Dynamical supersymmetry breaking

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Supersymmetry is one of the most plausible and theoretically motivated frameworks for extending the standard model. However, any supersymmetry in Nature must be a broken symmetry. Dynamical supersymmetry breaking (DSB) is an attractive idea for incorporating supersymmetry into a successful description of Nature. The study of DSB has recently enjoyed dramatic progress, fueled by advances in our understanding of the dynamics of supersymmetric field theories. These advances have allowed for direct analysis of DSB in strongly coupled theories, and for the discovery of new DSB theories, some of which contradict early criteria for DSB. The authors review these criteria, emphasizing recently discovered exceptions. They also describe, through many examples, various techniques for directly establishing DSB by studying the infrared theory, including both older techniques in regions of weak coupling and new techniques in regions of strong coupling. Finally, they present a list of representative DSB models, their main properties, and the relations among them.

CONTENTS

I. INTRODUCTION

Supersymmetry (SUSY), which rotates bosons into fermions and vice versa, is a beautiful theoretical idea. But Nature is certainly not supersymmetric. If it were, we would see a fermionic partner for each known gauge boson, and a scalar partner for each known fermion, with degenerate masses. But experimentalists have been looking for "superpartners" long and hard, and so far in vain, pushing the limits on superpartner masses to roughly above a 100 GeV. Thus any discussion of supersymmetry in Nature is necessarily a discussion of *broken* supersymmetry.

Still, even broken supersymmetry is theoretically more appealing than no supersymmetry at all. First, supersymmetry provides a solution to the gauge hierarchy problem. Without supersymmetry, the scalar Higgs mass is quadratically divergent, so that the natural scale for it is the fundamental scale of the theory, e.g., the Planck scale, many orders of magnitude above the electroweak scale. In a supersymmetric theory, the mass of the scalar Higgs is tied to the mass of its fermionic superpartner. Since fermion masses are protected by chiral symmetries, the Higgs mass can naturally be around the electroweak scale, and radiative corrections do not destabilize this hierarchy. This success is not spoiled even when explicit supersymmetry-breaking terms are added to the Lagrangian of the theory, as long as these terms are

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"soft," that is, they introduce only logarithmic divergences, but no quadratic divergences, into scalar masses. The appearance of explicit supersymmetry-breaking terms in the low-energy effective theory can be theoretically justified if the underlying theory is supersymmetric, yet the vacuum state breaks supersymmetry spontaneously.

While *spontaneously* broken supersymmetry would explain the stability of the gauge hierarchy against radiative corrections, it still does not explain the origin of the hierarchy, that is, the origin of the small mass ratios in the theory. Indeed, if supersymmetry were broken at the classical level (tree level), the scale of the soft terms would be determined by explicit mass parameters in the supersymmetric Lagrangian, and one would still have to understand why such parameters are so much smaller than the Planck scale. However, the origin of the hierarchy can be understood if supersymmetry is broken *dynamically* (Witten, 1981a). By "dynamical supersymmetry breaking" (DSB) we mean that supersymmetry is broken spontaneously in a theory that possesses supersymmetric vacua at the tree level, with the breaking triggered by dynamical effects. The crucial point about DSB is that if supersymmetry is unbroken at tree level, supersymmetric nonrenormalization theorems (Ferrara, Iliopoulos, and Zumino, 1974; Wess and Zumino, 1974; Grisaru, Rocek, and Siegel, 1979) imply that it remains unbroken to all orders in perturbation theory, and can therefore only be broken by nonperturbative effects, which are suppressed by roughly $e^{-8\pi^2/g^2}$, where g is the coupling. The electroweak scale is related to the size of the soft supersymmetry-breaking terms, and thus it is proportional to the supersymmetry-breaking scale. The latter is suppressed by the exponential above, and can easily be of the correct size, about 17 orders of magnitude below the Planck scale.

In addition, supersymmetry, or more precisely, local supersymmetry, provides the only known framework for a consistent description of gravity, in the context of string theory. If indeed the underlying fundamental physics is described by string theory, one can contemplate two qualitatively different scenarios. One is that SUSY is directly broken by stringy effects. Then, however, the SUSY-breaking scale is generically around the string scale (barring new and better understanding of string vacua), and thus the gauge hierarchy problem is not solved by supersymmetry. Therefore we shall focus here on a second possible scenario, namely, that in the low-energy limit, string theory gives rise to an effective field theory, and supersymmetry is spontaneously broken by the dynamics of this low-energy effective theory.

The aim of this review is to describe the phenomenon of dynamical supersymmetry breaking in field theories with $\mathcal{N}=1$ global supersymmetry. ($\mathcal{N}$ counts the number of supersymmetries. For $\mathcal{N}=1$ there are four supersymmetry charges, and this is the smallest amount of supersymmetry allowed in four dimensions.)

The restriction on $\mathcal{N}$ comes from the fact that only $\mathcal{N}=1$ supersymmetry has chiral matter, which we need in the low-energy theory if it is to contain the standard model. Moreover, theories with $\mathcal{N}>1$ supersymmetry are believed to have an exact moduli space and thus are not expected to exhibit dynamical supersymmetry breaking.

The restriction to global supersymmetry still allows us to answer most of the questions we would be interested in. This situation is quite analogous to studying the breaking of a gauged bosonic symmetry in, say, a theory with scalar matter. In that case we can determine the pattern of symmetry breaking just by studying the scalar potential. Similarly, we shall be able to determine whether supersymmetry is broken and, if the theory is weakly coupled, what the vacuum energy and the light spectrum are. From our perspective, the most relevant consequence of "gauging" supersymmetry is the analogue of the Higgs mechanism by which the massless fermion accompanying supersymmetry breaking, the Goldstino, is eaten by the gravitino.

As we shall see, supersymmetry is broken if and only if the vacuum energy is nonzero. Furthermore, as we mentioned above, if supersymmetry is unbroken at tree level, it can only be broken by nonperturbative effects. Thus studying supersymmetry breaking requires understanding the nonperturbative dynamics of gauge theories in the infrared. Fortunately in recent years there has been tremendous progress in understanding the dynamics of supersymmetric field theories.

The potential of a supersymmetric theory is determined by two quantities: the Kähler potential, which contains the kinetic terms for the matter fields, and the superpotential, a holomorphic function of the matter fields that controls their Yukawa interactions. Holomorphy, together with the symmetries of the theory, may be used to determine the physical degrees of freedom and the superpotential of the infrared theory (Seiberg, 1994, 1995). Since the latter two are precisely the ingredients needed for studying supersymmetry breaking, this progress has fueled the discovery of many new supersymmetry-breaking theories, as well as new techniques for establishing supersymmetry breaking.

The structure of this article is as follows. We start by describing general properties of supersymmetry breaking and studying examples of tree-level breaking in Sec. II. In Sec. III, we discuss indirect methods for finding theories with dynamical supersymmetry breaking, and for establishing supersymmetry breaking. As we shall see, these methods direct the search for supersymmetry breaking towards chiral theories with no flat directions, preferably possessing an anomaly-free R symmetry. These criteria do not amount to necessary conditions for supersymmetry breaking, and we shall point out "loopholes" in the indirect methods that allow the possibility of supersymmetry breaking in theories that violate all of the above requirements. Recent developments have led to the discovery of such SUSY-breaking theories, and we shall postpone the discussion of representative examples to Sec. VI. Still, some of the indirect methods we shall describe, most notably, the breaking of a global symmetry in a theory with no flat directions, provide the most convincing evidence for supersymmetry breaking in theories that cannot be directly analyzed.

In Secs. IV and V we turn to theories that can be directly analyzed in the infrared. In the early 1980s, such studies were limited to a semiclassical analysis in regions of weak coupling, and we shall describe such analyses in Sec. IV. The main development in recent years has been the better understanding of supersymmetric theories in regions of strong coupling, and we shall study supersymmetry breaking in such theories in Sec. V. In both cases, the analysis of supersymmetry breaking involves two ingredients. The first is identifying the correct degrees of freedom of the theory, in terms of which the Kähler potential is nonsingular. In all the theories we shall study, in the interior of the moduli space, these are either the confined variables or the variables of a dual theory.[1] Indeed, we shall see that duality (Seiberg, 1995)—the fact that different UV theories may lead to the same infrared physics—can be a useful tool for establishing supersymmetry breaking, as one can sometimes pick a more convenient theory in which to study whether supersymmetry is broken or not. A second, related ingredient is finding the exact superpotential in terms of the light, physical degrees of freedom.

Having established these two ingredients, at low energies one then typically has a theory of chiral superfields, with all gauge dynamics integrated out, with a known superpotential. The problem of establishing supersymmetry breaking is then reduced to solving a system of equations to check whether or not the superpotential can be extremized.

Although the results we shall use on the infrared degrees of freedom and the exact superpotential apply to supersymmetric theories, they may still be used to argue for supersymmetry breaking, since the theories we shall study in this way are obtained by perturbing a supersymmetric theory. For a sufficiently small perturbation, the scale of supersymmetry breaking can be made sufficiently small, so that we can work above this scale and still use known results on the infrared supersymmetric theory. If supersymmetry is indeed broken in the theory, the breaking should persist even as the perturbation is increased. Otherwise, the theory undergoes a phase transition as some coupling is varied, being supersymmetric in one region and nonsupersymmetric in another. However, one does not expect supersymmetric theories to undergo phase transitions as couplings are varied (Intriligator and Seiberg, 1994; Seiberg and Witten, 1994a, 1994b).

Having learned various techniques for the analysis of DSB, we use these in Sec. VI to study a few examples of theories that break supersymmetry dynamically even though they violate some of the criteria described in Sec. III.

Perhaps the most disappointing aspect of the recent progress in our understanding of supersymmetry breaking is that it still has not yielded any organizing principle to the study and classification of supersymmetry-breaking theories. In Sec. VII we shall describe one method for generating new supersymmetry-breaking theories from known theories. However, this is far from a full, systematic classification. Nor can we tell immediately, without detailed analysis, whether a specific theory breaks supersymmetry or not. For these reasons we find it useful to present a rough survey of known models in Sec. VII, pointing out their main features, the relations between them, and, where applicable, their relevant properties for model-building purposes. While we shall see many different mechanisms by which supersymmetry is broken in these examples, the breaking is almost always the consequence of the interplay between instanton effects and a tree-level superpotential.

Another area requiring further study is the analysis of supersymmetry-breaking vacua, and their symmetries and light spectra, in strongly coupled theories. One may hope that recently discovered realizations of supersymmetric gauge theories as extended brane configurations in string theory will lead to further progress in this direction, as well as to some organizing principle for DSB. Indeed, several DSB models have been realized as D-brane configurations in string/M theory; see for example de Boer *et al.* (1998) and Lykken, Poppitz, and Trivedi (1999). Moreover, in some cases the dynamical effects leading to supersymmetry breaking were understood in stringy language (de Boer *et al.*, 1998). However, this approach has yet to lead to results that cannot be directly obtained in a field-theory analysis.

We limit ourselves in this review to the theoretical analysis of supersymmetry breaking in different models. We do not discuss the questions of whether, and how, this breaking can feed down to the standard model. Ideally, a simple extension of the standard model would break supersymmetry by itself, generating an acceptable superpartner spectrum. Unfortunately this is not the case. In simple supersymmetry-breaking extensions of the standard model without new gauge interactions, nonperturbative effects would probably be too small to generate soft terms of the correct size (Affleck, Dine, and Seiberg, 1985). Moreover, unless some of the scalars obtained their masses either radiatively, or from nonrenormalizable operators, some superpartners would be lighter than the lightest lepton or quark (Dimopoulos and Georgi, 1981). Thus supersymmetry must be broken by a new, strongly interacting sector, and then communicated to the standard model either by supergravity effects, in which case the soft terms are generated by higher-dimension operators or occur at the loop level, or by gauge interactions, in which case the soft terms occur at the loop level. These different possibilities introduce different requirements on the SUSY-breaking sector. For example, gravity mediation often requires singlet fields that participate in the SUSY-breaking. Simple models of gauge mediation require a large unbroken global symmetry at the minimum of the SUSY-breaking theory, in which the standard-model gauge group can be embedded. Several of the supersymmetry-breaking

[1] At the boundary of moduli space, the microscopic degrees of freedom will often be more appropriate.

models discovered recently have some of these desired properties, and thus allow for improved phenomenological models for the communication of supersymmetry breaking.

Another issue that is important for phenomenological applications of DSB that we shall not address is the cosmological constant problem. In globally supersymmetric theories, fermionic and bosonic contributions to the vacuum energy cancel each other and the cosmological constant vanishes. Upon supersymmetry breaking this is no longer true, and the cosmological constant is comparable to the scale of supersymmetry breaking. While in a framework of local supersymmetry a further cancellation is possible, significant fine tuning is required. Eventually, a microscopic understanding of such fine tuning is needed in any successful phenomenological application of dynamical supersymmetry breaking.

In the body of this review we assume that the reader is familiar with the general properties of supersymmetric field theories and rely heavily on symmetries and a number of exact nonperturbative results obtained in recent years. The reader who is just beginning the study of supersymmetry should first consult the Appendix, where we briefly present basic facts about supersymmetry and relevant results to make our presentation self-contained. For a more complete introduction to SUSY see, for example, Bagger and Wess (1991) and Nilles (1984). Several excellent reviews of the recent progress in the study of strongly coupled SUSY gauge theories exist; see, for example, Intriligator and Seiberg (1996), Peskin (1997), and Shifman (1997).

In Appendix Sec. 1 we introduce notations and basic formulas for the Lagrangians of supersymmetric theories. The knowledge of these results is necessary in every section of the review. In Sec. 2 we discuss a method for finding the D-flat directions of a SUSY gauge theory (directions along which the gauge-interaction terms in the scalar potential vanish), and the parametrization of D-flat directions in terms of gauge-invariant operators. These results are necessary for the study of supersymmetry breaking in non-Abelian gauge theories, which we discuss starting in Sec. III.D. In Appendix Secs. 3–7 we turn to SUSY QCD with different numbers of flavors. While the results we present are used directly in various places in Secs. IV–VII, the discussion in the Appendix also illustrates techniques in the analysis of the dynamics of general SUSY gauge theories and are applicable to models with matter transforming in general representations of the gauge group. The discussion also provides simple examples of phenomena such as theories with no quantum moduli spaces, deformed quantum moduli spaces, confinement without chiral symmetry breaking, and duality. We shall encounter these phenomena in various theories throughout the review. References to analyses of the dynamics of theories other than $SU(N)$ are collected in Appendix Sec. 8.

We also note that the reader who is interested only in a general knowledge of DSB can skip Secs. VI and VII. Section VII.C can be used independently of the rest of the article as a guide to DSB models.

Finally, we note that the interested reader can find several useful reviews of dynamical supersymmetry breaking which have appeared in the past couple of years. Short introductions to recent developments can be found in Skiba (1997), Nelson (1998), Poppitz (1998), and Thomas (1998). Most notably, the review by Poppitz and Trivedi (1998), although smaller in scope than the present review, emphasizes recent developments in the field. It also contains a discussion of supersymmetry breaking in quantum-mechanical systems. Shifman and Vainshtein (1999) give an excellent introduction to instanton techniques and discuss their application for supersymmetry breaking. The review by Giudice and Rattazzi (1998) focuses on applications of DSB to building models of gauge-mediated supersymmetry breaking.

II. GENERALITIES

A. Vacuum energy—the order parameter of SUSY breaking, and *F* and *D* flatness

A positive vacuum energy is a necessary and sufficient condition for spontaneous SUSY breaking. This follows from the fact that the Hamiltonian of the theory is related to the absolute square of the SUSY generators (see Appendix Sec. 1):

$$H=\frac{1}{4}(\overline{Q}_1Q_1+Q_1\overline{Q}_1+\overline{Q}_2Q_2+Q_2\overline{Q}_2). \tag{1}$$

The energy is then either positive or zero. Furthermore, a state that is annihilated by Q_α has zero energy, and conversely, a zero-energy state is annihilated by Q_α. Thus the vacuum energy serves as an order parameter for supersymmetry breaking.

Therefore the study of supersymmetry breaking requires knowledge of the scalar potential of the theory. It is convenient to formulate the theory in $\mathcal{N}=1$ superspace, where space-time (bosonic) coordinates are supplemented by anticommuting (fermionic) coordinates.[2] In this formulation fields of different spins related by supersymmetry are combined in the supersymmetry multiplets, superfields. Matter fields form chiral superfields, while gauge bosons and their spin 1/2 superpartners form (real) vector superfields. In the superspace formulation, physics, and in particular, the scalar potential, is determined by two functions of the superfields, the superpotential and the Kähler potential. The superpotential encodes Yukawa-type interactions in the theory; in particular it contributes to the scalar potential. The superpotential is an analytic function of the superfields. This fact, together with symmetries and known (weakly coupled) limits, often allows us to determine the exact nonperturbative superpotential of the theory. The Kähler potential, on the other hand, is a real function of superfields, and can be reliably calculated

[2]For more detail see Appendix Sec. 1.

only when a weakly coupled description of the theory exists. From our perspective, the Kähler potential is important in two respects. First, it gives rise to gauge-interaction terms in the scalar potential. Second, it determines the kinetic terms of the matter fields and thus modifies scalar interactions arising from the superpotential. Assuming a canonical (quadratic in the fields) Kähler potential, the scalar potential is

$$V=\sum_a (D^a)^2+\sum_i F_i^2, \tag{2}$$

where the sum runs over all gauge indices a and all matter fields ϕ_i. In Eq. (2), the D terms and F terms are auxiliary components of vector and chiral superfields, respectively. D terms and F terms are not dynamical and one should solve their equations of motion. In particular, F terms are given by derivatives of the superpotential (for more details, and the analogous expressions for D terms, see Appendix Sec. 1):

$$F_i=\frac{\partial W}{\partial \phi_i}.$$

For supersymmetry to remain unbroken, there has to be some field configuration for which both the F terms and the D terms vanish.[3] In fact, generically such configurations exist not only at isolated points but on a subspace of the field space. This subspace is often referred to as the moduli space of the theory.

Classically one could set all superpotential couplings to zero. Then the moduli space of the theory is the set of "D-flat directions," along which the D terms vanish. A particularly useful parametrization of D-flat directions, which we discuss in Appendix Sec. 2, can be given in terms of the gauge-invariant operators of the theory (Luty and Taylor, 1996). Even when small tree-level superpotential couplings are turned on, the vacua will lie near the D-flat directions. It is convenient therefore to analyze SUSY gauge theories in two stages. First find the D-flat directions, then analyze the F terms along these directions. The latter have classical contributions from the tree-level superpotential, and may "lift" some, or all, of the D-flat directions. Since a classical superpotential is a polynomial in the fields, F terms typically grow for large scalar vacuum expectation values (VEV), and vanish at the origin.[4]

As mentioned in the Introduction, a key point in the study of SUSY breaking is the fact that, due to the supersymmetric nonrenormalization theorems (Ferrara, Iliopoulos, and Zumino, 1974; Wess and Zumino, 1974; Grisaru, Rocek, and Siegel, 1979; Seiberg, 1993), the moduli space remains unmodified in perturbation theory. If the classical potential vanishes for some choice of VEV's, it remains exactly zero to all orders in perturbation theory. Thus only nonperturbative effects may generate a nonzero potential and lift the classical zeros. Indeed, nonperturbative effects can modify the moduli space; they can lift the moduli space completely; or, finally, the quantum moduli space may coincide with the classical one.

There are numerous possibilities, then, for the behavior of the theory. If a theory breaks supersymmetry, it has some ground state of positive energy at some point in field space (or it may, in principle, have several ground states at different points).[5] Alternatively, the theory may remain supersymmetric, with either one ground state of zero energy at some point in field space, or a few ground states at isolated points, or with a continuum of ground states, corresponding to completely flat directions that are not lifted either classically or nonperturbatively. It is also possible that the theory does not have a stable vacuum state. In such a case, while a supersymmetric vacuum does not exist, the energy can become arbitrarily small along some direction on the moduli space.[6] While such a theory can still be given a cosmological interpretation (Affleck, Dine, and Seiberg, 1984a), we shall not consider it a SUSY-breaking theory for our purposes.[7]

Note that supersymmetric theories are very different, in this respect, from other theories. In a nonsupersymmetric theory, multiple ground states are usually related by a symmetry and are therefore physically equivalent. On the other hand, different ground states of a supersymmetric theory may describe completely different physics. For example, classically, one flat direction of an $SU(3)$ gauge theory with two flavors is [see Appendix Sec. 4 for details]

[3]Here we implicitly assumed that the Kähler potential is a regular function of the fields and has no singularities. For example, if the derivatives of the Kähler potential vanish, the scalar potential can be nonzero even if all F and D terms vanish; see Eq. (A6).

[4]Superpotential terms that are linear in the fields are an important exception. They lead to potentials that are nonzero even at the origin. Such terms necessarily involve gauge singlets and require the introduction of some mass scale. However, a linear term can be generated dynamically. We shall encounter examples of trilinear, or higher, superpotential terms that become linear after confinement.

[5]In this latter case the ground states are nondegenerate even if they appear to have the same energy in a certain approximation. This is because low-energy physics is nonsupersymmetric, and the vacuum energy receives quantum corrections (on top of the nonperturbative effects that led to the nonvanishing energy in the first place). Since different nonsupersymmetric vacua are nonequivalent, these quantum corrections lift the degeneracy.

[6]We shall call such directions in the moduli space "runaway" directions, and study them carefully in Secs. VI.A–VI.B.

[7]We also note that the runaway moduli may be stabilized, and supersymmetry broken, due to Kähler potential effects when the theory is coupled to gravity (Dvali and Kakushadze, 1998). However, since the typical VEV's in this case will be of Planck size, the vacuum will be determined by the details of the microscopic theory at M_P, and the dynamical supersymmetry breaking is not calculable in the low-energy effective-field theory.

$$f=\overline{f}=\begin{pmatrix} v & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \tag{3}$$

where f and $\overline{f}$ are the scalar components of the $SU(3)$ fundamentals and antifundamentals, respectively. For any given choice of v, the low-energy theory is an $SU(2)$ gauge theory, whose gauge coupling depends on v.[8] Note also that the flat directions of SUSY theories may extend to infinity, unlike the customary compact flat directions of bosonic global symmetries. Furthermore, in the case of other global symmetries, the existence of a flat direction is usually associated with spontaneous breaking of the symmetry, with the massless Goldstone bosons corresponding to motions along the flat direction. This is not the case with SUSY. The reason, of course, is that the SUSY generators do not correspond to motions in field space. Theories with unbroken SUSY may have degenerate vacua precisely because SUSY is unbroken. And, as we shall see in the next section, theories with spontaneously broken SUSY have the analog of Goldstone particles even when they only have a single ground state.

B. The Goldstino

The breaking of any bosonic global symmetry is accompanied by the appearance of massless Goldstone bosons that couple linearly to the symmetry current. Similarly a theory with broken supersymmetry contains a massless fermion, which is usually referred to as a "Goldstone fermion," or, in short, "Goldstino," that couples linearly to the SUSY current (Salam and Strathdee, 1974, Witten, 1981a).

The Goldstino coupling to the SUSY current can be expressed as

$$J^{\mu}_{\alpha}=f\,\sigma^{\mu\dot{\beta}}_{\ \ \alpha}\,\psi^{G}_{\dot{\beta}}+\cdots, \tag{4}$$

where $\psi^{G}_{\dot{\beta}}$ is the Goldstino and, as we shall see momentarily, f is a constant that is nonzero when SUSY is broken. The ellipsis in Eq. (4) stands for terms quadratic in the fields and for potential derivative terms. Conservation of the SUSY current then implies that the Goldstino is massless. To justify Eq. (4), note that, for broken SUSY (Witten, 1981a),

$$\int d^4x\,\partial_\eta\langle 0|T\,J^{\eta}_{\alpha}(x)\,J^{\nu}{}_{\dot{\beta}}(0)|0\rangle=\langle 0|\{Q_\alpha,J^{\nu}{}_{\dot{\beta}}(0)\}|0\rangle\neq 0. \tag{5}$$

If indeed there is a massless fermion coupling to the current as in Eq. (4), then the left-hand side of Eq. (5) is equal to

$$f^2\,\sigma^{\eta\dot{\alpha}}_{\ \ \alpha}\,\sigma^{\nu\beta}_{\ \ \dot{\beta}}\int d^4x\,\partial_\eta\langle 0|T\,\psi^{G}_{\dot{\alpha}}(x)\,\psi^{G}_{\beta}(0)|0\rangle$$
$$=f^2\,\sigma^{\eta\dot{\alpha}}_{\ \ \alpha}\,\sigma^{\nu\beta}_{\ \ \dot{\beta}}[-ip_\eta\,G(p)_{\dot{\alpha}\beta}]_{p\to 0}=f^2\,\sigma^{\nu}{}_{\alpha\dot{\beta}}, \tag{6}$$

where $G(p)_{\dot{\alpha}\beta}$ is the Goldstino propagator. So indeed f is nonzero. Note that a fermion with derivative coupling to the current would not contribute to the right-hand side of Eq. (6) because of the additional factors of the momentum in the numerator. In fact, since from the SUSY algebra $\langle 0|\{Q_\alpha,J^{\nu}{}_{\dot{\beta}}(0)\}|0\rangle=2E\sigma^{\nu}{}_{\alpha\dot{\beta}}$, with E the energy, we have $f^2=2E$.

To see the appearance of the Goldstino more concretely, consider the SUSY current

$$J^{\mu}_{\alpha}\sim\sum_{\phi}\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)}(\delta\phi)_\alpha, \tag{7}$$

where the sum is over all fields, and $(\delta\phi)_\alpha$ is the shift of the field ϕ under a SUSY transformation. Because of Lorentz symmetry, the only linear terms in Eq. (7) come from vacuum expectation values of $(\delta\phi)$. Examining the SUSY transformations of the chiral and vector multiplets of $\mathcal{N}=1$ SUSY (see for example Bagger and Wess, 1991), we see from Lorentz invariance that the only fields whose SUSY transformations contain Lorentz-invariant objects, which can develop VEV's, are the matter fermion ψ_i, whose SUSY transformation gives F_i, and the gauge fermion λ^a, whose transformation gives D^a. One then finds

$$\psi^G\sim\sum\langle F_i\rangle\,\psi_i+\frac{1}{\sqrt{2}}\sum\langle D^a\rangle\,\lambda^a, \tag{8}$$

so that the Goldstino is a linear combination of the chiral and gauge fermions whose auxiliary fields F and D acquire VEV's. Note that Eq. (8) actually holds only with a canonical (quadratic) Kähler potential, otherwise derivatives of the Kähler potential enter as well. We can use this to argue that a nonvanishing F VEV or D VEV is a necessary condition for SUSY breaking. When SUSY is broken, there is a massless fermion, the Goldstino, that transforms inhomogeneously under the action of the SUSY generators. But the only Lorentz-invariant objects that appear in the SUSY variations of the $\mathcal{N}=1$ multiplets, and that therefore may obtain VEV's, are the auxiliary fields F and D. Thus for SUSY breaking to occur, some F or D fields should develop VEV's.

In light of the above, it would first seem that if SUSY is relevant to nature, we should observe the massless Goldstino. However, we ultimately need to promote SUSY to a local symmetry to incorporate gravity into the full theory. In the framework of local supersymme-

[8]Here we consider the classical vacua of the theory. Quantum mechanically, the flat directions (3) are lifted, and the theory does not have a stable vacuum. Yet in many models nonequivalent quantum vacua exist.

try, the massless Goldstino becomes the longitudinal component of the gravitino, much like the case of the Higgs mechanism. The gravitino then has a coupling to ordinary matter other than the gravitational interaction, by virtue of its Goldstino component. It should come as no surprise, then, that the gravitino mass, as well as its coupling to matter fields, are related to the SUSY-breaking scale. This has important phenomenological implications. In particular, in models with low-scale SUSY breaking, the gravitino is very light, and the decay of other superpartners into the gravitino may be observed in collider searches for supersymmetry (Stump, Wiest, and Yuan, 1996; Dimopoulos *et al.*, 1998).

C. Tree-level breaking

1. O'Raifeartaigh models

One of the simplest models of spontaneous supersymmetry breaking was proposed by O'Raifeartaigh (1975) and is based on a theory of chiral superfields. Supersymmetry in the model is broken at tree level: while the Lagrangian of the model is supersymmetric, even the classical potential is such that a supersymmetric vacuum state does not exist.

In addition to giving the simplest example of spontaneously broken supersymmetry, the study of O'Raifeartaigh models will be useful for our later studies of DSB, as the low-energy description of many dynamical models we encounter will be given by an O'Raifeartaigh-type model.

Before writing down the simplest example of an O'Raifeartaigh model, let us describe the general properties of such models. First, we shall restrict our attention to superpotentials with only positive exponents of the fields. We shall later analyze a number of models in which the low-energy description involves superpotentials containing negative exponents of the fields. Such terms, however, are generated by the nonperturbative dynamics of the underlying (strongly coupled) microscopic theory, and are not appropriate in the tree-level superpotential we consider here.

Second, since the superpotential is a polynomial in the fields, at least one of the fields in this model needs to appear linearly in the superpotential, or there will be a supersymmetric vacuum at the origin of the moduli space.

It would prove useful, for future purposes, to pay special attention to the R symmetry of the model. [For the definition of an R symmetry, see Appendix Sec. 1]. If this symmetry is unbroken by the superpotential, there is then at least one field of R charge 2. More generally, consider a model containing fields $\phi_i^c, i=1,\ldots,k$ with R charge 2, and fields ϕ_a^n, $a=1,\ldots,l$ with R charge 0. (For convenience, we shall call them charged and neutral, respectively, even though various components of the superfields transform differently under R symmetry.) The most general superpotential respecting the R symmetry can be written as

$$W=\sum_{i=1}^{k} \phi_i^c f_i(\phi_a^n), \tag{9}$$

and, for supersymmetry to break, at least one of the f_i's, say f_1, contains a constant term, independent of the fields. The equations of motion for the R-charged fields $\partial W/\partial \phi_i^c = f_i(\phi_a^n)=0$ give k equations for l unknowns ϕ_a^n. If $k>l$ there are no solutions for generic functions f_i, the F-term conditions cannot all be satisfied, and supersymmetry is broken.

We can modify these models by adding fields with R charges $0<Q_R<2$. Since such fields cannot couple to the fields ϕ_i^c while preserving the R symmetry, they will not change the above discussion, and supersymmetry remains broken. If, on the other hand, fields with negative R charges are added to the model, the total number of variables on which the f_i's depend increases, and in general supersymmetry is unbroken. Finally, we should note that adding to the superpotential explicit R-symmetry-violating couplings that do not involve fields of R charge 2 will not modify the above discussion. However, R-symmetry-violating terms that include fields of R charge 2 will generically lead to the restoration of supersymmetry.

It is also useful to look at the equations of motion for the R-neutral fields. First, note that their VEV's are fixed by minimizing the part of the scalar potential arising from the F terms of the R-charged fields (the remaining terms in the scalar potential vanish at least when $\langle \phi_i^c \rangle = 0$ for all i). Therefore there are l F-term equations depending on k independent variables ϕ_i^c. In a SUSY-breaking model $k>l$, so there are $k-l$ linear combinations of the fields ϕ_i^c that are left undetermined. Thus O'Raifeartaigh models necessarily possess directions of flat (nonzero) potential in the tree-level approximation (Zumino, 1981; Einhorn and Jones, 1983; Polchinski, 1983). As we shall discuss later, this is not a generic situation in models of DSB.

The simplest example of an O'Raifeartaigh model requires two fields with R charge 2, one field with R charge 0 ($k=2$, $l=1$), and has the superpotential

$$W=\phi_1(M_1^2-\lambda_1\phi^2)+m_2\phi\phi_2. \tag{10}$$

It is easy to see that the F-term conditions for ϕ_1 and ϕ_2 are incompatible. We can directly minimize the scalar potential

$$V=|M_1^2-\lambda_1\phi^2|^2+|m_2\phi|^2+|m_2\phi_2-2\lambda_1\phi_1\phi|^2. \tag{11}$$

The first two terms in Eq. (11) determine the value of ϕ at the minimum. In the limit $m_2^2/(\lambda_1 M_1^2) \gg 1$ the minimum is found at $\phi=0$, while for small $m_2^2/(\lambda_1 M_1^2)$ we find $\phi=(M_1^2-m_2^2/2)^{1/2}/\lambda_1$. Note that ϕ_1 and ϕ_2 appear only in the last term in Eq. (11). This term should be set to zero for the potential to be extremal with respect to ϕ_1 and ϕ_2. This is achieved when $\phi_2=-2\lambda_1\phi\phi_1/m_2$, and therefore at tree level the linear combination

$m_2\phi_1-2\lambda_1\langle\phi\rangle\phi_2$ is arbitrary, as was expected from the previous discussion. Equivalently, we can parametrize different vacua by the expectation values of ϕ_1. Note that different vacua are physically nonequivalent; in particular the spectrum depends on $\langle\phi_1\rangle$.

It is easy to find the tree-level spectrum of the model. For any choice of parameters it contains a massless fermion, the Goldstino. The spectrum also contains the scalar field associated with the flat direction whose mass arises entirely due to radiative corrections (however, there are no quadratic divergences since the action of the theory is supersymmetric, although supersymmetry is not realized linearly). All other states are massive. Another important feature of this spectrum is that the supertrace of the mass matrix squared, $S\mathrm{Tr}m_i^2$, vanishes. This property of the spectrum holds for any model with tree-level supersymmetry breaking (Ferrara, Girardello, and Palumbo, 1979).

Since supersymmetry is broken, the vacuum degeneracy is lifted in perturbation theory. Huq (1976) calculated the one-loop corrections to the potential of this model. They are given by

$$\Delta V(\phi_1)=\sum_i\frac{(-1)^F}{64\pi^2}m_i(\phi_1)^4\ln\left(\frac{m_i(\phi_1)^2}{\mu^2}\right),\tag{12}$$

where the sum is over all massive fields (and the masses depend on the ϕ_1 VEV). Huq (1976) found that the corrections generate a positive mass for ϕ_1 and that the nonsupersymmetric vacuum is located at $\phi_1=0$ with unbroken R symmetry. He also analyzed a model with an $SU(3)$ global symmetry and a model constructed by Fayet (1975) with $SU(2)\times U(1)$ symmetry, and in both cases found that the tree-level modulus acquires positive mass due to one-loop corrections to the Kähler potential, leading to the unique vacuum with unbroken R symmetry. In fact, this conclusion is not surprising. In the model discussed above, quantum corrections to the vacuum energy come from the renormalization of the mass parameter M_1^2. Due to the holomorphy of the superpotential, these are completely determined by the wave function renormalization of ϕ_1 and, since the model is infrared free, necessarily generate a positive contribution to the scalar potential. It is important to note that in modifications of the model that include gauge fields, there may be a negative contribution to the potential. The balance of the two perturbative effects may produce a stable minimum at large values of the modulus VEV (Witten, 1981b).

2. Fayet-Iliopoulos breaking

Another useful example of tree-level supersymmetry breaking is given by a model with $U(1)$ gauge interactions (Fayet and Illiopoulos, 1974). In this model supersymmetry breaking is driven by D-term contributions to the potential, but depending on the parameters of the Lagrangian, the nonzero vacuum energy comes either entirely from D-term contributions or from both D and F terms. To understand how D terms can drive supersymmetry breaking, we recall that the Kähler potential can be written as a function of the gauge-invariant combination of fields

$$K=f(\phi^\dagger e^V\phi,\mathcal{W}^\dagger\mathcal{W},S),\tag{13}$$

where ϕ represents matter fields transforming in some representation of the gauge group, V is a vector superfield whose supersymmetric field strength is $\mathcal{W}$, and S represents gauge-singlet fields. In a non-Abelian theory this is the only possible form of field dependence in the Kähler potential. In an Abelian theory, however, the D term of the vector superfield V is invariant under the gauge and supersymmetry transformations by itself. Thus, if one does not require parity invariance, the lowest-order Kähler potential of a $U(1)$ gauge theory can be written as[9]

$$K=Q^\dagger e^VQ+\overline{Q}^\dagger e^{-V}\overline{Q}+\xi_{FI}V.\tag{14}$$

This Kähler potential, together with superpotential mass terms for the matter fields, leads to the following scalar potential:

$$V=\frac{g^2}{2}(|Q|^2-|\overline{Q}|^2+\xi)^2+m^2(|Q|^2+|\overline{Q}|^2).\tag{15}$$

It is easy to see that the vacuum energy determined by this potential is necessarily positive and supersymmetry is broken. When $g^2\xi<m^2$ both scalar fields have positive mass and their VEV's vanish. The positive contribution to the vacuum energy comes entirely from the D term in the potential. The scalar mass matrix has eigenvalues $m_\pm^2=m^2\pm g^2\xi$. The gauge symmetry is unbroken, and thus the gauge boson remains massless. The matter fermions retain their mass m, while the gaugino remains massless and plays the role of the Goldstino. [In accord with the fact that here $\langle F_i\rangle=0$ and $\langle D\rangle\neq0$; see Eq. (8).]

When $g^2\xi>m^2$, the field $\overline{Q}$ has negative mass and acquires a VEV. At the minimum of the potential $Q=0$ and $\overline{Q}=v$, where $v=(2\xi-4m^2/g^2)^{1/2}$. We see that both the gauge symmetry and supersymmetry are broken. Moreover, both the D term and the F term are nonvanishing and supersymmetry breaking is of the mixed type. One can easily find that the spectrum of the model contains one vector field and one real scalar field of mass squared $\frac{1}{2}g^2v^2$, one complex scalar of mass squared $2m^2$, two fermions of mass $(m^2+\frac{1}{2}g^2v^2)^{1/2}$, and a massless Goldstone fermion that is a linear combination of the Goldstino and the positively charged fermion

$$\tilde{\lambda}=\frac{1}{\sqrt{m^2+\frac{1}{2}g^2v^2}}\left(m\lambda+\frac{igv}{\sqrt{2}}\psi_Q\right).\tag{16}$$

[9]We restrict our attention to two matter multiplets with charges ±1.

III. INDIRECT CRITERIA FOR DSB

As we have seen in Sec. II.A, the fact that the vacuum energy is the relevant order parameter immediately points the way in our quest for SUSY breaking: we should study the zeros of the scalar potential. This indeed is what we shall undertake to do in Secs. IV and onward. Unfortunately, directly studying the zeros of the potential will not always be possible, or easy. In this section we review several alternate "indirect" methods that are useful in the search for supersymmetry breaking.

A. The Witten index

Supersymmetry breaking is related to the existence of zero-energy states. Rather than looking at the total number of zero-energy states, it is often useful to consider the Witten index (Witten, 1982), which measures the difference between the number of bosonic and fermionic states of zero energy,

$$\mathrm{Tr}(-1)^F \equiv n_B^0 - n_F^0. \tag{17}$$

If the Witten index is nonzero, there is at least one state of zero energy, and supersymmetry is unbroken. If the index vanishes, supersymmetry may either be broken, with no states of zero energy, or it may be unbroken, with identical numbers of fermionic and bosonic states of zero energy.

The Witten index is a topological invariant of the theory. In this lies its usefulness. It may be calculated for some convenient choice of the parameters of the theory, and in particular for weak coupling, but the result is valid generally. To see this, note that in a finite volume, fermionic and bosonic states of positive energy are paired by the action of the SUSY generator:

$$Q|b_E\rangle \sim \sqrt{E}|f_E\rangle \quad Q|f_E\rangle \sim \sqrt{E}|b_E\rangle, \tag{18}$$

where $|b_E\rangle$ ($|f_E\rangle$) is a bosonic (fermionic) state of energy E.[10] (Recall that states of zero energy are annihilated by Q and are therefore not paired.) Thus, under "mild" variations of the parameters of the theory, states may move to zero energy and from zero energy, but they always do so in Bose-Fermi pairs, leaving the Witten index unchanged.

Let us be a bit more precise now about what is meant by "mild" variations. As long as a parameter of the theory, which is originally nonzero, is varied to a different nonzero value, we do not expect the Witten index to change, since different states can only move between different energy levels in pairs. The danger lies in the appearance of new states of zero energy. This can happen if the asymptotic (in field space) behavior of the potential changes, which may happen if some parameter of the theory is set to zero, or is turned on. In that case, states may "come in" from infinity or "move out" to infinity.

The index of several theories was calculated by Witten (1982). In particular, Witten found that the index of a pure supersymmetric Yang-Mills theory is nonzero.[11] Thus these theories do not break supersymmetry spontaneously. An important corollary is that supersymmetric Yang-Mills theories with massive matter (and no massless matter) do not break supersymmetry either. The reason is that, at least in weak coupling, one can take all masses to be large, so that there are no massless states in these theories beyond those of the pure supersymmetric Yang-Mills theory, and so the value of the index is the same as in the pure supersymmetric Yang-Mills theory.

What happens when the mass of the matter fields is taken to zero? The theory with zero mass has flat directions, along which the potential is classically zero (away from these flat directions the potential behaves as the fourth power of the field strength). In contrast, the theory with mass for all matter fields has no classical flat directions, with the potential growing at least quadratically for large fields. Thus, as the mass is taken to zero, the asymptotic behavior of the potential changes, and the Witten index may change too. In fact, the index is ill defined in the presence of flat directions, since zero modes associated with the flat directions lead to a continuous spectrum of states. (Indeed, to calculate the index of any theory one needs to consider the theory in a finite volume so that the resulting spectrum is discrete.) We therefore cannot say anything about supersymmetry breaking in massless, nonchiral theories based on the Witten index of the pure supersymmetric Yang-Mills theory.

Consider for example supersymmetric QCD with N colors and F flavors of mass m, which we discuss in Appendix Secs. 4–7. As explained there, in the presence of mass terms $m^i_j Q_i \cdot \overline{Q}^j$, the theory has N vacua at

$$M_i^j \equiv Q_i \cdot \overline{Q}^j = \Lambda^{(3N-F)/N} (\det m)^{1/N} m_i^{-1\,j}, \tag{19}$$

[10] This in fact justifies including only zero-energy states in Eq. (17). States of nonzero energy do not contribute.

[11] For SU and SP groups the index is equal to $r+1$, where r is the rank of the group. This is the same as the number of gaugino condensates for these groups. More generally, the index equals the dual coxeter number of the group, which is different from $r+1$ for some groups, notably some of the SO groups (Witten, 1998). The fact that the number of gaugino condensates does not always equal $r+1$, which was believed to be the value of the index, remained a puzzle until its recent resolution by Witten (1998). This puzzle partly motivated the conjecture that supersymmetric Yang-Mills theories have a vacuum with no gaugino condensate (Kovner and Shifman, 1997). This possibility would have far-reaching consequences for supersymmetry breaking. The vast majority of theories that break SUSY do so by virtue of a superpotential generated by gaugino condensation. A vacuum with no gaugino condensate would mean an extra ground state, or an entire branch of ground states, with zero energy and unbroken SUSY.

corresponding to the N roots of unity. This is in agreement with the Witten index N of pure $SU(N)$ gauge theory. Consider now the massless limit $m^i_j \to 0$. For $F < N$, the vacua (19) all tend to infinity. The theory has no ground state at finite field VEV's. The potential is nonzero in any finite region of field space and slopes to zero at infinity. The massless limit of the theory is therefore not well defined. For $F > N$, by taking $m^i_j \to 0$ in different ways, any value of M^j_i may be attained. The massless theory has an entire moduli space of vacua, parametrized by M^j_i. The $N = F$ case is more subtle, but in this case too, the theory has a moduli space of vacua. Thus the ground states of the massless $SU(N)$ theory with F flavors are drastically different from those of the massive theory. In these examples we explicitly see how zero-energy states can disappear to infinity, or come in from infinity. Again, this is possible because the asymptotic behavior of the potential changes as the mass tends to zero. The theory including mass terms has no flat directions. Asymptotically the potential rises at least quadratically. The massless theory has classical flat directions. Quantum mechanically, they are completely lifted for $F < N$, and the potential asymptotes to zero as a fractional power of the field. For $F \geqslant N$ flat directions remain even quantum mechanically. In any case, adding mass terms changes the asymptotic behavior of the potential.

In the examples above, the massive theory was supersymmetric (with zero-energy states at finite fields VEV's) and the massless theory was either supersymmetric (with a continuum of vacua) or not well defined (with no ground state). It is natural to ask whether there exist vectorlike (parity-conserving) theories that break SUSY as the relevant masses are taken to zero. The answer to this question is affirmative, as we shall see in an explicit example in Sec. VI.A.2. It is useful to understand the general properties of the potential in such a theory. As before, we expect the fully massive theory to have a nonzero Witten index. The only way to obtain supersymmetry breaking as the masses are taken to zero is if the masses change the asymptotic behavior of the potential. Suppose that for any finite value of the mass parameter m, the theory possesses a supersymmetric vacuum at some VEV $v_0(m)$, which moves away to infinity as some of the masses are taken to zero. Clearly, for small finite masses, the directional derivative of the potential with respect to the modulus is negative for large values of $v < v_0(m)$. (The theory may have various minima for finite values of v, but we are interested in the asymptotic behavior of the potential at large v.) In the absence of a phase transition at zero mass, such a directional derivative will remain nonpositive in the limit $m \to 0$. However, there are still two possibilities. First, it is possible that the directional derivative is negative for any finite VEV and only vanishes in the double limit, $m \to 0, v \to \infty$. In such a case the theory does not have a stable vacuum. However, it is also possible that the derivative vanishes in the limit $m \to 0$ for sufficiently large but otherwise arbitrary v. If this is the case, the asymptotic behavior of the potential changes, and it becomes a nonzero constant asymptotically far along the flat direction, so supersymmetry is broken. However, because the potential is flat, running effects cannot be neglected. Indeed, as we shall argue in Sec. VI.A.2, such effects may lift the vacuum degeneracy and determine the true nonsupersymmetric vacuum.

To summarize, pure supersymmetric Yang-Mills theories, as well as vectorlike theories with masses for all matter fields, have a nonzero index and do not break supersymmetry. When some masses are taken to zero, the resulting theories have classical flat directions, and therefore the asymptotic behavior of the potential is different from that of the massive theory. The Witten index may then change discontinuously and differ, if it is well defined, from that of the massive theory.

What about chiral (parity-violating) theories? In such theories, at least some of the matter fields cannot be given mass. Thus these theories cannot be obtained by deforming a massive vectorlike theory, and there is no *a priori* reason to expect, based on existing computations of the Witten index, that these theories are supersymmetric. Indeed, most known examples of supersymmetry breaking are chiral.

B. Global symmetries and supersymmetry breaking

In this section we shall discuss the connection between global symmetries and supersymmetry breaking, which motivates two criteria for supersymmetry breaking. While these are useful guidelines for finding supersymmetry-breaking theories, they are not strict rules, and we shall encounter several exceptions in the following.

Consider first a theory with an exact, nonanomalous global symmetry, and no flat directions. If the global symmetry is spontaneously broken, there is a massless scalar field, the Goldstone boson, with no potential. With unbroken supersymmetry, the Goldstone boson is part of a chiral supermultiplet that contains an additional massless scalar, again with no potential. This scalar describes motions along a flat direction of zero potential. But this contradicts our initial assumption that there are no flat directions. To avoid the contradiction we should drop the assumption of unbroken supersymmetry. This gives a powerful tool for establishing supersymmetry breaking (Affleck, Dine, and Seiberg, 1984b, 1985): If a theory has a spontaneously broken global symmetry and no flat directions, the theory breaks supersymmetry.

We have assumed here that the additional massless scalar corresponds to motions along a *noncompact* flat direction. This is often the case since, in supersymmetric theories, the superpotential is invariant under the complexified global symmetry, with the Goldstone boson corresponding to the imaginary part of the relevant order parameter, and its supersymmetric partner corresponding to the real part of the order parameter.[12]

In general, deciding whether a global symmetry is broken requires detailed knowledge of the potential of the theory, and is at least as hard as determining whether the vacuum energy vanishes. However, if a theory is strongly coupled at the scale at which supersymmetry might be broken, one cannot directly answer either of these questions.[13] Still, in some cases one may argue, based on 't Hooft anomaly-matching conditions ('t Hooft, 1980), that a global symmetry is broken.

If a global symmetry is unbroken in the ground state, then the massless fermions of the low-energy theory should reproduce the global triangle anomalies of the microscopic theory ('t Hooft, 1980). Thus there should be a set of fields, with appropriate charges under the global symmetry, that give a solution to the anomaly-matching conditions. This fact may be used when trying to determine whether a theory confines, and how its global symmetries are realized in the vacuum. For example, if the gauge invariants that can be constructed out of the microscopic fields of the theory saturate the anomaly-matching conditions for some subgroup of the global symmetry of the microscopic theory, it is plausible that the theory confines and that the relevant symmetry subgroup remains unbroken in the vacuum. In contrast, if all possible solutions to the anomaly-matching conditions are very complicated, that is, they require a large set of fields, it is plausible to conclude that the global symmetry is spontaneously broken.

In the case of an R symmetry there is another way to determine whether it is spontaneously broken. In many theories, the scale of supersymmetry breaking is much lower than the strong-coupling scale, so that supersymmetry breaking can be studied in a low-energy effective theory involving chiral superfields only, with all gauge dynamics integrated out. In fact, the low-energy theory is an O'Raifeartaigh-like model (with possibly negative exponents of the fields in the superpotential arising from nonperturbative effects in the microscopic description). In some cases it is easy to see that the origin is excluded from the moduli space, because, for example, the potential diverges there. Then typically some terms appearing in the superpotential obtain VEV's. Since all terms in the superpotential have R charge 2 this implies that R symmetry is broken. Obviously such an argument is not applicable to other global symmetries because the superpotential is necessarily neutral under non-R symmetries. So it is often easier to prove that an R symmetry is broken than to prove that a non-R symmetry is broken. If the theory has no flat directions one can then conclude that supersymmetry is broken.

It is not surprising that R symmetries, which do not commute with supersymmetry, should play a special role in supersymmetry breaking. Let us discuss this role further, following Nelson and Seiberg (1994).

In what follows we shall assume that the gauge dynamics were integrated out. Suppose we have a low-energy theory with a superpotential $W(\{X_i\})$, where X_i are chiral fields and $i=1\ldots n$. For supersymmetry to remain unbroken, the superpotential should be extremal with respect to all fields,

$$\frac{\partial W}{\partial X_i}=0.$$

If the theory has no symmetries, the number of unknowns X_i equals the number of equations. Similarly, if the theory has a global symmetry that commutes with supersymmetry, the number of equations equals the number of unknowns. To see this note that in this case, the superpotential can only depend on chiral field combinations that are invariant under the symmetry. Therefore if there are k symmetry generators, the superpotential depends on $n-k$ invariant quantities [for example, for a $U(1)$ symmetry these could be $X_i/X_1^{q_i/q_1}$, where $i=2,\ldots,n$, and q_i,q_1 are the $U(1)$ charges of X_i,X_1, respectively], while the remaining k fields do not appear in the superpotential. Thus for supersymmetry to remain unbroken the superpotential should be extremal with respect to $n-k$ variables, leading to $n-k$ equations in $n-k$ unknowns. Thus generically there is a solution and supersymmetry is unbroken.

In contrast, suppose the theory has an R symmetry that is spontaneously broken. Then there is a field X with R charge $q\neq 0$, which gets a nonzero VEV. The superpotential then can be written as

$$W=X^{2/q}f(Y_i=X_i^q/X^{q_i}),$$

where q_i is the charge of X_i. For supersymmetry to be unbroken we need

$$\frac{\partial f}{\partial Y_i}=0,$$

and

$$f=0.$$

Thus there is one more equation than unknowns, and generically we do not expect a solution. Roughly speaking, what we mean by "generically" is that the superpotential is a generic function of the fields, that is, it contains all terms allowed by the symmetries. We shall return to this point shortly.

If the extremum of the superpotential were determined by a system of homogeneous linear equations, the above discussion would lead us to conclude that an R symmetry is a necessary condition for supersymmetry breaking, and a spontaneously broken R symmetry is a sufficient condition. While this conclusion generally

[12]The possibility that the low-energy theory is a theory of Goldstone bosons and massless chiral fields such that the supersymmetric scalar partner of any Goldstone is also a Goldstone is ruled out. Such theories can only be coupled to gravity for discrete values of Newton's constant (Bagger and Witten, 1982), and so cannot describe the low-energy behavior of renormalizable gauge theories.

[13]The most obvious example is a theory that does not possess any adjustable parameters and has only one scale, like an $SU(5)$ model of Sec. III.D.

holds for theories in which the superpotential is a generic function consistent with all the symmetries, there may be exceptions to this rule. This is because the F-flatness conditions are given by a system of nonlinear equations that may contain negative powers of fields (arising from the dynamical superpotential) as well as terms independent of fields (arising from linear terms, either generated dynamically or included in the tree-level superpotential). Such a system of equations is not guaranteed to have solutions. In fact, we have already argued in Sec. II.C.1 that we can add explicit R-symmetry-breaking terms to an O'Raifeartaigh model without restoring supersymmetry. Later we shall encounter other examples of supersymmetry-breaking models without R symmetry.

Let us make a bit more precise what we mean by a generic superpotential. The superpotential contains two parts. One is generated dynamically, and certainly does not contain all terms allowed by the symmetries (Seiberg, 1993). In particular, such terms could involve arbitrarily large negative powers of the fields. The other part is the classical superpotential, which is a polynomial (of some degree d) in the fields, that preserves some global symmetry. Here what we mean by "generic" is that no term with dimension smaller than or equal to d that is allowed by this global symmetry was omitted from the superpotential. On the other hand, the tree-level superpotential can still be considered generic if the operators with dimension higher than d are omitted. Indeed, in a renormalizable Lagrangian with a stable vacuum we do not expect Planck-scale VEV's. The analysis by Nelson and Seiberg (1994) shows that the inclusion of nonrenormalizable operators can only produce additional minima with Planck-scale VEV's, that is in a region of field space where our approximation of global supersymmetry is not sufficient anyway. On the other hand, in models in which a stable vacuum appears only after the inclusion of nonrenormalizable terms of dimension d, the typical expectation values will depend on the Planck scale (or other large scale) but often will remain much smaller than it. As long as the expectation values are small compared to the Planck scale, these minima will remain stable local minima even if operators of dimension higher than d are added.

We shall encounter several examples of theories that break supersymmetry even though they do not possess an R symmetry. In some cases, while the microscopic theory does not have an R symmetry, there is an effective, spontaneously broken R symmetry in the low-energy theory. In other cases, there is not even an effective R symmetry. In one example, SUSY will be broken even though the tree-level superpotential is generic and does not preserve any R symmetry, and there is no effective R symmetry.

C. Gaugino condensation

Let us now introduce a criterion for SUSY breaking that is based on gaugino condensation (Meurice and Veneziano, 1984; Amati, Rossi, and Veneziano, 1985). Suppose that a certain chiral superfield (or a linear combination of chiral superfields) does not appear in the superpotential, yet all the moduli are stabilized. In such a case the Konishi anomaly (Clark, Piguet, and Sibold, 1979; Konishi, 1984) implies

$$\bar{\mathcal{D}}^2(\bar{\Phi} e^V \Phi) \sim \mathrm{Tr} W^2, \tag{20}$$

where $\mathcal{D}$ is a supersymmetric covariant derivative [see Appendix Sec. 1], Φ is a chiral superfield, and V is the vector superfield. It is instructive to consider Eq. (20) in component form. It is given by an anomalous commutator with the supersymmetry generator Q,

$$\{Q, \psi_\Phi \phi\} \sim \lambda\lambda, \tag{21}$$

where ψ_Φ and ϕ are the fermionic and scalar components of Φ, respectively, and λ is the gaugino. From this equation we see that the vacuum energy is proportional to the lowest component of W^2, that is, to $\langle \mathrm{Tr}\lambda\lambda \rangle$. Therefore if the gaugino condensate forms one can conclude that supersymmetry is broken. Note that if the fields Φ and $\bar{\Phi}$ appear in the superpotential, the right-hand side of Eqs. (20) and (21) can be modified and the gaugino condensate may form without violating supersymmetry. For example, if there is a superpotential mass term, Eq. (21) becomes

$$\{Q, \psi_\Phi \phi\} = m \bar{\phi} \phi + \frac{1}{32\pi^2} \lambda\lambda. \tag{22}$$

The latter equation is compatible with supersymmetry and determines the VEV's of the scalar fields in terms of the gaugino condensate.

This criterion is related to the global symmetry arguments of Affleck, Dine, and Seiberg (1985), since if a gaugino condensate develops in a theory possessing an R symmetry, this symmetry is spontaneously broken. In the absence of flat directions, the Affleck-Dine-Seiberg argument leads to the conclusion that SUSY is broken.

D. Examples

We shall now demonstrate the techniques described in Secs. III.B and III.C by a few examples.

1. Spontaneously broken global symmetry: the $SU(5)$ model

Consider an $SU(5)$ gauge theory with one antisymmetric tensor (**10**) A, and one antifundamental $\bar{F}$ (Affleck, Dine, and Seiberg, 1984b; Meurice and Veneziano, 1984). The global symmetry of the theory is $U(1) \times U(1)_R$, under which we can take the charges of the fields to be $A(1,1)$ and $\bar{F}(-3,-9)$. No gauge invariants can be made out of A and $\bar{F}$. Thus there are no flat directions, and classically the theory has a unique vacuum at the origin. The theory is strongly coupled near the origin, and we have no way to determine the behavior of the quantum theory. Because there are no chiral gauge invariants, the theory does not admit any superpotential. If supersymmetry is broken, the only possible scale for its breaking is the strong-coupling scale of $SU(5)$.

Following Affleck, Dine, and Seiberg (1984b) we shall now use R symmetry to argue that supersymmetry is indeed broken [in the following subsection we shall consider a gaugino condensation argument put forth by Meurice and Veneziano (1984)]. Assuming the theory confines, the massless gauge-invariant fermions of the confined theory should reproduce the triangle anomalies generated in the microscopic theory. Affleck, Dine, and Seiberg (1984b) showed that the minimal number of fermions required, with $U(1)$ and $U(1)_R$ charges under 50, is five. This makes it quite implausible that the full global symmetry remains unbroken. But if the global symmetry is spontaneously broken and there are no flat directions, the theory breaks supersymmetry by the arguments of Sec. III.B.

2. Gaugino condensation: the $SU(5)$ model

We now would like to apply the gaugino condensate argument to the $SU(5)$ model discussed in the previous subsection. Since the gaugino condensate serves as an order parameter for supersymmetry breaking, we need to establish that it is nonzero. To do that we follow Meurice and Veneziano (1984) and consider the correlation function

$$\Pi(x,y,z)=\langle T(\lambda^2(x),\lambda^2(y),\chi(z))\rangle, \tag{23}$$

where

$$\chi=\epsilon^{abcde}\lambda_a^{a'}\lambda_{a'}^{b'}\overline{F}^{c'}A_{b'c'}A_{bc}A_{de}, \tag{24}$$

and $\overline{F}$ and A are scalar components of $\overline{5}$ and 10, respectively.

In the limit $x,y,z \ll \Lambda^{-1}$ one can show that the condensate $\Pi \sim \Lambda^{13}$. Then one can take the limit of large x, y, and z and, using cluster decomposition properties, argue that $\langle\lambda\lambda\rangle \neq 0$. As a result supersymmetry must be broken.

3. R symmetry and SUSY breaking

In the example of the $SU(5)$ model we could not explicitly verify the spontaneous breaking of the global symmetry, and had to rely on the use of 't Hooft anomaly-matching conditions to establish supersymmetry breaking. Now we consider examples in which we can explicitly show that R symmetry is spontaneously broken, and use that to establish supersymmetry breaking. While we shall be content to consider only models with tree-level breaking, similar arguments can be applied to a number of dynamical models discussed later.

Consider first the O'Raifeartaigh model of Eq. (10). There is an R symmetry under which ϕ_1,ϕ_2 have R charge 2, and ϕ has R charge zero. There is also a discrete Z_2 symmetry under which ϕ_1 is neutral, while ϕ_2 and ϕ change sign. The superpotential is the generic one consistent with the symmetries and supersymmetry is broken. We can add to the superpotential ϕ to some even power so that the R symmetry is broken. Still, supersymmetry is broken. However, the superpotential is no longer generic, and we can add the term ϕ_1^2 without breaking the remaining Z_2 symmetry. This term will restore supersymmetry.

If we do not wish to impose a discrete symmetry, the most general superpotential is

$$W=\sum_{i=1}^{2} M_i^2\phi_i+m_i\phi_i\phi+\lambda_i\phi_i\phi^2. \tag{25}$$

The model with this superpotential still breaks supersymmetry unless $M_1/M_2=m_1/m_2=\lambda_1/\lambda_2$. Note that if the parameters are chosen so that this latter equality is satisfied, the superpotential is independent of one linear combination of the fields, $\tilde{\phi}=(m_2\phi_1-m_1\phi_2)/(m_1^2+m_2^2)$, and thus is not generic.

As another example, consider the superpotential

$$W=P_1X_1+P_2X_2+A(X_1X_2-\Lambda^2)+\alpha X_1X_2. \tag{26}$$

For $\alpha=0$ the theory has a $U(1)\times U(1)_R$ symmetry with $A(0,2)$, $X_1(1,0)$, $X_2(-1,0)$, $P_1(-1,2)$, $P_2(1,2)$. Supersymmetry is broken whether or not $\alpha=0$. For $\alpha\neq 0$ the R symmetry is broken and the potential is no longer generic. Terms such as P_1P_2, A^2, which respect the $U(1)$, could restore supersymmetry. This example is a simple version of the low-energy theory of the example we shall study in Sec. VI.C.

4. Generalizations of the $SU(5)$ model

To conclude our presentation of the basic examples of supersymmetry breaking we construct an infinite class of models generalizing the $SU(5)$ model discussed earlier (Meurice and Veneziano, 1984; Affleck, Dine, and Seiberg, 1985). These models have an $SU(2N+1)$ gauge group, with matter transforming as an antisymmetric tensor A and $2N-3$ antifundamentals $\overline{F}_i$, $i=1,\ldots,2N-3$ (Affleck, Dine, and Seiberg, 1985). For $N>2$, all these models have D-flat directions, which are all lifted by the most general R-symmetry-preserving superpotential

$$W=\lambda_{ij}A\overline{F}_i\overline{F}_j, \tag{27}$$

for the appropriate choice of the matrix of coupling constants.

First, note that for small superpotential coupling the model possesses almost flat directions, and as a result, part of the dynamics can be analyzed directly. Yet the scale of the unbroken gauge dynamics in the low-energy theory is comparable to the scale of SUSY breaking and the model is not calculable. To analyze supersymmetry breaking it is convenient to start from the theory without the tree-level superpotential. In this case the models have classical flat directions along which the effective theory reduces to the $SU(5)$ theory with an antisymmetric and an antifundamental as well as the light modulus parametrizing the flat direction. We already know that in this effective theory SUSY is broken with a vacuum energy $\sim\Lambda_L^4$. The low-energy scale depends on the VEV of the modulus as in Eq. (A21), and thus there is a potential

$$V(\phi)\sim\Lambda_L^4\sim\phi^{-(4/13)(4N-8)}. \tag{28}$$

When small Yukawa couplings are turned on, the flat directions are stabilized by the balance between the tree-level contribution of order $\lambda^2\phi^4$ and the dynamical potential (28). The minimum of the potential then occurs for

$$\phi\sim\lambda^{-(13/2)(4N+5)}\Lambda \quad \text{with} \quad E_{vac}\sim\lambda^{8[(N-2)/(4N+5)]}\Lambda^4. \tag{29}$$

We found that the potential is stabilized at finite value of the modulus and the effective low-energy description is given in terms of the SUSY-breaking theory. Thus supersymmetry must be broken in the full theory. We also note that at the minimum, R symmetry is broken, giving us additional evidence for supersymmetry breaking.

Before concluding this section we comment on the analogous theories with $SU(2N)$ gauge groups (Affleck, Dine, and Seiberg, 1985). In this case the tree-level superpotential allowed by symmetries (including R symmetry) does not lift all the classical flat directions. On the other hand, a dynamical superpotential is generated, pushing the theory away from the origin. Thus the model does not have a stable ground state. It is possible to lift all flat directions by adding R-symmetry-breaking terms to the tree-level superpotential. While lifting the flat directions, these terms lead to the appearance of a stable supersymmetric vacuum.

IV. DIRECT ANALYSIS: CALCULABLE MODELS

As we have seen, SUSY breaking is directly related to the zero-energy properties of the theory, namely, the ground-state energy and the appearance of the massless Goldstino. Fortunately, then, to establish SUSY breaking, we only need to understand the low-energy behavior of the theory in question. As we shall see, many models can be described, in certain regions of the moduli space, by a low-energy O'Raifeartaigh-like effective theory. The question of whether SUSY is broken simply amounts to the question of whether all F terms can vanish simultaneously. The tricky part, of course, is obtaining the correct low-energy theory. This involves a number of related ingredients: establishing the correct degrees of freedom, and determining the superpotential and the Kähler potential. In many cases, holomorphy and symmetries indeed determine the superpotential, but the same is not true for the Kähler potential. However, if all we care about is whether the energy vanishes or not, it suffices to know that the Kähler potential is not singular as a function of the fields that make up the low-energy theory. This, in turn, is related to whether or or not we chose the correct degrees of freedom of our low-energy theory.

We shall divide our discussion into two parts. In this section we shall consider weakly coupled theories. By tuning some parameters in the superpotential to be very small, we can typically drive some of the fields to large expectation values, with the gauge symmetry completely broken, so that all gauge bosons are very heavy. We can then neglect gauge interactions and write down, as advertised, a low-energy O'Raifeartaigh-type model. Since the theory is weakly coupled, we shall also be able to calculate the Kähler potential and thus completely determine the low-energy theory including the ground-state energy, the composition of the Goldstino, and the masses of low-lying states.

But we can also, in many cases, analyze the theory near the origin in field space, where the theory is very strongly coupled. If the theory confines, then below the confinement scale, we are again left with an O'Raifeartaigh-type model. In Sec. V, we shall see examples of this kind. In some cases, though we shall not be able to analyze the theory in question, we shall be able to analyze a dual theory that, as we just described, undergoes confinement. In all these cases, however, we shall not be able to calculate the Kähler potential. Thus, while we shall ascertain that SUSY is broken, the details of the low-energy theory, and in particular the vacuum energy, the unbroken global symmetry, and the masses of low-lying states, remain unknown.

In the examples we encounter, supersymmetry is broken due to a variety of effects. Still, it is always the consequence of the interplay between, on the one hand, a tree-level superpotential, which gives rise to a nonzero potential everywhere except at the origin in field space, and, on the other hand, nonperturbative effects, either in the form of instantons or gaugino condensation, that generate a potential that is nonzero at the origin.

A. The 3–2 model

Probably the simplest model of dynamical supersymmetry breaking is the 3–2 model of Affleck, Dine, and Seiberg (1985). Here we shall choose the parameters of the model so that the low-energy effective theory is weakly coupled, and thus the model is calculable. In this weakly coupled regime, the main ingredient leading to supersymmetry breaking in the model is an instanton-generated superpotential. In Sec. VI.A.2, we shall analyze the same model in a strongly coupled regime in which SUSY is broken through the quantum deformation of the moduli space (which is again the result of instanton effects). We shall also discuss numerous generalizations of the 3–2 model.

The model is based on an $SU(3)\times SU(2)$ gauge group with the following matter content [we also show charges under the global $U(1)\times U(1)_R$ symmetry of the model]:

	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
Q	3	2	1/3	1
$\bar{u}$	$\bar{3}$	1	−4/3	−8
$\bar{d}$	$\bar{3}$	1	2/3	4
L	1	2	−1	−3.

(30)

Using methods described in Appendix Sec. 2 we can easily determine the classical moduli space of the model. In the absence of a tree-level superpotential, it is given by

$$Q_{if}=\overline{Q}^{if}=\begin{pmatrix} a & 0 \\ 0 & b \\ 0 & 0 \end{pmatrix},\quad L=(0,\sqrt{a^2-b^2}), \tag{31}$$

where $\overline{Q}=(\overline{u},\overline{d})$, and i and f are $SU(3)$ color and flavor indices, respectively. For generic values of a and b the gauge group is completely broken, and there are three light chiral fields. While it is not difficult to diagonalize the mass matrix and find the light degrees of freedom, it is convenient to use an alternative parametrization of the classical moduli space in terms of the composite operators

$$X_1=Q_{i\alpha}\overline{d}^i L_\beta \epsilon^{\alpha\beta},\quad X_2=Q_{i\alpha}\overline{u}^i L_\beta \epsilon^{\alpha\beta},\quad Y=\det(Q\overline{Q}), \tag{32}$$

where Greek indices denote $SU(2)$ gauge indices. The most general renormalizable superpotential that preserves the $U(1)\times U(1)_R$ global symmetry is

$$W_{tree}=\lambda Q\overline{d}L=\lambda X_1. \tag{33}$$

This superpotential lifts all classical flat directions.

Let us now analyze the quantum theory. To do that we choose $\lambda^2 \ll g_2^2 \ll g_3^2$. The former inequality implies that the minimum of the scalar potential lies very close to the D-flat direction. To simplify the analysis we shall, in fact, impose D-flatness conditions.[14] The latter inequality guarantees that effects due to the $SU(2)$ nonperturbative dynamics are exponentially suppressed compared to those due to the $SU(3)$ dynamics. In particular, at scales below Λ_3 and much bigger than Λ_2, the $SU(2)$ gauge theory is weakly coupled and its dynamics can be neglected. $SU(3)$, on the other hand, confines, so we can write down an effective theory in terms of its mesons $M_\alpha^f=Q_\alpha\cdot\overline{Q}^f$, subject to the nonperturbative superpotential

$$W_{np}=\frac{2\Lambda_3^7}{\det(Q\overline{Q})}, \tag{34}$$

which is generated by an $SU(3)$ instanton. The reader may now note that in this effective theory, $SU(2)$ appears anomalous; it has three doublets, $M^{f=1,2}$, L. However, this is not too surprising, because the superpotential (34) drives the fields Q, $\overline{Q}$ away from the origin, so that $SU(2)$ is broken everywhere. Indeed, as discussed in the Appendix, an $SU(3)$ gauge theory with two flavors has no moduli space.

In fact, we can already conclude that supersymmetry is broken. Since the superpotential (34) drives the fields Q, $\overline{Q}$ away from the origin, the R symmetry of the model is spontaneously broken. Combining this with the fact that the model has no flat directions, we see, based on the arguments of Sec. III.B, that the theory breaks supersymmetry.

Let us go on to analyze supersymmetry breaking in the theory in more detail. As we saw above, in the absence of a tree-level superpotential, the theory has a "runaway" vacuum with $Q,\overline{Q}\to\infty$. This is precisely what allows us to find a calculable minimum in this model. The tree-level superpotential lifts all classical flat directions. Any minimum would result from a balance between W_{tree}, which rises at infinity, and W_{np}, which is singular at the origin. If we choose λ to be very small, the minimum would occur for large Q, $\overline{Q}$ VEV's, so that the gauge symmetry is completely broken, and gauge interactions are negligible. Thus for a small $\lambda \ll 1$ we can conclude that the light degrees of freedom can still be described by the gauge-invariant operators X_1, X_2, and Y (in the following we shall see additional arguments supporting the fact that X_1, X_2, and Y are indeed the appropriate degrees of freedom). Furthermore, the superpotential in this limit is given by

$$W=\frac{2\Lambda_3^7}{Y}+\lambda X_1. \tag{35}$$

We now see explicitly that supersymmetry is broken, since $W_{X_1}=\lambda\neq 0$. Note that this conclusion depends crucially on the fact that we have the full list of massless fields. In general, care should be taken in drawing a conclusion about supersymmetry breaking based on the presence of a linear term for a composite field in the superpotential. If at some special points additional fields become massless, the Kähler metric is singular, and the potential $V=W_i K_{ij*}^{-1} W_{j*}$ may vanish even if all W_i are nonzero. Moreover, if the theory has classical flat directions, it is possible that the Kähler potential (written in terms of composites) has singularities at the boundaries of moduli space, with some fields going to infinity and possibly others to the origin. As a result supersymmetry may be restored at the origin.

As we saw above, for the choice of parameters $\Lambda_3 \gg \Lambda_2$, $\lambda \ll 1$, the theory is weakly coupled. The Kähler potential of the low-energy theory is therefore the canonical Kähler potential in terms of the elementary fields Q, $\overline{u}$, $\overline{d}$, and L, projected on the D-flat direction. In terms of the gauge-invariant operators it is given by (Affleck, Dine, and Seiberg, 1985; Bagger, Poppitz, and Randall, 1994)

$$K=24\frac{A+Bx}{x^2}, \tag{36}$$

where $A=1/2(X_1^\dagger X_1+X_2^\dagger X_2)$, $B=1/3\sqrt{Y^\dagger Y}$, and

$$x\equiv 4\sqrt{B}\cos\left(\frac{1}{3}\arccos\frac{A}{B^{3/2}}\right). \tag{37}$$

We therefore have all the ingredients of the low-energy theory, including the superpotential and the Kähler po-

[14] However, in this and in other calculable models it is easy to take D-term corrections to the scalar potential into account in numerical calculations.

tential. This allows us to explicitly minimize the scalar potential. For details of the analysis, we refer the reader to Affleck, Dine, and Seiberg (1985) and Bagger, Poppitz, and Randall (1994). Here we just give some qualitative results. It is possible to work in terms of either the elementary fields or the gauge-invariant fields. Simple dimensional analysis shows that the minimum occurs for elementary field VEV's $v \sim \lambda^{-1/7}\Lambda_3$, and that the vacuum energy is of order $\lambda^{5/14}\Lambda_3$. Explicitly we find that at the minimum $X_2=0$, so that the global $U(1)$ symmetry is unbroken (not surprising, as points of higher symmetry are extremal). The massless spectrum contains the Goldstino, a massless fermion of $U(1)$ charge -2, which saturates the $U(1)$ anomaly, as well as a massless scalar that is the Goldstone boson of the broken R symmetry (usually known as the R axion).

This concludes our discussion of the calculable minimum of the 3–2 model, but let us make a few more comments.

First, the above analysis of supersymmetry breaking did not involve the strong dynamics of $SU(2)$. It is interesting to see therefore the effect of turning off the $SU(2)$ gauge interactions. We then have an $SU(3)$ gauge theory with two flavors, plus two singlets $L_{\alpha=1,2}$, and with the superpotential (33). Classically, this superpotential leaves a set of flat directions. Up to global symmetry transformations [which now include an $SU(2)$ global symmetry], these flat directions are parametrized by L_1 and $Q_2\overline{u}$. The nonperturbative dynamics lead to runaway towards a supersymmetric vacuum at infinity along this direction.[15] This dangerous direction is no longer D flat when the $SU(2)$ is turned on.

Second, even though so far we have concentrated on the limit $\Lambda_3 \gg \Lambda_2$, it is possible to derive the exact superpotential of the 3–2 model for any choice of couplings, and to use it to establish supersymmetry breaking. Note first that the complete list of independent gauge invariants is X_1, X_2, Y, and $Z \equiv Q^3 L$ (we suppress all indices). The latter vanishes classically, or more precisely in the limit $\Lambda_2 \to 0$. In the limit $\Lambda_3 \gg \Lambda_2$, $\lambda=0$, the superpotential is given by Eq. (34). In the limit $\Lambda_2 \gg \Lambda_3$, $\lambda=0$, the theory is an $SU(2)$ gauge theory with two flavors and a quantum constraint that can be implemented in the superpotential using a Lagrange multiplier A, as $A(Z-\Lambda_2^4)$.[16] The most general superpotential that respects all the symmetries of the theory is

$$W=\frac{2\Lambda_3^7}{Y}f(t,z')+A(Z-\Lambda_2^4)g(t,z'), \tag{38}$$

where $t \equiv \lambda X_1 Y/\Lambda_3^7$, $z' \equiv Z/\Lambda_2^4$ are the only dimensionless field combinations neutral under all symmetries. In the limit Λ_3, $\lambda \to 0$ (for which any value of t can be attained) we find an $SU(2)$ theory with four doublets (and a set of noninteracting singlet fields). Since the exact superpotential for this theory is known we find $g(t,z') \equiv 1$. Similarly, in the limit $\Lambda_2 \to 0$ we find $f(t,z')=1+t$. The exact superpotential is then

$$W=\frac{2\Lambda_3^7}{Y}+A(Z-\Lambda_2^4)+\lambda X_1 . \tag{39}$$

It is clear from this superpotential that Z obtains a mass. To see if the mass is large we need to know the Kähler potential for this field. In the limit of weakly coupled $SU(2)$ and $\lambda \ll 1$ both gauge groups are strongly broken, and the Kähler potential is close to the classical one. Since classically Z vanishes, the projection of the classical Kähler potential on it also vanishes [see Eq. (36)]. For small but nonvanishing Λ_2 the Kähler potential of Z is suppressed by some function of Λ_2/v. Restoring the canonical normalization for the kinetic term, we find that the mass of Z is enhanced by the inverse of this function. We were therefore justified in keeping only X_1, X_2, and Y as the light fields.

Looking at Eq. (39), we can conclude that supersymmetry is broken for any choice of the parameters of the theory. However, unlike in the limit $\Lambda_3 \gg \Lambda_2$, in general the theory is strongly coupled, and we have no control over the Kähler potential. Therefore while we may be able to estimate the scale of supersymmetry breaking we cannot say anything about the vacuum, e.g., we cannot establish the pattern of symmetry breaking.

In the above, we first showed that supersymmetry is broken in a specific limit, and later realized that it is always broken. Indeed, we do not expect a theory to break supersymmetry for some choice of parameters and to develop a supersymmetric minimum for other choices. The reason is that no phase transitions are expected to occur in supersymmetric theories as their parameters are varied (Intriligator and Seiberg, 1994; Seiberg and Witten, 1994a, 1994b). If a theory is supersymmetric for some choice of parameters, it remains supersymmetric for any choice. This allows us to establish supersymmetry breaking by considering a convenient limit. In some theories, including the 3–2 model, we can establish supersymmetry breaking in different limits. The details of supersymmetry breaking, such as the vacuum energy and the source of the breaking, may be very different in the different limits.

Finally, another interesting feature of the 3–2 model is the possibility of gauging the global $U(1)$ symmetry, provided that a new field E^+, with $U(1)$ charge $+2$, is added to cancel the $U(1)^3$ anomaly. With the addition of this field, the analysis of dynamical supersymmetry breaking does not change, since new classical flat directions do not appear, nor are new tree-level superpotential terms allowed. This possibility proved to be useful in phenomenological model building (Dine, Nelson, and Shirman, 1995). For our purposes, however, the importance of this $U(1)$ is in the observation (Dine *et al.*, 1996) that with the addition of E^+, the matter content of the model falls into complete $SU(5)$ representations, and in fact they are the same representations that are required for DSB in the $SU(5)$ model of Sec. III.D. In the following section we shall discuss another simple and

[15]We shall discuss the quantum behavior of SUSY QCD coupled to singlet fields in detail in Sec. VI.A.1.

[16]Actually, with $SU(3)$ turned off, Z in this superpotential should be understood as the Pfaffian of the $SU(2)$ mesons.

calculable model of DSB based on an $SU(4)\times U(1)$ gauge group and again see that the matter fields form complete $SU(5)$ representations. In Sec. VII.A we shall introduce a method of constructing large classes of DSB models based on this observation. This method will lead us to an infinite class of models generalizing the 3–2 model. Many other calculable and noncalculable generalizations will be discussed in Sec. VII.

B. The 4–1 model

Another example of a calculable DSB model is the 4–1 model constructed by Dine *et al.* (1996) and Poppitz and Trivedi (1996). Consider an $SU(4)\times U(1)$ gauge group with matter transforming as an antisymmetric tensor of $SU(4)$, A_2 [where the subscript indicates $U(1)$ charge], a fundamental F_{-3}, an antifundamental $\overline{F}_{-1}$, and an $SU(4)$ singlet S_4.

For a range of parameters of the model, the scale of the gauge dynamics will be below the SUSY-breaking scale. Thus one could analyze supersymmetry breaking in terms of the microscopic variables (Dine *et al.*, 1996). Indeed, in terms of the microscopic variables the Kähler potential of the light degrees of freedom is nearly canonical and it is easy to calculate the vacuum energy. We shall, however, analyze this model in terms of the gauge-invariant polynomials. Again for convenience we shall work in a regime in which the couplings are arranged hierarchically, with the superpotential Yukawa coupling the smallest, and with the $U(1)$ coupling weak at the $SU(4)$ strong-coupling scale.[17] The $SU(4)$ moduli space is given by the fields $M=F\overline{F}$, $X=\mathrm{Pf}A$, and S. The model possesses a nonanomalous R symmetry, and the unique superpotential allowed by the symmetries is

$$W=\frac{\Lambda_4^5}{\sqrt{MX}}+\lambda\, S\, M. \tag{40}$$

The tree-level term in the superpotential lifts all classical flat directions [note that one more condition on the $SU(4)$ moduli is imposed by the $U(1)$ D term]. Due to the nonperturbative superpotential the vacuum cannot lie in the origin of the moduli space of the theory. As a result, the R symmetry is spontaneously broken at the minimum of the potential and supersymmetry is broken.

Let us argue that the nonperturbative term in the superpotential (40) is indeed generated. We would also like to establish that the model is calculable, namely, that for some choices of parameters, the corrections to the classical Kähler potential are small near the minimum. To this end, neglect the tree-level superpotential and consider a region of the classical moduli space with $M,S^2\gg X$. In this region the gauge group is broken down to an $SU(3)$ subgroup. Apart from the light modulus that controls the scale of the unbroken gauge group there is one $SU(3)$ flavor in the fundamental representation coming from the components of A. In this effective theory the nonperturbative superpotential is generated by gaugino condensation

$$W=\frac{\Lambda_3^4}{\sqrt{q\overline{q}}}, \tag{41}$$

where q and $\overline{q}$ denote light $SU(3)$ fields, and Λ_3^4 is an $SU(3)$ scale. Using the scale-matching condition

$$\Lambda_3^4=\frac{\Lambda_4^5}{\sqrt{M}}, \tag{42}$$

we easily recognize the superpotential (40).

Furthermore we see that the effective $SU(3)$ also possesses a flat direction along which q and $\overline{q}$ acquire VEV's and the gauge group is broken down to $SU(2)$. The strong scale of this $SU(2)$ tends to zero along the flat direction. While the tree-level superpotential stabilizes the theory at finite VEV's, the corrections to the classical scalar potential that scale as Λ_2/v (where v is the typical VEV) are negligible for sufficiently small λ and therefore the model is calculable. The vacuum energy in this model was explicitly calculated by Dine *et al.* (1996).

It is worth noting that one can add an R-symmetry-breaking (and nonrenormalizable) term, $M\mathrm{Pf}A$, to the superpotential (40) (Poppitz and Trivedi, 1996). We shall discuss DSB models without R symmetry in more detail in Sec. VI.C.

V. DIRECT ANALYSIS: STRONGLY COUPLED THEORIES

A. Supersymmetry breaking through confinement

In previous sections we have seen that, at the classical level, supersymmetric gauge theories without explicit mass terms possess a zero-energy minimum at least at the origin of field space. Classical tree-level superpotentials may lift the moduli space, but the supersymmetric vacuum at the origin survives. In traditional calculable models of DSB (such as the 3–2 model) the vacuum at the origin is lifted due to a dynamical superpotential generated by nonperturbative effects. On the other hand, in the $SU(5)$ model, no superpotential can be generated, and supersymmetry is broken by the confining dynamics. Unfortunately the low-energy spectrum of the $SU(5)$ model is not known, and thus the main arguments for DSB are based on the complexity of the solutions to the 't Hooft anomaly-matching conditions. It would be very useful to investigate a model in which supersymmetry breaking is generated by the confining dynamics, with a known low-energy spectrum. In fact a very simple and instructive model of this type was constructed by Intriligator, Seiberg, and Shenker (1995). This model clearly illustrates the fact that a crucial ingredient in studying supersymmetry breaking is the knowledge of the correct degrees of freedom of the low-energy theory. Supersymmetry breaking in this theory

[17]Since both Yukawa and $U(1)$ couplings become weaker in the infrared we can just choose them to be sufficiently small in the ultraviolet.

hinges on whether the theory confines or has an interacting Coulomb phase at the origin. It seems very plausible that the theory indeed confines and that supersymmetry is broken as a result.

The model is based on an $SU(2)$ gauge theory with a single matter field $q_{\alpha\beta\gamma}$ in a three-index symmetric representation. The model is chiral, since the quadratic invariant q^2 vanishes by the Bose statistics of the superfields. It also possesses an R symmetry under which q has the charge 3/5. Moreover, the model is asymptotically free, and thus nontrivial infrared dynamics may lift the supersymmetric vacuum at the origin of field space. It is therefore a candidate model of DSB according to traditional criteria for supersymmetry breaking.

The only nontrivial gauge-invariant polynomial that can be constructed out of q is $u=q^4$ (with appropriate contraction of indices). This composite parametrizes the only flat direction of the theory along which the gauge group is completely broken. R symmetry and holomorphy restrict any effective superpotential to be of the form $W=a\Lambda^{-1/3}u^{5/6}$, where Λ is the dynamical scale of the theory. This superpotential, however, does not have a sensible behavior as $\Lambda\to 0$, since for large u/Λ^4 the moduli space should be close to the classical one with $W\to 0$. Therefore $a=0$. This means that the quantum theory also has a moduli space of degenerate vacua. The moduli space may be lifted by the tree-level nonrenormalizable superpotential

$$W=\frac{\lambda}{M}u, \tag{43}$$

where λ is a constant of order 1. In the presence of the nonrenormalizable term, the model can be thought of as a low-energy effective description of a more fundamental theory, which is valid below the scale M. Choosing $\Lambda \ll M$, there is a region of moduli space, with $\Lambda^4 \ll u \ll M^4$, in which the gauge dynamics are weak and we have a good description of the physics in terms of an effective theory of chiral superfields.

In the presence of the nonrenormalizable term, holomorphy and symmetries restrict the exact superpotential to be

$$W=\frac{\lambda}{M}uf(t=\Lambda^2 u/M^6), \tag{44}$$

where the function f is given by the sum of instanton contributions. In the allowed region, $|t|\ll 1$, $f\approx 1$ and we can use the classical superpotential.

Naively, the linear superpotential for u leads to supersymmetry breaking since $F_u\neq 0$. One should remember, however, that as u is a composite field, its Kähler potential may be quite complicated. In particular, the Kähler potential may be singular at some points in the moduli space, potentially leading to supersymmetry restoration.

To determine the behavior of the Kähler potential, consider first the theory for large expectation values of u. In this regime the description of the model should be semiclassical and thus the Kähler potential scales as

$$\mathcal{K}\sim Q^\dagger Q\sim (u^\dagger u)^{1/4}. \tag{45}$$

Indeed this Kähler potential is singular at $u=0$. The singularity reflects the fact that at $u=0$ the gauge bosons become massless and must be included in the effective description. There are two plausible alternatives for the nature of the singularity in the quantum theory. It is possible that the theory is in a non-Abelian Coulomb phase. On the other hand, it is possible that the singularity is smoothed out quantum mechanically. Intriligator, Seiberg, and Shenker (1995) argued that this latter option is probably realized since the massless composite field u satisfies the 't Hooft anomaly-matching conditions, which is quite nontrivial.[18] We shall assume that this is indeed the case. Then R symmetry, smooth behavior near the origin, and semiclassical behavior at infinity imply that the Kähler potential is a (smooth) function of $u^\dagger u/|\Lambda|^8$ satisfying

$$K=|\Lambda|^2 k(u^\dagger u/|\Lambda|^8)\sim\begin{cases} u^\dagger u/|\Lambda|^6, & u^\dagger u\ll\Lambda^8\\ (u^\dagger u)^{1/4}, & u^\dagger u\gg\Lambda^8.\end{cases} \tag{46}$$

Combining this form of the Kähler potential with the superpotential of Eq. (44), with $f\equiv 1$ we find that the scalar potential

$$V=(K_{u^\dagger u})^{-1}|W_u|^2=(K_{u^\dagger u})^{-1}\left|\frac{\lambda}{M}\right|^2 \tag{47}$$

necessarily breaks supersymmetry with a vacuum energy of order

$$E\sim\frac{|\Lambda|^6}{M^2}. \tag{48}$$

At this point we should comment on several other interesting properties of the model. Before adding the tree-level superpotential (43), the effective description of the confined theory in terms of the u modulus possesses an accidental global $U(1)$ symmetry (as the Kähler potential does not depend on the phase of u). This $U(1)$ is anomalous in terms of the elementary degrees of freedom. The tree-level superpotential explicitly breaks the R symmetry of the model, as well as the accidental $U(1)$. However, in the low-energy description there is an effective R symmetry that is a combination of the $U(1)_R$ and accidental $U(1)$ symmetries. Since R symmetry in the macroscopic description is explicitly broken by the tree-level superpotential, higher-order terms can generically correct Eq. (43). These terms explicitly violate the effective R symmetry of the low-energy description. According to our analysis in Sec. III.B this leads to the appearance of supersymmetric vacua, but these vacua will lie outside the region of validity $|u|<M^4$ of our analysis, and the nonsupersymmetric minimum will remain a (metastable) local minimum of the potential.

[18]See, however, Brodie, Cho, and Intriligator (1998) for a class of theories in which the existence of simple solutions to the anomaly-matching conditions suggests that the theories confine, yet the theories in fact do not confine.

B. Establishing supersymmetry breaking through a dual theory

In this section we shall encounter a class of theories, with $SU(N)\times SU(N-2)$ gauge symmetry, that break supersymmetry for odd N. By directly studying these theories, we can show that they have calculable, supersymmetry-breaking minima for a certain choice of parameters. But we cannot show that there is no supersymmetric minimum, simply because we cannot analyze the low-energy theory in all regions of the moduli space. However, we shall be able to construct a Seiberg dual of the original theory that can be reliably analyzed at low energy. As we shall see, the dual theory breaks supersymmetry. We can then conclude that the original theory breaks supersymmetry as well. The reason is that at least as long as supersymmetry is unbroken, the two duals should have the same physics at zero energy. It is therefore impossible for one of them to be supersymmetric, with a vacuum at zero energy, and for the other one to break supersymmetry, with a nonzero-energy vacuum.

Furthermore, it is possible that the two dual theories actually agree not just at zero energy but in a small, finite energy window. In the theories at hand, the scale of supersymmetry breaking is proportional to some superpotential coupling and can be tuned to be small enough so that it is within this energy window. It is important to stress, however, that we only use the dual to establish that supersymmetry is broken. The details of supersymmetry breaking may be different between the original theory and its dual.

While we mostly concentrate on the application of duality to establish supersymmetry breaking, one could adopt a different point of view and use duality to construct new models of DSB starting with known models. Generically new models constructed in such a way will describe completely different yet nonsupersymmetric infrared physics.

Let us turn now to our example. The theory we start with is an $SU(N)\times SU(N-2)$ $(N\geqslant 5)$ gauge theory with fields $Q_{i\alpha}$, transforming as $(N,N-2)$ under the gauge groups, $N-2$ fields $\overline{L}_I^i$, transforming as $(\overline{N},\mathbf{1})$, and N fields $\overline{R}_A^\alpha$ that transform as $(\mathbf{1},\overline{N-2})$. We denote the gauge indices of $SU(N)$ and $SU(N-2)$ by i and α, respectively, while $I=1\dots N-2$ and $A=1\dots N$ are flavor indices. Note that these theories are chiral—no mass terms can be added for any of the matter fields.

In the following, we shall outline only the main stages of the analysis. For details we refer the reader to Poppitz, Shadmi, and Trivedi (1996b). In particular, we omit numerical factors and some scale factors throughout this section.

The classical moduli space of the theory is given by the gauge invariants $Y_{IA}=\overline{L}_I\cdot Q\cdot\overline{R}_A$, $\overline{b}^{AB}=(\overline{R}^{N-2})^{AB}$, and $\overline{\mathcal{B}}=Q^{N-2}\cdot\overline{L}^{N-2}$ (when appropriate, all indices are contracted with ϵ tensors), subject to the classical constraints $Y_{IA}\overline{b}^{AB}=0$ and $\overline{b}^{AB}\overline{\mathcal{B}}\sim(Y^{N-2})^{AB}$.

To lift all classical flat directions, we can add the superpotential

$$W_{tree}=\lambda^{IA}Y_{IA}+\alpha_{AB}\overline{b}^{AB}, \tag{49}$$

with $\lambda_{IA}=\lambda\,\delta_{IA}$ for $A\leqslant N-2$, and zero otherwise. α_{AB} is an antisymmetric matrix whose nonzero elements are $\alpha_{12}=\dots=\alpha_{(N-2)(N-1)}=\alpha$ for odd N, and $\alpha_{12}=\dots=\alpha_{(N-1)N}=\alpha$ for even N.[19] Note that the second term in Eq. (49) is nonrenormalizable for $N\geqslant 6$ but has dimension 4 for $N=5$.

As it turns out, there is an important difference between the theories with even and odd N. For odd N, the superpotential (49) preserves an R symmetry, and one may expect supersymmetry to break. For even N, there is no R symmetry that is preserved by Eq. (49), so supersymmetry is most likely unbroken. Both of these statements are indeed borne out by direct analysis, as we shall see.

It is also easy to check that if we set $\alpha_{AB}=0$ in Eq. (49), all flat directions are lifted except for the "baryon" directions $\overline{b}^{AB}$.

To analyze the quantum theory, we can start with the limit $\Lambda_N\gg\Lambda_{N-2}$, where Λ_N and Λ_{N-2} are the strong-coupling scales of $SU(N)$ and $SU(N-2)$, respectively. $SU(N)$ has $N-2$ flavors, so gaugino condensation in an unbroken $SU(2)$ subgroup generates the superpotential

$$W\sim\left(\frac{\Lambda_N^{2N+2}}{\overline{\mathcal{B}}}\right)^{1/2}. \tag{50}$$

Thus there is no moduli space. Below Λ_N, $SU(N-2)$ appears anomalous. It is also partially broken. This is reminiscent of the situation we encountered in the 3–2 model. However, there, because of the $SU(3)$ superpotential, the $SU(2)$ was completely broken. In contrast, here the $SU(N-2)$ is not completely broken, so there are some strong dynamics associated with the unbroken group. It is therefore very hard (or impossible) to analyze the theory (except for a special choice of parameters, for which it has a calculable minimum, as we shall see later). Fortunately we can turn to a dual theory in which the low-energy dynamics are under control.[20]

Before we do that, one comment is in order. It is already clear from Eq. (50) that the electric theory has no moduli space. In addition, with $\alpha=0$, the theory has classical flat directions. If these are not lifted quantum mechanically, the superpotential (50) pushes some fields to large VEV's along these directions. Then for $\alpha\ll 1$ we can find a calculable minimum. This indeed is the case. We shall return to this calculable minimum towards the end of the section. First, however, we would like to show that the theory has no supersymmetric vacua. To do that, we turn to the dual theory.

[19] In fact, one can lift all flat directions with other choices for λ^{IA} and α_{AB}; see Poppitz, Shadmi, and Trivedi (1996b).

[20] The appearance of the superpotential (50) can be seen in the dual theory as well (Poppitz, Shadmi, and Trivedi, 1996b).

We construct the dual theory in the limit $\Lambda_{N-2} \gg \Lambda_N$. However, it is expected to give a valid description of the original theory in the infrared for any Λ_{N-2}/Λ_N (Poppitz, Shadmi, and Trivedi, 1996a). The dual theory is obtained by dualizing the $SU(N-2)$. This can be thought of as the process of first turning off the $SU(N)$ coupling, so that we are left with an $SU(N-2)$ with N flavors. Dualizing this theory we find an $SU(2)$ theory with N flavors. Finally, we switch the $SU(N)$ coupling back on in this dual theory.

The dual theory then has $SU(N)\times SU(2)$ gauge symmetry, with the following field content:

	$SU(N)$	$SU(2)$
q^i_ν	$\overline{N}$	2
$\overline{r}^{A\nu}$	1	2
$\frac{1}{\mu}M_{iA}$	N	1
$\overline{L}^i_I$	$\overline{N}$	1

(51)

The $SU(2)$ singlets M_{iA} correspond to the $SU(N-2)$ mesons $q_i \cdot R_A$, and μ is a mass scale that relates the strong-coupling scales of $SU(N-2)$ and $SU(2)$: $\Lambda_{N-2}^{2N-6}\Lambda_2^{6-N} \sim \mu^N$.

In addition, the dual theory has a Yukawa superpotential:

$$W = \frac{1}{\mu} M_{\alpha A}\overline{r}^A \cdot q^\alpha. \tag{52}$$

Note that in this dual theory, $SU(2)$ has N flavors, so naively it is in the dual regime. We shall soon see, however, that the combination of the Yukawa superpotential (52) and the $SU(N)$ dynamics drives the theory into the confining regime.

To see that, note that $SU(N)$ now has N flavors, and therefore a quantum modified moduli space. Below the $SU(N)$ confining scale,[21] we can write down an effective theory in terms of the $SU(N)$ mesons $N_{A\nu} \sim M_{iA} q^i_\nu$ and $Y_{IA} \sim M_{iA}\overline{L}^i_I$, and the $SU(N)$ baryons $\mathcal{B} \sim \det(M_{\alpha A})$ and

$$\overline{\mathcal{B}}' \sim q^2 \cdot \overline{L}^{N-2} \sim Q^{N-2} \cdot \overline{L}^{N-2} \sim \overline{\mathcal{B}},$$

where in the last equation we used the baryon map of supersymmetric QCD, Eq. (A38). Here we omit various scales as well as numerical coefficients.

In terms of these variables, the $SU(2)$ still has $2N$ doublets, N_A and $\overline{r}^A$, but the superpotential (52) now turns into

$$W \sim N_A \cdot \overline{r}^A, \tag{53}$$

which gives masses to all $SU(2)$ doublets. Thus indeed, as $SU(N)$ confines, the Yukawa couplings turn into mass terms and drive $SU(2)$ into the confining regime.

[21] This scale is not Λ_N. Rather, it is a combination of Λ_N, Λ_{N-2}, and μ (Poppitz, Shadmi, and Trivedi, 1996b).

Since $SU(2)$ is now confining, with mass terms for all its doublets, we should work in terms of its mesons, all of which obtain VEV's. A convenient way of keeping track of the correct VEV's is to add the superpotential

$$W \sim \left(\frac{\mathrm{Pf}V}{\Lambda_{2L}^{6-N}}\right)^{1/(N-2)}. \tag{54}$$

Here Λ_{2L} is the $SU(2)$ scale after $SU(N)$ confines, and V stands collectively for the $SU(2)$ mesons $[N^2]$, $[\overline{r}^2]$, and $[N \cdot \overline{r}]$. We use brackets to indicate that these mesons should now be thought of as single fields.

In addition, recall that the $SU(N)$ dynamics lead to a constraint that can be implemented through the superpotential

$$A([N^2]\cdot Y^{N-2} - \mathcal{B}\overline{\mathcal{B}} - \overline{\Lambda}_{NL}^{2N}), \tag{55}$$

where A is a Lagrange multiplier and $\overline{\Lambda}_{NL}$ is the $SU(N)$ scale.

Combining Eqs. (55), (54), and (53) with the tree-level superpotential, which now has the form

$$\lambda^{IA} Y_{IA} + \alpha_{AB}[\overline{r}^A \cdot \overline{r}^B], \tag{56}$$

we have the complete superpotential. Note that in the last step we used the supersymmetric QCD baryon map $\overline{b}^{AB} \sim \overline{r}^A \cdot \overline{r}^B$.

We now have a low-energy field theory with all gauge dynamics integrated out. This low-energy theory consists of the fields $\mathcal{B}$, $\overline{\mathcal{B}}$, Y_{IA}, $[N^2]$, $[r^2]$, and $[N \cdot r]$, with a superpotential that is given by adding Eqs. (53) through (56). We can then check whether all F terms vanish simultaneously. This is a rather tedious task, and we refer the interested reader to Poppitz, Shadmi, and Trivedi (1996b). As the analysis shows, for odd N, no solution exists, and supersymmetry is broken. For even N a solution does exist.

It is interesting to see what happens before adding the tree-level superpotential. In that case, the F equations have no solution for finite field VEV's. Furthermore, for $\alpha=0$ and $\lambda \neq 0$, the different F terms tend to zero as some of the baryons $\overline{b}^{AB}$ tend to infinity.

One difficulty that we glossed over in the above discussion is related to the fact that after $SU(N)$ confines, the $SU(2)$ scale Λ_{2L} is field dependent. If this scale vanishes, additional fields may become massless, and the Kähler potential in terms of the degrees of freedom we have kept so far may become singular. To resolve this issue, we can add a heavy $SU(N)$ flavor. In this case no scale is field dependent, and the analysis confirms the results stated above. In particular, we find that supersymmetry is broken for odd N. For further details see Poppitz, Shadmi, and Trivedi (1996b).

To summarize, while we could not in general analyze the original $SU(N)\times SU(N-2)$ theory, we were able to show that it breaks supersymmetry for odd N by studying its dual $SU(N)\times SU(2)$ theory.

To complete our discussion of this theory, we now turn to the calculable minimum we mentioned earlier. This minimum can be studied in the electric theory itself, so duality plays no role in the analysis.

As we have already mentioned, with $\alpha=0$, all F terms asymptote to zero along the baryonic flat direction. Let us now see this in the electric theory. We choose a particular baryon direction with $R_A^i = v\,\delta_A^i$. This corresponds to $\overline{b}^{(N-1)N} \sim v^{N-2}$, with all other $\overline{b}^{AB}=0$. Along this direction, $SU(N-2)$ is completely broken, so for large v we can neglect its effects. Furthermore, the first term in Eq. (49) gives masses λv to all $SU(N)$ flavors (in the following we set $\lambda=1$ for convenience). At low energies we are thus left with a pure $SU(N)$ whose scale Λ_{NL} satisfies $\Lambda_{NL}^{3N} \sim v^{N-2}\Lambda_N^{2N+2}$. Gaugino condensation in this theory then leads to a superpotential

$$W \sim (v^{N-2})^{1/N} \sim (\overline{b}^{(N-1)N})^{1/N}. \tag{57}$$

We thus have a low-energy theory in terms of the baryons $\overline{b}^{AB}$, with the superpotential Eq. (57) (for $\alpha=0$), so the F term for $\overline{b}^{(N-1)N}$ behaves as $F \sim (\overline{b}^{(N-1)N})^{1/N-1}$, which goes to zero as $\overline{b}^{(N-1)N} \to \infty$. But whether this is a runaway direction or not depends on the Kähler potential. In fact, it can be argued (Shirman, 1996) that the Kähler potential is canonical in terms of the elementary fields $\overline{R}^A$, up to small corrections. Thus if the F terms for $\overline{R}^A$ tend to zero along this direction, there is a runaway minimum at infinity. Indeed, these F terms behave as $v^{[(N-2)/N]-1} = v^{-2/N}$ and asymptote to zero as $v \to \infty$.

Adding now a small $\alpha \neq 0$, the potential can be stabilized as $v \to \infty$, with a supersymmetry-breaking minimum for large values of v. Note that for $N \geqslant 6$, the baryon term in Eq. (49) is nonrenormalizable, so α is naturally small. In fact, as was shown in Poppitz and Trivedi (1997), this minimum can be analyzed using a simple σ-model approach and is interesting for model-building purposes, as there is a large unbroken global symmetry at the minimum in which, *a priori* at least, one can embed the standard-model gauge group.

C. Integrating matter in and out

In Sec. III.D we discussed the supersymmetry-breaking $SU(5)$ model with matter in the antisymmetric tensor and in the antifundamental representations. This model does not possess any classical or dynamical superpotential and does not have flat directions. We gave two arguments establishing DSB. One was based on the complexity of solutions to 't Hooft anomaly-matching conditions, while the other was based on the formation of the gaugino condensate. We also discussed generalizations of the $SU(5)$ model.

Here we shall use the same class of theories to illustrate another method of analysis that is useful in noncalculable models (Murayama, 1995; Poppitz and Trivedi, 1996). In this method we modify the model of interest to make it calculable through the introduction of extra vectorlike matter. When these vectorlike matter fields are massless, the models typically possess flat directions along which the gauge group is broken and the theory is in a weak-coupling regime. For small masses of these matter fields the weak-coupling approximation is still reliable and the theory remains calculable. Thus the modified theory with small masses allows a direct analysis of supersymmetry breaking. Then we consider the limit of infinite vectorlike matter mass. In this limit the vectorlike matter decouples and we are left with the original theory. If supersymmetry is broken in the modified theory with the additional light fields, holomorphy arguments ensure that it is broken for any finite values of masses. Moreover, since the theories that we have in mind do not have classical flat directions both for finite and infinite masses, we expect that the asymptotic behavior of the scalar potential (and therefore the Witten index) remains unchanged in the infinite mass limit. Thus the assumption that no phase transition occurs when going to the infinite mass limit leads us to the conclusion that supersymmetry is broken in the original strongly coupled model. We stress that this approach gives strong evidence for supersymmetry breaking, yet does not help in understanding the strongly coupled SUSY-breaking vacuum. The reason is that as the mass of the vectorlike matter becomes large, $m \sim \Lambda$, control of the Kähler potential is lost and the theory becomes noncalculable.

We now discuss the application of this method to the models at hand, following Poppitz and Trivedi (1996). We consider models with an $SU(2N+1)$ gauge group, an antisymmetric tensor $A_{\alpha\beta}$, $2N-3+N_f$ antifundamentals $\overline{Q}_i^\alpha$, $(i=1,\ldots,2N-3+N_f)$, and N_f fundamentals Q_α^a, $(a=1,\ldots,N_f)$. It is convenient to start with the case $N_f=3$ and then to integrate out vectorlike matter. The classical moduli space is described by the following gauge-invariant operators:

$$\begin{aligned} M_i^a &= \overline{Q}_i^\alpha Q_\alpha^a, \\ X_{ij} &= A_{\alpha\beta}\overline{Q}_i^\alpha \overline{Q}_j^\beta, \\ Y^a &= \epsilon^{\alpha_1,\ldots,\alpha_{2N+1}} A_{\alpha_1,\alpha_2}\cdots A_{\alpha_{2N-1}\alpha_{2N}} Q^a_{\alpha_{2N+1}}, \\ Z &= \epsilon^{\alpha_1,\ldots,\alpha_{2N+1}} A_{\alpha_1\alpha_2}\cdots A_{\alpha_{2N-3,2N-2}} Q^a_{\alpha_{2N-1}} \\ &\quad \times Q^b_{\alpha_{2N}} Q^c_{\alpha_{2N+1}} \epsilon_{abc}. \end{aligned} \tag{58}$$

These moduli overcount by one the number of the massless degrees of freedom at a generic point of the moduli space, and thus are related by a single constraint that easily follows from the Bose statistics of the superfields

$$Y \cdot M^2 \cdot X^{N-1} - \frac{k}{3} Z \,\mathrm{Pf} X = 0, \tag{59}$$

where appropriate contraction of indices is assumed. Vacuum expectation values of the moduli (58) satisfying the constraint (59) describe nonequivalent classical vacuum states. The Kähler potential of the theory written in terms of the gauge-invariant composites is singular at the origin. As usual this singularity reflects the fact that the gauge symmetry is restored at the origin of the moduli space and additional massless degrees of freedom descend into the low-energy theory. In complete analogy with supersymmetric QCD [see Eq. (A28)] this constraint is modified by nonperturbative effects,

$$Y\cdot M^2\cdot X^{N-1}-\frac{k}{3}Z\mathrm{Pf}X=\Lambda^{4N+2}. \qquad (60)$$

As a result, the origin of field space where the gauge symmetry is completely restored does not belong to the quantum moduli space. The Kähler potential is nonsingular in any finite region of the moduli space, and we have good control of the physics. We note in passing that the Kähler potential may still become singular at the boundaries of the (D-flat) moduli space, where a subgroup of the original $SU(2N+1)$ gauge group remains unbroken (corresponding to the situation with some moduli VEV's vanishing while other VEV's tend to infinity). In Sec. VI.B. we shall carefully consider models in which the physics in such boundary regions is important. For the time being we note that as long as the classical superpotential of the theory lifts all flat directions, such boundary regions are not accessible and we do not need to worry about them.

Having understood the properties of the moduli space, we turn to the tree-level superpotential. The full superpotential of the theory with three vectorlike flavors can be written as

$$W_3=L\left(Y\cdot M^2\cdot X^{k-1}-\frac{N}{3}Z\mathrm{Pf}X-\Lambda^{2(2N+1)}\right)+m_a^iM_i^a+\lambda^{ij}X_{ij}, \qquad (61)$$

where m_a^i is a rank three-mass matrix, and the matrix of Yukawa couplings λ is chosen so that all classical flat directions are lifted. We can now vary the masses m_a^i to move in the parameter space between the $N_f=3$ and $N_f=0$ theories. However, it is useful to first choose only one mass eigenvalue to be large, so that the effective description is of two light vectorlike flavors. In such a case the superpotential takes the form

$$W_2=\frac{\Lambda_{(2)}^{4N+3}}{\epsilon_{ac}Y^aM_{i_1}^c\epsilon^{i_1\ldots i_{2N-1}}X_{i_2i_3}\cdots X_{i_{2N-2}i_{2N-1}}}+m_a^iM_i^a+\lambda^{ij}X_{ij}, \qquad (62)$$

where the low-energy scale $\Lambda_{(2)}$ is given by the usual scale-matching condition, $\Lambda_{(2)}^{4N+3}=m\Lambda^{4N+2}$, and the tree-level terms include only fields of the $N_f=2$ model. We note that the nonperturbative term in Eq. (62) is generated by a one-instanton term in the gauge theory. By solving the equations of motion for the mesons M_i^a it is easy to see that the F-flatness conditions cannot be satisfied. Together with the regularity of the Kähler potential in any finite region of the moduli space and the absence of classical flat directions, this implies supersymmetry breaking (Poppitz and Trivedi, 1996). We note that for small masses $m\ll\Lambda$ and couplings $\lambda\ll 1$ the theory is in a semiclassical regime and the low-energy theory is calculable. As the masses are increased, control of the Kähler potential and, as a result, calculability are lost, yet supersymmetry remains broken. For large masses, $m\gg\Lambda$, the effective description is given by the $N_f=0$ models, so that we have given an additional argument for supersymmetry breaking in these noncalculable theories.

VI. VIOLATIONS OF INDIRECT CRITERIA FOR DSB

So far we have concentrated on models satisfying the Affleck-Dine-Seiberg (1985) criteria for dynamical SUSY breaking. These criteria restricted model-building efforts to chiral models with R symmetries and with no flat directions. In recent years a number of nonchiral models, models with classical flat directions, and models with no R symmetry have been shown to break supersymmetry dynamically. In this section we shall discuss such examples in turn.

A. Nonchiral models

1. SUSY QCD with singlets

We shall start the discussion of nonchiral models with SUSY QCD coupled to gauge-singlet fields. We shall vary the number of flavors in the theory and analyze the quantum behavior along the classical flat directions. We should warn the reader that generically these models do not break supersymmetry. However, this analysis will lead us to the nonchiral Intriligator-Thomas-Izawa-Yanagida model of DSB (Intriligator and Thomas, 1996a; Izawa and Yanagida, 1996) discussed in the following subsection. Along the way we shall develop useful techniques for the analysis of flat directions and illustrate them in additional examples in Sec. VI.B.

Consider an $SU(N)$ gauge theory with N_f flavors coupled to a single gauge-singlet field through the superpotential

$$W=S\,Q_i\cdot\overline{Q}_i. \qquad (63)$$

This superpotential lifts one classical flat direction of SUSY QCD, namely, $M_{ij}=v\,\delta_{ij}$. On the other hand, there is a flat direction along which S is nonvanishing. Along this direction the VEV of S plays the role of a mass for the quark superfields.[22]

For large S the effective theory is pure supersymmetric Yang-Mills with an effective strong-coupling scale $\Lambda_{SYM}^{3N}=S^{N_f}\Lambda^{3N-N_f}$, where S denotes the expectation value and Λ is the original $SU(N)$ scale. Gaugino condensation in the effective theory generates the superpotential

$$W=\Lambda_{SYM}^3=S^{N_f/N}\Lambda^{(3N-N_f)/N}. \qquad (64)$$

The superpotential (64) gives an effective description at scales much smaller than $\langle S\rangle$, yet the fluctuations of S itself remain massless, and Eq. (64) can be considered as an effective superpotential for this modulus, leading to the scalar potential

[22]Note that unlike the case with a tree-level mass term, the superpotential preserves a nonanomalous R symmetry, which is only broken spontaneously by the S VEV.

$$V=\Lambda^{2(3N-N_f)/N}|S|^{2(N_f-N)/N}. \tag{65}$$

Note that this effective description is valid only for $S \gg \Lambda$. We see that for $N_f<N$ this potential slopes to zero at infinity, and the vacuum energy is arbitrarily small for large S, exactly in the region in which our effective description is reliable. For $N_f \geq N$ the potential for S is nonvanishing at infinity (Affleck, Dine, and Seiberg, 1985). Of course, the stabilization of this direction in the case $N_f \geq N_c$ does not imply supersymmetry breaking (or even the existence of a stable vacuum) in the full model. First, the analysis performed so far is not valid near the origin of field space. In addition there are many unlifted mesonic and baryonic flat directions. Yet this suggests a way to stabilize other flat directions. Namely, one could couple the quarks to N_f^2 gauge-singlet fields[23]

$$W=\sum_{ij}^{k} \lambda_{ij} S_{ij} Q_i \cdot \overline{Q}_j , \tag{66}$$

where the matrix of Yukawa coupling constants has maximal rank, and in the following we shall choose it to be $\lambda_{ij}=\lambda\, \delta_{ij}$.[24] The superpotential (66) lifts all mesonic flat directions $M_{ij}=Q_i\overline{Q}_j$. If baryonic branches of the moduli space exist, they can be lifted by introducing additional nonrenormalizable couplings to singlets, but we shall leave these directions aside for the moment.

Along the singlet flat directions all quark superfields generically become massive, and the effective theory is pure supersymmetric Yang-Mills with the superpotential

$$W=\lambda^{N_f/N}\tilde{S}^{N_f/N}\Lambda^{(3N-N_f)/N}, \tag{67}$$

where $\tilde{S}=(\det S)^{1/N_f}$. This is just a direct generalization of Eq. (64), and we see that the flat direction is stabilized quantum mechanically for $N_f \geq N$. A somewhat more careful analysis would show that the stabilization happens for all directions S_{ij} as we shall see below.

To better understand the quantum behavior of the model, we shall repeat the above analysis in more detail. We write the scalar potential in the form[25]

$$V=\sum_{i}^{N_f}\left(\left|\frac{\partial W}{\partial Q_i}\right|^2+\left|\frac{\partial W}{\partial \overline{Q}_i}\right|^2\right)+\sum_{ij}^{N_f}\left|\frac{\partial W}{\partial S_{ij}}\right|^2, \tag{68}$$

where W includes all possible nonperturbative contributions. A supersymmetric minimum in the model exists if all three terms in Eq. (68) vanish. The first two contributions in this potential reproduce the scalar potential of SUSY QCD with N_f flavors and with the mass matrix $m_{ij}=\lambda\, S_{ij}$. We can therefore use Eq. (A24) of the Appendix to find the meson expectation values for which these terms vanish:

$$M_{ij}=[\det(\lambda S)\Lambda^{3N-N_f}]^{1/N}\left(\frac{1}{\lambda S}\right)_{ij}. \tag{69}$$

Note that analyticity requires that Eq. (69) is satisfied for all values of N_f. We can now substitute this solution back into Eq. (68):

$$V=\sum_{ij}^{N_f}\left|\frac{\partial W}{\partial S_{ij}}\right|^2=|\lambda|^2\sum_{ij}|M_{ij}|^2$$
$$=|\lambda|^{2(N_f/N)}|\det(S)\Lambda^{3N-N_f}|^{2/N}\sum_{ij}\left|\left(\frac{1}{S}\right)_{ij}\right|^2. \tag{70}$$

It is easy to see that this term is minimized by $S_{ij}=\tilde{S}\delta_{ij} \equiv (\det S)^{1/N_f}\delta_{ij}$. Therefore the scalar potential for the lightest modulus $\tilde{S}$ is

$$V=|\lambda^{N_f}\Lambda^{3N-N_f}\tilde{S}^{N_f-N}|^{2/N}. \tag{71}$$

This is just the potential that could be derived from Eq. (67) and we again see that the flat direction is lifted if $N_f \geq N$. In fact we can now make a stronger statement. When $N_f=N+1$ the model is *s*-confining (Csaki, Schmaltz, and Skiba, 1997c), and near the origin has a weakly coupled description in terms of composite (mesonic and baryonic) degrees of freedom. As a result the potential (70) is reliable near the origin, and we see that supersymmetry is restored there. When $N_f>N+1$ the weakly coupled description is given in terms of the dual gauge theory, and it is also possible to show that a supersymmetric vacuum exists at the origin.

The most interesting case for our purposes is $N_f=N$, where the vacuum energy is independent of the value of $\tilde{S}$ in the approximation that the Kähler potential is classical. This statement is equivalent to the statement that the energy is constant and nonvanishing everywhere on the mesonic branch of the moduli space. So far we have not considered the baryonic flat directions. In fact, in the model with $N_f=N$ flavors and the superpotential (66) the potential slopes to zero along the baryonic directions. However, it is easy to see that a simple modification leads to DSB (Intriligator and Thomas, 1996a; Izawa and Yanagida, 1996). This modification requires the introduction of two additional gauge-singlet fields with nonrenormalizable couplings to the $SU(N)$ baryons (in the $N=2$ case the new couplings are renormalizable).

2. The Intriligator-Thomas-Izawa-Yanagida model

Let us concentrate on a particular case with $SU(2)$ gauge group with two flavors of matter fields in the fundamental representation (four doublets Q_i, $i=1,\ldots,4$). Because the matter fields are in the pseudoreal representation, the superpotential (66) with N_f^2 singlets does

[23] In this case, the singlets transform under the global $SU(N_f)_L \times SU(N_f)_R$ group, and the chiral symmetry is preserved by the superpotential. It is spontaneously broken by the S_{ij} VEV's.

[24] Recall that the only fact we need to know about the Kähler potential to establish supersymmetry breaking is that it is nonsingular in the appropriate variables. As a result we can further rescale λ to one by field redefinitions, but it would be useful for us to keep it explicit.

[25] This potential, of course, is further modified by corrections to the Kähler potential.

not lift all the mesonic flat directions.[26] Two mesonic flat directions remain and lead to a supersymmetric minimum at infinity in direct analogy with the baryonic flat directions for general N. A slight modification of the theory with $N_f^2+2=6$ singlets lifts all mesonic flat directions

$$W=\sum_{ij} \lambda S_{ij} M_{ij}, \tag{72}$$

where $M_{ij}=Q_i\cdot Q_j$, and S_{ij} transform in the antisymmetric representation of the global $SU(4)_F$ symmetry.[27] Furthermore, we notice that near the origin of the moduli space, the theory has a weakly coupled description in terms of the mesons M_{ij}. Thus our preceding discussion immediately leads us to the conclusion that supersymmetry is broken.

Let us understand qualitatively the mechanism of supersymmetry breaking. The nonperturbative dynamics generate the quantum constraint

$$Pf(M)=\Lambda_2^4. \tag{73}$$

This quantum constraint modifies the moduli space. While the origin $M_{ij}=0$ belongs to the classical moduli space, it does not lie on the quantum moduli space. On the other hand, the S F-terms only vanish at the origin, $M_{ij}=0$. Supersymmetry is therefore broken because the F-flatness conditions are incompatible with the quantum moduli space.

It is often convenient to impose the quantum constraint through a Lagrange multiplier in the superpotential. Then the full superpotential is

$$W=\lambda\, S\, M + A\,(\mathrm{Pf}M-\Lambda_2^4), \tag{74}$$

where A is the Lagrange multiplier field. For $\lambda \ll 1$ the vacuum will lie close to the $SU(2)$ quantum moduli space. Thus one can consider the superpotential (72) as a small perturbation around the vacuum of the $N_f=N_c$ supersymmetric QCD with masses $m=\lambda\langle S\rangle$.[28] In this approximation the scalar potential is given again by Eq. (70) and is minimized when (up to symmetry transformations)

$$\tilde{S}=S_{12}=S_{34},$$
$$S_{13}=S_{14}=S_{23}=S_{24}=0, \tag{75}$$
$$M_{12}=M_{34}=\frac{1}{\lambda\tilde{S}}\left(\frac{\Lambda_2^4}{\mathrm{Pf}(\lambda S)}\right)=\Lambda_2^4.$$

The vacuum energy is then given by

$$V=|\lambda|^2\Lambda_2^4. \tag{76}$$

Thus we have a nonchiral (left-right symmetric) model[29] that breaks supersymmetry. Indeed, as we mentioned in Sec. III.A, the Witten index can change discontinuously if the asymptotic behavior of the classical potential changes. Consider modifying the Intriligator-Thomas-Izawa-Yanagida model by turning on a mass term for the singlet mS^2. For sufficiently large mass, the low-energy effective theory is pure supersymmetric Yang-Mills, and the Witten index $\mathrm{Tr}(-1)^F\neq 0$. This is therefore true for any nonvanishing value of m. As the limit $m\to 0$ is taken, the asymptotic behavior of the potential changes (there is now a classical flat direction with $S\neq 0$) and the Witten index vanishes. Note that, in accord with our discussion in Sec. III.A, the potential (76) is flat along the S-flat direction.

At the level of analysis we have performed so far, there is a pseudoflat direction parametrized by $\tilde{S}$. Since $\tilde{S}$ is the only light field in the low-energy theory and the superpotential (74) is exact, this direction would be exactly flat if the Kähler potential for S were canonical. However, quantum contributions to the Kähler potential lift the degeneracy. For sufficiently small λ and large $\lambda\langle S\rangle$ it is possible to show (Arkani-Hamed and Murayama, 1998; Dimopoulos *et al.*, 1998) by renormalization group arguments that the quantum corrections due to the wave-function renormalization of S are calculable and lead to a logarithmic growth of the potential at large S. It is possible to construct modifications of the Intriligator-Thomas-Izawa-Yanagida models with *calculable* (but not necessarily global) supersymmetry breaking (Murayama, 1997; Dimopoulos *et al.*, 1998). This is achieved by gauging a subgroup of the global symmetry under which S transforms. As a result, the wave-function renormalization of S as well as the vacuum energy depend on both the Yukawa and gauge coupling. For an appropriate choice of parameters, a local minimum of the potential exists for a large S VEV realizing Witten's (1981b) idea of inverted hierarchy in a model with dynamical supersymmetry breaking.

On the other hand, the exact superpotential of the theory (74) is of an O'Raifeartaigh type. Thus it is natural to ask whether there exists a region of the model's parameters such that near the origin of the moduli space (where, in particular, the singlet VEV is zero), the

[26]Note that the superpotential (66) does not preserve the global $SU(4)_F$ symmetry of the $SU(2)$ with four doublets.

[27]For simplicity we have chosen λ to respect global $SU(4)$ symmetry, but this is not necessary.

[28]This approximation is equivalent to satisfying the equation of motion for the Lagrange multiplier A.

[29]Practically, what is usually meant by a nonchiral model is that all fields can be given masses. This issue is a bit subtle in the Intriligator-Thomas-Izawa-Yanagida model, as quark mass terms can be absorbed by a redefinition of the singlets. Still, one may first give masses to the singlets and integrate them out, and then introduce quark masses, so that ultimately all fields become heavy.

strong coupling dynamics decouple and the potential is calculable. Indeed, Chacko, Luty, and Ponton (1998) have argued that for sufficiently small coupling λ, and $\tilde{S} \ll \Lambda_2/\lambda$, the contributions of the strong dynamics to the scalar potential are small compared with the contributions of the light particles of mass $\lambda\langle S\rangle$. The latter contribution is calculable, and it was found in Chacko, Luty, and Ponton (1998) that there exists a minimum of the potential at $\tilde{S}=0$. Moreover, it was argued that the calculability breaks down only when the Yukawa coupling λ has nonperturbative strength. Finally, another minimum of the potential may exist at $\tilde{S} \sim \mathcal{O}(\Lambda_2/\lambda)$; however, this possibility cannot be verified at present, since the strong-coupling dynamics are important in this region.

We should also mention several obvious but useful generalizations of the Intriligator-Thomas-Izawa-Yanagida model. Consider an $SP(N)$ gauge group with $N+1$ flavors of matter fields in the fundamental representation. This theory has an $SU(2N+2)$ flavor symmetry. When the quarks are coupled to gauge-singlet fields transforming in the antisymmetric representation of the flavor symmetry group, supersymmetry is broken in exactly the same way as in the $SU(2)$ model. A slightly more complicated generalization is based on an $SU(N)$ gauge group with $N_f=N$ flavors. In this case the baryonic operators B and $\bar{B}$ are required to parametrize the quantum moduli space. Therefore the superpotential (72) will not be sufficient for supersymmetry breaking. In particular there will be a supersymmetric solution $M_{ij}=0$, $B\bar{B}=\Lambda_N^{2N}$. Supersymmetry is broken if two additional fields, X and $\bar{X}$, with superpotential couplings $\lambda_1\bar{X}B+\lambda_2 X\bar{B}$ are added to the superpotential. To enforce this structure of the superpotential one can gauge baryon number. We should note that in the case of the $SU(N)$ models, the renormalization-group argument we used to show that the potential grows at large singlet VEV's is not applicable to the X and $\bar{X}$ directions. Other models with quantum-modified moduli spaces can also break supersymmetry when each invariant appearing in the constraint is coupled to a gauge-singlet Csaki-Schmaltz-Skiba (1997b).

Before closing this section we would like to reanalyze, following Intriligator and Thomas (1996a), the familiar 3–2 model of Affleck, Dine, and Seiberg (1985) discussed in Sec. IV.A in a different limit, $\Lambda_2 \gg \Lambda_3$. We shall see that the description of supersymmetry breaking is quite different in this limit. First, note that from the point of view of the $SU(2)$ gauge group we have the matter content of the Intriligator-Thomas-Izawa-Yanagida model, namely, four $SU(2)$ doublets (three Q's and L) and six singlets ($\bar{u}$ and $\bar{d}$). The superpotential couplings of the 3–2 models are not sufficient to lift all classical flat directions, and in addition to the "singlets" there is an $SU(2)$ meson that can acquire a VEV. (Of course all these flat directions, including the "singlet" ones, are lifted by $SU(3)$ D terms as we learned in Sec. IV.A.) Let us parametrize the $SU(2)$ mesons by $M_{ij}=Q_iQ_j$, $M_{i4}=Q_iL$, where the summation over $SU(2)$ indices is suppressed. In these variables the superpotential of the model is

$$W=\mathcal{A}(PfM-\Lambda_2^4)+\lambda\bar{d}_iM_{i4}, \tag{77}$$

where $\mathcal{A}$ is a Lagrange multiplier. Extremizing the superpotential with respect to $\bar{d}$ we find that the scalar potential contains terms

$$V=\sum_i^3 |M_{i4}|^2+\cdots. \tag{78}$$

By an $SU(3)$ rotation we can set $M_{14}=M_{24}=0$. Thus supersymmetry is restored if it is possible that $M_{34}=\epsilon^2 \to 0$. In turn this requirement and the quantum constraint (73) mean that

$$M_{12}=\frac{\Lambda_2^4}{\epsilon^2}\to\infty. \tag{79}$$

At large expectation values the quantum moduli space approaches the classical one. Thus Eq. (79) can only be satisfied if the model possesses classical flat directions. But as we already know, when the $SU(3)$ D-flatness conditions are imposed, the model does not have flat directions. Therefore supersymmetry must be broken. The natural expectation values of the (canonically normalized) fields at the minimum of the potential are of order $\mathcal{O}(\Lambda_2)$; therefore the quantum corrections to the Kähler potential are significant and one can only estimate the vacuum energy in this limit, $V\sim|\lambda^2\Lambda^4|$.

B. Quantum removal of flat directions

In the previous section we encountered the Intriligator-Thomas-Izawa-Yanagida model, which breaks supersymmetry even though it has a classical flat direction. Quantum mechanically, the potential becomes nonzero and flat (up to logarithmic corrections) far along this flat direction. It is in fact possible for quantum effects to completely "lift" classical flat directions, generating a growing potential along these directions. Thus it is possible for theories with classical flat directions to break supersymmetry, with a stable, supersymmetry-breaking minimum. We shall now build upon the insights gained in our analysis of supersymmetric QCD in the previous section to develop a method for determining when classical flat directions are lifted quantum mechanically. We shall also discuss some examples in which this happens.

As will become clear from our discussion, a crucial requirement for the quantum removal of flat directions is that some gauge dynamics become strong along the flat direction. In many models, the opposite happens; that is, the gauge group is completely broken along the flat direction and the dynamics become weaker as S increases. However, it may be the case that along the flat direction some gauge group remains unbroken, and fields charged under it obtain masses proportional to S. Then the dynamics associated with this gauge group become strong and may lift the flat direction.

We should stress that in this section we shall be asking two separate questions. First, we shall ask if quantum effects can stabilize the potential along a given flat direction. It is most convenient to answer this question by finding a set of degrees of freedom that give a weakly coupled description of the theory in the region of interest on the moduli space. However, if the quantum stabilization of the potential indeed happens, the vacuum may well lie in the genuinely strongly coupled region. Thus an affirmative answer to the first question is not sufficient to give an affirmative answer to our second question, which is to determine whether supersymmetry is broken in the model. To answer this second question we shall need to consider the properties of the exact superpotential in the strong-coupling region.

Following Shirman (1996), consider a model with classical flat directions. Assume for simplicity that there is a single modulus S. In the approximation of a canonical Kähler potential, the scalar potential of the model can be written as

$$V=V_r+V_S=\sum\left|\frac{\partial W}{\partial\phi_i}\right|^2+\left|\frac{\partial W}{\partial S}\right|^2. \qquad (80)$$

The applicability of this scalar potential is restricted by the assumption that the Kähler potential is canonical. However, for large enough S VEV's the description of the physics often simplifies, and in fact it may be possible to find a description in which the theory (or a sector of the theory) is weakly coupled. In such a limit, it is convenient to analyze the theory in two steps. First, one considers a "reduced" theory with the scalar potential V_r, where S plays a role of the fixed parameter. One then studies the behavior of the scalar potential of the "reduced" theory as a function of S as well as contributions of V_S. Let us consider various possibilities.

1. A SUSY-breaking reduced theory

Suppose that the potential V_r in the reduced theory along the flat direction is nonzero, so that the reduced theory breaks supersymmetry. If V_r is an increasing function of S, it is clear that the flat direction is stabilized. Typically, $V_r\to 0$ as $S\to 0$, but even if V_r tends to a nonvanishing constant one cannot conclude at this stage that supersymmetry is broken in the full theory. This is because the theory is typically in a strong-coupling regime near the origin of the moduli space, and therefore the assumption of a canonical Kähler potential as well as the separation of the scalar potential into the sum of two terms is not justified. An example of a model with such behavior is an $SU(4)\times SU(3)\times U(1)$ model of Csaki, Randall, and Skiba (1996); see Sec. VII.A. Classically there is a flat direction along which the gauge group is broken down to $SU(4)\times U(1)$ and the matter is the same as in the 4–1 model discussed in Sec. IV.B. The strong-coupling scale of the effective $SU(4)$ gauge group grows with the modulus, and the flat direction is stabilized (Shirman, 1996). Additional analysis performed by Csaki, Randall, and Skiba (1996) shows that there is no supersymmetric vacuum at the origin, thus allowing them to conclude that SUSY is broken. This mechanism of quantum removal of classical flat directions is quite generic for discarded generator models (see Sec. VII.A).

Another possibility is that V_r is a decreasing function of the modulus leading to a runaway behavior at moderate values of S. In this case contributions from V_S should be included in the analysis. Since the stable vacuum (if it exists at all) will be found at large values of S, the separation of the scalar potential into two terms is well justified. We can therefore conclude that as long as V_S stabilizes the flat direction, SUSY is broken. If, however, $V_S\to 0$ as $S\to\infty$ the theory does not have a stable vacuum. A very basic example of such behavior is the antisymmetric tensor models discussed in Sec. III.D.4. In these models the effective theory along the classical flat direction is SUSY breaking $SU(5)$ with the scale vanishing at the boundary of the moduli space. The theory does not have a stable vacuum. Introducing a tree-level superpotential lifts all classical flat directions, stabilizes the potential, and breaks supersymmetry. Note that there is no weak-coupling description anywhere on the moduli space of the model. This means that the separation of the scalar potential into V_r and V_S is not *a priori* justified. However, V_r arises from nonperturbative effects in the Kähler potential while V_S arises from the tree-level superpotential. As a result there are no interference effects between V_r and V_S and we can separate the potential into two positive definite terms.

2. A supersymmetric reduced theory

Now we would like to consider models in which $V_r=0$ has solutions for all values of S (or for a set of moduli). In these cases we have to analyze the behavior of V_S subject to the condition that $V_r=0$ is satisfied. It is instructive to consider as examples two classes of analogous models with the gauge groups $SP(N/2)\times SU(N-1)$ (Intriligator and Thomas, 1996a) and $SU(N)\times SU(N-1)$ (Poppitz, Shadmi, and Trivedi, 1996a).

We begin with the model of Intriligator and Thomas (1996a). The matter content is $Q\sim(N,N-1)$, $L\sim(N,1)$, $R_a\sim(1,\overline{N-1})$, with the tree-level superpotential

$$W_{tree}=\lambda QL\overline{R}_2+\frac{1}{M}\sum_{a,b>2}^{N}\lambda_{ab}Q^2\overline{R}_a\overline{R}_b. \qquad (81)$$

This superpotential leaves classical flat directions associated with the $SU(N-1)$ antibaryons $\overline{b}^a=(\overline{Q}^{N-1})^a=v^{N-1}$ (we shall denote the R VEV's by v). The exact superpotential was found in Intriligator and Thomas (1996a) and was used to show that there is no supersymmetric vacuum in the finite region of moduli space. Here we shall confine ourselves to discussing the physics along the classical baryonic flat directions.

Without loss of generality, we can consider the flat direction $S\equiv\overline{b}^1=v^{N-1}$. Along this direction, $SU(N-1)$ is completely broken. On the other hand, all SP flavors get masses proportional to v through the tree-level superpotential, so that we are left with a pure $SP(N/2)$ that gets stronger for larger S. Gaugino condensation in this group then produces the superpotential

$$W_S \sim S^{2/(N+2)}, \tag{82}$$

leading to the potential

$$V_S = \left|\frac{\partial W}{\partial S}\right|^2 \sim S^{-2N/(N+2)}. \tag{83}$$

(Note that here $V_r = 0$.) Actually, we can obtain this result starting from the exact superpotential. However, at scales $S \gg \Lambda_1$ the relevant degrees of freedom are the elementary ones, so we should consider the behavior of the potential in terms of v,

$$V_S \sim v^{2(N-4)/(N+2)}. \tag{84}$$

We see that for $N > 4$ it increases along the classical flat direction. Thus the classical flat direction is stabilized quantum mechanically. The analysis of the theory in the finite region of the field space (Intriligator and Thomas, 1996a) shows that SUSY is broken.

We would like to compare these results with the behavior of the model of Poppitz, Shadmi, and Trivedi (1996a) based on an $SU(N) \times SU(N-1)$ gauge group with matter in the fundamental representations: $Q \sim (N, N-1)$, $\overline{L}_i \sim (\overline{N}, 1)$, and $\overline{R}_a (1, \overline{N-1})$, where $i = 1 \cdots N-1$, and $a = 1 \cdots N$. The tree-level superpotential is given by

$$W_{tree} = \sum_{ia} \lambda_{ia} Q \overline{L}_i \overline{R}_a + \alpha_a \overline{b}^a, \tag{85}$$

where $\overline{b}^a = (\overline{R}^{N-1})^a$ is an antibaryon of $SU(N-1)$. This superpotential lifts all flat directions as long as the couplings are chosen so that (Poppitz, Shadmi, and Trivedi 1996a) λ_{ia} has maximal rank and

$$\lambda_{ia} \alpha_a \neq 0. \tag{86}$$

Since we are interested in understanding the physics along the flat directions we shall set $\alpha_a = 0$. Then there are classical flat directions parametrized by the $SU(N-1)$ antibaryons, in complete analogy with the $SU(N-1) \times SP(N/2)$ model discussed above. Again, along the direction $S \equiv \overline{b}^N$, $SU(N-1)$ is broken, and all flavors of $SU(N)$ obtain mass. $SU(N)$ gaugino condensation generates the potential

$$V_S \sim S^{-2[(N-1)/N]}. \tag{87}$$

Again, this potential can also be obtained from the exact superpotential of the theory, which was obtained by Poppitz, Shadmi, and Trivedi (1996a). In terms of the VEV of the elementary field $\overline{R}$ we then have

$$V_S \sim v^{-2/N}.$$

Unlike the case of the Intriligator-Thomas model, the runaway behavior persists and the model does not have a stable vacuum state. Turning on α_a according to Eq. (86), all flat directions are lifted. This, together with the analysis of Poppitz, Shadmi, and Trivedi (1996a), which shows that there is no supersymmetric minimum in the finite region of moduli space, allows us to conclude that supersymmetry is broken.

As we mentioned in the beginning of this subsection, the key ingredient in quantum lifting of flat directions is that the dynamics of some gauge group become strong along the flat direction. This in fact happens in both the $SU \times SP$ and the $SU \times SU$ examples we saw above, as, along the relevant flat direction, one group factor remains unbroken and fields charged under it obtain masses. However, the numerical factors are such that the potential grows along the flat direction in the first example, and slopes to zero in the second.

While general criteria for the determination of the quantum behavior along classical flat directions do not exist, we have illustrated several techniques that are useful for the analysis. We should also stress that we have concentrated on the simplest examples with a single modulus. In more general situations it is not sufficient to perform an analysis for each flat direction separately, assuming that other moduli are stabilized. One should do a complete analysis allowing all moduli to obtain independent VEV's consistent with D and F flatness conditions. In particular one should study the moduli that do not appear in the tree-level superpotential.

C. Supersymmetry breaking with no *R* symmetry

In Sec. III.B we discussed the relation between SUSY breaking and R symmetries. We saw that theories with a spontaneously broken R symmetry and no flat directions break SUSY. We also saw that if R breaking terms are added to the superpotential, SUSY is typically restored. We emphasized that both these statements assume that the superpotential is generic, that is, all terms allowed by the symmetries appear.

In this section we shall encounter a theory with the most general renormalizable superpotential allowed by symmetries, which breaks supersymmetry even though it does not have an R symmetry. Furthermore, unlike the theory of Sec. V.A, it does not possess an effective R symmetry in the low-energy description. As we shall see, the reason supersymmetry is broken is that the dynamical superpotential is not generic.

The model we describe is an $SU(4) \times SU(3)$ gauge theory studied by Poppitz, Shadmi, and Trivedi (1996a), which is the first in a series of $SU(N) \times SU(N-1)$ models that we have already discussed from a different perspective in Sec. VI.B. Here we only state some of the results. The matter content is $Q \sim (4,3)$, $\overline{L}_i \sim (\overline{4}, 1)$, and $\overline{R}_a \sim (1, \overline{3})$ with $i = 1 \ldots 3$, $a = 1 \ldots 4$. We can add the classical superpotential

$$W = \lambda \sum_{i=1}^{3} Q \cdot \overline{L}_i \cdot \overline{R}_i + \lambda' Q \cdot \overline{L}_1 \cdot \overline{R}_4 + \alpha (\overline{R}^3)^1 + \beta (\overline{R}^3)^4, \tag{88}$$

with appropriate contractions of the gauge indices [in particular, $(\overline{R}^3)^a$ stands for the $SU(3)$ "baryon" with the field $\overline{R}_a$ omitted]. This superpotential does not preserve any R symmetry. It is the most general renormalizable superpotential preserving an $SU(2)$ global symmetry that rotates $\overline{L}_2,\overline{L}_3$ together with $\overline{R}_2,\overline{R}_3$. In addition, it lifts all the classical flat directions of the model. If we add nonrenormalizable terms to this superpotential, (supersymmetric) minima will appear at Planckian field strength. These extra minima will not destabilize the nonsupersymmetric minima we shall be discussing.[30]

As was shown by Poppitz, Shadmi, and Trivedi (1996a), the theory breaks supersymmetry. This can be established by carefully analyzing the low-energy theory. In the limit that the $SU(3)$ dynamics is stronger, $SU(3)$ confines, giving a low-energy theory in which $SU(4)$ has four flavors. After $SU(4)$ confines one has an O'Raifeartaigh-like theory, with the fields $Y_{ia} = Q\cdot\overline{L}_i\cdot\overline{R}_a$, $\overline{b}^a=(\overline{R}^3)^a$, $\overline{B}=Q^3L^3$, $P_a=Q^3(Q\cdot\overline{R}_a)$, and $B=\det(Q\cdot\overline{R})$ (the last two vanish classically). Taking into account the dynamically generated superpotential we find that the full superpotential is given by

$$W=\frac{P_a\overline{b}^a-B}{\Lambda_3^5}+A\,(P\cdot Y^3-\overline{B}\,B-\Lambda_4^9\Lambda_3^5)+\lambda\sum_{i=1}^{3}Y_{ii}+\lambda'\,Y_{14}+\alpha\,\overline{b}^1+\beta\,\overline{b}^4, \tag{89}$$

where A is a Lagrange multiplier and Λ_4,Λ_3 are the scales of $SU(4),SU(3)$, respectively. This superpotential does not preserve any effective R symmetry in terms of the variables of the low-energy theory. Still, as was shown by Poppitz, Shadmi, and Trivedi (1996a), supersymmetry is broken. The crucial point is that the Lagrange multiplier A only appears linearly in Eq. (89). If the superpotential contained terms with higher powers of A, supersymmetry would be restored. Note that the superpotential (89) is reminiscent of the superpotential (10) of the simplest O'Raifeartaigh model, with A playing the role of ϕ_1. In the absence of an R symmetry, one cannot rule out the presence of higher powers of ϕ_1 in Eq. (10), whereas in the dynamically generated superpotential (89), A only appears linearly.

Other examples have been found that break supersymmetry even though the microscopic theory does not have an R symmetry. These include, among others, the 4–3–1 model of Leigh, Randall, and Rattazzi (1997) and the 4–1 model with the superpotential term *M*Pf*A* (Poppitz and Trivedi, 1996), as well as, as we have mentioned, the model of Intriligator, Seiberg, and Shenker (1995). In most of these examples, either the tree-level superpotential is not generic or there is an effective R symmetry in the low-energy theory.

[30]Other models in this class (with $N>4$) are nonrenormalizable, and we do not have a reason to neglect nonrenormalizable operators. For $N\leqslant 6$ the generalization of Eq. (88) still gives the most general superpotential up to operators whose dimension is smaller than or equal to the dimension of the baryon. Therefore the expected SUSY-breaking minimum is still a stable local minimum. In models with $N>6$, the most general superpotential with no R symmetry and operators whose dimension does not exceed the dimension of the baryon operator will generically preserve supersymmetry.

VII. DSB MODELS AND MODEL-BUILDING TOOLS

So far we have discussed several important models that illustrate the main methods and subtleties in the analysis of DSB. Many more models (in fact many infinite classes of models) have been constructed in recent years. The methods of analysis we have described can be used for these models. In fact, often both the method of analysis and the dynamics itself are analogous to one or the other models discussed in previous sections. Thus it is not practical to present a detailed investigation of every known model of DSB.

On the other hand, in many cases the dynamics are not well understood beyond the conclusion that SUSY must be broken, and further investigation of the dynamics as well as the connection between different mechanisms and models of SUSY breaking may lead to better understanding of the general conditions for DSB. Therefore in this section we shall give a list of known models, briefly discussing how SUSY is broken. We shall emphasize the relations between various models and give a partial classification. In addition we shall introduce a useful model-building method that can be used to construct new models.

We shall also discuss in this section supersymmetry breaking in theories with anomalous $U(1)$'*s* that give an example of dynamical SUSY breaking through the Fayet-Iliopolous mechanism.

A. Discarded generator models

Let us recall the observation that both the 4–1 and the 3–2 models have a gauge group that is a subgroup of $SU(5)$, while the matter content (after adding E^+ in the 3–2 model) falls into antisymmetric tensor and antifundamental representations—exactly as needed for DSB in $SU(5)$. Based on this observation, Dine *et al.* (1996) proposed the following method of constructing new DSB models. Take a known model of dynamical supersymmetry breaking without classical flat directions and discard some of the group generators. This reduces the number of D-flatness conditions and therefore leads to the appearance of flat directions. On the other hand, the most general tree-level superpotential allowed by the smaller symmetry may lift all the moduli. It is also possible that a unique nonperturbative superpotential will be allowed in such a "reduced" model. This construction is guaranteed to yield anomaly-free chiral models that often possess a nonanomalous R symmetry, and thus are good candidates for dynamical supersymmetry breaking. If a model constructed using this prescription breaks supersymmetry it is often calculable, since for small superpotential couplings it typically possesses almost flat directions along which the effective description may be weakly coupled.

In fact, the 3–2 and 4–1 models are the simplest examples of two infinite classes of DSB models based on $SU(2N-1)\times SU(2)\times U(1)$ (Dine *et al.*, 1996) and on $SU(2N)\times U(1)$ (Dine *et al.*, 1996; Poppitz and Trivedi, 1996) gauge groups, which can be constructed by using the discarded generator method.

To construct these theories one starts with an $SU(2N+1)$ theory with matter transforming as an antisymmetric tensor A and $2N-3$ antifundamentals $\overline{F}$. Then one requires gauge invariance under, for example, an $SU(2N-1)\times SU(2)\times(1)$ subgroup, with the $U(1)$ generator being

$$T=\mathrm{diag}[2,\ldots,2,-(2N-1),-(2N-1)]. \tag{90}$$

Under this group the matter fields decompose as

$$A(\overline{\boxminus}\,1,4),F(\square,2,3-2N),S(1,1,2-4N),$$
$$\overline{F}^a(\overline{\square},1,-2),\phi^a(1,2,2N-1),a=1,\ldots,2N-3. \tag{91}$$

The most general superpotential consistent with the symmetries is

$$W=\gamma_{ab}A\overline{F}^a\overline{F}^b+\eta_{ab}S\phi^a\phi^b+\lambda_a F\overline{F}^a\phi^a. \tag{92}$$

This superpotential lifts all classical flat directions. Models of this class have nonanomalous R symmetry and supersymmetry is broken. It is interesting to observe that the coupling η_{ab} in the superpotential above could be set to zero without restoring supersymmetry. While for $\eta=0$ classical flat directions appear, they are lifted by quantum effects.

The construction of the $SU(2N)\times U(1)$ DSB series is quite analogous. The matter fields in this class of models are

$$A_2,\quad F_{1-n},\quad \overline{F}^a_{-1},\quad S^a_n,\quad a=1,\ldots,2N-3, \tag{93}$$

where subscripts denote $U(1)$ charges and superscripts are flavor indices. With the most general superpotential allowed by symmetries, the models break supersymmetry.

Clearly one can consider many other subgroups of $SU(2N+1)$, and in fact several other classes of broken generator models were constructed: $SU(2N-2)\times SU(3)\times U(1)$ models (Csaki, Randall, and Skiba, 1996; Chou, 1997), and $SU(2N-3)\times SU(4)\times U(1)$ and $SU(2N-4)\times SU(5)\times U(1)$ models (Csaki *et al.*, 1996). While these models are similar by construction to those we discussed above, the supersymmetry-breaking dynamics are quite different, and various models in this class can have confinement, dual descriptions, and quantum removal of classical flat directions. Since we have already considered the simplest and most illuminating examples of these phenomena in DSB models, we shall not give a detailed discussion of all possible discarded generator models. We shall restrict ourselves to the mention of the $SP(2)\times U(1)$ model by Csaki, Schmaltz, and Skiba (1997a). This model is interesting because it is an example of the discarded generator model in which the rank of the gauge group is reduced compared to the "parent" theory. The matter fields in this model are

	$SP(2)$	$U(1)$
A	$\boxminus$	2
Q_1	$\square$	-3
Q_2	$\square$	-1
S_1	1	2
S_2	1	4

(94)

Nonrenormalizable couplings are required to lift all flat directions. The full superpotential is

$$W=\frac{\Lambda^7(Q_1Q_2)}{2(A)^2(Q_1Q_2)^2-(Q_1AQ_2)^2}+Q_1Q_2S_2+Q_1AQ_2S_1, \tag{95}$$

where the first term is generated dynamically.

The existence of a general method for constructing discarded generator models suggests that there may exist a unified description of these models. In fact, Leigh, Randall, and Rattazzi (1997) found exactly such a description. It is based on the antisymmetric tensor models supplemented by a chiral field Σ in the adjoint representation of the gauge group. We are interested in finding an effective description of the discarded generator models, or more generally of the models with $U(1)^{k-1}\times\Pi_{i=1}^{k}SU(n_i)$ gauge groups (where $\Sigma_{i=1}^{k}n_i=2N+1$) and the light matter given by decomposing the antisymmetric tensor and antifundamentals of $SU(2N+1)$ under the unbroken gauge group. The adjoint Σ needs to be heavy in such a vacuum. This can be achieved by introducing the superpotential for the adjoint

$$W_\Sigma=\sum_{i=2}^{k+1}\frac{s_i}{i}\,\mathrm{Tr}\,\Sigma^i. \tag{96}$$

We shall be most interested in the case $k=2$. For generic coefficients s_i there are several discrete vacua in which Σ is heavy and the model contains matter in desired representations. Note that in the most symmetric vacuum $\Sigma=0$, the low-energy physics is described by the antisymmetric tensor model, and SUSY is broken. For supersymmetry to be broken in other vacua one needs to lift the classical flat directions associated with the light fields, which requires the following tree-level superpotential:

$$W=\frac{1}{2}m\Sigma^2+\frac{1}{3}s_3\,\mathrm{Tr}\,\Sigma^3+\lambda_1^{ij}\overline{F}_iA\overline{F}_j+\lambda_2^{ij}\overline{F}_iA\Sigma\overline{F}_j+\lambda_3^{ij}\overline{F}_i\Sigma A\Sigma\overline{F}_j. \tag{97}$$

This superpotential is chosen so that in each vacuum of interest it exactly reproduces the superpotential needed for supersymmetry breaking. Leigh, Randall, and Rattazzi (1997) showed that in the full model supersymmetry is broken for any value of the adjoint mass including $m=0$. This latter conclusion at first seems quite unusual, since the one-loop beta function coefficient of the model is $b_0=2N+4$. In the $SU(2N+1)$ model this might suggest that, at least in the absence of the superpotential the theory is in a non-Abelian Coulomb phase. However, the analysis of Leigh, Randall, and Rattazzi (1997) showed that the superpotential is indeed quite relevant and, when certain requirements on the Yukawa couplings are satisfied, SUSY is broken. A similar construction with $k>2$ (that is, models with more than two non-Abelian factors and/or more than one Abelian factor in the gauge group) was shown (Leigh, Randall, and Rattazzi, 1997) not to break supersymmetry.

B. Supersymmetry breaking from an anomalous $U(1)$

Theories with an anomalous $U(1)$ provide a simple mechanism for supersymmetry breaking (Binetruy and Dudas, 1996; Dvali and Pomarol, 1996). Such theories contain a Fayet-Iliopoulos term, so supersymmetry can be broken just as in the Fayet-Iliopoulos model we discussed in Sec. II.C.2. In fact, the anomalous $U(1)$ theories discussed below are the only known examples in which the Fayet-Iliopoulos mechanism of supersymmetry breaking can be realized dynamically. In the absence of any superpotential, at least one field with an appropriate $U(1)$ charge develops a VEV to cancel the Fayet-Iliopoulos term. One can then introduce an (effective) superpotential mass term for this field so that some F term and the D term cannot vanish simultaneously and supersymmetry is broken.

In our discussion of the Fayet-Iliopoulos model in Sec. II.C.2 we simply put in a tree-level Fayet-Iliopoulos term by hand. It is well known that a $U(1)$ D term can be renormalized at one loop (Fischler *et al.*, 1981; Witten, 1981a). Such renormalization is proportional to the sum of the charges of the matter fields and therefore vanishes unless the theory is anomalous. Indeed, in many string models, the low-energy field theory contains an anomalous $U(1)$, whose anomalies are cancelled by shifts of the dilaton-axion superfield, through the Green-Schwarz mechanism (Green and Schwarz, 1984). A Fayet-Iliopoulos term is generated for this $U(1)$ by string loops. As far as the low-energy field theory is concerned, we can treat this Fayet-Iliopoulos term as if it were put in by hand. The only subtleties associated with supersymmetry breaking involve the dilaton superfield.

Consider a theory with an anomalous $U(1)$ gauge symmetry with

$$\delta_{GS}=\frac{1}{192\pi^2}\sum_i q_i, \tag{98}$$

where q_i denote the $U(1)$ charges of the different fields of the theory. The dilaton superfield S then transforms as

$$S\to S+i\frac{\delta_{GS}}{2}\alpha, \tag{99}$$

under the $U(1)$ transformation $A_\mu\to A_\mu+\partial_\mu\alpha$, where A_μ is the $U(1)$ vector boson. To be gauge invariant, the dilaton Kähler potential is then of the form

$$K=K(S+S^*-\delta_{GS}V), \tag{100}$$

where V is the $U(1)$ vector superfield. This then gives the Fayet-Iliopoulos term

$$\xi^2=-\frac{\delta_{GS}}{2}\frac{\partial K}{\partial S}. \tag{101}$$

Following Arkani-Hamed, Dine, and Martin (1998), we shall consider the model of Binetruy and Dudas (1996), which has, in addition to the anomalous $U(1)$, an $SU(N)$ gauge symmetry. The model contains the field ϕ, an $SU(N)$ singlet with $U(1)$ charge -1 (assuming $\delta_{GS}>0$), and one flavor of $SU(N)$, that is, fields Q and $\overline{Q}$ transforming as (N,q) and $(\overline{N},\overline{q})$ under $SU(N)\times U(1)$. Working in terms of the $SU(N)$ meson $M=Q\overline{Q}$, the superpotential is given by

$$W=mM\left(\frac{\phi}{M_P}\right)^{q+\overline{q}}+(N-1)\left(\frac{\Lambda^{3N-1}}{M}\right)^{1/(N-1)}, \tag{102}$$

where the first term is a tree-level term and the second term is generated dynamically by $SU(N)$ instantons. The potential will also contain contributions from the $U(1)$ D term, which is given by

$$D=-g^2[(q+\overline{q})|M|-|\phi|^2+\xi^2], \tag{103}$$

where g is the $U(1)$ gauge coupling. Minimizing the potential, one finds that supersymmetry is broken. ϕ wants to develop a VEB to cancel ξ^2, but because of the $SU(N)$ dynamics, the meson t develops a VEV, which then generates, through the first term in Eq. (102), a mass term for ϕ, so that the potential does not vanish.

It is important to recall, though, that the $SU(N)$ scale depends on the dilaton superfield. This dependence is most easily fixed by requiring that the second term in Eq. (102) is $U(1)$ invariant, giving

$$\Lambda^{3N-1}=M_P^{3N-1}e^{-2(q+\overline{q})S/\delta_{GS}}. \tag{104}$$

One can then minimize the potential in terms of t, ϕ, and the dilaton S. At the minimum, the D term as well as the t, ϕ, and dilaton F -terms are nonzero.

Note that if the dilaton superfield is neglected in the above analysis, the theory seems to have no Goldstino, as the gaugino and matter fermions obtain masses either by the Higgs mechanism or through the superpotential. In fact, as was shown by Arkani-Hamed, Dine, and Martin (1998), the Goldstino in these theories is a combination of the gaugino, the matter fermion, and the dilatino. In this basis the Goldstino wave function is given by

$$\left(\frac{D}{\sqrt{2}g},F_i,\sqrt{\frac{\partial^2K}{\partial S^2}}F_S\right), \tag{105}$$

where D, F_i, and F_S stand for the D term, the ith matter field F term, and the dilaton F term at the minimum, respectively.

C. List of models and literature guide

Finally, in this section we shall present an extensive (but certainly incomplete) list of models known to break supersymmetry, with references to the original papers in which these models were introduced. While some of the models discussed below have been studied in great detail, frequently it is only known that a given model breaks supersymmetry, but the low-energy spectrum and the properties of the vacuum have not been studied. In addition to the examples presented below, many other models have appeared in the literature. New supersymmetry-breaking theories can be constructed from the known models in a variety of ways. Moreover, for phenomenological purposes it is often sufficient to find a model with a local nonsupersymmetric minimum. While establishing the existence of a local nonsupersymmetric minimum may sometimes be more difficult than establishing the absence of any supersymmetric vacuum, the methods involved in the analysis are essentially the same, and we shall not discuss such models here.

• $SU(5)$ with an antisymmetric tensor and an antifundamental (Meurice and Veneziano, 1984; Affleck, Dine, and Seiberg, 1985). The arguments of Affleck, Dine, and Seiberg (1985) were based on the difficulty of satisfying 't Hooft anomaly-matching conditions, while the Meurice and Veneziano (1984) argument was based on gaugino condensation. The model is not calculable. Murayama (1995) and Poppitz and Trivedi (1996) have used the method of integrating in and out vectorlike matter to give additional arguments for DSB in the model. Pouliot (1996) constructed a dual of the $SU(5)$ model and showed that the dual breaks SUSY at tree level. For the discussion of the model in the present review see Sec. III.D and V.C.

• $SU(2N+1)$ with an antisymmetric tensor A, $2N-3$ antifundamentals $\overline{F}$, and the superpotential

$$W=\lambda_{ij}A\overline{F}_i\overline{F}_j, \tag{106}$$

where λ has the maximal rank (Affleck, Dine, and Seiberg, 1985). These are generalizations of the $SU(5)$ model. The integrating in and out method was used by Poppitz and Trivedi (1996) to further analyze these models. See Sec. III.D and also Sec. VII.A.

• $SO(10)$ with a single matter multiplet in the spinor representation [**16** of $SO(10)$; Affleck, Dine, and Seiberg, 1984c]. The analysis of supersymmetry breaking in this model is very similar to that of the noncalculable $SU(5)$ model. Indeed, the $SU(5)$ model may be constructed from the $SO(10)$ model by using the discarded generator method. Murayama (1995) discussed DSB in this model in the presence of an extra field in the vector **10** representation of $SO(10)$. For a small mass of the extra field, the theory is calculable, and assuming no phase transition, SUSY remains broken when the vector is integrated out. Pouliot and Strassler (1996) considered the same theory by adding an arbitrary number $N>5$ of vector fields and constructing the dual $SU(N-5)$ theory. They showed that the dual breaks SUSY when masses for the vectors are turned on. All these arguments can only be used as additional evidence of DSB in the $SO(10)$ model, but do not allow one to analyze the vacuum and low-energy spectrum of the theory.

• The two-generation $SU(5)$ model (Affleck, Dine, and Seiberg, 1984d; Meurice and Veneziano, 1984): $SU(5)$ with two antisymmetric tensors and two antifundamentals, with the superpotential

$$W=\lambda A_1\overline{F}_1\overline{F}_2. \tag{107}$$

Affleck, Dine, and Seiberg (1984d) showed that the model is calculable. In fact, historically this is the first calculable model with supersymmetry breaking driven by an instanton-induced superpotential. The vacuum and the low-energy spectrum of the model were analyzed in detail by ter Veldhuis (1996). ter Veldhuis (1998) also analyzed generalizations of this model, which include extra vectorlike matter with a mass term in the superpotential.

• The $3-2$ model (Affleck, Dine, and Seiberg, 1985); $SU(3)\times SU(2)$ with

$$Q(3,2),\ \overline{u}(\overline{3},1),\ \overline{d}(\overline{3},1),\ L(1,2), \tag{108}$$

with the superpotential

$$W=\lambda QL\overline{d}. \tag{109}$$

This model is calculable. The analysis of the vacuum and low-energy spectrum can be found in Affleck, Dine, and Seiberg (1985) and Bagger, Poppitz, and Randall (1994). The model possesses a global $U(1)$ symmetry that can be gauged without restoring SUSY; the relevant details of the vacuum structure in this case can be found in Dine, Nelson, and Shirman (1995). See Secs. IV.A and VI.A.2.

• Discarded generator models. These include $SU(n_1)\times SU(n_2)\times U(1)$ and $SU(2N)\times(1)$ subgroup of $SU(2N+1)$ (with $n_1+n_2=2N+1$) with matter given by the decomposition of the antisymmetric tensor and $2N-3$ antifundamentals of $SU(2N+1)$ under the appropriate gauge group. This construction was proposed in Dine *et al.* (1996). For details see Sec. VII.A. The two smallest models in this class are the $3-2$ model (Sec. IV.A) and the $4-1$ model (Sec. IV.B). The $SU(2N)\times U(1)$ models were first constructed in Dine *et al.* (1996) and Poppitz and Trivedi (1996); the $SU(2N-1)\times SU(2)\times U(1)$ models can be found in Dine *et al.* (1996); the $SU(2N-2)\times SU(3)\times U(1)$ models are considered in Csaki, Randall, and Skiba (1996)[31]; finally, the $SU(2N-3)\times SU(4)\times U(1)$ models are discussed in Csaki *et al.* (1996). A unified description of this class of models, as well as of the noncalculable $SU(2N+1)$ models, is given in Leigh, Randall, and Rattazzi (1997). Another example of the models in this class is the $SP(2)\times U(1)$ model of Csaki, Randall, and Skiba (1997a).

• $SU(2N+1)\times SU(2)$ (Dine *et al.*, 1996) with

$$Q\sim(\Box,\Box),\ L\sim(1,\Box),\ Q_i\sim(\overline{\Box},1),\ i=1,2, \tag{110}$$

and with a superpotential similar to that of the $3-2$ model. These models are obvious generalizations of the $3-2$ model. The dynamics in this class of models are

[31] See also Chou (1997).

very similar to those of the 3−2 model. For detailed analysis see Intriligator and Thomas (1996b). The low-energy physics of the $SU(5)\times SU(2)$ model in this class, in the limit of a strong $SU(2)$, is described by the noncalculable $SU(5)$ model (Intriligator and Thomas, 1996b) to which we have paid so much attention in this review.

• $SU(7)\times SP(1)$ and $SU(9)\times SP(2)$ (Intriligator and Thomas, 1996b). These models are obtained by dualizing the $SU(7)\times SU(2)$ and $SU(9)\times SU(2)$ models of the previous paragraph. The matter content is

$$A(\begin{smallmatrix}\Box\\\Box\end{smallmatrix},1),\ F(\Box,1),\ \overline{P}(\overline{\Box},\Box),$$
$$L(1,\Box),\ \overline{U}(\overline{\Box},1),\ \overline{D}(\overline{\Box},1), \tag{111}$$

and the superpotential

$$W=A\overline{P}\overline{P}+F\overline{P}L. \tag{112}$$

Note that these models can be constructed starting from the antisymmetric tensor models of Affleck, Dine, and Seiberg (1985), by gauging a maximal global symmetry and adding matter to cancel all anomalies with the most general superpotential.

• $SU(2N+1)\times U(1)$ with

$$A\sim(A,1),\ F\sim(\Box,2),\ \overline{F}_i\sim(\overline{\Box},1),\ D\sim(1,2), \tag{113}$$

where $i=1,\dots,N-2$ (Dine *et al.*, 1996). To lift all flat directions, a nonrenormalizable superpotential is required:

$$W=\sum_{i,j=1}^{2N-2}\gamma_{ij}A\overline{F}_i\overline{F}_j+\lambda\overline{F}_{2N-1}FD$$
$$+\frac{1}{M}\sum_{i,j=1}^{2N-2}\alpha_{ij}\overline{F}_i\overline{F}_jFF. \tag{114}$$

• $SU(2N+1)\times SP(M)$, $N\geqslant M-1$ with

$$Q\sim(\Box,\Box),\ \overline{Q}_i\sim(\overline{\Box},1),\ L\sim(1,\Box), \tag{115}$$

where $i=1,\dots,2M$ is a flavor index (Dine *et al.*, 1996). These are generalizations of the 3–2 model with a nonrenormalizable superpotential.

The $SU(2N+1)$ dynamics generate a dynamical superpotential (Dine *et al.*, 1996). In addition, a quantum constraint is generated by the $SP(M)$ dynamics for $N=M$. The tree-level superpotential

$$W=\lambda\overline{Q}_2QL+\sum_{i,j>2}^{2M}\gamma_{ij}Q^2\overline{Q}_j\overline{Q}_j \tag{116}$$

lifts all flat directions, and supersymmetry is broken. For details see Dine *et al.* (1996) and Intriligator and Thomas (1996a).

For $N=M+1$, the tree-level superpotential (116) does not lift all classical flat directions, yet they are lifted by nonperturbative effects (Intriligator and Thomas, 1996a; Shirman, 1996) and SUSY is broken. We discussed this model in Sec. VI.B. [Note that in that section, we used a different notation for SP and referred to this theory as $SU(N-1)\times SP(N/2)$.]

It is also useful to note that for $M+1<N$, the $SP(M)$ dynamics can have a dual description. Intriligator and Thomas (1996b) argued that the dual description with $SU(2N+1)\times SP(N-M-1)$ gauge group and matter content, which includes the symmetric tensor of $SU(2N+1)$ as well as (anti)fundamentals and bifundamentals, breaks SUSY. When $N>3M+2$ it is only the dual description that is asymptotically free and can be interpreted as a microscopic theory.

An interesting modification of these models (Luty and Terning, 1998) is an $SU(2N+1)\times SP(N+1)$ theory with

$$Q\sim(\Box,\Box),\ \overline{Q}_i\sim(\overline{\Box},1),\ L_a\sim(1,\Box), \tag{117}$$

where $i=1,\dots,2(N+1)$ and $a=1,\dots,2N+1$. We note that this version of the model possesses an $SU(2N+1)\times U(1)\times U(1)_R$ global symmetry. Luty and Terning (1998) considered only the renormalizable superpotential

$$W=\lambda\overline{Q}QL, \tag{118}$$

where λ has maximal rank. They showed that while this model has a large number of classical flat directions, all of them are lifted quantum mechanically. One can now add mass terms for some flavors of the $SP(N+1)$ fields L_a. For an appropriately chosen mass matrix, supersymmetry remains broken. Choosing a mass matrix of maximal rank and integrating out the massive matter, we recover the nonrenormalizable model discussed above.

• Nonrenormalizable $SU(2N)\times U(1)$ model (Dine *et al.*, 1996) with chiral superfields transforming under the gauge group and a global $SU(2N-4)$ symmetry as

$$A\sim(\begin{smallmatrix}\Box\\\Box\end{smallmatrix},2N-4,1),\ \overline{F}\sim[\overline{\Box},-(2N-2),\overline{\Box}],$$
$$S\sim(1,2N,\Box\Box). \tag{119}$$

The superpotential required to stabilize all flat directions,

$$W=A\overline{F}\overline{F}S, \tag{120}$$

explicitly breaks the global symmetry down to a subgroup. The anomaly-free $SU(N-2)$ subgroup of the global symmetry can be gauged without restoring SUSY.

• Nonrenormalizable $SU(N)\times U(1)$ models (Dine *et al.*, 1996). The matter content is [we also give charges under a maximal global $SU(N-3)$ symmetry]

$$A\sim(\begin{smallmatrix}\Box\\\Box\end{smallmatrix},2-N,1),\ N\sim(M,1,1),\ \overline{N}_i\sim(\overline{\Box},N-1,\overline{\Box}),$$
$$S_i\sim(1,-N,\Box)),\ S_{ij}\sim(1,-N,\begin{smallmatrix}\Box\\\Box\end{smallmatrix}), \tag{121}$$

where i, $j=1,\dots,N-3$. The superpotential

$$W=\lambda_i\overline{N}_iNS_i+\gamma_{ij}A\overline{N}_i\overline{N}_jS_{ij} \tag{122}$$

lifts all flat directions while preserving a global symmetry. Note that for $N=4$, S_{ij} does not exist and this is just the 4–1 model of Sec. IV.B.

• $SU(N)\times SU(N-1)$ (Poppitz, Shadmi, and Trivedi, 1996a) and $SU(N)\times SU(N-2)$ (Poppitz, Shadmi, and Trivedi, 1996b). The $SU(N)\times SU(N-1)$ models were discussed in Sec. VI.B. We discussed SUSY breaking without R symmetry in these models in Sec. VI.C. The $SU(N)\times SU(N-2)$ models, which are similar by construction but have very different dynamics, are discussed in Sec. V.B. Both classes of models have calculable minima with an unbroken global symmetry [$SU(N-2)$ and $SP(N-3)$, respectively; Poppitz and Trivedi, 1997; Shadmi, 1997; Arkani-Hamed *et al.*, 1998].

• Intriligator-Thomas-Izawa-Yanagida models (Intriligator and Thomas, 1996a; Izawa and Yanagida, 1996) and their modifications. We discussed these models in Sec. VI.A.2. They are based on an $SU(N)$ [$SP(N)$] gauge group with N ($N+1$) flavors of matter in the fundamental representation coupled to a set of gauge-singlet fields in such a way that all D-flat directions are lifted. Even after supersymmetry breaking these models possess a flat direction that is only lifted by (perturbative) corrections to the Kähler potential. In Shirman (1996), Arkani-Hamed and Murayama (1998), and Dimopoulos *et al.* (1998) it was argued that the perturbative corrections generate a growing potential for large VEV's along this direction. By gauging a subgroup of the global symmetry it is possible to obtain a modification of the model with a calculable local SUSY-breaking minimum at large VEV's (Murayama, 1997; Dimopoulos *et al.*, 1998). These models can be generalized in the following way. Take any model with a quantum modified constraint and couple all gauge-invariant operators to singlet fields. Since the quantum constraint becomes incompatible with the singlet F-term conditions, supersymmetry must be broken (Csaki, Schmaltz, and Skiba, 1997b). As an example consider an $SO(7)$ gauge group with five matter multiplets transforming in the spinor representation. The theory possesses an $SU(5)$ global symmetry and a $U(1)_R$ under which all matter fields are neutral. The gauge invariant composites transform as an antisymmetric tensor A and antifundamental $\overline{F}$ of the global symmetry. The quantum constraint is

$$A^5+A\overline{F}^4=\Lambda^{10}. \tag{123}$$

Coupling all gauge invariants to gauge singlets $\tilde{A}$ and F and implementing the constraint through the Lagrange multiplier λ we find

$$W=\tilde{A}A+F\overline{F}+\lambda(A^5+A\overline{F}^4-\Lambda^{10}), \tag{124}$$

and obviously SUSY is broken. We note that the model is nonrenormalizable.

• $SU(2)$ with one $I=3/2$ matter field (Intriligator, Seiberg, and Shenker, 1995). We discussed this theory in Sec. V.A. A nonrenormalizable tree-level superpotential lifts all classical flat directions. The theory confines, no superpotential is generated dynamically, and supersymmetry is broken since the tree-level superpotential cannot be extremized in terms of the confined field.

• $SU(7)$ with two symmetric tensors, S_a, $a=1,2$, six antifundamentals $\overline{Q}_i$, $i=1,\ldots,6$, and the tree-level superpotential

$$W=\sum_i^3 S^1\overline{Q}_{2i}\overline{Q}_{2i-1}+S^2\overline{Q}_{2i}\overline{Q}_{2i+1}, \tag{125}$$

where in summing over i we identify $7\sim 1$ (Nelson and Thomas, 1996; Csaki, Schmaltz, and Skiba, 1997b). This is another example of SUSY breaking through confinement, which we saw in the Intriligator-Seiberg-Shenker (1995) model in Sec. V.A. The superpotential lifts all classical flat directions while preserving a global anomaly-free $U(1)\times U(1)_R$ symmetry. $SU(7)$ dynamics lead to confinement, and generates the nonperturbative superpotential

$$W_{dyn}=\frac{1}{\Lambda^{13}}H^2N^2, \tag{126}$$

where $H^a_{ij}=S^a\overline{Q}_i\overline{Q}_j$ and $N_i=S^4\overline{Q}_i$. Near the origin of the moduli space the Kähler potential is canonical in terms of the composite fields. Solving the equations of motion for H^a_{ij} one finds that at least some of the composite fields acquire VEV's, breaking the global symmetry and therefore supersymmetry.

• $SO(12)\times U(1)$ and $SU(6)\times U(1)$ (Csaki, Schmaltz, and Skiba, 1997b). The matter content of the $SO(12)\times U(1)$ model is (32,1), (12,-4), (1,8), (1,2), (1,6). The matter content of the $SU(6)\times U(1)$ model is (20,1), (6,-3), ($\overline{6}$,-3), (1,4), (1,2). These models are constructed by starting with a nonchiral theory with a dynamical superpotential and gauging a global $U(1)$ symmetry (adding the necessary fields to make the full theory anomaly free) in a way that makes the theory chiral. Supersymmetry is broken by the interplay between a dynamically generated superpotential and the tree-level superpotential.

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APPENDIX: SOME RESULTS ON SUSY GAUGE THEORIES

In this Appendix we shall briefly review some results in supersymmetric gauge theories. Our main goal here is to introduce notations [which will mainly follow those of Bagger and Wess (1991)], and to summarize the results necessary to make the present review self-contained. Much more detailed reviews of the progress in our understanding of supersymmetric gauge theories exist in the literature, e.g., Intriligator and Seiberg (1996), Peskin (1997), and Shifman (1997).

1. Notations and superspace Lagrangian

We shall consider an effective low-energy theory of light degrees of freedom well above the possible scale of supersymmetry breaking. In this case the effective action will have linearly realized supersymmetry, and it is convenient to write an effective supersymmetric Lagrangian in $\mathcal{N}=1$ superspace, where four (bosonic) space-time coordinates are supplemented by four anticommuting (fermionic) coordinates θ_α and $\overline{\theta^\alpha}$, $\alpha=1,2$. The light matter fields combine into chiral superfields[32]

$$\Phi=\phi+\sqrt{2}\theta\psi+\Theta^2 F, \tag{A1}$$

while gauge bosons and their superpartners combine into vector superfields

$$V=-\theta\sigma^\mu\overline{\theta}A_\mu+i\theta^2\overline{\theta\lambda}-i\overline{\theta}^2\theta\lambda+\frac{1}{2}\theta^2\overline{\theta}^2D, \tag{A2}$$

where we have used the Wess-Zumino gauge.

The effective supersymmetric Lagrangian for a theory with gauge group G and matter fields Φ_i transforming in the representation r of the gauge group (with T^a being a generator of this representation) can be written as

$$\mathcal{L}=\int d^4\theta K(\Phi^\dagger,e^{V\cdot T}\Phi)+\frac{1}{g^2}\int d^2\theta\mathcal{W}^\alpha\mathcal{W}_\alpha+\text{H.c}$$
$$+\int d^2\theta W(\Phi)+\text{H.c.} \tag{A3}$$

The first term in Eq. (A3) is a Kähler potential that contains, among others, kinetic terms for the matter fields. The Kähler potential also contributes gauge-interaction terms to the scalar potential. The second term in Eq. (A3) is the kinetic term for the gauge fields. In particular, $\mathcal{W}^\alpha=-\frac{1}{4}\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_\alpha V$, where $\mathcal{D}$ is a superspace derivative, is a supersymmetric generalization of the gauge field strength $F^{\mu\nu}$. The last term in Eq. (A3) is the superpotential.

The superpotential is a holomorphic function of chiral superfields and obeys powerful nonrenormalization theorems. In particular, in perturbation theory the superpotential can only be modified by field rescalings (which can be absorbed into renormalization of the Kähler potential). Using holomorphy, symmetries of the theory, and known weakly coupled limits it is often possible to determine the superpotential exactly, including all nonperturbative effects. Similarly, the kinetic term for the gauge multiplet is a holomorphic function allowing one to obtain exact results on the renormalization of the gauge coupling.

The Kähler potential, on the other hand, can be a general real-valued function of $\Phi^\dagger$ and Φ consistent with symmetries. Classically it is given by

$$K=\Phi^\dagger e^{V\cdot T}\Phi, \tag{A4}$$

but quantum mechanically it is renormalized both perturbatively and nonperturbatively.

In studying the dynamical behavior of the supersymmetric theory it is often useful to remember that the Hamiltonian is determined by the supersymmetry generators

$$H=\frac{1}{4}(\overline{Q}_1Q_1+Q_1\overline{Q}_1+\overline{Q}_2Q_2+Q_2\overline{Q}_2). \tag{A5}$$

From Eq. (A5) we see that a supersymmetric vacuum state (a state annihilated by the supersymmetry charges) has vanishing energy. Therefore a particularly important role (especially in the analysis of supersymmetry breaking) is played by the scalar potential of the theory

$$V=\frac{1}{2}g^2\sum_a (D^a)^2+F_i^\dagger g_{ij}^{-1}F_j, \tag{A6}$$

where $g_{ij}=\partial^2K/\partial\Phi^{\dagger i}\partial\Phi^j$, and the auxiliary fields F and D are given in terms of the scalar fields by

$$F_i=\frac{\partial}{\partial\Phi_i}W,$$
$$D^a=\sum_i \Phi^{\dagger i}t^a\Phi^i, \tag{A7}$$

where in F_i one takes the derivatives of the superpotential with respect to the different superfields and then keeps only the lowest component, and in D^a, Φ^i stands for the scalar field of the Φ^i supermultiplet.

Typically a supersymmetric gauge theory possesses a set of directions in field space (called D-flat directions) along which $D^a=0$ for all a. Along some or all of these D-flat directions, the F-flatness conditions, $F_i=0$, can also be satisfied. The subspace of field space in which the scalar potential vanishes is called a moduli space and to a large degree determines the low-energy dynamics.

The study of nonperturbative effects in SUSY gauge theories relies heavily on the use of symmetries. An important role, especially in the applications to dynamical supersymmetry breaking, is played by an "R symmetry." We therefore pause to introduce this symmetry. Under an R symmetry, the fermionic coordinates rotate as

$$\theta\to e^{i\alpha}\theta. \tag{A8}$$

A chiral field with R charge q transforms under this symmetry as follows:

$$\Phi(x,\theta,\overline{\theta})\to e^{-iq\alpha}\Phi(x,e^{i\alpha}\theta,e^{-iq\alpha}\overline{\theta}). \tag{A9}$$

Note that different component fields transform differently under R symmetry, and thus it does not commute with supersymmetry. On the other hand, the vector superfield V is neutral under R symmetry (therefore the gaugino transforms as $\lambda\to e^{-i\alpha}\lambda$). Clearly there always exists an assignment of R-symmetry charges to the superfields such that the Kähler potential contribution to the action is invariant under R symmetry. On the other hand, the superpotential contributions to the action explicitly break R symmetry unless the superpotential has charge 2 under R symmetry.

In the following subsections we shall discuss methods for determining the classical moduli space. We shall also describe the quantum behavior of supersymmetric QCD with various choices of the matter content. At the end of this section we shall comment on analogous results for

[32]We shall usually use the same notation for a chiral superfield and its lowest scalar component.

models with different gauge groups and matter content. These results will provide us with the tools needed to analyze supersymmetry breaking.

2. D-flat directions

Classically, one could set all superpotential couplings to zero. Then the moduli space of the theory is determined by D-flatness conditions. Even when tree-level superpotential couplings are turned on but remain small, the vacuum states of the theory will lie near the solutions of D-flatness conditions (still in the classical approximation). It is convenient, therefore, to analyze SUSY gauge theories in two stages. First find a submanifold in the field space on which the D terms vanish, and then analyze the full theory, including both tree-level and nonperturbative contributions to the superpotential.

We start by describing a useful technique for finding the D-flat directions of a theory (Affleck, Dine, and Seiberg, 1984c, 1985) with $SU(N)$ gauge symmetry. Consider the $N\times N$ matrix

$$D^i_j=\phi^{\dagger l}(A^i_j)^k_l\phi_k, \tag{A10}$$

where $(A^i_j)^k_l$ are the real generators of $GL(N)$. For ϕ in the fundamental representation $(A^i_j)^k_l=\delta^i_l\delta^k_j$ [the generalization of (A^i_j) for a general multiindex representation is obvious]. It is easy to see that the vanishing of all D^a's is equivalent to the requirement that D^i_j be proportional to the unit matrix, $D^i_j\sim\delta^i_j$. To show this it is sufficient to note that $D^a=D^i_j\lambda^{aj}_i$, where λ^a are generators of $SU(N)$ in the fundamental representation.

Another way to parametrize the moduli space is by the use of gauge-invariant composite operators. It has been shown that a complete set of such operators is in one-to-one correspondence with the space of D-flat directions (Luty and Taylor, 1996). An important feature of this latter parametrization of the moduli space is that in some cases there exist gauge-invariant operators that vanish identically due to the Bose statistic of the superfields. For this reason they do not have counterparts in the "elementary" parametrization of the moduli space. However, due to quantum effects these operators typically describe light (composite) degrees of freedom of the low-energy theory and play an important role in the dynamics.

3. Pure supersymmetric $SU(N_c)$ theory

The Lagrangian of a pure supersymmetric Yang-Mills theory can be written as

$$\mathcal{L}=\frac{1}{4g_W^2}\int d^2\theta\mathcal{W}^\alpha\mathcal{W}_\alpha+\text{H.c.} \tag{A11}$$

The Wilsonian coupling constant in the Lagrangian can be promoted to a VEV of the background chiral superfield $1/g_W^2\rightarrow S=1/g_W^2-i(\Theta/8\pi^2)$. Since the physics is independent of shifts in Θ, the Wilsonian gauge coupling in Eq. (A11) receives corrections only at one loop. On the other hand, the gauge-coupling constant in the $1PI$ action receives contributions at all orders in perturbation theory. These two coupling constants can be related by field redefinitions (Shifman and Vainshtein, 1986, 1991). In the following we shall always use the Wilsonian action and work with Wilsonian coupling constants. We shall use functional knowledge of the exact beta functions only to establish the scaling dimensions of the composite operators in our discussion of duality in Appendix Sec. 7.

Supersymmetric Yang-Mills is a strongly interacting non-Abelian theory very much like QCD. In particular it is believed that it confines and develops a mass gap. By using symmetry arguments it is possible to show that if the gaugino condensate develops it has the form

$$\langle\lambda\lambda\rangle=\text{const}\times\Lambda^3_{SYM}=\text{const}\times\mu^3e^{-8\pi^2/N_cg^2}. \tag{A12}$$

In fact the constant can be exactly calculated (Novikov *et al.*, 1983; Shifman and Vainshtein, 1988). The theory has N_c supersymmetric vacuum states.

4. $N_f<N_c$: Affleck-Dine-Seiberg superpotential

As a next step one can consider an $SU(N_c)$ gauge theory with $N_f(<N_c)$ flavors of matter fields in the fundamental Q and antifundamental $\overline{Q}$ representations. This theory possesses a large nonanomalous global symmetry under which matter fields transform as follows:

	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
Q	N_f	1	1	$\frac{N_f-N_c}{N_f}$
$\overline{Q}$	1	$\overline{N}_f$	-1	$\frac{N_f-N_c}{N_f}$.

(A13)

Classically there are D-flat directions along which the scalar potential vanishes. Using the techniques described above we can parametrize these flat directions (up to symmetry transformations) by

$$Q=\begin{pmatrix} v_1 & & & \\ & v_2 & & \\ & & \cdots & \\ & & & v_{N_f} \\ \dots & \dots & \dots & \dots \end{pmatrix}=\overline{Q}. \tag{A14}$$

These flat directions can also be parametrized by the VEV's of the gauge-invariant operators $M_{ij}=Q_i\overline{Q}_j$. These composite degrees of freedom give a better (weakly coupled) description near the origin of moduli space where the theory is in a confined regime.

In this model a unique nonperturbative superpotential is allowed by the symmetries (Affleck, Dine, and Seiberg, 1984a)

$$W_{dyn}=\left(\frac{\Lambda^{3N_c-N_f}}{\det(Q\overline{Q})}\right)^{1/(N_c-N_f)}, \tag{A15}$$

where Λ is the renormalization group-invariant scale of the theory. It has been shown (Affleck, Dine, and Seiberg, 1984a; Cordes, 1986) that this superpotential is in fact generated by instanton effects for $N_f = N_c - 1$. It is generated by gaugino condensation in all other cases.

Before proving this last statement, let us pause for a moment and discuss the relation between the renormalization-group-invariant scales of the microscopic and effective theories. Suppose the microscopic theory is $SU(N_c)$ with N_f flavors. As has been mentioned above, the Wilsonian coupling (Shifman and Vainshtein, 1986, 1991) of the theory runs only at one loop

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M)} + \frac{b_0}{16\pi^2} \ln\left(\frac{\mu}{M}\right). \tag{A16}$$

Suppose also that at a scale v some fields in the theory become massive, and the physics below this scale is described by an $SU(N_c')$ gauge group with N_f' flavors. The Wilsonian coupling of the effective theory is

$$\frac{1}{g_L^2(\mu)} = \frac{1}{g_L^2(M)} + \frac{\tilde{b}_0}{16\pi^2} \ln\left(\frac{\mu}{M}\right). \tag{A17}$$

In Eqs. (A16) and (A17) $b_0 = 3N_c - N_f$ and $\tilde{b}_0 = 3N_c' - N_f'$ are the β-function coefficients. But the couplings should be equal at the scale $\mu = v$. This allows us to derive scale-matching conditions. For example, take an $SU(N_c)$ theory with N_f flavors. Its renormalization-group-invariant scale is given by

$$\Lambda^{3N_c - N_f} = \mu^{3N_c - N_f} \exp\left(-\frac{8\pi^2}{g^2}\right). \tag{A18}$$

If one of the matter fields is massive with mass $m \gg \Lambda$, the effective theory has $N_f - 1$ flavors and its scale is

$$\Lambda_L^{3N_c - N_f + 1} = \mu^{3N_c - N_f + 1} \exp\left(-\frac{8\pi^2}{g^2}\right). \tag{A19}$$

Requiring equality of couplings at the scale of the mass we find

$$\Lambda_L^{3N_c - N_f + 1} = m \Lambda^{3N_c - N_f}. \tag{A20}$$

In a general case the equation above becomes

$$\Lambda_L^{\tilde{b}_0} = v^{\tilde{b}_0 - b_0} \Lambda^{b_0}, \tag{A21}$$

where v represents a generic VEV and/or mass in the theory.

Now in the $N_f = N_c - 1$ theory with small masses $m \ll \Lambda$ an instanton calculation (Affleck, Dine, and Seiberg, 1984a; Cordes, 1986) is reliable and gives Eq. (A15). Due to the holomorphicity of the superpotential the result can be extrapolated into the region of moduli space where one flavor, say N_fth, is heavy, $m_{N_f N_f} \gg \Lambda$. It decouples from the low-energy effective theory. Solving the equations of motion for the heavy field and using the scale-matching condition (A21) one finds the superpotential (A15) for the effective theory with $N_f = N_c - 2$ flavors. In the low-energy effective theory this superpotential can be interpreted as arising from gaugino condensation. One can continue this procedure by induction and not only derive the superpotential for arbitrary $N_f < N_c$ but also fix the numerical coefficient in front of the superpotential. (This coefficient can be absorbed into the definition of Λ, and we shall set it to 1 most of the time.)

Even though the classical flat directions are lifted in the massless theory by the superpotential (A15), the scalar potential

$$V = \sum_i \left(\left| \frac{\partial W}{\partial Q_i} \right|^2 + \left| \frac{\partial W}{\partial \overline{Q}} \right|^2 \right) \tag{A22}$$

tends to zero as $Q = \overline{Q} \to \infty$ and as a result the theory does not possess a stable vacuum state. In our discussion of DSB we discuss examples in which flat directions may be not only lifted but stabilized due to nonperturbative effects (see Sec. VI.B).

One could lift classical flat directions by adding a mass term to the superpotential

$$W_{tree} = m_{ij} Q_i \overline{Q}_j. \tag{A23}$$

Note that this superpotential explicitly breaks the $U(1)_R$ symmetry. In Sec. III.B we argue that this is often a signal of unbroken supersymmetry. It is easy to find the supersymmetric vacua in this model. In terms of the meson fields they are given by

$$M_{ij} = [\det(m) \Lambda^{3N_c - N_f}]^{1/N_c} \left(\frac{1}{m_{ij}}\right). \tag{A24}$$

It is also worth noting that if some number of matter fields are massive they decouple from the low-energy theory and can be integrated out. Solving the equations of motion for the massive fields one can find the superpotential of Eq. (A15) and the solution (A24) for the VEV's of the remaining light fields.

5. $N_f = N_c$: quantum moduli space

Additional flat directions exist for $N_f = N_c$. The most general expression for the flat directions is

$$Q = \begin{pmatrix} a_1 & & & & \cdots \\ & a_2 & & & \cdots \\ & & \cdots & & \cdots \\ & & & a_{N_c} & \cdots \end{pmatrix}$$

$$\overline{Q} = \begin{pmatrix} b_1 & & & & \cdots \\ & b_2 & & & \cdots \\ & & \cdots & & \cdots \\ & & & b_{N_c} & \cdots \end{pmatrix} \tag{A25}$$

subject to the condition

$$|a_i|^2 - |b_i|^2 = v^2. \tag{A26}$$

As was mentioned above, the flat directions can be parametrized by VEV's of gauge-invariant polynomials. In this case new flat directions can be represented by fields with the quantum numbers of baryons[33] $B = Q^N$ and an-

[33]Summation over both color and flavor indices is implied.

tibaryons $\overline{B}=\overline{Q}^N$. Note, however, that due to the Bose statistics of the superfields, the gauge-invariant polynomials obey the constraint classically,

$$\det M - B\overline{B} = 0. \tag{A27}$$

Seiberg (1994, 1995) showed that this constraint is modified quantum mechanically:

$$\det(M) - B\overline{B} = \Lambda^{2N}. \tag{A28}$$

We refer the reader to Seiberg (1994), Intriligator and Seiberg (1996), Peskin (1997), and Shifman (1997) for a detailed explanation of this result.

It is often convenient to enforce this quantum-mechanical constraint by introducing a Lagrange multiplier term in the superpotential

$$W = A[\det(M) - B\overline{B} - \Lambda^{2N}] + m_{ij}M_{ij}. \tag{A29}$$

Once again the validity of this superpotential can be verified in the limit that some of the matter fields are heavy and decouple from the low-energy theory. Integrating them out leads to the superpotential (A15) for the light matter.

Naively, the Kähler potential of the $N_f = N_c$ theory is singular at the origin. This corresponds to the fact that at the origin, the full gauge group $SU(N_c)$ is restored and additional degrees of freedom become massless. This singular point, however, does not belong to the quantum moduli space. $SU(N_c)$ cannot be restored because of the constraint (A28), and the Kähler potential in terms of composite degrees of freedom is nonsingular. In the infrared mesons and baryons represent a good description of the theory. One of many nontrivial tests they pass is 't Hooft anomaly-matching conditions ('t Hooft, 1980). Far from the origin the quantum moduli space is very close to classical one and the elementary degrees of freedom should represent a good (weakly coupled) description of the theory.

6. $N_f = N_c + 1$

In this case there are N_f baryons and antibaryons transforming under the global $SU(N_f)_L \times SU(N_f)_R$ as $(N_f,1)$ and $(1,\overline{N}_f)$, respectively. Classically, the gauge invariants obey the constraints

$$\det(M) - B_i M_{ij}\overline{B}_j = 0,$$
$$B_i M_{ij} = M_{ij}\overline{B}_j = 0. \tag{A30}$$

These constraints are not modified quantum mechanically. One can easily see this by adding a tree-level superpotential $W_{tree} = \Sigma_{ij} m_{ij} M_{ij}$. Holomorphy guarantees that meson VEV's are given by Eq. (A24). Taking various limits of the mass matrix one can see that the mesons M_{ij} can have any values on the moduli space. This can also be shown for the baryons.

In terms of the elementary fields the Kähler potential is singular at the origin, reflecting the fact that $SU(N_c)$ is restored there and additional degrees of freedom become massless. In terms of composite degrees of freedom the Kähler potential is regular, and they represent a suitable infrared description of the theory. As in the case $N_f = N_c$, 't Hooft anomaly-matching conditions are satisfied by the effective description. In this model, the constraints can be implemented by the superpotential

$$W = \frac{1}{\Lambda^{2N_c - 1}}(B_i M_{ij}\overline{B}_j - \det M). \tag{A31}$$

Adding mass for one flavor correctly leads to the $N_f = N_c$ model.

7. $N_f > N_c + 1$: dual descriptions of the infrared physics

We shall start from the case $\frac{3}{2}N_c < N_f < 3N_c$. This theory flows to an infrared fixed point (Seiberg, 1995). Seiberg (1995) suggested that in the vicinity of the infrared fixed point the theory admits a dual, "magnetic," description with the same global symmetries but in terms of a theory with a different gauge group. This theory is based on the gauge group $SU(N_f - N_c)$ with N_f flavors of q and $\overline{q}$ transforming as fundamentals and antifundamentals, respectively, as well as gauge-singlet fields M, corresponding to the mesons of the original ("electric") theory. The global-symmetry charges are given by

	$SU(N_f)_L$	$\times SU(N_f)_R$	$\times U(1)_B$	$\times U(1)_R$
q	$\overline{N}_f$	1	$\frac{N_c}{N_f - N_c}$	$\frac{N_c}{N_f}$
$\overline{q}$	1	N_f	$-\frac{N_c}{N_f - N_c}$	$\frac{N_c}{N_f}$
M	N_f	$\overline{N}_f$	0	$2\frac{N_f - N_c}{N_c}$. (A32)

The magnetic theory also flows to a fixed point. However, in the magnetic theory a tree-level superpotential is allowed by symmetries

$$W = Mq\overline{q}. \tag{A33}$$

In the presence of this superpotential the theory flows to a new fixed point, which is identical to the fixed point of the "electric" theory.

At the fixed point the superconformal symmetry can be used to understand the behavior of the theory. For example, the scaling dimensions of the gauge-invariant operators are known. The exact beta function for the coupling in $1PI$ action (in the electric description) is given by (Novikov *et al.*, 1983; Shifman and Vainshtein, 1986, 1991)

$$\beta(g) = \frac{g^2}{16\pi^2}\frac{3N_c - N_f + N_f\gamma(g^2)}{1 - N_c(g^2/8\pi^2)}$$
$$\gamma(g) = -\frac{g^2}{16\pi^2}\frac{N_c^2 - 1}{N_c} + \mathcal{O}(g^4). \tag{A34}$$

At the zero of the β function the anomalous dimension is $\gamma = -3N_c/N_f + 1$, and one finds

$$D(Q\overline{Q}) = 2 + \gamma = 3\frac{N_f - N_c}{N_f}. \tag{A35}$$

The dimension of the baryon operators can be determined by exploiting the R symmetry (Seiberg, 1995)

$$D(B)=D(\overline{B})=\frac{3N_c(N_f-N_c)}{2N_f}. \quad (A36)$$

This allows one to determine the scaling of the Kähler potential near the fixed point both in the electric and in the magnetic description

$$K_e \sim (Q\overline{Q})^{2N_f/3(N_f-N_c)}, \quad (A37)$$
$$K_m \sim (q\overline{q})^{2N_f/3N_c}.$$

Let us summarize the correspondence between the electric and magnetic theories:

$$M_{ij}=Q_i\overline{Q}_j \to M_{ij},$$
$$W=m_{ij}M_{ij} \to W=m_{ij}M_{ij}+M_{ij}q_i\overline{q}_j,$$
$$b,\overline{b} \to B,\overline{B}. \quad (A38)$$

By performing a second duality transformation one can verify that in fact the magnetic meson is identified with the composite electric meson through the equations of motion.

The scales of the electric and the magnetic theories are related by

$$\Lambda^{3N_c-N_f}\tilde{\Lambda}^{3(N_f-N_c)-N_f}=(-1)^{N_f-N_c}\mu^{N_f}, \quad (A39)$$

where the scale μ is needed to map the composite electric meson $Q\overline{Q}$ into an elementary magnetic meson M. These fields have the same dimension at the infrared fixed point, but different dimensions in the ultraviolet.

If the number of flavors is $N_c+1<N_f<\frac{3}{2}N_c$ one can construct a dual description in a similar way. In that case only the electric description is asymptotically free and makes sense in the ultraviolet.

8. Other models

There are numerous generalizations of the results presented in previous subsections to theories with different gauge groups and matter fields. Here we mention some of the generalizations that will be useful for our discussion of supersymmetry breaking.

Results analogous to those for supersymmetry QCD can be found for $SP(N)$ theories with N_f flavors of matter fields transforming in the fundamental representation[34] (Intriligator and Pouliot, 1995). The one-loop β-function coefficient is given by

$$b_0=3(N_c+1)-N_f. \quad (A40)$$

This can be compared to the one-loop β function of the $SU(N)$ models given in Eq. (A16). In fact, one can find many of the results for $SP(N)$ theories by making the substitution $N_c \to N_c+1$ in the expressions for $SU(N)$ models (and rewriting determinants as Pfaffians). The theory does not have baryons for any number of flavors. The supersymmetric vacuum is given by

$$M_{ij}=Q_iQ_j=[\mathrm{Pf}(m)\Lambda^{3(N_c+1)}]^{1/(N_c+1)}\left(\frac{1}{m_{ij}}\right). \quad (A41)$$

We can easily see that for $N_f=N_c+1$, the quantum constraint is different from the classical one:

$$\mathrm{Pf}(M)=\Lambda^{2(N_c+1)}. \quad (A42)$$

If the number of flavors is $N_f>N_c+2$ there is a dual description analogous to the one for the $SU(N)$ theories.

We conclude this section by mentioning several other classes of models that are useful in the study of supersymmetry breaking. Poppitz and Trivedi (1996) studied quantum moduli space and exact superpotentials in $SU(N)$ gauge theories with matter in the antisymmetric tensor, fundamental, and antifundamental representations. In the case without fundamental fields Pouliot (1996) has constructed a dual by using Berkooz's (1996) deconfining trick. In Poppitz, Shadmi, and Trivedi (1996a) duality was studied in the product group theories and it was shown that dual models can be constructed by using single group duality.

[34] In our notation $SP(1)=SU(2)$, and N_f flavors correspond to $2N_f$ fields.

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