Anomalous g values of the electron and muon

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Recent advances in theory and experiment for the anomalous magnetic moments of the electron and muon are reviewed. Implications of these developments for fundamental physics are discussed. [S0034-6861(99)02402-2]

I. INTRODUCTION

Historically, the spin magnetic moment of the electron μ_e or its g value g_e has played a central role in modern physics, dating from its discovery in atomic optical spectroscopy and its subsequent incorporation in the Dirac theory of the electron, which predicted the value $g_e=2$. The experimental discovery in atomic microwave spectroscopy that g_e was larger than 2 by a multiplicative factor of about 1 part in 10^3, $g_e=2.00238(10)$, together with the discovery of the Lamb shift in hydrogen ($\mathcal{S}=2^2S_{1/2}-2^2P_{1/2}$), led to the development of modern quantum electrodynamics with its renormalization procedure. The theory enables us to calculate these effects precisely as finite radiative corrections. By now the experimental value of g_e-2 has been measured to about 4 ppb, and the theoretical value, which is expressed as a power series in the fine-structure constant α, has been evaluated to better than 1 ppb, assuming the value of α is known.

For the muon, as well, g_μ is greater than 2 by a multiplicative factor of about 1 part in 10^3. This was found experimentally shortly after the discovery of parity nonconservation in the weak interaction, which provided the basic tools for the measurement of g_μ. This result provided one of the crucial pieces of evidence that the muon behaves like a heavy electron, i.e., there is $\mu-e$ universality. By now the value of $g_\mu-2$ has been measured to 7 ppm. Treating the muon as a heavy electron, theorists have evaluated $g_\mu-2$ to within better than 1 ppm. The main difference between g_μ and g_e is that the lepton vacuum-polarization contributions are very different for the muon and the electron. Furthermore, because the muon has a heavier mass than that of the electron, higher-mass particles—some perhaps not yet discovered—contribute much more to g_μ than to g_e by a factor of $\sim(m_\mu/m_e)^2\simeq 4\times 10^4$.

The motivation for a continued study of electron and muon anomalous g values, $a\equiv(g-2)/2$, is twofold:

(1) Theoretically the anomalous g value is the simplest quantity calculable to an arbitrary precision. Note that quantities such as particle mass and the coupling constant $\propto$ are external parameters of the current standard theory and cannot be calculated from the theory itself. Precision measurements of a_e and a_μ therefore provide a crucial test of predictions of (renormalizable) quantum field theory. The firm theoretical basis for computing a_μ and a_e, taken together with more precise measurements of a_μ, will not only test the standard model further but may open up a window into the study of entirely new physics.

(2) The measurement and theory of a_e have become so precise that a_e gives the most stringent test of QED if α is known precisely. Unfortunately, no available α is known with sufficient precision to enable such a test. This means, however, that the theory and measurement of a_e together will lead to the most precise value of the fine-structure constant α currently available. Comparison of α derived from a_e with other high-precision measurements of α based on condensed-matter physics, atomic physics, and other means offers an intriguing opportunity to introduce a quantitative measure of the success of quantum theory, which is at the root of all physics developed in the twentieth century. This topic will be discussed in greater detail in Sec. VI.

II. ELECTRON $g-2$ EXPERIMENTS

The latest and most precise measurement of the electron $g-2$ value involves observation of microwave-induced transitions between Landau-Rabi levels of an electron in a magnetic field (Fig. 1) by Dehmelt and his

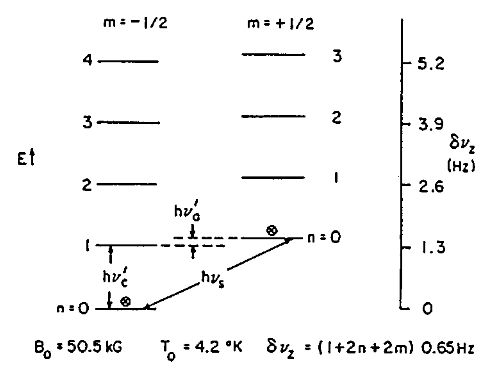

FIG. 1. Lowest Landau-Rabi levels for a geonium atom. The axial frequency (shown in the right-hand scale) corresponds to the coupling via the axial magnetic bottle field. The quantities ν_c' and ν_a' are perturbed values of ν_c and ν_a. The lowest state $(n=0)$ which is occupied by the electron or positron 80–90 % of the time differs by 1.3 Hz depending on the exact spin state. This is the signature used to indicate that a spin has flipped. From Van Dyck (1990).

collaborators (Van Dyck, Schwinburg, and Dehmelt, 1987; Van Dyck, 1990).

A single electron (or positron) moves in a Penning trap in a strong magnetic field of 5 T at a low temperature of 4 K, forming a "geonium" atom. Axial, cyclotron, and magnetron motions occur. The cyclotron frequency ω_c and the difference frequency ω_a (anomaly frequency) between the spin precession frequency ω_s and ω_c are measured. Their ratio determines a_e. The transitions are detected by changes in the axial frequency of the electron, observed through an induced voltage in an external circuit. This experiment has led to very precise values for electron and positron:

$$a_{\mathrm{e}^-}(\mathrm{expt}) = 1\,159\,652\,188.4\,(4.3)\times 10^{-12} \;\; (4 \;\; \mathrm{ppb}),$$

$$a_{\mathrm{e}^+}(\mathrm{expt}) = 1\,159\,652\,187.9\,(4.3)\times 10^{-12} \;\; (4 \;\; \mathrm{ppb}). \quad (1)$$

The values for a_{e^-} and a_{e^+} agree to within 1 ppb.

The statistical error in Eq. (1) is 0.62×10^{-12}, a systematic error of 1.3×10^{-12} is due to the uncertainty in a residual microwave power shift, and the largest uncertainty of 4×10^{-12} is assigned to a potential cavity-mode shift. This last error arises from a shift in the cyclotron frequency of the electron associated with image charges induced in the metallic Penning trap, an effect which depends on the cavity frequency modes and on the electron cyclotron frequency (Brown *et al.*, 1985a, 1985b).

Studies to improve the experimental precision for a_e focus on the understanding and control of this cavity influence on the cyclotron frequency. For this purpose Mittleman *et al.* (1995) have produced and studied a many-electron (kiloelectron) cluster in the trap, which magnifies the shift of the cyclotron frequency. Gabrielse and Tan (1994) are studying the use of a cylindrical cavity where the cyclotron frequency shift can be better understood and controlled. Eventual reduction of experimental uncertainty by about an order of magnitude is the goal.

III. MUON $g-2$ EXPERIMENTS

The muon $g-2$ value has been determined in a series of experiments at CERN (Bailey *et al.*, 1979; Farley and Picasso, 1990). In the latest experiment, polarized muons from pion decays are captured in a storage ring with a uniform magnetic field and a weak-focusing electric quadrupole field. For a muon momentum of 3.09 GeV/c and $\gamma = 29.3$ the muon spin motion is unaffected by the electric quadrupole field and the difference frequency ω_a is given by

$$\omega_a = \omega_s - \omega_c = \frac{eB}{mc} a_\mu, \quad (2)$$

in which ω_s is the spin precession frequency and ω_c the orbital cyclotron frequency. Measurements of ω_a and B thus determine a_μ.

The stored μ^+ in the ring decay to e^+ via the parity-violating weak decay $\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$, and the high-energy e^+ are emitted preferentially in the direction of the muon spin. Decay e^+ are detected with lead/scintillator detectors as a function of time after π injection. Of course μ^- can be treated in the same way. The time spectrum for the e^+ counts is given by

$$N_e = N_0 e^{-t/\gamma\tau_0}[1 + A\cos(\omega_a t + \phi)], \quad (3)$$

in which τ_0 is the muon lifetime at rest, γ is the relativistic time dilation factor, and A and ϕ are fitting parameters. The exponential muon decay is modulated at the frequency ω_a, which is determined from the fit of Eq. (3) to the data. The storage ring field B is measured by NMR.

The CERN results were

$$a_{\mu^-}(\mathrm{expt}) = 1\,165\,936\,(12)\times 10^{-9} \;\; (10 \;\; \mathrm{ppm}),$$

$$a_{\mu^+}(\mathrm{expt}) = 1\,165\,910\,(11)\times 10^{-9} \;\; (10 \;\; \mathrm{ppm}), \quad (4)$$

and for μ^+ and μ^- combined

$$a_\mu(\mathrm{expt}) = 1\,165\,923\,(8.5)\times 10^{-9} \;\; (7 \;\; \mathrm{ppm}), \quad (5)$$

in which the dominant error is statistical (Bailey *et al.*, 1979; Farley and Picasso, 1990). The largest systematic error of 1.5 ppm was due to uncertainty in the value of the magnetic field B.

At present a new experiment is in progress at Brookhaven National Laboratory with the goal of measuring a_μ to a precision of 0.35 ppm, which would represent an improvement by a factor of 20 over our present knowledge. The method of the BNL experiment is basically the same as that of the last CERN measurement of a_μ.

The important advances for the BNL experiment are

(1) An increase in primary proton-beam intensity by a factor of 200 with the present alternating-gradient synchroton as compared to the CERN PS used in the CERN experiment.

(2) A superferric magnet storage ring that provides a magnetic field of excellent stability and homogeneity, and an NMR system capable of field measurement to 0.1 ppm.

(3) A modern Pb/scintillating fiber detector system, incorporating a Loran frequency standard, capable of measuring time intervals with a precision of 20 ps.

(4) Muon as well as pion injection into the storage ring. Muon injection increases the number of stored muons and reduces background in the ring.

A photograph of the storage ring is shown in Fig. 2.

During 1997 a run for experimental checkout and initial data taking with pion injection was made. Figure 3 shows a time spectrum of decay positrons where the expected decay of the muons and the $g-2$ precession frequency are apparent. A total of 11.8 M e^+ with energy greater than 1.8 GeV were detected.

The value obtained for a_{μ^+} is

$$a_{\mu^+}(\mathrm{expt}) = 1\,165\,925\,(15)\times 10^{-9} \;\; (13 \;\; \mathrm{ppm}), \quad (6)$$

in which the dominant error is statistical (Carey *et al.*, 1998). This value agrees with the CERN value of Eq. (4).

IV. THEORY OF THE ELECTRON $g-2$

The current status of the theoretical calculation of a_e may be summarized as (Kinoshita, 1996)

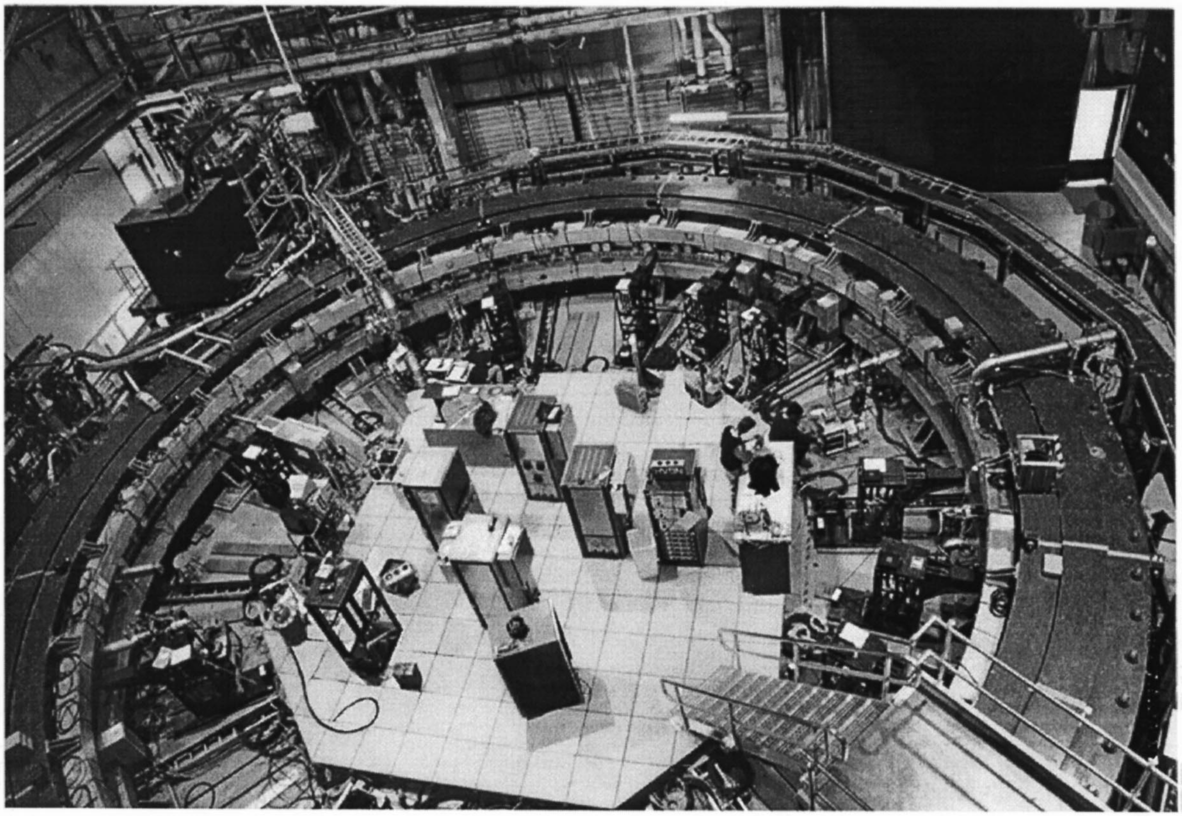

FIG. 2. The superferric C-magnet storage ring for the muon $g-2$ experiment at Brookhaven National Laboratory. The ring diameter is 14 m and the central field is 1.45 T. Twenty-four detectors are placed around the inside of the ring.

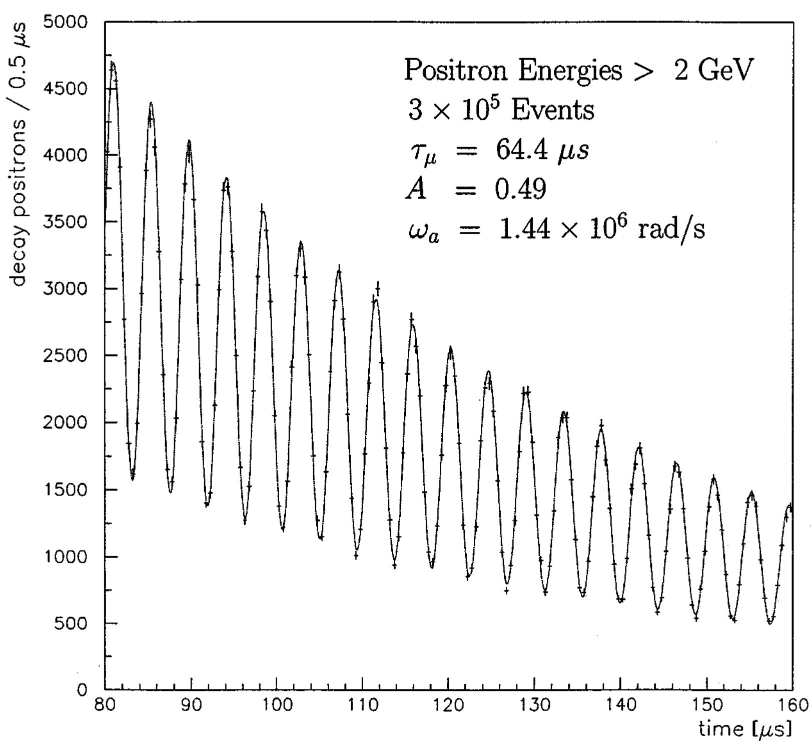

FIG. 3. A positron time spectrum fit by Eq. (3). Statistical errors are indicated.

$$a_e(\mathrm{th}) = 0.5\left(\frac{\alpha}{\pi}\right) - 0.328\,478\,965\ldots\left(\frac{\alpha}{\pi}\right)^2 + 1.181\,241\,456\ldots\left(\frac{\alpha}{\pi}\right)^3 - 1.509\,8\,(384)\left(\frac{\alpha}{\pi}\right)^4 + 4.393\,(27)\times 10^{-12}. \quad (7)$$

The analytic values of the α term and α^2 term have been known for a long time. The analytic value of the α^3 term has been obtained only recently (Laporta and Remeddi, 1996). It is in excellent agreement with the most recent numerical result, 1.181 259 (40), which was obtained shortly before the analytic result became available (see references in the review article of Kinoshita, 1996).

The α^4 term requires evaluation of 891 four-loop Feynman diagrams. This problem is so huge that analytic evaluation is prohibitively difficult even with the help of the fastest computers. Crude numerical evaluation of these integrals began around 1981 (for literature prior to 1990, see Kinoshita, 1990). It is only in the last few years that the calculation of this term began to move from a ''qualitative'' to a ''quantitative'' stage, thanks to the development of massively parallel computers. The coefficient of the α^4 term in Eq. (7) is the latest of the constantly improving values. Although it has a substantially higher precision than the best previous value, the old error estimate is used here pending completion of a more precise error analysis.

The last term of Eq. (7) consists of contributions from vacuum-polarization loops involving muons and taus and from hadronic and weak interactions. Evaluation of these quantities within the standard model gives

$$a_e(\mu\,\tau\,\mathrm{v.p.}) = 2.721\times 10^{-12},$$
$$a_e(\mathrm{hadronic\ v.p.}) = 1.642(27)\times 10^{-12},$$
$$a_e(\mathrm{weak}) = 0.030\times 10^{-12}. \quad (8)$$

Although the non-QED effect on the electron anomaly a_e is very small, it must be included in the theory of the electron $g-2$ in view of the forthcoming experiments. These contributions are estimated assuming the validity of the standard model and indeed require that the theory be renormalizable and incorporates $\mu-e$ universality (Kinoshita, 1996).

To compare the theory of a_e with experiment, it is necessary to know the value of α. Currently the best measurements of α, with a relative uncertainty of less than 1×10^{-7}, are those based on the quantum Hall effect, the ac Josephson effect, the muonium hyperfine structure, and the de Broglie wavelength of a neutron beam (Kruger *et al.*, 1995; Kinoshita, 1996; Jeffery *et al.*, 1997; Liu *et al.*, 1998):

$$\alpha^{-1}(\mathrm{q.\ Hall}) = 137.036\,003\,7(33)\quad [2.4\times 10^{-8}],$$
$$\alpha^{-1}(\mathrm{ac\ J}) = 137.035\,977\,0(77)\quad [5.6\times 10^{-8}],$$
$$\alpha^{-1}(\mathrm{M}) = 137.035\,996\,3(80)\quad [5.8\times 10^{-8}],$$
$$\alpha^{-1}(h/m_n) = 137.036\,010\,62(503)\quad [3.7\times 10^{-8}], \quad (9)$$

where numbers within the brackets represent fractional precisions. Substituting these values in Eq. (7), one finds

$$a_{\mathrm{e}}(\mathrm{q.\ Hall}) = 1\,159\,652\,153.5\ (1.2)\ \ (28.0)\times 10^{-12},$$
$$a_{\mathrm{e}}(\mathrm{ac\ J}) = 1\,159\,652\,379.1\ (1.2)\ \ (65.3)\times 10^{-12},$$
$$a_{\mathrm{e}}(\mathrm{M}) = 1\,159\,652\,216.0\ (1.2)\ \ (67.8)\times 10^{-12},$$
$$a_{\mathrm{e}}(h/m_n) = 1\,159\,652\,095.0\ (1.2)\ \ (42.7)\times 10^{-12}, \quad (10)$$

where the numbers enclosed in parentheses on each line are the uncertainty in the numerical integration result and in that of α used in the evaluation, respectively. The values in Eq. (10) are about -1.3, $+\ 2.9$, $+0.14$, and -2.2 standard deviations away from the measured value in Eq. (1).

V. THEORY OF THE MUON $g-2$

The standard model prediction of a_μ consists of three parts (Kinoshita and Marciano, 1990):

(i) Pure QED contribution. If one uses $\alpha(a_{\mathrm{e}})$ from Eq. (17) one finds

$$a_\mu(\mathrm{QED}) = 116\,584\,705.7\ (1.8)\times 10^{-11}. \quad (11)$$

Note that this does not agree with Eq. (4). This shows clearly that at least the effect of hadronic vacuum polarization must be taken into account. Furthermore, the goal of the new BNL muon $g-2$ experiment is to have the sensitivity to measure the weak-interaction effect. Hence, for comparison with experiment, a theory of the muon $g-2$ must deal with the strong and weak interactions as well as the electromagnetic interaction. The standard model satisfies this requirement.

(ii) Hadronic contribution, which itself consists of three parts:

(a) Hadronic vacuum-polarization contribution. This is obtained mainly from the measured hadron production cross section R in e^+e^- collisions. We quote here only the latest value that includes additional information obtained from the analysis of hadronic tau decay data (CLEO Collaboration, 1997; Davier and Höcker, 1998):

$$a_\mu(\mathrm{had_a}) = 6\,951\ (75)\times 10^{-11}. \quad (12)$$

However, the CVC predictions for the τ-lepton branching ratios based on e^+e^- data are systematically lower than observed in τ decays (Eidelman and Ivanchenko, 1998). If the e^+e^- data alone are used to evaluate $a_u(\mathrm{had}_a)$, the value of $a_u(\mathrm{had}_a)$ decreases by about 60×10^{-11} and its error increases by about 50% (Alemany *et al.*, 1998).

(b) Higher-order hadronic vacuum-polarization effect (Krause, 1997):

$$a_\mu(\mathrm{had_b}) = -101\ (6)\times 10^{-11}. \quad (13)$$

(c) Hadronic light-by-light scattering contribution (Hayakawa and Kinoshita, 1998):

$$a_\mu(\text{had}_c) = -79.2\ (15.4) \times 10^{-11}. \quad (14)$$

(iii) Electroweak contribution of up to two-loop order (Kukhto *et al.*, 1992; Czarnecki *et al.*, 1995, 1996; Peris *et al.*, 1995; Degrassi and Giudice, 1997):

$$a_\mu(\text{weak}) = 151\ \ (4) \times 10^{-11}. \quad (15)$$

Degrassi and Giudice (1997) employ an effective Lagrangian approach to derive the leading-logarithm two-loop electroweak contributions, which confirms the earlier explicit calculation of Kukhto *et al.* (1992), Czarnecki *et al.* (1995, 1996) and Peris *et al.* (1995). It estimates further the leading-logarithm three-loop electroweak contribution, which they find to be small. It also provides a useful parametrization for a certain class of new physics contribution to a_μ and estimates that the QED correction reduces such a new physics contribution by about 6%.

The sum of all these contributions, namely, the prediction of the standard model,

$$a_\mu(\text{th}) = 116\ 591\ 628\ (77) \times 10^{-11}\ \ (0.66\ \text{ppm}), \quad (16)$$

is in good agreement with the measurements in Eqs. (4) and (6).

The uncertainty in Eq. (16) comes mainly from the hadronic vacuum-polarization contribution from Eq. (12). It must be improved by at least a factor of 2 before we can extract the full useful information from the new high-precision measurement of a_μ. Fortunately, this contribution is calculable from the measured value of R in e^+e^- collisions. Future measurements of R at VEPP-2M, VEPP-4M, DAΦNE, and BEPS, as well as analysis of the hadronic tau decay data, will reduce the uncertainty of this contribution to a satisfactory level (CLEO Collaboration, 1997; Davier and Höcker, 1998).

The contribution of the hadronic light-by-light scattering effect in Eq. (14) is smaller but is potentially a source of a serious problem because it is difficult to express it in terms of experimentally accessible observables. Evaluation of this term in QCD has not yet been attempted. The best approach available is to estimate it within the framework of chiral perturbation theory and the $1/N_c$ expansion (Bijnens *et al.*, 1995, 1996; Hayakawa *et al.*, 1995, 1996). Recently, however, an important part of this term was improved significantly (Hayakawa and Kinoshita, 1998) using the information obtained from new measurements of the $P\gamma\gamma^*$ form factors (Gronberg *et al.*, 1998) where P stands for π^0, η, and η' mesons. The result of this work is included in Eq. (14).

VI. SOME IMPLICATIONS FOR FUNDAMENTAL PHYSICS

Because of the unusually high sensitivity of a precise experimental value of a_μ to physics beyond the standard model, theoretical predictions of the contributions to a_μ of speculative theories are of great interest. In general any new particles or interactions which couple to the muon or to the photon contribute to a_μ, whose value then provides a sum rule for physics. In comparison with experimental data from the higher-energy colliders (LEP II, Tevatron, LHC), an a_μ value with a precision of 0.35 ppm, as projected for the current BNL experiment, provides a comparable or greater sensitivity to a composite structure of the muon or W boson and also to the new particles in supersymmetric (SUSY) theories. For the muon a composite mass scale $\Lambda = 4$ TeV and for the W boson an anomalous magnetic moment $\kappa = 0.04$ would be observable. In supersymmetry theory a sparticle mass scale of about 130 GeV would be detected. Of course, any observation of physics beyond the standard model from a_μ would be indirect and would not by itself determine the process involved.

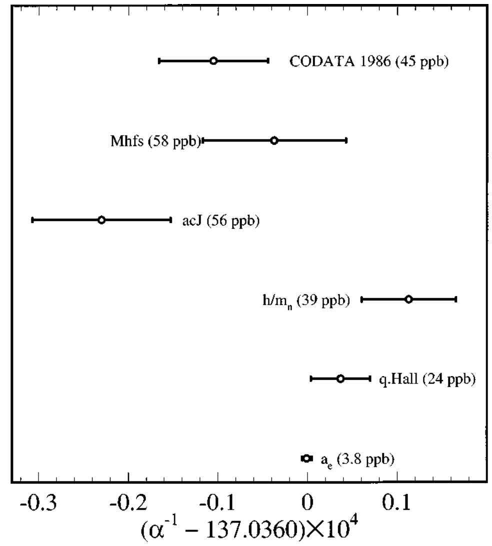

FIG. 4. Values of the fine-structure constant determined by various means. The CODATA 1986 value of α (Kinoshita, 1996) is included for comparison.

In the rest of this paper let us focus on a_e as a tool to test the validity of quantum mechanics. We note that the *intrinsic* uncertainty of theoretical values of a_e listed in Eq. (10) is already quite small, the overall uncertainty being dominated by those of α listed in Eq. (9). This means that we can obtain the most precise value of α from the theory and measurement of a_e. From the average of a_{e^-} and a_{e^+} in Eq. (1) and the theory one finds

$$\alpha^{-1}(a_e) = 137.035\ 999\ 58\ (14)\ (50)$$
$$= 137.035\ 999\ 58\ (52)\ [3.8 \times 10^{-9}], \quad (17)$$

where the uncertainties on the first line are from the α^4 term and the measurement uncertainty of a_e given in Eq. (1), respectively.

Continuing theoretical work on a_e will reduce the theoretical uncertainty by a factor of 2 to 3 in the near future. If the experimental precision is improved by an order of magnitude, the precision of $\alpha(a_e)$ will exceed 1 part in 10^9 (Gabrielse and Tan, 1994; Mittleman *et al.*, 1995). Besides these determinations of α, a powerful new approach using atom-beam interferometry of C_S is being developed (Weiss *et al.*, 1993). Another new approach is based on single-electron tunneling which has achieved a precision of 15 ppb in counting the number of electrons (Kinoshiita, 1996). Spectroscopic measurements of the He atom fine structure in the 2^3P state is also a promising source of a very precise α value. The best values of α available at present are shown in Fig. 4.

It is fortunate that many independent ways are available for measuring α with high precision. This offers an opportunity to examine the theoretical bases of all these measurements on an equal footing. The precision of these measurements requires that the underlying theories be valid to the same extent. The theories are based on quantum mechanics extended to include relativistic effects, radiative corrections, and renormalization with respect to the electroweak and strong interactions.

Currently, such a theoretical basis is fully satisfied only by $\alpha(a_e)$ and by α determined by the muonium hyperfine structure and other atomic measurements. Although the principle of neutron de Broglie wavelength measurement looks very simple, it requires determination of the *free* neutron mass from nuclear physics, which can be fully justified only within the context of renormalizable quantum field theory. The α determined in condensed-matter physics has another unsettled problem. It is argued that, although the theories of the ac Josephson effect and the quantum Hall effect start from the condensed-matter physics Hamiltonian with its usual simplifying approximations, their predictions may in fact be valid to a higher degree than that of $\alpha(a_e)$ because they are derived from the gauge invariance and one-valuedness of the wave function and are not dependent on specific approximations adopted in condensed-matter physics. It is important to note, however, that this assertion has not yet been proven rigorously. In particular, the theory of condensed-matter physics in the present form is not renormalizable. The NRQED method of Caswell and Lepage (1986) may provide an approach for establishing a sounder basis for condensed-matter physics. (Note that NRQED is not a nonrelativistic approximation to QED. Rather, it is a systematic expansion of QED in the electron velocity and is fully equivalent to QED on resummation.)

Currently, the standard model is the simplest theory to represent extended quantum mechanics, and within its context all measurements of α that can be reduced to those of the charge form factor or the magnetic form factor at zero-momentum transfer must give the same answer.

An expectation that the α's obtained from the charge form factor may be affected by short-range interactions by $\sim(\alpha/\pi)^2(m_e/m_\rho)^2 \simeq 2.4\times10^{-12}$, where m_ρ is the ρ meson mass, is not realized. This effect cannot be detected since it is absorbed by charge renormalization, which applies universally to all measurements of the charge form factor at threshold. The magnetic form factor, on the other hand, will be affected by the known short-range forces by $\sim 1.7\times10^{-12}$, which contributes about 1.5 ppb to $\alpha(a_e)$. But this effect is already taken into account in defining $\alpha(a_e)$. Thus $\alpha(a_e)$ determined from the magnetic form factor must have the same value as α's derived from the charge form factor. This equality is not affected by short-distance effects. This remark applies as well to α derived from the muonium hyperfine structure.

Effects beyond the standard model on $\alpha(a_e)$ can also be estimated using the measured a_μ insofar as the new interaction satisfies μ-e universality. Relative to known weak interactions, this effect will scale as $(m_W/m_X)^2$, where m_X is the mass scale of the new interaction. Such an effect will be too small to be significant at the present level of precision of $\alpha(a_e)$. Another useful constraint on a new interaction may come from a new measurement of the muon electric dipole moment.

The data shown in Fig. 4 cast some doubt on the likelihood that the α values determined by the different methods are the same. Improved precision in determining the α values, both experimental and theoretical, will provide a more sensitive test of the validity of (extended) quantum mechanics.

ACKNOWLEDGMENTS

The work of V. W. H. was supported in part by the U.S. Department of Energy and that of T. K. by the U.S. National Science Foundation.

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