Conductance viewed as transmission

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Electric current flow, in transport theory, has usually been viewed as the response to an applied electric field. Alternatively, current flow can be viewed as a consequence of the injection of carriers at contacts and their probability of reaching the other end. This approach has proven to be particularly useful for the small samples made by modern microelectronic techniques. The approach, some of its results, and related issues are described, but without an attempt to cover all the active subtopics in this field. [S0034-6861(99)00102-6]

CONTENTS

I. CONDUCTANCE CALCULATED FROM TRANSMISSION

Early quantum theories of electrical conduction were semiclassical. Electrons were accelerated according to Bloch's theorem; this was balanced by back scattering due to phonons and lattice defects. Cross sections for scattering, and band structures, were calculated quantum-mechanically, but the balancing process allowed only for occupation probabilities, not permitting a totally coherent process. Also, in most instances, scatterers at separate locations were presumed to act incoherently. Totally quantum-mechanical theories stem from the 1950s, and have diverse sources. Particularly intense concern with the need for more quantum mechanical approaches was manifested in Japan, and Kubo's formulation became the most widely accepted version. Quantum theory, as described by the Schrödinger equation, is a theory of conservative systems, and does not allow for dissipation. The Schrödinger equation readily allows us to calculate polarizability for atoms, molecules, or other isolated systems that do not permit electrons to enter or leave. Kubo's linear-response theory is essentially an extended theory of polarizability. Some supplementary handwaving is needed to calculate a dissipative effect such as conductance, for a sample with boundaries where electrons enter and leave (Anderson, 1997). After all, no theory that ignores the interfaces of a sample to the rest of its circuit can possibly calculate the resistance of such a sample of limited extent. Modern microelectronics has provided the techniques for fabricating very small samples. These permit us to study conductance in cases where the carriers have a totally quantum mechanically coherent history within the sample, making it essential to take the interfaces into account. Mesoscopic physics, concerned with samples that are intermediate in size between the atomic scale and the macroscopic one, can now demonstrate in manufactured structures much of the quantum mechanics we associate with atoms and molecules.

When scattering by a randomly placed set of point defects was under consideration, it quickly became customary in resistance calculations to evaluate the resistance after averaging over an ensemble of all possible defect placements. This removed the effects of quantum-mechanically coherent multiple scattering, which depends on the distance between the scatterers. This approach also made the unwarranted assumption that the variation of resistance between ensemble members was small. The approach made it impossible to ask about spatial variations of field and current within the sample. Unfortunately, as a result, the very existence of such questions, which distinguish between the ensemble average and the behavior of a particular sample, was ignored.

Electron transport theory has typically viewed the electric field as a cause and the current flow as a response. Circuit theory has had a broader approach, treating voltage sources and current sources on an equal footing. The approach to be emphasized in the following discussion is a generalization of the circuit theory alternative: Transport is a result of the carrier flow incident on the sample boundaries. The voltage distribution within the sample results from the self-consistent pileup of carriers.

The viewpoint stressed in this short note has been explained in much more detail in books and review papers. We can cite only a few: Beenakker and van Houten (1991), Datta (1995), Ferry and Goodnick (1997), Imry (1997). There are also many conference proceedings and special theme volumes related to this subject, e.g., Sohn *et al.*, 1997; Datta, 1998.

It seems obvious that the ease with which carriers penetrate through a sample should be closely related to its conductance. But this is a viewpoint that, with the exception of some highly specialized limiting cases, found slow acceptance. That the conductance of a single localized tunneling barrier, with a very small transmis-

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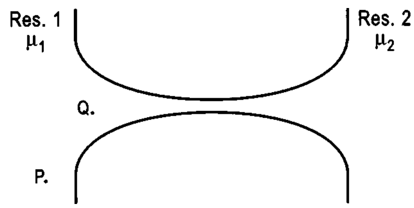

FIG. 1. Two reservoirs on each side of a perfect tube, at different electrochemical potentials μ_1 and μ_2. P is well inside reservoir 1; Q is in its entrance.

sion probability, is proportional to that probability was understood in the early 1930s (Landauer, 1994). In the case of a simple tunneling barrier it has always been apparent that the potential drop across the barrier is localized to the immediate vicinity of the barrier and not distributed over a region of the order of the mean free path in the surrounding medium. The implicit acceptance of a highly localized voltage drop across a single barrier did not, however, readily lead to a broader appreciation of spatially inhomogeneous transport fields in the presence of other types of scattering. The localized voltage drop across a barrier has been demonstrated with modern scanning tunneling microscopy (STM) probing methods (Briner *et al.*, 1996). We return to spatial variations in Sec. II.

Figure 1 shows an ideal conducting channel with no irregularities or scattering mechanisms along its length. A long perfect tube is tied to two large reservoirs via adiabatically tapered nonreflecting connectors. Carriers approaching a reservoir pass into that reservoir with certainty. The reservoirs are the electronic equivalent of a radiative blackbody; the electrons coming out of a reservoir are occupied according to the Fermi distribution that characterizes the deep interior of that reservoir. Assume, initially, that the tube is narrow enough so that only the lowest of the transverse eigenstates in the channel has its energy below the Fermi level. That makes the channel effectively one-dimensional. Take the zero temperature case and let the left reservoir be filled to up to level μ_1, higher than that of the right-hand reservoir, μ_2. Then in the range between μ_1 and μ_2 we have fully occupied states pouring from left to right. Thus the current is

$$j=-(\mu_1-\mu_2)ev(dn/d\mu), \tag{1.1}$$

where $dn/d\mu$ is the density of states (allowing for spin degeneracy) and v is the velocity component along the tube at the Fermi surface. Now $(\mu_1-\mu_2)=-e(V_1-V_2)$, where V is a voltage and e the magnitude of the electronic charge. Furthermore $dn/d\mu=1/\pi\hbar v$. Therefore the net current flow is given by $-(e/\pi\hbar)(\mu_1-\mu_2)$. The resulting conductance is

$$G=\frac{j}{V_1-V_2}=e^2/\pi\hbar. \tag{1.2}$$

This is the conductance of an ideal one-dimensional conductor. The conditions along the uniform part of the channel are the same; there is no potential drop there. The potential drop associated with the resistance specified in Eq. (1.2) occurs at the connections to the reservoir (Imry, 1986). Consider the left reservoir. Deep inside that reservoir there is a thermal equilibrium population. In the $1D$ channel only the right-moving electrons are present. The effective Fermi-level, or effective electrochemical potential, measures the level to which electrons are occupied. At point P in Fig. 1, deep inside the left-hand reservoir, the electron distribution is that characteristic of thermal equilibrium with the Fermi level μ_1. At point Q in Fig. 1, in the tapered part of the connection, the electron population shows some effect of the lowered density of electrons which have come out of the right hand reservoir with electrochemical potential μ_2. Thus there is a potential difference between P and Q. Along the ideal one-dimensional channel the electron population is equally controlled by both reservoirs, and the electrochemical potential there must be $\frac{1}{2}(\mu_1+\mu_2)$. Therefore the voltage drop specified by Eq. (1.2) is divided equally between the two tapered connectors. The physics we have just discussed is essential. Conductance can only be calculated after specifying the location where the potential is determined. The voltage specification deep inside the reservoir and the geometrical spreading, are essential aspects of the derivation of Eq. (1.2). Unfortunately, supposed derivations that ignore these geometrical aspects are common in the literature.

If we insert an obstacle into the channel, which transmits with probability T, the current will be reduced accordingly, and we find

$$G=(e^2/\pi\hbar)T. \tag{1.3}$$

Note that it does not matter whether the T in Eq. (1.3) is determined by a single highly localized barrier or by a more extended and complex potential profile. Expressions for the conductance with this same current, but with the potential measured within the narrow channel, on the two sides of the obstacle, also exist (Sec. 2.2 of Datta, 1995; Sec. 1.2.1 of Ferry and Goodnick, 1997, Chap. 5 of Imry, 1997). If, in that case, the potential is averaged over a region long enough to remove interference oscillations, then T in Eq. (1.3) is replaced by $T/(1-T)$.

The preceding discussion can easily be generalized to a channel that involves more than one transverse eigenstate with energy below the Fermi level (Imry, 1986). In that case we utilize the transmission matrix t of the scattering obstacle, which specifies the transmitted wave functions relative to the incident wave, utilizing the transverse eigenstates of the channel as a basis. This yields

$$G=(e^2/\pi\hbar)Tr(tt^\dagger). \tag{1.4}$$

In the particular case where we have N perfectly transmitting channels this becomes

$$G=N(e^2/\pi\hbar). \tag{1.5}$$

One of the earliest and most significant experimental verifications of this approach came from celebrated studies of quantum point contacts (QPC). These are narrow two-dimensional channels connecting wide reservoirs. The channel width can be controlled by externally applied gate voltages. As the conducting channel is widened, the number of transverse eigenstates below the Fermi level increases. Conductance steps corresponding to increasing values of N in Eq. (1.5) are clearly observed. The original 1988 experiments were carried out at Cambridge University and by a Delft-Philips collaboration (van Houten and Beenakker, 1996).

The material of this section has been extended in many directions; we list only a few. Büttiker (1986) describes the widely used results when more than two reservoirs are involved. Among a number of very diverse attempts to describe ac behavior we cite only one (Büttiker, 1993). In the ac case, however, the method of applying excitation to the sample matters. Moving the Fermi level of reservoirs up and down is one possibility; applying an electric field through an incident electromagnetic field is another. The discussion of systems that consist of incoherent semiclassical scatterers occurs repeatedly; we cite one with device relevance (Datta, Assad, and Lundstrom, in Datta (1998)). The extension to nonvanishing temperatures is contained in many of our broader citations. Nonlinearity has been treated repeatedly. The correction for reservoirs of limited lateral extent has been described by Landauer (1989). It must be stressed that the severe restrictions needed for the derivation of Eq. (1.4), i.e., the existence of ideal conducting tubes on both sides of the sample joined smoothly to the reservoirs, are only conditions for that particular expression. Transmission between reservoirs can be calculated under many other circumstances. Equation (1.4) has been applied to a wide variety of geometries. Many of the early experiments emphasized analogies to waveguide propagation. Transmission through cavities with classical chaotic motion has been studied extensively. Systems with superconducting interfaces and Andreev reflections have been examined; see Chap. 7 of Imry (1997). Three-dimensional narrow wires, resulting either from an STM geometry or from mechanically pulling wires, to or past their breaking point, have received extensive attention (Serena and Garcia, 1997).

The preceding discussion assumes that we can ascribe a transmission coefficient to electrons whose interactions while in the reservoir are neglected. That does not prevent a Hartree approximation Coulomb interaction along the conductor. Electron-electron interactions of almost any kind can exist *within* a sample, but that still permits us to discuss the transmission of uncorrelated electrons *through* that sample.

Feeding current from reservoirs, with the carriers coming from each side characterized by a thermal equilibrium distribution, is only one possible way of driving a sample. The exact distribution of arriving carriers, both in real space and in momentum, matters. A sample does not really have a unique resistance, independent of the way we attach to it. Wide reservoirs, connected to a narrower sample, and emitting a thermal equilibrium distribution, are a good approximation to many real experimental configurations.

Equation (1.4) describes conduction as a function of quantum mechanically coherent transmission. Current flow in the presence of a limited conductance is a dissipative process. Where are the dissipation and the irreversibility (Sec. 2.3 of Datta, 1995)? They are in the reservoirs; carriers returning to them from the sample eventually suffer inelastic collisions. These inelastic collisions give the carriers, when they later again reach the transmissive sample, the occupation probability characteristic of the reservoir. The inelastic collisions in the reservoir also serve to eliminate any phase memory of the carrier's earlier history. Thus the sample determines the size of the conductance, even though the irreversible process takes place elsewhere. For a narrow conductor, attached to reservoirs which can serve as effective heat sinks, this means that the energy is released where it can easily be carried away and allows surprisingly large currents. Frank *et al.* (1998) pass current through a carbon nanotube, which would heat it to 20 000 K if the dissipation occurred along the tube. Such large currents and the accompanying changes in the wave functions of the binding electrons may induce temporary atomic displacements (Sec. 14 of Sorbello, 1997).

II. SPATIAL VARIATION, CONDUCTANCE FLUCTUATIONS, LOCALIZATION

We have already emphasized that ensemble members differ and that transport fields are spatially inhomogeneous. Spatial variations of current and field exist for two reasons. First of all geometry and preparation can impose obvious patterns in space, as in a transistor or scanning tunneling microscope. But a random arrangement of point scatterers can also provide inhomogeneity with easy and hard paths through the sample. Why are spatial variations of interest? Calculating conductance from Eq. (1.4) does not require an understanding of the spatial variations within the sample. But spatial variations are vital in other contexts. We can actually probe spatial distributions (Briner *et al.*, 1996; Eriksson *et al.*, 1996). The notion, common in the middle 1980s, that transport could only be examined by very invasive extra conducting leads has been replaced by the awareness that there is a growing set of minimally disturbing probes, including, for example, electro-optic effects. Spatial variations also matter in nonlinear transport. In that case the transport field itself is part of the field that determines transmission through the sample, and a self-consistent analysis invoking Poisson's equation is needed. Datta *et al.* (1997), in an analysis of conduction through an organic molecule caught in an STM configuration, illustrate this. Spatial variations matter in high-frequency behavior. In that case there can be capacitive shorting across the resistively hard parts of the sample. Spatial variations matter for electromigration (Sorbello, 1997). Electromigration is the motion of lattice defects induced by electron transport. The moving defects probe their local environment, not a volume average. In that analysis care must be taken to include the spatial variation induced by that defect.

We have alluded to the localized voltage drop across a planar barrier. If, instead, we introduce a point scatterer, in the presence of a constant current flow, we can also expect an increase in transport field related to the defect's location. In fact, a planar barrier can be built from an array of point defects, and the two cases must show related behavior. In the planar case the localized voltage drop arises from the pileup of incident electrons on one side of the barrier and their removal from the other side, allowing for self-consistent screening of these piled-up charges. A similar pileup at a point defect will generate a dipole field, called the residual resistivity dipole (RRD), which has been studied for over four decades (Sorbello, 1997; Zwerger, 1997; also see Ref. 19 of Landauer's introductory chapter in Serena and Garcia, 1997). A volume with incoherent point scatterers will generate a set of dipole fields, one dipole per scatterer. The resulting space-average field is that given by other elementary semiclassical theories; there is no new result for the resistivity. We have only emphasized the strong spatial variation of the transport field. The current flow pattern is also spatially nonuniform (Zwerger, 1997), representing the fact that the incident carrier flux, scattered by a localized defect, has to be carried around that defect much as a current has to be carried around a macroscopic cavity.

A review should not be confined to progress, but can also list questions. The spatial variation of the field in the presence of randomly placed point scatterers, providing *coherent* multiple scattering, is not understood. A set of dipole fields can still be expected, but the size of each dipole can no longer depend only on the scattering action of a particular defect. The striking nonuniformity of the potential drop along a coherent disordered one-dimensional array has been demonstrated (Maschke and Schreiber, 1994).

An array of randomly placed point scatterers, acting incoherently, will allow for some variation in transmission depending on the carrier's path. In the presence of coherence this variation is sensitive not only to density fluctuations among the obstacles, but also to the exact relative phasing of scattered waves. At one extreme the random placement includes ensemble members that give a periodic, or almost periodic, arrangement and cause the electron to see an allowed band, giving excellent transmission. But the ensemble will also include members that in all or part of space simulate a forbidden gap, yielding exponentially small transmission. The relative phasing of scattered waves can be altered not only by changing atomic placement, but also by changing parameters, such as the Fermi energy or a magnetic field. The resulting variations of conductance have been widely observed (Washburn and Webb, 1992; Sec. 5.2 of Ferry and Goodnick, 1997) and are called universal conductance fluctuations (UCF). Relative phasing of waves, taking alternative paths, also plays a critical role in the observed oscillations in the conductance in a solid-state analog of the Aharonov-Bohm effect (Washburn and Webb, 1992).

Consider a one-dimensional ensemble of a fixed number of identical localized scatterers, allowing for all possible relative phases between adjacent scatterers at the Fermi wavelength. (The total length of the chains cannot be held fixed.) This is a particularly simple case of disorder. As already stated, this includes ensemble members that, over portions of the chain, simulate a forbidden band, resulting in an exponential decay of the wave function. (The forbidden band can be associated with a superlattice formed from the scatterers and in any case represents only a physically suggestive way of pointing to constructive interference in the buildup of reflections.) For an electron incident, say from the left, there will be no regions providing a compensating exponential increase to the right. An ensemble average of the resulting resistance, rather than conductance, weights the high-resistance ensemble members and can be shown to increase exponentially with the number of obstacles (Sec. 5.3 of Imry, 1997). The problem of treating such a highly dispersive ensemble (Azbel, 1983) was solved by Anderson *et al.* (1980), who emphasized $\ln(1+g^{-1})$, where $g=G\pi\hbar/e^2$, which behaves like a typical extensive quantity. The ensemble average of this quantity is proportional to the number of obstacles, and this quantity also has a mean-squared deviation which scales linearly with length. The exponential decay, with length, of transmission through a disordered array is a particularly simple example of *localization*; electrons cannot propagate as effectively as classical diffusion would suggest. In two or three dimensions a carrier can detour around a poorly transmissive region. As a result localization in higher dimensions is not as pronounced and is more complex.

Equation (1.4) tells us that the conductance can be considered to be a sum of contributions over the eigenvalues of $tt^\dagger$. These represent, effectively, channels that transmit independently. The relative phase of what is incident in different channels does not matter. The variation of transmission, which would depend on the exact choice of path in a semiclassical discussion, is now represented by the distribution of eigenvalues of $tt^\dagger$. For a sample long enough so that conduction is controlled by many random elastic-scattering events, producing diffusive carrier motion, but not long enough to exhibit localization, this distribution is bimodal (Beenakker, 1997). Most eigenvalues are very small, corresponding to channels that transmit very poorly. There is, however, a cluster of highly conducting channels, which transmit most of the current. Oakeshott and MacKinnon (1994) have modeled the striking nonuniformity of current flow, showing filamentary behavior, in a disordered block. This is a demonstration of the fact that the bimodal distribution is related to the distribution in real space. The bimodal distribution, the strong variation between ensemble members, and the geometrical nonuniformity have also been persistent themes in the work of Pendry (e.g., Pendry *et al.*, 1992), who stresses the importance of necklaces i.e., chains of sites that permit tunneling from one to the next, in the limit where localization matters.

III. ELECTRON INTERACTIONS

Both the thermodynamics and the transport properties of independent electrons propagating in a finite system with random scatterers are relatively well understood. It is also well known that for the infinite *homogeneous* electron gas the Hartree term for the interactions cancels the ionic background. The remaining exchange-correlation contributions still play an important role, especially in soluble 1D models (Emery, 1979), but a noninteracting model allows considerable progress in most higher-dimensional situations. A partial justification for this, with modified parameters, is provided by the Landau Fermi-liquid picture. In this description, the low-energy excitations of the interacting system are Fermion quasiparticles with a renormalized dispersion relation and a finite lifetime due to collisions. This is valid as long as the quasiparticle width is much smaller than its excitation energy, which is the case for homogenous systems at low enough excitation energies. However, when strong inhomogeneities exist, a rich variety of new phenomena opens up. Here we briefly consider the effects of disorder and finite size.

Disorder turns out to enhance the effects of the interactions, as explained by Altshuler and Aronov (1985). This enhancement is not only in the Hartree term, but also in greatly modified exchange and correlation contributions. These effects become very strong for low dimensions or strong disorder.

Singular behavior was found in the single-particle density of states (DOS) near the Fermi energy. For disordered 3D systems, the magnitude of this singularity (Altshuler and Aronov, 1985) is determined by the ratio of Fermi wavelength to mean free path. That is small for weak disorder, but increases markedly for stronger disorder. The situation is much more interesting for effectively 2D thin films and 1D narrow wires. There, when carried to low excitation energies, these corrections diverge, respectively, like the log and the inverse square root of $E-E_F$. Thus a more complete treatment, which is still lacking, is needed. These "zero-bias" DOS corrections should be ubiquitous. They have been observed experimentally and agree semiquantitatively with the theory, as long as they are not too large. Direct interaction-induced corrections to the conductivity in the metallic regime were also predicted by Altshuler and Aronov and confirmed by numerous experiments. The 2D case, realized in semiconductor heterojunctions and in thin metal films, is of special interest. The theory for noninteracting electrons (Abrahams *et al.*, 1979) predicts an insulating behavior as the temperature $\mathcal{T}\to 0$. The effect of electron-electron interactions is hard to treat fully. The calculations by Finkelstein (1983) show a window of possible metallic behavior characterized by a strong sensitivity to a parallel magnetic field. Several experimental studies suggest a similar metallic behavior in 2D (Lubkin, 1997), whose origin is still under debate.

Schmid (1974) found the interaction-induced lifetime broadening strongly enhanced by the disorder, varying in 3D as $(E-E_F)^{3/2}$ at $\mathcal{T}=0$ instead of the usual ballistic $(E-E_F)^2$ Landau result. These changes are stronger in 1D and 2D, where the expressions obtained for the disorder-dominated e-e scattering rate *diverge* at nonvanishing $\mathcal{T}$. However, physically meaningful scattering rates are finite. For example, Altshuler *et al.* (1982) found that the rate of dephasing of the relative phase between two different paths is regular and goes as $\mathcal{T}$ (with a small logarithmic prefactor) and $\mathcal{T}^{2/3}$ in 2D and 1D respectively. These results are crucial for numerous mesoscopic interference situations and agree quantitatively with experiments at temperatures that are not too low.

A particular case of strong inhomogeneity occurs when the electrons are confined to a small spatial range, such as a lattice site or a small grain or "quantum dot" (a two-dimensionally confined region of electrons). The deviation from charge neutrality is accompanied by an energy cost $e^2/2C$, where C is an effective capacitance. This energy, when it is large compared to the thermal energy, can prohibit double electronic occupancy for a hydrogenic impurity or exclude the electron transfer into or through a quantum dot. The latter phenomenon has been dubbed "the Coulomb blockade," and it is relevant to many experimental situations, including the optimistically named *single-electron transistor* (Chap. 4 of Ferry and Goodnick, 1997). Most interestingly, the correlation embodied by this strong inhibition of electrons to populate certain locations often results in subtle and dramatic phenomena. Those include a *Fermi-edge singularity* and the Kondo effect, both appearing (like the Altshuler-Aronov singularities) at low energies, near the Fermi level.

The Fermi-edge singularity is well known from x-ray absorption in metals. The attraction between the core hole left by the photoexcitation and the conduction electrons causes the absorption to diverge at its edge. Matveev and Larkin (1992) suggested that an analogous effect should exist for tunneling through a resonant impurity (or a quantum dot) state in a small tunnel junction. The hole left in that state by an electron tunneling out plays the same role as the above core hole. The interaction between this hole and the conduction electrons increases the transmission amplitude near E_F. The logarithmic divergence near threshold obtained from the simplest low-order perturbation theory is replaced in the full theory by a power-law singularity. Geim *et al.* (1994) observed this effect in impurity-assisted tunneling through small resonant tunneling diodes.

Another interesting near-Fermi-level structure is the Kondo resonance, due to the repulsion between the two opposite-spin electrons on the same impurity (or quantum dot) state and their hybridization with the conduction electrons. Again, for a quantum dot connected to two electrodes via tunnel junctions, this leads to a resonance in the zero-bias transmission. The splitting of this peak by a bias and its magnetic-field dependence were predicted by Meir *et al.* (1993). Very recently (Goldhaber-Gordon *et al.*, 1998) this effect was observed with a high-quality quantum dot.

IV. ELECTRON COHERENCE; PERSISTENT CURRENTS; OTHER FEATURES

It was pointed out in Sec. I that, in the absence of inelastic scattering in an intervening sample, the conductance between reservoirs is determined by the elastic scattering in that sample. In this case the ultimate source of irreversibility is in the inelastic scattering in the reservoirs. What happens if we eliminate the reservoirs and their inelastic scattering? We can do that by tying the leads to the sample to each other and creating a loop, considering the response of this quantum-mechanical system to an external magnetic flux through the ring. This will be done later in this section.

Inelastic scattering of the electron by other degrees of freedom of the ring acts like distributed coupling to external reservoirs. Inelastic scattering causes the electron waves to lose phase coherence; effects due to the interference between electron waves following alternative paths are eliminated (Chap. 3 of Imry, 1997; Chap. 6 of Ferry and Goodnick, 1997). Because the eigenstates of a closed system are determined by periodicity and boundary conditions, the energy levels of a bounded system with some inelastic scattering are no longer sharp and gradually lose their dependence on the periodicity condition when the inelastic scattering increases. For our ring, once the inelastic-scattering length exceeds the circumference of the ring, the waves can respond to the total set of boundary conditions. The ensuing flux sensitivity of the energy levels and their associated "persistent currents" are discussed below. The significant difference between the effects of elastic and inelastic scattering has been highlighted by recent research in disordered and mesoscopic systems. Equilibrium and transport experiments have determined the length the electron can propagate without losing phase coherence, often in good agreement with theory.

That the π electrons on a benzene-type ring molecule have a large orbital magnetic response has been known for more than half a century. The explanation in terms of "ring currents" was advanced by Pauling in 1936. Ordinary metals exhibit a small but measurable orbital diamagnetism, a purely quantum-mechanical phenomenon. However, the prediction that a metallic ring structure in the usual mesoscopic size range can support an equilibrium circulating current in response to an external flux was greeted with skepticism 15 years ago. This applied particularly to the diffusive regime with repeated elastic defect scattering for a carrier traversing the ring. When most of the flux is through the ring's opening, rather than the conductor, the response is periodic in the flux, with period $\Phi_0 = h/e$. This follows because such a flux Φ causes a phase change of $2\pi\Phi/\Phi_0$ in the phase of the eigenstates, upon taking an electron around the ring. Therefore the energy levels depend periodically on the flux. This leads to a dependence of the thermodynamic potential on the flux and hence, by thermodynamics, to an equilibrium current. Contrary to classical intuition, elastic scattering alone does not cause current decay.

There are now a number of experiments confirming the existence of persistent currents in single mesoscopic rings and also in rather large ensembles of rings, as summarized in Chap. 4, Sec. 2 of Imry (1997). In the latter case, the experiment measures the persistent current averaged over many rings. These rings differ through varying realizations of the random impurity potentials; an averaging over these differing realizations is effectively performed. The results of the single-ring experiments agree roughly with the theory for noninteracting electrons, and it can be shown (see p. 75 of Imry, 1997) that the interactions do not change the order of magnitude of the single-ring, sample-specific current. The situation is very different for the ensemble-averaged current. The periodicity in the flux was experimentally found to be $h/2e$, rather than h/e, in accordance with the theory for the ensemble-averaged persistent current for noninteracting electrons. However, these theories underestimate the magnitude of the measured current by more than two orders of magnitude. Introducing electron-electron interactions perturbatively gives a result with the required period $h/(2e)$, but still smaller than the experiment by a factor of 5 to 10. This clearly goes in the right direction, but there is still no definitive understanding of the magnitude of the measured ensemble-averaged persistent currents.

An interesting case in which persistent normal currents *may* exist on the millimeter length scale is provided by recent experiments on the magnetic response in a proximity-effect system. Very-low-temperature measurements of that response for a superconducting cylinder with a normal-metal coating (Mota *et al.*, 1994), revealed an unexpected strong paramagnetic moment in addition to the usual Meissner effect induced by the superconductor in the proximity layer. This paramagnetic moment is comparable to a diamagnetic moment. Whispering gallery modes of the normal electrons, bouncing around the outer perimeter of the normal layer, qualitatively and speculatively explain this magnetic moment as due to unusually large normal persistent currents flowing near that surface (Bruder and Imry, 1998).

Our subject has many more facets than we can discuss, or even list, in this short paper. We allude to only one subtopic. Noise measurements have developed into a surprisingly accurate probing method. Noise is more sensitive than the dc conductance to electron correlations. Schoelkopf *et al.* (1997) have studied the frequency dependence of noise in a current-carrying metallic conductor, with enough elastic scattering to give diffusive carrier motion, and find remarkable agreement with the simplest independent-electron models. Reznikov *et al.*, in Datta (1998), discuss recent measurements at the Weizmann Institute and by a CEA/Saclay-CNRS/Bagneux collaboration, using shot-noise measurements to demonstrate the effective $e/3$ charge of the tunneling entity in a fractional-quantum-Hall-effect experiment.

Many alternative views of quantum transport have been developed, e.g., the Keldysh formulation, adaptable to inelastic processes as treated in Chap. 8 of Datta (1995). Especially powerful is a block-scaling picture due to Thouless (p. 21 of Imry, 1997), which generalized 1D localization to finite cross-section wires. Later it was broadened into an intuitive and successful theory at higher dimension (Abrahams *et al.*, 1979).

Only the most settled aspects of electron-electron interaction have been discussed, slighting a number of fashionable efforts. The approach emphasized in this paper should permit some generalization to the case in which carrier interactions in the reservoir are critical. For a given potential difference between two reservoirs there is a maximum current that can be passed through a smooth and long laterally constricted connection. [The lateral dimension(s) of the connecting pipe can be less than the range of the electron interactions.] In the presence of irregularities only a portion of that maximum will pass.

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