The fractional quantum Hall effect

Horst L. Stormer

Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974
and Department of Physics and Department of Appl. Physics, Columbia University, New York, New York 10027

Daniel C. Tsui

Department of Electrical Engineering and Department of Physics, Princeton University, Princeton, New Jersey 08544-0430

Arthur C. Gossard

Material Department, University of Californiiia, Santa Barbara, Santa Barbara, California 93106

Two-dimensional electron systems in a high magnetic field behave very strangely. They exhibit rational fractional quantum numbers and contain exactly fractionally charged particles. Electrons seem to absorb magnetic flux quanta, altering their statistics and consuming the magnetic field. They condense into a manifold of novel ground states of boson and fermion character. These fascinating properties are not characteristic of any individual electron but rather emerge from the highly correlated motion of many. [S0034-6861(99)00802-8]

CONTENTS

I. INTRODUCTION

The fractional quantum Hall effect is an example of the new physics that has emerged from the enormous progress made during the past few decades in material synthesis and device processing. Driven by the increasing demands of the electronic and photonic industry for material control, ultrathin semiconductor layers of exceptional purity and smoothness are now being fabricated routinely. This technology has made it possible to realize two-dimensional (2D) electron systems of unprecedentedly low disorder, which have become an ideal laboratory in which to study many-particle physics in lower dimensions. In particular for 2D electrons in a magnetic field the discoveries have been stunning: An abundance of new energy gaps exists where one expected none. Hall resistances are quantized to exact rational fractions of the resistance quantum, while the magnetoresistance is vanishing. Huge external magnetic fields are apparently eliminated, and new particles with ballistic trajectories emerge. Theory has constructed a powerful and elegant model to account for these strange observations: Electrons condense into novel quantum liquids leading to rational fractional quantum numbers and to carriers with exactly fractional charge. Electrons absorb magnetic flux quanta, seemingly eliminating external magnetic fields. Particle statistics are altered from fermionic to bosonic and back to fermionic. And a novel, field-induced particle pairing mechanism is foreseen.

All these fascinating properties arise not from single electrons interacting with an external field, but rather from the strongly correlated motion of many electrons: It is not electron fission that leads to fractionally charged quasiparticles, but the interplay of some 10^{11} electrons per cm^2 that collaborate and create these bizarre objects.

The following pages are intended to provide a brief survey of the physics of strongly correlated 2D electron systems in the presence of a high magnetic field. By no means can it be considered an exhaustive review. For the sake of brevity many exciting topics and important contributions to the field have not been incorporated. The report focuses on the fractional quantum Hall effect and closely related phenomena. Rather than to instruct the expert, the aim is to sketch for the general, scientifically knowledgeable reader the strangeness of the experimental observations, reveal the simple beauty of the extracted concepts, and communicate the elegance of the still-evolving many-particle theory.

II. BACKGROUND

Nearly ideal two-dimensional electron systems can be realized by quantum-mechanically confining charge carriers in thin potential wells. They are fabricated by epitaxially growing high-quality single-crystal films of selected semiconductors with different energy band gaps (Stormer *et al.*, 1979). The carriers in such structures are free to move along the 2D plane, but their motion perpendicular to the planes is quantized. As a result, a low-density 2D metal of high perfection emerges ($n \sim 0.2$–4×10^{11} cm^{-2}). In the cleanest case of GaAs/AlGaAs heterostructures, the 2D carriers show low-temperature mean free paths as long as 0.1 mm (!).

 0034-6861/99/71(2)/298(8)/$16.60 ©1999 The American Physical Society

Application of a magnetic field normal to the plane further quantizes the in-plane motion into Landau levels at energies $E_i=(i+1/2)\hbar\omega_c$, where $\omega_c=eB/m^*$ represents the cyclotron frequency, B the magnetic field, and m^* the effective mass of electrons having charge e. The number of available states in each Landau level, $d=2eB/h$, is linearly proportional to B. The electron spin can further split the Landau level into two, each holding eB/h states per unit area. Thus the energy spectrum of the 2D electron system in a magnetic field is a series of discrete levels, each having a degeneracy of eB/h (Ando *et al.*, 1983).

At low temperature ($T\ll$Landau/spin splitting) and in a B field, the electron population of the 2D system is given simply by the Landau-level filling factor $\nu=n/d=n/(eB/h)$. As it turns out, ν is a parameter of central importance to 2D electron physics in high magnetic fields. Since $h/e=\phi_0$ is the magnetic-flux quantum, ν denotes the ratio of electron density to magnetic-flux density, or more succinctly, the number of electrons per flux quantum. Much of the physics of 2D electrons in a B field can be cast in terms of this filling factor.

Most of the experiments performed on 2D electron systems are electrical resistance measurements, although in recent years several more sophisticated experimental tools have been successfully employed. In electrical measurements, two characteristic voltages are measured as a function of B, which, when divided by the applied current, yield the magnetoresistance R_{xx} and the Hall resistance R_{xy} (see insert Fig. 1). While the former, measured along the current path, reduces to the regular resistance at zero field, the latter, measured across the current path, vanishes at $B=0$ and, in an ordinary conductor, increases linearly with increasing B. This Hall voltage is a simple consequence of the Lorentz force's acting on the moving carriers, deflecting them into the direction normal to current and magnetic field. According to this classical model, the Hall resistance is $R_{xy}=B/ne$, which has made it, traditionally, a convenient measure of n.

It is evident that in a B field current and voltage are no longer collinear. Therefore the resistivity $\hat{\rho}$ which is simply derived from R_{xx} and R_{xy} by taking into account geometrical factors and symmetry, is no longer a number but a tensor. Accordingly, conductivity $\hat{\sigma}$ and resistivity are no longer simply inverse to each other, but obey a tensor relationship $\hat{\sigma}=\hat{\rho}^{-1}$. As a consequence, for all cases of relevance to this review, the Hall conductance is indeed the *inverse* of the Hall resistance, but the magnetoconductance is under most conditions *proportional* to the magnetoresistance. Therefore, at vanishing resistance ($\rho\to 0$), the system behaves like an insulator ($\sigma\to 0$) rather than like an ideal conductor. We hasten to add that this relationship, although counterintuitive, is a simple consequence of the Lorentz force's acting on the electrons and is not at the origin of any of the phenomena to be reviewed.

Figure 1 shows a classical example of the characteristic resistances of a 2D electron system as a function of an intense magnetic field at a temperature of 85 mK.

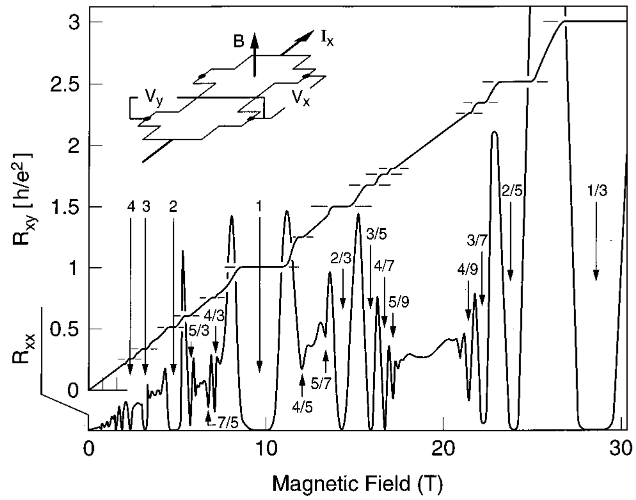

FIG. 1. Composite view showing the Hall resistance $R_{xy}=V_y/I_x$ and the magnetoresistance $R_{xx}=V_x/I_x$ of a two-dimensional electron system of density $n=2.33\times10^{11}$ cm^{-2} at a temperature of 85 mK, vs magnetic field. Numbers identify the filling factor ν, which indicates the degree to which the sequence of Landau levels is filled with electrons. Instead of rising strictly linearly with magnetic field, R_{xy} exhibits plateaus, quantized to $h/(\nu e^2)$ concomitant with minima of vanishing R_{xx}. These are the hallmarks of the integral ($\nu=i=$integer) quantum Hall effect (IQHE) and fractional ($\nu=p/q$) quantum Hall effect (FQHE). While the features of the IQHE are the results of the quantization conditions for individual electrons in a magnetic field, the FQHE is of many-particle origin. The insert shows the measurement geometry. $B=$magnetic field, $I_x=$current, $V_x=$longitudinal voltage, and $V_y=$transverse or Hall voltage. From Eisenstein and Stormer, 1990.

The striking observation, peculiar to 2D, is the appearance of steps in the Hall resistance R_{xy} and exceptionally strong modulations of the magnetoresistance R_{xx}, dropping to vanishing values. These are the hallmarks of the quantum Hall effects.

III. THE INTEGRAL QUANTUM HALL EFFECT

Integer numbers in Fig. 1 indicate the position of the integral quantum Hall effect (IQHE) (Von Klitzing, *et al.*, 1980). The associated features are the result of the discretization of the energy spectrum due to confinement to two dimensions plus Landau/spin quantization.

At specific magnetic fields B_i, when the filling factor $\nu=n/(eB/h)=i$ is an integer, an exact number of these levels is filled, and the Fermi level resides within one of the energy gaps. There are no states available in the vicinity of the Fermi energy. Therefore, at these singular positions in the magnetic field, the electron system is rendered incompressible, and its transport parameters (R_{xx},R_{xy}) assume quantized values (Laughlin, 1981). Localized states in the tails of each Landau/spin level, which are a result of residual disorder in the 2D system, extend the range of quantized transport from a set of precise points in B to finite ranges of B, leading at integer filling factors to the observed plateaus in the Hall resistance and stretches of vanishing magnetoresistance (Prange and Girvin, 1990; Chakraborty and Pietilainen, 1995).

In essence, the transport features are the result of transitions between alternating metallic and insulating behavior, i.e., from E_f within a Landau/spin band to E_f in a gap between Landau/spin bands. These IQHE states occur at integer filling factor i and display quantization of the Hall resistance to $h/(ie^2)$, as indicated in Fig. 1. They identify and exhaust all single-particle energy gaps. The IQHE is the result of the quantization conditions for noninteracting 2D electrons in a magnetic field.

IV. THE FRACTIONAL QUANTUM HALL EFFECT

Different from the IQHE, the fractional quantum Hall effect (FQHE; Tsui, *et al.*, 1982) occurs at fractional level filling and its quantum numbers are not integers but rational fractions p/q (see Fig. 1). Features at these fractional fillings cannot be explained in terms of single-electron physics. They occur when the Fermi energy resides *within* a highly degenerate Landau or spin level and imply the existence of energy gaps of many-particle origin.

The fractional quantum Hall effect is the result of the highly correlated motion of many electrons in 2D exposed to a magnetic field. Its driving force is the reduction of Coulomb interaction between the like-charged electrons. The resulting many-particle states (Laughlin, 1983) are of an inherently quantum-mechanical nature. Fractional quantum numbers and exactly fractionally charged quasiparticles are probably the most spectacular of its implications (Chakraborty and Pietilainen, 1995).

Today, the attachment of magnetic vortices to electrons has become the unifying principle underlying the multiple many-particle states of the FQHE (Read, 1994; DasSarma and Pinczuk, 1997). Laughlin's wave function describing the $\nu=1/3$ state is the prime example for this principle at work.

The presence of the magnetic field requires the many-electron wave function to assume as many zeroes within a unit area as there are magnetic flux quanta penetrating it. Each zero "heals" on the scale of a magnetic length $(l_0=\sqrt{\hbar/(eB)})$ and, limiting ourselves to the lowest Landau level, each such "hole" in the electron sheet represents an overall charge deficit of νe. Since the magnetic field also imparts a 2π phase twist to the wave function at the position of each such zero, these objects are termed vortices. In a certain sense, vortices are the embodiment of flux quanta in an electron system. A tiny coil threaded through the plane of the electrons and energized to generate just one magnetic flux quantum through its core would create one such vortex (Laughlin, 1984). Therefore, loosely speaking, vortices are often equated with flux quanta.

Just like electrons, vortices are delocalized in the plane. However, since electrons represent a charge accumulation and vortices a charge deficit, they attract each other. Considerable Coulomb energy can be gained by placing vortices onto electrons. At $\nu=1/3$ there exist three times as many vortices as there are electrons, each vortex representing a local charge deficit of $\frac{1}{3}e$.

Each electron must carry at least one vortex equivalent to one zero in the wave function to satisfy the Pauli principle. Additional vortex attachment is "optional," driven by Coulomb gain. Vortex attachment to an electron, representing a local depletion of companion electrons, is always energetically beneficial. The situation is somewhat reminiscent of the screening cloud around an electron in a regular metal, although in the case of the FQHE, such "screening" is very rigid and quantized in units of vortex charge.

The attachment of exactly three vortices to each electron is at the origin of the prominent $\nu=\frac{1}{3}$ FQHE state expressed by Laughlin's wave function as

$$\psi_{1/3}=\prod_{i<j}^{n}(Z_i-Z_j)^3\exp\left(-\frac{1}{4}\sum_{k}^{n}|Z_k|^2\right).$$

The $Z_{i,j,k}$'s represent the coordinates of n electrons in a complex 2D plane, which renders the wave function more compact. Normalization and magnetic length are set to unity. All electron-electron correlations derive from the first term, which is a product over all complex pair distances between electrons. The exponent 3 in each factor expresses in mathematical terms the attachment of three vortices exactly to the position of each electron. More generally, states at $\nu=1/q$ ($q=$odd) consist of electrons dressed by q vortices, and their wave function differs from the above only by the exponent, which changes from 3 to q. Only odd q are allowed, since only they guarantee antisymmetry of this electron wave function.

Any deviation from such a commensurate electron-vortex ratio comes at a considerable energetic cost. Creation of additional vortices, as induced by an increase in magnetic field, requires a finite amount of energy. This is the origin of the energy gap at $\nu=1/3$ and of the incompressibility of the electronic state at this filling factor. Unbound vortices become quasiholes in the sea of electrons, each carrying exactly $+1/3$ of an electronic charge (Laughlin, 1983). Gap formation, charged quasiparticles, and their localization at residual potential fluctuations are the ingredients required to account for the transport features in Fig. 1.

Returning to the representation of vortices as flux quanta, one can also regard the $\nu=1/3$ state as consisting of new objects: electrons to which three *vortex-generating* flux quanta have been attached (Kivelson *et al.*, 1992). This viewpoint has interesting conceptual consequences. First, since the total external magnetic field consists of exactly three flux quanta per electron, it appears that the entire magnetic field has been attached to electrons, reducing the magnetic field felt by these composite objects at $\nu=1/3$ to zero. Second, the flux carried by such objects has a dramatic effect on their statistics. Exchange of two such "magnetized" particles introduces an Aharanov-Bohm phase which turns them into bosons for an odd number of attached flux quanta while reverting them back to fermions for an even number of attached flux quanta. Thus the $\nu=1/3$ state consists of composite bosons created by the attachment of three flux quanta to each electron, which Bose condense in the apparent absence of an external magnetic field. This view, developed in conjunction with the composite fermion model (see below), links the physics of the $1/q$ FQHE state directly to the statistics of peculiar new particles.

Today, fractional charge and energy gap have been observed independently in several experiments (Clark *et al.*, 1988; Simmons *et al.*, 1989; Chang and Cunninghan, 1990; Kukushkin *et al.*, 1992; Pinczuk *et al.*, 1993; Goldman and Su, 1995; de-Picciotto *et al.*, 1997; Saminadayar *et al.*, 1997). The states at primary filling factor $\nu=1/q$, as well as at $\nu=1-1/q$ (electron-hole symmetry), are well understood. Their description in terms of a practically exact and remarkably succinct electronic wave function is a triumph of many-particle physics.

V. COMPOSITE FERMIONS IN THE FQHE

Composite fermions (CFs) are a new concept that provides us with a concise way of accounting for the so-called higher-order FQHE states at odd-denominator fractional filling factors, different from the primary states at $\nu=1/q$. They have also been used in an interpretation of the long-time enigmatic states at even-denominator fractional filling such as those at $\nu=1/2$. These will be addressed in the next section.

Higher-order FQHE states (e.g., $\nu=2/5$, 5/9, or 5/7) are abundant in Fig. 1, and the features they generate are very similar to those at $\nu=1/q$. This is particularly apparent for the states at $\nu=p/(2p\pm1)$ which converge towards half filling. Of course, the same principle of Coulomb energy optimization, that governs the physics at $\nu=1/q$, is expected to be at work. However, such higher-order states are much more difficult to describe in terms of many-electron wave functions than the states at $\nu=1/q$. An early hierarchical scheme (Haldane, 1983; Halperin, 1984; Laughlin, 1984), starting from the primary fractions, was able to rationalize the abundance of FQHE features and the odd-denominator rule: Just as the original electrons correlate to form primary states at $\nu=1/q$, at sufficient deviation from such filling factors, the so-created, charged quasielectrons or quasiholes can correlate and form new quantum liquids of quasiparticles at neighboring rational filling factors. This process can be repeated *ad infinitum*, eventually covering all odd-denominator fractions. However, wave functions proposed for such states at $\nu=p/q$ lack the simplicity of the Laughlin wave function at $\nu=1/q$, and they do not contain a simple underlying principle to generate a sequence of wave functions for the higher-order states.

Building on the notion of the transmutability of statistics in 2D (Wilczek, 1982; Halperin, 1984; Arovas *et al.*, 1985; Girvin and MacDonald, 1987; Laughlin, 1988; Lopez and Fradkin, 1991; Moore and Read, 1991; Zhang *et al.*, 1989), a new model for the generation of higher-order FQHE states was introduced by Jain (1989, 1990). It is best described for the concrete set of states at $\nu=p/(2p+1)$, starting at $\nu=1/3$ and converging towards $\nu=1/2$ (see Fig. 1). Again, vortex attachment to electrons plays a pivotal role. As elucidated in the previous section, the $\nu=1/3$ state consists of electrons to which three vortices have been attached by virtue of the electron-electron interaction and is expressed by Laughlin's wave function (exponent "3"). Formally, this wave function can be factorized into

$$\psi_{1/2}=\prod_{i<j}^{n}(Z_i-Z_j)^2\prod_{i<j}^{n}(Z_i-Z_j)^1\exp\left(-\frac{1}{4}\sum_{k}^{n}|Z_k|^2\right)$$

$$=\prod_{i<j}^{n}(Z_i-Z_j)^2\psi_1 .$$

The second factor turns out to represent exactly one fully filled Landau level, Ψ_1, in which each electron is provided with one vortex, the minimum needed to satisfy the Pauli principle (spin neglected). Hence, at least formally, the FQHE state at $\nu=\frac{1}{3}$ can also be viewed as an IQHE state at $\nu=1$. However, now each electron carries *two* attached vortices. In the flux-quanta attachment language, two out of three of the flux quanta per electron have been incorporated into the new particle, reducing the external magnetic field to an *effective magnetic field* of only one flux quantum per composite, which is equivalent to the field at $\nu=1$. These composites, carrying an even number of flux quanta, behave as fermions and fill exactly one Landau level.

While being of largely philosophical nature at $\nu=1/3$, generalization of this concept to the filling of more than one composite fermion Landau level has important formal implications. Such a generalized composite fermion model generates very good many-particle wave functions for the higher-order states at $\nu=p/(2p+1)$, as deduced from comparison with exact, few-particle numerical calculations (Dev and Jain, 1992). Furthermore, it has provided a rationale for the dominance of this particular sequence of states in experiment (see Fig. 1).

Exploiting the analogies between the FQHE at $\nu=p/(2p+1)$ and an IQHE of composite fermions at Landau level filling factor p, we can postulate an analogy between composite fermion gap energies (i.e., FQHE gaps of higher-order fractions, $\nu=p/q$) and Landau gaps in the electron case (Halperin *et al.*, 1993): Similar to electrons, for which the Landau gap opens linearly with the magnetic field, the FQHE gap energies are conjectured to open practically linearly with effective magnetic field B_{eff}. However, the mass value m^*, derived from such composite fermion cyclotron gaps ($\hbar eB_{\mathrm{eff}}/m^*$), is unrelated to the cyclotron mass of the electron. The composite fermion mass is exclusively a consequence of electron-electron interaction and, hence, for a given fraction, exclusively a function of electron density. This generation of mass solely from many-particle interactions is very peculiar.

Experimentally, such a quasilinear relationship between the energy gap and the effective magnetic field is indeed borne out (Du *et al.*, 1993; Leadley *et al.*, 1994; Manoharan *et al.*, 1994; Coleridge *et al.*, 1995). As an ex-

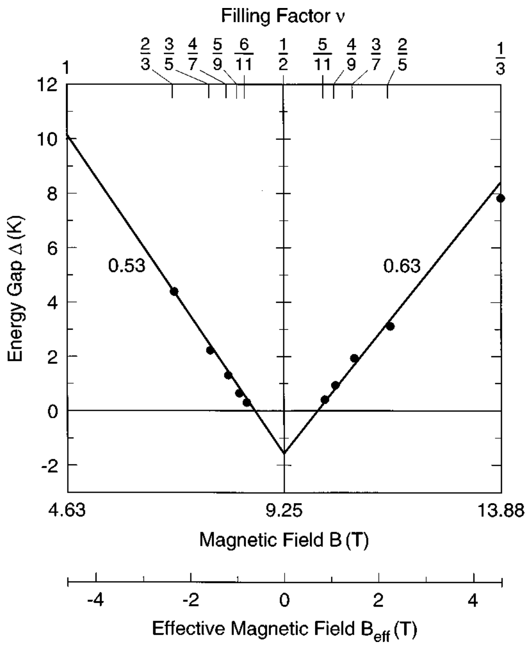

FIG. 2. Magnetic-field dependence of the energy gaps of fractional quantum Hall states at filling factors $\nu = p/(2p \pm 1)$ around $\nu = 1/2$, as determined from thermal activation energy measurements on a sample of density $n = 0.83 \times 10^{11}$ cm^{-2}. Lines are a guide to the eye. Gaps are clearly opening roughly linearly with an effective magnetic field, $B_{\rm eff} = B - B(\nu = 1/2)$, whose origin is at $\nu = 1/2$. It is reminiscent of the opening of Landau gaps for electrons around $B = 0$. Therefore the FQHE states around $\nu = 1/2$ can be viewed as arising from the Landau quantization of new particles, so-called composite fermions (CFs), which consist of electrons to which two magnetic flux quanta have been bound by virtue of the electron-electron interaction. Their mass, $m^* \sim 0.53$–$0.63 m_0$, determined from $\hbar e B_{\rm eff}/m^*$, is unrelated to the electron mass and of purely many-particle origin. The negative intercept at exactly $\nu = 1/2$ is believed to be a result of level broadening, which reduces the gaps by a fixed amount. From Du *et al.*, 1993.

ample, Fig. 2 shows measurements of the activation energies of the sequence of higher-order FQHE states in the vicinity of filling factor $\nu = 1/2$. The derived gap energies vary practically linearly with effective magnetic field, and masses of $\sim 0.6 m_0$ are obtained. The measured masses are about ten times larger than the electron mass in GaAs and increase in specimen with higher electron density n. The spirit of the composite fermion model has been most extensively tested for fractions in the vicinity of $\nu = 1/2$. However, in principle, the model is expected to hold for all odd-denominator rational filling factors, giving rise to multiple self-similarities in the experimental data. Studies of other such fractions are still in their infancy.

The composite fermion model has provided us with a rationale for the existence of higher-order FQHE states, the relative size of their energy gaps, and the appearance of particular sequences. It has also established a procedure by which to generate excellent many-particle wave functions as well as their excitations. How then are the FQHE and the IQHE related? Is the physics of the FQHE the same as the physics of the IQHE? Certainly not. The composite fermion model reveals a similarity in the mathematics between the sequence of IQHE states in terms of electrons and the sequence of FQHE states in terms of composite fermions. However, it also reveals a new class of strange particles, consisting of electrons dressed by vortices, which display a peculiar mass of purely many-particle origin. Excitations from such states carry an exactly fractional charge $e/3$, $e/5$, $e/7$, etc., and the Hall resistance exhibits odd rational fractional quantum numbers, such as 2/5, 3/13, and 5/23. These are truly remarkable characteristics of the fractional quantum Hall effect, having been brought into existence through the intricate cooperation of many electrons.

VI. EVEN-DENOMINATOR FILLING

What is the nature of the states at even-denominator filling? Compared to the odd-denominator FQHE states, even-denominator states appear uninspiring, showing hardly any variation in temperature-dependent transport. Yet, as it turns out, they are just as fascinating as their odd-denominator counterparts.

Early on, subtle features in the resistivity at $\nu = 1/2$ hinted at unanticipated interactions at half filling (Jiang *et al.*, 1989). However, it took a new probe, surface acoustic waves, to detect a distinctively different behavior at even-denominator filling as compared to odd-denominator filling.

As their name implies, surface acoustic waves are acoustic waves, typically at hundreds of MHz, that travel along the surface of a specimen. In the long-wavelength limit, velocity and attenuation of a surface acoustic wave are uniquely determined by material parameters of the semiconductor and the *dc* conductivity σ of the 2D electron gas. Actual surface acoustic wave data diverge markedly from those calculated from σ at $\nu = 1/2$ (see Fig. 3; Willett *et al.*, 1990). The origin of the discrepancy is found in the finite wavelength of the surface acoustic wave probe as compared to the *dc* transport measurements. Today we interpret this discrepancy as evidence for the existence of a novel Fermi system at $\nu = 1/2$ with a well-defined Fermi wave vector. Unlike the incompressible FQHE states away from half filling, such a Fermi system, having no energy gap, allows for infinitesimal excitations, and their specific wave vector dependence is qualitatively reflected in the surface acoustic wave data.

During the past few years, several experiments have unequivocally demonstrated the existence of a Fermi wave vector k_f at $\nu = 1/2$ (Kang *et al.*, 1993; Willett *et al.*, 1993; Goldman *et al.*, 1994; Smet *et al.*, 1996). They rely on the commensuration resonance between the classical cyclotron radius $r_c = \hbar k_f/(e B_{\rm eff})$ at filling factors somewhat off $\nu = 1/2$ and an externally imposed length. Remarkably, the relevant magnetic field, $B_{\rm eff} = B - B(\nu = 1/2)$, is the deviation of B from $\nu = 1/2$. The experiments reveal, in a very pictorial manner, the ballistic trajectories of the current-carrying objects. These particles, under the influence of $B_{\rm eff}$, execute large classical

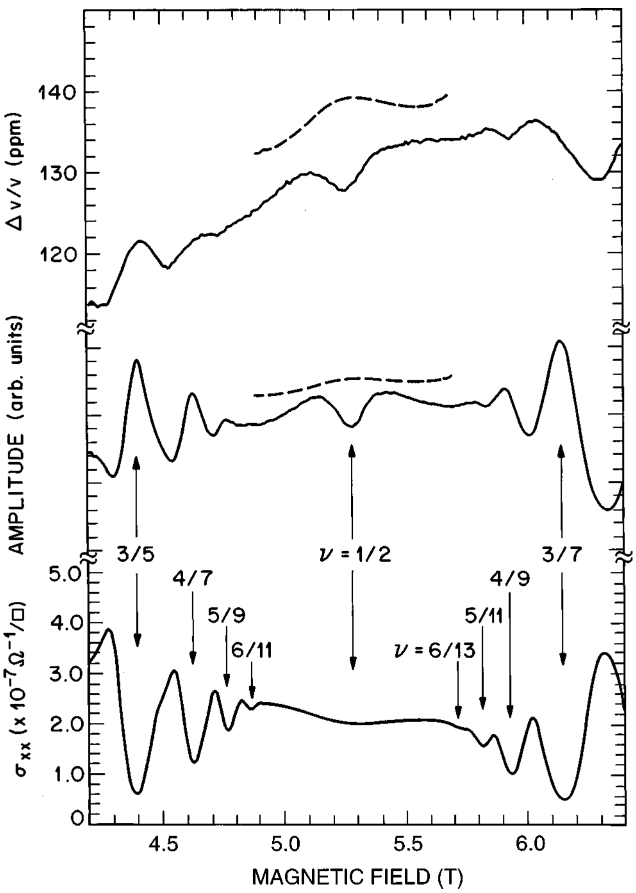

FIG. 3. Changes in amplitude and velocity ($\Delta v/v$) of surface acoustic waves through a 2D electron system as a function of magnetic field compared with its electrical conductivity σ_{xx}. Fractions ν indicate the Landau-level filling factor. Amplitude and $\Delta v/v$ agree closely (not shown) with the behavior calculated from σ_{xx} in most regions of the magnetic field, but deviate strongly at exactly $\nu=1/2$. This is indicated by the dashed curves, which are calculated in this regime from σ_{xx} and superimposed on the surface acoustic wave traces. These data provided evidence for exceptional properties at half filling. Today, we interpret this finding as being due to the formation of a novel Fermi system of composite fermions at exactly even-denominator fractional filling. The external magnetic field has been incorporated into the particles, and they move in the apparent absence of a magnetic field (see also Fig. 2). From Willett *et al.*, 1990.

cyclotron orbits, which degenerate into straight lines at exactly half filling. For them, the magnetic field seems to have vanished at $\nu=1/2$. How can a degenerate Fermi system emerge and what causes the field to vanish? Once again, an analogy between composite fermions and electrons offers an explanation.

As the filling factor converges towards $\nu=1/2$, composite fermions are populating more and more Landau levels. The effective magnetic field they are experiencing is the true external field reduced by two flux quanta per electron on account of the two vortices bound to each of them. At $\nu=1/2$ the effective field for composite fermions has dropped to zero, and the limit of infinitely many-Landau-level occupation has been reached. For electrons of fixed density, increasing Landau-level occupation is synonymous with convergence towards the electron Fermi sea at $B=0$. Hence one may conjecture that the state at $\nu=1/2$ represents a Fermi sea of composite fermions at $B_{\text{eff}}=0$, i.e., in the apparent absence of a magnetic field (Kalmeyer and Zhang, 1992; Halperin *et al.*, 1993). This model can account for many of the experimental observations, in particular the surface acoustic wave resonance data, the particle trajectories, and the geometrical resonances. Properties related to energy, such as particle mass at $\nu=1/2$, scattering rate, and a quantitative interpretation of the surface acoustic wave data, require further exploration, experimentally as well as theoretically. In fact, the Fermi system at $\nu=1/2$ has been conjectured to be of the marginal kind, exhibiting various singularities at E_f (Halperin *et al.*, 1993).

In the spirit of the above reasoning, a trial wave function for the state at $\nu=1/2$ has been presented (Rezayi and Read, 1994):

$$\psi_{1/2}=P\prod_{i<j}^{n}(Z_i-Z_j)^2\times\psi_\infty,$$

where Ψ_∞ denotes the electron Fermi sea at $B=0$. Composite fermions carry two vortices each (exponent 2) and are filling up a Fermi sea. The projection operator P ensures that the many-particle wave function resides wholly within the lowest Landau level, as required by the actual physical situation. What is the nature of this state, and what are its particles?

Just as at $\nu=1/3$, so also at $\nu=1/2$ vortex attachment to electrons reduces Coulomb interaction. The simplest and energetically most beneficial way to achieve this at $\nu=1/2$ is to place both vortices exactly onto the electron's position. Such a configuration represents a straight generalization of Laughlin's electronic wave function for the $\nu=1/3$ FQHE state. However, such a state violates the antisymmetry requirement for an electron wave function.

Barring exact vortex attachment, vortex proximity still remains beneficial. The electron system can maintain antisymmetry while reaping considerable Coulomb gain by making a slight adjustment. Of the two vortices per electron at $\nu=1/2$, one vortex is directly placed on each electron satisfying the requirements of the Pauli principle. The second vortex is being kept as close by as possible while obeying the antisymmetry requirement for the overall electron wave function. Successively increasing this separation in electron-vortex pairs, we find that a maximum distance r_f is reached which corresponds to k_f of the Fermi liquid. The resulting particles are no longer monopoles as they are at $\nu=1/3$ due to exact vortex attachment, but electron-vortex dipoles. Both "charges" of the dipole extend approximately one magnetic length and overlap strongly. This overlap considerably reduces their Coulomb energy.

Many of the properties of the state at $\nu=1/2$ can be visualized through such a simple picture, which regards the composite fermions at this filling factor as electrical

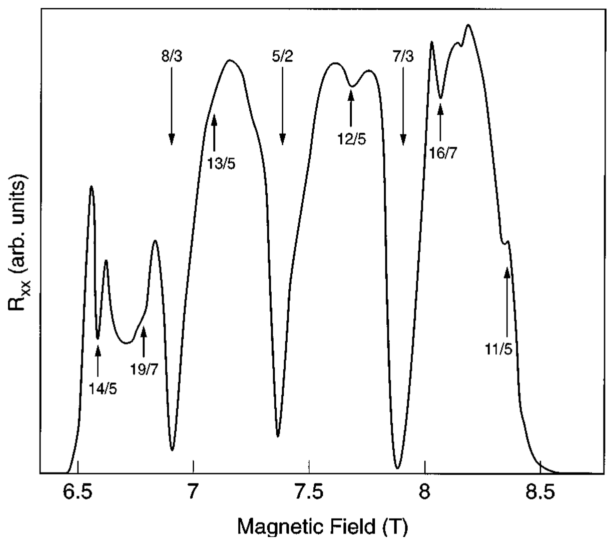

FIG. 4. Magnetoresistance data from the second Landau level (between $\nu=2$ and $\nu=3$) of a very high quality 2D electron system of density $n=4.45\times10^{11}$ cm^{-2} at 32 mK. Several FQHE states are visible. Most of them represent replicas, $\nu=2+p/q$, of FQHE states in the lowest Landau level at $\nu=p/q$. However, the state at $\nu=5/2$ is very unusual, showing the deep minimum in R_{xx} and plateau in R_{xy} (not shown) of a FQHE state while occurring at an *even-denominator* fraction. The origin of this state remains unclear. One possible explanation is the formation of composite fermions, as expected at even-denominator filling, which subsequently organize into pairs and condense into a state akin to superconductivity. From Du *et al.*, unpublished.

dipoles created from electrons and vortices. Even a mass can be derived from such a classical, albeit vortex-based, model. Again, the mass depends only on electron density and is unrelated to the mass of the electron.

A much more sophisticated reasoning than has been presented here for considering composite fermions as dipoles is currently being advanced (Shankar and Murthy, 1997; Lee, 1998; Pasquier and Haldane, 1998). It remains unclear to what degree it is valid and what all its implications are, in particular for such properties as the density of states in the vicinity of the Fermi energy.

The existence of such fascinating new Fermi systems is not limited to filling factor $\nu=1/2$. In fact, equivalent physics is expected to occur at all even-denominator fractions. Their study is just beginning, and data around $\nu=3/2$, 1/4, 3/4 reveal the anticipated composite fermion behavior. However, we already know of a major exception to the rule.

VII. COMPOSITE FERMION PAIRS?

At filling factor $\nu=5/2$, the lowest Landau level is totally filled and the lowest spin level of the second Landau level is half filled. The state at $\nu=5/2$ has always been puzzling. It has all the characteristics of a FQHE state, including energy gap and quantized Hall resistance, in spite of its even-denominator classification (see Fig. 4). Though it was discovered more than a decade ago (Willett *et al.*, 1987), its origin has never been clearly resolved (Rezayi and Haldane, 1988).

With the advent of the composite fermion model, the $\nu=5/2$ state has recently been revisited. An earlier proposal for the states at half filling (Moore and Read, 1991; Greiter, *et al.*, 1991) is now being examined as a contender for the state at $\nu=5/2$. This so-called "Pfaffian state," which has been characterized as a state of paired composite fermions, represents a very attractive option. If applicable, the state at $\nu=5/2$ can be thought of as arising in a two-step process. First, electrons in the second Landau level at half filling would form the "familiar" Fermi sea of composite fermions. These would then pair in a BCS-like fashion and condense into a novel "superconducting" ground state of composite fermions. Although this last analogy applies only in a very loose sense, the concept is truly exciting, and one should remain hopeful that further investigations into the state at $\nu=5/2$ may reveal an extraordinary new phase of 2D electrons (Morf, 1998).

CONCLUSION

Two-dimensional electrons in high magnetic fields have revealed to us totally new many-particle physics. Confined to a plane and exposed to a magnetic field, such electrons display an enormously diverse spectrum of fascinating new properties—fractional charge and fractional quantum numbers, new particles obeying either Bose or Fermi statistics, absorption of exceedingly high magnetic fields, apparent microscopic dipoles, and possibly a very unusual kind of particle pairing. They are just electrons—although lots of them. Indeed, "More is different" (Anderson, 1972).

ACKNOWLEDGMENTS

We should like to thank our numerous experimental collaborators for having been part of this quest, and we are grateful to many of our theorist colleagues for not giving up lecturing us on many-particle physics. In particular, we are indebted to L. N. Pfeiffer, K. W. West, and K. W. Baldwin, who created and processed most of the magnificent samples.

REFERENCES

Anderson, P. W., 1972, Science **177**, 393.
Ando, T., A. B. Fowler, and F. Stern, 1983, Rev. Mod. Phys. **54**, 437.
Arovas, D. P., J. R. Schrieffer, F. Wilczek, and A. Zee, 1985, Nucl. Phys. B **251**, 117.
Chakraborty, T., and P. Pietilainen, 1995, *The Quantum Hall Effects*, Springer Series in Solid State Sciences No. 85 (Springer, New York).
Chang, A. M., and J. E. Cunningham, 1990, Surf. Sci. **229**, 216.
Clark, R. G., J. R. Mallett, S. R. Haynes, J. J. Harris, and C. T. Foxon, 1988, Phys. Rev. Lett. **60**, 1747.
Coleridge, P. T., Z. W. Wasilewski, P. Zawadzki, A. S. Sachrajda, and H. A. Carmona, 1995, Phys. Rev. B **52**, 11 603.

DasSarma, S., and A. Pinczuk, 1997, Eds., *Perspectives in Quantum Hall Effects* (Wiley, New York).
de-Picciotto, R., M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, 1997, Nature (London) **389**, 162.
Dev, G., and J. K. Jain, 1992, Phys. Rev. Lett. **69**, 2843.
Du, R.-R., H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, 1993, Phys. Rev. Lett. **70**, 2944.
Du, R.-R., A. S. Yeh, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, 1998 (unpublished).
Eisenstein, J. P., and H. L. Stormer, 1990, Science **248**, 1461.
Girvin, S. M., and A. H. MacDonald, 1987, Phys. Rev. Lett. **58**, 1252.
Goldman, V. J., and B. Su, 1995, Science **267**, 1010.
Goldman, V. J., B. Su, and J. K. Jain, 1994, Phys. Rev. Lett. **72**, 2065.
Greiter, M., X.-G. Wen, and F. Wilczek, 1991, Phys. Rev. Lett. **66**, 3205.
Haldane, F. D. M., 1983, Phys. Rev. Lett. **51**, 605.
Halperin, B. I., 1984, Phys. Rev. Lett. **52**, 1983; **52**, 2390(E).
Halperin, B. I., P. A. Lee, and N. Read, 1993, Phys. Rev. B **47**, 7312.
Jain, J. K., 1989, Phys. Rev. Lett. **63**, 199.
Jain, J. K., 1990, Phys. Rev. B **41**, 7653.
Jiang, H. W., H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, 1989, Phys. Rev. B **40**, 12 013.
Kalmeyer, V., and S. I. Chang, 1992, Phys. Rev. B **46**, 9889.
Kang, W., H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, 1993, Phys. Rev. Lett. **71**, 3850.
Kivelson, S., D.-H. Lee, and S. C. Zhang, 1992, Phys. Rev. B **48**, 2223.
Kukushkin, I. V., N. J. Pulsford, K. von Klitzing, R. J. Haug, S. Koch, and V. B. Timofeev, 1992, Europhys. Lett. **18**, 62.
Laughlin, R. B., 1981, Phys. Rev. B **22**, 5632.
Laughlin, R. B., 1983, Phys. Rev. Lett. **50**, 1395.
Laughlin, R. B., 1984, in *Two-Dimensional Systems, Heterostructures, and Superlattices*, edited by G. Bauer, F. Kuchar, and H. Heinrich, Springer Series in Solid State Sciences, No. 53 (Springer, Berlin/New York).
Laughlin, R. B., 1988, Phys. Rev. Lett. **60**, 2677.
Leadley, D. R., R. J. Nicholas, C. T. Foxon, and J. J. Harris, 1994, Phys. Rev. Lett. **72**, 1906.
Lee, D.-H., 1998, Phys. Rev. Lett. **80**, 4745.
Lopez, A., and E. Fradkin, 1991, Phys. Rev. B **46**, 5246.
Manoharan, H. C., M. Shayegan, and S. J. Klepper, 1994, Phys. Rev. Lett. **73**, 3270.
Moore, G., and N. Read, 1991, Nucl. Phys. B **360**, 362.
Morf, R., 1998, Phys. Rev. Lett. **80**, 1505.
Pasquier, V., and F. D. M. Haldane, 1998, Nucl. Phys. B **515**, (F), in press.
Pinczuk, A., B. S. Dennis, L. N. Pfeiffer, and K. W. West, 1993, Phys. Rev. Lett. **70**, 3983.
Prange, R. E., and S. M. Girvin, 1990, Eds., *The Quantum Hall Effect*, 2nd Ed. (Springer, New York).
Read, N., 1994, Semicond. Sci. Technol. **9**, 1859.
Rezayi, E., and F. D. M. Haldane, 1988, Phys. Rev. Lett. **60**, 956.
Rezayi, E., and N. Read, 1994, Phys. Rev. Lett. **71**, 900.
Saminadayar, L., D. C. Glattli, Y. Jin, and B. Etienne, 1997, Phys. Rev. Lett. **79**, 2526.
Shankar, R., and G. Murthy, 1997, Phys. Rev. Lett. **79**, 4437.
Simmons, J. A., H. P. Wei, L. W. Engel, D. C. Tsui, and M. Shayegan, 1989, Phys. Rev. Lett. **63**, 1731.
Smet, J. H., D. Weiss, R. H. Blick, G. Lutjering, K. von Klitzing, R. Fleischmann, R. Ketzmerick, T. Geisel, and G. Weimann, 1996, Phys. Rev. Lett. **77**, 2272.
Stormer, H. L., R. Dingle, A. C. Gossard, W. Wiegmann, and M. D. Sturge, 1979, Solid State Commun. **29**, 705.
Tsui, D. C., H. L. Stormer, and A. C. Gossard, 1982, Phys. Rev. Lett. **48**, 1559.
von Klitzing, K., G. Dorda, and M. Pepper, 1980, Phys. Rev. Lett. **45**, 494.
Wilczek, F., 1982, Phys. Rev. Lett. **48**, 1144.
Willett, R. L., J. P. Eisenstein, H. L. Stormer, D. C. Tsui, A. C. Gossard, and J. H. English, 1987, Phys. Rev. Lett. **59**, 1776.
Willett, R. L., M. A. Paalanen, R. R. Ruel, K. W. West, L. N. Pfeiffer, and D. J. Bishop, 1990, Phys. Rev. Lett. **54**, 112.
Willett, R. L., R. R. Ruel, K. W. West, and L. N. Pfeiffer, 1993, Phys. Rev. Lett. **71**, 3846.
Zhang, S. C., H. Hanson, and S. A. Kivelson, 1989, Phys. Rev. Lett. **62**, 82.