Nobel Lecture: Interplay of disorder and interaction in two-dimensional electron gas in intense magnetic fields*

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In this lecture, I would like to briefly go through the physics that I have learned in the years since I ventured into what we nowadays call research of semiconductor electronics in low dimensions, or in my case, more simply the electronic properties of two-dimensional systems (Ando *et al.*, 1983). To summarize, electrons confined to the interface of two different semiconductors normally behave like an ordinary gas of particles in two dimensions. But, when taken to extreme conditions of low temperature and high magnetic field, they show new physics phenomena manifesting the interplay of electron-electron interactions and the interaction of the electrons with imperfections in the semiconductors. Let me first recall the events in my earlier research that led me to the journey that Art Gossard, Horst Stormer, and I took in our adventure towards the discovery of the fractional quantum Hall effect (FQHE) (Tsui *et al.*, 1982).

PROLOGUE

I joined Bell Laboratories in the spring of 1968. I was sufficiently naive that I foolhardily convinced myself to leave behind the more familiar band structures and Fermi surfaces of metal physics, which I had become comfortable with through my years of research as a graduate student, and to try something different, e.g., surfaces or interfaces, and preferably some many-body interaction physics. I read about Anderson localization, the Mott transition, and the notion that disorder and electron-electron interaction were the richest and most challenging problems in solid-state physics. But I did not have the foggiest idea on how to do what to get started.

Fortunately, I was advised to talk to John Rowell, who had at the time just completed the Rowell-McMillan electron-phonon interaction work using tunneling in superconductivity. John told me to look into point-contact tunneling into the high-T_c superconductors of those days and suggested that, instead of using tungsten whiskers, I should experiment with semiconductor tips which have built-in surface depletion layers as tunnel barriers. My effort to tunnel into the superconductors of niobium and vanadium compounds was unsuccessful, but the project forced me to learn some physics of semiconductor surfaces and interfaces. In fact, I

was able to demonstrate experimentally the electric-field quantization of the surface space-charge layer, first proposed by Schrieffer in the fifties, by doing a tunneling experiment on InAs to observe directly the quantized energy levels and the Landau levels of the resulting two-dimensional (2D) electrons (Tsui, 1970). However, the most exciting part of this effort was my discovery, in writing the paper on this work, of the beautiful work on the Si metal-oxide-semiconductor field-effect transistor (Si MOSFET) done by the IBM group in Yorktown Heights (Fang *et al.*, 1983). They laid a solid foundation for the development and growth of 2D electron physics in the subsequent decades.

Based on the IBM work, Jim Allen and I made a temperature dependence study of the inversion layer conductance in Si MOSFETs to look for the 2D Anderson localization-delocalization transition. By varying the gate voltage on the device, we were able to move the Fermi level into the band tail and observe directly a transition from the two-dimensional electron gas (2DEG) behavior to the behavior of an insulator. But, to our disappointment, we were unable to obtain quantitative agreement with theory. We thought interaction might be the cause and came to the conclusion that to enhance the interaction we should apply a magnetic field, which would change the individual electron's kinetic energy into the cyclotron energy. This should be especially effective in the extreme quantum limit when the cyclotron diameter was less than the average electron-electron separation.

In a 1976 paper, Kawaji and Wakabayashi (1976) reported the observation of localized states in the energy gap between two Landau levels. This discovery was a most important milestone on the path to the quantized Hall effect. In response to their work, I studied at the Frances Bitter National Magnet Laboratory on the MIT campus the conductance in the extreme quantum limit, when all electrons occupy the lowest Landau level, and saw some structures. Phil Anderson, after hearing this from John Rowell, asked to see the data. But by the time I showed them to him in the Bell Labs tearoom, I had repeated the experiment and found them to be sample specific. I told this to Phil and he made a cryptic remark under his breath that there should be some commensuration energy anyway. I reasoned: given that n is the 2D electron density and that the magnetic field B (applied perpendicular to the 2D plane) is expressed in terms of average flux density $n_\phi = B/\varphi_0$ (and $\varphi_0 = h/e$ is the Dirac flux quantum), the ratio n/n_φ is the Landau-level filling factor ν. For $n > n_\varphi$, an integer i number of Landau levels are filled at commensuration and the cy-
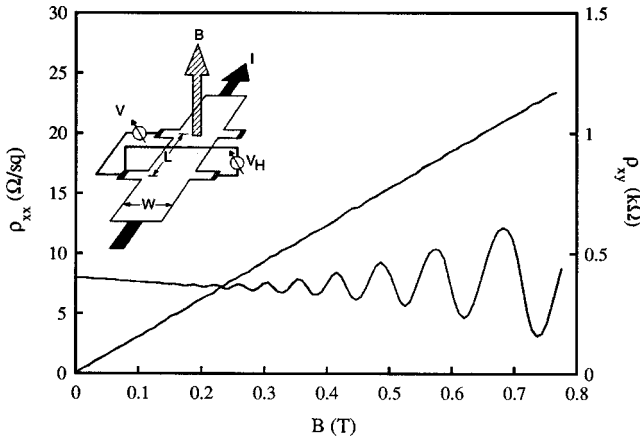
FIG. 1. Magneto-transport coefficients ρ_{xx} and ρ_{xy} of a 2DEG in GaAs/Al$_x$Ga$_{1-x}$As at 0.35 K in moderately low B. The insert shows the measurement geometry. The magnetic field B is perpendicular to the plane of the 2DEG and to the current I. The voltages V and V_H are measured along and perpendicular to I, respectively, $\rho_{xx}=(V/L)/(I/W)$ is the resistance across a square, independent of the square size, and $\rho_{xy}=V_H/I$ is the Hall resistance independent of the sample width. Data taken by A. Majumdar.

clotron energy, separating the filled from the empty levels, is the commensuration energy. I assumed he meant: In the $n<n_\varphi$ extreme quantum limit, at commensuration when $\nu=n/n_\varphi=1/i$, some interaction energy might become dominant to drive the 2D system into some new ground state. I was not brave enough to ask him: "What do you mean?" But I felt affirmed that I should continue to concentrate on the extreme quantum limit.

Indeed, with the advent of molecular beam epitaxy (Cho, 1995) and the invention of modulation doping to produce highly perfect 2D electron systems (Stormer *et al.*, 1979), it soon became quite clear to Art Gossard, Horst Stormer, and me that where we wanted to go to look for new many-body interaction physics should be a highest-mobility 2DEG sample placed in a most intense magnetic field.

TWO-DIMENSIONAL MAGNETO-TRANSPORT

In the presence of a perpendicular magnetic field, the energy levels of a two-dimensional electron collapse, as a result of Landau quantization of its cyclotron orbits, into discrete Landau levels separated by the cyclotron energy quantum. Scattering broadens the Landau levels and gives rise to 2D magneto-transport described by the Ando-Uemura theory (Ando and Uemura, 1979). Figure 1 is an example showing the quantum oscillations in the diagonal resistivity ρ_{xx}, reflecting the broadened Landau-level structure of the 2DEG, and the Hall resistance ρ_{xy}, well known from the Drude model. However, when the 2DEG is taken to the extreme condition of high B and low T, much more striking features appear, showing the interplay of disorder and electron-electron interaction in the system. More specifically, different physics phenomena are observed in three distinctly dif-
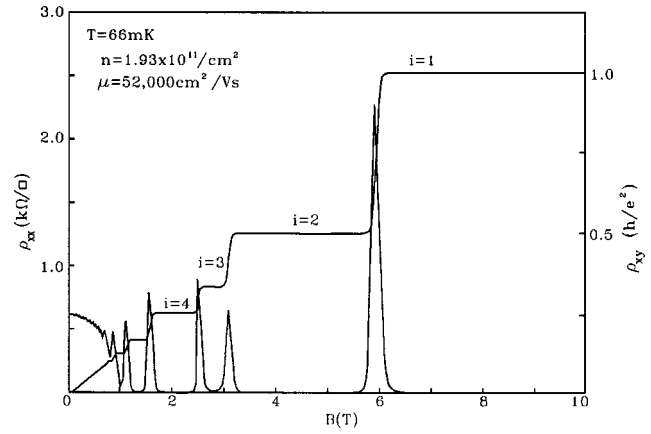

FIG. 2. ρ_{xx} and ρ_{xy} of a relatively low-mobility 2DEG in GaAs/Al$_x$Ga$_{1-x}$As. The plateaus in ρ_{xy} are quantized in the natural conductance unit e^2/h with integer quantum numbers $i=1,2,\ldots$. Data taken by H. P. Wei.

ferent physical regimes. The first is the disorder-dominant regime, when the sample is dirty with low 2D electron mobility (e.g., $\mu<10^5$ cm^2/V sec in the case of GaAs). The striking features in the data constitute the integral quantum Hall effect (IQHE) (von Klitzing *et al.*, 1980), which is understood in terms of the physics of independent electrons and their localization in the presence of random impurities in the semiconductors. The fractional quantum Hall effect (FQHE) is observed in high-mobility samples in the regime where the electron-electron interaction dominates. It manifests the many-body interaction physics of the 2DEG in the intense B field. Furthermore, even in the cleanest samples, the FQHE series terminates into an insulator in the high-B limit. This insulator is believed to be an electron crystal pinned by defects to the semiconductor. The third regime is this high-μ and high-B-field limit, where disorder and interaction play equally important roles and need to be treated on an equal footing.

QUANTUM PHASE TRANSITIONS IN IQHE

Quantization of the Hall resistance in the natural conductance unit e^2/h is currently understood in terms of the existence of an energy gap, separating the excited states from the ground state, and localized states inside the gap. In the IQHE case, where the quantum numbers are integers identified with the number of completely filled Landau levels, the energy gap is the Landau gap of a cyclotron energy quantum. The accurate quantization was shown by Laughlin, using a gedanken experiment, to be a consequence of charge quantization. He showed that the experiment in effect measures the charge carried by the excited electron. The localized states arise from disorder in the 2D system, and the data, as shown in Fig. 2, show the localization-delocalization phase transitions. In other words, for B in the plateau regions, the states at E_F are localized, and in between, delocalized. As T is decreased, the range of B for the existence of delocalized states decreases and the transition regions
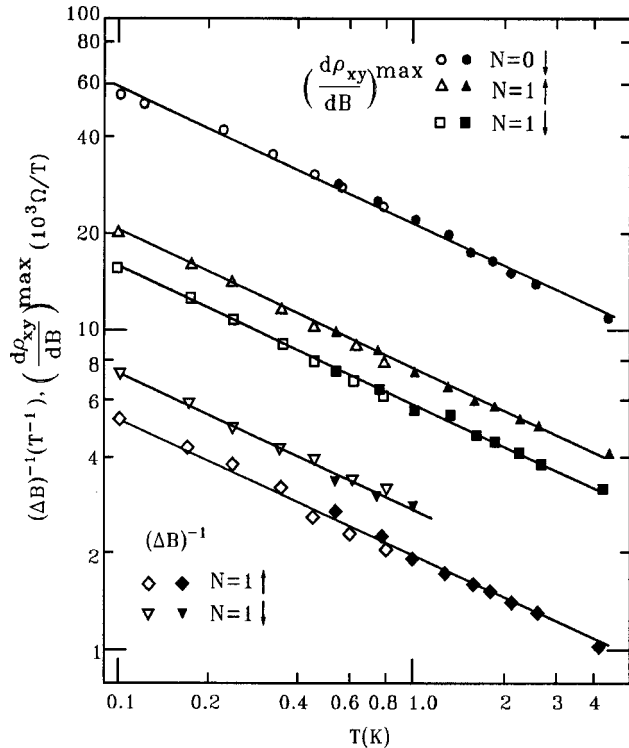
FIG. 3. Data on narrowing of the plateau-to-plateau transition regions from an $In_xGa_{1-x}As/InP$ sample with $n = 3.3 \times 10^{11}/cm^2$ and $\mu = 3.4 \times 10^4$ cm²/V s. The upper portion shows the maximum in $(d\rho_{xy}/dB)$ for the $i = 1 \rightarrow 2$, $2 \rightarrow 3$, and $3 \rightarrow 4$ transitions (i.e., the $N = 0\downarrow$, $1\uparrow$, and $1\downarrow$, respectively); the lower portion shows the inverse half-width $(\Delta B)^{-1}$ of the ρ_{xx} peak for the $i = 2 \rightarrow 3$ and $3 \rightarrow 4$ transitions. N is the Landau-level quantum number. From Wei *et al.*, 1988.

between the plateaus narrow. In the limit $T \rightarrow 0$, ρ_{xy} approaches a staircase. The underlying physics is the Anderson localization-delocalization quantum phase transition, and the experiment is simply a magnificent display of 2D quantum critical phenomena, as first put forward by Pruisken (1985).

Quantum phase transitions take place at $T = 0$. To relate them to experiments relies on the finite T behavior of the system at sufficiently close to the transition, which is governed by special rules, derived from simple scaling arguments. In our case, this boils down to narrowing of the plateau-to-plateau transition regions following a power-law dependence on T, with the temperature exponent a universal constant. Figure 3 shows data from Wei *et al.* (1988), where the narrowing of the transition regions is measured by the maximum in $d\rho_{xy}/dB$ and by the inverse half width $(\Delta B)^{-1}$ of the ρ_{xx} peak. The data show a power-law dependence $\sim T^{-\kappa}$ and the temperature exponent $\kappa = 0.42 \pm 0.04$, independent of the Hall plateaus involved. Furthermore, the hallmark of a quantum phase transition is that the finite frequency f behavior of the system and its finite T behavior are similar: both follow a power-law dependence and are characterized by the same exponent. Crossover from one to the other occurs around $hf = kT$. Engel *et al.* (1993) studied the microwave conductance in the frequency range from 0.2 GHz to 16 GHz in a dilution refrigerator and were
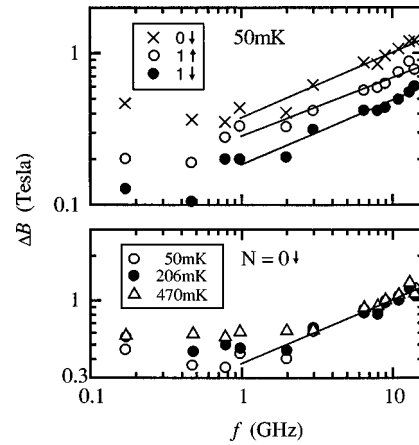

FIG. 4. Microwave frequency dependence of the width of the plateau-to-plateau transition regions. Upper panel: Half-width ΔB of $\sigma_{xx}(f)$ for the $i = 1 \rightarrow 2$, $2 \rightarrow 3$, and $3 \rightarrow 4$ transitions (i.e., the $N = 0\downarrow$, $N = 1\uparrow$, and $N = 1\downarrow$, respectively). Lines are least-squares fits of data for $f \gtrsim 0.97$ GHz to $\Delta B \sim f^\gamma$, with $\gamma = 0.43$, 0.38, and 0.42, respectively. Lower panel: ΔB of $\sigma_{xx}(f)$ for the $i = 1 \rightarrow 2$ transition at three different T's. The line is a fit to $\Delta B \sim f^{0.43}$. From Engel *et al.*, 1993.

able to go from $f < kT/h$, where the scaling is dominated by T, to $f > kT/h$, where frequency scaling should hold. Their data are shown in Fig. 4. They show a power-law dependence on f in the $f > kT/h$ limit with a frequency exponent equal to the T exponent within the experimental error bar, and are consistent with a crossover around $hf = kT$.

THE FQHE

The second regime, where the electron-electron interaction dominates, is accessible using high-mobility samples. In this regime, the FQHE becomes observable, and a large number of plateaus in ρ_{xy} and concomitant ρ_{xx} minima are apparent in the data (Figs. 5 and 6), even after the IQHE structures are exhausted in the $\nu < 1$ extreme quantum limit. These plateaus, as determined from their resistance values, are fractionally quantized, and they occur around the same fractions of Landau-level filling. At such fractional fillings, the single-electron levels are highly degenerate and there is no energy gap across E_F to possibly give rise to Hall resistance quantization. Horst Stormer and Bob Laughlin will discuss the new many-body interaction physics manifested in the phenomenon and the broader implications of it in their lectures. Here, I simply want to mention the so-called odd-denominator rule, that all the fraction quantum numbers are odd-denominator fractions, and to point out that there is now a firmly established exception at $\nu = 5/2$. Over a decade ago, Willett *et al.* (1987) reported the observation of a deep minimum in ρ_{xx} and a clear deviation of the Hall resistance from its classical line around $\nu = 5/2$ filling, suggestive of an even-denominator fraction 5/2 FQHE. Very recently, Wei Pan and Jian-Sheng Xia, working with the University of Florida micro-kelvin group in Gainesville, cooled
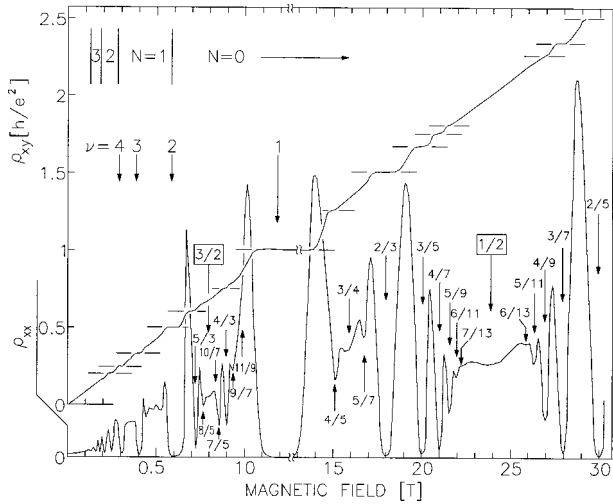
FIG. 5. ρ_{xx} and ρ_{xy} of a 2DEG in GaAs/Al$_x$Ga$_{1-x}$As with n $=3.0\times10^{11}$/cm^2 and $\mu=1.3\times10^6$ cm^2/V s. The fractionally quantized Hall resistance plateaus are indicated by the horizontal bars and the odd-denominator fractions marking the concomitant vanishing ρ_{xx}. The use of a hybrid magnet with fixed base field required composition of this figure from four different traces (breaks at $\simeq12$ T). Temperatures were ≈150 mK except for the high-field Hall trace at $T=85$ mK. The high-field ρ_{xx} trace is reduced in amplitude by a factor of 2.5 for clarity. N is the Landau-level quantum number. Filling factor ν is indicated. From Willett *et al.*, 1987.

the 2DEG below 10 mK and were able to observe a Hall plateau quantized to better than two parts in 10^6, thus making the FQHE nature of the ground state at $\nu=5/2$ unequivocal.

THE MAGNETIC-FIELD-INDUCED CRYSTAL REGIME

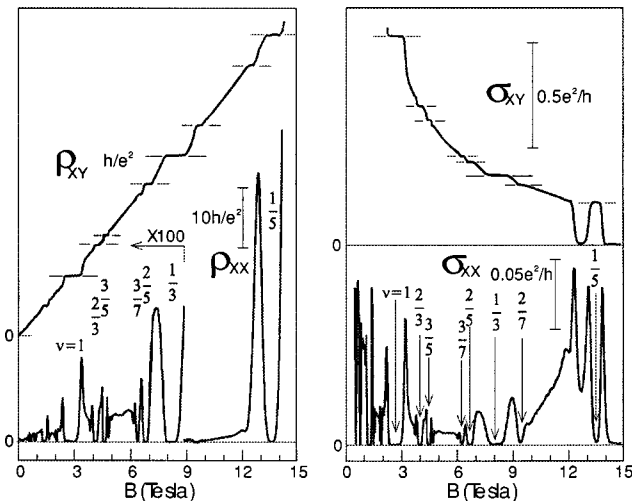Finally, there is the third regime, where disorder and interaction are equally important. This is the small-

FIG. 6. Magneto-transport coefficients of a 2DEG in GaAs/Al$_x$Ga$_{1-x}$As with $n=6.5\times10^{10}$/cm^2 and $\mu=1.5$ $\times10^6$ cm^2/V s. Left panel: ρ_{xx} and ρ_{xy} at 40 mK. Right panel: σ_{xx} and σ_{xy} obtained by inverting the ρ_{xx} and ρ_{xy} data. The vanishing of σ_{xy} together with σ_{xx} at $B\sim12.8$ T and $B>14$ T indicates insulating behavior. Data taken by Y. P. Li.
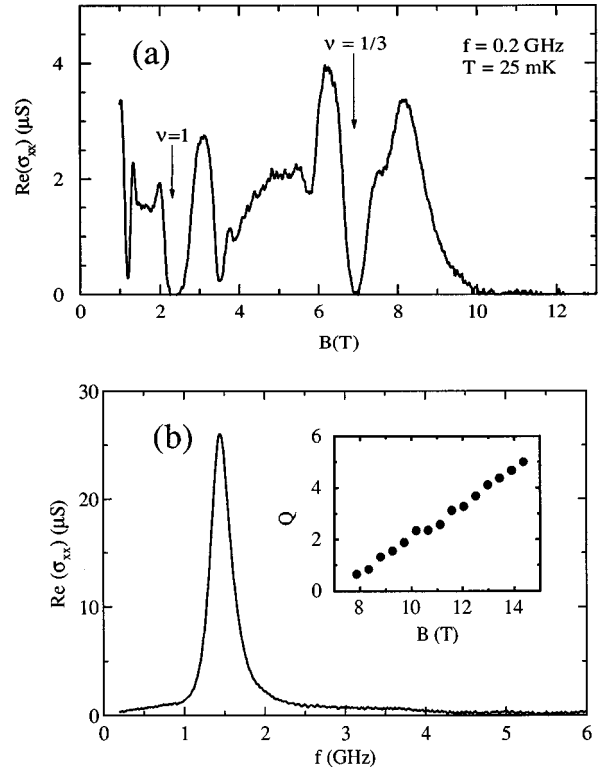
FIG. 7. Behavior of a two-dimensional hole gas (2DHG): (a) Real part of diagonal conductivity vs magnetic field of a two-dimensional hole gas in GaAs/Al$_x$Ga$_{1-x}$As with a hole density of 5.5×10^{10}/cm^2 and $\mu=3.5\times10^5$ cm^2/V s. The 2DHG becomes insulating for $B\gtrsim10$ T. (b) Resonant microwave absorption in the insulating phase of the 2DHG at $B=13$ T and $T=25$ mK. The insert shows the quality factor of the resonance Q vs B. From Li *et al.*, 1997.

filling limit after the FQHE series terminates into an insulating phase. In the absence of disorder, the ideal 2DEG is predicted to be a 2D electron crystal at sufficiently small fillings. But, in real physical systems, there is always disorder that can alter the ground state in fundamental ways. To date, insulating behavior is seen in the highest-mobility 2D electrons for $\nu<1/5$, and 2D holes for $\nu<1/3$. This insulating phase in the cleanest 2D systems we have is attributed to crystallization of the 2D electron and 2D hole gases under an intense B field. The crystal, being pinned to the semiconductor by defects in the semiconductor, cannot slide to conduct electricity.

Experimentally, this is a challenging regime. It requires all the best: the highest-mobility samples, lowest T, and most intense B. Since it is an insulator, dc transport is limited. Microwave measurements, more appropriate at first sight, are notoriously hard and can be plagued with pitfalls. Consequently, unequivocal experiments are few and there is very little direct information on the crystalline nature of the ground state. A great many of the properties of this insulating phase still remain unknown and unexplored. Chi-Chun Li and Lloyd Engel (Li *et al.*, 1997) recently improved their microwave absorption experiment and obtained data (Fig. 7) showing a sharp conductance resonance in the insulating phase at ~1.5 GHz, a frequency consistent with that expected of a pinning mode of the crystal. It is surprising,

however, that the resonance is sharp, with a quality factor Q much larger than 1. The Q, as seen in the inset of Fig. 7(b), increases with increasing B to the highest B studied. Thus it appears that there is nontrivial fundamental physics hidden in this regime, and I look forward to further experimental efforts.

ACKNOWLEDGMENTS

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