Superfluidity

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The original observation of the phenomenon, or more precisely the complex of phenomena, known as "superfluidity" was made simultaneously in liquid 4-He in 1938 by two groups, Kapitza in Moscow and Allen and Misener in Cambridge. It had been known for some years previously that liquid helium (which, until the early 1950s when the light isotope 3-He began to be produced in experimentally useful quantities from nuclear reactors, was synonymous with liquid 4-He) did not freeze under its own vapor pressure down to the lowest attainable temperatures, and during the early- and mid-1930s it had become clear that some peculiar things happened at and below a characteristic temperature (~2.17 K), which became known as the "lambda temperature." Stimulated by measurements that seemed to show that below the lambda temperature the heat flow was not simply proportional to the temperature gradient, Allen and Misener, and simultaneously Kapitza, decided to measure the resistance to the flow of liquid helium clamped in narrow channels and subjected to a pressure drop. They found that while the so-called He-I phase, i.e., helium above the lambda temperature, showed a behavior that could be described in terms of a conventional viscosity, below the lambda point (in the so-called He-II phase) the liquid flowed so easily that if the concept of viscosity was applicable at all, it would have to be at least a factor of 1500 smaller than in the He-I phase. It was this anomalous behavior for which Kapitza coined the term "superfluidity." Actually, as we shall see below, this "ability to flow without apparent friction" in the kind of geometry employed in the Moscow and Oxford experiments, while spectacular, is not the conceptually simplest manifestation of superfluidity.

Within a few months of the experimental observation Fritz London came up with a qualitative explanation that has stood the test of time. The He atom is composed of an even number of elementary particles (2 protons, 2 neutrons, and 2 electrons) and thus according to the general precepts of quantum field theory, the many-body wave function of the system should be symmetric under the exchange of any two atoms; in technical language, the system should obey "Bose statistics." Fourteen years earlier Albert Einstein had studied the thermodynamic behavior of a gas of noninteracting atoms of this type, and had shown that below a characteristic temperature, which depends on the mass and density, it should manifest a peculiar behavior, which is nowadays known as Bose-Einstein condensation (BEC); a finite fraction of all the atoms (and at zero temperature, all of them) should occupy a *single* one-particle state. At the time Einstein made this suggestion this behavior was widely suspected of being a pathology of the noninteracting gas, which would disappear as soon as the interatomic interactions were taken into account. However, London now resurrected it and, noting that for a noninteracting gas with the mass and density of 4-He, the BEC phenomenon would occur at 3.3 K, suggested that this was exactly what was going on at the observed lambda transition (2.17 K). Very soon thereafter Laszlo Tisza pushed the idea further by suggesting that the anomalous flow behavior seen in the He-II phase could be qualitatively understood in terms of a "two-fluid" model in which the "condensate" (that is, those atoms which occupy the "special" one-particle state) behaves completely without friction, while the rest behave qualitatively like an ordinary liquid. One striking prediction that he was able to make on this basis was of a new type of collective excitation in which the two components—the condensate and the rest—oscillate out of phase.

A major landmark in the history of superfluidity was the appearance in 1941 of a paper by Lev Landau in which he developed in a quantitative way the "two-fluid" description of liquid He-II. (It seems likely that because of wartime conditions, Landau was unaware of Tisza's earlier, more qualitative work.) It is interesting that in this paper Landau never explicitly introduced the idea of BEC (indeed, he seems to have been opposed to it, regarding it as a pathology of the noninteracting gas), but rather posited, on intuitive grounds, various properties of the "ground state" of a Bose liquid, which with hindsight can in fact be seen to be natural consequences of the BEC phenomenon (see below). This paper marks the first explicit introduction into condensed-matter physics of the seminal notion of a "quasiparticle," that is, an excitation of the system from the ground state, which is characterized by a definite energy and momentum, and such that, at least at sufficiently low temperatures, the total energy, momentum, etc., of the system can be regarded as the sum of that carried by the quasiparticles. Landau identified the quasiparticles of a Bose liquid as of two types: quantized sound waves or phonons, with an energy ε, which depends on momentum p as $\varepsilon = cp$ (c = speed of sound), and "rotons," which he regarded as corresponding to quantized rotational motion and to which he originally assigned an energy spectrum $\varepsilon(p) = \Delta + p^2/2\mu$ (later modified, see below). An immediate prediction of this ansatz was that in the limit of low temperatures ($T \ll \Delta$) the rotons give negligible contribution to the specific heat, which in this regime is entirely due to the phonons and is proportional to T^3 (just as in an ordinary insulating crystalline solid).

To construct a quantitative theory of the flow properties of He-II, Landau postulated that it consisted of two components: the "superfluid" component, which he identified, in an intuitive way, with the part of the liquid that remained in its ground state, and a "normal" component, which corresponded to the quasiparticles. The superfluid component was conceived as carrying zero entropy and flowing irrotationally (i.e., its velocity v_s satisfied the condition curl $v_s = 0$); by contrast, the normal component behaved like any other viscous liquid. From these apparently minimal postulates Landau was able to derive a complete, quantitative theory of two-fluid hydrodynamics. It made, in particular, three remarkable predictions: (1) If the liquid (or more precisely the superfluid component of it) flows relative to the walls of the vessel containing it at a velocity smaller than velocity v_c (nowadays known as the Landau critical velocity) given by the minimum value of $\varepsilon(p)/p$ (usually this is the speed of sound c), then it may be able to do so without dissipation; otherwise the flow will be unstable against creation of quasiparticles. (2) If the boundary conditions rotate slowly (as, for example, in a rotating bucket), then only the fraction ρ_n of the liquid which corresponds to the normal component will rotate with them; Landau gave a formula for ρ_n in terms of the excitation spectrum. (3) It should be possible (as had also been suggested by Tisza) to set up an oscillation (nowadays known as "second sound") in which the normal and superfluid components oscillate out of phase; we now know (though Landau originally did not) that in liquid helium such a wave corresponds to substantial oscillations in temperature but only a very slight variation in pressure. Predictions (2) and (3) were verified within a few years in experiments carried out in the Soviet Union, by Andronikashvili and by Peshkov, respectively; prediction (3), though of fundamental importance conceptually, proved much more difficult to verify explicitly, and it is only comparatively recently that a direct measurement of the Landau critical velocity has been made, with the flow in question being relative not to the walls of the vessel but to ions moving through it (arguably the only case to which Landau's argument actually applies in its original form without a string of caveats).

While Landau's two-fluid hydrodynamics provides a conceptual basis for superfluidity, which still stands today, it is phenomenological in the sense that both the properties of the superfluid and the nature of the excitation spectrum are postulated in an intuitive way rather than being explicitly demonstrated to be a consequence of the Bose statistics obeyed by the atoms. This lacuna was partially filled in 1946 in a paper by N. N. Bogoliubov, which may for practical purposes be taken as ushering in the area of research known today as the "many-body problem." Bogoliubov considered a dilute gas of atoms obeying Bose statistics and interacting via an interatomic interaction, which is weakly repulsive. He *assumed* that such a system, like the completely free Bose gas, would undergo the phenomenon of BEC, and then, using a series of controlled approximations, was able to show that while the energy spectrum for large momentum p corresponds approximately to the simple excitation of free atoms from the condensate $[\epsilon(p) = p^2/2m]$, at smaller momenta it has precisely the phonon-like form $\epsilon(p) = cp$ postulated by Landau, where the velocity of sound c is derived from the bulk compressibility in the standard way. (However, in Bogoliubov's work there is no obvious trace of the second, "roton" branch of the excitation spectrum postulated by Landau.) This work was subsequently refined and extended by Lee, Huang, Yang, Girardeau and others, and actually turns out to be applicable in more or less its original form to the recently stabilized BEC alkali gases (see below).

While Bogoliubov's results were extremely suggestive, they referred to a dilute system, which is rather far from real-life liquid He-II (where the atoms are so closely packed as to be sampling both the attractive and the repulsive parts of the van der Waals interaction virtually all of the time). Thus a number of attempts were made to treat the realistic helium problem by variational or related methods; a particularly successful attack on the problem was made in 1956 by Feynman and Cohen on the basis of Feynman's earlier work. Among other things, this work predicted that the excitation spectrum of real liquid He-II should go over from the "phonon-like" behavior $\epsilon(p) = cp$ at small momenta predicted by Bogoliubov to a "roton-like" form $\epsilon(p) = \Delta + (p - p_0)^2/2m$, at larger values of the momentum. (This revision of his original hypothesis had actually been advanced a few years earlier by Landau himself, on the basis of experimental measurements of the temperature-dependence of the second-sound velocity.) Actually, in the early 1950s the use of reactor sources permitted for the first time experiments on the scattering of neutrons from various materials including liquid 4-He. The neutrons essentially measure the energy distribution of a particular kind of excitation, namely the density fluctuations, which have given momentum p; what is seen is that for a given p the energy is indeed approximately unique (so that the "quasiparticle" hypothesis indeed seems to be valid), and furthermore, that the spectrum has exactly the general form predicted by the Landau-Feynman-Cohen ansatz.

Rather than reviewing further in historical sequence the important advances made throughout the 50s, 60s, and 70s in the study of superfluid 4-He, it may be useful at this point to stand back and try to give a brief overview of our current understanding of the subject, bringing in the relevant experiments to illustrate them as we go. This understanding is, in some sense, a coherent amalgam of the ideas of London on the one hand, and Landau on the other, as refined and amplified by many subsequent workers. It should be remarked that these ideas developed in parallel with similar considerations concerning superconductivity, and indeed from a modern point of view superconductivity *is* nothing but superfluidity occurring in a charged system (or vice versa)—an idea which was extensively exploited by Fritz London in his 1950 two-volume book *Superfluids*, which covers both subjects.

The fundamental assumption that underlies the modern theory of superfluidity in a simple Bose system such as liquid 4-He is that the superfluid phase is characterized by what one might call "generalized BEC." By this I mean the following: we assume that at any given time t it is possible to find a complete orthonormal basis (which may itself depend on time) of single-particle states such that *one and only one* of these states is occupied by a finite fraction of all the particles, while the number of particles in any other single-particle state is of order 1 or less. (In technical language: at any given time the one-particle density matrix has exactly one eigenvalue N_0 which is of order N, while all the other eigenvalues are of order unity or less.) The corresponding single-particle wave function $\chi_0(r,t)$ is then called the "condensate wave function," and the N_0 particles occupying it, the "condensate." It is not necessary that the number N_0 be equal to the total number of particles N in the system, even at zero temperature, and indeed it seems almost certain that in real-life liquid 4-He, the $T=0$ condensate fraction N_0/N is only in the region of 10% (see below).

Just why this state of affairs should be realized is quite a subtle question. First, why should there be macroscopic $[O(N)]$ occupation of *any* single-particle state? The only case for which a totally rigorous argument can be given (at least to my knowledge) is the one originally considered by Einstein, namely a completely noninteracting gas in thermal equilibrium. While it can be shown that a calculation that starts from the BEC state of the noninteracting gas and does perturbation theory in the interatomic interactions leads to a finite value (generally less than 100%) of the condensate fraction in thermal equilibrium, there is no general proof that an arbitrary system of Bose particles must show BEC at $T=0$, and indeed the existence of the solid phase of 4-He is a clear counterexample to this hypothesis. Whether the crystalline solid and the Bose-condensed liquid exhaust the possible $T=0$ phases of such a system is, as far as I know, an open question. For nonequilibrium states the situation is even less clear.

An even trickier question is why, given that macroscopic occupation occurs, it occurs only in a *single* one-particle state. A relatively straightforward argument shows that, at least within the Hartree-Fock approximation, macroscopic occupation of more than one state is always energetically unfavorable *provided* the effective low-energy interaction is repulsive, as is believed to be the case for 4-He. For the case of an attractive interaction the problem is complicated by the fact that in the thermodynamic limit, as usually understood, ($N\to\infty, V\to\infty; N/V\to$const) the system is unstable against a collapse in real space; for the finite geometries which are of interest in the case of the alkali gases, the issue is, at this time, controversial. Also, even in the repulsive case, it is not entirely obvious that one can exclude "multiple condensates" in certain nonequilibrium conditions.

Given that BEC occurs in the sense defined above, i.e., that at any given time there exists *one and only one* single-particle state $\chi_0(r,t)$ that is macroscopically occupied, the conceptual basis for superfluidity is quite simple. We write $\chi_0(r,t)=|\chi_0(r,t)|\exp i\phi(r,t)$, and define the *superfluid velocity* $v_s(r,t)$ by the prescription

$$v_s(r,t)\equiv\frac{\hbar}{m}\nabla\phi(r,t). \tag{1}$$

This immediately leads to the result $\nabla\times v_s=0$, i.e., the "superfluid" flow is irrotational. Moreover, we observe that no "ignorance" is associated with the single state χ_0, and thus the entropy must be carried entirely by the "normal" component, i.e., the particles occupying single-particle states other than χ_0. (Obviously, this argument can be made more precise.) These two observations provide the basis for Landau's phenomenological two-fluid hydrodynamics. However, it should be emphasized that the "superfluid density" ρ_s, which occurs in the latter is, in general, *not* simply given by N_0/V, where N_0 is the number of particles condensed into χ_0; indeed, in the case of liquid 4-He, it is believed that as $T\to0$, ρ_s tends to the total density N/V, while N_0 remains only about 10% of N.

For a simply connected region of space in which $|\chi_0|$ is everywhere nonzero, the application of Stokes' theorem to the curl of Eq. (1) leads at once to the conclusion that the integral of v_s around any closed curve is zero. A more interesting application of Eq. (1) is to the case in which there is a line, or more generally, a region infinite in one dimension, on which $|\chi_0(r,t)|$ vanishes. This may happen either because the liquid is physically excluded from this region, as in the example considered below, or because, while atoms are present in the region in question, the particular single-particle state into which BEC has taken place happens to have a nodal line there. In either case we can consider the integral of Eq. (1) around a circuit that encloses the one-dimensional region in question, while we are no longer entitled to use Stokes' theorem to conclude that this integral is zero, the fact that the phase of the wave function χ_0 must be single-valued modulo 2π leads to the *Onsager-Feynman quantization condition*

$$\oint v_s\cdot dl=nh/m. \tag{2}$$

In a region of space which is, from a purely geometrical point of view, simply connected, Eq. (2) can be satisfied by a "vortex," that is, a pattern of flow in which $v_s\sim1/r$, where r is the perpendicular distance from the "core"; the singularity which formally appears at the core is physically irrelevant because by hypothesis $|\chi_0|$ vanishes there and thus v_s is not defined. The statics and dynamics of vortices is, of course, a subject that has been extensively studied in *classical* hydrodynamics; but in that case the circulation, while independent of path, can take any value and, in addition, vortices tend to be stable only under nonequilibrium conditions. By contrast, in a superfluid system the circulation is quantized according to Eq. (2) (it is actually found that the only values of n of interest are $n=\pm1$, since vortices with higher values of n are unstable against decay into these), and in addition, for reasons we shall see, vortices can be metastable, even under equilibrium conditions, for essentially astronomical times.

The most interesting application of Eq. (2), and the most clear-cut definition of the various phenomena which together constitute what we call superfluidity, occurs in a literally multiply connected geometry, let us say for definiteness the annular region between two concentric cylinders. In the following I consider such a geometry, with the mean radius of the annulus denoted R and its thickness d taken small compared to R; I neglect corrections of relative order d/R. The superfluid velocity $v_s(rt)$ is not itself a directly observable quantity, and in practice we are interested in the value of the mass current $J(rt)$. With Landau we argue that in (stable or metastable) equilibrium this quantity should be given by an expression of the form

$$J(r,t)=\rho_s v_s(rt)+\rho_n v_n(rt), \tag{3}$$

where the "superfluid" and "normal" densities ρ_s and $\rho_n \equiv \rho-\rho_s$ are functions only of temperature, and where the "normal velocity" $v_n(rt)$ is assumed to behave just like the velocity field of a normal (nonsuperfluid) liquid; in particular, in equilibrium v_n should be zero in the frame of reference in which the walls of the vessel are at rest. In the following I mean by the scalar quantities v, v_s, and J, the tangential (circumferential) components of the respective vectors.

Consider two different thought-experiments, in each of which the cylinders are rotated synchronously with angular velocity omega; we note from Eq. (2) that a natural unit in which to measure omega is the angular velocity corresponding to $n=1$, that is $\omega_c=\hbar/mR^2$. In the first experiment, we start with the liquid above the lambda-temperature T_λ, rotate the cylinders with some *small* angular velocity ω and wait for thermal equilibrium to be established. Since for $T>T_\lambda$ the helium behaves like any other ("normal") liquid, e.g., H_2O, we see that in the rotating equilibrium the fluid velocity will be simply ωR and the total angular momentum $I_{cl}\omega$, where the classical moment of inertia I_{cl} is just NmR^2. Now, while continuing to rotate the container, we cool the liquid through T_λ. Below T_λ, the "superfluid fraction" is finite and moves, according to Eq. (3), with the superfluid velocity v_s. However, v_s is constrained by the quantization condition and in general *cannot* be taken equal to ωR. In fact, a simple statistical-mechanical argument shows that the lowest free energy is obtained when n takes the value closest to ω/ω_s; for $\omega \ll \omega_c$ this is obviously zero. Consequently, in Eq. (3) the superfluid component no longer contributes to the circulating current. Meanwhile, the quantity v_n is still given by ωR, and consequently the total angular momentum is reduced by a factor $\rho_n(T)/\rho$. Thus by ramping the temperature up and down below T_λ, the angular momentum can be *reversibly* increased or decreased; in particular, for $T\to 0$ it tends to zero in the laboratory frame (or more accurately in frame of the fixed stars) even though the vessel is still rotating. At larger values of $\omega(>\omega_c/2)$ the superfluid will contribute to the angular momentum an amount $\sim n\omega_c$, where n is the nearest integer to ω/ω_c; thus for $\omega=0.75\omega_c$, for example, the apparent velocity of the liquid may *exceed* that of the container. This remarkable effect, which turns out to be a close analog of the Meissner effect in superconductors, was originally predicted by F. London and eventually observed (in effect) by Hess and Fairbank in 1967; it is essential to appreciate that it is a manifestation of the *equilibrium* behavior of the system and has nothing to do with long relaxation times.

A second experiment, which is at first sight, closely related to the above but is conceptually quite different, goes as follows: we again start above T_λ, but now with the liquid rotating at a much higher angular velocity $\omega \gg \omega_c$, so that, as above, velocity v is ωR. We next cool, still rotating, through T_λ; according to the prescription given above, the superfluid component will take the quantized value of circulation which makes n closest to ω/ω_c; but since ω/ω_c is very large this means that the fractional change is proportional to ω_c/ω and in practice unobservably small, and the angular momentum is to all intents and purposes $I_{cl}\omega$. Finally, still keeping the temperature below T_λ, we stop the rotation of the container. What happens?

It should be strongly emphasized that in contrast to the "Hess-Fairbank" experiment discussed above, the present problem does not concern the nature of the thermodynamic equilibrium state under the new (final) conditions; the latter rather obviously corresponds to zero circulating current. Rather, the question concerns the *degree of metastability* of the circulating-current state. In practice we find that when we stop the rotation, the contribution of the normal component to Eq. (3) rapidly relaxes to zero, but the superfluid contribution persists for a time, which, except under very special conditions, is effectively infinite, and moreover can be reversibly increased or decreased by sweeping the temperature up and down (but never allowing it to exceed T_λ). In other words, the system preserves the value of the superfluid circulation [Eq. (2)] that it originally had, even though it is clearly not the equilibrium one. This is the phenomenon of *metastable superflow*, which should be carefully distinguished from the (equilibrium) Hess-Fairbank effect. Unfortunately, the term "persistent currents," frequently used in the literature, is ambiguous and tends to confuse these two conceptually very different effects. It is amusing that the phenomenon of "frictionless flow" originally discovered by Kapitza, Allen, and Misener may, depending on the parameters, be a manifestation of either of these effects.

Unlike the Hess-Fairbank effect, which can be understood at least qualitatively in terms of the behavior of a single atom under the same conditions, a viable explanation of the phenomenon of metastable superflow requires explicit consideration of the effects of the interatomic interactions; indeed, it is believed that a noninteracting Bose gas, even in the BEC state, would *not* display this behavior. Crudely speaking, the argument goes as follows: to go continuously from a state in which a macroscopic number N_0 of atoms occupies the state corresponding to a finite value of n, say n_0, in

Eq. (2) to one in which the same N_0 atoms occupy the state $n=0$, we must do one of two things: either we scatter particles one by one out of the state $n=n_0$ and into $n=0$, thereby creating, at intermediate times, a state in which *two* single-particle states are simultaneously macroscopically occupied, or we keep N_0 particles in a single one-particle wave function but modify the latter so as to go continuously from χ_{n_0} at $t=-\infty$ to χ_0 at $t=+\infty$. *Provided there is no extra "internal" quantum number* and the low-energy effective interatomic interaction is repulsive (as is the case for 4-He), it is straightforward to show that for not too large values of n_0 both of these "paths" involve surmounting a free-energy barrier, which except for T extremely close to T_λ, is so enormous that the chance of doing so is negligible even on astronomical timescales. When T is extremely close to T_λ, this energy barrier (which scales as ρ_s and hence vanishes in the limit $T\rightarrow T_\lambda$) becomes surmountable with difficulty, and indeed it is found experimentally that there is a measurable relaxation of superflow in this regime.

Thus a theory based on Eqs. (1)–(3) and the considerations of the last paragraph can account not only qualitatively but, as it turns out, quantitatively for the main phenomena of superfluidity in 4-He. (In addition, it predicts other characteristic phenomena, such as the Josephson effect, which have been searched for and found, but there is no space to discuss this topic here.) However, there is one feature of this whole scenario that might leave one with a feeling of slight disquiet: in the sixty years since London's original proposal, while there has been almost universal belief that the key to superfluidity is indeed the onset of BEC at the lambda-temperature it has proved very difficult, if not impossible, to verify the existence of the latter phenomenon directly. The main evidence for it comes from high-energy neutron scattering and, very recently, from the spectrum of atoms evaporated from the surface of the liquid, and while both are certainly consistent with the existence of a condensate fraction of approximately 10%, neither can be said to establish it beyond all possible doubt.

All the above refers to our best-known superfluid, liquid 4-He below the lambda-temperature. However, that is not the end of the story. In 1972 it was discovered that the light isotope of helium, 3-He (which is also liquid under its own vapor pressure down to the lowest temperatures) possesses, below the much lower temperature of 3 mK, not one but three anomalous phases, each of which appears to display most of the properties expected of a superfluid, so that these new phases are usually referred to collectively as "superfluid 3-He." In this case, since the 3-He atom obeys Fermi rather than Bose statistics, the mechanism of superfluidity cannot be simple BEC as in 4-He. Rather, it is believed that, just as in metallic superconductors, the fermions pair up to form "Cooper pairs"—a sort of giant diatomic quasi-molecule whose characteristic "radius" is very much larger than the typical interatomic distance—and that these molecules, being composed of two fermions, effectively obey Bose statistics and can thus undergo BEC. However, it should be emphasized that, at least within the context of the traditional theory, the formation of the Cooper pairs and the process of BEC are not two independent phenomena, rather they occur simultaneously and are intimately connected. A microscopic theory that is a generalization of the BCS theory of superconductivity can be constructed for these new phases, and in fact, over the last 25 years has had a remarkable degree of quantitative as well as qualitative success in explaining their properties, to the extent that we can now claim an understanding of these materials which is more quantitative than that which we at present have of the apparently simpler system 4-He.

Although not all the phenomena that accompany the onset of superfluidity in 4-He have been explicitly demonstrated in the low-temperature phases of 3-He, the general pattern is sufficiently similar that there is a fair degree of confidence that the underlying scenario is parallel in the two cases, with the role of the condensate wave function in 4-He being played by the center-of-mass wave function of the Cooper pairs in 3-He. However, there is one very important difference: as well as their center-of-mass degree of freedom, the pairs in 3-He turn out to have also *internal* degrees of freedom; if one thinks of them as like diatomic molecules, they turn out, crudely speaking, to possess total spin $S=1$ and also "intrinsic" orbital angular momentum $L=1$, and the corresponding vectors can be oriented, *prima facie*, in arbitrary directions. (By contrast, the Cooper pairs in traditional superconductors have $L=S=0$ and thus do not possess any interesting internal degrees of freedom.) A crucial aspect of BEC in such a system is that the "condensed" pairs should not only all possess the same center-of-mass wave function, *they should also all behave identically as regards their internal degrees of freedom*.

Now, one might at first sight think that the arguments given regarding the Hess-Fairbank effect and the metastability of superflow which, *prima facie*, refer only to the center-of-mass behavior, would be qualitatively unaffected by the presence or absence of internal degrees of freedom. This is indeed so with regard to the Hess-Fairbank effect, and in one of the three phases (the B phase) it is also true in the context of metastability of superflow. However, with regard to the other two phases, the situation is more intriguing: it turns out that the nature of the internal degree of freedom in these phases is such that once it is taken into account, at least within the simplest approximation, the argument that any attempt to deform the condensate wave function so as to pass continuously from $n=n_0$ to $n=0$ *no longer holds*, so that within such an approximation superflow is no longer stable for $|n|>1$. (For $n=\pm 1$, for a subtle reason, it is still metastable) In real life superflow does appear to be metastable in all the new phases, but both the experimental and the theoretical situation is considerably more complicated than in 4-He.

Finally, it may be remarked that there now exists a third electrically neutral laboratory system, which is generally expected to show behavior characteristic of a superfluid, namely various monatomic alkali gases (87-Rb, 23-Na, 7-Li, and also, very recently, 1-H) at ultralow temperatures. These atoms possess an odd number of electrons, thus an even total number of fermions, and so should obey Bose statistics, and under appropriately extreme conditions, display the phenomenon of BEC and the resulting superfluid behavior. However, because of the nature of the "confinement" of these systems (usually by magnetic or laser traps) the situation with regard to BEC and superfluidity is reversed with respect to 4-He: The onset of BEC should be spectacular in the form of a dramatic change in the density profile, while that of superfluidity should be much more subtle and difficult to observe. Indeed, since June 1995 many experiments have seen such a change of profile, or closely related effects, in these systems at μK or nK temperatures, and their low-temperature states universally believed to exhibit BEC; but, at least at this time, the evidence for superfluidity is still quite circumstantial.

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