String theory, supersymmetry, unification, and all that

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String theory and supersymmetry are theoretical ideas that go beyond the standard model of particle physics and show promise for unifying all forces. After a brief introduction to supersymmetry, the authors discuss the prospects for its experimental discovery in the near future. They then show how the magic of supersymmetry allows us to solve certain quantum field theories exactly, thus leading to new insights about field theory dynamics related to electric-magnetic duality. The discussion of superstring theory starts with its perturbation expansion, which exhibits new features including "stringy geometry." The authors then turn to more recent nonperturbative developments. Using new dualities, all known superstring theories are unified, and their strong-coupling behavior is clarified. A central ingredient is the existence of extended objects called branes. [S0034-6861(99)01402-6]

I. INTRODUCTION

The standard model of particle physics (see the article by Gaillard, Grannis, and Sciulli in this volume) is a beautiful theory that accounts for all known phenomena up to energies of order 100 GeV. Its consistency relies on the intricacies of quantum field theory (see Wilczek's article), and its agreement with experiment is spectacular. However, there are many open problems with the standard model. In particular, we would like to know what lies beyond the standard model. What is the physics at energies above 100 GeV?

One suggestion for physics at nearby energies of order 1 TeV ($=1000$ GeV), which we shall review below, is supersymmetry. At higher energies the various interactions of the standard model can be unified into a grand unified theory. Finally, at energies of the order of the Planck energy, $M_P c^2 = (c\hbar/G)^{1/2}c^2 \sim 10^{19}$ GeV, the theory must be modified. This energy scale is determined on dimensional grounds using Newton's constant G, the speed of light c, and Planck's constant $\hbar$. It determines the characteristic energy scale of any theory that incorporates gravitation in a relativistic and quantum-mechanical setting. At this energy scale the gravitational interactions become strong and cannot be neglected. How to combine the elaborate structure of quantum field theory and the standard model with Einstein's theory of gravity—general relativity—is one of the biggest challenges in theoretical physics today. String theory is the only viable attempt to achieve this!

There are various problems that arise when one attempts to combine general relativity and quantum field theory. The field theorist would point to the breakdown of renormalizability—the fact that short-distance singularities become so severe that the usual methods for dealing with them no longer work. By replacing point-like particles with one-dimensional extended strings, as the fundamental objects, *superstring theory* certainly overcomes the problem of perturbative nonrenormalizability. A relativist might point to a different set of problems including the issue of how to understand the causal structure of space-time when the metric has quantum-mechanical fluctuations. There are also a host of problems associated with black holes, such as the fundamental origin of their thermodynamic properties and an apparent loss of quantum coherence. The latter, if true, would imply a breakdown in the basic structure of quantum mechanics. The relativist's set of issues cannot be addressed properly in a perturbative setup, but recent discoveries are leading to nonperturbative understandings that should help in addressing them. Most string theorists expect that the theory will provide satisfying resolutions of these problems without any revision in the basic structure of quantum mechanics. Indeed, there are indications that someday quantum mechanics will be viewed as an implication of (or at least a necessary ingredient of) superstring theory.

String theory arose in the late 1960s in an attempt to describe strong nuclear forces. In 1971 it was discovered that the inclusion of fermions requires world-sheet supersymmetry. This led to the development of space-time supersymmetry, which was eventually recognized to be a generic feature of consistent string theories—hence the name *superstrings*. String theory was a quite active subject for about five years, but it encountered serious theoretical difficulties in describing the strong nuclear forces, and QCD came along as a convincing theory of the strong interaction. As a result the subject went into decline and was abandoned by all but a few diehards for over a decade. In 1974 two of the diehards (Joël Scherk and John Schwarz) proposed that the problems of string theory could be turned into virtues if it were used as a framework for realizing Einstein's old dream of unification, rather than as a theory of hadrons and strong nuclear forces. In particular, the massless spin-two particle in the string spectrum, which had no sensible hadronic interpretation, was identified as the graviton and shown to interact at low energies precisely as required by general relativity. One implication of this change in viewpoint was that the characteristic size of a string be-

came the Planck length, $L_P = \hbar/cM_P = (\hbar G/c^3)^{1/2}$ $\sim 10^{-33}$ cm, some 20 orders of magnitude smaller than previously envisaged. More refined analyses lead to a string scale L_S that is a couple of orders of magnitude larger than the Planck length. In any case, experiments at existing accelerators cannot resolve distances shorter than about 10^{-16} cm, which explains why the point-particle approximation of ordinary quantum field theories is so successful.

II. SUPERSYMMETRY

Supersymmetry is a symmetry relating bosons and fermions, according to which every fermion has a bosonic superpartner and vice versa. For example, fermionic quarks are partners of bosonic *squarks*. By this we mean that quarks and squarks belong to the same irreducible representation of the supersymmetry. Similarly, bosonic gluons (the gauge fields of QCD) are partners of fermionic *gluinos*. If supersymmetry were an unbroken symmetry, particles and their superpartners would have exactly the same mass. Since this is certainly not the case, supersymmetry must be a broken symmetry (if it is relevant at all). In supersymmetric theories containing gravity, such as supergravity and superstring theories, supersymmetry is a gauge symmetry. Specifically, the superpartner of the graviton, called the *gravitino*, is the gauge particle for local supersymmetry.

A. Fermionic dimensions of space-time

Another presentation of supersymmetry is based on the notion of *superspace*. We do not change the structure of space-time but we add structure to it. We start with the usual four coordinates, $X^\mu = t, x, y, z$, and add four odd dimensions, θ_α ($\alpha = 1, \ldots, 4$). These odd dimensions are fermionic and anticommute:

$$\theta_\alpha \theta_\beta = -\theta_\beta \theta_\alpha.$$

They are quantum dimensions that have no classical analog, which makes it difficult to visualize or to understand them intuitively. However, they can be treated formally.

The fact that the odd directions are anticommuting has important consequences. Consider a function of superspace,

$$\Phi(X, \theta) = \phi(X) + \theta_\alpha \psi_\alpha(X) + \cdots + \theta^4 F(X).$$

Since the square of any θ is zero and there are only four different θ's, the expansion in powers of θ terminates at the fourth order. Therefore a function of superspace includes only a finite number of functions of X (16 in this case). Hence we can replace any function of superspace $\Phi(X, \theta)$ with the component functions $\phi(X), \psi(X), \ldots$. These include bosons $\phi(X), \ldots$ and fermions $\psi(X), \ldots$. This is one way of understanding the pairing between bosons and fermions.

A supersymmetric theory looks like an ordinary theory with degrees of freedom and interactions that satisfy certain symmetry requirements. Indeed, a super-
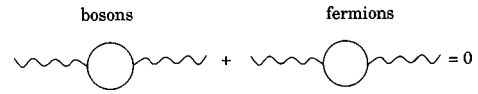

FIG. 1. Boson-fermion cancellation in some loop diagrams.

symmetric quantum field theory is a special case of a more generic quantum field theory rather than being a totally different kind of theory. In this sense, supersymmetry by itself is not a very radical proposal. However, the fact that bosons and fermions come in pairs in supersymmetric theories has important consequences. In some loop diagrams, like those in Fig. 1, the bosons and the fermions cancel each other. This boson-fermion cancellation is at the heart of most of the applications of supersymmetry. If superpartners are present in the TeV range, this cancellation solves the gauge hierarchy problem (see below). This cancellation is also one of the underlying reasons for our ability to analyze supersymmetric theories exactly.

B. Supersymmetry in the TeV range

There are several indications (discussed below) that supersymmetry is realized in the TeV range, so that the superpartners of the particles of the standard model have masses of the order of a few TeV or less. This is an important prediction, because the next generation of experiments at Fermilab and CERN will explore the energy range where at least some of the superpartners are expected to be found. Therefore, within a decade or two, we should know whether supersymmetry exists at this energy scale. If supersymmetry is indeed discovered in the TeV range, this will amount to the discovery of the new odd dimensions and will be a major change in our view of space and time. It would be a remarkable success for theoretical physics—predicting such a deep notion without any experimental input!

1. The gauge hierarchy problem

The *gauge hierarchy problem* is essentially a problem of dimensional analysis. Why is the characteristic energy of the standard model, which is given by the mass of the W boson $M_W \sim 100$ GeV, so much smaller than the characteristic scale of gravity, the Planck mass $M_P \sim 10^{19}$ GeV? It should be stressed that in quantum field theory this problem is not merely an aesthetic problem, but also a serious technical problem. Even if such a hierarchy is present in some approximation, radiative corrections tend to destroy it. More explicitly, divergent loop diagrams restore dimensional analysis and move $M_W \to M_P$.

The main theoretical motivation for supersymmetry at the TeV scale is the hierarchy problem. As we mentioned, in supersymmetric theories some loop diagrams vanish—or become less divergent—due to cancellations between bosons and fermions. In particular the loop diagram restoring dimensional analysis is canceled as in Fig. 1. Therefore, in its simplest form, supersymmetry solves the technical aspects of the hierarchy problem. More so-

phisticated ideas, known as dynamical supersymmetry breaking, also solve the aesthetic problem.

2. The supersymmetric standard model

The minimal supersymmetric extension of the standard model (the MSSM) contains superpartners for all the particles of the standard model, as we have already indicated. Some of their coupling constants are determined by supersymmetry and the known coupling constants of the standard model. Most of the remaining coupling constants and the masses of the superpartners depend on the details of supersymmetry breaking. These parameters are known as *soft breaking terms*. Various phenomenological considerations already put strong constraints on these unknown parameters but there is still a lot of freedom in them. If supersymmetry is discovered, the new parameters will be measured. These numbers will be extremely interesting as they will give us a window into physics at higher energies.

The MSSM must contain two electroweak doublets of Higgs fields. Whereas a single doublet can give mass to all quarks and charged leptons in the standard model, the MSSM requires one doublet to give mass to the charge-2/3 quarks and another to give mass to the charge-1/3 quarks and charged leptons. Correspondingly, electroweak symmetry breaking by the Higgs mechanism involves two Higgs fields' obtaining vacuum expectation values. The ratio, called $\tan \beta$, is an important phenomenological parameter. In the standard model the Higgs mass is determined by the Higgs vacuum expectation value and the strength of Higgs self-coupling (coefficient of the ϕ^4 term in the potential). In supersymmetry the latter is related to the strength of the gauge interactions. This leads to a prediction for the mass of the lightest Higgs boson h in the MSSM. In the leading semiclassical approximation one can show that $M_h \lesssim M_Z |\cos 2\beta|$, where $M_Z \sim 91$ GeV is the mass of the Z boson. Due to the large mass of the top quark, radiative corrections to this bound can be quite important. A reasonably safe estimate is that $M_h \lesssim 130$ GeV, which should be compared to current experimental lower bounds of about 80 GeV. The discovery of a relatively light Higgs boson, which might precede the discovery of any superparticles, would be encouraging for supersymmetry. However, it should be pointed out that there are rather mild extensions of the MSSM in which the upper bound is significantly higher.

It is useful to assign positive R parity to the known particles (including the Higgs) of the standard model and negative R parity to their superpartners. For reasonable values of the new parameters (including the soft breaking terms) R parity is a good symmetry. In this case the lightest supersymmetric particle (called the LSP) is absolutely stable. It could be an important constituent of the dark matter of the universe.

3. Supersymmetric grand unification

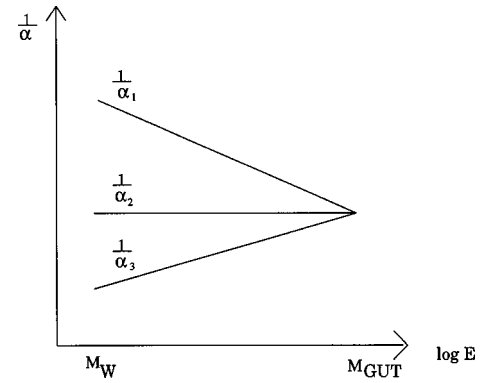The second motivation for supersymmetry in the TeV range comes from the idea of gauge unification.

FIG. 2. Coupling-constant unification in supersymmetric theories.

Recent experiments have yielded precise determinations of the strengths of the SU(3)×SU(2)×U(1) gauge interactions—the analogs of the fine-structure constant for these interactions. They are usually denoted by α_3, α_2, and α_1 for the three factors in SU(3)×SU(2)×U(1). In quantum field theory these values depend on the energy at which they are measured in a way that depends on the particle content of the theory. Using the measured values of the coupling constants and the particle content of the standard model, one can extrapolate to higher energies and determine the coupling constants there. The result is that the three coupling constants do not meet at the same point. However, when one repeats this extrapolation with the particles belonging to the minimal supersymmetric extension of the standard model, the three gauge-coupling constants meet at a point, M_{GUT}, as sketched in Fig. 2. At that point the strengths of the various gauge interactions become equal and the interactions can be unified into a *grand unified theory*. Possible grand unified theories embed the known SU(3)×SU(2)×U(1) gauge group into SU(5) or SO(10).

How much significance should we assign to this result? Two lines must meet at a point. Therefore, there are only two surprises here. The first is that the third line intersects the same point. The second more qualitative one is that the unification scale, M_{GUT}, is at a reasonable value. Its value is consistent with the experimental bound from proton decay, and it is a couple of orders of magnitude below the Planck scale, where gravity would need to be taken into account. One could imagine that that there are other modifications of the standard model that achieve the same thing, so this is far from a proof of supersymmetry, but it is certainly encouraging circumstantial evidence. It is an independent indication that superpartner masses should be around a TeV.

C. Supersymmetric quantum field theories

Quantum field theory is notoriously complicated. It is a nonlinear system of an infinite number of coupled degrees of freedom. Therefore, until recently when the power of supersymmetry began to be exploited, there were few exact results for quantum field theories (except in two dimensions). However, it has been realized re-
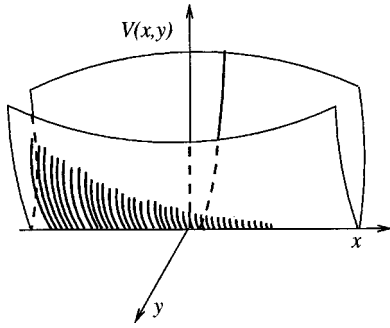
FIG. 3. Typical potential in supersymmetric theories exhibiting "accidental vacuum degeneracy."

cently that a large class of physical quantities in many supersymmetric quantum field theories can be computed exactly by analytic methods!

The main point is that these theories are very constrained. The dependence of some observables on the parameters of the problem is so constrained that there is only one solution that satisfies all of the consistency conditions. More technically, because of supersymmetry some observables vary holomorphically (complex analytically) with the coupling constants, which are complex numbers in these theories. Due to Cauchy's theorem, such analytic functions are determined in terms of very few data: the singularities and the asymptotic behavior. Therefore, if supersymmetry requires an observable to depend holomorphically on the parameters, and we know the singularities and the asymptotic behavior, we can determine the exact answer. The boson-fermion cancellation, which we mentioned above in the context of the hierarchy problem, can also be understood as a consequence of a constraint following from holomorphy.

1. Families of vacua

Another property of many supersymmetric theories that makes them tractable is that they have a family of inequivalent vacua. To understand this fact we should contrast it with the situation in a ferromagnet, which has a continuum of vacua, labeled by the common orientation of the spins. These vacua are all equivalent; i.e., the physical observables in one of these vacua are exactly the same as in any other. The reason is that these vacua are related by a symmetry. The system must choose one of them, which leads to spontaneous symmetry breaking.

We now study a situation with inequivalent vacua in contrast to the ferromagnet. Consider the case in which degrees of freedom, called x and y, have the potential $V(x,y)$ shown in Fig. 3. The vacua of the system correspond to different points along the valley of the potential, $y=0$ with arbitrary x. However, as we tried to make clear in the figure, these points are inequivalent—there is no symmetry that relates them. More explicitly, the potential is shallow around the origin but becomes steep for large x. Such *accidental degeneracy* is usually lifted by quantum effects. For example, if the system corresponding to the potential in the figure has no fermions, the zero-point fluctuations around the different vacua

would be different. They would lead to a potential along the valley, pushing the minimum to the origin. However, in a supersymmetric theory the zero-point energy of the fermions exactly cancels that of the bosons, and the degeneracy is not lifted. The valleys persist in the full quantum theory. Again, we see the power of the boson-fermion cancellation. We see that a supersymmetric system typically has a continuous family of vacua. This family, or manifold, is referred to as a *moduli space of vacua*, and the modes of the system corresponding to motion along the valleys are called *moduli*.

The analysis of supersymmetric theories is usually simplified by the presence of these manifolds of vacua. Asymptotically, far along the flat directions of the potential, the analysis of the system is simple and various approximation techniques are applicable. Then, by using the asymptotic behavior along several such flat directions, as well as the constraints from holomorphy, one obtains a unique solution. This is a rather unusual situation in physics. We perform approximate calculations, which are valid only in some regime, and this gives us the exact answer. This is a theorist's heaven—exact results with approximate methods!

2. Electric-magnetic duality

Once we know how to solve such theories, we can analyze many examples. The main lesson that has been learned is the fundamental role played by *electric-magnetic duality*. It turns out to be the underlying principle controlling the dynamics of these systems.

When faced with a complicated system with many coupled degrees of freedom it is common in physics to look for weakly coupled variables that capture most of the phenomena. For example, in condensed-matter physics we formulate the problem at short distance in terms of interacting electrons and nuclei. The desired solution is the macroscopic behavior of the matter and its possible phases. It is described by weakly coupled effective degrees of freedom. Usually they are related in a complicated, and in most cases unknown, way to the microscopic variables. Another example is hydrodynamics, where the microscopic degrees of freedom are molecules and the long-distance variables are properties of a fluid that are described by partial differential equations.

In one class of supersymmetric field theories, the long-distance behavior is described by a set of weakly coupled effective degrees of freedom. These are composites of the elementary degrees of freedom. As the characteristic length scale becomes longer, the interactions between these effective degrees of freedom become weaker, and the description in terms of them becomes more accurate. In other words, the long-distance theory is a "trivial" theory in terms of the composite effective degrees of freedom.

In another class of examples there are no variables in terms of which the long-distance theory is simple—the theory remains interacting. Because it is scale invariant, it is at a nontrivial fixed point of the renormalization group. In these situations there are two (or more) dual

TABLE I. Dual electric and magnetic descriptions.

	Electric	Magnetic
Coupling	strong	weak
Fluctuations	large	small
Phase	confinement	Higgs

descriptions of the physics leading to identical results for the long-distance interacting behavior.

In both classes of examples an explicit relation between the two sets of variables is not known. However, there are several reasons to consider these pairs of descriptions as being electric-magnetic duals of one another. The original variables at short distance are referred to as the electric degrees of freedom and the other set of long-distance variables as the magnetic ones. These two dual descriptions of the same theory give us a way to address strong-coupling problems. When the electric variables are strongly coupled, they fluctuate rapidly and their dynamics are complicated (see Table I). However, then the magnetic degrees of freedom are weakly coupled. They do not fluctuate rapidly and their dynamics is simple. In the first class of examples the magnetic degrees of freedom are the macroscopic ones, which are free at long distance. They are massless bound states of the elementary particles. In the second class of examples there are two valid descriptions of the long-distance theory: electric and magnetic. Since both of them are interacting, neither of them gives a ''trivial'' description of the physics. However, as one of them becomes more strongly coupled, the other becomes more weakly coupled (see Table I).

Finally, using this electric-magnetic duality we can find a simple description of complicated phenomena associated with the phase diagram of the theories. For example, as the electric degrees of freedom become strongly coupled, they can lead to confinement. In the magnetic variables, this is simply the Higgs phenomenon (superconductivity), which is easily understood in weak coupling. The electric-magnetic relations are summarized in Table I:

Apart from the ''practical'' application to solving quantum field theories, the fact that a theory can be described in terms of either electric or magnetic variables has deep consequences:

(i)   For theories belonging to the first class of examples it is natural to describe the magnetic degrees of freedom as composites of the elementary electric ones. The magnetic particles typically include massless gauge particles reflecting a new magnetic gauge symmetry. These massless composite gauge particles are associated with a new gauge symmetry which is not present in the fundamental electric theory. Since this gauge symmetry is not a symmetry of the original, short-distance theory, it is generated by the dynamics rather than being ''put in by hand.'' We see that, in this sense, *gauge invariance cannot be fundamental.*
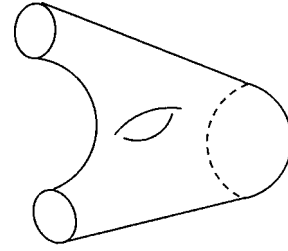

FIG. 4. An example of a string world sheet with two initial strings, one final string, and a handle.

(ii)   For theories of the second class the notion of elementary particle breaks down. *There is no invariant way of choosing which degrees of freedom are elementary and which are composite.* The magnetic degrees of freedom can be regarded as composites of the electric ones and vice versa.

III. SUPERSTRING THEORY

A. Perturbative string theory

All superstring theories contain a massless scalar field, called the *dilaton* ϕ, that belongs to the same supersymmetry multiplet as the graviton. In the semiclassical approximation, this field defines a flat direction in the moduli space of vacua, so that it can take any value ϕ_0. Remarkably, this determines the string coupling constant $g_S = e^{\phi_0}$, which is a dimensionless parameter on which one can base a perturbation expansion. The perturbation expansions are power-series expansions in powers of the string coupling constant like those customarily used to carry out computations in quantum field theory.

1. Structure of the string world sheet and the perturbation expansion

A string's space-time history is described by functions $X^\mu(\sigma, \tau)$ that map the string's two-dimensional *world sheet* (σ, τ) into space-time X^μ. There are also other world-sheet fields that describe other degrees of freedom, such as those associated with supersymmetry and gauge symmetries. Surprisingly, *classical* string theory dynamics is described by a conformally invariant 2D *quantum* field theory. What distinguishes one-dimensional strings from higher-dimensional analogs (discussed later) is the fact that this 2D theory is renormalizable. Perturbative quantum string theory can be formulated by the Feynman sum-over-histories method. This amounts to associating a genus h Riemann surface (a closed and orientable two-dimensional surface with h handles) to a Feynman diagram with h loops. It contains a factor of g_S^{2h}. For example, the string world sheet in Fig. 4 has one handle.

The attractive features of this approach are that there is just one diagram at each order h of the perturbation expansion and that each diagram represents an elegant (though complicated) finite-dimensional integral that is

ultraviolet finite. In other words, they do not give rise to the severe short-distance singularities that plague other attempts to incorporate general relativity in a quantum field theory. The main drawback of this approach is that it gives no insight into how to go beyond perturbation theory.

2. Five superstring theories

In 1984-1985 a series of discoveries convinced many theorists that superstring theory is a very promising approach to unification. This period is now sometimes referred to as *the first superstring revolution*. Almost overnight, the subject was transformed from an intellectual backwater to one of the most active areas of theoretical physics, which it has remained ever since. By the time the dust settled, it was clear that there are five different superstring theories, each requiring ten dimensions (nine space and one time), and that each has a consistent perturbation expansion. The five theories are denoted type I, type IIA, type IIB, $E_8 \times E_8$ heterotic (HE, for short), and $SO(32)$ heterotic (HO, for short). The type-II theories have two supersymmetries in the ten-dimensional sense, while the other three have just one. The type-I theory is special in that it is based on unoriented open and closed strings, whereas the other four are based on oriented closed strings. Type-I strings can break, whereas the other four are unbreakable. The type-IIA theory is nonchiral (i.e., it is parity conserving), and the other four are chiral (parity violating).

3. Compactification of extra dimensions

To have a chance of being realistic, the six extra space dimensions must somehow curl up into a tiny geometrical space as in Kaluza-Klein theory. The linear size of this space is presumably comparable to the string scale L_S. Since space-time geometry is determined dynamically (as in general relativity), only geometries that satisfy the dynamical equations are allowed. Among such solutions, one class stands out: The $E_8 \times E_8$ heterotic (HE) string theory, compactified on a particular kind of six-dimensional space, called a Calabi-Yau manifold, has many qualitative features at low energies that resemble the supersymmetric extension of the standard model of elementary particles. In particular, the low-mass fermions occur in suitable representations of a plausible unifying gauge group. Moreover, they occur in families whose number is controlled by the topology of the Calabi-Yau manifold. These successes have been achieved in a perturbative framework and are necessarily qualitative at best, since nonperturbative phenomena are essential to an understanding of supersymmetry breaking and other important details.

4. T duality and stringy geometry

The basic idea of T duality can be illustrated by considering a compact spatial dimension consisting of a circle of radius R. In this case there are two kinds of excitations to consider. The first, which is not unique to string theory, is due to the quantization of the momentum along the circle. These Kaluza-Klein excitations contribute $(n/R)^2$ to the energy squared, where n is an integer. The second kind are winding-mode excitations, which arise due to a closed string's being wound m times around the circular dimension. They are unique to string theory, though there are higher-dimensional analogs. When one lets $T = (2\pi L_S^2)^{-1}$ denote the fundamental string tension (energy per unit length), the contribution of a winding mode to the energy squared is $(2\pi R m T)^2$. T duality exchanges these two kinds of excitations by mapping $m \leftrightarrow n$ and $R \leftrightarrow L_S^2/R$. This is part of an exact map between a T-dual pair of theories A and B.

We see that the underlying geometry is ambiguous—there is no way to tell the difference between a compactification on a circle of radius R and a compactification on a circle of radius L_S^2/R. This ambiguity is clearly related to the fact that the objects used to probe the circle are extended objects—strings—which can wind around the circle.

One implication of this ambiguity is that usual geometric concepts break down at short distances, and classical geometry is replaced by *stringy geometry*, which is described mathematically by 2D conformal field theory. It also suggests a generalization of the Heisenberg uncertainty principle according to which the best possible spatial resolution Δx is bounded below not only by the reciprocal of the momentum spread, Δp, but also by the string size, which grows with energy. This is the best one can do using fundamental strings as probes. However, by probing with certain nonperturbative objects called D-branes, which we shall discuss later, it is sometimes possible (but not in the case of the circle discussed above) to do better.

A closely related phenomenon is that of mirror symmetry. In the example of the circle above the topology was not changed by T duality. Only the size was transformed. In more complicated compactifications, such as those on Calabi-Yau manifolds, there is even an ambiguity in the underlying topology—there is no way to tell on which of two mirror pairs of Calabi-Yau manifolds the theory is compactified. This ambiguity can be useful because it is sometimes easier to perform some calculations with one Calabi-Yau manifold than with its mirror manifold. Then, using mirror symmetry, we can infer what the answers are for different compactifications.

Two pairs of ten-dimensional superstring theories are T dual when compactified on a circle: the type-IIA and IIB theories and the HE and HO theories. The two edges of Fig. 5 labeled T connect vacua related by T duality. For example, if the IIA theory is compactified on a circle of radius R_A, leaving nine noncompact dimensions, this is equivalent to compactifying the IIB theory on a circle of radius $R_B = L_S^2/R_A$. The T duality relating the two heterotic theories, HE and HO, is essentially the same, though there are additional technical details in this case.

Another relation between theories is the following. A compactification of the type-I theory on a circle of radius R_I turns out to be related to a certain compactifi-
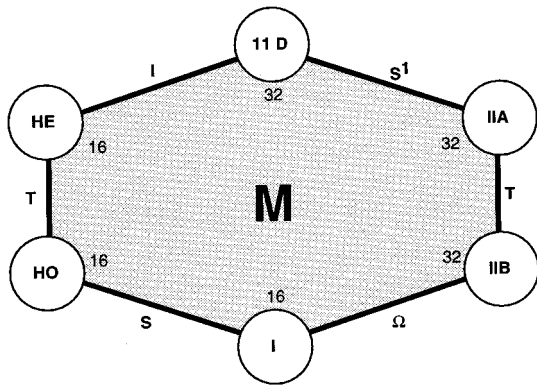
FIG. 5. The M theory moduli space.

cation of the type-IIA theory on a line interval I with size proportional to L_S^2/R_I. The line interval can be thought of as a circle with some identification of points $I = S^1/\Omega$. Therefore we can say that the type-I theory on a circle of radius R_I is obtained from the type IIA on a circle of radius L_S^2/R_I by acting with Ω. Since by T duality the IIA theory on a circle of radius L_S^2/R_I is the same as the IIB theory on a circle of radius R_I, we conclude that upon compactification on a circle type I is obtained from IIB by the action of Ω. By taking R_I to infinity we ensure that this relation is also true in ten dimensions. This is the reason for the edge denoted by Ω in Fig. 5.

These dualities reduce the number of (apparently) distinct superstring theories from five to three, or if we also use Ω to two. The point is that the two members of each pair are continuously connected by varying the compactification radius from zero to infinity. Like the string coupling constant, the compactification radius arises as the value of a scalar field. Therefore varying this radius is a motion in the moduli space of quantum vacua rather than a change in the parameters of the theory.

B. Nonperturbative string theory

The *second superstring revolution* (1994-??) has brought nonperturbative string physics within reach. The key discoveries were various dualities, which show that what was viewed previously as five distinct superstring theories is in fact five different perturbative expansions of a single underlying theory about five different points in the moduli space of consistent vacua! It is now clear that there is a unique theory, though it allows many different vacua. A sixth special vacuum involves an 11-dimensional Minkowski space-time. Another lesson we have learned is that, nonperturbatively, objects of more than one dimension (membranes and higher p-branes) play a central role. In most respects they appear to be on an equal footing with strings, but there is one big exception: a perturbation expansion cannot be based on p-branes with $p > 1$.

A schematic representation of the relationship between the five superstring vacua in 10D and the 11D vacuum is given in Fig. 5. The idea is that there is some large moduli space of consistent vacua of a single underlying theory—here denoted by M. The six limiting points, represented as circles, are special in the sense that they are the ones with (super) Poincaré invariance in ten or eleven dimensions. The letters on the edges refer to the type of duality relating a pair of limiting points. The numbers 16 or 32 refer to the number of unbroken supersymmetries. In 10D the minimal spinor has 16 real components, so the conserved supersymmetry charges (or *supercharges*) correspond to just one spinor in three cases (type I, HE, and HO). Type-II superstrings have two such spinorial supercharges. In 11D the minimal spinor has 32 real components.

1. S duality

Suppose now that a pair of theories (A and B) are S dual. This means that if $f_A(g_S)$ denotes any physical observable of theory A, where g_S is the coupling constant, then there is a corresponding physical observable $f_B(g_S)$ in theory B such that $f_A(g_S) = f_B(1/g_S)$. This duality relates one theory at weak coupling to the other at strong coupling. It generalizes the electric-magnetic duality of certain field theories, discussed in Sec. II.C.2. S duality relates the type-I theory to the HO theory and the IIB theory to itself. This determines the strong-coupling behavior of these three theories in terms of weakly coupled theories. Varying the strength of the string coupling also corresponds to a motion in the moduli space of vacua.

The edge connecting the HO vacuum and the type-I vacuum is labeled by S in Fig. 5, since these two vacua are related by S duality. It had been known for a long time that the two theories had the same gauge symmetry [SO(32)] and the same kind of supersymmetry, but it was unclear how they could be equivalent, because type-I strings and heterotic strings are very different. It is now understood that SO(32) heterotic strings appear as nonperturbative excitations in the type-I description.

2. M theory and the eleventh dimension

The understanding of how the remaining two superstring theories, type IIA and HE, behave at strong coupling came as quite a surprise. In each case there is an 11th dimension whose size R becomes large at strong string coupling g_S. In the IIA case the 11th dimension is a circle, whereas in the HE case it is a line interval. The strong-coupling limit of either of these theories gives an 11-dimensional Minkowski space-time. The 11-dimensional description of the underlying theory is called *M theory*.[1]

The 11D vacuum, including 11D supergravity, is characterized by a single scale—the 11D Planck scale L_P. It

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[1]The letter M could stand for a variety of things such as magic, mystery, meta, mother, or membrane.

is proportional to $G^{1/9}$, where G is the 11D Newton constant. The connection to type-IIA theory is obtained by taking one of the ten spatial dimensions to be a circle (S^1 in the diagram) of radius R. As we pointed out earlier, the type-IIA string theory in 10D has a dimensionless coupling constant g_S, given by the value of the dilaton field, and a length scale L_S. The relationship between the parameters of the 11D and IIA descriptions is given by

$$L_P^3 = R L_S^2, \tag{1}$$

$$R = L_S g_S. \tag{2}$$

Numerical factors (such as 2π) are not important for present purposes and have been dropped. The significance of these equations will emerge later. However, one point can be made immediately. The conventional perturbative analysis of the IIA theory is an expansion in powers of g_S with L_S fixed. The second relation implies that this is an expansion about $R=0$, which accounts for the fact that the 11D interpretation was not evident in studies of perturbative string theory. The radius R is a modulus—the value of a massless scalar field with a flat potential. One gets from the IIA point to the 11D point by continuing this value from zero to infinity. This is the meaning of the edge of Fig. 5 labeled S^1.

The relationship between the HE vacuum and 11D is very similar. The difference is that the compact spatial dimension is a line interval (denoted I in Fig. 5) instead of a circle. The same relations in Eqs. (1) and (2) apply in this case. This compactification leads to an 11D spacetime that is a slab with two parallel 10D faces. One set of E_8 gauge fields is confined to each face, whereas the gravitational fields reside in the bulk. One of the important discoveries in the first superstring revolution was a mechanism that cancels quantum-mechanical anomalies in the Yang-Mills and Lorentz gauge symmetries. This mechanism works only for SO(32) and $E_8 \times E_8$ gauge groups. There is a nice generalization of this 10D anomaly cancellation mechanism to the setting of 11 dimensions with a 10D boundary. It works only for E_8 gauge groups!

3. p-branes and D-branes

In addition to the strings the theory turns out to contain other objects, called *p-branes*. A p-brane is an extended object in space with p spatial dimensions. (The term p-brane originates from the word membrane, which describes a 2-brane.) For example, the 11D M theory turns out to contain two basic kinds of p-branes with $p=2$ and $p=5$, called the M2-brane and the M5-brane. A simpler example of a brane is readily understood in the type-IIA theory when it is viewed as a compactification of the 11D theory on a circle. Eleven-dimensional particles with momentum around the circle appear as massive particles in 10D, whose masses are proportional to $1/R$. Since they are point particles, they are referred to as 0-branes. Using Eq. (2), we find $1/R = 1/L_S g_S$, and we see that in the perturbative string region, where $g_S \ll 1$, these 0-branes are much heavier

than the ordinary string states whose masses are of order $1/L_S$. The type-IIA string in ten dimensions can be identified as the M2-brane wrapping the compact circle.

These p-branes are crucial in the various dualities discussed above—since they are states in the theory, they should be mapped correctly under T and S dualities. This is particularly interesting for S duality, which maps the fundamental string of one theory to a heavy 1-brane of the other. For example, the heterotic string is such a heavy 1-brane in the weakly coupled type-I theory. We therefore see that the notion of an elementary (or fundamental) string is ill defined. The string that appears fundamental at one boundary of Fig. 5 is a heavy brane at another boundary and vice versa. We have already encountered a similar phenomenon in our discussion of electric-magnetic duality in field theory, where there was an ambiguity in the notion of elementary objects.

A special class of p-branes is called *Dirichlet p-branes* (or *D-branes* for short). The name derives from the boundary conditions assigned to the ends of open strings. The usual open strings of the type-I theory have Neumann boundary conditions at their ends. More generally, in type-II theories, one can consider an open string with boundary conditions at the end given by $\sigma = 0$:

$$\frac{\partial X^\mu}{\partial \sigma} = 0, \quad \mu = 0, 1, \ldots, p,$$

$$X^\mu = X_0^\mu, \quad \mu = p+1, \ldots, 9,$$

and similar boundary conditions at the other end. The interpretation of these equations is that strings end on a p-dimensional object in space—a D-brane. The description of D-branes as a place where open strings can end leads to a simple picture of their dynamics. For weak string coupling this enables the use of perturbation theory to study nonperturbative phenomena!

D-branes have found many interesting applications. One of the most remarkable of these concerns the study of black holes. Specifically, D-brane techniques can be used to count the quantum microstates associated with classical black-hole configurations and to show that in suitable limits the entropy (defined by $S = \log N$, where N is the number of quantum states the system can be in) agrees with the Bekenstein–Hawking prediction: 1/4 the area of the event horizon. For further details, see the article by Horowitz and Teukolsky in this volume.

D-branes also led to new insights and new results in quantum field theory, arising from the realization that the open strings which end on D-branes are described at low energies by a local quantum field theory "living" on the brane. The dynamics of quantum field theories on different branes must be compatible with the various dualities. One can use this observation to test the dualities. Alternatively, assuming the various string dualities and the consistency of the theory, one can easily derive known results in quantum field theory from a new perspective as well as many new results.

IV. CONCLUSION

During the last 30 years the structure of string theory has been explored both in perturbation theory and non-

perturbatively with enormous success. A beautiful and consistent picture has emerged. The theory has also motivated many other developments, such as supersymmetry, which are interesting in their own right. Many of the techniques that have been used to obtain exact solutions of field theories were motivated by string theory. Similarly, many applications to mathematics have been discovered, mostly in the areas of topology and geometry. The rich structure and the many applications are viewed by many people as indications that we are on the right track. However, the main reason to be interested in string theory is that it is the only known candidate for a consistent quantum theory of gravity.

There are two main open problems in string theory. The first is that the underlying conceptual principles of the theory—the analog of curved space-time and general covariance for gravity—are not yet understood. Unlike other fields, string theory is not yet a mature field with a stable framework. Instead, the properties of the theory are being discovered with the hope that eventually they will lead to an understanding of the principles and the framework. The various revolutions that the field has undergone in recent years have completely changed our perspective on the theory. It is likely that there will be a few other revolutions and our perspective will change again. Indeed, fascinating connections to large-N gauge theories are currently being explored, which appear to be very promising. In any case, the field is developing very rapidly and it is clear that an article about string theory for the next centenary volume will look quite different from this one.

The second problem, which is no less important, is that we should like to make contact with experiment. We need to find unambiguous experimental confirmation of the theory. Supersymmetry would be a good start.

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