CODATA recommended values of the fundamental physical constants: 2002*

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This paper gives the 2002 self-consistent set of values of the basic constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology (CODATA) for international use. Further, it describes in detail the adjustment of the values of the subset of constants on which the complete 2002 set of recommended values is based. Two noteworthy additions in the 2002 adjustment are recommended values for the bound-state rms charge radii of the proton and deuteron and tests of the exactness of the Josephson and quantum-Hall-effect relations $K_J=2e/h$ and $R_K=h/e^2$, where K_J and R_K are the Josephson and von Klitzing constants, respectively, e is the elementary charge, and h is the Planck constant. The 2002 set replaces the previously recommended 1998 CODATA set. The 2002 adjustment takes into account the data considered in the 1998 adjustment as well as the data that became available between 31 December 1998, the closing date of that adjustment, and 31 December 2002, the closing date of the new adjustment. The differences between the 2002 and 1998 recommended values compared to the uncertainties of the latter are generally not unreasonable. The new CODATA set of recommended values may also be found on the World Wide Web at physics.nist.gov/constants.

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GLOSSARY

AMDC	Atomic Mass Data Center, Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM), Orsay, France
$A_r(X)$	relative atomic mass of X: $A_r(X) = m(X)/m_u$
A_{90}	conventional unit of electric current: $A_{90} = V_{90}/\Omega_{90}$
Å*	Ångström-star: $\lambda(WK\alpha_1) = 0.209\,010\,0$ Å*
a_e	electron magnetic moment anomaly: $a_e = (\lvert g_e\rvert - 2)/2$
a_μ	muon magnetic moment anomaly: $a_\mu = (\lvert g_\mu\rvert - 2)/2$
BIPM	International Bureau of Weights and Measures, Sèvres, France
BNL	Brookhaven National Laboratory, Upton, New York, USA
BNM	Bureau national de métrologie, France
CCM	Consultative Committee for Mass and Related Quantities of the CIPM
CERN	European Laboratory for Particle Physics, Geneva, Switzerland
CIPM	International Committee for Weights and Measures
CODATA	Committee on Data for Science and Technology of the International Council for Science (ICSU, formerly the International Council of Scientific Unions)
CPT	combined charge conjugation, parity inversion, and time reversal
c	speed of light in vacuum
d	deuteron (nucleus of deuterium D, or ^{2}H)
d_{220}	{220} lattice spacing of an ideal crystal of naturally occurring silicon
$d_{220}(x)$	{220} lattice spacing of crystal X of naturally occurring silicon
E_b	binding energy
e	symbol for either member of the electron-positron pair; when necessary, e^- or e^+ is used to indicate the electron or positron
e	elementary charge: absolute value of the charge of the electron
F	Faraday constant: $F = N_A e$
FSU	Friedrich-Schiller University, Jena, Germany
$\mathcal{F}_{90}$	$\mathcal{F}_{90} = (F/A_{90})$ A
G	Newtonian constant of gravitation
g	local acceleration of free fall
g_d	deuteron g-factor: $g_d = \mu_d/\mu_N$
g_e	electron g-factor: $g_e = 2\mu_e/\mu_B$
g_p	proton g-factor: $g_p = 2\mu_p/\mu_N$
g'_p	shielded proton g-factor: $g'_p = 2\mu'_p/\mu_N$
$g_X(Y)$	g-factor of particle X in the ground ($1S$) state of hydrogenic atom Y
g_μ	muon g-factor: $g_\mu = 2\mu_\mu/(e\hbar/2m_\mu)$
GSI	Gesellschaft für Schweironenforschung, Darmstadt, Germany
Harvard	Harvard University, Cambridge, Massachusetts, USA
h	helion (nucleus of ^{3}He)
h	Planck constant; $\hbar = h/2\pi$
HUST	Huazhong University of Science and Technology, Wuhan, People's Republic of China
ILL	Institut Max von Laue-Paul Langevin, Grenoble, France
IMGC	Istituto di Metrologia "G. Colonnetti," Torino, Italy
IUPAC	International Union of Pure and Applied Chemistry
IRMM	Institute for Reference Materials and Measurements, Geel, Belgium
KRISS	Korea Research Institute of Standards and Science, Taedok Science Town, Republic of Korea
KR/VN	KRISS-VNIIM collaboration
K_J	Josephson constant: $K_J = 2e/h$
K_{J-90}	conventional value of the Josephson constant K_J: $K_{J-90} = 483\,597.9$ GHz V^{-1}
k	Boltzmann constant: $k = R/N_A$
LAMPF	Clinton P. Anderson Meson Physics Facility at Los Alamos National Laboratory, Los Alamos, New Mexico, USA
LANL	Los Alamos National Laboratory, Los Alamos, New Mexico, USA
LCIE	Laboratoire des Industries Électriques, Fontenay-aux-Roses, France, later part of BNM
LKB	Laboratoire Kastler-Brossel, Paris, France
LK/SY	LKB and BNM-SYRTE collaboration
MIT	Massachusetts Institute of Technology, Cambridge, Massachusetts, USA
MPQ	Max-Planck-Institute für Quantenoptik, Garching, Germany
MSL	Measurement Standards Laboratory, Industrial Research, Lower Hutt, New Zealand
$M(X)$	molar mass of X: $M(X) = A_r(X)M_u$
Mu	muonium (μ^+e^- atom)
M_u	molar mass constant: $M_u = 10^{-3}$ kg mol^{-1}
m_u	unified atomic mass constant: $m_u = m(^{12}C)/12$
m_X, $m(X)$	mass of X (for the electron e, proton p, and other elementary particles, the first symbol is used, i.e., m_e, m_p, etc.)
N_A	Avogadro constant
N/P/I	NMIJ-PTB-IRMM combined result
NIM	National Institute of Metrology, Beijing, People's Republic of China
NIST	National Institute of Standards and Technology, Gaithersburg, Maryland and Boulder, Colorado, USA
NMIJ	National Metrology Institute of Japan, National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba, Japan [formed as of 1 April 2001 from the

	National Research Laboratory of Metrology (NRLM) and other laboratories]
NML	National Measurement Laboratory, Commonwealth Scientific and Industrial Research Organization (CSIRO), Lindfield, Australia
NMR	nuclear magnetic resonance
NPL	National Physical Laboratory, Teddington, UK
NRLM	see NMIJ
n	neutron
PTB	Physikalisch-Technische Bundesanstalt, Braunschweig and Berlin, Germany
p	proton
QED	quantum electrodynamics
$Q(\chi^2\|\nu)$	probability that an observed value of chi-square for ν degrees of freedom would exceed χ^2
R	molar gas constant
$\overline{R}$	ratio of muon anomaly difference frequency to free proton NMR frequency
R_B	Birge ratio: $R_B=(\chi^2/\nu)^{1/2}$
R_d	bound-state rms charge radius of the deuteron
R_K	von Klitzing constant: $R_K=h/e^2$
R_{K-90}	conventional value of the von Klitzing constant R_K: $R_{K-90}=25\,812.807\ \Omega$
R_p	bound-state rms charge radius of the proton
R_∞	Rydberg constant: $R_\infty=m_e c\alpha^2/2h$
$r(x_i,x_j)$	correlation coefficient of estimated values x_i and x_j: $r(x_i,x_j)=u(x_i,x_j)/[u(x_i)u(x_j)]$
S_c	self-sensitivity coefficient
SI	International System of Units
Stanford	Stanford University, Stanford, California, USA
SYRTE	Systèmes de référence Temps Espace of BNM [renamed from Laboratorie Primaire du Temps et des Fréquences (LPTF) of BNM as of 1 January 2001]
T	thermodynamic temperature
TR&D	Tribotech Research and Development Company, Moscow, Russian Federation
Type A	evaluation of uncertainty by the statistical analysis of series of observations
Type B	evaluation of uncertainty by means other than the statistical analysis of series of observations
t_{90}	Celsius temperature on the International Temperature Scale of 1990 (ITS-90)
USussex	University of Sussex, Sussex, UK
UVA	University of Virginia, Charlottesville, Virginia, USA
UWash	University of Washington, Seattle, Washington, USA
UWup	University of Wuppertal, Wuppertal, Germany
UZur	University of Zurich, Zurich, Switzerland
u	unified atomic mass unit: $1\ \mathrm{u}=m_u=m(^{12}\mathrm{C})/12$
$u(x_i)$	standard uncertainty (i.e., estimated standard deviation) of an estimated value x_i of a quantity X_i (also simply u)
$u_r(x_i)$	relative standard uncertainty of an estimated value x_i of a quantity X_i: $u_r(x_i)=u(x_i)/\|x_i\|$, $x_i\neq 0$ (also simply u_r)
$u(x_i,x_j)$	covariance of estimated values x_i and x_j
$u_r(x_i,x_j)$	relative covariance of estimated values x_i and x_j: $u_r(x_i,x_j)=u(x_i,x_j)/(x_i x_j)$
$V_m(\mathrm{Si})$	molar volume of naturally occurring silicon
VNIIM	D. I. Mendeleyev All-Russian Research Institute for Metrology, St. Petersburg, Russian Federation
V_{90}	conventional unit of voltage based on the Josephson effect and K_{J-90}: $V_{90}=(K_{J-90}/K_J)\ \mathrm{V}$
WGAC	Working Group on the Avogadro Constant of the CIPM Consultative Committee for Mass and Related Quantities (CCM)
W_{90}	conventional unit of power: $W_{90}=V_{90}^2/\Omega_{90}$
XROI	combined x-ray and optical interferometer
$\mathrm{xu(CuK}\alpha_1)$	Cu x unit: $\lambda(\mathrm{CuK}\alpha_1)=1537.400\ \mathrm{xu(CuK}\alpha_1)$
$\mathrm{xu(MoK}\alpha_1)$	Mo x unit: $\lambda(\mathrm{MoK}\alpha_1)=707.831\ \mathrm{xu(MoK}\alpha_1)$
$x(\mathrm{X})$	amount-of-substance fraction of X
Yale	Yale University, New Haven, Connecticut, USA
α	fine-structure constant: $\alpha=e^2/4\pi\epsilon_0\hbar c\approx 1/137$
α	alpha particle (nucleus of ^{4}He)
$\Gamma'_{X-90}(\mathrm{lo})$	$\Gamma'_{X-90}(\mathrm{lo})=(\gamma'_X A_{90})\ \mathrm{A}^{-1}$, X=p or h
$\Gamma'_{p-90}(\mathrm{hi})$	$\Gamma'_{p-90}(\mathrm{hi})=(\gamma'_p/A_{90})\ \mathrm{A}$
γ_p	proton gyromagnetic ratio: $\gamma_p=2\mu_p/\hbar$
γ'_p	shielded proton gyromagnetic ratio: $\gamma'_p=2\mu'_p/\hbar$
γ'_h	shielded helion gyromagnetic ratio: $\gamma'_h=2\|\mu'_h\|/\hbar$
$\Delta\nu_{Mu}$	muonium ground-state hyperfine splitting
δ_e	additive correction to the theoretical expression for the electron magnetic moment anomaly a_e
δ_{Mu}	additive correction to the theoretical expression for the ground-state hyperfine splitting of muonium $\Delta\nu_{Mu}$
$\delta_X(n\mathrm{L}_j)$	additive correction to the theoretical expression for an energy level of either hydrogen H or deuterium D with quantum numbers n, L, and j
δ_μ	additive correction to the theoretical expression for the muon magnetic moment anomaly a_μ
ϵ_0	electric constant: $\epsilon_0=1/\mu_0 c^2$
$\doteq$	symbol used to relate an input datum to its observational equation
$\lambda(\mathrm{XK}\alpha_1)$	wavelength of $\mathrm{K}\alpha_1$ x-ray line of element X
λ_{meas}	measured wavelength of the 2.2 MeV capture γ ray emitted in the reaction $\mathrm{n+p\rightarrow d+}\gamma$

μ	symbol for either member of the muon-antimuon pair; when necessary, μ^- or μ^+ is used to indicate the negative muon or positive muon
μ_B	Bohr magneton: $\mu_B = e\hbar/2m_e$
μ_N	nuclear magneton: $\mu_N = e\hbar/2m_p$
$\mu_X(Y)$	magnetic moment of particle X in atom Y
μ_0	magnetic constant: $\mu_0 = 4\pi \times 10^{-7}$ N/A^2
μ_X, μ'_X	magnetic moment, or shielded magnetic moment, of particle X
ν	degrees of freedom of a particular adjustment
$\nu(f_p)$	difference between muonium hyperfine-splitting Zeeman transition frequencies ν_{34} and ν_{12} at a magnetic flux density B corresponding to the free-proton NMR frequency f_p
σ	Stefan-Boltzmann constant: $\sigma = \pi^2 k^4/60\hbar^3 c^2$
τ	symbol for either member of the tau-antitau pair; when necessary, τ^- or τ^+ is used to indicate the negative tau or positive tau
χ^2	the statistic "chi square"
Ω_{90}	conventional unit of resistance based on the quantum Hall effect and R_{K-90}: $\Omega_{90} = (R_K/R_{K-90})\ \Omega$

I. INTRODUCTION

A. Background

The Task Group on Fundamental Constants of the Committee on Data for Science and Technology (CODATA) was established in 1969 to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all the relevant data available at a given point in time. CODATA itself, established three years earlier as an interdisciplinary committee of the international Council for Science, seeks to improve the quality, reliability, processing, management, and accessibility of data of importance to science and technology.

This report, prepared by the authors under the auspices of the Task Group, gives the 2002 CODATA set of recommended values of the constants and describes in detail the 2002 adjustment of the values of the subset of constants on which it is based. The 2002 set replaces its immediate predecessor, the 1998 set, which resulted from the 1998 adjustment also carried out by the authors under Task Group auspices. The detailed report of the 1998 adjustment was published in the April 2000 issue of this journal (Mohr and Taylor, 2000), and for convenience is referred to as "CODATA-98" throughout this article. [Essentially the same paper was published in the *Journal of Physical and Chemical Reference Data* (Mohr and Taylor, 1999).]

The two sets of recommended values of the constants provided by CODATA prior to the 1998 set are the 1973 set (Cohen and Taylor, 1973), which was the first from CODATA, and the 1986 set (Cohen and Taylor, 1987). Of course, as discussed in CODATA-98, there had been many other adjustments of the values of the constants during the previous decades; these extend back to the origin of such endeavors, the pioneering effort of R. T. Birge in the late 1920s, which was published starting on page one of the first issue of this journal (Birge, 1929). Also discussed in CODATA-98 is the Task Group's conclusion that, because data that impact our knowledge of the values of the constants become available nearly continuously, and because the Web allows the rapid and wide diffusion of information, 13 years between adjustments is no longer acceptable. By taking advantage of the high degree of computerization incorporated by the authors in the 1998 adjustment, the Task Group decided to issue a new set of recommended values at least every four years.

The 2002 CODATA set of recommended values of the constants, which first became available 9 December 2003 at http://physics.nist.gov/constants (a Web site of the NIST Fundamental Constants Data Center), is the initial set of constants resulting from the new four-year cycle, as is this report. In anticipation of the new schedule, sufficient detail was included in CODATA-98 to allow the reports that describe future adjustments to be more succinct and to focus mainly on the new results that become available between adjustments. The many references to CODATA-98 throughout this text are a reflection of this approach.

In keeping with the above, we note that Sec. I.A of CODATA-98 discusses the motivation for carrying out adjustments of the values of the constants and the benefits that result from doing so; and Sec. I.B describes our approach to the topics, "units, quantity symbols, numerical values, calculations," which in fact is quite straightforward. In brief, we use the units of the International System of Units (SI) (Taylor, 1995; BIPM, 1998); the generally accepted symbols for quantities and units and their generally accepted typefaces (Cohen and Giacomo, 1987; IEC, 1992; ISO, 1993b; Mills *et al.* 1993); and recognize that a quantity A can be written as $A = \{A\}[A]$, where $\{A\}$ is the numerical value of A when A is expressed in the unit $[A]$ (ISO, 1993b). This leads to $\{A\} = A/[A]$, where $A/[A]$ is interpreted to mean the ratio of the quantity A to a quantity of the same kind with the value 1 $[A]$.

With regard to our numerical calculations, the reader should bear in mind that most are carried out with more digits than are displayed in the text in order to avoid rounding errors. It is especially critical to recognize this fact in connection with correlation coefficients near unity, for which we only show three digits for simplicity. Data with more figures are available on request.

B. Uncertainties

Our approach to uncertainty assignment, a most important concern in adjustments of the values of the constants, is discussed at length in Sec. I.C of CODATA-98.

A key point is that we follow the approach of the *Guide to the Expression of Uncertainty in Measurement* (ISO, 1993a; Taylor and Kuyatt, 1994), which in fact has been used in the precision measurement-fundamental constants field for many years.

The standard uncertainty $u(y)$ (or simply u) of a result y is taken to represent the estimated standard deviation (the square root of the variance) of y. If the result y is obtained as a function of estimated values x_i of other quantities, then the standard uncertainty $u(y)$ is obtained by combining the individual standard uncertainty components $u(x_i)$, and covariances $u(x_i,x_j)$ where appropriate, using the law of propagation of uncertainty. The relative standard uncertainty of a result y, $u_r(y)$ (or simply u_r), is defined by $u_r(y)=u(y)/|y|$, if $y\neq 0$, with an analogous definition for individual components of uncertainty. Further, the evaluation of a standard uncertainty by the statistical analysis of series of observations is termed a *Type A evaluation*, while an evaluation by means other than the statistical analysis of series of observations is termed a *Type B evaluation*.

A numerical result is usually written as, for example,

$$y = 1234.567\,89(12) \times 10^{-10}\ \mathrm{U}\ [9.7 \times 10^{-8}], \qquad (1)$$

where U represents a unit symbol and the number in parentheses is the numerical value of the standard uncertainty of y referred to the last digits of the quoted value. The number in square brackets is $u_r(y)$. In general, numbers with more than four digits on either side of the decimal point are written with the figures in groups of three starting from the decimal point. An exception is that when there is a single separated figure followed by a two-figure standard uncertainty in parentheses, the single figure is grouped with the previous three figures; thus 1.234 5678(12). Also, a numerical value written as, for example, 12 345.6(1.2) means that the standard uncertainty of the figures 5.6 is 1.2.

C. Data categorization, selection, and evaluation procedures

Our approach to the subjects of data categorization, selection, and evaluation procedures is thoroughly covered in Secs. I.D and I.E of CODATA-98. In brief, we treat essentially all quantities on an equal footing—we abandon the earlier categories of "stochastic input data" and "auxiliary constants." Nevertheless, as in the 1998 adjustment, in a few instances a constant that enters the analysis of input data is taken as a fixed quantity rather than an adjusted quantity. In the extreme case, the constant is treated as fixed everywhere it appears, because the data that enter the adjustment have a negligible effect on its value. In the intermediate case, a constant is taken as fixed in some contexts and a variable in others, because in those contexts in which it is taken as a fixed quantity the data that enter the adjustment again have a negligible effect on its value. However, in such cases, rather than use arbitrary values for the fixed constants, we effectively use the 2002 recommended values by iterating the least-squares adjustment several times and replacing the fixed values by the adjusted values after each iteration.

Our criteria for the selection of data for the 2002 adjustment are similar to those for the 1998 adjustment. First, any datum considered for the 2002 adjustment had to be nominally available by 31 December 2002. However, in a few cases, because of the exceptional importance of the datum, the 31 December 2002 closing date was extended. Second, as in the 1998 adjustment, each datum considered for the 2002 adjustment had to have a standard uncertainty u sufficiently small that its weight $w=1/u^2$ was nontrivial in comparison with the weight of other directly measured values of the same quantity. Thus, in most cases, a result was not considered if its standard uncertainty was more than about five times the standard uncertainty of other similar results.

Third, sufficient information to allow a valid standard uncertainty to be assigned to a datum had to be published in an archival journal. In general, a Ph.D. thesis with no other publication is not a sufficient basis for including the result of an experiment or calculation that is otherwise unpublished. On the other hand, in the case of an established group with an ongoing project, a private communication or article in a conference proceedings can be considered sufficient documentation, at the discretion of the Task Group.

Finally, we note that the data evaluation procedures used in the 2002 adjustment are the same as those used in the 1998 adjustment, namely, we employ in our analysis the standard least-squares algorithm for correlated data, as described in Appendix E of CODATA-98.

D. Time variation of the constants

There has been increased interest recently in the subject of the possible temporal variation of the values of certain fundamental constants, no doubt engendered by the publication over the last several years of results, based on measurements of the absorption line spectra of light reaching earth from distant quasars after passing through gas surrounding an intervening galaxy, that appear to show that some 10^{10} years ago the value of the fine-structure constant α was smaller than the current value by approximately the fractional amount 6×10^{-6}. [See, for example, Webb *et al.* (2003) as well as the recent review of the subject of the time variation of the values of the constants by Uzan (2003). Results of two recent laboratory tests of the temporal variation of the values of combinations of different constants, based on the comparison of ultraprecise atomic clocks, are reported by Bize *et al.* (2003) and Marion *et al.* (2003). See also the paper by Karshenboim (2000b).]

It should be recognized, however, that although this is a quite important subject with implications for physics at its most fundamental level, experiments show that any current temporal variation in the values of the constants is extremely small and thus irrelevant as far as the set of recommended values of the constants is concerned. (At present, the most accurately measured fundamental con-

TABLE I. Some exact quantities relevant to the 2002 adjustment.

Quantity	Symbol	Value
Speed of light in vacuum	c, c_0	299 792 458 m s^{-1}
Magnetic constant	μ_0	$4\pi\times10^{-7}$ N A^{-2} = 12.566 370 614... $\times10^{-7}$ N A^{-2}
Electric constant	ϵ_0	$(\mu_0 c^2)^{-1}$ = 8.854 187 817... $\times10^{-12}$ F m^{-1}
Relative atomic mass of ^{12}C	$A_r(^{12}C)$	12
Molar mass constant	M_u	10^{-3} kg mol^{-1}
Molar mass of ^{12}C, $A_r(^{12}C)M_u$	$M(^{12}C)$	12×10^{-3} kg mol^{-1}
Conventional value of Josephson constant	K_{J-90}	483 597.9 GHz V^{-1}
Conventional value of von Klitzing constant	R_{K-90}	25 812.807 Ω

stants have relative standard uncertainties that exceed 10^{-12}, which is still much larger than current limits on the temporal variation in the values of the constants over a decade.)

E. Outline of paper

This report is structured in a similar way to CODATA-98. Section II discusses special quantities and units such as the speed of light in vacuum and the conventional values K_{J-90} and R_{K-90} of the Josephson and von Klitzing constants, respectively.

Section III, together with Appendixes A–E, arguably the most important portions of the paper, are devoted to the review of all the available experimental and theoretical data that might be relevant to the 2002 adjustment. Although only four years separate the 31 December 1998 and 31 December 2002 closing dates of the 1998 and 2002 adjustments, there are many new and important results to consider. The new experimental information includes improved values of the relative atomic masses of ^{3}He, ^{4}He, ^{16}O, ^{36}Ar, and ^{133}Cs; a more accurate value of the $1S_{1/2}-2S_{1/2}$ transition frequency in hydrogen; a new result for the bound-state rms charge radius of the proton R_p; highly accurate measurements related to the bound-state g-factor of the electron in hydrogenlike ^{12}C and ^{16}O, which provide a more accurate value of the relative atomic mass of the electron $A_r(e)$; a new, quite accurate measurement of the muon magnetic moment anomaly a_μ; an accurate value of $h/m(Cs)$, where h is the Planck constant and $m(Cs)$ is the mass of the ^{133}Cs atom, which provides a new, highly accurate value of α; a result for the molar volume of silicon $V_m(Si)$ and important new experimental findings concerning previous measurements of the {220} lattice spacing of particular crystals of silicon; new measurements of the Newtonian constant of gravitation G; and new measurements of the fine structure of ^{4}He.

The new theoretical information includes significant advances in the theory of hydrogen and deuterium energy levels (Appendix A), the electron and muon magnetic moment anomalies a_e and a_μ (Appendixes B and C), the electron bound-state g-factor in hydrogenic ions with nuclear-spin quantum number $i=0$ (Appendix D), the ground-state hyperfine splitting of muonium $\Delta\nu_{Mu}$ (the μ^+e^- atom; Appendix E), and the fine structure of ^{4}He.

It should be noted that, to avoid confusion, we follow our 1998 practice of identifying a result by its year of formal publication rather than the year the result became available.

Section IV describes the analysis of the data. Their consistency and potential impact on the determination of the 2002 recommended values is examined by first comparing directly measured values of the same quantity, then comparing directly measured values of different quantities through a third quantity such as α or h that can be inferred from the values of the directly measured quantities, and finally by the method of least squares. Based on these investigations, the final set of input data to be used in the 2002 adjustment is selected. (Note that summaries of the values of α and h, including relevant section and equation numbers, may be found in Tables XV and XVI, respectively.)

A new addition to the analysis is an investigation, discussed in Appendix F, of the effect of relaxing the assumptions $K_J=2e/h$ and $R_K=h/e^2$ (e is the elementary charge), which are a basic part of the theory of the Josephson and quantum Hall effects.

Section V presents, in several tables, the 2002 CODATA recommended values of the basic constants and conversion factors of physics and chemistry, including the covariance matrix of a selected group of constants.

Section VI concludes the main text with a comparison of the 2002 and 1998 recommended values of the constants, a discussion of the implications for physics and metrology of the 2002 values and adjustment, and suggestions for future work that could significantly advance our knowledge of the values of the constants.

II. SPECIAL QUANTITIES AND UNITS

Some important special quantities and units are briefly recalled in the following sections; see CODATA-98 for details. Table I gives those special quantities whose numerical values are exactly defined.

A. Speed of light in vacuum c and realization of the meter

The current definition of the meter—"The meter is the length of path traveled by light in vacuum in 1/299 792 458 of a second"—fixes the speed of light in vacuum to be exactly $c=299\,792\,458$ m/s (BIPM, 1998). In practice, the most widely used method to realize the meter is by means of one of the radiations and its stated frequency or wavelength recommended by the International Committee for Weights and Measures (CIPM, *Comité international des poids et mesure*) given in its periodically revised *Mise en pratique* of the definition of the meter. The current version was published in 2003 (Quinn, 2003); the recent reviews of Helmcke and Riehle (2001) and Helmcke (2003) are also relevant. It is also noteworthy, as pointed out by Task-Group member S. Karshenboim, that the relative standard uncertainties assigned to some of the CIPM recommended transition frequencies are comparable to the limits of the time variation of certain combinations of constants obtained by comparing the frequencies of ultraprecise atomic clocks (see Sec. I.D).

B. Magnetic constant μ_0 and electric constant ϵ_0

The SI definition of the ampere (BIPM, 1998), together with the equation that gives the force per length between two straight, parallel conductors in vacuum, of infinite length and negligible cross section, implies that the magnetic constant μ_0, also called the permeability of vacuum, is an exact quantity with the value given in Table I. Since the electric constant, also called the permittivity of vacuum, is related to μ_0 by $\epsilon_0=1/\mu_0 c^2$, it too is an exact quantity. [Note that here and in the rest of this paper, we use the convention that $a/bc\equiv a/(bc)$ for equations written in the text or tables.]

C. Electron volt eV, unified atomic mass unit u, and related quantities

The electron volt, which is the kinetic energy acquired by an electron traversing a potential difference of 1 V in vacuum, is related to the joule by $1\ \mathrm{eV}=(e/\mathrm{C})\ \mathrm{J}\approx 1.60\times10^{-19}$ J, where e is the elementary charge and e/C is the numerical value of e when e is expressed in the unit coulomb.

The unified atomic mass unit u is $\frac{1}{12}$ times the mass $m(^{12}\mathrm{C})$ of a free (noninteracting) neutral atom of carbon 12 at rest and in its ground state: $1\ \mathrm{u}=m_\mathrm{u}=\frac{1}{12}m(^{12}\mathrm{C})\approx 1.66\times10^{-27}$ kg, where the quantity m_u is the atomic mass constant.

The relative atomic mass $A_\mathrm{r}(\mathrm{X})$ of an elementary particle, atom, or more generally an entity X, is defined by $A_\mathrm{r}(\mathrm{X})=m(\mathrm{X})/m_\mathrm{u}$, where $m(\mathrm{X})$ is the mass of X. Thus $A_\mathrm{r}(\mathrm{X})$ is the numerical value of $m(\mathrm{X})$ when $m(\mathrm{X})$ is expressed in u, and evidently $A_\mathrm{r}(^{12}\mathrm{C})=12$. [For particles such as the electron e and proton p, the symbol m_X rather than $m(\mathrm{X})$ is used to denote the mass.]

One mole is the amount of substance of a collection of as many specified entities X as there are atoms in 0.012 kg of carbon 12, where it is understood that the carbon atoms are free, neutral, at rest, and in their ground state. This number of atoms per mole is the Avogadro constant $N_\mathrm{A}\approx 6.02\times10^{23}\ \mathrm{mol}^{-1}$. The amount of substance corresponding to N specific entities is $n=N/N_\mathrm{A}$. For convenience we introduce the molar mass constant M_u defined by $M_\mathrm{u}=N_\mathrm{A}m_\mathrm{u}=10^{-3}\ \mathrm{kg\,mol}^{-1}$, and the molar mass $M(\mathrm{X})$ of entity X is $M(\mathrm{X})=A_\mathrm{r}(\mathrm{X})M_\mathrm{u}$.

D. Josephson effect and Josephson constant K_J

When a Josephson device, or junction, is irradiated with electromagnetic radiation of frequency f, its current vs voltage curve exhibits current steps at precisely quantized Josephson voltages U_J. The voltage of the nth step, n an integer, is related to f by $U_\mathrm{J}(n)=nf/K_\mathrm{J}$, where K_J is the Josephson constant.

An extensive body of experimental and theoretical evidence supports the relation

$$K_\mathrm{J}=\frac{2e}{h}\approx 483\,598\ \mathrm{GHz/V}, \tag{2}$$

where as usual e is the elementary charge and h is the Planck constant.

The Josephson effect was predicted by Brian Josephson over 40 years ago (Josephson, 1962), hence it is a rather mature field. Current research of interest here focuses on improving practical voltage standards based on the Josephson effect, including arrays of thousands of Josephson junctions in series that enable present-day Josephson voltage standards to be programmable and to provide highly accurate voltages at the 10 V level; see the recent reviews by Hamilton (2000), Behr *et al.* (2002), and Kohlmann *et al.* (2003).

The comparison of different kinds of arrays, and the Josephson voltage standards of different laboratories, is an important aspect of the current research effort. Three recent array comparisons are worthy of special mention. Krasnopolin *et al.* (2002) showed that the voltage difference of two superconductor-insulator-normal metal-insulator-superconductor or SINIS arrays of 4086 $\mathrm{Nb/Al/AlO}_x\mathrm{/Al/AlO}_x\mathrm{/Al/Nb}$ junctions (nominally two halves of a 1 V array with 8192 junctions), each array generating about 0.6 V, was $<1.2\times10^{-17}$ V. Jeanneret *et al.* (2001) showed that the 1 V output of a superconductor-normal metal-superconductor or SNS array of 32 768 Nb/PdAu/Nb junctions and of a commercial superconductor-insulator-superconductor or SIS array of 20 208 $\mathrm{Nb/AlO}_x\mathrm{/Nb}$ junctions differed by the fractional amount $1.4(3.4)\times10^{-10}$. And Klushin *et al.* (2002) showed, within the 2×10^{-8} standard uncertainty of the experiment, the equivalency of K_J in the high-temperature superconductor $\mathrm{YBa_2Cu_3O_7}$ and of K_J in a SINIS array of the previously described type. This was done by comparing the approximate 10 mV output of 136 of the 512 junctions of a $\mathrm{YBa_2Cu_3O_7}$ bicrystal array to a voltage of 10 mV generated by the SINIS array.

[For other recent comparisons, see Reymann *et al.* (1999); Reymann *et al.* (2001); Behr *et al.* (2003); Lo-Hive *et al.* (2003).] In summary, all of the results obtained during the last four years continue to support the view that K_J is a constant of nature and equal to $2e/h$.

E. Quantum Hall effect and von Klitzing constant R_K

For a fixed current I through a quantum Hall effect device of the usual Hall-bar geometry—either a heterostructure or metal oxide semiconductor field-effect transistor (MOSFET)—there are regions in the curve of Hall voltage U_H vs applied magnetic flux density B for a heterostructure, or of U_H vs gate voltage U_g for a MOSFET, where the Hall voltage U_H remains constant as B or U_g is varied. These regions of constant U_H are called quantized Hall resistance plateaus, because in the limit of zero dissipation in the direction of current flow, the Hall resistance of the ith plateau $R_H(i)=U_H(i)/I$ is quantized: $R_H=R_K/i$, where i is an integer and R_K is the von Klitzing constant, after Klaus von Klitzing, who discovered the quantum Hall effect nearly 25 years ago (von Klitzing *et al.*, 1980). We are concerned here only with the integer quantum Hall effect. As for the Josephson-effect relation $K_J=2e/h$, there is an impressive body of evidence, both experimental and theoretical, that supports the relation

$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha} \approx 25\,813\ \Omega, \tag{3}$$

where as usual α is the fine-structure constant.

Being nearly 25 years old, the integer quantum Hall effect, like the Josephson effect, is a mature subject. However, considerable theoretical work is still underway to better understand the rather remarkable exactness of the quantization of the Hall resistance and the exactness of Eq. (3). The fact that R_K appears to be completely independent of sample geometry, sample material, step number i, etc., is a continuing surprise. A review for the nonexpert of progress in understanding the quantum Hall effect in terms of a topological invariant known as a Chern number is given by Avron *et al.* (2003).

The focus of the current experimental work on the quantum Hall effect of interest here continues to be the establishment of the criteria that must be met to accurately measure R_K, as well as the use of the quantum Hall effect in metrology, especially as a practical standard of resistance; comparisons of the resistance standards of different laboratories remains a part of the latter effort. For some recent results regarding quantum Hall effect measurement criteria, see Jeckelmann *et al.* (2001) and Delahaye and Jeckelmann (2003). Reviews of the application of the quantum Hall effect to metrology, including the currently quite active field of the ac measurement of the quantized Hall resistance $R_H(i)$, as well as the calibration of capacitance standards in terms of R_K, are given by Witt (1998); Piquemal (1999); Jeckelmann (2001); Jeckelmann and Jeanneret (2001, 2003); Bachmair *et al.* (2003). Results of some recent comparisons of quantum Hall effect resistance standards are reported by Delahaye *et al.* (2000), Satrapinski *et al.* (2001), and Nakanishi *et al.* (2002). Intriguing work to create large arrays of quantum Hall effect devices in parallel to provide highly accurate quantized resistances that are large submultiples of R_K, for example, $R_K/100$ and $R_K/200$, has been carried out by Poirier *et al.* (2002). As for the Josephson constant $K_J=2e/h$, all of the results obtained during the last four years continue to support the view that R_K is a constant of nature and equal to h/e^2.

F. Conventional Josephson and von Klitzing constants K_{J-90} and R_{K-90} and conventional electric units

On 1 January 1990, to establish worldwide uniformity in the measurement of voltage and resistance and other electric quantities, the CIPM introduced new, practical representations of the volt V and ohm Ω based on the Josephson effect and quantum Hall effect, respectively, and conventional (that is, adopted) values of K_J and R_K. These assigned exact values are (Quinn, 1989)

$$K_{J-90} = 483\,597.9\ \text{GHz/V}, \tag{4a}$$

$$R_{K-90} = 25\,812.807\ \Omega. \tag{4b}$$

For the purpose of least-squares adjustments of the values of the constants, it is useful to interpret the CIPM's adoption of K_{J-90} and R_{K-90} as establishing conventional, practical units of voltage and resistance V_{90} and Ω_{90} defined by

$$K_J = 483\,597.9\ \text{GHz}/V_{90}, \tag{5a}$$

$$R_K = 25\,812.807\ \Omega_{90}. \tag{5b}$$

(V_{90} and Ω_{90} are printed in italic type in recognition of the fact that they are physical quantities.) The conventional units V_{90} and Ω_{90}, which are readily realized in the laboratory (relative uncertainty of $<10^{-9}$ at the 1 V level and relative uncertainty approaching 10^{-9} at the 1 Ω level, respectively), are related to the SI units V and Ω by

$$V_{90} = \frac{K_{J-90}}{K_J}\ \text{V}, \tag{6a}$$

$$\Omega_{90} = \frac{R_K}{R_{K-90}}\ \Omega, \tag{6b}$$

which follow from Eqs. (4) and (5).

Other conventional electric units follow directly from V_{90} and Ω_{90}, for example, the conventional units of electric current I and power P: $A_{90}=V_{90}/\Omega_{90}$ and $W_{90}=V_{90}^2/\Omega_{90}$. Equations analogous to Eqs. (6a) and (6b) can be readily written for A_{90} and W_{90}, and of course for other conventional electric units as well.

It follows from the above discussion that for a voltage U,

$$U = \frac{U}{V_{90}} V_{90} = \frac{U}{V_{90}} \frac{K_{J-90}}{K_J} \text{ V}. \quad (7)$$

That is, the numerical value of U when U is expressed in the SI unit V, is equal to the numerical value of U when U is expressed in the conventional unit V_{90} multiplied by the ratio K_{J-90}/K_J. Similar expressions apply to other electric quantities and may be found in CODATA-98. In the 2002 adjustment, as in the 1998 adjustment, all electric-unit–dependent input data are expressed in terms of conventional electric units.

G. Acceleration of free fall *g*

As noted in CODATA-98, the acceleration of free fall, also called the acceleration due to gravity, is not really a constant: its fractional variation with height near the Earth's surface is $-3\times10^{-7}/\text{m}$, its fractional variation from equator to pole is about 0.5%, and it can have significant fractional variations over a day at a fixed latitude, for example, of order 2×10^{-7} at 40° latitude, due mostly to the varying influences of the moon and sun. For reference purposes, principally to calculate the now obsolete unit kilogram force, a conventional value called "standard acceleration due to gravity," given by $g_n = 9.806\,65\ \text{m/s}^2$, was adopted in 1903 by the General Conference on Weights and Measures (CGPM, *Conférence général des poids et mesures*) (BIPM, 1998).

As also noted in CODATA-98, highly accurate and portable absolute gravimeters are commercially available that allow g to be measured with $u_r < 10^{-8}$. This comparatively small uncertainty has been demonstrated by periodic international comparisons of gravimeters at the International Bureau of Weights and Measures (BIPM, *Bureau international des poids et mesures*, Sèvres, France); the two most recent comparisons, the fifth and sixth, were carried out in 1997 (Robertsson *et al.*, 2001) and 2001 (Vitushkin *et al.*, 2002). Further, a comparison at Stanford University (Peters *et al.*, 2001) of the value of g measured with a type of commercial gravimeter in common use and the value measured with an atom interferometer yielded the result $|\Delta g|/g = 7(7)\times10^{-9}$.

The most accurate measurement of a fundamental constant requiring a reliable value of g is the most recent watt-balance determination of $K_J^2 R_K$, in which a force is determined from the weight of a standard of mass and the value of g at the site of the experiment (see Sec. III.G.2). Although the uncertainty $u_r = 8.7\times10^{-8}$ assigned to the resulting value of $K_J^2 R_K$ is at least a factor of 10 larger than the uncertainty in g as measured by an absolute gravimeter, the future goal of determining $K_J^2 R_K$ with $u_r < 10^{-8}$ by a number of laboratories in their quest to find a replacement for the artifact international prototype of the kilogram will certainly challenge the existing gravimeter technology. (A number of papers on various aspects of absolute gravimetry may be found in a special issue of *Metrologia*, Vol. 39, No. 5, 2002, including a review by Faller, 2002).

TABLE II. Values of the relative atomic masses of various neutral atoms as given in the 1995 update to the 1993 atomic mass evaluation together with the defined value for ^{12}C.

Atom	Relative atomic mass $A_r(X)$	Relative standard uncertainty u_r
^{1}H	1.007 825 032 14(35)	3.5×10^{-10}
^{2}H	2.014 101 777 99(36)	1.8×10^{-10}
^{3}He	3.016 029 309 70(86)	2.8×10^{-10}
^{4}He	4.002 603 2497(10)	2.5×10^{-10}
^{12}C	12	(exact)
^{16}O	15.994 914 6221(15)	9.4×10^{-11}
^{28}Si	27.976 926 5327(20)	7.0×10^{-11}
^{29}Si	28.976 494 719(30)	1.0×10^{-9}
^{30}Si	29.973 770 218(45)	1.5×10^{-9}
^{36}Ar	35.967 546 28(27)	7.6×10^{-9}
^{38}Ar	37.962 732 16(53)	1.4×10^{-8}
^{40}Ar	39.962 383 1232(30)	7.6×10^{-11}
^{107}Ag	106.905 0930(60)	5.6×10^{-8}
^{109}Ag	108.904 7555(34)	3.1×10^{-8}
^{133}Cs	132.905 4469(32)	2.4×10^{-8}

III. REVIEW OF DATA

We review in this portion of the paper the experimental data relevant to the 2002 adjustment of the values of the constants. As discussed in Appendix E of CODATA-98, in a least-squares analysis of the fundamental constants the numerical data, both experimental and theoretical, also called *observational data* or *input data*, are expressed as functions of a set of independent variables called *adjusted constants*. The functions that relate the input data to the adjusted constants are called *observational equations*, and the least-squares procedure provides best estimated values, in the least-squares sense, of the adjusted constants. The focus of this Review of Data section is thus the identification and discussion of the input data and observational equations of interest for the 2002 adjustment. Although not all observational equations that we use are explicitly given in the text, all are summarized in Tables XIX and XXI of Sec. IV.B.

A. Relative atomic masses

The relative atomic masses $A_r(X)$ of a number of particles and atoms are important for the 2002 adjustment, as they were for the 1998 adjustment. In the current effort, the relative atomic masses of the electron $A_r(e)$, proton $A_r(p)$, neutron $A_r(n)$, helion $A_r(h)$ (nucleus of the ^{3}He atom), alpha particle $A_r(\alpha)$, hydrogenic ^{16}O ion $A_r(^{16}O^{7+})$, and cesium-133 atom $A_r(^{133}Cs)$ are included in the set of adjusted constants. The relevant data are summarized in Tables II–IV, and are discussed in the following sections. However, those data that were also used in the 1998 adjustment are only briefly touched

TABLE III. Values of the relative atomic masses of various particles and neutral atoms that have become available since the 1995 update to the 1993 atomic mass evaluation. The first two entries were used in the 1998 CODATA adjustment.

Particle or atom	Relative atomic mass $A_r(X)$	Relative standard uncertainty u_r
e	0.000 548 579 9111(12)	2.1×10^{-9}
p	1.007 276 466 89(14)	1.4×10^{-10}
^{3}He	3.016 029 3184(58)	1.9×10^{-9}
^{4}He	4.002 603 254 152(56)	1.4×10^{-11}
^{16}O	15.994 914 619 51(16)	1.0×10^{-11}
^{36}Ar	35.967 545 105(29)	8.1×10^{-10}
^{133}Cs	132.905 451 931(27)	2.0×10^{-10}

upon, since they are thoroughly reviewed in CODATA-98.

1. Atomic mass evaluation

Values of $A_r(X)$ for the neutral atoms of interest in the 2002 adjustment are listed in Table II, and the significant covariances of these values are given in Table V of CODATA-98. These data are taken from the 1995 update by Audi and Wapstra (1995) of their 1993 atomic mass evaluation (Audi and Wapstra, 1993). A more extensive version of the 1995 update can be found at www-csnsm.in2p3.fr/amdc, the Web site of the Atomic Mass Data Center (AMDC), Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM), Orsay, France. Results of the 2003 atomic mass evaluation (AME2003) of Audi *et al.* (2003); Wapstra *et al.* (2003) appeared in late 2003 and hence were not available in time for the 2002 CODATA adjustment. In place of the AMDC 1995 values for $A_r(^3\text{He})$, $A_r(^4\text{He})$, $A_r(^{16}\text{O})$, $A_r(^{36}\text{Ar})$, and $A_r(^{133}\text{Cs})$ listed in Table II, we use the more recent values listed in Table III, to which the following comments apply. [The values we use for $A_r(e)$ and $A_r(p)$ are also listed in Table III and are discussed in Secs. III.A.4.a and III.A.4.b]

a. $A_r(^3\text{He})$

The value of $A_r(^3\text{He})$ resulting from the 1995 update and listed in Table II is to a large extent determined by

TABLE IV. Ground-state ionization energies for ^{1}H and ^{2}H, and for neutral and ionized ^{3}He, ^{4}He, ^{12}C, and ^{16}O, where E represents E_I or E_b as appropriate (see text).

Atom or ion	Ionization energy (10^7 m^{-1})	Ionization energy (eV)	$10^9 E/m_u c^2$
^{1}H	1.096 787 717	13.5984	14.5985
^{2}H	1.097 086 146	13.6021	14.6025
^{3}He I	1.983 002	24.5861	26.3942
^{3}He II	4.388 892	54.4153	58.4173
^{3}He total	6.371 894	79.0014	84.8115
^{4}He I	1.983 107	24.5874	26.3956
^{4}He II	4.389 089	54.4178	58.4199
^{4}He total	6.372 195	79.0051	84.8155
^{12}C I	0.908 204	11.2603	12.0884
^{12}C II	1.966 647	24.3833	26.1766
^{12}C III	3.862 410	47.8878	51.4096
^{12}C IV	5.201 784	64.4939	69.2370
^{12}C V	31.624 23	392.0905	420.9265
^{12}C VI	39.520 614	489.9931	526.0293
^{12}C total	83.083 89	1030.1089	1105.8674
^{16}O I	1.098 37	13.6181	14.6196
^{16}O II	2.832 71	35.1211	37.7041
^{16}O III	4.430 85	54.9355	58.9757
^{16}O IV	6.243 82	77.4135	83.1068
^{16}O V	9.186 57	113.8989	122.2755
^{16}O VI	11.140 10	138.1196	148.2775
^{16}O VII	59.630 73	739.3268	793.7000
^{16}O VIII	70.283 935	871.4097	935.4968
^{16}O total	164.847 08	2043.8432	2194.1559

the value for $A_r(^3\mathrm{He})$ obtained by the University of Washington, Seattle, WA, USA group (Van Dyck *et al.*, 1993, 1995). During the course of subsequent work to determine $A_r(^{16}\mathrm{O})$ and $A_r(^4\mathrm{He})$ with unprecedented accuracy (discussed below), it was discovered that the magnetic flux density in the earlier University of Washington Penning trap mass spectrometer varied unexpectedly during the course of a run, which could last many days. Consequently, the cyclotron frequency of the helium-3 (or other) ion was not being measured in the same B as was the cyclotron frequency of the reference carbon-12 ion (Van Dyck *et al.*, 2001; Van Dyck, 2003a). Thus, at the suggestion of Van Dyck (2003a), we "correct" for this effect by increasing the University of Washington group's result for $A_r(^3\mathrm{He})$ by the same fractional amount that their new result for $A_r(^4\mathrm{He})$ has increased relative to its earlier value, which was presumably obtained when the variation in B that influenced the earlier value of $A_r(^3\mathrm{He})$ was still present. The scaled result is $A_r(^3\mathrm{He}) = 3.016\,029\,313\,28(98)$ $[3.2\times10^{-10}]$. This scaled value is then combined with the result $A_r(^3\mathrm{He}) = 3.016\,029\,3235(28)$ $[9.4\times10^{-10}]$ reported by the SMILETRAP (see subsequent paragraph) group (Fritioff *et al.*, 2001) in the following way: Each is assumed to be equally likely and each is modeled by a normal distribution with expectation and standard deviation (standard uncertainty) as quoted, except that the standard deviation of the University of Washington value is increased to that of the SMILETRAP value in order to account for the possibility that the scaling does not completely eliminate the error. The value given in Table III is then simply the expectation and standard deviation of the equally weighted sum of the two distributions.

SMILETRAP is a hyperbolic Penning trap mass spectrometer at the Manne Siegbahn Laboratory (MSL), Stockholm, Sweden. It operates at room temperature in a magnetic flux density $B = 4.7$ T and is connected to an electron-beam ion source called CRYSIS, which is able to produce highly charged ions of nearly any stable element. Relative atomic masses are determined from the ratio of the cyclotron frequency of the ion of interest to that of another ion of known relative atomic mass used as a reference, most often $^1\mathrm{H}_2^+$. In the latter case, the value of $A_r(^1\mathrm{H}_2^+)$ is calculated using the University of Washington value of $A_r(\mathrm{p})$ in Eq. (11), the hydrogen ionization energy, the molecular binding energy, and the average molecular vibrational energy. The cyclotron resonance frequencies are determined by a method in which the ions are ejected from the trap and their time of flight to a distant detector is measured. The SMILETRAP result for $A_r(^3\mathrm{He})$ is based on measurements of the cyclotron frequency ratio $f_c(^3\mathrm{He}^{2+})/f_c(^1\mathrm{H}_2^+)$. The influence of changes in B on the cyclotron frequency ratio is reduced to below the fractional amount 10^{-10} by measuring the ratio in less than 2 minutes. For a detailed description of SMILETRAP, see Bergströom, Carlberg, *et al.* (2002).

b. $A_r(^4\mathrm{He})$

The value of $A_r(^4\mathrm{He})$ in Table III is that obtained by the University of Washington group (Van Dyck, 2003b) using the significantly improved mass spectrometer discussed in CODATA-98 in connection with the group's determination of $A_r(\mathrm{p})$, which was the spectrometer's first significant application. It is based on measurements of the cyclotron frequency ratio $f_c(^4\mathrm{He}^{2+})/f_c(^{12}\mathrm{C}^{6+})$ carried out in eight runs from November 2000 to August 2002, each run lasting between 8 and 18 days. The resolution of the spectrometer is about 1 part in 10^{11} and is limited mainly by the approximately 5×10^{-12}/h fractional stability of the 6.0 T magnetic flux density in the Penning trap over the hundreds of hours duration of a typical run. The largest fractional correction applied to the data was $-202(9)\times10^{-12}$ to account for the effect of the image charge in the trap electrodes.

The value $A_r(^4\mathrm{He}) = 4.002\,603\,2568(13)$ $[3.2\times10^{-10}]$ has been reported by the SMILETRAP group based on measurements of the cyclotron frequency ratio $f_c(^4\mathrm{He}^{2+})/f_c(^1\mathrm{H}_2^+)$ (Fritioff *et al.*, 2001). In addition, the result $A_r(^4\mathrm{He}) = 4.002\,603\,2489(22)$ $[5.5\times10^{-10}]$ has been reported by researchers working at the University of Mainz, Germany (Brunner *et al.* 2001). They employed a hyperbolic, room-temperature Penning trap mass spectrometer, with $B = 7$ T, and a time-of-flight method to detect the cyclotron resonances similar to that used in SMILETRAP. The cyclotron frequency ratios determined in this experiment were $f_c(^4\mathrm{He}^+)/f_c(^1\mathrm{H}_2^+)$ and $f_c(^4\mathrm{He}^+)/f_c(^2\mathrm{H}_2^+)$, where the relative atomic masses $A_r(^1\mathrm{H}_1^+)$ and $A_r(^2\mathrm{H}_2^+)$ of the reference ions were calculated using the 1998 CODATA recommended values for $A_r(\mathrm{p})$ and $A_r(\mathrm{d})$ and the known values of the relevant ionization and binding energies. We note that the SMILETRAP value of $A_r(^4\mathrm{He})$ exceeds the University of Washington value by $2.0u_{\mathrm{diff}}$, while the University of Mainz value is smaller than the University of Washington value by $2.4u_{\mathrm{diff}}$, where u_{diff} is the standard uncertainty of the difference. However, the two values are not included in the 2002 adjustment, because their uncertainties exceed the uncertainty of the University of Washington value by factors of 23 and 39, respectively.

c. $A_r(^{16}\mathrm{O})$

The result for $A_r(^{16}\mathrm{O})$ listed in Table III is due to the University of Washington group (Van Dyck *et al.*, 2001; Van Dyck, 2003a) and is based on measurements, carried out using the group's new mass spectrometer, of the cyclotron frequency ratios $f_c(^{16}\mathrm{O}^{6+})/f_c(^{12}\mathrm{C}^{4+})$ and $f_c(^{16}\mathrm{O}^{6+})/f_c(^{12}\mathrm{C}^{6+})$; the data were obtained in nine runs during the period June 1999 to July 2000, with each run lasting between 7 and 14 days. The result is in acceptable agreement with that from the 1995 update in Table II, but the latter has an uncertainty more than a factor of nine larger.

d. $A_r(^{36}Ar)$

The result for $A_r(^{36}Ar)$ in Table III is due to the SMILETRAP group (Fritioff and Douysset, 2003). Although a value of $A_r(^{36}Ar)$ is required to calculate the relative atomic masses of the argon gas samples used in the National Institute of Standards and Technology (NIST), Gaithersburg, MD, USA and the National Physical Laboratory (NPL), Teddington, UK measurements of the molar gas constant R (see Secs. III.N.1 and III.N.2, its value is not especially critical.

e. $A_r(^{133}Cs)$

The value for $A_r(^{133}Cs)$ in Table III is that obtained by the MIT (Massachusetts Institute of Technology, Cambridge, MA, USA) group (Bradley *et al.*, 1999). The MIT mass spectrometer is an orthogonally compensated Penning trap in a magnetic flux density $B=8.5$ T. Cyclotron frequency measurements are made on single ions, which allows long observation times and provides freedom from ion-ion interactions. Data are generally acquired between 1:30 A.M. and 5:30 A.M. when a nearby subway is out of service. For the determination of $A_r(^{133}Cs)$, cyclotron frequency ratios $f_c(^{133}Cs^{3+})/f_c(CO_2^+)$ and $f_c(^{133}Cs^{2+})/f_c(C_5H_6^+)$, both of which are near unity, were obtained during five nights and four nights of measurements, respectively. Unexplained night-to-night variations of the data led Bradley *et al.* (1999) to increase the statistical uncertainties so that they reflected the observed day-to-day scatter. The result for $A_r(^{133}Cs)$ that can be deduced from the value of the ratio $A_r(^{133}Cs)/A_r(p)$ reported by the SMILETRAP group (Carlberg *et al.*, 1999) is in agreement with the MIT result, but it has a relative standard uncertainty $u_r=2.1\times10^{-9}$, which is a factor of 10 larger than that of the MIT result. Hence, because its uncertainty is not competitive, the SMILETRAP value is not included in the 2002 adjustment.

2. Observational equations for neutral atoms

As given in CODATA-98, the expression for the relative atomic mass $A_r(^AX)$ of a neutral atom ^AX in terms of the relative atomic mass of an ion of the atom formed by the removal of n electrons may be written as

$$A_r(^AX) = A_r(^AX^{(Z-n)+}) + nA_r(e) - \frac{E_b(^AX) - E_b(^AX^{(Z-n)+})}{m_u c^2}. \quad (8)$$

Here A is the mass number, Z is the atomic number (proton number), $E_b(^AX)/m_u c^2$ is the relative-atomic-mass equivalent of the total binding energy of the Z electrons of the atom, $E_b(^AX^{n+})/m_u c^2$ is the relative-atomic-mass equivalent of the binding energy of the $^AX^{n+}$ ion, and m_u is the atomic mass constant (see Sec. II.C). For a fully stripped atom, that is, for $n=Z$, $A_r(^AX^{n+})$ becomes $A_r(N)$, where N represents the nucleus of the atom, and $E_b(^AX^{n+})/m_u c^2=0$. Thus Eq. (8) can be written as

$$A_r(^AX) = A_r(N) + ZA_r(e) - \frac{E_b(^AX)}{m_u c^2}. \quad (9)$$

As appropriate, either Eq. (8) or Eq. (9) is used as the template for the observational equation for input data $A_r(^1H)$, $A_r(^2H)$, $A_r(^3He)$, $A_r(^4He)$, and $A_r(^{16}O)$. The observational equation for input datum $A_r(^{133}Cs)$ is simply $A_r(^{133}Cs) \doteq A_r(^{133}Cs)$. (Here the symbol $\doteq$ is used, because, in general, an observational equation does not express an equality; see Sec. IV.B.) It should be noted that, as discussed in the following section, the values of the quantities $E_b(^AX)/m_u c^2$ and $E_b(^AX^{n+})/m_u c^2$ can be taken as exact numbers.

3. Binding energies

Table IV lists the ionization energies E_I required in the present work. In that table, the value quoted for each ion is the energy required to remove one electron from the ground state and leave the atom or ion in the ground state of the next higher charge state. The total ionization energy of a given atom, or binding energy E_b, is the sum of the individual ionization energies for that atom.

With one exception, the wave numbers for the ionization energies and/or binding energies for ^1H, ^2H, ^3He, ^4He, and ^{12}C are as given in the corresponding table (Table III) of CODATA-98; their origins are discussed there in connection with that table. (The improved theory of hydrogen and deuterium energy levels discussed in Appendix A and the new, 2002 recommended values of the constants do not lead to changes in the wave numbers for ^1H and ^2H from those given in the 1998 table.) The exception is the wave number for ^{12}C V, for which we use the improved result of Drake (1988) rather than the value given by Kelly (1987). The wave numbers for ^{16}O I to ^{16}O VI are taken from the compilation of Moore and Gallagher (1993), the wave number for ^{16}O VII is the improved value of Drake (1988), and that for ^{16}O VIII is rescaled from the value given by Kelly (1987), which is taken from Erickson (1977), in the manner described in CODATA-98.

For informational purposes, column two of the table lists the various energies in eV, obtained using the 2002 recommended value for the factor that relates wave number in m^{-1} to the equivalent energy expressed in eV. The last column of Table IV gives the relative atomic mass equivalents of the various energies as obtained using the 2002 recommended value for the factor that relates wave number in m^{-1} to the equivalent energy expressed in u [recall from Sec. II.C that $m(X)=A_r(X)$ u]. The uncertainties of these two conversion factors are negligible in this application (see Table XXXII). Further, no uncertainties are given for the various energies in Table IV, because they are inconsequential compared to the uncertainties of the quantities with which they are used. Indeed, binding energies represent sufficiently

small corrections that the number of significant digits shown in the last column of the table is more than adequate.

4. Electron and proton relative atomic masses $A_r(e)$ and $A_r(p)$

a. $A_r(e)$

The most accurate value of $A_r(e)$ available at the time of the 1998 adjustment was obtained from a Penning trap measurement, carried out by the University of Washington group (Farnham *et al.*, 1995) of the ratio of the cyclotron frequency of a fully ionized carbon-12 atom $f_c(^{12}C^{6+})=6eB/2\pi m(^{12}C^{6+})$ to the cyclotron frequency of an electron $f_c(e)=eB/2\pi m_e$ in the same magnetic flux density B. This ratio leads to the value

$$A_r(e) = 0.000\,548\,579\,9111(12)\ [2.1\times10^{-9}], \qquad (10)$$

which we take as an input datum in the 2002 adjustment. [In the 1998 adjustment, the frequency ratio itself was taken as the input datum, but taking $A_r(e)$ as the input datum is, for all practical purposes, equivalent.] More recently, experimental and theoretical work related to the g-factor of the electron in both hydrogenic ^{12}C and hydrogenic ^{16}O has led to a value of $A_r(e)$ with an uncertainty smaller by about a factor of 4. This new work is discussed in Sec. III.C.3 and Appendix D.

b. $A_r(p)$

The most accurate value of $A_r(p)$ available at the time of the 1998 adjustment, and it remains the most accurate value available today, is that obtained from a Penning trap measurement of the ratio of the cyclotron frequency of a proton $f_c(p)$ to that of a four-times ionized carbon-12 atom $f_c(^{12}C^{4+})$ in the same magnetic flux density, also carried out by the University of Washington group (Van Dyck *et al.*, 1999). This ratio leads to

$$A_r(p) = 1.007\,276\,466\,89(14)\ [1.4\times10^{-10}], \qquad (11)$$

which we also take as an input datum in the 2002 adjustment. [As for the electron, in the 1998 adjustment the frequency ratio itself was taken as the input datum, but taking $A_r(p)$ as the input datum is, for all practical purposes, equivalent.] As pointed out in CODATA-98, this value is consistent with the value deduced from the result for $A_r(^1H)$ given in Table II with the aid of Eq. (9):

$$A_r(p) = 1.007\,276\,466\,83(35)\ [3.5\times10^{-10}]. \qquad (12)$$

Mention should also be made of the result $A_r(p)=1.007\,276\,466\,86(21)\ [2.1\times10^{-10}]$, statistical (Type A) uncertainty only, obtained by the SMILETRAP group from measurements of cyclotron frequency ratios $f_c(^AX^{n+})/f_c(^1H_2^+)$, where ^AX was ^{12}C, ^{28}Si, ^{14}N, ^{20}Ne, or ^{40}Ar, and n was between 4 and 16, depending on ^AX (Bergström, Fritioff, *et al.*, 2002). The quoted value is based only on data for which $n/A<\frac{1}{2}$ in order to avoid the possible influence of contaminant ions for which $n/A=\frac{1}{2}$. We do not include this result in the 2002 adjustment, because the principal purpose of the measurements was not to obtain an independent value of $A_r(p)$, but to use any observed difference between the SMILETRAP result and the University of Washington result in Eq. (11) to help assess the systematic effects that might be present in the SMILETRAP mass spectrometer (Bergström, Fritioff, *et al.*, 2002).

5. Neutron relative atomic mass $A_r(n)$

The datum that most affects the value of the relative atomic mass of the neutron $A_r(n)$, in the sense that its uncertainty makes the largest contribution to the uncertainty of $A_r(n)$, is the binding energy of the neutron in the deuteron $S_n(d)$. This binding energy is determined by measuring the wavelength of the 2.2 MeV γ ray in the reaction $n+p\rightarrow d+\gamma$ in terms of the d_{220} lattice spacing of a particular silicon crystal, the latter corrected to the commonly used reference conditions $t_{90}=22.5\ ^\circ C$ and $p=0$, that is, in vacuum (see Sec. III.I). The result for the wavelength-to-lattice spacing ratio obtained from 141 Bragg-angle measurements carried out in 1995 and 1998 using a flat crystal spectrometer of the GAMS4 diffraction facility at the high-flux reactor of the Institut Max von Laue-Paul Langevin (ILL), Grenoble, France, in a NIST and ILL collaboration, is (Kessler *et al.*, 1999)

$$\frac{\lambda_{meas}}{d_{220}(ILL)} = 0.002\,904\,302\,46(50)\ [1.7\times10^{-7}], \qquad (13)$$

where $d_{220}(ILL)$ is the {220} lattice spacing of the silicon crystals of the ILL GAMS4 spectrometer at $t_{90}=22.5\ ^\circ C$ and $p=0$. As shown in CODATA-98, taking into account the relativistic kinematics of the reaction, one obtains the following observational equation for the measured quantity, or input datum, $\lambda_{meas}/d_{220}(ILL)$:

$$\frac{\lambda_{meas}}{d_{220}(ILL)} \doteq \frac{\alpha^2 A_r(e)}{R_\infty d_{220}(ILL)} \frac{A_r(n)+A_r(p)}{[A_r(n)+A_r(p)]^2 - A_r^2(d)}, \qquad (14)$$

where all seven quantities on the right-hand side are adjusted constants.

As part of their effort to determine $S_n(d)$, Kessler *et al.* (1999) also compared the {220} lattice spacings of the ILL silicon crystals to those of three other silicon crystals in order to link the ILL crystals to a crystal whose lattice spacing has been measured in meters via a combined x-ray and optical interferometer (see Sec. III.I.2), thereby providing a value of $d_{220}(ILL)$ in meters. The results of the comparisons, which are also taken as input data, are

$$\frac{d_{220}(ILL) - d_{220}(W17)}{d_{220}(ILL)} = -8(22)\times10^{-9}, \qquad (15)$$

$$\frac{d_{220}(ILL) - d_{220}(MO^*)}{d_{220}(ILL)} = 86(27)\times10^{-9}, \qquad (16)$$

$$\frac{d_{220}(\mathrm{ILL}) - d_{220}(\mathrm{NR3})}{d_{220}(\mathrm{ILL})} = 34(22) \times 10^{-9}. \quad (17)$$

The three crystals are labeled WASO 17, MO*, and NRLM3, where for simplicity in equations, WASO *n* is abbreviated as W*n* and NRLM3 as NR3. It should be noted that in CODATA-98, the crystal MO* of the Istituto di Metrologia "G. Colonnetti" (IMGC), Torino, Italy, was labeled MO*4; and the crystal NRLM3 of the National Research Laboratory of Metrology (NRLM), Tsukuba, Japan, was labeled SH1. The change in MO*4 is for reasons of consistency—in general, we identify a crystal sample only by the label of the boule from which it comes; and the change in SH1 is in recognition of the fact that the boule from which it comes is labeled NRLM3. As of 1 April 2001, NRLM was joined by other Japanese laboratories and renamed the National Metrology Institute of Japan (NMIJ). As a consequence, in keeping with our practice in other similar cases, all former NRLM measurements are referred to in this paper as NMIJ measurements. However, to minimize confusion, the crystal designation NRLM*n* is still used.

Results for lattice-spacing differences of various crystals similar to those obtained at NIST have been reported by researchers from the Physikalisch-Technische Bundesanstalt (PTB), Braunschweig, Germany, and are given in Sec. III.I. It should also be noted that to include the three fractional lattice-spacing differences in Eqs. (15)–(17) in the 2002 adjustment, all four of the {220} lattice spacings in those three equations—d_{220}(ILL), d_{220}(W17), d_{220}(MO*), and d_{220}(NR3)—are taken as adjusted constants. Finally, we note that the fractional differences given in Eqs. (15)–(17) are correlated—the correlation coefficient of any two is approximately 0.5.

B. Hydrogenic transition frequencies and the Rydberg constant R_∞

The Rydberg constant is related to other constants by

$$R_\infty = \alpha^2 \frac{m_e c}{2h}. \quad (18)$$

It can be determined to highest accuracy by comparing measured resonant frequencies of transitions in hydrogen (H) and deuterium (D) to the theoretical expressions for the energy-level differences.

In view of the continuing increase in accuracy of the experiments and calculations, it is of interest to revisit the question of the relationship between the center of the observed absorption resonance and the theoretical calculation of the energy-level difference under investigation. It was established by Low (1952) that the difference between the theoretically calculated separation of the energy levels, including radiative corrections, and the line center of the observed absorption resonance is very small. Recently Labzowsky *et al.* (2001) investigated nonresonant corrections that could introduce a shift of the observed line center of the 1S-2S transition in H and D relative to the theoretical energy level separation by several Hz. Such a shift could ultimately influence the determination of the Rydberg constant. Subsequent work by Jentschura and Mohr (2002) showed that the field-free shift is indeed orders of magnitude below 1 Hz, in accord with the original estimate of Low (1952). Other related effects have been investigated, but they are also negligible (Labzowsky *et al.*, 2002a, 2002b).

The 1998 CODATA recommended value for R_∞ was obtained from transition frequency measurements carried out by researchers from the following laboratories:

- Max Planck Institut für Quantenoptik (MPQ) in Garching, Germany;
- Laboratoire Kastler-Brossel (LKB), Ecole Normale Supérieure et Université Pierre et Marie Curie, Paris, France;
- Systèmes de réfèrence Temps Espace (SYRTE), Bureau National de Métrologie (BNM), Paris, France [Laboratoire Primaire du Temps et des Fréquences (LPTF) of BNM prior to 1 January 2001];
- Yale University, New Haven, Connecticut, USA;
- Harvard University, Cambridge, Massachusetts, USA;
- University of Sussex, Sussex, UK.

These measurements were reviewed in CODATA-98, hence the results that have not been superseded are not reviewed here. These unchanged results, together with the more recent values discussed below, are listed in Table V. In many cases, the measured transition frequencies from the same laboratory are correlated, and these correlations are taken into account in the 2002 adjustment as they were in the 1998 adjustment. The corresponding correlation coefficients are given in Table XII of Sec. IV.

1. Max Planck Institut für Quantenoptik

The value of the $1S_{1/2}-2S_{1/2}$ hydrogen transition frequency used in the 1998 adjustment was that reported by Udem *et al.* (1997):

$$\nu_H(1S_{1/2} - 2S_{1/2}) = 2\,466\,061\,413\,187.34(84)\ \mathrm{kHz} \quad [3.4 \times 10^{-13}]. \quad (19)$$

It was obtained by using longitudinal Doppler-free two-photon spectroscopy of a cold atomic beam; the required light at 243 nm was produced by doubling the frequency of an ultrastable 486 nm dye laser. The $1S(F=1) \rightarrow 2S(F=1)$ resonance frequency was compared to the frequency of a cesium atomic clock using a phase-coherent laser frequency chain.

Members of the MPQ group continued the effort to determine this critical transition frequency to higher accuracy, and Reichert *et al.* (2000) reported

$$\nu_H(1S_{1/2} - 2S_{1/2}) = 2\,466\,061\,413\,187.29(37)\ \mathrm{kHz} \quad [1.5 \times 10^{-13}]. \quad (20)$$

The reduction in uncertainty was achieved by employing

TABLE V. Summary of measured transition frequencies ν considered in the present work for the determination of the Rydberg constant R_∞ (H is hydrogen and D is deuterium).

Authors	Laboratory[a]	Frequency interval(s)	Reported value ν/kHz	Rel. stand. uncert. u_r
Niering *et al.* (2000)	MPQ	$\nu_H(1S_{1/2}-2S_{1/2})$	2 466 061 413 187.103(46)	1.9×10^{-14}
Weitz *et al.* (1995)	MPQ	$\nu_H(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 797 338(10)	2.1×10^{-6}
		$\nu_H(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 490 144(24)	3.7×10^{-6}
		$\nu_D(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	4 801 693(20)	4.2×10^{-6}
		$\nu_D(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	6 494 841(41)	6.3×10^{-6}
Huber *et al.* (1998)	MPQ	$\nu_D(1S_{1/2}-2S_{1/2})-\nu_H(1S_{1/2}-2S_{1/2})$	670 994 334.64(15)	2.2×10^{-10}
de Beauvoir *et al.* (1997)	LKB/SYRTE	$\nu_H(2S_{1/2}-8S_{1/2})$	770 649 350 012.0(8.6)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{3/2})$	770 649 504 450.0(8.3)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{5/2})$	770 649 561 584.2(6.4)	8.3×10^{-12}
		$\nu_D(2S_{1/2}-8S_{1/2})$	770 859 041 245.7(6.9)	8.9×10^{-12}
		$\nu_D(2S_{1/2}-8D_{3/2})$	770 859 195 701.8(6.3)	8.2×10^{-12}
		$\nu_D(2S_{1/2}-8D_{5/2})$	770 859 252 849.5(5.9)	7.7×10^{-12}
Schwob *et al.* (1999)	LKB/SYRTE	$\nu_H(2S_{1/2}-12D_{3/2})$	799 191 710 472.7(9.4)	1.2×10^{-11}
		$\nu_H(2S_{1/2}-12D_{5/2})$	799 191 727 403.7(7.0)	8.7×10^{-12}
		$\nu_D(2S_{1/2}-12D_{3/2})$	799 409 168 038.0(8.6)	1.1×10^{-11}
		$\nu_D(2S_{1/2}-12D_{5/2})$	799 409 184 966.8(6.8)	8.5×10^{-12}
Bourzeix *et al.* (1996)	LKB	$\nu_H(2S_{1/2}-6S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 197 604(21)	4.9×10^{-6}
		$\nu_H(2S_{1/2}-6D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 699 099(10)	2.2×10^{-6}
Berkeland *et al.* (1995)	Yale	$\nu_H(2S_{1/2}-4P_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 664 269(15)	3.2×10^{-6}
		$\nu_H(2S_{1/2}-4P_{3/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 035 373(10)	1.7×10^{-6}
Hagley and Pipkin (1994)	Harvard	$\nu_H(2S_{1/2}-2P_{3/2})$	9 911 200(12)	1.2×10^{-6}
Lundeen and Pipkin (1986)	Harvard	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 845.0(9.0)	8.5×10^{-6}
Newton *et al.* (1979)	U. Sussex	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 862(20)	1.9×10^{-5}

[a]MPQ: Max-Planck-Institut für Quantenoptik, Garching. LKB: Laboratoire Kastler-Brossel, Paris. SYRTE: Systèmes de référence Temps Espace, Paris, formerly Laboratoire Primaire du Temps et des Fréquences (LPTF).

a new technique in which a femtosecond pulse train of a mode-locked Ti:sapphire laser provides a wide comb of equally spaced frequencies that can be used to measure the interval between two widely separated optical frequencies (Udem *et al.*, 1999a, 1999b). The relative standard uncertainty of this result is dominated by two components: the 1.3×10^{-13} relative uncertainty of the ac Stark shift (Type B evaluation) and the 5.7×10^{-14} statistical relative uncertainty of the measurements (Type A evaluation), with the latter due mainly to the instability of the 10 MHz output frequency of the cesium clock used in the experiment. This reference frequency plays a crucial role, because it determines the repetition rate of the pulses and hence the spacing of the equidistant frequencies of the comb.

The uncertainty of the hydrogen $1S_{1/2}-2S_{1/2}$ transition frequency was subsequently reduced by nearly an order of magnitude when the commercial cesium clock employed in the previous experiment was replaced by a laser-cooled cesium atom fountain clock (Lemonde *et al.*, 2000; Santarelli *et al.*, 1999). This transportable clock of the BNM-SYRTE, which was compared to a second BNM-SYRTE cesium fountain clock before and after the measurements at Garching, provided a reference frequency with significantly improved stability and accuracy. The increased stability made it possible to carry out measurements in a reasonable integration time over a range of light powers in order to extrapolate the measured transition frequency to zero laser intensity. Based on this and other improvements, Niering *et al.* (2000) obtained the result

$$\nu_H(1S_{1/2}-2S_{1/2}) = 2\,466\,061\,413\,187.103(46)\ \text{kHz}\ [1.9\times10^{-14}], \quad (21)$$

which is the value used for this transition in the current adjustment and listed in Table V.

2. Laboratoire Kastler-Brossel and Systèmes de réfèrence Temps Espace

A comprehensive final report by the LKB/BNM-SYRTE collaboration describing their joint work and the earlier work by the LKB group has been published (de Beauvoir *et al.* 2000); the reported transition frequencies and uncertainties are the same as those used in the 1998 adjustment, with the following exception:

$$\nu_H(2S_{1/2} - 8S_{1/2}) = 770\,649\,350\,012.0(8.6)\ \text{kHz} \quad [1.1 \times 10^{-11}]. \tag{22}$$

This revised result, which is used in the current adjustment and is listed in Table V, differs by only 1 in the last figure, and so the change is inconsequential. We note that there is a minor change in two of the uncertainties listed in the final report. However, the original values quoted in CODATA-98 and given in Table V are the correct values (Biraben, 2003).

3. Other data

A number of other potentially relevant results have been reported, but are not included in either the 1998 adjustment or current adjustment for a variety of reasons.

A high-precision measurement of transition frequencies between circular Rydberg states of hydrogen with principle quantum number n in the range 27–30 has been carried out at the Massachusetts Institute of Technology (MIT) and described in the Ph.D. thesis of De Vries (2002). Such measurements lead to a value of the Rydberg constant R_∞ that is largely independent of the measurements made in lower levels of hydrogen and deuterium. Since the circular Rydberg states, that is, states with the largest angular momentum l and magnetic quantum number m for a given n, are far from the nucleus, they depend relatively weakly on nuclear size and QED effects, which means that the theoretical relation between the transition frequencies and R_∞ can be calculated relatively easily (Lutwak *et al.*, 1997).

The circular Rydberg states have a weak allowed radiative decay channel only to the next lower circular state, which has the experimental advantage that the states have long lifetimes and corresponding narrow widths. Another experimental advantage of such transitions is that the frequencies are of the order of 300 GHz, so that an extensive chain or a complex frequency comb to relate optical frequencies to the cesium clock frequency is not needed.

The current value of R_∞ derived from the MIT experiment is (De Vries, 2002)

$$R_\infty c = 3\,289\,841\,960\,306(69)\ \text{kHz} \quad [2.1 \times 10^{-11}], \tag{23}$$

which is consistent with the 2002 CODATA recommended value. Nevertheless, the MIT result is not included in the 2002 adjustment, because the final analysis of the experiment, including the effects of dipole interactions on the results, was not available at the time of the adjustment. However, such a paper is anticipated (De Vries, 2003; Kleppner, 2003).

Also not included is the result 1 057 852(15) kHz for the classic hydrogen Lamb shift obtained by van Wijngaarden *et al.* (1998) from the anisotropy of emitted photons in an applied electric field, because it is viewed as a test of the anisotropy method rather than an independent determination. An important concern is that the relationship between the observed anisotropy and the Lamb shift is based only on lowest-order calculations and the magnitude of higher-order corrections has not been elucidated (Hillery and Mohr, 1980). At the time of the 1998 adjustment there was serious disagreement between the theoretical value of the Lamb shift in He^+ and the experimental result obtained using the anisotropy method, which gave rise to further concern. Although a more recent measurement of the anisotropy in He^+ has led to a new value of the Lamb shift that reduces the discrepancy (van Wijngaarden *et al.*, 2000), the comparison with theory is still problematic (Pachucki, 2001). In any case, the question of higher-order corrections needs to be addressed, as was done for the 1S-2S resonance measurements in H and D.

Based on the discussion in CODATA-98, the result 1 057 851.4(1.9) kHz for the Lamb shift in hydrogen reported by Pal'chikov *et al.* (1985) is also omitted.

Earlier results in H and D are omitted for at least one of the following four reasons:

(i) The uncertainty of the result is large compared to the uncertainty of a more recent measurement of the same interval that employed more advanced technology.

(ii) The uncertainty of the result is large compared to the uncertainty of the value predicted by a least-squares adjustment that incorporates both modern measurements of other intervals and the relevant theory.

(iii) The result disagrees significantly with a more recent measurement of the same interval that employed more advanced technology.

(iv) The result disagrees significantly with the value predicted by a least-squares adjustment that incorporates both modern measurements of other intervals and the relevant theory.

If (i) or (ii) is true, then the datum would not provide useful information for the adjustment. On the other hand, if (iii) and (iv) are true, then the datum is assumed to be in error. [Note that for the earlier results, there happens to be no case where (iii) applies but (iv) does not. Summaries and discussion of such results are given by Taylor *et al.* (1969), Cohen and Taylor (1973), and Pipkin (1990). The result of Safinya *et al.* (1980) is corrected by Hagley and Pipkin (1994).]

4. Nuclear radii

The theoretical expressions for the finite nuclear size contributions to hydrogenic energy levels in Appendix A are given in terms of the bound-state nuclear rms charge radius R_N with $N \to p$ or $N \to d$ for H or D. The values of R_p and R_d that are used as input data in the 2002 adjustment are determined from elastic electron-nucleon–scattering experiments.

The world data on elastic electron-deuteron scattering, consisting of some 340 data points below 10 GeV/c momentum transfer, have been used by Sick and Trautmann (1998) in a thorough analysis that in-

cludes Coulomb distortion to determine the deuteron rms charge radius. As discussed in CODATA-98, this leads to

$$R_d = 2.130(10)\ \text{fm}, \tag{24}$$

which was considered in the preliminary analysis in the 1998 adjustment.

There is a long history of determinations of the proton radius based on the analysis of electron-scattering data. Recent work that takes into account Coulomb and recoil corrections has yielded (Sick, 2003)

$$R_p = 0.895(18)\ \text{fm}. \tag{25}$$

In 1998, the electron-scattering data for the proton and deuteron charge radii were not included in the final least-squares adjustment, because there were apparent inconsistencies between the value of the proton radius obtained from the scattering data and the value based solely on the spectroscopic data. However, in the current adjustment, due to a significant reduction in the uncertainty of the relevant bound-state theory as described in Appendix A, the proton and deuteron charge radii derived from the spectroscopic data are more precise and appear to be more reliable than in the 1998 adjustment. Therefore the values of the charge radii of the proton and deuteron obtained in the current adjustment, based on both the electron-scattering data and the spectroscopic data, are included in the 2002 CODATA set of recommended values of the constants.

C. Magnetic moments and *g*-factors

The magnetic moment of any of the three charged leptons (e, μ, τ) is written as

$$\boldsymbol{\mu} = g\frac{e}{2m}\boldsymbol{s}, \tag{26}$$

where g is the g-factor of the particle, m is its mass, and $\boldsymbol{s}$ is its spin. In Eq. (26), e is the elementary charge and is positive. For the negatively charged leptons (e^-, μ^-, and τ^-), g is negative, and for the corresponding antiparticles (e^+, μ^+, and τ^+), g is positive. *CPT* invariance implies that the masses and absolute values of the g-factors are the same for each particle-antiparticle pair.

These leptons have eigenvalues of spin projection $s_z = \pm\hbar/2$, and in the case of the electron and positron it is conventional to write, based on Eq. (26),

$$\mu_e = \frac{g_e}{2}\mu_B, \tag{27}$$

where $\mu_B = e\hbar/2m_e$ is the Bohr magneton.

For nucleons or nuclei with spin $\boldsymbol{I}$, the magnetic moment can be written as

$$\boldsymbol{\mu} = g\frac{e}{2m_p}\boldsymbol{I}, \tag{28}$$

or

$$\mu = g\mu_N i. \tag{29}$$

In Eq. (29), $\mu_N = e\hbar/2m_p$ is the nuclear magneton, defined in analogy with the Bohr magneton, and i is the spin quantum number of the nucleus defined by $\boldsymbol{I}^2 = i(i+1)\hbar^2$ and $I_z = -i\hbar, \ldots, (i-1)\hbar, i\hbar$, where I_z is the spin projection. However, in some publications moments of nucleons are expressed in terms of the Bohr magneton with a corresponding change in the definition of the g-factor.

Magnetic moments, magnetic moment ratios, and g-factors of various particles which impact the determination of other constants of interest are discussed in the following sections, and the relevant data are summarized in Table VI. (The shielded gyromagnetic ratios of some of the same particles are discussed in Sec. III.D.) Also given in Table VI are values of quantities of interest that may be inferred from the data, as discussed in connection with each experiment. Each inferred value is indented for clarity and is given only for comparison purposes. In practice, the source data and not the inferred values are used as input data for the 2002 adjustment.

1. Electron magnetic moment anomaly a_e

The electron magnetic moment anomaly a_e is defined as

$$a_e = \frac{|g_e| - 2}{2} = \frac{|\mu_e|}{\mu_B} - 1, \tag{30}$$

where $g_e = 2\mu_e/\mu_B$ is the g-factor of the electron and μ_e is its magnetic moment.

a. University of Washington

The measurements of the electron and positron anomalies carried out at the University of Washington by Van Dyck *et al.* (1987) yield the value

$$a_e = 1.159\,652\,1883(42) \times 10^{-3} \quad [3.7 \times 10^{-9}], \tag{31}$$

based on the analysis described in CODATA-98. This analysis includes the assumption that *CPT* invariance holds for the electron-positron system.

b. Theory

As discussed in CODATA-98, a value of the fine-structure constant α can be obtained from the University of Washington weighted-mean experimental value of a_e, given in Eq. (31), by determining the value $\alpha(a_e)$ for which $a_e(\text{exp}) = a_e(\text{th})$, where $a_e(\text{th})$ is the theoretical expression for a_e as a function of α. The theory of a_e is briefly summarized in Appendix B. We have

$$a_e(\text{th}) = a_e(\text{QED}) + a_e(\text{weak}) + a_e(\text{had}), \tag{32}$$

with

TABLE VI. Summary of data related to magnetic moments of various particles, and inferred values of various quantities.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
a_e	$1.159\,652\,1883(42)\times10^{-3}$	3.7×10^{-9}	UWash-87	III.C.1.a (31)
$\alpha^{-1}(a_e)$	137.035 998 80(52)	3.8×10^{-9}		III.C.1.b (35)
$\overline{R}$	0.003 707 2048(25)	6.7×10^{-7}	BNL-02	III.C.2.a (38)
a_μ	$1.165\,920\,33(79)\times10^{-3}$	6.8×10^{-7}		III.C.2.a (39)
α^{-1}	137.035 81(15)	1.1×10^{-6}		III.C.2.b (45)
$\mu_{e^-}(H)/\mu_p(H)$	−658.210 7058(66)	1.0×10^{-8}	MIT-72	III.C.4.a (56)
μ_{e^-}/μ_p	−658.210 6860(66)	1.0×10^{-8}		III.C.4.a (57)
$\mu_d(D)/\mu_{e^-}(D)$	$-4.664\,345\,392(50)\times10^{-4}$	1.1×10^{-8}	MIT-84	III.C.4.b (58)
μ_d/μ_{e^-}	$-4.664\,345\,548(50)\times10^{-4}$	1.1×10^{-8}		III.C.4.b (59)
$\mu_{e^-}(H)/\mu'_p$	−658.215 9430(72)	1.1×10^{-8}	MIT-77	III.C.4.c (60)
μ_{e^-}/μ'_p	−658.227 5971(72)	1.1×10^{-8}		III.C.4.c (61)
μ'_h/μ'_p	−0.761 786 1313(33)	4.3×10^{-9}	NPL-93	III.C.4.d (62)
μ_n/μ'_p	−0.684 996 94(16)	2.4×10^{-7}	ILL-79	III.C.4.e (65)
$\Delta\nu_{Mu}$	4 463 302.88(16) kHz	3.6×10^{-8}	LAMPF-82	III.C.5.a (70)
$\nu(f_p)$	627 994.77(14) kHz	2.2×10^{-7}	LAMPF-82	III.C.5.a (71)
μ_{μ^+}/μ_p	3.183 3461(11)	3.6×10^{-7}		III.C.5.a (73)
m_μ/m_e	206.768 220(74)	3.6×10^{-7}		III.C.5.a (74)
α^{-1}	137.036 019(24)	1.8×10^{-7}		III.C.5.a (75)
$\Delta\nu_{Mu}$	4 463 302 765(53) Hz	1.2×10^{-8}	LAMPF-99	III.C.5.b (76)
$\nu(f_p)$	668 223 166(57) Hz	8.6×10^{-8}	LAMPF-99	III.C.5.b (77)
μ_{μ^+}/μ_p	3.183 345 14(39)	1.2×10^{-7}		III.C.5.b (79)
m_μ/m_e	206.768 283(25)	1.2×10^{-7}		III.C.5.b (80)
α^{-1}	137.035 9997(84)	6.1×10^{-8}		III.C.5.b (81)

$$a_e(\mathrm{QED}) = C_e^{(2)}\left(\frac{\alpha}{\pi}\right) + C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_e^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots, \tag{33}$$

where the coefficients $C_e^{(2n)}$, as well as a_e(weak) and a_e(had), are given in that appendix. As also indicated there, the standard uncertainty of a_e(th) is

$$u[a_e(\mathrm{th})] = 1.15\times10^{-12} = 0.99\times10^{-9}a_e \tag{34}$$

and is mainly due to the uncertainty of the coefficients $C_e^{(8)}$ and $C_e^{(10)}$.

Equating the theoretical expression with a_e(exp) given in Eq. (31) yields

$$\alpha^{-1}(a_e) = 137.035\,998\,80(52) \quad [3.8\times10^{-9}], \tag{35}$$

which is the value included in Table VI. The uncertainty of a_e(th) is significantly smaller than the uncertainty of a_e(exp), and thus the uncertainty of this inferred value of α is determined mainly by the uncertainty of a_e(exp). This result has the smallest uncertainty of any value of alpha currently available. Nevertheless, it should be noted that the value of α in CODATA-98 inferred from the University of Washington result in Eq. (31) is $\alpha^{-1}(a_e) = 137.035\,999\,58(52)$ [3.8×10^{-9}], which exceeds the value in Eq. (35) by the fractional amount 5.7×10^{-9}. The reason for this change is the revision of a_e(th), as explained in Appendix B.

2. Muon magnetic moment anomaly a_μ

In a manner similar to that for the electron [see Eq. (30)], the muon magnetic moment anomaly a_μ is defined as

$$a_\mu = \frac{|g_\mu| - 2}{2} = \frac{|\mu_\mu|}{e\hbar/2m_\mu} - 1, \tag{36}$$

where as usual $g_\mu = 2\mu_\mu/(e\hbar/2m_\mu)$ is the g-factor of the muon and μ_μ is its magnetic moment. The basic principle of the experimental determination of a_μ is similar to that used to determine a_e and involves measuring the anomaly difference frequency $f_a = f_s - f_c$, where $f_s = |g_\mu|(e\hbar/2m_\mu)B/h$ is the muon spin-flip (often called pre-

cession) frequency in the applied magnetic flux density B and where $f_c = eB/2\pi m_\mu$ is the corresponding muon cyclotron frequency. However, instead of eliminating B by measuring f_c as is done for the electron, B is determined from proton nuclear-magnetic-resonance (NMR) measurements. As a consequence, a value of μ_μ/μ_p is required to deduce the value of a_μ from the data. The relevant equation is

$$a_\mu = \frac{\overline{R}}{|\mu_\mu/\mu_p| - \overline{R}}, \tag{37}$$

where $\overline{R} = f_a/\bar{f}_p$, and $\bar{f}_p$ is the free-proton NMR frequency corresponding to the average flux density seen by the muons in their orbits in the muon storage ring used in the experiment. (In the corresponding experiment for the electron, a Penning trap is employed rather than a storage ring.)

a. Brookhaven

In the 1998 adjustment, two independent experimental values of $\overline{R}$ were considered: $\overline{R} = 0.003\,707\,213(27)$ $[7.2\times10^{-6}]$ reported by Bailey *et al.* (1979) and obtained from the third CERN (European Laboratory for Particle Physics, Geneva, Switzerland) $g-2$ experiment; and $\overline{R} = 0.003\,707\,220(48)$ $[13\times10^{-6}]$ reported by Carey *et al.* (1999) and obtained from the BNL (Brookhaven National Laboratory, Upton, New York, USA) initial 1997 engineering run of an entirely new experiment. The CERN value is based on μ^+ as well as μ^- data, while the BNL result is based on only μ^+ data. The initial BNL run also employed, as did all the CERN runs, pion injection rather than muon injection into the muon storage ring. [For brief descriptions of the CERN and BNL experiments, see CODATA-98. A quite extensive description of the BNL effort may be found in the review by Hughes and Sichtermann (2003). Discussions of some key components and techniques of this ambitious experiment may also be found in the reports of Fei *et al.* (1997); Dhawan *et al.* (2000); Sedykh *et al.* (2000); Danby *et al.* (2001); Redin *et al.* (2001); Orlov *et al.* (2002); Yamamoto *et al.* (2002); Efstathiadis *et al.* (2003); Semertzidis *et al.* (2003).]

At the time of the 1998 adjustment, the value of the muon anomaly implied by the theoretical expression for a_μ was assigned a relative standard uncertainty $u_r = 5.5\times10^{-7}$, more than an order of magnitude smaller than the uncertainty of the value of a_μ that could be deduced from either the CERN or BNL result for $\overline{R}$. Thus neither value of $\overline{R}$ was included in the final adjustment used to determine the 1998 recommended values of the constants; the recommended value was based entirely on theory.

The last four years have seen significant changes in both the theoretical and experimental status of a_μ. First, as discussed in Appendix C, the uncertainty we have now assigned to the theoretical expression for a_μ is $u_r = 8.5\times10^{-7}$, which is to be compared to the value $u_r = 5.5\times10^{-7}$ assigned in 1998. Second, as discussed in the following paragraphs, the BNL result for $\overline{R}$ that we use as an input datum in the 2002 adjustment has an uncertainty $u_r = 6.7\times10^{-7}$. As a consequence, because this uncertainty is over a factor of 10 smaller than the uncertainty $u_r = 7.2\times10^{-6}$ of the CERN result for $\overline{R}$, the latter is not considered further. (We note, nonetheless, that the CERN and BNL results agree.)

In the 1997 BNL measurements, only about 25×10^{-6} of the daughter muons from pion decay of the pions injected into the muon storage ring were actually stored in the ring. However, in August 1998 a fast muon "kicker" was commissioned at BNL which allowed the direct injection of muons into the ring. Other important improvements in the BNL experiment were also implemented, including better stability and homogeneity of the storage-ring magnetic flux density B. The result of the 1998 run, again using μ^+ data, is (Brown *et al.*, 2000) $\overline{R} = 0.003\,707\,201(19)$ $[5.0\times10^{-6}]$, a significant improvement over the 1997 result. Data continued to be collected in 1999, 2000, and 2001, with the 1999 and 2000 data being obtained with μ^+ and the 2001 data with μ^-. Some experimental improvements were incorporated in the 1999 run, but the principal new feature of the 1999 data was a factor of 20 increase in the data collected. On the other hand, a higher count rate and much larger data set required careful consideration of a number of additional factors in the data analysis. The final result obtained from the 1999 run is (Brown *et al.*, 2001) $\overline{R} = 0.003\,707\,2043(48)$ $[1.3\times10^{-6}]$, and represents nearly a factor of 4 reduction in uncertainty compared to the result from the 1998 run.

The experimental improvements in the 2000 run included operating the BNL alternating gradient synchrotron (AGS) with 12 beam bunches rather than six, contributing to an increase in the data collected by about a factor of 4 compared to the 1999 run; a sweeper magnet in the AGS beam line to reduce background; added muon loss detectors to improve the determination of muon losses with time; and a new superconducting inflector magnet which improved the homogeneity of the magnetic flux density in the muon storage region. The value of $\overline{R}$ from the 2000 run, together with the results from the 1998 and 1999 runs, is given as (Bennett *et al.*, 2002; Sichtermann, 2003)

$$\overline{R} = 0.003\,707\,2048(25) \quad [6.7\times10^{-7}], \tag{38}$$

which we take as the BNL input datum for $\overline{R}$ in the 2002 adjustment. [The BNL a_μ collaboration does not include the result from the 1997 engineering run in its averages (Sichtermann, 2004). The result from the 2001 run, which as noted above used μ^- data, did not become available until January 2004, and hence could not be included in the 2002 adjustment. It is reported to be (Bennett *et al.*, 2004) $\overline{R}(\mu^-) = 0.003\,707\,2083(26)$ $[7.0\times10^{-7}]$. The combined value of $\overline{R}$ from the 1998, 1999, 2000, and 2001 runs, assuming *CPT* invariance so that $\overline{R}(\mu^+) = \overline{R}(\mu^-)$, is given as (Bennett *et al.*, 2004) $\overline{R} = 0.003\,707\,2063(20)$ $[5.3\times10^{-7}]$.]

There is a possible question of the effect of a muon electric dipole moment (edm) on the interpretation of the data resulting from experiments to measure a_μ (Feng *et al.*, 2003). However, it can be argued that a muon edm of sufficient size to have an observable impact would imply, if simple mass scaling is assumed, an electron edm larger than current experimental limits (Roberts, 2003).

Based on Eq. (37), the BNL value of $\overline{R}$ in Eq. (38) together with the 2002 recommended value of μ_μ/μ_p, the uncertainty of which is inconsequential in this application, implies

$$a_\mu = 1.165\,920\,33(79) \times 10^{-3} \quad [6.8 \times 10^{-7}]. \tag{39}$$

We also note that with the aid of Eq. (68) in Sec. III.C.5, Eq. (37) can be written as

$$\overline{R} = -\frac{a_\mu(\alpha,\delta_\mu)}{1+a_e(\alpha,\delta_e)}\frac{m_e}{m_\mu}\frac{\mu_{e^-}}{\mu_p}, \tag{40}$$

where we have used the relations $g_e=-2(1+a_e)$ and $g_\mu=-2(1+a_\mu)$ and replaced a_e and a_μ with their complete theoretical expressions $a_e(\alpha,\delta_e)$ and $a_\mu(\alpha,\delta_\mu)$, which are discussed in Appendix B and Appendix C, respectively. Equation (40) is, in fact, the observational equation for the input datum $\overline{R}$.

b. Theory

As just indicated, the theory of a_μ is reviewed in Appendix C. We have

$$a_\mu(\text{th}) = a_\mu(\text{QED}) + a_\mu(\text{weak}) + a_\mu(\text{had}), \tag{41}$$

with

$$a_\mu(\text{QED}) = C_\mu^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\mu^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\mu^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\mu^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\mu^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots, \tag{42}$$

where the coefficients $C_\mu^{(2n)}$, as well as a_μ(weak) and a_μ(had), are given in that appendix.

As also indicated there, the standard uncertainty of a_μ(th) is

$$u[a_\mu(\text{th})] = 100 \times 10^{-11} = 85 \times 10^{-8} a_\mu, \tag{43}$$

and is primarily due to the uncertainty of a_μ(had). Evaluation of the theoretical expression for a_μ using the 2002 recommended value of α, the uncertainty of which is negligible in this context, yields

$$a_\mu = 1.165\,9188(10) \times 10^{-3} \quad [8.5 \times 10^{-7}], \tag{44}$$

which may be compared to the value in Eq. (39) deduced from the BNL result for $\overline{R}$ given in Eq. (38). We see that the experimental value exceeds the theoretical value by $1.2u_{\text{diff}}$, where u_{diff} is the standard uncertainty of the difference. The agreement is not unreasonable.

The consistency between theory and experiment may also be examined by considering the value of α obtained by equating the theoretical expression for a_μ with the BNL experimental value, as was done for a_e in the previous section. The result is

$$\alpha^{-1} = 137.035\,81(15) \quad [1.1 \times 10^{-6}], \tag{45}$$

which is the value included in Table VI.

3. Electron *g*-factor in hydrogenic ions $g_e(X)$

For a ground-state hydrogenic ion ${}^A X^{(Z-1)+}$ with mass number A, atomic number (proton number) Z, nuclear-spin quantum number $i=0$, and g-factor $g_{e^-}({}^A\text{X}^{(Z-1)+})$ in an applied magnetic flux density B, the ratio of the electron's spin-flip (often called precession) frequency $f_s = |g_{e^-}({}^A\text{X}^{(Z-1)+})|(e\hbar/2m_e)B/h$ to the cyclotron frequency of the ion $f_c=(Z-1)eB/2\pi m({}^A\text{X}^{(Z-1)+})$ in the same magnetic flux density is

$$\frac{f_s\left({}^A X^{(Z-1)+}\right)}{f_c\left({}^A X^{(Z-1)+}\right)} = -\frac{g_{e^-}\left({}^A X^{(Z-1)+}\right)}{2(Z-1)}\frac{A_r\left({}^A X^{(Z-1)+}\right)}{A_r(e)}, \tag{46}$$

where as usual, A_r(X) is the relative atomic mass of particle X. If the frequency ratio f_s/f_c is determined experimentally with high accuracy, and $A_r({}^A\text{X}^{(Z-1)+})$ of the ion is also accurately known, then this expression can be used to determine an accurate value of A_r(e), assuming the bound-state electron g-factor can be calculated from QED theory with sufficient accuracy; or the g-factor can be determined if A_r(e) is accurately known from another experiment. In fact, a broad program involving workers from a number of European laboratories has been underway since about the mid-1990s to measure the frequency ratio and calculate the g-factor for different ions, most notably (to date) $^{12}C^{5+}$ and $^{16}O^{7+}$. The measurements themselves are being performed at the GSI (Gesellschaft für Schwerionenforschung, Darmstadt, Germany) by GSI and University of Mainz researchers, and we discuss the experimental determinations of f_s/f_c for $^{12}C^{5+}$ and $^{16}O^{7+}$ at GSI in the following sections. The theoretical expressions for the bound-state electron g-factors of these two ions are reviewed in Appendix D.

a. $g_e(^{12}C^{5+})$

Many papers documenting the progress of the GSI-Mainz collaboration have been published over the last decade. In this section we focus on the experiment as it currently exists and the measurement of the ratio $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$. An in-depth review of the double Penning trap technique that allows the frequency ratios f_s/f_c to be determined with relative standard uncertainties $u_r \approx 6\times10^{-10}$ and its application to $^{12}C^{5+}$ is given by Häffner *et al.* (2003) (see also Werth *et al.*, 2001, 2002).

In brief, the experiment employs two Penning traps of identical geometry. Together they consist of 13 coaxial cylindrical electrodes of inner diameter 7 mm with axis in the vertical or z direction. The two traps are separated by 1 cm and are situated in a magnetic flux density $B=3.8$ T, also in the z direction, generated by a superconducting magnet. The ambient temperature of the

traps is 4 K. The ring electrode in the upper trap is made of nickel in order to introduce an inhomogeneity in B in the trap. Because this allows the trap to be used to analyze the spin direction of the electron in the ion, it is called the "analysis trap." The ring electrode of the lower trap (as well as all other electrodes of both traps) is made of pure copper, hence this trap sees a homogeneous B and is called the "precision trap."

After the preparation of a single $^{12}C^{5+}$ ion in the analysis trap, the cooling of its natural modes to the trap's ambient temperature, and the optimization of the trapping potentials of both traps, the measurement cycle begins by determining the direction of the electron's spin. This is done by inducing a spin flip via an applied microwave field of about 104 MHz and observing the corresponding 0.7 Hz change in the 364 kHz frequency of the ion's axial motion in the trap. The $^{12}C^{5+}$ ion is then adiabatically transferred a distance of 3 cm in less than 1 s to the precision trap, and a spin flip is attempted to be induced via an applied microwave field with a frequency near f_s. Simultaneous with the attempt to induce a spin flip, the cyclotron frequency of the ion is measured. Finally, the ion is transferred back to the analysis trap, where the spin direction of the electron is again detected in order to determine if in fact a spin flip took place while the ion was in the precision trap. Because the spin-flip frequency and cyclotron frequency are measured at the same time, fluctuations in B cancel to first order.

The desired ratio $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$ is obtained by carrying out many measurements as just described for a given value of the frequency of the microwave radiation applied in the precision trap, varying the frequency around $f_s(^{12}C^{5+})$, and plotting the probability that a spin flip will occur vs the ratio of the frequency of the microwave radiation to the measured cyclotron frequency. These latter ratios are first corrected for the cyclotron energy at which they were obtained, based on an extrapolation procedure designed to eliminate their dependence on cyclotron energy. The probability that a spin flip will occur at a given microwave frequency is simply the number of times it was observed to occur divided by the number of times it was attempted. The frequency ratio at the center of the fitted resonance curve of probability vs frequency ratio is then corrected for a number of relatively small effects to obtain the final result. As a fraction of the ratio, these are -3×10^{-10} for extrapolation to zero axial energy, 7×10^{-11} for the zero point of the cyclotron energy, and -7×10^{-11} for ground loops (Werth, 2003). The value reported is (Beier *et al.*, 2002; Häffner *et al.*, 2003; Werth, 2003)

$$\frac{f_s(^{12}C^{5+})}{f_c(^{12}C^{5+})} = 4376.210\,4989(23) \quad [5.2\times10^{-10}], \tag{47}$$

which we take as an input datum in the 2002 adjustment.

The search for, and investigation of, possible systematic effects in the experiment was quite thorough; the total relative standard uncertainty from such effects (Type B evaluation) is about 3×10^{-10}, with the three largest components arising from determination of the cyclotron energy (1.8×10^{-10}), extrapolation to zero microwave power (1.8×10^{-10}), and spectrum analyzer time base (0.9×10^{-10}). These should be compared to the statistical component of relative standard uncertainty (Type A) of 4.3×10^{-10}.

From Eq. (46) and Eq. (8), we have

$$\frac{f_s(^{12}C^{5+})}{f_c(^{12}C^{5+})} = -\frac{g_{e^-}(^{12}C^{5+})}{10A_r(e)}\times\left[12-5A_r(e) + \frac{E_b(^{12}C)-E_b(^{12}C^{5+})}{m_u c^2}\right], \tag{48}$$

which is the basis for the observational equation for the $^{12}C^{5+}$ frequency-ratio input datum.

Evaluation of this expression using the result for $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$ in Eq. (47), the theoretical result for $g_{e^-}(^{12}C^{5+})$ in Table XXXIX in Appendix D, and the relevant binding energies in Table IV, yields

$$A_r(e) = 0.000\,548\,579\,909\,31(29) \quad [5.3\times10^{-10}], \tag{49}$$

a result that is consistent with the University of Washington result in Eq. (10) but has about a factor of 4 smaller uncertainty.

b. $g_e(^{16}O^{7+})$

The GSI measurement of the frequency ratio $f_s(^{16}O^{7+})/f_c(^{16}O^{7+})$ is very similar to the GSI measurement of the frequency ratio $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$ just described. In the case of $^{16}O^{7+}$, the shift in the 369 kHz axial frequency of the ion in the analysis trap for a spin flip is 0.47 Hz. The reported result is (Verdú *et al.*, 2002, 2003; Werth, 2003)

$$\frac{f_s(^{16}O^{7+})}{f_c(^{16}O^{7+})} = 4164.376\,1836(31) \quad [7.5\times10^{-10}], \tag{50}$$

which is also taken as an input datum in the current adjustment.

The statistical component of relative standard uncertainty (Type A) of the oxygen frequency ratio is 1.6 times as large as the similar component for carbon (7.4×10^{-10} compared to 4.3×10^{-10}), while the relative standard uncertainty component due to systematic effects (Type B) is 2.8 times *smaller* than the corresponding carbon component (1.1×10^{-10} compared to 2.9×10^{-10}). The two largest fractional corrections applied to the oxygen frequency ratio are -4.8×10^{-10} for extrapolation to zero axial energy and -2.4×10^{-10} for extrapolation to zero microwave amplitude.

In analogy with what was done above with the ratio $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$, from Eq. (46) and Eq. (8), we have

$$\frac{f_s(^{16}O^{7+})}{f_c(^{16}O^{7+})} = -\frac{g_{e^-}(^{16}O^{7+})}{14A_r(e)}A_r(^{16}O^{7+}) \tag{51}$$

with

$$A_r(^{16}O) = A_r(^{16}O^{7+}) + 7A_r(e) - \frac{E_b(^{16}O) - E_b(^{16}O^{7+})}{m_u c^2}, \quad (52)$$

which are the basis for the observational equations for the oxygen frequency ratio and $A_r(^{16}O)$, respectively. The first expression, evaluated using the result for $f_s(^{16}O^{7+})/f_c(^{16}O^{7+})$ in Eq. (50) and the theoretical result for $g_{e^-}(^{16}O^{7+})$ in Table XL in Appendix D, in combination with the second expression, evaluated using the value of $A_r(^{16}O)$ in Table III and the relevant binding energies in Table IV, yields

$$A_r(e) = 0.000\,548\,579\,909\,57(43) \quad [7.8 \times 10^{-10}], \quad (53)$$

a value that is consistent with both the University of Washington value in Eq. (10) and the value in Eq. (49) obtained from $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$.

c. Relations between $g_e(^{12}C^{5+})$ and $g_e(^{16}O^{7+})$

It should be noted that the GSI frequency ratios for $^{12}C^{5+}$ and $^{16}O^{7+}$ are correlated. Based on the detailed uncertainty budget of the two results (Werth, 2003), we find

$$r\left[\frac{f_s(^{12}C^{5+})}{f_c(^{12}C^{5+})}, \frac{f_s(^{16}O^{7+})}{f_c(^{16}O^{7+})}\right] = 0.035. \quad (54)$$

Finally, as a consistency test, it is of interest to compare the experimental and theoretical values of the ratio of $g_{e^-}(^{12}C^{5+})$ to $g_{e^-}(^{16}O^{7+})$ (Karshenboim and Ivanov, 2002). The main reason is that the experimental value of the ratio is only weakly dependent on the value of $A_r(e)$. The theoretical value of the ratio, $g_{e^-}(^{12}C^{5+})/g_{e^-}(^{16}O^{7+}) = 1.000\,497\,273\,23(13)$ $[1.3 \times 10^{-10}]$, is given in Eq. (D34) of Appendix D and takes into account the covariance of the two theoretical values. The experimental value of the ratio can be obtained by combining Eqs. (47), (48), (50)–(52), and (54) and using the 2002 recommended value for $A_r(e)$. [Because of the weak dependence of the experimental ratio on $A_r(e)$, the value used is not at all critical.] The result is

$$\frac{g_{e^-}(^{12}C^{5+})}{g_{e^-}(^{16}O^{7+})} = 1.000\,497\,273\,70(90) \quad [9.0 \times 10^{-10}], \quad (55)$$

in agreement with the theoretical value.

4. Magnetic moment ratios

A number of magnetic moment ratios are of interest for the 2002 adjustment. They have been discussed in detail in CODATA-98, and no competitive new datum has become available by the 31 December 2002 closing date of the 2002 adjustment. The results of measurements and the inferred values of various quantities are summarized in the following paragraphs and in Table VI. The inferred moment ratios depend on the relevant theoretical binding corrections that relate the g-factor measured in the bound state to the corresponding free-particle g-factor. These ratios of bound to free g-factors are discussed in Appendix D.

a. Electron to proton magnetic moment ratio μ_e/μ_p

The ratio μ_e/μ_p is obtained from measurements of the ratio of the magnetic moment of the electron to the magnetic moment of the proton in the 1S state of hydrogen $\mu_{e^-}(H)/\mu_p(H)$. We use the value obtained by Winkler *et al.* (1972) at MIT:

$$\frac{\mu_{e^-}(H)}{\mu_p(H)} = -658.210\,7058(66) \quad [1.0 \times 10^{-8}], \quad (56)$$

where a minor typographical error in the original publication has been corrected (Kleppner, 1997). The free-particle ratio μ_e/μ_p follows from the bound-particle ratio and the relation

$$\frac{\mu_{e^-}}{\mu_p} = \frac{g_p(H)}{g_p}\left(\frac{g_{e^-}(H)}{g_{e^-}}\right)^{-1}\frac{\mu_{e^-}(H)}{\mu_p(H)} = -658.210\,6860(66) \quad [1.0 \times 10^{-8}], \quad (57)$$

where the bound-state g-factor ratios (and all others needed in this section) are given in Table XLI in Appendix D. The stated standard uncertainty is due entirely to the uncertainty of the experimental value of $\mu_{e^-}(H)/\mu_p(H)$, because the bound-state corrections are taken as exact.

b. Deuteron to electron magnetic moment ratio μ_d/μ_e

From measurements of the ratio $\mu_d(D)/\mu_{e^-}(D)$ in the 1S state of deuterium, Phillips *et al.* (1984) at MIT obtained

$$\frac{\mu_d(D)}{\mu_{e^-}(D)} = -4.664\,345\,392(50) \times 10^{-4} \quad [1.1 \times 10^{-8}]. \quad (58)$$

Although this result has not been published, as in CODATA-98, we include it as an input datum, because the method is described in detail by Winkler *et al.* (1972) in connection with their measurement of $\mu_{e^-}(H)/\mu_p(H)$. The free-particle ratio is given by

$$\frac{\mu_d}{\mu_{e^-}} = \frac{g_{e^-}(D)}{g_{e^-}}\left(\frac{g_d(D)}{g_d}\right)^{-1}\frac{\mu_d(D)}{\mu_{e^-}(D)} = -4.664\,345\,548(50) \times 10^{-4} \quad [1.1 \times 10^{-8}]. \quad (59)$$

c. Electron to shielded proton magnetic moment ratio μ_e/μ_p'

Following CODATA-98, based on the measurement of the ratio of the electron moment in the 1S state of hydrogen to the shielded proton moment at 34.7 °C by Phillips *et al.* (1977) at MIT, and temperature-dependence measurements of the shielded proton moment by Petley and Donaldson (1984) at NPL, we have

$$\frac{\mu_{e^-}(\mathrm{H})}{\mu'_p} = -658.215\,9430(72) \quad [1.1\times10^{-8}], \tag{60}$$

where the prime indicates that the protons are in a spherical sample of pure H_2O at 25 °C surrounded by vacuum. Hence

$$\frac{\mu_{e^-}}{\mu'_p} = \left(\frac{g_{e^-}(\mathrm{H})}{g_{e^-}}\right)^{-1}\frac{\mu_{e^-}(\mathrm{H})}{\mu'_p} = -658.227\,5971(72) \quad [1.1\times10^{-8}]. \tag{61}$$

d. Shielded helion to shielded proton magnetic moment ratio μ'_h/μ'_p

The ratio of the magnetic moment of the helion h, the nucleus of the ^{3}He atom, to the magnetic moment of the proton in H_2O was determined in a high-accuracy nuclear-magnetic-resonance (NMR) experiment by Flowers *et al.* (1993) at NPL with the result

$$\frac{\mu'_h}{\mu'_p} = -0.761\,786\,1313(33) \quad [4.3\times10^{-9}]. \tag{62}$$

(The prime on the symbol for the helion moment indicates that the helion is not free, but is bound in a helium atom. Although the exact shape and temperature of the gaseous ^{3}He sample is unimportant, we assume that it is spherical, at 25 °C, and surrounded by vacuum.)

In view of the importance of the value of the ratio $\mu_{e^-}(\mathrm{H})/\mu'_p$ in this and the 1998 adjustments, it is of interest to obtain an independent estimate of its value from the result in Eq. (62) and the preliminary result

$$\frac{\mu_{e^-}(\mathrm{H})}{\mu'_h} = 864.042\,9614(48) \quad [5.5\times10^{-9}] \tag{63}$$

provided by Flowers (2002), based on an NMR measurement in helium-3 and an electron-spin-resonance measurement in atomic hydrogen, as described by Flowers *et al.* (1997); Flowers *et al.* (1999); Flowers *et al.* (2002). The quoted uncertainty is statistical (Type A) only, but it is expected that uncertainties from systematic effects (Type B) would not add to it significantly. The estimate is

$$\frac{\mu_{e^-}(\mathrm{H})}{\mu'_p} = \frac{\mu_{e^-}(\mathrm{H})}{\mu'_h}\frac{\mu'_h}{\mu'_p} = -658.215\,9449(46) \quad [7.0\times10^{-9}]. \tag{64}$$

This result is, in fact, consistent with the value in Eq. (60).

e. Neutron to shielded proton magnetic moment ratio μ_n/μ'_p

Based on the measurement of Greene *et al.* (1977, 1979) carried out at ILL, as reviewed in CODATA-98, we have

$$\frac{\mu_n}{\mu'_p} = -0.684\,996\,94(16) \quad [2.4\times10^{-7}]. \tag{65}$$

5. Muon-proton magnetic moment ratio μ_μ/μ_p and muon-electron mass ratio m_μ/m_e from muonium

Measurements of the frequencies of transitions between Zeeman energy levels in muonium (μ^+e^- atom) can yield a value of μ_μ/μ_p and a value of the muonium ground-state hyperfine splitting $\Delta\nu_{Mu}$ that are only weakly dependent on theoretical input. The relevant expression for the magnetic moment ratio is

$$\frac{\mu_{\mu^+}}{\mu_p} = \frac{\Delta\nu_{Mu}^2 - \nu^2(f_p) + 2s_e f_p \nu(f_p)}{4s_e f_p^2 - 2f_p\nu(f_p)}\left(\frac{g_{\mu^+}(\mathrm{Mu})}{g_{\mu^+}}\right)^{-1}, \tag{66}$$

where $\Delta\nu_{Mu}$ and $\nu(f_p)$ are the sum and difference of two measured transition frequencies, f_p is the free-proton NMR reference frequency corresponding to the magnetic flux density used in the experiment, $g_{\mu^+}(\mathrm{Mu})/g_{\mu^+}$ is the bound-state correction for the muon in muonium given in Table XLI of Appendix D, and

$$s_e = \frac{\mu_{e^-}}{\mu_p}\frac{g_{e^-}(\mathrm{Mu})}{g_{e^-}}, \tag{67}$$

where $g_{e^-}(\mathrm{Mu})/g_{e^-}$ is the bound-state correction for the electron in muonium given in the same table.

The muon-to-electron mass ratio m_μ/m_e and the muon-to-proton magnetic moment ratio μ_μ/μ_p are related by

$$\frac{m_\mu}{m_e} = \left(\frac{\mu_e}{\mu_p}\right)\left(\frac{\mu_\mu}{\mu_p}\right)^{-1}\left(\frac{g_\mu}{g_e}\right). \tag{68}$$

The theoretical expression for the hyperfine splitting $\Delta\nu_{Mu}(\mathrm{th})$ is discussed in Appendix E and may be written as

$$\Delta\nu_{Mu}(\mathrm{th}) = \frac{16}{3}cR_\infty\alpha^2\frac{m_e}{m_\mu}\left(1+\frac{m_e}{m_\mu}\right)^{-3}\mathcal{F}(\alpha, m_e/m_\mu) = \Delta\nu_F\mathcal{F}(\alpha, m_e/m_\mu), \tag{69}$$

where the function $\mathcal{F}$ depends on α and m_e/m_μ only weakly compared to the dependence of $\Delta\nu_F$ on these quantities. It follows from Eq. (69) that, given experimental values of $\Delta\nu_{Mu}$ and m_μ/m_e, one can calculate a value of α by equating $\Delta\nu_{Mu}(\mathrm{exp})$ with $\Delta\nu_{Mu}(\mathrm{th})$; or similarly, given values of $\Delta\nu_{Mu}(\mathrm{exp})$ and α, one can calculate a value of m_μ/m_e.

The two most precise measurements of muonium Zeeman transition frequencies were carried out at the Clinton P. Anderson Meson Physics Facility at Los Alamos (LAMPF), USA and were reviewed in CODATA-98.

a. LAMPF 1982

The results obtained by Mariam (1981) and Mariam *et al.* (1982) may be summarized as follows:

$$\Delta\nu_{Mu} = 4\,463\,302.88(16)\ \mathrm{kHz} \quad [3.6\times10^{-8}], \tag{70}$$

$$\nu(f_p) = 627\,994.77(14)\ \mathrm{kHz} \quad [2.2\times10^{-7}], \tag{71}$$

$$r[\Delta\nu_{\mathrm{Mu}}, \nu(f_{\mathrm{p}})] = 0.23, \tag{72}$$

where f_{p} is very nearly 57.972 993 MHz, corresponding to the flux density of about 1.3616 T used in the experiment, and $r[\Delta\nu_{\mathrm{Mu}}, \nu(f_{\mathrm{p}})]$ is the correlation coefficient of $\Delta\nu_{\mathrm{Mu}}$ and $\nu(f_{\mathrm{p}})$. This value of f_{p}, the data in Eqs. (70)–(72), the 2002 recommended values of R_∞, $\mu_{\mathrm{e}}/\mu_{\mathrm{p}}$, g_{e}, and g_μ, together with Eqs. (66)–(69) yield

$$\frac{\mu_{\mu^+}}{\mu_{\mathrm{p}}} = 3.183\,3461(11) \quad [3.6\times10^{-7}], \tag{73}$$

$$\frac{m_\mu}{m_{\mathrm{e}}} = 206.768\,220(74) \quad [3.6\times10^{-7}], \tag{74}$$

$$\alpha^{-1} = 137.036\,019(24) \quad [1.8\times10^{-7}]. \tag{75}$$

(Note that all significant correlations are taken into account in this and similar calculations.)

b. LAMPF 1999

The results obtained by Liu *et al.* (1999) may be summarized as follows:

$$\Delta\nu_{\mathrm{Mu}} = 4\,463\,302\,765(53)\ \mathrm{Hz} \quad [1.2\times10^{-8}], \tag{76}$$

$$\nu(f_{\mathrm{p}}) = 668\,223\,166(57)\ \mathrm{Hz} \quad [8.6\times10^{-8}], \tag{77}$$

$$r[\Delta\nu_{\mathrm{Mu}}, \nu(f_{\mathrm{p}})] = 0.19, \tag{78}$$

where f_{p} is exactly 72.320 000 MHz, corresponding to the flux density of approximately 1.7 T used in the experiment, and $r[\Delta\nu_{\mathrm{Mu}}, \nu(f_{\mathrm{p}})]$ is the correlation coefficient of $\Delta\nu_{\mathrm{Mu}}$ and $\nu(f_{\mathrm{p}})$. For these data, the analysis described above yields

$$\frac{\mu_{\mu^+}}{\mu_{\mathrm{p}}} = 3.183\,345\,14(39) \quad [1.2\times10^{-7}], \tag{79}$$

$$\frac{m_\mu}{m_{\mathrm{e}}} = 206.768\,283(25) \quad [1.2\times10^{-7}], \tag{80}$$

$$\alpha^{-1} = 137.035\,9997(84) \quad [6.1\times10^{-8}]. \tag{81}$$

c. Combined LAMPF results

By carrying out a least-squares adjustment using only the LAMPF 1982 and LAMPF 1999 data, the 2002 recommended values of the quantities identified above Eq. (73), together with Eqs. (66)–(69), we obtain

$$\frac{\mu_{\mu^+}}{\mu_{\mathrm{p}}} = 3.183\,345\,24(37) \quad [1.2\times10^{-7}], \tag{82}$$

$$\frac{m_\mu}{m_{\mathrm{e}}} = 206.768\,276(24) \quad [1.2\times10^{-7}], \tag{83}$$

$$\alpha^{-1} = 137.036\,0017(80) \quad [5.8\times10^{-8}], \tag{84}$$

where this value of α may be called the muonium value of the fine-structure constant and denoted as $\alpha^{-1}(\Delta\nu_{\mathrm{Mu}})$.

It is worth noting that the uncertainty of the value of the mass ratio m_μ/m_{e} given in Eq. (83) is about four times the uncertainty of the 2002 recommended value. The reason is that taken together, the experimental value of and theoretical expression for the hyperfine splitting essentially determine only the value of the product $\alpha^2 m_\mu/m_{\mathrm{e}}$, as is evident from Eq. (69). In the full adjustment the value of α is determined by other data with an uncertainty significantly smaller than that of the value in Eq. (84), which in turn determines the value of m_μ/m_{e} with a smaller uncertainty than that of Eq. (83).

d. Other values

Other values of μ_μ/μ_{p} and m_μ/m_{e} were discussed in CODATA-98 and they generally agree with the values reviewed above. The most accurate of these is the value of the ratio μ_μ/μ_{p} that follows from the NMR measurements of Klempt *et al.* (1982) made on positive muons stopped in spherical targets at the Swiss Institute for Nuclear Research, Villigen, Switzerland (SIN, now the Paul Scherrer Institute or PSI):

$$\frac{\mu_{\mu^+}}{\mu_{\mathrm{p}}} = 3.183\,3442(17) \quad [5.3\times10^{-7}]. \tag{85}$$

Although this result was initially included in the least-squares analyses used to investigate the consistency of the data considered for the 1998 adjustment, it was excluded from the 1998 final adjustment because its contribution to the determination of the 1998 recommended values of μ_μ/μ_{p} and m_μ/m_{e} was inconsequential. The next most accurate value of either μ_μ/μ_{p} or m_μ/m_{e} had a relative standard uncertainty $u_{\mathrm{r}}=2.6\times10^{-6}$, and thus neither this value nor any of the remaining other values were at all competitive.

More recently, using Doppler-free two-photon pulsed laser spectroscopy, Meyer *et al.* (2000) obtained an improved value of the 1S-2S transition frequency in muonium with a relative standard uncertainty $u_{\mathrm{r}}=4.0\times10^{-9}$. From a comparison of the measured frequency and the known theoretical expression for the 1S-2S transition, they deduced the value $m_\mu/m_{\mathrm{e}}=206.768\,38(17)$ [8.2×10^{-7}]. This result also agrees with the values reviewed above, but its uncertainty is still too large to make the muonium 1S-2S transition a competitive method of obtaining the mass ratio.

For completeness, we note that a value of m_μ/m_{e} can, in principle, be obtained from the measured value of the quantity $\overline{R}$ (see Sec. III.C.2) and the value of a_μ predicted by the theoretical expression in Eq. (C28) of Appendix C for the muon magnetic moment anomaly. The relevant relation is

$$\frac{m_\mu}{m_{\mathrm{e}}} = \frac{2a_\mu}{g_{\mathrm{e}}\overline{R}}\frac{\mu_{\mathrm{e}^-}}{\mu_{\mathrm{p}}}. \tag{86}$$

If a_μ were known exactly, the current best value of $\overline{R}$ [see Eq. (38)] would yield a value of m_μ/m_{e} with $u_{\mathrm{r}}=6.8\times10^{-7}$. However, neither $\overline{R}$ nor a_μ(th) is sufficiently

TABLE VII. Summary of data related to shielded gyromagnetic ratios of the proton and helion, and inferred values of α and h.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
Γ'_{p-90}(lo)	$2.675\,154\,05(30)\times10^8$ s^{-1} T^{-1}	1.1×10^{-7}	NIST-89	III.D.1.a (91)
α^{-1}	137.035 9880(51)	3.7×10^{-8}		III.D.1.a (93)
Γ'_{p-90}(lo)	$2.675\,1530(18)\times10^8$ s^{-1} T^{-1}	6.6×10^{-7}	NIM-95	III.D.1.b (94)
α^{-1}	137.036 006(30)	2.2×10^{-7}		III.D.1.b (97)
Γ'_{p-90}(hi)	$2.675\,1525(43)\times10^8$ s^{-1} T^{-1}	1.6×10^{-6}	NIM-95	III.D.1.b (95)
h	$6.626\,071(11)\times10^{-34}$ J s	1.6×10^{-6}		III.D.1.b (99)
Γ'_{p-90}(hi)	$2.675\,1518(27)\times10^8$ s^{-1} T^{-1}	1.0×10^{-6}	NPL-79	III.D.1.c (100)
h	$6.626\,0730(67)\times10^{-34}$ J s	1.0×10^{-6}		III.D.1.c (101)
Γ'_{h-90}(lo)	$2.037\,895\,37(37)\times10^8$ s^{-1} T^{-1}	1.8×10^{-7}	KR/VN-98	III.D.2.a (102)
α^{-1}	137.035 9853(82)	6.0×10^{-8}		III.D.2.a (104)
Γ'_{h-90}(lo)	$2.037\,897\,29(72)\times10^8$ s^{-1} T^{-1}	3.5×10^{-7}	VNIIM-89	III.D.2.b (105)
α^{-1}	137.035 942(16)	1.2×10^{-7}		III.D.2.b (106)

well known at present to yield a competitive value of the mass ratio.

D. Shielded gyromagnetic ratios γ'

The gyromagnetic ratio γ of a bound particle of spin quantum number i and magnetic moment μ is given by

$$\gamma = \frac{2\pi f}{B} = \frac{\omega}{B} = \frac{|\mu|}{i\hbar}, \tag{87}$$

where f is the precession (that is, spin-flip) frequency and ω is the angular precession frequency of the particle in the magnetic flux density B. The SI unit of γ is $s^{-1}\ T^{-1} = C\ kg^{-1} = A\ s\ kg^{-1}$. In this section we summarize measurements of the gyromagnetic ratio of the shielded proton,

$$\gamma'_p = \frac{2\mu'_p}{\hbar}, \tag{88}$$

and of the shielded helion,

$$\gamma'_h = \frac{2|\mu'_h|}{\hbar}, \tag{89}$$

where, as in previous sections that dealt with magnetic moment ratios involving these particles, the protons are those in a spherical sample of pure H_2O at 25 °C surrounded by vacuum; and the helions are those in a spherical sample of low-pressure, pure ^{3}He gas at 25 °C surrounded by vacuum. Also as was assumed in these previous sections, B is the flux density in vacuum before the sample is introduced and the sources of B are infinitely far from the sample.

In practice, two methods are used to determine the shielded gyromagnetic ratio γ' of a particle. In the low-field method, B is of the order of 1 mT and is usually generated by a single-layer precision solenoid carrying an electric current I. The flux density B is calculated from the dimensions of the solenoid and the current: $B = \mu_0 k_s I$, where k_s is the measured solenoid constant and has the dimension of reciprocal length. In the high-field method, B is of the order of 0.5 T, is generated by an electromagnet or a permanent magnet, and is measured in terms of the force F_e it produces on a straight conducting wire of length l carrying an electric current I: $B = F_e/lI$.

In either case the measured current I can be expressed in terms of the product $K_J R_K$, but B depends on I differently in the two cases, as explained in more detail in CODATA-98. In essence, the low-field experiments determine $\gamma'/K_J R_K$ and the high-field experiments determine $\gamma' K_J R_K$. This leads to the relations

$$\gamma' = \Gamma'_{90}(\text{lo})\frac{K_J R_K}{K_{J-90} R_{K-90}} \tag{90a}$$

$$\gamma' = \Gamma'_{90}(\text{hi})\frac{K_{J-90} R_{K-90}}{K_J R_K}, \tag{90b}$$

where Γ'_{90}(lo) and Γ'_{90}(hi) are the experimental values of γ' in SI units that would result from the low- and high-field experiments, respectively, if $K_J = K_{J-90}$ and $R_K = R_{K-90}$. The quantities Γ'_{90}(lo) and Γ'_{90}(hi) are the input data used in the adjustment, but the observational equations take into account the fact that $K_{J-90} \neq K_J$ and $R_{K-90} \neq R_K$

The gyromagnetic ratio experiments of interest were discussed in detail in CODATA-98, and only a brief summary is given here. The results, together with the value of α inferred from each low-field experiment and the value of h inferred from each high-field experiment, are collected in Table VII. Although some of these val-

ues of Γ' are not included in the final least-squares adjustment from which the 1998 recommended values were obtained, they are included as input data in trial adjustments in 2002 to see whether they are pertinent to tests of the exactness of the relations $K_J = 2e/h$ and $R_K = h/e^2$; these tests are described in Appendix F.

1. Proton p

A number of national metrology institutes have long histories of measuring the gyromagnetic ratio of the shielded proton, motivated, in part, by their need to monitor the stability of their practical unit of current based on groups of standard cells and standard resistors.

a. NIST: Low field

The most recent National Institute of Standards and Technology low-field measurement was reported by Williams *et al.* (1989). Their result, including the corrections discussed in CODATA-98, is

$$\Gamma'_{p-90}(\text{lo}) = 2.675\,154\,05(30) \times 10^8\ \text{s}^{-1}\ \text{T}^{-1} \quad [1.1 \times 10^{-7}], \tag{91}$$

where $\Gamma'_{p-90}(\text{lo})$ is related to γ'_p by Eq. (90a).

As discussed in CODATA-98, the value of α that may be inferred from this result follows from the expression

$$\Gamma'_{p-90}(\text{lo}) = \frac{K_{J-90} R_{K-90} g_{e^-}}{4\mu_0 R_\infty} \frac{\mu'_p}{\mu_{e^-}} \alpha^3, \tag{92}$$

which assumes the validity of the relations $K_J = 2e/h$ and $R_K = h/e^2$. Using the 2002 recommended values for the other relevant quantities, the uncertainties of which are significantly smaller than the uncertainty of the NIST experimental result, we find

$$\alpha^{-1} = 137.035\,9880(51) \quad [3.7 \times 10^{-8}], \tag{93}$$

where the uncertainty is about one-third the uncertainty of the NIST value of $\Gamma'_{p-90}(\text{lo})$ because of the cube-root dependence of alpha on $\Gamma'_{p-90}(\text{lo})$.

b. NIM: Low field and high field

The latest low- and high-field measurements by researchers at the National Institute of Metrology (NIM), Beijing, People's Republic of China, yielded (Liu *et al.*, 1995)

$$\Gamma'_{p-90}(\text{lo}) = 2.675\,1530(18) \times 10^8\ \text{s}^{-1}\ \text{T}^{-1} \quad [6.6 \times 10^{-7}], \tag{94}$$

$$\Gamma'_{p-90}(\text{hi}) = 2.675\,1525(43) \times 10^8\ \text{s}^{-1}\ \text{T}^{-1} \quad [1.6 \times 10^{-6}], \tag{95}$$

which include the corrections discussed in CODATA-98 and have a correlation coefficient of

$$r(\text{lo},\text{hi}) = -0.014, \tag{96}$$

where $\Gamma'_{p-90}(\text{hi})$ is related to γ'_p by Eq. (90b).

Based on Eq. (92), we find that the value of α that may be inferred from the NIM low-field result is

$$\alpha^{-1} = 137.036\,006(30) \quad [2.2 \times 10^{-7}]. \tag{97}$$

Similarly, based on the relation

$$\Gamma'_{p-90}(\text{hi}) = \frac{c\alpha^2 g_{e^-}}{2K_{J-90} R_{K-90} R_\infty} \frac{\mu'_p}{\mu_{e^-}} \frac{1}{h} \tag{98}$$

as given in CODATA-98, the value of h that may be inferred from the NIM high-field result is

$$h = 6.626\,071(11) \times 10^{-34}\ \text{J s} \quad [1.6 \times 10^{-6}]. \tag{99}$$

In both cases the 2002 recommended values for the other relevant quantities have been used; their uncertainties are negligible compared to the NIM values of $\Gamma'_{p-90}(\text{lo})$ and $\Gamma'_{p-90}(\text{hi})$.

c. NPL: High field

The most accurate high-field γ'_p experiment was carried out at NPL by Kibble and Hunt (1979), with the result

$$\Gamma'_{p-90}(\text{hi}) = 2.675\,1518(27) \times 10^8\ \text{s}^{-1}\ \text{T}^{-1} \quad [1.0 \times 10^{-6}], \tag{100}$$

which includes the corrections discussed in CODATA-98. This leads to the inferred value

$$h = 6.626\,0730(67) \times 10^{-34}\ \text{J s} \quad [1.0 \times 10^{-6}], \tag{101}$$

based on Eq. (98). [It should be noted that various input data in the 2002 adjustment such as the one in Eq. (100) depend on the same NIST quantum Hall effect and/or calculable capacitor measurements; nevertheless, their covariances are negligible.]

2. Helion h

As an alternative to water samples, gaseous helium samples have been used in various magnetic-resonance experiments, leading to a value for the gyromagnetic ratio of the shielded helion γ'_h. Although we have defined γ'_h to correspond to 25 °C (see Sec. III.C.4.d), the temperature dependence of the shielded helion gyromagnetic ratio is expected to be significantly less than that of the shielded proton gyromagnetic ratio. Thus small variations in temperature from 25 °C need not be considered.

a. KRISS/VNIIM: Low field

The experiment carried out at the Korea Research Institute of Standards and Science (KRISS), Taedok Science Town, Republic of Korea, in a collaborative effort with researchers from the Mendeleyev All-Russian Research Institute for Metrology (VNIIM), St. Petersburg, Russian Federation, was discussed in detail in CODATA-98 (Kim *et al.*, 1995; Park *et al.*, 1999; Shifrin, Khorev, *et al.*, 1998; Shifrin, Park, *et al.*, 1998; Shifrin, *et al.*, 1999). The result of this work can be expressed as

TABLE VIII. Summary of data related to the Josephson constant K_J, the von Klitzing constant R_K, and the Faraday constant F, and inferred values of α and h.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
K_J	483 597.91(13) GHz V^{-1}	2.7×10^{-7}	NML-89	III.E.1 (107)
h	$6.626\,0684(36)\times10^{-34}$ J s	5.4×10^{-7}		III.E.1 (109)
K_J	483 597.96(15) GHz V^{-1}	3.1×10^{-7}	PTB-91	III.E.2 (110)
h	$6.626\,0670(42)\times10^{-34}$ J s	6.3×10^{-7}		III.E.2 (111)
R_K	25 812.808 31(62) Ω	2.4×10^{-8}	NIST-97	III.F.1 (113)
α^{-1}	137.036 0037(33)	2.4×10^{-8}		III.F.1 (114)
R_K	25 812.8071(11) Ω	4.4×10^{-8}	NML-97	III.F.2 (115)
α^{-1}	137.035 9973(61)	4.4×10^{-8}		III.F.2 (116)
R_K	25 812.8092(14) Ω	5.4×10^{-8}	NPL-88	III.F.3 (117)
α^{-1}	137.036 0083(73)	5.4×10^{-8}		III.F.3 (118)
R_K	25 812.8084(34) Ω	1.3×10^{-7}	NIM-95	III.F.4 (119)
α^{-1}	137.036 004(18)	1.3×10^{-7}		III.F.4 (120)
R_K	25 812.8081(14) Ω	5.3×10^{-8}	BNM-01	III.F.5 (121)
α^{-1}	137.036 0023(73)	5.3×10^{-8}		III.F.5 (122)
$K_J^2R_K$	$6.036\,7625(12)\times10^{33}$ J^{-1} s^{-1}	2.0×10^{-7}	NPL-90	III.G.1 (124)
h	$6.626\,0682(13)\times10^{-34}$ J s	2.0×10^{-7}		III.G.1 (125)
$K_J^2R_K$	$6.036\,761\,85(53)\times10^{33}$ J^{-1} s^{-1}	8.7×10^{-8}	NIST-98	III.G.2 (126)
h	$6.626\,068\,91(58)\times10^{-34}$ J s	8.7×10^{-8}		III.G.2 (127)
$\mathcal{F}_{90}$	96 485.39(13) C mol^{-1}	1.3×10^{-6}	NIST-80	III.H.1 (132)
h	$6.626\,0658(88)\times10^{-34}$ J s	1.3×10^{-6}		III.H.1 (133)

$$\Gamma'_{h-90}(\mathrm{lo}) = 2.037\,895\,37(37)\times10^{8}\ \mathrm{s^{-1}\ T^{-1}} \quad [1.8\times10^{-7}]. \tag{102}$$

The value of α that may be inferred from this result follows from the relation

$$\Gamma'_{h-90}(\mathrm{lo}) = -\frac{K_{J-90}R_{K-90}g_{e^-}}{4\mu_0R_\infty}\frac{\mu'_h}{\mu_{e^-}}\alpha^3, \tag{103}$$

which is analogous to Eq. (92). We find

$$\alpha^{-1} = 137.035\,9853(82) \quad [6.0\times10^{-8}]. \tag{104}$$

b. VNIIM: Low field

The gyromagnetic ratio of the shielded helion was also determined at VNIIM itself (Tarbeev *et al.*, 1989). Based on the review of the experiment in CODATA-98, we have

$$\Gamma'_{h-90}(\mathrm{lo}) = 2.037\,897\,29(72)\times10^{8}\ \mathrm{s^{-1}\ T^{-1}} \quad [3.5\times10^{-7}], \tag{105}$$

from which one may infer

$$\alpha^{-1} = 137.035\,942(16) \quad [1.2\times10^{-7}] \tag{106}$$

based on Eq. (103).

E. Josephson constant K_J

In this section we consider measurements of the Josephson constant K_J in its SI unit Hz/V. In the following three sections we consider measurements of the von Klitzing constant R_K in its SI unit Ω, the quantity $K_J^2R_K$ in its SI unit J^{-1} s^{-1}, and the Faraday constant F in the unit A_{90} s mol^{-1}, where A_{90} is the conventional unit of current based on the Josephson and quantum Hall effects and the conventional values K_{J-90} and R_{K-90} (see Sec. II.F). Since all of these measurements involve K_J and/or R_K, the results are grouped in Table VIII, together with the values of α and h that may be inferred from the data, assuming the validity of the relations $K_J=2e/h$ and $R_K=h/e^2$. The new result for R_K from BNM is discussed in some detail below. The other results were reviewed in CODATA-98, and are only briefly summarized here. Although some of these other data were not included in the final least-squares adjustment from which the 1998 recommended values were obtained, they are included as input data in 2002 trial adjustments to possibly provide information on the validity of the assumed relations.

The quantity K_J is measured by comparing, either directly or indirectly, a Josephson voltage $U_J(n)=nf/K_J$

(see Sec. II.D) to a high voltage U whose value is known in terms of the SI unit of voltage V. In practice, the latter quantity, the ratio U/V, is determined by counterbalancing an electrostatic force arising from the voltage U with a known gravitational force.

1. NML: Hg electrometer

The determination of K_J at the National Measurement Laboratory (NML) of the Commonwealth Scientific and Industrial Research Organization (CSIRO), Lindfield, Australia, was carried out using an apparatus called the liquid-mercury electrometer (Clothier *et al.*, 1989). As discussed in CODATA-98, this experiment yielded the result

$$K_J = 483\,594[1 + 8.087(269) \times 10^{-6}]\ \text{GHz/V} = 483\,597.91(13)\ \text{GHz/V} \quad [2.7 \times 10^{-7}]. \tag{107}$$

Assuming the validity of the relation $K_J = 2e/h$ and recalling that $\alpha = e^2/4\pi\epsilon_0\hbar = \mu_0 c e^2/2h$, we have

$$h = \frac{8\alpha}{\mu_0 c K_J^2}. \tag{108}$$

This expression, the NML value of K_J in Eq. (107), and the 2002 recommended value of α, which has a much smaller relative uncertainty, yields the inferred value

$$h = 6.626\,0684(36) \times 10^{-34}\ \text{J s} \quad [5.4 \times 10^{-7}]. \tag{109}$$

2. PTB: Capacitor voltage balance

The determination of K_J at PTB was carried out by using a voltage balance consisting of two coaxial cylindrical electrodes (Sienknecht and Funck, 1985, 1986; Funck and Sienknecht, 1991). Taking into account the correction associated with the reference capacitor used in the PTB experiment as described in CODATA-98, the result of the PTB determination is

$$K_J = 483\,597.96(15)\ \text{GHz/V} \quad [3.1 \times 10^{-7}], \tag{110}$$

from which we infer, using Eq. (108),

$$h = 6.626\,0670(42) \times 10^{-34}\ \text{J s} \quad [6.3 \times 10^{-7}]. \tag{111}$$

F. von Klitzing constant R_K

The quantity R_K is measured by comparing, either directly or indirectly, a quantized Hall resistance $R_H(i) = R_K/i$ (see Sec. II.E) to a resistance R whose value is known in terms of the SI unit of resistance Ω. In practice, the latter quantity, the ratio R/Ω, is determined by means of a calculable cross capacitor.

The calculable cross capacitor is based on a theorem in electrostatics discovered in the 1950s (Thompson and Lampard, 1956; Lampard, 1957). The theorem allows one to construct a cylindrical capacitor (Thompson, 1959) whose capacitance, to high accuracy, depends only on its length. The electric constant $\epsilon_0 = 1/\mu_0 c^2$ is also required but is exactly known, since in the SI, μ_0 and c are exactly known.

The uncertainty of R_K is determined mainly by the quality and implementation of the design of the calculable capacitor and the impedance chain used to compare its capacitance to the resistance R. Of particular importance is the determination of the difference in ac and dc values of R, since the impedance measurements are carried out at ac (for example, $\omega = 10^4$ rad/s or approximately 1592 Hz) and the quantized Hall resistance measurements are carried out at dc. Recent work tends to confirm that the difference in the ac and dc values of the resistances of the special kinds of resistors used in calculable capacitor experiments is reasonably well understood (Belliss, 2000; Elmquist, 2000; Elmquist *et al.*, 2001; Boháček, 2002).

As noted in Sec. II.E, if one assumes the validity of the relation $R_K = h/e^2$, then R_K and the fine-structure constant α are related by

$$\alpha = \mu_0 c/2R_K. \tag{112}$$

Hence the relative uncertainty of the value of α that may be inferred from a particular experimental value of R_K is the same as the relative uncertainty of that value.

1. NIST: Calculable capacitor

The result obtained at NIST and reported in 1997 by Jeffery *et al.* (1997) (see also Jeffery *et al.*, 1998) is

$$R_K = 25\,812.8[1 + 0.322(24) \times 10^{-6}]\ \Omega = 25\,812.808\,31(62)\ \Omega \quad [2.4 \times 10^{-8}], \tag{113}$$

which, as discussed in CODATA-98, is viewed as superseding the NIST result reported in 1989 by Cage *et al.* (1989). Work by Jeffery *et al.* (1999) provides additional support for the uncertainty budget of the NIST calculable capacitor. The value of α that may be inferred from the NIST 1997 value of R_K is, from Eq. (112),

$$\alpha^{-1} = 137.036\,0037(33) \quad [2.4 \times 10^{-8}]. \tag{114}$$

2. NML: Calculable capacitor

Based on measurements carried out from December 1994 to April 1995 and a complete reassessment of uncertainties associated with their apparatus, Small *et al.* (1997) reported the result

$$R_K = R_{K-90}[1 + 0.4(4.4) \times 10^{-8}] = 25\,812.8071(11)\ \Omega \quad [4.4 \times 10^{-8}]. \tag{115}$$

Because of problems associated with the 1989 NML value of R_K, only the result reported in 1997 is used in the 2002 adjustment, as was the case in the 1998 adjustment. The value of α it implies is

$$\alpha^{-1} = 137.035\,9973(61) \quad [4.4 \times 10^{-8}]. \tag{116}$$

3. NPL: Calculable capacitor

The NPL calculable cross capacitor is similar in design to those of NIST and NML. The result for R_K reported in 1988 by Hartland *et al.* (1988) is

$$R_K = 25\,812.8[1 + 0.356(54) \times 10^{-6}]\ \Omega$$
$$= 25\,812.8092(14)\ \Omega \quad [5.4 \times 10^{-8}], \tag{117}$$

and the value of α that one may infer from it is

$$\alpha^{-1} = 137.036\,0083(73) \quad [5.4 \times 10^{-8}]. \tag{118}$$

The 1988 NPL value of R_K given in Eq. (117) and used in the 2002 and 1998 adjustments supersedes earlier and less accurate NPL values obtained when the apparatus was in a less developed state, as discussed in CODATA-98.

4. NIM: Calculable capacitor

The NIM calculable cross capacitor differs markedly from the version used at NIST, NML, and NPL. The four bars that comprise the capacitor are horizontal rather than vertical and the length that determines its known capacitance is fixed rather than variable. The NIM result for R_K, as reported in 1995 by Zhang *et al.* (1995), is

$$R_K = 25\,812.8084(34)\ \Omega \quad [1.3 \times 10^{-7}], \tag{119}$$

which implies

$$\alpha^{-1} = 137.036\,004(18) \quad [1.3 \times 10^{-7}]. \tag{120}$$

As in the case of the NPL experiment, the NIM value of R_K used in the 2002 and 1998 adjustments as given in Eq. (119) supersedes a less accurate value obtained when the apparatus was in an earlier stage of development (see CODATA-98).

5. BNM: Calculable capacitor

The value of R_K obtained at BNM by Trapon *et al.* (2001, 2003) is

$$R_K = 25\,812.8081(14)\ \Omega \quad [5.3 \times 10^{-8}], \tag{121}$$

which implies

$$\alpha^{-1} = 137.036\,0023(73) \quad [5.3 \times 10^{-8}]. \tag{122}$$

This value of R_K is the final result of an effort that evolved over a 15-year period at LCIE (*Laboratoire des Industries Électriques*, Fontenay-aux-Roses, France). LCIE had long been responsible for fundamental electrical metrology in France and eventually became part of BNM, after which time it was referred to as BNM-LCIE. As of 1 July 2001, French fundamental electrical metrology work was transferred to the National Testing Laboratory (LNE, *Laboratoire National d'Essais*, Paris, France), which is also part of BNM and is referred to as BNM-LNE. For simplicity, we give the identification BNM-01 to this new value of R_K.

The BNM Thompson-Lampard calculable capacitor is unique among all calculable capacitors in that it consists of five 450 mm long, 75.5 mm diameter horizontal bars (electrodes) arranged at the corners of a regular pentagon. The change in capacitance between one bar and the two furthest adjacent bars is 3/8 pF when the movable guard electrode at the center of the pentagonal group of bars is displaced by ≈138 mm. An 8:3 ratio capacitance bridge is used to calibrate a 1 pF capacitor in terms of the five 3/8 pF values of capacitance obtained from the five possible connection configurations of the bars.

A series of impedance bridges—two-terminal-pair, four-terminal-pair, quadrature—and intermediate reference capacitors with capacitances of 10 pF, 100 pF, 1000 pF, and 10 000 pF are used to determine, in terms of the known capacitance of the 1 pF capacitor, the ac resistances of a set of three pairs of resistors with resistances of 10, 20, and 40 kΩ. The measurements extending from the calculable capacitor to the three resistor pairs are carried out at the frequencies $f \approx 1600$ Hz ($\omega = 10\,000$ rad/s), $f \approx 800$ Hz ($\omega = 5000$ rad/s), and $f \approx 400$ Hz ($\omega = 2500$ rad/s), respectively. (Note that the capacitive reactance of each of the final pair of 10 000 pF capacitances in the BNM chain at each of these angular frequencies is 10 kΩ, 20 kΩ, and 40 kΩ, respectively.) The ac resistance of each resistor at its measured frequency is corrected to its dc value by comparison with a special coaxial resistor whose ac-dc resistance difference is calculable. A cryogenic current comparator is used to compare each resistance to the $i=2$, 12 906.4 Ω quantized Hall resistance, thereby determining R_K. However, the three values of R_K are not the same because of the frequency dependence of the capacitance of the calculable capacitor, corrections having been applied to eliminate all other frequency dependences in the measurement chain. (The frequency dependence of the capacitance of the calculable capacitor is mainly due to the inductance of the reed relays employed to connect the bars that comprise the capacitor to the 8:3 ratio capacitance bridge or to ground.) To eliminate this last frequency dependence, the three values of R_K are appropriately extrapolated to zero frequency.

The five largest relative-standard-uncertainty components contributing to the total relative standard uncertainty $u_r = 5.3 \times 10^{-8}$ of the BNM result for R_K is the $\approx 2.8 \times 10^{-8}$ statistical (Type A) uncertainty, and the following four components arising from systematic effects (Type B): 3×10^{-8} to account for the possible lateral movement of the compensating spike at the end of the movable guard electrode; 2.4×10^{-8} to account for possible imperfections in the shapes and positions of the five bars that comprise the calculable capacitor; 1.5×10^{-8} for the ratio of the 10:1 capacitance bridge (used five times); and $\approx 1.3 \times 10^{-8}$ for the possible imperfect correction of the frequency dependences of the 10 kΩ, 20 kΩ, and 40 kΩ resistors.

As discussed in considerable detail by Trapon *et al.* (2003), many significant improvements have been incorporated in the BNM apparatus since Delahaye *et al.* (1987) reported the first result for R_K obtained from it,

which had an assigned standard uncertainty $u_r = 2.2 \times 10^{-7}$. These include a new laser and means to align it for measuring the displacement of the movable guard electrode, together with an improved vacuum to reduce the correction for the index of refraction of air; the addition of a fixed guard electrode at the end of the five bars opposite the movable guard electrode to hold the interferometer, and a star-shaped electrode near the compensating spike at the end of the movable electrode that significantly improves the linearity of the change in capacitance with the position of this electrode; detailed evaluation of the effect of the five bars not being perfectly cylindrical and the near-elimination of the effect via a new compensating spike affixed to the end of the movable electrode; and better understanding of the frequency dependence of the calculable capacitor and its connections. Thus, the new BNM value of R_K, although obtained with a unique form of the original Thompson-Lampard calculable capacitor, is viewed as being on an equal footing with other values obtained with calculable capacitors of more traditional form.

6. Other values

As noted in CODATA-98, three values of R_K based directly on calculable capacitor measurements with quoted relative standard uncertainties of 22×10^{-8}, 26×10^{-8}, and 32×10^{-8} were not considered in the 1998 adjustment because of their comparatively large uncertainties. (In fact, the first of these is the 1987 result from the BNM experiment just described.) Although some of the values listed in the previous sections that were not included in the 1998 final least-squares adjustment are being reconsidered in the present adjustment to test the validity of the relations $K_J = 2e/h$ and $R_K = h/e^2$, the three other values referred to in this section are still not sufficiently competitive to be considered in the 2002 adjustment.

G. Product $K_J^2 R_K$

A value of the product $K_J^2 R_K$ is of importance to the determination of the Planck constant h, because if one assumes that the relations $K_J = 2e/h$ and $R_K = h/e^2$ are valid, then

$$h = \frac{4}{K_J^2 R_K}. \tag{123}$$

The product $K_J^2 R_K$ is determined by comparing electrical power known in terms of a Josephson voltage and quantized Hall resistance to the equivalent mechanical power known in the SI unit $W = m^2\ kg\ s^{-3}$. The comparison is carried out using an apparatus known as a moving-coil watt balance, as discussed in CODATA-98. To date two laboratories, NPL and NIST, have determined $K_J^2 R_K$ using this method. For a recent review of watt-balance experiments, see Eichenberger *et al.*, 2003.

1. NPL: Watt balance

A practical approach that allows $K_J^2 R_K$ to be determined with high accuracy based on the above idea was first proposed by Kibble at NPL (Kibble, 1975). Shortly after Kibble's original proposal of 1975, Kibble and Robinson (1977) carried out a feasibility study of this idea based on experience with the NPL apparatus that was used to determine γ'_p by the high-field method (Kibble and Hunt, 1979).

The NPL result used in the 1998 adjustment was reported in 1990 by Kibble *et al.* (1990) and may be written as

$$\begin{aligned} K_J^2 R_K &= K_{J-NPL}^2 R_{K-NPL}[1 + 16.14(20) \times 10^{-6}] \\ &= 6.036\,7625(12) \times 10^{33}\ J^{-1}\ s^{-1} \quad [2.0 \times 10^{-7}], \end{aligned} \tag{124}$$

where $K_{J-NPL} = 483\,594\ GHz/V$ and $R_{K-NPL} = 25\,812.809\,2\ \Omega$. The value of h that may be inferred from the 1990 NPL value of $K_J^2 R_K$ is, according to Eq. (123),

$$h = 6.626\,0682(13) \times 10^{-34}\ J\ s \quad [2.0 \times 10^{-7}]. \tag{125}$$

Based on the experience gained in the experiment that led to the above value of $K_J^2 R_K$, NPL researchers designed and constructed a new apparatus that is expected to eventually yield a result for $K_J^2 R_K$ with $u_r \approx 10^{-8}$ (Robinson and Kibble, 1997; Kibble and Robinson, 2003). The apparatus has the cylindrical symmetry of the NIST watt balance that yielded the result for $K_J^2 R_K$ discussed in the following section, and although it uses the same balance beam as in the previous NPL apparatus, little else from that experiment is retained in the new experiment.

Over 1000 measurements in vacuum were carried out with the new NPL watt balance between January 2000 and November 2001. Many were made in an effort to identify the cause of an observed fractional change in the value of $K_J^2 R_K$ of about 3×10^{-7} that occurred in mid-April 2000 (Robinson and Kibble, 2002). A change in the alignment of the apparatus is suspected of being the cause of the shift. We do note that the value of h implied by the mean of these measurements is in much better agreement with the value of h inferred from the result for the molar volume of silicon $V_m(Si)$ discussed in Sec. III.J than it is with the above NPL 1990 value of h and the similarly obtained NIST 1998 value of h given in the next section. [The $V_m(Si)$ inferred value of h exceeds the NPL and NIST watt balance combined value of h by the surprisingly large fractional amount 1.1×10^{-6}; see Sec. IV.A.] However, Robinson and Kibble (2002); Robinson (2003) stress that it is "inadvisable to draw any conclusions from this fact," because modifications to the apparatus and studies of a variety of systematic effects, especially those associated with the alignment of the apparatus, are still underway.

2. NIST: Watt balance

Work on a moving-coil watt balance at NIST began shortly after Kibble made his 1975 proposal. A first result was reported by NIST researchers in 1989 with $u_r = 1.3 \times 10^{-6}$. Significant improvements were then made to the apparatus and the final result from this phase of the NIST effort was reported in 1998 by Williams *et al.* (1998):

$$K_J^2 R_K = K_{J-90}^2 R_{K-90}[1 - 0.008(87) \times 10^{-6}] = 6.036\,761\,85(53) \times 10^{33}\ \mathrm{J^{-1}\,s^{-1}} \quad [8.7 \times 10^{-8}]. \tag{126}$$

This is the value used in the 1998 and 2002 adjustments; the 1989 NIST result is consistent with the 1998 value, but has an uncertainty about 15 times larger.

The value of h implied by the result in Eq. (126) is

$$h = 6.626\,068\,91(58) \times 10^{-34}\ \mathrm{J\,s} \quad [8.7 \times 10^{-8}]. \tag{127}$$

Based on the lessons learned in the decade-long effort that led to their 1998 value of $K_J^2 R_K$, the NIST watt-balance researchers initiated a new program with the goal of measuring $K_J^2 R_K$ with $u_r \approx 10^{-8}$. Major changes in the watt balance have since been implemented and little remains of the earlier apparatus except the superconducting magnet used to generate the required radial magnetic flux density and the wheel used as the balance. An important change in the experiment is that, in the new apparatus, the entire balance mechanism and moving coils are in vacuum, which virtually eliminates index-of-refraction corrections in the laser position measurement and a buoyancy correction for the force exerted by the standard mass. A brief progress report is given by Steiner *et al.* (2002), and the results of a study of hysteresis and related effects arising from mechanical, magnetic, and thermal sources are reported by Schwarz *et al.* (2001).

3. Other values

Although there is no published value of $K_J^2 R_K$ other than the two discussed above, two additional laboratories have watt-balance experiments in progress: the Swiss Federal Office of Metrology and Accreditation (METAS, *Metrologie und Akkreditierung Schweiz*, Bern-Wabern, Switzerland); and the BNM. METAS has a functioning apparatus, as described by Beer *et al.* (1999, 2001, 2003) (see also Courteville *et al.*, 2000; Beer *et al.*, 2002). The BNM experiment is currently only in the planning and design stage, and will involve several laboratories of the BNM (Genevès, 2002). In addition, the BIPM is considering initiating a watt-balance experiment (Quinn, 2002).

H. Faraday constant *F*

The Faraday constant F is equal to the Avogadro constant N_A times the elementary charge e, $F = N_A e$; its SI unit is coulomb per mol, $\mathrm{C\,mol^{-1}} = \mathrm{A\,s\,mol^{-1}}$. It determines the amount of substance $n(\mathrm{X})$ of an entity X that is deposited or dissolved during electrolysis by the passage of a quantity of electricity, or charge, $Q = It$, due to the flow of a current I in a time t. In particular, the Faraday constant F is related to the molar mass $M(\mathrm{X})$ (see Sec. II.C) and valence z of entity X by

$$F = \frac{ItM(\mathrm{X})}{z m_d(\mathrm{X})}, \tag{128}$$

where $m_d(\mathrm{X})$ is the mass of entity X dissolved as the result of transfer of charge $Q = It$ during the electrolysis. It follows from the relations $F = N_A e$, $e^2 = 2\alpha h/\mu_0 c$, $m_e = 2R_\infty h/c\alpha^2$, and $N_A = A_r(\mathrm{e}) M_u/m_e$, where $M_u = 10^{-3}$ kg mol^{-1} (see Sec. II.C), that

$$F = \frac{A_r(\mathrm{e}) M_u}{R_\infty}\left(\frac{c}{2\mu_0}\frac{\alpha^5}{h}\right)^{1/2}. \tag{129}$$

Since, according to Eq. (128), F is proportional to the current I, and I is inversely proportional to the product $K_J R_K$ if the current is determined in terms of the Josephson and quantum Hall effects, we may write

$$\mathcal{F}_{90} = \frac{K_J R_K}{K_{J-90} R_{K-90}} \frac{A_r(\mathrm{e}) M_u}{R_\infty}\left(\frac{c}{2\mu_0}\frac{\alpha^5}{h}\right)^{1/2}, \tag{130}$$

where $\mathcal{F}_{90}$ is the experimental value of F in SI units that would result from the Faraday experiment if $K_J = K_{J-90}$ and $R_K = R_{K-90}$. The quantity $\mathcal{F}_{90}$ is the input datum used in the adjustment, but the observational equation takes into account the fact that $K_{J-90} \neq K_J$ and $R_{K-90} \neq R_K$. If one assumes the validity of the expressions $K_J = 2e/h$ and $R_K = h/e^2$, Eq. (130) can be written as

$$\mathcal{F}_{90} \doteq \frac{cM_u}{K_{J-90} R_{K-90}} \frac{A_r(\mathrm{e})\alpha^2}{R_\infty h}, \tag{131}$$

which in that case is the observational equation for the measured value of $\mathcal{F}_{90}$.

1. NIST: Ag coulometer

There is one high-accuracy experimental value of $\mathcal{F}_{90}$ available, that from NIST. The NIST experiment of Bower and Davis (1980) used the silver dissolution coulometer pioneered by Craig *et al.* (1960) in their earlier determination of F, also at NIST. It is based on the anodic dissolution by electrolysis of silver, which is monovalent, into a solution of perchloric acid containing a small amount of silver perchlorate. The basic chemical reaction is $\mathrm{Ag} \rightarrow \mathrm{Ag^+} + \mathrm{e^-}$ and occurs at the anode, which in the NIST work was a highly purified silver bar.

As discussed in CODATA-98, the NIST measurement by Bower and Davis (1980) leads to the result

$$\mathcal{F}_{90} = 96\,485.39(13)\ \mathrm{C\,mol^{-1}} \quad [1.3 \times 10^{-6}], \tag{132}$$

which supersedes the earlier and similar 1960 NIST result reported by Craig *et al.* (1960). The value of h that may be inferred from Eq. (131) using the 1980 result in Eq. (132) for $\mathcal{F}_{90}$ and the recommended values from the 2002 adjustment for the other quantities is

$$h = 6.626\,0658(88) \times 10^{-34}\ \text{J s} \quad [1.3 \times 10^{-6}], \qquad (133)$$

where the uncertainties of the other quantities are negligible compared to the uncertainty of $\mathcal{F}_{90}$.

2. Other values

As in the 1998 adjustment, the two other available values of the Faraday constant, which have relative standard uncertainties of about 1×10^{-5}, are not considered competitive (Marinenko and Taylor, 1968; Cohen and Taylor, 1973; Koch, 1980).

I. {220} lattice spacing of silicon d_{220}

A value of the {220} lattice spacing of a silicon crystal in meters is relevant to the 2002 adjustment not only for its role in determining $A_r(n)$ (see Sec. III.A.5), but also because, together with the measured value of $h/m_n d_{220}(W04)$, where h/m_n is the quotient of the Planck constant and neutron mass and W04 is abbreviated from WASO 04, it can provide a useful value of the fine-structure constant α (see Sec. III.K.1). Further, together with the measured value of the molar volume of silicon $V_m(Si)$, it can provide a useful value of h (see Sec. III.J).

Various aspects of silicon and its crystal plane spacings of interest here are reviewed in CODATA-98 (for more recent reviews, see Becker, 2001a, 2003b; Mana, 2001). Some points worth noting are that silicon is a cubic crystal with eight atoms per face-centered cubic unit cell of edge length (or lattice parameter) $a = 543$ pm with $d_{220} = a/\sqrt{8}$. The three naturally occurring isotopes of Si are ^{28}Si, ^{29}Si, and ^{30}Si, and the amount-of-substance fractions $x(^{28}\text{Si})$, $x(^{29}\text{Si})$, and $x(^{30}\text{Si})$ of natural silicon are approximately 0.92, 0.05, and 0.03, respectively.

Although the {220} lattice spacing of Si is not a fundamental constant in the usual sense, for practical purposes one can consider a, and hence d_{220}, of an impurity-free, crystallographically perfect or "ideal" silicon crystal under specified conditions, principally of temperature, pressure, and isotopic composition, to be an invariant of nature. The reference temperature and pressure currently adopted are $t_{90} = 22.5\ ^\circ\text{C}$ and $p = 0$ (that is, vacuum), where t_{90} is Celsius temperature on the International Temperature Scale of 1990 (ITS-90) (Preston-Thomas, 1990). However, no reference values for $x(^A\text{Si})$ have as yet been adopted, because the variation of a due to the variation of the isotopic composition of the crystals used in high-accuracy experiments is taken to be negligible at the current level of experimental uncertainty in a. A much larger effect on a is the impurities that the silicon crystal contains—mainly oxygen (O) and carbon (C)—and corrections must be applied to convert the {220} lattice spacing $d_{220}(X)$ of a real crystal X to the {220} lattice spacing d_{220} of an "ideal" crystal.

1. d_{220} difference measurements

To relate the lattice spacings of crystals used in different experiments, highly accurate measurements are made of the fractional difference $[d_{220}(x) - d_{220}(\text{ref})]d_{220}(\text{ref})$ of the {220} lattice spacing of a sample of crystal X and that of a reference crystal "ref." Such fractional differences obtained at NIST by Kessler *et al.* (1999) that we take as input data are given in Sec. III.A.5, Eqs. (15)–(17). The following are the fractional differences obtained at PTB by Martin *et al.* (1998) that we also take as input data:

$$\frac{d_{220}(\text{W4.2a}) - d_{220}(\text{W04})}{d_{220}(\text{W04})} = -1(21) \times 10^{-9}, \qquad (134)$$

$$\frac{d_{220}(\text{W17}) - d_{220}(\text{W04})}{d_{220}(\text{W04})} = 22(22) \times 10^{-9}, \qquad (135)$$

$$\frac{d_{220}(\text{MO}^*) - d_{220}(\text{W04})}{d_{220}(\text{W04})} = -103(28) \times 10^{-9}, \qquad (136)$$

$$\frac{d_{220}(\text{NR3}) - d_{220}(\text{W04})}{d_{220}(\text{W04})} = -23(21) \times 10^{-9}. \qquad (137)$$

As for the three similar NIST input data, these four input data are correlated, with correlation coefficients of about 0.4. It should also be noted that in order to include these four input data in the 2002 adjustment, as in the 1998 adjustment, the quantities $d_{220}(\text{W4.2a})$ and $d_{220}(\text{W04})$ must be taken to be adjusted constants in addition to the four similar quantities in Eqs. (15)–(17).

To relate $d_{220}(\text{W04})$ to the {220} lattice spacing d_{220} of an "ideal" silicon crystal, we take as an input datum

$$\frac{d_{220} - d_{220}(\text{W04})}{d_{220}(\text{W04})} = 10(11) \times 10^{-9} \qquad (138)$$

given by Becker *et al.* (2003), who obtained it by taking into account the known C, O, and nitrogen (N) impurities in WASO 04. However, following what was done in the 1998 adjustment, we have included an additional component of uncertainty of 1×10^{-8} to account for the possibility that, even after correction for C, O, and N impurities, the crystal WASO 04 does not meet all of the criteria for an ideal crystal. [The value used in the 1998 adjustment was $15(11) \times 10^{-9}$, but it did not include a correction for nitrogen.] In order to include this fractional difference in the 2002 adjustment, the quantity d_{220} is also taken as an adjusted constant.

2. X-ray/optical interferometer measurements of $d_{220}(X)$

Included as input data in the 1998 CODATA adjustment were three measurements of $d_{220}(X)$ in meters carried out at different laboratories using a combined x-ray and optical interferometer or "XROI" with crystals from different silicon boules. One measurement was made at PTB and published in 1981 (Becker *et al.*, 1981), the second was made at IMGC and published in 1994 (Basile *et al.*, 1994), and the third was made at NMIJ and

published in 1997 (Nakayama and Fujimoto, 1997). These experiments as well as the basic principles of the XROI, which is a device that enables x-ray fringes of unknown period $d_{220}(\mathrm{X})$ to be compared with optical fringes of known period, are fully discussed in CODATA-98 and hence are not discussed further here.

While the values obtained at PTB and IMGC are consistent with each other, they disagree with the value obtained at NMIJ. On the other hand, the NMIJ value is in better agreement with other data in the 1998 adjustment. During the last four years, considerable work has been carried out at both IMGC and NMIJ, and also at IMGC in a collaborative effort involving both IMGC and NMIJ researchers, to improve the measurement of $d_{220}(\mathrm{X})$, motivated in part by the above situation. The results of this work, which were published in early 2004 (Cavagnero *et al.*, 2004), did not become available until late in 2003, long after the 31 December 2002 closing date of the 2002 adjustment. However, sufficient information was available and made known to the Task Group by the time of its 4 July 2003 meeting, the aim of which was to finalize the 2002 input data and its treatment, for the Task Group to decide that the uncertainties assigned to the PTB and IMGC results may have been underestimated and that only the NMIJ result of Nakayama and Fujimoto (1997),

$$d_{220}(\mathrm{NR3}) = 192\,015.587(11)\ \mathrm{fm} \quad [5.6\times10^{-8}], \tag{139}$$

should be included in the 2002 adjustment. We note also that an investigation by Fujimoto *et al.* (2000) supports the data-averaging procedure used by Nakayama and Fujimoto (1997) to obtain their value of $d_{220}(\mathrm{NR3})$. [Recent work indicates that significant issues concerning the measurement of d_{220} are not yet resolved and require further investigation (Mana, 2004).]

The observational equation for the NMIJ lattice-spacing result is simply

$$d_{220}(\mathrm{NR3}) \doteq d_{220}(\mathrm{NR3}). \tag{140}$$

J. Molar volume of silicon V_m(Si)

The definition of the molar volume of silicon $V_m(\mathrm{Si})$ and its relationship to the Avogadro constant N_A as well as other constants is discussed in CODATA-98 and may be summarized by the following equations:

$$m(\mathrm{Si}) = \rho(\mathrm{Si})\frac{a^3}{n}, \tag{141}$$

$$V_m(\mathrm{Si}) = \frac{M(\mathrm{Si})}{\rho(\mathrm{Si})} = \frac{A_r(\mathrm{Si})M_u}{\rho(\mathrm{Si})}, \tag{142}$$

$$N_A = \frac{V_m(\mathrm{Si})}{a^3/n} = \frac{A_r(\mathrm{Si})M_u}{\sqrt{8}d_{220}^3\rho(\mathrm{Si})}, \tag{143}$$

$$V_m(\mathrm{Si}) \doteq \frac{\sqrt{2}cM_uA_r(\mathrm{e})\alpha^2d_{220}^3}{R_\infty h}. \tag{144}$$

These expressions are to be understood in the context of an impurity-free, crystallographically perfect, "ideal" silicon crystal at the reference conditions $t_{90}=22.5\ ^\circ\mathrm{C}$ and $p=0$, and of isotopic composition in the range normally observed for crystals used in high-accuracy experiments. Thus $m(\mathrm{Si})$, $V_m(\mathrm{Si})$, $M(\mathrm{Si})$, and $A_r(\mathrm{Si})$ are the mean mass, mean molar volume, mean molar mass, and mean relative atomic mass, respectively, of the silicon atoms in such a crystal, and $\rho(\mathrm{Si})$ is the crystal's macroscopic mass density. Further, a is the edge length of the crystal's cubic unit cell, $d_{220}=a/\sqrt{8}$ is the {220} lattice spacing of the crystal, $n=8$ is the number of silicon atoms per unit cell, and $M_u=10^{-3}$ kg/mol is the molar mass constant and hence $M(\mathrm{Si})=A_r(\mathrm{Si})M_u$. Equation (144) is the observational equation for a measured value of $V_m(\mathrm{Si})$ and is based on the relations $m_e=2R_\infty h/c\alpha^2$ and $N_A=A_r(\mathrm{e})M_u/m_e$. [From Eq. (143) we see that N_A is also equal to the quotient of the mean molar volume of silicon and the mean volume of a silicon atom.]

It follows from the definition of $V_m(\mathrm{Si})$ given in Eq. (142) that the experimental determination of the molar volume of silicon requires (i) measurement of the amount-of-substance ratios $n(^{29}\mathrm{Si})/n(^{28}\mathrm{Si})$ and $n(^{30}\mathrm{Si})/n(^{28}\mathrm{Si})$ of a nearly perfect silicon crystal—and hence amount-of-substance fractions $x(^A\mathrm{Si})$—and then calculation of $A_r(\mathrm{Si})$ from the well-known values of $A_r(^A\mathrm{Si})$ in Table II; and (ii) measurement of the macroscopic mass density $\rho(\mathrm{Si})$ of the crystal. Of course, the crystal must be carefully characterized structurally to ensure that it approximates an ideal crystal to a sufficient degree, and it must also be carefully characterized chemically so that appropriate corrections can be applied to account for the effect of impurities.

An extensive international effort has been under way since at least the early 1990s to reduce the relative standard uncertainty of the measured value of the Avogadro constant N_A to $u_r\approx10^{-8}$ so that serious consideration can be given to replacing the current artifact-based definition of the SI unit of mass—the international prototype of the kilogram—by a definition based on an invariant of nature. [Recall that determining N_A by measuring $V_m(\mathrm{Si})$ as described above, and d_{220} using x rays, is called the x-ray-crystal-density or XRCD method.] This effort is being coordinated by the Working Group on the Avogadro Constant (WGAC) of the Consultative Committee for Mass and Related Quantities (CCM, *Comité consultatif pour la masse et les grandeurs apparentées*) of the CIPM. The WGAC, which has representatives from all major research groups working in areas relevant to the determination of N_A, is currently chaired by P. Becker of PTB.

In spite of the quite significant advances made in the 1990s in measuring $V_m(\mathrm{Si})$, which include improved understanding of the imperfections of real silicon crystals, the Task Group decided not to include any values of $V_m(\mathrm{Si})$ as input data in the 1998 adjustment. This deci-

sion was reached in collaboration with, and had the full support of, the WGAC. The reason for this exclusion, as discussed in CODATA-98, was troublesome inconsistencies among a number of values of $V_m(Si)$. The most problematic of these was the value of $V_m(Si)$ obtained by NMIJ researchers using crystal NRLM3—it exceeds the weighted-mean value obtained at NMIJ and other laboratories from a variety of different crystals by about the fractional amount 3.5×10^{-6} (Fujii *et al.*, 1999; De Bièvre *et al.*, 2001). Although no convincing explanation for this inconsistency has yet been found, the WGAC has concluded that the NMIJ result for $V_m(Si)$ from crystal NRLM3 is anomalous and hence should be discarded from future consideration, and that the best value for $V_m(Si)$ based on all the data available through about mid-2000, obtained from 15 different crystals, is (Becker, 2001b; De Bièvre *et al.*, 2001)

$$V_m(Si) = 12.058\,8207(54)\times10^{-6}\ m^3\ mol^{-1} \quad [4.5\times10^{-7}]. \tag{145}$$

[It is important to recognize that because the fractional variation of d_{220} due to the observed variation of the isotopic composition of the silicon crystals employed in high-accuracy experiments may be considered negligible at the current level of uncertainty of such experiments, Eq. (143) implies that after correction for impurities, values of $V_m(Si)$ obtained using different crystals should be very nearly invariant.]

Work to improve the determination of $V_m(Si)$ has continued to the present with new measurements of $\rho(Si)$ being carried out at IMGC, NMIJ, NML, and PTB with newly prepared silicon crystals, and with amount-of-substance ratio measurements required to determine $A_r(Si)$ continuing to be made at the Institute for Reference Materials and Measurements (IRMM), European Commission, Geel, Belgium. Because this institution is the only one making such measurements, its role is crucial and all current determinations of $V_m(Si)$ are highly correlated. A detailed description of some of the most important aspects of the absolute isotopic ratio mass-spectrometry technique used by IRMM to determine $n(^{29}Si)/n(^{28}Si)$ and $n(^{30}Si)/n(^{28}Si)$ is given by De Bièvre, Lenaers, *et al.* (1995). Differential measurements that lend some support to the validity of the IRMM absolute measurements of isotopic composition have been carried out at the Institute of Mineral Resources (IMR), of the Chinese Academy of Geological Sciences, Beijing, Peoples's Republic of China (De Bièvre *et al.*, 2001).

At the request of the Task Group, the WGAC has recommended a value of $V_m(Si)$ for use as an input datum in the 2002 adjustment. This value is (Becker, 2003a; Fujii, 2003)

$$V_m(Si) = 12.058\,8257(36)\times10^{-6}\ m^3\ mol^{-1} \quad [3.0\times10^{-7}] \tag{146}$$

with identification N/P/I-03 in recognition of the work done by researchers at NMIJ, PTB, and IRMM to obtain this result. (It should be noted that Task Group-member and WGAC-member K. Fujii of NMIJ serves as liaison between the two groups, and the authors worked closely with him in the derivation of this result.) The following is a brief discussion of the measurements that have led to the WGAC consensus value in Eq. (146).

1. NMIJ result for $V_m(Si)$

The new NMIJ result for $V_m(Si)$ is reported by Fujii (2003) and Fujii *et al.* (2003). The density standards used as references in this work were 1 kg silicon spheres S4 and S5 prepared from boule, or ingot, NRLM3 and whose absolute mass densities were determined in 1994 from optical interferometric measurements of their diameters and measurements of their masses in terms of the SI unit of mass by weighing in both air and vacuum. The samples for which $V_m(Si)$ was determined in the new measurement campaign are 1 kg silicon spheres S1 and S2 fabricated from ingot NRLM1, sphere S3 fabricated from ingot NRLM2, spheres S6 and S7 fabricated from ingot NRLM4, and two rectangular parallelepiped samples A1-2-1 and A4-2-1 also prepared from ingot NRLM4. The mass densities of all seven of these samples were determined from 14 comparisons using hydrostatic weighing or the pressure-of-flotation method, with each comparison involving two or three samples and with some involving spheres S4 and S5 whose densities are known.

The molar masses of 19 samples taken from ingots NRLM1, NRLM2, and NRLM4 were measured at IRMM by isotopic mass spectrometry in which synthetic isotope mixtures prepared gravimetrically from highly enriched ^{28}Si, ^{29}Si, and ^{30}Si compounds are used to calibrate the spectrometer. During the 1990s, improvements in the mass spectrometer and improved understanding of various effects such as time-dependent mass fractionation as prescribed by kinetic gas theory, adsorption and desorption, and back diffusion, have led to reductions in the uncertainty of IRMM silicon molar mass measurements (De Bièvre, Valkiers, *et al.*, 1995; Gonfiantini *et al.*, 1997). More recently, however, the $BaSiF_6$ compounds bearing the synthesized amount-of-substance ratios $n(^{A}Si)/n(^{28}Si)$ used for spectrometer calibration have shown chemical instabilities, which have led to an increase in uncertainty to the current value $u_r=1.4\times10^{-7}$ and the use of crystal WASO 17.2, whose molar mass had been measured previously, as the reference crystal in terms of which IRMM measurements of the unknown molar masses of silicon samples are now made. The molar mass measurements of the 16 NRLM4 samples, as well as the six samples from ingot WASO 04 discussed below in connection with the new PTB determination of $V_m(Si)$, were carried out in this way.

The NRLM1 molar mass sample is taken from between spheres S1 and S2, the two NRLM2 molar mass samples are taken from either side of sphere S3, and the 16 NRLM4 molar mass samples are taken from judiciously selected locations throughout the NRLM4 ingot. Because of the reduced uncertainty of the IRMM molar mass values for the NRLM4 samples, the large number of samples, and the fact that they were selected from

sites throughout the ingot, the preferred NMIJ value of $V_m(Si)$ is based on only the NRLM4 data. The result is (Fujii, 2003)

$$V_m(Si) = 12.058\,8272(30) \times 10^{-6}\ m^3\ mol^{-1} \quad [2.5 \times 10^{-7}]. \tag{147}$$

In obtaining this value, the NMIJ researchers took into account the isotopic gradient in the ingot as determined from the molar masses and positions of the 16 NRLM4 molar mass samples. Corrections were, of course, also applied to account for the C and O impurities in the ingot. The quoted value was deduced by calculating the molar volume of each of the four NRLM4 samples—S6, S7, A1-2-1, and A4-2-1—and then calculating the weighted mean of these four values, taking into account their covariances, and multiplying each element of the initial covariance matrix by the square of the Birge ratio $R_B = 1.36$ associated with the weighted mean. This led to an increase in the initial standard uncertainty of the weighted mean by the factor 1.36. (Note that $R_B = \sqrt{\chi^2/\nu}$, where χ^2 is the statistic "chi square," $\nu = N - M = 4 - 1 = 3$ is the degrees of freedom, with N the number of values and M the number of unknowns, and the expected value of χ^2 is assumed to be ν; see Appendix E of CODATA-98.)

2. PTB result for $V_m(Si)$

The new PTB result for $V_m(Si)$ is reported by Becker (2003a) and Becker *et al.* (2003). The sample for which $V_m(Si)$ was determined is sphere AVO#1, which is one of three 1 kg spheres prepared at NML from ingot WASO 04. [Researchers at IMGC and NML are determining $V_m(Si)$ using spheres AVO#2 and AVO#3, respectively, and all of the NMIJ spheres were manufactured at NML as well.] The WASO 04 ingot, which is well characterized with respect to impurities and crystallographic imperfections such as vacancies and self-interstitials, was originally about 165 cm long and 100 mm in diameter, with AVO#1 being prepared from its 54 cm to 64 cm segment. The volume of AVO#1 was measured using a newly developed interferometer with spherical symmetry that allows the spherical diameters over a 60° segment of the sphere to be measured simultaneously from Fizeau interference patterns. The measurements were carried out in vacuum at $t_{90} = 20\ °C$, and considerable care was taken to properly account for the effect of the sphere's oxide layer and other surface layers on the determination of its mean radius as well as its mass. The mass of AVO#1 was obtained from two series of weighings in air, one carried out for 20 days using a Pt-Ir standard of mass and the other for 8 days using a stainless steel standard. The relative standard uncertainty of the mass density of sphere AVO#1 after correction for impurities, surface layers, and the application of corrections to convert its mass density to the reference conditions $t_{90} = 22.5\ °C$ and $p = 0$, is given as $u_r = 9.0 \times 10^{-8}$.

The molar mass of sphere AVO#1 is based on IRMM measurements of six WASO 04 samples, two taken from the 54 cm to 64 cm segment of the WASO 04 ingot, two from the 75 cm to 85 cm segment, and two from the 108 cm to 118 cm segment. These segments correspond to those portions of the ingot from which the three spheres were fabricated. Unfortunately, the scatter between the two values of molar mass obtained for a given pair of samples is rather larger than one would expect. To account for this scatter, an additional component of uncertainty, equal to the standard uncertainty of the molar masses of all six samples, which corresponds to the fractional amount 4.1×10^{-7}, is combined with the relative standard uncertainty 1.4×10^{-7} for an individual IRMM molar mass measurement to obtain the relative standard uncertainty $u_r[M(AVO\#1)] = 4.1 \times 10^{-7}$ for the molar mass of sphere AVO#1. The final result for $V_m(Si)$ is the quotient of the molar mass and mass density and is (Becker, 2003a; Becker *et al.*, 2003; Fujii, 2003)

$$V_m(Si) = 12.058\,8199(53) \times 10^{-6}\ m^3\ mol^{-1} \quad [4.4 \times 10^{-7}]. \tag{148}$$

Because the molar mass values used in both the NMIJ and PTB determinations of $V_m(Si)$ result from IRMM measurements based on WASO 17.2, the two values are correlated with a correlation coefficient of 0.161 and a covariance of $2.572 \times 10^{-18}\ (m^3\ mol^{-1})^2$. [The fixed component of relative standard uncertainty of any IRMM molar mass measurement is 1.33×10^{-7}, which gives rise to a covariance of $13.95 \times 10^{-18}\ (m^3\ mol^{-1})^2$ for any two IRMM measurements of molar mass.]

3. WGAC value of $V_m(Si)$

The WGAC consensus value for $V_m(Si)$ to be used in the 2002 adjustment, which is given in Eq. (146), is the weighted mean of the NMIJ and PTB values in Eqs. (147) and (148), respectively, taking into account their covariance of $2.572 \times 10^{-18}\ (m^3\ mol^{-1})^2$, and multiplying each element of the initial covariance matrix by the square of the Birge ratio $R_B = 1.30$ associated with the weighted mean (see above discussion). The initial standard uncertainty of the weighted mean is therefore increased by the factor 1.30. We note that this value for the molar volume of silicon exceeds the earlier WGAC value in Eq. (145) by the fractional amount 4.1×10^{-7}.

The value of h that can be inferred from Eq. (144), the new WGAC consensus value for $V_m(Si)$ in Eq. (146) (identification N/P/I-03), together with the 2002 recommended values for $A_r(e)$, α, R_∞, and d_{220}, is

$$h = 6.626\,0762(21) \times 10^{-34}\ J\ s \quad [3.2 \times 10^{-7}]. \tag{149}$$

A comparison of this value of h with those in Tables VII and VIII shows that it is generally not in good agreement with the other values.

K. Quotient of Planck constant and particle mass $h/m(X)$

The relation $R_\infty = \alpha^2 m_e c/2h$ leads to

TABLE IX. Summary of data related to the quotient $h/m_n d_{220}$(W04), the {220} lattice spacing of silicon, and the quotient h/m(Cs), together with inferred values of α.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
$h/m_n d_{220}$(W04)	2060.267 004(84) m s^{-1}	4.1×10^{-8}	PTB-99	III.K.1 (151)
d_{220}(NR3)	192 015.587(11) fm	5.6×10^{-8}	NMIJ-97	III.I.2 (139)
α^{-1}	137.036 0015(47)	3.4×10^{-8}		III.K.1 (153)
h/m(Cs)	3.002 369 430(46)$\times10^{-9}$ m^2 s^{-1}	1.5×10^{-8}	Stanford-02	III.K.2 (160)
α^{-1}	137.036 0001(11)	7.7×10^{-9}		III.K.2 (162)

$$\alpha = \left[\frac{2R_\infty}{c}\frac{A_r(\mathrm{X})}{A_r(\mathrm{e})}\frac{h}{m(\mathrm{X})}\right]^{1/2}, \tag{150}$$

where A_r(X) is the relative atomic mass of particle X with mass m(X) and A_r(e) is the relative atomic mass of the electron. Because c is exactly known, the relative standard uncertainty of R_∞ and A_r(e) are less than 7×10^{-12} and 5×10^{-10}, respectively, and the uncertainty of A_r(X) for many particles and atoms is less than that of A_r(e), Eq. (150) can provide a value of α with a competitive uncertainty if h/m(X) is determined with a sufficiently small uncertainty. Here we discuss the determination of h/m(X) for the neutron n and for the ^{133}Cs atom.

1. Quotient h/m_n

The PTB determination of h/m_n, the result of an approximately 25-year effort, was discussed at length in CODATA-98. In brief, the de Broglie relation $p=m_n v=h/\lambda$ was used to determine $h/m_n=\lambda v$ for the neutron by measuring both the de Broglie wavelength λ and the corresponding velocity v of slow neutrons. The measurements were carried out at the ILL high-flux reactor after initial investigations at the PTB reactor. The de Broglie wavelength, $\lambda\approx0.25$ mm, of slow neutrons was determined using back reflection from a silicon crystal, and the velocity, $v\approx1600$ m/s, of the neutrons was determined by a special time-of-flight method. The final result of the experiment is (Krüger *et al.* 1999)

$$\frac{h}{m_n d_{220}(\mathrm{W04})} = 2060.267\,004(84)\ \mathrm{m\,s^{-1}} \quad [4.1\times10^{-8}], \tag{151}$$

where as before, d_{220}(W04) is the {220} lattice spacing of the crystal WASO 04 at $t_{90}=22.5\,^\circ$C in vacuum. This result is correlated with the PTB fractional lattice-spacing differences given in Eqs. (134)–(137)—the correlation coefficients are about 0.2.

The observational equation for the PTB result, which follows from Eq. (150), is

$$\frac{h}{m_n d_{220}(\mathrm{W04})} \doteq \frac{A_r(\mathrm{e})}{A_r(\mathrm{n})}\frac{c\alpha^2}{2R_\infty d_{220}(\mathrm{W04})}. \tag{152}$$

The value of α that can be inferred from this relation and the PTB value of $h/m_n d_{220}$(W04), the 2002 recommended values of R_∞, A_r(e), and A_r(n), the NIST and PTB fractional lattice-spacing differences in Eqs. (15)–(17) and Eqs. (134)–(137), and the NMIJ result for d_{220}(NR3) given in Eq. (139), is

$$\alpha^{-1} = 137.036\,0015(47) \quad [3.4\times10^{-8}], \tag{153}$$

which is also included in Table IX.

2. Quotient $h/m(^{133}\mathrm{Cs})$

The atomic recoil frequency shift of photons absorbed and emitted by cesium atoms is being measured at Stanford University in order to determine the quotient $h/m(^{133}\mathrm{Cs})$ and thus the fine-structure constant (Peters *et al.* 1997; Wicht *et al.*, 2002).

The atomic recoil frequency shift follows from energy and momentum conservation. In its simplest form, when an atom at rest decays, the momentum of the emitted photon is balanced by a recoil of the atom, and the emitted photon has less energy than the energy difference of the atomic levels, because part of the transition energy appears as the kinetic energy of the recoiling atom. This can be extended to a process that involves both absorption and emission, as in the case where a photon of frequency ν_1 propagating in the x direction is absorbed by an atom of mass m_1 initially at rest and a second photon of frequency ν_2 is emitted by the atom in the $-x$ direction. In the final state, the atom has mass m_2 and a recoil momentum of magnitude p. The mass difference is taken into account in order to include the case in which the absorbed and emitted photons correspond to transitions between different hyperfine levels of the ground state and a common excited state, where the hyperfine frequency difference is $\Delta mc^2/h$, with $\Delta m=m_2-m_1$. The frequency shift due to the recoil is $\Delta\nu=\nu_1-\nu_2-\Delta mc^2/h$. Conservation of momentum and energy for this process yields

$$\Delta\nu = \frac{h\nu_{\rm eff}^2}{2m_2c^2}\left[1-\left(\frac{\Delta\nu}{\nu_{\rm eff}}\right)^2\right] \approx \frac{h\nu_{\rm eff}^2}{2m_2c^2}, \tag{154}$$

where $\nu_{\rm eff}=\nu_1+\nu_2$. Hence, for the Stanford experiment,

$$\frac{h}{m(^{133}\mathrm{Cs})} = \frac{c^2\Delta\nu_{\rm Cs}}{2\nu_{\rm eff}^2}, \tag{155}$$

where the mass in the denominator is taken to be the mass of the cesium atom in its ground state with negligible loss of accuracy.

This recoil frequency shift leads to spectral doubling in saturation absorption spectroscopy, as predicted by Kol'chenko *et al.* (1968) and optically resolved by Hall *et al.* (1976). Hall *et al.* (1976) also pointed out that the splitting provides a measure of h/m.

Equation (154) assumes that the atoms are initially at rest, and a nonzero initial velocity v_x leads to a correction to the recoil frequency shift $\Delta\nu$ of order $\nu v_x/c$. To eliminate the contribution linear in the atom's initial velocity, the Stanford experiment uses atom interferometry, where instead of using π-pulse counterpropagating beams of photons it employs two pairs of $\pi/2$ pulses to create two interferometers (Bordé, 1989) in order to make a differential measurement possible. This leads to a Ramsey interference pattern in the probability that atoms arrive in the initial state at the final intersection point of the interferometer as a function of the frequency of the second pair of $\pi/2$ pulses for each interferometer, which is independent of the initial velocity of the atoms. In this case, the separation of the centers of the two interference patterns is twice the recoil frequency. In this approach, the effect of gravity is also eliminated.

Additional refinements are included in the experiment to improve the resolution: Between the two pairs of $\pi/2$ pulses, $N\approx 30$ additional π pulses are applied that increase the recoil frequency shift to $(N+1)\Delta\nu$. The experiment is done in an atomic fountain to provide long measurement times. To eliminate losses due to radiative decay, the ground- and excited-state combination is replaced by a combination of two hyperfine levels in the ground state, and two-photon Raman transitions are employed. Finally, the experiment uses a method of adiabatically transferring momentum to the atoms (Gaubatz *et al.*, 1988).

The Stanford program includes an extensive study of corrections due to possible systematic effects. The largest component of uncertainty in the final result for $h/m(^{133}\mathrm{Cs})$ arises from the possible deviation from 1 of the index of refraction of the cloud of cold cesium atoms. This effect has been checked experimentally by varying the density of cold cesium atoms with the result that it introduces a relative standard uncertainty component $u_{\rm r}=14\times10^{-9}$ (Type B) in the recoil frequency, which corresponds to $u_{\rm r}=7\times10^{-7}$ in the derived value of α. An independent check of this correction by a numerical simulation is in progress, and it may be possible for the uncertainty of α in this experiment to be reduced to $u_{\rm r}=3.1\times10^{-9}$ (Wicht *et al.*, 2002).

The result of the Stanford experiment for the recoil frequency shift $\Delta\nu_{\rm Cs}$ is (Wicht *et al.*, 2002)

$$\frac{\Delta\nu_{\rm Cs}}{2} = 15\,006.276\,88(23)\ \mathrm{Hz} \quad [1.5\times10^{-8}]. \tag{156}$$

In order to obtain the ratio $h/m(^{133}\mathrm{Cs})$ from Eq. (155) and this result, a value of the effective frequency $\nu_{\rm eff}$ is needed. For the Stanford experiment, this frequency corresponds to the sum of the energy difference between the ground-state hyperfine level with $F=3$ and the $6P_{1/2}$-state $F=3$ hyperfine level and the energy difference between the ground-state hyperfine level with $F=4$ and the same $6P_{1/2}$ hyperfine level.

These transition frequencies have been determined accurately by Udem *et al.* (1999a), who used a phase-coherent optical frequency measurement technique based on ultrashort light pulses. This experiment compared the cesium transition frequencies to the fourth harmonic of a transportable CH_4-stabilized 3.39 μm He–Ne laser that was calibrated against a cesium atomic clock at the PTB. The remaining difference between the 4×88.4 THz=354 THz harmonic and the 335 THz frequency of the cesium transitions was measured with a frequency comb spanning about 244 000 modes of a Kerr-lens, mode-locked laser. The results of this measurement are

$$\nu[6S_{1/2}(F=3)-6P_{1/2}(F=3)] = 335\,120\,562\,838(41)\ \mathrm{kHz}, \tag{157}$$

$$\nu[6S_{1/2}(F=4)-6P_{1/2}(F=3)] = 335\,111\,370\,206(41)\ \mathrm{kHz}, \tag{158}$$

which sum to

$$\nu_{\rm eff} = 670\,231\,933\,044(81)\ \mathrm{kHz} \quad [1.2\times10^{-10}]. \tag{159}$$

The frequencies in Eqs. (157) and (158) have a dominant component of uncertainty from the Zeeman shift of the levels, and may be highly correlated. The uncertainty of the value in Eq. (159) allows for this possible correlation, and hence it is almost the linear sum of the uncertainties of the individual frequencies.

Evaluation of Eq. (155) with the frequencies in Eqs. (156) and (159) yields

$$\frac{h}{m(^{133}\mathrm{Cs})} = 3.002\,369\,430(46)\times10^{-9}\ \mathrm{m^2\,s^{-1}} \quad [1.5\times10^{-8}], \tag{160}$$

where the uncertainty is almost entirely due to the uncertainty in the recoil frequency shift $\Delta\nu$. The value of the quotient in Eq. (160) is taken as an input datum in the 2002 adjustment, and the corresponding observational equation is

$$\frac{h}{m(^{133}\mathrm{Cs})} \doteq \frac{A_{\rm r}(\mathrm{e})}{A_{\rm r}(^{133}\mathrm{Cs})}\frac{c\alpha^2}{2R_\infty}. \tag{161}$$

This equation also depends on R_∞ and the mass ratio $A_{\rm r}(\mathrm{e})/A_{\rm r}(^{133}\mathrm{Cs})$; the determination of these relative

atomic masses is discussed in Sec. III.A.1.e for cesium and in Secs. III.A.4.a and III.C.3 for the electron.

Evaluation of Eq. (150) for α with the Stanford result for $h/m(^{133}\mathrm{Cs})$ together with the 2002 recommended values of R_∞, $A_r(\mathrm{e})$, and $A_r(^{133}\mathrm{Cs})$, whose uncertainties are inconsequential in this application, yields

$$\alpha^{-1} = 137.036\,0001(11) \quad [7.7\times10^{-9}], \tag{162}$$

where the dominant component of uncertainty arises from the measured value of the recoil frequency shift, in particular, the component of uncertainty due to a possible index-of-refraction effect. The uncertainty of this value of α is smaller than the uncertainty of any other value except that of $\alpha[a_e]$, and it exceeds that uncertainty by only a factor of about 2.

L. Hyperfine structure

The ground-state hyperfine transition frequencies of hydrogen, muonium, and positronium, $\Delta\nu_H$, $\Delta\nu_{Mu}$, and $\Delta\nu_{Ps}$, respectively, are proportional to $\alpha^2R_\infty c$, hence, in principle, a value of α can be obtained by equating an experimental value for a splitting to its corresponding theoretical expression. In fact, a value of α with a relative standard uncertainty $u_r=5.8\times10^{-8}$ is deduced in this way in Sec. III.C.5.c from data on muonium.

For hydrogen, the relative standard uncertainty of the experimental value of $\Delta\nu_H$ is about 10^{-12} (Ramsey, 1990). However, the relative uncertainty of the theory is of the order of 10^{-6}, and thus the hydrogen hyperfine splitting does not provide a competitive value for α. The main sources of uncertainty in the theory are the rms electric charge and magnetic moment radii and the polarizability of the proton, which are used to calculate the effect of the finite size and internal structure of the proton on the theoretical value of $\Delta\nu_H$. These quantities, which can be deduced from experiment, are not well known compared to the measured value of the hyperfine splitting, although recent work has provided a significantly improved result for the nonpolarization contribution (Friar and Sick, 2004; see also Eides *et al.*, 2001b; Faustov and Martynenko, 2002c).

It is also not now possible to obtain a useful value of α from $\Delta\nu_{Ps}$. The relative standard uncertainty of the most accurate experimental value is 3.6×10^{-6}. Although progress has been made in recent years in the theoretical calculation of $\Delta\nu_{Ps}$, its relative uncertainty due to uncalculated terms is of the order of 10^{-6}. (See, for example, Czarnecki *et al.*, 1999, 2001b; Kniehl and Penin, 2000; Melnikov and Yelkhovsky, 2001; Markushin, 2002.)

M. Fine structure

As in the case of hyperfine splittings (see the previous section), fine-structure transition frequencies are proportional to $\alpha^2R_\infty c$ and could be used to deduce a value of α. Data related to the fine structure of hydrogen and deuterium are discussed in Sec. III.B in connection with the Rydberg constant. These data are included in the adjustment because of their influence on the adjusted value of R_∞. However, the value of α that can be derived from these data is not competitive.

The accuracy of the experimental determination of fine-structure transition frequencies involving hydrogen or deuterium 2P states is limited by the large natural widths of the levels. On the other hand, the 2^3P_J states of ^{4}He cannot decay to the ground 1^1S_0 state by allowed electric dipole transitions, so their levels are relatively narrow. Because the transition frequencies corresponding to the differences in energy of the three 2^3P levels can be both measured and calculated with reasonable accuracy, the fine structure of ^{4}He has long been viewed as a potential source of a reliable value of α.

The three frequencies of interest are $\nu_{01}\approx29.6$ GHz, $\nu_{12}\approx2.29$ GHz, and $\nu_{02}\approx31.9$ GHz, which correspond to the intervals $2^3P_1-2^3P_0$, $2^3P_2-2^3P_1$, and $2^3P_2-2^3P_0$, respectively. Improvements in experiment have been especially significant during the last decade (for a review of the early work, see Pichanick and Hughes, 1990). For example, the group at York University, Toronto, Canada has reported the value (George *et al.*, 2001)

$$\nu_{01} = 29\,616\,950.9(9)\ \mathrm{kHz} \quad [3.0\times10^{-8}], \tag{163}$$

and three other groups are carrying out similar measurements: one at Harvard University (Roach *et al.*, 1998), one at the European Laboratory for Nonlinear Spectroscopy (LENS), Firenze, Italy (Pastor *et al.*, 2001), and one at the University of North Texas (Castillega *et al.*, 2000). If the theoretical expression for ν_{01} were exactly known, the result of George *et al.* (2001) given in Eq. (163) would yield a value of α with $u_r=1.5\times10^{-8}$.

The past decade has seen significant progress in the theory of the 2^3P_J transition frequencies. Of particular interest are the recent papers of Drake (2002) and Pachucki and Sapirstein (2002, 2003). In fact, the relative standard uncertainty $u_r=6.1\times10^{-9}$ of the theoretical value for ν_{01} quoted by Drake (2002) taken at face value, together with the uncertainty of the experimental value in Eq. (163), would yield a relative uncertainty for the inferred value of α of 1.5×10^{-8}. However, as is pointed out by Drake (2002) and Pachucki and Sapirstein (2002, 2003), because of the considerable complexity of the calculations and the history of their evolution, results that have not been confirmed by independent evaluations should be taken as tentative. This situation can be compared to that of the theory of the anomalous magnetic moment of the electron a_e. In both cases, extremely complex and difficult calculations are involved. However, in the case of the helium fine structure, it is necessary to identify in the context of the more complex formulation of the bound-state problem the terms to be calculated to reach a given level of uncertainty, while in the case of a_e, the set of Feynman integrals to be evaluated is explicitly known.

The present situation for the ^{4}He fine structure is further complicated by the fact that there is poor agreement between the theoretical and experimental values of ν_{12} (Castillega *et al.*, 2000; Storry *et al.*, 2000; Drake,

2002; Pachucki and Sapirstein, 2002, 2003). This suggests that there is a problem with theory and/or experiment which must be resolved before a meaningful value of α can be obtained from the helium fine structure (Drake, 2002; Pachucki and Sapirstein, 2003). Therefore we do not include ^{4}He fine-structure data in the 2002 adjustment.

N. Molar gas constant *R*

As discussed in CODATA-98, the square of the speed of sound $c_a^2(p,T)$ of a real gas at pressure p and thermodynamic temperature T can be written as (Colclough, 1973)

$$c_a^2(p,T) = A_0(T) + A_1(T)p + A_2(T)p^2 + A_3(T)p^3 + \cdots, \tag{164}$$

where $A_1(T)$ is the first acoustic virial coefficient, $A_2(T)$ is the second, etc. In the limit $p \to 0$, we have

$$c_a^2(0,T) = A_0(T) = \frac{\gamma_0 RT}{A_r(\mathrm{X})M_u}, \tag{165}$$

where the expression on the right-hand side is the square of the speed of sound for an unbounded ideal gas, and where $\gamma_0 = c_p/c_V$ is the ratio of the specific-heat capacity of the gas at constant pressure to that at constant volume, $A_r(\mathrm{X})$ is the relative atomic mass of the atoms or molecules of the gas, and $M_u = 10^{-3}$ kg mol^{-1}. For a monatomic ideal gas, $\gamma_0 = 5/3$.

The 1998 recommended value of R was based on measurements of the speed of sound in argon. (For a recent review, see Gavioso, 2001.) Values of $c_a^2(p,T_{tw})$, where $T_{tw} = 273.16$ K is the triple point of water, were obtained at various pressures and extrapolated to $p=0$ in order to determine $A_0(T_{tw}) = c_a^2(0,T_{tw})$ and hence R from the relation

$$R = \frac{c_a^2(0,T_{tw})A_r(\mathrm{Ar})M_u}{\gamma_0 T_{tw}}, \tag{166}$$

which follows from Eq. (165). These measurements were carried out in two independent experiments, one done in the 1970s at NPL and the other done in the 1980s at NIST. The relative standard uncertainties of the values of R obtained in the two experiments are $u_r = 8.4 \times 10^{-6}$ and $u_r = 1.8 \times 10^{-6}$, respectively. Because the work of both laboratories is fully reviewed in CODATA-98, and nothing has occurred since then that would change the values of R, we give only a brief summary here. [In fact, the relative atomic masses of the elements Xe and N are required to correct for impurities in the argon samples used in the NIST and NPL experiments, and the IUPAC recommended value for each of these elements (Coplen, 2001) has changed slightly since the 1998 adjustment. However, the effect on the NIST and NPL values of R is negligible. Similarly, the use of the improved value of the relative atomic mass of ^{36}Ar in Table III to calculate the relative atomic mass of the NIST and NPL samples leads to insignificant changes in the values of R.]

Since R cannot be expressed as a function of any other of our adjusted constants, we take R itself as an adjusted constant and the relation

$$R \doteq R \tag{167}$$

as the observational equation for the NIST and NPL measured values of R.

1. NIST: Speed of sound in argon

In the NIST experiment of Moldover *et al.* (1988), a spherical acoustic resonator at a temperature $T = T_{tw}$ filled with argon, of fixed dimensions (180 mm inside diameter), and operated near five different radially symmetric modes at frequencies in the range 2.4 kHz–9.5 kHz was used to determine $c_a^2(p,T_{tw})$. The volume of the resonator, the value of which enters the relation between the speed of sound in the argon and the resonant frequencies of the resonator, was measured by determining the mass of the amount of mercury of precisely known density necessary to fill the resonator at the temperature T_{tw}. Seventy data points for $c_a^2(p,T_{tw})$ vs p obtained from measurements of the frequencies of the five modes at each of 14 different values of p in the range 25 kPa–0.5 MPa were used to extrapolate to $p=0$. The final NIST result for the molar gas constant is

$$R = 8.314\,471(15)\ \mathrm{J\ mol^{-1}\ K^{-1}} \quad [1.8 \times 10^{-6}]. \tag{168}$$

The mercury employed to determine the volume of the spherical resonator was traceable to the mercury whose density was measured by Cook (1961; see also Cook and Stone, 1957). The mercury employed in the NML Hg electrometer determination of K_J (see Sec. III.E.1) was also traceable to the same mercury. Consequently the NIST value of R and the NML value of K_J are correlated with the non-negligible correlation coefficient 0.068.

2. NPL: Speed of sound in argon

In contrast to the dimensionally fixed, multiple-frequency spherical acoustic resonator used in the NIST experiment, the NPL experiment employed a variable path length, 5.6 kHz fixed-frequency cylindrical acoustic interferometer to measure $c_a^2(p,T_{tw})$. A transducer of frequency $f = 5.6$ kHz was located at the bottom end of a 30 mm diameter cylindrical vertical cavity; it excited and monitored the cavity's resonant frequencies as the acoustic reflector forming the top of the cavity was moved and its displacement measured by means of an optical interferometer. Resonances were separated by $\Delta l = \lambda/2$, where Δl is the change in length of the cavity and λ is the wavelength of the standing wave in the cavity. The speed of sound was calculated from the known value of f and the value of λ, which was determined from the measured separations of five resonances.

The final result of the first NPL determination of R reported by Quinn *et al.* (1976), obtained in the pressure range 30 kPa–200 kPa, was subsequently found to be in

error because a nonlinearity of the transducer had been overlooked. The correction for this error was applied to the original 98 data points (together with some additional corrections for some relatively minor effects), and 48 new data points were obtained in the pressure range 200 kPa–1.3 MPa with an improved apparatus. All 146 data points were then used to extrapolate to $p=0$. The final NPL result for the molar gas constant is (Colclough *et al.*, 1979)

$$R = 8.314\,504(70)\ \text{J mol}^{-1}\,\text{K}^{-1} \quad [8.4\times10^{-6}]. \tag{169}$$

Although both the NIST and NPL values of R are based on the same values of $A_r(^{40}\text{Ar})$, $A_r(^{38}\text{Ar})$, and $A_r(^{36}\text{Ar})$, the uncertainties of these relative atomic masses are sufficiently small that the covariance of the two values of R is negligible.

3. Other values

The most important of the historical values of R have been reviewed by Colclough (1984; see also Quinn *et al.*, 1976 and CODATA-98). However, because of the large uncertainties of these early values, they were not considered for use in either the 1986 or the 1998 CODATA adjustments, and we exclude them from the 2002 adjustment as well.

Recently, a value of R obtained from measurements of the speed of sound in argon at a temperature of about 20 °C, using a spherical acoustic resonator of about 140 mm inside diameter similar to that employed at NIST by Moldover *et al.* (1988), was reported by He and Liu (2002) at the Xián Jiaotong University, Xián, People's Republic of China. Their result is

$$R = 8.314\,39(30)\ \text{mol}^{-1}\,\text{K}^{-1} \quad [3.6\times10^{-5}]. \tag{170}$$

The resonator was operated at five frequencies in the range 3.6 kHz–14.2 kHz, and the pressure of the argon gas was in the range 200 kPa–800 kPa. The quoted uncertainty is dominated by the 2.7×10^{-5} and 2.3×10^{-5} relative standard uncertainty components arising from the measurement of the temperature of the argon gas and the diameter of the resonator, respectively. Because the uncertainty of this new result is 20 times that of the NIST result, it is not considered further. We do note that the two values are in agreement.

O. Boltzmann constant k

The Boltzmann constant is related to the molar gas constant R and other adjusted constants by

$$k = \frac{2R_\infty h}{cA_r(\text{e})M_u\alpha^2}R = \frac{R}{N_A}. \tag{171}$$

The 1998 recommended value was, in essence, obtained from this relation and has a relative standard uncertainty $u_r=1.7\times10^{-6}$. No competitive directly measured value of k was available for the 1998 adjustment, and the situation remains unchanged for the current adjustment. However, we do take this opportunity to recall the discussion given in CODATA-98 of a potentially useful approach to the direct determination of k, inasmuch as a significant improvement in theory has been achieved in the last four years that makes it more attractive.

The approach is based on the virial expansion of the Clausius-Mossotti equation for a real gas of atoms of amount of substance n occupying a volume V (see Pendrill, 1996):

$$\frac{\epsilon-\epsilon_0}{\epsilon+2\epsilon_0} = \frac{n}{V}A_\epsilon\left(1+\frac{n}{V}B_\epsilon+\frac{n^2}{V^2}C_\epsilon+\cdots\right). \tag{172}$$

Here ϵ is the permittivity of the gas, ϵ_0 is the exactly known electric constant (see Sec. II.B), A_ϵ is the molar polarizability of the atoms, and B_ϵ, C_ϵ, etc. are the dielectric virial coefficients. The molar polarizability A_ϵ is related to the molar gas constant R, the Boltzmann constant k, and the static electric dipole polarizability of the atoms α_0 by

$$A_\epsilon = \frac{R\alpha_0}{3\epsilon_0 k}. \tag{173}$$

Hence a measurement of A_ϵ/R together with a theoretical value for α_0 yields a value of k.

As discussed in CODATA-98, A_ϵ/R can be measured by means of dielectric-constant gas thermometry, and the value with the smallest uncertainty obtained to date is that of Luther *et al.* (1996), who used ^{4}He over the temperature range 4.2 K–27 K:

$$\frac{A_\epsilon}{R} = 6.221\,12(19)\times10^{-8}\ \text{K Pa}^{-1} \quad [3.0\times10^{-5}]. \tag{174}$$

Ab initio calculations of $\alpha_0^*(^4\text{He})=\alpha_0(^4\text{He})/4\pi\epsilon_0 a_0^3(1+m_e/m_\alpha)^3$, the static electric dipole polarizability of the 1^1S ground state of the ^{4}He atom $\alpha_0(^4\text{He})$ expressed in the ^{4}He reduced atomic unit of electric polarizability, have been carried out over the years by a number of workers (a_0 is the Bohr radius and m_e/m_α is the electron to alpha-particle mass ratio). In terms of this calculated value and the experimentally determined value of A_ϵ/R for ^{4}He, Eq. (173) yields

$$k = \frac{4\pi a_0^3(1+m_e/m_\alpha)^3}{3}\frac{\alpha_0^*(^4\text{He})}{(A_\epsilon/R)_{^4\text{He}}}. \tag{175}$$

Recently, an accurate value of $\alpha_0^*(^4\text{He})$ has been reported by Pachucki and Sapirstein (2001), who calculated relativistic and QED corrections to the nonrelativistic value and confirmed earlier calculations of the nonrelativistic value. Their result is

$$\alpha_0^*(^4\text{He}) = 1.383\,191(2) \quad [1.4\times10^{-6}], \tag{176}$$

where the uncertainty is meant to account for uncalculated QED contributions. The nonrelativistic value and the relativistic correction of Pachucki and Sapirstein (2001) were confirmed by Cencek *et al.* (2001), who did not calculate the QED correction. The earlier result of Bhatia and Drachman (1998) for the relativistic correc-

tion differs from the recent calculations by about 0.4% of the correction, while the still earlier result of Johnson and Cheng (1996) differs by about 5%. The nonrelativistic values obtained in the recent work are in agreement with the earlier calculation of Bhatia and Drachman (1994), and if the mass polarization term is omitted, they are in agreement with the calculation of Yan *et al.* (1996).

The value of k that follows from Eqs. (174)–(176) is

$$k = 1.380\,65(4) \times 10^{-23} \quad [3.0 \times 10^{-5}], \tag{177}$$

based on the 2002 recommended values of a_0 and m_e/m_α, whose uncertainties are negligible in this context.

The uncertainty of this value of k is essentially due to the experimental uncertainty in Eq. (174) and is too large for this value to be included in the 2002 adjustment. Nevertheless, it is in excellent agreement with the 2002 recommended value, and the small theoretical uncertainty improves the prospect of this being a competitive method for determining the Boltzmann constant.

P. Stefan-Boltzmann constant σ

The Stefan-Boltzmann constant is related to c, h, and the Boltzmann constant k by

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2}, \tag{178}$$

which, with the aid of Eq. (171), can be expressed in terms of the molar gas constant and other adjusted constants as

$$\sigma = \frac{32\pi^5 h}{15c^6}\left(\frac{R_\infty R}{A_r(e)M_u\alpha^2}\right)^4. \tag{179}$$

The 1998 recommended value was essentially obtained from this relation and has a relative standard uncertainty $u_r = 7.0 \times 10^{-6}$. No competitive directly measured value of σ was available for the 1998 adjustment, and the situation remains unchanged for the 2002 adjustment.

Q. Newtonian constant of gravitation G

Because there is no known quantitative theoretical relationship between the Newtonian constant of gravitation G and other fundamental constants, and because the currently available experimental values of G are independent of all of the other data relevant to the 2002 adjustment, these experimental values contribute only to the determination of the 2002 recommended value of G and can be considered independently from the other data.

The 1998 CODATA recommended value of G is

$$G = 6.673(10) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} \quad [1.5 \times 10^{-3}], \tag{180}$$

which has essentially the same numerical value as the 1986 CODATA recommended value $G = 6.672\,59(85) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ $[1.3 \times 10^{-4}]$ (Cohen and Taylor, 1987) but has a significantly increased uncertainty. The 1986 value is that obtained at NIST by Luther and Towler (1982) in a collaboration between NIST and the University of Virginia (NIST-UVA), but with its uncertainty doubled—the Task Group believed that the uncertainty of the 1986 recommended value should reflect the fact that, historically, measurements of G have been difficult to carry out and the result of Luther and Towler (1982) was possibly not final. (A detailed description of the Luther-Towler experiment, which employed a rather classic torsion balance operated in the dynamic mode and the time-of-swing method, is given in CODATA-98.)

The 1998 recommended value was selected by the Task Group after a careful review of the status of measurements of G. A number of points were considered, one of the most important being the existence of a highly credible value of G from researchers at the PTB (Michaelis *et al.*, 1996) that was in substantial disagreement with the 1986 recommended value. However, in the past four years, new results for G have been obtained that have led the Task Group to conclude that the PTB result should not be taken into account in the determination of the 2002 recommended value. In fact, experimental investigations of several critical aspects of the PTB determination of G recently carried out and reported by PTB researchers (Michaelis *et al.*, 2004) have led these authors to conclude that the PTB result for G and its uncertainty cannot be considered correct.

Table X summarizes the various measurements of G that we consider, and Fig. 1 compares them graphically. For reference purposes, the 1986, 1998, and 2002 CODATA recommended values are also included in the table and the 2002 value is included in the figure. The following comments apply to these data, but it should be noted that the LANL-97 and TR&D-98 values are unchanged from CODATA-98, the HUST-99 value has been corrected from its CODATA-98 value, the UWash-00 value is entirely new, the BIPM-01, UWup-02, UZur-02, and MSL-03 values replace the earlier values from these same experiments given in the corresponding table in CODATA-98, and since descriptions of all of these experiments are given in CODATA-98 except that of Gundlach and Merkowitz (2000) at the University of Washington, only it is described in any detail. [Note that, although it was included in the corresponding table in CODATA-98, the result of Schwarz *et al.* (1998, 1999) is not included in Table X because of its relatively large uncertainty.]

For simplicity, in the following text, we write G as a numerical factor multiplying G_0, where

$$G_0 = 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}. \tag{181}$$

(a) *LANL-97.* The experiment of Bagley and Luther (1997) at the Los Alamos National Laboratory is in many ways similar to the NIST-UVA experiment of Luther and Towler (1982), although the torsion fiber anelasticity problem pointed out by Kuroda

TABLE X. Summary of the results of measurements of the Newtonian constant of gravitation relevant to the 2002 adjustment together with the 1986, 1998, and 2002 CODATA recommended values. (See the text for brief discussions of the experiments.)

Item	Source	Identification	Method	$\frac{10^{11}\ G}{\mathrm{m^3\ kg^{-1}\ s^{-2}}}$	Rel. stand. uncert u_r
	1986 CODATA Adjustment	CODATA-86		6.672 59(85)	1.3×10^{-4}
	1998 CODATA Adjustment	CODATA-98		6.673(10)	1.5×10^{-3}
a.	Bagley and Luther (1997)	LANL-97	fiber torsion balance, dynamic mode	6.674 0(7)	1.0×10^{-4}
b.	Karagioz *et al.* (1998)	TR&D-98	fiber torsion balance, dynamic mode	6.672 9(5)	7.5×10^{-5}
c.	Luo (2003); Luo *et al.* (1999)	HUST-99	fiber torsion balance, dynamic mode	6.670 9(7)	1.0×10^{-4}
d.	Gundlach and Markowitz (2000, 2002)	UWash-00	fiber torsion balance, dynamic compensation	6.674 255(92)	1.4×10^{-5}
e.	Quinn *et al.* (2001)	BIPM-01	strip torsion balance, compensation mode, static deflection	6.675 59(27)	4.0×10^{-5}
f.	Kleinevoß (2002); Kleinvoß *et al.* (2002)	UWup-02	suspended body, displacement	6.674 22(98)	1.5×10^{-4}
g.	Schlamminger *et al.* (2002)	UZur-02	stationary body, weight change	6.674 07(22)	3.3×10^{-5}
h.	Armstrong and Fitzgerald (2003)	MSL-03	strip torsion balance, compensation mode	6.673 87(27)	4.0×10^{-5}
	2002 CODATA Adjustment	CODATA-02		6.6742(10)	1.5×10^{-4}

(1995) was taken into account. This result is now considered final and as superseding the 1982 NIST-UVA result.

(b) *TR&D-98*. The long-term researchers involved in the determination of G published by Karagioz *et al.* (1998) are now at the Tribotech Research and Development Company (TR&D), Moscow. Although no new results have since been reported by the TR&D group, group members have published a number of papers relevant to the determination of G in the last few years (Karagioz *et al.*, 1999, 2001; Izmailov *et al.*, 2001, 2002; Kudryavitskii *et al.*, 2001).

(c) *HUST-99*. The group at the Huazhong University of Science and Technology, Wuhan, has reported no new result for G since the publication of Luo *et al.* (1999). However, Luo (2003) has informed us that the two cylindrical stainless steel source, or field, masses used in the HUST experiment have been reweighed and the mass values obtained are consistent with those given by Luo *et al.* (1999), which were previously reported by Chen *et al.* (1984). However, the values are for the masses in vacuum and not in air, as they are used in the HUST experiment. This has led to an air buoyancy correction, first suggested by R. S. Davis and T. J. Quinn of the BIPM, and a 1.5×10^{-5} fractional increase in the original HUST-99 value of G. The result given in Table X is the corrected value. Some recent papers relevant to the determination of G published by members of the HUST group are Hu *et al.* (2001, 2002); Luo *et al.* (2001); Wang *et al.* (2001); Wu *et al.* (2003); Zhao *et al.* (2003).

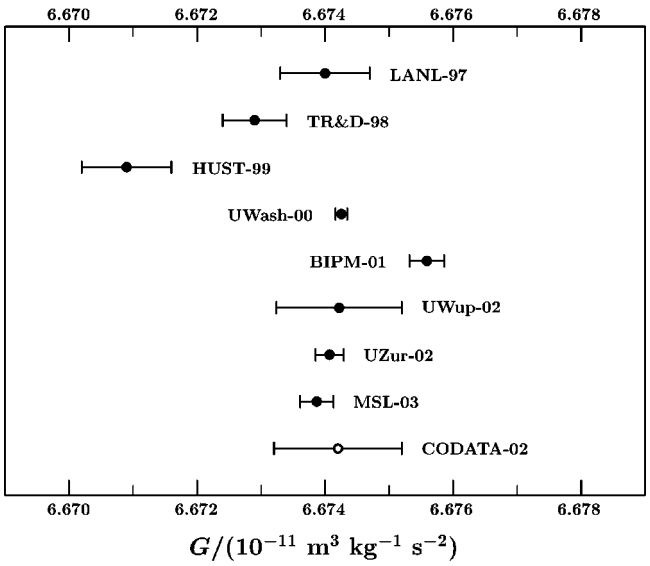

FIG. 1. Values of the Newtonian constant of gravitation G. See Glossary for the source abbreviation.

(d) *UWash-00*. The University of Washington, Seattle, result for G in Table X is reported by Gundlach and Merkowitz (2000, 2002) and has the smallest assigned uncertainty of any value ever published. The significantly reduced uncertainty is achieved by using a test mass in the form of a torsion pendulum of unique design and a comparably novel measurement method. The pendulum is a chrome-gold–coated rectangular Pyrex plate of thickness $t \approx 1.5$ mm, width $w \approx 76$ mm, and height $h \approx 42$ mm, and with a mass of ≈ 12 g. It is suspended from a 41.5 cm long, 17 μm diameter uncoated tungsten torsion fiber attached to the center of the long edge of the pendulum. The upper end of the fiber is attached to a magnetic damper, in the form of a disk centered between two permanent ring magnets, that is used to reduce pendulumlike motions of the test mass. The disk is attached to a 2.5 cm long, 76 μm diameter tungsten "prehanger" fiber, which is itself supported by vertical and rotational translation stages outside the vacuum chamber enclosing the pendulum system. (The importance of the damper is made clear at the end of this discussion.)

The vacuum chamber is fastened to a well-controlled turntable that, when activated, rotates the torsion pendulum between four stainless steel field, or attractor, masses ≈ 125 mm in diameter and of mass ≈ 8.1 kg. Two of the attractor masses rest on a horizontal plate around and just above the pendulum, and two on a horizontal plate around and just below the pendulum. The masses are symmetrically positioned on either side of the pendulum, one over the other (≈ 13.5 cm vertical separation between the centers of the spheres), and the length of the torsion fiber is coincident with the axes of the plates supporting the attractor masses, the turntable, and the cylindrical vacuum chamber. The two plates are attached to each other and to a second coaxial turntable completely independent of the turntable to which the vacuum chamber, and hence torsion pendulum, is attached.

As the pendulum turntable is rotated at a constant rate, for example, one revolution per 20 min, the fixed attractor masses subject the pendulum to a sinusoidal gravitational torque that leads to a small, twice-per-revolution angular deflection of the pendulum. When a feedback system that couples the angular position of the pendulum to the motion of the turntable is activated, the rotation rate of the turntable is altered so as to drive the deflection angle of the pendulum to zero. As a consequence, the resulting angular acceleration of the turntable, determined from the second derivative with respect to time of the turntable's angular position, is very nearly equal to the gravitational angular acceleration of the pendulum arising from the attractor masses. Since the torsion fiber does not experience any significant twist, fiber anelasticity problems are of minimal concern. The angular acceleration $\alpha(\phi)$ of the pendulum, where ϕ is the angle between the pendulum and attractor masses, is directly proportional to $G \sin 2\phi$, where, because of the carefully chosen geometry of the apparatus, the constant of proportionality depends only on the width w and thickness t of the flat-plate pendulum, the mass of the attractor masses, and the ≈ 16.5 cm radial distance of the attractor masses from the axis of the torsion pendulum, that is, the fiber. Corrections to the acceleration are small and easily calculated, and depend on only w and t.

Another important innovation is the rotation of the attractor mass turntable with angular velocity $\omega_a(t) = \omega_d + \omega_p(t)$, where $\omega_p(t)$ is the angular velocity of the torsion pendulum turntable. The angular velocity difference between the two turntables $\omega_d = \dot{\phi}$ is held constant at a convenient value. The gravitational constant G is obtained from the amplitude of the $\alpha(\phi)$ signal that varies as $\sin(2\omega_d t)$. This allows one to distinguish the gravitational acceleration of the pendulum due to nearby stationary masses, the signal of which varies as $\sin(2\omega_p t)$, from that due to the attractor masses. Shifting the frequency of the desired gravitational signal to a relatively higher frequency suppresses the troublesome $1/f$ noise characteristic of a torsion balance and of gravitational background noise. Thus, as emphasized by the University of Washington researchers, their new approach represents three major advances: (i) reduction of the metrology required to determine the exact dimensions and mass distribution of the pendulum, or test mass system; (ii) reduction of the $1/f$ noise from a heavily loaded torsion fiber, the unavoidable movement of nearby objects (for example, automobiles and people), seismic vibrations, and thermal, vacuum, and other fluctuations; and (iii) elimination of problems arising from twisting the torsion fiber.

The data on which the University of Washington result in Table X is based were obtained with two different sets of four spheres—the first set was used from 10 March 2000 to 1 April 2000, the second from 3 April 2000 to 18 April 2000. Three different values of G resulted from each set, each value being from a combination of a pair of attractor mass configurations that together eliminate the effect of accelerations due to the attractor plates and turntable themselves. The three values in a set were obtained using different, optimally chosen orientations of the spherical attractor masses in order to reduce the influence of imperfections in the shape of the spheres and possible nonuniform densities. For most of the data, $\omega_p(t) \approx 5.3$ mrad/s and $\omega_d \approx 20.01$ mrad/s so that the desired signal occurred at ≈ 6.37 mHz. A wide variety of values of ω_p and ω_d were examined but no dependence on these angular velocities was found. The statistical component of relative standard uncertainty in the final result is 5.8×10^{-6} (Type A), and the compo-

nent due to systematic effects is 12.5×10^{-6} (Type B), the four largest individual components being 7.1×10^{-6}, 6.9×10^{-6}, 5.2×10^{-6}, and $4.0.\times10^{-6}$ from measurements of the separation of the attractor masses on the same plate, temperature, the vertical separation of the attractor masses, and the thickness and flatness of the pendulum, respectively.

The value of G reported initially by Gundlach and Merkowitz (2000) is $G=6.674\,215(92)G_0$ [1.4×10^{-5}]. Subsequently, in the course of preparing a more detailed description of the experiment, Gundlach and Merkowitz (2002) identified an additional fractional correction of 6.0×10^{-6} due to a torque arising from the magnetic damper; the torque twisted the prehanger fiber and therefore the torsion fiber supporting the pendulum by about 5 nrad and in such a direction as to lower the measured value of $\alpha(\phi)$ by the same fractional amount, that is, 6.0×10^{-6}.

(e) *BIPM-01*. Only a first result of the BIPM experiment to measure G was available at the time of the 1998 adjustment, and that was from only one of the three possible methods of operation of the BIPM torsion balance indicated by Quinn *et al.* (2001): (i) electrostatic servo-control, or compensation mode; (ii) free, or static, deflection mode; and (iii) change in period of oscillation, or dynamic mode. A key ingredient of the BIPM balance is a thin, heavily loaded copper-beryllium alloy torsion strip that serves as the balance's suspension element and for which anelasticity effects are greatly reduced. (The stiffness of the torsional strip, required for the measurement of G using the balance in the deflection mode, is determined from the measured oscillation period of the balance.) The BIPM-01 result for G in Table X is the weighted mean of the values

$$G_{\rm cm}=6.675\,53(40)G_0 \quad [6.0\times10^{-5}], \tag{182}$$

$$G_{\rm dm}=6.675\,65(45)G_0 \quad [6.7\times10^{-5}], \tag{183}$$

with a correlation coefficient of −0.18, obtained from operation of the balance in the compensation mode and deflection mode, respectively (Quinn *et al.*, 2001). An important feature of the compensation or servo-controlled mode is the control of the balance via 1 kHz ac voltages applied between the test masses and pairs of thin vertical cylindrical copper electrodes about 1 mm from the test masses; and calibration of the electrostatic torque constant directly in SI units. This is done by measuring, also at 1 kHz, all $dC_{ij}/d\theta$, where θ is the displacement angle of the test mass system and C_{ij} is the capacitance between one copper electrode and the other copper electrode of a pair, the test masses, and the vacuum can of the balance. Operating the servo system at the same frequency used to calibrate the electrostatic torque constant eliminates the possibility of an error arising from frequency-dependent losses that could make the calibration of the torque constant at a high frequency invalid at the essentially zero frequency of a dc servo system. Quinn *et al.* (2001), who also paid special attention to the density uniformity of both the source and test masses, believe that the agreement of the two separate results, obtained by two substantially independent methods, strongly support the validity of their final result. This experiment is continuing with a rebuilt and much improved apparatus and should yield a result for G with a significantly reduced uncertainty (Quinn *et al.*, 2002).

(f) *UWup-02*. The final result of the experiment at the University of Wuppertal, Wuppertal, Germany is reported by Kleinevoß (2002) in his Ph.D. thesis and in a preprint by Kleinvoß *et al.* (2002). It is based on measurements carried out from January 2000 to May 2001 and as in earlier work (Kleinevoß *et al.*, 1999), with the two cylindrical 576 kg brass field masses in a horizontal position and symmetrically placed on opposite sides of a Fabry-Pérot microwave resonator suspended between them and with the axes of the resonator and field masses coincident. The two reflectors of the resonator are independently suspended by 2.6 m long tungsten wires, and the change in the length of the resonator arising from the change in position of the field masses is determined from the change in the resonance frequency of the resonator, which is in the range 20 GHz–26 GHz. The improvements incorporated in the apparatus for this new series of measurements include a better system for controlling the positions of the field masses, an active heating system which reduces temperature variations to $\Delta T\approx0.1$ K, and ancillary measurements which allow corrections to be applied for resonator tilt arising from the deformation of the laboratory floor when the field masses are moved; depending on the position of the masses, the fractional correction to G varies from 0.87×10^{-4} to 10.4×10^{-4}. The final University of Wuppertal value of G given in Table X is the combination of 12 different values obtained from measurements made with the field masses in six different positions and at the two resonator frequencies 23 GHz and 22 GHz. The statistical component of relative standard uncertainty (Type A) of the quoted result is 0.73×10^{-4}, while the component arising from systematic effects (Type B) is 1.27×10^{-4}.

The Wuppertal apparatus has been moved to DESY (Deutches Elektronen-Synchrotron), Hamburg, Germany, and it is expected that improvements in the experiment will eventually

lead to a value of G with a reduced uncertainty (Meyer, 2004).

(g) *UZur-02*. Table X gives the final result of the University of Zurich measurement of G, which was carried out at the Paul Scherrer Institute, Villigen, Switzerland. In this experiment, a modified commercial single-pan balance is used to measure the change in the difference in weight of two cylindrical test masses when the position of two source masses is changed. Reported by Schlamminger *et al.* (2002), the final result is the weighted mean, with correlations appropriately taken into account, of three values obtained from three series of measurements performed in August–September 2001, January–February 2002, and April–May 2002 and denoted Cu, Ta I, and Ta II, respectively. The designation Cu means that the test masses were gold plated copper, and the designation Ta means that they were tantalum. The position of the field masses was the same for the Cu and Ta II series of measurements and different for the Ta I series. The three values obtained are $G=6.674\,403\,G_0$, $G=6.674\,409\,G_0$, and $G=6.674\,410\,G_0$, respectively, all in good agreement. The component of the 3.3×10^{-5} relative standard uncertainty of the final University of Zurich result due to the nonlinearity of the balance was reduced to 2.1×10^{-5}, nearly an order of magnitude smaller than the previously estimated value for this component, by use of a new method which averages out the nonlinearity *in situ*. This was done by measuring the $\approx$800 μg mass difference signal at many different working points in the calibration interval using two sets of 16 individual wire weights. The other significant component of relative standard uncertainty, also equal to 2.1×10^{-5}, arises from the mass distribution of the liquid-mercury-filled stainless steel tanks that serve as the field masses. Additional effects taken into account include water adsorption by the test masses due to small changes in temperature and magnetic forces.

In a Comment on the paper of Schlamminger *et al.* (2002), Datta *et al.* (2003) point out an apparently remarkable correlation of the data with the lunar phase, possibly providing evidence for tidal effects. However, in their reply to the Comment, Kündig and Schlamminger (2003) convincingly argue that, although their data may "represent a possible, perhaps somewhat unusual statistical distribution," the data "show no evidence for tidal effects of the magnitude suggested in the Comment."

(h) *MSL-03*. The final result of the determination of G at the Measurement Standards Laboratory, Industrial Research, Lower Hutt, New Zealand is reported by Armstrong and Fitzgerald (2003). The MSL torsion balance is similar to the BIPM balance operated in the compensation mode in that an electrostatic torque generated by an electrometer, integral with the balance, is applied to the tungsten-fiber-suspended test mass (a 532 g horizontal copper cylinder of diameter 19 mm and length 220 mm) to compensate for the gravitational torque produced by two large, vertical, cylindrical field masses. Because the test mass remains stationary, the strip torsion fiber supporting it does not twist, and fiber anelasticity is not a problem. In the MSL experiment, the electrostatic torque constant is determined in a separate experiment by measuring the angular acceleration α of the test mass when the field masses are removed and a voltage U_A is applied to the electrometer. This is done by giving the entire torsion balance the same acceleration as the test mass, which keeps the fiber from twisting, and measuring the angular position of the balance as a function of time. This approach significantly reduces the dependence of the measured value of G on the length, mass, and density uniformity of the test mass. The improvements made to the MSL apparatus for the final series of measurements include remote mounting of the vacuum system to allow continuous pumping when the gravitational signal is measured, a magnetic damper to damp out pendulum modes of oscillation of the test mass, and copper field masses in addition to the previously used stainless steel masses. Further, a new measurement procedure was implemented that leads to more uncertainty components being purely statistical (Type A) in nature. The NML final result for G in Table X is obtained from four separate values, $G=6.673\,59(44)\,G_0$, $G=6.673\,98(46)\,G_0$, $G=6.673\,99(45)\,G_0$, and $G=6.673\,92(49)\,G_0$, where the first three were obtained using the new copper field masses and the last using the stainless steel masses, and where the uncertainties are statistical (Type A) only. The four values are in agreement. The uncertainty of the final result in Table X consists of 3.3×10^{-5} and 2.3×10^{-5} Type A and Type B relative standard uncertainty components, respectively, and is about 2.5 times smaller than the uncertainty of the previous MSL result, $G=6.6742(7)\,G_0\,[1.0\times10^{-4}]$ (Fitzgerald and Armstrong, 1999). The new and earlier values are consistent.

Although the situation with G has improved considerably since the 1998 adjustment, the eight available input data in Table X (items *a* to *h*) are not in complete agreement, as can be seen from the table and Fig. 1; their weighted mean is $G=6.674\,232(75)\,G_0\,[1.1\times10^{-5}]$, with $\chi^2=57.7$ for $\nu=N-M=8-1=7$ degrees of freedom and a Birge ratio of $R_B=\sqrt{\chi^2/\nu}=2.87$ (see Appendix E of CODATA-98). The eight normalized residuals $r_i=(G_i-\hat{G})/u(G_i)$ are −0.33, −2.66, −4.76, 0.25, 5.03, −0.01, −0.74, and −1.39, respectively. The value $|r_i|\approx5$ for the

HUST-99 and BIPM-01 values are obviously problematic. If each uncertainty $u(G_i)$ is multiplied by the value of the Birge ratio 2.87 so that χ^2 is reduced to its expected value of 7 and R_B to 1, then $G = 6.674\,23(22)\ G_0\ [3.2\times10^{-5}]$. If the two input data with the largest residuals are deleted, namely, the HUST-99 and BIPM-01 values of G, then the weighted mean is $G = 6.674\,159(79)\ G_0\ [1.2\times10^{-5}]$ with $\chi^2 = 8.9$, $\nu = 5$, and $R_B = 1.33$. In this case, the only datum with a significant normalized residual is the TR&D-98 value of G, for which $r_i = -2.52$. If it is also deleted, then the weighted mean of the remaining five input data is $G = 6.674\,191(80)\ G_0\ [1.2\times10^{-5}]$, with $\chi^2 = 2.4$, $\nu = 4$, and $R_B = 0.77$.

Finally, if the UWash-00 result for G, which has the smallest assigned uncertainty of any of the eight values, is deleted from the initial group, the weighted mean of the remaining seven data is $G = 6.674\,19(13)\ G_0\ [2.0\times10^{-5}]$, with $\chi^2 = 57.5$, $\nu = 6$, and $R_B = 3.10$. The normalized residuals are −0.27, −2.57, −4.69, 5.20, 0.03, −0.53, and −1.22, respectively. If each uncertainty $u(G_i)$ is multiplied by $R_B = 3.10$ so that χ^2 is reduced to its expected value of 6 and R_B to 1, we have $G = 6.674\,19(41)\ G_0\ [6.1\times10^{-5}]$.

Based on these various weighted means, all of which round to $G = 6.6742\ G_0$, as well as their uncertainties, the relatively poor agreement of the data, and the historic and apparently continuing difficulty of assigning an uncertainty to a measured value of G that adequately reflects its true reliability, the Task Group has taken

$$G = 6.6742(10)\times10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}} \quad [1.5\times10^{-4}] \tag{184}$$

as the 2002 recommended value.

This value exceeds the 1998 recommended value in Eq. (180) by the fractional amount 1.8×10^{-4}, which is only about 1/8 times the latter's relative standard uncertainty $u_r = 1.5\times10^{-3}$, and its uncertainty is a factor of 10 smaller than that of the 1998 recommended value. The BIPM-01 result for G, which is the highest of the eight in Table X, lies above the 2002 recommended value G_{02} by $1.4\,u(G_{02})$, where $u(G_{02}) = 0.0010\ G_0$ is the standard uncertainty of the 2002 recommended value G_{02}. The HUST-99 result for G, which is the lowest of the eight, lies below G_{02} by $3.3\,u(G_{02})$, and the UWash-00 result for G, which has the smallest assigned uncertainty of any of the eight values, lies above G_{02} by only $0.06\,u(G_{02})$. The uncertainty $u(G_{02})$ is less than five times the uncertainty $0.000\,22\ G_0$ of the weighted mean of all eight values, as obtained when each of the uncertainties $u(G_i)$ is multiplied by the Birge ratio of the weighted mean, 2.87 (see above). These considerations provide further support for the recommended value.

R. X-ray units

The three most important units that have historically been used to express the wavelengths of x-ray lines are the copper $K\alpha_1$ x unit, symbol xu($CuK\alpha_1$), the molybdenum $K\alpha_1$ x unit, symbol xu($MoK\alpha_1$), and the ångstrom star, symbol Å*. These units are defined by assigning an exact conventional value to the wavelength of the $CuK\alpha_1$, $MoK\alpha_1$, and $WK\alpha_1$ x-ray lines when each is expressed in its corresponding unit:

$$\lambda(\mathrm{CuK}\alpha_1) = 1\,537.400\ \mathrm{xu(CuK}\alpha_1), \tag{185}$$

$$\lambda(\mathrm{MoK}\alpha_1) = 707.831\ \mathrm{xu(MoK}\alpha_1), \tag{186}$$

$$\lambda(\mathrm{WK}\alpha_1) = 0.209\,010\,0\ \text{Å}^*. \tag{187}$$

The experimental work that determines the best values of these three units was reviewed in CODATA-98, and the relevant data may be summarized as follows:

$$\frac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{W4.2a})} = 0.802\,327\,11(24) \quad [3.0\times10^{-7}], \tag{188}$$

$$\frac{\lambda(\mathrm{WK}\alpha_1)}{d_{220}(\mathrm{N})} = 0.108\,852\,175(98) \quad [9.0\times10^{-7}], \tag{189}$$

$$\frac{\lambda(\mathrm{MoK}\alpha_1)}{d_{220}(\mathrm{N})} = 0.369\,406\,04(19) \quad [5.3\times10^{-7}], \tag{190}$$

$$\frac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{N})} = 0.802\,328\,04(77) \quad [9.6\times10^{-7}], \tag{191}$$

where d_{220}(W4.2a) and d_{220}(N) denote the 220 lattice spacings, at the standard reference conditions $p = 0$ and $t_{90} = 22.5\ ^\circ\mathrm{C}$, of particular silicon crystals used in the measurements. [The result in Eq. (188) is from a collaboration between researchers from Friedrich-Schiller University (FSU), Jena, Germany and the PTB (Härtwig *et al.*, 1991).] The lattice spacing d_{220}(N) is connected to crystals of known lattice spacing through the relation

$$\frac{d_{220}(\mathrm{W17}) - d_{220}(\mathrm{N})}{d_{220}(\mathrm{W17})} = 7(17)\times10^{-9}. \tag{192}$$

The correlation coefficients of this fractional difference and the other NIST fractional differences given in Eqs. (15)–(17) are in the range −0.37 to 0.15.

In order to obtain best values in the least-squares sense for xu($CuK\alpha_1$), xu($MoK\alpha_1$), and Å*, we take these units to be adjusted constants. Thus the observational equations for the data of Eqs. (188)–(191) are

$$\frac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \frac{1\,537.400\ \mathrm{xu(CuK}\alpha_1)}{d_{220}(\mathrm{N})}, \tag{193}$$

$$\frac{\lambda(\mathrm{MoK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \frac{707.831\ \mathrm{xu(MoK}\alpha_1)}{d_{220}(\mathrm{N})}, \tag{194}$$

$$\frac{\lambda(\mathrm{WK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \frac{0.209\,010\,0\ \text{Å}^*}{d_{220}(\mathrm{N})}, \tag{195}$$

$$\frac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{W4.2a})} \doteq \frac{1\,537.400\ \mathrm{xu}(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{W4.2a})}, \tag{196}$$

where $d_{220}(\mathrm{N})$ is taken to be an adjusted constant and $d_{220}(\mathrm{W17})$ and $d_{220}(\mathrm{W4.2a})$ are adjusted constants as well.

S. Other quantities

As pointed out in Sec. I.C, there are a few cases in the 1998 adjustment where an inexact constant that enters the analysis of input data is taken to be a fixed quantity rather than an adjusted quantity, because the input data have a negligible effect on its value. Three such constants, used in the calculation of the theoretical expressions for the electron and muon magnetic moment anomalies a_e and a_μ (see Appendixes B and C), are the mass of the tau lepton m_τ, the Fermi coupling constant G_F, and sine squared of the weak mixing angle $\sin^2\theta_W$. The values we adopt for these constants are based on the most recent report of the Particle Data Group (Hagiwara *et al.*, 2002):

$$m_\tau c^2 = 1776.99(29)\ \mathrm{MeV} \quad [1.6\times10^{-4}], \tag{197}$$

$$\frac{G_F}{(\hbar c)^3} = 1.166\,39(1)\times10^{-5}\mathrm{GeV}^{-2} \quad [8.6\times10^{-6}], \tag{198}$$

$$\sin^2\theta_W = 0.222\,15(76) \quad [3.4\times10^{-3}]. \tag{199}$$

Note, however, that the uncertainty assigned to $m_\tau c^2$ by the Particle Data Group is unsymmetrical and equal to +0.29 MeV, −0.26 MeV. For simplicity and because it is not at all critical, we have symmetrized the uncertainty by taking it to be 0.29 MeV. Also, the definition of $\sin^2\theta_W$ depends on the renormalization prescription used. We take as its definition $\sin^2\theta_W = s_W^2 \equiv 1-(m_W/m_Z)^2$ based on the on-shell scheme, where m_W and m_Z are, respectively, the masses of the $W^\pm$ and Z^0 bosons, because this definition is conceptually simple and is that employed in the calculation of the electroweak contributions to a_e and a_μ (Czarnecki *et al.*, 1996). The recommended value for the mass ratio of these bosons is $m_W/m_Z = 0.881\,96(43)$, which leads to our adopted value of $\sin^2\theta_W$ given above. On the other hand, the value recommended by the Particle Data Group (Hagiwara *et al.*, 2002) is based on a particular variant of the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme, which gives the much more accurate value $\sin^2\hat{\theta}_W(M_Z) = 0.231\,13(15)$.

IV. ANALYSIS OF DATA

Here we examine the previously discussed input data for their mutual compatibility and their potential role in determining the 2002 recommended values of the constants. Based on this analysis, the data are selected for the final least-squares adjustment from which the recommended values are obtained. (Because the data on the Newtonian constant of gravitation G are independent of the other data and are analyzed in Sec. III.Q, they are not examined further.) The mutual compatibility of the input data is evaluated by directly comparing different measurements of the same quantity and by comparing the values of a single fundamental constant inferred from measurements of different quantities. The potential role of a particular input datum is evaluated by carrying out a multivariate analysis of the data using the least-squares method as summarized in Appendix E of CODATA-98. A particular measurement of a quantity is not included in the final adjustment if the multivariate analysis yields an adjusted value for that quantity with a significantly smaller uncertainty than that of the measured value itself. (The cutoff point for a particular measurement of a quantity to contribute to the adjustment is when its uncertainty is approximately ten times as large as the uncertainty of that quantity as predicted by the adjustment. The formal measure we use is the "self-sensitivity coefficient" of an input datum, S_c. In general, it must be greater than 0.01 in order for the datum to be included in the final least-squares adjustment used to obtain the recommended values of the constants; further details may be found in CODATA-98.)

The input data are given in Tables XI and XIII and their covariances are given as correlation coefficients in Tables XII and XIV. The δ's given in Tables XI and XIII are quantities added to corresponding theoretical expressions to account for the uncertainties of those expressions, as discussed in Appendixes A–E. It should be noted that the value of the Rydberg constant R_∞ depends only weakly on the data in Table XIII.

A. Comparison of data

Among the Rydberg constant data, the classic Lamb shift is the only quantity with more than one measured value, while among the other data, there are eight different quantities with more than one measured value. In Tables XI and XIII the item numbers for the members of such groups of data (*A*14, *B*24, *B*26–*B*31, and *B*47) have a decimal point with an additional digit to label each member.

In fact, all of these data were directly compared in the 1998 adjustment, except the recent BNM result for R_K, item *B*30.5. The four earlier results for R_K (items *B*30.1–*B*30.4) and the BNM result are all in agreement. Calculation of the weighted mean of all five values yields for the mean value of R_K and its corresponding value of α

$$R_K = 25\,812.808\,18(47)\Omega \quad [1.8\times10^{-8}], \tag{200}$$

$$\alpha^{-1} = 137.036\,0030(25) \quad [1.8\times10^{-8}], \tag{201}$$

with $\chi^2 = 1.47$ for degrees of freedom $\nu = N - M = 4$, where N is the number of measurements and M is the number of unknowns, and Birge ratio $R_B = \sqrt{\chi^2/\nu} = 0.61$ (see Appendix E of CODATA-98). The normalized residuals,

TABLE XI. Summary of principal input data for the determination of the 2002 recommended value of the Rydberg constant R_∞. [The notation for the additive corrections $\delta_X(nL_j)$ in this table has the same meaning as the notation δ^X_{nLj} in Appendix A, Sec. A.12.]

Item number	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec.
*A*1	$\nu_H(1S_{1/2}-2S_{1/2})$	2 466 061 413 187.103(46) kHz	1.9×10^{-14}	MPQ-00	III.B.1
*A*2	$\nu_H(2S_{1/2}-8S_{1/2})$	770 649 350 012.0(8.6) kHz	1.1×10^{-11}	LK/SY-97	III.B.2
*A*3	$\nu_H(2S_{1/2}-8D_{3/2})$	770 649 504 450.0(8.3) kHz	1.1×10^{-11}	LK/SY-97	III.B.2
*A*4	$\nu_H(2S_{1/2}-8D_{5/2})$	770 649 561 584.2(6.4) kHz	8.3×10^{-12}	LK/SY-97	III.B.2
*A*5	$\nu_H(2S_{1/2}-12D_{3/2})$	799 191 710 472.7(9.4) kHz	1.2×10^{-11}	LK/SY-98	III.B.2
*A*6	$\nu_H(2S_{1/2}-12D_{5/2})$	799 191 727 403.7(7.0) kHz	8.7×10^{-12}	LK/SY-98	III.B.2
*A*7	$\nu_H(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 797 338(10) kHz	2.1×10^{-6}	MPQ-95	III.B.1
*A*8	$\nu_H(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 490 144(24) kHz	3.7×10^{-6}	MPQ-95	III.B.1
*A*9	$\nu_H(2S_{1/2}-6S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 197 604(21) kHz	4.9×10^{-6}	LKB-96	III.B.2
*A*10	$\nu_H(2S_{1/2}-6D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 699 099(10) kHz	2.2×10^{-6}	LKB-96	III.B.2
*A*11	$\nu_H(2S_{1/2}-4P_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 664 269(15) kHz	3.2×10^{-6}	Yale-95	III.B
*A*12	$\nu_H(2S_{1/2}-4P_{3/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 035 373(10) kHz	1.7×10^{-6}	Yale-95	III.B
*A*13	$\nu_H(2S_{1/2}-2P_{3/2})$	9 911 200(12) kHz	1.2×10^{-6}	Harv-94	III.B
*A*14.1	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 845.0(9.0) kHz	8.5×10^{-6}	Harv-86	III.B
*A*14.2	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 862(20) kHz	1.9×10^{-5}	USus-79	III.B
*A*15	R_p	0.895(18) fm	2.0×10^{-2}	Rp-03	III.B.4
*A*16	$\nu_D(2S_{1/2}-8S_{1/2})$	770 859 041 245.7(6.9) kHz	8.9×10^{-12}	LK/SY-97	III.B.2
*A*17	$\nu_D(2S_{1/2}-8D_{3/2})$	770 859 195 701.8(6.3) kHz	8.2×10^{-12}	LK/SY-97	III.B.2
*A*18	$\nu_D(2S_{1/2}-8D_{5/2})$	770 859 252 849.5(5.9) kHz	7.7×10^{-12}	LK/SY-97	III.B.2
*A*19	$\nu_D(2S_{1/2}-12D_{3/2})$	799 409 168 038.0(8.6) kHz	1.1×10^{-11}	LK/SY-98	III.B.2
*A*20	$\nu_D(2S_{1/2}-12D_{5/2})$	799 409 184 966.8(6.8) kHz	8.5×10^{-12}	LK/SY-98	III.B.2
*A*21	$\nu_D(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	4 801 693(20) kHz	4.2×10^{-6}	MPQ-95	III.B.1
*A*22	$\nu_D(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	6 494 841(41) kHz	6.3×10^{-6}	MPQ-95	III.B.1
*A*23	R_d	2.130(10) fm	4.7×10^{-3}	Rd-98	III.B.4
*A*24	$\nu_D(1S_{1/2}-2S_{1/2})-\nu_H(1S_{1/2}-2S_{1/2})$	670 994 334.64(15) kHz	2.2×10^{-10}	MPQ-98	III.B.1
*A*25	$\delta_H(1S_{1/2})$	0.0(1.7) kHz	$[5.3\times10^{-13}]$	theory	App. A
*A*26	$\delta_H(2S_{1/2})$	0.00(21) kHz	$[2.6\times10^{-13}]$	theory	App. A
*A*27	$\delta_H(3S_{1/2})$	0.00(12) kHz	$[3.2\times10^{-13}]$	theory	App. A
*A*28	$\delta_H(4S_{1/2})$	0.000(43) kHz	$[2.1\times10^{-13}]$	theory	App. A
*A*29	$\delta_H(6S_{1/2})$	0.000(18) kHz	$[2.0\times10^{-13}]$	theory	App. A
*A*30	$\delta_H(8S_{1/2})$	0.0000(83) kHz	$[1.6\times10^{-13}]$	theory	App. A
*A*31	$\delta_H(2P_{1/2})$	0.00(63) kHz	$[7.7\times10^{-13}]$	theory	App. A
*A*32	$\delta_H(4P_{1/2})$	0.000(79) kHz	$[3.9\times10^{-13}]$	theory	App. A
*A*33	$\delta_H(2P_{3/2})$	0.00(63) kHz	$[7.7\times10^{-13}]$	theory	App. A
*A*34	$\delta_H(4P_{3/2})$	0.000(79) kHz	$[3.9\times10^{-13}]$	theory	App. A
*A*35	$\delta_H(8D_{3/2})$	0.0000(25) kHz	$[4.8\times10^{-14}]$	theory	App. A
*A*36	$\delta_H(12D_{3/2})$	0.000 00(74) kHz	$[3.2\times10^{-14}]$	theory	App. A
*A*37	$\delta_H(4D_{5/2})$	0.000(20) kHz	$[9.7\times10^{-14}]$	theory	App. A
*A*38	$\delta_H(6D_{5/2})$	0.0000(59) kHz	$[6.4\times10^{-14}]$	theory	App. A
*A*39	$\delta_H(8D_{5/2})$	0.0000(25) kHz	$[4.8\times10^{-14}]$	theory	App. A
*A*40	$\delta_H(12D_{5/2})$	0.000 00(73) kHz	$[3.2\times10^{-14}]$	theory	App. A
*A*41	$\delta_D(1S_{1/2})$	0.0(1.5) kHz	$[4.5\times10^{-13}]$	theory	App. A
*A*42	$\delta_D(2S_{1/2})$	0.00(17) kHz	$[2.1\times10^{-13}]$	theory	App. A
*A*43	$\delta_D(4S_{1/2})$	0.000(41) kHz	$[2.0\times10^{-13}]$	theory	App. A
*A*44	$\delta_D(8S_{1/2})$	0.0000(81) kHz	$[1.6\times10^{-13}]$	theory	App. A
*A*45	$\delta_D(8D_{3/2})$	0.0000(21) kHz	$[4.2\times10^{-14}]$	theory	App. A
*A*46	$\delta_D(12D_{3/2})$	0.000 00(64) kHz	$[2.8\times10^{-14}]$	theory	App. A

TABLE XI. (*Continued.*)

Item number	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec.
*A*47	$\delta_D(4D_{5/2})$	0.000(17) kHz	$[8.4\times10^{-14}]$	theory	App. A
*A*48	$\delta_D(8D_{5/2})$	0.0000(21) kHz	$[4.1\times10^{-14}]$	theory	App. A
*A*49	$\delta_D(12D_{5/2})$	0.000 00(63) kHz	$[2.7\times10^{-14}]$	theory	App. A

[a]The values in brackets are relative to the frequency corresponding to the binding energy of the level.

TABLE XII. Non-negligible correlation coefficients $r(x_i,x_j)$ of the input data related to R_∞ in Table XI. For simplicity, the two items of data to which a particular correlation coefficient corresponds are identified by their item numbers in Table XI.

$r(A2,A3)=0.348$	$r(A5,A20)=0.114$	$r(A25,A27)=0.544$	$r(A30,A44)=0.991$
$r(A2,A4)=0.453$	$r(A6,A9)=0.028$	$r(A25,A28)=0.610$	$r(A31,A32)=0.049$
$r(A2,A5)=0.090$	$r(A6,A10)=0.055$	$r(A25,A29)=0.434$	$r(A33,A34)=0.049$
$r(A2,A6)=0.121$	$r(A6,A16)=0.151$	$r(A25,A30)=0.393$	$r(A35,A36)=0.786$
$r(A2,A9)=0.023$	$r(A6,A17)=0.165$	$r(A25,A41)=0.954$	$r(A35,A45)=0.962$
$r(A2,A10)=0.045$	$r(A6,A18)=0.175$	$r(A25,A42)=0.936$	$r(A35,A46)=0.716$
$r(A2,A16)=0.123$	$r(A6,A19)=0.121$	$r(A25,A43)=0.517$	$r(A36,A45)=0.716$
$r(A2,A17)=0.133$	$r(A6,A20)=0.152$	$r(A25,A44)=0.320$	$r(A36,A46)=0.962$
$r(A2,A18)=0.142$	$r(A7,A8)=0.105$	$r(A26,A27)=0.543$	$r(A37,A38)=0.812$
$r(A2,A19)=0.098$	$r(A7,A21)=0.210$	$r(A26,A28)=0.609$	$r(A37,A39)=0.810$
$r(A2,A20)=0.124$	$r(A7,A22)=0.040$	$r(A26,A29)=0.434$	$r(A37,A40)=0.810$
$r(A3,A4)=0.470$	$r(A8,A21)=0.027$	$r(A26,A30)=0.393$	$r(A37,A47)=0.962$
$r(A3,A5)=0.093$	$r(A8,A22)=0.047$	$r(A26,A41)=0.921$	$r(A37,A48)=0.745$
$r(A3,A6)=0.125$	$r(A9,A10)=0.141$	$r(A26,A42)=0.951$	$r(A37,A49)=0.745$
$r(A3,A9)=0.023$	$r(A9,A16)=0.028$	$r(A26,A43)=0.511$	$r(A38,A39)=0.807$
$r(A3,A10)=0.047$	$r(A9,A17)=0.031$	$r(A26,A44)=0.317$	$r(A38,A40)=0.807$
$r(A3,A16)=0.127$	$r(A9,A18)=0.033$	$r(A27,A28)=0.338$	$r(A38,A47)=0.744$
$r(A3,A17)=0.139$	$r(A9,A19)=0.023$	$r(A27,A29)=0.241$	$r(A38,A48)=0.740$
$r(A3,A18)=0.147$	$r(A9,A20)=0.028$	$r(A27,A30)=0.218$	$r(A38,A49)=0.740$
$r(A3,A19)=0.102$	$r(A10,A16)=0.056$	$r(A27,A41)=0.516$	$r(A39,A40)=0.806$
$r(A3,A20)=0.128$	$r(A10,A17)=0.061$	$r(A27,A42)=0.518$	$r(A39,A47)=0.741$
$r(A4,A5)=0.121$	$r(A10,A18)=0.065$	$r(A27,A43)=0.286$	$r(A39,A48)=0.961$
$r(A4,A6)=0.162$	$r(A10,A19)=0.045$	$r(A27,A44)=0.177$	$r(A39,A49)=0.737$
$r(A4,A9)=0.030$	$r(A10,A20)=0.057$	$r(A28,A29)=0.270$	$r(A40,A47)=0.741$
$r(A4,A10)=0.060$	$r(A11,A12)=0.083$	$r(A28,A30)=0.244$	$r(A40,A48)=0.737$
$r(A4,A16)=0.165$	$r(A16,A17)=0.570$	$r(A28,A41)=0.578$	$r(A40,A49)=0.961$
$r(A4,A17)=0.180$	$r(A16,A18)=0.612$	$r(A28,A42)=0.581$	$r(A41,A42)=0.972$
$r(A4,A18)=0.191$	$r(A16,A19)=0.123$	$r(A28,A43)=0.980$	$r(A41,A43)=0.540$
$r(A4,A19)=0.132$	$r(A16,A20)=0.155$	$r(A28,A44)=0.198$	$r(A41,A44)=0.333$
$r(A4,A20)=0.166$	$r(A17,A18)=0.667$	$r(A29,A30)=0.174$	$r(A42,A43)=0.538$
$r(A5,A6)=0.475$	$r(A17,A19)=0.134$	$r(A29,A41)=0.410$	$r(A42,A44)=0.333$
$r(A5,A9)=0.021$	$r(A17,A20)=0.169$	$r(A29,A42)=0.413$	$r(A43,A44)=0.184$
$r(A5,A10)=0.041$	$r(A18,A19)=0.142$	$r(A29,A43)=0.228$	$r(A45,A46)=0.717$
$r(A5,A16)=0.113$	$r(A18,A20)=0.179$	$r(A29,A44)=0.141$	$r(A47,A48)=0.748$
$r(A5,A17)=0.123$	$r(A19,A20)=0.522$	$r(A30,A41)=0.371$	$r(A47,A49)=0.748$
$r(A5,A18)=0.130$	$r(A21,A22)=0.011$	$r(A30,A42)=0.373$	$r(A48,A49)=0.741$
$r(A5,A19)=0.090$	$r(A25,A26)=0.979$	$r(A30,A43)=0.206$	

TABLE XIII. Summary of principal input data for the determination of the 2002 recommended values of the fundamental constants (R_∞ and G excepted).

Item number	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec. and Eq.
*B*1	$A_r(^1\mathrm{H})$	1.007 825 032 14(35)	3.5×10^{-10}	AMDC-95	III.A.1
*B*2	$A_r(^2\mathrm{H})$	2.014 101 777 99(36)	1.8×10^{-10}	AMDC-95	III.A.1
*B*3	$A_r(^3\mathrm{He})$	3.016 029 3184(58)	1.9×10^{-9}	UWash-03	III.A.1.a
*B*4	$A_r(^4\mathrm{He})$	4.002 603 254 152(56)	1.4×10^{-11}	UWash-03	III.A.1.b
*B*5	$A_r(^{16}\mathrm{O})$	15.994 914 619 51(16)	1.0×10^{-11}	UWash-03	III.A.1.c
*B*6	$A_r(^{133}\mathrm{Cs})$	132.905 451 931(27)	2.0×10^{-10}	MIT-99	III.A.1.e
*B*7	$A_r(\mathrm{e})$	0.000 548 579 9111(12)	2.1×10^{-9}	UWash-95	III.A.4.a (10)
*B*8	$A_r(\mathrm{p})$	1.007 276 466 89(14)	1.4×10^{-10}	UWash-99	III.A.4.b (11)
*B*9	a_e	$1.159\,652\,1883(42)\times10^{-3}$	3.7×10^{-9}	UWash-87	III.C.1.a (31)
*B*10	δ_e	$0.0(1.1)\times10^{-12}$	$[9.9\times10^{-10}]$	theory	App. B (B22)
*B*11	$\overline{R}$	0.003 707 2048(25)	6.7×10^{-7}	BNL-02	III.C.2.a (38)
*B*12	δ_μ	$0(10)\times10^{-10}$	$[8.5\times10^{-7}]$	theory	App. C (C29)
*B*13	$f_s(\mathrm{C})/f_c(\mathrm{C})$	4376.210 4989(23)	5.2×10^{-10}	GSI-02	III.C.3.a (47)
*B*14	δ_C	$0.0(1.8)\times10^{-10}$	$[9.0\times10^{-11}]$	theory	App. D (D29)
*B*15	$f_s(\mathrm{O})/f_c(\mathrm{O})$	4164.376 1836(31)	7.5×10^{-10}	GSI-02	III.C.3.b (50)
*B*16	δ_O	$0.0(4.3)\times10^{-10}$	$[2.2\times10^{-10}]$	theory	App. D (D32)
*B*17	$\mu_{e^-}(\mathrm{H})/\mu_p(\mathrm{H})$	−658.210 7058(66)	1.0×10^{-8}	MIT-72	III.C.4.a (56)
*B*18	$\mu_d(\mathrm{D})/\mu_{e^-}(\mathrm{D})$	$-4.664\,345\,392(50)\times10^{-4}$	1.1×10^{-8}	MIT-84	III.C.4.b (58)
*B*19	$\mu_{e^-}(\mathrm{H})/\mu'_p$	−658.215 9430(72)	1.1×10^{-8}	MIT-77	III.C.4.c (60)
*B*20	μ'_h/μ'_p	−0.761 786 1313(33)	4.3×10^{-9}	NPL-93	III.C.4.d (62)
*B*21	μ_n/μ'_p	−0.684 996 94(16)	2.4×10^{-7}	ILL-79	III.C.4.e (65)
*B*22	ν(58 MHz)	627 994.77(14) kHz	2.2×10^{-7}	LAMPF-82	III.C.5.a (71)
*B*23	ν(72 MHz)	668 223 166(57) Hz	8.6×10^{-8}	LAMPF-99	III.C.5.b (77)
*B*24.1	$\Delta\nu_{Mu}$	4 463 302.88(16) kHz	3.6×10^{-8}	LAMPF-82	III.C.5.a (70)
*B*24.2	$\Delta\nu_{Mu}$	4 463 302 765(53) Hz	1.2×10^{-8}	LAMPF-99	III.C.5.b (76)
*B*25	δ_{Mu}	0(101) Hz	$[2.3\times10^{-8}]$	theory	App. E (E16)
*B*26.1	Γ'_{p-90}(lo)	$2.675\,154\,05(30)\times10^8\ \mathrm{s^{-1}\,T^{-1}}$	1.1×10^{-7}	NIST-89	III.D.1.a (91)
*B*26.2[b]	Γ'_{p-90}(lo)	$2.675\,1530(18)\times10^8\ \mathrm{s^{-1}\,T^{-1}}$	6.6×10^{-7}	NIM-95	III.D.1.b (94)
*B*27.1	Γ'_{p-90}(hi)	$2.675\,1525(43)\times10^8\ \mathrm{s^{-1}\,T^{-1}}$	1.6×10^{-6}	NIM-95	III.D.1.b (95)
*B*27.2	Γ'_{p-90}(hi)	$2.675\,1518(27)\times10^8\ \mathrm{s^{-1}\,T^{-1}}$	1.0×10^{-6}	NPL-79	III.D.1.c (100)
*B*28.1[b]	Γ'_{h-90}(lo)	$2.037\,895\,37(37)\times10^8\ \mathrm{s^{-1}\,T^{-1}}$	1.8×10^{-7}	KR/VN-98	III.D.2.a (102)
*B*28.2[b]	Γ'_{h-90}(lo)	$2.037\,897\,29(72)\times10^8\ \mathrm{s^{-1}\,T^{-1}}$	3.5×10^{-7}	VNIIM-89	III.D.2.b (105)
*B*29.1[c]	K_J	483 597.91(13) GHz $\mathrm{V^{-1}}$	2.7×10^{-7}	NML-89	III.E.1 (107)
*B*29.2[c]	K_J	483 597.96(15) GHz $\mathrm{V^{-1}}$	3.1×10^{-7}	PTB-91	III.E.2 (110)
*B*30.1	R_K	25 812.808 31(62) Ω	2.4×10^{-8}	NIST-97	III.F.1 (113)
*B*30.2[b]	R_K	25 812.8071(11) Ω	4.4×10^{-8}	NML-97	III.F.2 (115)
*B*30.3[b]	R_K	25 812.8092(14) Ω	5.4×10^{-8}	NPL-88	III.F.3 (117)
*B*30.4[b]	R_K	25 812.8084(34) Ω	1.3×10^{-7}	NIM-95	III.F.4 (119)
*B*30.5[b]	R_K	25 812.8081(14) Ω	5.3×10^{-8}	BNM-01	III.F.5 (121)
*B*31.1[c]	$K_J^2R_K$	$6.036\,7625(12)\times10^{33}\ \mathrm{J^{-1}\,s^{-1}}$	2.0×10^{-7}	NPL-90	III.G.1 (124)
*B*31.2[c]	$K_J^2R_K$	$6.036\,761\,85(53)\times10^{33}\ \mathrm{J^{-1}\,s^{-1}}$	8.7×10^{-8}	NIST-98	III.G.2 (126)
*B*32	$\mathcal{F}_{90}$	96 485.39(13) $\mathrm{C\,mol^{-1}}$	1.3×10^{-6}	NIST-80	III.H.1 (132)
*B*33	$h/m(\mathrm{Cs})$	$3.002\,369\,430(46)\times10^{-9}\ \mathrm{m^2\,s^{-1}}$	1.5×10^{-8}	Stanford-02	III.K.2 (160)
*B*34	$h/m_n d_{220}$(W04)	2060.267 004(84) $\mathrm{m\,s^{-1}}$	4.1×10^{-8}	PTB-99	III.K.1 (151)
*B*35	λ_{meas}/d_{220}(ILL)	0.002 904 302 46(50) $\mathrm{m\,s^{-1}}$	1.7×10^{-7}	NIST-99	III.A.5 (13)
*B*36	$1-d_{220}(\mathrm{W17})/d_{220}(\mathrm{ILL})$	$-8(22)\times10^{-9}$	$[2.2\times10^{-8}]$	NIST-99	III.A.5 (15)
*B*37	$1-d_{220}(\mathrm{MO}^*)/d_{220}(\mathrm{ILL})$	$86(27)\times10^{-9}$	$[2.7\times10^{-8}]$	NIST-99	III.A.5 (16)
*B*38	$1-d_{220}(\mathrm{NR3})/d_{220}(\mathrm{ILL})$	$34(22)\times10^{-9}$	$[2.2\times10^{-8}]$	NIST-99	III.A.5 (17)

TABLE XIII. (*Continued.*)

Item number	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec. and Eq.
*B*39	$1-d_{220}(\mathrm{N})/d_{220}(\mathrm{W17})$	$7(17)\times10^{-9}$	$[1.7\times10^{-8}]$	NIST-97	III.R (192)
*B*40	$d_{220}(\mathrm{W4.2a})/d_{220}(\mathrm{W04})-1$	$-1(21)\times10^{-9}$	$[2.1\times10^{-8}]$	PTB-98	III.I.1 (134)
*B*41	$d_{220}(\mathrm{W17})/d_{220}(\mathrm{W04})-1$	$22(22)\times10^{-9}$	$[2.2\times10^{-8}]$	PTB-98	III.I.1 (135)
*B*42	$d_{220}(\mathrm{MO}^*)/d_{220}(\mathrm{W04})-1$	$-103(28)\times10^{-9}$	$[2.8\times10^{-8}]$	PTB-98	III.I.1 (136)
*B*43	$d_{220}(\mathrm{NR3})/d_{220}(\mathrm{W04})-1$	$-23(21)\times10^{-9}$	$[2.1\times10^{-8}]$	PTB-98	III.I.1 (137)
*B*44	$d_{220}/d_{220}(\mathrm{W04})-1$	$10(11)\times10^{-9}$	$[1.1\times10^{-8}]$	PTB-03	III.I.1 (138)
*B*45	$d_{220}(\mathrm{NR3})$	192 015.587(11) fm	5.6×10^{-8}	NMIJ-97	III.I.2 (139)
*B*46[c]	$V_m(\mathrm{Si})$	$12.058\,8257(36)\times10^{-6}$ m^3 mol^{-1}	3.0×10^{-7}	N/P/I-03	III.J (146)
*B*47.1	R	8.314 471(15) J mol^{-1} K^{-1}	1.8×10^{-6}	NIST-88	III.N.1 (168)
*B*47.2	R	8.314 504(70) J mol^{-1} K^{-1}	8.4×10^{-6}	NPL-79	III.N.2 (169)
*B*48	$\lambda(\mathrm{CuK}\alpha_1)/d_{220}(\mathrm{W4.2a})$	0.802 327 11(24)	3.0×10^{-7}	FSU/PTB-91	III.R (188)
*B*49	$\lambda(\mathrm{WK}\alpha_1)/d_{220}(\mathrm{N})$	0.108 852 175(98)	9.0×10^{-7}	NIST-79	III.R (189)
*B*50	$\lambda(\mathrm{MoK}\alpha_1)/d_{220}(\mathrm{N})$	0.369 406 04(19)	5.3×10^{-7}	NIST-73	III.R (190)
*B*51	$\lambda(\mathrm{CuK}\alpha_1)/d_{220}(\mathrm{N})$	0.802 328 04(77)	9.6×10^{-7}	NIST-73	III.R (191)

[a]The numbers in brackets are the uncertainties relative to the quantities a_e, a_μ, $g_{e^-}(^{12}C^{5+})$, $g_{e^-}(^{16}O^{7+})$, $\Delta\nu_{Mu}$, or d_{220} as appropriate.
[b]Datum not included in the final least-squares adjustment that provides the recommended values of the constants.
[c]Datum included in the final least-squares adjustment with an expanded uncertainty.

$r_i=(R_{K,i}-\hat{R}_K)/u(R_{K,i})$, for the five values are 0.21, −0.94, 0.73, 0.06, and −0.09 and their weights in the calculation of their weighted mean are 0.58, 0.17, 0.12, 0.02, and 0.12.

The results of the other 1998 direct comparisons may be summarized as follows. All of the data are in good agreement, except the two values of $\Gamma'_{h-90}(\mathrm{lo})$; the KRISS/VNIIM 1998 result, item *B*28.1, is smaller than the VNIIM 1989 result, item *B*28.2, by $2.4u_{diff}$, where u_{diff} is the standard uncertainty of their difference. Moreover, no datum has a weight of less than 0.03 in the weighted mean of its group of measurements.

The consistency of measurements of various quantities of different types is shown by comparing the values of the fine-structure constant α or the Planck constant h inferred from the measured values of the quantities. Such inferred values of α and h are given throughout Sec. III, and the results are summarized and further discussed here.

The consistency of a significant fraction of the data of Table XIII is indicated in Table XV and Figs. 2 and 3, which give the values of α inferred from that data. (Only values of α with $u_r<10^{-7}$ are included in Fig. 2.) Most of the values of α are in reasonable agreement, implying that most of the data from which they are obtained are reasonably consistent. An exception is the value of α implied by the VNIIM-89 result for $\Gamma'_{h-90}(\mathrm{lo})$, item *B*28.2, which disagrees by about 3.5 combined standard deviations (3.5σ) with the two values of α that have the smallest uncertainty. However, item *B*28.2 has such a large uncertainty that its contribution to the determination of α is inconsequential. A datum of possible concern is the NIST-89 result for $\Gamma'_{p-90}(\mathrm{lo})$, item *B*26.1, which disagrees at about the 2.2σ level and its uncertainty places it at the margin of contributing to the adjusted value of α. Although the inferred values of α of the ten input data following the NIST value for $\Gamma_{p-90}(\mathrm{lo})$ in Table XV have still larger uncertainties, some of these

TABLE XIV. Non-negligible correlation coefficients $r(x_i,x_j)$ of the input data in Table XIII. For simplicity, the two items of data to which a particular correlation coefficient corresponds are identified by their item numbers in Table XIII.

$r(B1,B2)=0.314$	$r(B29.1,B47.1)=0.068$	$r(B36,B38)=0.516$	$r(B40,B42)=0.372$
$r(B13,B15)=0.035$	$r(B34,B40)=0.258$	$r(B36,B39)=-0.375$	$r(B40,B43)=0.502$
$r(B14,B16)=0.951$	$r(B34,B41)=0.241$	$r(B37,B38)=0.421$	$r(B41,B42)=0.347$
$r(B22,B24.1)=0.227$	$r(B34,B42)=0.192$	$r(B37,B39)=0.125$	$r(B41,B43)=0.469$
$r(B23,B24.2)=0.195$	$r(B34,B43)=0.258$	$r(B38,B39)=0.153$	$r(B42,B43)=0.372$
$r(B26.2,B27.1)=-0.014$	$r(B36,B37)=0.421$	$r(B40,B41)=0.469$	

TABLE XV. Comparison of the input data in Table XIII through inferred values of the fine-structure constant α in order of increasing standard uncertainty.

Primary source	Item number	Identification	Sec. and Eq.	α^{-1}	Relative standard uncertainty u_r
a_e	*B*9	UWash-87	III.C.1.b (35)	137.035 998 80(52)	3.8×10^{-9}
h/m(Cs)	*B*33	Stanford-02	III.K.2 (162)	137.036 0001(11)	7.7×10^{-9}
R_K	*B*30.1	NIST-97	III.F.1 (114)	137.036 0037(33)	2.4×10^{-8}
$h/m_n d_{220}$(W04)	*B*34	PTB-99			
d_{220}(NR3)	*B*45	NMIJ-97	III.K.1 (153)	137.036 0015(47)	3.4×10^{-8}
Γ'_{p-90}(lo)	*B*26.1	NIST-89	III.D.1.a (93)	137.035 9880(51)	3.7×10^{-8}
R_K	*B*30.2	NML-97	III.F.2 (116)	137.035 9973(61)	4.4×10^{-8}
R_K	*B*30.5	BNM-01	III.F.5 (122)	137.036 0023(73)	5.3×10^{-8}
R_K	*B*30.3	NPL-88	III.F.3 (118)	137.036 0083(73)	5.4×10^{-8}
Γ'_{h-90}(lo)	*B*28.1	KR/VN-98	III.D.2.a (104)	137.035 9853(82)	6.0×10^{-8}
$\Delta\nu_{Mu}$	*B*24.2	LAMPF-99	III.C.5.b (81)	137.035 9997(84)	6.1×10^{-8}
Γ'_{h-90}(lo)	*B*28.2	VNIIM-89	III.D.2.b (106)	137.035 942(16)	1.2×10^{-7}
R_K	*B*30.4	NIM-95	III.F.4 (120)	137.036 004(18)	1.3×10^{-7}
$\Delta\nu_{Mu}$	*B*24.1	LAMPF-82	III.C.5.a (75)	137.036 019(24)	1.8×10^{-7}
Γ'_{p-90}(lo)	*B*26.2	NIM-95	III.D.1.b (97)	137.036 006(30)	2.2×10^{-7}
$\overline{R}$	*B*11	BNL-02	III.C.2.b (45)	137.035 81(15)	1.1×10^{-6}

data do make significant contributions to the determination of other constants.

To further compare the data of Table XV, we put aside the value of α from item *B*11 (last entry of Table XV) because of its very large uncertainty and omit the value of α from item *B*28.2 because of its large uncertainty and incompatibility with all the other values. We then have, again in order of increasing uncertainty,

$$\alpha^{-1}[a_e] = 137.035\,998\,80(52) \quad [3.8\times10^{-9}], \tag{202}$$

$$\alpha^{-1}[h/m(\mathrm{Cs})] = 137.036\,0001(11) \quad [7.7\times10^{-9}], \tag{203}$$

$$\alpha^{-1}[R_K] = 137.036\,0030(25) \quad [1.8\times10^{-8}], \tag{204}$$

$$\alpha^{-1}[\Gamma'_{p,h-90}(\mathrm{lo})] = 137.035\,9875(43) \quad [3.1\times10^{-8}], \tag{205}$$

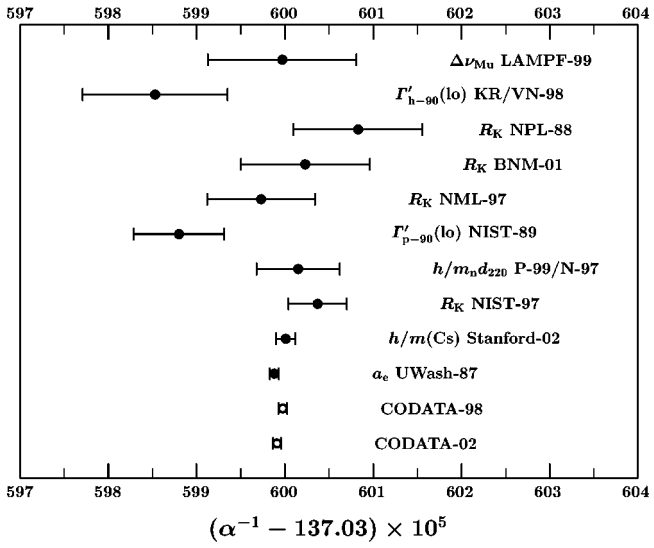

FIG. 2. Values of the fine-structure constant α implied by the input data in Table XIII, in order of decreasing uncertainty from top to bottom, and the 1998 and 2002 CODATA recommended values of α (see Table XV). See Glossary for source abbreviations. Here, P-99/N-97 indicates the PTB-97 result for $h/m_n d_{220}$(W04) together with the NMIJ result for d_{220}(NR3).

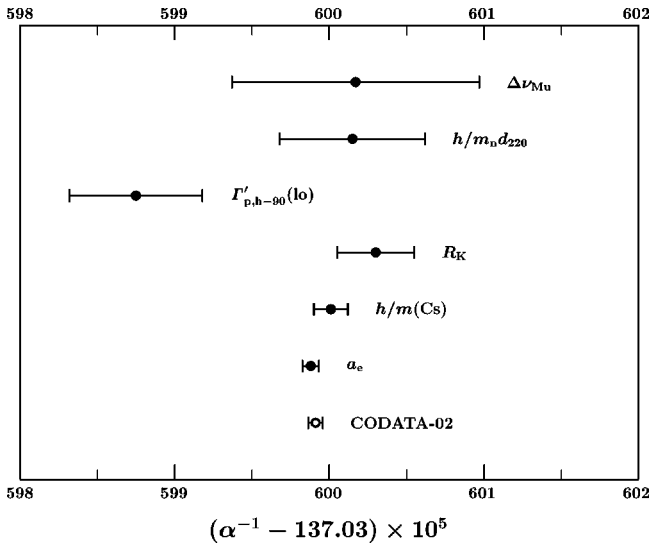

FIG. 3. Values of the fine-structure constant α implied by the input data in Table XIII, taken as a weighted average when more than one measurement of a given type is considered [see Eqs. (202)–(207)], in order of decreasing uncertainty from top to bottom, and the 2002 CODATA recommended value of α.

TABLE XVI. Comparison of the input data in Table XIII through inferred values of the Planck constant h in order of increasing standard uncertainty.

Primary source	Item number	Identification	Sec. and Eq.	h/(J s)	Relative standard uncertainty u_r
$K_J^2R_K$	*B*31.2	NIST-98	III.G.2 (127)	$6.626\,068\,91(58)\times10^{-34}$	8.7×10^{-8}
$K_J^2R_K$	*B*31.1	NPL-90	III.G.1 (125)	$6.626\,0682(13)\times10^{-34}$	2.0×10^{-7}
V_m(Si)	*B*46	N/P/I-03	III.J.3 (149)	$6.626\,0762(21)\times10^{-34}$	3.2×10^{-7}
K_J	*B*29.1	NML-89	III.E.1 (109)	$6.626\,0684(36)\times10^{-34}$	5.4×10^{-7}
K_J	*B*29.2	PTB-91	III.E.2 (111)	$6.626\,0670(42)\times10^{-34}$	6.3×10^{-7}
Γ'_{p-90}(hi)	*B*27.2	NPL-79	III.D.1.c (101)	$6.626\,0730(67)\times10^{-34}$	1.0×10^{-6}
$\mathcal{F}_{90}$	*B*32	NIST-80	III.H.1 (133)	$6.626\,0658(88)\times10^{-34}$	1.3×10^{-6}
Γ'_{p-90}(hi)	*B*27.1	NIM-95	III.D.1.b (99)	$6.626\,071(11)\times10^{-34}$	1.6×10^{-6}

$$\alpha^{-1}[h/m_n d_{220}] = 137.036\,0015(47) \quad [3.4\times10^{-8}], \tag{206}$$

$$\alpha^{-1}[\Delta\nu_{Mu}] = 137.036\,0017(80) \quad [5.8\times10^{-8}]. \tag{207}$$

Here $\alpha^{-1}[a_e]$, $\alpha^{-1}[h/m(\mathrm{Cs})]$, and $\alpha^{-1}[h/m_n d_{220}]$ are as given in Table XV; $\alpha^{-1}[R_K]$ is the weighted-mean value from the five von Klitzing constant measurements, items *B*30.1 to *B*30.5, and is given in Eq. (201); $\alpha^{-1}[\Gamma'_{p,h-90}(\mathrm{lo})]$ is the weighted-mean value from the three low-field gyromagnetic ratio measurements, items *B*26.1, *B*26.2, and *B*28.1 (the experiments are similar and the three inferred values of α agree); and $\alpha^{-1}[\Delta\nu_{Mu}]$ is the muonium value obtained from the LAMPF-82 and LAMPF-99 experiments, items *B*24.1 and *B*24.2, and is given in Eq. (84) of Sec. III.C.5.c. The six values of α are graphically compared in Fig. 3. It is clear from the figure that even if all of the data of Table XIII were retained, the 2002 recommended value of α would be determined to a great extent by a_e, as in the 1998 adjustment.

The consistency of many of the data of Table XIII is indicated in Table XVI and Figs. 4 and 5, which give the values of h inferred from that data. All but one of the values of h are in good agreement, indicating that the data from which they are obtained are generally consistent. However, the value of h from the WGAC consensus result for the molar volume of silicon V_m(Si), item *B*46, disagrees at about the 3.3σ level with the values of h with the smallest uncertainty. Moreover, the input value of V_m(Si) has an uncertainty that is sufficiently small that it contributes to the adjusted value of h with a weight of about 0.06.

In analogy with the discussion of the inferred values of α, we further compare the data of Table XVI by writing, again in order of increasing uncertainty,

$$h[K_J^2R_K] = 6.626\,068\,79(53)\times10^{-34}\ \mathrm{J\ s} \quad [8.0\times10^{-8}], \tag{208}$$

$$h[V_m(\mathrm{Si})] = 6.626\,0762(21)\times10^{-34}\ \mathrm{J\ s} \quad [3.2\times10^{-7}], \tag{209}$$

$$h[K_J] = 6.626\,0679(27)\times10^{-34}\ \mathrm{J\ s} \quad [4.1\times10^{-7}], \tag{210}$$

$$h[\Gamma'_{p-90}(\mathrm{hi})] = 6.626\,0725(57)\times10^{-34}\ \mathrm{J\ s} \quad [8.6\times10^{-7}], \tag{211}$$

$$h[\mathcal{F}_{90}] = 6.626\,0658(88)\times10^{-34}\ \mathrm{J\ s} \quad [1.3\times10^{-6}]. \tag{212}$$

Here $h[V_m(\mathrm{Si})]$ and $h[F_{90}]$ are as given in Table XVI; $h[K_J^2R_K]$ is the weighted-mean value of h from the two measurements of $K_J^2R_K$, items *B*31.1 and *B*31.2, which agree; $h[K_J]$ is the weighted-mean value of h from the two measurements of K_J, items *B*29.1 and *B*29.2, which also agree; and $h[\Gamma'_{p-90}(\mathrm{hi})]$ is the weighted-mean value of h obtained from the two measurements of $\Gamma'_{p-90}(\mathrm{hi})$, items *B*27.1 and *B*27.2, which are consistent as well. The

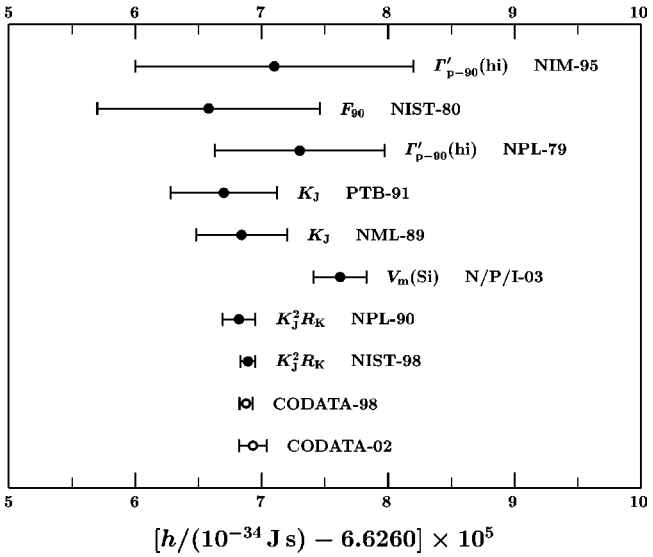

FIG. 4. Values of the Planck constant h implied by the input data in Table XIII, in order of decreasing uncertainty from top to bottom, and the 1998 and 2002 CODATA recommended values of h (see Table XVI).

TABLE XVII. Values of $A_r(e)$ implied by the input data in Table XIII in order of increasing standard uncertainty.

Primary source	Item number	Identification	Sec. and Eq.	$A_r(e)$	Relative standard uncertainty u_r
$f_s(C)/f_c(C)$	*B*13	GSI-02	III.C.3.a (49)	0.000 548 579 909 31(29)	5.3×10^{-10}
$f_s(O)/f_c(O)$	*B*15	GSI-02	III.C.3.b (53)	0.000 548 579 909 57(43)	7.8×10^{-10}
$A_r(e)$	*B*7	UWash-95	III.A.4.a (10)	0.000 548 579 9111(12)	2.1×10^{-9}

five values of h are graphically compared in Fig. 5. The disagreement between $V_m(Si)$ and the other input data of Table XVI appears to be a more significant problem than the disagreement of both the VNIIM-89 result for $\Gamma'_{h-90}(lo)$ and the NIST-89 result for $\Gamma'_{p-90}(lo)$ with the other input data, as discussed above, and warrants special attention.

We conclude this section by listing in Table XVII the three values of $A_r(e)$ implied by the input data consisting of $A_r(^{16}O)$, $A_r(e)$, $f_s(C)/f_c(C)$, δ_C, $f_s(O)/f_c(O)$, and δ_O, items *B*5, *B*7, and *B*13 to *B*16 of Table XIII. The values of $A_r(e)$ in the table are in reasonable agreement, which suggests that the associated input data are consistent.

B. Multivariate analysis of data

The multivariate analysis of the data is based on the fact that measured quantities can be expressed as theoretical functions of fundamental constants. These expressions, or observational equations, are written in terms of a particular independent subset of the constants whose members are here called *adjusted constants*. The goal of the analysis is to find the values of the adjusted constants that predict values for the measured data that best agree with the data themselves in the least-squares sense (see Appendix E of CODATA-98).

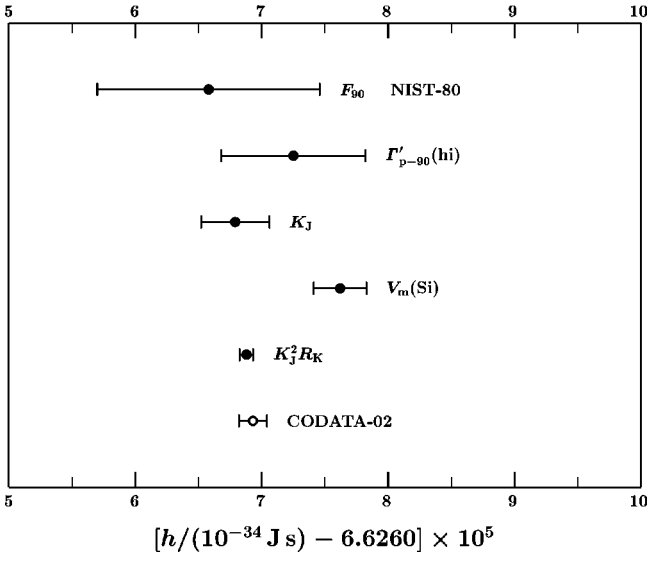

FIG. 5. Values of the Planck constant h implied by the input data in Table XIII, taken as a weighted average when more than one measurement of a given type is considered [see Eqs. (208)–(212)], in order of decreasing uncertainty from top to bottom, and the 2002 CODATA recommended value of h.

The symbol $\doteq$ is used to indicate that an observed value of an input datum of the particular type shown on the left-hand side is ideally given by the function of the adjusted constants on the right-hand side; however, the two sides are not necessarily equal, because the equation is one of an overdetermined set relating the data to the adjusted constants. The best estimate of a quantity is given by its observational equation evaluated with the least-squares estimated values of the adjusted constants on which it depends.

In essence, we follow the least-squares approach of Aitken (1934; also see Sheppard, 1912), who treated the case in which the input data are correlated. The 112 input data of Tables XI and XIII are of 100 distinct types and are expressed as functions of the 61 adjusted constants listed in Tables XVIII and XX. The observational equations that relate the input data to the adjusted constants are given in Tables XIX and XXI.

Note that the various binding energies $E_b(X)/m_uc^2$ in Table XXI, such as in the equation for item *B*1, are treated as fixed quantities with negligible uncertainties. Similarly, the bound-state g-factor ratios in this table, such as in the equation for item *B*17, are treated in the same way. Further, the frequency f_p is not an adjusted constant but is included in the equation for items *B*22 and *B*23 to indicate that they are functions of f_p. Finally, the observational equation for items *B*22 and *B*23, based on Eqs.(66)–(68) of Sec. III.C.5, includes the functions $a_e(\alpha,\delta_e)$ and $a_\mu(\alpha,\delta_\mu)$ as well as the theoretical expression for input data of type *B*24, $\Delta\nu_{Mu}$. The latter expression is discussed in Appendix E and is a function of R_∞, α, m_e/m_μ, $a_\mu(\alpha,\delta_\mu)$, and δ_{Mu}.

A number of adjustments were carried out to gauge the compatibility of the input data in Tables XI and XIII (together with their covariances in Tables XII and XIV) and to assess their influence on the values of the adjusted constants. The results of 11 of these are given in Tables XXII–XXIV and are discussed in the following paragraphs. (Because the adjusted value of the Rydberg constant R_∞ is essentially the same for all six adjustments summarized in Table XXII and equal to that of adjustment 4 of Table XXIV, the value of R_∞ is not listed in Table XXII. It should also be noted that adjustment 4 of all three tables is the same adjustment.)

- *Adjustment 1*. This initial adjustment is based on all of the input data. From Table XXIII we see that item *B*28.2, with its normalized residual of 3.52, is the

TABLE XVIII. The 28 adjusted constants (variables) used in the least-squares multivariate analysis of the Rydberg-constant data given in Table XI. These adjusted constants appear as arguments of the functions on the right-hand side of the observational equations of Table XIX. [The notation for hydrogenic energy levels $E_X(nL_j)$ and for additive corrections $\delta_X(nL_j)$ in this table have the same meaning as the notations E^X_{nLj} and δ^X_{nLj} in Appendix A, Sec. A.12.]

Adjusted constant	Symbol
Rydberg constant	R_∞
Bound-state proton rms charge radius	R_p
Additive correction to $E_H(1S_{1/2})/h$	$\delta_H(1S_{1/2})$
Additive correction to $E_H(2S_{1/2})/h$	$\delta_H(2S_{1/2})$
Additive correction to $E_H(3S_{1/2})/h$	$\delta_H(3S_{1/2})$
Additive correction to $E_H(4S_{1/2})/h$	$\delta_H(4S_{1/2})$
Additive correction to $E_H(6S_{1/2})/h$	$\delta_H(6S_{1/2})$
Additive correction to $E_H(8S_{1/2})/h$	$\delta_H(8S_{1/2})$
Additive correction to $E_H(2P_{1/2})/h$	$\delta_H(2P_{1/2})$
Additive correction to $E_H(4P_{1/2})/h$	$\delta_H(4P_{1/2})$
Additive correction to $E_H(2P_{3/2})/h$	$\delta_H(2P_{3/2})$
Additive correction to $E_H(4P_{3/2})/h$	$\delta_H(4P_{3/2})$
Additive correction to $E_H(8D_{3/2})/h$	$\delta_H(8D_{3/2})$
Additive correction to $E_H(12D_{3/2})/h$	$\delta_H(12D_{3/2})$
Additive correction to $E_H(4D_{5/2})/h$	$\delta_H(4D_{5/2})$
Additive correction to $E_H(6D_{5/2})/h$	$\delta_H(6D_{5/2})$
Additive correction to $E_H(8D_{5/2})/h$	$\delta_H(8D_{5/2})$
Additive correction to $E_H(12D_{5/2})/h$	$\delta_H(12D_{5/2})$
Bound-state deuteron rms charge radius	R_d
Additive correction to $E_D(1S_{1/2})/h$	$\delta_D(1S_{1/2})$
Additive correction to $E_D(2S_{1/2})/h$	$\delta_D(2S_{1/2})$
Additive correction to $E_D(4S_{1/2})/h$	$\delta_D(4S_{1/2})$
Additive correction to $E_D(8S_{1/2})/h$	$\delta_D(8S_{1/2})$
Additive correction to $E_D(8D_{3/2})/h$	$\delta_D(8D_{3/2})$
Additive correction to $E_D(12D_{3/2})/h$	$\delta_D(12D_{3/2})$
Additive correction to $E_D(4D_{5/2})/h$	$\delta_D(4D_{5/2})$
Additive correction to $E_D(8D_{5/2})/h$	$\delta_D(8D_{5/2})$
Additive correction to $E_D(12D_{5/2})/h$	$\delta_D(12D_{5/2})$

most discrepant datum. Although its self-sensitivity coefficient S_c is only 0.002, it is responsible for 21% of χ^2. These results are in keeping with the discussion of this datum in the previous section.

- *Adjustment 2*. This adjustment is obtained from adjustment 1 by deleting item *B*28.2. Because of the comparatively large uncertainty of this datum, the change in the adjusted value of α is quite small. We omit *B*28.2 from the final 2002 adjustment because of its minimal contribution to the determination of the recommended values and its discrepant nature.

Inspection of Table XXIII shows that in adjustment 2, item *B*46, with its normalized residual of −3.17, is now the most discrepant datum; it is responsible for 22% of χ^2, but it has a quite significant self-sensitivity coefficient: $S_c=0.173$. The large residual is a reflection of the disagreement among the data that primarily determine the Planck constant h, as is apparent from Table XVI and Figs. 4 and 5. These data are items *B*29.1 and *B*29.2, the NML-89 and PTB-91 results for K_J, and items *B*31.1 and *B*31.2, the NPL-90 and NIST-98 results for $K_J^2R_K$, in addition to item *B*46.

The problem that this discrepancy poses has been given careful consideration by the Task Group, taking into account the following points:

(i) The N/P/I-03 result for V_m(Si), item *B*46, represents an international effort over many years involving numerous researchers in more than half a dozen different institutes. As such, it must be viewed as a credible result even though it disagrees with a number of equally credible results. Therefore it must be included in the 2002 adjustment with a reasonable weight.

(ii) The disagreement of item *B*46 with items *B*29.1, *B*29.2, *B*31.1, and *B*31.2 naturally raises a question about all five data. This question is made more significant by the recent (although highly preliminary) report on the new NPL watt-balance experiment (see Sec. III.G.1).

(iii) Item *B*46 is in greatest disagreement with a number of other input data whose treatment involves either one or both of the relations $K_J=2e/h$ and $R_K=h/e^2$. This raises the question of whether relaxing the assumption that these relations are exact would reduce or possibly even eliminate the disagreement. To address this issue, the effect of assuming $K_J=(2e/h)(1+\varepsilon_J)$ and/or $R_K=(h/e^2)(1+\varepsilon_K)$, where ε_J and ε_K are unknown correction factors to be taken as additional adjusted constants, is discussed in Appendix F. The conclusion is that this generalization of the adjustments does not remove the disagreement or provide statistically significant evidence that the relations $K_J=2e/h$ and $R_K=h/e^2$ are not exact.

In view of points (i)–(iii), the Task Group decided that the best way to deal with this problem is to include in the final adjustment all five items of input data under consideration, but with their uncertainties weighted by a common multiplicative factor chosen so that the largest absolute value of a normalized residual among these data is 1.5. This ensures that the discrepancy is reduced to a quite acceptable level and that the final recommended value of the Planck constant h has an uncertainty that reflects the problematic nature of the current situation. The multiplicative factor turns out to be 2.325.

- *Adjustment 3*. This adjustment is obtained from adjustment 2 by weighting the uncertainties of input data *B*29.1, *B*29.2, *B*31.1, *B*31.2, and *B*46 with the above factor. The effect on the value of α and its uncertainty is quite minor, as one would expect, since α is mainly determined by items *B*9 and *B*33, the Uwash-87 result for a_e and the Stanford-02 result for h/m(Cs). The impact on the value of h itself is also quite small, because the uncertainties of the data that play the dominant role in determining h have all been weighted by the same factor. However, this results in the uncertainty of the value of h from adjust-

TABLE XIX. Observational equations that express the input data related to R_∞ in Table XI as functions of the adjusted constants in Table XVIII. The numbers in the first column correspond to the numbers in the first column of Table XI. The expressions for the energy levels of hydrogenic atoms are discussed in Appendix A. As pointed out in Sec. A.12 of that Appendix, $E_{\rm X}(n{\rm L}_j)/h$ is in fact proportional to cR_∞ and independent of h, hence h is not an adjusted constant in these equations. [The notation for hydrogenic energy levels $E_{\rm X}(n{\rm L}_j)$ and for additive corrections $\delta_{\rm X}(n{\rm L}_j)$ in this table have the same meaning as the notations $E^{\rm X}_{n{\rm L}j}$ and $\delta^{\rm X}_{n{\rm L}j}$ in Appendix A, Sec. A.12.] See Sec. IV.B for an explanation of the symbol $\doteq$.

Type of input datum	Observational equation
*A*1–*A*6 *A*13, *A*14	$\nu_{\rm H}(n_1{\rm L}_{1j_1}-n_2{\rm L}_{2j_2}) \doteq [E_{\rm H}(n_2{\rm L}_{2j_2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}(n_2{\rm L}_{2j_2}))$ $-E_{\rm H}(n_1{\rm L}_{1j_1};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}(n_1{\rm L}_{1j_1}))]/h$
*A*7–*A*12	$\nu_{\rm H}(n_1{\rm L}_{1j_1}-n_2{\rm L}_{2j_2})-\frac{1}{4}\nu_{\rm H}(n_3{\rm L}_{3j_3}-n_4{\rm L}_{4j_4}) \doteq \{E_{\rm H}(n_2{\rm L}_{2j_2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}(n_2{\rm L}_{2j_2}))$ $-E_{\rm H}(n_1{\rm L}_{1j_1};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}(n_1{\rm L}_{1j_1}))$ $-\frac{1}{4}[E_{\rm H}(n_4{\rm L}_{4j_4};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}(n_4{\rm L}_{4j_4}))$ $-E_{\rm H}(n_3{\rm L}_{3j_3};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}(n_3{\rm L}_{3j_3}))]\}/h$
*A*15	$R_{\rm p} \doteq R_{\rm p}$
*A*16–*A*20	$\nu_{\rm D}(n_1{\rm L}_{1j_1}-n_2{\rm L}_{2j_2}) \doteq [E_{\rm D}(n_2{\rm L}_{2j_2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}(n_2{\rm L}_{2j_2}))$ $-E_{\rm D}(n_1{\rm L}_{1j_1};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}(n_1{\rm L}_{1j_1}))]/h$
*A*21–*A*22	$\nu_{\rm D}(n_1{\rm L}_{1j_1}-n_2{\rm L}_{2j_2})-\frac{1}{4}\nu_{\rm D}(n_3{\rm L}_{3j_3}-n_4{\rm L}_{4j_4}) \doteq \{E_{\rm D}(n_2{\rm L}_{2j_2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}(n_2{\rm L}_{2j_2}))$ $-E_D(n_1{\rm L}_{1j_1};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}(n_1{\rm L}_{1j_1}))$ $-\frac{1}{4}[E_{\rm D}(n_4{\rm L}_{4j_4};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}(n_4{\rm L}_{4j_4}))$ $-E_{\rm D}(n_3{\rm L}_{3j_3};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}(n_3{\rm L}_{3j_3}))]\}/h$
*A*23	$R_{\rm d} \doteq R_{\rm d}$
*A*24	$\nu_{\rm D}({\rm 1S}_{1/2}-{\rm 2S}_{1/2})-\nu_{\rm H}({\rm 1S}_{1/2}-{\rm 2S}_{1/2}) \doteq \{E_{\rm D}({\rm 2S}_{1/2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}({\rm 2S}_{1/2}))$ $-E_{\rm D}({\rm 1S}_{1/2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm d}),R_{\rm d},\delta_{\rm D}({\rm 1S}_{1/2}))$ $-[E_{\rm H}({\rm 2S}_{1/2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}({\rm 2S}_{1/2}))$ $-E_{\rm H}({\rm 1S}_{1/2};R_\infty,\alpha,A_{\rm r}({\rm e}),A_{\rm r}({\rm p}),R_{\rm p},\delta_{\rm H}({\rm 1S}_{1/2}))]\}/h$
*A*25–*A*40	$\delta_{\rm H}(n{\rm L}_j) \doteq \delta_{\rm H}(n{\rm L}_j)$
*A*41–*A*49	$\delta_{\rm D}(n{\rm L}_j) \doteq \delta_{\rm D}(n{\rm L}_j)$

ment 3 being larger than that of the value of h from adjustment 2 by about the same weighting factor of 2.3.

- *Adjustment 4.* As measured by their self-sensitivity coefficients $S_{\rm c}$ in adjustment 3, a number of input data do not contribute significantly to the determination of the adjusted constants. In adjustment 4 we omit those six additional input data with $S_{\rm c}<0.01$ unless they are a subset of the data of an experiment that provides input data with $S_{\rm c}>0.01$. These six input data are *B*26.2, *B*28.1, and *B*30.2–*B*30.5, which are the NIM-95 result for $\Gamma'_{\rm p-90}({\rm lo})$, the KR/VN-98 result for $\Gamma'_{\rm h-90}({\rm lo})$, and the NML-97, NPL-88, NIM-95, and BNM-01 results for $R_{\rm K}$. The respective values of $S_{\rm c}$ in adjustment 3 are 0.001, 0.007, 0.006, 0.004, 0.001, and 0.004. Deleting such marginal data follows the practice of the 1998 CODATA adjustment. In this context, it is interesting to note that because of the weighting of the uncertainties of the five input data as discussed above, three input data that were omitted from the 1998 adjustment because their values of $S_{\rm c}$ were below the cutoff of 0.01, now have values of $S_{\rm c}>0.01$ and thus are included in the 2002 adjustment. These are input data *B*27.1, *B*27.2, and *B*32, the NIM-95 and the NPL-79 results for $\Gamma'_{\rm p-90}({\rm hi})$, and the NIST-80 result for $\mathcal{F}_{90}$. The respective values of $S_{\rm c}$ in adjustment 4 are 0.011, 0.029, and 0.017.

In adjustment 4, the NIST result for $\Gamma'_{\rm p-90}({\rm lo})$, item *B*26.1, has the largest normalized residual of any input datum; its value of $r_i=2.20$ is responsible for 15% of χ^2. In fact, the situation with item *B*26.1 in the 2002 adjustment is essentially the same as it was in the 1998 adjustment. Following what was done there, we retain datum *B*26.1 with no added weighting of its uncertainty, because its normalized residual is viewed as being within the acceptable range for use in the final adjustment while its value of $S_{\rm c}$ exceeds 0.01. We therefore use adjustment 4 to determine the 2002 CODATA recommended values of the constants.

- *Adjustments 5 and 6.* Adjustments 5 and 6 test the robustness of the final adjustment, that is, adjustment 4, while adjustments 7–11 probe various aspects of the data related to R_∞. Adjustment 5 is obtained from adjustment 4 by deleting the two most accurate input data that determine the adjusted value of

TABLE XX. The 33 adjusted constants (variables) used in the least-squares multivariate analysis of the input data in Table XIII. These adjusted constants appear as arguments of the functions on the right-hand side of the observational equations of Table XXI.

Adjusted constant	Symbol
Electron relative atomic mass	$A_r(e)$
Proton relative atomic mass	$A_r(p)$
Neutron relative atomic mass	$A_r(n)$
Deuteron relative atomic mass	$A_r(d)$
Helion relative atomic mass	$A_r(h)$
Alpha-particle relative atomic mass	$A_r(\alpha)$
$^{16}O^{7+}$ relative atomic mass	$A_r(^{16}O^{7+})$
^{133}Cs relative atomic mass	$A_r(^{133}Cs)$
Fine-structure constant	α
Additive correction to a_e(th)	δ_e
Additive correction to a_μ(th)	δ_μ
Additive correction to g_C(th)	δ_C
Additive correction to g_O(th)	δ_O
Electron-proton magnetic moment ratio	μ_{e^-}/μ_p
Deuteron-electron magnetic moment ratio	μ_d/μ_{e^-}
Electron to shielded proton magnetic moment ratio	μ_{e^-}/μ'_p
Shielded helion to shielded proton magnetic moment ratio	μ'_h/μ'_p
Neutron to shielded proton magnetic moment ratio	μ_n/μ'_p
Electron-muon mass ratio	m_e/m_μ
Additive correction to $\Delta\nu_{Mu}$(th)	δ_{Mu}
Planck constant	h
Molar gas constant	R
Copper Kα_1 x unit	xu(CuKα_1)
Molybdenum Kα_1 x unit	xu(MoKα_1)
Ångstrom star	Å*
d_{220} of Si crystal ILL	d_{220}(ILL)
d_{220} of Si crystal N	d_{220}(N)
d_{220} of Si crystal WASO 17	d_{220}(W17)
d_{220} of Si crystal WASO 04	d_{220}(W04)
d_{220} of Si crystal WASO 4.2a	d_{220}(W4.2a)
d_{220} of Si crystal MO*	d_{220}(MO*)
d_{220} of Si crystal NR3	d_{220}(NR3)
d_{220} of an ideal Si crystal	d_{220}

α—items $B9$ and $B33$, the Uwash-87 result for a_e and the Stanford-02 result for h/m(Cs). Similarly, adjustment 6 is obtained from adjustment 4 by deleting the two most accurate input data that determine the adjusted value of h—items $B31.1$ and $B31.2$, the NPL-90 and NIST-98 results for $K_J^2R_K$. Examination of Table XXII shows that the values of α and h from both adjustments 5 and 6 agree with those from Adjustment 4. This agreement indicates that the less accurate data are consistent with the more accurate data, which reflects well on the reliability of the 2002 CODATA recommended values of the constants.

We now turn our attention to Table XXIV and the data that primarily determine R_∞ and the bound-state rms charge radii of the proton and deuteron, R_p and R_d. In this table, the values of R_∞, R_p, and R_d from adjustment 4, which are the 2002 recommended values of these constants, serve as reference values.

It is useful to recall that in the 1998 least-squares adjustment of the constants, it was found that the normalized residuals of each $\delta_X(nS_{1/2})$, $n=1,2,3,4,6,8$, was in the narrow range $-1.410<r_i<-1.406$, indicating a systematic deviation between theory and experiment corresponding to $126/n^3$ kHz for $nS_{1/2}$ states. The most likely sources for this difference were believed to be a deviation of the value of the proton rms charge radius and/or the deuteron rms charge radius predicted by the spectroscopic data from the values deduced from scattering experiments, an uncalculated contribution to the energy levels from the two-photon QED correction that exceeded the estimated uncertainty for this term, or a combination of these. In the end, it was decided that the preferred way of treating the Rydberg-constant data was to omit as input data the values of the proton and deuteron radii obtained from the scattering data and to allow the values of R_p and R_d to be determined entirely by the spectroscopic data. At the same time, it was decided not to provide recommended values of R_p and R_d because of the observed systematic deviation between theory and experiment.

Fortunately, the situation in the 2002 least-squares adjustment is much improved due to advances made in the theory of H and D energy levels, as discussed in Appendix A, and an improved value of the proton radius deduced from the scattering data, as discussed in Sec. III.B.4. The normalized residuals of $\delta_X(nS_{1/2})$, $n=1,2,3,4,6,8$, show no systematic deviation and are all less than 0.02. In view of this apparent consistency, recommended values of R_p and R_d are included in the 2002 recommended set of constants.

- *Adjustment 7.* This adjustment is obtained from adjustment 4 by deleting the scattering-data input values for R_p and R_d, items $A15$ and $A23$. As can be seen from Table XXIV, the effect of these deletions on the magnitudes and uncertainties of R_∞, R_p, and R_d is small, indicating that the H and D spectroscopic data and theory play the dominant role in determining the recommended values of R_∞, R_p, and R_d.
- *Adjustments 8 and 9.* Adjustment 8 is obtained from adjustment 4 by deleting the scattering-data input value of R_p, item $A15$; and adjustment 9 is obtained from adjustment 4 by deleting the scattering-data input value of R_d, item $A23$. An examination of Table XXIV shows that the values of R_∞, R_p, and R_d from adjustments 8 and 9 again agree with the values from adjustment 4.
- *Adjustments 10 and 11.* These adjustments indicate the consistency of the hydrogen and deuterium data,

TABLE XXI. Observational equations that express the input data in Table XIII as functions of the adjusted constants in Table XX. The numbers in the first column correspond to the numbers in the first column of Table XIII. For simplicity, the lengthier functions are not explicitly given. See Sec. IV.B for an explanation of the symbol $\doteq$.

Type of input datum	Observational equation	Sec.
*B*1	$A_r(^1\mathrm{H}) \doteq A_r(\mathrm{p}) + A_r(\mathrm{e}) - E_b(^1\mathrm{H})/m_u c^2$	III.A.2
*B*2	$A_r(^2\mathrm{H}) \doteq A_r(\mathrm{d}) + A_r(\mathrm{e}) - E_b(^2\mathrm{H})/m_u c^2$	III.A.2
*B*3	$A_r(^3\mathrm{He}) \doteq A_r(\mathrm{h}) + 2A_r(\mathrm{e}) - E_b(^3\mathrm{He})/m_u c^2$	III.A.2
*B*4	$A_r(^4\mathrm{He}) \doteq A_r(\alpha) + 2A_r(\mathrm{e}) - E_b(^4\mathrm{He})/m_u c^2$	III.A.2
*B*5	$A_r(^{16}\mathrm{O}) \doteq A_r(^{16}\mathrm{O}^{7+}) + 7A_r(\mathrm{e}) - [E_b(^{16}\mathrm{O}) - E_b(^{16}\mathrm{O}^{7+})]/m_u c^2$	III.C.3.b
*B*6	$A_r(^{133}\mathrm{Cs}) \doteq A_r(^{133}\mathrm{Cs})$	
*B*7	$A_r(\mathrm{e}) \doteq A_r(\mathrm{e})$	
*B*8	$A_r(\mathrm{p}) \doteq A_r(\mathrm{p})$	
*B*9	$a_e \doteq a_e(\alpha, \delta_e)$	App. B
*B*10	$\delta_e \doteq \delta_e$	
*B*11	$\overline{R} \doteq -\dfrac{a_\mu(\alpha,\delta_\mu)}{1+a_e(\alpha,\delta_e)} \dfrac{m_e}{m_\mu} \dfrac{\mu_{e^-}}{\mu_p}$	III.C.2.a
*B*12	$\delta_\mu \doteq \delta_\mu$	
*B*13	$\dfrac{f_s(^{12}\mathrm{C}^{5+})}{f_c(^{12}\mathrm{C}^{5+})} \doteq -\dfrac{g_C(\alpha,\delta_C)}{10A_r(\mathrm{e})}\left[12 - 5A_r(\mathrm{e}) + \dfrac{E_b(^{12}\mathrm{C}) - E_b(^{12}\mathrm{C}^{5+})}{m_u c^2}\right]$	III.C.3.a
*B*14	$\delta_C \doteq \delta_C$	
*B*15	$\dfrac{f_s(^{16}\mathrm{O}^{7+})}{f_c(^{16}\mathrm{O}^{7+})} \doteq -\dfrac{g_O(\alpha,\delta_O)}{14A_r(\mathrm{e})} A_r(^{16}\mathrm{O}^{7+})$	III.C.3.b
*B*16	$\delta_O \doteq \delta_O$	
*B*17	$\dfrac{\mu_{e^-}(\mathrm{H})}{\mu_p(\mathrm{H})} \doteq \dfrac{g_{e^-}(\mathrm{H})}{g_{e^-}} \left(\dfrac{g_p(\mathrm{H})}{g_p}\right)^{-1} \dfrac{\mu_{e^-}}{\mu_p}$	III.C.4.a
*B*18	$\dfrac{\mu_d(\mathrm{D})}{\mu_{e^-}(\mathrm{D})} \doteq \dfrac{g_d(\mathrm{D})}{g_d} \left(\dfrac{g_{e^-}(\mathrm{D})}{g_{e^-}}\right)^{-1} \dfrac{\mu_d}{\mu_{e^-}}$	III.C.4.b
*B*19	$\dfrac{\mu_{e^-}(\mathrm{H})}{\mu'_p} \doteq \dfrac{g_{e^-}(\mathrm{H})}{g_{e^-}} \dfrac{\mu_{e^-}}{\mu'_p}$	III.C.4.c
*B*20	$\dfrac{\mu'_h}{\mu'_p} \doteq \dfrac{\mu'_h}{\mu'_p}$	
*B*21	$\dfrac{\mu_n}{\mu'_p} \doteq \dfrac{\mu_n}{\mu'_p}$	
*B*22, *B*23	$\nu(f_p) \doteq \nu\left(f_p; R_\infty, \alpha, \dfrac{m_e}{m_\mu}, \dfrac{\mu_{e^-}}{\mu_p}, \delta_e, \delta_\mu, \delta_{Mu}\right)$	III.C.5
*B*24	$\Delta\nu_{Mu} \doteq \Delta\nu_{Mu}\left(R_\infty, \alpha, \dfrac{m_e}{m_\mu}, \delta_\mu, \delta_{Mu}\right)$	App. E
*B*25	$\delta_{Mu} \doteq \delta_{Mu}$	
*B*26	$\Gamma'_{p-90}(\mathrm{lo}) \doteq -\dfrac{K_{J-90}R_{K-90}[1+a_e(\alpha,\delta_e)]\alpha^3}{2\mu_0 R_\infty}\left(\dfrac{\mu_{e^-}}{\mu'_p}\right)^{-1}$	III.D.1.a

TABLE XXI. (*Continued.*)

Type of input datum	Observational equation	Sec.
*B*27	$\Gamma'_{\mathrm{p-90}}(\mathrm{hi}) \doteq -\dfrac{c[1+a_{\mathrm{e}}(\alpha,\delta_{\mathrm{e}})]\alpha^2}{K_{\mathrm{J-90}}R_{\mathrm{K-90}}R_\infty h}\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_{\mathrm{p}}}\right)^{-1}$	III.D.1.b
*B*28	$\Gamma'_{\mathrm{h-90}}(\mathrm{lo}) \doteq \dfrac{K_{\mathrm{J-90}}R_{\mathrm{K-90}}[1+a_{\mathrm{e}}(\alpha,\delta_{\mathrm{e}})]\alpha^3}{2\mu_0 R_\infty}\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_{\mathrm{p}}}\right)^{-1}\dfrac{\mu'_{\mathrm{h}}}{\mu'_{\mathrm{p}}}$	III.D.2.a
*B*29	$K_{\mathrm{J}} \doteq \left(\dfrac{8\alpha}{\mu_0 ch}\right)^{1/2}$	III.E.1
*B*30	$R_{\mathrm{K}} \doteq \dfrac{\mu_0 c}{2\alpha}$	III.F
*B*31	$K_{\mathrm{J}}^2 R_{\mathrm{K}} \doteq \dfrac{4}{h}$	III.G
*B*32	$\mathcal{F}_{90} \doteq \dfrac{cM_{\mathrm{u}}A_{\mathrm{r}}(\mathrm{e})\alpha^2}{K_{\mathrm{J-90}}R_{\mathrm{K-90}}R_\infty h}$	III.H
*B*33	$\dfrac{h}{m(^{133}\mathrm{Cs})} \doteq \dfrac{A_{\mathrm{r}}(\mathrm{e})}{A_{\mathrm{r}}(^{133}\mathrm{Cs})}\dfrac{c\alpha^2}{2R_\infty}$	III.K.2
*B*34	$\dfrac{h}{m_{\mathrm{n}}d_{220}(\mathrm{W04})} \doteq \dfrac{A_{\mathrm{r}}(\mathrm{e})}{A_{\mathrm{r}}(\mathrm{n})}\dfrac{c\alpha^2}{2R_\infty d_{220}(\mathrm{W04})}$	III.K.1
*B*35	$\dfrac{\lambda_{\mathrm{meas}}}{d_{220}(\mathrm{ILL})} \doteq \dfrac{\alpha^2 A_{\mathrm{r}}(\mathrm{e})}{R_\infty d_{220}(\mathrm{ILL})}\dfrac{A_{\mathrm{r}}(\mathrm{n})+A_{\mathrm{r}}(\mathrm{p})}{[A_{\mathrm{r}}(\mathrm{n})+A_{\mathrm{r}}(\mathrm{p})]^2-A_{\mathrm{r}}^2(\mathrm{d})}$	III.A.5
*B*36–*B*39	$1-\dfrac{d_{220}(\mathrm{Y})}{d_{220}(\mathrm{X})} \doteq 1-\dfrac{d_{220}(\mathrm{Y})}{d_{220}(\mathrm{X})}$	
*B*40–*B*43	$\dfrac{d_{220}(\mathrm{X})}{d_{220}(\mathrm{Y})}-1 \doteq \dfrac{d_{220}(\mathrm{X})}{d_{220}(\mathrm{Y})}-1$	
*B*44	$\dfrac{d_{220}}{d_{220}(\mathrm{W04})}-1 \doteq \dfrac{d_{220}}{d_{220}(\mathrm{W04})}-1$	
*B*45	$d_{220}(\mathrm{NR3}) \doteq d_{220}(\mathrm{NR3})$	
*B*46	$V_{\mathrm{m}}(\mathrm{Si}) \doteq \dfrac{\sqrt{2}cM_{\mathrm{u}}A_{\mathrm{r}}(\mathrm{e})\alpha^2 d_{220}^3}{R_\infty h}$	III.J
*B*47	$R \doteq R$	
*B*48, *B*51	$\dfrac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{X})} \doteq \dfrac{1537.400\ \mathrm{xu}(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{X})}$	III.R
*B*49	$\dfrac{\lambda(\mathrm{WK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \dfrac{0.209\,010\,0\ \text{Å}^*}{d_{220}(\mathrm{N})}$	III.R
*B*50	$\dfrac{\lambda(\mathrm{MoK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \dfrac{707.831\ \mathrm{xu}(\mathrm{MoK}\alpha_1)}{d_{220}(\mathrm{N})}$	III.R

TABLE XXII. Summary of the results of some of the least-squares adjustments used to analyze all of the input data given in Tables XI–XIV. The values of α and h are those obtained in the adjustment, N is the number of input data, M is the number of adjusted constants, $\nu=N-M$ is the degrees of freedom, and $R_B=\sqrt{\chi^2/\nu}$ is the Birge ratio.

Adj.	N	M	ν	χ^2	R_B	α^{-1}	$u_r(\alpha^{-1})$	h/(J s)	$u_r(h)$
1	112	61	51	58.5	1.07	137.035 999 14(45)	3.3×10^{-9}	$6.626\,069\,21(50)\times10^{-34}$	7.6×10^{-8}
2	111	61	50	46.1	0.96	137.035 999 18(45)	3.3×10^{-9}	$6.626\,069\,21(50)\times10^{-34}$	7.6×10^{-8}
3	111	61	50	36.0	0.85	137.035 999 11(45)	3.3×10^{-9}	$6.626\,0693(11)\times10^{-34}$	1.7×10^{-7}
4	105	61	44	31.2	0.84	137.035 999 11(46)	3.3×10^{-9}	$6.626\,0693(11)\times10^{-34}$	1.7×10^{-7}
5	103	61	42	29.7	0.84	137.036 0004(23)	1.6×10^{-8}	$6.626\,0693(11)\times10^{-34}$	1.7×10^{-7}
6	103	61	42	29.9	0.84	137.035 999 09(46)	3.3×10^{-9}	$6.626\,0723(29)\times10^{-34}$	4.4×10^{-7}

both experimental and theoretical; only the data on H are included in adjustment 10, and only the data on D are included in adjustment 11. However, in each case, input datum *A*24, the MPQ-98 result for the H-D isotope shift, is omitted. It is clear from Table XXIV that the values of R_∞ resulting from the two adjustments are in agreement with each other and with those from adjustments 4–9. Further, the value of R_p from adjustment 10 and the value of R_d from adjustment 11 also agree with their counterparts from adjustments 4–9. These adjustments provide additional evidence that the experimental and theoretical H and D data are consistent.

This concludes the "Analysis of Data" section of our report; the next section provides the 2002 recommended values.

V. THE 2002 CODATA RECOMMENDED VALUES

A. Calculation details

The 2002 recommended values of the constants, as indicated in Sec. IV.B, are based on adjustment 4 of Tables XXII–XXIV. To reiterate, adjustment 4 is obtained by deleting input data *B*26.2, *B*28.1, *B*28.2, and *B*30.2–*B*30.5 from the initially considered 112 input data of Tables XI and XIII and by weighting the uncertainties of input data *B*29.1, *B*29.2, *B*31.1, *B*31.2, and *B*46 by the multiplicative factor 2.325. The correlation coefficients of the data, as given in Tables XII and XIV, are also taken into account. The 105 final input data are expressed in terms of the 61 adjusted constants of Tables XVIII and XX, corresponding to $\nu=N-M=44$ degrees of freedom. For this final adjustment, $\chi^2=31.2$, $R_B=\sqrt{\chi^2/\nu}=0.84$, and $Q(31.2|44)=0.93$, where $Q(\chi^2|\nu)$ is the probability that the observed value of χ^2 for ν degrees of freedom would have exceeded that observed value (see Appendix E of CODATA-98). Each input datum included in the final adjustment has a self-sensitivity coefficient $S_c>0.01$, or is a subset of the data of an experiment that provides an input datum or input data with $S_c>0.01$. The four input data with the largest absolute normalized residuals $|r_i|$ are *B*26.1, *B*46, *B*7, and *B*30.1; their respective values of r_i are 2.20, −1.50, 1.43, and 1.39. Three other input data have values of r_i of 1.11, −1.11, and 1.05; all other values of $|r_i|$ are less than 1.

The output of adjustment 4 is the set of best estimated values, in the least-squares sense, of the 61 adjusted constants together with their variances and covariances. All of the 2002 recommended values, including their uncertainties, are obtained from these 61 constants, together with (i) those constants that have defined values such as c and μ_0; (ii) the value of G deduced in Sec. III.Q; and (iii) the values of m_τ, G_F, and $\sin^2\theta_W$ given in Sec. III.S. The calculational details are straightforward and are explained fully in Sec. V.B of CODATA-98.

B. Tables of values

The 2002 CODATA recommended values of the basic constants and conversion factors of physics and chemis-

TABLE XXIII. Normalized residuals r_i and self-sensitivity coefficients S_c that result from the six least-squares adjustments summarized in Table XXII for the four input data whose absolute values of r_i in adjustment 1 exceed 1.50. [S_c is a measure of how the least-squares estimated value of a given type of input datum depends on a particular measured or calculated value of that type of datum; see Appendix E of CODATA-98 (Mohr and Taylor, 2000).]

Item number	Input quantity	Identification	Adj. 1		Adj. 2		Adj. 3		Adj. 4		Adj. 5		Adj. 6	
			r_i	S_c	r_i	S_c	r_i	S_c	r_i	S_c	r_i	S_c	r_i	S_c
*B*28.2	Γ'_{h-90}(lo)	VNIIM-89	3.52	0.002										
*B*46	V_m(Si)	N/P/I-03	−3.18	0.173	−3.17	0.173	−1.50	0.083	−1.50	0.083	−1.42	0.100	−0.86	0.416
*B*26.1	Γ'_{p-90}(lo)	NIST-89	2.19	0.017	2.21	0.017	2.19	0.017	2.20	0.018	2.45	0.205	2.20	0.018
*B*28.1	Γ'_{h-90}(lo)	KR/VN-98	1.68	0.007	1.69	0.007	1.68	0.007						

TABLE XXIV. Summary of the results of some of the least-squares adjustments used to analyze the input data related to R_∞. The values of R_∞, R_p, and R_d are those obtained in the indicated adjustment, N is the number of input data, M is the number of adjusted constants, $\nu = N - M$ is the degrees of freedom, and $R_B = \sqrt{\chi^2/\nu}$ is the Birge ratio.

Adj.	N	M	ν	χ^2	R_B	R_∞/m^{-1}	$u_r(R_\infty)$	R_p/fm	R_d/fm
4	105	61	44	31.2	0.84	10 973 731.568 525(73)	6.6×10^{-12}	0.8750(68)	2.1394(28)
7	103	61	42	29.0	0.83	10 973 731.568 511(82)	7.5×10^{-12}	0.8736(77)	2.1389(32)
8	104	61	43	29.7	0.83	10 973 731.568 490(78)	7.1×10^{-12}	0.8717(74)	2.1381(30)
9	104	61	43	30.2	0.84	10 973 731.568 546(76)	6.9×10^{-12}	0.8769(71)	2.1402(29)
10	87	36	51	27.1	0.87	10 973 731.568 559(85)	7.8×10^{-12}	0.8782(80)	
11	72	28	44	20.9	0.86	10 973 731.568 39(13)	1.1×10^{-11}		2.1285(93)

try, including the values of related quantities, are given in Tables XXV–XXXII. These tables are nearly identical in form to their 1998 counterparts; the difference is that four new recommended values have been added to Table XXVI and one of these reappears in Table XXX.The new constants are $\hbar c$ in Mev fm, the Planck temperature T_P, and the proton and deuteron bound-state rms charge radii R_p and R_d.

Table XXV is a highly abbreviated list containing the values of the constants and conversion factors most

TABLE XXV. An abbreviated list of the CODATA recommended values of the fundamental constants of physics and chemistry based on the 2002 adjustment.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c, c_0	299 792 458	$\mathrm{m\ s^{-1}}$	(exact)
magnetic constant	μ_0	$4\pi\times10^{-7}$	$\mathrm{N\ A^{-2}}$	
		$=12.566\,370\,614\ldots\times10^{-7}$	$\mathrm{N\ A^{-2}}$	(exact)
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817\ldots\times10^{-12}$	$\mathrm{F\ m^{-1}}$	(exact)
Newtonian constant of gravitation	G	$6.6742(10)\times10^{-11}$	$\mathrm{m^3\ kg^{-1}\ s^{-2}}$	1.5×10^{-4}
Planck constant	h	$6.626\,0693(11)\times10^{-34}$	J s	1.7×10^{-7}
$h/2\pi$	$\hbar$	$1.054\,571\,68(18)\times10^{-34}$	J s	1.7×10^{-7}
elementary charge	e	$1.602\,176\,53(14)\times10^{-19}$	C	8.5×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,72(18)\times10^{-15}$	Wb	8.5×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,733(26)\times10^{-5}$	S	3.3×10^{-9}
electron mass	m_e	$9.109\,3826(16)\times10^{-31}$	kg	1.7×10^{-7}
proton mass	m_p	$1.672\,621\,71(29)\times10^{-27}$	kg	1.7×10^{-7}
proton-electron mass ratio	m_p/m_e	1836.152 672 61(85)		4.6×10^{-10}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,568(24)\times10^{-3}$		3.3×10^{-9}
inverse fine-structure constant	α^{-1}	137.035 999 11(46)		3.3×10^{-9}
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	10 973 731.568 525(73)	$\mathrm{m^{-1}}$	6.6×10^{-12}
Avogadro constant	N_A, L	$6.022\,1415(10)\times10^{23}$	$\mathrm{mol^{-1}}$	1.7×10^{-7}
Faraday constant $N_A e$	F	96 485.3383(83)	$\mathrm{C\ mol^{-1}}$	8.6×10^{-8}
molar gas constant	R	8.314 472(15)	$\mathrm{J\ mol^{-1}\ K^{-1}}$	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6505(24)\times10^{-23}$	$\mathrm{J\ K^{-1}}$	1.8×10^{-6}
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,400(40)\times10^{-8}$	$\mathrm{W\ m^{-2}\ K^{-4}}$	7.0×10^{-6}
Non-SI units accepted for use with the SI				
electron volt: (e/C) J	eV	$1.602\,176\,53(14)\times10^{-19}$	J	8.5×10^{-8}
(unified) atomic mass unit: 1 u $= m_u = \frac{1}{12}m(^{12}\mathrm{C})$ $=10^{-3}\ \mathrm{kg\ mol^{-1}}/N_A$	u	$1.660\,538\,86(28)\times10^{-27}$	kg	1.7×10^{-7}

TABLE XXVI. The CODATA recommended values of the fundamental constants of physics and chemistry based on the 2002 adjustment.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
		UNIVERSAL		
speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	(exact)
magnetic constant	μ_0	$4\pi \times 10^{-7}$	N A^{-2}	
		$=12.566\,370\,614\ldots \times 10^{-7}$	N A^{-2}	(exact)
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817\ldots \times 10^{-12}$	F m^{-1}	(exact)
characteristic impedance of vacuum $\sqrt{\mu_0/\epsilon_0} = \mu_0 c$	Z_0	376.730 313 461…	Ω	(exact)
Newtonian constant of gravitation	G	$6.6742(10) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	1.5×10^{-4}
	$G/\hbar c$	$6.7087(10) \times 10^{-39}$	(GeV/c^2)$^{-2}$	1.5×10^{-4}
Planck constant	h	$6.626\,0693(11) \times 10^{-34}$	J s	1.7×10^{-7}
in eV s		$4.135\,667\,43(35) \times 10^{-15}$	eV s	8.5×10^{-8}
$h/2\pi$	$\hbar$	$1.054\,571\,68(18) \times 10^{-34}$	J s	1.7×10^{-7}
in eV s		$6.582\,119\,15(56) \times 10^{-16}$	eV s	8.5×10^{-8}
$\hbar c$ in MeV fm		197.326 968(17)	MeV fm	8.5×10^{-8}
Planck mass $(\hbar c/G)^{1/2}$	m_P	$2.176\,45(16) \times 10^{-8}$	kg	7.5×10^{-5}
Planck temperature $(\hbar c^5/G)^{1/2}/k$	T_P	$1.416\,79(11) \times 10^{32}$	K	7.5×10^{-5}
Planck length $\hbar/m_P c = (\hbar G/c^3)^{1/2}$	l_P	$1.616\,24(12) \times 10^{-35}$	m	7.5×10^{-5}
Planck time $l_P/c = (\hbar G/c^5)^{1/2}$	t_P	$5.391\,21(40) \times 10^{-44}$	s	7.5×10^{-5}
		ELECTROMAGNETIC		
elementary charge	e	$1.602\,176\,53(14) \times 10^{-19}$	C	8.5×10^{-8}
	e/h	$2.417\,989\,40(21) \times 10^{14}$	A J^{-1}	8.5×10^{-8}
magnetic-flux quantum $h/2e$	Φ_0	$2.067\,833\,72(18) \times 10^{-15}$	Wb	8.5×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,733(26) \times 10^{-5}$	S	3.3×10^{-9}
inverse of conductance quantum	G_0^{-1}	12 906.403 725(43)	Ω	3.3×10^{-9}
Josephson constant[a] $2e/h$	K_J	$483\,597.879(41) \times 10^9$	Hz V^{-1}	8.5×10^{-8}
von Klitzing constant[b] $h/e^2 = \mu_0 c/2\alpha$	R_K	25 812.807 449(86)	Ω	3.3×10^{-9}
Bohr magneton $e\hbar/2m_e$	μ_B	$927.400\,949(80) \times 10^{-26}$	J T^{-1}	8.6×10^{-8}
in eV T^{-1}		$5.788\,381\,804(39) \times 10^{-5}$	eV T^{-1}	6.7×10^{-9}
	μ_B/h	$13.996\,2458(12) \times 10^9$	Hz T^{-1}	8.6×10^{-8}
	μ_B/hc	46.686 4507(40)	m^{-1} T^{-1}	8.6×10^{-8}
	μ_B/k	0.671 7131(12)	K T^{-1}	1.8×10^{-6}
nuclear magneton $e\hbar/2m_p$	μ_N	$5.050\,783\,43(43) \times 10^{-27}$	J T^{-1}	8.6×10^{-8}
in eV T^{-1}		$3.152\,451\,259(21) \times 10^{-8}$	eV T^{-1}	6.7×10^{-9}
	μ_N/h	7.622 593 71(65)	MHz T^{-1}	8.6×10^{-8}
	μ_N/hc	$2.542\,623\,58(22) \times 10^{-2}$	m^{-1} T^{-1}	8.6×10^{-8}
	μ_N/k	$3.658\,2637(64) \times 10^{-4}$	K T^{-1}	1.8×10^{-6}
		ATOMIC AND NUCLEAR		
		General		
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,568(24) \times 10^{-3}$		3.3×10^{-9}
inverse fine-structure constant	α^{-1}	137.035 999 11(46)		3.3×10^{-9}
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	10 973 731.568 525(73)	m^{-1}	6.6×10^{-12}

TABLE XXVI. (*Continued.*)

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
	$R_\infty c$	$3.289\,841\,960\,360(22)\times10^{-15}$	Hz	6.6×10^{-12}
	$R_\infty hc$	$2.179\,872\,09(37)\times10^{-18}$	J	1.7×10^{-7}
$R_\infty hc$ in eV		13.605 6923(12)	eV	8.5×10^{-8}
Bohr radius $\alpha/4\pi R_\infty=4\pi\epsilon_0\hbar^2/m_e e^2$	a_0	$0.529\,177\,2108(18)\times10^{-10}$	m	3.3×10^{-9}
Hartree energy $e^2/4\pi\epsilon_0 a_0=2R_\infty hc$ $=\alpha^2 m_e c^2$	E_h	$4.359\,744\,17(75)\times10^{-18}$	J	1.7×10^{-7}
in eV		27.211 3845(23)	eV	8.5×10^{-8}
quantum of circulation	$h/2m_e$	$3.636\,947\,550(24)\times10^{-4}$	$m^2\,s^{-1}$	6.7×10^{-9}
	h/m_e	$7.273\,895\,101(48)\times10^{-4}$	$m^2\,s^{-1}$	6.7×10^{-9}
		Electroweak		
Fermi coupling constant[c]	$G_F/(\hbar c)^3$	$1.166\,39(1)\times10^{-5}$	GeV^{-2}	8.6×10^{-6}
weak mixing angle[d] θ_W (on-shell scheme) $\sin^2\theta_W=s_W^2$ $\equiv1-(m_W/m_Z)^2$	$\sin^2\theta_W$	0.222 15(76)		3.4×10^{-3}
		Electron, e^-		
electron mass	m_e	$9.109\,3826(16)\times10^{-31}$	kg	1.7×10^{-7}
in u, $m_e=A_r(e)$ u (electron relative atomic mass times u)		$5.485\,799\,0945(24)\times10^{-4}$	u	4.4×10^{-10}
energy equivalent	$m_e c^2$	$8.187\,1047(14)\times10^{-14}$	J	1.7×10^{-7}
in MeV		0.510 998 918(44)	MeV	8.6×10^{-8}
electron-muon mass ratio	m_e/m_μ	$4.836\,331\,67(13)\times10^{-3}$		2.6×10^{-8}
electron-tau mass ratio	m_e/m_τ	$2.875\,64(47)\times10^{-4}$		1.6×10^{-4}
electron-proton mass ratio	m_e/m_p	$5.446\,170\,2173(25)\times10^{-4}$		4.6×10^{-10}
electron-neutron mass ratio	m_e/m_n	$5.438\,673\,4481(38)\times10^{-4}$		7.0×10^{-10}
electron-deuteron mass ratio	m_e/m_d	$2.724\,437\,1095(13)\times10^{-4}$		4.8×10^{-10}
electron to alpha-particle mass ratio	m_e/m_α	$1.370\,933\,555\,75(61)\times10^{-4}$		4.4×10^{-10}
electron charge to mass quotient	$-e/m_e$	$-1.758\,820\,12(15)\times10^{11}$	$C\,kg^{-1}$	8.6×10^{-8}
electron molar mass $N_A m_e$	$M(e), M_e$	$5.485\,799\,0945(24)\times10^{-7}$	$kg\,mol^{-1}$	4.4×10^{-10}
Compton wavelength $h/m_e c$	λ_C	$2.426\,310\,238(16)\times10^{-12}$	m	6.7×10^{-9}
$\lambda_C/2\pi=\alpha a_0=\alpha^2/4\pi R_\infty$	$\lambda\!\!\!^{-}_C$	$386.159\,2678(26)\times10^{-15}$	m	6.7×10^{-9}
classical electron radius $\alpha^2 a_0$	r_e	$2.817\,940\,325(28)\times10^{-15}$	m	1.0×10^{-8}
Thomson cross section $(8\pi/3)r_e^2$	σ_e	$0.665\,245\,873(13)\times10^{-28}$	m^2	2.0×10^{-8}
electron magnetic moment	μ_e	$-928.476\,412(80)\times10^{-26}$	$J\,T^{-1}$	8.6×10^{-8}
to Bohr magneton ratio	μ_e/μ_B	−1.001 159 652 1859(38)		3.8×10^{-12}
to nuclear magneton ratio	μ_e/μ_N	−1838.281 971 07(85)		4.6×10^{-10}
electron magnetic moment anomaly $\lvert\mu_e\rvert/\mu_B-1$	a_e	$1.159\,652\,1859(38)\times10^{-3}$		3.2×10^{-9}
electron g-factor $-2(1+a_e)$	g_e	−2.002 319 304 3718(75)		3.8×10^{-12}
electron-muon magnetic moment ratio	μ_e/μ_μ	206.766 9894(54)		2.6×10^{-8}
electron-proton magnetic moment ratio	μ_e/μ_p	−658.210 6862(66)		1.0×10^{-8}
electron to shielded proton magnetic moment ratio (H_2O, sphere, 25 °C)	μ_e/μ'_p	−658.227 5956(71)		1.1×10^{-8}

TABLE XXVI. (*Continued.*)

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
electron-neutron magnetic moment ratio	μ_e/μ_n	960.920 50(23)		2.4×10^{-7}
electron-deuteron magnetic moment ratio	μ_e/μ_d	−2143.923 493(23)		1.1×10^{-8}
electron to shielded helion[e] magnetic moment ratio (gas, sphere, 25 °C)	μ_e/μ'_h	864.058 255(10)		1.2×10^{-8}
electron gyromagnetic ratio $2\lvert\mu_e\rvert/\hbar$	γ_e	$1.760\,859\,74(15)\times10^{11}$	$s^{-1}\,T^{-1}$	8.6×10^{-8}
	$\gamma_e/2\pi$	28 024.9532(24)	$MHz\,T^{-1}$	8.6×10^{-8}
		Muon, μ^-		
muon mass	m_μ	$1.883\,531\,40(33)\times10^{-28}$	kg	1.7×10^{-7}
in u, $m_\mu=A_r(\mu)$ u (muon relative atomic mass times u)		0.113 428 9264(30)	u	2.6×10^{-8}
energy equivalent	$m_\mu c^2$	$1.692\,833\,60(29)\times10^{-11}$	J	1.7×10^{-7}
in MeV		105.658 3692(94)	MeV	8.9×10^{-8}
muon-electron mass ratio	m_μ/m_e	206.768 2838(54)		2.6×10^{-8}
muon-tau mass ratio	m_μ/m_τ	$5.945\,92(97)\times10^{-2}$		1.6×10^{-4}
muon-proton mass ratio	m_μ/m_p	0.112 609 5269(29)		2.6×10^{-8}
muon-neutron mass ratio	m_μ/m_n	0.112 454 5175(29)		2.6×10^{-8}
muon molar mass $N_A m_\mu$	$M(\mu), m_\mu$	$0.113\,428\,9264(30)\times10^{-3}$	$kg\,mol^{-1}$	2.6×10^{-8}
muon Compton wavelength $h/m_\mu c$	$\lambda_{C,\mu}$	$11.734\,441\,05(30)\times10^{-15}$	m	2.5×10^{-8}
$\lambda_{C,\mu}/2\pi$	$\lambda\!\!\!^{-}_{C,\mu}$	$1.867\,594\,298(47)\times10^{-15}$	m	2.5×10^{-8}
muon magnetic moment	μ_μ	$-4.490\,447\,99(40)\times10^{-26}$	$J\,T^{-1}$	8.9×10^{-8}
to Bohr magneton ratio	μ_μ/μ_B	$-4.841\,970\,45(13)\times10^{-3}$		2.6×10^{-8}
to nuclear magneton ratio	μ_μ/μ_N	−8.890 596 98(23)		2.6×10^{-8}
muon magnetic moment anomaly $\lvert\mu_\mu\rvert/(e\hbar/2m_\mu)-1$	a_μ	$1.165\,919\,81(62)\times10^{-3}$		5.3×10^{-7}
muon g-factor $-2(1+a_\mu)$	g_μ	−2.002 331 8396(12)		6.2×10^{-10}
muon-proton magnetic moment ratio	μ_μ/μ_p	−3.183 345 118(89)		2.8×10^{-8}
		Tau, τ^-		
tau mass[f]	m_τ	$3.167\,77(52)\times10^{-27}$	kg	1.6×10^{-4}
in u, $m_\tau=A_r(\tau)$ u (tau relative atomic mass times u)		1.907 68(31)	u	1.6×10^{-4}
energy equivalent	$m_\tau c^2$	$2.847\,05(46)\times10^{-10}$	J	1.6×10^{-4}
in MeV		1776.99(29)	MeV	1.6×10^{-4}
tau-electron mass ratio	m_τ/m_e	3477.48(57)		1.6×10^{-4}
tau-muon mass ratio	m_τ/m_μ	16.8183(27)		1.6×10^{-4}
tau-proton mass ratio	m_τ/m_p	1.893 90(31)		1.6×10^{-4}
tau-neutron mass ratio	m_τ/m_n	1.891 29(31)		1.6×10^{-4}
tau molar mass $N_A m_\tau$	$M(\tau), M_\tau$	$1.907\,68(31)\times10^{-3}$	$kg\,mol^{-1}$	1.6×10^{-4}
tau Compton wavelength $h/m_\tau c$	$\lambda_{C,\tau}$	$0.697\,72(11)\times10^{-15}$	m	1.6×10^{-4}
$\lambda_{C,\tau}/2\pi$	$\lambda\!\!\!^{-}_{C,\tau}$	$0.111\,046(18)\times10^{-15}$	m	1.6×10^{-4}
		Proton, p		
proton mass	m_p	$1.672\,621\,71(29)\times10^{-27}$	kg	1.7×10^{-7}

TABLE XXVI. (*Continued.*)

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
in u, $m_p = A_r(p)$ u (proton relative atomic mass times u)		1.007 276 466 88(13)	u	1.3×10^{-10}
energy equivalent	$m_p c^2$	$1.503\,277\,43(26)\times10^{-10}$	J	1.7×10^{-7}
in MeV		938.272 029(80)	MeV	8.6×10^{-8}
proton-electron mass ratio	m_p/m_e	1836.152 672 61(85)		4.6×10^{-10}
proton-muon mass ratio	m_p/m_μ	8.880 243 33(23)		2.6×10^{-8}
proton-tau mass ratio	m_p/m_τ	0.528 012(86)		1.6×10^{-4}
proton-neutron mass ratio	m_p/m_n	0.998 623 478 72(58)		5.8×10^{-10}
proton charge to mass quotient	e/m_p	$9.578\,833\,76(82)\times10^{7}$	C kg^{-1}	8.6×10^{-8}
proton molar mass $N_A m_p$	$M(p), M_p$	$1.007\,276\,466\,88(13)\times10^{-3}$	kg mol^{-1}	1.3×10^{-10}
proton Compton wavelength $h/m_p c$	$\lambda_{C,p}$	$1.321\,409\,8555(88)\times10^{-15}$	m	6.7×10^{-9}
$\lambda_{C,p}/2\pi$	$\lambda\!\!\!^{-}_{C,p}$	$0.210\,308\,9104(14)\times10^{-15}$	m	6.7×10^{-9}
proton rms charge radius	R_p	$0.8750(68)\times10^{-15}$	m	7.8×10^{-3}
proton magnetic moment	μ_p	$1.410\,606\,71(12)\times10^{-26}$	J T^{-1}	8.7×10^{-8}
to Bohr magneton ratio	μ_p/μ_B	$1.521\,032\,206(15)\times10^{-3}$		1.0×10^{-8}
to nuclear magneton ratio	μ_p/μ_N	2.792 847 351(28)		1.0×10^{-8}
proton g-factor $2\mu_p/\mu_N$	g_p	5.585 694 701(56)		1.0×10^{-8}
proton-neutron magnetic moment ratio	μ_p/μ_n	−1.459 898 05(34)		2.4×10^{-7}
shielded proton magnetic moment (H_2O, sphere, 25 °C)	μ'_p	$1.410\,570\,47(12)\times10^{-26}$	J T^{-1}	8.7×10^{-8}
to Bohr magneton ratio	μ'_p/μ_B	$1.520\,993\,132(16)\times10^{-3}$		1.1×10^{-8}
to nuclear magneton ratio	μ'_p/μ_N	2.792 775 604(30)		1.1×10^{-8}
proton magnetic shielding correction $1-\mu'_p/\mu_p$ (H_2O, sphere, 25 °C)	σ'_p	$25.689(15)\times10^{-6}$		5.7×10^{-4}
proton gyromagnetic ratio $2\mu_p/\hbar$	γ_p	$2.675\,222\,05(23)\times10^{8}$	s^{-1} T^{-1}	8.6×10^{-8}
	$\gamma_p/2\pi$	42.577 4813(37)	MHz T^{-1}	8.6×10^{-8}
shielded proton gyromagnetic ratio $2\mu'_p/\hbar$ (H_2O, sphere, 25 °C)	γ'_p	$2.675\,153\,33(23)\times10^{8}$	s^{-1} T^{-1}	8.6×10^{-8}
	$\gamma'_p/2\pi$	42.576 3875(37)	MHz T^{-1}	8.6×10^{-8}
		Neutron, n		
neutron mass	m_n	$1.674\,927\,28(29)\times10^{-27}$	kg	1.7×10^{-7}
in u, $m_n = A_r(n)$ u (neutron relative atomic mass times u)		1.008 664 915 60(55)	u	5.5×10^{-10}
energy equivalent	$m_n c^2$	$1.505\,349\,57(26)\times10^{-10}$	J	1.7×10^{-7}
in MeV		939.565 360(81)	MeV	8.6×10^{-8}
neutron-electron mass ratio	m_n/m_e	1838.683 6598(13)		7.0×10^{-10}
neutron-muon mass ratio	m_n/m_μ	8.892 484 02(23)		2.6×10^{-8}
neutron-tau mass ratio	m_n/m_τ	0.528 740(86)		1.6×10^{-4}
neutron-proton mass ratio	m_n/m_p	1.001 378 418 70(58)		5.8×10^{-10}
neutron molar mass $N_A m_n$	$M(n), M_n$	$1.008\,664\,915\,60(55)\times10^{-3}$	kg mol^{-1}	5.5×10^{-10}
neutron Compton wavelength $h/m_n c$	$\lambda_{C,n}$	$1.319\,590\,9067(88)\times10^{-15}$	m	6.7×10^{-9}
$\lambda_{C,n}/2\pi$	$\lambda\!\!\!^{-}_{C,n}$	$0.210\,019\,4157(14)\times10^{-15}$	m	6.7×10^{-9}

TABLE XXVI. (*Continued.*)

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
neutron magnetic moment	μ_n	$-0.966\,236\,45(24)\times10^{-26}$	J T^{-1}	2.5×10^{-7}
to Bohr magneton ratio	μ_n/μ_B	$-1.041\,875\,63(25)\times10^{-3}$		2.4×10^{-7}
to nuclear magneton ratio	μ_n/μ_N	$-1.913\,042\,73(45)$		2.4×10^{-7}
neutron g-factor $2\mu_n/\mu_N$	g_n	$-3.826\,085\,46(90)$		2.4×10^{-7}
neutron-electron magnetic moment ratio	μ_n/μ_e	$1.040\,668\,82(25)\times10^{-3}$		2.4×10^{-7}
neutron-proton magnetic moment ratio	μ_n/μ_p	$-0.684\,979\,34(16)$		2.4×10^{-7}
neutron to shielded proton magnetic moment ratio (H_2O, sphere, 25 °C)	μ_n/μ'_p	$-0.684\,996\,94(16)$		2.4×10^{-7}
neutron gyromagnetic ratio $2\lvert\mu_n\rvert/\hbar$	γ_n	$1.832\,471\,83(46)\times10^{8}$	s^{-1} T^{-1}	2.5×10^{-7}
	$\gamma_n/2\pi$	29.164 6950(73)	MHz T^{-1}	2.5×10^{-7}
		Deuteron, d		
deuteron mass	m_d	$3.343\,583\,35(57)\times10^{-27}$	kg	1.7×10^{-7}
in u, $m_d=A_r(d)$ u (deuteron relative atomic mass times u)		2.013 553 212 70(35)	u	1.7×10^{-10}
energy equivalent	m_dc^2	$3.005\,062\,85(51)\times10^{-10}$	J	1.7×10^{-7}
in MeV		1875.612 82(16)	MeV	8.6×10^{-8}
deuteron-electron mass ratio	m_d/m_e	3670.482 9652(18)		4.8×10^{-10}
deuteron-proton mass ratio	m_d/m_p	1.999 007 500 82(41)		2.0×10^{-10}
deuteron molar mass N_Am_d	$M(d)$, M_d	$2.013\,553\,212\,70(35)\times10^{-3}$	kg mol^{-1}	1.7×10^{-10}
deuteron rms charge radius	R_d	$2.1394(28)\times10^{-15}$	m	1.3×10^{-3}
deuteron magnetic moment	μ_d	$0.433\,073\,482(38)\times10^{-26}$	J T^{-1}	8.7×10^{-8}
to Bohr magneton ratio	μ_d/μ_B	$0.466\,975\,4567(50)\times10^{-3}$		1.1×10^{-8}
to nuclear magneton ratio	μ_d/μ_N	0.857 438 2329(92)		1.1×10^{-8}
deuteron-electron magnetic moment ratio	μ_d/μ_e	$-4.664\,345\,548(50)\times10^{-4}$		1.1×10^{-8}
deuteron-proton magnetic moment ratio	μ_d/μ_p	0.307 012 2084(45)		1.5×10^{-8}
deuteron-neutron magnetic moment ratio	μ_d/μ_n	$-0.448\,206\,52(11)$		2.4×10^{-7}
		Helion, h		
helion mass[e]	m_h	$5.006\,412\,14(86)\times10^{-27}$	kg	1.7×10^{-7}
in u, $m_h=A_r(h)$ u (helion relative atomic mass times u)		3.014 932 2434(58)	u	1.9×10^{-9}
energy equivalent	m_hc^2	$4.499\,538\,84(77)\times10^{-10}$	J	1.7×10^{-7}
in MeV		2808.391 42(24)	MeV	8.6×10^{-8}
helion-electron mass ratio	m_h/m_e	5495. 885 269(11)		2.0×10^{-9}
helion-proton mass ratio	m_h/m_p	2.993 152 6671(58)		1.9×10^{-9}
helion molar mass N_Am_h	$M(h)$, M_h	$3.014\,932\,2434(58)\times10^{-3}$	kg mol^{-1}	1.9×10^{-9}
shielded helion magnetic moment (gas, sphere, 25 °C)	μ'_h	$-1.074\,553\,024(93)\times10^{-26}$	J T^{-1}	8.7×10^{-8}
to Bohr magneton ratio	μ'_h/μ_B	$-1.158\,671\,474(14)\times10^{-3}$		1.2×10^{-8}
to nuclear magneton ratio	μ'_h/μ_N	$-2.127\,497\,723(25)$		1.2×10^{-8}

TABLE XXVI. (*Continued.*)

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
shielded helion to proton magnetic moment ratio (gas, sphere, 25 °C)	μ_h'/μ_p	−0.761 766 562(12)		1.5×10^{-8}
shielded helion to shielded proton magnetic moment ratio (gas/H_2O, spheres, 25 °C)	μ_h'/μ_p'	−0.761 786 1313(33)		4.3×10^{-9}
shielded helion gyromagnetic ratio $2\lvert\mu_h'\rvert/\hbar$ (gas, sphere, 25 °C)	γ_h'	$2.037\,894\,70(18)\times10^{8}$	$s^{-1}\,T^{-1}$	8.7×10^{-8}
	$\gamma_h'/2\pi$	32.434 1015(28)	$MHz\,T^{-1}$	8.7×10^{-8}
		Alpha particle, α		
alpha-particle mass	m_α	$6.644\,6565(11)\times10^{-27}$	kg	1.7×10^{-7}
in u, $m_\alpha=A_r(\alpha)$ u (alpha-particle relative atomic mass times u)		4.001 506 179 149(56)	u	1.4×10^{-11}
energy equivalent	$m_\alpha c^2$	$5.971\,9194(10)\times10^{-10}$	J	1.7×10^{-7}
in MeV		3727.379 17(32)	MeV	8.6×10^{-8}
alpha particle to electron mass ratio	m_α/m_e	7294.299 5363(32)		4.4×10^{-10}
alpha particle to proton mass ratio	m_α/m_p	3.972 599 689 07(52)		1.3×10^{-10}
alpha-particle molar mass $N_A m_\alpha$	$M(\alpha)$, M_α	$4.001\,506\,179\,149(56)\times10^{-3}$	$kg\,mol^{-1}$	1.4×10^{-11}
		PHYSICOCHEMICAL		
Avogadro constant	N_A, L	$6.022\,1415(10)\times10^{23}$	mol^{-1}	1.7×10^{-7}
atomic mass constant	m_u	$1.660\,538\,86(28)\times10^{-27}$	kg	1.7×10^{-7}
$m_u=\frac{1}{12}m(^{12}C)=1$ u $=10^{-3}\,kg\,mol^{-1}/N_A$				
energy equivalent	$m_u c^2$	$1.492\,417\,90(26)\times10^{-10}$	J	1.7×10^{-7}
in MeV		931.494 043(80)	MeV	8.6×10^{-8}
Faraday constant[g] $N_A e$	F	96 485.3383(83)	$C\,mol^{-1}$	8.6×10^{-8}
molar Planck constant	$N_A h$	$3.990\,312\,716(27)\times10^{-10}$	$J\,s\,mol^{-1}$	6.7×10^{-9}
	$N_A hc$	0.119 626 565 72(80)	$J\,m\,mol^{-1}$	6.7×10^{-9}
molar gas constant	R	8.314 472(15)	$J\,mol^{-1}\,K^{-1}$	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6505(24)\times10^{-23}$	$J\,K^{-1}$	1.8×10^{-6}
in $eV\,K^{-1}$		$8.617\,343(15)\times10^{-5}$	$eV\,K^{-1}$	1.8×10^{-6}
	k/h	$2.083\,6644(36)\times10^{10}$	$Hz\,K^{-1}$	1.7×10^{-6}
	k/hc	69.503 56(12)	$m^{-1}\,K^{-1}$	1.7×10^{-6}
molar volume of ideal gas RT/p				
$T=273.15$ K, $p=101.325$ kPa	V_m	$22.413\,996(39)\times10^{-3}$	$m^3\,mol^{-1}$	1.7×10^{-6}
Loschmidt constant N_A/V_m	n_0	$2.686\,7773(47)\times10^{25}$	m^{-3}	1.8×10^{-6}
$T=273.15$ K, $p=100$ kPa	V_m	$22.710\,981(40)\times10^{-3}$	$m^3\,mol^{-1}$	1.7×10^{-6}
Sackur-Tetrode constant (absolute entropy constant)[h] $\frac{5}{2}+\ln[(2\pi m_u kT_1/h^2)^{3/2}kT_1/p_0]$				
$T_1=1$ K, $p_0=100$ kPa	S_0/R	−1.151 7047(44)		3.8×10^{-6}
$T_1=1$ K, $p_0=101.325$ kPa		−1.164 8677(44)		3.8×10^{-6}
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,400(40)\times10^{-8}$	$W\,m^{-2}\,K^{-4}$	7.0×10^{-6}

TABLE XXVI. (*Continued.*)

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
first radiation constant $2\pi hc^2$	c_1	$3.741\,771\,38(64)\times10^{-16}$	W m^2	1.7×10^{-7}
first radiation constant for spectral radiance $2hc^2$	c_{1L}	$1.191\,042\,82(20)\times10^{-16}$	W m^2 sr^{-1}	1.7×10^{-7}
second radiation constant hc/k	c_2	$1.438\,7752(25)\times10^{-2}$	m K	1.7×10^{-6}
Wien displacement law constant $b=\lambda_{max}T=c_2/4.965\,114\,231\ldots$	b	$2.897\,7685(51)\times10^{-3}$	m K	1.7×10^{-6}

[a]See Table XXVIII for the conventional value adopted internationally for realizing representations of the volt using the Josephson effect.
[b]See Table XXVIII for the conventional value adopted internationally for realizing representations of the ohm using the quantum Hall effect.
[c]Value recommended by the Particle Data Group (Hagiwara *et al.*, 2002).
[d]Based on the ratio of the masses of the W and Z bosons m_W/m_Z recommended by the Particle Data Group (Hagiwara *et al.*, 2002). The value for $\sin^2\theta_W$ they recommend, which is based on a particular variant of the modified minimal subtraction ($\overline{MS}$) scheme, is $\sin^2\hat{\theta}_W(M_Z)=0.231\,24\,(24)$.
[e]The helion, symbol h, is the nucleus of the ^{3}He atom.
[f]This and all other values involving m_τ are based on the value of $m_\tau c^2$ in MeV recommended by the Particle Data Group (Hagiwara *et al.*, 2002), but with a standard uncertainty of 0.29 MeV rather than the quoted uncertainty of −0.26 MeV, +0.29 MeV.
[g]The numerical value of F to be used in coulometric chemical measurements is 96 485.336(16) [1.7×10^{-7}] when the relevant current is measured in terms of representations of the volt and ohm based on the Josephson and quantum Hall effects and the internationally adopted conventional values of the Josephson and von Klitzing constants K_{J-90} and R_{K-90} given in Table XXXVIII.
[h]The entropy of an ideal monoatomic gas of relative atomic mass A_r is given by $S=S_0+\frac{3}{2}R\ln A_r-R\ln(p/p_0)+\frac{5}{2}R\ln(T/\mathrm{K})$.

commonly used. Table XXVI is a much more extensive list of values categorized as follows: universal; electromagnetic; atomic and nuclear; and physicochemical. The atomic and nuclear category is subdivided into ten subcategories: general; electroweak; electron, e^-; muon, μ^-; tau, τ^-; proton, p; neutron, n; deuteron, d; helion, h; and alpha particle, α. Table XXVII gives the variances, covariances, and correlation coefficients of a selected group of constants. (Application of the covariance matrix is discussed in Appendix E of CODATA-98.) Table XXVIII gives the internationally adopted values of various quantities; Table XXIX lists the values of a number of x-ray related quantities; Table XXX lists the values of various non-SI units; and Tables XXXI and XXXII give the values of various energy equivalents.

All of the values given in Tables XXV–XXXII are available on the Web pages of the Fundamental Constants Data Center of the NIST Physics Laboratory at physics.nist.gov/constants. This electronic version of the 2002 CODATA recommended values of the constants also includes a much more extensive correlation coefficient matrix. Indeed, the correlation coefficient of any two constants listed in the tables is accessible on the Web site, as well as the automatic conversion of the value of an energy-related quantity expressed in one unit to the corresponding value expressed in another

TABLE XXVII. The variances, covariances, and correlation coefficients of the values of a selected group of constants based on the 2002 CODATA adjustment. The numbers in boldface above the main diagonal are 10^{16} times the numerical values of the relative covariances; the numbers in boldface on the main diagonal are 10^{16} times the numerical values of the relative variances; and the numbers in italics below the main diagonal are the correlation coefficients.[a]

	α	h	e	m_e	N_A	m_e/m_μ	F
α	**0.111**	**0.057**	**0.084**	**−0.165**	**0.165**	**−0.217**	**0.248**
h	*0.010*	**292.154**	**146.105**	**292.040**	**−292.040**	**−0.111**	**−145.935**
e	*0.029*	*1.000*	**73.094**	**145.938**	**−145.938**	**−0.164**	**−72.843**
m_e	*−0.029*	*0.999*	*0.998*	**292.370**	**−292.370**	**0.323**	**−146.432**
N_A	*0.029*	*−0.999*	*−0.998*	*−1.000*	**292.371**	**−0.322**	**146.433**
m_e/m_μ	*−0.249*	*−0.002*	*−0.007*	*0.007*	*−0.007*	**6.845**	**−0.486**
F	*0.087*	*−0.995*	*−0.993*	*−0.998*	*0.998*	*−0.022*	**73.590**

[a]The relative covariance is $u_r(x_i,x_j)=u(x_i,x_j)/(x_ix_j)$, where $u(x_i,x_j)$ is the covariance of x_i and x_j; the relative variance is $u_r^2(x_i)=u_r(x_i,x_i)$; and the correlation coefficient is $r(x_i,x_j)=u(x_i,x_j)/[u(x_i)u(x_j)]$.

TABLE XXVIII. Internationally adopted values of various quantities.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
relative atomic mass[a] of ^{12}C	$A_r(^{12}C)$	12		(exact)
molar mass constant	M_u	1×10^{-3}	kg mol^{-1}	(exact)
molar mass of ^{12}C	$M(^{12}C)$	12×10^{-3}	kg mol^{-1}	(exact)
conventional value of Josephson constant[b]	K_{J-90}	483 597.9	GHz V^{-1}	(exact)
conventional value of von Klitzing constant[c]	R_{K-90}	25 812.807	Ω	(exact)
standard atmosphere		101 325	Pa	(exact)
standard acceleration of gravity[d]	g_n	9.806 65	m s^{-2}	(exact)

[a]The relative atomic mass $A_r(X)$ of particle X with mass $m(X)$ is defined by $A_r(X)=m(X)/m_u$, where $m_u=m(^{12}C)/12=M_u/N_A=1$ u is the atomic mass constant, M_u is the molar mass constant, N_A is the Avogadro constant, and u is the unified atomic mass unit. Thus the mass of particle X is $m(X)=A_r(X)$ u and the molar mass of X is $M(X)=A_r(X)M_u$.

[b]This is the value adopted internationally for realizing representations of the volt using the Josephson effect.

[c]This is the value adopted internationally for realizing representations of the ohm using the quantum Hall effect.

[d]The value given was adopted by the 3rd General Conference on Weights and Measures (CGPM), 1903 (BIPM, 1998), and is the conventional value used to calculate the now obsolete unit kilogram force.

unit (in essence, an automated version of Tables XXXI and XXXII.

VI. SUMMARY AND CONCLUSION

We conclude by comparing the 2002 and 1998 CODATA recommended values of the constants and identifying those new results that have contributed most to the differences between them. This is followed by a brief discussion of some of the conclusions that can be drawn from the 2002 recommended values and adjustment. We then look to the future and identify experimental and theoretical work that can advance our knowledge of the values of the constants.

A. Comparison of 2002 and 1998 CODATA recommended values

Table XXXIII compares the recommended values of a representative group of constants as given by the 2002 and 1998 CODATA adjustments. As in the similar table in CODATA-98, most of the regularities exhibited in Table XXXIII are due to the functional dependence of the derived constants on the Rydberg constant R_∞, the fine-structure constant α, the Planck constant h, and the molar gas constant R, all of which are adjusted constants, and the fact that for both the 2002 and 1998 adjustments $u_r(R)\gg u_r(h)\gg u_r(\alpha)\gg u_r(R_\infty)$. This dependence is such that the uncertainty of a particular derived constant is determined mainly by the uncertainty of α, h, or R. For example, the first five quantities after α in the table are calculated from expressions that contain a factor α^a (and in some cases R_∞), where $a=-1$, 2, 3, or 6. In an analogous way, the 14 quantities m_e through μ_p are obtained from expressions that contain a factor h^a, where $a=-1$, $-\frac{1}{2}$ or $\frac{1}{2}$, and other constants with relative standard uncertainties rather less than $u_r(h)$. Similarly, the four quantities k through σ are calculated from ex-

TABLE XXIX. Values of some x-ray-related quantities based on the 2002 CODATA adjustment of the values of the constants.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
Cu x unit: $\lambda(CuK\alpha_1)/1537.400$	xu(CuKα_1)	$1.002\,077\,10(29)\times10^{-13}$	m	2.8×10^{-7}
Mo x unit: $\lambda(MoK\alpha_1)/707.831$	xu(MoKα_1)	$1.002\,099\,66(53)\times10^{-13}$	m	5.3×10^{-7}
Ångstrom star: $\lambda(WK\alpha_1)/0.209\,010\,0$	Å*	$1.000\,015\,09(90)\times10^{-10}$	m	9.0×10^{-7}
lattice parameter[a] of Si (in vacuum, 22.5 °C)	a	$543.102\,122(20)\times10^{-12}$	m	3.6×10^{-8}
{220} lattice spacing of Si $a/\sqrt{8}$ (in vacuum, 22.5 °C)	d_{220}	$192.015\,5965(70)\times10^{-12}$	m	3.6×10^{-8}
molar volume of Si $M(Si)/\rho(Si)=N_Aa^3/8$ (in vacuum, 22.5 °C)	$V_m(Si)$	$12.058\,8382(24)\times10^{-6}$	m^3 mol^{-1}	2.0×10^{-7}

[a]This is the lattice parameter (unit cell edge length) of an ideal single crystal of naturally occurring Si free of impurities and imperfections, and is deduced from measurements on extremely pure and nearly perfect single crystals of Si by correcting for the effects of impurities.

TABLE XXX. The values in SI units of some non-SI units based on the 2002 CODATA adjustment of the values of the constants.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
		Non-SI units accepted for use with the SI		
electron volt: (e/C) J	eV	$1.602\,176\,53(14)\times10^{-19}$	J	8.5×10^{-8}
(unified) atomic mass unit: 1 u = $m_u = \frac{1}{12}m(^{12}C)$ = 10^{-3} kg mol^{-1}/N_A	u	$1.660\,538\,86(28)\times10^{-27}$	kg	1.7×10^{-7}
		Natural units (n.u.)		
n.u. of velocity:				
speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	(exact)
n.u. of action:				
reduced Planck constant ($h/2\pi$)	$\hbar$	$1.054\,571\,68(18)\times10^{-34}$	J s	1.7×10^{-7}
in eV s		$6.582\,119\,15(56)\times10^{-16}$	eV s	8.5×10^{-8}
in MeV fm	$\hbar c$	197.326 968(17)	MeV fm	8.5×10^{-8}
n.u. of mass:				
electron mass	m_e	$9.109\,3826(16)\times10^{-31}$	kg	1.7×10^{-7}
n.u. of energy	$m_e c^2$	$8.187\,1047(14)\times10^{-14}$	J	1.7×10^{-7}
in MeV		0.510 998 918(44)	MeV	8.6×10^{-8}
n.u. of momentum	$m_e c$	$2.730\,924\,19(47)\times10^{-22}$	kg m s^{-1}	1.7×10^{-7}
in MeV/c		0.510 998 918(44)	MeV/c	8.6×10^{-8}
n.u. of length ($\hbar/m_e c$)	$\lambda\!\!\!^{-}_C$	$386.159\,2678(26)\times10^{-15}$	m	6.7×10^{-9}
n.u. of time	$\hbar/m_e c^2$	$1.288\,088\,6677(86)\times10^{-21}$	s	6.7×10^{-9}
		Atomic units (a.u.)		
a.u. of charge:				
elementary charge	e	$1.602\,176\,53(14)\times10^{-19}$	C	8.5×10^{-8}
a.u. of mass:				
electron mass	m_e	$9.109\,3826(16)\times10^{-31}$	kg	1.7×10^{-7}
a.u. of action:				
reduced Planck constant ($h/2\pi$)	$\hbar$	$1.054\,571\,68(18)\times10^{-34}$	J s	1.7×10^{-7}
a.u. of length:				
Bohr radius (bohr) ($\alpha/4\pi R_\infty$)	a_0	$0.529\,177\,2108(18)\times10^{-10}$	m	3.3×10^{-9}
a.u. of energy:				
Hartree energy (hartree) ($e^2/4\pi\epsilon_0 a_0 = 2R_\infty hc = \alpha^2 m_e c^2$)	E_h	$4.359\,744\,17(75)\times10^{-18}$	J	1.7×10^{-7}
a.u. of time	$\hbar/E_h$	$2.418\,884\,326\,505(16)\times10^{-17}$	s	6.6×10^{-12}
a.u. of force	E_h/a_0	$8.238\,7225(14)\times10^{-8}$	N	1.7×10^{-7}
a.u. of velocity (αc)	$a_0 E_h/\hbar$	$2.187\,691\,2633(73)\times10^{6}$	m s^{-1}	3.3×10^{-9}
a.u. of momentum	$\hbar/a_0$	$1.992\,851\,66(34)\times10^{-24}$	kg m s^{-1}	1.7×10^{-7}
a.u. of current	$eE_h/\hbar$	$6.623\,617\,82(57)\times10^{-3}$	A	8.5×10^{-8}
a.u. of charge density	e/a_0^3	$1.081\,202\,317(93)\times10^{12}$	C m^{-3}	8.6×10^{-8}
a.u. of electric potential	E_h/e	27.211 3845(23)	V	8.5×10^{-8}
a.u. of electric field	E_h/ea_0	$5.142\,206\,42(44)\times10^{11}$	V m^{-1}	8.6×10^{-8}
a.u. of electric field gradient	E_h/ea_0^2	$9.717\,361\,82(83)\times10^{21}$	V m^{-2}	8.6×10^{-8}
a.u. of electric dipole moment	ea_0	$8.478\,353\,09(73)\times10^{-30}$	C m	8.6×10^{-8}
a.u. of electric quadrupole moment	ea_0^2	$4.486\,551\,24(39)\times10^{-40}$	C m^2	8.6×10^{-8}
a.u. of electric polarizability	$e^2a_0^2/E_h$	$1.648\,777\,274(16)\times10^{-41}$	C^2 m^2 J^{-1}	1.0×10^{-8}
a.u. of 1st hyperpolarizability	$e^3a_0^3/E_h^2$	$3.206\,361\,51(28)\times10^{-53}$	C^3 m^3 J^{-2}	8.7×10^{-8}
a.u. of 2nd hyperpolarizability	$e^4a_0^4/E_h^3$	$6.235\,3808(11)\times10^{-65}$	C^4 m^4 J^{-3}	1.7×10^{-7}

TABLE XXX. (*Continued.*)

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
a.u. of magnetic flux density	$\hbar/ea_0^2$	$2.350\,517\,42(20)\times10^{5}$	T	8.6×10^{-8}
a.u. of magnetic dipole moment ($2\mu_B$)	$\hbar e/m_e$	$1.854\,801\,90(16)\times10^{-23}$	J T^{-1}	8.6×10^{-8}
a.u. of magnetizability	$e^2a_0^2/m_e$	$7.891\,036\,60(13)\times10^{-29}$	J T^{-2}	1.7×10^{-8}
a.u. of permittivity ($10^7/c^2$)	e^2/a_0E_h	$1.112\,650\,056\ldots\times10^{-10}$	F m^{-1}	(exact)

pressions that contain a factor R^a, where $a=-1$, 1, or 4.

A number of clarifying comments can be made about Table XXXIII.

(i) The 2002 uncertainties of α and those constants highly dependent upon it, such as R_K through σ_e, are 92% of their corresponding 1998 uncertainties. This is mainly because of the influence on the value of α of the new experimental result for h/m(Cs) obtained from atom recoil and interferometry (see Sec. III.K.2). The 1.3 standard uncertainty shift from the 1998 values is due to the fact that an error was found in the calculation of the eighth-order coefficient $A_1^{(8)}$ in the theoretical expression for the electron magnetic moment anomaly a_e(th), which leads to a fractional increase of 5.7×10^{-9} in the value of α implied by the experimental result for a_e (see Sec. III.C.1.a and Appendix B). Since the recommended value of α is to a large extent determined by a_e(th) together with the experimental result, this increase is reflected in the 2002 recommended value.

(ii) The uncertainties of h and those constants highly dependent upon it, such as m_e through μ_p, are over twice as large as their 1998 uncertainties. The principal reason for this increase is the disagreement of the WGAC consensus value of the molar volume of silicon V_m(Si) with the two watt-balance results for $K_J^2R_K$ and with the Hg electrometer and voltage-balance determinations of K_J; this inconsistency led the Task Group to weight by the multiplicative factor 2.325 the uncertainties of these five input data in the final least-squares adjustment used to determine the 2002 recommended values, and these five results play the dominant role in determining the 2002 value of h. The approximate shift of one standard uncertainty in the 1998 values of most of the h-dependent constants is due to the newly included result for V_m(Si).

(iii) The uncertainty of the Newtonian constant of gravitation G has decreased by a factor of 10. The explanation is that a number of new determinations of G were completed since the 31 December 1998 closing date of the 1998 adjustment, and the results of those new experiments were in sufficiently good agreement to convince the Task Group that a highly discrepant but credible result, which influenced the uncertainty of the 1998 recommended value of G, should not influence the 2002 value. (Recall that the 1986 CODATA recommended value of G was retained as the 1998 recommended value, but with u_r increased to 1.5×10^{-3}, a factor of about 12, to partially account for the existence of the highly discrepant result.)

(iv) The 12% reduction in the uncertainty of R_∞ and the slight change in the 1998 value are due to (a) improvements in the theory of hydrogen and deuterium energy levels (see Appendix A); (b) an improved value of the proton bound-state rms charge radius deduced from existing scattering data (see Sec. III.B.4); and (c) an improved measurement of the $1S_{1/2}-2S_{1/2}$ hydrogen transition frequency with an uncertainty smaller by nearly a factor of 18 than that of the result available in 1998 (see Sec. III.B.1).

(v) The reduction in the uncertainty of m_e/m_p and of A_r(e) by about a factor of 4.7 and the shift in the values of these constants by −1.3 standard uncertainties arises from the new values of A_r(e) implied by the highly accurate measurements of the frequency ratios $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$ and $f_s(^{16}C^{7+})/f_c(^{16}C^{7+})$ [see Secs. III.C.3.a and III.C.3.b] and the highly accurate theory of the g-factor of the electron in hydrogenlike ions with nuclear spin quantum number $i=0$ (see Appendix D).

(vi) The 14% reduction in the uncertainty of m_e/m_μ and the large shift in the value of this ratio can be ascribed to improvements in the theory of the muonium ground-state hyperfine splitting $\Delta\nu_{Mu}$, in particular, to the discovery of some problems with certain aspects of the earlier theory, especially the recoil correction to $\Delta\nu_{Mu}$(th) (see Appendix E).

(vii) The large increase in uncertainty of the relative atomic mass A_r(h) of the helion (nucleus of the ^{3}He atom) and shift in its value is due to the discovery of an error in the earlier experimental result for $A_r(^3\text{He})$, the difficulty of correcting for the error *a posteriori*, and the disagreement between the corrected value and a more recent result from a different laboratory (see Sec. III.A.1.a). However, as can be seen from Table XXXIII, this problem does not reflect itself in the recommended value of the mass of the helion m_h in kg, which has a much larger uncertainty due to its dependence on the value of the Planck constant h.

(viii) The nearly factor-of-18 reduction in the uncertainty of the relative atomic mass of the alpha particle $A_r(\alpha)$ and the rather large shift in its value is explained by a new result from a significantly improved experiment; the latter uncovered an error in the measurement that led to the 1998 value (see Sec. III.A.1.b).

TABLE XXXI. The values of some energy equivalents derived from the relations $E=mc^2=hc/\lambda=h\nu=kT$, and based on the 2002 CODATA adjustment of the values of the constants; 1 eV$=(e/\mathrm{C})$ J, 1 u$=m_\mathrm{u}=\frac{1}{12}m(^{12}\mathrm{C})=10^{-3}$ kg mol$^{-1}/N_\mathrm{A}$, and $E_\mathrm{h}=2R_\infty hc=\alpha^2 m_\mathrm{e}c^2$ is the Hartree energy (hartree).

	Relevant unit			
	J	kg	m^{-1}	Hz
1 J	(1 J)$=1$ J	(1 J)$/c^2=1.112\,650\,056\ldots\times10^{-17}$ kg	(1 J)$/hc=5.034\,117\,20(86)\times10^{24}$ m^{-1}	(1 J)$/h=1.509\,190\,37(26)\times10^{33}$ Hz
1 kg	(1 kg)$c^2=8.987\,551\,787\ldots\times10^{16}$ J	(1 kg)$=1$ kg	(1 kg)$c/h=4.524\,438\,91(77)\times10^{41}$ m^{-1}	(1 kg)$c^2/h=1.356\,392\,66(23)\times10^{50}$ Hz
1 m^{-1}	(1 m^{-1})$hc=1.986\,445\,61(34)\times10^{-25}$ J	(1 m^{-1})$h/c=2.210\,218\,81(38)\times10^{-42}$ kg	(1 m^{-1})$=1$ m^{-1}	(1 m^{-1})$c=299\,792\,458$ Hz
1 Hz	(1 Hz)$h=6.626\,0693(11)\times10^{-34}$ J	(1 Hz)$h/c^2=7.372\,4964(13)\times10^{-51}$ kg	(1 Hz)$/c=3.335\,640\,951\ldots\times10^{-9}$ m^{-1}	(1 Hz)$=1$ Hz
1 K	(1 K)$k=1.380\,6505(24)\times10^{-23}$ J	(1 K)$k/c^2=1.536\,1808(27)\times10^{-40}$ kg	(1 K)$k/hc=69.503\,56(12)$ m^{-1}	(1 K)$k/h=2.083\,6644(36)\times10^{10}$ Hz
1 eV	(1 eV)$=1.602\,176\,53(14)\times10^{-19}$ J	(1 eV)$/c^2=1.782\,661\,81(15)\times10^{-36}$ kg	(1 eV)$/hc=8.065\,544\,45(69)\times10^{5}$ m^{-1}	(1 eV)$/h=2.417\,989\,40(21)\times10^{14}$ Hz
1 u	(1 u)$c^2=1.492\,417\,90(26)\times10^{-10}$ J	(1 u)$=1.660\,538\,86(28)\times10^{-27}$ kg	(1 u)$c/h=7.513\,006\,608(50)\times10^{14}$ m^{-1}	(1 u)$c^2/h=2.252\,342\,718(15)\times10^{23}$ Hz
1 E_h	(1 E_h)$=4.359\,744\,17(75)\times10^{-18}$ J	(1 E_h)$/c^2=4.850\,869\,60(83)\times10^{-35}$ kg	(1 E_h)$/hc=2.194\,746\,313\,705(15)\times10^{7}$ m^{-1}	(1 E_h)$/h=6.579\,683\,920\,721(44)\times10^{15}$ Hz

TABLE XXXII. The values of some energy equivalents derived from the relations $E=mc^2=hc/\lambda=h\nu=kT$, and based on the 2002 CODATA adjustment of the values of the constants; 1 eV$=(e/\mathrm{C})$ J, 1 u$=m_\mathrm{u}=\frac{1}{12}m(^{12}\mathrm{C})=10^{-3}$ kg mol$^{-1}/N_\mathrm{A}$, and $E_\mathrm{h}=2R_\infty hc=\alpha^2 m_\mathrm{e}c^2$ is the Hartree energy (hartree).

Relevant unit				
	K	eV	u	E_h
1 J	(1 J)$/k=7.242\,963(13)\times10^{22}$ K	(1 J)$=6.241\,509\,47(53)\times10^{18}$ eV	(1 J)$/c^2=6.700\,5361(11)\times10^{9}$ u	(1 J)$=2.293\,712\,57(39)\times10^{17}$ E_h
1 kg	(1 kg)$c^2/k=6.509\,650(11)\times10^{39}$ K	(1 kg)$c^2=5.609\,588\,96(48)\times10^{35}$ eV	(1 kg)$=6.022\,1415(10)\times10^{26}$ u	(1 kg)$c^2=2.061\,486\,05(35)\times10^{34}$ E_h
1 m^{-1}	(1 m^{-1})$hc/k=1.438\,7752(25)\times10^{-2}$ K	(1 m^{-1})$hc=1.239\,841\,91(11)\times10^{-6}$ eV	(1 m^{-1})$h/c=1.331\,025\,0506(89)\times10^{-15}$ u	(1 m^{-1})$hc=4.556\,335\,252\,760(30)\times10^{-8}$ E_h
1 Hz	(1 Hz)$h/k=4.799\,2374(84)\times10^{-11}$ K	(1 Hz)$h=4.135\,667\,43(35)\times10^{-15}$ eV	(1 Hz)$h/c^2=4.439\,821\,667(30)\times10^{-24}$ u	(1 Hz)$h=1.519\,829\,846\,006(10)\times10^{-16}$ E_h
1 K	(1 K)$=1$ K	(1 K)$k=8.617\,343(15)\times10^{-5}$ eV	(1 K)$k/c^2=9.251\,098(16)\times10^{-14}$ u	(1 K)$k=3.166\,8153(55)\times10^{-6}$ E_h
1 eV	(1 eV)$/k=1.160\,4505(20)\times10^{4}$ K	(1 eV)$=1$ eV	(1 eV)$/c^2=1.073\,544\,171(92)\times10^{-9}$ u	(1 eV)$=3.674\,932\,45(31)\times10^{-2}$ E_h
1 u	(1 u)$c^2/k=1.080\,9527(19)\times10^{13}$ K	(1 u)$c^2=931.494\,043(80)\times10^{6}$ eV	(1 u)$=1$ u	(1 u)$c^2=3.423\,177\,686(23)\times10^{7}$ E_h
1 E_h	(1 E_h)$/k=3.157\,7465(55)\times10^{5}$ K	(1 E_h)$=27.211\,3845(23)$ eV	(1 E_h)$/c^2=2.921\,262\,323(19)\times10^{-8}$ u	(1 E_h)$=1$ E_h

TABLE XXXIII. Comparison of the 2002 and 1998 CODATA adjustments of the values of the constants by the comparison of the corresponding recommended values of a representative group of constants. Here D_r is the 2002 value minus the 1998 value divided by the standard uncertainty u of the 1998 value (i.e., D_r is the change in the value of the constant from 1998 to 2002 relative to its 1998 standard uncertainty).

Quantity	2002 rel. standard uncert. u_r	Ratio 2002 u_r to 1998 u_r	D_r
α	3.3×10^{-9}	0.9	1.3
R_K	3.3×10^{-9}	0.9	−1.3
a_0	3.3×10^{-9}	0.9	1.3
λ_C	6.7×10^{-9}	0.9	1.3
r_e	1.0×10^{-8}	0.9	1.3
σ_e	2.0×10^{-8}	0.9	1.3
h	1.7×10^{-7}	2.2	1.1
m_e	1.7×10^{-7}	2.2	0.9
m_h	1.7×10^{-7}	2.2	1.0
m_α	1.7×10^{-7}	2.2	1.0
N_A	1.7×10^{-7}	2.2	−1.0
E_h	1.7×10^{-7}	2.2	1.1
c_1	1.7×10^{-7}	2.2	1.1
e	8.5×10^{-8}	2.2	1.1
K_J	8.5×10^{-8}	2.2	−1.0
F	8.6×10^{-8}	2.1	−0.8
γ'_p	8.6×10^{-8}	2.1	−0.7
μ_B	8.6×10^{-8}	2.1	1.3
μ_N	8.6×10^{-8}	2.1	1.3
μ_e	8.6×10^{-8}	2.1	−1.3
μ_p	8.7×10^{-8}	2.1	1.3
R	1.7×10^{-6}	1.0	0.0
k	1.8×10^{-6}	1.0	0.1
V_m	1.7×10^{-6}	1.0	0.0
c_2	1.7×10^{-6}	1.0	0.0
σ	7.0×10^{-6}	1.0	0.0
G	1.5×10^{-4}	0.1	0.2
R_∞	6.6×10^{-12}	0.9	−0.3
m_e/m_p	4.6×10^{-10}	0.2	−1.3
m_e/m_μ	2.6×10^{-8}	0.9	−2.9
A_r(e)	4.4×10^{-10}	0.2	−1.3
A_r(p)	1.3×10^{-10}	1.0	0.0
A_r(n)	5.5×10^{-10}	1.0	−0.3
A_r(d)	1.7×10^{-10}	1.0	0.0
A_r(h)	1.9×10^{-9}	6.8	10.2
$A_r(\alpha)$	1.4×10^{-11}	0.1	4.5
d_{220}	3.6×10^{-8}	1.2	2.1
g_e	3.8×10^{-12}	0.9	0.2
g_μ	6.2×10^{-10}	1.0	−5.9
μ_p/μ_B	1.0×10^{-8}	1.0	0.2
μ_p/μ_N	1.0×10^{-8}	1.0	0.5
μ_n/μ_N	2.4×10^{-7}	1.0	0.0
μ_d/μ_N	1.1×10^{-8}	1.0	0.5
μ_e/μ_p	1.0×10^{-8}	1.0	0.2
μ_n/μ_p	2.4×10^{-7}	1.0	0.0
μ_d/μ_p	1.5×10^{-8}	1.0	0.0

(ix) The large shift in the 1998 value of the g-factor of the muon g_μ is due to the discovery of a sign error in the calculation of the hadronic light-by-light vacuum polarization contribution to the theoretical expression a_μ(th) for the muon magnetic moment anomaly a_μ, advances in the calculation of the fourth-order hadronic vacuum polarization contribution $a_\mu^{(4)}$(had) to a_μ(th), and a new, highly accurate experimental result for a_μ (see Appendix C and Sec III.C.2).

In summary, a number of new results, both experimental and theoretical, obtained between the closing date of the 1998 adjustment (31 December 1998) and the closing date of the 2002 adjustment (31 December 2002) have led to reductions in uncertainties of some recommended values, the most important no doubt being the factor of 10 reduction in the uncertainty of G. On the other hand, a few of these same new results have led to large shifts in the 1998 values themselves, while other new results have actually led to *increases* in the uncertainties of some constants. Most prominent in this category is the result for V_m(Si), which has led to more than a doubling of the uncertainty of the Planck constant h and those derived constants that strongly depend upon it.

In most cases new information leads to smaller uncertainties. However, in this case, the larger uncertainties that new information has engendered are presumably a more accurate reflection of reality. Thus, one can argue that our knowledge has actually advanced, even though the uncertainties have increased. And of course, it should always be borne in mind that the recommended values of the constants represent the information available at a given point in time.

B. Some implications for physics and metrology of the 2002 CODATA recommended values and adjustment

The focus of this report, as was the case for CODATA-98, has been the thorough review and analysis of the experimental and theoretical data currently available for the determination of the values of the fundamental constants, not what the data can tell us about the basic theories and experimental methods of physics. Nevertheless, a number of useful conclusions relevant to these topics may be drawn from the 2002 values and adjustment. For conciseness, we focus on those conclusions that are new or that are different from those drawn from the 1998 adjustment.

- *Conventional electric units*. As pointed out in Sec. II.F, the adoption of the conventional values $K_{J-90}=483\,597.9$ GHz/V and $R_{K-90}=25\,812.807\ \Omega$ for the Josephson and von Klitzing constants can be interpreted as establishing conventional, practical units of voltage and resistance, V_{90} and Ω_{90}, given by $V_{90}=(K_{J-90}/K_J)$ V and $\Omega_{90}=(R_K/R_{K-90})\ \Omega$. Other conventional electric units follow from V_{90} and Ω_{90}, for example, $A_{90}=V_{90}/\Omega_{90}$, $C_{90}=A_{90}$ s, $W_{90}=A_{90}V_{90}$, $F_{90}=C_{90}/V_{90}$, and $H_{90}=\Omega_{90}$ s, which are the conventional, practical units of current, charge, power, ca-

pacitance, and inductance, respectively (Taylor and Mohr, 2001).

The relations between the 2002 recommended values of K_J and R_K and the values of K_{J-90} and R_{K-90}, namely,

$$K_J = K_{J-90}[1 - 4.3(8.5) \times 10^{-8}], \quad (213)$$

$$R_K = R_{K-90}[1 + 1.74(33) \times 10^{-8}], \quad (214)$$

together with the expressions that relate the practical units to the corresponding SI units, lead to

$$V_{90} = [1 + 4.3(8.5) \times 10^{-8}]\ \mathrm{V}, \quad (215)$$

$$\Omega_{90} = [1 + 1.74(33) \times 10^{-8}]\ \Omega, \quad (216)$$

$$A_{90} = [1 + 2.6(8.6) \times 10^{-8}]\ \mathrm{A}, \quad (217)$$

$$C_{90} = [1 + 2.6(8.6) \times 10^{-8}]\ \mathrm{C}, \quad (218)$$

$$W_{90} = [1 + 6.9(17.1) \times 10^{-8}]\ \mathrm{W}, \quad (219)$$

$$F_{90} = [1 + 1.74(33) \times 10^{-8}]\ \mathrm{F}, \quad (220)$$

$$H_{90} = [1 + 1.74(33) \times 10^{-8}]\ \mathrm{H}. \quad (221)$$

One can see from Eqs. (215) and (216), for example, that the practical unit of voltage V_{90} exceeds V by the fractional amount $4.3(8.5) \times 10^{-8}$, and that the practical unit of resistance Ω_{90} exceeds Ω by $1.74(33) \times 10^{-8}$. The implication is that measured voltages U and resistances R traceable to the Josephson effect and K_{J-90} and to the quantum Hall effect and R_{K-90}, respectively, are too small relative to the SI by these same fractional amounts. Fortunately, such differences from the SI are not significant for the vast majority of measurements and are well within the current uncertainties of 40×10^{-8} for V_{90}/V and 10×10^{-8} for Ω_{90}/Ω as assigned by the CIPM's Consultative Committee for Electricity and Magnetism (CCEM, *Comité consultatif d'électricité et magnetism*) (Quinn, 1989, 2001). Nevertheless, corrections to account for the deviations could be necessary in those few cases where consistency with the SI is critical. Analogous statements apply to the other five practical electric units of Eqs. (217)–(221).

- *Hydrogenic energy levels and p and d bound-state rms charge radii*. Advances in the theory of H and D energy levels and an improved value of the proton radius R_p has eliminated the systematic deviation between theory and experiment observed in the 1998 adjustment and has allowed the Task Group to provide recommended values for R_p and R_d. This is a major step forward and increases our confidence in the current formulation of bound-state QED theory. The validity of certain aspects of the theory is also reinforced by the agreement between an earlier result for the relative atomic mass of the electron $A_r(e)$ obtained solely from cyclotron resonance measurements in a Penning trap and two new values that depend critically on the theory of the g-factor of the electron in hydrogenic ^{12}C and ^{16}O.
- *Josephson and quantum Hall effects*. The investigations summarized in Appendix F, which take into account all 112 input data initially considered for inclusion in the 2002 adjustment, provide no statistically significant evidence that the basic Josephson and quantum Hall effect relations $K_J = 2e/h$ and $R_K = h/e^2$ are not exact. This formal study increases our confidence in the theory of two of the most important phenomena of condensed-matter physics.
- *Newtonian constant of gravitation*. The new results for G that have become available in the last four years and their general agreement have led the Task Group to conclude that an earlier, credible result in quite significant disagreement with all other values of G can now be safely omitted from the 2002 adjustment. Although the new values have allowed a factor-of-10 reduction in the uncertainty of the recommended value of G, they are still not completely consistent, thereby implying that some problems in experiments to measure G still remain.
- *Electron and muon magnetic moment anomalies*. The comparison of the value of α inferred from a_e with values of α deduced from measurements of other quantities, such as R_K and the proton gyromagnetic ratio-related quantity $\Gamma'_{p-90}(lo)$, has long been viewed as an important test for QED theory, since $a_e(th)$ on which $\alpha[a_e]$ depends is strongly dominated by pure QED contributions. In the past, there was no value of α with an uncertainty that was competitive with $u_r(\alpha[a_e])$. This has recently changed; we now have a value of α from $h/m(Cs)$, the latter determined from atom recoil and atom interferometry, with an uncertainty only about twice as large as that of $\alpha[a_e]$. The two values agree, thus adding to our confidence in the QED theory of a_e.

Because of the large hadronic and weak contributions to $a_\mu(th)$, the comparison of the experimentally determined value of a_μ with the value of a_μ deduced from $a_\mu(th)$ together with an accurate value of α such as $\alpha[a_e]$ is viewed as a test of the Standard Model and a means for possibly uncovering "new physics," rather than a test of QED. At present, the uncertainty of the fourth-order hadronic contribution $a_\mu^{(4)}(had)$ to $a_\mu(th)$ dominates the latter's uncertainty; it contributes a relative standard uncertainty component $u_r = 8.4 \times 10^{-7}$ to $a_\mu(th)$,

which exceeds the uncertainty $u_r=6.7\times10^{-7}$ of the experimental value of a_μ. One cause of the large uncertainty of $a_\mu^{(4)}$(had) is the inconsistency between the value obtained for it using e^+e^- annihilation data alone and the value obtained for it using such data together with data on the hadronic decays of the τ. Until this issue is fully resolved and new experimental data become available that lead to a significantly reduced uncertainty for $a_\mu^{(4)}$(had), any disagreement between a_μ(th) and the experimental value, the uncertainty of which may eventually be reduced to about $u_r=4\times10^{-7}$, will be difficult to interpret.

- *Molar volume and lattice spacing of silicon*. The most problematic input datum by far in the 2002 adjustment is the molar volume of silicon V_m(Si). The disagreement of this quite credible result with the two watt-balance results for $K_J^2R_K$ and the Hg-electrometer result and the voltage-balance result for K_J, all four of which are equally credible, is troublesome; it raises possible doubt about all five results and hence the 2002 recommended value of h and those other constants highly dependent upon it. Further, the disagreement raises questions about a number of experimental techniques, for example, determining the diameter of a sphere via optical interferometry, determining the isotopic composition of samples of naturally occurring silicon via absolute isotopic ratio mass spectrometry, controlling the alignment of complex electromechanical systems such as a watt balance, and a whole host of other mechanical, electrical, and optical techniques that are assumed to be well understood. Clearly, there is much work to be done to resolve this problem.

  The experimental discovery of a possible error in two of the three XROI determinations of the d_{220} lattice spacing of particular silicon crystals, all three of which were used as input data in the 1998 adjustment, and the subsequent omission of these two apparently problematic data from the 2002 adjustment, has eliminated the scatter in the values of α implied by the accurate measurement of the quantity $h/m_n d_{220}$(W04). Further, using this remaining value of d_{220}, the agreement of the value of α inferred from $h/m_n d_{220}$(W04) with other values of α is now good.

- *Gyromagnetic ratios of proton and helion*. In the initial least-squares adjustment with all 112 input data, the absolute values of the normalized residuals $|r_i|$ of three of the four available results for the proton and helion gyromagnetic ratio-related quantities Γ'_{p-90}(lo) and Γ'_{h-90}(lo) are among the four largest values of $|r_i|$. In the final adjustment on which the 2002 recommended values are based, the datum with the largest residual is one of the two results for Γ'_{p-90}(lo). The cause of the apparent problem in measuring these quantities is unknown, but may be related to the determination of the dimensions of the single-layer precision solenoid used to produce the calculable magnetic flux density required in each experiment and/or to understanding the distribution of the current in its windings.

C. Outlook and suggestions for future work

In the corresponding section of CODATA-98, we made extensive suggestions for future work based on what we believed to be the principal weaknesses of the 1998 adjustment. In fact, most of those suggestions remain valid, because, as for the 1998 adjustment, a key weakness of the 2002 adjustment is the lack of redundancy in the input data: α, h, and R are major players in the determination of the values of many constants, yet the adjusted value of each is still to a large extent determined by a pair of input data or a single input datum. An additional problem in the 2002 adjustment is the significant disagreement of the new value of V_m(Si) with other data. If successively met, the following needs for new work—succinctly presented as a "wish list"—should resolve the key issues and advance our knowledge of the values of the basic constants and conversion factors of physics and chemistry.

- *Relative atomic masses*
  A measurement of $A_r(^3\text{He})$ with $u_r\leqslant2\times10^{-10}$.
- *Fine-structure constant*
  An independent calculation of the eighth-order coefficient $A_1^{(8)}$ in the theoretical expression for a_e.
  A second measurement of a_e with $u_r\leqslant5\times10^{-9}$.
  One or more values of α obtained by a completely different method with $u_r\leqslant5\times10^{-9}$, for example, by atom recoil or by improved measurements and theoretical calculations of the fine structure of ^{4}He.
- *Planck constant*
  One or more watt-balance determinations of $K_J^2R_K$ with $u_r\leqslant5\times10^{-8}$.
  One or more determinations of V_m(Si) with $u_r\leqslant10^{-7}$, including a measurement of the molar mass of silicon that is completely independent of the current measurement.
- *Rydberg constant*
  One or more measurements of a transition frequency in hydrogen or deuterium with $u_r<10^{-12}$ (other than the already well-known $1S_{1/2}-2S_{1/2}$ frequency), thereby providing a value of R_∞ with approximately the same relative uncertainty.
  A measurement of the proton radius with $u_r\approx10^{-3}$ by means of spectroscopy in muonic hydrogen ($p\,\mu^-$ atom) which would also lead to an improved value of R_∞.
- *Molar gas constant*
  One or more velocity-of-sound measurements of R with $u_r\leqslant2\times10^{-6}$, preferably using a gas other than

argon.
One or more measurements of k with $u_r \leqslant 2\times10^{-6}$ (such a result would yield a value of R with essentially the same uncertainty and represent a heretofore unrealized approach to determining R).

- *Newtonian constant of gravitation*
One or more measurements of G with $u_r \leqslant 15\times10^{-6}$.
- *Muon magnetic moment anomaly*
New experimental data and improved theory that will reduce the contribution of the standard uncertainty of $a_\mu^{(4)}$(had) to the standard uncertainty of a_μ(th) to $\leqslant 2\times10^{-7}a_\mu$.
- *Proton and helion gyromagnetic ratios*
One or more measurements of Γ'_{p-90}(lo) and/or Γ'_{h-90}(lo) with $u_r \leqslant 5\times10^{-8}$.
- *Josephson and quantum Hall effects*
Experimental tests of the exactness of the Josephson and quantum Hall effect relations $K_J=2e/h$ and $R_K=h/e^2$ with $u_r \leqslant 1\times10^{-8}$ using a method such as the "closing of the metrological triangle" in which one compares a current derived from Josephson and quantum Hall effect devices to a current derived from a single-electron tunneling device [see, for example, Piquemal and Genevés (2000)].

This is by no means a trivial list of suggested work—it will require years of effort by a large number of dedicated researchers and considerable innovation. Nevertheless, if the past accomplishments of those who work in the precision measurement-fundamental constants field are any indication of future accomplishments, its completion is not beyond reasonable hope. As noted in CODATA-98, the reason such an endeavor is highly worthwhile is aptly summarized by the now over 70-year-old observation of F. K. Richtmyer (1932): "…the whole history of physics proves that a new discovery is quite likely to be found lurking in the next decimal place."

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We warmly thank our many colleagues in the precision measurement-fundamental constants field for promptly answering our numerous questions about their experiments and theoretical calculations, and for providing us results prior to formal publication.

APPENDIX A: THEORY RELEVANT TO THE RYDBERG CONSTANT

This appendix gives a brief summary of the theory of the energy levels of the hydrogen atom relevant to the determination of the Rydberg constant R_∞ based on measurements of transition frequencies. It is an updated version of an earlier review by one of the authors (Mohr, 1996) and a subsequent review in CODATA-98. In this appendix, information to completely determine the theoretical values for the energy levels used in the current adjustment is provided. Results that were included in CODATA-98 are given with minimal discussion, and the emphasis is on results that have become available since then. For brevity, references to most historical works are not included. Eides *et al.* (2001b) have recently provided a comprehensive review of the relevant theory.

It should be noted that the theoretical values of the energy levels of different states are highly correlated. For example, for S states, the uncalculated terms are primarily of the form of an unknown common constant divided by n^3. This fact is taken into account by calculating covariances between energy levels in addition to the uncertainties of the individual levels as discussed in detail in Sec. A.12. To provide the information needed to calculate the covariances, where necessary we distinguish between components of uncertainty that are proportional to $1/n^3$, denoted by u_0, and components of uncertainty that are essentially random functions of n, denoted by u_n.

The energy levels of hydrogenlike atoms are determined mainly by the Dirac eigenvalue, QED effects such as self-energy and vacuum polarization, and nuclear size and motion effects. We consider each of these contributions in turn.

1. Dirac eigenvalue

The binding energy of an electron in a static Coulomb field (the external electric field of a point nucleus of charge Ze with infinite mass) is determined predominantly by the Dirac eigenvalue

$$E_D = \left[1 + \frac{(Z\alpha)^2}{(n-\delta)^2}\right]^{-1/2} m_e c^2, \tag{A1}$$

where n is the principal quantum number,

$$\delta = |\kappa| - [\kappa^2 - (Z\alpha)^2]^{1/2}, \tag{A2}$$

and κ is the angular momentum-parity quantum number ($\kappa=-1,1,-2,2,-3$ for $S_{1/2}$, $P_{1/2}$, $P_{3/2}$, $D_{3/2}$ and $D_{5/2}$ states, respectively). States with the same principal quantum number n and angular momentum quantum number $j=|\kappa|-\frac{1}{2}$ have degenerate eigenvalues. The nonrelativistic orbital angular momentum is given by $l=|\kappa+\frac{1}{2}|-\frac{1}{2}$. (Although we are interested only in the case where the nuclear charge is e, we retain the atomic number Z in order to indicate the nature of various terms.)

Corrections to the Dirac eigenvalue that approximately take into account the finite mass of the nucleus m_N are included in the more general expression for atomic energy levels, which replaces Eq. (A1) (Barker and Glover, 1955; Sapirstein and Yennie, 1990):

$$E_M = Mc^2 + [f(n,j)-1]m_r c^2 - [f(n,j)-1]^2\frac{m_r^2 c^2}{2M} + \frac{1-\delta_{l0}}{\kappa(2l+1)}\frac{(Z\alpha)^4 m_r^3 c^2}{2n^3 m_N^2} + \cdots, \tag{A3}$$

where

$$f(n,j)=\left[1+\frac{(Z\alpha)^2}{(n-\delta)^2}\right]^{-1/2}, \tag{A4}$$

$M=m_e+m_N$, and $m_r=m_e m_N/(m_e+m_N)$ is the reduced mass.

2. Relativistic recoil

Relativistic corrections to Eq. (A3) associated with motion of the nucleus are considered relativistic-recoil corrections. The leading term, to lowest order in $Z\alpha$ and all orders in m_e/m_N, is (Erickson, 1977; Sapirstein and Yennie, 1990)

$$E_S=\frac{m_r^3}{m_e^2 m_N}\frac{(Z\alpha)^5}{\pi n^3}m_e c^2\left\{\frac{1}{3}\delta_{l0}\ln(Z\alpha)^{-2}-\frac{8}{3}\ln k_0(n,l)-\frac{1}{9}\delta_{l0}-\frac{7}{3}a_n-\frac{2}{m_N^2-m_e^2}\delta_{l0}\left[m_N^2\ln\left(\frac{m_e}{m_r}\right)-m_e^2\ln\left(\frac{m_N}{m_r}\right)\right]\right\}, \tag{A5}$$

where

$$a_n=-2\left[\ln\left(\frac{2}{n}\right)+\sum_{i=1}^{n}\frac{1}{i}+1-\frac{1}{2n}\right]\delta_{l0}+\frac{1-\delta_{l0}}{l(l+1)(2l+1)}. \tag{A6}$$

To lowest order in the mass ratio, higher-order corrections in $Z\alpha$ have been extensively investigated; the contribution of the next two orders in $Z\alpha$ can be written as

$$E_R=\frac{m_e}{m_N}\frac{(Z\alpha)^6}{n^3}m_e c^2[D_{60}+D_{72}Z\alpha\ln^2(Z\alpha)^{-2}+\cdots], \tag{A7}$$

where for $nS_{1/2}$ states (Pachucki and Grotch, 1995; Eides and Grotch, 1997b)

$$D_{60}=4\ln 2-\frac{7}{2} \tag{A8}$$

and for states with $l\geqslant 1$ (Golosov *et al.*, 1995; Elkhovskiĭ, 1996; Jentschura and Pachucki, 1996)

$$D_{60}=\left[3-\frac{l(l+1)}{n^2}\right]\frac{2}{(4l^2-1)(2l+3)}. \tag{A9}$$

[As usual, the first subscript on the coefficient refers to the power of $Z\alpha$ and the second subscript to the power of $\ln(Z\alpha)^{-2}$.] The next coefficient in Eq. (A7) has been calculated recently with the result (Melnikov and Yelkhovsky, 1999; Pachucki and Karshenboim, 1999)

$$D_{72}=-\frac{11}{60\pi}\delta_{l0}. \tag{A10}$$

The relativistic recoil correction used in the 2002 adjustment is based on Eqs. (A5)–(A10). Numerical values for the complete contribution of Eq. (A7) to all orders in $Z\alpha$ have been obtained by Shabaev *et al.* (1998). While these results are in general agreement with the values given by the power-series expressions, the difference between them for S states is about three times larger than expected [based on the uncertainty quoted by Shabaev *et al.* (1998) and the estimated uncertainty of the truncated power series which is taken to be one-half the contribution of the term proportional to D_{72}, as suggested by Eides *et al.* (2001b)]. This difference is not critical, and we allow for the ambiguity by assigning an uncertainty for S states of 10% of the contribution given by Eq. (A7). This is sufficiently large that the power series value is consistent with the numerical all-order calculated value. For the states with $l\geqslant 1$, we assign an uncertainty of 1% of the contribution in Eq. (A7). The covariances of the theoretical values are calculated by assuming that the uncertainties are predominately due to uncalculated terms proportional to $(m_e/m_N)/n^3$.

3. Nuclear polarization

Another effect involving specific properties of the nucleus, in addition to relativistic recoil, is nuclear polarization. It arises from interactions between the electron and nucleus in which the nucleus is excited from the ground state to virtual higher states.

For hydrogen, the result that we use for the nuclear polarization is (Khriplovich and Sen'kov, 2000)

$$E_P(\mathrm{H})=-0.070(13)h\frac{\delta_{l0}}{n^3}\ \mathrm{kHz}. \tag{A11}$$

Larger values for this correction have been reported by Rosenfelder (1999) and Martynenko and Faustov (2000), but apparently they are based on an incorrect formulation of the dispersion relations (Khriplovich and Sen'kov, 2000; Eides *et al.*, 2001b).

For deuterium, to a good approximation, the polarizability of the nucleus is the sum of the proton polarizability, the neutron polarizibility (Khriplovich and Sen'kov, 1998), and the dominant nuclear structure polarizibility (Friar and Payne, 1997a), with the total given by

$$E_P(\mathrm{D})=-21.37(8)h\frac{\delta_{l0}}{n^3}\ \mathrm{kHz}. \tag{A12}$$

We assume that this effect is negligible in states of higher l.

4. Self-energy

The second-order (in e, first order in α) level shift due to the one-photon electron self-energy, the lowest-order radiative correction, is given by

$$E_{SE}^{(2)}=\frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}F(Z\alpha)m_e c^2, \tag{A13}$$

where

TABLE XXXIV. Bethe logarithms $\ln k_0(n,l)$ relevant to the determination of R_∞.

n	S	P	D
1	2.984 128 556		
2	2.811 769 893	−0.030 016 709	
3	2.767 663 612		
4	2.749 811 840	−0.041 954 895	−0.006 740 939
6	2.735 664 207		−0.008 147 204
8	2.730 267 261		−0.008 785 043
12			−0.009 342 954

$$F(Z\alpha) = A_{41}\ln(Z\alpha)^{-2} + A_{40} + A_{50}(Z\alpha) + A_{62}(Z\alpha)^2\ln^2(Z\alpha)^{-2} + A_{61}(Z\alpha)^2\ln(Z\alpha)^{-2} + G_{\rm SE}(Z\alpha)(Z\alpha)^2, \tag{A14}$$

with (Erickson and Yennie, 1965)

$$A_{41} = \frac{4}{3}\delta_{l0},$$

$$A_{40} = -\frac{4}{3}\ln k_0(n,l) + \frac{10}{9}\delta_{l0} - \frac{1}{2\kappa(2l+1)}(1-\delta_{l0}),$$

$$A_{50} = \left(\frac{139}{32} - 2\ln 2\right)\pi\delta_{l0}, \tag{A15}$$

$$A_{62} = -\delta_{l0},$$

$$A_{61} = \left[4\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right) + \frac{28}{3}\ln 2 - 4\ln n - \frac{601}{180} - \frac{77}{45n^2}\right]\delta_{l0} + \left(1-\frac{1}{n^2}\right)\left(\frac{2}{15}+\frac{1}{3}\delta_{j(1/2)}\right)\delta_{l1} + \frac{96n^2-32l(l+1)}{3n^2(2l-1)(2l)(2l+1)(2l+2)(2l+3)}(1-\delta_{l0}).$$

Bethe logarithms $\ln k_0(n,l)$ that appear in Eq. (A15) needed for this work are given in Table XXXIV (Drake and Swainson, 1990).

The function $G_{\rm SE}(Z\alpha)$ in Eq. (A14) gives the higher-order contribution (in $Z\alpha$) to the self-energy, and the values for $G_{\rm SE}(\alpha)$ that we use here are listed in Table XXXV. For the states with $n=1$ and $n=2$, the values in the table are based on direct numerical evaluations by Jentschura *et al.* (1999, 2001). The values of $G_{\rm SE}(\alpha)$ for higher-n states are based on the low-Z limit of this function, $G_{\rm SE}(0)=A_{60}$, in the cases where it is known, together with extrapolations of the results of complete numerical calculations of $F(Z\alpha)$ [see Eq. (A14)] at higher Z (Kotochigova *et al.*, 2002; Le Bigot, Jentschura, Mohr, and Indelicato, 2003). There is a long history of calculations of A_{60} (Eides *et al.*, 2001b), leading up to the accurate values of A_{60} for the 1S and 2S states obtained by Pachucki (1992, 1993c, 1999). Values for P and D states subsequently have been reported by Jentschura and Pachucki (1996) and Jentschura *et al.* (1997, 2003). Extensive numerical evaluations of $F(Z\alpha)$ at higher Z, which in turn yield values for $G_{\rm SE}(Z\alpha)$, have been done by Mohr (1992); Mohr and Kim (1992); Indelicato and Mohr (1998); and Le Bigot (2001).

TABLE XXXV. Values of the function $G_{\rm SE}(\alpha)$.

n	$S_{1/2}$	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$	$D_{5/2}$
1	−30.290 24(2)				
2	−31.185 15(9)	−0.9735(2)	−0.4865(2)		
3	−31.01(6)				
4	−30.87(5)	−1.165(2)	−0.611(2)		0.031(1)
6	−30.82(8)				0.034(2)
8	−30.80(9)			0.008(5)	0.034(2)
12				0.009(5)	0.035(2)

The dominant effect of the finite mass of the nucleus on the self-energy correction is taken into account by multiplying each term of $F(Z\alpha)$ by the reduced-mass factor $(m_{\rm r}/m_{\rm e})^3$, except that the magnetic moment term $-1/[2\kappa(2l+1)]$ in A_{40} is instead multiplied by the factor $(m_{\rm r}/m_{\rm e})^2$. In addition, the argument $(Z\alpha)^{-2}$ of the logarithms is replaced by $(m_{\rm e}/m_{\rm r})(Z\alpha)^{-2}$ (Sapirstein and Yennie, 1990).

The uncertainty of the self-energy contribution to a given level arises entirely from the uncertainty of $G_{\rm SE}(\alpha)$ listed in Table XXXV and is taken to be entirely of type u_n.

5. Vacuum polarization

The second-order vacuum polarization level shift, due to the creation of a virtual electron-positron pair in the exchange of photons between the electron and the nucleus, is

$$E^{(2)}_{\rm VP} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}H(Z\alpha)m_{\rm e}c^2, \tag{A16}$$

where the function $H(Z\alpha)$ is divided into the part corresponding to the Uehling potential, denoted here by $H^{(1)}(Z\alpha)$, and the higher-order remainder $H^{(\rm R)}(Z\alpha) = H^{(3)}(Z\alpha) + H^{(5)}(Z\alpha) + \cdots$, where the superscript denotes the order in powers of the external field. The individual terms are expanded in a power series in $Z\alpha$ as

$$H^{(1)}(Z\alpha) = V_{40} + V_{50}(Z\alpha) + V_{61}(Z\alpha)^2\ln(Z\alpha)^{-2} + G^{(1)}_{\rm VP}(Z\alpha)(Z\alpha)^2, \tag{A17}$$

$$H^{(\rm R)}(Z\alpha) = G^{(\rm R)}_{\rm VP}(Z\alpha)(Z\alpha)^2, \tag{A18}$$

with

$$V_{40} = -\frac{4}{15}\delta_{l0},$$

$$V_{50} = \frac{5}{48}\pi\delta_{l0}, \tag{A19}$$

TABLE XXXVI. Values of the function $G_{\rm VP}^{(1)}(\alpha)$.

n	$S_{1/2}$	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$	$D_{5/2}$
1	−0.618 724				
2	−0.808 872	−0.064 006	−0.014 132		
3	−0.814 530				
4	−0.806 579	−0.080 007	−0.017 666		−0.000 000
6	−0.791 450				−0.000 000
8	−0.781 197			−0.000 000	−0.000 000
12				−0.000 000	−0.000 000

$$V_{61} = -\frac{2}{15}\delta_{l0}.$$

The part $G_{\rm VP}^{(1)}(Z\alpha)$ arises from the Uehling potential, and is readily calculated numerically (Mohr, 1982; Kotochigova *et al.*, 2002); values are given in Table XXXVI. The higher-order remainder $G_{\rm VP}^{(\rm R)}(Z\alpha)$ has been considered by Wichmann and Kroll, and the leading terms in powers of $Z\alpha$ are (Wichmann and Kroll, 1956; Mohr, 1975, 1983)

$$G_{\rm VP}^{(\rm R)}(Z\alpha) = \left(\frac{19}{45} - \frac{\pi^2}{27}\right)\delta_{l0} + \left(\frac{1}{16} - \frac{31\pi^2}{2880}\right)\pi(Z\alpha)\,\delta_{l0} + \cdots. \tag{A20}$$

Higher-order terms omitted from Eq. (A20) are negligible.

In a manner similar to that for the self-energy, the leading effect of the finite mass of the nucleus is taken into account by multiplying Eq. (A16) by the factor $(m_\tau/m_e)^3$ and including a multiplicative factor of (m_e/m_τ) in the argument of the logarithm in Eq. (A17).

There is also a second-order vacuum polarization level shift due to the creation of virtual particle pairs other than the e^+e^- pair. The predominant contribution for nS states arises from $\mu^+\mu^-$, with the leading term being (Eides and Shelyuto, 1995; Karshenboim, 1995)

$$E_{\mu\rm VP}^{(2)} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}\left(-\frac{4}{15}\right)\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_\tau}{m_e}\right)^3 m_e c^2. \tag{A21}$$

The next-order term in the contribution of muon vacuum polarization to nS states is of relative order $Z\alpha m_e/m_\mu$ and is therefore negligible. The analogous contribution $E_{\tau\rm VP}^{(2)}$ from $\tau^+\tau^-$ (−18 Hz for the 1S state) is also negligible at the level of uncertainty of current interest.

For the hadronic vacuum polarization contribution, we take the result given by Friar, Martorell, and Sprung (1999) that utilizes all available e^+e^- scattering data:

$$E_{\rm had\ VP}^{(2)} = 0.671(15)E_{\mu\rm VP}^{(2)}, \tag{A22}$$

where the uncertainty is of type u_0.

The muonic and hadronic vacuum polarization contributions are negligible for P and D states.

6. Two-photon corrections

Corrections from two virtual photons, of order α^2, have been calculated as a power series in $Z\alpha$:

$$E^{(4)} = \left(\frac{\alpha}{\pi}\right)^2\frac{(Z\alpha)^4}{n^3}m_e c^2 F^{(4)}(Z\alpha), \tag{A23}$$

where

$$F^{(4)}(Z\alpha) = B_{40} + B_{50}(Z\alpha) + B_{63}(Z\alpha)^2\ln^3(Z\alpha)^{-2} + B_{62}(Z\alpha)^2\ln^2(Z\alpha)^{-2} + B_{61}(Z\alpha)^2\ln(Z\alpha)^{-2} + B_{60}(Z\alpha)^2 + \cdots. \tag{A24}$$

The leading term B_{40} is well known:

$$B_{40} = \left[\frac{3\pi^2}{2}\ln 2 - \frac{10\pi^2}{27} - \frac{2179}{648} - \frac{9}{4}\zeta(3)\right]\delta_{l0} + \left[\frac{\pi^2\ln 2}{2} - \frac{\pi^2}{12} - \frac{197}{144} - \frac{3\zeta(3)}{4}\right]\frac{1-\delta_{l0}}{\kappa(2l+1)}. \tag{A25}$$

The second term has been calculated by Pachucki (1993b, 1994); Eides and Shelyuto (1995); Eides *et al.* (1997) with the result

$$B_{50} = -21.5561(31)\,\delta_{l0}. \tag{A26}$$

The next coefficient, as obtained by Karshenboim (1993); Manohar and Stewart (2000); Yerokhin (2000); and Pachucki (2001), is

$$B_{63} = -\frac{8}{27}\delta_{l0}. \tag{A27}$$

For S states the coefficient B_{62} has been found to be

$$B_{62} = \frac{16}{9}\left[\frac{71}{60} - \ln 2 + \gamma + \psi(n) - \ln n - \frac{1}{n} + \frac{1}{4n^2}\right], \tag{A28}$$

where $\gamma = 0.577\cdots$ is Euler's constant and ψ is the psi function (Abramowitz and Stegun, 1965). The difference $B_{62}(1) - B_{62}(n)$ was calculated by Karshenboim (1996) and confirmed by Pachucki (2001) who also calculated the n-independent additive constant. For P states the calculated value is (Karshenboim, 1996)

TABLE XXXVII. Values of N used in the 2002 adjustment.

n	N
1	17.855672(1)
2	12.032209(1)
3	10.449810(1)
4	9.722413(1)
6	9.031832(1)
8	8.697639(1)

$$B_{62} = \frac{4}{27}\frac{n^2-1}{n^2}. \tag{A29}$$

This result has been confirmed by Jentschura and Nándori (2002) who also show that for D and higher angular momentum states $B_{62}=0$.

The single-logarithm coefficient B_{61} for S states is given by (Pachucki, 2001)

$$B_{61} = \frac{39\,751}{10\,800} + \frac{4N(n)}{3} + \frac{55\pi^2}{27} - \frac{616\ln 2}{135} + \frac{3\pi^2\ln 2}{4} + \frac{40\ln^2 2}{9} - \frac{9\zeta(3)}{8} + \left(\frac{304}{135} - \frac{32\ln 2}{9}\right) \times \left[\frac{3}{4} + \gamma + \psi(n) - \ln n - \frac{1}{n} + \frac{1}{4n^2}\right], \tag{A30}$$

where $N(n)$ is a term that was numerically evaluated for the 1S state by Pachucki (2001). Jentschura (2003) has evaluated $N(n)$ for excited S states with $n=2$ to $n=8$, has made an improved evaluation for $n=1$, and has given an approximate fit to the calculated results in order to extend them to higher n. Values of the function $N(n)$ for the states of interest here are given in Table XXXVII. There are no results yet for P or D states for B_{61}. Based on the relative magnitude of A_{61} for the S, P, and D states, we take as uncertainties $u_n(B_{61})=5.0$ for P states and $u_n(B_{61})=0.5$ for D states. [Recent work done after the completion of the 2002 adjustment has indicated that there may be an additional contribution to B_{61} and/or B_{60} (Pachucki, 2004; Yerokhin *et al.*, 2004). The only effect of such a contribution would be to change the recommended values of the proton and deuteron bound-state rms charge radii at a level that is likely to be less than half of the quoted uncertainty.]

The two-loop Bethe logarithm $b_{\rm L}$, which is expected to be the dominant part of the no-log term B_{60}, has been calculated for the 1S and 2S states by Pachucki and Jentschura (2003) who obtained

$$b_{\rm L} = -81.4(3) \quad \text{1S state}, \tag{A31a}$$

$$b_{\rm L} = -66.6(3) \quad \text{2S state}. \tag{A31b}$$

An additional contribution for S states,

TABLE XXXVIII. Values of $b_{\rm L}$ and B_{60} used in the 2002 adjustment.

n	$b_{\rm L}$	B_{60}
1	−81.4(3)	−61.6(9.2)
2	−66.6(3)	−53.2(8.0)
3	−61.7(5.0)	−50.1(9.0)
4	−59.2(5.0)	−48.4(8.8)
6	−56.7(5.0)	−46.7(8.6)
8	−55.5(5.0)	−45.8(8.5)

$$b_{\rm M} = \frac{10}{9}N, \tag{A32}$$

was derived by Pachucki (2001), where N is given in Table XXXVII as a function of the state n. These contributions can be combined to obtain an estimate for the coefficient B_{60} for S states:

$$B_{60} = b_{\rm L} + \frac{10}{9}N + \cdots, \tag{A33}$$

where the dots represent uncalculated contributions to B_{60} which are at the relative level of 15% (Pachucki and Jentschura, 2003). In order to obtain an approximate value for B_{60} for S states with $n \geqslant 3$, we employ a simple extrapolation formula,

$$b_{\rm L} = a + \frac{b}{n}, \tag{A34}$$

with a and b fitted to the 1S and 2S values of $b_{\rm L}$, and we include a component of uncertainty $u_0(b_{\rm L})=5.0$. The results for $b_{\rm L}$, along with the total estimated values of B_{60} for S states, is given in Table XXXVIII. For P states, there is a calculation of fine-structure differences (Jentschura and Pachucki, 2002), but because of the uncertainty in B_{61} for P states, we do not include this result. We assume that for both the P and D states, the uncertainty attributed to B_{61} is sufficiently large to account for the uncertainty in B_{60} and higher-order terms as well.

As in the case of the order α self-energy and vacuum polarization contributions, the dominant effect of the finite mass of the nucleus is taken into account by multiplying each term of the two-photon contribution by the reduced-mass factor $(m_{\rm r}/m_{\rm e})^3$, except that the magnetic moment term, the second line of Eq. (A25), is instead multiplied by the factor $(m_{\rm r}/m_{\rm e})^2$. In addition, the argument $(Z\alpha)^{-2}$ of the logarithms is replaced by $(m_{\rm e}/m_{\rm r})(Z\alpha)^{-2}$.

7. Three-photon corrections

The leading contribution from three virtual photons is assumed to have the form

$$E^{(6)} = \left(\frac{\alpha}{\pi}\right)^3 \frac{(Z\alpha)^4}{n^3} m_e c^2 [C_{40} + C_{50}(Z\alpha) + \cdots], \quad \text{(A35)}$$

in analogy with Eq. (A23) for two photons. The level shifts of order $(\alpha/\pi)^3(Z\alpha)^4 m_e c^2$ that contribute to C_{40} can be characterized as the sum of a self-energy correction, a magnetic moment correction, and a vacuum polarization correction. The self-energy correction arises from the slope of the Dirac form factor, and it has recently been calculated by Melnikov and van Ritbergen (2000) who obtained

$$E^{(6)}_{\rm SE} = \left(\frac{\alpha}{\pi}\right)^3 \frac{(Z\alpha)^4}{n^3} m_e c^2 \left[-\frac{868 a_4}{9} + \frac{25\zeta(5)}{2} - \frac{17\pi^2\zeta(3)}{6} - \frac{2929\zeta(3)}{72} - \frac{217\ln^4 2}{54} - \frac{103\pi^2 \ln^2 2}{270} + \frac{41\,671\pi^2 \ln 2}{540} + \frac{3899\pi^4}{6480} - \frac{454\,979\pi^2}{9720} - \frac{77\,513}{46\,656}\right]\delta_{l0}, \quad \text{(A36)}$$

where ζ is the Riemann zeta function and $a_4 = \sum_{n=1}^{\infty} 1/(2^n n^4) = 0.517\,479\,061\ldots$. The magnetic moment correction comes from the known three-loop electron anomalous magnetic moment (Laporta and Remiddi, 1996), and is given by

$$E^{(6)}_{\rm MM} = \left(\frac{\alpha}{\pi}\right)^3 \frac{(Z\alpha)^4}{n^3} m_e c^2 \left[-\frac{100 a_4}{3} + \frac{215\zeta(5)}{24} - \frac{83\pi^2\zeta(3)}{72} - \frac{139\zeta(3)}{18} - \frac{25\ln^4 2}{18} + \frac{25\pi^2 \ln^2 2}{18} + \frac{298\pi^2 \ln 2}{9} + \frac{239\pi^4}{2160} - \frac{17\,101\pi^2}{810} - \frac{28\,259}{5184}\right]\frac{1}{\kappa(2l+1)}, \quad \text{(A37)}$$

and the vacuum polarization correction is (Baikov and Broadhurst, 1995; Eides and Grotch, 1995b)

$$E^{(6)}_{\rm VP} = \left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^4}{n^3} m_e c^2 \left[-\frac{8135\zeta(3)}{2304} + \frac{4\pi^2 \ln 2}{15} - \frac{23\pi^2}{90} + \frac{325\,805}{93\,312}\right]\delta_{l0}. \quad \text{(A38)}$$

The total for C_{40} is

$$C_{40} = \left[-\frac{568 a_4}{9} + \frac{85\zeta(5)}{24} - \frac{121\pi^2\zeta(3)}{72} - \frac{84\,071\zeta(3)}{2304} - \frac{71\ln^4 2}{27} - \frac{239\pi^2 \ln^2 2}{135} + \frac{4787\pi^2 \ln 2}{108} + \frac{1591\pi^4}{3240} - \frac{252\,251\pi^2}{9720} + \frac{679\,441}{93\,312}\right]\delta_{l0} + \left[-\frac{100 a_4}{3} + \frac{215\zeta(5)}{24} - \frac{83\pi^2\zeta(3)}{72} - \frac{139\zeta(3)}{18} - \frac{25\ln^4 2}{18} + \frac{25\pi^2 \ln^2 2}{18} + \frac{298\pi^2 \ln 2}{9} + \frac{239\pi^4}{2160} - \frac{17\,101\pi^2}{810} - \frac{28\,259}{5184}\right]\frac{1-\delta_{l0}}{\kappa(2l+1)}. \quad \text{(A39)}$$

An uncertainty in the three-photon correction is assigned by taking $u_0(C_{50}) = 30\delta_{l0}$ and $u_n(C_{63}) = 1$, where C_{63} is defined by the usual convention.

The dominant effect of the finite mass of the nucleus is taken into account by multiplying C_{40} in Eq. (A39) by the reduced-mass factor $(m_r/m_e)^3$ for $l=0$ or by the factor $(m_r/m_e)^2$ for $l \neq 0$.

The contribution from four photons is expected to be of order

$$\left(\frac{\alpha}{\pi}\right)^4 \frac{(Z\alpha)^4}{n^3} m_e c^2, \quad \text{(A40)}$$

which is about 10 Hz for the 1S state and is negligible at the level of uncertainty of current interest.

8. Finite nuclear size

At low Z, the leading contribution due to the finite size of the nucleus is

$$E^{(0)}_{\rm NS} = \mathcal{E}_{\rm NS}\delta_{l0}, \quad \text{(A41)}$$

with

$$\mathcal{E}_{\rm NS} = \frac{2}{3}\left(\frac{m_r}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_N}{\lambda\!\!\!^{-}_C}\right)^2, \quad \text{(A42)}$$

where R_N is the bound-state root-mean-square (rms) charge radius of the nucleus and $\lambda\!\!\!^{-}_C$ is the Compton wavelength of the electron divided by 2π. The leading higher-order contributions have been examined by Friar (1979b); Friar and Payne (1997b); Karshenboim (1997; see also Borisoglebsky and Trofimenko, 1979; Mohr, 1983). The expressions that we employ to evaluate the nuclear size correction are the same as those discussed in more detail in CODATA-98.

For S states the leading and next-order corrections are given by

$$E_{\rm NS} = \mathcal{E}_{\rm NS}\left\{1 - C_\eta \frac{m_r}{m_e}\frac{R_N}{\lambda\!\!\!^{-}_C} Z\alpha - \left[\ln\left(\frac{m_r}{m_e}\frac{R_N}{\lambda\!\!\!^{-}_C}\frac{Z\alpha}{n}\right) + \psi(n) + \gamma - \frac{(5n+9)(n-1)}{4n^2} - C_\theta\right](Z\alpha)^2\right\}, \quad \text{(A43)}$$

where C_η and C_θ are constants that depend on the details of the assumed charge distribution in the nucleus. The values used here are $C_\eta = 1.7(1)$ and $C_\theta = 0.47(4)$ for hydrogen or $C_\eta = 2.0(1)$ and $C_\theta = 0.38(4)$ for deuterium.

For the $P_{1/2}$ states in hydrogen the leading term is

$$E_{\rm NS} = \mathcal{E}_{\rm NS}\frac{(Z\alpha)^2(n^2-1)}{4n^2}. \quad \text{(A44)}$$

For $P_{3/2}$ states and D states the nuclear-size contribution is negligible.

9. Nuclear-size correction to self-energy and vacuum polarization

In addition to the direct effect of finite nuclear size on energy levels, its effect on the self-energy and vacuum polarization contributions must also be considered. This same correction is sometimes called the radiative correction to the nuclear-size effect.

For the self-energy, the additional contribution due to the finite size of the nucleus is (Pachucki, 1993a; Eides and Grotch, 1997c; Milstein *et al.* 2002, 2003a)

$$E_{\rm NSE} = \left(4\ln 2 - \frac{23}{4}\right)\alpha(Z\alpha)\mathcal{E}_{\rm NS}\delta_{l0}, \quad \text{(A45)}$$

and for the vacuum polarization it is (Friar, 1979a; Hylton, 1985; Eides and Grotch, 1997c)

$$E_{\rm NVP} = \frac{3}{4}\alpha(Z\alpha)\mathcal{E}_{\rm NS}\delta_{l0}. \quad \text{(A46)}$$

For the self-energy term, higher-order size corrections for S states (Milstein *et al.*, 2002) and size corrections for P states have been calculated (Jentschura, 2003; Milstein *et al.*, 2003b), but these corrections are negligible for the current work, and are not included. The D-state corrections are assumed to be negligible.

10. Radiative-recoil corrections

The dominant effect of nuclear motion on the self-energy and vacuum polarization has been taken into account by including appropriate reduced-mass factors. The additional contributions beyond this prescription are termed radiative-recoil effects with leading terms given by

$$E_{\rm RR} = \frac{m_{\rm r}^3}{m_{\rm e}^2 m_{\rm N}}\frac{\alpha(Z\alpha)^5}{\pi^2 n^3}m_{\rm e}c^2\delta_{l0}\left[6\zeta(3) - 2\pi^2\ln 2 + \frac{35\pi^2}{36} - \frac{448}{27} + \frac{2}{3}\pi(Z\alpha)\ln^2(Z\alpha)^{-2} + \cdots\right]. \quad \text{(A47)}$$

The leading constant term in Eq. (A47) is the sum of the analytic result for the electron-line contribution (Czarnecki and Melnikov, 2001; Eides *et al.*, 2001a) and the vacuum polarization contribution (Eides and Grotch, 1995a; Pachucki, 1995). This term agrees with the numerical value (Pachucki, 1995) used in CODATA-98. The log-squared term has been calculated by Melnikov and Yelkhovsky (1999); and Pachucki and Karshenboim (1999).

For the uncertainty, we take a term of order $(Z\alpha)\ln(Z\alpha)^{-2}$ relative to the square brackets in Eq. (A47) with numerical coefficients 10 for u_0 and 1 for u_n. These coefficients are roughly what one would expect for the higher-order uncalculated terms.

11. Nucleus self-energy

An additional contribution due to the self-energy of the nucleus has been given by Pachucki (1995):

$$E_{\rm SEN} = \frac{4Z^2\alpha(Z\alpha)^4}{3\pi n^3}\frac{m_{\rm r}^3}{m_{\rm N}^2}c^2\left[\ln\left(\frac{m_{\rm N}}{m_{\rm r}(Z\alpha)^2}\right)\delta_{l0} - \ln k_0(n,l)\right]. \quad \text{(A48)}$$

This correction has also been examined by Eides *et al.* (2001b), who consider how it is modified by the effect of structure of the proton. The structure effect leads to an additional model-dependent constant in the square brackets in Eq. (A48).

To evaluate the nucleus self-energy correction, we use Eq. (A48) and assign an uncertainty u_0 that corresponds to an additive constant of 0.5 in the square brackets for S states. For P and D states, the correction is small and its uncertainty, compared to other uncertainties, is negligible.

12. Total energy and uncertainty

The total energy $E^{\rm X}_{n{\rm L}j}$ of a particular level (where L = S, P, … and X = H, D) is the sum of the various contributions listed above plus an additive correction $\delta^{\rm X}_{n{\rm L}j}$ that accounts for the uncertainty in the theoretical expression for $E^{\rm X}_{n{\rm L}j}$. Our theoretical estimate of the value of $\delta^{\rm X}_{n{\rm L}j}$ for a particular level is zero with a standard uncertainty of $u(\delta^{\rm X}_{n{\rm L}j})$ equal to the square root of the sum of the squares of the individual uncertainties of the contributions, since, as they are defined above, the contributions to the energy of a given level are independent. (Components of uncertainty associated with the fundamental constants are not included here, because they are determined by the least-squares adjustment itself.) Thus we have for the square of the uncertainty, or variance, of a particular level

$$u^2(\delta^{\rm X}_{n{\rm L}j}) = \sum_i \frac{u_{0i}^2({\rm XL}j) + u_{ni}^2({\rm XL}j)}{n^6}, \quad \text{(A49)}$$

where the individual values $u_{0i}({\rm XL}j)/n^3$ and $u_{ni}({\rm XL}j)/n^3$ are the components of uncertainty from each of the contributions, labeled by i, discussed above. [The factors of $1/n^3$ are isolated so that $u_{0i}({\rm XL}j)$ is explicitly independent of n.]

The covariance of any two δ's follows from Eq. (F7) of Appendix F of CODATA-98. For a given isotope X, we have

$$u(\delta^{\rm X}_{n_1{\rm L}j}, \delta^{\rm X}_{n_2{\rm L}j}) = \sum_i \frac{u_{0i}^2({\rm XL}j)}{(n_1 n_2)^3}, \quad \text{(A50)}$$

which follows from the fact that $u(u_{0i},u_{ni})=0$ and $u(u_{n_1 i},u_{n_2 i})=0$ for $n_1 \neq n_2$. We also set

$$u(\delta^{\mathrm{X}}_{n_1\mathrm{L}_1 j_1},\delta^{\mathrm{X}}_{n_2\mathrm{L}_2 j_2})=0, \tag{A51}$$

if $\mathrm{L}_1 \neq \mathrm{L}_2$ or $j_1 \neq j_2$.

For covariances between δ's for hydrogen and deuterium, we have for states of the same n

$$u(\delta^{\mathrm{H}}_{n\mathrm{L}j},\delta^{\mathrm{D}}_{n\mathrm{L}j}) = \sum_{i=i_c} \frac{u_{0i}(\mathrm{HL}j)u_{0i}(\mathrm{DL}j)+u_{ni}(\mathrm{HL}j)u_{ni}(\mathrm{DL}j)}{n^6}, \tag{A52}$$

and for $n_1 \neq n_2$

$$u(\delta^{\mathrm{H}}_{n_1\mathrm{L}j},\delta^{\mathrm{D}}_{n_2\mathrm{L}j}) = \sum_{i=i_c} \frac{u_{0i}(\mathrm{HL}j)u_{0i}(\mathrm{DL}j)}{(n_1 n_2)^3}, \tag{A53}$$

where the summation is over the uncertainties common to hydrogen and deuterium. In most cases, the uncertainties can in fact be viewed as common except for a known multiplicative factor that contains all of the mass dependence. We assume

$$u(\delta^{\mathrm{H}}_{n_1\mathrm{L}_1 j_1},\delta^{\mathrm{D}}_{n_2\mathrm{L}_2 j_2})=0, \tag{A54}$$

if $\mathrm{L}_1 \neq \mathrm{L}_2$ or $j_1 \neq j_2$.

The values of $u(\delta^{\mathrm{X}}_{n\mathrm{L}j})$ of interest for the 2002 adjustment are given in Table XI of Sec. IV, and the nonnegligible covariances of the δ's are given in the form of correlation coefficients in Table XII of that section. These coefficients are as large as 0.991.

Since the transitions between levels are measured in frequency units (Hz), in order to apply the above equations for the energy level contributions we divide the theoretical expression for the energy difference ΔE of the transition by the Planck constant h to convert it to a frequency. Further, since we take the Rydberg constant $R_\infty = \alpha^2 m_e c/2h$ (expressed in m^{-1}) rather than the electron mass m_e to be an adjusted constant, we replace the group of constants $\alpha^2 m_e c^2/2h$ in $\Delta E/h$ by cR_∞.

13. Transition frequencies between levels with $n=2$

As an indication of the consistency of the theory summarized above and the experimental data, we list values of the transition frequencies between levels with $n=2$ in hydrogen. These results are based on values of the constants obtained in a variation of the 2002 least squares adjustment in which the measurements of the directly related transitions (items $A13$, $A14.1$, and $A14.2$ in Table XI are not included. The results are

$$\nu_{\mathrm{H}}(2\mathrm{P}_{1/2}-2\mathrm{S}_{1/2}) = 1\,057\,844.5(2.6)\ \mathrm{kHz} \quad [2.4\times10^{-6}],$$

$$\nu_{\mathrm{H}}(2\mathrm{S}_{1/2}-2\mathrm{P}_{3/2}) = 9\,911\,197.1(2.6)\ \mathrm{kHz} \quad [2.6\times10^{-7}],$$

$$\nu_{\mathrm{H}}(2\mathrm{P}_{1/2}-2\mathrm{P}_{3/2}) = 10\,969\,041.57(89)\ \mathrm{kHz} \quad [8.1\times10^{-8}]. \tag{A55}$$

APPENDIX B: THEORY OF ELECTRON MAGNETIC MOMENT ANOMALY

This appendix gives a brief summary of the current theory of a_e, the magnetic moment anomaly of the electron. A more detailed discussion and additional references can be found in CODATA-98. A summary of the theory of a_μ, the muon anomaly, is given in Appendix C. As indicated in Sec. III.C.1, Eq. (30), a_e is defined according to

$$a_e = \frac{|g_e|-2}{2} = \frac{|\mu_e|}{\mu_B} - 1. \tag{B1}$$

The theoretical expression for a_e may be written as

$$a_e(\mathrm{th}) = a_e(\mathrm{QED}) + a_e(\mathrm{weak}) + a_e(\mathrm{had}), \tag{B2}$$

where the terms denoted by QED, weak, and had account for the purely quantum electrodynamic, predominantly electroweak, and predominantly hadronic (that is, strong-interaction) contributions to a_e, respectively. The QED contribution may be written as (Kinoshita *et al.*, 1990).

$$a_e(\mathrm{QED}) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau). \tag{B3}$$

The term A_1 is mass independent and the other terms are functions of the indicated mass ratios. For these terms the lepton in the numerator of the mass ratio is the particle under consideration, while the lepton in the denominator of the ratio is the virtual particle that is the source of the vacuum polarization that gives rise to the term.

Each of the four terms on the right-hand side of Eq. (B3) is expressed as a power series in the fine-structure constant α:

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + \cdots. \tag{B4}$$

The fine-structure constant α is proportional to the square of the elementary charge e, and the order of a term containing $(\alpha/\pi)^n$ is $2n$ and its coefficient is called the $2n$th-order coefficient.

The second-order coefficient is known exactly, and the fourth- and sixth-order coefficients are known analytically in terms of readily evaluated functions:

$$A_1^{(2)} = \frac{1}{2}, \tag{B5}$$

$$A_1^{(4)} = -0.328\,478\,965\,579\ldots, \tag{B6}$$

$$A_1^{(6)} = 1.181\,241\,456\ldots. \tag{B7}$$

A total of 891 Feynman diagrams give rise to the eighth-order coefficient $A_1^{(8)}$, and only a few of these are known analytically. However, in an effort begun in the 1970s, Kinoshita and collaborators have calculated $A_1^{(8)}$

numerically (for a summary of some of this work, see Kinoshita, 2001a, 2001b). The value of $A_1^{(8)}$ used in the 1998 adjustment was –1.5098(384). Recently an error in the program employed in the evaluation of a gauge-invariant 18-diagram subset of the 891 diagrams was discovered in the course of carrying out an independent calculation to check this value (Kinoshita and Nio, 2003). The corrected program together with improved precision in the numerical integration for all diagrams leads to the tentative value $A_1^{(8)} = -1.7366(60)$ (Kinoshita, 2002), where the shift from the earlier value is predominately due to the correction of the error. As a result of this recent work, Kinoshita and Nio (2003) report that the integrals from all 891 Feynman diagrams have now been verified by independent calculation and/or checked by analytic comparison with lower-order integrals. Nevertheless, because the precision of the numerical evaluation of some integrals is still being improved and a closer examination is being made of the uncertainty of the numerical evaluation of other integrals, we retain the uncertainty estimate of the earlier reported value of $A_1^{(8)}$ for the 2002 adjustment. Thus the value we adopt is

$$A_1^{(8)} = -1.7366(384). \tag{B8}$$

The 0.0384 standard uncertainty of $A_1^{(8)}$ contributes a standard uncertainty to a_e(th) of $0.96\times10^{-9}a_e$, which may be compared to the $3.7\times10^{-9}a_e$ uncertainty of the experimental value [see Eq. (31), Sec. III.C.1.a]. We also note that work is in progress on analytic calculations of eighth-order integrals. See, for example, Laporta (2001) and Mastrolia and Remiddi (2001).

Little is known about the tenth-order coefficient $A_1^{(10)}$ and higher-order coefficients. To evaluate the contribution to the uncertainty of a_e(th) due to lack of knowledge of $A_1^{(10)}$, we follow CODATA-98 to obtain $A_1^{(10)} = 0.0(3.8)$. Because the 3.8 standard uncertainty of $A_1^{(10)}$ contributes a standard uncertainty component to a_e(th) of only $0.22\times10^{-9}a_e$, the uncertainty contributions to a_e(th) from all other higher-order coefficients are assumed to be negligible.

The mass-dependent coefficients of possible interest and corresponding contributions to a_e(th), based on the 2002 recommended values of the mass ratios, are

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,70(27)\times10^{-7} \rightarrow 2.418\times10^{-9}a_e, \tag{B9}$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,63(60)\times10^{-9} \rightarrow 0.009\times10^{-9}a_e, \tag{B10}$$

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,58(28)\times10^{-6} \rightarrow -0.080\times10^{-9}a_e, \tag{B11}$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5819(19)\times10^{-8} \rightarrow -0.001\times10^{-9}a_e, \tag{B12}$$

where the standard uncertainties of the coefficients are due to the uncertainties of the mass ratios. However, the contributions are so small that the uncertainties of the mass ratios are negligible. It may also be noted that the contributions from $A_3^{(6)}(m_e/m_\mu, m_e/m_\tau)$ and all higher-order mass-dependent terms are negligible as well.

For the electroweak contribution we have

$$a_e(\text{weak}) = 0.0297(5)\times10^{-12} = 0.0256(5)\times10^{-9}a_e, \tag{B13}$$

as in CODATA-98.

The hadronic contribution is

$$a_e(\text{had}) = 1.671(19)\times10^{-12} = 1.441(17)\times10^{-9}a_e, \tag{B14}$$

and is the sum of the following three contributions: $a_e^{(4)}(\text{had}) = 1.875(18)\times10^{-12}$ obtained by Davier and Höcker (1998); $a_e^{(6a)}(\text{had}) = -0.225(5)\times10^{-12}$ given by Krause (1997); and $a_e^{(\gamma\gamma)}(\text{had}) = 0.0210(36)\times10^{-12}$ obtained by multiplying the corresponding result for the muon given in Appendix C by the factor $(m_e/m_\mu)^2$, since $a_e^{(\gamma\gamma)}(\text{had})$ is assumed to vary approximately as m_μ^2. [Note that this value differs from the value $-0.0185(36)\times10^{-12}$ used in CODATA-98 because of the change in $a_\mu^{(\gamma\gamma)}(\text{had})$ discussed in Appendix C.] The contribution a_e(had), although larger than a_e(weak), is not yet of major significance.

For our least-squares adjustment, we require a_e(th) as a function of α. Since the dependence on α of any contribution other than a_e(QED) is negligible, we obtain a convenient form for the function by combining terms in a_e(QED) that have like powers of α/π. This leads to the following summary of the above results:

$$a_e(\text{th}) = a_e(\text{QED}) + a_e(\text{weak}) + a_e(\text{had}), \tag{B15}$$

where

$$a_e(\text{QED}) = C_e^{(2)}\left(\frac{\alpha}{\pi}\right) + C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_e^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots, \tag{B16}$$

with

$$C_e^{(2)} = 0.5,$$

$$C_e^{(4)} = -0.328\,478\,444\,00,$$

$$C_e^{(6)} = 1.181\,234\,017,$$

$$C_e^{(8)} = -1.7366(384),$$

$$C_e^{(10)} = 0.0(3.8), \tag{B17}$$

and where

$$a_e(\text{weak}) = 0.030(1)\times10^{-12} \tag{B18}$$

and

$$a_e(\text{had}) = 1.671(19) \times 10^{-12}. \quad \text{(B19)}$$

The standard uncertainty of a_e(th) from the uncertainties of the terms listed above, other than that due to α, is

$$u[a_e(\text{th})] = 1.15 \times 10^{-12} = 0.99 \times 10^{-9} a_e, \quad \text{(B20)}$$

and is dominated by the uncertainty of the coefficient $C_e^{(8)}$.

For the purpose of the least-squares calculations carried out in Sec. IV.B, we define an additive correction δ_e to a_e(th) to account for the lack of exact knowledge of a_e(th), and hence the complete theoretical expression for the electron anomaly is

$$a_e(\alpha, \delta_e) = a_e(\text{th}) + \delta_e. \quad \text{(B21)}$$

Our theoretical estimate of δ_e is zero and its standard uncertainty is $u[a_e(\text{th})]$:

$$\delta_e = 0.0(1.1) \times 10^{-12}. \quad \text{(B22)}$$

APPENDIX C: THEORY OF MUON MAGNETIC MOMENT ANOMALY

This appendix gives a brief summary of the current theory of the magnetic moment anomaly of the muon a_μ. A more detailed discussion and additional references can be found in CODATA-98. A similar summary of the theory of the electron anomaly a_e is given in Appendix B. As indicated in Sec. III.C.2, Eq. (36), a_μ is defined according to

$$a_\mu = \frac{|g_\mu| - 2}{2} = \frac{|\mu_\mu|}{e\hbar/2m_\mu} - 1. \quad \text{(C1)}$$

As for the electron, the theoretical expression for a_μ may be written as

$$a_\mu(\text{th}) = a_\mu(\text{QED}) + a_\mu(\text{weak}) + a_\mu(\text{had}), \quad \text{(C2)}$$

where the terms denoted by QED, electroweak, and had account for the purely quantum-electrodynamic, predominately electroweak, and predominately hadronic (that is, strong-interaction) contributions to a_μ, respectively. Also in the same manner as for the electron, the QED contribution may be written as (Kinoshita *et al.*, 1990).

$$a_\mu(\text{QED}) = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau). \quad \text{(C3)}$$

The mass-dependent terms are a function of the indicated mass ratios, and we again note that for these terms the lepton in the numerator of the mass ratio is the particle under consideration, while the lepton in the denominator of the ratio is the virtual particle that is the source of the vacuum polarization that gives rise to the term. As for the electron, each of the four terms on the right-hand side of Eq. (C3) is expressed as a power series in the fine-structure constant α:

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + \cdots. \quad \text{(C4)}$$

The mass-independent term A_1, which is given in Appendix B, is the same for all three charged leptons.

The relevant mass-dependent terms and the corresponding contributions to a_μ(th), based on the 2002 recommended values of the mass ratios, are

$$A_2^{(4)}(m_\mu/m_e) = 1.094\,258\,3111(84) \rightarrow 506\,386.4606(39) \times 10^{-8} a_\mu, \quad \text{(C5)}$$

$$A_2^{(4)}(m_\mu/m_\tau) = 0.000\,078\,064(25) \rightarrow 36.126(12) \times 10^{-8} a_\mu, \quad \text{(C6)}$$

$$A_2^{(6)}(m_\mu/m_e) = 22.868\,380\,02(20) \rightarrow 24\,581.766\,49(21) \times 10^{-8} a_\mu, \quad \text{(C7)}$$

$$A_2^{(6)}(m_\mu/m_\tau) = 0.000\,360\,58(21) \rightarrow 0.387\,59(22) \times 10^{-8} a_\mu, \quad \text{(C8)}$$

$$A_2^{(8)}(m_\mu/m_e) = 126.51(41) \rightarrow 315.9(1.0) \times 10^{-8} a_\mu, \quad \text{(C9)}$$

$$A_2^{(10)}(m_\mu/m_e) = 930(170) \rightarrow 5.4(1.0) \times 10^{-8} a_\mu, \quad \text{(C10)}$$

$$A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) = 0.000\,527\,66(17) \rightarrow 0.567\,20(18) \times 10^{-8} a_\mu, \quad \text{(C11)}$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.079(3) \rightarrow 0.1973(75) \times 10^{-8} a_\mu. \quad \text{(C12)}$$

These contributions and their uncertainties, as well as the values (including uncertainties) of a_μ(weak) and a_μ(had) given below, should be compared with the approximate $70 \times 10^{-8} a_\mu$ standard uncertainties of the μ^+ and μ^- experimental values of a_μ from BNL (see Sec. III.C.2.a) and the original $35 \times 10^{-8} a_\mu$ uncertainty goal of the BNL effort. The only nontrivial change in these coefficients and their contributions from those given in CODATA-98 is the one for $A_2^{(8)}$, which was previously $A_2^{(8)}(m_\mu/m_e) = 127.50(41) \rightarrow 318.3(1.0) \times 10^{-8} a_\mu$. The slight shift in value is due to the elimination of the program error and improved precision in numerical integration discussed in connection with the term $A_1^{(8)}$ in Appendix B (Kinoshita and Nio, 2003). It may also be noted that the contribution to this coefficient due to Baikov and Broadhurst (1995) used in obtaining the 1998 value has been confirmed by Kinoshita and Nio (1999). [The results $A_2^{(8)}(m_\mu/m_e) = 132.6823(72)$ and $A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.0376(9)$ recently reported by Kinoshita and Nio (2004) did not become available until early 2004, and thus could not be included in the 2002

adjustment. We do note that the increase in the value of $A_2^{(8)}(m_\mu/m_e)$ by about 6.2 implied by the new result produces an increase in the theoretical value of a_μ that is only about 1/6 of the total uncertainty of that value.]

The electroweak contribution to a_μ(th) can be characterized by the number of closed loops in the relevant Feynman diagrams:

$$a_\mu(\text{weak}) = a_\mu^{(1\ell)}(\text{weak}) + a_\mu^{(2\ell)}(\text{weak}) + \cdots, \tag{C13}$$

where 1ℓ indicates one loop, 2ℓ indicates two loops, etc. The dominant contribution to a_μ(weak) arises from one-loop diagrams involving W and Z bosons; the contribution from the Higgs boson is negligible for any reasonable estimated value of its mass.

The value $a_\mu(\text{weak}) = 153(3) \times 10^{-11} = 131(3) \times 10^{-8} a_\mu$ was used in the 1998 adjustment and was due to Degrassi and Giudice (1998). They calculated the dependence of the coefficients of the leading logarithmic terms of a particular part of $a_\mu^{(2\ell)}$ on $\sin^2\theta_W = 1 - (m_W/m_Z)^2$, where θ_W is the weak mixing angle and m_W/m_Z is the ratio of the mass of the $W^\pm$ to the mass of the Z^0, and the leading logarithmic terms of a particular part of the three-loop contribution $a_\mu^{(3\ell)}$. These additional terms provided small corrections to the value $a_\mu(\text{weak}) = 151(4) \times 10^{-11}$ obtained from the work of Czarnecki *et al.* (1996) as discussed in CODATA-98.

More recently, Knecht, Perrottet, *et al.* (2002) reanalyzed the two-loop electroweak contribution from light-quark (i.e., u, d, and s) triangle diagrams, with the end result $a_\mu(\text{weak}) = 152(1) \times 10^{-11}$, where the uncertainty does not include a component for higher-order electroweak effects. Subsequently, Czarnecki *et al.* (2003) reviewed and refined the calculation of a_μ(weak). They computed previously neglected two-loop contributions that are suppressed by the factor $1 - 4\sin^2\theta_W$, calculated the long-distance corrections to the quark triangle diagrams, and reevaluated the three-loop leading short-distance log terms. Their result,

$$a_\mu(\text{weak}) = 154(2) \times 10^{-11}, \tag{C14}$$

is the value used in the 2002 adjustment. The quoted standard uncertainty consists of a 1×10^{-11} component for uncertainty in the large distance hadronic correction and a 2×10^{-11} component for uncertainty in the value of the Higgs mass. The electroweak contribution to a_μ(th) is significant, but its uncertainty is of little consequence.

The hadronic contribution to a_μ(th) may be written as

$$a_\mu(\text{had}) = a_\mu^{(4)}(\text{had}) + a_\mu^{(6a)}(\text{had}) + a_\mu^{(\gamma\gamma)}(\text{had}) + \cdots, \tag{C15}$$

where $a_\mu^{(4)}$(had) and $a_\mu^{(6a)}$(had) arise from hadronic vacuum polarization and are of order $(\alpha/\pi)^2$ and $(\alpha/\pi)^3$, respectively; and $a_\mu^{(\gamma\gamma)}$(had) arises from hadronic light-by-light vacuum polarization. [The *a* in the superscript of $a_e^{(6a)}$(had) indicates that $a_\mu^{(\gamma\gamma)}$(had), which is also of sixth order, is not included.] Because the determination of a_μ(had) is a very active field of research, the value can be expected to change over time as it has in the past. In fact, new information has become available since the 2002 adjustment was completed in the Fall of 2003.

The value of $a_\mu^{(4)}$(had) used in the 1998 adjustment was $a_\mu^{(4)}(\text{had}) = 6924(62) \times 10^{-11}$, as given by Davier and Höcker (1998). It was based on theory and experimental data from both the production of hadrons in e^+e^- collisions and the decay of the τ into hadrons. The quoted standard uncertainty was due to uncertainties in both the theory and experimental data and was by far the largest component contributing to the uncertainty of a_μ(th). In the last several years, there has been additional work relevant to this quantity, motivated mainly by the improved experimental determination of a_μ at BNL, as discussed in Sec. III.C.2.a, and by new measurements of e^+e^- annihilation and hadronic decays of the τ.

An analysis of the available information has been given by Davier (2003a; 2003b), who find

$$a_\mu^{(4)}(\text{had:}e^+e^-) = 6963(72) \times 10^{-11}, \tag{C16}$$

$$a_\mu^{(4)}(\text{had:}e^+e^-,\tau) = 7110(58) \times 10^{-11}, \tag{C17}$$

where e^+e^- and e^+e^-, τ respectively indicate that the result is based on an analysis that does not include or does include data from τ decays into hadrons. The quoted standard uncertainty of each arises from uncertainties in both the experimental data and the theoretical analysis. These results can be compared to a number of other recent calculations. de Trocóniz and Ynduráin (2002) obtain $a_\mu^{(4)}(\text{had:}e^+e^-) = 6932(96) \times 10^{-11}$ and $a_\mu^{(4)}(\text{had:}e^+e^-,\tau) = 6952(64) \times 10^{-11}$. Groote (2002) reports $a_\mu^{(4)}(\text{had:}e^+e^-,\tau) = 6941(70) \times 10^{-11}$, Narison (2002) finds $a_\mu^{(4)}(\text{had:}e^+e^-) = 7016(119) \times 10^{-11}$ and $a_\mu^{(4)}(\text{had:}e^+e^-,\tau) = 7036(76) \times 10^{-11}$, Hagiwara *et al.* (2003) give $a_\mu^{(4)}(\text{had:}e^+e^-) = 6831(62) \times 10^{-11}$, and Jegerlehner (2003) gives the value $a_\mu^{(4)}(\text{had:}e^+e^-) = 6836(86) \times 10^{-11}$. In addition, Geshkenbeĭn (2003) reports the value $a_\mu^{(4)}(\text{had}) = 6780(70) \times 10^{-11}$ based on a theoretical model supplemented with the masses and widths of a few low-energy resonances. Also, a small part of the hadronic contribution as been considered by Achasov and Kiselev (2002). In view of the limitations of both the theory and experimental data, these results are in reasonable overall agreement. [This statement also applies to the very recent results reported by Ezhela *et al.* (2003); de Trocóniz and Ynduráin (2004); Ghozzi and Jegerlehner (2004); Hagiwara *et al.* (2004).]

However, at a more detailed level, the results of Davier *et al.* (2003a, 2003b) in Eqs. (C16) and (C17) show a significant difference depending on whether or not τ-decay data are used to replace e^+e^- in certain parts of the calculation. These authors point out that this discrepancy arises mainly from the difference in the $\pi\pi$ spectral functions obtained from the e^+e^- or τ data used in the calculation. They also believe that this discrepancy apparently went unnoticed in other analyses, because those analyses were based on preliminary e^+e^- data not corrected for vacuum polarization and final-state radiation; further, they express concern about

whether isospin symmetry breaking was correctly taken into account for the τ data in at least one of the other calculations.

For the 2002 adjustment, we adopt the value

$$a_\mu^{(4)}(\text{had}) = 7036(98) \times 10^{-11} = 6035(84) \times 10^{-8} a_\mu, \tag{C18}$$

which encompasses both of the results of Davier *et al.* (2003a, 2003b) as given above in Eqs. (C16) and (C17) and reflects the estimates of the other authors. We obtained this value, which we believe is a reasonable representation of the current knowledge of $a_\mu^{(4)}(\text{had})$, by the following procedure: Either result of Davier *et al.* (2003a, 2003b) is assumed to be equally likely and each is modeled by a normal distribution with expectation and standard deviation as quoted in Eqs. (C16) and (C17), respectively. The value in Eq. (C18) is then simply the expectation and standard deviation of the equally weighted sum of the two individual distributions.

In keeping with our comment above about the active nature of current research aimed at determining $a_\mu(\text{had})$, we note that all estimates of $a_\mu^{(4)}(\text{had}:e^+e^-)$ are plagued by the considerable statistical uncertainties and uncertainties due to systematic effects in the measurements of the cross section for $e^+e^- \to \pi^+\pi^-$ in the energy range below 1 GeV. However, the work currently underway at the DAΦNE accelerator, Frascati, Italy, on cross-section measurements of the radiative process $e^+e^- \to \pi^+\pi^-\gamma$, from which the cross section for $e^+e^- \to \pi^+\pi^-$ can be deduced, could reduce these uncertainties significantly (Di Falco, 2003; Aloisio *et al.*, 2004). Hence improvement in $a_\mu(\text{had})$ may be expected.

For $a_\mu^{(6a)}(\text{had})$ we take the value calculated by Alemany *et al.* (1998)

$$a_\mu^{(6a)}(\text{had}) = -100(6) \times 10^{-11}, \tag{C19}$$

which is an update of and very nearly the same as the value obtained by Krause (1997) used in the 1998 adjustment.

The value for $a_\mu^{(\gamma\gamma)}(\text{had})$ used in the 1998 adjustment was $a_\mu^{(\gamma\gamma)}(\text{had}) = -79.2(15.4) \times 10^{-11}$ as quoted by Hayakawa and Kinoshita (1998), which was consistent with $a_\mu^{(\gamma\gamma)}(\text{had}) = -92(32) \times 10^{-11}$ of Bijnens *et al.* (1996). Both of these estimates include the effects of the π^0, η, and η' mesons, which together give the dominant contribution to $a_\mu^{(\gamma\gamma)}(\text{had})$, as well as other contributions.

Recently, Knecht and Nyffeler (2002); Knecht, Nyffeler *et al.* (2002) found that this dominant contribution had the wrong sign, a fact subsequently confirmed by a number of other authors: the erratum to Hayakawa and Kinoshita (1998); Hayakawa and Kinoshita (2001); Bartoš *et al.* (2002); Bijnens *et al.* (2002); Blokland *et al.* (2002a). For the 2002 adjustment, we employ the updated value (Hayakawa and Kinoshita, 1998 erratum; 2001)

$$a_\mu^{(\gamma\gamma)}(\text{had}) = 89.6(15.4) \times 10^{-11} \tag{C20}$$

for the light-by-light contribution. [Subsequently, Melnikov and Vainshtein (2004) have obtained the value $a_\mu^{(\gamma\gamma)}(\text{had}) = 136(25) \times 10^{-11}$.]

By summing Eqs. (C18)–(C20), one obtains

$$a_\mu(\text{had}) = 7026(100) \times 10^{-11} = 6026(85) \times 10^{-8} a_\mu. \tag{C21}$$

Clearly, the uncertainty of $a_\mu(\text{had})$ is the dominant contribution to the uncertainty of $a_\mu(\text{th})$.

The foregoing analysis is based on the Standard Model of particle physics. When the result of the BNL measurements of a_μ with a relative uncertainty of 1.3×10^{-6} was reported in 2001 (Brown *et al.*, 2001), it differed from the theoretical value that was current at the time by 2.6 standard deviations of the difference. This apparent discrepancy has led to numerous theoretical papers that have attempted to explain it in terms of "new physics," such as supersymmetry. [See, for example, Czarnecki and Marciano (2001).] Although there are a number of theoretical models that would produce a contribution of the appropriate magnitude, there is none that has been confirmed by an independent experiment. Thus, at this time, we do not include physics beyond the Standard Model.

Following the same procedure as with $a_e(\text{th})$ in Appendix B, by adding terms in $a_\mu(\text{QED})$ that have like powers of α/π, including the results for A_1 given in that appendix, we summarize the theory of a_μ as follows:

$$a_\mu(\text{th}) = a_\mu(\text{QED}) + a_\mu(\text{weak}) + a_\mu(\text{had}), \tag{C22}$$

where

$$a_\mu(\text{QED}) = C_\mu^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\mu^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\mu^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\mu^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\mu^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots, \tag{C23}$$

with

$$C_\mu^{(2)} = 0.5,$$

$$C_\mu^{(4)} = 0.765\,857\,410(27),$$

$$C_\mu^{(6)} = 24.050\,509\,71(43),$$

$$C_\mu^{(8)} = 124.85(41),$$

$$C_\mu^{(10)} = 930(170), \tag{C24}$$

and where

$$a_\mu(\text{weak}) = 154(2) \times 10^{-11} \tag{C25}$$

and

$$a_\mu(\text{had}) = 7026(100) \times 10^{-11}. \tag{C26}$$

The standard uncertainty of $a_\mu(\text{th})$ from the uncertainties of the terms listed above, other than that due to α, is

$$u[a_\mu(\text{th})] = 100 \times 10^{-11} = 85 \times 10^{-8} a_\mu, \tag{C27}$$

and is primarily due to the uncertainty of a_μ(had).

In a manner similar to that for a_e(th), for the purpose of the least-squares calculations carried out in Sec. IV.B, we define an additive correction δ_μ to a_μ(th) to account for the lack of exact knowledge of a_μ(th), and hence the complete theoretical expression for the muon anomaly is

$$a_\mu(\alpha, \delta_\mu) = a_\mu(\text{th}) + \delta_\mu. \tag{C28}$$

Our theoretical estimate of δ_μ is zero and its standard uncertainty is $u[a_\mu(\text{th})]$:

$$\delta_\mu = 0(10) \times 10^{-10}. \tag{C29}$$

Although a_μ(th) and a_e(th) have common components of uncertainty, due mainly to the uncertainty of $A_1^{(8)}$, $u[a_\mu(\text{th})]$ is so large due to the uncertainty of a_μ(had) that the covariance of δ_μ and δ_e is negligible.

APPENDIX D: THEORY OF BOUND-STATE *g*-FACTORS

The experiments on magnetic moments and *g*-factors of interest in this paper are carried out on hydrogen, deuterium, muonium, hydrogenic carbon ($^{12}C^{5+}$), and hydrogenic oxygen ($^{16}O^{7+}$), each in the ground (1S) state. To use the results of these experiments in the 2002 adjustment, we employ theoretical expressions that give predictions for the moments and *g*-factors of the bound particles in terms of adjusted constants. Bound-state *g*-factors are defined by considering the contribution to the Hamiltonian from the interaction of the atom with an applied magnetic flux density B. For example, for hydrogen, in the framework of the Pauli approximation, we have

$$\begin{aligned}\mathcal{H} &= \beta(\text{H})\boldsymbol{\mu}_{e^-} \cdot \boldsymbol{\mu}_p - \boldsymbol{\mu}_{e^-}(\text{H}) \cdot \boldsymbol{B} - \boldsymbol{\mu}_p(\text{H}) \cdot \boldsymbol{B} \\ &= \frac{2\pi}{\hbar} \Delta\nu_H \boldsymbol{s} \cdot \boldsymbol{I} - g_{e^-}(\text{H}) \frac{\mu_B}{\hbar} \boldsymbol{s} \cdot \boldsymbol{B} - g_p(\text{H}) \frac{\mu_N}{\hbar} \boldsymbol{I} \cdot \boldsymbol{B},\end{aligned} \tag{D1}$$

where β(H) characterizes the strength of the hyperfine interaction, $\Delta\nu_H$ is the ground-state hyperfine frequency, $\boldsymbol{s}$ is the spin of the electron, and $\boldsymbol{I}$ is the spin of the nucleus, that is, the proton. The individual cases of interest are discussed in the following sections.

1. Bound electron in $^{12}C^{5+}$ and in $^{16}O^{7+}$

In this section, we consider an electron in the 1S state of hydrogenlike carbon 12 (atomic number $Z=6$, nuclear-spin quantum number $i=0$) or in the 1S state of hydrogenlike oxygen 16 (atomic number $Z=8$, nuclear-spin quantum number $i=0$) within the framework of relativistic bound-state theory. The measured quantity is the transition frequency between the two Zeeman levels of the atom in an externally applied magnetic field.

The energy of a free electron with spin projection s_z in a magnetic flux density B in the z direction is

TABLE XXXIX. Theoretical contributions and total for the *g* factor of the electron in hydrogenic carbon 12 based on the 2002 recommended values of the constants.

Contribution	Value	Source
Dirac g_D	−1.998 721 354 39(1)	Eq.(D6)
$\Delta g_{SE}^{(2)}$	−0.002 323 672 45(9)	Eq. (D10)
$\Delta g_{VP}^{(2)}$	0.000 000 008 51	Eq. (D14)
$\Delta g^{(4)}$	0.000 003 545 74(16)	Eq. (D18)
$\Delta g^{(6)}$	−0.000 000 029 62	Eq. (D19)
$\Delta g^{(8)}$	0.000 000 000 10	Eq. (D20)
Δg_{rec}	−0.000 000 087 64(1)	Eqs. (D21),(D23)
Δg_{ns}	−0.000 000 000 41	Eq. (D25)
$g_{e^-}(^{12}C^{5+})$	−2.001 041 590 16(18)	Eq. (D26)

$$E_{s_z} = -g_{e^-}\frac{e}{2m_e}s_z B, \tag{D2}$$

and hence the spin-flip energy difference is

$$\Delta E = -g_{e^-}\mu_B B. \tag{D3}$$

(In keeping with the definition of the *g*-factor in Sec. III.C, the quantity g_{e^-} is negative.) The analogous expressions for the ions considered here are

$$\Delta E_b(\text{X}) = -g_{e^-}(\text{X})\mu_B B, \tag{D4}$$

which defines the bound-state electron *g*-factor in the case where there is no nuclear spin, and where X is either $^{12}C^{5+}$ or $^{16}O^{7+}$.

The main theoretical contributions to g_{e^-}(X) can be categorized as follows:

- Dirac (relativistic) value g_D;
- radiative corrections Δg_{rad};
- recoil corrections Δg_{rec};
- nuclear size corrections Δg_{ns}.

Thus we write

$$g_{e^-}(\text{X}) = g_D + \Delta g_{rad} + \Delta g_{rec} + \Delta g_{ns} + \cdots, \tag{D5}$$

where terms accounting for other effects are assumed to be negligible at the current level of uncertainty of the relevant experiments (relative standard uncertainty $u_r \approx 6\times10^{-10}$; see Sec. III.C.3.a). These theoretical contributions are discussed in the following paragraphs; numerical results based on the 2002 recommended values are summarized in Tables XXXIX and XL. In the 2002 adjustment α in g_D is treated as a variable, but the constants in the rest of the calculation of the *g*-factors are taken as fixed quantities, because in that context their uncertainties are negligible.

Breit (1928) obtained the exact value

TABLE XL. Theoretical contributions and total for the g-factor of the electron in hydrogenic oxygen 16 based on the 2002 recommended values of the constants.

Contribution	Value	Source
Dirac g_D	−1.997 726 003 06(2)	Eq. (D6)
$\Delta g^{(2)}_{SE}$	−0.002 324 442 15(9)	Eq. (D10)
$\Delta g^{(2)}_{VP}$	0.000 000 026 38	Eq. (D14)
$\Delta g^{(4)}$	0.000 003 546 62(42)	Eq. (D18)
$\Delta g^{(6)}$	−0.000 000 029 62	Eq. (D19)
$\Delta g^{(8)}$	0.000 000 000 10	Eq. (D20)
Δg_{rec}	−0.000 000 117 02(1)	Eqs. (D21),(D23)
Δg_{ns}	−0.000 000 001 56(1)	Eq. (D25)
$g_{e^-}(^{16}O^{7+})$	−2.000 047 020 31(43)	Eq. (D26)

$$g_D = -\frac{2}{3}[1+2\sqrt{1-(Z\alpha)^2}] = -2\left[1-\frac{1}{3}(Z\alpha)^2-\frac{1}{12}(Z\alpha)^4-\frac{1}{24}(Z\alpha)^6+\cdots\right] \tag{D6}$$

from the Dirac equation for an electron in the field of a fixed-point charge of magnitude Ze, where the only uncertainty is that due to the uncertainty in α.

The radiative corrections may be written as

$$\Delta g_{rad} = -2\left[C_e^{(2)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)+C_e^{(4)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)^2+\cdots\right], \tag{D7}$$

where the coefficients $C_e^{(2n)}(Z\alpha)$, corresponding to n virtual photons, are slowly varying functions of $Z\alpha$. These coefficients are defined in direct analogy with the corresponding coefficients for the free electron $C_e^{(2n)}$ given in Appendix B, so that

$$\lim_{Z\alpha\to 0} C_e^{(2n)}(Z\alpha) = C_e^{(2n)}. \tag{D8}$$

The coefficient $C_e^{(2)}(Z\alpha)$ has been calculated to second order in $Z\alpha$ by Grotch (1970a), who finds

$$C_e^{(2)}(Z\alpha) = C_e^{(2)}+\frac{1}{12}(Z\alpha)^2+\cdots = \frac{1}{2}+\frac{1}{12}(Z\alpha)^2+\cdots. \tag{D9}$$

This result has been confirmed by Faustov (1970) and Close and Osborn (1971), as well as by others.

The terms listed in Eq. (D9) do not provide a value of $C_e^{(2)}(Z\alpha)$ which is sufficiently accurate at the level of uncertainty of the current experimental results. However, Yerokhin *et al.* (2002a, 2002b) have recently calculated numerically the self-energy contribution $C^{(2)}_{e,SE}(Z\alpha)$ to the coefficient to all orders in $Z\alpha$ over a wide range of Z. These results are in general agreement with, but are more accurate than, the earlier results of Beier (2000) and Beier *et al.* (2000). Other calculations of the self-energy have been carried out by Blundell *et al.* (1997b); Persson *et al.* (1997); and Goidenko *et al.* (2002). For $Z=6$ and $Z=8$ the calculation of Yerokhin *et al.* (2002b) gives

$$C^{(2)}_{e,SE}(6\alpha) = 0.500\,183\,609(19),$$
$$C^{(2)}_{e,SE}(8\alpha) = 0.500\,349\,291(19), \tag{D10}$$

where we have converted their quoted result to conform with our notation convention, taking into account the value of α employed in their calculation.

The lowest-order vacuum polarization correction is conveniently considered as consisting of two parts. In one the vacuum polarization loop modifies the interaction between the bound electron and the Coulomb field of the nucleus, and in the other the loop modifies the interaction between the bound electron and the external magnetic field. The first part, sometimes called the "wave-function" correction, has been calculated numerically by Beier *et al.* (2000), with the result (in our notation)

$$C^{(2)}_{e,VPwf}(6\alpha) = -0.000\,001\,840\,3431(43),$$
$$C^{(2)}_{e,VPwf}(8\alpha) = -0.000\,005\,712\,028(26). \tag{D11}$$

Each of these values is the sum of the Uehling potential contribution and the higher-order Wichmann-Kroll contribution, which were calculated separately.

The values in Eq. (D11) are consistent with the result of an evaluation of the correction in powers of $Z\alpha$. Terms to order $(\alpha/\pi)(Z\alpha)^7$ have been calculated for the Uehling potential contribution (Karshenboim, 2000a; Karshenboim *et al.*, 2001a, 2001b); and an estimate of the leading-order $(\alpha/\pi)(Z\alpha)^6$ term of the Wichmann-Kroll contribution has been given by Karshenboim *et al.* (2001b) based on a prescription of Karshenboim (2000a). To the level of uncertainty of interest here, the values from the power series are the same as the numerical values in Eq. (D11). [Note that for the Wichmann-Kroll term, the agreement between the power-series results and the numerical results is improved by an order of magnitude if an additional term in the power series for the energy level (Mohr, 1975) used in Karshenboim's prescription is included.]

For the second part of the lowest-order vacuum polarization correction, sometimes called the "potential" correction, Beier *et al.* (2000) found that the Uehling potential contribution is zero. They also calculated the Wichmann-Kroll contribution numerically over a wide range of Z. Their value at low Z is very small and only an uncertainty estimate of 3×10^{-10} in g is given because of poor convergence of the partial wave expansion. The reduction in uncertainty (by a factor of 30 for carbon) employed by Beier *et al.* (2002) for this term, based on the assumption that it is of the order of $(\alpha/\pi)(Z\alpha)^7$, is not considered here, because the reference quoted for this estimate (Karshenboim *et al.*, 2001b) does not explicitly discuss this term. Yerokhin *et al.* (2002b) ob-

tained numerical values for this contribution for carbon and oxygen by a least-squares fit to the values of Beier *et al.* (2000) at higher Z.

Subsequently, Karshenboim and Milstein (2002) analytically calculated the Wichmann-Kroll contribution to the potential correction to lowest order in $Z\alpha$. Their result in our notation is

$$C^{(2)}_{e,VPp}(Z\alpha) = \frac{7\pi}{432}(Z\alpha)^5 + \cdots . \tag{D12}$$

This result, together with the numerical values from Beier (2000), yields

$$C^{(2)}_{e,VPp}(6\alpha) = 0.000\,000\,007\,9595(69),$$

$$C^{(2)}_{e,VPp}(8\alpha) = 0.000\,000\,033\,235(29), \tag{D13}$$

which are used in the present analysis. We obtained these results by fitting a function of the form $[a+bZ\alpha + c(Z\alpha)^2](Z\alpha)^5$ to the point in Eq. (D12) and two values of the complete function calculated by Beier (2000) (separated by about 10 calculated values) and evaluating the fitted function at $Z=6$ or 8. This was done for a range of pairs of points from Beier (2000), and the results in Eq. (D13) are the the apparent limit of the values as the lower Z member of the pair used in the fit approaches either 6 or 8 as appropriate. [This general approach is described in more detail by Le Bigot, Jentschura, Mohr, Indelicato, and Soff (2003).]

The total one-photon vacuum polarization coefficients are given by the sum of Eqs. (D11) and (D13):

$$\begin{aligned} C^{(2)}_{e,VP}(6\alpha) &= C^{(2)}_{e,VPwf}(6\alpha) + C^{(2)}_{e,VPp}(6\alpha) \\ &= -0.000\,001\,832\,384(11), \\ C^{(2)}_{e,VP}(8\alpha) &= C^{(2)}_{e,VPwf}(8\alpha) + C^{(2)}_{e,VPp}(8\alpha) \\ &= -0.000\,005\,678\,793(55). \end{aligned} \tag{D14}$$

The total for the one-photon coefficient $C^{(2)}_e(Z\alpha)$, given by the sum of Eqs. (D10) and (D14), is

$$C^{(2)}_e(6\alpha) = C^{(2)}_{e,SE}(6\alpha) + C^{(2)}_{e,VP}(6\alpha) = 0.500\,181\,777(19),$$

$$C^{(2)}_e(8\alpha) = C^{(2)}_{e,SE}(8\alpha) + C^{(2)}_{e,VP}(8\alpha) = 0.500\,343\,613(19), \tag{D15}$$

where in this case, following Beier *et al.* (2000), the uncertainty is simply the sum of the individual uncertainties in Eqs. (D10) and (D14). The total one-photon contribution $\Delta g^{(2)}$ to the g-factor is thus

$$\begin{aligned} \Delta g^{(2)} &= -2C^{(2)}_e(Z\alpha)\left(\frac{\alpha}{\pi}\right) \\ &= -0.002\,323\,663\,93(9) \quad \text{for } Z=6 \\ &= -0.002\,324\,415\,77(9) \quad \text{for } Z=8. \end{aligned} \tag{D16}$$

The separate one-photon self energy and vacuum polarization contributions to the g-factor are given in Tables XXXIX and XL.

Evaluations by Eides and Grotch (1997a) using the Bargmann-Michel-Telegdi equation and by Czarnecki *et al.* (2001a) using an effective-potential approach yield

$$C^{(2n)}_e(Z\alpha) = C^{(2n)}_e\left(1 + \frac{(Z\alpha)^2}{6} + \cdots\right) \tag{D17}$$

as the leading binding correction to the free-electron coefficients $C^{(2n)}_e$ for any n. For $n=1$, this result was already known, as is evident from Eq. (D9). We include this correction for the two-photon term, that is, for $n=2$, which gives

$$\begin{aligned} C^{(4)}_e(Z\alpha) &= C^{(4)}_e\left(1 + \frac{(Z\alpha)^2}{6} + \cdots\right) \\ &= -0.328\,583(14) \quad \text{for } Z=6 \\ &= -0.328\,665(39) \quad \text{for } Z=8, \end{aligned} \tag{D18}$$

where $C^{(4)}_e = -0.328\,478\,444\ldots$. The uncertainty is due to uncalculated terms and is obtained by assuming that the unknown higher-order terms for $n=2$, represented by the dots in Eq. (D17), are the same as the higher-order terms for $n=1$ as can be deduced by comparing the numerical results given in Eq. (D15) to those resulting from the expression in Eq. (D9). This is the same general approach as that employed by Beier *et al.* (2002).

The three-photon term is calculated in a similar way but the uncertainty due to uncalculated higher-order terms is negligible:

$$\begin{aligned} C^{(6)}_e(Z\alpha) &= C^{(6)}_e\left(1 + \frac{(Z\alpha)^2}{6} + \cdots\right) \\ &= 1.1816\ldots \quad \text{for } Z=6 \\ &= 1.1819\ldots \quad \text{for } Z=8, \end{aligned} \tag{D19}$$

where $C^{(6)}_e = 1.181\,234\ldots$. For the four-photon correction, at the level of uncertainty of current interest, only the free-electron coefficient is necessary:

$$C^{(8)}_e(Z\alpha) \approx C^{(8)}_e = -1.7366(384). \tag{D20}$$

The preceding corrections Δg_D and Δg_{rad} are based on the approximation that the nucleus of the hydrogenic atom has an infinite mass. The recoil correction to the bound-state g-factor associated with the finite mass of the nucleus is denoted by Δg_{rec}, which we write here as the sum $\Delta g^{(0)}_{rec} + \Delta g^{(2)}_{rec}$ corresponding to terms that are zero order and first order in α/π, respectively. For $\Delta g^{(0)}_{rec}$, we have

$$\begin{aligned} \Delta g^{(0)}_{rec} &= \left\{ -(Z\alpha)^2 + \frac{(Z\alpha)^4}{3[1+\sqrt{1-(Z\alpha)^2}]^2} \right. \\ &\quad \left. -(Z\alpha)^5 P(Z\alpha) \right\} \frac{m_e}{m_N} + \mathcal{O}\left(\frac{m_e}{m_N}\right)^2 \\ &= -0.000\,000\,087\,71(1)\ldots \quad \text{for } Z=6 \\ &= -0.000\,000\,117\,11(1)\ldots \quad \text{for } Z=8, \end{aligned} \tag{D21}$$

where m_N is the mass of the nucleus. The mass ratios, obtained from the 2002 adjustment, are $m_e/m(^{12}C^{6+})$

$=0.000\,045\,727\,5\ldots$ and $m_e/m(^{16}O^{8+})=0.000\,034\,306\,5\ldots$. In Eq. (D21), the first term in the braces was calculated by Grotch (1970b). Shortly thereafter, this term and higher-order terms were obtained by Faustov (1970), Close and Osborn (1971), Grotch (1971), Grotch and Hegstrom (1971), and Hegstrom (1971) [see also Hegstrom (1969) and Grotch (1970a)]. The second and third terms in the braces were calculated by Shabaev and Yerokhin (2002) based on the formulation of Shabaev (2001) [see also Yelkhovsky (2001)]. Shabaev and Yerokhin have numerically evaluated the function $P(Z\alpha)$ over a wide range of Z, with the result $P(6\alpha)=10.493\,95(1)$ for hydrogenic carbon and $P(8\alpha)=9.300\,18(1)$ for hydrogenic oxygen.

An additional term of the order of the mass ratio squared has been considered by various authors. Earlier calculations of this term for atoms with a spin-one-half nucleus, such as muonium, have been done by Close and Osborn (1971) and Grotch and Hegstrom (1971) (see also Eides and Grotch, 1997a). Their result for this term is

$$(1+Z)(Z\alpha)^2\left(\frac{m_e}{m_N}\right)^2. \tag{D22}$$

Eides (2002); Eides and Grotch (1997a) find that this correction to the g-factor is independent of the spin of the nucleus, so Eq. (D22) gives the correction for carbon and oxygen, as well as atoms with a spin-one-half nucleus. On the other hand, Martynenko and Faustov (2001, 2002) find that the correction of this order depends on the spin of the nucleus and give a result with the factor $1+Z$ replaced by $Z/3$ for a spin-zero nucleus. In view of this discrepancy, we include a contribution to $\Delta g_{rec}^{(0)}$ in Eq. (D21) that is the average of the two quoted results with an uncertainty of half of the difference between them.

For $\Delta g_{rec}^{(2)}$, we have

$$\begin{aligned}\Delta g_{rec}^{(2)} &= \frac{\alpha}{\pi}\frac{(Z\alpha)^2}{3}\frac{m_e}{m_N}+\cdots\\ &= 0.000\,000\,000\,06\ldots \quad \text{for } Z=6\\ &= 0.000\,000\,000\,09\ldots \quad \text{for } Z=8.\end{aligned} \tag{D23}$$

There is a small correction to the bound-state g-factor due to the finite size of the nucleus:

$$\Delta g_{ns}=\frac{8}{3}(Z\alpha)^4\left(\frac{R_N}{\lambda\!\!\!^{-}_C}\right)^2+\cdots, \tag{D24}$$

where R_N is the bound-state nuclear rms charge radius and $\lambda\!\!\!^{-}_C$ is the Compton wavelength of the electron divided by 2π. In Eq. (D24), the term shown is the nonrelativistic approximation given by Karshenboim (2000a). This term and the dominant relativistic correction have been calculated by Glazov and Shabaev (2002). We take $R_N=2.4705(23)$ fm and $R_N=2.6995(68)$ from the compilation of Angeli (1998) for the values of the ^{12}C and ^{16}O nuclear radii, respectively, which, based on Glazov and Shabaev (2002), yields

$$\begin{aligned}\Delta g_{ns} &= -0.000\,000\,000\,41 \quad \text{for } ^{12}C,\\ \Delta g_{ns} &= -0.000\,000\,001\,56(1) \quad \text{for } ^{16}O.\end{aligned} \tag{D25}$$

The theoretical value for the g-factor of the electron in hydrogenic carbon 12 or oxygen 16 is the sum of the individual contributions discussed above and summarized in Tables XXXIX and XL:

$$\begin{aligned}g_{e^-}(^{12}C^{5+}) &= -2.001\,041\,590\,16(18) \quad [9.0\times10^{-11}],\\ g_{e^-}(^{16}O^{7+}) &= -2.000\,047\,020\,31(43) \quad [2.2\times10^{-10}].\end{aligned} \tag{D26}$$

For the purpose of the least-squares calculations carried out in Sec. IV.B, we define $g_C(\text{th})$ to be the sum of g_D as given in Eq. (D6), the term $-2(\alpha/\pi)C_e^{(2)}$, and the numerical values of the remaining terms in Eq. (D5) as given in Table XXXIX without the uncertainties. The standard uncertainty of $g_C(\text{th})$ from the uncertainties of these latter terms is

$$u[g_C(\text{th})]=1.8\times10^{-10}=9.0\times10^{-11}|g_C(\text{th})|. \tag{D27}$$

The uncertainty in $g_C(\text{th})$ due to the uncertainty in α enters the adjustment primarily through the functional dependence of g_D and the term $-2(\alpha/\pi)C_e^{(2)}$ on α. Therefore this particular component of uncertainty is not explicitly included in $u[g_C(\text{th})]$. To take the uncertainty $u[g_C(\text{th})]$ into account we employ as the theoretical expression for the g-factor

$$g_C(\alpha,\delta_C)=g_C(\text{th})+\delta_C, \tag{D28}$$

where the input value of the additive correction δ_C is taken to be zero and its standard uncertainty is $u[g_C(\text{th})]$:

$$\delta_C=0.0(1.8)\times10^{-10}. \tag{D29}$$

Analogous considerations apply for the g-factor in oxygen:

$$u[g_O(\text{th})]=4.3\times10^{-10}=2.2\times10^{-10}|g_O(\text{th})|, \tag{D30}$$

$$g_O(\alpha,\delta_O)=g_O(\text{th})+\delta_O \tag{D31}$$

$$\delta_O=0.0(4.3)\times10^{-10}. \tag{D32}$$

Since the uncertainties of the theoretical values of the carbon and oxygen g-factors arise primarily from the same sources, the quantities δ_C and δ_O are highly correlated. Their covariance is

$$u(\delta_C,\delta_O)=741\times10^{-22}, \tag{D33}$$

which corresponds to a correlation coefficient of $r(\delta_C,\delta_O)=0.95$.

The theoretical value of the ratio of the two g-factors, which is relevant to the discussion in Sec. III.C.3.c, is

$$\frac{g_{e^-}(^{12}C^{5+})}{g_{e^-}(^{16}O^{7+})} = 1.000\,497\,273\,23(13), \tag{D34}$$

where the covariance is taken into account in calculating the uncertainty, and for this purpose includes the contribution due to the uncertainty in α.

2. Ratio of bound-particle to free-particle *g*-factors

Other theoretical *g*-factor-related quantities of interest in the 2002 adjustment are the ratio of the *g*-factor of the electron in the ground state of hydrogen to that of the free electron $g_{e^-}(H)/g_{e^-}$; the ratio of the *g*-factor of the proton in hydrogen to that of the free proton $g_p(H)/g_p$; the analogous ratios for the electron and deuteron in deuterium, $g_{e^-}(D)/g_{e^-}$ and $g_d(D)/g_d$, respectively; and the analogous ratios for the electron and positive muon in muonium, $g_{e^-}(Mu)/g_{e^-}$ and $g_{\mu^+}(Mu)/g_{\mu^+}$, respectively. All six of these ratios were discussed in detail in CODATA-1998, but small additional corrections are included in the following expressions.

For the electron in hydrogen, we have

$$\frac{g_{e^-}(H)}{g_{e^-}} = 1 - \frac{1}{3}(Z\alpha)^2 - \frac{1}{12}(Z\alpha)^4 + \frac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right) + \frac{1}{2}(Z\alpha)^2\frac{m_e}{m_p} + \frac{1}{2}\left(A_1^{(4)} - \frac{1}{4}\right)(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^2 - \frac{5}{12}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_p} + \cdots, \tag{D35}$$

where $A_1^{(4)}$ is given in Eq. (B6). All the terms in Eq. (D35) arise from expanding the ratio of the individual *g*-factors, based on the expressions in Appendix B and Sec. D.1 of Appendix D (Karshenboim and Ivanov, (2003). For the proton in hydrogen, we have

$$\frac{g_p(H)}{g_p} = 1 - \frac{1}{3}\alpha(Z\alpha) - \frac{97}{108}\alpha(Z\alpha)^3 + \frac{1}{6}\alpha(Z\alpha)\frac{m_e}{m_p}\frac{3+4a_p}{1+a_p} + \cdots, \tag{D36}$$

where the third term on the right-hand side is a recently derived relativistic shielding correction (Moore, 1999; Pyper, 1999; Pyper and Zhang, 1999; Karshenboim and Ivanov, 2003), and the proton magnetic moment anomaly a_p is defined by

$$a_p = \frac{\mu_p}{(e\hbar/2m_p)} - 1 \approx 1.793. \tag{D37}$$

For deuterium, similar expressions apply for the electron

$$\frac{g_{e^-}(D)}{g_{e^-}} = 1 - \frac{1}{3}(Z\alpha)^2 - \frac{1}{12}(Z\alpha)^4 + \frac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right) + \frac{1}{2}(Z\alpha)^2\frac{m_e}{m_d} + \frac{1}{2}\left(A_1^{(4)} - \frac{1}{4}\right)(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^2 - \frac{5}{12}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_d} + \cdots, \tag{D38}$$

and deuteron

$$\frac{g_d(D)}{g_d} = 1 - \frac{1}{3}\alpha(Z\alpha) - \frac{97}{108}\alpha(Z\alpha)^3 + \frac{1}{6}\alpha(Z\alpha)\frac{m_e}{m_d}\frac{3+4a_d}{1+a_d} + \cdots, \tag{D39}$$

where the deuteron magnetic moment anomaly a_d is defined by

$$a_d = \frac{\mu_d}{(e\hbar/m_d)} - 1 \approx -0.143. \tag{D40}$$

In the case of muonium Mu, since the the electron-nucleus mass ratio is larger than in hydrogen or deuterium, some additional higher-order terms are included here (Karshenboim and Ivanov, 2002). For the electron in muonium, we have

$$\frac{g_{e^-}(Mu)}{g_{e^-}} = 1 - \frac{1}{3}(Z\alpha)^2 - \frac{1}{12}(Z\alpha)^4 + \frac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right) + \frac{1}{2}(Z\alpha)^2\frac{m_e}{m_\mu} + \frac{1}{2}\left(A_1^{(4)} - \frac{1}{4}\right)(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^2 - \frac{5}{12}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_\mu} - \frac{1}{2}(1+Z) \times (Z\alpha)^2\left(\frac{m_e}{m_\mu}\right)^2 + \cdots, \tag{D41}$$

and for the muon in muonium, the ratio is

$$\frac{g_{\mu^+}(Mu)}{g_{\mu^+}} = 1 - \frac{1}{3}\alpha(Z\alpha) - \frac{97}{108}\alpha(Z\alpha)^3 + \frac{1}{2}\alpha(Z\alpha)\frac{m_e}{m_\mu} + \frac{1}{12}\alpha(Z\alpha)\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_\mu} - \frac{1}{2}(1+Z)\alpha(Z\alpha) \times \left(\frac{m_e}{m_\mu}\right)^2 + \cdots. \tag{D42}$$

The numerical values of the corrections in Eqs. (D35)–(D42), based on the 2002 adjusted values of the relevant constants, are listed in Table XLI. Uncertainties are not given for these ratios because they are negligible at the level of uncertainty of the relevant experiments.

3. Comparison of theory and experiment

As a test of the theory, a comparison of the experimental values for various *g*-factor ratios with the theoretically predicted values was given in CODATA-98; in

TABLE XLI. Theoretical values for various bound-particle to free-particle g factor ratios relevant to the 2002 adjustment based on the 2002 recommended values of the constants.

Ratio	Value
$g_{e^-}(\mathrm{H})/g_{e^-}$	$1-17.7054\times10^{-6}$
$g_p(\mathrm{H})/g_p$	$1-17.7354\times10^{-6}$
$g_{e^-}(\mathrm{D})/g_{e^-}$	$1-17.7126\times10^{-6}$
$g_d(\mathrm{D})/g_d$	$1-17.7461\times10^{-6}$
$g_{e^-}(\mathrm{Mu})/g_{e^-}$	$1-17.5926\times10^{-6}$
$g_{\mu^+}(\mathrm{Mu})/g_{\mu^+}$	$1-17.6254\times10^{-6}$

each case, the terms employed in the bound g-factor expressions provided reasonable agreement. To the extent that the relative atomic mass of the electron $A_r(e)$ is known, the more recent work on $^{12}C^{5+}$ and $^{16}O^{7+}$ provides a consistency check through the comparison of the experimental and theoretical values of $g_{e^-}(^{12}C^{5+})$ and $g_{e^-}(^{16}O^{7+})$, as discussed in Sec. III.C.3.

APPENDIX E: THEORY OF MUONIUM GROUND-STATE HYPERFINE SPLITTING

This appendix gives a brief summary of the present theory of $\Delta\nu_{Mu}$, the ground-state hyperfine splitting of muonium (μ^+e^- atom), with a focus on results that have become available since the preparation of CODATA-98. Although a complete presentation of the theory is given here, references to the original literature for results included in CODATA-98 are generally not repeated.

The dominant part of the splitting is given by the Fermi formula:

$$\Delta\nu_F = \frac{16}{3}cR_\infty Z^3\alpha^2\frac{m_e}{m_\mu}\left[1+\frac{m_e}{m_\mu}\right]^{-3}. \tag{E1}$$

(Note that although the charge of the muon is e, some of the expressions in this appendix correspond to a muon with charge Ze in order to indicate the nature of various terms.) The full theoretical expression may be written as

$$\Delta\nu_{Mu}(\mathrm{th}) = \Delta\nu_D + \Delta\nu_{rad} + \Delta\nu_{rec} + \Delta\nu_{r\text{-}r} + \Delta\nu_{weak} + \Delta\nu_{had}, \tag{E2}$$

where the terms labeled D, rad, rec, r-r, weak, and had account for the Dirac (relativistic), radiative, recoil, radiative-recoil, electroweak, and hadronic (that is, strong-interaction) contributions to the hyperfine splitting, respectively.

The contribution $\Delta\nu_D$, given by the Dirac equation, is

$$\Delta\nu_D = \Delta\nu_F(1+a_\mu)\left[1+\frac{3}{2}(Z\alpha)^2+\frac{17}{8}(Z\alpha)^4+\cdots\right], \tag{E3}$$

where a_μ is the muon magnetic moment anomaly.

The radiative corrections are of the form

$$\Delta\nu_{rad} = \Delta\nu_F(1+a_\mu)\left[D^{(2)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)+D^{(4)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)^2+D^{(6)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)^3+\cdots\right], \tag{E4}$$

where the functions $D^{(2n)}(Z\alpha)$ are contributions associated with n virtual photons. The functions $D^{(2n)}(Z\alpha)$ are as follows:

$$D^{(2)}(Z\alpha) = A_1^{(2)} + \left(\ln 2-\frac{5}{2}\right)\pi Z\alpha + \left[-\frac{2}{3}\ln^2(Z\alpha)^{-2} + \left(\frac{281}{360}-\frac{8}{3}\ln 2\right)\ln(Z\alpha)^{-2} + 16.9037\ldots\right]\times(Z\alpha)^2 + \left[\left(\frac{5}{2}\ln 2-\frac{547}{96}\right)\ln(Z\alpha)^{-2}\right]\pi(Z\alpha)^3 + G(Z\alpha)(Z\alpha)^3, \tag{E5}$$

where $A_1^{(2)}=\frac{1}{2}$, as given in Appendix B. The number 16.9037… includes both self-energy and vacuum polarization contributions. The self-energy contribution has been calculated to high accuracy by Pachucki (1996), who obtained the result 17.1223…. The value 17.1227(11) subsequently reported by Nio and Kinoshita (1997), based on a nonrelativistic QED method, is in agreement with the earlier value. [The apparent disagreement of separate pieces of the two calculations noted by Nio and Kinoshita (1997) is due to the fact that in making the transformation from the regularization scheme actually used in his calculation to that presented in his printed paper, Pachucki omitted a term in the low-energy part and the negative of that term in the high-energy part, with no effect on the total (Pachucki, 2002)].

The function $G(Z\alpha)$ accounts for all higher-order contributions in powers of $Z\alpha$, and can be divided into parts that correspond to a self-energy Feynman diagram and a vacuum polarization diagram, $G(Z\alpha)=G_{SE}(Z\alpha)+G_{VP}(Z\alpha)$. Blundell *et al.* (1997a) have obtained $G_{SE}(\alpha)=-12.0(2.0)$ by extrapolating numerical results for higher Z to $Z=1$. Nio has obtained a preliminary result for the fifth and last contribution to the value of $G_{SE}(Z\alpha)$ at $Z=0$ using a nonrelativistic QED approach yielding a total result of $G_{SE}(0)=-15.9(1.6)$ (Nio, 2001, 2002). More recently, Yerokhin and Shabaev (2001) have carried out a numerical calculation and extrapolation to low Z of the self-energy contribution similar to that of Blundell *et al.* (1997a), with the result $G_{SE}(\alpha)=-14.3(1.1)$. In view of the differences among these three results and the uncertainty in the extrapolation procedure, we adopt the value

$$G_{SE}(\alpha) = -14(2), \tag{E6}$$

which is the simple mean and standard deviation of the three values.

The vacuum polarization part $G_{VP}(Z\alpha)$ has been calculated to several orders of $Z\alpha$ by Karshenboim *et al.* (1999, 2000). Their result corresponds to

$$G_{\rm VP}(\alpha) = 7.227(9), \tag{E7}$$

where the uncertainty is meant to account for neglected higher-order Uehling-potential terms; it corresponds to less than 0.1 Hz, and is thus entirely negligible.

For $D^{(4)}(Z\alpha)$ we have

$$D^{(4)}(Z\alpha) = A_1^{(4)} + 0.7717(4)\pi Z\alpha + \left[-\frac{1}{3}\ln^2(Z\alpha)^{-2} - 0.6390\ldots\ln(Z\alpha)^{-2} + 10(2.5)\right](Z\alpha)^2 + \cdots, \tag{E8}$$

where $A_1^{(4)}$ is given in Appendix B. The uncertainty 0.0004 in the second term of Eq. (E8), which arises from numerical integration, corresponds to less than 0.3 Hz and is negligible. In Eq. (E8), the coefficient of the term $\ln(Z\alpha)^{-2}$, calculated analytically, and the estimate of the constant term are due to Nio (1995) and Kinoshita and Nio (1998). This equation differs from the analogous Eq. (D6) in CODATA-98, because we have moved terms included there as reduced mass effects to the radiative recoil category $\Delta\nu_{\rm r-r}$. This is consistent with the classification of terms given by Eides *et al.* (2001b) in their comprehensive review of the theory of light hydrogenlike atoms. Finally,

$$D^{(6)}(Z\alpha) = A_1^{(6)} + \cdots, \tag{E9}$$

where only the leading contribution is given for the sixth-order term, because no binding correction has yet been calculated ($A_1^{(6)}$ is given in Appendix B). Higher-order functions $D^{(2n)}(Z\alpha)$ with $n>3$ are expected to be negligible.

The recoil contribution is given by

$$\Delta\nu_{\rm rec} = \Delta\nu_{\rm F}\frac{m_e}{m_\mu}\left(-\frac{3}{1-(m_e/m_\mu)^2}\ln\left(\frac{m_\mu}{m_e}\right)\frac{Z\alpha}{\pi} + \frac{1}{(1+m_e/m_\mu)^2}\left\{\ln(Z\alpha)^{-2} - 8\ln 2 + \frac{65}{18} + \left[\frac{9}{2\pi^2}\ln^2\left(\frac{m_\mu}{m_e}\right) + \left(\frac{27}{2\pi^2}-1\right)\ln\left(\frac{m_\mu}{m_e}\right) + \frac{93}{4\pi^2} + \frac{33\zeta(3)}{\pi^2} - \frac{13}{12} - 12\ln 2\right]\frac{m_e}{m_\mu}\right\}(Z\alpha)^2 + \left\{-\frac{3}{2}\ln\left(\frac{m_\mu}{m_e}\right)\ln(Z\alpha)^{-2} - \frac{1}{6}\ln^2(Z\alpha)^{-2} + \left(\frac{101}{18} - 10\ln 2\right)\ln(Z\alpha)^{-2} + 40(10)\right\}\frac{(Z\alpha)^3}{\pi}\right) + \cdots. \tag{E10}$$

This equation is an updated version of Eq. (D8) in CODATA-98. The new terms in square brackets are given by Blokland *et al.* (2002b) and are in agreement with the numerical calculation of Pachucki (1997, 2002). [The relevance of the work of Pachucki (1997) was noted by Hill (2001). The terms of higher order in m_e/m_μ given by Blokland *et al.* (2002b) contribute less than 1 Hz and are omitted here.] The number −57(22) in Eq. (D8) of CODATA-98 arose from a partial calculation of both a term proportional to $\ln(Z\alpha)^{-2}$ and a constant term (Nio, 1995; Kinoshita and Nio, 1998). Here the coefficient of the log-term contribution to that number has been replaced by $(101/18-10\ln 2)$, which was calculated independently by Hill (2001) and by Melnikov and Yelkhovsky (2001). This replacement leads to a significant increase of 223 Hz in the theoretical value of the hyperfine frequency $\Delta\nu_{\rm Mu}$. The estimate of the constant term 40(10) by Nio (1995) and Kinoshita and Nio (1998) has been retained. The term proportional to $\ln^2(Z\alpha)^{-2}$ in the above equation and the similar term in Eq. (E5) have been confirmed by Manohar and Stewart (2000).

The radiative-recoil contribution is

$$\Delta\nu_{\rm r-r} = \Delta\nu_{\rm F}\left(\frac{\alpha}{\pi}\right)^2\frac{m_e}{m_\mu}\left\{\left[-2\ln^2\left(\frac{m_\mu}{m_e}\right) + \frac{13}{12}\ln\left(\frac{m_\mu}{m_e}\right) + \frac{21}{2}\zeta(3) + \frac{\pi^2}{6} + \frac{35}{9}\right] + \left[\frac{4}{3}\ln^2\alpha^{-2} + \left(\frac{16}{3}\ln 2 - \frac{341}{180}\right)\ln\alpha^{-2} - 40(10)\right]\pi\alpha + \left[-\frac{4}{3}\ln^3\left(\frac{m_\mu}{m_e}\right) + \frac{4}{3}\ln^2\left(\frac{m_\mu}{m_e}\right)\right]\frac{\alpha}{\pi}\right\} - \Delta\nu_{\rm F}\alpha^2\left(\frac{m_e}{m_\mu}\right)^2 \times\left(6\ln 2 + \frac{13}{6}\right) + \cdots, \tag{E11}$$

where, for simplicity, the explicit dependence on Z is not shown. This equation differs in the following ways from the earlier version of the radiative-recoil correction given by Eq. (D9) in CODATA-98. The term proportional to $\ln\alpha^{-2}$ has been added, based on independent calculations by Hill (2001) and by Melnikov and Yelkhovsky (2001). It replaces the partial result of Nio (1995) and Kinoshita and Nio (1998) that was included as a reduced-mass correction in $\Delta\nu_{\rm rad}$ and contributed to the number −86(18) in Eq. (D6) of CODATA-98; the replacement leads to a net change in the theoretical value of the hyperfine frequency of +55 Hz. The term proportional to −40(10) in Eq. (E11) is due to Nio (1995) and Kinoshita and Nio (1998). It was also previously included in the number −86(18). Finally, the number 43.1 in Eq. (D9) of CODATA-98 (corresponding to 12 Hz), a partial result for the $\ln(m_\mu/m_e)$ and constant terms (Li *et al.*. 1993), is omitted here, because there are many uncalculated terms of the same order; the corresponding uncertainty is included in the estimate of uncalculated terms given below.

The electroweak contribution due to the exchange of a Z^0 boson is (Eides, 1996)

$$\Delta\nu_{\rm weak} = -65\ {\rm Hz}. \tag{E12}$$

The hadronic vacuum polarization contribution used in CODATA-98 was $\Delta\nu_{\rm had} = 240(7)$ Hz, as given by Faus-

tov *et al.* (1999). Since then a number of new calculations have been completed. Here we use the result of Eidelman *et al.* (2002),

$$\Delta\nu_{\rm had} = 236(4)\ {\rm Hz}, \tag{E13}$$

which takes into account new experimental data on the cross section for $e^-e^+ \to \pi^+\pi^-$ and on the ϕ meson leptonic width. It is a refinement of the result of Czarnecki *et al.* (2002), includes a higher-order hadronic contribution of 5(2) Hz, and is based on a convincing analysis. A comparable result, $\Delta\nu_{\rm had}=244(7)$ Hz, has been given by Faustov and Martynenko (2002a, 2002b). Narison (2002) has obtained the lowest-order result $\Delta\nu_{\rm had}=232.5(3.2)$ Hz, which is consistent with the other results if the higher-order hadronic contribution is added.

The standard uncertainty of $\Delta\nu_{\rm Mu}$(th), not including the uncertainties of the quantities R_∞, α, $m_{\rm e}/m_\mu$, and a_μ, consists of the following components:

$\Delta\nu_{\rm rad}$: 8 Hz $[0.2\times10^{-8}]$ due to the uncertainty 2 of $G_{\rm SE}(\alpha)$ in the function $D^{(2)}(Z\alpha)$; 6 Hz $[0.1\times10^{-8}]$ due to the uncalculated Wichmann-Kroll contribution of order $\alpha(Z\alpha)^3$, assuming a probable error of order 1 in $G_{\rm VP}$ [Czarnecki *et al.* (2002) give a somewhat larger estimate for this uncertainty component as well as for others]; 3 Hz $[0.1\times10^{-8}]$ from the uncertainty 2.5 of the number 10 in the function $D^{(4)}(Z\alpha)$.

$\Delta\nu_{\rm rec}$: 53 Hz $[1.2\times10^{-8}]$ due to 2 times the uncertainty 10 of the number 40 (the factor of 2 is added, because the estimate of the uncertainty 10 is linked to a calculation that has been found to be problematic); 51 Hz $[1.1\times10^{-8}]$ due to a recoil correction of order $\Delta\nu_{\rm F}(m_{\rm e}/m_\mu)\times(Z\alpha)^3\ln(m_\mu/m_{\rm e})$ suggested by the partial calculation of Hill (2001); 9 Hz $[0.2\times10^{-8}]$ to reflect a possible uncalculated recoil contribution of order $\Delta\nu_{\rm F}(m_{\rm e}/m_\mu)\times(Z\alpha)^4\ln^2(Z\alpha)^{-2}$.

$\Delta\nu_{\rm r-r}$: 53 Hz $[1.2\times10^{-8}]$ due to 2 times the uncertainty 10 of the number −40 (as above); 41 Hz $[0.9\times10^{-8}]$ to reflect a possible uncalculated radiative-recoil contribution of order $\Delta\nu_{\rm F}(m_{\rm e}/m_\mu)(\alpha/\pi)^3\ln(m_\mu/m_{\rm e})$ and nonlogarithmic terms, based on the partial calculations of Li *et al.* (1993) and Eides *et al.* (2002, 2003).

$\Delta\nu_{\rm had}$: 4 Hz $[0.1\times10^{-8}]$.

Note that the uncertainties arising from the uncalculated terms are standard uncertainties based on hypothetical numerical coefficients suggested by analogous calculated terms in $\Delta\nu_{\rm Mu}$(th). These terms are taken to be probable errors (approximately 50% confidence level) and are multiplied by the factor 1.48 to convert them to standard uncertainties. Any contribution to $\Delta\nu_{\rm Mu}$(th) not explicitly included in Eqs. (E3)–(E13) or reflected in the uncertainty evaluation is either known to be or is assumed to be less than about 5 Hz $[0.1\times10^{-8}]$, and therefore negligible at the level of uncertainty of current interest.

Combining the above components of uncertainty, we obtain for the standard uncertainty of the theoretical expression for the muonium hyperfine splitting $\Delta\nu_{\rm Mu}$(th),

$$u[\Delta\nu_{\rm Mu}({\rm th})] = 101\ {\rm Hz} \quad [2.3\times10^{-8}], \tag{E14}$$

compared with the CODATA-98 value of 123 Hz. For the least-squares calculations, we use as the theoretical expression for the hyperfine splitting,

$$\Delta\nu_{\rm Mu}\left(R_\infty, \alpha, \frac{m_{\rm e}}{m_\mu}, \delta_\mu, \delta_{\rm Mu}\right) = \Delta\nu_{\rm Mu}({\rm th}) + \delta_{\rm Mu}, \tag{E15}$$

where $\delta_{\rm Mu}$ is assigned, *a priori*, the value

$$\delta_{\rm Mu} = 0(101)\ {\rm Hz} \tag{E16}$$

in order to account for the uncertainty of the theoretical expression.

The theory summarized above predicts

$$\Delta\nu_{\rm Mu} = 4\,463\,302\,905(272)\ {\rm Hz} \quad [6.1\times10^{-8}], \tag{E17}$$

based on values of the constants obtained from a variation of the 2002 least-squares adjustment that omits as input data the two LAMPF measured values of $\Delta\nu_{\rm Mu}$. The main source of uncertainty is the mass ratio $m_{\rm e}/m_\mu$ that appears in the theoretical expression as an overall factor. [See the text following Eq. (D14) of Appendix D of CODATA-98 for an explanation of why the relative uncertainty of the predicted value of $\Delta\nu_{\rm Mu}$ in Eq. (E17) is smaller than the relative uncertainty in the electron-muon mass ratio as given in Eq. (83) of Sec. III.C.5.c.] Although this result is 238 Hz larger than the corresponding 1998 result, both are in agreement with the two experimental values of $\Delta\nu_{\rm Mu}$.

APPENDIX F: TEST OF THE JOSEPHSON AND QUANTUM HALL EFFECT RELATIONS

As discussed in Sec. IV.B, the inconsistencies among certain input data, in particular, the disagreement between the WGAC consensus value of the molar volume of silicon $V_{\rm m}$(Si) and the two moving-coil watt-balance results for the product $K_{\rm J}^2R_{\rm K}$, where $K_{\rm J}$ is the Josephson constant and $R_{\rm K}$ is the von Klitzing constant, and also between $V_{\rm m}$(Si) and a Hg-electrometer result for $K_{\rm J}$ and a voltage-balance result for the same constant, naturally lead one to consider whether relaxing the assumptions $K_{\rm J}=2e/h$ and $R_{\rm K}=h/e^2$ would reduce or possibly even eliminate the inconsistencies. Although theory and experiment strongly support the validity of these relations (see Secs. II.D and II.E), nevertheless, our analysis of the input data for the 2002 adjustment would be incomplete without an investigation of this possibility.

To this end, in this appendix we carry out a study, which is reminiscent of a similar study reported a number of years ago (Taylor and Cohen, 1991), of the effect of taking $K_{\rm J}$ and $R_{\rm K}$ to be simply phenomenological constants that appear in the basic Josephson and quantum Hall effect expressions $U_{\rm J}(n)=nf/K_{\rm J}$ and $R_{\rm H}(i)=R_{\rm K}/i$ (see Secs. II.D and II.E). This may readily be done by writing

TABLE XLII. Generalized observational equations that express input data *B*26.1 to *B*32 in Table XIII as functions of the adjusted constants in Table XX with the additional adjusted constants ε_J and ε_K as given in Eqs. (F1) and (F2). The numbers in the first column correspond to the numbers in the first column of Table XIII. For simplicity, the lengthier functions are not explicitly given. See Sec. IV.B for an explanation of the symbol $\doteq$.

Type of input datum	Generalized observational equation
*B*26*	$\Gamma'_{p-90}(\mathrm{lo}) \doteq -\dfrac{K_{J-90}R_{K-90}[1+a_e(\alpha,\delta_e)]\alpha^3}{2\mu_0 R_\infty(1+\varepsilon_J)(1+\varepsilon_K)}\left(\dfrac{\mu_{e^-}}{\mu'_p}\right)^{-1}$
*B*27*	$\Gamma'_{p-90}(\mathrm{hi}) \doteq -\dfrac{c[1+a_e(\alpha,\delta_e)]\alpha^2}{K_{J-90}R_{K-90}R_\infty h}(1+\varepsilon_J)(1+\varepsilon_K)\left(\dfrac{\mu_{e^-}}{\mu'_p}\right)^{-1}$
*B*28*	$\Gamma'_{h-90}(\mathrm{lo}) \doteq \dfrac{K_{J-90}R_{K-90}[1+a_e(\alpha,\delta_e)]\alpha^3}{2\mu_0 R_\infty(1+\varepsilon_J)(1+\varepsilon_K)}\left(\dfrac{\mu_{e^-}}{\mu'_p}\right)^{-1}\dfrac{\mu'_h}{\mu'_p}$
*B*29*	$K_J \doteq \left(\dfrac{8\alpha}{\mu_0 ch}\right)^{1/2}(1+\varepsilon_J)$
*B*30*	$R_K \doteq \dfrac{\mu_0 c}{2\alpha}(1+\varepsilon_K)$
*B*31*	$K_J^2 R_K \doteq \dfrac{4}{h}(1+\varepsilon_J)^2(1+\varepsilon_K)$
*B*32*	$\mathcal{F}_{90} \doteq \dfrac{cM_u A_r(e)\alpha^2}{K_{J-90}R_{K-90}R_\infty h}(1+\varepsilon_J)(1+\varepsilon_K)$
*B*52*	$\varepsilon_J \doteq \varepsilon_J$
*B*53*	$\varepsilon_K \doteq \varepsilon_K$

$$K_J = \frac{2e}{h}(1+\varepsilon_J) = \left(\frac{8\alpha}{\mu_0 ch}\right)^{1/2}(1+\varepsilon_J), \tag{F1}$$

$$R_K = \frac{h}{e^2}(1+\varepsilon_K) = \frac{\mu_0 c}{2\alpha}(1+\varepsilon_K), \tag{F2}$$

where ε_J and ε_K are unknown correction factors to be taken as additional adjusted constants. Their initial input values are assigned the value zero but with a sufficiently large uncertainty that their output values resulting from a least-squares adjustment are in fact determined by the other input data and not by their initial values. (Adjustments may, of course, also be carried out in which only one of the relations is relaxed.) If it turns out that the adjusted values of ε_J and ε_K are statistically compatible with zero, then one can conclude that there is no experimental evidence that the relations $K_J = 2e/h$ and $R_K = h/e^2$ are invalid. If, on the other hand, either the adjusted value of ε_J or of ε_K were found to differ from zero in a statistically significant way, then doubt about the exactness of the associated relation would be engendered.

The observational equations in Table XXI that are altered as a result of relaxing the assumptions $K_J = 2e/h$ and $R_K = h/e^2$ are *B*26 to *B*32. Based on Eqs. (F1) and (F2), they take the form given in Table XLII, where we have used an asterisk to differentiate between these new observational equations and their counterparts in Table XXI, which are based on the assumptions $K_J = 2e/h$ and $R_K = h/e^2$.

We have carried out a number of least-squares adjustments starting with all 112 items of input data given in Tables XI and XIII, plus the initial input values of ε_J and ε_K assigned as described above, for a total of 114, and using the 61 adjusted constants of Tables XVIII and XX, plus ε_J and ε_K, for a total of 63. (The correlation coefficients in Tables XII and XIV are, of course, also taken into account.) The results of these adjustments are summarized in Table XLIII, in which we give, in addition to the adjusted values of α, h, ε_K, and ε_J, the normalized residuals r_i of the four input data with the largest values of $|r_i|$, each of which initially exceeds 1.50 (see Table XXIII): the VNIIM-89 result for $\Gamma'_{h-90}(\mathrm{lo})$, item *B*28.2; the N/P/I-03 result for $V_m(\mathrm{Si})$, item *B*46; the NIST-89 result for $\Gamma'_{p-90}(\mathrm{lo})$, item *B*26.1; and the KR/VN-98 result for $\Gamma'_{h-90}(\mathrm{lo})$, item *B*28.1. These are included as additional indicators of whether relaxing the assumptions $K_J = 2e/h$ and/or $R_K = h/e^2$ reduce the inconsistencies among the data.

The adjusted value of R_∞ is not included in Table XLIII, because it remains essentially unchanged from one adjustment to the next and equal to the 2002 recommended value. For adjustments (i)–(iv), the number of input data is $N = 114$, the number of adjusted constants is

TABLE XLIII. Summary of the results of several least-squares adjustments carried out to investigate the effect of assuming the relations for K_J and R_K given in Eqs. (F1) and (F2). The values of α, h, ε_K, and ε_J are those obtained in the indicated adjustments. The quantity $R_B=\sqrt{\chi^2/\nu}$ is the Birge ratio and r_i is the normalized residual of the indicated input datum (see Table XIII). These four data have the largest $|r_i|$ of all the input data and are the only data in Adjustment (i) with $|r_i|>1.50$.

Adj.	R_B	α^{-1}	h/(J s)	ε_K	ε_J	$r_{B28.2}$	r_{B46}	$r_{B26.1}$	$r_{B28.1}$
(i)	1.07	137.035 999 14(45)	$6.626\,069\,21(50)\times10^{-34}$	0(0)	0(0)	3.52	–3.18	2.19	1.68
(ii)	1.05	137.035 999 21(45)	$6.626\,0679(11)\times10^{-34}$	0(0)	$-108(80)\times10^{-9}$	3.22	–3.77	1.24	1.09
(iii)	1.06	137.035 999 05(46)	$6.626\,069\,32(51)\times10^{-34}$	$18(18)\times10^{-9}$	0(0)	3.56	–3.14	2.33	1.76
(iv)	1.04	137.035 999 11(46)	$6.626\,0678(11)\times10^{-34}$	$23(19)\times10^{-9}$	$-126(81)\times10^{-9}$	3.23	–3.82	1.27	1.11
(v)	0.96	137.035 999 18(45)	$6.626\,069\,21(50)\times10^{-34}$	0(0)	0(0)		–3.17	2.21	1.69
(vi)	0.96	137.035 999 22(45)	$6.626\,0686(11)\times10^{-34}$	0(0)	$-49(82)\times10^{-9}$		–3.44	1.78	1.43
(vii)	0.95	137.035 999 08(46)	$6.626\,069\,34(51)\times10^{-34}$	$21(18)\times10^{-9}$	0(0)		–3.13	2.37	1.79
(viii)	0.94	137.035 999 11(46)	$6.626\,0685(11)\times10^{-34}$	$24(19)\times10^{-9}$	$-67(83)\times10^{-9}$		–3.49	1.81	1.44

$M=63$, and the degrees of freedom is $\nu=N-M=51$; for adjustments (v)–(viii), $N=113$, $M=63$, and $\nu=50$. The values of χ^2 for the eight adjustments are 58.5, 56.7, 57.5, 55.1, 46.1, 45.7, 44.8, and 44.1. An entry 0(0) in the ε_K column means that it is assumed $R_K=h/e^2$ in the corresponding adjustment; similarly, an entry 0(0) in the ε_J column means that it is assumed $K_J=2e/h$ in the corresponding adjustment. The following comments apply to the adjustments of Table XLIII.

The initial adjustment, adjustment (i), is essentially identical to adjustment 1 of Tables XXII and XXIII in Sec. IV.B. Adjustment (ii) is obtained from adjustment (i) by relaxing the relation $K_J=2e/h$, adjustment (iii) is obtained from adjustment (i) by relaxing the relation $R_K=h/e^2$, and adjustment (iv) is obtained from adjustment (i) by relaxing both of these relations. Adjustments (v)–(viii) are obtained from adjustments (i)–(iv), respectively, by deleting input datum *B*28.2.

The results of these eight adjustments show that there is no statistically significant evidence that ε_J and ε_K are not equal to zero, and hence that the relations $K_J=2e/h$ and $R_K=h/e^2$ are not exact. There is also no dramatic reduction in the absolute values of the normalized residuals of items *B*28.2 and *B*46, thereby indicating that the significant disagreement of these data with the other input data cannot be explained by the assumptions $K_J=2e/h$ and $R_K=h/e^2$ being invalid. We do note the modest reduction in the normalized residuals of items *B*26.1 and *B*28.1 in those four adjustments in which the relation $R_K=h/e^2$ is relaxed, but since the size of these residuals in the four adjustments in which this relation is not relaxed are viewed as being in the acceptable range, this reduction is not of great significance.

It should be mentioned that in this exercise, all data relevant to the determination of the value of the Planck constant h are included in each adjustment. This is a necessary condition for the results to be meaningful, because if enough data were excluded, we would be certain to have small normalized residuals and obtain predictions for nonzero values of ε_J and ε_K.

We also note the comparatively narrow range in which the adjusted values of α lie and their closeness to the 2002 recommended value, $\alpha^{-1}=137.035\,999\,11(46)$. It can be explained by the fact that the input data that play the dominant role in the determination of α, namely the experimental result and theoretical expression for the electron magnetic moment anomaly a_e and the experimental result for h/m(Cs), do not depend on the Josephson and quantum Hall effects. This is, of course, not the case for the data that play the principal role in the determination of the Planck constant h, hence the adjusted values of h vary over a much wider range. However, it is interesting to note that these values of h are not inconsistent with the 2002 recommended value, $h=6.626\,0693(11)\times10^{-34}$ J s.

We conclude this appendix by recalling that historically, a "Josephson value of α" was calculated from a measurement of K_J in terms of a laboratory unit of voltage V_{LAB} (assuming that $K_J=2e/h$), a measured value of the proton gyromagnetic ratio-related quantity Γ'_p(lo) expressed in terms of the same unit of voltage, and a measurement in ohms of the laboratory unit of resistance Ω_{LAB} in terms of which Γ'_p(lo) is also expressed (Taylor *et al.*, 1969). Such a calculation may now be carried out as well using a measured value of Γ'_h(lo), the counterpart quantity for the helion (nucleus of the ^{3}He atom). The current relevant expressions are

$$\alpha^{-1}[\text{Jos-p}]=\left[\frac{K_{J-90}R_{K-90}g_e c}{8R_K R_\infty \Gamma'_{p-90}(\text{lo})}\left(\frac{\mu_e}{\mu'_p}\right)^{-1}\right]^{1/2}, \tag{F3}$$

$$\alpha^{-1}[\text{Jos-h}]=\left[-\frac{K_{J-90}R_{K-90}g_e c}{8R_K R_\infty \Gamma'_{h-90}(\text{lo})}\left(\frac{\mu_e}{\mu'_p}\right)^{-1}\left(\frac{\mu'_h}{\mu'_p}\right)\right]^{1/2}, \tag{F4}$$

where $g_e=-2(1+a_e)$ is the electron g-factor and R_K is the von Klitzing constant, but here it is not assumed to be equal to $h/e^2=\mu_0 c/2\alpha$ but simply a phenomenological constant measured in ohms. As before, K_{J-90} and

R_{K-90} are the conventional values of the Josephson and von Klitzing constants and the subscript 90 on $\Gamma'_p(\text{lo})$ and $\Gamma'_h(\text{lo})$ indicate that these quantities are measured in the conventional electric units established by the adoption of the conventional values of K_J and R_K (see Sec. II.F).

Evaluation of these expressions with the 2002 recommended values of g_e, R_∞, μ_e/μ'_p, and μ'_h/μ'_p, the uncertainties of which are inconsequential in this context, the value $R_K = 25\,812.808\,18(47)\ \Omega\ [1.8\times10^{-8}]$, which is the weighted-mean value given in Eq. (200) of Sec. IV.A, and the weighted mean of the NIST-89 and NIM-95 results for $\Gamma'_{p-90}(\text{lo})$, items *B*26.1 and *B*26.2 of Table XIII, or the KR/VN-98 result for $\Gamma'_{h-90}(\text{lo})$, item *B*28.1 of the same table, yields

$$\alpha^{-1}[\text{Jos-p:NIST-89/NIM-95}] = 137.035\,9810(76) \quad [5.6\times10^{-8}], \tag{F5}$$

$$\alpha^{-1}[\text{Jos-h:KR/VN-98}] = 137.035\,976(12) \quad [9.0\times10^{-8}]. \tag{F6}$$

[The other value of $\Gamma'_{h-90}(\text{lo})$ in that table item *B*28.2, is omitted from the calculation, because it has been shown in Sec. IV that it is highly discrepant.] The combined value of Eqs. (F5) and (F6), which we recommend for use whenever a "Josephson value of α" is required for comparison purposes, is

$$\alpha^{-1}[\text{Jos}] = 137.035\,9798(66) \quad [4.8\times10^{-8}]. \tag{F7}$$

This value is smaller than the value $\alpha^{-1}[a_e]$ given in Eq. (35) of Sec. III.C.1.b, which has the smallest uncertainty of any value of α, by $2.9u_{\text{diff}}$, where as before, u_{diff} is the combined standard certainty of the difference.

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