Black holes

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Black holes are among the most intriguing objects in modern physics. They power quasars and other active galactic nuclei and also provide key insights into quantum gravity. We review the observational evidence for black holes and briefly discuss some of their properties. We also describe some recent developments involving cosmic censorship and the statistical origin of black-hole entropy. [S0034-6861(99)04902-8]

I. INTRODUCTION

Black holes are predicted by general relativity to be formed whenever sufficient mass is compressed into a small enough volume. In Newtonian language, the escape velocity from the surface becomes greater than the speed of light, so that nothing can escape. In general relativity, a black hole is defined as a region of spacetime that cannot communicate with the external universe. The boundary of this region is called the surface of the black hole, or the event horizon.

It appears impossible to compress matter on earth sufficiently to form a black hole. But in nature, gravity itself can compress matter if there is not enough pressure to resist the inward attractive force. When a massive star reaches the endpoint of its thermonuclear burning phase, nuclear reactions no longer supply thermal pressure, and gravitational collapse will proceed all the way to a black hole. By contrast, the collapse of a less massive star halts at high density when the core is transformed entirely into nuclear matter. The envelope of the star is blown off in a gigantic supernova explosion, leaving the core behind as a nascent neutron star.

The "modern" history of the black hole begins with the classic paper of Oppenheimer and Snyder (1939). They calculated the collapse of a homogeneous sphere of pressureless gas in general relativity. They found that the sphere eventually becomes cut off from all communication with the rest of the universe. Ultimately, the matter is crushed to infinite density at the center. Most previous discussions of the exterior gravitational field of a spherical mass had not taken into account the fact that the apparent singularity in the solution at the Schwarzschild radius was merely a coordinate artifact. Einstein himself claimed that one need not worry about the "Schwarzschild singularity" since no material body could ever be compressed to such a radius (Einstein, 1939). His error was that he considered only bodies in equilibrium. Even the usually sober Landau had been bothered by the prospect of continued gravitational collapse implied by the existence of a maximum stable mass for neutron stars and white dwarfs. To circumvent this, he believed at one time that " . . . all stars heavier than $1.5M_\odot$ certainly possess regions in which the laws of quantum mechanics . . . are violated" (Landau, 1932).

Despite the work of Oppenheimer and Snyder, black holes were generally ignored until the late 1950s, when Wheeler and his collaborators began a serious investigation of the problem of gravitational collapse (Harrison *et al.*, 1965). It was Wheeler (1968) who coined the term "black hole." The discovery of quasars, pulsars, and compact x-ray sources in the 1960s finally gave observational impetus to the subject and ushered in the "golden age" of black-hole research.

Black holes are now believed to exist with a variety of masses. A current estimate for the dividing line between progenitor stars that produce neutron stars and those that produce black holes is around $25M_\odot$. The resulting black holes are expected to have masses in the range $3-60\ M_\odot$. As discussed below, there is also good astrophysical evidence for supermassive black holes, with masses of order $10^6-10^9\ M_\odot$. There are a number of scenarios that could produce such large black holes: the gravitational collapse of individual supermassive gas clouds, the growth of a seed black hole capturing stars and gas from a dense star cluster at the center of a galaxy, or the merger of smaller black holes produced by collapse. There has also been speculation that black holes with a very wide range of masses might have been produced from density fluctuations in the early universe, but so far there is no convincing evidence for the existence of such primordial black holes.

This article provides just an overview of the astrophysical evidence for black holes, and discusses some recent theoretical developments in black-hole research. For a more complete discussion of the basic properties of black holes, see the books by Misner, Thorne, and Wheeler (1973), Shapiro and Teukolsky (1983), or Wald (1984).

II. OBSERVATIONAL EVIDENCE FOR BLACK HOLES

A. The maximum mass of neutron stars

Neutron stars of small enough mass can exist happily in equilibrium, but beyond a certain critical mass, the inward pull of gravity overwhelms the balancing pressure force—the star is unstable and will collapse to a black hole. This provides one of the key observational signatures of a black hole astronomically: look for a system containing a dark, compact object. If you can determine that the mass of the object is greater than the maximum allowed mass of a neutron star, then it must be a black hole.

The value of the maximum neutron star mass is uncertain theoretically because we do not understand nuclear physics well enough to calculate it reliably (see, e.g., Baym, 1995). Current conventional nuclear equations of state predict a maximum mass around $2M_\odot$ (see, e.g., the discussion and references in Cook, Shapiro, and Teukolsky, 1994, or Baym, 1995). (For some "unconventional" possibilities, see Brown and Bethe, 1994; Bahcall, Lynn, and Selipsky, 1990; Miller, Shahbaz, and Nolan, 1998.)

Because of these uncertainties, astrophysicists generally rely on a calculation that assumes we understand nuclear physics up to some density ρ_0 and then varies the pressure-density relation over all possibilities beyond this point to maximize the resulting mass (Rhoades and Ruffini, 1974). This procedure yields an upper limit to the maximum mass of

$$M_{\rm max} \simeq 3.2 M_\odot \left(\frac{4.6\times 10^{14}\ \mathrm{g\,cm^{-3}}}{\rho_0} \right)^{1/2}. \tag{1}$$

Kalogera and Baym (1996) have redone the Rhoades-Ruffini calculation with more up-to-date physics and obtained essentially the same numbers: a coefficient of $2.9M_\odot$ for a preferred matching density of 5.4×10^{14} g cm^{-3}. Rotation increases the amount of matter that can be supported against collapse, but even for stars rotating near breakup speed, the effect is only about 25% (see, e.g., Cook, Shapiro, and Teukolsky, 1994). The Rhoades-Ruffini calculation assumes the causality condition that the speed of sound is less than the speed of light: $dP/d\rho \leqslant c^2$. Abandoning this assumption increases the coefficient in Eq. (1) from 3.2 to 5.2 (Hartle and Sabbadini, 1977, and references therein). But it is not clear that this can be done without the material of the star becoming spontaneously unstable (Bludman and Ruderman, 1970; but see also Hartle, 1978). In summary, circumventing these mass limits would require us to accept some unconventional physics—much more unconventional than black holes!

B. Observational signatures of black holes

A black hole is the most compact configuration of matter possible for a given mass. The size of a black hole of mass M is given by the Schwarzschild radius, the radius of the event horizon:

$$R_S = \frac{2GM}{c^2} = 3\ \mathrm{km} \left(\frac{M}{M_\odot} \right). \tag{2}$$

One way of verifying the compactness of a candidate black hole is to measure the speed of matter in orbit around it, which is expected to approach c near the horizon. This test is feasible since accretion flows of orbiting gas are common around gravitating objects in astrophysics. In a few objects, direct evidence for high orbital speeds is obtained by measuring the Doppler broadening of spectral lines from the accreting gas. More often, black-hole candidates exhibit gas outflows, or jets, with relativistic speeds. Another indication of compactness comes from observations of strong x-ray emission from the accreting gas, which imply high temperatures $> 10^9$ K. Such temperatures are easily achieved by accretion onto a black hole or a neutron star, both of which have sufficiently deep potential wells.

When the radiation (typically x rays) from a compact object varies on a characteristic time scale t, without contrived conditions the size of the object must be less than ct. If this size limit is comparable to R_S (determined from an independent mass estimate) then the object is potentially a black hole. For solar-mass black holes, this implies looking for variability on the scale of less than a millisecond.

The demonstration of compactness alone, however, is not sufficient to identify a black hole; a neutron star, with a radius of about $3R_S$, is only slightly larger than a black hole of the same mass. Clear evidence that $M > M_{\rm max}$ is needed in addition to compactness.

Any gravitating object has a maximum luminosity, the Eddington limit, given by

$$L_{\rm Edd} \simeq 10^{38}\ \mathrm{erg\,s^{-1}} \left(\frac{M}{M_\odot} \right) \tag{3}$$

(see, e.g., Shapiro and Teukolsky, 1983). Above this luminosity, the outward force due to escaping radiation on the accreting gas overwhelms the attractive force due to gravity, and accretion is no longer possible. Thus the observed luminosity sets a lower limit on the mass of the accreting object, which can often suggest the presence of a black hole.

C. Supermassive black holes in galactic nuclei

Quasars emit immense amounts of radiation, up to $\sim 10^{46}$ erg s^{-1}, from very small volumes. They are members of a wider class of objects, active galactic nuclei, all of which generally radiate intensely.

Nearly all active galactic nuclei emit substantial fractions of their radiation in x rays, and some emit the bulk of their radiation in even more energetic γ rays. Rapid variability of the flux has been observed in some active galactic nuclei. Many also have relativistic jets. These are all signatures of a compact relativistic object. If the observed radiation is powered by accretion, as is generally assumed, then the Eddington limit [Eq. (3)] implies masses in the range 10^6–$10^{10} M_\odot$. This is well above the maximum mass of a neutron star, and so active galactic nuclei are considered secure black-hole candidates. Menou, Quataert, and Narayan (1998) give a summary of the current best supermassive black-hole candidates at the centers of nearby galaxies.

Direct evidence for the existence of a central relativistic potential well has come from the recent detection of broad iron fluorescence lines in x rays in a few active galactic nuclei. The line broadening can be interpreted as a combination of Doppler broadening and gravitational redshift. A spectacular example is the galaxy MCG-6-30-15, where a very broad emission line has been observed. The data can be interpreted as suggesting that the central mass is a rapidly rotating black hole, but this is still tentative. (See Menou, Quataert, and Narayan, 1998, for a discussion and references for this source and many others. See Rees, 1998, for a general discussion of astrophysical evidence for black holes.)

D. Black holes in x-ray binaries

In an x-ray binary, one of the stars is compact and accretes gas from the outer layers of its companion. Because of angular momentum conservation in the rotating system, gas cannot flow directly onto the compact object. Instead, it spirals towards the compact object and heats up because of viscous dissipation, producing x rays. In many cases, the compact star is known to be a neutron star, but there are also a number of excellent black-hole candidates.

The mass of the x-ray-emitting star M_X can be constrained by observations of the spectral lines of the secondary star. The Doppler shifts of these lines give an estimate of the radial velocity v_r of the secondary as it orbits the x-ray star. Combining v_r with the orbital period P of the binary and using Kepler's third law yields the "mass function" of the compact object,

$$f(M_X) \equiv \frac{M_X \sin^3 i}{(1+q)^2} = \frac{P v_r^3}{2\pi G} \tag{4}$$

(see, e.g., Shapiro and Teukolsky, 1983). The mass function does not give M_X directly because of its dependence on the unknown inclination i of the binary orbit and the ratio q of the two masses. However, it is a firm lower limit on M_X. Therefore, mass functions above $3M_\odot$ suggest the presence of black holes. Additional observational data—absence or presence of eclipses, for instance, or information on the nature of the secondary star—can help to constrain i or q, so that a likely value of M_X can often be determined. The best stellar-mass black-hole candidates currently known are summarized in Menou, Quataert, and Narayan (1998).

The first black-hole candidate discovered in this way was Cyg X–1. Although its mass function is not very large, there are good observations that set limits on i and q and suggest that M_X is definitely greater than 3–4 $M_\odot$, with the likely value being 7–20 $M_\odot$. Even stronger evidence is provided by other x-ray binaries for which $f(M_X) > 3M_\odot$. Without any further astrophysical assumptions, one can be pretty sure that these objects are not neutron stars. Currently, the most compelling black-hole candidate is V404 Cyg, with a mass function of $6M_\odot$.

Many of these sources show the key observational signatures of black holes described in Sec. II.B. Some display rapid variability in their x-ray emission. Many occasionally reach high luminosities, implying masses greater than that of a neutron star via the Eddington limit Eq. (3). A few exhibit relativistic jets.

E. Conclusive evidence for black holes

All the methods for finding black holes described above are indirect. They essentially say that there is a lot of mass in a small volume. Direct proof that a candidate object is a black hole requires a demonstration that the object has the spacetime geometry predicted by Einstein's theory. For example, we would like to have evidence for an event horizon, the one feature that is unique to a black hole.

One possible approach is via accretion theory (see Menou, Quataert, and Narayan, 1998, for a review). Two kinds of accretion are important for flow onto compact objects. The first is accretion from a thin disk. The accreting gas quickly radiates whatever energy is released through viscous dissipation. The gas stays relatively cool and so the disk remains thin, each gas element orbiting the central mass at the Keplerian velocity. Unlike the Newtonian case, the gravitational field of a compact mass in general relativity has a final stable circular orbit. The inner edge of the disk extends up to this radius. Observations such as those of the iron fluorescence lines described above provide information on the radius of the inner edge of the accretion disk. Since the radius of the last stable circular orbit depends on the spin of central mass, we may be able to measure the spin of black holes in this way.

Thin disks have oscillatory modes whose details depend on general relativity. Quasiperiodic oscillations have been detected in several x-ray binaries, and can be used to probe the spacetime geometry ("diskoseismology"; see Rees, 1998, for a review and references). In addition, if the disk is tilted with respect to the spin axis of the central mass, it will precess because of frame dragging (Lense-Thirring effect). This produces a periodic modulation of the x-ray luminosity, which may already have been seen in a few cases.

The second important kind of accretion is advection-dominated accretion flow. Here, the accreting gas advects most of the energy released by viscosity to the center. The gas becomes relatively hot and quasispherical. The spectrum is quite different from that of a thin disk. Advection-dominated accretion flows appear to be present in both galactic nuclei and in x-ray binaries when the accretion rate is relatively low. In an advection-dominated accretion flow, what happens to the energy advected to the center depends on the nature of the central object. If it is a black hole, the energy simply disappears behind the event horizon. If it is a neutron star or any object with a surface, the energy is reradiated from the surface and will dominate the spectrum. For those black-hole candidates that seem to be accreting in advection-dominated accretion flows, the evidence is that they lack surfaces. While not yet conclusive because of modeling uncertainties, this is the most direct evidence yet that black holes with event horizons are present in nature.

Is there any hope of a clean observation of black-hole geometry without the complications of dirty astrophysics? The best hope is from the observation of gravitational waves from black-hole collisions (see the article by Weiss in this volume). Laser interferometers now under construction, such as LIGO, VIRGO, and GEO (see, e.g., Abramovici *et al.*, 1992; Thorne, 1994) will be sensitive to black hole–black hole and black hole–neutron star collisions with black-hole masses up to a few tens of solar masses. The predicted event rate for such collisions is highly uncertain: estimates range from about one per year for the initial LIGO detector and thousands per year for the upgraded LIGO (Siggurdson and Hernquist, 1993; Lipunov, Postnov, and Prokhorov, 1997; Bethe and Brown, 1998), to essentially zero (Zwart and Yungelson, 1998). If nature is kind and we do detect such events, the wave form encodes a great deal of information about the spacetime geometry. The part of the wave form from the highly nonlinear merger phase is currently being calculated with large-scale supercomputer simulations (see, e.g., Finn, 1997), and it is expected that comparison of such calculations with observations should yield not only the masses and spins of the colliding objects, but also a check that the wave form is consistent with general relativity. The final part of the wave form is a "ring down," like a damped harmonic oscillator. This has been calculated by perturbation theory, and should provide another strong test.

There is also good reason to believe that, when two galaxies each containing supermassive black holes merge, the black holes will spiral together and coalesce. The frequency of the gravitational waves emitted is too low to be detectable on earth, where the waves would be swamped by seismic noise. However, such events should be readily detectable by a laser interferometer in space, such as the proposed LISA detector (see, e.g., Bender *et al.*, 1996)

III. BLACK-HOLE UNIQUENESS

The solution of Einstein's equations that describes a spherical black hole was discovered by Karl Schwarzschild only a few months after Einstein published the final form of general relativity:

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2\,(d\theta^2+\sin^2\theta\,d\phi^2). \tag{5}$$

(Here, and for the remainder of our discussion, we use units with $c = G = 1$.) This metric turns out to be the only spherically symmetric solution in the absence of matter. In general relativity, as in Newtonian gravity, the vacuum gravitational field outside any spherically symmetric object is the same as that of a point mass. The event horizon occurs at $r = 2M$ [cf. Eq. (2)]. Although the metric components are singular there, they can be made regular by a simple change of coordinates. In contrast, the singularity at $r = 0$ is real. An observer falling into a Schwarzschild black hole will be ripped apart by infinite tidal forces at $r = 0$.

One might expect that solutions of Einstein's equations describing realistic black holes that form in nature and settle down to equilibrium would be very complicated. After all, a black hole can be formed from collapse of all kinds of matter configurations, with arbitrary multipole distributions, magnetic fields, distributions of angular momentum, and so on. For most situations, after the black hole has settled down, it can be described by a solution of Einstein's vacuum field equations. Remarkably, one can show that the only stationary solution of this equation that is asymptotically flat and has a regular event horizon is a generalization of Eq. (5) known as the Kerr metric. This solution has only two parameters: the mass M and angular momentum J. All other information about the precursor state of the system is radiated away during the collapse. Astrophysical black holes are not expected to have a large electric charge since free charges are rapidly neutralized by plasma in an astrophysical environment. Nevertheless, there is an analog of this uniqueness theorem for charged black holes: all stationary solutions of the Einstein-Maxwell equations that are asymptotically flat and have a regular event horizon are known, and depend only on M, J and the charge Q.

The simplicity of the final black-hole state is summarized by Wheeler's aphorism, "A black hole has no hair." This is supported not only by the above uniqueness theorems, but also by results showing that if one couples general relativity to simple matter fields, e.g., free scalar fields, there are no new stationary black-hole solutions. However, it has recently been shown that if more complicated matter is considered, new black-hole solutions can be found. Examples include Einstein-Yang-Mills black holes, black holes inside magnetic monopoles, and charged black holes coupled to a scalar "dilaton." Even these new black holes are characterized by only a few parameters, so the spirit of Wheeler's aphorism is maintained. (For a recent review and references, see Bekenstein, 1997.)

IV. COSMIC CENSORSHIP

In the late 1960s, a series of powerful results were established in general relativity showing that, under generic conditions, gravitational collapse produces infinite gravitational fields, i.e., infinite spacetime curvature (see, e.g., Hawking and Ellis, 1973). However, these "singularity theorems" do not guarantee the existence of an event horizon. It is known that uniform-density, spherically symmetric gravitational collapse produces a black hole (the Oppenheimer-Snyder solution), and small perturbations do not change this. It is conceivable, however, that highly nonspherical collapse or, e.g., the collision of two black holes could produce singularities that are not hidden behind event horizons. These regions of infinite curvature would be visible to distant observers and hence are called "naked" singularities. Penrose (1969) proposed that naked singularities could not form in realistic situations, a hypothesis that has become known as cosmic censorship. If this is violated, general relativity could break down outside black holes, and would not be sufficient to predict the future evolution. On the positive side, this would open up the possibility of direct observations of quantum gravitational effects. Establishing whether cosmic censorship holds is perhaps the most important open question in classical general relativity today.

Despite almost 30 years of effort, we are still far from a general proof of cosmic censorship. (For a recent review and references, see Wald, 1997.) This seems to require analysis of the late time evolution of Einstein's equation in the strong-field regime. The much simpler problem of determining the global evolution of relatively weak (but still nonlinear) gravitational waves was achieved only in the late 1980s, and was hailed as a technical tour-de-force. In light of this, progress has been made by studying simpler systems, trying to find counterexamples, and by numerical simulations. The simpler systems are usually general relativity with one or two symmetries imposed. For example, cosmic censorship has been established for a class of solutions with two commuting symmetries. One class of potential counterexamples consists of time-symmetric initial data containing a minimal surface S. Assuming cosmic censorship, one can show that the area of this minimal surface must be related to the total mass M by $A(S) \leqslant 16\pi M^2$. Unsuccessful attempts were made to find initial data that violate this inequality. Recently, a general proof of this inequality has been found, showing that no counterexamples of this type exist. Numerical simulations of nonspherical collapse have found some indication that cosmic censorship may be violated in certain situations (Shapiro and Teukolsky, 1991), and suggest that any theorem might need careful specification of what is meant by "generic" initial data.

Perhaps the most effort and the most interesting results have come from studying spherically symmetric collapse. It was shown in the early 1970s that naked singularities could form in inhomogeneous dust collapse, but it was quickly realized that these "shell-crossing" or "shell-focusing" singularities also occurred in the absence of gravity and just reflected an unrealistic model of matter. It was believed at the time that any description of matter that did not produce singularities in flat spacetime would not produce naked singularities when coupled to gravity. This has recently been shown to be false. Consider spherically symmetric scalar fields coupled to gravity. If the initial amplitude is small, the waves will scatter and disperse to infinity. If the initial amplitude is large, the waves will collapse to form a black hole. As one continuously varies the amplitude, there is a critical value that divides these two outcomes. It has been shown that, at this critical value, the evolution produces a naked singularity. This is not believed to be a serious counterexample to cosmic censorship since it is not generic. But it again indicates that a true formulation of cosmic censorship is rather subtle.

Studies of spherical scalar-field collapse near the critical amplitude $\mathcal{A}_0$ have yielded a surprising result. The mass of the resulting black hole, for $\mathcal{A} > \mathcal{A}_0$, is

$$M_{\rm BH} \sim |\mathcal{A} - \mathcal{A}_0|^\gamma, \tag{6}$$

where γ is a universal exponent that is independent of the initial wave profile. Gravitational collapse of other matter fields, or axisymmetric gravitational waves, exhibit similar behavior (with a different exponent). Furthermore, the solution with $\mathcal{A} = \mathcal{A}_0$, exhibits a type of scale invariance. These properties are similar to critical phenomena in condensed matter systems. They are not yet fully understood, but may turn out to be related to thermodynamic properties of black holes, which we discuss next. For recent reviews of critical phenomena in gravitational collapse, see Gundlach (1998) and Choptuik (1998).

V. QUANTUM BLACK HOLES

For an equilibrium black hole, one can define a quantity called the surface gravity κ which can be thought of as the force that must be exerted on a rope at infinity to hold a unit mass stationary near the horizon of a black hole. During the early 1970s, it was shown that black holes have the following properties:

(0) The surface gravity is constant over the horizon, even for rotating black holes that are not spherically symmetric.

(1) If one throws a small amount of mass into a stationary black hole characterized by M, Q, and J, it will settle down to a new stationary black hole. The change in these three quantities satisfies

$$\delta M = \frac{\kappa \delta A}{8\pi} + \Omega\, \delta J, \tag{7}$$

where A is the area of the event horizon and Ω is the angular velocity of the horizon.

(2) The area of a black hole cannot decrease during physical processes.

It was immediately noticed that there was a close similarity between these "laws of black-hole mechanics" and the usual laws of thermodynamics, with κ proportional to the temperature and A proportional to the entropy. However, it was originally thought that this could only be an analogy, since if a black hole really had a nonzero temperature, it would have to radiate and everyone knew that nothing could escape from a black hole. This view changed completely when Hawking (1975) showed that if matter is treated quantum mechanically, black holes do radiate. This showed that black holes are indeed thermodynamic objects with a temperature and entropy given by

$$T_{\rm bh} = \frac{\hbar \kappa}{2\pi}, \quad S_{\rm bh} = \frac{A}{4\hbar}. \tag{8}$$

This turns out to be an enormous entropy, much larger than the entropy of a corresponding amount of ordinary matter. For a review of black-hole thermodynamics, see Wald (1998).

In all other contexts, we know that thermodynamics is the result of averaging over a large number of different microscopic configurations with the same macroscopic properties. So it is natural to ask, What are the microstates of a black hole that are responsible for its thermodynamic properties? This question has recently been answered in both of the dominant approaches to quantum gravity today: string theory and canonical quantization of general relativity. We will focus on the situation in string theory, since this is further developed. (String theory is discussed in more detail in the article by Schwarz and Seiberg in this volume.) Briefly, string theory is based on the idea that elementary particles are not pointlike, but are actually different excitations of a one-dimensional extended object—the string. Strings interact by a simple splitting and joining interaction that turns out to reproduce the standard interactions of elementary particles. The strength of the interactions is governed by a string coupling constant g. A crucial ingredient in string theory is that it is supersymmetric. In any supersymmetric theory, the mass and charge satisfy an inequality of the form $M \geqslant cQ$ for some constant c. States that saturate this bound are called BPS (Bogolmonyi-Prasad-Sommerfield) states and have the special property that their mass does not receive any quantum corrections.

Now consider all BPS states in string theory with a given large charge Q. At weak string coupling g, these states are easy to describe and count. Now imagine increasing the string coupling. This increases the force of gravity, and causes these states to become black holes. Charged black holes also satisfy the inequality $M \geqslant cQ$ and, when equality holds, the black holes are called extremal. So the BPS states all become extremal black holes. But there is only one black hole for a given mass and charge, so the BPS states all become identical black holes. This is the origin of the thermodynamic properties of black holes. When one compares the number of BPS states N to the area of the event horizon, one finds that, in the limit of large charge,

$$N = e^{S_{\mathrm{bh}}}, \tag{9}$$

in precise agreement with black-hole thermodynamics. This agreement has been shown to hold for near-extremal black holes as well, where the mass is slightly larger than cQ.

Extremal black holes have zero Hawking temperature and hence do not radiate. But near-extremal black holes do radiate approximately thermal radiation at low temperature. Similarly, the interactions between near-BPS states in string theory produce radiation. Remarkably, it turns out that the radiation predicted in string theory agrees precisely with that coming from black holes. This includes deviations from the black-body spectrum, which arise from two very different sources in the two cases. In the black-hole case, the deviations occur because the radiation has to propagate through the curved spacetime around the black hole. This gives rise to an effective potential that results in a frequency-dependent "grey-body factor" in the radiation spectrum. The string calculation at weak coupling is done in flat spacetime, so there are no curvature corrections. Nevertheless, there are deviations from a purely thermal spectrum because there are separate left- and right-moving degrees of freedom along the string. Remarkably, the resulting spectra agree. Progress has also been made in understanding the entropy of black holes far from extremality. In both string theory and a canonical quantization of general relativity, there are calculations of the entropy of neutral black holes up to an undetermined numerical coefficient. For reviews of these developments in string theory, see Horowitz (1998) or Maldacena (1996). For the canonical quantization results, see Ashtekar *et al.* (1998).

VI. CONCLUSIONS

Black holes connect to a wide variety of fields of physics. They are invoked to explain high-energy phenomena in astrophysics, they are the subject of analytic and numerical inquiry in classical general relativity, and they may provide key insights into quantum gravity. We also seem to be on the verge of verifying that these objects actually exist in nature with the spacetime properties given by Einstein's theory. Finding absolutely incontrovertible evidence for a black hole would be the capstone of one of the most remarkable discoveries in the history of science.

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