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The fundamental constants and their variation: observational and theoretical status

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This article describes the various experimental bounds on the variation of the fundamental constants of nature. After a discussion of the role of fundamental constants, their definition and link with metrology, it reviews the various constraints on the variation of the fine-structure constant, the gravitational, weak- and strong-interaction couplings and the electron-to-proton mass ratio. The review aims (1) to provide the basics of each measurement, (2) to show as clearly as possible why it constrains a given constant, and (3) to point out the underlying hypotheses. Such an investigation is of importance in comparing the different results and in understanding the recent claims of the detection of a variation of the fine-structure constant and of the electron-to-proton mass ratio in quasar absorption spectra. The theoretical models leading to the prediction of such variation are also reviewed, including Kaluza-Klein theories, string theories, and other alternative theories. Cosmological implications of these results are also discussed. The links with the tests of general relativity are emphasized.

CONTENTS

I. INTRODUCTION

The development of physics relied considerably on the Copernican principle, which states that we are not living in a particular place in the universe and that the laws of physics do not differ from one point in spacetime to another. This contrasts with the Aristotelian point of view, in which the laws on Earth and in heavens differ. It is, however, natural to question this assumption. Indeed, it is difficult to imagine a change in the form of physical laws (e.g., a Newtonian gravitation force behaving on Earth as the inverse of the square of the distance and, somewhere else as another power). A smooth change in the physical constants is much easier to conceive.

Comparing and reproducing experiments is also at the foundation of the scientific approach, which makes sense only if the laws of nature do not depend on time and space. This hypothesis of the constancy of the constants plays an important role in astronomy and cosmology, in

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particular with respect to the look-back time measured by the redshift. Ignoring the possibility of varying constants could lead to a distorted view of our universe and, if such a variation were established, corrections would have to be applied. It is thus important to investigate this possibility, especially as the measurements become more and more precise. Obviously, the constants have not undergone huge variations on Solar System scales and geological time scales, and one is looking for tiny effects. The values of the constants are also central to physics. One might hope to explain them dynamically, as predicted by some high-energy theories. Testing for the constancy of the constants is thus part of testing general relativity. It is analogous to leaving behind the Newtonian description of mechanics, in which space and time were just a static background for the evolution of matter, and adopting the relativistic description, in which spacetime becomes a dynamical quantity determined by the Einstein equations (Damour, 2001).

Before discussing the properties of the constants of nature, we must have an idea of which constants to consider. Some physical constants play more important roles than others. Following Levy-Leblond (1979), we can define three classes of fundamental constants, *class A* being the class characteristic of particular objects, *class B* being the class characteristic of a class of physical phenomena, and *class C* being the class of universal constants. Indeed, the status of a constant can change with time. For instance, the velocity of light was initially a type-A constant (describing a property of light) then became a type-B constant when it was realized that the velocity of light was related to electromagnetic phenomena, and, ended up as a type-C constant (entering many laws of physics from electromagnetism to relativity, including the notion of causality). It has even become a much more fundamental constant since it enters in the definition of the meter (Petley, 1983). A more conservative definition of a fundamental constant would thus be that it is *any parameter* that cannot be calculated with our present knowledge of physics, i.e., a free parameter of a current theory. Each free parameter of a theory is in fact a challenge to future theories, to explain its value.

How many fundamental constants should we consider? The set of constants that are conventionally considered as fundamental (Flowers and Petley, 2001) consists of the electron charge e, the electron mass m_e, the proton mass m_p, the reduced Planck constant $\hbar$, the velocity of light in vacuum c, the Avogadro constant N_A, the Boltzmann constant k_B, the Newton constant G, and the permeability and permittivity of space, ε_0 and μ_0. The latter has a fixed value in the SI system of units ($\mu_0 = 4\pi\times10^{-7}$ H m^{-1}) that is implicit in the definition of the ampere; ε_0 is then fixed by the relation $\varepsilon_0\mu_0 = c^{-2}$. The inclusion of N_A in the former list has been much debated (see, for example, Birge, 1929). In comparision, the minimal standard model of particle physics plus gravitation that describes the four known interactions depends on 20 free parameters (Cahn, 1996; Hogan, 2000): the Yukawa coefficients determining the masses of the six quark (u,d,c,s,t,b) and three lepton (e,μ,τ) flavors, the Higgs mass and vacuum expectation value, three angles and a phase of the Cabibbo-Kobayashi-Maskawa matrix, a phase for the QCD vacuum and three coupling constants g_S, g_W, g_1 for the gauge group $SU(3)\times SU(2)\times U(1)$, respectively. Below the Z mass, g_1 and g_W combine to form the electromagnetic coupling constant

$$g_{EM}^{-2} = \frac{5}{3} g_1^{-2} + g_W^{-2}. \tag{1}$$

The number of free parameters indeed depends on the physical model at hand (see Weinberg, 1983a). This issue has to be disconnected from the number of required fundamental dimensionful constants. Duff, Okun and Veneziano (2002) recently debated this question, respectively, arguing for none, three, and two (see also Wignall, 2000). Arguing for no fundamental constants leads one to consider them simply as conversion parameters. Some of them are, like the Boltzmann constant, but others play a deeper role in the sense that when a physical quantity becomes of the same order as this constant, new phenomena appear; this is the case, for example, of $\hbar$ and c that are associated, respectively, with quantum and relativistic effects. Okun (1991) considered that only three fundamental constants are necessary, the underlying reason being that, in the international system of units which has seven base units and 17 derived units, four of the seven base units are in fact derived (ampere, kelvin, mole, and candela). The three remaining base units (meter, second, and kilogram) are then associated with three fundamental constants (c, $\hbar$, and G). They can be seen as limiting quantities: c is associated with the maximum velocity and $\hbar$ with the unit quantum of angular momentum and sets a minimum of uncertainty, whereas G is not directly associated with any physical quantity [see Martins (2002), who argues that G is the limiting potential for a mass that does not form a black hole]. In the framework of quantum field theory + general relativity, it seems that this set of three constants has to be considered, and it allows us to classify the physical theories (see Fig. 1). However, Veneziano (1986) argued that, in the framework of string theory, one requires only two dimensionful fundamental constants, c and the string length λ_s. The use of $\hbar$ seems unnecessary since it combines with the string tension to give λ_s. In the case of the Goto-Nambu action $S/\hbar = (T/\hbar)\int d(\text{Area}) \equiv \lambda_s^{-2}\int d(\text{Area})$, and the Planck constant is just given by λ_s^{-2}. In this view, $\hbar$ has not disappeared but has been promoted to the role of a UV cutoff that removes both the infinities of quantum-field theory and singularities of general relativity. This situation is analogous to that of pure quantum gravity (Novikov and Zel'dovich, 1982) for which $\hbar$ and G never appear separately but only in the combination $\ell_{Pl} = \sqrt{G\hbar/c^3}$ so that only c and ℓ_{Pl} are needed. Volovik (2002) proposed an analogy with quantum liquids. There, an observer knows both the effective and microscopic physics so that he can judge whether the fundamental constants of the effective theory remain fundamental constants of the micro-

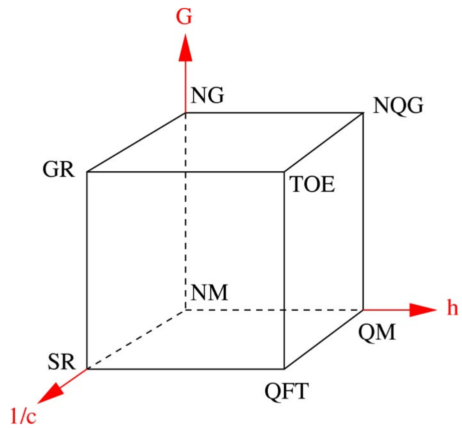

FIG. 1. The cube of physical theories as presented by Okun (1991). At the origin stands the part of Newtonian mechanics (NM) that does not take gravity into account. NG, QM, and SR then stand for Newtonian gravity, quantum mechanics, and special relativity, which, respectively, introduce the effect of one of the constants. Special relativity "merges," respectively, with quantum mechanics and Newtonian gravity to give quantum field theory (QFT) and general relativity (GR). Bringing quantum mechanics and Newtonian gravity together leads to nonrelativistic quantum gravity, and all theories together give the theory of everything (TOE). From Okun (1991) (Color in online edition).

scopic theory. The status of a constant depends on the considered theory (effective or microscopic) and, more interestingly, on the observer measuring them, i.e., on whether this observer belongs to the world of low-energy quasiparticles or to the microscopic world.

Resolving this issue is indeed far beyond the scope of this paper and can probably be considered more of an epistemological question than a physical one. But, as the discussion above shows, the answer depends on the theoretical framework considered [see also Cohen-Tannoudji (1995) for arguments in favor of considering the Boltzmann constant as a fundamental constant]. A more pragmatic approach is then to choose a theoretical framework so that the set of undetermined fixed parameters is fully known, and then to wonder why they have the values they have and whether they are constant.

We review in this paper both the status of experimental constraints on the variation of fundamental constants and the theoretical motivations for considering such variations. In Sec. II, we recall the argument of Dirac, who initiated the consideration of time-varying constants, and we briefly discuss how this is linked to anthropic arguments. Then, since the fundamental constants are entangled with the theory of measurement, we make some very general comments on the consequences of metrology. In Secs. III and IV, we review the observational constraints, respectively, on the variation of the fine structure and of gravitational constants. Indeed, we have to keep in mind that the obtained constraints depend on underlying assumptions about a certain set of other constants. We summarize briefly in Sec. V the constraints on other constants, and we give, in Sec. VI, some hints of the theoretical motivations arising mainly from grand unified theories, Kaluza-Klein, and string theories. We also discuss a number of cosmological models taking these variations into account. For recent shorter reviews, the reader is referred to Varshalovich *et al.* (2000a), Chiba (2001), Martins (2002), and Uzan (2002).

Notations: In this work, we use SI units and the following values of the fundamental constants today:[1]

$$c = 299\,792\,458\ \mathrm{m\,s^{-1}}, \tag{2}$$

$$\hbar = 1.054\,571\,596(82)\times 10^{-34}\ \mathrm{J\,s}, \tag{3}$$

$$G = 6.673(10)\times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}, \tag{4}$$

$$m_{\mathrm{e}} = 9.109\,381\,88(72)\times 10^{-31}\ \mathrm{kg}, \tag{5}$$

$$m_{\mathrm{p}} = 1.672\,621\,58(13)\times 10^{-27}\ \mathrm{kg}, \tag{6}$$

$$m_{\mathrm{n}} = 1.674\,927\,16(13)\times 10^{-27}\ \mathrm{kg}, \tag{7}$$

$$e = 1.602\,176\,462(63)\times 10^{-19}\ \mathrm{C} \tag{8}$$

for the velocity of light, the reduced Planck constant, the Newton constant, the masses of the electron, proton and neutron, and the charge of the electron. We also define

$$q^2 \equiv \frac{e^2}{4\pi\varepsilon_0} \tag{9}$$

and the following dimensionless ratios:

$$\alpha_{\mathrm{EM}} \equiv \frac{q^2}{\hbar c} \sim 1/137.035\,999\,76(50), \tag{10}$$

$$\alpha_{\mathrm{W}} \equiv \frac{G_{\mathrm{F}} m_{\mathrm{p}}^2 c}{\hbar^3} \sim 1.03\times 10^{-5}, \tag{11}$$

$$\alpha_{\mathrm{S}}(E) \equiv \frac{g_{\mathrm{s}}^2(E)}{\hbar c}, \tag{12}$$

$$\alpha_{\mathrm{G}} \equiv \frac{G m_{\mathrm{p}}^2}{\hbar c} \sim 5\times 10^{-39}, \tag{13}$$

$$\mu \equiv \frac{m_{\mathrm{e}}}{m_{\mathrm{p}}} \sim 5.446\,17\times 10^{-4}, \tag{14}$$

$$x \equiv g_{\mathrm{p}} \alpha_{\mathrm{EM}}^2 \mu \sim 1.62\times 10^{-7}, \tag{15}$$

$$y \equiv g_{\mathrm{p}} \alpha_{\mathrm{EM}}^2 \sim 2.977\times 10^{-4}, \tag{16}$$

which characterize, respectively, the strength of the electromagnetic, weak, strong, and gravitational forces, and the electron-proton mass ratio $g_{\mathrm{p}} \simeq 5.585$ is the proton gyromagnetic factor. Note that Eq. (12) extends between two quantities that depend strongly on energy; this will be discussed in more detail in Sec. V. We introduce the notations

$$a_0 = \frac{\hbar}{m_{\mathrm{e}} c\, \alpha_{\mathrm{EM}}} = 0.529\,177\,1\ \ \text{Å}, \tag{17}$$

$$-E_I = \frac{1}{2} m_{\mathrm{e}} c^2 \alpha_{\mathrm{EM}}^2 = 13.605\,80\ \mathrm{eV}, \tag{18}$$

[1]See http://physics.nist.gov/cuu/Constants/ for an up-to-date list of the recommended values of the constants of nature.

$$R_\infty = -\frac{E_I}{hc} = 1.097\,373\,156\,854\,9(83)\times 10^{7}\ \mathrm{m}^{-1}, \quad (19)$$

respectively, for the Bohr radius, the hydrogen ionization energy, and the Rydberg constant.

While working in cosmology, we assume that the universe is described by a Friedmann-Lemaître spacetime

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j, \quad (20)$$

where t is the cosmic time, a the scale factor, and γ_{ij} the metric of the spatial sections. We define the redshift as

$$1+z \equiv \frac{a_0}{a} = \frac{\nu_e}{\nu_0}, \quad (21)$$

where a_0 is the value of the scale factor today, while ν_e and ν_0 are, respectively, the frequencies at emission and today. We decompose the Hubble constant today as

$$H_0^{-1} = 9.7776\times 10^9 h^{-1}\ \mathrm{yr}, \quad (22)$$

where $h = 0.68\pm 0.15$ is a dimensionless number, and the density of the universe today is given by

$$\rho_0 = 1.879\times 10^{-26}\Omega h^2\ \mathrm{kg\,m}^{-3}. \quad (23)$$

II. GENERALITIES

A. From Dirac numerological principle to anthropic arguments

The question of the constancy of the constants of physics was probably first addressed by Dirac (1937, 1938, 1979) who expressed, in his "Large Numbers hypothesis," the opinion that very large (or small) dimensionless universal constants cannot be pure mathematical numbers and must not occur in the basic laws of physics. He suggested, on the basis of this numerological principle, that these large numbers should rather be considered as variable parameters characterizing the state of the universe. Dirac formed the five dimensionless ratios α_{EM}, α_{W}, α_{G}, $\delta \equiv H_0\hbar/m_{\mathrm{p}}c^2 \sim 2h\times 10^{-42}$, and $\epsilon \equiv G\rho_0/H_0^2 \sim 5h^{-2}\times 10^{-4}$, and he then asked the question of which of these ratios was constant as the universe evolved. Usually, only δ and ϵ vary as the inverse of the cosmic time (note that with the value of the density chosen by Dirac, the universe is not flat, so that $a\propto t$ and $\rho \propto t^{-3}$). Dirac then noticed that $\alpha_{\mathrm{G}}\mu/\alpha_{\mathrm{EM}}$, representing the relative magnitude of electrostatic and gravitational forces between a proton and an electron, was of the same order as $H_0 e^2/m_{\mathrm{e}}c^2 = \delta\alpha_{\mathrm{EM}}/\mu$, representing the age of the universe in atomic time, so that the five previous numbers can be "harmonized" if one assumes that α_{G} and δ vary with time and scale as the inverse of the cosmic time.[2] This implies that the intensity of all gravitational effects decrease with a rate of about $10^{-10}\ \mathrm{yr}^{-1}$.

[2]The ratio $\delta\alpha_{\mathrm{EM}}/\mu$ represents roughly the inverse of the number of times an electron orbits around a proton during the age of the universe. Already, this suggested a link between microphysics and cosmological scales.

Chandrasekhar (1937) and Kothari (1938) were the first to point out that some astronomical consequences of this statement may be detectable. Similar ideas were expressed by Milne (1935).

Dicke (1961) pointed out that in fact the density of the universe is determined by its age, this age being related to the time needed to form galaxies, stars, heavy nuclei, etc. This led him to formulate that the presence of an observer in the universe places constraints on the physical laws that can be observed. In fact, what is meant by observer is the existence of (highly?) organized systems, and the anthropic principle can be seen as a rephrasing of the question "why is the universe the way it is?" (Hogan, 2000). Carter (1974, 1976, 1983), who actually coined the term "anthropic principle," showed that the numerological coincidence found by Dirac can be derived from physical models of stars and the competition between the weakness of gravity and nuclear fusion. Carr and Rees (1979) then showed how one can scale up from atomic to cosmological scales only by using combinations of α_{EM}, α_{G}, and $m_{\mathrm{e}}/m_{\mathrm{p}}$.

The first implementation of Dirac's phenomenological idea into a field-theory framework (i.e., modifying Einstein gravity and incorporating nongravitational forces and matter) was proposed by Jordan (1937, 1939, 1955). He realized that the constants have to become dynamical fields and used the action

$$S = \int \sqrt{-g}\,d^4\mathbf{x}\,\phi^{\eta}\left[R - \xi\left(\frac{\nabla\phi}{\phi}\right)^2 - \frac{\phi}{2}F^2\right], \quad (24)$$

η and ξ being two parameters. Fierz (1956) realized that with such a Lagrangian, atomic spectra would be spacetime dependent, and he proposed to fix eta to the value -1 to prevent such a spacetime dependence. This led to the definition of a one-parameter (ξ) class of scalar-tensor theories, which has been further explored by Brans and Dicke (1961) (with the change of notation $\xi \to \omega$), who emphasized the connection with Mach's principle. In this Jordan-Fierz-Brans-Dicke theory, the gravitational constant is replaced by a scalar field that can vary both in space and time. It follows that, for cosmological solutions, $G\propto t^{-n}$, $H\propto t^{-1}$, and $\rho\propto t^{n-2}$, where n is expressible in terms of an arbitrary parameter ω_{BD} as $n^{-1} = 2 + 3\omega_{\mathrm{BD}}/2$. Einsteinian gravity is recovered when $\omega_{\mathrm{BD}}\to\infty$. This predicts that $\alpha_{\mathrm{G}}\propto t^{-n}$ and $\delta\propto t^{-1}$, whereas α_{EM}, α_{W}, and ϵ are kept constant. This kind of theory was further generalized to obtain various functional dependences for G in the formalization of scalar-tensor theories of gravitation (see, e.g., Damour and Esposito-Farèse, 1992).

Dirac's idea was revived after Teller (1948) argued that the decrease of G contradicts paleontological evidence [see also Pochoda and Schwarzschild (1964) and Gamow (1967c) for evidence based on the nuclear resources of the Sun]. Gamow (1967a, 1967b) proposed that α_{EM} might vary as t in order to save the "elegant" (according to him) idea of Dirac (see also Stanyukovich, 1962). In both Gamow's (1967a, 1967b) and Dirac's (1937) theories, the ratio $\alpha_{\mathrm{G}}/\alpha_{\mathrm{EM}}$ decreases as t^{-1}. Teller (1948) remarked that $\alpha_{\mathrm{EM}}^{-1} \sim -\ln H_0 t_{\mathrm{Pl}}$, so that

$\alpha_{\rm EM}^{-1}$ would become the logarithm of a large number. Landau (1955), de Witt (1964), and Isham *et al.* (1971) advocated that such a dependence may arise if the Planck length provides a cutoff to the logarithmic divergences of quantum electrodynamics. In this latter class of models $\alpha_{\rm EM}\propto 1/\ln t$, $\alpha_{\rm G}\propto t^{-1}$, $\delta\propto t^{-1}$, and $\alpha_{\rm W}$ and ϵ remain constant. Dyson (1967), Peres (1967), and then Davies (1972) showed, using geological data of the abundance of rhenium and osmium and the stability of heavy nuclei, that these two hypotheses were ruled out observationally (see Sec. III for details on the experimental results). Modern theories of high-energy physics offer new arguments to reconsider the variation of the fundamental constants (see Sec. VI). The most important outcome of Dirac's proposal and of the following assimilated theories [including a later version of Dirac's (1974) theory in which there is matter creation either where old matter was present or uniformly throughout the universe] is that the hypothesis of the constancy of the fundamental constants can and must be checked experimentally. Further details on this early history can be found in Barrow and Tipler (1986).

A way to reconcile some of the large numbers is to consider the energy dependence of the couplings as determined by the renormalization group (see, e.g., Itzykson and Zuber, 1980). For instance, concerning the fine-structure constant, the energy dependence arises from vacuum polarization that tends to screen the charge. This screening is less important at small distances, and the charge appears bigger, so that the effective coupling constant grows with energy. It follows from this approach that the three gauge groups get unified into a larger grand-unification group, so that the three couplings $\alpha_{\rm EM}$, $\alpha_{\rm W}$, and $\alpha_{\rm S}$ stem from the same dimensionless number $\alpha_{\rm GUT}$. This might explain some large numbers and answer some of Dirac's concerns (Hogan, 2000) but indeed, it does not explain the weakness of gravity that has become known as the hierarchy problem.

Let us come back briefly to the anthropic considerations and show that they allow us to set an interval of admissible values for some constants. Indeed, the anthropic principle does not determine whether the constants are varying or not, but it gives an insight into how special our universe is. In such an approach, one studies the effect of small variations of a constant around its observed value and tries to find a phenomenon highly dependent on this constant. This does not ensure that there is no other set of constants (very different than the one observed today) for which an organized universe may exist. It just tells us about the stability in a neighborhood of the location of our universe in the parameter space of physical constants. Rozental (1988) argued that requiring that the lifetime of the proton $\tau_p\sim\alpha_{\rm EM}^{-2}(\hbar/m_{\rm p}c^2)\exp(1/\alpha_{\rm EM})\sim 10^{32}$ yr be larger than the age of the universe $t_{\rm u}\sim c/H_0\sim 10^{17}$ s implies that $\alpha_{\rm EM}<1/80$. On the other hand, if we believe in a grand unified theory, this unification has to take place below the Planck scale, implying that $\alpha_{\rm EM}>1/170$, this bound depending on assumptions on the particle content. Similarly, requiring that the electromagnetic repulsion be much smaller than the attraction by strong interaction in nuclei (which is necessary to have nuclei) leads to $\alpha_{\rm EM}<1/20$. The thermonuclear reactions in stars are efficient if $k_{\rm B}T\sim\alpha_{\rm EM}m_{\rm p}c^2$ and the temperature of a star of radius $R_{\rm S}$ and mass $M_{\rm S}$ can roughly be estimated as $k_{\rm B}T\sim GM_s m_{\rm p}/R_{\rm S}$, which leads to the estimate $\alpha_{\rm EM}\sim 10^{-3}$. One can indeed think of many other examples to place such bounds. From the previous considerations, we maintain that the most stringent is

$$1/170<\alpha_{\rm EM}<1/80. \tag{25}$$

It is difficult to believe that these arguments can lead to much sharper constraints. They are illustrative and give a hint that the constants may not be "random" parameters without giving any explanation for their values.

Rozental (1988) also argued that the existence of hydrogen and the formation of complex elements in stars (mainly the possibility of the reaction $3\alpha\rightarrow{}^{12}$C) set constraints on the values of the strong-coupling constant. The production of ^{12}C in stars requires a triple tuning: (i) the decay lifetime of ^{8}Be, of order 10^{-6} s, is four orders of magnitude longer than the time for two α particles to scatter, (ii) an excited state of the carbon lies just above the energy of $^8{\rm Be}+\alpha$, and finally (iii) the energy level of ^{16}O at 7.1197 MeV is nonresonant and below the energy of $^{12}{\rm C}+\alpha$, of order 7.1616 MeV, which ensures that most of the carbon synthesized is not destroyed by the capture of an α particle (see Livio *et al.*, 1989). Oberhummer *et al.* (2000) showed that outside a window of, respectively, 0.5% and 4% of the values of the strong and electromagnetic forces, the stellar production of carbon or oxygen will be reduced by a factor 30–1000 (see also Pochet *et al.*, 1991; Jeltema and Sher, 1999). Concerning the gravitational constant, galaxy formation requires $\alpha_{\rm G}<10^4$. Other such constraints on the other parameters listed in the previous section can be obtained (see, e.g., Barrow, Sandvik, and Magueijo, 2002b for further discussions).

B. Metrology

The introduction of constants in physical law is closely related to the existence of systems of units. For instance, Newton's law states that the gravitational force between two masses is proportional to each mass and inversely proportional to their separation. To transform the proportionality to an equality, one requires the use of a quantity with dimension of ${\rm m^3\,kg^{-1}\,s^{-2}}$ independent of the separation between the two bodies, of their mass, of their composition (equivalence principle), and of the position (local position invariance). With another system of units, this constant could have been simply anything.

The determination of the laboratory value of constants relies mainly on the measurements of lengths, frequencies, times, etc. (see Petley, 1985, for a treatise on the measurement of constants, and Flowers and Petley, 2001, for a recent review). Hence, any question on the variation of constants is linked to the definition of the system of units and to the theory of measurement. The choice of base units affects the possible time variation of constants.

The behavior of atomic matter is mainly determined by the value of the electron mass and of the fine-structure constant. The Rydberg energy sets the (nonrelativistic) atomic levels, the hyperfine structure involves higher powers of the fine-structure constant, and molecular modes (including vibrational, rotational, etc. modes) depend on the ratio m_e/m_p. As a consequence, if the fine-structure constant is spacetime dependent, the comparison between several devices such as clocks and rulers will also be spacetime dependent. This dependence will also differ from one clock to another so that *metrology becomes both device and spacetime dependent*.

Besides this first metrological problem, the choice of units has implications on the permissible variations of certain dimensionful constant. As an illustration, we follow Petley (1983) who discusses the implication of the definition of the meter. The definition of the meter via a prototype platinum-iridium bar depends on the interatomic spacing in the material used in the construction of the bar. Atkinson (1968) argued that, at first order, it mainly depends on the Bohr radius of the atom so that this definition of the meter fixes the combination (17) as constant. Another definition was based on the wavelength of the orange radiation from krypton-86 atoms. It is likely that this wavelength depends on the Rydberg constant and on the reduced mass of the atom so that it ensures that $m_e c^2 \alpha_{EM}^2/2\hbar$ is constant. The more recent definition of the meter as the length of the path traveled by light in vacuum during a time of 1/299 792 458 of a second imposes the constancy of the speed of light[3] c. Identically, the definitions of the second as the duration of 9 192 631 770 periods of the transition between two hyperfine levels of the ground state of cesium-133 or of the kilogram via an international prototype, respectively, require that $m_e^2 c^2 \alpha_{EM}^4/\hbar$ and m_p be fixed.

Since the definition of a system of units and the value of the fundamental constants (and thus the status of their constancy) are entangled, and since the measurement of any dimensionful quantity is in fact the measurement of a ratio to standards chosen as units, *it only makes sense to consider the variation of dimensionless ratios*.

In theoretical physics, we often use the fundamental constants as units (see McWeeny, 1973, for the relation between natural units and SI units). The International System of units (SI) is more appropriate to human-size measurements, whereas natural systems of units are more appropriate to the physical systems to which they refer. For instance, $\hbar$, c, and G allows us to construct the Planck mass, time, and length, which are of great use as units while studying high-energy physics, and the same can be done from $\hbar$, e, m_e, and ε_0 to construct a unit

[3]Note that the velocity of light is not assigned a fixed value *directly*, but rather the value is fixed as a consequence of the definition of the meter.

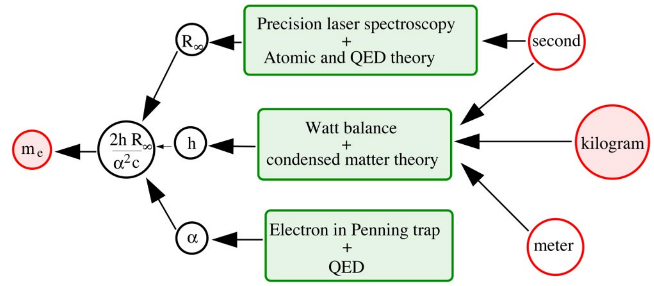

FIG. 2. Sketch of the experimental and theoretical chain leading to the determination of the electron mass. Note that, as expected, the determination of α_{EM} requires no dimensional input. From Mohr and Taylor (2001) (Color in online edition).

mass (m_e), length ($4\pi\varepsilon_0 h^2/m_e e^2$), and time ($2\varepsilon_0 h^3/\pi m_e e^4$). A physical quantity can always be decomposed as the product of a label representing a standard quantity of reference and a numerical value representing the number of times the standard has to be taken to build the required quantity. It follows that a given quantity X can be expressed as $X = k_1 F_1(\mathrm{m,kg,s,}\ldots)$ with k_1 a dimensionless quantity and F_1 a function of the base units (here SI) to some power. Let us decompose X as $X = k_2 F_2(\hbar, e, c, \ldots)$ where k_2 is another dimensionless constant and F_2 a function of a sufficient number of fundamental constants to be consistent with the initial base units. The time variation of X is given by

$$\frac{d\ln X}{dt} = \frac{d\ln k_1}{dt} + \frac{d\ln F_1}{dt} = \frac{d\ln k_2}{dt} + \frac{d\ln F_2}{dt}.$$

Since only dk_1/dt or dk_2/dt can be measured, it is necessary to have chosen a system of units, the constancy of which is assumed (i.e., that either $dF_1/dt = 0$ or $dF_2/dt = 0$) in order to draw any conclusion concerning the time variation of X, in the same way as the description of a motion needs to specify a reference frame.

To illustrate the importance of the choice of units and the entanglement between experiment and theory while measuring a fundamental constant, let us sketch how one determines m_e in the SI system (following Mohr and Taylor, 2001), that is, in the kilogram (see Fig. 2). The kilogram is defined by a platinum-iridium bar to which we have to compare the mass of the electron. The key to this measurement is to express the electron mass as $m_e = 2hR_\infty/\alpha_{EM}^2 c$. From the definition of the second, R_∞ is determined by precision laser-spectroscopy on hydrogen and deuterium and the theoretical expression for the $1s$-$2s$ hydrogen transition as $\nu = (3/4)R_\infty c[1-\mu + 11\alpha_{EM}^2/48 + (56\alpha_{EM}^3)/(9\pi)\ln\alpha_{EM} + \cdots]$ arising from QED. The fine-structure constant is determined by comparing theory and experiment for the anomalous magnetic moment of the electron (involving, again, QED). Finally, the Planck constant is determined by a Watt balance, comparing Watt electrical to Watt mechanical power (involving classical mechanics and classical electromagnetism only: h enters through the current and voltage calibration based on two condensed-matter phenomena, Josephson and quantum Hall effects, so that it involves the theories of these two effects).

As a conclusion, let us recall that (i) in general, the values of the constants are not determined by a direct measurement but by a chain of measurements involving both theoretical and experimental steps, (ii) they depend on our theoretical understanding, (iii) the determination of a self-consistent set of values of the fundamental constants results from an adjustment to achieve the best match between theory and a defined set of experiments (see, e.g., Birge, 1929), (iv) the system of units plays a crucial role in the measurement chain, since, for instance, in atomic units, the mass of the electron could have been obtained directly from a mass ratio measurement (which is even more precise), and (v) the test of the variability of the constants fortunately does not require us to have *a priori* a high-precision value of the considered constant.

In the following, we will thus focus on the variation of dimensionless ratios which, for instance, characterize the relative magnitude of two forces, and are independent of the choice of the system of units and of the choice of standard rulers or clocks. Let us note that some (hopeless) attempts to constrain the time variation of dimensionful constants have been tried and will be briefly discussed in Sec. V.F. This does not, however, mean that a physical theory cannot have dimensionful varying constants. For instance, a theory of varying fine-structure constants can be implemented either as a theory with varying electric charge or varying speed of light.

C. Overview of the methods

Before going into the details of the constraints, it is worth taking some time to discuss the kind of experiments or observations that we need to consider and what we can hope to infer from them.

As emphasized in the previous section, we can only measure the variation of dimensionless quantities (such as the ratio of two wavelengths, two decay rates, two cross sections, etc.); the idea is to pick a physical system that depends strongly on the value of a set of constants so that a small variation will have dramatic effects. The general strategy is thus to constrain the spacetime variation of an observable quantity as precisely as possible and then to relate it to a set of fundamental constants.

Basically, we can group all the methods into three classes: (i) *atomic methods*, including atomic clocks, quasar absorption spectra, and observation of the cosmic microwave background (CMB) radiation where one compares ratios of atomic-transition frequencies (the CMB observation depends on the dependence of the recombination process on $\alpha_{\rm EM}$); (ii) *nuclear methods*, including nucleosynthesis, α and β decay, and Oklo reactor (for which the observables are, respectively, abundances, lifetimes, and cross sections); and (iii) *gravitational methods*, including the test of the violation of the universality of free fall, where one constrains the relative acceleration of two bodies, stellar evolution, etc.

These methods are either *experimental* (e.g., atomic clocks), for which one can have a better control of the systematics, *observational* (e.g., geochemical, astrophysical, and cosmological observations), or *mixed* (α and β decay, universality of free fall). This sets the time scales on which a possible variation can be measured. For instance, in the case of the fine-structure constant (see Sec. III), one expects to be able to constrain a relative variation of $\alpha_{\rm EM}$ of order 10^{-8} [geochemical (Oklo)], 10^{-5} [astrophysical (quasars)], 10^{-3}–10^{-2} (cosmological methods), and 10^{-13}–10^{-14} (laboratory methods) on time scales of order 10^9 yr, 10^9–10^{10} yr, 10^{10} yr, and 1–12 months, respectively. This brings up the question of the comparison and of the compatibility of the different measurements since one will have to take into account, for example, the rate of change of $\alpha_{\rm EM}$, which is often assumed to be constant. In general, this requires us to specify a model both to determine the law of evolution and the links between the constants. Long-time-scale experiments allow the testing of a slow-drift evolution, while short-time-scale experiments enable the testing of the possibility of a rapidly varying constant.

The next step is to convert the bound on the variation of some measured physical quantities (decay rate, cross section, etc.) into a bound on some constants. It is clear that, in general (for atomic and nuclear methods at least), it is impossible to consider the electromagnetic, weak, and strong effects independently, so that this latter step involves some assumptions.

Atomic methods are mainly based on the comparison of the wavelengths of different transitions. The nonrelativistic spectrum depends mainly on R_∞ and μ, the fine structure on $R_\infty\alpha_{\rm EM}^2$, and the hyperfine structure on $g_{\rm p}R_\infty\alpha_{\rm EM}^2$. Extending this to molecular spectra to include rotational and vibrational transitions allows us to have access to μ. It follows that we can hope to disentangle the observations of the comparisons of different transitions to constrain on the variation of $(\alpha_{\rm EM},\mu,g_{\rm p})$. The exception is CMB, which involves a dependence on $\alpha_{\rm EM}$ and $m_{\rm e}$ mainly due to the Thomson-scattering cross section and the ionization fraction. Unfortunately, the effect of these parameters has to be distinguished from the dependence on the usual cosmological parameters, which renders the interpretation more difficult.

The internal structure and mass of the proton and neutron are completely determined by strong gauge fields and quarks interacting together. Provided we can ignore the quark masses and electromagnetic effects, the whole structure is only dependent on an energy scale $\Lambda_{\rm QCD}$. It follows that the stability of the proton greatly depends on the electromagnetic effects and the masses $m_{\rm u}$ and $m_{\rm d}$ of the up and down quarks. In nuclei, the interaction of hadrons can be thought to be mediated by pions of mass $m_\pi^2 \sim m_{\rm p}(m_{\rm u}+m_{\rm d})$. Since the stability of the nucleus mainly results from the balance between this attractive nuclear force, the nucleon degeneracy pressure, and the Coulomb repulsion, it will mainly involve $m_{\rm u}$, $m_{\rm d}$, $\alpha_{\rm EM}$.

Big bang nucleosynthesis depends on G (expansion rate), $G_{\rm F}$ (weak interaction rates), $\alpha_{\rm S}$ (binding of light elements), and $\alpha_{\rm EM}$ (via the electromagnetic contribution to $m_{\rm n}-m_{\rm p}$, but one will also have to take into account the contribution of a possible variation of the mass of the quarks, m_u and m_d). Besides, if m_n-m_p falls below m_e, the β decay of the neutron is no longer energetically possible. The abundance of helium is mainly sensitive to the freeze-out temperature, and the neutron lifetime and heavier-element abundances to the nuclear rates.

All *nuclear methods* involve a dependence on the mass of the nuclei of charge Z and atomic number A,

$$m(A,Z)=Zm_p+(A-Z)m_n+E_S+E_{EM},$$

where E_S and E_{EM} are, respectively, the strong and electromagnetic contributions to the binding energy. The Bethe-Weizäcker formula gives

$$E_{EM}=98.25\frac{Z(Z-1)}{A^{1/3}}\alpha_{EM}\ \text{MeV}. \tag{26}$$

If we decompose m_p and m_n as (see Gasser and Leutwyler, 1982) $m_{(p,n)}=u_3+b_{(u,d)}m_u+b_{(d,u)}m_d+B_{(p,n)}\alpha_{EM}$ where u_3 is the pure QCD approximation of the nucleon mass (b_u, b_d, and $B_{(n,p)}/u_3$ being pure numbers), it reduces to

$$\begin{aligned}m(A,Z)=&(Au_3+E_S)+(Zb_u+Nb_d)m_u\\&+(Zb_d+Nb_u)m_d\\&+\left(ZB_p+NB_n+98.25\frac{Z(Z-1)}{A^{1/3}}\ \text{MeV}\right)\alpha_{EM},\end{aligned} \tag{27}$$

with $N=A-Z$, the neutron number. This depends on our understanding of the description of the nucleus and can be more sophisticated. For an atom, one would have to add the contribution of the electrons, Zm_e. Equation (27) depends on strong, weak, and electromagnetic quantities. The numerical coefficients $B_{(n,p)}$ are given explicitly by Gasser and Leutwiller (1982) as

$$B_p\alpha_{EM}=0.63\ \text{MeV},\quad B_n\alpha_{EM}=-0.13\ \text{MeV}. \tag{28}$$

It follows that it is generally difficult to discern the effect of each parameter and compare the different methods. For instance, comparing the constraint on μ obtained from electromagnetic methods to the constraints on α_S and G_F from nuclear methods requires some theoretical input, such as a theory to explain the fermion masses. Moreover, most of the theoretical models predict a variation of the coupling constants from which one has to infer the variation of μ, etc.

For macroscopic bodies, the mass has also a negative contribution

$$\Delta m(G)=-\frac{G}{2c^2}\int\frac{\rho(\vec r)\rho(\vec r\,')}{|\vec r-\vec r\,'|}d^3\vec r d^3\vec r\,' \tag{29}$$

from the gravitational binding energy. As a conclusion, from Eqs. (27) and (29), we expect the mass to depend on all the coupling constants, $m(\alpha_{EM},\alpha_W,\alpha_S,\alpha_G,\text{ etc.})$.

This has a profound consequence concerning the motion of any body. Let α be any fundamental constant, assumed to be a scalar function and having a time variation of cosmological origin so that in the privileged cosmological rest frame it is given by $\alpha(t)$. A body of mass m moving at velocity $\vec v$ will experience an anomalous acceleration

$$\delta\vec a\equiv\frac{1}{m}\frac{dm\vec v}{dt}-\frac{d\vec v}{dt}=\frac{\partial\ln m}{\partial\alpha}\dot\alpha\vec v. \tag{30}$$

Now, in the rest frame of the body, α has a spatial dependence $\alpha[(t'+\vec v\cdot\vec r\,'/c^2)/\sqrt{1-v^2/c^2}]$ so that, as long as $v\ll c$, $\nabla\alpha=(\dot\alpha/c^2)\vec v$. The anomalous acceleration can thus be rewritten as

$$\delta\vec a=-\left(\frac{\alpha}{m}\frac{\delta mc^2}{\delta\alpha}\right)\nabla\ln\alpha. \tag{31}$$

In the most general case, for a nonrelativistic body,

$$\delta\vec a=-\left(\frac{\alpha}{m}\frac{\delta mc^2}{\delta\alpha}\right)\left(\frac{\nabla\alpha}{\alpha}+\frac{\dot\alpha}{\alpha}\frac{\vec v}{c^2}\right). \tag{32}$$

It reduces to Eq. (30) in the appropriate limit, and the additional gradient term will be produced by local matter sources. This anomalous acceleration is generated by the change in the (electromagnetic, gravitational, etc.) binding energy (Dicke, 1964, 1969; Eardley, 1975; Haugan, 1979; Nordtvedt, 1990). Besides, the α dependence is *a priori* composition dependent [see, e.g., Eq. (27)]. As a consequence, any variation of the fundamental constants will entail a violation of the universality of free fall: the total mass of the body being space dependent, an anomalous force appears if energy is to be conserved. The variation of the constants, deviation from general relativity, and violation of the weak equivalence principle are in general expected together, for example, if there exists a new interaction mediated by a massless scalar field.

Gravitational methods include the constraints that can be derived from the test of the theory of gravity, such as the test of the universality of free fall, the motion of the planets in the solar system, and stellar and galactic evolutions. They are based on the comparison of two time scales, the first (gravitational time) dictated by gravity (ephemeris, stellar ages, etc.) and the second (atomic time) determined by any system not determined by gravity (e.g., atomic clocks) (Canuto and Goldman, 1982). For instance, planet ranging, neutron star binary observations, primordial nucleosynthesis, and paleontological data allow one to constrain the relative variation of G to a level, respectively, of 10^{-12}–10^{-11}, 10^{-13}–10^{-12}, 10^{-12}, and 10^{-10} per year.

Attacking the full general problem is a hazardous and dangerous task, so we will first describe the constraints obtained in the literature by focusing on the fine-structure constant and the gravitational constant, and we will then extend to some other (less studied) combinations of the constants. A complementary approach is to predict the mutual variations of different constants in a given theoretical model (see Sec. VI).

III. FINE-STRUCTURE CONSTANT

A. Geological constraints

1. The Oklo phenomenon

Oklo is a prehistoric, natural fission reactor that operated about 2×10^9 yr ago (corresponding to a redshift of ~0.14) during a few million years in the Oklo uranium mine in Gabon. This phenomenon was discovered by the French Commissariat à l'Énergie Atomique in 1972 while monitoring for uranium ores (see Naudet, 1974, Maurette, 1976, and Petrov, 1977, for early studies, and Naudet, 2000, for a general review). Two billion years ago, uranium was naturally enriched (due to the difference of decay rate between ^{235}U and ^{238}U) and ^{235}U represented about 3.68% of the total uranium (compared with 0.72% today). Besides, in Oklo the concentration of neutron absorbers that prevent the neutrons from being available for the chain fission was low; water played the role of moderator and slowed down fast neutrons so that they could interact with other ^{235}U, and the reactor was large enough so that the neutrons did not escape faster than they were produced.

From isotopic abundances of the yields, one can extract information about the nuclear reactions at the time the reactor was operational and reconstruct the reaction rates at that time. One of the key quantities measured is the ratio $^{149}_{62}$Sm/$^{147}_{62}$Sm of two light isotopes of samarium, which are not fission products. This ratio, of order of 0.9 in normal samarium, is about 0.02 in Oklo ores. This low value is explained by the depletion of $^{149}_{62}$Sm by thermal neutrons to which it was exposed while the reactor was active.

Shlyakhter (1976) pointed out that the capture cross section of thermal neutron by $^{149}_{62}$Sm,

$$n+{}^{149}_{62}\mathrm{Sm}\rightarrow{}^{150}_{62}\mathrm{Sm}+\gamma, \tag{33}$$

is dominated by a capture resonance of a neutron of energy of about 0.1 eV. The existence of this resonance is a consequence of a near cancellation between the electromagnetic repulsive force and the strong interaction.

To obtain a constraint, one first needs to measure the neutron-capture cross section of $^{149}_{62}$Sm at the time of the reaction and to relate it to the energy of the resonance. One has finally to translate the constraint on the variation of this energy to a constraint on the time variation of the considered constant.

The cross section of the neutron capture (33) is strongly dependent on the energy of a resonance at $E_r=97.3$ meV and is well described by the Breit-Wigner formula

$$\sigma_{(n,\gamma)}(E)=\frac{g_0\pi}{2}\frac{\hbar^2}{m_\mathrm{n}E}\frac{\Gamma_\mathrm{n}\Gamma_\gamma}{(E-E_r)^2+\Gamma^2/4}, \tag{34}$$

where $g_0\equiv(2J+1)(2s+1)^{-1}(2I+1)^{-1}$ is a statistical factor that depends on the spin of the incident neutron $s=1/2$, of the target nucleus I, and of the compound nucleus J; for the reaction (33), we have $g_0=9/16$. The total width $\Gamma\equiv\Gamma_\mathrm{n}+\Gamma_\gamma$ is the sum of the neutron partial width $\Gamma_\mathrm{n}=0.533$ meV (at E_r) and of the radiative partial width $\Gamma_\gamma=60.5$ meV.

The effective absorption cross section is defined by

$$\hat\sigma(E_r,T)=\frac{1}{v_0}\frac{2}{\sqrt{\pi}}\int\sigma_{(n,\gamma)}(E)\times\sqrt{\frac{2E}{m_\mathrm{n}}}\frac{e^{-E/k_\mathrm{B}T}}{(k_\mathrm{B}T)^{3/2}}\sqrt{E}dE, \tag{35}$$

where the velocity $v_0=2200\ \mathrm{m\,s^{-1}}$ corresponds to an energy $E_0=25.3$ meV and the effective neutron flux is similarly given by

$$\hat\phi=v_0\frac{2}{\sqrt{\pi}}\int\sqrt{\frac{2E}{m_\mathrm{n}}}\frac{e^{-E/k_\mathrm{B}T}}{(k_\mathrm{B}T)^{3/2}}\sqrt{E}dE. \tag{36}$$

The samples of the Oklo reactors were exposed (Naudet, 1974) to an integrated effective fluence $\int\hat\phi dt$ of about 10^{21} neutron $\mathrm{cm^{-2}}=1\ \mathrm{kb^{-1}}$. This implies that any process with a cross section smaller than 1 kb can be neglected in the computation of the abundances; this includes neutron capture by $^{144}_{62}$Sm and $^{148}_{62}$Sm. On the other hand, the fission of $^{235}_{92}$U and the capture of a neutron by $^{143}_{60}$Nd and by $^{149}_{62}$Sm with respective cross sections $\sigma_5\simeq0.6$ kb, $\sigma_{143}\sim0.3$ kb, and $\sigma_{149}\geqslant70$ kb are the dominant processes. It follows that the equations of evolution for the number densities N_{147}, N_{148}, N_{149}, and N_{235} of $^{147}_{62}$Sm, $^{148}_{62}$Sm, $^{149}_{62}$Sm, and $^{235}_{92}$U are (Damour and Dyson, 1996; Fujii *et al.*, 2000)

$$\frac{dN_{147}}{dt}=-\hat\sigma_{147}\hat\phi N_{147}+\hat\sigma_{f235}\hat\phi N_{235}, \tag{37}$$

$$\frac{dN_{148}}{dt}=\hat\sigma_{147}\hat\phi N_{147}, \tag{38}$$

$$\frac{dN_{149}}{dt}=-\hat\sigma_{149}\hat\phi N_{149}+\hat\sigma_{f235}\hat\phi N_{235}, \tag{39}$$

$$\frac{dN_{235}}{dt}=-\sigma_5^*N_{235}, \tag{40}$$

where the system has to be closed by using a modified absorption cross section $\sigma_5^*=\sigma_5(1-C)$ (see references in Damour and Dyson, 1996). This system can be integrated under the assumption that the cross sections are constant and the result compared with the natural abundances of the samarium to extract the value of $\hat\sigma_{149}$ at the time of the reaction. Shlyakhter (1976) first claimed that $\hat\sigma_{149}=55\pm8$ kb (as cited by Dyson, 1978). Damour and Dyson (1996) reanalyzed several samples of Oklo data and found that $57\ \mathrm{kb}\leqslant\hat\sigma_{149}\leqslant93$ kb. Fujii *et al.* (2000) found that $\hat\sigma_{149}=91\pm6$ kb.

By comparing these measurements to the current value of the cross section and using Eq. (35), one can transform it into a constraint on the variation of the resonance energy. This step requires an estimation of the neutron temperature. It can be obtained by using information from the abundances of other isotopes such as lutetium and gadolinium. Shlyakhter (1976) deduced that $|\Delta E_r|<20$ meV but assumed the much-too-low temperature of $T=20\,^\circ$C. Damour and Dyson (1996) allowed the temperature to vary between $180\,^\circ$C and $700\,^\circ$C and deduced the conservative bound -120 meV$<\Delta E_r<90$ meV; Fujii *et al.* (2000) obtained two branches, the first compatible with a null variation $\Delta E_r=9\pm11$ meV and the second indicating a nonzero effect $\Delta E_r=-97\pm8$ meV for $T=200$–$400\,^\circ$C, arguing that the first branch was favored.

Damour and Dyson (1996) related the variation of E_r to the fine-structure constant by taking into account that the radiative capture of the neutron by $^{149}_{62}$Sm corresponds to the existence of an excited quantum state $^{150}_{62}$Sm (so that $E_r=E^*_{150}-E_{149}-m_{\rm n}$) and by assuming that the nuclear energy is independent of $\alpha_{\rm EM}$. It follows that the variation of $\alpha_{\rm EM}$ can be related to the difference in Coulomb energy of these two states. The computation of this latter quantity is difficult and requires a relation to the mean-square radii of the protons in the isotopes of samarium; Damour and Dyson (1996) showed that the Bethe-Weizäcker formula (26) overestimates by about a factor the 2 the $\alpha_{\rm EM}$ sensitivity to the resonance energy. It follows from this analysis that

$$\alpha_{\rm EM}\frac{\Delta E_r}{\Delta\alpha_{\rm EM}}\simeq-1.1\ \ {\rm MeV}, \tag{41}$$

which, once combined with the constraint on ΔE_r, implies

$$-0.9\times10^{-7}<\Delta\alpha_{\rm EM}/\alpha_{\rm EM}<1.2\times10^{-7} \tag{42}$$

at 2σ level, corresponding to the range -6.7×10^{-17} yr$^{-1}<\dot\alpha_{\rm EM}/\alpha_{\rm EM}<5.0\times10^{-17}$ yr^{-1} if $\dot\alpha_{\rm EM}$ is assumed constant. This tight constraint arises from the large amplification between the resonance energy ($\sim$0.1 eV) and the sensitivity ($\sim$1 MeV). Fujii *et al.* (2000) reanalyzed the data including gadolinium and found the favored result $\dot\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.2\pm0.8)\times10^{-17}$ yr^{-1}, which corresponds to

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.36\pm1.44)\times10^{-8} \tag{43}$$

and another branch $\dot\alpha_{\rm EM}/\alpha_{\rm EM}=(4.9\pm0.4)\times10^{-17}$ yr^{-1}. The first bound is favored given the constraint on the temperature of the reactor. Nevertheless, the nonzero result cannot be eliminated, even using results from gadolinium abundances (Fujii, 2002). Note, however, that spliting the analysis into two branches seems to be at odds with the aim of obtaining a constraint. Starting from the bound on ΔE_r derived by Damour and Dyson (1996), Olive *et al.* (2002) made some theoretical assumptions about the quark-mass dependence of ΔE_r and derived from these assumptions a tighter limit on the variation of $\alpha_{\rm EM}$ of order $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\lesssim10^{-9}$.

Earlier studies include the original work by Shlyakhter (1976) who found that $|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|<10^{-17}$ yr^{-1}, corresponding to

$$|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<1.8\times10^{-8}. \tag{44}$$

In fact, he stated that the variation of the strong-interaction coupling constant was given by $\Delta g_{\rm S}/g_{\rm S}\sim\Delta E_r/V_0$, where $V_0\simeq50$ MeV is the depth of a square potential well. Arguing that the Coulomb force increases the average internuclear distance by about 2.5% for $A\sim150$, he concluded that $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\sim20\Delta g_{\rm S}/g_{\rm S}$, leading to $|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|<10^{-17}$ yr^{-1}. Irvine (1983a, 1983b) quoted the bound $|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|<5\times10^{-17}$ yr^{-1}. The analysis of Sisterna and Vucetich (1990) used, according to Damour and Dyson (1996), an ill-motivated finite-temperature description of the excited state of the compound nucleus. Most of the studies focus on the effect of the fine-structure constant mainly because the effects of its variation can be well controlled, but one would also have to take the effect of the variation of the Fermi constant, or, identically, $\alpha_{\rm W}$ (see Sec. V.A). Horváth and Vucetich (1988) interpreted the results from Oklo in terms of null-redshift experiments.

2. α decay

The fact that α decay can be used to put constraints on the time variation of the fine-structure constant was pointed out by Wilkinson (1958) and then revived by Dyson (1972, 1978). The main idea is to extract the $\alpha_{\rm EM}$ dependence of the decay rate and to use geological samples to bound its time variation.

The decay rate λ of the α decay of a nucleus ^A_ZX of charge Z and atomic number A,

$$^{A+4}_{Z+2}{\rm X}\to{}^A_Z{\rm X}+{}^4_2{\rm He}, \tag{45}$$

is governed by the penetration of the Coulomb barrier described by the Gamow theory and well approximated by

$$\lambda\simeq\Lambda(\alpha_{\rm EM},v)e^{-4\pi Z\alpha_{\rm EM}c/v}, \tag{46}$$

where v is the escape velocity of the α particle and where Λ is a function that depends slowly on $\alpha_{\rm EM}$ and v. It follows that the variation of the decay rate with respect to the fine-structure constant is well approximated by

$$\frac{d\ln\lambda}{d\alpha_{\rm EM}}\simeq-4\pi Z\frac{c}{v}\left(1-\frac{1}{2}\frac{d\ln\Delta E}{d\ln\alpha_{\rm EM}}\right), \tag{47}$$

where $\Delta E\equiv2mv^2$ is the decay energy. Considering that the total energy is the sum of the nuclear energy $E_{\rm nuc}$ and of the Coulomb energy $E_{\rm EM}/80\ {\rm MeV}\simeq Z(Z-1)A^{-1/3}\alpha_{\rm EM}$, and that the former does not depend on $\alpha_{\rm EM}$, one deduces that

$$\frac{d\ln\Delta E}{d\ln\alpha_{\rm EM}}\simeq\left(\frac{\Delta E}{0.6\ {\rm MeV}}\right)^{-1}f(A,Z) \tag{48}$$

with $f(A,Z)\equiv[(Z+2)(Z+1)(A+4)^{-1/3}-Z(Z-1)A^{-1/3}]$. It follows that the sensitivity of the decay rate on the fine-structure constant is given by

$$s\equiv\frac{d\ln\lambda}{d\ln\alpha_{\rm EM}}\simeq4\pi Z\frac{c}{v}\alpha_{\rm EM}\left\{\left(\frac{0.3\ \ {\rm MeV}}{\Delta E}\right)f(A,Z)-1\right\}. \tag{49}$$

This result can be qualitatively understood since an increase of $\alpha_{\rm EM}$ induces an increase in the height of the Coulomb barrier at the nuclear surface while the depth of the nuclear potential below the top remains the same. It follows that the α particle escapes with greater energy but is at the same energy below the top of the barrier. Since the barrier becomes thinner at a given energy below its top, the penetrability increases. This computation indeed neglects the effect of a variation of $\alpha_{\rm EM}$ on the nucleus that can be estimated to be dilated by about 1% if $\alpha_{\rm EM}$ increases by 1%.

Wilkinson (1958) considered the most favorable α-decay reaction, which is the decay of ${}^{238}_{92}$U,

$$ {}^{238}_{92}\mathrm{U} \rightarrow {}^{235}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}, \tag{50} $$

for which $\Delta E \simeq 4.27$ MeV ($s \simeq 540$). By comparing the geological dating of the Earth by different methods, he concluded that the decay constants λ of ^{238}U, ^{235}U, and ^{232}Th have not changed by more than a factor of 3 or 4 during the last $3-4\times10^9$ years, from which it follows that $|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|<2\times10^{-12}$ yr^{-1} and thus

$$ |\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<8\times10^{-3}. \tag{51} $$

This constraint was revised by Dyson (1972), who claimed that the decay rate has not changed by more than 20% during the past 2×10^9 years, which implies

$$ |\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<4\times10^{-4}. \tag{52} $$

These data were recently revisited by Olive *et al.* (2002). Using laboratory and meteoric data for ^{147}Sm ($\Delta E \simeq 2.31$ MeV, $s \simeq 770$) for which $\Delta\lambda/\lambda$ was estimated to be of order 7.5×10^{-3}, they concluded that

$$ |\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<10^{-5}. \tag{53} $$

3. Spontaneous fission

α-emitting nuclei are classified into four generically independent decay series (the thorium, neptunium, uranium, and actinium series). The uranium series is the longest known series. It begins with ${}^{238}_{92}$U, passes a second time through $Z=92$ (${}^{234}_{92}$U) as a consequence of an α-β decay, then passes by five α decays and finishes by an α-β-β decay to end with ${}^{206}_{82}$Pb. The longest-lived member is ${}^{238}_{92}$U with a half-life of 4.47×10^9 yr, which is four orders of magnitude larger than the second-longest-lived elements. ${}^{238}_{92}$U thus determines the time scale of the whole series.

The expression of the lifetime in the case of spontaneous fission can be obtained from Gamow's theory of α decay by replacing the charge Z with the product of the charges of the two fission products.

Gold (1968) studied the fission of ${}^{238}_{92}$U with a decay time of 7×10^{-17} yr^{-1}. He obtained a sensitivity [Eq. (49)] of $s=120$. Ancient rock samples allow us to conclude, after comparison of rock samples dated by potassium-argon and rubidium-strontium, that the decay time of ${}^{238}_{92}$U has not varied by more than 10% in the last 2×10^9 yr. Indeed, the main uncertainty comes from the dating of the rock. Gold (1968) concluded on that basis that

$$ |\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<4.66\times10^{-4}, \tag{54} $$

which corresponds to $|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|<2.3\times10^{-13}$ yr^{-1} if one assumes that $\dot\alpha_{\rm EM}$ is constant. This bound is indeed comparable, in order of magnitude, to the one obtained by α-decay data.

Chitre and Pal (1968) compared the uranium-lead and potassium-argon dating methods governed, respectively, by α and β decay to date stony meteoric samples. Both methods have different $\alpha_{\rm EM}$ dependence (see below), and they concluded that

$$ |\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<(1-5)\times10^{-4}. \tag{55} $$

Dyson (1972) argued on a similar basis that the decay rate of ${}^{238}_{92}$U has not varied by more than 10% in the past 2×10^9 yr, so that

$$ |\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<10^{-3}. \tag{56} $$

4. β decay

Dicke (1959) stressed that the comparison of the rubidium-strontium and potassium-argon dating methods to uranium and thorium rates constrains the variation of $\alpha_{\rm EM}$. He concluded that there was no evidence to rule out a time variation of the β-decay rate.

Peres (1967) discussed qualitatively the effect of a fine-structure constant increasing with time, arguing that the nuclei chart would have then been very different in the past since the stable heavy element would have had N/Z ratios much closer to unity (because the deviation from unity is mainly due to the electrostatic repulsion between protons). For instance, ^{238}U would be unstable against double-β decay to ^{238}Pu. One of the arguments to claim that $\alpha_{\rm EM}$ has almost not varied lies in the fact that ^{208}Pb existed in the past as ^{208}Rn, which is a gas, so that the lead ores on Earth would be uniformly distributed.

As far as long-lived isotopes for which the decay energy ΔE is small are concerned, we can use a nonrelativistic approximation for the decay rate

$$ \lambda = \Lambda_{\pm}(\Delta E)^{p_\pm} \tag{57} $$

for, respectively, β^- decay and electron capture. $\Lambda_\pm$ are functions that depend smoothly on $\alpha_{\rm EM}$ and can thus be considered constant, and $p_+=\ell+3$ and $p_-=2\ell+2$ are the degrees of forbiddenness of the transition. For high-Z nuclei with small decay energy ΔE, the exponent p becomes $p=2+\sqrt{1-\alpha_{\rm EM}^2 Z^2}$ and is independent of ℓ. It follows that the sensitivity [Eq. (49)] becomes

$$ s = p\,\frac{d\ln\Delta E}{d\ln\alpha_{\rm EM}}. \tag{58} $$

The second factor can be estimated exactly as in Eq. (48) for α decay but with $f(A,Z)=\pm(2Z+1)A^{-1/3}[0.6\ \mathrm{MeV}/\Delta E]$, the $-$ and $+$ signs corresponding, respectively, to β decay and electron capture.

The laboratory-determined decay rates of rubidium to strontium by β decay,

$$ {}^{87}_{37}\mathrm{Rb} \rightarrow {}^{87}_{38}\mathrm{Sr} + \overline{\nu}_e + e^-, \tag{59} $$

and to potassium to argon by electron capture,

$$^{40}_{19}\mathrm{K}+e^{-}\rightarrow^{40}_{18}\mathrm{Ar}+\nu_e, \tag{60}$$

are, respectively, $1.41\times10^{-11}\,\mathrm{yr}^{-1}$ and $4.72\times10^{-10}\,\mathrm{yr}^{-1}$. The decay energies are, respectively, $\Delta E=0.275$ MeV and $\Delta E=1.31$ MeV, so that $s\simeq-180$ and $s\simeq-30$. Peebles and Dicke (1962) compared these laboratory-determined values with their abundances in rock samples after dating by the uranium-lead method and with meteorite data (dated by uranium-lead and lead-lead). They concluded that the variation of α_{EM} with α_{G} cannot be ruled out by comparison to meteorite data. Later, Yahil (1975) used the concordance of the K-Ar and Rb-Sr geochemical ages to place the limit

$$|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}|<1.2 \tag{61}$$

over the past 10^{10} yr.

The case of the decay of osmium to rhenium by electron emission

$$^{187}_{75}\mathrm{Re}\rightarrow^{187}_{76}\mathrm{Os}+\overline{\nu}_e+e^{-} \tag{62}$$

was first considered by Peebles and Dicke (1962). They noted that the very small value of its decay energy $\Delta E\simeq2.5$ keV makes it a very sensitive indicator of the variation of α_{EM}. In that case $p\simeq2.8$, so that $s\simeq-18\,000$. It follows that a change of about 10^{-2}% of α_{EM} will induce a change in the decay energy of order of the keV, that is, of the order of the decay energy itself. With a time-decreasing α_{EM}, the decay rate of rhenium will have slowed down and then osmium will have become unstable. Peebles and Dicke (1962) did not have reliable laboratory determination of the decay rate to set any constraint. Dyson (1967) compared the isotopic analysis of molybdenite ores, the isotopic analysis of 14 iron meteorites, and laboratory measurements of the decay rate. Assuming that the variation of the decay energy comes entirely from the variation of α_{EM}, he concluded that

$$|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}|<9\times10^{-4} \tag{63}$$

during the past 3×10^{9} years. In a reanalysis (Dyson, 1972), he concluded that the rhenium decay-rate had not changed by more than 10% in the past 10^{9} years, so that

$$|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}|<5\times10^{-6}. \tag{64}$$

Using a better determination of the decay rate of $^{187}_{75}\mathrm{Re}$ based on the growth of $^{187}\mathrm{Os}$ over a four-year period into a large source of osmium free rhenium, Lindner *et al.* (1986) deduced that

$$\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}=(-4.5\pm9)\times10^{-4} \tag{65}$$

over a 4.5×10^{9} yr period. This was recently updated (Olive *et al.*, 2002) to take into account the improvements in the analysis of the meteorite data that now show that the half-life has not varied by more than 0.5% in the past 4.6 Gyr (i.e., a redshift of about 0.45). This implies that

$$|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}|<3\times10^{-7}. \tag{66}$$

We just reported the values of the decay rates as used at the time of the studies. One might want to update these constraints by using new results of the measurements of the decay rate. Even so, they will not, in general, be competitive with the bounds obtained by other methods. These results can also be altered if the neutrinos are massive.

5. Conclusion

All the geological studies are on time scales of order of the age of the Earth (typically $z\sim0.1$–0.15, depending on the values of the cosmological parameters).

The Oklo results are probably the most powerful geochemical data to study the variation of the fine-structure constant, but one has to understand and to model carefully the correlations of the variation of α_{W} and g_{S} as well as the effect of μ (see the recent study by Olive *et al.*, 2002). This difficult but necessary task remains to be done.

The β-decay results depend on the combination $\alpha_{\mathrm{EM}}^{s}\alpha_{\mathrm{W}}^{2}$ and have the advantage of not depending on G. They may be considered more as historical investigations than as competitive methods to constrain the variation of the fine-structure constant, especially in view of the Oklo results. The dependence and use of this method on α_{S} was studied by Broulik and Trefil (1971) and Davies (1972) (see Sec. V.B).

B. Atomic spectra

The previous bounds on the fine-structure constant assume that other constants like the Fermi constant do not vary. The use of atomic spectra may offer cleaner tests, since we expect them to depend mainly on combinations of α_{EM}, μ, and g_{p}.

We start by recalling some basics concerning atomic spectra in order to describe the modeling of the spectra of many-electron systems, which is of great use while studying quasar absorption spectra. We then focus on laboratory experiments and the results from quasar absorption spectra.

1. α_{EM} dependence of atomic spectra

As an example, let us briefly recall the spectrum of the hydrogen atom (see, e.g., Cohen-Tannoudji *et al.*, 1986). As long as we neglect the effect of the spins and work in the nonrelativistic approximation, the spectrum is simply obtained by solving the Schrödinger equation with Hamiltonian

$$H_0=\frac{\mathbf{P}^2}{2m_{\mathrm{e}}}-\frac{e^2}{4\pi\varepsilon_0 r}, \tag{67}$$

the eigenfunction of which is of the form $\psi_{nlm}=R_n(r)Y_{lm}(\theta,\phi)$, where n is the principal quantum number. This solution has an energy

$$E_n=-\frac{E_I}{n^2}\left(1+\frac{m_{\mathrm{e}}}{m_{\mathrm{p}}}\right)^{-1} \tag{68}$$

independent of the quantum numbers l and m satisfying $0\leqslant l<n$, $|m|\leqslant l$ and where E_I is defined by Eq. (18). It follows that there are n^2 states with the same energy. The spectroscopic nomenclature refers to a given energy level by the principal quantum number and a letter designating the quantum number l $(s,p,d,f,g,\ldots$, respectively, for $l=0,1,2,3,4,\ldots)$.

This analysis neglects relativistic effects, which are expected to be typically of order $\alpha_{\rm EM}^4$ (since, in the Bohr model, $v/c=\alpha_{\rm EM}$ for the orbit $n=1$), to give the *fine structure* of the spectrum. The derivation of this fine-structure spectrum requires the solution of the Dirac equation for a particle in a potential $-q^2/r$ and then the development of the solution in the nonrelativist limit. Here, we simply use a perturbative approach in which the Hamiltonian of the system is expanded in v/c as

$$H=H_0+W, \tag{69}$$

where the corrective term W has different contributions. In what follows, we neglect for simplicity $m_{\rm e}/m_{\rm p}$. The spin-orbit interaction is described by

$$W_{\rm S.O.}=\frac{\alpha_{\rm EM}}{2m_{\rm e}c^2}\frac{\hbar c}{r^3}\frac{g_{\rm e}}{2}\mathbf{L}\cdot\mathbf{S}, \tag{70}$$

where $g_{\rm e}$ is the electron gyromagnetic factor. At lowest order, the QED loop correction gives $(g_{\rm e}-2)/2=\alpha_{\rm EM}/2\pi+\cdots$ (Schwinger, 1948) and the nonelectromagnetic contributions are smaller than 10^{-10}. It could be considered as an additional parameter of the problem (see Armendáriz-Picon, 2002). Since r is of order of the Bohr radius, it follows that $W_{\rm S.O.}\sim\alpha_{\rm EM}^2H_0$. The splitting is indeed small: for instance, it is of order 4×10^{-5} eV between the levels $2p_{3/2}$ and $2p_{1/2}$, where we have added in indices the total electron angular-moment quantum number J. The second correction arises from the $(v/c)^2$-relativistic terms and is of the form

$$W_{\rm rel}=-\frac{\mathbf{P}^4}{8m_{\rm e}^3c^2}; \tag{71}$$

it is easy to see that its amplitude is also of order $W_{\rm rel}\sim\alpha_{\rm EM}^2H_0$. The third and last correction, known as the Darwin term, arises from the fact that, in the Dirac equation, the interaction between the electron and the Coulomb field is local. But, the nonrelativist approximation leads to a nonlocal equation for the electron spinor that is sensitive to the field on a zone of order of the Compton wavelength centered in $\mathbf{r}$. It follows that

$$W_{\rm D}=\frac{\pi\hbar^2q^2}{m_{\rm e}^2c^2}\delta(\mathbf{r}). \tag{72}$$

The average in an atomic state is of order $\langle W_{\rm D}\rangle=\pi\hbar^2q^2/(2m_{\rm e}^2c^2)|\psi(\mathbf{0})|^2\sim m_{\rm e}c^2\alpha_{\rm EM}^4\sim\alpha_{\rm EM}^2H_0$. In conclusion, all the relativistic corrections are of order $\alpha_{\rm EM}^2\sim(v/c)^2$. The energy of a fine-structure level is

$$E_{nlJ}=m_{\rm e}c^2-\frac{E_I}{n^2}-\frac{m_{\rm e}c^2}{2n^4}\left(\frac{n}{J+1/2}-\frac{3}{4}\right)\alpha_{\rm EM}^4+\cdots$$

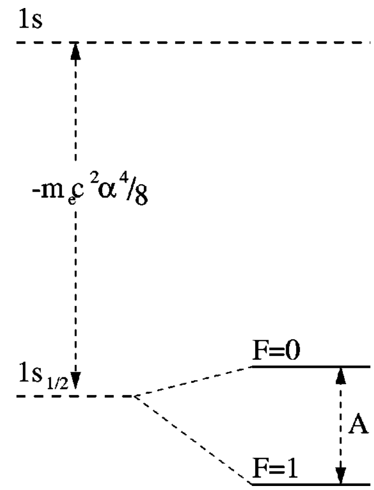

FIG. 3. Hyperfine structure of the $n=1$ level of the hydrogen atom. The fine-structure Hamiltonian induces a shift of $-m_{\rm e}c^2\alpha_{\rm EM}^4/8$ of the level $1s$. J can only take the value $+1/2$. The hyperfine Hamiltonian (74) induces a splitting of the level $1s_{1/2}$ into the two hyperfine levels $F=0$ and $F=+1$. The transition between these two levels corresponds to the 21-cm ray with $Ah^2=1\,420\,405\,751.768\pm0.001$ Hz and is of first importance in astronomy.-

and is independent[4] of the quantum number l.

A much finer effect, referred to as *hyperfine structure*, arises from the interaction between the spins of the electron, $\mathbf{S}$, and the proton, $\mathbf{I}$. They are associated, respectively, to the magnetic moments

$$\mathbf{M}_S=\frac{q\hbar}{2m_{\rm e}}\frac{g_{\rm e}}{2}\frac{\mathbf{S}}{\hbar},\quad \mathbf{M}_I=-g_{\rm p}\frac{q\hbar}{2m_{\rm p}}\frac{\mathbf{I}}{\hbar}. \tag{73}$$

Note that at this stage, the spectrum becomes dependent on the strong interaction via $g_{\rm p}$ (and via g_I in more general cases). This effect can be taken into account by adding the Hamiltonian

$$W_{\rm hf}=-\frac{\mu_0}{4\pi}\left\{\frac{q}{r^3}\mathbf{L}\cdot\mathbf{M}_I+\frac{8\pi}{3}\mathbf{M}_I\cdot\mathbf{M}_S\delta(\mathbf{r})+\frac{1}{r^3}[3(\mathbf{M}_S\cdot\mathbf{n})(\mathbf{M}_I\cdot\mathbf{n})-\mathbf{M}_I\cdot\mathbf{M}_S]\right\}, \tag{74}$$

where $\mathbf{n}$ is the unit vector pointing from the proton to the electron. The order of magnitude of this effect is typically $e^2\hbar^2/(m_{\rm e}m_{\rm p}c^2r^3)$, hence roughly 2000 times smaller than the effect of the spin-orbit coupling. It splits each fine level into a series of hyperfine levels labeled by $F\in[|J-I|,I+J]$. For instance, for the levels $2s_{1/2}$ and $2p_{1/2}$, we have $J=1/2$, and F can take the two values 0 and 1; for the level $2p_{3/2}$, $J=3/2$ and $F=1$ or $F=2$, etc. (see Fig. 3 for an example). This description neglects the quantum aspect of the electromagnetic field; one effect of the coupling of the atom to this field

[4]This is valid to all orders in $\alpha_{\rm EM}$, and the Dirac equation directly gives $E_{nlJ}=m_{\rm e}c^2[1+\alpha_{\rm EM}^2\{n-J-1/2+\sqrt{(J+1/2)^2-\alpha_{\rm EM}^2}\}^{-2}]^{-1/2}$.

is to lift the degeneracy between the levels $2s_{1/2}$ and $2p_{1/2}$. This is called the Lamb shift.

In more complex situations, the computation of the spectrum of a given atom has to take all these effects into account, but the solution of the Schrödinger equation depends on the charge distribution and has to be performed numerically.

The easiest generalization concerns hydrogenlike atoms of charge Z, for which the spectrum can be obtained by replacing e^2 by $(Ze)^2$ and $m_{\rm p}$ by $Am_{\rm p}$. For an external electron in a many-electron atom, the electron density near the nucleus is given (see, e.g., Dzuba *et al.*, 1999a, 2000) by $Z_a^2 Z/(n_* a_0)^3$, where Z_a is the effective charge felt by the external electron outside the atom, and n_* is an effective principal quantum number defined by $E_{n_*} = -E_I Z_a^2/n_*^2$. It follows that the relativistic corrections to the energy level are given by

$$\Delta E_{n_*,l,J} = \frac{E_I}{n_*^4} Z_a^2 Z^2 \alpha_{\rm EM}^2 \left[\frac{n_*}{J+1/2} - \frac{Z_a}{Z}\left(1 - \frac{Z_a}{4Z}\right)\right].$$

Such a formula does not account for many-body effects, and one expects in general a formula of the form $\Delta E_{n_*,l,J} = E_{n_*} Z^2 \alpha_{\rm EM}^2 [1/J+1/2 - C(Z,J,l)]/n_*$. Dzuba *et al.* (1999b) developed a method to compute the atomic spectra of many-electron atoms including relativistic effects. It is based on many-body perturbation theory (Dzuba *et al.*, 1996), including electron-electron correlations, and uses a correlation-potential method for the atom (Dzuba *et al.*, 1983).

Laboratory measurements can provide these spectra but only for the value of $\alpha_{\rm EM}$ today, $\alpha_{\rm EM}^{(0)}$. In order to detect a variation of $\alpha_{\rm EM}$, one needs to compute them for different values of $\alpha_{\rm EM}$. Dzuba *et al.* (1999a, 2000) describe the energy levels within one fine-structure multiplet as

$$E = E_0 + Q_1\left[\left(\frac{\alpha_{\rm EM}}{\alpha_{\rm EM}^{(0)}}\right)^2 - 1\right] + Q_2\left[\left(\frac{\alpha_{\rm EM}}{\alpha_{\rm EM}^{(0)}}\right)^4 - 1\right] + K_1 \mathbf{L}\cdot\mathbf{S}\left(\frac{\alpha_{\rm EM}}{\alpha_{\rm EM}^{(0)}}\right)^2 + K_2(\mathbf{L}\cdot\mathbf{S})^2\left(\frac{\alpha_{\rm EM}}{\alpha_{\rm EM}^{(0)}}\right)^4, \tag{75}$$

where E_0, Q_1, and Q_2 describe the configuration center. The terms in $\mathbf{L}\cdot\mathbf{S}$ induce the spin-orbit coupling, second-order spin-orbit interaction, and the first order of the Breit interaction. Experimental data can be fitted to get K_1 and K_2, and then numerical simulations determine Q_1 and Q_2. The result is conveniently written as

$$\omega = \omega_0 + q_1 x + q_2 y, \tag{76}$$

with $x \equiv [\alpha_{\rm EM}/\alpha_{\rm EM}^{(0)}]^2 - 1$ and $y \equiv [\alpha_{\rm EM}/\alpha_{\rm EM}^{(0)}]^4 - 1$. As an example, let us cite the result of Dzuba *et al.* (1999b) for Fe II,

$$\begin{aligned}
6d:\ & J=9/2, \quad \omega = 38\,458.9871 + 1394x + 38y,\\
& J=7/2, \quad \omega = 38\,660.0494 + 1632x + 0y,\\
6f:\ & J=11/2, \quad \omega = 41\,968.0642 + 1622x + 3y,\\
& J=9/2, \quad \omega = 42\,114.8329 + 1772x + 0y,\\
& J=7/2, \quad \omega = 42\,237.0500 + 1894x + 0y,\\
6p:\ & J=7/2, \quad \omega = 42\,658.2404 + 1398x - 13y
\end{aligned} \tag{77}$$

with the frequency in $\rm cm^{-1}$ for transitions from the ground state. An interesting case is Ni II (Dzuba *et al.*, 2001b), which has large relativistic effects of opposite signs,

$$\begin{aligned}
2f:\ & J=7/2, \quad \omega = 57\,080.373 - 300x,\\
6d:\ & J=5/2, \quad \omega = 57\,420.013 - 700x,\\
6f:\ & J=5/2, \quad \omega = 58\,493.071 + 800x.
\end{aligned} \tag{78}$$

Such results are particularly useful to compare with spectra obtained from quasar-absorption systems such as in the analysis by Murphy *et al.* (2001c). These results were recently revisited by Dzuba *et al.* (2001a).

In conclusion, the key points are that the spectra of atoms depend mainly on μ, $\alpha_{\rm EM}$, and $g_{\rm p}$ and contain terms both in $\alpha_{\rm EM}^2$ and $\alpha_{\rm EM}^4$, and that typically

$$H = \alpha_{\rm EM}^2 \tilde{H}_0 + \alpha_{\rm EM}^4 \tilde{W}_{\rm fine} + g_{\rm p}\mu^2 \alpha_{\rm EM}^4 \tilde{W}_{\rm hyperfine}, \tag{79}$$

so that by comparing different kinds of transitions in different atoms one can hope to measure these constants despite the fact that $\alpha_{\rm S}$ plays a role via the nuclear-magnetic moment. Note that $\tilde{W}_{\rm fine}$ and $\tilde{W}_{\rm hyperfine}$ depend on $g_{\rm e}$ and μ, which can have some implications while setting constraints (Armendáriz-Picon, 2002). We describe in the next section the laboratory experiments and then turn to the measurement of quasar absorption spectra.

2. Laboratory experiments

Laboratory experiments are based on the comparison either of different atomic clocks or of an atomic clock with ultrastable oscillators. They are thus based only on the quantum-mechanical theory of the atomic spectra. They also have the advantage of being more reliable and reproducible, thus allowing for better control of the systematics and better statistics. Their evident drawback is their short time scale, fixed by the fractional stability of the least precise standards. This time scale is of order of a month to a year, so that the obtained constraints are restricted to the instantaneous variation today, but this can be compensated for by the extreme sensitivity. Laboratory experiments involve the comparison of ultrastable oscillators of different compositions or atomic clocks of different species. Solid resonators, electronic, fine-structure and hyperfine-structure transitions, respectively, give access to $R_\infty/\alpha_{\rm EM}$, R_∞, $R_\infty\alpha_{\rm EM}^2$, and $g_{\rm p}\mu R_\infty \alpha_{\rm EM}^2$.

Turneaure and Stein (1974) compared cesium atomic clocks with superconducting microwave cavity oscillators. The frequency of the cavity-controlled oscillators was compared during 10 days to one of a cesium beam. The relative drift rate was $(-0.4\pm3.4)\times10^{-14}\ \rm day^{-1}$. The dimensions of the cavity depend on the Bohr radius of the atom, while the cesium-clock frequency depends on $g_{\rm p}\mu\alpha_{\rm EM}^2$ (hyperfine transition). It follows that $\nu_{\rm Cs}/\nu_{\rm cavity} \propto g_{\rm p}\mu\alpha_{\rm EM}^3$ so that

$$\frac{d}{dt}\ln(g_{\rm p}\mu\alpha_{\rm EM}^3)<4.1\times10^{-12}\ {\rm yr}^{-1}. \tag{80}$$

Godone *et al.* (1993) compared the frequencies of cesium and magnesium atomic beams. The cesium clock, used to define the second in the SI system of units, is based on the *hyperfine transition* $F=3$, $m_{\rm F}=0\rightarrow F=4$, $m_{\rm F}=0$ in the ground state $6^2s_{1/2}$ of ^{133}Cs, with frequency given, at lowest order and neglecting relativistic and quantum electrodynamic corrections, by

$$\nu_{\rm Cs}=\frac{32cR_\infty Z_s^3\alpha_{\rm EM}^2}{3n^3}g_I\mu\sim9.2\ {\rm GHz}, \tag{81}$$

where Z_s is the effective nuclear charge and g_I is the cesium-nucleus gyromagnetic ratio. The magnesium clock is based on the frequency of the *fine-structure* transition $3p_1\rightarrow3p_0$, $\Delta m_j=0$ in the metastable triplet of ^{24}Mg,

$$\nu_{\rm Hg}=\frac{cR_\infty Z_s^4\alpha_{\rm EM}^2}{6n^3}\sim601\ {\rm GHz}. \tag{82}$$

It follows that

$$\frac{d}{dt}\ln\frac{\nu_{\rm Cs}}{\nu_{\rm Hg}}=\left[\frac{d}{dt}\ln(g_I\mu)\right]\times(1\pm10^{-2}). \tag{83}$$

The experiment led to the bound

$$\left|\frac{d}{dt}\ln(g_{\rm p}\mu)\right|<5.4\times10^{-13}\ {\rm yr}^{-1} \tag{84}$$

after using the constraint $d\ln(g_{\rm p}/g_I)/dt<5.5\times10^{-14}\ {\rm yr}^{-1}$ (Demidov *et al.*, 1992). When combined with the astrophysical result by Wolfe *et al.* (1976) on the constraint of $g_{\rm p}\mu\alpha_{\rm EM}^2$ (see Sec. V.D), it can be deduced that

$$|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|<2.7\times10^{-13}\ {\rm yr}^{-1}. \tag{85}$$

We note that relativistic corrections were neglected.

Prestage *et al.* (1995) compared the rates of different atomic clocks based on hyperfine transitions in alkali atoms with different atomic numbers. The frequency of the hyperfine transition between $I\pm1/2$ states is given by (see, e.g., Vanier and Audouin, 1989)

$$\nu_{\rm alkali}=\frac{8}{3}\left(I+\frac{1}{2}\right)\alpha_{\rm EM}^2g_IZ\frac{z^2}{n_*^3}\left(1-\frac{d\Delta_n}{dn}\right)F_{\rm rel}(\alpha_{\rm EM}Z)\times(1-\delta)(1-\epsilon)\mu R_\infty c, \tag{86}$$

where z is the charge of the remaining ion once the valence electron has been removed and $\Delta_n=n-n_*$. The term $(1-\delta)$ is the correction to the potential with respect to the Coulomb potential and $(1-\epsilon)$ a correction for the finite size of the nuclear magnetic dipole moment. It is estimated that $\delta\simeq4-12\ \%$ and $\epsilon\simeq0.5\%$. $F_{\rm rel}(\alpha_{\rm EM}Z)$ is the Casimir relativistic contribution to the hyperfine structure, and one takes advantage of the increasing importance of $F_{\rm rel}$ as the atomic number increases (see Fig. 4). It follows that

$$\frac{d}{dt}\ln\frac{\nu_{\rm alkali}}{\nu_{\rm H}}=\frac{\dot\alpha_{\rm EM}}{\alpha_{\rm EM}}\frac{d\ln F_{\rm rel}(\alpha_{\rm EM}Z)}{d\ln\alpha_{\rm EM}}, \tag{87}$$

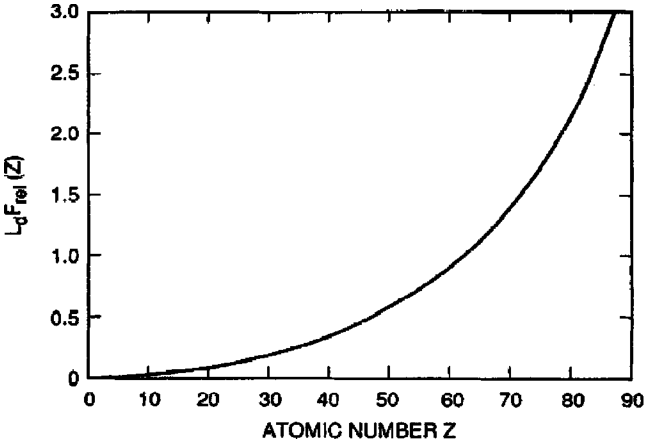

FIG. 4. The correction function $F_{\rm rel}$. From Prestage *et al.* (1995).

where $\nu_{\rm H}$ is the frequency of a H maser; when comparing two alkali atoms

$$\frac{d}{dt}\ln\frac{\nu_{\rm alkali1}}{\nu_{\rm alkali2}}=\frac{\dot\alpha_{\rm EM}}{\alpha_{\rm EM}}\left(\left.\frac{d\ln F_{\rm rel}}{d\ln\alpha_{\rm EM}}\right|_1-\left.\frac{d\ln F_{\rm rel}}{d\ln\alpha_{\rm EM}}\right|_2\right). \tag{88}$$

The comparison of different alkali clocks was performed, and the comparison of Hg^+ ions with a cavity-tuned H maser over a period of 140 days led to the conclusion that

$$|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|<3.7\times10^{-14}\ {\rm yr}^{-1}. \tag{89}$$

This method constrains in fact the variation of the quantity $\alpha_{\rm EM}g_{\rm p}/g_I$. One delicate point is the evaluation of the correction function and the form used by Prestage *et al.* (1995) $[F_{\rm rel}\sim1+11(Z\alpha_{\rm EM})^2/6+\cdots]$ that differs from the 1s $[F_{\rm rel}\sim1+3(Z\alpha_{\rm EM})^2/2+\cdots]$ and 2s $[F_{\rm rel}\sim1+17(Z\alpha_{\rm EM})^2/8+\cdots]$ results for hydrogenlike atoms (Breit, 1930). The question of the accuracy of these computations is currently debated [see, e.g., Karshenboim (2000)]. They should not affect the final constraint by more than 10–50 % and will not change its order of magnitude.

Sortais *et al.* (2001) compared a rubidium to a cesium clock over a period of 24 months and deduced that $d\ln(\nu_{\rm Rb}/\nu_{\rm Cs})/dt=(1.9\pm3.1)\times10^{-15}\ {\rm yr}^{-1}$, hence improving the frequency uncertainty by a factor 20 relative to Prestage *et al.* (1995). Assuming that $g_{\rm p}$ is constant and staying within the Prestage *et al.* (1995) framework, they deduced

$$\dot\alpha_{\rm EM}/\alpha_{\rm EM}=(4.2\pm6.9)\times10^{-15}\ {\rm yr}^{-1} \tag{90}$$

if all the drift can be attributed to the Casimir relativistic correction $F_{\rm rel}$. These sensitivities have been revised by Karshenboim (2000).

All the results and characteristics of these experiments are summed up in Table I. Recently, Braxmaier *et al.* (2001) proposed a new method to test the variability of $\alpha_{\rm EM}$ and μ using electromagnetic resonators filled with a dielectric. The index of the dielectric depending on both $\alpha_{\rm EM}$ and μ, the comparison of two oscillators could lead to an accuracy of $4\times10^{-15}\ {\rm yr}^{-1}$. Torgerson (2000) proposed a comparison of atom-stabilized optical

TABLE I. The different atomic clock experiments. We recall the transitions which are compared and the constraint on the time variation obtained. SCO refers to superconductor cavity oscillator and the reference to Breakiron (1993) is cited in Prestage *et al.* (1995). fs and hfs refer, respectively, to fine structure and hyperfine structure.

Reference	Experiment	Constant	Duration	Limit (yr^{-1})
(Turneaure and Stein, 1974)	hfs of Cs vs SCO	$g_p \mu \alpha_{EM}^3$	12 days	$<1.5\times10^{-12}$
(Godone *et al.*, 1993)	hfs of Cs vs fs of Mg	$g_p \mu$	1 year	$<2.5\times10^{-13}$
(Demidov *et al.*, 1992)	hfs of Cs vs hfs of H	$\alpha_{EM} g_p / g_I$	1 year	$<5.5\times10^{-14}$
(Breakiron, 1993)	hfs of Cs vs hfs of H	$\alpha_{EM} g_p / g_I$		$<5\times10^{-14}$
(Prestage *et al.*, 1995)	hfs of HG^+ vs hfs of H	$\alpha_{EM} g_p / g_I$	140 days	$<2.7\times10^{-14}$
(Sortais *et al.*, 2001)	hfs of Cs vs hfs of Rb		24 months	$(4.2\pm6.9)\times10^{-15}$

frequency using an optical resonator. In an explicit example using indium and thalium, it is argued that a precision of $\dot{\alpha}_{EM}/\alpha_{EM} \sim 10^{-18}/t$, t being the time of the experiment, might be reached. Note that to go beyond a precision of 10^{-17} the clocks have to be located at the same place since this is roughly the order of magnitude, for instance, of the Channel tide on the determination of the geoid at Paris.

Finally, let us note that similar techniques were used to test local Lorentz invariance (Lamoreaux *et al.*, 1986; Chupp *et al.*, 1989) and CPT symmetry (Bluhm *et al.*, 2002). In the former case, the breakdown of local Lorentz invariance would cause shifts in the energy levels of atoms and nuclei that depend on the orientation of the quantization axis of the state with respect to a universal velocity vector, and thus on the quantum numbers of the state.

3. Astrophysical observations

The observation of spectra of distant astrophysical objects encodes information about the atomic energy levels at the position and time of emission. As long as one sticks to the nonrelativistic approximation, the atomic transition energies are proportional to the Rydberg energy and all transitions have the same α_{EM} dependence, so that the variation will affect all the wavelengths by the same factor. Such a uniform shift of the spectra cannot be distinguished from a Doppler effect due to the motion of the source or to the gravitational field where it sits.

The idea is to compare different absorption lines from different species or, equivalently, the redshift associated with them. According to the lines compared, one can extract information about different combinations of the constants at the time of emission (see Table II).

While performing this kind of observation, one has to take into account and control a number of problems and systematic effects.

(1) Errors in the determination of laboratory wavelengths to which the observations are compared must be acknowledged.
(2) While comparing wavelengths from different atoms, one has to take into account that they may be located in different regions of the cloud with different velocities and hence with different Doppler redshift.
(3) One has to ensure that there is no light blending.
(4) The differential isotopic saturation has to be controlled. Usually quasar absorption systems are expected to have lower heavy-element abundances (Prochoska and Wolfe, 1996, 1997, 2000). The spatial inhomogeneity of these abundances may also play a role.
(5) One must be aware that hyperfine splitting can induce a saturation similar to isotopic abundances.
(6) The fact that variation of the velocity of the Earth during the integration of a quasar spectrum can induce differential Doppler shift must be recognized.
(7) One should allow for atmospheric dispersion across the spectral direction of the spectrograph slit, which can stretch the spectrum. It was shown that this can only mimic a negative $\Delta\alpha_{EM}/\alpha_{EM}$ (Murphy *et al.*, 2001a).
(8) It should be noted that the presence of a magnetic field will shift the energy levels by Zeeman effect.
(9) One must check temperature variations during the observation, which will change the air refractive index in the spectrograph.
(10) Instrumental effects such as variations of the intrinsic instrument profile have to be controlled.

The effect of these possible systematic errors is discussed by Murphy *et al.* (2001a). In the particular case of the comparison of hydrogen and molecular lines, Wiklind and Combes (1997) argued that the detection of the variation of μ was limited to $\Delta\mu/\mu \simeq 10^{-5}$. One possibility of reducing the systematics is to look at atoms having relativistic corrections of different signs (see Sec. III.B) since the systematics are not expected, *a priori*, to simulate the correlation of the shift of different lines of a multiplet [see, e.g., the example of Ni II (Dzuba *et al.*,

TABLE II. Comparison of absorption lines and the combinations of the fundamental constants that can be constrained.

Comparison	Constant
Fine-structure doublet	α_{EM}
Hyperfine H vs optical	$g_p \mu \alpha_{EM}^2$
Hyperfine H vs fine structure	$g_p \mu \alpha_{EM}$
Rotational vs vibrational modes of molecules	μ
Rotational modes vs hyperfine H	$g_p \alpha_{EM}^2$
Fine-structure doublet vs hyperfine H	g_p

2001a, 2001b)]. Besides the systematics, statistical errors were important in early studies but have now enormously decreased.

An efficient method is to observe fine-structure doublets for which

$$\Delta\nu=\frac{\alpha_{\rm EM}^2 Z^4 R_\infty}{2n^3}\,{\rm cm}^{-1}, \tag{91}$$

$\Delta\nu$ being the frequency splitting between the two lines of the doublet and $\bar{\nu}$ the mean frequency (Bethe and Salpeter, 1977). It follows that $\Delta\nu/\bar{\nu}\propto\alpha_{\rm EM}^2$ and thus $\Delta\ln\lambda|_z/\Delta\ln\lambda|_0=[1+\Delta\alpha_{\rm EM}/\alpha_{\rm EM}]^2$. It can be inverted to give $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ as a function of $\Delta\lambda$ and $\bar{\lambda}$ as

$$\left(\frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}}\right)(z)=\frac{1}{2}\left[\left(\frac{\Delta\lambda}{\bar{\lambda}}\right)_z\bigg/\left(\frac{\Delta\lambda}{\bar{\lambda}}\right)_0-1\right]. \tag{92}$$

As an example, it takes the following form for Si IV (Varshalovich, Potekhin *et al.*, 1996):

$$\left(\frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}}\right)(z)=77.55\left(\frac{\Delta\lambda}{\bar{\lambda}}\right)_z-0.5. \tag{93}$$

Since the observed wavelengths are redshifted as $\lambda_{\rm obs}=\lambda_{em}(1+z)$, it reduces to

$$\left(\frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}}\right)(z)=77.55\frac{\Delta z}{1+\bar{z}}. \tag{94}$$

As a conclusion, by measuring the two wavelengths of the doublet and comparing the result to laboratory values, one can measure the time variation of the fine-structure constant. This method has been applied to different systems and is the only one that gives a direct measurement of $\alpha_{\rm EM}$.

The first to realize that the fine and hyperfine structures can help to disentangle the redshift effect from a possible variation of $\alpha_{\rm EM}$ was Savedoff (1956), and Wilkinson (1958) pointed out that "the interpretation of redshift of spectral lines probably implies that atomic constants have not changed by more than 10^{-9} parts per year."

Savedoff (1956) used the data by Minkowski and Wolson (1956) of the spectral lines of H, N II, O I, O II, Ne III, and N V for the radio source Cygnus A of redshift $z\sim0.057$. Using the data for the fine-structure doublet of N II and Ne III and assuming that the splitting was proportional to $\alpha_{\rm EM}^2(1+z)$ led him to

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(1.8\pm1.6)\times10^{-3}. \tag{95}$$

Bahcall and Salpeter (1965) used the fine-structure splitting of the O III and Ne III emission lines in the spectra of the quasistellar radio sources 3C 47 and 3C 147. Bahcall *et al.* (1967) used the observed fine structure of Si II and Si IV in the quasistellar radio sources 3C 191 to deduce that

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-2\pm5)\times10^{-2} \tag{96}$$

at a redshift $z=1.95$. Gamow (1967a) criticized these measurements and suggested that the observed absorption lines were not associated with the quasistellar source but were instead produced in the intervening galaxies. But Bahcall *et al.* (1967) showed in the particular example of 3C 191 that the excited fine-structure states of Si II were seen to be populated in the spectrum of this object and that the photon fluxes required to populate these states were orders of magnitude too high to be obtained in intervening galaxies.

Bahcall and Schmidt (1967) then used the emission lines of the O III multiplet of the spectra of five radio galaxies with redshift of order $z\sim0.2$ to improve the former bound to

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(1\pm2)\times10^{-3}, \tag{97}$$

considering only statistical errors.

Wolfe *et al.* (1976) studied the spectrum of AO 0235 +164, a BL Lac object with redshift $z\sim0.5$. From the comparison of the hydrogen hyperfine frequency with the resonance line for Mg^+, they obtained a constraint on $g_{\rm p}\mu\alpha_{\rm EM}^2$ (see Sec. V.D). From the comparison with the Mg^+ fine-structure separations they constrained $g_{\rm p}\mu\alpha_{\rm EM}$, and the Mg^+ fine-structure doublet splitting gave

$$|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<3\times10^{-2}. \tag{98}$$

Potekhin and Varshalovich (1994) extended this method based on the absorption lines of alkalilike atoms and compared the wavelengths of a catalog of transitions $2s_{1/2}$-$2p_{3/2}$ and $2s_{1/2}$-$2p_{1/2}$ for a set of five elements. The advantages of such a method are that (1) it is based on the measurement of the difference of wavelengths that can be measured much more accurately than (broader) emission lines, and (2) these transitions correspond to transitions from a single level and are thus not affected by differences in the radial-velocity distributions of different ions. They used data on 1414 absorption doublets of C IV, N V, O VI, Mg II, Al III, and Si IV and obtained

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(2.1\pm2.3)\times10^{-3} \tag{99}$$

at $z\sim3.2$ and $|d\ln\alpha_{\rm EM}/dz|<5.6\times10^{-4}$ between $z=0.2$ and $z=3.7$ at 2σ level. In these measurements, Si IV, the most widely spaced doublet, is the most sensitive to a change in $\alpha_{\rm EM}$. The use of a large number of systems allows one to reduce statistical error and to obtain a redshift dependence after averaging over the celestial sphere. Note, however, that averaging on shells of constant redshift implies that we average over *a priori* noncausally connected regions in which the value of the fine-structure constant may *a priori* be different. This result was further constrained by Varshalovitch and Potekhin (1994), who extended the catalog to 1487 pairs of lines and got

$$|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<1.5\times10^{-3} \tag{100}$$

at $z\sim3.2$. It was also shown that the fine-structure splitting was the same in eight causally disconnected regions at $z=2.2$ at a 3σ level.

Cowie and Songaila (1995) improved the previous analysis to get

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.3\pm1.9)\times10^{-4} \tag{101}$$

for quasars between $z=2.785$ and $z=3.191$. Varshalovich, Potekhin, *et al.* (1996) used the fine-structure doublet of Si IV to get

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(2\pm7)\times10^{-5} \tag{102}$$

at 2σ for quasars between $z=2.8$ and $z=3.1$ (see also Varshalovich, Panchuk, and Ivanchik, 1996).

Varshalovich *et al.* (2000a, 2000b) studied the doublet lines of Si IV, C IV, and Ng II and focused on the fine-structure doublet of Si IV to get

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-4.5\pm4.3[{\rm stat}]\pm1.4[{\rm syst}])\times10^{-5} \tag{103}$$

for $z=2-4$. An update of this analysis (Ivanchik *et al.*, 1999) with 20 absorption systems between $z=2$ and $z=3.2$ gave

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-3.3\pm6.5[{\rm stat}]\pm8[{\rm syst}])\times10^{-5}. \tag{104}$$

Murphy *et al.* (2001d) used the same method with 21 Si IV absorption systems toward 8 quasars with redshift $z\sim2-3$ to get

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.5\pm1.3)\times10^{-5}, \tag{105}$$

hence improving the previous constraint by a factor of 3.

Recently Dzuba *et al.* (1999a, 1999b) and Webb *et al.* (1999) introduced a new method referred to as the *many-multiplet* method, in which one correlates the shift of the absorption lines of a set of multiplets of different ions. It is based on the parametrization (76) of the computation of atomic spectra. One advantage is that the correlation between different lines can reduce the systematics. That one can compare the transitions from different ground states is an improvement; using ions with very different atomic mass also increases the sensitivity because the difference between ground-states' relativistic corrections can be very large and even of opposite sign (see the example of Ni II by Dzuba *et al.*, 2001a, 2001b).

Webb *et al.* (1999) analyzed one transition of the Mg II doublet and five Fe II transitions from three multiplets. The limit of accuracy of the method is set by the frequency interval between Mg II 2796 and Fe II 2383, which induces a fractional change of $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\sim10^{-5}$. Using the simulations by Dzuba *et al.* (1999a, 1999b), one can deduce that a change in $\alpha_{\rm EM}$ induces a large change in the spectrum of Fe II and a small one for Mg II (the magnitude of the effect being mainly related to the atomic charge). The method is then to measure the shift of the Fe II spectrum with respect to that of Mg II. This comparison increases the sensitivity compared with methods using only alkali doublets. Using 30 absorption systems toward 17 quasars, they obtained

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.17\pm0.39)\times10^{-5}, \tag{106}$$

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-1.88\pm0.53)\times10^{-5}, \tag{107}$$

respectively, for $0.6<z<1$ and $1<z<1.6$. There is no signal of a variation of $\alpha_{\rm EM}$ for redshift smaller than 1 but a 3.5σ deviation for redshifts larger than 1 and particularly in the range $z\sim0.9-1.2$. The summary of these measurements are depicted in Fig. 5. A possible explanation is a variation of the isotopic ratio, but the change of ^{26}Mg/^{24}Mg would need to be substantial to explain the result (Murphy *et al.*, 2001a). Calibration effects can also be important since Fe II and Mg II lines are situated in a different order of magnitude of the spectra.

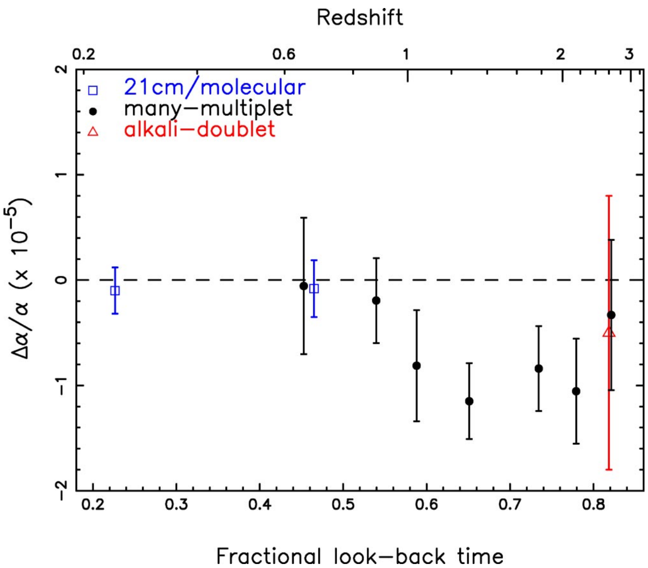

FIG. 5. $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ as a function of the look-back time computed with the cosmological parameters $(\Omega_m,\Omega_\Lambda)=(0.3,0.7)$ and $h=0.68$. □, data by Murphy *et al.* (2001d) assuming $g_{\rm p}$ constant; △, Si IV systems by Murphy *et al.* (2001b); ●, Mg II and Fe II systems for redshifts smaller than 1.6 (Webb *et al.*, 2001) and higher redshifts come from Murphy *et al.* (2001c). From Webb *et al.* (2001) (Color in online edition).

Murphy *et al.* (2001c) extended this technique of fitting of the absorption lines to the species Mg I, Mg II, Al II, Al III, Si II, Cr II, Fe II, Ni II, and Zn II for 49 absorption systems towards 28 quasars with redshift $z\sim0.5-3.5$ and got

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.2\pm0.3)\times10^{-5}, \tag{108}$$

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-1.2\pm0.3)\times10^{-5}, \tag{109}$$

respectively, for $0.5<z<1$ and $1<z<1.8$ at 4.1σ. The low redshift part is a reanalysis of the data by Webb *et al.* (1999). Over the whole sample $(z=0.5-1.8)$ it gives the constraint

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.7\pm0.23)\times10^{-5}. \tag{110}$$

Webb *et al.* (2001) reanalyzed their initial sample and included new optical QSO (quasistellar object) data to have 28 absorption systems with redshift $z=0.5-1.8$, plus 18 damped Lyman-α absorption systems toward 13 QSO, and, in addition, 21 Si IV absorption systems toward 13 QSO. The analysis used mainly the multiplets of Ni II, Cr II, and Zn II, and Mg I, Mg II, Al II, Al III, and Fe II were also included. One improvement compared with the analysis by Webb *et al.* (1999) is that the "q" coefficient of Ni II, Cr II, and Zn II in Eq. (76) vary both in magnitude and sign so that lines shift in opposite directions. The data were reduced to get 72 individual estimates of $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ spanning a large range of redshift. From the Fe II and Mg II sample they obtained

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.7\pm0.23)\times10^{-5} \tag{111}$$

for $z=0.5-1.8$; from the Ni II, Cr II, and Zn II they got

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.76\pm0.28)\times10^{-5} \tag{112}$$

for $z=1.8-3.5$ at a 4σ level. The evaluation of the confidence level is not made explicit by the authors and does not seem to take into account the systematic effects (in the example of two effects, it was argued that these effects amplify the deviation from $\Delta\alpha_{\rm EM}=0$ so that they would enhance the significance of the result). It probably refers only to the statistical confidence level. The fine structure of Si IV gave

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=(-0.5\pm1.3)\times10^{-5} \tag{113}$$

for $z=2-3$.

This series of results is of great importance since all other constraints are just upper bounds. Note that they are incompatible with both Oklo ($z\sim0.14$) and meteorite data ($z\sim0.45$) if the variation is linear with time. Such a nonzero detection, if confirmed, will have tremendous implications concerning our understanding of physics. Among the first questions that arise are those of whether this variation is compatible with other bounds (e.g., testing for the universality of free fall), the level of detection needed by the other experiments when the level of variation by Webb *et al.* (2001) is known, the amplitude of the variation of the other constants, and whether any systematic effects have been forgotten. For instance, the fact that Mg II and Fe II are *a priori* not in the same region of the cloud was not modeled; this could increase the errors even if it is difficult to think that it can mimic the observed variation of $\alpha_{\rm EM}$. If one forgets the two points arising from HI 21 cm and molecular absorption systems (□ in Fig. 5), the best fit of the data of Fig. 5 does not seem to favor today's value of the fine-structure constant. This could indicate an unknown systematic effect. Besides, if the variation of $\alpha_{\rm EM}$ is linear then these observations are incompatible with the Oklo results.

C. Cosmological constraints

1. Cosmic microwave background

The cosmic microwave background (CMB) radiation is composed of the photons emitted at the time of the recombination of hydrogen and helium when the universe was about 300 000 years old [see, e.g., Durrer (2001) or Hu and Dodelson (2002) for recent reviews on CMB radiation physics]. This radiation is observed to be a black body with a temperature $T=2.723$ K with small anisotropies of order of μK. The temperature fluctuation in a direction (ϑ,φ) is usually decomposed on a basis of spherical harmonics as

$$\frac{\delta T}{T}(\vartheta,\varphi)=\sum_{\ell}\sum_{m=-\ell}^{m=+\ell}a_{\ell m}Y_{\ell m}(\vartheta,\varphi). \tag{114}$$

The angular power-spectrum multipole $C_\ell=\langle|a_{lm}|^2\rangle$ is the coefficient of the decomposition of the angular correlation function on Legendre polynomials. Given a model of structure formation and a set of cosmological parameters, this angular power spectrum can be computed and compared to observational data in order to constrain this set of parameters.

Prior to recombination, the photons are tightly coupled to the electrons, but after recombination they can be considered mainly as free particles. Changing the fine-structure constant modifies the strength of the electromagnetic interaction. Thus the only effect on CMB anisotropies arises from the change in the differential optical depth of photons due to the Thomson scattering

$$\dot{\tau}=x_{\rm e}n_{\rm e}c\sigma_{\rm T}, \tag{115}$$

which enters in the collision term of the Boltzmann equation describing the evolution of the photon distribution function, where $x_{\rm e}$ is the ionization fraction (i.e., the number density of free electrons with respect to their total number density $n_{\rm e}$). The first dependence of the optical depth on the fine-structure constant arises from the Thomson-scattering cross section given by

$$\sigma_{\rm T}=\frac{8\pi}{3}\frac{\hbar^2}{m_{\rm e}^2c^2}\alpha_{\rm EM}^2, \tag{116}$$

and the scattering by free protons can be neglected since $m_{\rm e}/m_{\rm p}\sim5\times10^{-4}$. The second and more subtle dependence comes from the ionization fraction. Recombination proceeds via two-photon emission from the $2s$ level or via the Ly-α photons that are redshifted out of the resonance line (Peebles, 1968) because recombination to the ground state can be neglected since it leads to immediate reionization of another hydrogen atom by the emission of a Ly-α photon. Following Ma and Bertschinger (1995) and Peebles (1968), and taking into account only the recombination of hydrogen, we see that the equation of evolution of the ionization fraction takes the form

$$\frac{dx_{\rm e}}{dt}=\mathcal{C}\left[\beta(1-x_{\rm e})\exp\left(-\frac{B_1-B_2}{k_{\rm B}T}\right)-\mathcal{R}n_{\rm p}x_{\rm e}^2\right].$$

$B_n=-E_I/n^2$ is the energy of the nth hydrogen atomic level, β is the ionization coefficient, $\mathcal{R}$ the recombination coefficient, $\mathcal{C}$ the correction constant due to the redshift of Ly-α photons and to two-photon decay, and $n_p=n_e$ is the number of protons. β is related to $\mathcal{R}$ by the principle of detailed balance so that

$$\beta=\mathcal{R}\left(\frac{2\pi m_{\rm e}k_{\rm B}T}{h^2}\right)\exp\left(-\frac{B_2}{k_{\rm B}T}\right). \tag{117}$$

The recombination rate to all other excited levels is

$$\mathcal{R}=\frac{8\pi}{c^2}\left(\frac{k_{\rm B}T}{2\pi m_{\rm e}}\right)^{3/2}\sum_{n,l}^{*}(2l+1)e^{B_n/k_{\rm B}T}\times\int_{B_n/k_{\rm B}T}^{\infty}\sigma_{nl}\frac{y^2dy}{e^y-1},$$

where σ_{nl} is the ionization cross section for the (n,l) excited level of hydrogen. The asterisk indicates that the sum needs to be regularized and the $\alpha_{\rm EM}$, $m_{\rm e}$ dependence of the ionization cross section is complicated to extract. It can, however, be shown to behave as $\sigma_{nl} \propto \alpha_{\rm EM}^{-1} m_{\rm e}^{-2} f(h\nu/B_1)$.

Finally, the factor $\mathcal{C}$ is given by

$$\mathcal{C}=\frac{1+K\Lambda_{2s}(1-x_e)}{1+K(\beta+\Lambda_{2s})(1-x_e)}, \tag{118}$$

where Λ_{2s} is the rate of decay of the $2s$ excited level to the ground state via two photons; it scales as $m_{\rm e}\alpha_{\rm EM}^8$. The constant K is given in terms of the Ly-α photon $\lambda_\alpha=16\pi\hbar/(3m_{\rm e}\alpha_{\rm EM}^2 c)$ by $K=n_p\lambda_\alpha^3/(8\pi H)$, and scales as $m_{\rm e}^{-3}\alpha_{\rm EM}^{-6}$.

Changing $\alpha_{\rm EM}$ will thus have two effects: first, it changes the temperature at which the last scattering happens, and second it changes the residual ionization after recombination. Both effects influence the CMB temperature anisotropies [see Kaplinghat *et al.* (1999) and Battye *et al.* (2001) for discussions]. The last scattering can roughly be determined by the maximum of the visibility function $g=\dot\tau\exp(-\tau)$, which measures the differential probability for a photon to be scattered at a given redshift. Increasing $\alpha_{\rm EM}$ shifts g to higher redshift, at which the expansion rate is faster, so that the temperature and x_e decrease more rapidly, resulting in a narrower g. This induces a shift of the C_ℓ spectrum to higher multipoles and an increase of the values of the C_ℓ. The first effect can be understood by the fact that pushing the last-scattering surface to a higher redshift leads to a smaller sound horizon at decoupling. The second effect results from a smaller Silk damping.

Hannestad (1999) and then Kaplinghat *et al.* (1999) implemented these equations in a Boltzmann code, taking into account only the recombination of hydrogen and neglecting helium, and showed that coming satellite experiments such as MAP[5] and Planck[6] should provide a constraint on $\alpha_{\rm EM}$ at recombination with a precision $|\dot\alpha_{\rm EM}/\alpha_{\rm EM}|\leq 7\times10^{-13}\,{\rm yr}^{-1}$, which corresponds to a sensitivity $|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|\sim10^{-2}-10^{-3}$ at a redshift of about $z\sim1000$. Avelino *et al.* (2000a) studied the dependence of the position of the first acoustic peak on $\alpha_{\rm EM}$. Hannestad (1999) chose the underlying ΛCDM (Λ cold dark matter) model $(\Omega,\Omega_{\rm b},\Lambda,h,n,N_\nu,\tau,\alpha_{\rm EM})=(1,0.08,0,0.5,1,3,0,\alpha_{\rm EM}^{(0)})$ and performed an eight-parameter fit to determine the precision to which the parameters could be extracted. Kaplinghat *et al.* (1999) worked with the parameters $(h,\Omega_{\rm b},\Lambda,N_\nu,Y_{\rm p},\alpha_{\rm EM})$. They showed that the precision of $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ varies from 10^{-2} if the maximum observed CMB multipole is of order 500–1000 to 10^{-3} if one observes multipoles higher than 1500.

Avelino *et al.* (2000a) claim that BOOMERanG and MAXIMA data favor a value of $\alpha_{\rm EM}$ smaller by a few percent in the past (see also Martins *et al.*, 2002), and Battye *et al.* (2001) showed that the fit to current CMB data is improved by allowing $\Delta\alpha_{\rm EM}\neq0$. Battye *et al.* also pointed out that the evidence of a variation of the fine-structure constant can be thought of as favoring a delayed recombination model (assuming $\Omega=1$ and $n=1$). Avelino *et al.* (2001) then performed a joint analysis of nucleosynthesis and CMB data and did not find any evidence for a variation of $\alpha_{\rm EM}$ at the 1σ level at either epoch. They considered $\Omega_{\rm b}$ and $\Delta\alpha_{\rm EM}$ as independent, and the marginalization over one of the two parameters led to

$$-0.09<\Delta\alpha_{\rm EM}/\alpha_{\rm EM}<0.02 \tag{119}$$

at a 68% confidence level. Martins *et al.* (2002) concluded that MAP and Planck will allow one to set, respectively, a 2.2% and 0.4% constraint at 1σ if all other parameters are marginalized. Landau *et al.* (2001) concluded from the study of BOOMERanG, MAXIMA, and COBE data in spatially flat models with adiabatic primordial fluctuations that, at the 2σ level,

$$-0.14<\Delta\alpha_{\rm EM}/\alpha_{\rm EM}<0.03. \tag{120}$$

All these works assume that only $\alpha_{\rm EM}$ is varying, but, as can been seen from Eqs. (114) to (118), one has to assume the constancy of the electron mass. Battye *et al.* (2001) show that the changes in the fine-structure constant and in the mass of the electron are degenerate according to $\Delta\alpha_{\rm EM}\approx0.39\Delta m_{\rm e}$, but that this degeneracy is broken for multipoles higher than 1500. The variation of the gravitational constant can also have similar effects on the CMB (Riazuelo and Uzan, 2002). All the authors also assume the $\alpha_{\rm EM}$ dependence of $\mathcal{R}$ was negligible, and Battye *et al.* (2001) checked that the helium recombination was negligible in the range of $\Delta\alpha_{\rm EM}$ considered.

In conclusion, strong constraints on the variation of $\alpha_{\rm EM}$ can be obtained from the CMB only if the cosmological parameters are independently known. This method is thus noncompetitive unless one has strong bounds on $\Omega_{\rm b}$ and h (and the result will always be conditional to the model of structure formation) and assumptions about the variation of other constants such as the electron mass and gravitational constant are made.

2. Nucleosynthesis

The amount of ^{4}He produced during the Big Bang nucleosynthesis is mainly determined by the neutron-to-proton ratio at the freeze-out of the weak interactions that interconvert neutrons and protons. The result of Big Bang nucleosynthesis (BBN) thus depends on G, $\alpha_{\rm W}$, $\alpha_{\rm EM}$, and $\alpha_{\rm S}$, respectively, through the expansion rate, the neutron-to-proton ratio, the neutron-proton mass difference, and the nuclear reaction rates, besides the standard parameters such as, for example, the number of neutrino families. The standard BBN scenario (see, e.g., Malaney and Mathews, 1993; Reeves, 1994) proceeds in three main steps.

(1) For $T>1\,$MeV, $(t<1\,$s) is the first stage during which the neutrons, protons, electrons, positrons,

[5]See http://map.gsfc.nasa.gov/
[6]See http://astro.estec.esa.nl/SA-general/Projects/Planck/

and neutrinos are kept in statistical equilibrium by the (rapid) weak interaction

$$n\leftrightarrow p+e^-+\overline{\nu_e},\quad n+\nu_e\leftrightarrow p+e^-,$$
$$n+e^+\leftrightarrow p+\overline{\nu_e}. \tag{121}$$

As long as statistical equilibrium holds, the neutron-to-proton ratio is

$$(n/p)=e^{-Q/k_\mathrm{B}T}, \tag{122}$$

where $Q\equiv(m_\mathrm{n}-m_\mathrm{p})c^2=1.29$ MeV. The abundance of the other light elements is given by (Kolb and Turner, 1993)

$$Y_A=g_A\left(\frac{\zeta(3)}{\sqrt{\pi}}\right)^{A-1}2^{(3A-5)/2}A^{5/2}\times\left[\frac{k_\mathrm{B}T}{m_\mathrm{N}c^2}\right]^{3(A-1)/2}\eta^{A-1}Y_\mathrm{p}^ZY_\mathrm{n}^{A-Z}e^{B_A/k_\mathrm{B}T}, \tag{123}$$

where g_A is the number of degrees of freedom of the nucleus ${}^A_Z\mathrm{X}$, m_N is the nucleon mass, η the baryon-photon ratio, and $B_A\equiv[Zm_\mathrm{p}+(A-Z)m_\mathrm{n}-m_A]c^2$ the binding energy.

(2) Around $T\sim0.8$ MeV ($t\sim2$ s), the weak interactions freeze-out at a temperature T_f determined by the competition between the weak interaction rates and the expansion rate of the universe, and thus they are determined by $\Gamma_\mathrm{w}(T_\mathrm{f})\sim H(T_\mathrm{f})$, that is,

$$G_\mathrm{F}^2(k_\mathrm{B}T_\mathrm{f})^5\sim\sqrt{GN_*}(k_\mathrm{B}T_\mathrm{f})^2, \tag{124}$$

where G_F is the Fermi constant and N_* the number of relativistic degrees of freedom at T_f. Below T_f, the number of neutrons and protons changes only from the neutron β decay between T_f to $T_\mathrm{N}\sim0.1$ MeV when $p+n$ reactions proceed faster than their inverse dissociation. T_N is determined by demanding that the relative number of photons with energy larger that the deuteron binding energy, E_D, is smaller than one, so that $n_\gamma/n_p\sim\exp(E_\mathrm{D}/T_\mathrm{N})\sim1$.

(3) For $0.05\ \mathrm{MeV}<T<0.6\ \mathrm{MeV}$ ($3\ \mathrm{s}<t<6\ \mathrm{min}$), the synthesis of light elements occurs only by two-body reactions. This requires the deuteron to be synthesized ($p+n\rightarrow D$) and the photon density must be low enough for the photodissociation to be negligible. This happens roughly when

$$\frac{n_\mathrm{d}}{n_\gamma}\sim\eta^2\exp(-E_\mathrm{D}/T_\mathrm{N})\sim1, \tag{125}$$

with $\eta\sim3\times10^{-10}$. The abundance of ^{4}He by mass, Y_p, is then well estimated by

$$Y_\mathrm{p}\simeq2\frac{(n/p)_\mathrm{N}}{1+(n/p)_\mathrm{N}} \tag{126}$$

with

$$(n/p)_\mathrm{N}=(n/p)_\mathrm{f}\exp(-t_\mathrm{N}/\tau_\mathrm{n}), \tag{127}$$

$t_\mathrm{N}\propto G^{-1/2}T_\mathrm{N}^{-2}$ and $\tau_\mathrm{n}^{-1}=1.636G_\mathrm{F}^2(1+3g_A^2)m_\mathrm{e}^5/(2\pi^3)$, with $g_A\simeq1.26$ being the axial/vector coupling of the nucleon. Assuming that $E_\mathrm{D}\propto\alpha_\mathrm{S}^2$, this gives a dependence $t_\mathrm{N}/\tau_\mathrm{p}\propto G^{-1/2}\alpha_\mathrm{S}^2G_\mathrm{F}^2$ (see Sec. V.B).
The helium abundance depends thus mainly on Q, T_f, and T_N (and hence mainly on the neutron lifetime, τ_n), and the abundances of the other elements depend also on the nuclear reaction rates.

The light element abundances are thus sensible to the freeze-out temperature, which depends on G_F, G, on the proton-neutron mass difference Q, and on the values of the binding energies B_A, so that they mainly depend on α_EM, α_W, α_S, α_G, and the mass of the quarks. An increase in G or N_* results in a higher expansion rate and thus to an earlier freeze-out, i.e., a higher T_f. A decrease in G_F, corresponding to a longer neutron lifetime, leads to a decrease of the weak interaction rates and also results in a higher T_f. It implies, assuming uncorrelated variations, that $|\Delta G/G|<0.25$ (see Sec. IV) and $|\Delta G_\mathrm{F}/G_\mathrm{F}|<6\times10^{-2}$ (see Sec. V.A).

Initially, the radiative and Coulomb corrections for the weak reactions (121) were computed by Dicus *et al.* (1982) and shown to have a very small influence on the abundances.

The constraints on the variation of these quantities were first studied by Kolb *et al.* (1986), who calculated the dependence of primordial ^{4}He on G, G_F, and Q. They studied the influence of independent changes of the former parameters and showed that the helium abundance was mostly sensitive in the change in Q. Other abundances are less sensitive to the value of Q, mainly because ^{4}He has a larger binding energy; its abundances are less sensitive to the weak reaction rate and more to the parameters fixing the value of (n/p). To extract the constraint on the fine-structure constant, one needs a particular model for the α_EM dependence of Q. Kolb *et al.* (1986) decomposed Q as

$$Q=\alpha_\mathrm{EM}Q_\alpha+\beta Q_\beta, \tag{128}$$

where the first part represents the electromagnetic contribution and the second part corresponds to all nonelectromagnetic contributions. Assuming that Q_α and Q_β are constant and that the electromagnetic contribution is the dominant part of Q, they deduced that $Q/Q_0\simeq\alpha_\mathrm{EM}/\alpha_\mathrm{EM}^{(0)}$ and thus that $(n/p)\simeq(n/p)_0[1-q_0T_\mathrm{f}\alpha_\mathrm{EM}/\alpha_\mathrm{EM}^{(0)}]$. To consider the effect of the dependent variation of G, G_F, and α_EM, the time variation of these constants was related to the time variation of the volume of an internal space of characteristic size R for a ten-dimensional superstring model as well as for Kaluza-Klein models (see Sec. VI for details on these models).[7] They concluded that

$$|\Delta\alpha_\mathrm{EM}/\alpha_\mathrm{EM}|<10^{-2} \tag{129}$$

and showed that, if one requires that the abundances of ^{2}H and ^{3}He remain unchanged, it is impossible to com-

[7]Their hypothesis on the variation of the Fermi constant is questionable, see Sec. V.A for details.

pensate for the change in $\alpha_{\rm EM}$ through a change in the baryon-to-photon ratio. Indeed, the result depends strongly on the hypothesis of the functional dependence. Khare (1986) then showed that the effect of the extra dimensions can be canceled if the primordial neutrinos are degenerate. This approach was generalized by Vayonakis (1988), who considered the ten-dimensional limit of superstrings, by Barrow (1987), and by Coley (1990) for the case Kaluza-Klein theories.

Campbell and Olive (1995) kept track of the changes in $T_{\rm f}$ and Q separately and deduced that

$$\frac{\Delta Y_{\rm p}}{Y_{\rm p}} \simeq \frac{\Delta T_{\rm f}}{T_{\rm f}} - \frac{\Delta Q}{Q}. \qquad (130)$$

They used this to study the constraints on $G_{\rm F}$ (see Sec. V.A).

Bergström *et al.* (1999) extended the original work by Kolb *et al.* (1986) by considering other nuclei. They assumed the dependence of Q on $\alpha_{\rm EM}$

$$Q \simeq (1.29 - 0.76\Delta\alpha_{\rm EM}/\alpha_{\rm EM})\ \ \text{MeV} \qquad (131)$$

that relies on a change of quark masses due to strong and electromagnetic energy binding. Since the abundances of other nuclei depend mostly on the weak interaction rates, they studied the dependence of the thermonuclear rates on $\alpha_{\rm EM}$. In the nonrelativistic limit, it is obtained as the thermal average of the cross section times the relative velocity times the number densities. The key point is that, for charged particles, the cross section takes the form

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta(E)}, \qquad (132)$$

where $\eta(E)$ arises from the Coulomb barrier and is given in terms of the charges and the reduced mass μ of the two particles as

$$\eta(E) = \alpha_{\rm EM} Z_1 Z_2 \sqrt{\frac{\mu c^2}{2E}}. \qquad (133)$$

The factor $S(E)$ has to be extrapolated from experimental nuclear data, which allowed Bergström *et al.* (1999) to determine the $\alpha_{\rm EM}$ dependence of all the relevant reaction rates. Let us note that the $\alpha_{\rm EM}$ dependence of the reduced mass μ and of $S(E)$ were neglected; the latter one is polynomial in $\alpha_{\rm EM}$ (Fowler *et al.*, 1975). Keeping all other constants fixed, assuming no exotic effects, and taking a lifetime of 886.7 s for the neutron, they deduced that

$$|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}| < 2\times 10^{-2}. \qquad (134)$$

In the low range of $\eta \sim 1.8\times 10^{-10}$, the ^{7}Li abundance does not depend strongly on $\alpha_{\rm EM}$, and ^{4}He has to be used to constrain $\alpha_{\rm EM}$. But it has to be noted that the observational status of the abundance of ^{4}He is still a matter of debate and that the theoretical prediction of its variation with $\alpha_{\rm EM}$ depends on the model-dependent ansatz (131). For the high range of $\eta \sim 5\times 10^{-10}$, the variation of ^{7}Li with $\alpha_{\rm EM}$ is rapid, due to the exponential Coulomb barrier, and limits the variation of $\alpha_{\rm EM}$.

Nollet and Lopez (2002) pointed out that Eq. (132) does not contain all the $\alpha_{\rm EM}$ dependence. They argue that (i) the factor S depends linearly on $\alpha_{\rm EM}$, (ii) when a reaction produces two charged particles there should be an extra $\alpha_{\rm EM}$ contribution arising from the fact that the particles need to escape the Coulomb potential, (iii) the reaction energies depend on $\alpha_{\rm EM}$, and (iv) radiative-capture-matrix elements are proportional to $\alpha_{\rm EM}$. The most secure constraint arising from D/H measurements and combining with CMB data to determine Ω_B gives

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (3\pm 7)\times 10^{-2} \qquad (135)$$

at 1σ level.

Ichikawa and Kawasaki (2002) included the effect of the quark mass by considering a joint variation of the different couplings as it appears from a dilaton. Q then takes the form

$$Q = a\,\alpha_{\rm EM}\Lambda_{\rm QCD} + b(y_{\rm d} - y_{\rm u})v, \qquad (136)$$

where a and b are two parameters and $y_{\rm d}$, $y_{\rm u}$ the Yukawa couplings. The neutron lifetime then behaves as

$$\tau_{\rm n} = (1/v y_{\rm e}^5) f^{-1}(Q/m_{\rm e}), \qquad (137)$$

with f a known function. Assuming that all the couplings vary due to the effect of a dilaton, such that the Higgs vacuum expectation value v remains fixed, they constrained the variation of this dilaton and deduced

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-2.24\pm 3.75)\times 10^{-4}. \qquad (138)$$

In all the studies, one assumes either that all other constants are fixed or that a functional dependence exists between them, as inspired from string theory. The bounds are of the same order of magnitude as those obtained from the CMB; they have the advantage of being at higher redshift, but suffer from the drawback of being model dependent.

3. Conclusion

Even if cosmological observations allow the testing of larger time scales, it is difficult to extract tight constraints on the variation of the fine-structure constant from them.

The CMB seems clean at first glance since the effect of the fine-structure constant is well decoupled from the effect of the weak- and strong-coupling constants. Still, it is entangled with the assumption of G. Besides, it was shown that degeneracy between some parameters exists, mainly between the fine-structure constant, the electron-to-proton mass ratio, the baryonic density, and the dark-energy equation of state (Huey *et al.*, 2002).

Nucleosynthesis is degenerate in the four fundamental coupling constants. In some specific models where the variation of these constants is linked, it constrains them. The helium abundance alone cannot definitively constrain the fine structure constant.

D. Equivalence principle

The equivalence principle is closely related to the development of the theory of gravity from Newton's theory to general relativity (see Will, 1993, 2001 for reviews). Its first aspect is the *weak-equivalence* principle stating that the weight of a body is proportional to its mass or, equivalently, that the trajectory of any freely falling body does not depend on its internal structure, mass, and composition. Einstein formulated a stronger equivalence principle (usually referred to as the *Einstein equivalence principle*) which states that (1) the weak-equivalence principle holds, (2) any nongravitational experiment is independent of the velocity of the laboratory rest frame (local Lorentz invariance), and (3) any such experiment is also independent of when and where it is performed (local position invariance).

If the Einstein equivalence principle is valid, then gravity can be described as the consequence of a curved spacetime and is a metric theory, examples of which are general relativity and the Brans-Dicke (1961) theory. This statement is not a "theorem," but there are a lot of indications to back it up (see Will, 1993, 2001). Note that superstring theory violates the Einstein equivalence principle since it introduces additional fields (e.g., dilaton, moduli, etc.) that have gravitational-strength couplings that violate the weak-equivalence principle. A time variation of a fundamental constant is in contradiction with the Einstein equivalence principle since it violates the local position invariance. Dicke (1957, 1964) was probably the first to try to use the experimental result of Eötvös *et al.* (1922) to argue that the strong-interaction constant was approximately position independent. All new interactions that appear in the extension of standard physics imply extra scalar or vector fields, and thus also imply an expected violation of the weak equivalence principle. The only exception is metric theories such as the class of tensor-scalar theories of gravitation, in which the dilaton couples universally to all fields and in which one can have a time variation of gravitational constant without a violation of the weak-equivalence principle (see, e.g., Damour and Esposito-Farèse, 1992).

The difference in acceleration between two bodies of different composition can be measured in Eötvös-type experiments (Eötvös *et al.*, 1922), in which the acceleration of various pairs of material in the Earth's gravitational field are compared. The results of this kind of laboratory experiment are presented as bounds on the parameter η,

$$\eta \equiv 2\frac{|\vec{a}_1-\vec{a}_2|}{|\vec{a}_1+\vec{a}_2|}. \tag{139}$$

The most accurate constraints on η are $\eta=(-1.9\pm2.5)\times10^{-12}$ between beryllium and copper (Su *et al.*, 1994) and $|\eta|<5.5\times10^{-13}$ between Earth-core-like and Moon-mantle-like materials (Baessler *et al.*, 1999). The Lunar Laser Ranging (LLR) experiment gives the bound $\eta=(3.2\pm4.6)\times10^{-13}$ (Williams *et al.*, 1996) and $\eta=(3.6\pm4)\times10^{-13}$ (Müller and Nordtvedt, 1998; Müller *et al.*, 1999). Note, however, that as pointed out by Nordtvedt (1988, 2001), the LLR measurement is ambiguous since the Earth and the Moon have (i) a different fraction of gravitational self-energy and (ii) a difference of composition (the core of the Earth having a larger Fe/Ni ratio than the Moon). This makes this test sensitive both to self-gravity and to nongravitational forms of energy. The experiment by Baessler *et al.* (1999) lifts the degeneracy by considering a miniature "Earth" and "Moon."

As explained in Sec. II.C, if the self-energy depends on position, the conservation of energy implies the existence of an anomalous acceleration. In the more general case where the long-range force is mediated by a scalar field ϕ, one has to determine the dependence $m_i(\phi)$ of the different particles. If it is different for neutron and proton, then the force will be composition dependent. At the Newtonian approximation, the interaction potential between two particles is of the form (Damour and Esposito-Farèse, 1992)

$$V(r)=-G(1+\alpha_{12}e^{-r/\lambda})\frac{m_1m_2}{r}, \tag{140}$$

with $\alpha_{12}\equiv f_1f_2$ and f_i defined as

$$f_i\equiv M_4\frac{\partial \ln m_i(\phi)}{\partial\phi}, \tag{141}$$

where $M_4^{-2}\equiv 8\pi G/\hbar c$ is the four-dimensional Planck mass. The coefficient α_{12} is thus not a fundamental constant and depends *a priori* on the chemical composition of the two test masses. It follows that

$$\eta_{12}=\frac{f_{\rm ext}|f_1-f_2|}{1+f_{\rm ext}(f_1+f_2)/2}\simeq M_4 f_{\rm ext}\left|\partial_\phi \ln\frac{m_1}{m_2}\right|. \tag{142}$$

To set any constraint, one has to determine the functions $f_i(\phi)$, which can only be made in a model-dependent approach [see, e.g., Damour (1996) for a discussion of the information that can be extracted in a model-independent way]. For instance, if ϕ couples to a charge Q, the additional potential is expected to be of the form

$$V(r)=-f_Q\frac{Q_1Q_2}{r}e^{-r/\lambda}, \tag{143}$$

with f_Q being a fundamental constant ($f_Q>0$ for scalar exchange and $f_Q<0$ for vector exchange). It follows that α_{12} depends explicitly of the composition of the two bodies as

$$\alpha_{12}=\xi_Q\frac{Q_1}{\mu_1}\frac{Q_2}{\mu_2}, \tag{144}$$

where $\mu_i\equiv m_i/m_{\rm N}$ and $\xi_Q=f_Q/Gm_{\rm N}^2$. Their relative acceleration in an external field $\vec{g}_{\rm ext}$ is

$$\Delta\vec{a}_{12}=\xi_Q\left(\frac{Q}{\mu}\right)_{\rm ext}\left[\frac{Q_1}{\mu_1}-\frac{Q_2}{\mu_2}\right]\vec{g}_{\rm ext}. \tag{145}$$

For instance, in the case of a fifth force induced by a dilaton or string moduli, Damour and Polyakov (1994a, 1994b) showed that there are three charges, $B=N+Z$, $D=N-Z$, and $E=Z(Z-1)B^{1/3}$, representing, respectively, the baryon number, the neutron excess, and a term proportional to the nuclear Coulomb energy. The

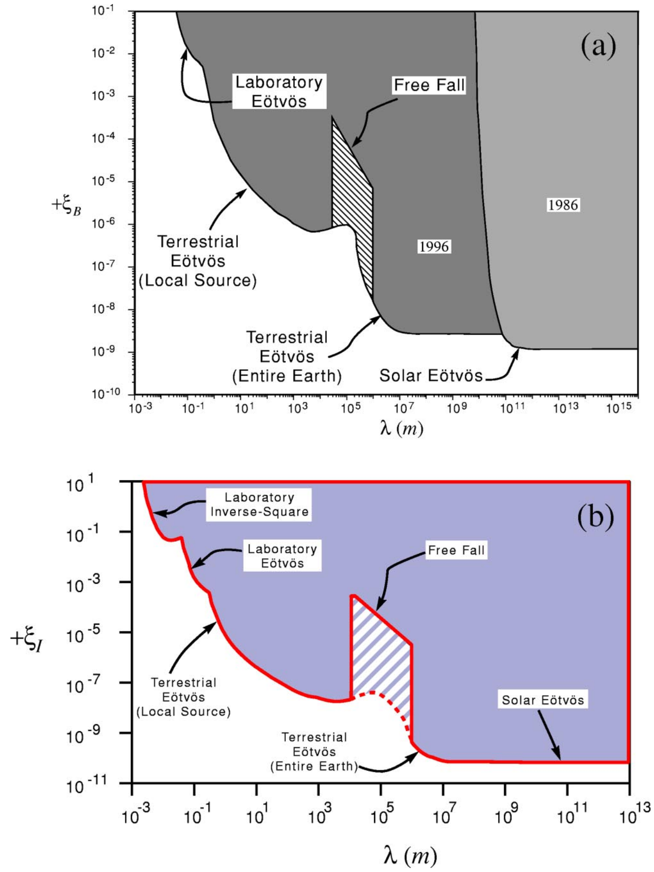

FIG. 6. Constraints on the coupling ξ_B (a) and ξ_I (b), respectively, to $N+Z$ and $N-Z$ as a function of the length scale λ. The shaded regions are excluded at 2σ. From Fischbach and Talmadge (1996) (Color in online edition).-

test of the equivalence principle results in an exclusion plot in the plane (ξ_Q, λ) (see Fig. 6).

To illustrate the link between the variation of the constants and the tests of relativity, let us consider the string-inspired model developed by Damour and Polyakov (1994a, 1994b), in which the fine-structure constant is given in terms of a function of the four-dimensional dilaton as $\alpha_{\rm EM}=B_F^{-1}(\phi)$. The QCD mass scale can be expressed in terms of the string mass scale, $M_s\sim 3\times 10^{17}$ GeV [see Sec. VI.B for details and Eq. (277)]. In the chiral limit, the (Einstein-frame) hadron mass is proportional to the QCD mass scale, so that

$$f_{\rm hadron}\simeq -\left(\ln\frac{M_s}{m_{\rm hadron}}+\frac{1}{2}\right)\frac{\partial \ln \alpha_{\rm EM}}{\partial \phi}. \tag{146}$$

With the expected form $\ln B_F(\phi)=-\kappa(\phi-\phi_m)^2/2$ (see Sec. VI.B), the factor of the right-hand-side of the previous equation is of order $40\kappa(\phi-\phi_m)$. The exchange of the scalar field excitation induces a deviation from general relativity characterized, at post-Newtonian level, by the Eddington parameters

$$1-\gamma_{\rm Edd}\simeq 2(40\kappa)^2(\phi_0-\phi_m)^2, \tag{147}$$

$$\beta_{\rm Edd}-1\simeq(40\kappa)^3(\phi_0-\phi_m)^2/2. \tag{148}$$

Besides, the violation of the universality of free fall is given by $\eta_{12}=\hat\delta_1-\hat\delta_2$ with

$$\hat\delta_1=(1-\gamma_{\rm Edd})\left[c_2\left(\frac{B}{\mu}\right)_1+c_D\left(\frac{D}{\mu}\right)_1+0.943\times10^{-5}\left(\frac{E}{\mu}\right)_1\right] \tag{149}$$

obtained from expression (27) for the mass. In this expression, the third term is expected to dominate. We see in this example that the variation of the constants, the violation of the equivalence principle, and post-Newtonian deviation from general relativity have to be considered together.

Similarly, in an effective four-dimensional theory, the only consistent approach to make a Lagrangian parameter time dependent is to consider it as a field. The Klein-Gordon equation for this field $(\ddot\phi+3H\dot\phi+m^2\phi+\cdots=0)$ implies that ϕ is damped as $\dot\phi\propto a^{-3}$ if its mass is much smaller than the Hubble scale. Thus, in order to be varying during the last Hubble time, ϕ has to be very light, with typical mass $m\sim H_0\sim10^{-33}$ eV. This is analogous to the case of quintessence models (see Sec. V.E for details). As a consequence, ϕ has to be very weakly coupled to the standard model fields. To illustrate this, Dvali and Zaldarriaga (2002) [followed by a reanalysis by Chiba and Khori (2002) and Wetterich (2002)] expanded $\alpha_{\rm EM}$ around its value today as

$$\alpha_{\rm EM}=\alpha_{\rm EM}(0)+\lambda\frac{\phi}{M_4}+\mathcal{O}\left(\frac{\phi^2}{M_4^2}\right), \tag{150}$$

from which it follows, from Webb *et al.* (2001), that $\lambda\Delta\phi/M_4\sim10^{-7}$ during the last Hubble time. The change of the mass of the proton and of the neutron due to electromagnetic effects was obtained from Eqs. (27) and (28), but neglecting the last term. The extra Lagrangian for the field ϕ is thus

$$\delta L=\lambda\frac{\phi}{M_4}(B_{\rm p}p\bar p+B_{\rm n}n\bar n). \tag{151}$$

A test body composed of $n_{\rm n}$ neutrons and $n_{\rm p}$ protons will be characterized by a sensitivity

$$f_i=\frac{\lambda}{m_{\rm N}}(\nu_{\rm p}B_{\rm p}+\nu_{\rm n}B_{\rm n}), \tag{152}$$

where $\nu_{\rm n}$ $(\nu_{\rm p})$ is the ratio of neutrons (protons) and where it has been assumed that $m_{\rm n}\sim m_{\rm p}\sim m_{\rm N}$. Assuming[8] that $\nu_{\rm n,p}^{\rm Earth}\sim1/2$ and using the compactness of the Moon-Earth system $\partial\ln(m_{\rm Earth}/m_{\rm Moon})/\partial\ln\alpha_{\rm EM}\sim10^{-3}$, one gets $\eta_{12}\sim10^{-3}\lambda^2$. Dvali and Zaldarriaga (2002) obtained the same result by considering that $\Delta\nu_{\rm n,p}\sim6\times10^{-2}$–$10^{-1}$. This implies that $\lambda<10^{-5}$, which is compatible with the variation of $\alpha_{\rm EM}$ if $\Delta\phi/M_4>10^{-2}$ during the last Hubble period.

From cosmological investigations one can show that $(\Delta\phi/M_4)^2\sim(\rho_\phi+P_\phi)/\rho_{\rm total}$. If ϕ dominates the matter content of the universe, $\rho_{\rm total}$, then $\Delta\phi\sim M_4$ so that $\lambda\sim10^{-7}$, whereas if it is subdominant $\Delta\phi\ll M_4$ and $\lambda\gg10^{-7}$. In conclusion

[8]For copper $\nu_{\rm p}=0.456$, for uranium $\nu_{\rm p}=0.385$, and for lead $\nu_{\rm p}=0.397$.

$$10^{-7}<\lambda<10^{-5}. \quad (153)$$

This explains the tuning on the parameter λ.

An underlying approximation is that the ϕ dependence arises only from the electromagnetic self-energy. But, in general, one would expect that the dominant contribution to the hadron mass, the QCD contributions, also induces a ϕ dependence [as in the Damour and Polyakov (1994a, 1994b) approach].

In conclusion, the test of the equivalence principle offers a very precise test of the variation of constants (Damour, 2001). The LLR constraint $\eta \lesssim 10^{-13}$, i.e., $|\vec{a}_{\rm Earth}-\vec{a}_{\rm Moon}|\lesssim 10^{-14}$ cm s^{-2}, implies that on the size of the Earth orbit

$$|\nabla \ln \alpha_{\rm EM}| \lesssim 10^{-33}\text{–}10^{-32}\ \text{cm}^{-1}. \quad (154)$$

Extending this measurement to the Hubble size leads to the estimate $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\lesssim 10^{-4}\text{–}10^{-5}$. This indicates that if the claim by Webb *et al.* (2001) is correct, then it should induce a detectable violation of the equivalence principle in coming experiments such as MICROSCOPE[9] and STEP.[10] They will have an accuracy of, respectively, the levels $\eta \sim 10^{-15}$ and $\eta \sim 10^{-18}$. Indeed, this is a rough estimate in which $\dot{\alpha}_{\rm EM}$ is assumed to be constant, but this is also the conclusion indicated by the results of Bekenstein (1982) and Dvali and Zaldarriaga (2002).

Let us also note that this constraint has been discarded by some models (see Sec. VI.C), particularly while claiming that a variation of $\alpha_{\rm EM}$ of 10^{-5} was realistic (Barrow *et al.*, 2002a, 2002b; Sandvik *et al.*, 2002) [see however the recent study by Bekenstein (2002) and the discussion by Magueijo *et al.* (2002)].

IV. GRAVITATIONAL CONSTANT

As pointed out by Dicke and Peebles (1965), the importance of gravitation on large scales is due to the short range of the strong and weak forces and to the fact that the electromagnetic force becomes weak because of the global neutrality of matter. As they provide tests of the law of gravitation (e.g., planetary motions, light deflection), space science and cosmology also offer tests of the constancy of the gravitational constant.

Contrary to most of the other fundamental constants, as the precision of the measurements increased, the disparity between the measured values of G also increased. This led the CODATA[11] in 1998 to raise the relative uncertainty for G from 0.013% to 0.15% (Gundlach and Merkowitz, 2000). The following constraints assume that the mass of stars and/or planets is kept constant.

[9] See http://sci2.esa.int/Microscope/

[10] See http://einstein.stanford.edu/STEP/

[11] The CODATA is the COmmittee on Data for Science and Technology; see http://www.codata.org/

A. Paleontological and geophysical arguments

Dicke (1964) stressed that the Earth is such a complex system that it would be difficult to use it as a source of evidence for or against the existence of a time variation of the gravitational constant. Following Jordan (1955), he noted that, among the direct effects, a weakening of the gravitational constant induces a variation of the Earth surface temperature, an expansion of the Earth radius, and a variation of the length of the day (Jordan, 1955; Murphy and Dicke, 1964; Hoyle, 1972).

1. Earth surface temperature

Teller (1948) first emphasized that the Dirac hypothesis may be in conflict with paleontological evidence. His argument is based on the estimation of the temperature at the center of the Sun $T_\odot \propto GM_\odot/R_\odot$ using the virial theorem. The luminosity of the Sun is then proportional to the radiation energy gradient times the mean free path of a photon times the surface of the Sun, that is, $L_\odot \propto T_\odot^7 R_\odot^7 M_\odot^{-2}$, hence $L_\odot \propto T_\odot^7 M_\odot^5$. Computing the radius of the Earth orbit in Newtonian mechanics, assuming the conservation of angular momentum (so that $GM_\odot R_{\rm Earth}$ is constant), and stating that the Earth mean temperature is proportional to the fourth root of the energy received, he concluded that

$$T_{\rm Earth} \propto G^{2.25} M_\odot^{1.75}. \quad (155)$$

If $M_\odot$ is constant and G was 10% larger 300 million years ago, the Earth surface temperature should have been 20% higher, that is, close to the boiling temperature. This contradicts the existence of trilobites in the Cambrian era.

With even a smaller variation, Gamow (1967a) showed that even if it was safe at the Cambrian era, there was still a contradiction with bacteria and algae estimated to have lived 4×10^9 years ago. It follows that

$$|\Delta G/G| < 0.1 \quad (156)$$

over a 4×10^9 yr time scale. Eichendorf and Reinhardt (1977) reactualized Teller's argument in light of a new estimate of the age of the universe and new paleontological discoveries to get $|\dot{G}/G| < 2.0\times 10^{-11}$ yr^{-1} (cited by Petley, 1985).

When using such an argument, the heat balance of the atmosphere is affected by many factors (water vapor content, carbon dioxide content, and circulatory patterns, among others). This renders the extrapolation during several billion years very unreliable. For instance, the rise of the temperature implies that the atmosphere is at some stage composed mostly of water vapor so that its convective mechanism is expected to change in such a way as to increase the Earth albedo and thus to decrease the temperature!

2. Expanding Earth

Egeyed (1961) first remarked that paleomagnetic data could be used to calculate the Earth paleoradius for different geological epochs. Under the hypothesis that the area of continental material has remained constant while the bulk of the Earth has expanded, the determination of the difference in paleolatitudes between two sites of known separation gives a measurement of the paleoradius. Creer (1965) showed that data older than 3×10^8 years form a coherent group in $\dot{r}_{\rm Earth}$, and Wesson (1973) concluded from a compilation of data that an "expansion from a completely sial-covered globe of about 3700 km radius at a constant rate of 0.66 $\rm mm\cdot yr^{-1}$ over a 4.5×10^9 yr interval would give the continents a configuration as we now see them."

Dicke (1962b, 1964) related the variation of the Earth radius to a variation of the gravitational constant by

$$\Delta \ln r_{\rm Earth} = -0.1\Delta \ln G. \tag{157}$$

McElhinny *et al.* (1978) reestimated the paleoradius of the Earth and extended the analysis to the Moon, Mars, and Mercury. Starting from the hydrostatic equilibrium equation

$$\frac{dP}{dr} = -G\frac{\rho(r)M(r)}{r^2}, \tag{158}$$

where $M(r)$ is the mass within radius r, they generalized Dicke's result to get

$$\Delta \ln r_{\rm Earth} = -\alpha\Delta \ln G, \tag{159}$$

where α depends on the equation of state $P(\rho)$, for instance, $\alpha=1/(3n-4)$ for a polytropic gas, $P=C\rho^n$. In the case of small planets, one can work in a small gravitational self-compression limit and set $P=K_0(\rho/\rho_0-1)$. Equation (158) then gives $\alpha=(2/15)(\Delta\rho/\rho_0)$, $\Delta\rho$ being the density difference between the center and surface. This approximation is poor for the Earth, and more sophisticated models exist. They give $\alpha_{\rm Earth}=0.085\pm0.02$, $\alpha_{\rm Mars}=0.032$, $\alpha_{\rm Mercury}=0.02\pm0.05$, and $\alpha_{\rm Moon}=0.004\pm0.001$. Using the observational fact that the Earth has not expanded by more than 0.8% over the past 4×10^8 years, the Moon by 0.06% over the past 4×10^9 years, and Mars by 0.6%, they concluded that

$$-\dot{G}/G \lesssim 8\times10^{-12}\ {\rm yr}^{-1}. \tag{160}$$

Despite any real evidence in favor of an expanding Earth, the rate of expansion is also limited by another geophysical aspect, i.e., the deceleration of the Earth's rotation.

Dicke (1957) listed some other possible consequences on the scenario of the formation of the Moon and on the geomagnetic field, but none of them enable us to give serious constraints. The paleontological data give only poor limits on the variation of the gravitational constant, and even though the Earth keeps a memory of the early gravitational conditions, this memory is crude and geological data are not easy to interpret.

B. Planetary and stellar orbits

Vinti (1974) studied the dynamics of a two-body system in Dirac cosmology. He showed that the equation of motion

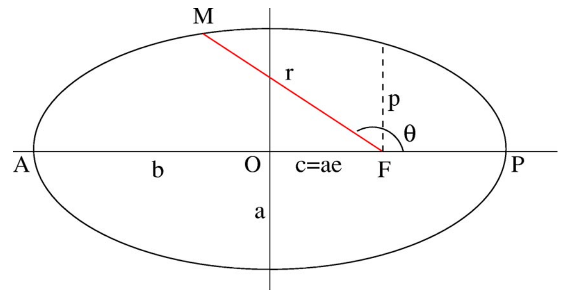

FIG. 7. The standard orbital parameters. a and b are the semimajor and semiminor axis, $c=ae$ the focal distance, p the semilatus rectum, θ the true anomaly. F is the focus, A and B the periastron and apoastron (see Murray and Dermott, 2000). It is easy to check that $b^2=a^2(1-e^2)$ and that $p=a(1-e^2)$, and one defines the frequency or mean motion as $n=2\pi/P$ where P is the period (Color in online edition).-

$$\frac{d^2\vec{r}}{dt^2} = -G_0\frac{k+t_0}{k+t}m\frac{\vec{r}}{r^3}, \tag{161}$$

where k is a constant and G_0 the gravitational constant today, can be integrated. For bounded orbits (see Fig. 7), the solution describes a growing ellipse with constant eccentricity e, pericenter argument ω, and a linearly growing semimajor axis $p(t)=(l^2/G_0m)(k+t)/(k+t_0)$, where l is the constant angular momentum, of equation

$$r = \frac{p(t)}{1+e\cos(\theta-\omega)}. \tag{162}$$

Similarly, Lynden-Bell (1982) showed that the equations of motion of the N-body problem can be transformed to the standard equation if G varies as t^{-1}.

It follows that in the Newtonian limit, the orbital period of a two-body system is

$$P = \frac{2\pi l}{(Gm)^2}\frac{1}{(1-e^2)^{3/2}}\left[1+\mathcal{O}\left(\frac{G^2m^2}{c^2l^2}\right)\right] \tag{163}$$

in which the correction terms represent the post-Newtonian corrections to the Keplerian relationship. It is typically of order 10^{-7} and 10^{-6}, respectively, for Solar system planetary orbits and for a binary pulsar. It follows that

$$\frac{\dot{P}}{P} = 3\frac{\dot{l}}{l} - 2\frac{\dot{G}}{G} - 2\frac{\dot{m}}{m}. \tag{164}$$

Only for the orbits of bodies for which the gravitational self-energy can be neglected does the previous equation reduce to

$$\frac{\dot{P}}{P} = -2\frac{\dot{G}}{G}. \tag{165}$$

This leads to two observable effects in the Solar system (Shapiro, 1964; Counselman and Shapiro, 1968). First, the scale of the Solar system changes, and second, if G evolves adiabatically as $G=G_0+\dot{G}_0(t-t_0)$, there will be a quadratically growing increment in the mean longitude of each body.

For a compact body, the mass depends on G as well as other post-Newtonian parameters. At first order in the post-Newtonian expansion, there is a negative contribution, Eq. (29), to the mass arising from the gravitational binding energy, and one cannot neglect $\dot{m}$ in Eq. (164). This is also the case if other constants are varying.

1. Early works

Early works mainly focus on the Earth-Moon system and try to relate a time variation of G to a variation of the frequency or mean motion ($n=2\pi/P$) of the Moon around the Earth. Arguments of an expanding Earth also raise interest in the determination of the Earth's rotation rate. One of the greatest problems is to evaluate and subtract the contribution of the spin-down of the Earth arising from the friction in the seas due to tides raised by the Moon [Van Flandern (1981) estimated that $\dot{n}_{\rm tidal}=(-28.8\pm1.5)''\,{\rm century}^{-2}$] and a contribution from the Moon recession.

The determination of ancient rotation rates can rely on paleontological data and ancient eclipse observations, as well as on measurements of star declinations (Newton, 1970, 1974). It can be concluded from these studies that there were about 400 days in a year during the Devonian era. Indeed, these studies contain a lot of uncertainties; for instance, Runcorn (1964) compared telescope observation from the 17th century to the ancient eclipse records and found a discrepancy of a factor 2. In other examples, Muller (1978) studied eclipses from 1374 B.C. to 1715 A.D. to conclude that

$$\dot{G}/G=(2.6\pm15)\times10^{-11}\ {\rm yr}^{-1}, \tag{166}$$

and Morrison (1973) used ephemeris from 1663 to 1972, including 40 000 Lunar occultations from 1943 to 1972, to deduce that

$$|\dot{G}/G|<2\times10^{-11}\ {\rm yr}^{-1}. \tag{167}$$

Paleontological data such as the growth rhythm found in fossil bivalves and corals also enable the setting of constraints on the Earth rotational history and the Moon orbit (Van Diggelen, 1976) [for instance, in the study by Scrutton (1965) the fossils showed marking so fine that the phases of the Moon were mirrored in the coral growth]. Blake (1977b) related the variation of the number of sidereal days in a sidereal year, $Y=n_E/n_S$, and in a sidereal month, $M=n_E/n_M$ (n_E, n_S, and n_M being, respectively, the orbital frequencies of the motion of the Earth, of the Moon around the Earth, and of the Earth around the Sun), to the variation of the Newton constant and the Earth momentum of inertia I as

$$(\gamma-1)\frac{\Delta Y}{Y}-\gamma\frac{\Delta M}{M}=\frac{\Delta I}{I}+2\frac{\Delta G}{G} \tag{168}$$

with $\gamma=1.9856$ being a calculated constant. The fossil data represent the number of Solar days in a tropical year and in a synodic month that can be related to Y and M so that one obtains a constraint on $\Delta I/I+2\Delta G/G$. Attributing the variation of I to the expansion of the Earth (Wesson, 1973), one can argue that $\Delta I/I$ represents only 10–20 % of the right-hand side of Eq. (168). Blake (1977b) concluded that

$$\dot{G}/G=(-0.5\pm2)\times10^{-11}\ {\rm yr}^{-1}. \tag{169}$$

Van Flandern (1971, 1975) studied the motion of the Moon from Lunar occultation observations from 1955 to 1974 using atomic time, which differs from the ephemeris time relying on the motion of the Earth around the Sun. He attributed the residual acceleration after correction of tidal effect to a variation of G, $\dot{n}^G_{\rm Moon}/2n^G_{\rm Moon}=(-8\pm5)\times10^{-9}\ {\rm century}^{-2}$ to claim that

$$\dot{G}/G=(-8\pm5)\times10^{-11}\ {\rm yr}^{-1}. \tag{170}$$

In a new analysis, Van Flandern (1981) concluded that $\dot{n}^G_{\rm Moon}/n^G_{\rm Moon}=(3.2\pm1.1)\times10^{-11}\ {\rm yr}^{-1}$, hence that G was increasing as

$$\dot{G}/G=(3.2\pm1.1)\times10^{-11}\ {\rm yr}^{-1}, \tag{171}$$

which has the opposite sign. In this comparison, the time scale of the atomic time is 20 years compared to that of the ephemeris at 200 years, but it is less precise. It follows that the comparison is not obvious and that these results are far from convincing. In this occultation method, one has to be sure that the proper motions of the stars are taken into account. One also has to assume that (1) the masses of the planets are not varying [see Eq. (164)], which can happen if, for instance, the strong- and fine-structure constants are varying, (2) the fine-structure constant is not varying when compared to atomic time, and (3) the effect of the changing radius of the Earth is not taken into account.

2. Solar system

Monitoring the separation of orbiting bodies offers a possibility to constrain the time variation of G. This accounts for comparing a gravitational time scale (set by the orbit) with an atomic time scale, and it is thus assumed that the variation of atomic constants is negligible for the time of the experiment.

Shapiro *et al.* (1971) compared radar-echo time delays between Earth, Venus, and Mercury with a cesium atomic clock between 1964 and 1969. The data were fitted to the theoretical equation of motion for the bodies in a Schwarzschild spacetime, taking into account the perturbations from the Moon and other planets. They concluded that

$$|\dot{G}/G|<4\times10^{-10}\ {\rm yr}^{-1}. \tag{172}$$

The data concerning Venus cannot be used due to imprecision in the determination of the portion of the planet reflecting the radar. This was improved to

$$|\dot{G}/G|<1.5\times10^{-10}\ {\rm yr}^{-1} \tag{173}$$

by including Mariner 9 and Mars Orbiter data (Reasenberg and Shapiro, 1976, 1978). The analysis was further extended (Shapiro, 1990) to give

$$\dot{G}/G=(-2\pm10)\times10^{-12}\ {\rm yr}^{-1}. \tag{174}$$

The combination of Mariner 10 and Mercury and Venus ranging data gives (Anderson *et al.*, 1992)

$$\dot{G}/G=(0.0\pm 2.0)\times 10^{-12}\ \text{yr}^{-1}. \quad (175)$$

The Lunar Laser Ranging (LLR) experiment has measured the position of the Moon with an accuracy of about 1 cm for 30 years. This was made possible by the American Apollo 11, 14, and 15 missions and Soviet-French Lunakhod 1 and 4, which landed retro-reflectors on the Moon that reflect laser pulses from the Earth [see Dickey *et al.* (1994) for a complete description]. Restricting data to positive values of $\omega_{\rm BD}$, Williams *et al.* (1976) deduced from the first six years of LLR that $\omega_{\rm BD}>29$, which, assuming a Brans-Dicke theory for a universe with flat spatial sections and no cosmological constant, implies

$$|\dot{G}/G|\lesssim 3\times 10^{-11}\ \text{yr}^{-1}. \quad (176)$$

Müller *et al.* (1991) used 20 years of data to improve this result to

$$|\dot{G}/G|<1.04\times 10^{-11}\ \text{yr}^{-1}, \quad (177)$$

the main error arising from the Lunar tidal acceleration. Dickey *et al.* (1994) improved this constraint to

$$|\dot{G}/G|<6\times 10^{-12}\ \text{yr}^{-1}, \quad (178)$$

and Williams *et al.* (1996) with 24 years of data concluded that

$$|\dot{G}/G|<8\times 10^{-12}\ \text{yr}^{-1}. \quad (179)$$

Reasenberg *et al.* (1979) considered the 14 months of data obtained from the ranging of the Viking spacecraft and deduced that $\omega_{\rm BD}>500$, which implies, under the same hypothesis as for Eq. (176),

$$|\dot{G}/G|<10^{-12}\ \text{yr}^{-1}. \quad (180)$$

Using all available astrometric data and in particular the ranging data from Viking landers on Mars, Hellings *et al.* (1983) deduced that

$$|\dot{G}/G|=(2\pm 4)\times 10^{-12}\ \text{yr}^{-1}. \quad (181)$$

The major contribution to the uncertainty is due to the modeling of the dynamics of the asteroids on the Earth-Mars range. Hellings *et al.* (1983) also tried to attribute their result to a time variation of the atomic constants. Using the same data but a different modeling of the asteroids, Reasenberg (1983) got

$$|\dot{G}/G|<3\times 10^{-11}\ \text{yr}^{-1}, \quad (182)$$

which was then improved by Chandler *et al.* (1993) to

$$|\dot{G}/G|<10^{-11}\ \text{yr}^{-1}. \quad (183)$$

All these measurements test more than just the time variation of the gravitational constant and offer a series of tests on the theory of gravitation; they also constrain parametrized post-Newtonian (PPN) parameters, geodetic precession, etc. (see Will, 1993).

3. Pulsars

Contrary to the Solar system case, the dependence of the gravitational binding energy cannot be neglected while computing the time variation of the period (Dicke, 1969; Eardley, 1975; Haugan, 1979). Here two approaches can be followed; either one sticks to a model (e.g., scalar-tensor gravity) and computes all the effects in this model or one has a more phenomenological approach and tries to put some model-independent bounds.

Eardley (1975) followed the first route and discussed the effects of a time variation of the gravitational constant on binary pulsars in the framework of the Brans-Dicke theory. In that case, both a dipole gravitational radiation and the variation of G induce a periodic variation in the pulse period. Nordtvedt (1990) showed the orbital period changes as

$$\frac{\dot{P}}{P}=-\left[2+\frac{2(m_1c_1+m_2c_2)+3(m_1c_2+m_2c_1)}{m_1+m_2}\right]\frac{\dot{G}}{G}, \quad (184)$$

where $c_i\equiv\delta\ln m_i/\delta\ln G$. He concluded that for the pulsar PSR 1913+16 ($m_1\simeq m_2$ and $c_1\simeq c_2$) one gets

$$\frac{\dot{P}}{P}=-[2+5c]\frac{\dot{G}}{G}, \quad (185)$$

the coefficient c being model dependent. As another application, he estimated that $c_{\rm Earth}\sim -5\times 10^{-10}$, $c_{\rm Moon}\sim -10^{-8}$, and $c_{\rm Sun}\sim -4\times 10^{-6}$, justifying the approximation (165) for the Solar system.

Damour *et al.* (1988) used the timing data of the binary pulsar PSR 1913+16. They implemented the effect of the time variation of G by considering the effect on $\dot{P}/P$ and making use of the transformation suggested by Lynden-Bell (1982) to integrate the orbit. They defined, in a phenomenological way, that $\dot{G}/G=-0.5\delta\dot{P}/P$, where $\delta\dot{P}$ is the part of the orbital period derivative that is not explained otherwise (by gravitational waves radiation damping). This theory-independent definition has to be contrasted with the theory-dependent result (185) by Nordtvedt (1990). They got

$$\dot{G}/G=(1.0\pm 2.3)\times 10^{-11}\ \text{yr}^{-1}. \quad (186)$$

Damour and Taylor (1991) reexamined the data of PSR 1913+16 and the upper bound

$$\dot{G}/G<(1.10\pm 1.07)\times 10^{-11}\ \text{yr}^{-1}. \quad (187)$$

Kaspi *et al.* (1994) used data from PSR B1913+16 and PSR B1855+09, respectively, to get

$$\dot{G}/G=(4\pm 5)\times 10^{-12}\ \text{yr}^{-1} \quad (188)$$

and

$$\dot{G}/G=(-9\pm 18)\times 10^{-12}\ \text{yr}^{-1}, \quad (189)$$

the latter case being more “secure” since the orbiting companion is not a neutron star.

All the previous results concern binary pulsars, but isolated ones can also be used. Heintzmann and Hillebrandt (1975) related the spin down of the pulsar JP1953 to a time variation of G. The spin down is a combined effect of electromagnetic losses, emission of gravitational waves, possible spin up due to matter accretion. Assuming that the angular momentum is conserved so that $I/P=\text{const}$, one deduces that

$$\frac{\dot{P}}{P_G}=\left(\frac{d\ln I}{d\ln G}\right)\frac{\dot{G}}{G}. \tag{190}$$

The observational spin down can be decomposed as

$$\frac{\dot{P}}{P_{\rm obs}}=\frac{\dot{P}}{P_{\rm mag}}+\frac{\dot{P}}{P_{\rm GW}}+\frac{\dot{P}}{P_G}. \tag{191}$$

Since $\dot{P}/P_{\rm mag}$ and $\dot{P}/P_{\rm GW}$ are positive definite, it follows that $\dot{P}/P_{\rm obs}\geqslant\dot{P}/P_G$, so that a bound on $\dot{G}$ can be inferred if the main pulse period is the period of rotation.

Heintzmann and Hillebrandt (1975) modeled the pulsar by a polytropic ($P\propto\rho^n$) white dwarf and deduced that $d\ln I/d\ln G=2-3n/2$, so that

$$|\dot{G}/G|<10^{-10}\ \text{yr}^{-1}. \tag{192}$$

Mansfield (1976) assumed a relativistic degenerate, zero-temperature polytropic star and got that, when $\dot{G}<0$,

$$0\leqslant-\dot{G}/G<6.8\times10^{-11}\ \text{yr}^{-1} \tag{193}$$

at a 2σ level. He also noted that a positive $\dot{G}$ induces a spin up counteracting the electromagnetic spin down, which can provide another bound if an independent estimate of the pulsar magnetic field can be obtained. Goldman (1990), following Eardley (1975), used the scaling relations $N\propto G^{-3/2}$ and $M\propto G^{-5/2}$ to deduce that $2d\ln I/d\ln G=-5+3d\ln I/d\ln N$. He used the data from the pulsar PSR 0655+64 to deduce that the rate of decrease of G was smaller than

$$0\leqslant-\dot{G}/G<5.5\times10^{-11}\ \text{yr}^{-1}. \tag{194}$$

C. Stellar constraints

In early works, Pochoda and Schwarzschild (1963), Ezer and Cameron (1966), Roeder and Demarque (1966), and then Gamow (1967c), Shaviv and Bahcall (1969), and Chin and Stothers (1975, 1976) studied Solar evolution in the presence of a time-varying gravitational constant. They came to the conclusion that under the Dirac hypothesis, the original nuclear resources of the Sun would have been depleted by now. This results from the fact that an increase of the gravitational constant is equivalent to an increase of the star density (because of the Poisson equation). With a slighter decrease rate, the Sun would be more evolved, so that its central helium content, temperature, and neutrino luminosity must be larger than in standard solar models.

A side effect of the change of luminosity is a change in the depth of the convection zone. This induces a modification of the vibration modes of the star and particularly to the acoustic waves, i.e., p modes (Demarque *et al.*, 1994). Demarque *et al.* (1994) considered an ansatz in which $G\propto t^{-\beta}$ and showed that $|\beta|<0.1$ over the last 4.5×10^9 years, which corresponds to

$$|\dot{G}/G|<2\times10^{-11}\ \text{yr}^{-1}. \tag{195}$$

Guenther *et al.* (1995) also showed that g modes could provide even much tighter constraints, but these modes are to this date very difficult to observe. Nevertheless, they concluded, using the claim of detection by Hill and Gu (1990), that

$$|\dot{G}/G|<4.5\times10^{-12}\ \text{yr}^{-1}. \tag{196}$$

Guenther *et al.* (1998) compared the p-mode spectra predicted by different theories with varying gravitational constants to the observed spectrum obtained by a network of six telescopes and deduced that

$$|\dot{G}/G|<1.6\times10^{-12}\ \text{yr}^{-1}. \tag{197}$$

The standard Solar model depends on a few parameters, and G plays an important role since stellar evolution is dictated by the balance between gravitation and other interactions. Astronomical observations determine very accurately $GM_\odot$, and a variation of G with $GM_\odot$ fixed induces a change of the pressue ($P=GM_\odot^2/R_\odot^2$) and density ($\rho=M_\odot/R_\odot^3$). Ricci and Villante (2002) studied the effect of a variation of G on the density and pressure profile of the Sun and concluded that present data cannot constrain G better than $10^{-2}\%$.

The late stages of stellar evolution are governed by the Chandrasekhar mass $(\hbar c/G)^{3/2}m_{\rm n}^{-2}$ determined mainly by the balance between the Fermi pressure of a degenerate electron gas and gravity. Assuming that the mean neutron star mass is given by the Chandrasekhar mass, one expects that $\dot{G}/G=-2\dot{M}_{\rm NS}/3M_{\rm NS}$. Thorsett (1996) used the observations of five neutron star binaries for which five Keplerian parameters can be determined (the binary period P_b, the projection of the orbital semimajor axis $a_1\sin i$, the eccentricity e, the time and longitude of the periastron T_0 and ω) as well as the relativistic advance of the angle of the periastron $\dot{\omega}$. Assuming that the neutron star masses vary slowly as $M_{\rm NS}=M_{\rm NS}^{(0)}-\dot{M}_{\rm NS}t_{\rm NS}$, that their age was determined by the rate at which P_b is increasing (so that $t_{NS}\simeq2P_b/\dot{P}_b$), and that the mass follows a normal distribution, Thorsett (1996) deduced that, at 2σ,

$$\dot{G}/G=(-0.6\pm4.2)\times10^{-12}\ \text{yr}^{-1}. \tag{198}$$

Analogously, the Chandrasekhar mass sets the characteristic of the light curves of supernovae (Riazuelo and Uzan, 2002).

Garcia-Berro *et al.* (1995) considered the effect of a variation of the gravitational constant on the cooling of white dwarfs and on their luminosity function. As first pointed out by Vila (1976), the energy of white dwarfs is entirely of gravitational and thermal origin, so that a variation of G will induce a modification of their energy balance. Restricted to cold white dwarfs with luminosity smaller than ten Solar luminosities, the luminosity can be related to the star binding energy B and gravitational energy $E_{\rm grav}$ as

$$L=-\frac{dB}{dt}+\frac{\dot{G}}{G}E_{\rm grav}, \tag{199}$$

which simply results from the hydrostatic equilibrium. Again, the variation of the gravitational constant intervenes via the Poisson equation and the gravitational potential. The cooling process is accelerated if $\dot{G}/G<0$, which then induces a shift in the position of the cutoff in the luminosity function. Garcia-Berro *et al.* (1995) concluded that

$$0\leqslant-\dot{G}/G<4\times10^{-11}\ \ {\rm yr}^{-1}. \tag{200}$$

The result depends on the details of the cooling theory, on whether the C/O white dwarf is stratified or not, and on the hypothesis of the age of the galactic disk. For instance, with no stratification of the C/O binary mixture, one would require $\dot{G}/G=-(2.5\pm0.5)\times10^{-11}\ \ {\rm yr}^{-1}$ if the Solar neighborhood has a value of 8 Gyr (i.e., one would require a variation of G to explain the data). In the case of the standard hypothesis of an age of 11 Gyr, one obtains $0\leqslant-\dot{G}/G<3\times10^{-11}\ \ {\rm yr}^{-1}$.

A time variation of G also modifies the main sequence time of globular clusters (Dicke 1962a; Roeder, 1967). Del'Innocenti *et al.* (1996) calculated the evolution of low-mass stars and deduced the age of the isochrones. The principal effect was a modification of the main-sequence evolutionary time scale while the appearance of the color-magnitude diagram remained undistorted within the observational resolution and theoretical uncertainties. Since the globular clusters must be younger than the universe, and assuming that their age was between 8 and 20 Gyr, they concluded that

$$\dot{G}/G=(-1.4\pm2.1)\times10^{-11}\ \ {\rm yr}^{-1}. \tag{201}$$

This analysis was also applied to clusters of galaxies by Dearborn and Schramm (1974). In that case, a lower gravitational constant allows the particle to escape from the cluster since the gravitational binding energy also decreases. They deduced that the decrease of G that allows the existence of clusters at the present epoch is

$$0\leqslant-\dot{G}/G<4\times10^{-11}\ \ {\rm yr}^{-1}. \tag{202}$$

D. Cosmological constraints

1. Cosmic microwave background

A time-dependent gravitational constant will have mainly three effects on the CMB angular power spectrum [see Riazuelo and Uzan (2002) for discussions within the framework of scalar-tensor gravity in which G is considered as a field].

(1) The variation of G modifies the Friedmann equation and therefore the age of the Universe (and, hence, the sound horizon). For instance, if G is larger at an earlier time, the age of the Universe is smaller at recombination, so that the peak structure is shifted towards higher angular scales.

(2) The amplitude of the Silk damping is modified. At small scales, viscosity and heat conduction in the photon-baryon fluid produce a damping of the photon perturbations (Silk, 1968). The damping scale is determined by the photon diffusion length at recombination, and therefore depends on the size of the horizon at this epoch, and hence depends on any variation of the Newton constant throughout the history of the Universe.

(3) The thickness of the last scattering surface is modified. In the same vein, the duration of recombination is modified by a variation of the Newton constant as the expansion rate is different. It is well known that CMB anisotropies are affected on small scales because the last scattering "surface" has a finite thickness. The net effect is to introduce an extra, roughly exponential damping term, with the cutoff length being determined by the thickness of the last scattering surface. When translating redshift into time (or length), one has to use the Friedmann equations, which are affected by a variation of the Newton constant. The relevant quantity to consider is the visibility function g. In the limit of an infinitely thin last scattering surface, τ goes from ∞ to 0 at recombination epoch. For standard cosmology, it drops from a large value to a much smaller one, and hence the visibility function still exhibits a peak, but is much broader.

Liddle *et al.* (1998) studied the transition from radiation domination to matter domination in Jordan-Brans-Dicke theory and its effect on CMB anisotropies. Chen and Kamionkowski (1999) investigated in more detail the CMB spectrum in Brans-Dicke theory and showed that CMB experiments such as MAP will be able to constrain these theories for $\omega_{\rm BD}<100$ if all parameters are to be determined by the same CMB experiment, $\omega_{\rm BD}<500$ if all parameters are fixed but the CMB normalization, and $\omega_{\rm BD}<800$ if one uses the polarization. For the Planck mission these numbers are, respectively, 800, 2500, and 3200.

As far as we are aware, no complete study of the impact of the variation of the gravitational constant (e.g., in scalar-tensor theory) on the CMB has yet been performed. Note that, to compute the CMB anisotropies, one needs not only the value of G at the time of decoupling but also its complete time evolution up to now, since it will affect the integrated Sachs-Wolfe effect.

2. Nucleosynthesis

As explained in detail in Sec. III.C.2, changing the value of the gravitational constant affects the freeze-out temperature $T_{\rm f}$. A larger value of G corresponds to a higher expansion rate. This rate is determined by the combination $G\rho$, and in the standard case the Friedmann equations imply that $G\rho t^2$ is constant. The density ρ is determined by the number N_* of relativistic particles at the time of nucleosynthesis, so that nucleosynthesis allows us to put a bound on the number of neutrinos N_ν. Equivalently, assuming the number of neutrinos to be three leads to the conclusion that G has not varied by more than 20% since nucleosynthesis. But, allowing for a change both in G and N_ν allows for a wider range of variation. Contrary to the fine-structure constant, the role of G is less involved.

Steigmann (1976) used nucleosynthesis to put constraints on the Dirac theory. Barrow (1978) assumed that $G \propto t^{-n}$ and obtained from the helium abundances that $-5.9\times10^{-3}<n<7\times10^{-3}$, which implies that

$$|\dot{G}/G|<(2\pm9.3)h\times10^{-12}\ \mathrm{yr}^{-1}, \tag{203}$$

assuming a flat universe. This corresponds in terms of the Brans-Dicke parameter to $\omega_{\mathrm{BD}}>25$, which is a much smaller bound than the ones obtained today. Yang *et al.* (1979) included the computation of the deuterium and lithium. They improved the result by Barrow (1978) to $n<5\times10^{-3}$, which corresponds to $\omega_{\mathrm{BD}}>50$, and also pointed out that the constraint is tighter if there are extra neutrinos. It was further improved by Rothman and Matzner (1982) to $|n|<3\times10^{-3}$, implying

$$|\dot{G}/G|<1.7\times10^{-13}\ \mathrm{yr}^{-1}. \tag{204}$$

Accetta *et al.* (1990) studied the dependence of the abundances of D, ^{3}He, ^{4}He, and ^{7}Li upon the variation of G and concluded that

$$-0.3<\Delta G/G<0.4, \tag{205}$$

which corresponds roughly to $9\times10^{-3}<n<8\times10^{-3}$ and to $|\dot{G}/G|<9\times10^{-13}\ \mathrm{yr}^{-1}$.

All previous investigations assumed that the other constants were kept fixed and that physics was unchanged. Kolb *et al.* (1986) assumed a correlated variation of G, α_{EM}, and G_{F} and got a bound on the variation of the radius of the extra dimensions.

The case of Brans-Dicke (1961) theory, in which only the gravitational constant varied, was well studied. Casas *et al.* (1992a, 1992b) concluded from the study of helium and deuterium abundances that $\omega_{\mathrm{BD}}>380$ when $N_\nu=3$ (see also Damour and Gundlach, 1991, and Serna *et al.*, 1992) and $\omega_{\mathrm{BD}}>50$ when $N_\nu=2$.

Kim and Lee (1995) calculated the allowed value for the gravitational constant, electron chemical potential, and entropy consistent with observations up to lithium-7 and argued that beryllium-9 and boron-11 abundances are very sensitive to a change in G. Kim *et al.* (1998) further included neutrino degeneracy. The degeneracy of the electron-neutrino not only increases the radiation density but also influences the weak interaction rates so that it cannot be absorbed in a variation of G. It was shown that a higher gravitational constant can be balanced by a higher electron-neutrino degeneracy, so that the range of electron chemical potential, G was wider.

Damour and Pichon (1999) extended these investigations by considering a two-parameter family of scalar-tensor theories of gravitation involving a nonlinear scalar field-matter coupling function. They concluded that, even in the cases where the scalar-tensor theory before BBN was far from general relativity, BBN enables the setting of quite tight constraints on the observable deviations from general relativity today.

Let us also note the work by Carroll and Kaplinghat (2002), in which they tried to constrain the expansion history of our universe in a model-independent way during nucleosynthesis. They assumed changes in the gravitational dynamics and not in the particle physics processes. For that purpose, the expansion rate at the time of nucleosynthesis is approximated as $H(T)=(T/1\ \mathrm{MeV})^\alpha H_1$ in order to infer the constraints on (α,H_1). This is a simple way to compare an alternative to cosmology with data.

V. OTHER CONSTANTS

Up to now, we have detailed the results concerning the two most studied constants, α_{G} and α_{EM}. But, as we emphasized, if α_{EM} is varying one also expects a variation of other constants such as α_{S} and α_{W}. There are many theoretical reasons for that. First, in Kaluza-Klein or string-inspired models, all constants are varying due either to the dilaton or the extra dimensions (see Sec. VI for details).

Another argument lies in the fact that if we believe in grand unified theories, there exists an energy scale Λ_{GUT} at which all the (nongravitational) couplings unify,

$$\frac{8}{3}\alpha_{\mathrm{EM}}(\Lambda_{\mathrm{GUT}})=\alpha_{\mathrm{W}}(\Lambda_{\mathrm{GUT}})=\alpha_{\mathrm{S}}(\Lambda_{\mathrm{GUT}})\equiv\alpha_{\mathrm{GUT}}. \tag{206}$$

The value of the coupling constants at any energy scale smaller than Λ_{GUT} is obtained from the renormalization-group equations. It follows that a time variation of α_{EM} induces a time variation of α_{GUT} and thus of α_{W} and α_{S}. In such a framework, the varying parameters would then be α_{GUT}, $\Lambda_{\mathrm{GUT}}/M_4$, and the Yukawa couplings.

The strong coupling at an energy scale E is related to the QCD scale Λ_{QCD} by

$$\alpha_{\mathrm{S}}(E)=-\frac{2\pi}{\beta_0\ln(E/\Lambda_{\mathrm{QCD}})}, \tag{207}$$

with $\beta_0=-11+2n_{\mathrm{f}}/3$, n_{f} being the number of quark flavors. It follows that

$$\Delta\ln\Lambda_{\mathrm{QCD}}/E=\ln\left(\frac{E}{\Lambda_{\mathrm{QCD}}}\right)\frac{\Delta\alpha_{\mathrm{S}}}{\alpha_{\mathrm{S}}}. \tag{208}$$

The time variation of α_{S} is thus not the same at all energy scales. In the chiral limit, in which the quarks are massless, the proton mass is proportional to the QCD energy scale, $m_{\mathrm{p}}\propto\Lambda_{\mathrm{QCD}}$, so that a change in α_{S} (or in α_{GUT}) induces a change in μ and we have

$$\Delta m_{\mathrm{p}}/m_{\mathrm{p}}=\Delta\Lambda_{\mathrm{QCD}}/\Lambda_{\mathrm{QCD}}. \tag{209}$$

The energy-scale evolution of the three coupling constants in a one-loop approximation takes the form

$$\alpha_i^{-1}(E)=\alpha_{\mathrm{GUT}}^{-1}-\frac{b_i}{2\pi}\ln\left(\frac{E}{\Lambda_{\mathrm{GUT}}}\right), \tag{210}$$

where the numerical coefficients depend on the choice of the considered gauge group. For instance, $b_i=(41/10,-19/16,-7)$ in the standard model (SM) and $b_i=(33/5,1,-3)$ in its minimal supersymmetric extension. In the case of supersymmetric models (SUSY), Eq. (210) has to be replaced by

$$\alpha_i^{-1}(E)=\left[\alpha_{\rm GUT}^{-1}-\frac{b_i^{\rm SUSY}}{2\pi}\ln\left(\frac{E}{\Lambda_{\rm GUT}}\right)\right]\Theta(E-\Lambda_{\rm SUSY})$$
$$+\left[\alpha_i^{-1}(\Lambda_{\rm SUSY})-\frac{b_i^{\rm SM}}{2\pi}\ln\left(\frac{E}{\Lambda_{\rm SUSY}}\right)\right]\Theta(\Lambda_{\rm SUSY}-E).$$

This SUSY threshold applies only to SUSY models, and analogous thresholds have to be taken into account such as those associated with the t, b, and c quarks (see Dent and Fairbairn, 2003). Using Eq. (210), one can work out the variation of all couplings once the grand unified group is chosen, whether supersymmetry is assumed or not. Note also that, theoretically, $\alpha_{\rm EM}$ is determined by running down from some high-energy cutoff Λ and by the electron mass that is the infrared cutoff. The effect of the variation of $m_{\rm e}/\Lambda$ has been shown to have little effect (Wetterich, 2002).

In the string-inspired model by Damour and Polyakov (1994a, 1994b), Eq. (146) [obtained from Eq. (277)] implies that

$$\Delta m_{\rm hadron}/m_{\rm hadron}\simeq 40\Delta\alpha_{\rm EM}/\alpha_{\rm EM}, \tag{211}$$

where the string-mass scale is kept fixed, as was first pointed out by Taylor and Veneziano (1988).

Recently Calmet and Fritzsch (2002a, 2002b), Dent and Fairbairn (2003), and Langacker *et al.* (2002) have tried to work out these relationships in different models and have confirmed the order of magnitude Eq. (211). Calmet and Fritzsch (2002a) computed low-energy effects of a time-varying fine-structure constant within a grand-unified-theory- (GUT) like theory with a constraint of the form (206) and focused their analysis on the nucleon mass. Nevertheless, they assumed that the mechanisms of electroweak and supersymmetry breaking as well as fermion mass generation were left unchanged, and thus that quarks, leptons, W, and Z masses do not vary. But, as seen from Eqs. (212) and (213) below, one cannot vary $g_{\rm W}$ with $M_{\rm W}$ being fixed without varying the Higgs vacuum expectation value, which induces a variation of the mass of the fermions. On this basis they concluded that the result by Webb *et al.* (2001) on the fine-structure constant implies that $\Delta m_{\rm p}/m_{\rm p}\simeq 38\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\simeq -4\times 10^{-4}$ (keeping the Planck mass constant) and that $\Delta y/y\sim -36\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\sim 9\times 10^{-5}$, which is above the current observational constraints (see Sec. V.D). Calmet and Fritzsch (2002b) considered different scenarios: (i) $\Lambda_{\rm GUT}$ is constant and $\alpha_{\rm GUT}$ time dependent, (ii) only $\Lambda_{\rm GUT}$ is time dependent, and (iii) both are varying. One needs to specify the quantities that are kept constant; M_4 seems a good candidate since these studies do not consider the gravitational sector but other choices such as the string-mass scale, etc., can be made. They concluded that the most "interesting" situation, in view of the variation of $\alpha_{\rm EM}$ and μ, is the second case. Langacker *et al.* (2002) pointed out that changes in the quark masses and in the Higgs vacuum expectation value were also expected, and they parametrized the effects of the variation of $\alpha_{\rm GUT}$ on the electroweak and Yukawa sector. They assumed that $\alpha_{\rm GUT}$ was the vacuum expectation value of a slowly varying scalar field. They concluded that $\Delta\Lambda_{\rm QCD}/\Lambda_{\rm QCD}\sim 34\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ (with a precision of about 20% on the numerical factor) and that a variation of the fine structure of the magnitude of the one observed by Webb *et al.* (2001) would imply $\Delta m_{\rm p}/m_{\rm p}\simeq -2.5\times 10^{-4}$. They also argued that $\Delta x/x\sim -32\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\sim 8\times 10^{-5}$, which is consistent with current bounds if one assumes the variation of the proton gyromagnetic factor to be negligible. Earlier, Sisterna and Vucetich (1990) tried to determine the compatibility of all the bounds by restricting their study to $\Lambda_{\rm QCD}$, $\alpha_{\rm EM}$, $G_{\rm F}$, and G, and then included the u, d, and s quark masses (Sisterna and Vucetich, 1991).

Since we do not have the theories of electroweak and supersymmetry breaking as well as the ones for the generation of fermion masses, the correlations between different low-energy observables remain model dependent. But, in this unification picture, one is able to derive stronger constraints. For instance, Olive *et al.* (2002) expressed the constraints from α, β decays, and Oklo as functions of $|\Delta\Lambda_{\rm QCD}/\Lambda_{\rm QCD}-\Delta v/v|\sim 50\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ to give the tighter constraints, respectively, of $<10^{-7}$, $<10^{-9}$, and $<10^{-10}$. The goal of this section is to discuss the constraints on some of the constants that are of importance while checking for consistency.

A. Weak interaction

Most of the studies on the variation either of $G_{\rm F}$ or $\alpha_{\rm W}$ concern BBN, Oklo, CMB, and geochemial dating.

The Fermi constant can be expressed in terms of $g_{\rm W}$ and of the mass of the boson W, $M_{\rm W}$, as

$$G_{\rm F}=\frac{g_{\rm W}^2}{8M_{\rm W}^2}. \tag{212}$$

In the standard model, $M_{\rm W}^2$ is simply the product of $g_{\rm W}^2/4$ by the Higgs vacuum expectation value $v^2\equiv\langle\phi\rangle^2$, so that

$$G_{\rm F}=1/2v^2. \tag{213}$$

Thus, at tree level, $G_{\rm F}$ is actually independent of the SU(2) coupling and is a direct measurement of the magnitude of the electroweak symmetry breaking. Note that a change in v is related to a change in the Yukawa couplings.

Kolb *et al.* (1986) considered the effect of the variations of the fundamental constants on nucleosynthesis. As detailed in Sec. III.C.2, they found the dependence of the helium abundance on G, $G_{\rm F}$, and Q, the variation of which was related to the variation of $\alpha_{\rm EM}$ (then related to the size of extra dimensions). Kolb *et al.* (1986) did not consider changes in $G_{\rm F}$ due to the variation in $M_{\rm W}$ and assumed that $\delta G_{\rm F}\propto\delta g_{\rm W}$. Since G, $\alpha_{\rm EM}$, and $G_{\rm F}$ were related to the volume of extra space, this study gives no bound on the variation of $G_{\rm F}$, assuming $m_{\rm e}$ fixed.

Dixit and Sher (1988) argued that the relation between G_F and g_W in the work by Kolb *et al.* (1985) was ill motivated and that the only way to vary G_F was to vary v. Changing v has four effects on BBN: it changes (1) all weak interaction rates, (2) m_e, (3) the quark masses and hence Q, and (4) the pion mass, which affects the strong nuclear force and the binding of the deuteron. Using results on the dependence of m_e and m_p on α_{EM} and v (Gasser and Leutwyler, 1982), they got

$$\frac{Q}{1\ \mathrm{MeV}}=1.293-0.9\frac{\Delta\alpha_{EM}}{\alpha_{EM}}+2.193\frac{\Delta v}{v}. \tag{214}$$

Besides, a change of 1% of the quark masses changes the pion mass by 0.5%, which implies that the deuteron binding energy changes also by 0.5% (Davies, 1972). They concluded that the helium abundance was given by

$$Y_p=0.240-0.31\Delta v/v+0.38\Delta\alpha_{EM}/\alpha_{EM} \tag{215}$$

and deduced that $\Delta v/v<0.032$ if α_{EM} is fixed (assuming either the four-dimensional Planck mass or a higher-dimensional Planck mass fixed) and $\Delta\alpha_{EM}/\alpha_{EM}<0.026$ if v is fixed. They also noted that the changes in Q, m_e, and G_F induced by v tend to cancel the change in α_{EM}, appearing larger only in Q.

Scherrer and Spergel (1993) followed the same path and focused on two cases: (1) one in which the Yukawa couplings are fixed so that both G_F and the fermion masses vary in parallel and (2) one in which the Yukawa couplings vary so that G_F changes while the fermion masses are kept constant. Considering the abundances of helium and assuming α_{EM} fixed, they deduced that

$$-0.22<\Delta G_F/G_F<0.01 \tag{216}$$

in the first case and

$$-0.01<\Delta G_F/G_F<0.09 \tag{217}$$

in the second case.

To finish with cosmological constraints, a change in G_F induces a change in m_e that can be constrained by the CMB. The electron mass appears in the expression of the Thomson cross section [Eq. (116)] and on the binding energy of hydrogen [Eq. (117)], which induces a change in the ionization fraction. Kujat and Scherrer (2000) implemented these changes as in Sec. III.C and showed that the upper limit on $\Delta m_e/m_e$ is of order 10^{-2}–10^{-3} (keeping the Planck mass constant) for a maximum multipole of $\ell\sim 500$–2500 if α_{EM} is assumed constant. The degeneracy with α_{EM} is broken at high multipoles, so that one can hope to detect a 1% variation with a maximum multipole of $\ell>1500$.

From Oklo data, Shlyakhter (1976) argued that the weak interaction contribution to the total energy of the nucleus is of order $10^{-5}(m_\pi/m_p)^2$, so that $\Delta g_W/g_W\sim 5\times 10^6\Delta g_S/g_S$, where g_S is defined through the depth of the nuclear potential (see Sec. V.B), to conclude that

$$|\Delta\alpha_W/\alpha_W|<4\times 10^{-3}. \tag{218}$$

But in fact, the change in α_W is much more difficult to model than the change in α_{EM}. Damour and Dyson (1996) used the estimate by Haugan and Will (1976) for the weak-interaction contribution to the nuclear ground-state energy of samarium $E(^{150}\mathrm{Sm})-E(^{149}\mathrm{Sm})\simeq 5.6$ eV to conclude that, if no subtle cancellation appears,

$$|\Delta\alpha_W/\alpha_W|<0.02. \tag{219}$$

Concerning geochemical data (see Sec. V.A), Dyson (1972) pointed out that all β-decay rates are proportional to α_W^2, so that all constraints are in fact dependent on the combination $\alpha_{EM}^s\alpha_W^2$, s being defined in Eq. (49). The degeneracy can be lifted by comparing different nuclei, e.g., $^{187}_{75}\mathrm{Re}$ ($s_{Re}=-18\,000$) and $^{40}_{19}\mathrm{K}$ ($s_K=-30$). The constancy of the decay rates of these two nuclei have approximatively the same accuracy. From the constancy of the ratio

$$\Delta\frac{\lambda_{Re}}{\lambda_K}=(s_{Re}-s_K)\frac{\lambda_{Re}}{\lambda_K}\frac{\Delta\alpha_{EM}}{\alpha_{EM}}$$

within a few parts in 10^{10} per year, one can deduce that, independently of any assumption of α_W,

$$|\Delta\alpha_{EM}/\alpha_{EM}|<2\times 10^{-5} \tag{220}$$

and thus that

$$|\Delta\alpha_W/\alpha_W|<10^{-1} \tag{221}$$

during the last 10^9 years.

Wilkinson (1958) studied the variation of α_W by using pleochroic halos, that is, spheres formed by α-ray tracks around specks of uranium-bearing mineral in mica. The intensities of the halos of different radii give a picture of the natural radioactive series integrated over geological time from which one can deduce the proportion of different daughter activities in the decay chain from uranium to lead. This series contain elements undergoing both α and β decay. For instance, Ac branches 1.2% by α decay and the rest by β decay. From 10^9-year-old samples, Wilkinson (1958) deduced that

$$|\Delta\alpha_W/\alpha_W|<10. \tag{222}$$

Let us also mention some works (Agrawal *et al.*, 1998a, 1998b) in which the mass scale of the standard model and the scale of electroweak symmetry breaking are constrained by means of the anthropic principle. Passarino (2001) investigated the effects of the time variation of the Higgs vacuum expectation value and showed that the classical equation of motion for the Higgs field in the standard model accepts time-dependent solutions.

B. Strong interaction

A very small number of works address this issue. Due to the strong energy dependence of α_S, it makes more sense to constrain the variation of Λ_{QCD}. Doing so has a lot of implications on the stability properties of nuclei, and it follows that most of the constraints arise from nuclear considerations. Let us remember that in the chiral limit, all dimensional parameters are proportional to Λ_{QCD} so that all dimensionless ratios will be, in this limit, pure numbers and thus insensitive to a change of the strong interaction. But quark masses will play an important role in the variation of dimensionless ratios and so have to be taken into account.

A change in the strong interaction affects the light elements, and (1) the most weakly bound nucleus, namely, the deuteron, can be unbound if it is weaker, (2) there may exist stable dineutron and diproton if it is stronger [and hydrogen would have been burned catastrophically at the beginning of the universe (Dyson, 1971)], (3) the rate of the proton capture ($p+n\to D+\gamma$) is altered, and (4) the neutron lifetime changes. All these effects influence the nucleosynthesis (Barrow, 1987). It will also be a catastrophe if the deuteron is not stable (by affecting the hydrogen burning properties in stars).

Most of the early studies considered these stability properties by modeling the nuclear force by a Yukawa approximation of the form $V(r)\sim g_S^2\exp(-m_\pi r)$. In the following, the cited bounds refer to such a definition of α_S together with Eq. (12). Davies (1972) studied the stability of two-nucleon systems in terms of α_{EM} and α_S, assuming that α_W remains fixed, and concluded that the diproton is not bounded if $\Delta\alpha_S/\alpha_S-\Delta\alpha_{EM}/\alpha_{EM}<0.034$. Rozental (1980) assumed that the depth of the potential well in the deuteron is proportional to α_S to state that a decrease of α_S of 10–15 % would make it unstable. An increase of α_S would render the diproton stable, so that $|\Delta\alpha_S/\alpha_S|<10^{-1}$ at nucleosynthesis. A previous and more detailed analysis by Davies (1972) yields $|\Delta\alpha_S/\alpha_S|<4\times10^{-2}$, and Pochet *et al.* (1991) concluded that $|\Delta\alpha_S/\alpha_S|<4\times10^{-2}$ for the deuteron to be stable and $|\Delta\alpha_S/\alpha_S|<6\times10^{-1}$ for the diproton to be unstable.

Concerning high-Z nuclei, Broulik and Trefil (1971) used the liquid-drop model of the nucleus and the observed half-lives and abundances of transuramium elements to constrain the variation of α_S/α_{EM}. In this model, the stability of a nucleus can be discussed by comparing the Coulomb repulsion between protons to the strong-interaction attraction modeled by a surface tension T proportional to α_S. With increasing atomic weight, the individual nucleons become progressively more weakly bound as the Coulomb force dominates. A nucleus is stable against spontaneous fission if

$$\frac{Z^2}{A}<\frac{40\pi}{3}\frac{r_0^2}{e^2}T. \tag{223}$$

If α_S/α_{EM} was larger, in the past some unstable nuclei would have been stable. The idea is thus to find unstable nuclei with long half-lives that do not occur naturally. The variation of α_S/α_{EM} would make them stable in the past but this must have occurred roughly more than about ten times their lifetime since otherwise they would be in detectable abundances. Assuming that α_{EM} is fixed, Broulik and Trefil concluded from data on $^{244}_{94}$Pu that $|\Delta\alpha_S/\alpha_S|<1.7\times10^{-3}$ on a time scale of about 7.6×10^8 yr. Unfortunately, four months later it was reported that $^{244}_{94}$Pu occurs naturally on Earth (Hoffmann *et al.*, 1971), making the argument invalid. Davies (1972) argued that the binding energy is expected to vary as α_S^2 (contrary to the ansatz by Broulik and Trefil, 1971) so that the previous bound becomes $|\Delta\alpha_S/\alpha_S|<8.5\times10^{-4}$.

Barrow (1987) studied the effect of the change of α_S on the BBN predictions in Kaluza-Klein and superstring theories in which all the couplings depend on the compaction radius. Assuming that probably the most sensitive parameter of BBN, the deuteron binding energy, scales as α_S, he concluded that

$$t_N/\tau_n\propto G^{-1/2}\alpha_S^2G_F^2, \tag{224}$$

which affects the helium abundances from Eqs. (126) and (127). Dent and Fairbairn (2003) discussed the deuteron, diproton, and dineutron systems on the basis of a nuclear potential arising from chiral symmetry breaking in QCD. They derived the dependence of their binding energy on the parameter $(m_u+m_d)/\Lambda_{QCD}$ (see, however, Beane *et al.*, 2002 for details on the computation of nuclear forces). Analogously, Flambaum and Shuryak (2001) concluded that $|\delta_\pi|<0.005$ between BBN and today where $\delta_\pi\equiv\delta\ln(m_\pi/\Lambda_{QCD})=\delta\ln[\sqrt{(m_u+m_d)/\Lambda_{QCD}}]$.

As detailed in Sec. III.A.1, Shlyakhter (1976) argued that the change in the energy of the resonance in $^{149}_{62}$Sm is related to a change in α_S by

$$\Delta\alpha_S/\alpha_S\sim2\Delta E_r/V_0 \tag{225}$$

and deduced that $|\Delta\alpha_S/\alpha_S|<3.8\times10^{-9}$. Clearly, this analysis is not very reliable. Flambaum and Shuryak (2001) estimated the variation of the resonance energy due to a variation of the pion mass and concluded that $\Delta E_r/E_r\sim3\times10^8|\delta_\pi|$, so that $|\delta_\pi|<7\times10^{-10}$.

Flambaum and Shuryak (2001) also argued that, in the worst case, all strong-interaction phenomena depend on $\Lambda_{QCD}+Km_S$ where K is some universal constant and m_S the strange quark mass, but a real study of the effect of m_S on all hadronic masses remains to be done. It also follows that the proton gyromagnetic factor can be time dependent and constrained by observations such as those presented in Sec. V.D.

C. Electron-to-proton mass ratio

An early limit on the variation of[12] μ was derived by Yahil (1975), who compared the concordance of K-Ar and Rb-Sr geochemical ages and deduced that $|\Delta\mu/\mu|<1.2$ over the past 10^{10} yr.

As first pointed out by Thompson (1975), molecular absorption lines can provide a test of the variation of μ. The energy difference between two adjacent rotational levels in a diatomic molecule is proportional to Mr^{-2}, r being the bond length and M the reduced mass, and the vibrational transition of the same molecule has, in first approximation, a $\sqrt{M}$ dependence. For molecular hydrogen, $M=m_p/2$, so that a comparison of an observed vi-

[12]In the literature μ refers either to m_e/m_p or to its inverse. In the present work we choose the first definition and harmonize the results of the different articles.

brorotational spectrum with its present analog will thus give information on the variation of m_p and m_n. Comparing pure rotational transitions with electronic transitions gives a measurement of μ.

Following Thompson (1975), the frequency of vibration-rotation transitions is, in the Born-Oppenheimer approximation, of the form

$$\nu \simeq E_I(c_{\rm elec} + c_{\rm vib}/\sqrt{\mu} + c_{\rm rot}/\mu), \tag{226}$$

where $c_{\rm elec}$, $c_{\rm vib}$, and $c_{\rm rot}$ are some numerical coefficients. Comparing the ratio of wavelengths of various electronic-vibration-rotational lines in the quasar spectrum and in the laboratory allows us to trace the variation of μ, since, at lowest order, Eq. (226) implies

$$\frac{\Delta E_{ij}(z)}{\Delta E_{ij}(0)} = 1 + K_{ij}\frac{\Delta\mu}{\mu} + \mathcal{O}\left(\frac{\Delta\mu^2}{\mu^2}\right). \tag{227}$$

Varshalovich and Levshakov (1993) used the observations of a damped Lyman-α system associated with the quasar PKS 0528-250 (which is believed to have molecular hydrogen in its spectrum) of redshift $z=2.811$ and deduced that

$$|\Delta\mu/\mu| < 4\times 10^{-3}. \tag{228}$$

A similar analysis was first tried by Foltz *et al.* (1988) but their analysis did not take into account the wavelength-to-mass sensitivity and hence their result seems not very reliable. Nevertheless, they concluded that

$$|\Delta\mu/\mu| < 2\times 10^{-4} \tag{229}$$

at $z=2.811$. Cowie and Songaila (1995) observed the same quasar and deduced that

$$\Delta\mu/\mu = (0.75 \pm 6.25)\times 10^{-4} \tag{230}$$

at 95% confidence level from the data on 19 absorption lines. Varshalovich and Potekhin (1995) calculated the coefficient K_{ij} with a better precision and deduced that

$$|\Delta\mu/\mu| < 2\times 10^{-4} \tag{231}$$

at 95% confidence level. Lanzetta *et al.* (1995) and Varshalovich, Panchuk, and Ivanchik (1996) used 59 transitions for H_2 rotational levels in PKS 0528-250 and got

$$\Delta\mu/\mu = (-1 \pm 1.2)\times 10^{-4} \tag{232}$$

at 2σ level. These results were confirmed by Potekhin *et al.* (1998) using 83 absorption lines to get

$$\Delta\mu/\mu = (-7.5 \pm 9.5)\times 10^{-5} \tag{233}$$

at a 2σ level.

More recently, Ivanchik *et al.* (2002) measured, with the Very Large Telescope (VLT), the vibrorotational lines of molecular hydrogen for two quasars with damped Lyman-α systems at, respectively, $z=2.3377$ and 3.0249 and also argued for the detection of a time variation of μ. Their most conservative result (the observational data were compared to two experimental data sets) is

$$\Delta\mu/\mu = (-5.7 \pm 3.8)\times 10^{-5} \tag{234}$$

at 1.5σ, and the authors cautiously point out that additional measurements are necessary to support this conclusion. 1.5σ is not really significant, and this may not survive further extended analysis. The result is also dependent on the laboratory dataset used for the comparison since it gave $\Delta\mu/\mu = (-12.2 \pm 7.3)\times 10^{-5}$ with another dataset.

As in the case of Webb *et al.* (1999, 2001), this measurement is very important in the sense that it is a nonzero detection that will have to be compared with other bounds. The measurements by Ivanchik *et al.* (2002) is indeed much larger than one would expect from the electromagnetic contributions. As seen above, with the change in any unified theory, the changes in the masses are expected to be larger than the change in $\alpha_{\rm EM}$. Typically, we expect $\Delta\mu/\mu \sim \Delta\Lambda_{\rm QCD}/\Lambda_{\rm QCD} - \Delta v/v \sim (30\text{–}40)\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$, so that it seems the detection by Webb *et al.* (2001) is too large by a factor of 10 to be compatible with it. Note that Calmet and Fritzsch (2002b) expect that $\Delta\mu/\mu \sim 22\times 10^{-5}$ if $\Lambda_{\rm GUT}/M_4$ is varying.

Wiklind and Combes (1997) observed the quasar PKS 1413+135 with redshift $z=0.247$ and used different transitions from the same molecule to constrain the variation of μ. They compared different lines of HCO^+, HCN, and CO and showed that the redshift differences are likely to be dominated by the velocity difference between the two species, which limits the precision of the measurements to $\Delta\mu/\mu \sim 10^{-5}$ at 3σ level. In one source (B3 1504+377), they observed a discrepancy of $\Delta\mu/\mu \sim 10^{-4}$.

Pagel (1977, 1983) used another method to constrain μ based on the measurement of the mass shift in the spectral lines of heavy elements. In that case, the mass of the nucleus can be considered as infinite, contrary to the case of hydrogen. A variation of μ will thus influence the redshift determined from hydrogen [see Eq. (68)]. Pagel compared the redshifts obtained from the spectrum of the hydrogen atom and metal lines for quasars of redshift ranging from 2.1 to 2.7. Since

$$\Delta z \equiv z_{\rm H} - z_{\rm metal} = (1+z)\frac{\Delta\mu}{1-\mu_0}, \tag{235}$$

he obtained

$$|\Delta\mu/\mu| < 4\times 10^{-1} \tag{236}$$

at 3σ level. This result is unfortunately not conclusive because heavy elements and hydrogen usually belong to different interstellar clouds with different radial velocities.

D. Proton gyromagnetic factor

As seen in Sec. III.B, the hyperfine structure induces a splitting dependent on $g_p\mu\alpha_{\rm EM}^2$. The ratio between the frequency ν_{21} of the hyperfine 21-cm absorption transition and optical resonance transition of frequency $\nu_{\rm opt}$ mainly depends on

$$\nu_{21}/\nu_{\rm opt} \propto \alpha_{\rm EM}^2 g_p \mu \equiv x. \tag{237}$$

By comparing the redshift of the same object determined from optical data and the 21-cm transition, one deduces that

$$\Delta z = z_{\rm opt} - z_{21} = (1+z)\Delta x/x. \tag{238}$$

Savedoff (1956) used the spectrum of Cygnus A and deduced that

$$\Delta x/x = (3\pm7)\times10^{-4} \tag{239}$$

at $z\sim0.057$. Wolfe *et al.* (1976) discovered a BL Lac object (AO 0235+164) having the same redshift determined either by the 21-cm absorption line or by the ultraviolet doublet of Mg^+. Using

$$\nu_{\rm H}/\nu_{\rm Mg} \propto g_{\rm p}\mu\alpha_{\rm EM}^2(1-3\mu+\cdots), \tag{240}$$

they concluded that

$$\Delta x/x = (5\pm10)\times10^{-5} \tag{241}$$

at redshift of $z=0.5$. They also got a constraint on the variation of $g_{\rm p}\mu$ by comparing the separation of the Mg II doublet to hydrogen to get $|\Delta g_{\rm p}\mu/g_{\rm p}\mu|<6\times10^{-2}$. Wolfe and Davis (1979) used the 21-cm absorption lines of neutral hydrogen in front of the quasar QSO 1331+170 at a redshift $z\sim2.081$. They determined that the cloud was at redshift $z\sim1.755$. The agreement between the 21-cm and optical redshifts is limited by the error in the determination of the optical redshift. They concluded that

$$|\Delta x/x| \leqslant 2\times10^{-4} \tag{242}$$

at a redshift $z\sim1.755$; another absorber at redshift $z\sim0.524$ around the quasar AO 0235+164 gives

$$|\Delta x/x| \leqslant 2.8\times10^{-4}. \tag{243}$$

Tubbs and Wolfe (1980) used a set of four quasars, among them MC3 1331+17, for which $z_{21}=1.77642\pm2\times10^{-5}$ is known with very high precision, and deduced that

$$|\Delta x/x| < 2\times10^{-4}. \tag{244}$$

Cowie and Songaila (1995) used the observations of neutral carbon absorption and fine structure to get the better optical redshift $z_{\rm opt}=1.776\,44\pm2\times10^{-5}$, which enables them to improve the constraint to

$$\Delta x/x = (7\pm11)\times10^{-6}. \tag{245}$$

Besides the uncertainty in the determination of the optical redshift, since the 21-cm optical depth depends sensitively on spin temperature while resonance-line optical depths do not, the two regions of absorption need not coincide. This induces an uncertainty $\Delta z=\pm(1+z)\times(\Delta v_{\rm opt}/c)$ into Eq. (237) [see e.g., Wolfe and Davis (1979) for a discussion].

Drinkwater *et al.* (1998) compared the hydrogen hyperfine structure to molecular absorption for three systems at redshift $z=0.24$, 0.67, and 0.68 and used CO absorption lines. This allowed them to constrain $y\equiv g_{\rm p}\alpha_{\rm EM}^2$ and they got

$$|\Delta y/y| < 5\times10^{-6}. \tag{246}$$

Assuming that the change in $g_{\rm p}$ and $\alpha_{\rm EM}$ are not correlated, they deduced that $|\Delta g_{\rm p}/g_{\rm p}|<5\times10^{-6}$ and

$$|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}| < 2.5\times10^{-6}. \tag{247}$$

If $g_{\rm p}$ is constant, then this constraint is incompatible with the claim by Webb *et al.* (2001). This is also the case in GUT-like theories (Calmet and Fritzsch, 2002a) that argue $\Delta y/y\sim-36\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$.

Varshalovich and Potekhin (1996) used the CO and hyperfine hydrogen redshift toward PKS 1413+135 ($z=0.247$) to get

$$\Delta y/y = (-4\pm6)\times10^{-5} \tag{248}$$

and PKS 1157+0.14 ($z=1.944$) for

$$\Delta y/y = (7\pm10)\times10^{-5}. \tag{249}$$

Murphy *et al.* (2001d) improved the precision of this measurement by fitting Voigt profiles to the H 21-cm profile instead of using published redshifts and got

$$\Delta y/y = (-0.2\pm0.44)\times10^{-5} \tag{250}$$

at $z=0.25$ and

$$\Delta y/y = (-0.16\pm0.54)\times10^{-5} \tag{251}$$

at $z=0.68$. With the same systems, Carrilli *et al.* (2000) found

$$|\Delta y/y| < 1.7\times10^{-5} \tag{252}$$

both at $z=0.25$ and $z=0.68$. Murphy *et al.* (2001d) argued that one can estimate the velocity to 1.2 km s^{-1} instead of the 10 km s^{-1} assumed by Carrilli *et al.* (2000), so that their results in fact lead to $\Delta y/y=(1\pm0.03)\times10^{-5}$ at $z=0.25$ and $\Delta y/y=(1.29\pm0.08)\times10^{-5}$ at $z=0.68$.

E. The particular case of the cosmological constant

The cosmological constant has also been losing its status as a constant. In this section, we briefly review the observations backing up this fact and then describe the theoretical models in favor of a time-dependent cosmological constant, providing some links with the variation of other fundamental constants.

The combination of recent astrophysical and cosmological observations [which include the luminosity distance-redshift relation up to $z\sim1$ from type-Ia supernovae (Perlmutter *et al.*, 1998; Riess *et al.*, 1998), the cosmic microwave background temperature anisotropies (de Bernardis *et al.*, 2000), and gravitational lensing (Mellier, 1999)] seems to indicate that the universe is accelerating and that about 70% of the energy density of the universe is made of matter with a negative pressure (i.e., having an equation of state $w\equiv P/\rho<0$).

There are many different candidates to account for this exotic type of matter. The most simple solution would be a cosmological constant (for which $w=-1$), but one will then have to face the well-known *cosmological constant problem* (Weinberg, 1989), i.e., the fact that the value of this cosmological constant inferred from the cosmological observations is extremely small—by about 120 orders of magnitude—compared with the energy scales of high-energy physics (such as Planck, GUT). Another solution is to argue for a (yet unknown) mechanism that makes the cosmological constant strictly vanish, and to find another matter candidate (referred to as "dark energy") able to explain the cosmological observations.

Among all the proposals (see, e.g., Binétruy, 2000 and Carroll, 2001 for a review), quintessence seems to be a promising mechanism. In these models, a scalar field is rolling down a runaway potential, decreasing to zero at infinity and hence acting as a fluid, with an effective equation of state in the range $-1\leqslant w\leqslant 1$ if the field is minimally coupled. Runaway potentials such as exponential potential and inverse power-law potentials

$$V(\phi)=M^{4+\alpha}/\phi^{\alpha}, \tag{253}$$

with $\alpha>0$ and M a mass scale, arise in models where supersymmetry is dynamically broken (Binétruy, 1999) and in which flat directions are lifted by nonperturbative effects. Note, however, that, in general, imposing the correct superpartner mass splittings implies that supergravity corrections to the quintessence mass are expected to be very large.

One of the underlying motivations to replace the cosmological constant by a scalar field comes from superstring models in which any dimensionful parameter is expressed in terms of the string mass scale and the vacuum expectation value of a scalar field. However, the requirement of slow roll (mandatory in order to have a negative pressure) and the fact that the quintessence field dominates today imply, if the minimum of the potential is zero, that (i) it is very light, roughly of order $\sim 10^{-33}$ eV (Carroll, 1998), and that (ii) the vacuum expectation value of the quintessence field today is of order of the Planck mass. It follows that coupling of this quintessence field leads to observable long-range forces and time dependence of the constant of nature.

Carroll (1998) considered the effect of the coupling of this very light quintessence field to ordinary matter via an interaction of the form $\beta_i(\phi/M)\mathcal{L}_i$ and to the electromagnetic field as $\phi F^{\mu\nu}\tilde{F}_{\mu\nu}$. Chiba and Kohri (2002) also argued that an ultralight quintessence field induces a time variation of the coupling constant if it is coupled to ordinary matter; they studied a coupling of the form $\phi F^{\mu\nu}F_{\mu\nu}$. Dvali and Zaldarriaga (2002) showed that it will be either detectable as a quintessence field or by tests of the equivalence principle, as also concluded by Wetterich (2002).

It was proposed that the quintessence field is also the dilaton (Uzan, 1999; Riazuelo and Uzan, 2000; Banerjee and Pavon, 2001; Esposito-Farèse and Polarski, 2001; Gasperini *et al.*, 2002). The same scalar field drives the time variation of the cosmological constant as well as of the gravitational constant, and it has the ability to also have tracking solutions (Uzan, 1999).

Another motivation for considering the link between a dynamical cosmological constant and the time variation of fundamental constants comes from the origin of the inverse power-law potential. As shown by Binétruy (1999), it can arise from supersymmetry breaking by nonperturbative effects such as gaugino condensation. The same kind of potential was also considered by Vayonakis (1988) while discussing the variation of the fundamental couplings in the framework of ten-dimensional supergravity.

The variation of fundamental constants also has other implications for the measurement of the cosmological constant. Riazuelo and Uzan (2002) considered the effect of the variation of the gravitational constant on supernovae data. Besides changing the luminosity distance-redshift relation, the variation of G changes the standard picture in two ways according to which type-Ia supernovae are standard candles. First, the thermonuclear energy release proportional to the synthesized nickel mass is changing (and hence the maximum of the light curve); second, the time scale of the supernovae explosion and thus the width of the light curve is also changed. Riazuelo and Uzan (2002) derived the modified magnitude-redshift relation to include the effect of the variation of G using a one-zone analytical model for the supernovae. This was confirmed by numerical simulations (Gaztañaga *et al.*, 2002).

Barrow and Magueijo (2001) considered the effect of a time-dependent fine-structure constant on the interpretation of the supernovae data. Their study was restricted to a class of varying speed-of-light theories (see Sec. VI.C) that have cosmological solutions very similar to quintessence. But only the effect on the Hubble diagram was studied; the influence of the change of the fine-structure constant on the thermonuclear burst of the supernovae, and hence on its light curve, was not considered at all.

Up to now there has been no observational evidence of a time variation of the cosmological constant. The measurement of the equation of state of the dark energy will be possible, it is hoped, very soon, the best candidate being the use of large-scale structure growth and weak gravitational lensing (Benabed and Bernardeau, 2001). But it seems that the variation of constants and the dark energy are somehow related (Banks *et al.*, 2002; Chiba and Khori, 2002; Dvali and Zaldarriaga, 2002; Fujii, 2002; Wetterich, 2002); at least they share the properties of being very light and appearing in many models with a runaway potential. The effects of the cosmological constant on the evolution of the fine-structure constant in scalar-tensor theories and their anthropic implications were investigated by Barrow, Sandvik, and Magueijo (2002a).

F. Attempts to constrain the variation of dimensionful constants

As emphasized in Sec. I, considering the variation of dimensionful constants is doubtful and seems meaningless, but such attempts have nevertheless been made. We briefly review and comment on them as follows. These investigations were motivated mainly by the construction of cosmological models alternative to the big bang scenario and in which the redshift needs to have another interpretation.

Bahcall and Salpeter (1965) proposed to look for a time variation of the Planck constant by comparing the light emitted by two quasars. Their idea was based on the observation that a prism was sensitive to the energy E of the photon and a diffraction grating to its wavelength λ, so that any difference in the comparison of the wavelengths of a particular spectral line could be attributed to a change in $\hbar$. Their study led to a null result in terms of experimental errors.

Noerdlinger (1973) [and later Blake (1977a)] tried to measure $E\lambda$. His argument was that the intensity of the Rayleigh-Jeans tail of the Planck spectrum of the CMB photon determines $k_B T$ whereas the turnover point of the spectrum determines $h\nu/k_B T$. It follows that one can determine the value of hc at the time of recombination, leading to the constraint $|\Delta \ln hc|<0.3$.

Further work as performed by Solheim *et al.* (1976) and Baum and Florentin-Nielsen (1976), who compared the light of nearby and distant galaxies in order to test the constancy of $E\lambda$. Bekenstein (1979) demonstrated that these experiments were meaningless since the constancy of $E\lambda$ was interpreted as the constancy of $\hbar c$; this latter fact was implicitly assumed in the two experiments since the wave vector and momentum of the photon were both propagated in a parallel manner. This is only possible if their proportionality factor $\hbar c$ is constant, ensuring the null result of the two experiments.

VI. THEORETICAL MOTIVATIONS

One general feature of higher-dimensional dimensional theories, such as Kaluza-Klein and string theories, is that the "true" constants of nature are defined in the full higher-dimensional theory. The effective four-dimensional constants depend on the value of some scalar fields and on the structure and sizes of the extra dimensions. Any evolution of these sizes either in time or space would lead to a spacetime dependence of the effective four-dimensional constants. If some of these fields are extremely light, they could give rise to variation of constants on cosmological time scales.

We present in Secs. VI.A and VI.B some results concerning Kaluza-Klein theories and string theories. We end in Sec. VI.C by describing some phenomenological approaches initiated by Bekenstein (1982).

A. Kaluza-Klein theories

The aim of the early model by Kaluza (1921) and Klein (1926) to consider a five-dimensional spacetime with one spatial extra dimension S^1 (assumed to be of radius R_{KK}) was to unify electromagnetism and gravity (for a review see, e.g., Overduin and Wesson, 1997). Starting from the five-dimensional Einstein-Hilbert Lagrangian

$$S_5=\frac{1}{2}\int d^5\mathbf{x}\sqrt{-g_5}M_5^3R_5, \quad (254)$$

we decompose the five-dimensional metric as

$$ds_5^2=g_{\mu\nu}dx^\mu dx^\nu+e^{2\sigma}(A_\mu dx^\mu+dy)^2. \quad (255)$$

This form still allows four-dimensional reparametrizations of the form $y'=y+\lambda(x^\mu)$ provided that $A'_\mu=A_\mu-\partial_\mu\lambda$, so that gauge transformations arise from the higher-dimensional coordinate transformations group. Any field ϕ can be decomposed as

$$\phi(x^\mu,y)=\sum_{n\in Z}\phi^{(n)}(x^\mu)e^{iny/R_{KK}}. \quad (256)$$

The five-dimensional Klein-Gordon equation for a massless field becomes

$$\nabla_\mu\nabla^\mu\phi^{(n)}=(n/R_{KK})^2\phi^{(n)} \quad (257)$$

so that $\phi^{(n)}$ has a mass $m_n=n/R_{KK}$. At energies small with respect to $m_{KK}=R_{KK}^{-1}$, only y-independent fields remain, and the physics is four dimensional. The effective action for the massless fields is obtained from the relation $R_5=R_4-2e^{-\sigma}\Delta e^{\sigma}-e^{2\sigma}F^2/4$ with $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ so that

$$S_4=\pi\int d^4\mathbf{x}\sqrt{-g}e^{2\sigma}R_{KK}M_5^3 \times[R_4-\partial_\mu\sigma\partial^\mu\sigma-\tfrac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}]. \quad (258)$$

The field equations do not determine the compaction radius and only the invariant radius $\rho=R_{KK}\exp(\sigma)$ distinguishes nonequivalent solutions (one can set R_{KK} to unity without loss of generality).

Setting $A_\mu=R_{KK}\tilde{A}_\mu$, the covariant derivative is $\partial_\mu+ip_yA_\mu=\partial_\mu+in\tilde{A}_\mu$, so that the charges are integers. The four-dimensional Yang-Mills coupling, identified as the coefficient $-1/4g_{YM}^2$ of $\tilde{F}^2$, and the gravitational constant are given by

$$M_4^2=2\pi\rho M_5^3,\quad 4g_{YM}^{-2}=M_4^2\rho^2/2, \quad (259)$$

$2\pi\rho$ being the volume of the extra space. Note that as long as one considers vacuum as in Eq. (255), there is a conformal undeterminacy that has to be lifted when adding matter fields. This generalizes to the case of D extra dimensions (see, e.g., Cremmer and Scherk, 1977, and Forgács and Horváth, 1979 for the case of two extra dimensions) to

$$G\propto\rho^{-D},\quad \alpha_i(m_{KK})=K_i(D)G\rho^2, \quad (260)$$

where the constants K_i depend only on the dimension and topology of the compact space (Weinberg, 1983b), so that the only fundamental constant of the theory is M_{4+D}. A theory of $\mathcal{M}_4\times\mathcal{M}_D$, where $\mathcal{M}_D$ is a D-dimensional compact space, generates a low-energy quantum field theory of the Yang-Mills type related to the isometries of $\mathcal{M}_D$ [for instance, Witten (1981) showed that for $D=7$, it can accommodate the Yang-Mills group $SU(3)\times SU(2)\times U(1)$]. The two main problems of these theories are that one cannot construct chiral fermions in four dimensions by compaction on a smooth manifold with such a procedure, and gauge theories in five dimensions or more are not renormalizable.

The expression for the structure constants at lower energy are obtained by the renormalization group (Marciano, 1984; Wu and Wang, 1986)

$$\alpha_i^{-1}(mc^2)=\alpha_i^{-1}(m_{\mathrm{KK}}c^2) - \frac{1}{\pi}\sum_j C_{ij}\left[\ln\frac{m_{\mathrm{KK}}}{m_j}-\theta(m-m_j)\ln\frac{m_j}{m}\right], \tag{261}$$

where the sum is over all leptons, quarks, gluons, etc., and the C_{ij} are constants that depend on the spin and group representation (Georgi *et al.*, 1974). Note, however, that this relation is obtained by considering the renormalization group in four dimensions, and does not take into account the contribution of the Kaluza-Klein modes in loops.

Chodos and Detweiler (1980) illustrated the effect of the fifth dimension by considering a five-dimensional vacuum solution of the Kasner form

$$ds^2=-dt^2+\sum_{i=1,\ldots,4}\left(\frac{t}{t_0}\right)^{2p_i}(dx^i)^2 \tag{262}$$

with $\Sigma p_i=\Sigma p_i^2=1$ and assuming compact spatial sections $0\leqslant x_i<L$. In order to ensure local isotropy and homogeneity, they choose the solutions $p_1=p_2=p_3=1/2$ and $p_4=-1/2$, so that the universe has four macroscopic spatial dimensions at the time t_0 and looks spatially three dimensional at a time $t\gg t_0$ with a small compact dimension of radius $(T_0/t)^{1/2}L$. Considering A_μ as a small metric perturbation, they deduced that

$$\alpha_{\mathrm{EM}}/\alpha_{\mathrm{G}}=t/t_0, \tag{263}$$

hence offering a realization of three Dirac large-number hypothesis. Freund (1982) studied $(4+D)$ Kaluza-Klein cosmologies starting both in a $(4+D)$-dimensional Einstein gravity or a $(4+D)$-dimensional Brans-Dicke gravity.

Using the expressions (260) and (261), Marciano (1984) related the time dependence of the different couplings and restricted his discussion to the cases where $\dot{K}_i$ and $\dot{m}_j$ vanish. In the case where $\dot{\alpha}_i(m_{\mathrm{KK}})=0$ (as studied in Chodos and Detweiler, 1980) one can relate the time variation of the gravitational and fine structure constant as

$$\frac{\dot{\alpha}_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}}=-\frac{\alpha_{\mathrm{EM}}}{2\pi}\sum_j\left(\frac{5}{3}C_{1j}+C_{2j}\right)\frac{\dot{G}}{G}. \tag{264}$$

In the case where $\dot{\alpha}_i(m_{\mathrm{KK}})\neq 0$ (as studied in Freund, 1982), it was shown that the time variation of α_{S} was enhanced at low energy, so that constraints on the time variation of $m_{\mathrm{e}}/m_{\mathrm{p}}$ provided a sensitive test. It was also claimed that, in the case of an oscillating m_{KK}, the amplitude of the oscillations would be damped by radiation in our three-dimensional spacetime due to oscillating charges, and that experimental bounds can be circumvented.

Kolb *et al.* (1986) used the variation (260) to constrain the time variation of the radius of the extra dimensions during primordial nucleosynthesis (see Sec. III.C.2) assuming that $G_{\mathrm{F}}\propto g_{\mathrm{W}}^2$. They deduced $|\Delta R_{\mathrm{KK}}/R_{\mathrm{KK}}|<1\%$. Barrow (1987) took the effects of the variation of $\alpha_{\mathrm{S}}\propto R_{\mathrm{KK}}^{-2}$ (see Sec. V.B) and deduced from the helium abundances that for $|\Delta R_{\mathrm{KK}}/R_{\mathrm{KK}}|<0.7\%$ and $|\Delta R_{\mathrm{KK}}/R_{\mathrm{KK}}|<1.1\%$, respectively, for $D=2$ and $D=7$ Kaluza-Klein theory, and that $|\Delta R_{\mathrm{KK}}/R_{\mathrm{KK}}|<3.4\times10^{-10}$ from the Oklo data.

It follows that the radius of the extra dimensions has to be stabilized, but no satisfactory and complete mechanism has yet been found. Li and Gott (1998) considered a five-dimensional Kaluza-Klein inflationary scenario, which is static in the internal dimension and expanding in the other dimensions, and solves the five-dimensional semiclassical Einstein equations including the Casimir effect. In particular, it was deduced that the effective four-dimensional cosmological constant is related to the fine-structure constant by $G\Lambda_{\mathrm{eff}}=(15g_*/2048\pi^7)\,\alpha_{\mathrm{EM}}^2$.

B. Superstring theories

There exist five anomaly free supersymmetric perturbative string theories, respectively known as type-I, type-IIA, type-IIB, SO(32) heterotic, and $E_8\times E_8$ heterotic theories (see, e.g., Polchinski, 1997). One of the definitive predictions of these theories is the existence of a scalar field, the dilaton, that couples directly to matter (Taylor and Veneziano, 1988) and whose vacuum expectation value determines the string coupling constant (Witten, 1984). There are two other excitations that are common to all perturbative string theories, a rank-two symmetric tensor (the graviton) $g_{\mu\nu}$ and a rank-two antisymmetric tensor $B_{\mu\nu}$. The field content then differs from one theory to another. It follows that the four-dimensional couplings are determined in terms of a string scale and various dynamical fields (dilaton, volume of compact space, etc.). When the dilaton is massless, we expect *three* effects: (i) a scalar admixture of a scalar component inducing deviations from general relativity in gravitational effects, (ii) a variation of the couplings, and (iii) a violation of the weak-equivalence principle. Our purpose is to show how the four-dimensional couplings are related to the string mass scale, to the dilaton, and to the structure of the extra dimensions mainly in the example of heterotic theories.

To be more specific, let us consider an example. The two *heterotic theories* originate from the fact that left- and right-moving modes of a closed string are independent. This reduces the number of supersymmetry to $N=1$, and the quantization of the left-moving modes requires that the gauge group be either SO(32) or $E_8\times E_8$ depending on the fermionic boundary conditions. The effective tree-level action is

$$S_H=\int d^{10}\mathbf{x}\sqrt{-g_{10}}e^{-2\Phi}\left[M_H^8\{R_{10}+4\Box\Phi-4(\nabla\Phi)^2\} - \frac{M_H^6}{4}F_{AB}F^{AB}+\cdots\right]. \tag{265}$$

When compacted on a six-dimensional Calabi-Yau space, the effective four-dimensional action takes the form

$$S_H=\int d^4\mathbf{x}\sqrt{-g_4}\phi\left[M_H^8\left\{R_4+\left(\frac{\nabla\phi}{\phi}\right)^2-\frac{1}{6}\left(\frac{\nabla V_6}{V_6}\right)^2\right\}-\frac{M_H^6}{4}F^2\right]+\cdots, \qquad (266)$$

where $\phi\equiv V_6 e^{-2\Phi}$ couples identically to the Einstein and Yang-Mills terms. It follows that

$$M_4^2=M_H^8\phi, \quad g_{\rm YM}^{-2}=M_H^6\phi \qquad (267)$$

at tree level. Note that to reach this conclusion, one has to assume that the matter fields [in the "dots" of Eq. (266)] are minimally coupled to g_4 (see, e.g., the discussion by Maeda, 1988).

The strongly coupled SO(32) heterotic string theory is equivalent to the weakly coupled type-I string theory. *Type-I superstring* admits open strings, the boundary conditions of which divide the number of supersymmetries by two. It follows that the tree-level effective bosonic action is $N=1$, $D=10$ supergravity which takes the form, in the string frame,

$$S_I=\int d^{10}\mathbf{x}\sqrt{-g_{10}}M_I^6 e^{-\Phi}\left[e^{-\Phi}M_I^2R_{10}-\frac{F^2}{4}+\cdots\right], \qquad (268)$$

where the dots contains terms describing the dynamics of the dilaton, fermions, and other form fields. At variance with Eq. (265), the field Φ couples differently to the gravitational and Yang-Mills terms because the graviton and Yang-Mills fields are, respectively, excitation of closed and open strings. It follows that M_I can be lowered even to the weak scale by simply having $\exp(\Phi)$ be small enough. Type-I theories require $D9$-branes for consistancy. When V_6 is small, one can use T duality (to render V_6 large, which allows us to use a quantum-field-theory approach) and turn the $D9$-brane into a $D3$-brane so that

$$S_I=\int d^{10}\mathbf{x}\sqrt{-g_{10}}e^{-2\Phi}M_I^8R_{10}-\int d^4\mathbf{x}\sqrt{-g_4}e^{-\Phi}\frac{1}{4}F^2+\cdots, \qquad (269)$$

where the second term describes the Yang-Mills fields localized on the $D3$-brane. It follows that

$$M_4^2=e^{-2\Phi}V_6M_I^8, \quad g_{\rm YM}^{-2}=e^{-\Phi} \qquad (270)$$

at tree level. If one compacts the $D9$-brane on a six-dimensional orbifold instead of a six-torus, and if the brane is localized at an orbifold fixed point, then gauge fields couple to fields M_i living only at these orbifold fixed points with a (calculable) tree-level coupling c_i so that

$$M_4^2=e^{-2\Phi}V_6M_I^8, \quad g_{\rm YM}^{-2}=e^{-\Phi}+c_iM_i. \qquad (271)$$

The coupling to the field c_i is *a priori* nonuniversal. At strong coupling, the ten-dimensional $E_8\times E_8$ heterotic theory becomes M theory on $R^{10}\times S^1/Z_2$ (Hořava and Witten, 1996). The gravitational field propagates in the 11-dimensional space, while the gauge fields are localized on two ten-dimensional branes.

At one loop, one can derive the couplings by including Kaluza-Klein excitations to get (see, e.g., Dudas, 2000)

$$g_{\rm YM}^{-2}=M_H^6\phi-\frac{b_a}{2}(RM_H)^2+\cdots \qquad (272)$$

when the volume is large compared to the mass scale, and in that case the coupling is no longer universal. Otherwise, one would get a more complicated function. Obviously, the four-dimensional effective gravitational and Yang-Mills couplings depend on the considered superstring theory or on the compaction scheme but in any case they depend on the dilaton.

Wu and Wang (1986) studied the cosmological behavior of the theory [Eq. (265)] assuming a ten-dimensional metric of the form $\mathrm{diag}(-1,R_3(t)^2\tilde{g}_{ij}(x),R_6(t)^2\tilde{g}_{mn}(y))$, where R_3 and R_6 are scale factors of the external and internal spaces. The rate of evolution of the size of the internal space was related to the time variation of the gravitational constant. The effect of a potential for the size of the internal space was also studied.

Maeda (1988) considered the $(N=1,D=10)$ supergravity model derived from the heterotic superstring theory in the low energy limit and assumed that ten-dimensional spacetime is compacted on a 6-torus of radius $R(x^\mu)$, so that the effective four-dimensional theory described by Eq. (266) is of the Brans-Dicke type, with $\omega=-1$. Assuming that ϕ has a mass μ and couples to the matter fluid in the universe as $S_{\rm matter}=\int d^{10}\mathbf{x}\sqrt{-g_{10}}\exp(-2\Phi)\mathcal{L}_{\rm matter}(g_{10})$, the reduced four-dimensional matter action is

$$S_{\rm matter}=\int d^4\mathbf{x}\sqrt{-g}\,\phi\mathcal{L}_{\rm matter}(g). \qquad (273)$$

The cosmological evolution of ϕ and R can then be computed; Maeda (1988) deduced that $\dot\alpha_{\rm EM}/\alpha_{\rm EM}\simeq 10^{10}(\mu/1\ {\rm eV})^{-2}\ {\rm yr}^{-1}$. In this approach, there is an ambiguity in the way to introduce the matter fluid.

Vayonakis (1988) considered the same model but assumed that supersymmetry is broken by nonperturbative effects such as gaugino condensation. In this model, and contrary to the work by Maeda (1988), ϕ is stabilized and the variation of the constants arises mainly from the variation of R in a runaway potential.

Damour and Polyakov (1994a, 1994b) argued that the effective action for the massless modes taking into account the full string loop expansion is of the form

$$S=\int d^4\mathbf{x}\sqrt{-\hat g}\Big[M_s^2\{B_g(\Phi)\hat R+4B_\Phi(\Phi)[\hat\Box\Phi-(\hat\nabla\Phi)^2]\}-B_F(\Phi)\frac{k}{4}\hat F^2-B_\psi(\Phi)\bar{\hat\psi}\hat{\not{D}}\hat\psi+\cdots\Big] \qquad (274)$$

in the string frame, M_s being the string mass scale. The functions B_i are not known but can be expanded as

$$B_i(\Phi)=\mathrm{e}^{-2\Phi}+c_0^{(i)}+c_1^{(i)}e^{2\Phi}+c_2^{(i)}e^{4\Phi}+\cdots \tag{275}$$

in the limit $\Phi\to-\infty$, so that these functions can exhibit a local maximum. After a conformal transformation $[g_{\mu\nu}=CB_g\hat{g}_{\mu\nu},\psi=(CB_g)^{-3/4}B_\psi^{1/2}\hat{\psi}]$, the action in Einstein frame takes the form

$$S=\int\frac{d^4\mathbf{x}}{16\pi G}\sqrt{-g}\left[R-2(\nabla\phi)^2-\frac{k}{4}B_F(\phi)F^2-\bar{\psi}\not{D}\psi+\cdots\right], \tag{276}$$

from which it follows that the Yang-Mills coupling behaves as $g_{\rm YM}^{-2}=kB_F(\phi)$. This also implies that the QCD mass scale is given by

$$\Lambda_{\rm QCD}\sim M_s(CB_g)^{-1/2}e^{-8\pi^2kB_F/b}, \tag{277}$$

where b depends on the matter content. It follows that the mass of any hadron, proportional to $\Lambda_{\rm QCD}$ in first approximation, depends on the dilaton, $m_A(B_g,B_F,\ldots)$. With the ansatz (275), $m_A(\phi)$ can exhibit a minimum ϕ_m that is an attractor of the cosmological evolution that drives the dilaton towards a regime where it decouples from matter. But one needs to assume that, for this mechanism to apply, and particularly to avoid violation of the equivalence principle at an unacceptable level, all the minima are the same, which can be implemented by setting $B_i=B$. Expanding $\ln B$ around its maximum ϕ_m as $\ln B\propto-\kappa(\phi-\phi_m)^2/2$, Damour and Polyakov (1994a, 1994b) constrained the set of parameters $(\kappa,\phi_0-\phi_m)$ using the different observational bounds. This toy model allows one to address the unsolved problem of the dilaton stabilization, to study all the experimental bounds together, and to relate them in a quantitative manner (e.g., by deriving a link between equivalence-principle violations and time variation of $\alpha_{\rm EM}$).

Damour, Piazza and Veneziano (2002a, 2002b) extended this model to a case where the coupling functions have a smooth finite limit for infinite value of the bare-string coupling, so that $B_i=C_i+\mathcal{O}(e^{-\phi})$. The dilaton runs away toward its attractor at infinity during a stage of inflation. The amplitude of residual dilaton interaction is related to the amplitude of the primordial density fluctuations, and it can induce a variation of the fundamental constants provided it couples to dark matter or dark energy. It is concluded that, in this framework, the largest allowed variation of $\alpha_{\rm EM}$ is of order 2×10^{-6}, which is reached for a violation of the universality of free fall of order 10^{-12}.

Kolb *et al.* (1986) argued that in ten-dimensional superstring models, $G\propto R^{-6}$ and $\alpha_{\rm EM}\propto R^{-2}$, which led them to deduce that $|\Delta R/R|<0.5\%$. This was revised by Barrow (1987), who included the effect of $\alpha_{\rm S}$ to discern that helium abundances impose $|\Delta R/R|<0.2\%$. Recently Ichikawa and Kawasaki (2002) considered a model in which all the couplings vary due to the dilaton dynamics and constrained the variation of the dilaton field from nucleosynthesis as $-1.5\times10^{-4}<\sqrt{16\pi G}\Delta\phi<6.0\times10^{-4}$. From the Oklo data, Barrow (1987) concluded that $|\Delta R/R|<1.5\times10^{-10}$.

To conclude, superstring theories offer a theoretical framework to discuss the value of the fundamental constants since they become expectation values of some fields. This is a first step towards their understanding, but as yet no complete and satisfactory mechanism for the stabilization of the extra dimension and dilaton is known.

C. Other investigations

Bekenstein (1982) formulated a framework independent of string theory to incorporate a varying fine-structure constant. Working in units in which $\hbar$ and c are constant, he adopted a classical description of the electromagnetic field and made a set of assumptions to obtain a reasonable modification of Maxwell equations to account for the effect of the variation of the elementary charge (for instance, to incorporate the problem of charge conservation that usually derive from Maxwell equations). His eight postulates are that (1) for a constant $\alpha_{\rm EM}$, electromagnetism is described by Maxwell theory and the coupling of the potential vector A_ν to matter is minimal, (2) the variation of $\alpha_{\rm EM}$ results from dynamics, (3) the dynamics of electromagnetism and $\alpha_{\rm EM}$ can be obtained from an invariant action that is (4) locally gauge invariant, (5) electromagnetism is causal and (6) its action is time-reversal invariant, (7) the shortest length scale is the Planck length, and (8) gravitation is described by a metric theory that satisfies Einstein equations.

Assuming that the charges of all particles vary in the same way, one can set $e=e_0\epsilon(x^\mu)$ where $\epsilon(x^\mu)$ is a dimensionless universal field (the theory governing ϵ should be invariant under $\epsilon\to\mathrm{const}\times\epsilon$ through a redefinition of e_0). The electromagnetic tensor generalizes to

$$F_{\mu\nu}=\epsilon^{-1}\nabla_{[\mu}(\epsilon A_{\nu]}) \tag{278}$$

and the electromagnetic action is given by

$$S_{\rm EM}=\frac{-1}{16\pi}\int F_{\mu\nu}F^{\mu\nu}\sqrt{-g}d^4\mathbf{x}. \tag{279}$$

The dynamics of ϵ can be shown to derive from the action

$$S_\epsilon=\frac{-1}{2}\frac{\hbar c}{\ell^2}\int\frac{\partial_\mu\epsilon\partial^\mu\epsilon}{\epsilon^2}\sqrt{-g}d^4\mathbf{x}, \tag{280}$$

where ℓ is length scale, which needs to be small enough to be compatible with the observed scale invariance of electromagnetism ($\ell_{\rm Pl}<\ell<10^{-15}$–$10^{-16}$ cm around which electromagnetism merges with the weak interaction). Finally, the matter action for point particles of mass m takes the form $S_m=\Sigma\int[-mc^2+(e/c)u^\mu A_\mu]\gamma^{-1}\delta^3(x^i-x^i(\tau))d^4\mathbf{x}$ where γ is the Lorentz factor and τ the proper time.

Varying the total action gives the electromagnetic equation

$$\nabla_\mu(\epsilon^{-1}F^{\mu\nu})=4\pi j^\nu \tag{281}$$

and the equation for the dynamics of ϵ

$$\Box\epsilon=\frac{\ell^2}{\hbar c}\left[\epsilon\frac{\partial\sigma}{\partial\epsilon}-\frac{1}{8\pi}F_{\mu\nu}F^{\mu\nu}\right], \tag{282}$$

with $\sigma=\Sigma mc^2\gamma^{-1}\delta^3(x^i-x^i(\tau))/\sqrt{-g}$. The Maxwell equation (281) is the same as electromagnetism in a material medium with dielectric constant ϵ^{-2} and permeability ϵ^2 [this was the original description proposed by Lichnérowicz (1955) and Fierz (1956); see also Dicke (1964)].

On cosmological scales, it can be shown that the dynamical equation for ϵ can be cast under the form

$$(a^3\dot\epsilon/\epsilon)^{\cdot}=-a^3\zeta\frac{\ell^2}{\hbar c}\rho_m c^2, \tag{283}$$

where $\zeta=\mathcal{O}(10^{-2})$ is a dimensionless (and approximately constant) measuring of the fraction of mass in Coulomb energy for an average nucleon compared with the free proton mass, and ρ_m is the matter density. Since $\rho_m\propto a^{-3}$, Eq. (283) can be integrated to relate $(\dot\epsilon/\epsilon)_0$ to $\ell/\ell_{\rm Pl}$ and the cosmological parameters. In order to integrate this equation, Bekenstein assumed that ζ was constant, which was a reasonable assumption at low redshift. Livio and Stiavelli (1998) extended this analysis and got $\zeta=1.2\times10^{-2}(X+4/3Y)$ where X and Y are the mass fraction of hydrogen and helium.

Replacing the quantity in the brackets of the right-hand side of Eq. (282) by $\zeta\rho_m c^2$ with $\zeta=\mathcal{O}(10^{-2})$, the static form Eq. (282) is analogous to the standard Poisson equation, so that $\ln\epsilon$ is proportional to the gravitational potential

$$\ln\epsilon=\frac{\zeta}{4\pi c^2}\frac{\ell}{\ell_{\rm Pl}}\Phi, \tag{284}$$

from which it follows that a test body of mass m and of electromagnetic energy $E_{\rm EM}$ experiences an acceleration of $\vec a=-\nabla\Phi-M^{-1}(\partial E_{\rm EM}/\partial\epsilon)\nabla\epsilon$.

From the comparison of the results of the spatial and cosmological variation of ϵ, Bekenstein (1982) concluded, given his assumptions on the couplings, that $\alpha_{\rm EM}$ "is a parameter, not a dynamical variable." This problem was recently bypassed by Olive and Pospelov (2002), who generalized the model to allow additional coupling of a scalar field $\epsilon^{-2}=B_F(\phi)$ to nonbaryonic dark matter (as first proposed by Damour *et al.*, 1990) and the cosmological constant. They argue that in certain classes of dark-matter models, and particularly in supersymmetric ones, it is natural to expect that ϕ would couple more strongly to dark matter than to baryon. For instance, by supersymmetrizing the Bekenstein model, ϕ will get a coupling to the kinetic term of the gaugino of the form $M_*^{-1}\phi\bar\chi\partial\chi$, so that, assuming the gaugino is a large fraction of the stable, lightest supersymmetric particle, the coupling to dark matter would then be of order 10^3–10^4 times larger. Such a factor could almost reconcile the constraint arising from the test of the universality of free fall with the order of magnitude of the cosmological variation. This generalization of the Bekenstein model relies on an action of the form

$$\begin{aligned}S=&\frac{1}{2}M_4^2\int R\sqrt{-g}d^4\mathbf{x}-\int\left[\frac{1}{2}M_*^2\partial_\mu\phi\partial^\mu\phi\right.\\&\left.+\frac{1}{4}B_F(\phi)F_{\mu\nu}F^{\mu\nu}\right]\sqrt{-g}d^4\mathbf{x}\\&-\int\left\{\sum\bar N_i[i\not{D}-m_iB_{N_i}(\phi)]N_i\right.\\&\left.+\frac{1}{2}\bar\chi\partial\chi\right\}\sqrt{-g}d^4\mathbf{x}-\int\left[M_4^2B_\Lambda(\phi)\Lambda\right.\\&\left.+\frac{1}{2}M_\chi B_\chi(\phi)\chi^T\chi\right]\sqrt{-g}d^4\mathbf{x},\end{aligned} \tag{285}$$

where the sum is over proton $[\not{D}=\gamma^\mu(\partial_\mu-ie_0A_\mu)]$ and neutron $[\not{D}=\gamma^\mu\partial_\mu]$. The functions B can be expanded (since one focuses on small variations of the fine-structure constant and thus of ϕ) as $B_X=1+\zeta_X\phi+\xi_X\phi^2/2$. It follows that $\alpha_{\rm EM}(\phi)=e_0^2/4\pi B_F(\phi)$, so that $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}=\zeta_F\phi+(\xi_F-2\zeta_F^2)\phi^2/2$. This framework extends the analysis by Bekenstein (1982) to a four-dimensional parameter space $(M_*,\zeta_F,\zeta_m,\zeta_\Lambda)$. It contains the Bekenstein model ($\zeta_F=-2$, $\zeta_\Lambda=0$, $\zeta_m\sim10^{-4}\xi_F$), a Jordan-Brans-Dicke model ($\zeta_F=0$, $\zeta_\Lambda=-2\sqrt{2/2\omega+3}$, $\xi_m=-1/\sqrt{4\omega+6}$), and a stringlike model ($\zeta_F=-\sqrt2$, $\zeta_\Lambda=\sqrt2$, $\zeta_m=\sqrt2/2$), so that $\Delta/\alpha_{\rm EM}/\alpha_{\rm EM}=3$ and the Bekenstein model is supersymmetrized ($\zeta_F=-2$, $\zeta_\chi=-2$, $\zeta_m=\zeta_\chi$, so that $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}\sim5/\omega$). In all the models, the universality of free fall sets a strong constraint on $\zeta_F/\sqrt\omega$ (with $\omega\equiv M_*/2M_4^2$), and the authors show that a small set of models are compatible with the cosmological variation claimed by Webb *et al.* (2001) and the equivalence-principle tests.

The constraint arising from the universality of free fall can be fulfilled if one sets by hand $B_F-1\propto[\phi-\phi(0)]^2$, where $\phi(0)$ is the value of the field today. It then follows that cosmological evolution will drive the system toward a state in which ϕ is almost stabilized today but allows for cosmological variation of the constants of nature. In their two-parameter extension, Livio and Stiavelli (1998) found that only $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ variations of 8×10^{-6} and 9×10^{-7}, respectively, for $z<5$ and $z<1.6$ were compatible with Solar system experiments.

The formalism developed by Bekenstein (1982) was also applied to the the strong interaction (Chamoun *et al.*, 2000, 2001) by simply adding a term $f_{abc}A_\mu^bA_\nu^c$ to describe the gluon tensor field $G_{\mu\nu}^a$, f_{abc} being the structure constants of the non-Abelian group. It was also implemented in the braneworld context (e.g., Youm, 2001), and Magueijo *et al.* (2001) studied the effect of a varying fine-structure constant on a complex scalar field undergoing an electromagnetic U(1) symmetry breaking in this framework. Armendáriz-Picón (2002) derived the most general low-energy action, including a real scalar field that is local, invariant under space inversion and time reversal, diffeomormism invariant, and has a U(1) gauge invariance. This form includes the previous form

(285) of Bekenstein's theory as well as scalar-tensor theories and the long-wavelength limit of bimetric theories.

Recently Sandvik *et al.* (2002) claimed to have generalized the Bekenstein model by simply redefining $a_\mu \equiv \epsilon A_\mu$, $f_{\mu\nu} \equiv \partial_{[\mu} a_{\nu]}$, and $\psi \equiv \ln \epsilon$ so that the covariant derivative becomes $D_\mu \equiv \partial_\mu + i e_0 a_\mu$. It follows that the total action, including the Einstein-Hilbert action for gravity, the actions (279) and (280) for the modified electromagnetism and normal matter, takes the form

$$S=\int \sqrt{-g}\, d^4\mathbf{x}(\mathcal{L}_{\rm grav}+\mathcal{L}_{\rm mat}+\mathcal{L}_\psi+\mathcal{L}_{\rm EM}e^{-2\psi}), \quad (286)$$

with $\mathcal{L}_\psi = -(\omega_{\rm SBM}/2)\partial_\mu\psi\partial^\mu\psi$, so that the Einstein equation is among the "standard" Einstein equations with an additive stress-energy tensor for the scalar field ψ. Indeed, Bekenstein (1982) did not take into account the effect of ϵ (or ψ) in the Friedmann equation and studied only the time variation of ϵ in a matter-dominated universe. In that sense, Sandvik *et al.* (2002) extended the analysis of Bekenstein (1982) by solving the coupled system of Friedmann and Klein-Gordon equations. They studied numerically the function of $\zeta/\omega_{\rm SBM}$ (with $\omega_{\rm SBM} = \ell_{\rm Pl}^2/\ell^2$) and showed that cosmological and astrophysical data can be explained by $\omega_{\rm SBM}=1$ if ζ ranges between 0.02% and 0.1% (that is about one order of magnitude smaller than Bekenstein's value based on the argument that dark matter has to be taken into account). An extension of the discussion of the cosmological scenarios was performed in Barrow, Sandvik, and Magueijo (2002a), and it was shown that $\alpha_{\rm EM}$ is constant during the radiation era, then evolves logarithmically with the cosmic time during the matter era, and then tends toward a constant during a curvature or cosmological-constant era. The scalar-tensor case with both varying G and $\alpha_{\rm EM}$ was considered by Barrow, Magueijo, and Sandvik (2002a, 2000b).

Sandvik *et al.* (2002), following Barrow and O'Toole (2001), estimated the spatial variations to be of order $\Delta \ln \alpha_{\rm EM} \sim 4.8\times 10^{-4} GM/c^2 r$ (Magueijo, 2001) to conclude that on a cosmological scale $\Delta \ln \alpha_{\rm EM} \sim 10^{-8}$ if $GM/c^2 r \sim 10^{-4}$, as expected on cosmological scales if $(\delta T/T)_{\rm CMB} \sim GM/c^2 r$. On the Earth-orbit scale, this leads, assuming a constant gradient, to the rough estimate $|\nabla \ln \alpha_{\rm EM}| \sim \Delta \ln \alpha_{\rm EM}/2R_{\rm Earth} \sim 10^{-23}$–$10^{-22}$ cm^{-1}, which is about ten orders of magnitude higher than the constraint (154) arising from the test of the universality of free fall. Nevertheless, Magueijo *et al.* (2002) reanalyzed the violation of the universality of free fall and claimed that the theory is still compatible with equivalence-principle tests provided that $\zeta_m \lesssim 1$ for dark matter. This arises probably from the fact that only $\alpha_{\rm EM}$ is varying while other constants are fixed, so that the dominant factor in Eq. (146) is absent.

Wetterich (2002) considered the effect of the scalar field responsible for the acceleration of the universe (the "cosmon") on the couplings arising from the coupling of the cosmon to the kinetic term of the gauge field as $Z_F(\phi)F^2/4$. Focusing on grand unified theory, so that one gets a coupling of the form $\mathcal{L}=Z_F(\phi)\mathrm{Tr}(F^2)/4 + iZ_\psi(\phi)\bar\psi \not{D}\psi$, and assuming a runaway expotenial potential, he related the variation of $\alpha_{\rm EM}$, $\alpha_{\rm S}$, the nucleon masses to the arbitrary function Z_F and to the ϕ-dependent electroweak scale.

Chacko *et al.* (2002) proposed that the variation of the fine-structure constant could be explained by a late second-order phase transition at $z\sim 1-3$ (that is, around $T\sim 10^{-3}$ eV) inducing a change in the vacuum expectation value of a scalar field. This can be implemented, for instance, in supersymmetric theories with low-energy symmetry-breaking scale. This will induce a variation of the masses of electrically charged particles. From the renormalization group equation $\alpha_{\rm EM}^{-1} = \alpha_{\rm EM}^{-1}(\Lambda) + \Sigma_i (b_{i+1}/2\pi)\ln(m_{i+1}/m_i)$, and assuming that $\alpha_{\rm EM}^{-1}(\Lambda)$ was fixed, one would require that $\Sigma\, \delta m_i/m_i = \mathcal{O}(10^{-2})$ to explain the observations by Webb *et al.* (2001), so that the masses have to increase. Note that it will induce a time variation of the Fermi constant. Such models can occur in a large class of supersymmetric theories. Unfortunately, it is yet incomplete and its viability depends on the existence of an adjustment mechanism for the cosmological constant. But it offers a new way of thinking about the variation of the constants at odds with the previous analysis involving a rolling scalar field.

Motivated by resolving the standard cosmological puzzles (horizon, flatness, cosmological constant, entropy, homogeneity problems) without inflation, Albrecht and Magueijo (1999) introduced a cosmological model in which the speed of light is varying. Earlier related attempts were investigated by Moffat (1993a, 1993b). Albrecht and Magueijo (1999) postulated that the Friedmann equations are kept unchanged, from which it follows that matter conservation has to be changed and designated by a term proportional to $\dot c/c$. The flatness and horizon problems are not solved by a period of accelerated expansion, so that, contrary to inflation, it does not offer any explanation for the initial perturbations (see, however, Harko and Mak, 1999). Albrecht and Magueijo (1999) considered an abrupt change in the velocity of light, as may happen during a phase transition. It was extended to scenarios in which both c and G were proportional to some power of the scale factor by Barrow (1999) (see also Barrow and Magueigo, 1999a, 1999b). The link between this theory and the Bekenstein theory was investigated by Barrow and Magueijo (1998). Magueijo *et al.* (2002) investigated the test of universality of free fall. A Lagrangian formulation would probably require the introduction of an "ether" vector field to break local Lorentz invariance as was used in, for example, Lemoine *et al.* (2002) [see also Levin and Freese (1994), Drummond (1999), and Basset *et al.* (2000) for alternative formulations of the varying-speed-of-light theory]. Note also that c refers at least to three different quantities: the speed $c_{\rm EM}$ of propagation of the electromagnetic waves, the speed $c_{\rm L}$ entering the Lorentz transformation and related to the concept of causality, and the speed $c_{\rm g}$ of propagation of gravitation. In the standard picture, $c_{\rm EM}=c_{\rm L}=c_{\rm g}$, so that one has to specify clearly what is meant by a varying-speed-of-light theory, since one can let one, or two, or three of these

"c" vary. As lucidly explained in Bekenstein (2002), c appears at least in four contexts in a physical action. Two different "c" can be promoted to different powers of the same scalar field (e.g., Magueijo, 2000), or of different fields.

Clayton and Moffat (1998) implemented a varying-speed-of-light model by considering a bimetric theory of gravitation in which one metric $g_{\mu\nu}$ describes the standard gravitational vacuum, whereas a second metric $g_{\mu\nu}+\beta\psi_\mu\psi_\nu$, β being a dimensionless constant and ψ^μ a dynamical vector field, describes the geometry in which matter is propagating (see also Bekenstein, 1993). When choosing $\psi_\mu=\partial_\mu\phi$, this reduces to the models developed by Clayton and Moffat (2000, 2001). Some cosmological implications were discussed by Moffat (2001, 2003), but no study of the constraints arising from Solar-system experiments have been taken into account. Note that Dirac (1979) also proposed that a varying G can be reconciled with Einstein's theory of gravity if the space metric was different from the "atomic" metric. Landau and Vucetich (2001) investigated the constraints arising from the violation of the charge conservation. Other realizations arise from the brane world picture in which our universe is a three-dimensional brane embedded in a higher-dimensional spacetime. Kiritsis (1999) showed that when a test brane is moving in a black-hole bulk spacetime (Kehagias and Kiritsis, 1999), the velocity of light varies according to the distance between the brane and the black hole. Alexander (2000) generalizes this model (see also Steer and Parry, 2002) by including rotation and expansion of the bulk, so that the speed of light gets stabilized at late time. Carter *et al.* (2001) nevertheless showed that even if a Newton-like force is recovered on small scales such models are very constrained at the post-Newtonian level. Brane models allowing for the scalar field in the bulk naturally predict a time-variable gravitational constant (see, e.g., Brax and Davis, 2001).

D. A new cosmological constant problem?

The question of the compatibility between an observed variation of the fine-structure constant and particle-physics models was put forward by Banks *et al.* (2002). As seen above, in the low-energy limit, the change of the fine-structure constant can be implemented by coupling a scalar field to the photon kinetic term $F^{\mu\nu}F_{\mu\nu}$, but this implies that the vacuum energy computed in this low-energy limit must depend on $\alpha_{\rm EM}$. Estimating that

$$\Delta\Lambda_{\rm vac}\sim\Lambda^4\Delta\alpha_{\rm EM} \tag{287}$$

leads to a variation of order $\Delta\Lambda_{\rm vac}\sim10^{28}\ ({\rm eV})^4$ for a variation $\Delta\alpha_{\rm EM}\sim\mathcal{O}(10^{-4})$ and for $\Lambda=\Lambda_{\rm QCD}\sim100$ MeV. Indeed, this contrasts with the average energy density of the universe of about $10^4\ {\rm eV}^4$ during the matter era, so that the universe was dominated by the cosmological constant at $z\sim3$, which is at odds with observations. It was thus concluded that this imposes

$$|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}|<10^{-28}. \tag{288}$$

Contrary to the standard cosmological-constant problem, the vacuum zero-point energy to be removed is time dependent and one can only remove it for a fixed value of $\alpha_{\rm EM}$. Whereas the cosmological constant problem involves the fine tuning of a parameter, this now implies the fine tuning of a function!

It follows that a varying $\alpha_{\rm EM}$ cannot be naturally explained in a field-theory approach. A possible way out would be to consider that the field is in fact an axion (see Carroll, 1998; Choi, 2000; Banks and Dine, 2001). Some possible links with Heisenberg relations and quantum mechanics were also investigated by Rañada (2003). The resolution of the cosmological constant problem may also provide the missing elements to understand the variation of the constants. It is hoped that both problems can be solved by string theory.

VII. CONCLUSIONS

The experimental and observational constraints on the variation of the fine-structure and gravitational constants, of the electron-to-proton mass ratio and different combinations of the proton gyromagnetic factor and the two previous constants, as well as bounds on $\alpha_{\rm W}$ are summarized in Tables III, IV, and V.

The development of high-energy physics theories such as multidimensional and string theories provides new motivations to consider the time variation of the fundamental constants. The observation of the variability of these constants constitutes one of the very few hopes to test directly the existence of extra dimensions and to test these high-energy physics models. In the long run, it may help to discriminate between different effective potentials for the dilaton and/or the dynamics of the internal space. But indeed, independently of these motivations, the understanding of the value of the fundamental constants of nature and the discussion of their status as constant remains a central question of physics in general: questioning the free parameters of a theory amounts to questioning the theory itself. It is a basic and direct test of the law of gravity.

As we have shown, proving that a fundamental constant has changed is not an obvious task, mainly because observations usually entangle a set of constants and because the bounds presented in the literature often assume the constancy of a set of parameters. But, in GUT, Kaluza-Klein, and string-inspired models, one expects all the couplings to vary simultaneously. Better analysis of the degeneracies, as started by Sisterna and Vucetich (1990, 1991) (see also Landau and Vucetich, 2002), are really needed before drawing definite conclusions, but such analyses are also dependent on the progress in our understanding of the fundamental interactions and particularly of the QCD theory and on the generation of fermion masses.

Other progress requires (model-dependent) investigations of the compatibility of the different bounds. It has also to be remembered that arguing about the nonexist-

TABLE III. Summary of the constraints on the variation of the fine-structure constant $\Delta\alpha_{EM}/\alpha_{EM}$. The confidence levels (C.L.) are those cited by the authors (x means that no C.L. was given so that the bound could be taken as a 1σ bound). For the details concerning these C.L., one should refer to the text (particularly to know what is taken into account by the C.L.).

Reference	Constraint	C.L.	Redshift	Time (10^9 yr)	Method
(Savedoff, 1956)	$(1.8\pm1.6)\times10^{-3}$	x	0.057		Cygnus A (N II, Ne III)
(Wilkinson, 1958)	$(0\pm8)\times10^{-3}$	x		3–4	α decay
(Bahcall *et al.*, 1967)	$(-2\pm5)\times10^{-2}$	x	1.95		QSO (Si II, Si IV)
(Bahcall and Schmidt, 1967)	$(1\pm2)\times10^{-3}$	x	0.2		radio galaxies (O III)
(Dyson, 1967)	$(0\pm9)\times10^{-4}$	x		3	Re/Os
(Gold, 1968)	$(0\pm4.66)\times10^{-4}$	x		2	Fission
(Chitre and Pal, 1968)	$(0\pm3^{+2}_{-2})\times10^{-4}$	x		1	Fission
(Dyson, 1972)	$(0\pm4)\times10^{-4}$	x		2	α decay
(Dyson, 1972)	$(0\pm1)\times10^{-3}$	x		2	Fission
(Dyson, 1972)	$(0\pm5)\times10^{-6}$	x		1	Re/Os
(Shlyakhter, 1976)	$(0\pm1.8)\times10^{-8}$	x		1.8	Oklo
(Wolfe *et al.*, 1976)	$(0\pm3)\times10^{-2}$	x	0.524		QSO (Mg I)
(Irvine, 1983a)	$(0\pm9)\times10^{-8}$	x		1.8	Oklo
(Lindner *et al.*, 1986)	$(-4.5\pm9)\times10^{-6}$	2σ		4.5	Re/Os
(Kolb *et al.*, 1986)	$(0\pm1)\times10^{-4}$	x	10^8		BBN
(Potekhin and Varshalovich, 1994)	$(2.1\pm2.3)\times10^{-3}$	2σ	3.2		QSO (C IV, Si IV,...)
(Varshalovich and Potekhin, 1994)	$(0\pm1.5)\times10^{-3}$	2σ	3.2		QSO (C IV, Si IV,...)
(Cowie and Songaila, 1995)	$(-0.3\pm1.9)\times10^{-4}$	2σ	2.785–3.191		QSO
(Prestage *et al.*, 1995)	$(0\pm1.42)\times10^{-14}$	x	0	140 days	Atomic clocks
(Damour and Dyson, 1996)	$(0.15\pm1.05)\times10^{-7}$	2σ		1.8	Oklo
(Varshalovich, Potekhin, *et al.*, 1996)	$(2\pm7)\times10^{-5}$	2σ	2.8–3.1		QSO (Si IV)
(Bergström *et al.*, 1999)	$(0\pm2)\times10^{-2}$	x	10^8		BBN
(Webb *et al.*, 1999)	$(-0.17\pm0.39)\times10^{-5}$	3.5σ	0.6–1		QSO (Mg II, Fe II)
(Webb *et al.*, 1999)	$(-1.88\pm0.53)\times10^{-5}$	3.5σ	1–1.6		QSO (Mg II, Fe II)
(Ivanchik *et al.*, 1999)	$(-3.3\pm6.5\pm8)\times10^{-5}$	2σ	2–3.5		QSO (Si IV)
(Fujii *et al.*, 2000)	$(-0.36\pm1.44)\times10^{-7}$	2σ		1.8	Oklo
(Varshalovich *et al.*, 2000a)	$(-4.5\pm4.3\pm1.4)\times10^{-5}$	2σ	2–4		QSO (Si IV)
(Avelino *et al.*, 2001)	$(-3.5\pm5.5)\times10^{-2}$	1σ	10^3		CMB
(Landau *et al.*, 2001)	$(-5.5\pm8.5)\times10^{-2}$	2σ	10^3		CMB
(Webb *et al.*, 2001)	$(-0.7\pm0.23)\times10^{-5}$	4σ	0.5–1.8		QSO (Fe II, Mg II)
(Webb *et al.*, 2001)	$(-0.76\pm0.28)\times10^{-5}$	4σ	1.8–3.5		QSO (Ni II, Cr II, Zn II)
(Webb *et al.*, 2001)	$(-0.5\pm1.3)\times10^{-5}$	4σ	2–3		QSO (Si IV)
(Murphy *et al.*, 2001c)	$(-0.2\pm0.3)\times10^{-5}$	4.1σ	0.5–1		QSO (Mg I, Mg II,...)
(Murphy *et al.*, 2001c)	$(-1.2\pm0.3)\times10^{-5}$	4.1σ	1–1.8		QSO (Mg I, Mg II,...)
(Murphy *et al.*, 2001c)	$(-0.7\pm0.23)\times10^{-5}$	4.1σ	0.5–1.8		QSO (Mg I, Mg II,...)
(Murphy *et al.*, 2001b)	$(-0.5\pm1.3)\times10^{-5}$	2σ	2–3		QSO (Si IV)
(Sortais *et al.*, 2001)	$(8.4\pm13.8)\times10^{-15}$	2σ		24 months	Atomic clock
(Nollet and Lopez, 2002)	$(3\pm7)\times10^{-2}$	1σ	10^8		BBN
(Ichikawa and Kawasaki, 2002)	$(-2.24\pm3.75)\times10^{-4}$	x	10^8		BBN
(Olive *et al.*, 2002)	$(0\pm1)\times10^{-7}$	x		1.8	Oklo
(Olive *et al.*, 2002)	$(0\pm3)\times10^{-7}$	x	~0.45	4.6	Re/Os
(Olive *et al.*, 2002)	$(0\pm1)\times10^{-5}$	x			α decay

ence of something to set constraints (e.g., Broulik and Trefil, 1971) is very dangerous. From an observational point of view, one needs to study further the systematics (and remember some erroneous claims such as those by Van Flandern, 1975) and to propose new experiments (see, e.g., Karshenboim, 2000, 2001, who proposed experiments based on the hyperfine structure of deuterium and ytterbium-171 as well as atoms with magnetic moment; Torgerson, 2000 who proposed the comparison of optical-frequency references; Braxmaier *et al.*, 2001; Sor-

TABLE IV. Summary of the constraints on the time variation of the Newton constant G. The constraints labeled by * refer to bounds obtained under the assumption that G was decreasing (that is, $-\dot{G}/G<0$). G being dimensionful, these bounds assume in general that the masses of stars and/or planets or some other mass scale are kept constant (see text for details). The confidence levels (C.L.) are those cited by the authors (x means that no C.L. was given so that the bound could be taken as a 1σ bound).

Reference	Constraint (yr^{-1})	C.L.	Method
(Teller, 1948)	$(0\pm2.5)\times10^{-11}$	x	Earth temperature
(Shapiro *et al.*, 1971)	$(0\pm4)\times10^{-10}$	x	Planetary ranging
(Morrison, 1973)	$(0\pm2)\times10^{-11}$	x	Lunar occultations
(Dearborn and Schramm, 1974)*	$<4\times10^{-11}$	x	Clusters of galaxies
(Van Flandern, 1975)	$(-8\pm5)\times10^{-11}$	x	Lunar occultations
(Heintzmann and Hillebrandt, 1975)	$(0\pm1)\times10^{-10}$	x	Pulsar spin-down
(Reasenberg and Shapiro, 1976)	$(0\pm1.5)\times10^{-10}$	x	Planetary ranging
(Mansfield, 1976)*	$<(3.4\pm3.4)\times10^{-11}$	2σ	Pulsar spin-down
(Williams *et al.*, 1976)	$(0\pm3)\times10^{-11}$	1.1σ	Planetary ranging
(Blake, 1977b)	$(-0.5\pm2)\times10^{-11}$	x	Earth radius
(Muller, 1978)	$(2.6\pm1.5)\times10^{-11}$	x	Solar eclipses
(McElhinny *et al.*, 1978)*	$<8\times10^{-12}$	x	Planetary radii
(Barrow, 1978)	$(2\pm9.3)h\times10^{-12}$	x	BBN
(Reasenberg *et al.*, 1979)*	$<10^{-12}$	x	Viking ranging
(Van Flandern, 1981)	$(3.2\pm1.1)\times10^{-11}$	x	Lunar occultation
(Rothman and Matzner, 1982)	$(0\pm1.7)\times10^{-13}$	x	BBN
(Hellings *et al.*, 1983)	$(2\pm4)\times10^{-12}$	x	Viking ranging
(Reasenberg, 1983)	$(0\pm3)\times10^{-11}$	x	Viking ranging
(Damour *et al.*, 1988)	$(1.0\pm2.3)\times10^{-11}$	2σ	PSR 1913+16
(Shapiro, 1990)	$(-2\pm10)\times10^{-12}$	x	Planetary ranging
(Goldman, 1990)*	$(2.25\pm2.25)\times10^{-11}$	x	PSR 0655+64
(Accetta *et al.*, 1990)	$(0\pm9)\times10^{-13}$	2σ	BBN
(Müller *et al.*, 1991)	$(0\pm1.04)\times10^{-11}$	x	Lunar laser ranging
(Anderson *et al.*, 1992)	$(0.0\pm2.0)\times10^{-12}$	x	Planetary ranging
(Damour and Taylor, 1991)	$(1.10\pm1.07)\times10^{-11}$	x	PSR 1913+16
(Chandler *et al.*, 1993)	$(0\pm1)\times10^{-11}$	x	Viking ranging
(Dickey *et al.*, 1994)	$(0\pm6)\times10^{-12}$	x	Lunar laser ranging
(Kaspi *et al.*, 1994)	$(4\pm5)\times10^{-12}$	2σ	PSR B1913+16
(Kaspi *et al.*, 1994)	$(-9\pm18)\times10^{-12}$	2σ	PSR B1855+09
(Demarque *et al.*, 1994)	$(0\pm2)\times10^{-11}$	x	Heliosismology
(Guenther *et al.*, 1995)	$(0\pm4.5)\times10^{-12}$	x	Heliosismology
(Garcia-Berro *et al.*, 1995)*	$(2\pm2)\times10^{-11}$	1σ	White dwarf
(Williams *et al.*, 1996)	$(0\pm8)\times10^{-12}$	x	Lunar laser ranging
(Thorsett, 1996)	$(-0.6\pm4.2)\times10^{-12}$	2σ	Pulsar statistics
(Del'Innocenti *et al.*, 1996)	$(-1.4\pm2.1)\times10^{-11}$	1σ	Globular clusters
(Guenther *et al.*, 1998)	$(0\pm1.6)\times10^{-12}$	x	Heliosismology

tais *et al.*, 2001, who improved the sensitivity of frequency standards; the coming satellite experiments ACES, MICROSCOPE, and STEP, etc.). On local scales, the test of the universality of free fall sets drastic constraints, and one can hope to use similar methods on cosmological scales from the measurements of weak gravitational lensing (Uzan and Bernardeau, 2001) or from structure formation (Martins *et al.*, 2002). The complementarity between local experiments and geoastrophysical observations is necessary since these methods test different time scales and are mainly sensitive either to rapid oscillations or a slow drift of the constants.

Based on recent astrophysical observations of quasars, it has been claimed that the fine-structure constant has evolved. These measurements are nonzero detections and thus very different in consequences compared with other bounds. They raise the question of their compatibility with the bounds obtained from other physical systems, such as the test of the universality of free fall and Oklo, but also, in a more theoretical aspect, they raise the question of the understanding of such a late-time variation that does not seem to be natural from a field-theory point of view. Theoretically, one expects *all* constants to vary, and the level of their variation is also worth investigating. For instance, constraints on the

TABLE V. Summary of the constraints on the variation of the constant k. We use the notation $\mu \equiv m_e/m_p$, $x \equiv \alpha_{EM}^2 g_p \mu$, and $y \equiv \alpha_{EM}^2 g_p$.

Reference	Constant	Constraint	redshift	Time (10^9 yr)	Method
(Yahil, 1975)	μ	(0 ± 1.2)		10	Rb-Sr, K-Ar
(Pagel, 1977)	μ	$(0\pm4)\times10^{-1}$	2.1–2.7		QSO
(Foltz *et al.*, 1988)	μ	$(0\pm2)\times10^{-4}$	2.811		QSO
(Varshalovich and Levshakov, 1993)	μ	$(0\pm4)\times10^{-3}$	2.811		QSO
(Cowie and Songaila, 1995)	μ	$(0.75\pm6.25)\times10^{-4}$	2.811		QSO
(Varshalovich and Potekhin, 1995)	μ	$(0\pm2)\times10^{-4}$	2.811		QSO
(Varshalovich, Potekhin, *et al.*, 1996a)	μ	$(0\pm2)\times10^{-4}$	2.811		QSO
(Varshalovich, Panchuk, and Ivanchik, 1996)	μ	$(-1\pm1.2)\times10^{-4}$	2.811		QSO
(Potekhin *et al.*, 1998)	μ	$(-7.5\pm9.5)\times10^{-5}$	2.811		QSO
(Ivanchik *et al.*, 2002)	μ	$(-5.7\pm3.8)\times10^{-5}$	2.3–3		QSO
(Savedoff, 1956)	x	$(3\pm7)\times10^{-4}$	0.057		Cygnus A
(Wolfe *et al.*, 1976)	x	$(5\pm10)\times10^{-5}$	~0.5		QSO (Mg I)
(Wolfe and Davis, 1979)	x	$(0\pm2)\times10^{-4}$	1.755		QSO
(Wolfe and Davis, 1979)	x	$(0\pm2.8)\times10^{-4}$	0.524		QSO
(Tubbs and Wolfe, 1980)	x	$(0\pm1)\times10^{-4}$	1.776		QSO
(Cowie and Songaila, 1995)	x	$(7\pm11)\times10^{-6}$	1.776		QSO
(Varshalovich and Potekhin, 1996)	y	$(-4\pm6)\times10^{-5}$	0.247		QSO
(Varshalovich and Potekhin, 1996)	y	$(-7\pm10)\times10^{-5}$	1.94		QSO
(Drinkwater *et al.*, 1998)	y	$(0\pm5)\times10^{-6}$	0.25, 0.68		QSO
(Carilli *et al.*, 2000)	y	$(0\pm3.4)\times10^{-5}$	0.25, 0.68		QSO
(Murphy *et al.*, 2001d)	y	$(-0.2\pm0.44)\times10^{-5}$	0.25		QSO
(Murphy *et al.*, 2001d)	y	$(-0.16\pm0.54)\times10^{-5}$	0.68		QSO
(Wolfe *et al.*, 1976)	$g_p\mu$	$(0\pm0.68)\times10^{-2}$	0.524		QSO
(Turneaure and Stein, 1974)	$g_p\mu\alpha_{EM}^3$	$(0\pm9.3)\times10^{-16}$		12 days	Atomic clocks
(Godone *et al.*, 1993)	$g_p\mu$	$(0\pm5.4)\times10^{-13}$		1 year	Atomic clocks
(Wilkinson, 1958)	α_W	$(0\pm1)\times10^{1}$		1	Fission
(Dyson, 1972)	α_W	$(0\pm1)\times10^{-1}$		1	β decay
(Shlyakhter, 1976)	α_W	$(0\pm4)\times10^{-3}$		1.8	Oklo
(Damour and Dyson, 1996)	α_W	$(0\pm2)\times10^{-2}$		1.8	Oklo

variation of μ and y from quasar spectra are incompatible with the result by Webb *et al.* (2001) in a GUT framework, nor are they compatible with the Oklo or Re/Os results if the variation is linear with time or with the bound by Murphy *et al.* (2001b). The recent bound by Olive *et al.* (2002) at $z\sim0.45$ emphasizes the difficulty of achieving the deviation claimed by Webb *et al.* (2001), and the need to study the compatibility. One would also need to study the implication of these measurements for the other experiments and try to determine their expected level of detection. Both results arise from the observation of quasar-absorption spectra; it is important to ensure that all systematics are taken into account and are confirmed by independent teams, using, for instance, the VLT which offers a better signal-to-noise and spectral resolution.

The step from the standard model+general relativity to string theory allows for dynamical constants and thus starts to address the question of why the constants have the value they have. Unfortunately, no complete and satisfactory stabilization mechanism is yet known; we have to understand why, if confirmed, the constants are still varying and whether such a variation induces a new cosmological constant problem.

The study of the variation of the constants offers a new link between astrophysics, cosmology, and high-energy physics complementary to primordial cosmology. It is deeply related to the test of the law of gravitation, both of the deviations from general relativity, and the violation of the weak-equivalence principle. But much work is yet needed to both disentangle the observations and relate them to theoretical models.

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