_{TT}$ is proportional to $\delta q \delta \bar{q}$ with even chirality. However, the asymmetry is small requiring very large luminosity samples because of the large background from gluon-induced processes in unpolarized scattering. The most promising process for measuring this double-spin asymmetry is perhaps Drell-Yan production.

Transverse single-spin asymmetries $\mathcal{A}_N$ are also being studied with a view to extracting information about transversity distributions. Here the focus is on single-hadron production with a transversely polarized proton beam or target in *pp* and *ep* collisions. The key process is

$$A(p,\vec{s}_t) + B(p') \to C(l) + X, \quad (157)$$

where C is typically a pion produced at large transverse momentum l_t.

Several mechanisms for producing these transverse single-spin asymmetries have been discussed in the literature. The asymmetries $\mathcal{A}_N$ are power suppressed in QCD. Leading l_t behavior of the produced pion can occur from the Collins (1993b) and Sivers (1991) effects plus twist-3 mechanisms (Qiu and Sterman, 1999). The Collins effect involves the chiral-odd twist-2 transversity distribution in combination with a chiral-odd fragmentation function for the high-l_t pion in the final state. It gives a possible route to measuring transversity. The Sivers effect is associated with intrinsic quark transverse momentum in the initial state. The challenge is to disentangle these effects from experimental data.

Factorization for transverse single-spin processes in proton-proton collisions has been derived by Qiu and Sterman (1999) in terms of the convolution of a twist-2 parton distribution from the unpolarized hadron, a twist-3 quark-gluon correlation function from the polarized hadron, and a short-distance partonic hard part calculable in perturbative QCD. We refer to a paper by Anselmino *et al.* (2005) for a discussion of factorization for processes such as the Collins and Sivers effects involving unintegrated transverse-momentum-dependent parton and fragmentation functions.

We next outline the Collins and Sivers effects. The Collins effect (1993b) uses properties of fragmentation to probe transversity. The idea is that a pion produced in fragmentation will have some transverse momentum with respect to the momentum k of the transversely polarized fragmenting parent quark. One finds a correlation of the form $i\vec{s}_t \cdot (\vec{l}^{(\pi)} \times \vec{k}_t)$. The Collins fragmentation function associated with this correlation is chiral odd and T even. It combines with the chiral-odd transversity distribution to contribute to the transverse single-spin asymmetry.

For the Sivers effect (1991) the k_t distribution of a quark in a transversely polarized hadron can generate an azimuthal asymmetry through the correlation $\vec{s}_t \cdot (\vec{p} \times \vec{k}_t)$. In this process final-state interaction of the active quark produces the asymmetry before it fragments into hadrons (Brodsky *et al.*, 2002; Yuan, 2003; Bachetta *et al.*, 2004; Burkardt and Hwang, 2004). This process involves a k_t unintegrated quark distribution function in the transversely polarized proton. The dependence on intrinsic quark transverse momentum means that this Sivers process is related to quark orbital angular momentum in the proton (Burkardt, 2002). The Sivers distribution function is chiral even and T odd. The possible role of quark orbital angular momentum in understanding transverse single-spin asymmetries is also discussed by Boros *et al.* (1993).

The Sivers process is associated with the gauge link in operator definitions of the parton distributions. The gauge-link factor is trivial and equal to 1 for the usual k_t integrated parton distributions measured in inclusive polarized deep-inelastic scattering. However, for k_t unintegrated distributions the gauge link survives in a transverse direction at light-cone component $\xi^- = \infty$. The gauge link plays a vital role in the Sivers process (Burkardt, 2005). Without it (e.g., in the pre-QCD "naive" parton model) time-reversal invariance implies vanishing Sivers effect (Ji and Yuan, 2002; Belitsky *et al.*, 2003a). The Sivers distribution function has the interesting property that it has the opposite sign in deep-inelastic scattering and Drell-Yan reactions (Collins, 2002). It thus violates the universality of parton distribution functions.

The FermiLab experiment E704 found large transverse single-spin asymmetries $\mathcal{A}_N$ for π and Λ production in proton-antiproton collisions at center-of-mass energy $\sqrt{s} = 20$ GeV (Adams *et al.*, 1991a, 1991b; Bravar *et al.*, 1996). Large transverse single-spin asymmetries have also been observed in recent data from the STAR Collaboration at RHIC in proton-proton collisions at center-of-mass energy $\sqrt{s} = 200$ GeV (Adams *et al.*, 2004)—see Fig. 19 which also shows various theoretical predictions. In a recent paper Anselmino *et al.* (2005) take into account intrinsic parton motion in the distribution and fragmentation functions as well as in the elementary dynamics and argue that the Collins mechanism may be strongly suppressed at large Feynman x_F in this process. The Sivers effect is not suppressed and remains a candidate to explain the data. Higher-twist contributions (Qiu and Sterman, 1999) from quark-gluon correlations may also be important.

The HERMES experiment has taken measurements of charged-pion production in *ep* scattering with transverse target polarization (Airapetian *et al.*, 2005b). These data have been analyzed for possible contributions from the Collins and Sivers effects. The azimuthal distribution of the final-state pions with respect to the virtual-photon axis is expected to carry information

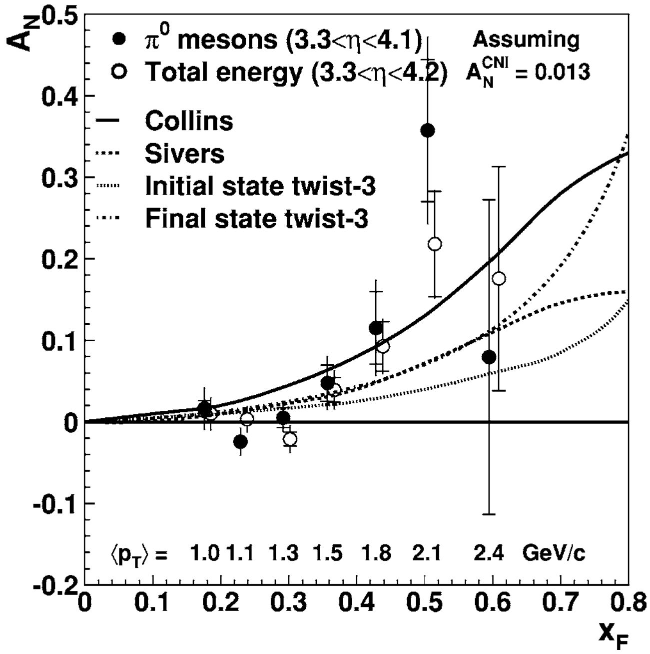

FIG. 19. Recent STAR results for the asymmetry A_N in $pp \to \pi^0 X$ in the forward Feynman-x_F region (Adams *et al.*, 2004).

about transversity through the Collins effect and about intrinsic transverse momentum in the proton through the Sivers effect. In this analysis one first writes the transverse single-spin asymmetry $\mathcal{A}_N$ as the sum

$$\mathcal{A}_N(x,z) = \mathcal{A}_N^{\text{Collins}} + \mathcal{A}_N^{\text{Sivers}} + \cdots, \tag{158}$$

where

$$\mathcal{A}_N^{\text{Collins}} \propto |\vec{s}_t| \sin(\phi + \phi_S) \frac{\sum_q e_q^2 \delta q(x) H_1^{\perp,q}(z)}{\sum_q e_q^2 q(x) D_q^{\pi}(z)} \tag{159}$$

and

$$\mathcal{A}_N^{\text{Sivers}} \propto |\vec{s}_t| \sin(\phi - \phi_S) \frac{\sum_q e_q^2 f_{1T}^{\perp,q} D_q^{\pi}(z)}{\sum_q e_q^2 q(x) D_q^{\pi}(z)} \tag{160}$$

denote the contributions from the Collins and Sivers effects. Here ϕ is the angle between the lepton direction and the $(\gamma^* \pi)$ plane and ϕ_S is the angle between the lepton direction and the transverse target spin; $H_1^{\perp q}$ is the Collins function for a quark of flavor q, $f_{1T}^{\perp q}$ is the Sivers distribution function, and D_q^{π} is the regular spin-independent fragmentation function. When one projects out the two terms with different azimuthal angular dependence the HERMES analysis suggests that both the Collins and Sivers effects are present in the data. Furthermore, the analysis suggests the puzzling result that the "favored" (for $u \to \pi^+$) and "unfavored" (for $d \to \pi^+$) Collins fragmentation functions may contribute with equal weight (and opposite sign) (Airapetian *et al.*, 2005b).

Other processes and experiments will help to clarify the importance of the Collins and Sivers processes. Additional studies of the Collins effect have been proposed in e^+e^- collisions using the high-statistics data samples of BABAR and BELLE. The aim is to measure two relevant fragmentation functions: the Collins function $H_1^{\perp}$ and the interference fragmentation functions $\delta\hat{q}^{h_1,h_2}$. For the first, one measures the fragmentation of a transversely polarized quark into a charged pion and the azimuthal distribution of the final-state pion with respect to the initial quark momentum (jet axis). For the second, one measures the fragmentation of transversely polarized quarks into pairs of hadrons in a state which is the superposition of two different partial-wave amplitudes; e.g., π^+, π^- pairs in the ρ and σ invariant-mass region (Collins *et al.*, 1994; Jaffe *et al.*, 1998). The high luminosity and particle identification capabilities of detectors at B factories makes these measurements possible.

The Sivers distribution function might be measurable through the transverse single-spin asymmetry $\mathcal{A}_N$ for D-meson production generated in $p^{\uparrow}p$ scattering (Anselmino *et al.*, 2004). Here the underlying elementary processes guarantee the absence of any polarization in the final partonic state so that there is no contamination from Collins-like terms. Large dominance of the $gg \to c\bar{c}$ process at low and intermediate x_F offers a unique opportunity to measure the gluonic Sivers distribution function. The gluonic Sivers function could also be extracted from back-to-back correlations in the azimuthal angle of jets in collisions of unpolarized and transversely polarized proton beams at RHIC (Boer and Vogelsang, 2004).

Measurements with transversely polarized targets have a bright future and are already yielding surprises. The results promise to be interesting and to teach us about transversity and about the role of quark transverse momentum in the structure of the proton and fragmentation processes.

XI. DEEPLY VIRTUAL COMPTON SCATTERING AND EXCLUSIVE PROCESSES

A. Orbital angular momentum

So far in this review we have concentrated on intrinsic spin in the proton. The orbital angular momentum structure of the proton is also of considerable interest and much effort has gone into devising ways to measure it. The strategy involves the use of hard exclusive reactions and the formalism of generalized parton distributions (GPDs) which describes deeply virtual Compton scattering and deeply virtual meson production. Possible hints of quark orbital angular momentum are also suggested by recent form-factor measurements at Jefferson Laboratory (Jones *et al.*, 2000; Gayou *et al.*, 2002)—see Fig. 20. The ratio of the spin-flip Pauli form factor to the Dirac form factor is observed to have a $1/\sqrt{Q^2}$ behavior in the measured region in contrast with the $1/Q^2$ behavior predicted by QCD counting rules (helicity conserva-

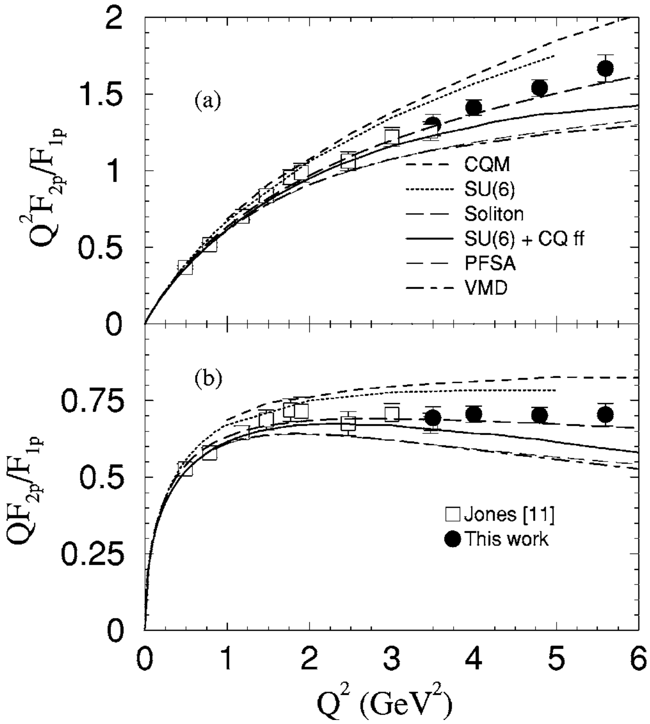

FIG. 20. Jefferson Laboratory data on the the ratio of the proton's Pauli to Dirac form factors (Gayou *et al.*, 2002).

tion neglecting angular momentum), viz., $F_1 \sim 1/Q^4$ and $F_2 \sim 1/Q^6$ (Brodsky and Lepage, 1980). However, these data can also be fit with the formula

$$\frac{F_2(Q^2)}{F_1(Q^2)} = \frac{\mu_A}{1 + (Q^2/c)\ln^b(1 + Q^2/a)} \tag{161}$$

(with $\mu_A = 1.79$, $a = 4m_\pi^2 = 0.073$ GeV^2, $b = -0.5922$, $c = 0.9599$ GeV^2) prompting the question at which Q^2 the counting-rule prediction is supposed to work and at which Q^2 higher-twist effects can be neglected (Brodsky, 2002). At this point it is interesting to recall that the simple counting-rule prediction fails to describe the large-x behavior of $\Delta d/d$ in the presently measured JLab kinematics—Sec. IX.A.

A $1/Q$ behavior for $F_2(Q^2)/F_1(Q^2)$ is found in a light-front cloudy bag calculation (Miller, 2002; Miller and Frank, 2002) and in quark models with orbital angular momentum (Ralston *et al.*, 2002; Ralston and Jain, 2004). A new perturbative QCD calculation which takes into account orbital angular momentum (Belitsky *et al.*, 2003b) gives $F_2/F_1 \sim (\log^2 Q^2/\Lambda^2)/Q^2$ and also fits the Jefferson Laboratory data well. The planned 12-GeV upgrade at Jefferson Laboratory will enable us to measure these nucleon form factors at higher Q^2 and the inclusive spin asymmetries at values of Bjorken x closer to 1, and thus probe deeper into the kinematic regions where QCD counting rules should apply. These data promise to be very interesting!

Deeply virtual Compton scattering provides a possible experimental tool for accessing the quark total angular momentum J_q in the proton through the physics of generalized parton distributions (GPDs) (Ji, 1997a, 1997b).

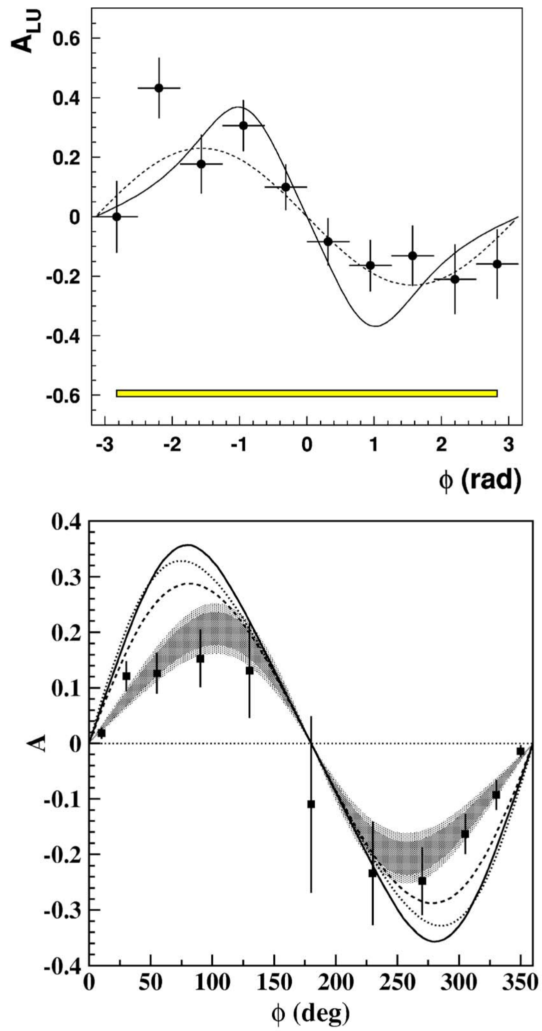

FIG. 21. (Color online) Recent data from HERMES (top) and the CLAS experiment at Jefferson Laboratory (bottom) in the realm of deeply virtual Compton scattering Bethe-Heitler interference. The sin ϕ azimuthal dependence of the single-spin asymmetry is clearly visible in the data (Airapetian *et al.*, 2001; Stepanyan *et al.*, 2001).

The form factors which appear in the forward limit ($t \to 0$) of the second moment of the spin-independent generalized quark parton distribution in the (leading-twist) spin-independent part of the deeply virtual Compton scattering amplitude project out the quark total angular momentum defined through the proton matrix element of the QCD angular momentum tensor. We explain this physics below.

Deeply virtual Compton scattering studies have to be careful to choose the kinematics so that they are not saturated by a large Bethe-Heitler background where the emitted real photon is radiated from the electron rather than the proton. The HERMES and Jefferson Laboratory experiments measure in the kinematics where they expect to be dominated by the deeply virtual Compton scattering–Bethe-Heitler interference term and observe the sin ϕ azimuthal angle and helicity dependence expected for this contribution—see Fig. 21.

First measurements of the single-spin asymmetry have been reported by Airapetian *et al.* (2001) and Stepanyan *et al.* (2001), which have the characteristics expected from the deeply virtual Compton scattering–Bethe-Heitler interference.

B. Generalized parton distributions

For exclusive processes such as deeply virtual Compton scattering or hard meson production the generalized parton distributions involve nonforward proton matrix elements (Radyushkin, 1997; Ji, 1998; Vanderhaeghen *et al.*, 1998; Goeke *et al.*, 2001; Diehl, 2003). The important kinematic variables are the virtuality of the hard photon Q^2, the momenta $p-\Delta/2$ of the incident proton and $p+\Delta/2$ of the outgoing proton, the invariant four-momentum transferred to the target $t=\Delta^2$, the average nucleon momentum P, the generalized Bjorken variable $k^+=xP^+$, and the light-cone momentum transferred to the target proton $\xi=-\Delta^+/2p^+$. The generalized parton distributions are defined as the light-cone Fourier transform of the point-split matrix element

$$\begin{aligned}\frac{P_+}{2\pi}\int dy^- e^{-ixP^+y^-}\langle p'|\bar{\psi}_\alpha(y)\psi_\beta(0)|p\rangle_{y^+=y_\perp=0} \\ =\frac{1}{4}\gamma^-_{\alpha\beta}\bigg[H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+u(p) \\ +E(x,\xi,\Delta^2)\bar{u}(p')\sigma^{+\mu}\frac{\Delta_\mu}{2M}u(p)\bigg] \\ +\frac{1}{4}(\gamma_5\gamma^-)_{\alpha\beta}\bigg[\tilde{H}(x,\xi,\Delta^2)\bar{u}(p')\gamma^+\gamma_5u(p) \\ +\tilde{E}(x,\xi,\Delta^2)\bar{u}(p')\gamma_5\frac{\Delta^+}{2M}u(p)\bigg].\end{aligned} \tag{162}$$

(Here we work in the light-cone gauge $A_+=0$ so that the path-ordered gauge link becomes trivial and equal to 1 to maintain gauge invariance throughout.)

The physical interpretation of the generalized parton distributions (before worrying about possible renormalization effects and higher-order corrections) is the following. Expanding out the quark field operators in Eq. (162) in terms of light-cone quantized creation and annihilation operators one finds that for $x>\xi$ $(x<\xi)$ the GPD is the amplitude required to take a quark (antiquark) of momentum $k-\Delta/2$ out of the proton and reinsert a quark (antiquark) of momentum $k+\Delta/2$ into the proton some distance along the light cone to reform the recoiling proton. In this region the GPD is a simple generalization of the usual parton distributions studied in inclusive and semi-inclusive scattering. In the remaining region $-\xi<x<\xi$ the GPD involves taking out (or inserting) a $q\bar{q}$ pair with momentum $k-\Delta/2$ and $-k-\Delta/2$ (or $k+\Delta/2$ and $-k+\Delta/2$), respectively. Note that the GPDs are interpreted as probability amplitudes rather than densities.

In the forward limit the GPDs H and $\tilde{H}$ are related to the forward parton distributions studied in (polarized) deep-inelastic scattering:

$$\begin{aligned}H(x,\xi,\Delta^2)|_{\xi=\Delta^2=0}=q(x), \\ \tilde{H}(x,\xi,\Delta^2)|_{\xi=\Delta^2=0}=\Delta q(x),\end{aligned} \tag{163}$$

whereas the GPDs E and $\tilde{E}$ have no such analog. In the fully renormalized theory the spin-dependent distributions $\tilde{H}$ and $\tilde{E}$ will be sensitive to the physics of the axial anomaly and, in this case, it is not easy to separate off an "anomalous component" because the nonforward matrix elements of the gluonic Chern-Simons current are non-gauge-invariant even in the light-cone gauge $A_+=0$. Integrating over x the first moments of the GPDs are related to the nucleon form factors:

$$\begin{aligned}\int_{-1}^{+1} dxH(x,\zeta,\Delta^2)=F_1(\Delta^2), \\ \int_{-1}^{+1} dxE(x,\zeta,\Delta^2)=F_2(\Delta^2), \\ \int_{-1}^{+1} dx\tilde{H}(x,\zeta,\Delta^2)=G_A(\Delta^2), \\ \int_{-1}^{+1} dx\tilde{E}(x,\zeta,\Delta^2)=G_P(\Delta^2).\end{aligned} \tag{164}$$

Here F_1 and F_2 are the Dirac and Pauli form factors of the nucleon, and G_A and G_P are the axial and induced-pseudoscalar form factors, respectively. (The dependence on ξ drops out after integration over x.)

The GPD formalism allows one, in principle, to extract information about quark angular momentum from hard exclusive reactions (Ji, 1997a). The current associated with Lorentz transformations is

$$M_{\mu\nu\lambda}=z_\nu T_{\mu\lambda}-z_\lambda T_{\mu\nu}, \tag{165}$$

where $T_{\mu\nu}$ is the QCD energy-momentum tensor. Thus the total angular momentum operator is related to the energy-momentum tensor through the equation

$$J^z_{q,g}=\left\langle p',\tfrac{1}{2}\left|\int d^3z(\vec{z}\times\vec{T}_{q,g})^z\right|p,\tfrac{1}{2}\right\rangle. \tag{166}$$

The form factors corresponding to the energy-momentum tensor can be projected out by taking the second moment with respect to x of the GPD. One finds Ji's sum rule for the total quark angular momentum

$$J_q=\frac{1}{2}\int_{-1}^{+1} dxx[H(x,\zeta,\Delta^2=0)+E(x,\zeta,\Delta^2=0)]. \tag{167}$$

The gluon "total angular momentum" could then be obtained through

$$\sum_q J_q + J_g = \frac{1}{2}. \tag{168}$$

In principle, it could also be extracted from precision measurements of the Q^2 dependence of hard exclusive processes like deeply virtual Compton scattering and meson production at next-to-leading-order accuracy where the quark GPDs mix with glue under QCD evolution.

To obtain information about the orbital angular momentum L_q we need to subtract the value of the intrinsic spin measured in polarized deep-inelastic scattering (or a future precision measurement of νp elastic scattering) from the total quark angular momentum J_q. This means that L_q is scheme dependent with different schemes corresponding to different physics content depending on how the scheme handles information about the axial anomaly, large-k_t physics, and any possible subtraction at infinity in the dispersion relation for g_1. The quark total angular momentum J_q is anomaly-free in QCD so that QCD axial anomaly effects occur with equal magnitude and opposite sign in L_q and S_q (Shore and White, 2000; Bass, 2002a). The quark orbital angular momentum L_q is measured by the proton matrix element of $[\bar{q}(\vec{z}\times\vec{D})_3 q](0)$. The gauge-covariant derivative means that L_q becomes sensitive to gluonic degrees of freedom in addition to the axial anomaly—for a recent discussion see Jaffe (2001). A first attempt to extract the valence contributions to the energy-momentum form factors entering Ji's sum rule is reported by Diehl *et al.* (2005).

The study of GPDs is being pioneered in experiments at HERMES, Jefferson Laboratory, and COMPASS. Proposals and ideas exist for dedicated studies using a 12-GeV CEBAF machine, a possible polarized *ep* collider (EIC) in connection with RHIC or JLab, and a high-luminosity polarized proton-antiproton collider at GSI. To extract information about quark total angular momentum one needs high luminosity, plus measurements over a range of kinematics Q^2, x, and Δ (bearing in mind the need to make reliable extrapolations into unmeasured kinematics). There is a challenging program to disentangle the GPDs from the formalism and to undo the convolution integrals which relate the GPDs to measured cross sections, and to check (experimentally) the kinematics where twist 2 dominates. Varying the photon or meson in the final state will give access to different spin-flavor combinations of GPDs even with unpolarized beams and targets. Besides yielding possible information about the spin structure of the proton, measurements of hard exclusive processes will, in general, help to constrain our understanding of the structure of the proton.

XII. POLARIZED PHOTON STRUCTURE FUNCTIONS

Deep-inelastic scattering from photon targets reveals many novel effects in QCD. The unpolarized photon structure function has been well studied both theoretically and experimentally. The polarized photon spin structure function is an ideal (theoretical) laboratory for studying the QCD dynamics associated with the axial anomaly.

The photon structure functions are observed experimentally in $e^+e^- \to$ hadrons where, for example, a hard photon (large Q^2) probes the quark structure of a soft photon ($P^2 \sim 0$). For any virtuality P^2 of the target photon the measured structure functions receive a contribution from contact photon-photon fusion and also a hadronic piece, which is commonly associated with vector-meson dominance of the soft target photon. The hadronic term scales with Q^2 while the contact term behaves as $\ln Q^2$ as we let Q^2 tend to ∞. This result was discovered by Witten (1977) for the unpolarized structure function F_2^γ and extended to the polarized case by Manohar (1989) and Sasaki (1980). The $\ln Q^2$ scaling behavior mimics the leading-order box-diagram prediction but the coefficient of the logarithm receives a finite renormalization in QCD. From the viewpoint of the renormalization group the essential detail discovered by Witten is that the coefficient functions of the photonic and singlet hadronic operators will mix under QCD evolution. The hadronic matrix elements are of leading order in α while the photon operator matrix elements are O(1). Since the hadronic coefficient functions are O(1) and the photon coefficient functions start at O(α), the photon structure functions receive leading-order contributions in α from both the hadronic and photonic channels.

In polarized scattering the first moment of g_1^γ is especially interesting. First, consider a real-photon target (and assume no fixed-pole correction). The first moment of g_1^γ vanishes

$$\int_0^1 dx g_1^\gamma(x,Q^2) = 0 \tag{169}$$

for a real-photon target independent of the virtuality Q^2 of the photon that it is probed with (Bass, 1992a; Bass *et al.*, 1998). This result is nonperturbative. To understand this, consider the real photon as the beam and the virtual photon as the target. Next apply the Gerasimov-Drell-Hearn sum rule. The anomalous magnetic moment of a photon vanishes to all orders because of Furry's theorem from which one obtains the sum rule. The sum rule (169) holds to all orders in perturbation theory and at every twist (Bass *et al.*, 1998).

The interplay of QCD and QED dynamics here can be seen through the axial anomaly equation

$$\partial^\mu J_{\mu 5} = 2m_q \bar{q} i\gamma_5 q + \frac{\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{\alpha}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{170}$$

(including the QED anomaly). The gauge-invariantly renormalized axial-vector current can then be written as the sum of the partially conserved current plus QCD and Abelian QED Chern-Simons currents

$$J_{\mu5} = J_{\mu5}^{\rm con} + K_\mu + k_\mu, \tag{171}$$

where k_μ is the anomalous Chern-Simons current in QED.

The vanishing first moment of $\int_0^1 dx g_1^\gamma$ is the sum of a contact term $-(\alpha/\pi)\Sigma_q e_q^2$ measured by the QED Chern-Simons current and a hadronic term associated with the two QCD currents in Eq. (171). The contact term is associated with high-k_t leptons and two-quark-jet events (and no beam jet) in the final state. For the gluonic contribution associated with polarized glue in the hadronic component of the polarized photon, the two-quark-jet cross section is associated with an extra soft "beam jet."

For a virtual-photon target one expects the first moment to exhibit similar behavior to that suggested by the tree box graph amplitude—that is, for $P^2 \gg m^2$ the first moment tends to equal just the QED anomalous contribution and the hadron term vanishes. However, here the mass scale m^2 is expected to be set by the ρ-meson mass corresponding to a typical hadronic scale and vector-meson dominance of the soft photon instead of the light-quark mass or the pion mass (Shore and Veneziano, 1993a, 1993b). Measurements of g_1^γ might be possible with a polarized $e\gamma$ collider (De Roeck, 2001). The virtual-photon target could be investigated through the study of resolved photon contributions to polarized deep-inelastic scattering from a nucleon target (Stratmann, 1998). Target mass effects in the polarized virtual-photon structure function are discussed by Baba *et al.* (2003).

XIII. CONCLUSIONS AND OPEN QUESTIONS

The exciting challenge of understanding the spin structure of the proton has produced many unexpected surprises in experimental data and inspired much theoretical activity and new insight into QCD dynamics and the interplay between spin and chiral or axial U(1) symmetry breaking in QCD.

There is a vigorous global program in experimental spin physics spanning (semi-)inclusive polarized deep-inelastic scattering, photoproduction experiments, exclusive measurements over a broad kinematical region, polarized proton-proton collisions, fragmentation studies in e^+e^- collisions, and νp elastic scattering.

In this review we surveyed the present (and near future) experimental situation and the new theoretical understanding that spin experiments have inspired. New experiments (planned and underway) will surely produce more surprises and exciting new challenges for theorists as we continue our quest to understand the internal structure of the proton and QCD confinement-related dynamics.

We conclude with a summary of key issues and open problems in QCD spin physics where the next generation of experiments should yield vital information:

- What happens to "spin" in the transition from current to constituent quarks through dynamical axial U(1) symmetry breaking?
- How large is the gluon spin polarization in the proton? If Δg is indeed large, what would this mean for models of the structure of the nucleon? What dynamics could produce a large Δg?
- Are there fixed-pole corrections to spin sum rules for polarized photon-nucleon scattering? If yes, which ones?
- Is gluon topology important in the spin structure of the proton?
- What is the x and k_t dependence of the (negative) polarized strangeness extracted from inclusive and semi-inclusive polarized deep-inelastic scattering?
- How (if at all) do the effective intercepts for small-x physics change in the transition region between polarized photoproduction and polarized deep-inelastic scattering?
- In which kinematics, if at all, does the magnitude of the isosinglet component of g_1 at small x exceed the magnitude of the isovector component?
- How does $\Delta d/d$ behave at x very close to 1?
- Does perturbative QCD factorization work for spin-dependent processes? That is, will the polarized quark and gluon distributions extracted from the next generation of experiments prove to be process independent?
- Is the small value of $g_A^{(0)}$ extracted from polarized deep-inelastic scattering "target independent," e.g., through topological charge screening?
- Can we find and observe processes in the η' nucleon interaction which are also sensitive to the dynamics which underlies the singlet axial charge?
- How large is quark (and gluon) "orbital angular momentum" in the proton?
- Transversity measurements are sensitive to k_t-dependent effects in the proton and fragmentation processes. The difference between the C-odd transversity distribution $\delta q(x)$ and the C-even spin distribution $\Delta q(x)$, viz., $(\delta q - \Delta q)(x)$, probes relativistic dynamics in the proton. Precision measurements at large Bjorken x where just the valence quarks contribute would allow a direct comparison and teach us about relativistic effects in the confinement region.

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