Top-quark condensation

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Top-quark condensation, in particular the minimal framework in which the neutral Higgs scalar is (predominantly) an effective $\bar{t}t$ condensate of the standard model, is reviewed. Computational approaches are compared and similarities, differences, and deficiencies pointed out. Extensions of the minimal framework, including scenarios with two composite Higgs doublets, additional neutrino condensates, and $\bar{t}t$ condensation arising from four-fermion interactions with enlarged symmetries, are described. Possible renormalizable models of underlying physics potentially responsible for the condensation, including topcolor-assisted technicolor frameworks, are discussed. Phenomenological implications of top condensate models are outlined. Outstanding theoretical issues and problems for future investigation are pointed out. Progress in the field after this article was accepted has been briefly covered in a Note added at the end. [S0034-6861(99)00903-4]

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 0034-6861/99/71(3)/513(62)/$27.40 ©1999 The American Physical Society

I. INTRODUCTION

The most important outstanding problem in the standard model of strong and electroweak interactions is the source of the electroweak symmetry breaking (EWSB) and the related problem of fermion mass hierarchies. The EWSB mechanism should explain the masses of electroweak gauge bosons Z and W, and at the same time the generation of masses of fermions, particularly the heavy quarks. An understanding of the origin of EWSB is expected to provide a window to physics beyond the standard model.

The standard model, as currently defined, contains an elementary $SU(2)_L$-doublet Higgs scalar sector that is appended to the model in an *ad hoc* way. In order to accommodate the gauge-boson masses M_Z and M_W while keeping the model formally renormalizable, one adds a Higgs-field mass term and a quartic self-interaction term to the Lagrangian density. Parameters associated with those terms are adjusted to obtain a nonzero electroweak vacuum expectation value (VEV) for the neutral CP-even component of the Higgs field. The VEV is then responsible for the nonzero Z and W masses. This is called the Higgs mechanism. Further, when Higgs-fermion-antifermion Yukawa terms are added to the model with *ad hoc* coupling strengths, the nonzero VEV leads to the masses of fermions.

The outlined Higgs mechanism of the standard model is unsatisfactory in several ways. The origin of a Higgs field is not explained in a fundamental way. This is disturbing since the residual Higgs particle ($\mathcal{H}$) has yet to be observed. Further, emergence of the required nonzero VEV is obtained simply by adjusting parameters in the Higgs potential. Another disturbing feature is that the known values of fermion masses are obtained in an *ad hoc* manner, by adjusting phenomenologically introduced Yukawa parameters. Moreover, the Higgs sector of the standard model requires extreme fine tuning in order to preserve the perturbative renormalizability of the model. That is, loop-induced corrections to the mass of the Higgs (and hence to its VEV) grow violently ($\propto\Lambda^2$) when the ultraviolet cutoff Λ of the theory increases. To maintain the renormalizability in a formal sense (i.e., with $\Lambda\to\infty$), an extremely fine tuning is needed to cancel the various Λ^2 terms, or, equivalently, the bare-mass parameter $M_\Phi^2(\Lambda)$ of the Higgs isodoublet must be fine tuned at each loop order in the perturbative expansion.

One solution to some of the above problems is the introduction of supersymmetry. In that case fine tuning of the bare-mass parameters need be performed only once, at the tree level. The mentioned Λ^2 terms are then not present at higher (loop) levels, due to the cancellation of the Λ^2 radiative contributions of particles and their superpartners. This solves to a large extent the problem of fine tuning. In that case, the cutoff is of the order of the grand unified scale $E_{\mathrm{GUT}}\sim10^{16}$ GeV. Thus a "great desert" in an energy interval $[E_{\mathrm{SUSY}},E_{\mathrm{GUT}}]$ $\sim[10^4\,\mathrm{GeV},10^{16}\,\mathrm{GeV}]$ generally emerges in these scenarios. While eliminating several of the free parameters of the standard model, supersymmetry introduces many new parameters and elementary particles that have not (yet) been observed. The Higgs field is generally elementary in most supersymmetric frameworks.

The Higgs sector of the standard model appears to be just an effective Ginzburg-Landau-type description of low-energy (standard-model) physics represented by a composite (nonelementary) isodoublet scalar field or fields. This is the basic idea of top-quark condensation, as well as of technicolor models. In such frameworks, the Higgs isodoublet is a condensate of a fermion-antifermion pair or pairs, the constituents being predominantly t_R, t_L, and b_L, or pairs of technifermions. The focus of the present review article is models involving primarily $\overline{t}t$ condensation, although scenarios including technicolor are also discussed.

Models involving $\overline{t}t$ condensation generally start with minimal assumptions about physics beyond the standard model. They assume that the underlying physics above a compositeness scale Λ leads at energies $\mu\sim\Lambda$ to effective four-quark interactions strong enough to induce quark-antiquark condensation into composite Higgs fields, leading thus to an effective standard model at $\mu<\Lambda$. The attraction of such effective dynamical frameworks lies in the simplicity of their assumptions and in the fact that these frameworks can connect the dynamical generation of the heavy top-quark mass and all or part of the (dynamical) EWSB. In principle, such models can even lead to dynamical generation of the lighter fermion masses. Unfortunately, the more we want to explain, the more assumptions about the four-fermion interactions we have to make.

To explain the origin of the aforementioned effective four-fermion terms, several models of underlying dynamics have been constructed in the literature. In most cases, these models are renormalizable, and some are quite promising. The present article does not provide a detailed review and discussion of such models. Instead, it concentrates more on effective models of relatively simple four-fermion interactions and on methods of calculating the dynamical generation of fermionic masses and dynamical EWSB. Since the physical mechanism that such methods investigate is nonperturbative, it is understandable that the methods are still relatively crude, and many outstanding questions remain regarding their applicability. In the opinion of this author, these questions are very pressing. To make further progress, a reliable and systematic method of investigating condensation mechanisms is required, particularly when the compositeness scale Λ is low ($<10^8$ GeV).

In Sec. II, we review the simplest (minimal) framework, in which $\bar{t}t$ condensation alone is assumed to be responsible for the generation of the full dynamical EWSB and top-quark mass. We also discuss studies of that phenomenon involving perturbative renormalization-group (RG) equations. These methods are used in the literature to investigate condensation frameworks and were applied prominently in the original minimal framework. The actual dynamics of condensation in such an approach is either not investigated directly or examined only schematically—in the lowest, quark-loop, approximation. The latter approximation then provides compositeness boundary conditions for RG equations at an energy $\mu \sim \Lambda$, due to a strong attraction in the top-quark sector. For very large values of Λ ($\Lambda \gtrsim 10^8$ GeV in the minimal framework), low-energy predictions $m_t^{\rm phys}$ and $m_H^{\rm phys}$ do not depend on details of the actual physical condensation mechanism, due to the infrared fixed-point behavior of the relevant perturbative RG equations. This feature makes the method relevant for the $\bar{t}t$ condensation program, but only for large values of Λ. The method can be used for any realization of an effective strongly attractive interaction, not just four-fermion terms.

In Sec. III, we describe a different approach for studying quark condensation effects. This method employs Dyson-Schwinger integral equations to relate dynamically generated quark masses to the strengths of the four-quark terms, as well as the Pagels-Stokar equations relating the dynamical quark masses to the decay constant F_π of the Nambu-Goldstone bosons. Pagels-Stokar relations are closely related to the Bethe-Salpeter equations for the bound state of Nambu-Goldstone bosons. The method is illustrated within the minimal $\bar{t}t$ condensation framework (where F_π = the electroweak VEV), and within its extension involving also $\bar{b}b$ condensation. The approach described in this section, in contrast to the renormalization-group method of Sec. II, addresses the strong dynamics of the condensation mechanism more directly. However, the integral Dyson-Schwinger equations are complicated, and a full Dyson-Schwinger+Pagels-Stokar analysis has been performed in the literature only in the leading-N_c (quark-loop+QCD) approximation. Further, in any known systematic approximation scheme (e.g., $1/N_c$ expansion), this method is fraught with the problem of gauge noninvariance.

In Sec. IV, we discuss next-to-leading-order (NTLO) effects in the $1/N_c$ expansion, within the minimal condensation framework. The method of calculating and minimizing the effective potential yields a "hard bare-mass" (i.e., nonvariational) version of the Dyson-Schwinger equation, usually called the gap equation. This equation, combined with subsequent renormalization of the bare dynamical "hard mass" $m_t(\Lambda)$, is expected to be a good approximation to the corresponding variational NTLO version of the Dyson-Schwinger equation involving a running dynamical mass $\Sigma_t(\bar{p}^2) \equiv m_t(|\bar{p}|)$, as long as the compositeness scale Λ is not very high. A fully systematic NTLO version of the Bethe-Salpeter equation and its application to the $\bar{t}t$ condensate framework has not appeared in the literature. Inclusion of the latter equation would be needed for a full NTLO analysis of the condensation. Besides a fully nonperturbative renormalization-group approach (proposed in the literature, see Sec. II.E, but not yet applied to $\bar{t}t$ condensation), the $1/N_c$ expansion—an extension of approaches of the approaches of Dyson-Schwinger+Pagels-Stokar type beyond the leading-N_c level—appears to be at this time the only viable approach to calculating condensation effects when the compositeness scale is not very high: $\Lambda < 10^8$ GeV.

In Sec. V, we compare and comment on the various approaches to calculating condensation effects. We also review work on the mass-dependent perturbative RG approach and discuss possible contributions of four-quark interactions with dimension higher than six.

The minimal $\bar{t}t$ condensation framework appears to be ruled out, since it predicts too high a mass m_t when the full dynamical $\bar{t}t$-induced EWSB is implemented. For that reason, many extensions of the minimal framework have been investigated in the literature—see Secs. VI–IX. Extensions usually involve, in addition to $\bar{t}t$, other condensates which also contribute to the EWSB (i.e., to the electroweak VEV $v \approx 246$ GeV). This feature can bring the mass $m_t^{\rm dyn}$ down to acceptable values. Beyond understanding m_t and EWSB dynamically, other reasons for investigating extensions include the hope of understanding and/or predicting dynamically the mass spectra of fermions other than t, Cabibbo-Kobayashi-Maskawa (CKM) mixing, parity violation, larger spectra of composite particles, and breaking of higher gauge symmetries.

In Sec. VI, we review generalizations of the minimal condensation framework that do not involve an extension of the standard-model gauge group—among them effective four-quark scenarios with two Higgs doublets, colored composite scalars, and effective scenarios involving, in addition to heavy quarks, leptonic condensation.

In Sec. VII, we discuss work on effective four-fermion interaction models with extended symmetries and the resulting dynamical symmetry breaking patterns.

In Sec. VIII, we review those extensions of the minimal framework which embed $\bar{t}t$ condensation in fully renormalizable frameworks of underlying physics. At some compositeness scale $\sim\Lambda$, these models become strongly coupled and effectively lead to strong four-fermion terms and thus to condensation. We include a discussion of topcolor-assisted technicolor. These models combine features of $\bar{t}t$ condensation and of technicolor, the latter being predominantly responsible for the dynamical electroweak symmetry breaking and for giving masses $\sim\Lambda$ to the topcolor gauge sector, leading to effective four-quark terms and to $\bar{t}t$ condensation and $m_t^{\rm dyn}$.

In Sec. IX, we discuss phenomenological predictions of scenarios involving $\bar{t}t$ condensation, including topcolor-assisted technicolor scenarios. We also describe some outstanding theoretical questions arising in condensation frameworks.

Section X contains a short summary, outlines prospects for further development, and discusses some remaining unresolved problems.

II. THE METHOD OF RENORMALIZATION-GROUP EQUATIONS

A. The starting Lagrangian—truncated top-mode standard model

Bardeen, Hill, and Lindner (1990), motivated by an idea of Nambu (1989)[1] and by the work of Miransky, Tanabashi, and Yamawaki (1989a, 1989b),[2] considered top-quark condensation induced by a truncated four-quark interaction involving only the heaviest generation of quarks,

$$\mathcal{L}^{(\Lambda)} = \mathcal{L}^{0}_{\text{kin}} + G(\bar{\Psi}^{ia}_{L} t^{a}_{R})(\bar{t}^{b}_{R}\Psi^{ib}_{L}), \tag{2.1}$$

where a and b are color and i isospin indices, and $\Psi^{T}_{L} = (t_L, b_L)$. $\mathcal{L}^{0}_{\text{kin}}$ represents the usual gauge-invariant kinetic terms for fermions and gauge bosons. The four-quark term is also invariant under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ of the standard model; here, the subscript c denotes the color of the strong interaction, L refers to the left-handed sector, and Y to the hypercharge sector of the electroweak interactions. As denoted above, the model assumes a finite upper cutoff scale Λ where $\bar{t}t$ condensation is supposed to occur, and all constants and fields in Eq. (2.1) are the "bare" quantities in this theory: $G = G(\Lambda)$, $\Psi_L = \Psi^{(\Lambda)}_{L}$, etc. The four-quark term is assumed to have a strong-coupling parameter $\Lambda^2 G$ responsible for creation of a composite Higgs doublet Φ:

$$\Phi \propto \frac{1}{2}\begin{pmatrix} \bar{b}^{a}(1+\gamma_5)t^{a} \\ -\bar{t}^{a}(1+\gamma_5)t^{a} \end{pmatrix} = i\tau_2(\bar{t}_R\Psi_L)^{\dagger T},$$
$$\tilde{\Phi} \equiv i\tau_2\Phi^{\dagger T} \propto \bar{t}_R\Psi_L. \tag{2.2}$$

Other four-quark terms omitted from Eq. (2.1) are assumed to be unimportant. That assumption is based on the belief that t, being by far the heaviest known quark, is perhaps the only one responsible for generating its own mass and possibly also for the electroweak symmetry breaking ($\langle \mathcal{H} \rangle_0 \approx 246\,\text{GeV}$). The discussion in Sec. III will substantiate this assumption. The framework described by Eq. (2.1), called also the (truncated) top-mode standard model, is a specific version of the Nambu–Jona-Lasinio–Vaks–Larkin (NJLVL) model (Nambu and Jona-Lasinio, 1961; Vaks and Larkin, 1961), but regarded here not as a low energy QCD framework ($\Lambda \sim 10^2\,\text{MeV}$), but a framework for dynamical electroweak symmetry breaking (DEWSB, $\Lambda \gtrsim 1\,\text{TeV}$).[3] The underlying physics responsible for the four-quark term (2.1) is not specified. It could, for example, be a theory with massive gauge bosons ($M \sim \Lambda$) that couple strongly to the third-generation quarks.

The Lagrangian density (2.1) of the truncated top-mode standard model can be rewritten with an additional, auxiliary, scalar $SU(2)_L$ isodoublet Φ, by adding to Eq. (2.1) a quadratic term,

$$\begin{aligned}\mathcal{L}^{(\Lambda)}_{\text{new}} &= \mathcal{L}^{(\Lambda)}_{\text{old}} - [M_0\tilde{\Phi}^{i\dagger} + \sqrt{G}\bar{\Psi}^{ia}_{L}t^{a}_{R}][M_0\tilde{\Phi}^{i} + \sqrt{G}\bar{t}^{b}_{R}\Psi^{ib}_{L}] \\ &= i\bar{\Psi}\not{\partial}\Psi - M_0\sqrt{G}[\bar{\Psi}_L\tilde{\Phi}t_R + \bar{t}_R\tilde{\Phi}^{\dagger}\Psi_L] - M_0^2\tilde{\Phi}^{\dagger}\tilde{\Phi} \\ &\quad + \cdots,\end{aligned} \tag{2.3}$$

where

$$\tilde{\Phi} = i\tau_2\Phi^{\dagger T}, \quad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\mathcal{G}^{(+)} \\ \mathcal{H} + i\mathcal{G}^{(0)} \end{pmatrix},$$

$$\mathcal{G}^{(\pm)} = \frac{1}{\sqrt{2}}(\mathcal{G}^{(1)} \pm i\mathcal{G}^{(2)}), \tag{2.4}$$

and the dots in Eq. (2.3) denote the terms containing electroweak gauge bosons and gluons. Addition of such a term changes the generating functional only by a source-independent factor (Kikkawa, 1976; Kugo, 1976), and is therefore assumed to lead to physics equivalent to Eq. (2.1).[4] Here, $\mathcal{H}$, $\mathcal{G}^{(0)}$, $\mathcal{G}^{(1)}$, and $\mathcal{G}^{(2)}$ are the Higgs and the three real Nambu-Goldstone components of the auxiliary complex isodoublet field Φ, and M_0 is an unspecified bare-mass parameter (usually taken as $M_0 \sim \Lambda$) for Φ at $\mu \sim \Lambda$. Physical results will turn out to be independent of the specific value of M_0. These auxiliary fields become through quantum effects the physical bound-state Higgs and the "scalar" longitudinal components of electroweak bosons at energies $\mu > \Lambda$. Equations of motion for the scalars in Eq. (2.3) indeed reveal the composite structure (2.2):

$$\Phi = \frac{\sqrt{G}}{2M_0}\begin{pmatrix} \bar{b}^{a}(1+\gamma_5)t^{a} \\ -\bar{t}^{a}(1+\gamma_5)t^{a} \end{pmatrix} = \frac{\sqrt{G}}{M_0}(i\tau_2)(\bar{t}_R\Psi_L)^{\dagger T}$$
$$\Rightarrow\ \tilde{\Phi} = -\frac{\sqrt{G}}{M_0}(\bar{t}_R\Psi_L). \tag{2.5}$$

[1]A brief discussion of Nambu's "bootstrap" idea for $\bar{t}t$ condensation is included in Sec. VI.B.

[2]The work and methods of Miransky, Tanabashi, and Yamawaki (1989a, 1989b) are discussed in Sec. III.

[3]In this article, we shall refer to six-dimension four-fermion contact interactions, i.e., those without any derivatives, as NJLVL terms.

[4]This is true in the present case. However, in other types of NJLVL models, introduction of one auxiliary isodoublet Φ may not always lead to a physics equivalent to those four-fermion NJLVL interactions, as pointed out by Dudas (1993). Various *Ansätze* for the scalar sector constrain us in fact to various specific condensation scenarios. Only the scenario with the lowest value of the energy density (the vacuum) should materialize (see discussion in Sec. IX.B).

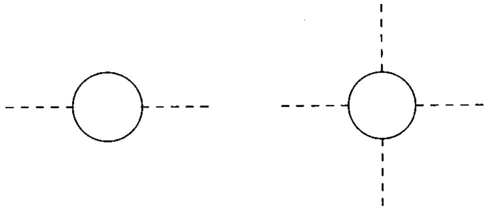

FIG. 1. The relevant graphs contributing to induced terms of composite scalars in the quark-loop approximation. Solid lines represent the top quark and dashed lines the (composite) scalars.

B. Quark-loop approximation

To illustrate how the truncated top-mode standard model transforms into the minimal standard model as the cutoff μ decreases below Λ, it is instructive to consider the quark-loop approximation, i.e., ignore loops of particles other than quarks, as well as radiative contributions of the composite scalar sector ("feedback" effects). The approximation amounts to truncating the corresponding $1/N_c$ expansions to the leading-N_c terms and without QCD, where $N_c=3$ is the number of quark colors. Integrating out the heavy-quark components at one-loop level in energy interval $[\mu, \Lambda]$ (see Fig. 1), ignoring terms ~ 1 when compared with $\ln(\Lambda^2/\mu^2)$, and ignoring masses of particles,[5] leads to additional (induced) terms in the Lagrangian density:

$$\mathcal{L}^{(\mu)}=i\bar{\Psi}\not\partial\Psi - M_0\sqrt{G}[\bar{\Psi}_L\tilde{\Phi}t_R+\text{H.c.}]+\Delta\mathcal{L}_{\text{gauge}} + Z_\Phi(\Lambda;\mu)(D_\nu\Phi)^\dagger D^\nu\Phi - M_\Phi^2(\Lambda;\mu)\Phi^\dagger\Phi - \frac{\lambda(\Lambda;\mu)}{2}(\Phi^\dagger\Phi)^2+\cdots. \tag{2.6}$$

The dots represent the old terms with gauge bosons, $\Delta\mathcal{L}_{\text{gauge}}$ are quark-loop corrections to gauge coupling constants, and D_ν are the covariant derivatives. In Eq. (2.6), the fields are still those from the theory with the Λ cutoff [Eq. (2.3)]. Λ- and μ-dependent parts of the two- and four-leg Green's functions[6] of Fig. 1 give

$$Z_\Phi(\Lambda;\mu)=\frac{N_c M_0^2 G}{(4\pi)^2}\ln\left(\frac{\Lambda^2}{\mu^2}\right),$$

$$M_\Phi^2(\Lambda;\mu)=M_0^2-\frac{2N_c M_0^2 G}{(4\pi)^2}(\Lambda^2-\mu^2), \tag{2.7}$$

$$\lambda(\Lambda;\mu)=\frac{2N_c(M_0^2G)^2}{(4\pi)^2}\ln\left(\frac{\Lambda^2}{\mu^2}\right). \tag{2.8}$$

[5]Based on the assumption that $m/\Lambda, m/\mu \ll 1$, where m is a typical particle mass. As pointed out by Bando *et al.* (1990), this is not correct for the bare mass of the scalar doublet $M_0 \sim\Lambda$, and the calculation has to be modified. Modification does not affect the leading-N_c quark-loop approximation, but it does affect the full analysis by RG equations. See Sec. V.D for details.

[6]Green's functions with more external legs are "finite" (Λ- and μ-independent for $\Lambda,\mu \gg E_{\text{ew}}$).

The Lagrangian density (2.6) can be brought into the canonical form of the minimal standard model by rescaling the scalar field $\Phi=\Phi^{(\mu)}/\sqrt{Z_\Phi(\Lambda;\mu)}$:

$$\mathcal{L}^{(\mu)}=i\bar{\Psi}\not\partial\Psi - g_t(\mu)[\bar{\Psi}_L\tilde{\Phi}^{(\mu)}t_R+\text{H.c.}]+\Delta\mathcal{L}_{\text{gauge}} +(D_\nu\Phi^{(\mu)})^\dagger D^\nu\Phi^{(\mu)} - m_\Phi^2(\mu)\Phi^{(\mu)\dagger}\Phi^{(\mu)} - \frac{\lambda(\mu)}{2}(\Phi^{(\mu)\dagger}\Phi^{(\mu)})^2+\cdots, \tag{2.9}$$

where

$$g_t(\mu)=\frac{M_0\sqrt{G}}{\sqrt{Z_\Phi(\Lambda;\mu)}}=\frac{(4\pi)}{\sqrt{N_c}}\frac{1}{\sqrt{\ln(\Lambda^2/\mu^2)}}, \tag{2.10}$$

$$m_\Phi^2(\mu)=\frac{M_\Phi^2(\Lambda;\mu)}{Z_\Phi(\Lambda;\mu)}=\left[\frac{8\pi^2}{N_c G}-(\Lambda^2-\mu^2)\right]\frac{2}{\ln(\Lambda^2/\mu^2)}, \tag{2.11}$$

$$\lambda(\mu)=\frac{\lambda(\Lambda;\mu)}{Z_\Phi^2(\Lambda;\mu)}=\frac{32\pi^2}{N_c\ln(\Lambda^2/\mu^2)}. \tag{2.12}$$

These parameters are to be interpreted as the running parameters of the minimal standard model at the "probe" (cutoff) energy $E=\mu$ in the quark-loop approximation,[7] and in addition they are now interdependent. Here we see explicitly that in these physical parameters the initial arbitrary scaling mass M_0 cancels out. For $\mu\ll\Lambda$, these parameters can be applied in either the unbroken ($m_\Phi^2>0$) or the broken phase ($m_\Phi^2<0$). The latter requires fine tuning of the parameter G close to $G_{\text{crit}}=(8\pi^2)/(N_c\Lambda^2)$: $G\gtrsim(8\pi^2)/(N_c\Lambda^2)$. In Sec. III, we shall see that this is equivalent to turning on a nonzero solution $m_t\neq 0$ of the so-called gap equation in the quark-loop approximation. Further, if we assume that $\mu\sim E_{\text{ew}}\ll\Lambda$ and that we have full DEWSB due to this composite Higgs only $[m_\Phi^2(E_{\text{ew}})<0]$, then the low-energy Higgs potential $V(\Phi)^{(\mu)}$ in Eq. (2.9) has its minimum at $\langle\Phi^{(\mu)}\rangle_0=(0,v/\sqrt{2})^T$. Here, v is the full electroweak vacuum expectation value (VEV): $v=246\,\text{GeV}$. At this point we have $m_\Phi^2=-\lambda v^2/2$. Then Eq. (2.10) implies

$$m_t^2=\frac{g_t^2(\mu=m_t)v^2}{2}=\frac{8\pi^2v^2}{N_c\ln(\Lambda^2/m_t^2)}$$

$$\Rightarrow\ v^2=m_t^2\frac{N_c}{8\pi^2}\ln\left(\frac{\Lambda^2}{m_t^2}\right), \tag{2.13}$$

$$m_H^2=\left.\frac{\partial^2 V(\mathcal{H};\mathcal{G}^{(j)}=0)^{(\mu=m_H)}}{\partial\mathcal{H}^2}\right|_{\mathcal{H}=v} =\lambda(\mu=m_H)v^2 =\frac{32\pi^2v^2}{N_c\ln(\Lambda^2/m_H^2)}. \tag{2.14}$$

Comparison of Eqs. (2.13) and (2.14) yields for the

[7]In the quark-loop approximation, quark fields do not evolve with energy μ: $\Psi^{(\mu)}=\Psi^{(\Lambda)}$ $(=\Psi)$.

Higgs and the top-quark masses the relation $m_H \approx 2m_t$, valid for $\Lambda \gg m_t, m_H$. This is a result of Nambu and Jona-Lasinio (1961) and of Vaks and Larkin (1961). In Sec. III we shall see that this is in fact the Pagels-Stokar relation in the quark-loop approximation. Note that in this approximation $m_t^2(\mu) = g_t^2(\mu)\langle\Phi^{(\mu)0}\rangle_0^2 = M_0^2 G\langle\Phi^{(\Lambda)0}\rangle_0^2$ is not evolving with μ. For $m_t = 175$ GeV, Eq. (2.13) yields $\Lambda \approx 3.6\times 10^{13}$ GeV.

C. Renormalization-group equations plus compositeness conditions

Bardeen, Hill, and Lindner (1990) performed a calculation in the truncated top-mode standard model (2.1)[8] by using a method involving perturbative RG equations plus a compositeness condition (at scale Λ). The latter condition was motivated by the quark-loop approximation described in Sec. II.B.[9] This calculation relies on the following assumption: The model behaves as the minimal standard model (with certain additional relations) at energies $\mu < \mu_*$, where μ_* is not far below the compositeness scale Λ: $\ln(\Lambda/\mu_*) \approx 1$. We can intuitively expect that this assumption implicitly necessitates large Λ and that the binding of the quark constituents into the composite scalar is very tight. It is expected that under such circumstances the quark-loop approximation, Eqs. (2.9)–(2.12), describes, at least qualitatively, the evolution of the physical parameters $g_t(\mu)$, $m_\Phi^2(\mu)$, and $\lambda(\mu)$ in the short interval $[\ln\mu_*, \ln\Lambda]$. In other words, since $\ln(\Lambda/\mu_*) \approx 1$, the approximate boundary conditions at $\mu = \mu_*$ for evolution of g_t and λ in the large interval $[\ln E_{\rm ew}, \ln\mu_*]$ can be read off from Eqs. (2.10) and (2.12) as

$$g_t(\mu)\big|_{\mu=\mu_*} \gg 1,$$
$$\left.\frac{\lambda(\mu)}{g_t^2(\mu)}\right|_{\mu=\mu_*} \sim 1 \quad \text{for} \quad \mu_* < \Lambda,\ \ln\left(\frac{\Lambda}{\mu_*}\right) \approx 1. \tag{2.15}$$

Bardeen, Hill, and Lindner then applied these boundary conditions to one-loop RG equations of the minimal standard model,

$$16\pi^2 \frac{dg_t(\mu)}{d\ln\mu} = \left[\left(N_{\rm c} + \frac{3}{2}\right)g_t^2(\mu) - 3\,\frac{(N_{\rm c}^2-1)}{N_{\rm c}}\,g_3^2(\mu) - \frac{9}{4}g_2^2(\mu) - \frac{17}{12}g_1^2(\mu)\right]g_t(\mu), \tag{2.16}$$

$$16\pi^2 \frac{d\lambda(\mu)}{d\ln\mu} = -4N_{\rm c}g_t^4(\mu) + 4N_{\rm c}\lambda(\mu)g_t^2(\mu) + 12\lambda^2(\mu) - \mathcal{A}(\mu)\lambda(\mu) + \mathcal{B}(\mu), \tag{2.17}$$

where g_3, g_2, and g_1 are the usual coupling parameters of $SU(3)_{\rm c}$, $SU(2)_L$, and $U(1)_Y$, respectively, satisfying their own one-loop RG equations:

$$16\pi^2 \frac{dg_j(\mu)}{d\ln\mu} = -C_j g_j^3(\mu), \tag{2.18}$$

$$C_3 = \frac{1}{3}(11N_{\rm c} - 2n_q), \quad C_2 = \frac{43}{6} - \frac{2}{3}n_q,$$
$$C_1 = -\frac{1}{6} - \frac{10}{9}n_q. \tag{2.19}$$

Here, n_q is the number of effective quark flavors (for $\mu > m_t$: $n_q = 6$), and $N_{\rm c}$ the number of colors ($N_{\rm c} = 3$). Expressions $\mathcal{A}$ and $\mathcal{B}$ in Eq. (2.17) are

$$\mathcal{A} = 9g_2^2 + 3g_1^2, \quad \mathcal{B} = \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_1^2 + \frac{3}{4}g_1^4. \tag{2.20}$$

Bardeen, Hill, and Lindner used for low-energy g_j's the values: $g_3^2(M_Z) \approx 1.44$, $g_2^2(M_Z) \approx 0.446$, and $g_1^2(M_Z) \approx 0.127$. They ignored contributions of g_b to the evolution of g_t and λ. Solutions of minimal standard-model RG equations (2.16) and (2.17) with compositeness boundary conditions (2.15) gave the renormalized masses

$$m_t^{\rm ren} = g_t(\mu = m_t^{\rm ren})v/\sqrt{2}, \quad m_H^{\rm ren} = \lambda^{1/2}(\mu = m_H^{\rm ren})v, \tag{2.21}$$

which, for very large $\Lambda \approx \mu_* > 10^8$ GeV, turned out to be rather stable against variations of boundary conditions (2.15). Specifically, the predicted $m_t^{\rm ren}$ and $m_H^{\rm ren}$ are rather insensitive if (a) $g_t^2(\mu_*)/(4\pi)$ is varied between ∞ and 1, and Λ is large ($\Lambda \sim \mu_* > 10^8$ GeV); (b) Compositeness scale Λ (or: μ_*; $\mu_* \sim \Lambda$) is varied on a logarithmic scale by quantities of order 1, and Λ is large ($\Lambda \sim \mu_* > 10^8$ GeV). This is known as infrared fixed-point behavior. This feature makes the application of the RG approach an important contribution to the $t\bar{t}$ condensation program. It has its origin in the presence of QCD contributions on the right of the RG equation (2.16) and was discussed earlier (Chang, 1974; Cabibbo *et al.*, 1979; Hill, 1981; Pendleton and Ross, 1981; Hill, Leung, and Rao, 1985) in a context independent of condensation. This behavior can be seen explicitly in Table I, which shows the results of Bardeen, Hill, and Lindner (1990), as well as those in the more primitive quark-loop approximation.

As can be seen in Table I, the (one-loop) RG results of Bardeen, Hill, and Lindner gave too high a mass $m_t^{\rm ren}$. The larger the Λ, the smaller the $m_t^{\rm ren}$. For $\Lambda \sim E_{\rm Planck}$ ($\sim 10^{19}$ GeV), they obtained $m_t^{\rm ren} \approx 218$ GeV, substantially higher than the measured $m_t^{\rm phys} \approx 170$–$180$ GeV (Abe *et al.*, 1995; Adachi *et al.*, 1995). Consideration of the two-loop RG equations for g_t and λ does not change these results significantly—it only in-

[8]We shall sometimes refer to the truncated top-mode standard-model framework as the minimal ($t\bar{t}$ condensation) framework, in accordance with terminology used in the literature. Strictly speaking, the minimal framework is more general—it is not restricted to the picture of four-quark interactions.

[9]The quark-loop arguments of Sec. II.B were first presented for the truncated top-mode standard model by Bardeen, Hill, and Lindner.

TABLE I. Predicted m_t and m_H in the quark-loop approximation [see Eqs. (2.13) and (2.14)] and by the full one-loop RG approach of Bardeen, Hill, and Lindner (1990).

$\Lambda \approx \mu_*$ [GeV]	10^{19}	10^{17}	10^{15}	10^{13}	10^{11}	10^{9}	10^{7}	10^{5}	10^{4}
m_t [GeV] (quark loop)	143	153	165	179.5	200	228	276	378	519
m_H [GeV] (quark loop)	289.5	309	333	364	406	468	571.5	814.5	1235
$m_t(m_t)$ [GeV] RGE	218	223	229	237	248	264	293	360	455
$m_H(m_H)$ [GeV] RGE	239	246	256	268	285	310	354	455	605

creases $m_t^{\rm ren}$ further by a few GeV, thus slightly exacerbating the problem (Lavoura, 1992; ter Veldhuis, 1992).

As an interesting point, we mention that solution (2.10) for $g_t(\mu)$, obtained in the quark-loop approximation, can also be obtained directly from RG equation (2.16) by keeping on the right of the equation only the term $N_{\rm c} g_t^3(\mu)$ and applying the boundary condition $g_t(\Lambda)=\infty$. Thus, in retrospect, we see that it is the leading-$N_{\rm c}$ Yukawa term $N_{\rm c} g_t^3(\mu)$ on the right of Eq. (2.16), which represents the quark-loop effects on the evolution of $g_t(\mu)$.

D. Marciano's approach

Independently of Bardeen, Hill, and Lindner (1990), Marciano (1989, 1990) introduced his own version of the RG approach to $\bar{t}t$ condensation. He investigated the behavior of $k_t(\mu) \equiv g_t^2(\mu)/(4\pi)$. He included in his calculation the one-loop QCD contribution and neglected the (small) contributions of the electroweak gauge bosons. The one-loop RG equation of the minimal standard model is then

$$\mu \frac{dk_t(\mu)}{d\mu} = \frac{1}{2\pi}\left(N_{\rm c}+\frac{3}{2}\right)k_t^2(\mu) - \frac{4}{\pi}\alpha_3(\mu)k_t(\mu), \tag{2.22}$$

where $N_{\rm c}=3$ is used throughout. Since in the massless version of this theory only the gauge coupling α_3 exists as an independent parameter, Marciano made the assumption that k_t is a function of α_3 only: $k_t = k_t[\alpha_3(\mu)]$. Therefore

$$\mu \frac{dk_t(\mu)}{d\mu} = \left[\mu \frac{d}{d\mu}\alpha_3\right]\left[\frac{dk_t}{d\alpha_3}\right]. \tag{2.23}$$

This is a version of the assumption that the compositeness (condensation) at a scale Λ causes a reduction of coupling parameters in the low-energy theory. A free constant of integration C appearing in the thus obtained formula for the family of solutions for $k_t(\mu)$ [cf. Eq. (2.24) below] is then assumed to be determined solely by the scale Λ of the new physics: $C=C(\Lambda)$. It is in this way that, in addition to the minimal-standard-model evolution of k_t (Eq. 2.22), certain asymptotic conditions at $\mu=\Lambda \gg E_{\rm ew}$ were imposed in order to take into account indirectly the effects of a new underlying physics responsible for the $\bar{t}t$ condensation. The leading-$N_{\rm c}$ case, when $N_{\rm c}+1.5 \mapsto N_{\rm c} \equiv 3$ in Eq. (2.22), was interpreted as a case of a less tightly bound $\bar{t}t$ scalar which does not contribute any "feedback" effects to its own binding. The full one-loop case, as given by Eq. (2.22), was interpreted as the case of a tightly bound (pointlike) $\bar{t}t$ condensate. Most of the investigation was focused on the latter case. In addition to the minimal-standard-model RG equation for $k_t \equiv g_t^2/(4\pi)$ and assumption (2.23) of the reduction of coupling parameters, Marciano used the one-loop RG equation for α_3 and consequently arrived at the general family of solutions[10]

$$k_t(\mu) = \frac{2}{9}\frac{\alpha_3^{8/7}(\mu)}{\alpha_3^{1/7}(\mu)-C},$$

$$\text{with } \alpha_3^{-1}(\mu) = \alpha_3^{-1}(m_t) + \frac{7}{2\pi}\ln(\mu/m_t). \tag{2.24}$$

Here, m_t is the renormalized mass $m_t(m_t)$, $\mu \geq m_t$, and $C=C(\Lambda)$ is the previously mentioned arbitrary constant of integration to be determined by assuming certain behavior on $k_t(\mu)$ in the asymptotic region of the onset of new physics (i.e., for large $\mu \sim \Lambda$). Specification of C then also determines $m_t = v\sqrt{2\pi k_t(m_t)}$, where $v \approx 246\,$GeV is the VEV. In the first work (Marciano, 1989), the solution $C=0$ ($\Rightarrow k_t = 2\alpha_3/9$) was proposed, leading to $m_t \approx 98\,$GeV. This solution corresponds formally to the solution of the Bardeen, Hill, and Lindner approach with $\Lambda=\infty$ and the effects of electroweak gauge bosons neglected. In the second work (Marciano, 1990), two other choices for C were advocated. The first choice was $C=\alpha_3^{1/7}(\Lambda)$, where Λ was the energy of the onset of new physics responsible for the condensation, thus leading to

$$k_t(\mu)\left(\equiv \frac{g_t^2(\mu)}{4\pi}\right) = \frac{2}{9}\frac{\alpha_3^{8/7}(\mu)}{[\alpha_3^{1/7}(\mu)-\alpha_3^{1/7}(\Lambda)]}. \tag{2.25}$$

In this solution the boundary condition is $k_t(\mu)\to\infty$ when $\mu\to\Lambda$, just like the boundary condition (2.15) of the Bardeen-Hill-Lindner approach, where it was motivated by the results of the diagrammatic approach in the quark-loop approximation [Eqs. (2.10)–(2.12)] of the truncated top-mode standard model (2.1). However, Marciano offered an alternative motivation for the asymptotic behavior of k_t, by employing a variant of the

[10]In leading-log approximation, all procedures *must* give Eq. (2.24). The constant of integration C depends on the underlying dynamics. Any prescription that does not give the form (2.24) is lacking in that it misses leading logs by some approximation.

Pagels-Stokar relation (Pagels and Stoker, 1979; cf. Eqs. (3.11) and (3.12)] with a variable ("running") lower integration bound

$$M_W^2(\mu)\left[\equiv\frac{\alpha_2(\mu)m_t^2(\mu)}{2k_t(\mu)}\equiv\pi\alpha_2(\mu)v^2(\mu)\right]$$

$$\simeq\frac{3}{8\pi}\int_{\mu^2}^{\Lambda^2}d\overline{p}^2\overline{p}^2\alpha_2(\overline{p})\frac{\Sigma_t^2(\overline{p}^2)}{[\overline{p}^2+\Sigma_t^2(\overline{p}^2)]^2}, \qquad (2.26)$$

where $\Sigma_t(\overline{p}^2)$ is the top-quark self-energy (the dynamical "running" mass) and the UV cutoff Λ indicates the new underlying physics. From this relation it can be seen that $M_W(\mu)\to 0$ when $\mu\to\Lambda$, and therefore this indicates $k_t(\mu)\to\infty$ when $\mu\to\Lambda$, i.e., the solution (2.25). However, Marciano also argued that this asymptotic boundary condition was not realistic and could sometimes, particularly for lower cutoffs $\Lambda<10^8$ GeV, even be misleading, because it extended the consideration of the one-loop RG equation (2.22) [or (2.16) of the full approach by Bardeen, Hill, and Lindner] into a highly nonperturbative region where perturbative unitarity bounds were also violated. Therefore he used for the boundary condition perturbative unitarity bounds (Marciano, Valencia, and Willenbrock, 1989) as an indicator of the onset of new underlying physics. Specifically, he used unitarity bounds originating from consideration of the $\overline{t}t\to\overline{t}t$ scattering: $k_t(\mu)\leqslant 2/3\Rightarrow k_t(\Lambda)=2/3$. This led him to a value of $C(\Lambda)$ (second choice) somewhat smaller than the value $C(\Lambda)=\alpha_3^{1/7}(\Lambda)$ [first choice, Eq. (2.25)]. The differences from the Bardeen, Hill, and Lindner approach arising from this boundary condition were substantial for lower cutoffs $\Lambda<10^8$ GeV, due to the disappearance of the infrared fixed-point behavior of the RG equations for these cutoffs. Marciano subsequently also took into account the gluon-cloud corrections

$$m_t^{\text{phys}}\approx m_t(\mu=m_t)\left[1+\frac{4}{3\pi}\alpha_3(\mu=m_t)\right]$$

$$\approx 1.047\times m_t(m_t) \qquad (2.27)$$

and obtained for $\Lambda\simeq 10^{15}$–10^{19} GeV the predictions $m_t^{\text{phys}}\simeq 214$–$202$ GeV, about 16 or 17 GeV below the Bardeen, Hill, and Lindner results of Table I. However, he pointed out that only a difference of about 4 GeV stems from using the boundary condition $k_t(\Lambda)=2/3$, and the rest from the different input parameters [he used $\alpha_3(M_W)=0.107$, while Bardeen, Hill, and Lindner used $\alpha_3(M_Z)=0.115$] and from his neglecting the contributions of the electroweak gauge bosons.

It should be stressed that the RG approach is applicable to any high-energy realization of strong attraction that may result in quark-antiquark condensation, not just to the NJLVL four-quark frameworks. This was apparently first pointed out by Marciano (1989, 1990). For example, the compositeness condition (2.15), although motivated in the work of Bardeen, Hill, and Lindner by the quark-loop approximation within an NJLVL type of picture [top-mode standard model, Eq. (2.1)], can be regarded as independent of such a specific model.

E. Other related work

Another approach not referring to any particular realization of the strong attraction is that of Paschos and Zakharov (1991). They used, at a leading-N_c level, only the fine-tuning condition $m_t\ll\Lambda$ and symmetries of the low-energy theory to show that $M_H\approx 2m_t$.

Fishbane, Norton, and Truong (1992) and subsequently Fishbane and Norton (1993), employing perturbative RG methods, investigated the infrared fixed-point structure of an NJLVL model with an internal $SU(N)$ "color" symmetry, with fermions f (one flavor) and without gauge bosons. Their model involved a composite scalar $\sigma\sim\overline{f}f$. They concluded the model supported the hypothesis that, for fine-tuning cases of dynamical symmetry breaking ($\mu\sim E_{\text{ew}}\ll\Lambda$), NJLVL-type models give definite Λ-independent predictions for M_σ/m_f, i.e., that the Bardeen, Hill, and Lindner approach is predictive in such cases.

Cooper and Perez-Mercader (1991) investigated the connection between results obtained when compositeness conditions are imposed (in analogy with Bardeen, Hill, and Lindner) and when an assumption of coupling parameter reduction is made in a theory (partially in analogy with Marciano). They investigated specifically a linear four-dimensional σ model with N "colors" which contained a Yukawa parameter g_f and the four-scalar self-coupling parameter λ. In the first approach, compositeness conditions $Z_3=Z_4=0$ at $\mu=\Lambda$ for these two parameters were imposed. In the second approach, the assumption $\lambda=f[g_f^2(\mu)]$ was made and RG equations were applied (cf. also Marciano's approach, Sec. II.D). The analysis was done in the fermion-loop (leading-N) approximation, and the authors demonstrated that both approaches consistently give the relationship between the two parameters: $\lambda(E_{\text{ew}})/g_f^2(E_{\text{ew}})=\alpha$ for $\Lambda\gg E_{\text{ew}}$, where α is a Λ-independent constant. The first approach gave a more specific result: $\alpha\equiv 2$.

Recently, Aoki *et al.* (1997; Aoki, 1997) proposed a systematic approximation scheme for the Wilsonian nonperturbative renormalization-group (nonperturbative RG) approach to dynamical symmetry breaking. The approximation is based on the local-potential approximation (LPA), in which the effective action $S(\phi;\Lambda)_{\text{eff}}$ of a physical system with a field ϕ (generic notation) and a finite ultraviolet cutoff Λ is written in terms of a local potential $V(\phi)$: $S_{\text{eff}}=\int d^4x[V(\phi)+\partial_\mu\phi\,\partial^\mu\phi/2]$. The authors expand $V(\phi)$ in powers of a (conveniently defined) polynomial of ϕ. The method appears to be promising because it describes the RG flow of the system nonperturbatively over the entire energy range, and the flow can be systematically refined within the LPA frame. In contrast to the previously described method of perturbative RG equations with compositeness boundary condition, the approach does not rely on the infrared fixed-point behavior of RG equations (its applicability is hence not restricted to large $\Lambda \gtrsim 10^8$ GeV), and it deals with four-fermion interaction terms directly. Therefore the method has some similarity with the Dyson-Schwinger and Pagels-Stokar equations (see Sec. III). Aoki *et al.* emphasize that, unlike the Dyson-Schwinger+Pagels-Stokar approach, their nonperturbative RG approach is gauge independent and does not lead to divergent series. The method has not yet been applied to frameworks involving $\bar{t}t$ condensation.

F. Conclusions

The minimal $\bar{t}t$ condensation framework is usually (but not necessarily) discussed in terms of an effective model with NJLVL four-quark interactions [Eq. (2.1)] involving t_R, t_L, and b_L, called the (truncated) top-mode standard model. The minimal framework is based on the assumption that top-quark condensation alone is responsible simultaneously for m_t and for the *full* (dynamical) EWSB, i.e., $\bar{t}t \sim \mathcal{H}$, where $\mathcal{H}$ is the only Higgs particle, and $\langle \mathcal{H} \rangle_0$ at low energies equals the full electroweak VEV, $v \approx 246$ GeV, needed to reproduce M_W and M_Z. The resulting effective theory at low energies is essentially the minimal standard model.

The method of perturbative RG equations in this minimal framework considers renormalization effects $(\delta g_t)_{\rm ren}$ and $(\delta\lambda)_{\rm ren}$ by applying compositeness boundary conditions (2.15) at a scale $\mu = \mu_* (\sim \Lambda)$ to RG equations of the minimal standard model for the Yukawa and scalar self-coupling parameters (g_t, λ). These compositeness conditions reflect a strong attraction in the top-quark sector and were motivated by the behavior of $g_t(\mu)$ and $\lambda(\mu)$ at high energies $\mu \sim \Lambda$ in the quark-loop approximation of the truncated top-mode standard model [cf. Eqs. (2.10), (2.12)]. However, the results are basically independent of any particular realization of the strong-attraction sector of the top quark. An implicit assumption is that the model starts behaving as (a special version of) the minimal standard model as we come down from Λ to energies μ_* which are still close to Λ: $\ln(\Lambda/\mu_*) \approx 1$. Stated differently, binding of the quark constituents in the composite scalars is very tight, in contrast to low-energy QCD. For very large compositeness scales ($\Lambda \gtrsim 10^8$ GeV), predictions of this approach turn out to be rather insensitive to the details of the actual condensation mechanism. It is this infrared fixed-point behavior of the RG equations that makes the application of the method an important contribution to the $\bar{t}t$ condensation program. Compositeness conditions (2.15) should be regarded only as a crude qualitative estimate of the dynamics in the strong-condensation sector. Therefore the method can be trusted only for high Λ's—in the minimal framework (2.1): $\Lambda \gtrsim 10^8$ GeV.

Nonperturbative renormalization-group approaches appear to be very promising for lower Λ's. However, they have not yet been applied to $\bar{t}t$ condensation.

III. THE METHOD OF DYSON-SCHWINGER AND PAGELS-STOKAR EQUATIONS

A. General top-mode standard model

We now turn to a different computational approach, which was first applied to top-quark condensation by Miransky, Tanabashi, and Yamawaki (1989a, 1989b)[11] and subsequently by King and Mannan (1990, 1991a).

Miransky, Tanabashi, and Yamawaki began by introducing a rather general top-mode standard model containing $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant four-quark NJLVL interactions,

$$\begin{aligned}\mathcal{L}_{4q}^{(\Lambda)} = \frac{4\pi^2}{N_c \Lambda^2} [& g^{(1)}_{\alpha\alpha';\beta\beta'} (\bar{\Psi}_L^{\alpha i} \Psi_R^{\alpha' j})(\bar{\Psi}_R^{\beta j} \Psi_L^{\beta' i}) \\ & + g^{(2)}_{\alpha\alpha';\beta\beta'} (\bar{\Psi}_L^{\alpha i} \Psi_R^{\alpha' j})(i\tau_2)^{ik}(i\tau_2)^{jl}(\bar{\Psi}_L^{\beta k} \Psi_R^{\beta' l}) \\ & + g^{(3)}_{\alpha\alpha';\beta\beta'} (\bar{\Psi}_L^{\alpha i} \Psi_R^{\alpha' j})(\tau_3)^{jk}(\bar{\Psi}_R^{\beta k} \Psi_L^{\beta' i})] + \text{H.c.}\end{aligned} \tag{3.1}$$

Coefficients $g^{(k)}_{\alpha\alpha';\beta\beta'}$ $(k=1,2,3)$ are "bare" dimensionless $\mathcal{O}(1)$ four-quark parameters in the above theory with Λ cutoff; $(\alpha, \alpha', \beta, \beta')$ and (i,j,k,l) are family and isospin indices, respectively. For example, $\Psi_R^{31} = t_R$, $\Psi_R^{32} = b_R$, etc. The color indices, omitted from Eq. (3.1), are distributed the same way as in the truncated top-mode standard model, Eq. (2.1), thus leading to the N_c enhancement of quark-loop-induced parameters, as in Eqs. (2.7) and (2.8). We shall return to this point later and show why $g^{(k)}_{\alpha\alpha';\beta\beta'} = \mathcal{O}(N_c^0)$.

The four-quark terms involving the third-generation quarks induce heavy quark masses (m_t, and possibly m_b) as well as DEWSB—thus they are of particular interest:

$$\begin{aligned}\mathcal{L}_{4q}^{(\Lambda)3\text{rd}} = \frac{8\pi^2}{N_c \Lambda^2} \{ & \kappa_t (\bar{\Psi}_L^{ia} t_R^a)(\bar{t}_R^b \Psi_L^{ib}) + \kappa_b (\bar{\Psi}_L^{ia} b_R^a)(\bar{b}_R^b \Psi_L^{ib}) \\ & + 2\kappa_{tb} [(\bar{t}_L^a t_R^a)(\bar{b}_L^b b_R^b) + (\bar{t}_R^a t_L^a)(\bar{b}_R^b b_L^b) \\ & - (\bar{t}_L^a b_R^a)(\bar{b}_L^b t_R^b) - (\bar{t}_R^a b_L^a)(\bar{b}_R^b t_L^b)]\},\end{aligned} \tag{3.2}$$

where $\kappa_t = (g^{(1)} + g^{(3)})_{33;33}$, $\kappa_b = (g^{(1)} - g^{(3)})_{33;33}$, and $\kappa_{tb} = g^{(2)}_{33;33}$. Color indices a,b are explicitly written in Eq. (3.2). The first term here is the truncated top-mode standard model, Eq. (2.1). We shall call the framework of Eq. (3.2) the general top-mode standard model, although not just $\bar{t}t$ but also $\bar{b}b$ condensation may occur in this framework (if $\kappa_b > \kappa_{\rm crit}$). Without loss of generality, we can regard $g^{(j)}_{33;33}$ $(j=1,3)$ as real. In addition, $\kappa_{tb} = g^{(2)}_{33;33}$ is also assumed to be real. All these parameters are "bare" parameters of a theory with a finite UV cutoff Λ: $\kappa_t \equiv \kappa_t(\mu = \Lambda)$, etc.

Later, in Sec. VI.A.2, it will be argued that the NJLVL Lagrangian density (3.2) in general leads at low energies to an effective standard model with two com-

[11]Independently of Nambu's proposal (Nambu, 1989).

posite Higgs doublets, as long as the four-quark parameters are near or above the critical values.

B. Dyson-Schwinger integral equations

In contrast to Bardeen, Hill, and Lindner, the authors Miransky, Tanabashi, and Yamawaki and King and Mannan investigated directly the actual condensation mechanism leading to $\bar{t}t$ condensation. The mathematical formalism employed in their approach is the Dyson-Schwinger integral equation for the dynamical mass functions $\Sigma_t(p^2)$ and $\Sigma_b(p^2)$ of the top and bottom quarks, and the Pagels-Stokar relation for the decay constants F_π of the electroweak Nambu-Goldstone bosons (Pagels and Stokar, 1979). In this approach, the propagator $S_i(p)$ of quark i ($i=t$ or b) in momentum space is

$$S_i^{-1}(p)=(-i)Z_i^{-1}(p^2)[\not{p}-\Sigma_i(p^2)], \tag{3.3}$$

where $Z_i(p^2)$ is the quark field renormalization constant. For the Landau gauge in the ladder approximation, $Z_i(p^2)=1$. In the improved ladder approximation[12] discussed below, it can be shown that $Z_i(p^2)\to 1$ for $|p^2|\gg E_{\rm ew}^2$; thus in actual calculations $Z_i(p^2)\equiv 1$ is used.

The Dyson-Schwinger equation was applied at the leading-N_c level, including QCD in the Landau gauge with one-loop running $\alpha_3(p)$, i.e., in an improved ladder approximation. The Dyson-Schwinger equation is essentially a variational version of the usual gap equation for the dynamically induced mass of heavy quarks. The general formalism leading to this equation in the context of nonperturbative dynamical symmetry breaking involves the Cornwall-Jackiw-Tomboulis effective action (effective potential) $V_{\rm CJT}$ or effective potential Σ (Cornwall, Jackiw, and Tomboulis, 1974). It is a functional of the dynamical "running" mass $\Sigma(p^2)$. The Dyson-Schwinger equation is obtained by requiring that the functional derivative of $V_{\rm CJT}$ be zero: $\delta V_{\rm CJT}[\Sigma]/\delta\Sigma(p^2)=0$. The analogous relation for the case of a nonrunning dynamical mass $\Sigma(p^2)=m$ is usually called a gap equation. A detailed derivation of the Dyson-Schwinger ladder equation, based on the Cornwall-Jackiw-Tomboulis formalism, within the context of the top-mode standard model was given by Kondo, Tanabashi, and Yamawaki (1993), and this derivation can also be applied to the case of the improved ladder approximation. A graphic representation of the Dyson-Schwinger equation is given in Fig. 2. After angular integration in the Wick-rotated Euclidean space and introducing a cutoff Λ for the (Euclidean) quark and gluonic momenta, the improved Dyson-Schwinger ladder equations in the t-b decoupled case ($\kappa_{tb}=0$) take the form

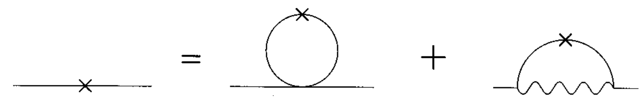

FIG. 2. Graphic illustration of the gap (Dyson-Schwinger) equation for a dynamically generated heavy-quark mass. The cross represents insertion of the dynamical "running" quark self-energy $\Sigma_i(\bar{p}^2)$ ($i=t$ or b). The first term on the right represents the four-quark interaction, and the second the gluon exchange (wavy line).

$$\Sigma_i(\bar{p}^2)=\frac{\kappa_i}{\Lambda^2}\int_0^{\Lambda^2}\frac{d\bar{q}^2\bar{q}^2\Sigma_i(\bar{q}^2)}{\bar{q}^2+\Sigma_i^2(\bar{q}^2)} +\int_0^{\Lambda^2}\frac{d\bar{q}^2\bar{q}^2\Sigma_i(\bar{q}^2)}{\bar{q}^2+\Sigma_i^2(\bar{q}^2)}K(\bar{p}^2;\bar{q}^2) \quad (i=t \text{ or } b). \tag{3.4}$$

Bars over momenta $\bar{p}$ and $\bar{q}$ mean that they are in the Euclidean metric. The function $K(\bar{p}^2;\bar{q}^2)$ originating from the gluon propagator in the Landau gauge is

$$K(\bar{p}^2;\bar{q}^2)=\frac{\lambda_{\rm QCD}[\max(\bar{p}^2;\bar{q}^2)]}{\max(\bar{p}^2;\bar{q}^2)},$$

$$\lambda_{\rm QCD}(\bar{p}^2)=\frac{3(N_c^2-1)}{8\pi N_c}\alpha_3(\bar{p}). \tag{3.5}$$

The QCD parameter $\alpha_3(\bar{p})\equiv g_3^2(\bar{p})/(4\pi)$ follows from the one-loop RG equations (2.18) and (2.19) ($j=3$),

$$\alpha_3(\bar{p})=\alpha_3(\mu_{\rm IR}) \quad (\bar{p}^2\leqslant\mu_{\rm IR}^2),$$

$$\alpha_3(\bar{p})=\frac{12\pi}{(11N_c-2n_q)}\frac{1}{\ln[\bar{p}^2/\Lambda_{\rm QCD}^2(n_q)]} \quad (\bar{p}^2\geqslant\mu_{\rm IR}^2). \tag{3.6}$$

Here, n_q is the number of effective flavors of quarks (for $|\bar{p}|>m_t$, $n_q=6$). For $N_c=3$ and $\alpha_3(180\,\text{GeV})=0.11$, we have $\Lambda_{\rm QCD}(n_q=6)\approx 51\,\text{MeV}$. An infrared cutoff $\mu_{\rm IR}\sim\Lambda_{\rm QCD}$ was introduced to avoid spurious divergence in the infrared. Details of the IR cutoff do not influence appreciably the results of the analysis. A nonzero solution of Eq. (3.4) exists if $\kappa_i>\kappa_{\rm crit}$.

In the quark-loop approximation ($\lambda_{\rm QCD}=0$), the Dyson-Schwinger equation (3.4) has one nonzero solution, $\Sigma_i=m_i^{(0)}$ if $\kappa_i\gtrsim 1=\kappa_{\rm crit}^{(0)}$, and this solution is not evolving with $\bar{p}^2$:

$$\Sigma_i(\bar{p}^2)=m_i^{(0)},$$

$$\kappa_i=[1-(m_i^{(0)2}/\Lambda^2)\ln(\Lambda^2/m_i^{(0)2}+1)]^{-1}$$

$$\text{for } \kappa_i>1 \quad (i=t,b). \tag{3.7}$$

Equations (3.7) are usually referred to as gap equations in the quark-loop (bubble) approximation. Here we see that $\kappa_i\sim 1=\mathcal{O}(N_c^0)$. Therefore the motivation for normalizing κ_t, κ_b, and κ_{tb} as we did in Eq. (3.2) can now be understood. From Eqs. (3.5) and (3.6) we see that

[12]The ladder approximation in this context means that only the leading-N_c contributions are calculated, i.e., quark-loop and one-gluon-exchange QCD contributions, and that QCD loop contributions are calculated by using a nonrunning QCD coupling parameter, e.g., $\alpha_3(\bar{p})\equiv\alpha_3(m_t)\approx 0.11$. The improved ladder approximation uses a running $\alpha_3(\bar{p})$ instead.

$K(\overline{p}^2,\overline{q}^2)\propto\lambda_{\rm QCD}=\mathcal{O}(N_c^0)$. Since $\kappa_i=\mathcal{O}(N_c^0)$, as we have just argued, we can see in retrospect that the (improved) ladder QCD contribution on the right side of Eq. (3.4) is formally of leading-N_c order, just as the numerically dominant quark-loop contribution is. The effect of this QCD contribution is that the dynamical mass solution $\Sigma_i(\overline{p}^2)$ becomes "running"—it is weakly dependent on the Euclidean momentum $\overline{p}$, with the asymptotic solution

$$\Sigma_i(\overline{p}^2)\approx m_i\left[\frac{\alpha_3(\overline{p})}{\alpha_3(m_i)}\right]^{c_m} = m_i\left[\frac{\ln(m_i^2/\Lambda_{\rm QCD}^2)}{\ln(\overline{p}^2/\Lambda_{\rm QCD}^2)}\right]^{c_m}, \qquad \text{for } m_i^2\ll\overline{p}^2\ll\Lambda^2, \tag{3.8}$$

where m_i is the low-energy ("renormalized") mass of the quark i $(i=t,b)$ and

$$c_m=\frac{9(N_c^2-1)}{2N_c(11N_c-2n_q)} \quad \left(=\frac{4}{7} \quad \text{for } N_c=3, n_q=6\right). \tag{3.9}$$

We shall comment more extensively on the effect of the QCD term in Eq. (3.4) on $\Sigma_i(\overline{p}^2)$ in Secs. V.B–V.C, where this approach is compared with the RG approach.

C. Generalized Pagels-Stokar relations

An essential aspect of the analysis by Miransky, Tanabashi, and Yamawaki (1989a, 1989b) and King and Mannan (1990, 1991a), was the use of generalized Pagels-Stokar relations for the decay constants F_{π^0} and $F_{\pi^\pm}$ of the composite Nambu-Goldstone bosons. If the heavy-quark condensation in the general top-mode standard model (3.2) is assumed to be responsible for all or most of the (dynamical) EWSB, then $F_{\pi^0}\approx F_{\pi^\pm}\approx v=246\,{\rm GeV}$ appears in mass formulas for electroweak gauge bosons,[13]

$$M_W^2=\frac{g_2^2}{4}F_{\pi^\pm}^2, \quad M_Z^2\cos^2\theta_W=\frac{g_2^2}{4}F_{\pi^0}^2. \tag{3.10}$$

The Pagels-Stokar relations relate these decay constants to mass functions of the heavy quarks and thus provide a correspondence between gauge-boson and heavy-quark masses. These relations are an approximation to a sum rule that originates from the amplitudes of the Bethe-Salpeter equation for the bound states of the Nambu-Goldstone bosons in the leading-N_c (improved ladder) approximation. In fact, Pagels-Stokar relations are obtained from Bethe-Salpeter amplitudes in the weak-coupling limit $\lambda_{\rm QCD}\to 0$ when Dyson-Schwinger equations (3.4) and Σ_i derived from them are kept unchanged. It turns out that this weak-coupling approximation changes F_π^2's by less than 3%, as discussed by Aoki *et al.* (1990; see also Aoki, 1991). Generalized Pagels-Stokar relations (generalized from the QCD-like to the general top-mode standard-model case), connecting $F_{\pi^\pm}$ and F_{π^0} with dynamical masses Σ_i in the Euclidean metric, were first given by Miransky, Tanabashi, and Yamawaki (1989a, 1989b):

$$\begin{aligned} F_{\pi^\pm}^2=&\frac{N_c}{8\pi^2}\int_0^{\Lambda^2}d\overline{p}^2\overline{p}^2[\overline{p}^2+\Sigma_t^2(\overline{p}^2)]^{-1}[\overline{p}^2+\Sigma_b^2(\overline{p}^2)]^{-1} \\ &\times\Bigg\{\left(1-\frac{\overline{p}^2}{4}\frac{d}{d\overline{p}^2}\right)[\Sigma_t^2(\overline{p}^2)+\Sigma_b^2(\overline{p}^2)] \\ &+\frac{\overline{p}^2}{2}[\Sigma_t^2(\overline{p}^2)-\Sigma_b^2(\overline{p}^2)] \\ &\times\Bigg[[\overline{p}^2+\Sigma_t^2(\overline{p}^2)]^{-1}\left(1+\frac{d}{d\overline{p}^2}\Sigma_t^2(\overline{p}^2)\right) \\ &-[\overline{p}^2+\Sigma_b^2(\overline{p}^2)]^{-1}\left(1+\frac{d}{d\overline{p}^2}\Sigma_b^2(\overline{p}^2)\right)\Bigg]\Bigg\}, \end{aligned} \tag{3.11}$$

$$\begin{aligned} F_{\pi^0}^2=&\frac{N_c}{8\pi^2}\int_0^{\Lambda^2}d\overline{p}^2\overline{p}^2\Bigg\{[\overline{p}^2+\Sigma_t^2(\overline{p}^2)]^{-2} \\ &\times\left[\Sigma_t^2(\overline{p}^2)-\frac{\overline{p}^2}{4}\frac{d}{d\overline{p}^2}\Sigma_t^2(\overline{p}^2)\right] \\ &+[\overline{p}^2+\Sigma_b^2(\overline{p}^2)]^{-2}\left[\Sigma_b^2(\overline{p}^2)-\frac{\overline{p}^2}{4}\frac{d}{d\overline{p}^2}\Sigma_b^2(\overline{p}^2)\right]\Bigg\}. \end{aligned} \tag{3.12}$$

Since the top-mode standard model contains no explicit custodial symmetry $SU(2)_V$ of the minimal-standard-model Higgs sector, one might worry lest the electroweak $\rho\equiv M_W^2/(M_Z^2\cos^2\theta_W)$ parameter become phenomenologically unacceptable in the case of maximal isospin violation in the heavy-quark sector, i.e., in the case of $\kappa_t>\kappa_{\rm crit}>\kappa_b$ when $\Sigma_t(\overline{p}^2)\neq 0$ and $\Sigma_b(\overline{p}^2)=0$. However, it can be seen that the Pagels-Stokar relations (3.11) and (3.12) reproduce approximately the same suppressed $\delta\rho\equiv(\rho-1)$ as the minimal standard model with protective custodial symmetry. For example, if we assume that the dynamical mass $\Sigma_t=m_t$ is not running [i.e., the quark-loop approximation, $\lambda_{\rm QCD}=0$, of the Dyson-Schwinger equation (3.4)] and $\Sigma_b=0$, we obtain from the above Pagels-Stokar relations

$$F_{\pi^\pm}^2({\rm q.l.})=\frac{N_c}{8\pi^2}m_t^2\left[\ln\frac{\Lambda^2}{m_t^2}+\frac{1}{2}\right],$$

$$F_{\pi^0}^2({\rm q.l.})=\frac{N_c}{8\pi^2}m_t^2\ln\frac{\Lambda^2}{m_t^2}. \tag{3.13}$$

Here, "(q.l.)" denotes the quark-loop approximation. These relations lead to

$$\delta\rho^{({\rm q.l.})}=\left(\frac{M_W^2}{M_Z^2\cos^2\theta_W}\right)^{({\rm q.l.})}-1=\frac{F_{\pi^\pm}^2({\rm q.l.})}{F_{\pi^0}^2({\rm q.l.})}-1 =\frac{N_c m_t^2}{16\pi^2F_{\pi^0}^2({\rm q.l.})}=\frac{1}{2\ln(\Lambda^2/m_t^2)}. \tag{3.14}$$

[13]For models in which the top (plus bottom) quark condensation is responsible for only a minor part of the DEWSB ($F_{\pi^0},F_{\pi^\pm}\ll v$), see Sec. VIII.F.

TABLE II. Predicted $\Sigma_t(\bar{p}^2=0)\approx m_t^{\rm phys}$ in the improved ladder approximation of the Dyson-Schwinger+Pagels-Stokar approach. The values are taken from King and Mannan (1990), who used the following QCD parameters: $\mu_{\rm IR}=0.30$ GeV, $\alpha_3(\mu_{\rm IR})\approx 3.93$, $\alpha_3(M_Z)=0.127$ [$\Rightarrow\Lambda_{\rm QCD}(n_q=6)\approx 70\,{\rm MeV}$]. For comparison, predictions in the quark-loop approximation and of the RG approach of Bardeen, Hill, and Lindner are also given.

Λ [GeV]	10^{19}	10^{17}	10^{15}	10^{13}	10^{11}	10^{9}	10^{7}	10^{5}	10^{4}
m_t [GeV] (quark loop)	143	153	165	179.5	200	228	276	378	519
$m_t=\Sigma_t(0)$ [GeV]	253	259	268	279	293	316	354	446	591
$m_t(m_t)$ [GeV] (RGE)	218	223	229	237	248	264	293	360	455

Therefore $\delta\rho^{(\rm q.l.)}\ll 1$ for large Λ. We see that the familiar expression (Veltman, 1977) of the minimal standard model for $\delta\rho$ is reproduced: $\delta\rho\approx G_F\sqrt{2}N_{\rm c}m_t^2/(4\pi)^2$ (note, $G_F\sqrt{2}=v^{-2}\approx F_\pi^{-2}$). Even the maximal isospin violation in the top-mode standard-model framework does not raise a problem of phenomenologically unacceptable $\delta\rho$. As a matter of fact, custodial $SU(2)_V$ symmetry is implicitly contained in this condensation mechanism, as suggested by Eq. (3.14).

D. Results of the approach

In the quark-loop approximation (no QCD), the two approaches of Miransky, Tanabashi, and Yamawaki and Bardeen, Hill, and Lindner give the same results, written in Table I in the line "m_t[GeV] (quark loop)" (see also discussion in Sec. V.C). However, the Dyson-Schwinger equations (3.4) also contain contributions of QCD loops, and the latter change the quark-loop predictions of Table I substantially, as can be seen explicitly from Table II. Solutions in Table II, in the line "$m_t=\Sigma_t(0)$," are for the simplest case of $\kappa_b<\kappa_{\rm crit}$ and $\kappa_{tb}=0$ ($\Sigma_b=0$, i.e., the truncated top-mode standard model). To obtain these results, the authors proceeded in the following way:

(1) At each chosen cutoff Λ, solutions $\Sigma_t(\bar{p}^2)$ of the Dyson-Schwinger equation (3.4) were found by iterative calculation, for various choices of four-quark coupling parameters κ_t.
(2) These solutions were then used in the Pagels-Stokar relations (3.11) and (3.12), with $\Sigma_b=0$.
(3) Demanding that decay constants F_{π^0} and $F_{\pi^\pm}$ be approximately equal to the full electroweak VEV, $v\approx 246\,{\rm GeV}$, led to fine tuning of κ_t to values near or slightly above the critical value $\kappa_{\rm crit}\approx 1-\lambda_{\rm QCD}(\Lambda)$, thus resulting in the predictions $m_t\approx\Sigma_t(0)$ given in Table II.

It turned out that the critical value satisfies $1-\kappa_{\rm crit}\sim\lambda_{\rm QCD}(\Lambda^2)\sim 1/\ln(\Lambda^2/\Lambda_{\rm QCD}^2)$. In this context, Mahanta (1990) pointed out, within the Dyson-Schwinger ladder approach ($i=t$), that the fine-tuning of the four-quark parameter κ_t to $\kappa_{\rm crit}$ for the hierarchy $m_t\ll\Lambda$ gets softened substantially by the presence of (QCD) gauge interaction. He showed, in an additional approximation of nonrunning α_3, that the fine tuning changes from $\sim m_t^2/\Lambda^2$ [no gauge interaction; cf. Eq. (3.8)] to $\sim(m_t^2/\Lambda^2)^b$, where $b=\sqrt{1-\alpha_3/\alpha_3^{\rm crit}}<1$. Here, $\alpha_3^{\rm crit}$ is the critical value of α_3 for the pure QCD-induced dynamical symmetry breaking: $\alpha_3^{\rm crit}=2\pi N_{\rm c}/[3(N_{\rm c}^2-1)]$.

King and Mannan (1991a) also investigated numerically the cases $\kappa_b>\kappa_{\rm crit}$ and $\kappa_{tb}\neq 0$. One aspect of the case $\kappa_{tb}\neq 0$ (and $\kappa_b<\kappa_{\rm crit}$) was pointed out by Miransky, Tanabashi, and Yamawaki (1989a, 1989b): the κ_{tb} term leads to a nonzero m_b via the "feed-down" effect coming from[14] the $\langle\bar{t}t\rangle_0$ expectation value: $m_b\approx -(8\pi^2)\kappa_{tb}\langle\bar{t}t\rangle_0/(N_{\rm c}\Lambda^2)\approx 2\kappa_{tb}m_t$. The κ_{tb} term, unlike the other two, breaks $U(1)_{\gamma_5}$ invariance [$q\mapsto\exp(i\alpha\gamma_5)q$]. Therefore, if $\kappa_{tb}=0$, the Lagrangian is invariant under $U(1)_{\gamma_5}$. This invariance plays the role of the Peccei-Quinn symmetry (Peccei and Quinn, 1977a, 1977b), which is explicitly broken only by the color anomaly (a strong Adler-Bell-Jackiw anomaly). Because of that explicit breaking, the resulting Nambu-Goldstone boson, called an axion, acquires a mass (it would be massless if the breaking were spontaneous). This mass is estimated (see Tanabashi, 1992; Miransky, 1993) to be about 2–3 MeV, for $m_t\approx 180\,{\rm GeV}$. However, such a visible axion is phenomenologically unacceptable (Peccei, 1989). Therefore $\kappa_{tb}\neq 0$ must be assumed, and this cross term can be adjusted to give an acceptable axion mass.

To summarize, the use of the improved Dyson-Schwinger ladder equations and Pagels-Stokar relations (Miransky, Tanabashi, and Yamawaki, 1989a, 1989b; King and Mannan, 1990, 1991a) gave higher predictions for m_t than the RG approach of Bardeen, Hill, and Lindner (1990). Even for very high $\Lambda\sim E_{\rm Planck}\sim 10^{19}\,{\rm GeV}$, the predicted mass is $m_t\approx 253\,{\rm GeV}$, i.e., 35 GeV higher than the one-loop RG prediction and 70–80 GeV higher than the experimental value. Further, as shown by King and Mannan (1990), when the dynamical structure (propagator) of a heavy gauge boson, whose exchange is assumed to generate the effective κ_t-four-quark contact term, is also taken into account, the predicted m_t remains practically unchanged. However, the effective critical coupling parameter $\kappa_t\approx\kappa_{\rm crit}$ needed for condensation increases in this case, offsetting the smaller gauge-boson propagator

[14] At the leading-$N_{\rm c}$ level, $\langle\bar{t}t\rangle_0=-[N_{\rm c}/(4\pi^2)]\times\int_0^{\Lambda^2}d\bar{q}^2\bar{q}^2 m_t(\bar{q}^2)[\bar{q}^2+m_t^2(\bar{q}^2)]^{-1}$.

$[\approx 1/(\bar{q}^2+\Lambda^2)<1/\Lambda^2]$. In Sec. V, we discuss connections, analogies, and differences between the Dyson-Schwinger+Pagels-Stokar and RG approaches, as well as compare these two methods with the "hard-mass" effective potential method. It should be stressed that all approaches must give increasingly similar results when increasingly similar assumptions are used.

E. Other related approaches

Gribov (1994) derived a relation similar to the Pagels-Stokar equation (3.12). He investigated the polarization operator of the $SU(2)_L$ bosons within a leading-N_c framework. His framework was based only on the assumption that the Landau pole λ of the $U(1)_Y$ coupling parameter was responsible for the composite structure of the Higgs ($\underline{\lambda}=\Lambda$). In his picture, the Higgs was predominantly a $\bar{t}t$ condensate, with admixtures of condensates of other fermions proportional to their masses. Incidentally, Gribov also derived in his leading-N_c framework the following remarkable Pagels-Stokar-type relation for the Higgs (*not W*) mass:

$$M_H^2=\frac{N_c}{2v^2\pi^2}\int_{m_t^2}^{\Lambda^2}\frac{d\bar{q}^2}{\bar{q}^2}m_t^4(\bar{q}^2), \tag{3.15}$$

which apparently can also be applied in the top-mode standard model type scenarios.

The question of the formal renormalizability of gauged NJLVL models with nonrunning gauge coupling constant has been investigated by Bardeen, Leung, and Love (1989) and by Kondo, Tanabashi, and Yamawaki (1993). They showed that in the ladder approximation of the Dyson-Schwinger approach such models are renormalizable. This means that they were able to construct a well-defined algorithm to express low-energy physical predictions as finite quantities even in the limit $\Lambda\to\infty$. The conclusion has been additionally confirmed by the work of Kondo *et al.* (1994), in which the flow of the (renormalized) Yukawa parameter and mass parameter were investigated in the corresponding theory with composite Higgs and gauge bosons. One possible drawback of these proofs was that they were confined to the case of a nonrunning gauge coupling and zero scalar self-coupling parameters. On the other hand, Yamawaki (1991) and later also Kondo, Shuto, and Yamawaki (1991) investigated explicit solutions of the Dyson-Schwinger equation for $\Sigma_t(\bar{p}^2)$ and their behavior in the asymptotic region for the case of a running gauge coupling parameter. They showed that such (QCD) gauge interactions are crucial for the behavior in the asymptotic region. Moreover, the decay constant F_π as determined by the Pagels-Stokar relation then remains finite even when $\Lambda\to\infty$. In this sense, they argued that QCD makes the NJLVL framework formally renormalizable. Krasnikov (1993) investigated the framework in $4-\epsilon$ dimensions, by including the scalar self-interactions, and argued that it also remains formally renormalizable in this case. Later, Harada *et al.* (1994) and Kugo (1996) discussed rigorously the relation between the gauged NJLVL model on the one hand, and the system of gauge bosons and (composite) Higgs with renormalized Yukawa parameter y on the other hand, in the general case of a running gauge coupling and scalar quartic self-coupling parameter λ, for a finite compositeness scale (cutoff) Λ. This allowed them to take the continuum limit $\Lambda\to\infty$ ($\epsilon\to 0$) and to show with RG methods that the two models (gauged NJLVL and gauge-Higgs-Yukawa) become equivalent and nontrivial in this limit when renormalized parameters (y,λ) take a specific critical value in the parameter plane. Therefore gauged NJLVL models are formally renormalizable.

One of the main arguments against the use of Dyson-Schwinger (ladder) equations in studying nonperturbative physics has been the gauge dependence of the resulting critical coupling parameter(s) and of the dynamical mass(es) $m=\Sigma(\bar{p}^2=m^2)$. While it has been assumed by many that investigation of the (improved) Dyson-Schwinger ladder equation gives the most reasonable results in the Landau gauge, arguments supporting this assumption have appeared only relatively recently (Kondo, 1992; Curtis and Pennington, 1993; and references therein). Curtis and Pennington worked within the strongly coupled QED and in the general covariant R_ξ gauge. They showed that application of a nonperturbative *Ansatz* for the fermion-photon vertex, satisfying the Ward-Takahashi and the (derivative) Ward identity, leads to values of α_{crit} and m^{dyn} that are only weakly gauge dependent and close to the corresponding results of calculations in the usual ladder ("rainbow") approximation in the Landau gauge.[15]

Recently, Blumhofer and Manus (1998) proposed treating the Dyson-Schwinger equation in a drastically different way—in Minkowski instead of Euclidean metric. The main motivation behind this approach lies in the possibly insurmountable difficulty of analytic continuation of the Euclidean dynamical quark mass $\Sigma_q(\bar{p}^2)$ to the timelike on-shell region $\bar{p}^2\mapsto -p^2=-m_q^2$. They worked in the framework of single exchange of strongly coupled gauge bosons (the ladder approximation), using the Landau gauge and the linear approximation for the denominator stemming from the quark propagator $[(k^2-\Sigma_q(k^2))^{-1}\mapsto(k^2-m_q^2)^{-1}]$ in the Dyson-Schwinger equation. When they applied the unsubtracted dispersion relation, valid for the *regular* solution $\Sigma_q(p^2)\propto 1/p^2$ [in the timelike region, $\Sigma_q(p^2)$ is in general complex], to the Dyson-Schwinger equation, they obtained a one-dimensional integral equation for

[15]Ladder ("rainbow") approximation means here above all that the fermion-boson vertex in the single-photon-exchange picture of the Dyson-Schwinger equation is taken to be that of the tree level: $\Gamma^\mu(k,p)\mapsto\gamma^\mu$. An improvement of such an approximation along this "rainbow" approach (i.e., using tree-level vertices) would be calculation of higher, e.g., next-to-leading in $1/N_c$, contributions to the Dyson-Schwinger equation—see Sec. IV. Such an expansion is systematic and gauge noninvariant, in contrast to the above approach, which is nearly gauge invariant and nonunique (nonsystematic) due to application of an *Ansatz*.

$I(p^2)\equiv \mathrm{Im}[\Sigma_q(p^2)]$. After introducing asymptotically free running of the gauge coupling $\alpha(p^2) \propto (\ln p^2/\Lambda_{\mathrm{IR}}^2)^{-1}$, they were able to solve the above-mentioned integral equation, even when the mass M of the exchanged gauge boson was zero. The authors pointed out that the results agree in the asymptotic region $p^2 \gg M^2$ with the corresponding regular solutions of the Euclidean metric approach. (Analytic continuation $\overline{p^2} \mapsto -\overline{p^2} < 0$ for $\overline{p^2} \gg M^2$, where M is effectively the cutoff, can be performed easily in the asymptotic region, since the solution there is explicitly known.) The authors further indicated how to use the *subtracted* dispersion relations to obtain the *irregular* solution [for QCD, $\Sigma_q(p^2)_{\mathrm{irreg}} \propto (\ln p^2/\Lambda_{\mathrm{IR}})^{-8/14}$], which corresponds to the actual running quark mass $m_q(p^2)$ [cf. Eq. (5.7) for RG-running $m_q(p^2)$ in QCD].

We defer to Sec. IV the discussion of work connected with investigations of Dyson-Schwinger (gap) equations beyond the (improved) ladder approximation.

IV. NEXT-TO-LEADING-ORDER (NTLO) EFFECTS IN THE $1/N_c$ EXPANSION

A. Effective potential as a function of a "hard mass"

With a view toward studying the next-to-leading-order (NTLO) contributions to the condensation mechanism, we present here a third approach. It makes use of an auxiliary field method to calculate the effective potential[16] V_{eff} as a function of a "hard mass" (nonrunning top-quark mass) σ. At the minimum of V_{eff} (vacuum), we have $\langle\hat{\sigma}\rangle_0 = m_t(\Lambda)$, where $m_t(\Lambda)$ is the dynamical bare mass. The mass m_t^{ren} can then be obtained by including renormalization effects in the energy interval below Λ. This approach is an alternative to solving the Dyson-Schwinger integral equation for $\Sigma_t(\overline{p^2})$, and it allows inclusion of the NTLO contributions in a systematic way.[17] The approach has been applied in a series of papers (Cvetič, Paschos, and Vlachos, 1996; Cvetič and Vlachos, 1996; Cvetič, 1997), at the NTLO level, specifically for the minimal framework of Eqs. (2.3) and (2.4). V_{eff} is calculated as a function of $\langle\hat{\mathcal{H}}\rangle = \mathcal{H}_0$ and $\langle\hat{\mathcal{G}}^{(j)}\rangle = \mathcal{G}_0^{(j)}$ via the path-integration formula[18] (Fukuda and Kyriakopoulos, 1975):

$$\exp[-\Omega V_{\mathrm{eff}}(\mathcal{H}_0,\mathcal{G}_0^{(j)})] = \mathrm{const}\times\int\prod_{j=0}^{2}\left[\mathcal{D}\mathcal{G}^{(j)}\delta\left(\int d^4\overline{y}\,\mathcal{G}^{(j)}(\overline{y})-\Omega\mathcal{G}_0^{(j)}\right)\right] \times\int\mathcal{D}\mathcal{H}\delta\left(\int d^4\overline{y}\,\mathcal{H}(\overline{y})-\Omega\mathcal{H}_0\right) \times\int\mathcal{D}\overline{\Psi}\,\mathcal{D}\Psi\exp\left[+\int d^4\overline{x}\,\mathcal{L}(\overline{x})\right]. \tag{4.1}$$

A Euclidean metric is used, $\hbar=1$, $\Omega=\int d^4\overline{x}$ (formally infinite), and $\mathcal{L}(\overline{x})$ is the Euclidean version of density (2.3) with cutoff Λ. Path integration over the quark degrees of freedom can be done exactly because it involves Gaussian integrals. Path integration over the scalar degrees of freedom can be performed subsequently in the sense of the $1/N_c$ expansion up to next-to-leading order, and even an analogous integration over the QCD (gluonic) degrees of freedom can be included. Details are given in the work of Cvetič (1997), where proper time cutoff and a Pauli-Villars regularization cutoff were used for the fermionic momenta, within the proper time regularization approach, as well as the covariant spherical cutoff (Λ_f). For the bosonic (scalar) momenta, which appear at the NTLO level, a spherical covariant cutoff Λ_b had to be used ($\Lambda_f \sim \Lambda_b \sim \Lambda$). In the following, we give results only for the case of spherical covariant cutoffs. V_{eff} can be expressed conveniently by dimensionless parameters,

$$\Xi_{\mathrm{eff}}(\varepsilon^2;a;\Lambda_b^2/\Lambda_f^2) \equiv 8\pi^2 V_{\mathrm{eff}}/(N_c\Lambda_f^4) = \Xi^{(0)} + \frac{1}{N_c}\Xi^{(1)} + \mathcal{O}\left(\frac{1}{N_c^2}\right), \tag{4.2}$$

$$\varepsilon^2 \equiv \frac{\sigma_0^2}{\Lambda_f^2} \equiv \frac{GM_0^2}{2\Lambda_f^2}\langle\mathcal{H}\rangle_0^2, \quad a \equiv \frac{GN_c\Lambda_f^2}{8\pi^2} (\equiv \kappa_t). \tag{4.3}$$

Parameter a is in fact κ_t of Sec. III [cf. Eq. (3.2)]. The leading-N_c part $\Xi^{(0)}$ is made up of the quark loop (q.l.) and of the QCD (gluonic) contributions,[19]

$$\Xi_{\mathrm{q.l.}}^{(0)}(\varepsilon^2;a) = \frac{\varepsilon^2}{a} - \int_0^1 d\overline{k^2}\,\overline{k^2}\ln\left(1+\frac{\varepsilon^2}{\overline{k^2}}\right), \tag{4.4}$$

$$\Xi_{\mathrm{gl}}^{(0)}(\varepsilon^2;\Lambda_b^2/\Lambda_f^2) = \frac{(N_c^2-1)}{4N_c}\int_0^{\Lambda_b^2/\Lambda_f^2} d\overline{p^2}\,\overline{p^2} \times\ln[1-a_{\mathrm{gl}}\mathcal{J}_{\mathrm{gl}}(\overline{p^2};\varepsilon^2)], \tag{4.5}$$

$$\Xi^{(0)} = \Xi_{\mathrm{q.l.}}^{(0)} + \Xi_{\mathrm{gl}}^{(0)}. \tag{4.6}$$

Here, $a_{\mathrm{gl}} \approx 3\alpha_3(m_t^{\mathrm{phys}})/\pi \approx 0.105$. The (QCD) gluon part (4.5) has its origin in exchanges of the gluons (in the Landau gauge). We note that $\Xi_{\mathrm{gl}}^{(0)}$ is really leading-N_c, i.e., $\Xi_{\mathrm{gl}}^{(0)} \sim N_c^0$, because $\alpha_3 \sim N_c^{-1}$ as seen from Eq. (3.6)

[16]The use of V_{eff} to investigate spontaneous symmetry breaking was introduced by Coleman and E. Weinberg (1973). S. Weinberg (1973) showed that cancellation of tadpole contributions in the vacuum (in the Landau gauge) is equivalent to the minimization of V_{eff}. The path-integral method for evaluating V_{eff} was worked out by Jackiw (1974).

[17]The Bethe–Salpeter equation for Nambu–Goldstone bosons at the NTLO level (or an NTLO analog of the Pagels-Stokar relation in the Miransky, Tanabashi, and Yamawaki approach) has not yet been investigated. Thus we cannot yet make predictions at the NTLO level for m_t^{ren} as a function of the cutoffs Λ alone.

[18]In the vacuum we have $\langle\hat{\sigma}\rangle_0 = \sqrt{G}M_0\langle\hat{\mathcal{H}}\rangle_0/\sqrt{2} = m_t(\Lambda)$ and $\langle\hat{\mathcal{G}}^{(j)}\rangle_0 = 0$.

[19]The QCD integrals were regulated by the proper time cutoffs $\tau_f \geq 1/\Lambda_f^2$ and $\tau_b \geq 1/\Lambda_b^2$.

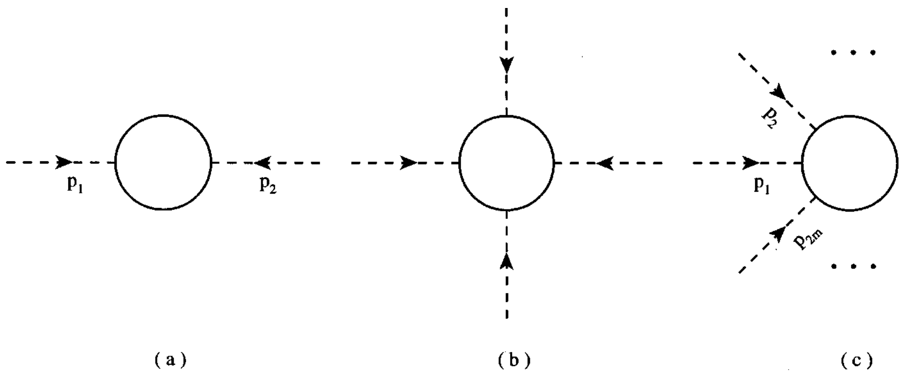

FIG. 3. One-loop one-particle irreducible diagrams giving the relevant Green's functions $\tilde{\Gamma}_H^{(2m;1)}(p_1,\ldots,p_{2m})$. The latter, for zero external momenta, enable us to form an infinite power series in $\mathcal{H}_0$ leading to the leading-N_c quark-loop part of the effective potential $V_{\rm eff}(\mathcal{H}_0)$. Solid lines represent massless top quarks, and dotted external lines the nondynamical Higgs particle. The couplings used are those of the top-mode standard-model Lagrangian density [Eq. (2.3)].

and thus $\ln[1-a_{\rm gl}\mathcal{J}_{\rm gl}]\propto -a_{\rm gl}\mathcal{J}_{\rm gl}\propto a_{\rm gl}\propto \alpha_3\sim N_c^{-1}$. The NTLO part $\Xi^{(1)}$ comes from exchanges of (composite) scalars:

$$\frac{1}{N_c}\Xi^{(1)}(\varepsilon^2;a;\Lambda_b^2/\Lambda_f^2) = \frac{1}{4N_c}\int_0^{\Lambda_b^2/\Lambda_f^2} d\overline{p}^2\,\overline{p}^2 \sum_X A_X \ln[1-a\mathcal{J}_X(\overline{p}^2;\varepsilon^2)]. \quad (4.7)$$

Fermionic ($\overline{k}$) and bosonic ($\overline{p}$) momenta were rescaled: $\overline{k}_{\rm new}=\overline{k}_{\rm old}/\Lambda_f$, $\overline{p}_{\rm new}=\overline{p}_{\rm old}/\Lambda_f$. X=H (Higgs), Gn, Gch (neutral and charged Nambu-Goldstone bosons). The corresponding multiplicity factors are $A_X=1,1,2$. $\mathcal{J}_X$ and $\mathcal{J}_{\rm gl}$ represent loop contributions of t and b to the dimensionless two-leg Green's functions for scalars and gluons.[20] We refer the reader to the above-mentioned references for details.

Alternatively, $\Xi^{(0)}_{\rm q.l.}$, $\Xi^{(0)}_{\rm gl}$, and $\Xi^{(1)}$ can be rederived diagrammatically by summing up terms corresponding to the one-particle irreducible Green's functions depicted in Figs. 3 (for $\Xi^{(0)}_{\rm q.l.}$) and 4 (for $\Xi^{(1)}$ and $\Xi^{(0)}_{\rm gl}$).

B. Gap equation and mass renormalization at NTLO level

Minimizing the part $\Xi^{(0)}_{\rm q.l.}$ (of $\Xi_{\rm eff}$) leads to the familiar gap equation in the leading-N_c approximation without QCD (the quark-loop approximation)—relating the cutoff Λ_f, the four-quark coupling strength G ($\Leftrightarrow a$), and the leading-N_c (quark-loop) mass $m_t^{(0)}$,

$$\left.\frac{\partial\Xi^{(0)}_{\rm q.l.}(\varepsilon^2;a)}{\partial\varepsilon^2}\right|_{\varepsilon^2=\varepsilon_0^2} = \left.\left[\frac{1}{a}-1+\varepsilon^2\ln(\varepsilon^{-2}+1)\right]\right|_{\varepsilon^2=\varepsilon_0^2}=0, \quad (4.8)$$

$$\Rightarrow\ a\left(\equiv\frac{GN_c\Lambda_f^2}{8\pi^2}\right)=[1-\varepsilon_0^2\ln(\varepsilon_0^{-2}+1)]^{-1}(\sim 1),$$

$$\varepsilon_0^2\equiv(m_t^{(0)}/\Lambda_f)^2. \quad (4.9)$$

This coincides with the quark-loop-approximated solution (3.7) of the Dyson-Schwinger equation, as it should [there are no renormalization corrections to $m_t^{(0)}$ at the quark-loop level, i.e., $m_t^{(0)}(\overline{p}^2)$ is standing]. Equation (4.9) shows that $a>1$ and $a\sim 1$ ($\sim N_c^0$). The NTLO information connecting the bare mass $m_t(\Lambda_f)$, the cutoff Λ_f, and the parameter G (or: a) can be obtained by minimizing $\Xi_{\rm eff}$ of Eqs. (4.2)–(4.7):

$$\left.\frac{\partial\Xi_{\rm eff}(\varepsilon^2;a;\Lambda_b^2/\Lambda_f^2)}{\partial\varepsilon^2}\right|_{\varepsilon^2=\varepsilon^2_{\rm gap}} = \frac{1}{a}-[1-\varepsilon^2_{\rm gap}\ln(\varepsilon^{-2}_{\rm gap}+1)] - a_{\rm gl}\frac{(N_c^2-1)}{4N_c}\int_0^{\Lambda_b^2/\Lambda_f^2} d\overline{p}^2\,\overline{p}^2\frac{\partial\mathcal{J}_{\rm gl}(\overline{p}^2;\varepsilon^2)}{\partial\varepsilon^2} \times\left.[1-a_{\rm gl}\mathcal{J}_{\rm gl}(\overline{p}^2;\varepsilon^2)]^{-1}\right|_{\varepsilon^2=\varepsilon^2_{\rm gap}} - \frac{1}{4N_c}\int_0^{\Lambda_b^2/\Lambda_f^2} d\overline{p}^2\,\overline{p}^2 \times\left\{\sum_X A_X\frac{\partial\mathcal{J}_X}{\partial\varepsilon^2}[1-\varepsilon^2\ln(\varepsilon^{-2}+1) -\mathcal{J}_X(\overline{p}^2;\varepsilon^2)]^{-1}\right\}\Bigg|_{\varepsilon^2=\varepsilon^2_{\rm gap}}=0, \quad (4.10)$$

where $\varepsilon^2_{\rm gap}(a;r)=m_t^2(\Lambda)/\Lambda_f^2$, with a and $r\equiv\Lambda_b/\Lambda_f$ the input parameters. Since this gap equation includes NTLO contributions, we have formally $\varepsilon^2_{\rm gap}=\varepsilon_0^2[1+\mathcal{O}(1/N_c)]$. An important point should be mentioned. Denominators in the above integrands of the NTLO part are propagators of the composite scalars in the leading-N_c approximation. Formally, as implied by Eq. (4.7), they should be written as $(a/2)[1-a\mathcal{J}_X(\overline{p}^2;\varepsilon^2)]^{-1}$ (X=H, Gn, Gch). However, they contain singularities (poles) in the interval $0<\overline{p}^2\leqslant 1$, because $1/N_c$ expansion does not automatically respect the Goldstone theorem (masslessness of Nambu-Goldstone

[20]They are represented by Fig. 3(a), where external momenta are in this case nonzero, and t in the loop is regarded as already having acquired the (dynamical) mass $m_t(\Lambda)=\varepsilon\Lambda_f$, while $m_b=0$.

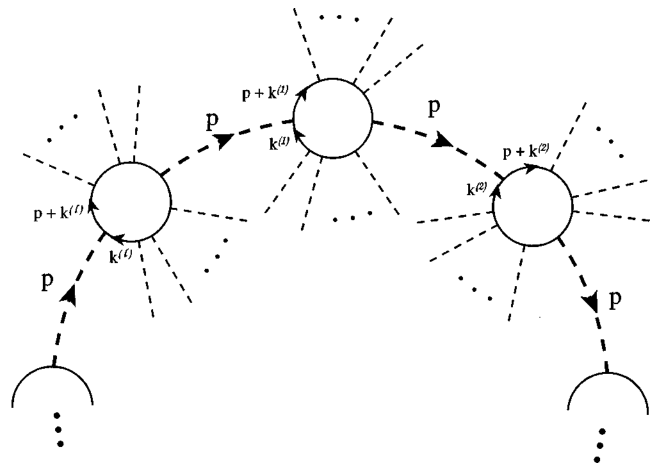

FIG. 4. The higher-loop one-particle irreducible diagrams that contribute to one-particle irreducible Green's functions at next-to-leading order in the $1/N_c$ expansion. Solid lines represent the massless top and bottom quarks, and dotted lines either (nondynamical) scalars or gluons. The couplings used are those of the top-mode standard-model Lagrangian density [Eq. (2.3)].

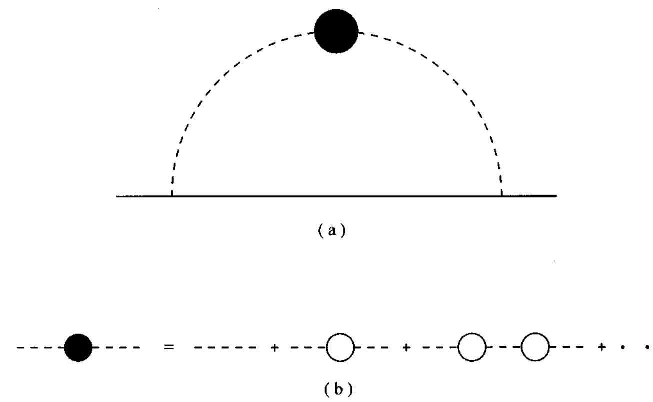

FIG. 5. The one-particle irreducible diagrams that give $\delta(\varepsilon^2)_{\rm ren}$. The t quarks in loops have mass $m_t(\Lambda) \equiv \Lambda_f \varepsilon_{\rm gap}$ as determined by the NTLO gap equation (4.10). The dashed line is the propagator of either the (nondynamical) scalar or the gluon. According to Fig. 5(b), the black spot makes the propagator of scalars dynamical.

bosons). This theorem is ensured in the NTLO gap equation (4.10) by replacing the above-mentioned propagators in $\partial \Xi^{(1)}/\partial \varepsilon^2$ by $(1/2)[1-\varepsilon^2 \ln(\varepsilon^{-2}+1) - \mathcal{J}_X(\overline{p}^2;\varepsilon^2)]^{-1}$. The difference between the inverse of the old and of the latter expression is $2\partial \Xi^{(0)}_{\rm q.l.}/\partial \varepsilon^2$ and is hence $\sim 1/N_c$ for the values of ε^2 near the new minimum: $\varepsilon^2 \approx \varepsilon^2_{\rm gap} = \varepsilon_0^2[1+\mathcal{O}(1/N_c)]$. Thus the NTLO expression $N_c^{-1}\partial \Xi^{(1)}/\partial \varepsilon^2$ has been formally changed by a next-to-next-to-leading-order ($\sim 1/N_c^2$) contribution, resulting in a fully legitimate modification of the NTLO gap equation. This change now makes all three propagators nonsingular in the interval $0<\overline{p}^2 \leq 1$; the integrals in Eq. (4.10) are finite and well defined. The Goldstone theorem is now explicitly respected at the NTLO level of the gap equation,[21] because $[1-\varepsilon^2 \ln(\varepsilon^{-2}+1) - \mathcal{J}_X(\overline{p}^2;\varepsilon^2)] \propto \overline{p}^2$ for X=Gn, Gch, when $\overline{p}^2 \to 0$. Similar problems were also discussed by Nikolov *et al.* (1996) in an $SU(2)$-invariant NJLVL model which they regarded as a model of low-energy QCD.

The renormalization $\varepsilon_{\rm gap}(a;r) \mapsto \varepsilon_{\rm ren}(a;r)$ allows one to obtain values of energies where the new physics responsible for the condensation is expected to set in: $\Lambda \sim \Lambda_f = m_t^{\rm ren}/\varepsilon_{\rm ren}(a;r) = \Lambda_f(a;r)$, where a and $r \equiv \Lambda_b/\Lambda_f$ are the input parameters, and $m_t^{\rm ren} \approx 170-180$ GeV. The QCD contribution is formally a leading-N_c one,

$$\delta(\varepsilon^2)^{\rm QCD}_{\rm ren} \approx \frac{9(N_c^2-1)}{2N_c(11N_c-12)} \frac{1}{\ln(m_t/\Lambda_{\rm QCD})} \times \varepsilon^2[\ln(\varepsilon^{-2}) + \ln(r^2) + 0.256 + \mathcal{O}(\varepsilon^2)], \tag{4.11}$$

where $\varepsilon = \varepsilon_{\rm gap}(a;r)$ [the solution of the NTLO gap equation (4.10)]. Contribution of the (composite) scalars is NTLO, but at the cutoffs $\Lambda \sim 1$ TeV it is numerically larger than Eq. (4.11):

$$\delta(\varepsilon^2)^{\rm (NTLO)}_{\rm ren} = \kappa_{1r}/N_c = (\kappa_{1r}^{\rm (H)} + \kappa_{1r}^{\rm (Gn)} + \kappa_{1r}^{\rm (Gch)})/N_c. \tag{4.12}$$

For details on integrals $\kappa_{1r}^{(X)}$, including the needed analytical continuation to the on-mass-shell values of the external quark momentum in Fig. 5(a) ($\overline{q}^2 = -q^2 = -\varepsilon^2_{\rm gap} < 0$), we refer the reader to the literature. Just as in Eq. (4.10), the dynamic scalar propagators in these integrals are modified to ensure the Goldstone theorem.

C. Numerical results: Leading-N_c vs NTLO gap equation

The NTLO contributions in the gap equation (4.10) represent mostly the so-called "feedback effects" of the composite scalars on their own binding. The basic conclusion following from the numerical results is that these NTLO contributions are strong and, for given values of the four-quark coupling parameter a ($\equiv \kappa_t$) and of the ratio $r \equiv \Lambda_b/\Lambda_f$ (<1),[22] change the leading-N_c predictions for the bare mass $m_t(\Lambda_f)$ so drastically that $1/N_c$ expansion may lose predictability unless the cutoffs are low, $\Lambda_f \sim \Lambda_b \lesssim 1$ TeV. This is illustrated in Table III. For given values of $r \equiv \Lambda_b/\Lambda_f = 1/\sqrt{2}, 1$, the other input param-

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[21]For more discussion on these points see Cvetič (1997). Note that an earlier work (Cvetič, Paschos, and Vlachos, 1996) did not include the composite Nambu-Goldstone bosons, and the integrands stemming from the composite Higgs exchanges were not regularized; as a result, for small $\varepsilon^2 \ll \varepsilon_0^2$ a singularity was encountered, but did not affect the basic conclusions of the paper.

[22]The diagrams of Fig. 4 suggest that $\overline{p}^2_{\rm max} \lesssim \overline{k}^2_{\rm max}$, implying the input values $\Lambda_b/\Lambda_f \lesssim 1$.

TABLE III. Quark (fermionic) and bosonic cutoffs Λ_f and Λ_b (in TeV), obtained for given $r \equiv \Lambda_b/\Lambda_f = 1/\sqrt{2}, 1$ and imposing the requirement that the ratio $\varepsilon_{gap}^2/\varepsilon_0^2 = [m_t(\Lambda)/m_t^{(0)}]^2$ not be smaller than 1/4, 1/3, 1/2 (a was adjusted accordingly). The corresponding ratios $m_t^{ren}/m_t^{(0)}$ are also given. Covariant spherical (S) cutoffs were employed. The value $m_t^{ren} = 180$ GeV was used (Abe *et al.*, 1995; Adachi *et al.*, 1995) to obtain $\Lambda_f(a;r) = m_t^{ren}/\varepsilon_{ren}(a;r)$.

a	Λ_b/Λ_f	$m_t(\Lambda)/m_t^{(0)}$	$m_t^{ren.}/m_t^{(0)}$	Λ_f	Λ_b
1.441	$1/\sqrt{2}$	0.500 ($=\sqrt{1/4}$)	0.448	1.037	0.733
1.520	$1/\sqrt{2}$	0.577 ($=\sqrt{1/3}$)	0.515	0.816	0.577
1.841	$1/\sqrt{2}$	0.707 ($=\sqrt{1/2}$)	0.624	0.506	0.358
1.959	1	0.500 ($=\sqrt{1/4}$)	0.397	0.737	0.737
2.292	1	0.577 ($=\sqrt{1/3}$)	0.450	0.548	0.548

eter a was adjusted so that $\varepsilon_{gap}^2(a;r)/\varepsilon_0^2(a) \equiv (m_t(\Lambda)/m_t^{(0)})^2$ attained values 1/4,1/3,1/2, where $\varepsilon_{gap}(a;r)$ solves the NTLO gap equation (4.10), and $\varepsilon_0(a)$ the leading-N_c quark-loop[23] gap equation (4.9).

In Table IV the resulting cutoffs are given when we allow a highly nonperturbative behavior of $1/N_c$ expansion: $\varepsilon_{gap}^{(NTLO)}/\varepsilon_0 = 0.05$. The cutoffs are then higher ($\sim$10 TeV). However, these cases are speculative, since the meaning of the $1/N_c$ expansion is then in serious danger. Such results could make sense if the next-to-next-to-leading terms of the gap equation (which were not calculated) are very small and provoke only a small relative change of the obtained small ε_{gap}^2 of the NTLO gap equation (4.10). While this scenario seems to be unlikely, it cannot be completely ruled out since the strong "feedback" effects might be represented almost exclusively by the NTLO contributions.

The contributions of the pure (i.e., transverse) components of the electroweak gauge bosons were not included in the discussed NTLO calculations. There are indications that these contributions are numerically not very important. The RG approach of Bardeen, Hill, and Lindner indicates that these contributions, at least those to the mass renormalization, are small because of relatively small $SU(2)_L \times U(1)_Y$ gauge coupling parameters.

D. Small-ε (large-Λ/m_t) expansion of the NTLO gap equation

Despite the just mentioned reservations about small-ε_{gap}^2 (large-Λ) solutions, it is instructive to investigate the latter to discern the nature of the phase transition between the symmetric and the broken phases at the NTLO level.[24]

[23]Although QCD contributions are formally part of the leading-N_c contributions, we consider them numerically as part of NTLO effects—because QCD contributions turn out here to be even smaller than the NTLO "feedback" contributions of the composite scalars.

TABLE IV. Results for the cutoffs (in TeV) in the case of highly nonperturbative behavior in the sense of the $1/N_c$ expansion: $m_t(\Lambda) = 0.05 m_t^{(0)} \ll m_t^{(0)}$. The results are given for two choices of cutoff ratios, Λ_b/Λ_f: 1, $1/\sqrt{2}$. The value $m_t^{ren} = 180$ GeV was used.

a	Λ_b/Λ_f	$m_t(\Lambda)/m_t^{(0)}$	$m_t^{ren.}/m_t^{(0)}$	Λ_f	Λ_b
1.303	$1/\sqrt{2}$	0.050	0.046	12.7	9.0
1.549	1	0.050	0.041	9.8	9.8

The small-ε^2 NTLO gap equation is obtained by performing in the relevant integrals expansions for small ε^2 and small $\bar{p}^2 \equiv \bar{q}_{boson}^2/\Lambda_f^2$. Specifically, such expansions in the case of the proper-time cutoff (PTC) for fermionic momenta give the gap equation[25]

$$\frac{\partial \Xi_{eff}}{\partial \varepsilon^2} \equiv \left[\frac{1}{a} - \frac{1}{a_{crit}} + \kappa(\varepsilon^2 \ln \varepsilon^{-2}) + \mathcal{O}\left(\frac{1}{N_c} \varepsilon^2 \ln \ln \varepsilon^{-2} \right) \right] = 0, \tag{4.13}$$

with the NTLO solution $\varepsilon^2 = \varepsilon_{gap}^2(a;r) \equiv m_t^2(\Lambda)/\Lambda_f^2 \ll 1$, and

$$a_{crit}^{-1}(r) = 1 - \tilde{a}_1^{-1}(r)/N_c, \quad \kappa(r) = 1 + \tilde{\kappa}_1(r)/N_c. \tag{4.14}$$

Here, $\tilde{a}_1^{-1}, \tilde{\kappa}_1(r) \sim 1$; $\tilde{a}_1^{-1}$ is independent and $\tilde{\kappa}_1$ is very moderately dependent on ε^2 ($\sim \ln \ln \varepsilon^{-2}$, see below). For r ($\equiv \Lambda_b/\Lambda_f) = 1$, we have

$$\tilde{a}_1^{-1} \approx 1.79202 \Rightarrow$$
$$a_{crit}^{-1} \approx 1 - (1.79202)/N_c \approx 0.403 \quad (\Rightarrow a_{crit} \approx 2.5), \tag{4.15}$$
$$\tilde{\kappa}_1 = (3 \ln \ln \varepsilon^{-2} - C),$$
$$8.7 \lesssim C \lesssim 10.8 \quad (\text{for } r=1; \text{PTC}). \tag{4.16}$$

For $m_t(\Lambda) \sim 10^2$ GeV and 10^{10} GeV$\lesssim \Lambda_f \lesssim 10^{16}$ GeV ($\Lambda_b = \Lambda_f$), we then obtain: $0 \lesssim \tilde{\kappa}_1 \lesssim 4 \Rightarrow 1 < \kappa < 2.4$. The bounds for the constant C can be systematically made narrower. When $r<1$, smaller $\tilde{a}_1^{-1}(r)$ and $a_{crit}(r)$ are obtained ($r \downarrow 0 \Rightarrow a_{crit} \downarrow 1$). Qualitatively similar results are expected when other quark momentum regularization prescriptions are employed and/or QCD contributions are included (the latter decrease a_{crit}).

Ensuring the respect of the Goldstone theorem at the NTLO level was crucial for obtaining the above results. Comparing Eqs. (4.13) and (4.8), we see that the NTLO contributions for small ε^2 do not change the structure of the leading-N_c (quark-loop) gap equation.[26] Thus, the

[24]The author acknowledges that W. A. Bardeen had suggested small-ε^2 expansion at the NTLO level for this reason and that E. A. Paschos had conveyed to him this idea of Bardeen. The results of this section have not appeared in the literature.

[25]No QCD was included. For details on the integrals of the NTLO gap equation in the proper-time-cutoff case, see Cvetič (1997) and Eqs. (41), (39), (B.4), (B.20), and (B.21) therein.

[26]$a_{crit} = 1 = \kappa$ at the quark-loop level in the case of covariant spherical cutoff and in the proper-time cutoff case.

phase transition remains second order. This means that when $a \downarrow a_{\rm crit}(r)$ continuously, then $\varepsilon^2_{\rm gap}(a;r) \downarrow 0$ continuously, too. For $a \leqslant a_{\rm crit}$, we have $\varepsilon_{\rm gap}=0$ $[m_t(\Lambda)=0]$. The question of the mass renormalization in the small-$\varepsilon^2_{\rm gap}$ (large-Λ) limit should also be investigated, probably under inclusion of the (minimal standard model) RG equations.

Comparing the results of this section with those of the previous, we can express the latter in this way: For given values of parameters of the new physics (a and $\Lambda \sim \Lambda_{\rm f} \sim \Lambda_{\rm b}$), and if $\Lambda \gg E_{\rm ew}$, we can probably predict with the $1/N_{\rm c}$ approach reasonably well and in a systematic way the critical parameters $a_{\rm crit}$ and κ [cf. Eqs. (4.14)–(4.16)]. However, predictions for $\varepsilon_{\rm gap} \equiv m_t(\Lambda)/\Lambda_{\rm f}$ and $\varepsilon_{\rm ren} \equiv m_t^{\rm ren}/\Lambda_{\rm f}$, ($\Rightarrow m_t^{\rm ren}$) appear to be in serious danger unless $\Lambda \lesssim 1$ TeV, i.e., unless we abandon the case of fine-tuning problem of $a \approx a_{\rm crit}(r)$. For example, from the last line of Table IV it follows that for high cutoffs $\Lambda_{\rm b}=\Lambda_{\rm f}=9.8$ GeV, we have to choose $a=1.549$, and[27] the leading-$N_{\rm c}$ (quark-loop) prediction $m_t^{(0)}$ is in this case drastically higher than $m_t^{\rm ren}$: $m_t^{(0)} \approx 24 m_t^{\rm ren} \approx 4.4$ TeV. This illustrates drastically the gravity of the fine-tuning problem within the $1/N_{\rm c}$ expansion approach. We shall return briefly to questions concerning the $1/N_{\rm c}$ expansion in Sec. IX.C.

E. Other work on NTLO effects in quark condensation mechanisms

In addition to the aforementioned work, other authors have investigated NTLO contributions in NJLVL frameworks without gauge bosons: Hands, Kocić, and Kogut (1991, 1993), Derkachov *et al.* (1993), Gracey (1993; 1994), Lurié and Tupper (1993), Dmitrašinović *et al.* (1995), Akama (1996), Dyatlov (1996), Nikolov *et al.* (1996), and Preparata and Xue (1996a, 1996b).

Nikolov *et al.* (1996) calculated NTLO contributions with an effective action formalism, Dmitrašinović *et al.* (1995) with diagrammatic methods. Both groups investigated an $SU(2)$ symmetric NJLVL model and regarded it as a framework of low-energy QCD,[28] where the cutoffs $\Lambda_{\rm b} \sim \Lambda_{\rm f} \sim 1$ GeV are relatively close to the constituent (dynamical) mass of the light quarks $m_q \approx 0.2$–0.4 GeV. They took care that their formalism respected the Goldstone theorem and calculated at the NTLO level (meson loop level) the gap equation which connects the NJLVL four-quark coupling parameter with m_q and the cutoffs $\Lambda_{\rm f}, \Lambda_{\rm b}$. In addition, they calculated at the NTLO level the pion decay constant f_π as a function of m_q and of cutoffs. The latter relation can be viewed as a sum rule for an NTLO version of the Bethe-Salpeter pion bound-state wave function, i.e., an NTLO extension of the Pagels-Stokar relation (3.13) for a nonrunning constituent mass m_q. In the chiral limit, this relation gave the Goldberger-Treiman relation $m_q = g_{\pi qq} f_\pi$, where $g_{\pi qq}$ is the pion-quark coupling parameter. Their NTLO relation for f_π, like Pagels-Stokar relations (3.11)–(3.13), does not involve the NJLVL four-quark coupling constant $a=\kappa_q$ ($\Leftrightarrow G$). Hence it can be regarded independently of the NTLO gap equation. Taking the experimental value $f_\pi \approx 93$ MeV, Nikolov *et al.* and Dmitrašinović *et al.* used only this relation, $f_\pi = f_\pi(m_q;\Lambda_{\rm f};\Lambda_{\rm b}/\Lambda_{\rm f})$, to find numerically, among other things, relations $m_q = m_q(\Lambda_{\rm f})$ at fixed ratios $\Lambda_{\rm b}/\Lambda_{\rm f}$, as well as (see Dmitrašinović *et al.*) relations $m_q = m_q(\Lambda_{\rm b}/\Lambda_{\rm f})$ for fixed values of $\Lambda_{\rm f}$. The latter relations show that m_q increases when NTLO effects are included (at a fixed value of $\Lambda_{\rm f}$), as pointed out by Dmitrašinović *et al.* (see Fig. 12 of that reference). For example, for $\Lambda_{\rm f}=0.8$ GeV, m_q can increase from 0.25 GeV (the leading-$N_{\rm c}$ result, i.e., for $\Lambda_{\rm b}/\Lambda_{\rm f}=0$) to 0.4 GeV (the NTLO result, for $\Lambda_{\rm b}/\Lambda_{\rm f} \approx 1$).

The works of Dmitrašinović *et al.* and of Nikolov *et al.*, although applied to low-energy QCD, have relevance for studies of $\bar{t}t$ condensation at the NTLO level. In particular, they may be relevant for a derivation of an NTLO sum rule analog of the (leading-$N_{\rm c}$) Pagels-Stokar relations described in Sec. III. Such a sum rule would allow us to predict $m_t^{\rm dyn}$ at NTLO level as a function of the cutoffs only. It could alternatively be derived by first considering the Bethe-Salpeter equation for the Nambu-Goldstone boson bound states at the NTLO level.[29] However, blindly applying the NTLO relation for f_π, obtained in the chiral limit by the two groups, to $\bar{t}t$ condensation may in some circumstances be misleading. Such an application would consist of replacing $f_\pi \approx 93$ MeV by the Nambu-Goldstone boson decay constant $f_\pi = \sqrt{2} F_\pi = v\sqrt{2} \approx 350$ GeV and $\Lambda \sim 1$ GeV$\mapsto \Lambda \gtrsim 1$ TeV. There are at least two reasons for caution. First, the model these authors describe is an $SU(2) \times SU(3)_c$-symmetric NJLVL model, while for the $\bar{t}t$ condensation the top-mode standard models (2.1) and (3.2) apply. The latter are NJLVL models with $SU(2)_L \times U(1)_Y \times SU(3)_c$ symmetry. Secondly, the two groups did not consider the effects of mass renormalization $m_q(\Lambda) \mapsto m_q(m_q)$ or, equivalently, the influence of RG evolution $m_q(\bar{p}^2)$ on the value of f_π. The latter effects may be important in $\bar{t}t$ condensation frameworks, especially if cutoffs are above 1 TeV. This contrasts with low-energy QCD. Further, besides the NJLVL four-quark interactions, gluon exchanges may play an important role in predictions of Pagels-Stokar relations for $m_t^{\rm phys}$ as a function of cutoffs Λ, at least when the Λ's are large, as can be seen in Sec. III. NTLO gap equation effects, discussed in Sec. IV.C, indicate that cutoffs should be low [~ 1 TeV] if $1/N_{\rm c}$ expansion for the bare-mass solution $m_t^2(\Lambda_{\rm f})/\Lambda_{\rm f}^2 \equiv \varepsilon^2_{\rm gap}(a;\Lambda_{\rm b}/\Lambda_{\rm f})$ is

[27]Covariant spherical cutoffs were used there, and QCD contributions included; $a=1.549$ is a bit above, but apparently close to $a_{\rm crit}$ in this case.

[28]$SU(2)$ and $SU(3)$ versions of NJLVL frameworks as models of low-energy QCD were reviewed by Klevansky (1992).

[29]The problem of incorporating NTLO effects systematically in the Bethe-Salpeter equation for Nambu-Goldstone bosons has to our knowledge not been investigated in the literature.

not to be endangered. For thus low cutoffs, the f_π relation of Dmitrašinović *et al.*, recalculated for the top-mode standard model (2.1), could give us at least an estimate of how NTLO contributions change the predicted mass $m_t^{\rm phys}$ as a function of cutoffs $\Lambda_{\rm f}$ and $\Lambda_{\rm b}$.

Hands, Kocić, and Kogut (1991, 1993), Derkachov *et al.* (1993), and Gracey (1993, 1994) calculated for various dimensions ($d \leqslant 4$) the NTLO contributions to critical exponents of the fields at the fixed points, i.e., at locations where the β function has a nontrivial zero. The implications of these works for physical predictions of four-dimensional NJLVL models with finite cutoffs are not clear and deserve investigation.

Akama (1996) investigated the NTLO contributions in the truncated top-mode standard model by considering the compositeness condition for the composite scalars. This condition says that the renormalization constants of the composite scalars and of their self-interactions are zero. Further, Lurié and Tupper (1993) had earlier investigated the compositeness condition by taking into account some of the NTLO effects. Akama, as well as Lurié and Tupper, concluded that the compositeness condition implies that NTLO contributions to the corresponding physical quantities for $N_{\rm c}=3$ are larger than the leading-$N_{\rm c}$ contributions and indicate that the $1/N_{\rm c}$ expansion is in trouble. The approaches used by these three authors are similar in spirit, but not identical, to the approach of Bardeen, Hill, and Lindner (1990). Akama, Lurié, and Tupper treated the top-mode standard model as a renormalizable Yukawa-type model without gauge bosons plus the compositeness condition. The implicit assumption was that the cutoffs are large: $\ln(\Lambda/E_{\rm ew}) \gg 1$. Thus their results are consistent with the conclusions of the papers described in Sec. IV.B and Sec. IV.C (Cvetič *et al.*, 1996; Cvetič, 1997): $\bar{t}t$ condensation can be described by the $1/N_{\rm c}$ expansion as long as $\Lambda \lesssim 1$ TeV, and this expansion may be endangered for $\Lambda > 1$ TeV.

Dyatlov (1996) investigated NTLO contributions to quark masses in an almost "flavor-democratic" $[U(1)^n \times U(1)^n \times SU(N_{\rm c})]$ NJLVL model. It was based partly on his phenomenological framework for dynamical generation of quark mass hierarchy and CKM mixing (Dyatlov, 1992, 1993). He ensured validity of the Goldstone theorem at the NTLO level. However, it appears that he accounted at the NTLO level only for effective renormalization contributions to the quark masses, i.e., contributions corresponding to the (1-PI) diagrams of Fig. 5(a). The tadpole-type (1-PR) NTLO diagrams in the gap equation which would correspond to the derivative $\partial \Xi^{(1)}/\partial \langle \mathcal{H} \rangle_0^2/N_{\rm c}$ [cf. Eqs. (4.7) and (4.10)] were apparently not included in his analysis (diagrams corresponding to $\Xi^{(1)}/N_{\rm c}$ are shown in Fig. 4).

Preparata and Xue (1996a, 1996b) studied analytically $\bar{q}q$ condensation in an NJLVL framework on a lattice with finite lattice constant the $a_{\rm Planck} \sim 10^{-33}$ cm. They employed a lattice version of a leading-$N_{\rm c}$ (quark-loop) gap equation. They then calculated the resulting vacuum energy $\Delta E_{\rm vac}$, via the effective Wilson action over the ground state, as a function of the number of quark generations $N_{\rm g}$ acquiring nonzero dynamical masses. When they included in $\Delta E_{\rm vac}$ the (NTLO) contributions of composite (pseudo)scalars, they were able to show that the energetically preferred configuration is that with $N_{\rm g}=1$. Thus there is only one (third) quark generation with dynamical masses. The conclusion depends critically on inclusion of NTLO contributions to $\Delta E_{\rm vac}$. It appears that the results of Preparata and Xue would become more consistent if they included the NTLO contributions in their gap equation as well. An intriguing feature of their analysis is the disappearance of (composite) Higgs bosons from the low-energy spectrum ($M_H \sim E_{\rm Planck}$).

In the framework of technicolor theories, Appelquist, Lane, and Mahanta (1988), Mahanta (1989), and Kamli and Ross (1992) investigated corrections beyond the (improved) ladder approximation in the variational version of the gap equation (i.e., the Dyson-Schwinger equation). Appelquist, Lane, and Mahanta (1988) performed a two-loop calculation for the dynamical fermion mass $\Sigma(\bar{p}^2)$ in the nonrunning limit of $\alpha_{\rm technicolor}$, using the linearized form of the Dyson-Schwinger equation [i.e., $\bar{p}^2 \gg \Sigma^2(\bar{p}^2)$ was taken for large $|\bar{p}| > \Lambda_{\rm technicolor} \sim E_{\rm ew}$]. They restricted themselves to two specific gauges (the Landau gauge $\xi=0$; and $\xi=-3$). They showed semianalytically that the two-loop corrections ($\sim \alpha^2_{\rm technicolor}$) under such assumptions result in negative corrections to $\alpha^{\rm crit}_{\rm technicolor}$ not larger than 20% [cf. Eq. (4.15) for the top-mode standard model, where the NTLO corrections to an analogous four-quark parameter $a_{\rm crit}$ are positive and about 150%]. Hence they argued that ladder (one-loop) results in technicolor theories can in general be trusted. Mahanta (1989) employed the same approximation of the linearized Dyson-Schwinger and nonrunning $\alpha_{\rm technicolor}$ and worked in the Landau gauge. He showed that $\Sigma(\bar{p}^2) \propto |\bar{p}|^{-1}$ to all orders in $\alpha_{\rm technicolor}$ when $\alpha_{\rm technicolor} = \alpha^{\rm crit}_{\rm technicolor}$. Thus the anomalous dimension γ_m of the technifermion condensate[30] $\langle \bar{\Psi}_{\rm technicolor} \Psi_{\rm technicolor} \rangle_0$ is relatively large $\gamma(\alpha^{\rm crit}_{\rm technicolor})=1$ [since $\Sigma(\bar{p}^2) \propto |\bar{p}|^{\gamma_m - 2}$]. This implies that the hierarchy of the extended technicolor ($\Lambda_{\rm ETC} \gg \Lambda_{\rm technicolor}$) that suppresses the flavor-changing neutral currents is not just an artifact of the ladder approximation.

Kamli and Ross (1992), on the other hand, investigated numerically the gauge dependence of the one-plus two-loop contributions to the Dyson-Schwinger equations in technicolor theories. They did not invoke any further approximations (in contrast to Appelquist *et al.* and Mahanta). They worked in the general covariant R_ξ gauge and showed that F_π (obtained by using the Pagels-Stokar relation which is *leading*-$N_{\rm c}$) is only moderately dependent on the ξ parameter, but the technifermion condensate $\langle \bar{\Psi}_{\rm technicolor} \Psi_{\rm technicolor} \rangle_0$ depends drasti-

[30] $\gamma_m(\alpha_{\rm technicolor})$ is defined by: $d \ln \langle \bar{\Psi}^{(\mu)}_{\rm technicolor} \Psi^{(\mu)}_{\rm technicolor} \rangle_0 / d \ln \mu \equiv \gamma_m[\alpha_{\rm technicolor}(\mu)]$.

cally on ξ. Hence they argued that the predictability of perturbative expansions in powers of $\alpha_{\text{technicolor}}$ in the Dyson-Schwinger approach is doubtful.

It should be stressed that the conclusions of Appelquist, Lane, and Mahanta; Mahanta; and Kamli and Ross do not apply to the top-mode-type standard models of $\bar{t}t$ condensation. These authors investigated exchanges of *massless* (techni)gluons and did not derive the NTLO terms of the $1/N_c$ ($1/N_{\text{technicolor}}$) expansion but rather the NTLO terms of an expansion in powers of α ($\alpha_{\text{technicolor}}$). Top-mode-type interactions like Eqs. (2.1) and (3.2) are usually assumed to arise from exchange of *massive* particles (e.g., massive gauge bosons with mass $M \sim \Lambda \gtrsim 1$ TeV). They lead to a tightly bound $\langle \bar{t}t \rangle_0$ condensate ($\gamma_m \simeq 2$), in contrast to the more weakly bound $\langle \bar{\Psi}_{\text{technicolor}} \Psi_{\text{technicolor}} \rangle_0$ ($\gamma_m \simeq 1$)—see also the discussion in Sec. IX.A.1.

Smith, Jain, and McKay (1995) investigated Dyson-Schwinger equations for quarks within the minimal standard model in the ladder approximation. Interestingly, they found out that exchanges of (elementary) scalars contribute to the heavy-quark self-energies $\Sigma_q(\bar{p}^2)$ more strongly than do exchanges of gluons, especially for large $|p| > m_q$ ($\sim 10^2$ GeV). This would suggest that in top-mode-type standard models exchanges of tightly bound composite scalars (Higgs $\sim \bar{t}t$, etc.) are more important than other (gluon) exchanges. This was explicitly confirmed in the work described in Secs. IV.C and IV.D.

V. COMPARISONS OF VARIOUS APPROACHES IN THE MINIMAL FRAMEWORK

A. Initial remarks

The next two subsections concentrate basically on relations between the two dominant computational approaches to the $\bar{t}t$ condensation mechanism: the RG approach (Bardeen, Hill, and Lindner; Sec. II) and the Dyson-Schwinger+Pagels-Stokar approach in the leading-N_c approximation (Miransky, Tanabashi, and Yamawaki; Sec. III). Several similarities and differences are pointed out. We should stress that the various computational approaches are equivalent in principle. Differences arise due to the approximations made, as is also suggested by the results discussed in this section. Advantages and drawbacks of the two methods have been mentioned in Secs. II and III. To reiterate briefly, the RG approch is simpler; it properly sums up leading-log effects and accounts for some of the effects beyond the leading N_c. The actual condensation mechanism is indirectly accounted for by a compositeness boundary condition. The predictions rely heavily on the infrared fixed-point behavior of the RG equations and are thus reliable for high compositeness scales $\Lambda \gtrsim 10^8$ GeV. The Dyson-Schwinger+Pagels-Stokar approach is more complicated; it has been worked out only at the leading-N_c level and suffers from gauge dependence. In principle, it deals with the condensation mechanism (e.g., NJLVL terms) directly and is also applicable for low Λ's.

B. Preliminaries—renormalization-group solutions in closed form

To compare the RG equation with the Dyson-Schwinger+Pagels-Stokar approach (in the leading-N_c approximation), it is useful to write down first an analytic approximate formula for the one-loop running Yukawa parameter $g_t(\mu)$ and mass $m_t(\mu)$ in the minimal standard model. This approximation consists of neglecting the small contributions from transverse components of the electroweak gauge bosons ($g_1 \mapsto 0$ and $g_2 \mapsto 0$). Using the one-loop RG solution (3.6) for $\alpha_3(\mu)$ (for $\mu \geqslant m_t^{\text{phys}}$), one can solve Eq. (2.16) for the Yukawa parameter $g_t(\mu)$ explicitly:

$$g_t^2(\mu) = \frac{(A-1)}{A} \frac{2(4\pi)^2}{(N_c+1.5)} \vartheta_s(\mu)^A \times \{[\vartheta_s(\mu)^{A-1} - \vartheta_s(m_t)^{A-1}] + c(m_t)\}^{-1}, \tag{5.1}$$

where

$$\vartheta_s(\mu) \equiv \frac{\alpha_3(\mu)}{4\pi} \frac{3(N_c^2-1)}{2N_c} = \frac{A}{2\ln(\mu^2/\Lambda_{\text{QCD}}^2)} \quad (\text{for } \mu > m_t^{\text{phys}}), \tag{5.2}$$

$$A(N_c) = \frac{9(N_c^2-1)}{N_c(11N_c-12)} \sim N_c^0,$$

$$c(m_t) = \frac{(A-1)}{A} \frac{2(4\pi)^2}{(N_c+1.5)} \frac{\vartheta_s(m_t)^A}{g_t^2(m_t)}, \tag{5.3}$$

and $\Lambda_{\text{QCD}} \approx 51$ MeV. Inserting this $g_t^2(\mu)$ into the one-loop minimal standard model RG equation for the evolution of the VEV $v(\mu)$ (Arason *et al.*, 1992), we obtain

$$16\pi^2 \frac{d \ln v(\mu)}{d \ln \mu} = -N_c g_t^2(\mu) + \frac{9}{4} g_2^2(\mu) + \frac{3}{4} g_1^2(\mu). \tag{5.4}$$

Again setting $g_1, g_2 \mapsto 0$, we obtain

$$v^2(\mu) \equiv \langle H^{(\mu)} \rangle_0^2 = v^2(m_t) \{1 - [\vartheta_s(m_t)^{A-1} - \vartheta_s(\mu)^{A-1}]/c(m_t)\}^{N_c/(N_c+1.5)}, \tag{5.5}$$

where $c(m_t)$ is given in Eq. (5.3). Combining Eqs. (5.1) and (5.5), we obtain $m_t^2(\mu) \equiv g_t^2(\mu) v^2(\mu)/2$

$$m_t^2(\mu)=m_t^2\left[\frac{\ln(\mu/\Lambda_{\rm QCD})}{\ln(m_t/\Lambda_{\rm QCD})}\right]^{-A}\left\{\frac{c(m_t)}{c(m_t)-[\vartheta_s(m_t)^{A-1}-\vartheta_s(\mu)^{A-1}]}\right\}^{[1-(N_c)/(N_c+1.5)]}. \tag{5.6}$$

The mass m_t denotes here throughout $m_t(m_t)$ (the renormalized mass); constant A and $c(m_t)$ are given in Eq. (5.3), and the QCD parameter ϑ_s in Eq. (5.2).

C. Comparisons at the leading-N_c level

When only quark loops are taken into account in the RG equation for g_t (2.16), the only term kept on the right of that equation is $N_c g_t^3(\mu)$ because this is the term corresponding to the quark-loop diagram of Fig. 6. Therefore, the RG structure (2.16) implies that the quark-loop approximation is leading-N_c with no QCD (and $g_1,g_2\mapsto 0$). The mass $m_t(\mu)$ in this case is nonrunning, as can be seen from the RG solution (5.6) if we use $A\mapsto 0$ (to exclude QCD) and replace $(N_c+1.5)$ with N_c. Hence the RG approach gives in the quark-loop approximation the same behavior for $m_t(\mu)$ as the approach with the Dyson-Schwinger equation in the quark-loop approximation (3.7). Moreover, the approach with the "hard-mass" effective potential and with subsequent mass renormalization (Sec. IV) gave mass renormalization effects (when QCD was switched off) only at the NTLO level of the $1/N_c$ expansion (4.12). Hence all three approaches at the level of the quark-loop approximation give consistently a nonrunning dynamical m_t.

Further, in the quark-loop approximation the Pagels-Stokar relation gave the same relation (3.13) between the condensation cutoff Λ, m_t, and F_π as did the RG approach. Namely, keeping in the RG equation (2.16) only quark-loop terms [$g_i\mapsto 0$ $(i=1,2,3)$ and $(N_c+1.5)\mapsto N_c$] and taking into account the compositeness condition $g_t(\Lambda)=\infty$, we arrive precisely at the quark-loop effective action solution (2.10), and the latter is in turn identical with Eq. (3.13) of the Dyson-Schwinger+Pagels-Stokar approach in the quark-loop approximation once we impose the minimal framework condition for the full DEWSB: $F_\pi=v(\approx 246$ GeV). The effective potential approach of Sec. IV, however, did not contain any relations analogous to the Pagels-Stokar equation and thus cannot be compared with it.

When including, in addition to quark loops, also the (one-gluon-exchange) QCD contributions in the Dyson-Schwinger+Pagels-Stokar approach, we remain formally in the leading-N_c approximation, as argued in Sec. III. The same is true in the RG approach. Namely, the QCD term on the right side of the RG equation for g_t (2.16) is of the same leading-N_c order as the quark-loop term $N_c g_t^3$, because $g_3^2=\mathcal{O}(1/N_c)$ [cf. Eq. (3.6)], and $g_t^2=\mathcal{O}(1/N_c)$ [the latter can be seen by solving the RG equation (2.16) explicitly in the quark-loop approximation between the Landau pole Λ and μ, giving exactly Eq. (2.10)]. The one-loop RG result (5.6) in this approximation [$(N_c+1.5)\mapsto N_c$; and $A=8/7$ to include QCD] gives the QCD-run mass $m_t(\mu)$,

$$m_t^2(\mu)=m_t^2(m_t)\left[\frac{\ln(\mu/\Lambda_{\rm QCD})}{\ln(m_t/\Lambda_{\rm QCD})}\right]^{-8/7}, \tag{5.7}$$

in agreement with the solution of the nonperturbative Dyson-Schwinger equations (3.8) and (3.9) in the asymptotic μ region $m_t(m_t)\ll\mu\ll\Lambda$. Why does the *perturbative* leading-N_c RG solution (5.7) agree with solution (3.8) of the *nonperturbative* leading-N_c Dyson-Schwinger approach (3.4)? A way to explain this: in the leading-N_c Dyson-Schwinger approach of Eq. (3.4), the truly nonperturbative physics is contained in the four-quark contribution ($\propto\kappa_t\sim 1$), which is not directly responsible for the running of $m_t(\mu)$—only the weaker QCD contribution causes $\Sigma_t\equiv m_t$ to run (slowly). In the approach of Sec. IV, QCD mass-renormalization effects are given in Eq. (4.11)—they are perturbative and can be reproduced to the leading order in $\ln(\varepsilon^{-2})$ and in α_3 from the RG solution (5.7):

$$\frac{m_t^2}{\Lambda^2}-\frac{m_t^2(\Lambda)}{\Lambda^2}=\frac{m_t^2}{\Lambda^2}\left[\frac{A}{\ln(m_t^2/\Lambda_{\rm QCD}^2)}\ln(\Lambda^2/m_t^2)+\cdots\right], \tag{5.8}$$

where $m_t=m_t(m_t)$ is the renormalized mass and $A=8/7$ for $N_c=3$. The dots represent corrections smaller than 50% (25%) when $\Lambda<5$ TeV ($\Lambda\leqslant 1$ TeV). Using Eq. (5.3) for A, we see that the dominant term in the above RG expansion is equal to the dominant $\ln(\varepsilon^{-2})$ term in Eq. (4.11), if we set there $\varepsilon=m_t^{\rm ren}/\Lambda$ and $\Lambda_{\rm b}=\Lambda_{\rm f}=\Lambda$. Therefore all three approaches at the leading-N_c level (i.e., quark loop+QCD) give similar behavior for $m_t(\mu)$.

Further, at the leading-N_c level, the RG approach and the Pagels-Stokar equation give approximately the same relation between the VEV, m_t, and the cutoff Λ. In the RG approach of Sec. II, this relation is given implicitly by Eq. (5.1), when, in addition, the RG compositeness condition is used, i.e., $g_t(\Lambda)=\infty\Rightarrow[\vartheta_s(\Lambda)^{A-1}-\vartheta_s(m_t)^{A-1}]+c(m_t)=0\Rightarrow$

$$g_t^2(m_t)\left(\equiv\frac{2m_t^2}{v^2}\right)=\frac{(A-1)}{A}\,\frac{2(4\pi)^2}{(N_c+1.5)}\,\frac{\vartheta_s(m_t)^A}{[\vartheta_s(m_t)^{A-1}-\vartheta_s(\Lambda)^{A-1}]}. \tag{5.9}$$

This analytic expression was first obtained by Marciano (1990), in the realistic case $N_c=3$ and $n_q=6$ ($A=8/7$); see Eq. (2.25). It was also reproduced by Gribov (1994) within his leading-N_c framework (see Sec. III.E). In the Dyson-Schwinger+Pagels-Stokar approach of Sec. III, Eq. (5.9) can be obtained in the following way: take for the dynamical mass $\Sigma_t(\bar{p}^2)=m_t(|\bar{p}|)$ the asymptotic solution (3.8) and (3.9) of the Dyson-Schwinger integral equation (3.4) [equal to the RG solution (5.7)] and insert it in the Pagels-Stokar relation (3.12). There, neglect the term with $d\Sigma_t/d\bar{p}^2$ due to the slow $\bar{p}^2$ dependence and set $\Sigma_b=0$. Neglect contributions from $\bar{p}^2 \lesssim m_t^2$, i.e., approximately $[\bar{p}^2+\Sigma_t^2(\bar{p}^2)]^{-2}\approx[\bar{p}^2]^{-2}$. The resulting integrand can be analytically integrated:

$$F_\pi^2 \approx \frac{N_c}{8\pi^2}\int_{m_t^2}^{\Lambda^2} d\bar{p}^2 \frac{\Sigma_t^2(\bar{p}^2)}{\bar{p}^2}$$

$$= \frac{N_c m_t^2}{8\pi^2}\int_{m_t^2}^{\Lambda^2}\frac{d\bar{p}^2}{\bar{p}^2}\left[\frac{\ln(\bar{p}^2/\Lambda_{\rm QCD}^2)}{\ln(m_t^2/\Lambda_{\rm QCD}^2)}\right]^{-A}$$

$$= \frac{N_c m_t^2}{16\pi^2}\frac{A}{(A-1)}\frac{[\vartheta_s(m_t)^{A-1}-\vartheta_s(\Lambda)^{A-1}]}{\vartheta_s(m_t)^A}. \quad (5.10)$$

This is now equivalent to the RG result (5.9) at the leading-N_c level [when $(N_c+1.5)\mapsto N_c$ in Eq. (5.9)], keeping in mind that $F_\pi=v$. We note that this equivalence proof at the leading-N_c level is valid only when the cutoff Λ is high: $\ln(\Lambda/m_t)\gg 1$. Anyway, the full RG approach of Bardeen, Hill, and Lindner (1990) has predictive power only for large $\Lambda \gtrsim 10^8$ GeV.

The RG approach of Bardeen, Hill, and Lindner (1990) and the Pagels-Stokar+Dyson-Schwinger approach of Miransky, Tanabashi, and Yamawaki (1989a, 1989b), at the leading-N_c level for which the latter method had been carried out, were shown analytically to be approximately equivalent by Blumhofer, Dawid, and Lindner (1995); this similarity was reemphasized by Yamawaki (1995). It was also shown earlier to hold quite well numerically by Barrios and Mahanta (1991), who obtained good agreement (up to a few percent) of the predicted masses m_t of both approaches at the leading-N_c level for almost all cutoffs, even for cutoffs as low as 100 TeV. However, we recall that the RG approach at the leading-N_c level does not show infrared fixed-point behavior at such low cutoffs.

In fact, it turns out that for $\Lambda \lesssim 10^8$ GeV the leading-N_c RG predictions for $m_t(m_t)$ decrease considerably (by several percent) if the compositeness condition is taken to be $k_t \equiv g_t^2(\Lambda)/(4\pi)=1$, 2/3, or 1/3, instead of ∞. This effect is more pronounced at lower Λ. The results of the Dyson-Schwinger+Pagels-Stokar approach, given in Table II [$m_t \approx \Sigma_t(0)$], do not suffer from the mentioned ambiguity problems of the RG approach stemming from the choice of various boundary values $g_t(\Lambda)$, not even for low Λ's. This is because the Dyson-Schwinger+Pagels-Stokar approach actually treats directly the top-mode standard-model physics of Eq. (2.1) or Eq. (3.2) responsible for the condensation. In contrast to the RG approach, the Dyson-Schwinger+Pagels-Stokar approach in the minimal framework predicts, for any given Λ, not just values $m_t(\mu)$ (and $m_t^{\rm phys}$), but also the value of the dimensionless four-quark parameter $a \equiv G\Lambda^2 N_c/(8\pi^2)$ of the top-mode standard model (2.1) [or κ_t of Eq. (3.2)]. However, we should keep in mind that the Dyson-Schwinger+Pagels-Stokar approach suffers from the problem of gauge noninvariance (see Sec. III.E) and the difficulties of extending it beyond the leading-N_c level (see Sec. IV).

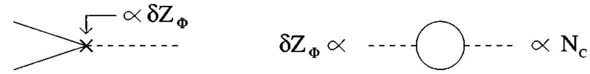

FIG. 6. Loop contribution of t to the evolution of g_t. This is the origin of the term $N_c g_t^3(\mu)$ on the right-hand side of the minimal-standard-model RG equation (2.16). Solid lines represent t, and dashed ones (neutral) scalars. δZ_Φ is the change in the scalar field renormalization when the cutoff is changed by $\delta\mu$. δZ_Φ is caused by the t-loop effect and is hence $\propto N_c$.

We have not yet discussed the mass M_H of the composite Higgs in the comparison of the RG and Dyson-Schwinger+Pagels-Stokar approaches. Chesterman, King, and Ross (1991) investigated relation between predictions of the RG and Dyson-Schwinger approaches for the mass M_H in the leading-N_c approximation. They inserted the solution $\Sigma_t(\bar{p}^2)$ of the (improved lattice) Dyson-Schwinger equation of the truncated top-mode standard model (2.1) into the Bethe-Salpeter amplitude for $\bar{t}t \to \bar{t}t$ scattering. In this amplitude, they included dominant QCD corrections in the Landau gauge and searched for poles $\bar{p}^2=-X^2$ in the scalar channel, identifying X with the physical mass M_H. As a consistency check, they showed that the pseudoscalar channel gives a pole at $\bar{p}^2=0$. Quantum chromodynamics corrections pushed down the prediction $M_H=2m_t$ of the quark-loop approximation. They compared the results with those of the Bardeen, Hill, and Lindner RG approach at the leading-N_c level, i.e., with the solutions $m_t^2(m_t)=g_t^2(m_t)v^2/2$ and $M_H^2=v^2\lambda(\mu=M_H)$ stemming from the leading-N_c version of minimal-standard-model RG equations (2.16) and (2.17):

$$16\pi^2\frac{dg_t(\mu)}{d\ln\mu}=\left[N_c g_t^2(\mu)-3\frac{(N_c^2-1)}{N_c}g_3^2(\mu)\right]g_t(\mu), \quad (5.11)$$

$$16\pi^2\frac{d\lambda(\mu)}{d\ln\mu}=-4N_c g_t^4(\mu)+4N_c\lambda(\mu)g_t^2(\mu), \quad (5.12)$$

where $N_c=3$ is the actual number of colors, $g_3^2=4\pi\alpha_3$ is the one-loop solution (3.6), and the usual Bardeen, Hill, and Lindner compositeness boundary conditions (2.15) are taken at $\mu=\mu_* \approx \Lambda$. Chesterman *et al.* considered only cutoffs $\Lambda \leq 10^7$ GeV, because at larger values they ran into numerical instabilities due to subtractions and cancellations of very large terms $\sim\Lambda^2$. They obtained values for M_H that differ from those of the leading-N_c Bardeen, Hill, and Lindner approach by less than 10% for $10^5\,{\rm GeV} \leq \Lambda \leq 10^7$ GeV, and for $\Lambda=10^4$ GeV they are lower by about 15%. These results are given in Table V. For comparison, the corresponding values of M_H as obtained from Gribov's leading-N_c sum rule (3.15) were also included in the table:

TABLE V. Predicted masses m_t and M_H in the leading-N_c approximation (quark loop+QCD). Entries on the first two lines are from the Dyson-Schwinger+Pagels-Stokar approach, and the Bethe–Salpeter amplitude of Chesterman, King, and Ross (1991), while the next two lines contain results of the leading-N_c RG approach of Eqs. (5.11) and (5.12). The results were taken from the aforementioned work of Chesterman, King, and Ross (1991), where apparently $\Lambda_{QCD}\approx 0.122$ GeV $[\Rightarrow \alpha_3(M_Z)\approx 0.136]$ was used. For comparison, the corresponding values of M_H as determined by Gribov's leading-N_c sum rule [Eqs. (3.15), (5.13)] were also included. All entries are in GeV.

Λ [GeV]	10^4	10^5	10^6	10^7
$m_t=\Sigma_t(0)$ (Dyson-Schwinger+Pagels-Stokar, l-N_c)	594	448	391	357
M_H (Bethe-Salpeter, l-N_c)	1062	769	634	617
$m_t(m_t)$ (RGE, l-N_c)	577	446	391	358
M_H (RGE, l-N_c)	1246	827	676	592
M_H (Gribov)	1068	778	655	578

$$M_H^2=\frac{7}{3\pi^2}\frac{m_t^4}{v^2}\ln\left(\frac{m_t}{\Lambda_{QCD}}\right)\left\{1-\left[\frac{\ln(m_t/\Lambda_{QCD})}{\ln(\Lambda/\Lambda_{QCD})}\right]^{9/7}\right\}, \tag{5.13}$$

where the shorthand notation $m_t(m_t)\equiv m_t$ is used.

D. Mass-dependent renormalization-group approach

After the paper by Bardeen, Hill, and Lindner (1990) had appeared, another group (Bando *et al.*, 1990) pointed out that their approach had a theoretical and practical deficiency because it employed the usual RG equations (2.16) and (2.17) of the $\overline{\text{MS}}$ scheme, which is mass independent. Bando *et al.* (1990) started with the cutoff-dependent Lagrangian density (2.9) of the minimal standard model. Since they included $g_b(\mu)$, however, their discussion also applies to the truncated case $g_b\mapsto 0$, which we illustrate here. Compositeness of the scalar doublet $\Phi^{(\mu)}$ in Eq. (2.9) can be discerned at a large-energy Λ analogous to the discussion in Sec. II, Eqs. (2.1) and (2.12), where it was discussed in the quark-loop approximation. In general, a dynamical scalar isodoublet $\Phi^{(\mu)}$ is composite up to a large Λ $[\ln(\Lambda/E_{ew})\gg 1]$, where it decomposes into constituents, if an auxiliary (nondynamical) scalar isodoublet Φ can be defined as $\Phi=\Phi^{(\mu)}/\sqrt{Z_\Phi(\Lambda;\mu)}\equiv\Phi^{(\mu)}/\sqrt{\epsilon(\Lambda;\mu)}$, with $\epsilon(\Lambda;\mu)\equiv Z_\Phi(\Lambda;\mu)$ having the following two properties:

(1) In the relation $\Phi^{(\mu_1)}\equiv\Phi^{(\mu_2)}\sqrt{Z_\Phi(\Lambda;\mu_1)}/\sqrt{Z_\Phi(\Lambda;\mu_2)}$, the ratio of square roots is for $\mu_1,\mu_2\ll\Lambda$ independent of Λ, leading to the usual minimal-standard-model renormalization prescription.

(2) $Z_\Phi(\Lambda;\mu)\to 0$ when $\mu\to\Lambda$; this is the usual compositeness condition, resulting in the disappearance of the dynamical kinetic term of the scalar degrees of freedom when the UV cutoff μ for the Lagrangian density approaches the energy Λ where the composite scalar is supposed to fall apart into its constituent parts (2.5).

Also the quark fields in Eq. (2.9) are, in general, energy (cutoff) dependent: $\Psi\mapsto\Psi^{(\mu)}$ (though not in the quark-loop approximation). Hence $\Psi^{(\mu_1)}=\Psi^{(\mu_2)}\sqrt{Z_\Psi(\mu_1)}/\sqrt{Z_\Psi(\mu_2)}$. The quark fields appearing in the truncated top-mode standard model (2.1)–(2.3) at $\mu=\Lambda$, where the scalars decompose, are then $\Psi^{(\Lambda)}=\Psi^{(\mu)}\sqrt{Z_\Psi(\Lambda)}/\sqrt{Z_\Psi(\mu)}$, with still existing kinetic terms—quarks remain dynamical and do not "decompose" at Λ. The Lagrangian density at $\mu<\Lambda$ is

$$\begin{aligned}\mathcal{L}^{(\mu)}=&\,i\bar{\Psi}^{(\mu)}\not{D}\Psi^{(\mu)}+(D_\nu\Phi^{(\mu)})^\dagger D^\nu\Phi^{(\mu)}\\&-m_\Phi^2(\mu)\Phi^{(\mu)\dagger}\Phi^{(\mu)}-\frac{\lambda(\mu)}{2}(\Phi^{(\mu)\dagger}\Phi^{(\mu)})^2-g_t(\mu)\\&\times[\bar{\Psi}_L^{(\mu)}\tilde{\Phi}^{(\mu)}t_R^{(\mu)}+\text{H.c.}]+\Delta\mathcal{L}_{\text{gauge}}^{(\mu)},\end{aligned}\tag{5.14}$$

where $\Psi^T=(t,b)$, and $\tilde{\Phi}^{(\mu)}=i\tau_2\Phi^{(\mu)\dagger T}$. D_ν and $\not{D}$ are the usual covariant derivatives, and $\Delta\mathcal{L}_{\text{gauge}}^{(\mu)}$ contains terms without the scalars and quarks. This density at μ can then be written in terms of the auxiliary (nondynamical) scalar isodoublet Φ:

$$\begin{aligned}\mathcal{L}^{(\mu)}=&\,i[Z_\Psi(\mu)/Z_\Psi(\Lambda)]\bar{\Psi}^{(\Lambda)}\not{D}\Psi^{(\Lambda)}\\&+\epsilon(\Lambda;\mu)(D_\nu\Phi)^\dagger D^\nu\Phi-\tilde{m}_\Phi^2\Phi^\dagger\Phi\\&-\frac{\tilde{\lambda}}{2}(\Phi^\dagger\Phi)^2-\tilde{g}_t[Z_\Psi(\mu)/Z_\Psi(\Lambda)]\\&\times[\bar{\Psi}_L^{(\Lambda)}\tilde{\Phi}t_R^{(\Lambda)}+\text{H.c.}]+\Delta\mathcal{L}_{\text{gauge}}^{(\mu)},\end{aligned}\tag{5.15}$$

Here, the quantities $\tilde{m}_\Phi$, $\tilde{\lambda}$, and $\tilde{g}_t$, are finite and independent of μ. The running parameters of Eq. (5.14) are related to them through

$$\begin{aligned}&g_t(\mu)=\tilde{g}_t\epsilon^{-1/2}(\Lambda;\mu),\quad m_\Phi^2(\mu)=\tilde{m}_\Phi^2\epsilon^{-1}(\Lambda;\mu),\\&\lambda(\mu)=\tilde{\lambda}\epsilon^{-2}(\Lambda;\mu),\end{aligned}\tag{5.16}$$

where we keep in mind that $\epsilon(\Lambda;\mu)\to 0$ when $\mu\to\Lambda$, as a consequence of compositeness. At $\mu=\Lambda$ ($\epsilon=0$) and for $\tilde{\lambda}=0$, the above Lagrangian density reduces to the truncated top-mode standard model (2.3), which is equivalent to the NJLVL type of interaction (2.1). Therefore the special case $\tilde{\lambda}=0$ represents the case of NJLVL-type $\bar{t}t$ condensation at high Λ $[\ln(\Lambda/E_{ew})\gg 1]$, and $\tilde{\lambda}\neq 0$ represents some other more general framework of $\bar{t}t$ condensation at high Λ.

The problem pointed out by Bando *et al.* (1990) is that

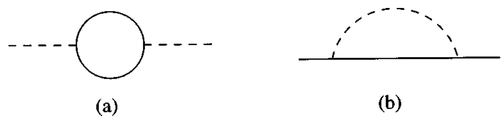

FIG. 7. Feynman diagrams responsible for the terms on the left-hand side of the RG equations (2.16) and (5.17): (a) the leading-N_c term $N_c g_t^3$ ($N_c g_t^2$); (b) the next-to-leading term $1.5 g_t^3$ ($1.5 g_t^2 f$). Solid lines are top quarks, dashed ones are (neutral) components of the composite isodoublet scalar $\Phi^{(\mu)}$.

$m_\Phi^2(\mu)$ becomes large when $g_t(\mu)$ becomes large for $\mu \to \Lambda$, as can be seen explicitly in Eq. (5.16). In fact, when $g_t(\mu) \to \infty$ for $\mu \to \Lambda$, then $m_\Phi^2(\mu) \to \infty$ in such a way that the ratio $[m_\Phi^2(\mu)/(\mu^2 g_t^2(\mu))]$. remains finite, equal to $[\tilde{m}_\Phi^2/(\Lambda^2 \tilde{g}_t^2)]$ in the limit. Therefore the effect of the scalar exchange of Fig. 7(b), which resulted in the NTLO term $1.5 g_t^3$ on the left of the RG equation (2.16), should be screened out for $\mu \sim \Lambda$ because of the decoupling due to the large mass $m_\Phi(\mu)$. This is not taken into account in the $\overline{\mathrm{MS}}$ scheme RG equation (2.16). Therefore Bando *et al.* used the mass-dependent renormalization devised by Georgi and Politzer (1976), in which contributions of the propagators of heavy particles are suppressed. The scalar exchange term was then suppressed by a factor $f(c) \sim c + \mathcal{O}(c^2)$, where $c \equiv \mu^2/m_\Phi^2(\mu) \to 0$ when $\mu \to \Lambda$. The modified one-loop RG equation for g_t then read

$$8\pi^2 \frac{d \ln g_t^2(\mu)}{d \ln \mu} = N_c g_t^2(\mu) + 1.5 g_t^2(\mu) f[c(\mu)] - G_t(\mu), \tag{5.17}$$

where

$$G_t(\mu) = 3\frac{(N_c^2-1)}{N_c} g_3^2(\mu) + \frac{9}{4} g_2^2(\mu) + \frac{17}{12} g_1^2(\mu). \tag{5.18}$$

In order to see from another perspective that a shielding factor $f[c(\mu)]$ is needed in Eq. (5.17), we also write down the RG equation for the quantity $c(\mu) = \mu^2/m_\Phi^2(\mu)$:

$$8\pi^2 \frac{d \ln c(\mu)}{d \ln \mu} = 16\pi^2 - [1 + c(\mu)] N_c g_t^2(\mu). \tag{5.19}$$

Figure 7(a) shows the origin of this RG equation and of the leading-N_c term $N_c g_t^2$ in Eq. (5.17) (see also Fig. 6). Incidentally, Eq. (5.19) tells us that $m_\Phi^2(\mu) \to \infty$ when $\mu \to \Lambda$, because $g_t^2(\mu) \sim [\ln(\Lambda/\mu)]^{-1}$ for $\mu \sim \Lambda$ by Eq. (5.17). Adding the RG equations (5.19) and (5.17) leads to

$$8\pi^2 \frac{d \ln[m_\Phi^2(\mu)/g_t^2(\mu)]}{d \ln \mu} = \frac{\mu^2}{m_\Phi^2(\mu)} N_c g_t^2(\mu) - 1.5 g_t^2(\mu) f[c(\mu)] + G_t(\mu). \tag{5.20}$$

From here we can see that the $\overline{\mathrm{MS}}$ case $[f(c) \equiv 1]$ implies $m_\Phi^2(\mu)/g_t^2(\mu) \to 0$ when $\mu \to \Lambda$. This is so because (a) $g_t^2(\mu) \propto [\ln(\Lambda/\mu)]^{-1}$ for $\mu \sim \Lambda$; (b) $\mu^2/m_\Phi^2(\mu) \to 0$ when $\mu \to \Lambda$; (c) the second term on the right-hand side of Eq. (5.20) consequently becomes dominant for $\mu \sim \Lambda$. However, from the compositeness conditions (5.16) we should have $m_\Phi^2(\mu)/g_t^2(\mu) \to \tilde{m}_\Phi^2/\tilde{g}_t^2$, which is finite and nonzero.[31] This contradiction shows from another perspective that the RG equation (2.16) of a mass-independent renormalization scheme is at $\mu \simeq \Lambda$ not consistent with a condensation scenario. On the other hand, if $f[c(\mu)] \sim c(\mu) \equiv \mu^2/m_\Phi^2(\mu) \to 0$ when $\mu \to \Lambda$, it is straightforward to check that Eq. (5.20) gives a finite nonzero ratio $m_\Phi^2(\mu)/g_t^2(\mu)$ when $\mu \to \Lambda$, in accordance with the compositeness conditions (5.16).

The precise behavior of the modification factor $f(c)$ depends on the choice of renormalization conditions for the scalar and the top-quark propagators. For a specific choice, Bando *et al.* (1990) obtained values of $m_t(m_t)$ which were, for $\Lambda = 10^{15}$ and 10^{10} GeV, higher than those of Bardeen, Hill, and Lindner by about 2.2% and 1.6%, respectively. However, for the lower cutoff $\Lambda = 10^4$ GeV, they obtained $m_t(m_t) = 405$ GeV, while the Bardeen, Hill, and Lindner value in this case is 455 GeV. Therefore it appears that the scalar mass decoupling effect is numerically important only for low cutoffs Λ, while for high cutoffs it is marginal—a consequence of the infrared fixed-point behavior of the RG equations for high cutoffs.

E. Possible effects of higher than six-dimensional operators

Soon after the appearance of the first work on $\bar{t}t$ condensation within the NJLVL models, several authors (Bastero-Gil and Pérez-Mercader, 1990; Suzuki, 1990a; Hasenfratz *et al.*, 1991; Mannan and King, 1991) questioned the validity of such a framework. They argued that four-quark effective interactions other than those of the NJLVL type (2.1) and (3.2), i.e., terms with dimension higher than six also contribute to the $\bar{t}t$ condensation. They argued that predictions for mass ratios and/or masses of the top quark m_t and of the composite Higgs boson M_H can, in principle, acquire any value, if only the coupling parameters of the higher than six-dimensional four-quark terms are chosen to be strong enough. Thus this generalized $\bar{t}t$ condensation mechanism becomes in principle, as unpredictive as the standard model. While Suzuki (1990a), and Hasenfratz *et al.* (1991) argued in the quark-loop approximation, Mannan and King (1991) extended Suzuki's work by including the (improved) ladder QCD contributions in the Dyson-Schwinger+Pagels-Stokar approach, and ter Veldhuis (1992) investigated two-loop RG equations using the quark-loop results of Suzuki as the UV boundary condition. Hasenfratz *et al.* emphasized their point additionally by the title of their paper ("The equivalence of the top quark condensate and the elementary Higgs

[31] In the truncated top-mode standard model (2.3), $\tilde{m}_\Phi = M_0$ and $\tilde{g}_t = M_0\sqrt{G}$, and thus the ratio is $1/G$ (finite and nonzero).

field''). Their paper had an immediate and strong impact in the $\bar{t}t$ condensation community. Further, a paper by Zinn-Justin (1991) argued similarly that large classes of composite models are not more predictive than those with explicit (elementary) bosons. To illustrate these claims, he considered the extended Gross-Neveu model near four dimensions.

Other authors (Badeen, 1990; Hill, 1991; Lindner, 1993) subsequently argued against the conclusions of Hasenfratz *et al.* One of the principal arguments is very transparent: the aforementioned additional coupling parameters are not arbitrary, but are determined by the underlying physics at energies $E \gtrsim \Lambda$. These parameters were defined as dimensionless, via multiplication with an appropriate power of Λ. Since the compositeness scale Λ is also the UV cutoff of the theory, it is the natural energy scale of the theory, and therefore it can be naively expected that the above-mentioned dimensionless parameters are ~ 1. For large cutoffs, when $\ln(\Lambda/m_t) \gg 1$, such parameters then modify predictions of the original (NJLVL) framework very little.

Strictly speaking, the effects of the additional operators can be calculated only when we know the underlying physics. Hill (1991) calculated approximately one such parameter (d in the notation by Hasenfratz *et al.*; he called it χ) in his renormalizable topcolor model of the underlying physics. Exchange of massive colorons ($M_B = \Lambda$) is responsible for the top-mode standard-model interaction term (2.1) of the NJLVL type. However, the box diagrams containing colorons contribute at leading N_c to an eight-dimensional four-quark operator and consequently to the χ parameter. His calculation gives $\chi \approx 1/8 = 0.125$, which is substantially smaller than 1. In the block-spin renormalization-group approach, even for cutoffs as low as 1 TeV, this would imply corrections to the masses m_t and M_H of only $\sim 10\%$, and for significantly higher cutoffs the corrections would be negligible. Hill thus expressed the belief that the conclusions of Hasenfratz *et al.* (1991) were not relevant in most of the realistic theories of underlying physics leading to $\bar{t}t$ condensation.

F. Deficiencies of the minimal framework—motivation for extensions

The minimal $\bar{t}t$ condensation framework represents an elegant solution to several problems that would otherwise be solved within the conventional minimal standard model in a rather *ad hoc* manner—it explains, in a dynamical way, simultaneously the mass of by far the heaviest known fermion (together with its Yukawa parameter) and the scalar sector leading to the EWSB. However, various approximate methods of calculation (perturbative RG equations, Dyson-Schwinger+Bethe-Salpeter in leading-N_c) consistently indicate that the phenomenologically acceptable values $\langle \mathcal{H} \rangle_0 \equiv v \approx 246$ GeV and $m_t \approx 175$ GeV cannot be simultaneously produced with this mechanism in the minimal framework. When the electroweak VEV v is adjusted to 246 GeV, m_t^{dyn} is too high by at least 20%, even when the compositeness scale Λ is very high ($\sim E_{\text{Planck}}$). An indication of this problem can be seen qualitatively by inspecting the quark-loop-approximated Pagels-Stokar relation (3.13)—when one requires $F_{\pi^0} = v$ ($=246$ GeV), one can obtain phenomenologically acceptable values of m_t in this crude approximation only for very high $\Lambda \sim 10^{13}$ GeV. Therefore there appears to be a need to relieve the $\bar{t}t$ condensate from the burden of being responsible for the entire EWSB, i.e., to allow $F_{\pi^0} < v$. This can be achieved either by introducing DEWSB frameworks with at least one additional condensate (e.g., $\bar{f}f$, where f is a fermion different from t), or by having an elementary Higgs in addition to the composite $\bar{t}t$ scalar. Both scalars together can then lead to the correct electroweak VEV $v \approx 246$ GeV, while keeping m_t^{dyn} at the value ≈ 175 GeV, and there is an additional possibility of reducing Λ substantially, thus even eliminating any need for fine tuning.

This almost automatically leads to extended frameworks of DEWSB. In Secs. VI and VII we discuss some *effective* extensions of the (effective) minimal framework which do not involve or do involve, respectively, an enlargement of symmetries. In Sec. VIII, on the other hand, we discuss some *renormalizable* models of underlying physics that, at lower energies, lead in general to extended effective frameworks of DEWSB and $\bar{t}t$ condensation.

VI. EXTENSIONS WITHOUT ENLARGING THE SYMMETRY GROUP

A. Composite two-Higgs-doublet scenarios

1. A general framework with more than one family

The basic idea that a NJLVL framework can lead to a composite Higgs doublet via $\bar{t}t$ condensation can be extended in a straightforward manner to scenarios with two composite Higgs doublets (composite 2HD). Suzuki (1990b) investigated this possibility for the general NJLVL scenario with the standard-model symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$, as written for example in Eq. (3.1). First he relabeled biquark combinations $\bar{q}_\alpha q_\beta$ with a single ''pair'' index $\mathcal{A} = (\alpha, \beta)$,

$$\mathcal{L}_{4q}^{(\Lambda)} = \sum_{\mathcal{A},\mathcal{B}=1}^{2n^2} G_{\mathcal{A}\mathcal{B}} J_{\mathcal{A}}^\dagger J_{\mathcal{B}},$$

$$\text{where} \quad J_{\mathcal{A}} = \begin{cases} \bar{\Psi}_L^{\alpha a} u_R^{\beta a} & (\mathcal{A} = 1, \ldots, n^2), \\ \bar{\Psi}_L^{c\alpha a} d_R^{c\beta a} & (\mathcal{A} = n^2+1, \ldots, 2n^2). \end{cases} \tag{6.1}$$

As in Eqs. (3.1) and (3.2), $\Psi^\alpha = (u^\alpha, d^\alpha)^T$, and (α, β) and a are family and color indices, respectively. $\Psi^{c\alpha} = (d^{c\alpha}, -u^{c\alpha})^T$ are the charge-conjugated fields of family α. Then the $2n^2 \times 2n^2$ Hermitian coupling matrix $G_{\mathcal{A}\mathcal{B}}$ was diagonalized by a unitary matrix K to obtain $\mathcal{L}_{4q}^{(\Lambda)} = G_D J_D^\dagger J_D$, with $J_D = K_{D\mathcal{A}} J_{\mathcal{A}}$, where D and $\mathcal{A}$ run from 1 to $2n^2$, and G_D are the (real) eigenvalues of $G_{\mathcal{A}\mathcal{B}}$.

Suzuki argued that one composite Higgs doublet is generated in the eigenchannel with the largest G_D (say, $G_{D=1}$), i.e., in the channel with the strongest attractive force. The mass eigenstate quarks are then obtained by diagonalizing the neutral component of $J_{D=1}=\Sigma K_{1\mathcal{A}} J_{\mathcal{A}}$ by using unitary rotations for the L and R components of the up-type and down-type quarks separately—a procedure analogous to that in the minimal standard model. In this way, the four-quark terms of the most attractive channel do not contribute any flavor-changing neutral currents (FCNC's), in accordance with experimental evidence. Suzuki then argued that, in this general framework, the second most attractive channel (corresponding to, say $D=2$) would analogously lead to the generation of the second composite Higgs doublet. This doublet, in the described general NJLVL framework, would generally give nonzero FCNC's in the basis of mass eigenstates of quarks. The author then discussed in detail the conditions under which the composite two-Higgs-doublet scenario results in suppressed FCNC's, using the quark-loop (bubble) approximation for gap equations and assuming $E_{\rm ew}/\Lambda \ll 1$.

2. The four-quark interaction picture vs the composite type-II 2HDSM picture

It can be shown that the general NJLVL-type of top-mode standard-model Lagrangian density for the third quark generation (3.2) leads at $\mu \ll \Lambda$ under certain conditions to an effective standard model with two Higgs doublets of type II. This is the model in which one Higgs isodoublet (Φ_u) is responsible for the generation of m_t, and the other (Φ_d) for the generation of m_b (Donoghue and Li, 1979; Hall and Wise, 1981):

$$\mathcal{L}^{(\mu)}_{\rm Yukawa}=-[\bar{\Psi}^{(\mu)}_L \tilde{\Phi}^{(\mu)}_u t^{(\mu)}_R+\text{H.c.}] -[\bar{\Psi}^{(\mu)}_L \Phi^{(\mu)}_d b^{(\mu)}_R+\text{H.c.}]. \tag{6.2}$$

To see this, let us start with the following "type-II 2HDSM" of Lagrangian density at the compositeness scale $\mu=\Lambda$, where Φ_u,Φ_d are auxiliary scalar isodoublet fields:

$$\mathcal{L}^{(\Lambda)}=-\mu_u^2 \Phi_u^\dagger \Phi_u-\mu_d^2 \Phi_d^\dagger \Phi_d-\mu_{ud}^2(\Phi_u^\dagger \Phi_d+\text{H.c.}) -[\bar{\Psi}_L \tilde{\Phi}_u t_R+\bar{\Psi}_L \Phi_d b_R+\text{H.c.}], \tag{6.3}$$

where $\Psi_L \equiv (t_L,b_L)^T$ and $\tilde{\Phi} \equiv i\tau_2 \Phi^{\dagger T}$. We omit the color indices and the UV energy cutoff superscripts (Λ). The Yukawa coupling parameters are normalized to 1 for convenience, by appropriate redefinition of Φ_u,Φ_d. If the mass parameters above satisfy certain constraints, the auxiliary isodoublets become dynamical—they develop kinetic and self-interaction terms and VEV's at $\mu<\Lambda$, thus leading to dynamical generation of m_t and m_b. The resulting low-energy theory is a composite type-II 2HDSM. To show the equivalence of Eq. (6.3) with the NJLVL density (3.2) of Sec. III, we employ equations of motion for the auxiliary fields, leading to

$$\tilde{\Phi}_u \equiv i\tau_2 \Phi_u^{\dagger T} =[-\mu_d^2(\bar{t}_R \Psi_L)+\mu_{ud}^2(i\tau_2)(\bar{b}_R \Psi_L)^{\dagger T}]/\mathcal{A}, \tag{6.4}$$

$$\Phi_d=[-\mu_u^2(\bar{b}_R \Psi_L)-\mu_{ud}^2(i\tau_2)(\bar{t}_R \Psi_L)^{\dagger T}]/\mathcal{A},$$
$$\text{with} \quad \mathcal{A}=[\mu_u^2 \mu_d^2-(\mu_{ud}^2)^2]. \tag{6.5}$$

In Eqs. (6.4) and (6.5), τ_2 is the second Pauli matrix acting on isospin components, and $(\bar{t}_R \Psi_L)^{\dagger T} =(\bar{t}_L t_R,\bar{b}_L t_R)^T$. Inserting Eqs. (6.4) and (6.5) into Eq. (6.3) leads to an equivalent Lagrangian density, which is of the NJLVL-type (i.e., four-quark interactions without derivatives):

$$\mathcal{L}^{(\Lambda)}=\{\mu_d^2(\bar{\Psi}^{ia}_L t^a_R)(\bar{t}^b_R \Psi^{ib}_L)+\mu_u^2(\bar{\Psi}^{ia}_L b^a_R)(\bar{b}^b_R \Psi^{ib}_L) +(-\mu_{ud}^2)[(\bar{t}^a_R t^a_L)(\bar{b}^b_R b^b_L) -(\bar{t}^a_R b^a_L)(\bar{b}^b_R t^b_L)+\text{H.c.}]\}/\mathcal{A}, \tag{6.6}$$

where the isospin (i) and color indices (a,b) are explicitly written. This density has the form (3.2) of Sec. III. Comparison gives relations between the parameters of Eqs. (3.2) and (6.3)

$$\kappa_t=\frac{\mathcal{B}}{\mu_u^2}, \quad \kappa_b=\frac{\mathcal{B}}{\mu_d^2}, \quad 2\kappa_{tb}=-\frac{\mathcal{B}\mu_{ud}^2}{\mu_u^2 \mu_d^2}$$
$$\text{where} \quad \mathcal{B}=\frac{N_c \Lambda^2}{8\pi^2}\frac{\mu_u^2 \mu_d^2}{[\mu_u^2 \mu_d^2-(\mu_{ud}^2)^2]}. \tag{6.7}$$

From the above formulas we see that

$$(8\pi^2)^2[\kappa_t \kappa_b-4\kappa_{tb}^2]=(N_c \Lambda^2)^2[\mu_u^2 \mu_d^2-(\mu_{ud}^2)^2]^{-1}(\neq 0). \tag{6.8}$$

This shows that the NJLVL-type of framework (3.2) can lead to a composite dynamical type-II 2HDSM only as long as $\kappa_t \kappa_b-4\kappa_{tb}^2 \neq 0$. When $\kappa_t \kappa_b-4\kappa_{tb}^2=0$ [note, $\kappa_t \neq 0$ and $\kappa_b=\kappa_{tb}=0$ is the case of truncated top-mode standard model (2.1)], this can lead only to an effective model with *one* (composite) scalar isodoublet—a composite minimal standard model (minimal framework).

Equations (6.4) and (6.5) imply that the generalized top-mode standard model (3.2) discussed in Sec. III leads to two dynamical composite scalar isodoublets made up of t and b quarks and with nonzero VEV's, as long as parameters κ_t, κ_b, and κ_{tb} satisfy some criticality bounds. In the framework of Eq. (3.2), conditions for the dynamic generation of heavy $m_t \sim 10^2$ GeV and light $m_b \approx 5$ GeV were investigated numerically by King and Mannan (1991a), who employed the Dyson-Schwinger+Pagels-Stokar approach (quark loop plus QCD contributions)—see discussion in Sec. III. This approach does not deal directly with the picture of dynamically generated Higgs doublets of type-II 2HDSM or minimal standard model, but only with the dynamically generated masses m_t and m_b. The results of King and Mannan show that the heavy $m_t^{\rm dyn}$ (i.e., a corresponding $\langle \Phi_u \rangle_0 \neq 0$ in the present picture) is generated when κ_t is at least a bit above a critical value $\kappa_{\rm crit}$

~ 1. The latter value is slightly dependent on κ_b and κ_{tb}, and both latter parameters satisfy $0 \leqslant \kappa_b < \kappa_t$ and $0 \leqslant \kappa_{tb} \ll \kappa_t$. Further, a smaller $m_b^{\rm dyn}$ (i.e., $\langle \Phi_d \rangle_0 \neq 0$ in the present picture) is generated when, for a given $\kappa_b < \kappa_t$, the mixing parameter κ_{tb} acquires a certain small positive value $\sim 10^{-2}$.

If the $U(1)_{\gamma_5}$-violating four-quark term in Eq. (3.2) is taken to be zero [$\kappa_{tb}=0$, hence by Eq. (6.7) $\mu_{ud}^2=0$], we see from Eqs. (6.4) and (6.5) that the first isodoublet has the neutral scalar component made up entirely of the $\bar{t}t$ condensate while the second has this component made up of the $(\bar{b}b)$ condensate. Therefore, in this case, the possible dynamically generated nonzero VEV's lead to $m_t^{\rm dyn}$ coming exclusively from the $\bar{t}t$ condensation and $m_b^{\rm dyn}$ from the $\bar{b}b$ condensation. The Dyson-Schwinger integral equations (3.4) in Sec. III in fact describe just this dynamic type-II 2HDSM scenario. King and Mannan (1991a) found out that, when $\kappa_{tb}=0$, *both* κ_t and κ_b have to be at least a bit above a certain common critical value $\kappa_{\rm crit} \sim 1$ for the generation of *both* nonzero[32] $m_t^{\rm dyn}$ and $m_b^{\rm dyn}$. In the present picture this means the generation of $\langle \Phi_u \rangle_0$, $\langle \Phi_d \rangle_0 \neq 0$ when $\mu_{tb}^2=0$. In the subcase $\kappa_t > \kappa_{\rm crit} > \kappa_b$ and $\kappa_{tb}=0$, we have $m_t \sim 10^2$ GeV and $m_b=0$, and equivalently $\langle \Phi_u \rangle_0 \neq 0$ and $\langle \Phi_d \rangle_0 = 0$. In this subcase, a question appears as to the physical picture of composite isodoublets—are there two dynamical composite isodoublets or only one at low energies ($\mu \ll \Lambda$)? The work discussed in Sec. III did not address this question, since the Dyson-Schwinger+Pagels-Stokar formalism employed there does not contain composite isodoublets explicitly. We argue here that in such a case, the relevant four-quark coupling κ_b, being too weak to lead to a nonzero VEV $\langle \Phi_d \rangle_0$, appears in general to be too weak to bind the composite auxiliary field $\Phi_d \sim \bar{b}_R \Psi_L$ into an existing and detectable dynamical field $\Phi_d^{(\mu)} \sim \bar{b}_R \Psi_L$ at lower energies $\mu < \Lambda$. We can also understand this claim at the level of the quark-loop approximation this way: We replace in Eqs. (2.6)–(2.12) of Sec. II the field t_R by b_R, the parameter $\kappa_t \equiv G N_c \Lambda^2/(8\pi^2)$ by κ_b, and the auxiliary field $\tilde{\Phi} \equiv \tilde{\Phi}_u$ by the auxiliary field Φ_d. Then those equations describe the $\Phi_d \sim \bar{b}_R \Psi_L$ condensation sector at the quark-loop level in our subcase. However, for κ_b *substantially* smaller than $\kappa_{\rm crit}=1$ $(0<1-\kappa_b \sim 1)$, relation (2.11) implies that the mass of the presumably dynamic zero-VEV isodoublet $\Phi_d^{(\mu)}$ at $\mu \sim E_{\rm ew}$ becomes as large as $\sim \Lambda$:

$$m^2_{\Phi_d}(\mu) = \left[\left(\frac{1}{\kappa_b} - 1 \right) \Lambda^2 + \mu^2 \right] \frac{2}{\ln(\Lambda^2/\mu^2)} \sim \frac{\Lambda^2}{\ln(\Lambda^2/\mu^2)} \quad [\text{roughly} \sim \Lambda^2]. \tag{6.9}$$

Despite the fact that quark loops induce kinetic terms for *both* Φ_u and Φ_d at $\mu < \Lambda$ [cf. $Z_\Phi(\Lambda;\mu)$ in Eqs. (2.6) and (2.7), where Φ is now Φ_u or Φ_d], having $m_{\Phi_d} \sim \Lambda$ means that $\Phi_d^{(\mu)} \sim \bar{b}_R \Psi_L$ is undetectable since the theory has, by definition, a UV cutoff at $\sim \Lambda$. From Eq. (6.9) we see that this is true as long as the value of κ_b is below and not very close to the critical value $\kappa_{\rm crit}$ ($\kappa_{\rm crit}=1$ in the quark-loop approximation).

When there is some small mixing ($\kappa_{tb} \neq 0$, hence $\mu_{ud}^2 \neq 0$), we see from Eqs. (6.4) and (6.5) that the first (second) neutral scalar, responsible for the mass m_t (m_b), has a small admixture of $\bar{b}b$ ($\bar{t}t$). The latter phenomenon is the "feed-down" mechanism mentioned already by King and Mannan (1991a) in their Dyson-Schwinger+Pagels-Stokar approach (see Sec. III): the $\bar{t}t$-condensate VEV $\langle \bar{t}t \rangle_0$, being largely responsible for large dynamical m_t, gives a (small) nonzero value m_b to the bottom quark through the κ_{tb} mixing term interaction.

Harada and Kitazawa (1991) investigated the question of whether the composite two-Higgs-doublet (2HD) scenario could lead to a breaking of $U(1)_{\rm em}$, i.e., whether the problem of $\langle \bar{t}b \rangle_0 \neq 0$ or $\langle \bar{b}t \rangle_0 \neq 0$ could occur. They considered the third generation of quarks, thus investigating the composite 2HD scenario of the general top-mode standard model (3.2). As shown above, this can lead, when $\kappa_t \kappa_b \neq 4\kappa_{tb}^2$, to the composite type-II 2HDSM. They then constructed the 2HD effective potential in the quark-loop approximation. The gap equations were obtained by minimizing it, and they found out that the resulting VEV's of the charged components of the Higgs doublets must be zero if $\kappa_t \neq \kappa_b$. For $\kappa_t = \kappa_b$, they argued that inclusion of the isospin-breaking $U(1)_Y$ interaction is of crucial importance, to see whether the quark condensates also align in this case in a $U(1)_{\rm em}$-conserving manner.

We implicitly assumed in Eqs. (3.2) and (6.3) that the mixing parameter κ_{tb}, and hence μ_{ud}^2, are real. Harada and Kitazawa (1991) also considered such a case. They claimed that we always have the freedom of redefining phases of the quark fields in such a way that these mixing parameters can be made real. However, we wish to stress that this claim may not be true—at least it should be investigated in detail. That is, if we assume that κ_{tb} and hence μ_{ud} are complex, we could end up with a composite type-II 2HDSM in which the two dynamically generated VEV's may acquire substantially different phases, thus possibly leading to a *CP* violation in the dynamical scalar sector.[33]

3. Renormalization-group analyses of the composite type-II 2HDSM

Luty (1990) investigated the composite type-II 2HDSM scenario within the same NJLVL framework of

[32] We note that when $\kappa_t, \kappa_b < \kappa_{\rm crit}$, King and Mannan (1991a) still obtained nonzero $m_t^{\rm dyn} = m_b^{\rm dyn} \approx 0.3$ GeV. This small value originates solely from QCD gluon exchange effects.

[33] See also the work of Andrianov *et al.* (1996), summarized toward the end of Sec. VI.A.4.

the top-mode standard model, by going beyond the quark-loop approximation. He employed the one-loop RG approach, in analogy with the original one-Higgs-doublet top-mode standard-model approach of Bardeen, Hill, and Lindner (1990). It turned out that the UV boundary conditions (compositeness conditions) for the Yukawa parameters $g_t(\mu)$ and $g_b(\mu)$ were completely analogous to those of the RG approach in the case of the minimal framework for $\bar{t}t$ condensation (2.15):

$$g_t(\mu)\to\infty, \quad g_b(\mu)\to\infty, \quad \text{when } \mu\to\Lambda. \tag{6.10}$$

In addition, Luty investigated one-loop RG equations governing the composite scalar sector (scalar self-interaction parameters λ_i, $i=1,\ldots,4$) and the resulting masses of the composite neutral and charged scalars and (neutral) pseudoscalar. Since the RG equations for the Yukawa couplings g_t and g_b are, at the one-loop level, decoupled from those of the scalar self-couplings, it suffices for our limited purposes of illustration to consider only the former,

$$16\pi^2\frac{dg_t(\mu)}{d\ln\mu}=\left[\left(N_c+\frac{3}{2}\right)g_t^2(\mu)+\frac{1}{2}g_b^2(\mu)-G_t(\mu)\right]g_t(\mu), \tag{6.11}$$

$$16\pi^2\frac{dg_b(\mu)}{d\ln\mu}=\left[\left(N_c+\frac{3}{2}\right)g_b^2(\mu)+\frac{1}{2}g_t^2(\mu)-G_t(\mu)+g_1^2(\mu)\right]g_b(\mu), \tag{6.12}$$

where $G_t(\mu)$ contains gauge coupling parameters g_j and is given in Eq. (5.18). The parameters g_j satisfy the RG equations (2.18), but with slightly changed constants C_j (for $j=1,2$) from those of the minimal standard model (2.19), due to the presence of two Higgs doublets,

$$\text{NH}=1\mapsto\text{NH}=2 \;\Rightarrow$$
$$C_1=-\frac{1}{3}-\frac{10}{9}n_q, \quad C_2=7-\frac{2}{3}n_q. \tag{6.13}$$

Again, n_q is the number of effective quark flavors (for $\mu>m_t$: $n_q=6$). It turns out that at the Landau poles ($\mu\to\Lambda$), where both g_t and g_b formally diverge, we have either $g_b/g_t\to0$ or $g_b/g_t\to\infty$, depending on whether at $E=E_{\text{ew}}$ the ratio $r\equiv g_b/g_t$ is smaller or larger, respectively, than a critical value $r_{\text{crit}}(m_t)=1-\delta(m_t)$.[34] Here δ is a very small positive number, nonzero due to the different g_1^2 terms in the RG equations (6.11) and (6.12). The difference in g_1^2 terms originates from the fact that t and b have different hypercharges Y. For example, for $m_t(m_t)=250$ GeV, numerical analysis gives $\delta\approx6\times10^{-3}$. Stated differently, for any given m_t, there is a critical ratio of VEV's $\tan\beta_{\text{crit}}(m_t)=(v_u/v_d)_{\text{crit}}\approx(m_t/m_b)r_{\text{crit}}$ below which g_b/g_t at $\mu=\Lambda$ becomes zero and above which it becomes infinity. These arguments suggest that the full compositeness in this framework, i.e., the compositeness of both Φ_u and Φ_d, can be achieved only if $g_t(\Lambda)\approx g_b(\Lambda)$, in which case then $g_t(\mu)\approx g_b(\mu)$ for any $\mu\leq\Lambda$ by Eqs. (6.11) and (6.12).

However, there is also another argument leading to the requirement $g_t(\mu)\approx g_b(\mu)$ for $\mu\leq\Lambda$. We denote the two composite doublet fields as $\Phi_u^{(\mu)}$ and $\Phi_d^{(\mu)}$, where μ is any chosen upper energy cutoff of the type-II 2HDSM. Renormalization-group equations for the running renormalization "constants" $Z_j(\mu)^{1/2}=\Phi_j^{(\mu)}/\Phi_j^{(E_{\text{tew}})}$ ($j=u,d$) can be derived in a straightforward manner from the diagrams of Fig. 7(a) of Sec. IV (with particles in the loop being either electroweak gauge bosons, top quarks, or bottom quarks):

$$8\pi^2\frac{d\ln Z_u(\mu)}{d\ln\mu}=-N_c g_t^2(\mu)+\frac{3}{4}g_1^2(\mu)+\frac{9}{4}g_2^2(\mu), \tag{6.14}$$

$$8\pi^2\frac{d\ln Z_d(\mu)}{d\ln\mu}=-N_c g_b^2(\mu)+\frac{3}{4}g_1^2(\mu)+\frac{9}{4}g_2^2(\mu). \tag{6.15}$$

Here $Z_j(\mu)^{1/2}$ are proportional to the corresponding running VEV's $v_j(\mu)$, and in Eqs. (6.14) and (6.15) we can replace[35] $Z_j(\mu)$ by $v_j^2(\mu)$ ($j=u,d$). Since the compositeness of the scalar isodoublets $\Phi_j^{(\mu)}$ requires their disappearance at $\mu=\Lambda$, this means $Z_j(\mu)\to0$ when $\mu\to\Lambda$. However, for any chosen m_t [$\equiv m_t(m_t)$], the following results from numerical analysis: when the ratio g_b/g_t at E_{ew} is not fine tuned to $r_{\text{crit}}(m_t)=1-\delta(m_t)$, or equivalently, when $\tan\beta\equiv v_u/v_d$ is not fine tuned to the corresponding $\tan\beta_{\text{crit}}(m_t)$, we end up with either $Z_u(\mu)\ll1$ and $Z_d(\mu)\sim1$, or $Z_u(\mu)\sim1$ and $Z_d(\mu)\ll1$, for $\mu\sim\Lambda$ ($\equiv\Lambda_{\text{pole}}$). Hence one of the scalar isodoublets is not composite in such cases. For example, if we set $m_t(m_t)=250\,\text{GeV}$, then $\tan\beta_{\text{crit}}\approx81.85$.

Thus we are led to the conclusion that RG compositeness conditions, i.e., $Z_j(\mu)\to0$ ($j=u,d$) [or $g_j(\mu)\to\infty$ ($j=t,b$)] as $\mu\to\Lambda$, can only be fulfilled when $g_t(\Lambda)\approx g_b(\Lambda)$, thus resulting in $g_t(E_{\text{ew}})\approx g_b(E_{\text{ew}})$. This is the reason why Luty (1990) used implicitly for the one-loop RG equations (6.11) and (6.12) the boundary condition $g_t(\Lambda)=g_b(\Lambda)$ ($\gg1$). This implied that $\tan\beta=v_u/v_d$ of the type-II 2HDSM must be very large: $\tan\beta\approx m_t(M_Z)/m_b(M_Z)\sim10^2$. This can also be seen explicitly in Table VI, where we have chosen as a compositeness condition for the one-loop RG equations (6.11) and (6.12): $g_t^2(\Lambda)/(4\pi)=g_b^2(\Lambda)/(4\pi)=1/3$, 2/3, 1 and 10 (="$\infty$"). The results of Luty for $m_t(m_t)$ are similar to those in the last line of Table VI.

Table VI shows that the RG approach to $\bar{t}t$ (and $\bar{b}b$) condensation in the type-II 2HDSM framework leads to phenomenologically unacceptable (too high) values of $m_t(m_t)$: $m_t(m_t)\gtrsim206\,\text{GeV}$ for $\Lambda\lesssim E_{\text{Planck}}$, similar to

[34]Here, "(m_t)" means that r_{crit} and δ are functions of the renormalized mass $m_t=m_t(E=m_t)$.

[35]We denote $v_j(\mu=E_{\text{ew}})$ simply as v_j. We shall use $E_{\text{ew}}=M_Z$.

TABLE VI. Mass $m_t(m_t)$ in the composite type-II two-Higgs-doublet standard model as a function of the cutoff $\mu_* = \Lambda$ and the chosen value $k_t\ [\equiv g_t^2(\Lambda)/(4\pi)] = k_b[\equiv g_b^2(\Lambda)/(4\pi)]$. Masses and energies are in GeV. In brackets [ ], the corresponding VEV ratios $\tan\beta = v_u/v_d$ (at $E = M_Z$) are given. Values used: $\alpha_3(M_Z) = 0.120$ [$\Rightarrow \alpha_3(180\ \text{GeV}) = 0.110$]; $m_b(m_b) = 4.3$ GeV [$\Rightarrow m_b(M_Z) \approx 3.175$ GeV]; the number of quark flavors $n_q = 6$ was taken, for $\mu \geq M_Z$.

$\Lambda \rightarrow$ / $k_t = k_b \downarrow$	10^{19}	10^{17}	10^{15}	10^{13}	10^{11}	10^{9}	10^{7}	10^{5}	10^{4}
2/3	205.4	208.9	213.3	219.1	226.9	238.2	255.8	288.5	319.0
	[66.68]	[67.94]	[69.50]	[71.50]	[74.18]	[77.95]	[83.77]	[94.12]	[103.28]
1	205.9	209.5	214.1	222.1	228.3	240.3	259.5	296.6	333.7
	[66.85]	[68.14]	[69.76]	[71.84]	[74.65]	[78.66]	[84.96]	[96.60]	[107.47]
10	206.7	210.5	215.5	221.9	230.9	244.3	266.7	313.4	367.0
	[67.15]	[68.51]	[70.23]	[72.47]	[75.53]	[79.99]	[87.24]	[101.61]	[116.49]

3the results of Bardeen, Hill, and Lindner (1990) for the minimal framework (see Table I). That is, since $g_t \approx g_b$, RG in Eq. (6.11) looks almost like RG Eq. (2.16) of the minimal framework, with $(N_c+1.5) \mapsto (N_c+2)$ and slight changes in evolution of g_1 and g_2, while the compositeness condition $g_t(\Lambda) = \infty$ is the same as in the minimal framework. Thus, for a given Λ, $g_t(E_{\text{ew}})$ is somewhat below but close to the value in the minimal framework. Also the corresponding VEV's in the two frameworks are $v \approx v_u$, due to $v_u/v_d \approx m_t/m_b \gg 1$ and the relation $v_u^2 + v_d^2 = v^2$. Therefore the resulting $m_t(m_t)$ in the two frameworks, for a given Λ, turn out to have values close to each other.

The composite type-II 2HDSM framework was also investigated by Froggatt, Knowles, and Moorhouse (1990, 1992), who performed an RG analysis similar to that of Luty (1990). In one of the discussed scenarios, Φ_d coupled also to τ_R. The compositeness conditions were $g_f^2(\mu_*)/\pi \sim 1$ $(f = t, b, \tau)$ and $\lambda_i(\mu_*)^2/\pi \sim 1$ $(i = 1, \ldots, 5)$ at $\mu_* = 10^{14}$–10^{15} GeV. The resulting infrared fixed-point behavior led them to predictions of dynamical masses of t, b, and τ $(m_b/m_\tau \approx 2.6)$ and of the composite scalars. In their first work (Froggatt, Knowles, and Moorhouse, 1990), unlike Luty, they did not include the Φ_u-Φ_d mixing term [see Eq. (6.3)] and consequently obtained a very light—phenomenologically unacceptable—composite neutral scalar: $M_{\eta^0} \lesssim m_b$. In the overlapping region of parameter space their results (1990) agree with those of Luty. Later (1992) they included the mixing term, thus in general raising M_{η^0} to an acceptable level. In particular, they proposed a decoupling scenario, in which the scalar mass-term parameters $\mu_u^2(\mu)$ and $\mu_d^2(\mu)$ at energies $\mu \sim E_{\text{ew}}$ are fine tuned to values $\sim E_{\text{ew}}$ in such a way that $\sqrt{v_u^2 + v_d^2} = v \approx 246$ GeV, and the mixing mass-term parameter is not fine tuned: $\mu_{ud}^2(E_{\text{ew}}) \sim \Lambda^2 \gg E_{\text{ew}}$ (Λ is the compositeness scale).[36] Then, only one composite (neutral) scalar $h^{(0)}$ retains a small mass $M_{h^0} \sim E_{\text{ew}}$, while the other four acquire masses $\sim\Lambda$ and decouple from the electroweak physics. Thus the low-energy physics behaves as the minimal standard model, RG equations having the minimal-standard-model form for $\mu < \Lambda$, but v_u can be substantially smaller than $v \approx 246$ GeV. In one of the discussed cases (case B), they set $v_u/v_d \approx 1$, thus $v_u \approx v/\sqrt{2}$, and consequently obtained a lower $m_t \approx 163$ GeV. However, we previously argued that abandoning the condition $v_u/v_d \approx m_t(M_Z)/m_b(M_Z) \gg 1$ and replacing it by a smaller v_u/v_d implies abandoning the condition $Z_d(\mu) \to 0$ when $\mu \to \Lambda_{\text{pole}}$ [if $Z_u(\mu) \to 0$ remains]. Therefore Φ_d is not (fully) composite in this scenario. This is the price to be paid for decreasing m_t^{dyn} to acceptable values 160–180 GeV. The mass m_t^{dyn} can be decreased only if the composite top-quark sector is not made (almost) fully responsible for the EWSB ($v_u \neq v = 246$ GeV).[37] Froggatt, Knowles, and Moorhouse included in their work (1992) an extensive analysis of radiative corrections of the scalar sector to the vacuum polarizations of vector bosons. They showed that these corrections are very close to those in the minimal standard model, because contributions of the four additional heavy scalars (with masses $M \sim \Lambda$) cancel as these scalars have almost degenerate masses ($|\Delta M| \ll E_{\text{ew}}$).

Mahanta (1992) discussed a possible CP-violation effect in the composite type-II 2HDSM, which arises when the four-quark parameter κ_{tb} in Eq. (3.2) [or equivalently, μ_{ud}^2 in Eq. (6.6)] is complex.[38] CP violation in this framework stems from the mixing of neutral CP-even and CP-odd composite scalars at low energies, and is manifested through the (t-loop-induced) electric dipole moment d_e of the electron. Mahanta employed essentially the one-loop RG approach in the quark-loop

[36]All the mass-term parameters $\mu_u^2(\mu)$, $\mu_d^2(\mu)$, $\mu_{ud}^2(\mu)$ at $\mu \sim E_{\text{ew}}$ are in general roughly $\sim\Lambda^2$ unless fine tuned—cf. $m_\Phi^2(\mu)$ in Eq. (2.11) for the case of *one* composite Higgs doublet.

[37]We shall see later that this is also a feature of the fully renormalizable topcollor-assisted technicollor frameworks (see Sec. VIII.F). A somewhat different mechanism for decreasing m_t works in a $\bar{t}t$ condensation framework within supersymmetric models (see Sec. VII.A).

[38]Of course, $\mathcal{L}^{(\Lambda)}$ is kept Hermitian, i.e., the corresponding κ_{tb} terms in Eq. (3.2) read in this case $2\kappa_{tb}[(\overline{t_L^a} t_R^a)(\overline{b_L^b} b_R^b) - (\overline{t_L^a} b_R^a)(\overline{b_L^b} t_R^b)] + \text{H.c.}$, and the μ_{ud}^2 terms in Eq. (6.6) are charged accordingly.

(bubble) approximation to construct from $\mathcal{L}^{(\Lambda)}$ of Eq. (6.6), with μ_{ud}^2 being complex, the effective density $\mathcal{L}^{(\mu)}$ of a type-II 2HDSM at low energies $\mu \sim v$, and he was subsequently able to impose an upper bound on a parameter $\propto \kappa_{tb}$ by requiring that d_e not exceed the phenomenological upper bound.

In this context, we mention another study (Cvetič and Kim, 1993) in which the (one-loop) RG evolution of Yukawa coupling parameters of the second and third quark generation in the type-II 2HDSM was investigated, with the purpose of seeing whether the model allows flavor-democratic structures (flavor democracy) at high energies. Indeed, the authors found that, in contrast to the minimal standard model, the Yukawa matrices in the up-type and the down-type sectors moved toward the flavor-democratic form[39] in a weak basis when the energy increased. Moreover, the CKM mixing matrix moved toward the identity matrix as the energy increased. The chosen values of the VEV ratio were $v_u/v_d = 0.5$, 1.0, and 5. Therefore at the pole energies $\mu = \Lambda$ the ratio g_b/g_t was zero, although both g_t and g_b diverged there. Equivalently, $Z_u(\Lambda) = 0$ and $Z_d(\Lambda) \sim 1$. Thus the model can be interpreted as having a composite Higgs doublet $\tilde{\Phi}_u \sim \overline{t}_R \Psi_L$ (with compositeness scale Λ) and another Higgs doublet Φ_d of unspecified nature. The mass $m_t \approx 180$ GeV is then of completely dynamic origin, and the EWSB of partially dynamic origin ($v \equiv \sqrt{v_u^2 + v_d^2} > v_u$). Interestingly, for low values of the VEV ratio $v_u/v_d = 0.5$, the pole (compositeness) energy is as low as ~ 10 TeV, for m_t $(= m_t^{\text{dyn}})$ ≈ 180 GeV.

4. Explaining large isospin breaking, $m_t \gg m_b$, in the composite type-II 2HDSM

The problem of explaining, by underlying physics, the large isospin breaking ($m_t \gg m_b \neq 0$) in the framework of the type-II composite two-Higgs-doublet standard model was discussed by Bando, Kugo, Maekawa, and Nakano (1991), Bando, Kugo, and Suehiro (1991), Nagoshi, Nakanishi, and Tanaka (1991), and King and Suzuki (1992). As discussed in Sec. VI.A.3, in the fully composite type-II 2HDSM, the problem of explaining the mass hierarchy $m_t \gg m_b$ can be replaced by the problem of explaining the VEV hierarchy $v_u \gg v_d$. In one of the schemes discussed by these authors, NJLVL interactions (3.2) with bare parameters $\kappa_{tb}(\Lambda) = 0$ and $\kappa_t(\Lambda) = \kappa_b(\Lambda)$ were investigated. Such interactions can originate from exchange of heavy gauge bosons that have equal coupling strengths to t and b. Since t and b have different hypercharges, the weak-hypercharge boson exchange causes the critical values $\kappa_t^{(\text{crit})}$ and $\kappa_b^{(\text{crit})}$ and the effective values $\kappa_t^{(\text{eff})}$ and $\kappa_b^{(\text{eff})}$ in the gap equation to differ slightly:[40]

$$\kappa_t^{(\text{eff})}(\Lambda) - \kappa_b^{(\text{eff})}(\Lambda) = \kappa_b^{(\text{crit})} - \kappa_t^{(\text{crit})} = g_1^2(\Lambda)/(16\pi^2), \tag{6.16}$$

where $g_1(E)$ is the running $U(1)_Y$ gauge coupling parameter. They showed in the quark-loop approximation that the difference between the VEV's then becomes enhanced,

$$v_u^2 - v_d^2 \approx 2N_c g_1^2(\Lambda)\Lambda^2/(16\pi^2)^2, \tag{6.17}$$

where $\sqrt{v_u^2 + v_d^2} = v \approx 246$ GeV and $\Lambda \gg E_{\text{ew}}$. This result is expected also to hold qualitatively beyond the quark-loop approximation. Relation (6.17) suggests that $v_u \gg v_d$, i.e., $v_u/v_d \gg 1$. Keeping in mind that $g_t \approx g_b$ (see Sec. VI.A.3), this explains why a very tiny $U(1)_Y$ splitting of the effective parameters at a large scale Λ can lead to $m_t^{\text{phys}} \gg m_b^{\text{phys}}$.

Large isospin breaking was investigated indirectly within the generalized top-mode standard model (3.2) by King and Mannan (1991a). No direct reference was made to the fact that this was a composite type-II 2HDSM, since the employed Dyson-Schwinger+Pagels-Stokar formalism (see Sec. III) does not use composite scalar fields. King and Mannan's calculations include quark-loop and QCD contributions, but not those of $U(1)_Y$. Therefore their bare four-quark parameters $\kappa_i(\Lambda)$ should be regarded as $\kappa_i^{(\text{eff})}(\Lambda)$ with $U(1)_Y$ contributions already contained in them. For very large[41] $\Lambda = 10^{15}$ GeV, already a tiny $U(1)_Y$ splitting $\kappa_t^{(\text{eff})} - \kappa_b^{(\text{eff})} < 10^{-6}$ resulted in $m_t^{\text{dyn}} \gg m_b^{\text{dyn}}$. However, Miransky, Tanabashi, and Yamawaki (1989a), within the Dyson-Schwinger+Pagels-Stokar approach, proposed that the cross term $\propto \kappa_{tb}(\Lambda)$ in Eq. (3.2) also be nonzero, leading to a "feed-down" effect—see the beginning of Sec. III.B. Mass m_t^{dyn} then feeds down through this term to yield $m_b \sim \kappa_{tb}(\Lambda) m_t \ll m_t$. King and Mannan subsequently showed numerically that in such a case there is no need to fine tune $\kappa_b^{(\text{eff})}(\Lambda)$ close to $\kappa_t^{(\text{eff})}(\Lambda)$, and that then $\kappa_{tb}^{(\text{eff})}(\Lambda) \sim 10^{-2}$.

Andrianov, Andrianov, and Yudichev (1996) investigated a framework in which there are two composite Higgs doublets arising from a generalized version of NJLVL interactions, which include higher-dimensional terms with two and four derivative insertions. For a special configuration of four-quark constants, they obtained a dynamically generated CP violation reflected in a complex ratio of the two dynamically generated VEV's. It appears that in their scenario both Higgs doublets are responsible for the dynamical generation of m_t and m_b, i.e., theirs is a composite 2HDSM of "type III."

Partly related to this context is the work of Fröhlich and Lavoura (1991), who investigated the case when NJLVL interactions (3.2) satisfy the relation $\kappa_t \kappa_b = 4\kappa_{tb}^2$. The resulting Lagrangian density can then be rewritten with *one* single auxiliary isodoublet, thus leading at $E \ll \Lambda$ to an effective minimal standard model. In contrast to the truncated top-mode standard model

[39] A matrix has the flavor democracy form when all its elements are equal.

[40] The bare effective coupling $\kappa^{(\text{eff})}$ $(\neq \kappa)$ incorporates $U(1)_Y$ gauge-boson exchange contributions from the entire energy interval $[E_{\text{ew}}, \Lambda]$, while the original bare coupling κ does not.

[41] For so large Λ, Eq. (6.17) is not applicable.

(2.1), their framework leads to a nonzero $m_b^{\rm dyn}$, in addition to $m_t^{\rm dyn}$. They calculated in the quark-loop approximation and obtained an interesting relation $M_H^2 \approx 4(m_t^2 - m_b^2)$, if $m_b \ll m_t$. They assumed the hierarchy $m_b \ll m_t$, while noting that m_b is in principle arbitrary in this framework. Mass m_t acquired values close to those of the truncated top-mode standard model (2.1) when $m_b \ll m_t$.

B. Two Higgs doublets—one elementary and one composite

Clague and Ross (1991) investigated a scenario in which multiple exchanges of an elementary Higgs between $\bar{t}$ and t lead to $\bar{t}t$ condensation (to a composite doublet $\Phi_{\rm comp}$) and to nonzero $m_t^{\rm dyn}$. This Higgs has a strong Yukawa coupling parameter h_t to t_R: $h_t^2(E)/(4\pi) \gtrsim 1$, for $E \sim E_{\rm ew}$. In the scenario, the $\langle \bar{t}t \rangle_0$ VEV induces the VEV $v = \langle (\phi_3)_{\rm el} \rangle_0 \approx 246$ GeV of the elementary Higgs doublet $\Phi_{\rm el}$. The authors calculated in the quark-loop (bubble) approximation, thus not dealing directly with composite scalars. Mass eigenstates of scalars were mixtures of components of both isodoublets $\Phi_{\rm el}$ and $\Phi_{\rm comp}$, with mass eigenvalues $M_s^{(1)} \sim m_t$, $M_s^{(2)} \sim m_\phi$ ($\gg m_t$). Here $m_\phi^2 > 0$ is the original mass parameter of $\Phi_{\rm el}$. The authors argued that such a $\bar{t}t$ binding, parametrized by an NJLVL interaction of the top-mode standard-model form (2.1), results in the dependence of the four-quark form factor G on the momentum transfer q^2: $G(q^2) = H/(q^2 - b m_\phi^2)$, where $H, b \sim 1$ ($H > 0$). The effective cutoff scale is $\Lambda \sim m_\phi$. For the case $G(q^2=0) > 0$ they obtained only slightly reduced m_t in comparison with the minimal framework in the corresponding approximation, i.e., too high m_t. When $G(q^2=0) < 0$ (i.e., $b > 0$), m_t could be substantially lowered (and m_ϕ as low as $E_{\rm ew}$). However, their modified gap equation implies that the average $\langle G(q^2) \rangle$ over the relevant momenta must be positive. Therefore it is unclear how the (otherwise interesting) case of $G(q^2) < 0$ for $q^2 \lesssim m_\phi^2$ ($\sim \Lambda^2$) can be reconciled with a motivation for working with effective models—namely, that the momenta above the cutoff do not appreciably affect the low-energy predictions of the model.

The work of Clague and Ross has some similarity with the original "bootstrap" idea of Nambu (1989). In this early work, which stimulated much of the subsequent research activity on $\bar{t}t$ condensation, Nambu proposed that the BCS mechanism (Bardeen, Cooper, and Schrieffer, 1957), and its low-energy effective description through a Ginzburg-Landau type of Hamiltonian, be incorporated into relativistic field theories. He showed that this possibility does exist and leads to theories that can contain composite scalars and have a structure he called "relativistic quasi-supersymmetry." He proposed two scenarios:

(1) A hierarchical chain of dynamical symmetry breaking ("tumbling")—composite scalars could mediate between massive fermion forces attractive enough to cause another symmetry breaking, and so on repeatedly down to the electroweak scales.

(2) Self-sustaining ("bootstrap") mechanism—composite scalars mediate forces between fermions leading to four-fermion interactions strong enough to explain the condensation (composite scalars responsible for their own existence). The dynamics should be self-consistent in the sense that it would not be necessary to look for any new physics (substratum) at a high energy generating the four-fermion interactions. This would imply insensitivity of the electroweak model to the UV cutoff Λ, and therefore the Λ^2-tadpole effects for the masses m_f should cancel, and preferably even $\ln \Lambda$ terms.[42]

The main difference between the picture of Nambu and that of Clague and Ross is that the latter assumed that the scalar responsible for binding together $\bar{t}$ and t is not the generated $\bar{t}t$ condensate itself, while Nambu assumed that it was (the "bootstrap" assumption).

In this context, we mention the observation of Tanabashi, Yamawaki, and Kondo (1990) that a strong Yukawa coupling parameter h_t ($\equiv g_t$) of an elementary scalar to the top quark may generate a $\bar{t}t$ condensate and that the VEV $\langle \bar{t}t \rangle_0$ induces a VEV v for $\Phi_{\rm el}$.

An approach somewhat different from that of Clague and Ross was later taken by Delépine, Gérard, and González Felipe (1996). They assumed that the strong Yukawa parameter g_t of the $\Phi_{\rm el}$ doublet induces an effective top-mode standard-model-type interaction (2.1); the four-quark parameter G was taken to be independent of the momentum transfer.[43] This led them to an additional $\Phi_{\rm comp}$ doublet of the top-mode standard-model type ($\bar{t}t$). Both $\Phi_{\rm el}$ and $\Phi_{\rm comp}$ couple to t_R. Delépine *et al.* applied a formalism of the effective potential $V_{\rm eff}$ (see Sec. IV), constructing $V_{\rm eff}$ as a function of both VEV's, in the quark-loop approximation and using simple spherical cutoff Λ. Their formalism allows *both* scalar doublets to develop appreciable VEV's—this is a major difference from the scenario of Clague and Ross. Neutral scalar mass eigenstates are linear combinations of the neutral scalar components of $\Phi_{\rm el}$ and $\Phi_{\rm comp}$. Taking $g_t^2/(4\pi) \simeq 1$ and $m_t = 174$ GeV resulted in the lightest Higgs particle being almost entirely composite and having a mass possibly as small as 80 GeV, if $\Lambda \sim 1$ TeV. Masses of electroweak gauge bosons could be reproduced even for Λ's this low, because *both* $\Phi_{\rm el}$ and $\Phi_{\rm comp}$ contribute to the EWSB. Their Lagrangians for $\Phi_{\rm el}$ and for $\Phi_{\rm comp}$ have identical forms. Formally, it is not possible to distinguish in their formalism between

[42]For earlier discussions of these points, without the condensation interpretation, see Decker and Pestieau (1979, 1989, 1992) and Veltman (1981).

[43]This appears to be contrary to what they suggest in their Eqs. (3) and (4).

the two isodoublets in the quark-loop approximation of $V_{\rm eff}$ (see Fig. 3 in Sec. IV) where the composite nature is not felt. Going beyond this approximation (i.e., to the next-to-leading terms—NTLO) in calculating $V_{\rm eff}$ would make the difference between $\Phi_{\rm el}$ and $\Phi_{\rm comp}$ formally manifest, because the diagrams needed to calculate the NTLO terms include the scalar propagators, which are dynamical and nondynamical, respectively (see Fig. 4, where they are nondynamical). Further, since in their framework both isodoublets contribute to m_t, this would lead at low energies to an effective 2HDSM of "type III." The Yukawa sector of such a model has no discrete symmetry, and thus suppression of the FCNC is not automatically ensured when including the lighter quark generations. Thus the framework of Delépine *et al.*, while giving interesting results at quark-loop level, also raises several interesting questions that deserve further investigation.

C. Colored composite scalars

To obtain a phenomenologically acceptable prediction for m_t in models involving $\bar{t}t$ condensation, several groups of authors have proposed NJLVL interactions which, while respecting the standard-model gauge symmetry, lead to ($\bar{t}t$-dominated) condensation into composite scalars with *color*. Such interactions must differ from those of Eqs. (2.1), (3.1) and (3.2)—the latter lead to formation of only *colorless* composite scalars (for $\kappa_t > \kappa_{\rm crit}$).

Babu and Mohapatra (1991) proposed an NJLVL model leading to a $\bar{t}t$-dominated composite scalar sector with one usual isodoublet Φ and two color-triplet isoscalars ω_1 and ω_2. At energies $\mu<\Lambda$, scalars become dynamical through quantum effects:

$$\mathcal{L}_{\rm Y.}^{(\mu)}=h_t(\mu)\bar{Q}_L^a\tilde{\Phi}t_R^a+f_1(\mu)Q_L^{aT}C^{-1}i\tau_2Q_L^b\omega_1^d\epsilon_{abd}+f_2(\mu)t_R^{aT}C^{-1}b_R^b\omega_2^d\epsilon_{abd}+\text{H.c.} \tag{6.18}$$

Here (a,b,d) are color indices; isospin indices and superscripts "(μ)" were suppressed. The authors then studied the RG equations for Yukawa parameters h_t, f_1, f_2, by imposing compositeness conditions $h_t, f_1, f_2\to\infty$ when $\mu\to\Lambda$. Only Φ develops a VEV ($\approx$246 GeV); colored scalars do not because the vacuum is colorless. Thus $m_t^{\rm dyn}=m_t(m_t)$ is determined only by $h_t(m_t)$. The renormalization-group equation for h_t is different from Eq. (2.16) of the minimal standard model—f_1 and f_2 give positive contributions to the slope $dh_t/d\ln\mu$. This results in lower values of $m_t(m_t)=h_t(m_t)v/\sqrt{2}$ than in the minimal framework. Acceptable values $m_t(m_t)\approx 170$ GeV can be obtained, for $\Lambda\sim 10^{17}$–10^{19} GeV. The mass m_b ($\ll m_t$) gets induced out of $m_t^{\rm dyn}$ as a one-loop radiative effect, depicted in Fig. 8. They also calculated the mass M_H of the colorless Higgs by considering the scalar potential of Φ, ω_1, and ω_2 and investigating one-loop RG equations for the self-coupling parameters λ_i appearing in the potential. The

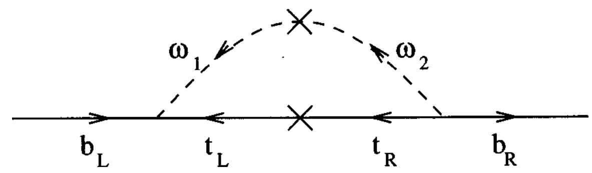

FIG. 8. Generation of m_b out of $m_t^{\rm dyn.}$, via mixing of the composite isosinglet color-triplet scalars ω_1 and ω_2. From Babu and Mohapatra (1991).

compositeness conditions were $\lambda_i\to\infty$ when $\mu\to\Lambda$. The resulting M_H were also lower than those of the minimal framework—e.g., $M_H\approx 166$ or 158 GeV for $\Lambda\sim 10^{17}$ or 10^{19} GeV, respectively. They subsequently included four-fermion interactions involving the third generation of leptons (no ν_R), leading to two additional composite color-triplet isosinglet scalars χ_1 and χ_2 and to a modified RG flow for h_t. The mass $m_t(m_t)$ was reduced: $m_t(m_t)\approx 153$–155 GeV for $\Lambda=10^{17}$–10^{19} GeV; $m_t(m_t)\approx 170$ GeV for $\Lambda\sim 10^{11}$ GeV.

Kundu, De, and Dutta-Roy (1993) investigated another $\bar{t}t$ condensation scenario with colored composite scalars. The starting density was used first by Dai *et al.* (1992),

$$\mathcal{L}_{4q}^{(\Lambda)}=\frac{g}{\Lambda^2}(\bar{Q}_L^a t_R^a)(\bar{t}_R^b Q_L^b)+\frac{g'}{\Lambda^2}(\bar{Q}_L^a\lambda_{ab}^{(\alpha)}t_R^b)(\bar{t}_R^d\lambda_{de}^{(\alpha)}Q_L^e), \tag{6.19}$$

where (a,b,d,e) are color indices, $\lambda^{(\alpha)}$ ($\alpha=1,\ldots,8$) are the $SU(3)_c$ Gell-Mann matrices satisfying $\mathrm{tr}(\lambda^{(\alpha)}\lambda^{(\beta)})=2\delta_{\alpha\beta}$, and $Q_L=(t_L,b_L)^T$. Isospin indices are omitted for simplicity. The first term is the truncated top-mode standard model (2.1). The second term is also invariant under the standard-model gauge group. If strong enough, it generates a composite color-octet isodoublet scalar χ by $\bar{t}t$-dominated condensation, as pointed out by Kundu, De, and Dutta-Roy (1993).

Dai *et al.* (1992) calculated m_t in this model by employing the ladder Dyson-Schwinger integral equation and the Bethe–Salpeter equation for the Nambu-Goldstone boson (instead of the Pagels-Stokar equation) for calculating the decay constant $F_\pi=v$. The formalism of the approach contains explicitly dynamical masses of quarks, but not composite scalars—hence Dai *et al.* did not refer to the latter. They argued that the g' term in Eq. (6.19) does not contribute in the ladder approximation to the Dyson-Schwinger equation, but they included it in the Bethe-Salpeter equation. For large g' ($g'=3g$)[44] they obtained values of m_t lower than those

[44]We note that $g'=3g/2$ corresponds to the case in which the origin of Eq. (6.19) is an exchange of massive ($M\sim\Lambda$) colorless vector bosons between the quarks—see identity (8.2) in Sec. VIII.B.

in the truncated top-mode standard model in the Dyson-Schwinger+Pagels-Stokar approach (see Sec. III, Table II) by 30–40 GeV for $\Lambda \sim 10^{11}-10^{19}$ GeV. Nonetheless, for $g'=3g$, the m_t's were still above 200 GeV, for $\Lambda \lesssim E_{\rm Planck}$. Since they used the ladder Dyson-Schwinger equation, their results at high Λ's seem to correspond partly to the RG approach in leading $N_{\rm c}$ (and with gauge couplings $g_1, g_2 \mapsto 0$). Hence it is to be expected that their m_t's are higher than those of the model in the RG approach.

Indeed, Kundu, De, and Dutta-Roy (1993) recalculated m_t in this model by using the RG approach. For $g' > g'_{\rm crit}$, they argued that the composite scalar sector also contains, beside the usual isodoublet Φ, a color-octet isodoublet field χ (with zero VEV). At low energies, t_R couples to both fields with Yukawa parameters g_t and g'_t, respectively. The renormalization-group equation for $g_t(\mu)$, modified by the presence of $g'_t(\mu)$, gives an appreciably higher slope $dg_t/d\ln\mu$ than in the minimal-standard-model case (2.16). Kundu *et al.* investigated the infrared fixed point of one-loop RG equations for g_t and g'_t, in conjunction with RG equations for the scalar self-coupling parameters (with $g_1, g_2 \mapsto 0$). Demanding in addition that the scalar potential have meaningful finite minima, they obtained the fixed-point solution $m_t^{\rm (f.p.)} \approx 145$ GeV and the scalar masses $M_{\chi^0} \approx 135$ GeV, $M_{\chi^+} \approx 74$ GeV, and $M_H \approx 209$ GeV. In analogy with the minimal framework, it appears that this $m_t^{\rm (f.p.)}$ value represents a solution of the RG equations to an accuracy of a few percent for compositeness scales $\Lambda \gtrsim 10^{10}$ GeV. The obtained value for m_t is substantially lower than those obtained by Dai *et al.* (1992), because of the different approach used.

Emergence of colored scalars with relatively low masses $\sim E_{\rm ew}$ may pose a problem when confronted with experimental evidence. Babu and Mohapatra did not address this question. Kundu *et al.* argued that the question is not alarming because the M_χ's are higher than the masses of light hadrons (thus not influencing light-hadron phenomenology) and that physical objects involving χ will be colorless.

D. Other structures of composite scalars

Arata and Konishi (1992) introduced a $\bar{t}t$ condensation model containing the isodoublet $\tilde{\Phi}_i \sim \bar{t}^a_R Q^{ia}_L$ and an isotriplet $\chi_{ij} = \chi_{ji} \sim (\bar{t}^a_R Q^{ia}_L)(\bar{t}^b_R Q^{jb}_L)$. They started with

$$\mathcal{L}^{(\Lambda)}_{4q} = G(\bar{Q}^i_L t_R)(\bar{t}_R Q^i_L) + G'(\bar{Q}^i_L t_R)(\bar{Q}^j_L t_R)(\bar{t}_R Q^i_L)(\bar{t}_R Q^j_L), \qquad (6.20)$$

where $Q_L = (t_L, b_L)^T$, i,j are isospin indices, and color indices (of the top-mode standard-model type) were suppressed. The first term is the truncated top-mode standard model. For $GN_{\rm c}\Lambda^2/(8\pi^2) > \kappa_{\rm crit}$ it leads to the emergence of $\tilde{\Phi}$. The eight-quark term leads, for strong enough $G'\Lambda^8$, to the emergence of χ. The main motivation for introducing this term, and hence χ, in the fact that values $m_t(m_t) \approx 220-230$ GeV of the minimal framework (for $\Lambda \gtrsim 10^{15}$ GeV) were irreconcilable with the experimental restrictions on the electroweak $\delta\rho$ parameter within the minimal standard model (m_t had not yet been measured at that time). The extended framework allowed $\langle \chi_{11} \rangle_0 \neq 0$, resulting in a reduction of the effective $\delta\rho$. When the value of $m_t(m_t)$ was 220–230 GeV and the VEV ratio was $\eta \equiv \langle \chi_{11} \rangle_0 / \langle \tilde{\Phi}_1 \rangle_0 \sim 10^{-1}$, acceptable values of $\delta\rho$ were obtained. The authors then used these values of η in a one-loop RG investigation of the scalar self-couplings, concluding that at least one neutral scalar mass eigenstate [a linear combination of ${\rm Re}(\tilde{\Phi}_1)$ and ${\rm Re}(\chi_{11})$], with $M_H < 300$ GeV, couples to quarks as in the minimal standard model. Other mass eigenstates turned out to couple to quarks only weakly. The evolution of $g_t(\mu)$ was almost the same as in the minimal framework, as was the VEV $\langle \tilde{\Phi}_1 \rangle_0$, resulting in the nowadays unacceptable mass $m_t(m_t) > 210$ GeV.

Achiman and Davidson (1991) investigated, within the NJLVL framework (3.2) involving $\bar{t}t$ condensation, the possibility that the Peccei-Quinn spontaneous symmetry breaking (Peccei and Quinn, 1977a, 1977b) is dynamical. They identified the global axial $U(1)_{\gamma_5}$ symmetry[45] of the first two terms in Eq. (3.2) with the color-anomalous Peccei-Quinn symmetry group $U(1)_{\rm Peccei\text{-}Quinn}$. In Eq. (3.2), the third term ($\propto \kappa_{tb}$) violates this symmetry explicitly. In their framework, this term is generated dynamically at an intermediate scale $E_{\rm Peccei\text{-}Quinn}$: $E_{\rm ew} \sim 10^2$ GeV $\ll E_{\rm Peccei\text{-}Quinn} \sim 10^{11}$ GeV $\ll \Lambda$, where Λ is the $\bar{t}t$ condensation scale. They proposed that the condensate responsible for this Peccei-Quinn dynamical symmetry breaking is made up of third-generation right-handed neutrinos:[46] $\Phi \sim \tilde{\nu}_R \nu_R \equiv \bar{\nu}^c_R \nu_R$. Since this condensate is not invariant under $U(1)_{\gamma_5}$ [$\Phi \mapsto e^{i2\alpha}\Phi$ when $f \mapsto \exp(i\alpha\gamma_5)f$], a nonzero VEV $\langle \Phi \rangle_0 \neq 0$ breaks Peccei-Quinn symmetry dynamically. The see-saw mechanism yields heavy right-handed third-generation neutrinos (with Majorana mass $M \sim E_{\rm Peccei\text{-}Quinn}$) and very light left-handed third-generation neutrinos and axions ($m_{\nu_L} \sim m_{\rm axion}$).

Kaus and Meshkov (1990) proposed a flavor-democratic NJLVL model:

$$\mathcal{L}^{(\Lambda)}_{4f} = -G \sum_{i,j,k,l} \bar{\psi}_i \psi_j \bar{\psi}_k \psi_l, \qquad (6.21)$$

where the sum is over "ur quarks" of a given electric charge (i.e., either up-type or down-type). The motivation was to explain masses and mass hierarchies of fermions in a semiphenomenological way. If the sum $(\psi_1 + \psi_2 + \psi_3)$ in the up-type sector is identified as the top quark Ψ_t, then the resulting interaction is $\propto |\bar{\Psi}_t \Psi_t|^2$. This yields a top-mode standard-model-type of model

[45]Fermions transform under $U(1)_{\gamma_5}$: $f \mapsto \exp(i\alpha\gamma_5)f$, i.e., $f_L \mapsto e^{-i\alpha} f_L$, $f_R \mapsto e^{+i\alpha} f_R$.

[46]$\tilde{\nu}^c_R$ is a shorthand notation for: $\bar{\nu}^c_R \equiv \overline{(\nu_R)^c}$.

that, for large enough G, leads to $\overline{t}t$ condensation and $m_t^{\mathrm{dyn}}\neq 0$. The other two up-type quarks remain massless. This leads to a 3×3 up-type quark mass matrix which, in a specific basis, has a completely flavor-democratic form (all nine elements equal). The $\overline{t}t$ condensation represents, in that basis, breaking of the original global $U(3)_L\times U(3)_R$ symmetry (masslessness) to the $S(3)_L\times S(3)_R$ symmetry, where $S(3)$ is the group of permutations of three elements. The masses $m_u, m_c\neq 0$ were explained phenomenologically via explicit breaking of the $S(3)_L\times S(3)_R$. The down-type quark mass matrix was constructed analogously and the hierarchy of the weak (CKM) mixing angles was taken into account in the procedure.

Lindner and Lüst (1991) proposed an explanation of how to obtain acceptable lower m_t^{dyn} without enlarging the standard-model symmetry of the NJLVL framework and without introducing an enlarged *low-mass* composite scalar sector. They argued that the minimal framework naturally allows a larger spectrum involving additional composite scalars and vectors, made up of $\overline{t}$, t, and b_L, and with masses $M>\mathcal{O}(E_{\mathrm{ew}})$. For example, the additional scalars $\tilde{H}^{(i)}$ can be radial excitations of the "ground" state $H\sim\overline{t}t$. They allowed the existence of colorless composite excited states $\tilde{H}^{(i)}$, $\tilde{B}^{(j)}$, and $\tilde{W}^{(k)}$—scalar isodoublets, vector isosinglets, and vector isotriplets, respectively. Although their RG analysis does not rely on any specific choice of the effective terms at the compositeness scale Λ, they motivated the model by various scalar and vector NJLVL terms. The authors then constructed RG equations for the Yukawa parameters g_t and $\tilde{g}_t^{(i)}$ (where $\tilde{g}_t^{(i)}$ corresponds to the coupling of $\tilde{H}^{(i)}$ to t_R) and for the gauge coupling parameters g_1, g_2, g_3, $\tilde{g}_B^{(j)}$, and $\tilde{g}_W^{(k)}$. At $\mu=\Lambda$ they imposed compositeness conditions: g_t, $\tilde{g}_t^{(i)}$, $\tilde{g}_B^{(j)}$, $\tilde{g}_W^{(k)}\to\infty$. In the RG equations they took into account the threshold effects of the heavy composite particles—by switching off these degrees of freedom of mass M for energies $\mu<M$. Although $M\gg E_{\mathrm{ew}}$, the presence of these degrees of freedom in the RG equation for g_t nonetheless influences the evolution of g_t substantially, leading to values $m_t(m_t)$ different from those of the minimal framework. In particular, emergence of excited scalars, i.e., of their Yukawa parameters $\tilde{g}_t^{(i)}$, increases the slope $dg_t^2/d\ln\mu$ and thus leads to smaller values of $g_t(E_{\mathrm{ew}})$ and $m_t(m_t)$ than in the minimal top-mode standard-model framework. The authors chose a form for the mass spectrum of the excited particles which is equidistant on the log scale and extends through the interval (M_1,Λ), where M_1 ($\gg E_{\mathrm{ew}}$) is the mass of the lowest excited state. At the thresholds, they apparently took $\tilde{g}_t^{(i)}(\mu)=\tilde{g}_t^{(i+1)}(\mu)$ $[\equiv\tilde{g}_t(\mu)]$ at $\mu=M_{i+1}$ $(i\geq 1)$. For $\tilde{g}_t(\Lambda)/g_t(\Lambda)$ they chose values 2, 1, and 0.5. The choice $\tilde{g}_t(\Lambda)/g_t(\Lambda)=2$ gave the lowest values for m_t. However, as they argued, this choice represents very strongly coupled excited scalars and is not likely within the NJLVL type of dynamic scenario. Lindner and Lüst obtained the lowest m_t's when the excited composite states were scalars only.[47] In such cases, for the choice $\tilde{g}_t(\Lambda)/g_t(\Lambda)=1$, $M_1=10^2 M_W$, and for the number of excited scalars $\tilde{N}_H=5{,}1$, acceptable values $m_t(m_t)\approx 167$ GeV ($m_t^{\mathrm{phys}}\approx 175$ GeV) were obtained at $\Lambda\sim 10^9$ GeV, and 10^{19} GeV, respectively. For much heavier excitations, $M_1=10^{10}M_W$, the value $m_t(m_t)\approx 167$ GeV can be reached for $\Lambda<E_{\mathrm{Planck}}$ only if $\tilde{N}_H>5$. If another realistic choice $\tilde{g}_t(\Lambda)/g_t(\Lambda)=0.5$ is made (weakly coupled excited scalar states), m_t becomes higher: $m_t(m_t)\gtrsim 190$ GeV, for $\Lambda<\Lambda_{\mathrm{GUT}}$ and $\tilde{N}_H\leq 5$ and $M_1\geq 10^2 M_W$.

E. Condensation including the fourth generation

The existence of a heavy fourth quark generation (t',b') has been proposed and/or assumed by many authors, among others by Marciano (1989, 1990); Bardeen, Hill, and Lindner (1990); King (1990); Barrios and Mahanta (1991); Chesterman, King, and Ross (1991). The purpose was to avoid the too high m_t of the minimal framework. Usually, in such extended frameworks, the fourth quark masses were considered to be heavy and (almost) degenerate: $m_{t'}\approx m_{b'}>m_t$. Approximate degeneracy was needed to keep the value of the electroweak $\delta\rho$ parameter acceptably low. Heaviness was needed in order to have the fourth generation primarily responsible for the DEWSB (for $v\approx 246$ GeV), thus making m_t less constrained from below and adjustable to the experimental value $m_t(m_t)\approx 170$ GeV. Some of the results, obtained by the RG approach, are given in Table VII Implicitly it was assumed that the fourth-generation leptons have masses substantially lower than $m_{t'}$.

Hill, Luty, and Paschos (1991) considered a model in which the *leptonic* sector played a crucial role in fourth-generation condensation. The model was partly motivated by the LEP+SLC limits on the mass of the fourth-generation neutrino eigenstate: $m_{\nu_4}>M_Z/2$. Further, the $\delta\rho$ constraints imply near degeneracy of the heavy fourth-generation fermions: $m_{t'}\approx m_{b'}$ and $m_{\nu_4}\approx m_{l_4}$, where l_4 is the charged fourth-generation lepton. Hence, if there are four generations, then the masses of the fourth-generation mass eigenstates t', b', ν_4, and l_4 are higher by at least a factor $\sim 10^2$ than those of other fermions except for the top quark. The authors therefore believed that these four heavy fermions play the central role in their own dynamical mass generation. Another motivation was to have a viable scenario with the cutoff Λ as low as ~ 1 TeV, thus allowing direct probes of the underlying physics by future LHC experiments. How-

[47]When including composite vector (gauge) fields, their framework implied even compositeness of (the transverse components of) Z, γ, and $W^{\pm}$: $g_1(\Lambda)$, $g_2(\Lambda)=\infty$. NJLVL frameworks leading to the effective minimal standard model with fully composite electroweak gauge bosons were investigated by Terazawa, Akama, and Chikashige (1976, 1977), D. Kahana and S. Kahana (1991, 1995), and Akama and Hattori (1997).

TABLE VII. Predicted renormalized masses (in GeV) of degenerate fourth-generation quarks and of the (composite) Higgs. The results are taken from Bardeen, Hill, and Lindner (1990) and King (1995).

Λ [GeV]	10^{19}	10^{15}	10^{11}	10^{7}	10^{4}	2×10^{3}
$m_{t'}=m_{b'}$	218	229	248	293	455	$\sim$1000
m_H	239	256	285	354	605	$\sim$2000

ever, the top quark did not play a direct role in their dynamical framework—its mass was fixed to be $m_t(m_t) \approx 130$ GeV by choosing the value of its Yukawa parameter in their RG analysis accordingly. The compositeness conditions were applied to the Yukawa parameters of the fourth-generation fermions and to those of the Majorana sector, but not to the top-quark Yukawa parameter. Although the dynamical framework of these authors is rich and interesting, discussing it in greater detail would take us beyond the limited scope of this review article which includes frameworks involving $\overline{t}t$ condensation.

F. Including the leptonic sector with third generation only

Martin (1991) proposed a scenario in which the third generation of quarks *and* the third generation of leptons participate in the condensation. The third-generation neutrino sector also included a right-handed component ν_R with a large Majorana mass term $\propto M_{\rm M}$. Martin assumed the hierarchy $E_{\rm ew} \ll M_{\rm M} \ll \Lambda$, where Λ is the compositeness scale. The Higgs particle turned out to be a combination of third-generation quark and lepton condensates: $\tilde{H} \sim k_t(\overline{t^a_R} Q^a_L) + k_\nu(\overline{\nu_R} L_L)$. Unlike Hill, Luty, and Paschos (1991), Martin did not devise any specific dynamical mechanism for generation of $M_{\rm M} \overline{\nu^c_R} \nu_R$; he just assumed its presence. For the third-generation Dirac mass $m_{\nu,{\rm D}}$, one needs an additional hierarchy $m_{\nu,{\rm D}} \sim E_{\rm ew} \ll M_{\rm M}$, since then the see-saw mechanism (Gell-Mann, Ramond, and Slansky, 1979; Yanagida, 1979) can yield an acceptably light eigenmass $m_\nu^{(2)} \approx m^2_{\nu,{\rm D}}/M_{\rm M} \ll E_{\rm ew}$. The inclusion of threshold effects in the RG equations then simplifies because $\nu_R \approx \nu^{(1)}$, where $\nu^{(1)}$ is the heavier mass eigenstate with mass $m_\nu^{(1)} \approx M_{\rm M}$. The heavy ν_R contributes to the evolution of the Dirac-Yukawa parameters $g_t(\mu)$ and $g_\nu^{\rm D}(\mu)$ in the energy interval $[M_{\rm M},\Lambda]$, by increasing the slopes $dg_t(\mu)/d\ln\mu$ and $dg_\nu^{\rm D}(\mu)/d\ln\mu$. The renormalization-group compositeness conditions for $\tilde{H}$ were

$$g_t(\mu) \to \infty, \quad g_\nu^{\rm D}(\mu) \to \infty \quad \text{when} \quad \mu \to \Lambda,$$

$$\text{and} \quad \frac{k_\nu}{k_t} \approx \lim_{\mu\to\Lambda} \frac{g_\nu^{\rm D}(\mu)}{g_t(\mu)} \neq 0 \quad (\sim 1). \tag{6.22}$$

Renormalized Dirac masses $m_t(m_t) = g_t(m_t) v/\sqrt{2}$ and $m_{\nu,{\rm D}} = g_\nu^{\rm D}(M_Z) v/\sqrt{2}$ were obtained from the RG equations under inclusion of Eq. (6.22). Both were $\sim E_{\rm ew}$. The hierarchy $m_{\nu,{\rm D}} \sim E_{\rm ew} \ll M_{\rm M}$ thus appears automatically. Results for various choices of $M_{\rm M}$ and Λ satisfying the assumed hierarchy $E_{\rm ew} \ll M_{\rm M} \ll \Lambda$ were presented. Some of them far exceeded the experimental upper bound $m_\nu^{(2)} < 24$ MeV for tau neutrinos. For given values of $M_{\rm M}$ and high enough values of Λ, it is in general possible to adjust the finite nonzero ratio $g_\nu^{\rm D}(\Lambda)/g_t(\Lambda)$ of the compositeness condition so that $m_t(m_t) \approx 170$ GeV ($m_t^{\rm phys} \approx 175$ GeV). Such results are included in Table VIII. The values of the Higgs masses were obtained by considering, in addition to the above-mentioned RG equations, the one-loop RG equations for the scalar self-coupling parameter $\lambda(\mu)$ modified by the presence of a heavy ν_R. From the table, we can see that the acceptable values $m_t(m_t) \approx 170$ GeV are possible here without resorting to very high cutoff values $\Lambda > E_{\rm Planck} \sim 10^{19}$ GeV. In fact, the values $\Lambda \gtrsim 10^8$ GeV appear to be acceptable, while much lower values lead to an unacceptable $m_\nu^{(2)} > 24$ MeV. When $\ln M_{\rm M}$ is rather close to $\ln\Lambda$, the needed ratio $g_\nu^{\rm D}(\Lambda)/g_t(\Lambda)$ $(= k_\nu/k_t)$ must be taken larger to attain the low value $m_t(m_t) = 170$ GeV. Then, $\tilde{H}$ has an appreciable admixture of the lepton condensate $\overline{\nu_R} L_L$.

TABLE VIII. Some values of the compositeness scale Λ and of the third-generation Majorana mass $M_{\rm M}$, which allow $m_t(m_t) \approx 170$ GeV. Corresponding values of Higgs mass M_H and of the light third-generation neutrino eigenmass $m_\nu^{(2)} \approx m^2_{\nu,{\rm D}}/M_{\rm M}$ are also given. The first line is excluded by experimental bounds $m_\nu^{(2)} < 24$ MeV. The numbers were read off or deduced from graphs of the work of Martin (1991).

Λ [GeV]	$M_{\rm M}$ [GeV]	m_t [GeV]	M_H [GeV]	$m_\nu^{(2)}$ [MeV]
10^{10}	10^{6}	170	235	87
10^{10}	10^{8}	170	225	1.7
10^{15}	10^{9}	170	210	5.1×10^{-2}
10^{15}	10^{11}	170	208	7.6×10^{-4}
10^{15}	10^{13}	170	205	1.5×10^{-5}

Thus the described scenario gives acceptable values of $m_t^{\rm dyn}$ and $m_\nu^{(2)}$, for compositeness scales Λ between 10^8 GeV and $E_{\rm Planck}$. In comparison with the minimal framework, at least two additional relatively free parameters $[M_{\rm M}$ and $g_\nu^{\rm D}(\Lambda)/g_t(\Lambda)]$[48] appear in the framework, thus making predictions of the model dependent on them, too.

VII. ENLARGING THE SYMMETRY OR THE GAUGE SYMMETRY GROUP

A. Top-quark condensation in supersymmetry

Top-quark condensation within the framework of the minimal supersymmetric standard model was first proposed by Bardeen, Clark, and Love (1990). The main motivation was to avoid fine tuning of the relevant four-quark parameter of the non-supersymmetric framework for large Λ, in view of the fact that supersymmetric (SUSY) models are free from Λ^2 terms in the two-point

[48]The minimal framework has only one free parameter (Λ).

Green's functions of the Higgs superfields. This is a consequence of cancellation between boson and corresponding fermion loops. The authors used an $SU(3)\times SU(2)\times U(1)$-invariant softly broken NJLVL type of interaction (Buchmüller and Love, 1982; Buchmüller and Ellwanger, 1984) at a compositeness scale Λ. The extra SUSY particles were assumed to decouple at a SUSY soft-breaking scale $\Delta_S<\Lambda$. The model becomes, at scales $\mu<\Lambda$ ($\mu>\Delta_S$), the minimal SUSY model (MSSM), but with both Higgs superfield multiplets being composite, made up of pairs of top- and bottom-quark SUSY chiral multiplets.

Couplings of these Higgs bosons to the fermion chiral multiplets in the mentioned energy intervals are analogous to the non-SUSY type-II 2HDSM discussed in Sec. VI.A. However, there is one crucial difference between the discussed composite minimal supersymmetric standard model and the composite type-II 2HDSM: in the latter, both composite Higgs doublets become dynamical at energies $\mu<\Lambda$, predominantly through quark-loop quantum effects; in the composite minimal supersymmetric standard model, the composite $H_2\equiv H_u$ develops a kinetic-energy term at $\mu<\Lambda$ analogous to the non-SUSY case, predominantly through quark superfield loops, while the composite $H_1\equiv H_d$ already has a canonical kinetic-energy term at the NJLVL level (at $\mu=\Lambda$). In contrast to the composite type-II 2HDSM in Sec. VI.A.3, in SUSY RG equations with third-generation quarks, the usual "non-SUSY" compositeness conditions are taken only for $H_2\equiv H_u$: $Z_u(\mu)\to 0$ when $\mu\to\Lambda$; thus $g_t(\mu)\to\infty$ when $\mu\to\Lambda$. There is no such compositeness boundary condition for $Z_d(\mu)$ or, equivalently, for $g_b(\mu)$ and $g_\tau(\mu)$, in contrast to the fully composite non-SUSY type-II 2HDSM.

Bardeen, Clark, and Love (1990) ignored fermion multiplet masses other than m_t, and derived the gap equation at the quark-multiplet-loop (bubble) level:

$$(a_S\equiv)\,\frac{GN_c\Delta_S^2}{8\pi^2}=\left[\left(1+\frac{m_t^2}{\Delta_S^2}\right)\ln\left(\frac{\Lambda^2}{\Delta_S^2+m_t^2}\right)-\frac{m_t^2}{\Delta_S^2}\ln\left(\frac{\Lambda^2}{m_t^2}\right)\right]^{-1}(\sim 1). \qquad (7.1)$$

Here G is the coupling strength of the SUSY NJLVL four-quark interaction, analogous to G of the non-SUSY case (2.1). Comparing Eq. (7.1) with its non-SUSY counterpart (4.9), we see that the low SUSY-breaking scale $\Delta_S\sim 1$ TeV ($\Delta_S\ll\Lambda$) ameliorates the familiar fine tuning of the high-Λ case—there is no longer any Λ^2 dependence. However, Eq. (7.1) implies $G\sim\Delta_S^{-2}$ ($\gg\Lambda^{-2}$), which may be somewhat unnatural since the physics responsible for the four-quark interaction (and hence for G) appears at scales $\mu\gtrsim\Lambda$ ($\gg\Delta_S$). Thus the composite SUSY framework does not explain the hierarchy $\Delta_S/\Lambda\ll 1$. The latter is imposed by hand.

In analogy with the non-SUSY framework, RG analysis within the minimal supersymmetric standard model, with the corresponding compositeness boundary condition, represents an improvement over the quark-multiplet-loop aproximation. In the interval $[\Delta_S,\Lambda]$, the one-loop minimal SUSY standard-model RG equation for g_t is

$$16\pi^2\frac{dg_t(\mu)}{d\ln\mu}=\left[6g_t^2(\mu)-\frac{16}{3}g_3^2(\mu)-3g_2^2(\mu)-\frac{13}{9}g_1^2(\mu)\right]g_t(\mu), \qquad (7.2)$$

and the compositeness condition is $g_t(\Lambda)=\infty$. Gauge coupling parameters g_j satisfy the SUSY-modified RG equations when compared with their non-SUSY counterparts within the minimal standard model, Eqs. (2.18) and (2.19): $C_j\mapsto\tilde{C}_j$; $\tilde{C}_3=3$, $\tilde{C}_2=-1$, $\tilde{C}_1=-11$. Evolution below Δ_S for the Yukawa and gauge coupling parameters is governed either by minimal-standard-model RG equations or by RG equations of the composite type-II 2HDSM, depending on values of the mass parameters in the scalar potential. At $\mu=\Delta_S$, continuity of the quark masses $m_q(\mu)$ [and of $g_j(\mu)$'s] has to be implemented.

Bardeen, Clark, and Love (1990) included in their work a one-loop RG analysis and chose the VEV ratio $\tan\beta(\equiv v_u/v_d)=\infty$ ($\Rightarrow v_d\approx 0$, $v_u\approx v$). The lowest m_t [$\equiv m_t(m_t)$], for a given value of Δ_S was obtained at the highest $\Lambda\sim E_{\rm Planck}\sim 10^{19}$ GeV. For $\Delta_S\sim 1$ TeV and $\Lambda\sim 10^{19}$ GeV, $m_t\approx 196$ GeV was obtained. When Δ_S was increased, m_t increased, too.

Subsequently, Bardeen *et al.* (1992) allowed in their RG analysis, in general, a nonzero VEV $\langle H_1\rangle_0\equiv v_1$ ($\equiv v_d$) ($\tan\beta<\infty$) and included the bottom-quark chiral multiplet and its Yukawa parameter g_b. Their results for $m_t(m_t)$, when $\Delta_S=1$ TeV, are given in Fig. 9, where it can be seen that, for $\tan\beta(\equiv v_u/v_d)\gtrsim 1$, acceptable values, $m_t(m_t)\approx 160$–180 GeV, can be obtained if $\Lambda\gtrsim 10^7$ GeV. For $\tan\beta\gg 1$ ($\Rightarrow v_u\approx v=246$ GeV), the m_t's are too large. Incidentally, it is straightforward to see why

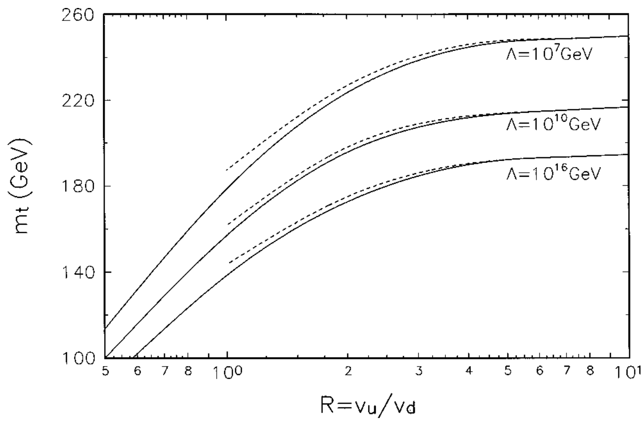

FIG. 9. Mass $m_t(m_t)$ as a function of the VEV ratio $R=v_u/v_d$, in the composite minimal supersymmetric standard model for the supersymmetric breaking scale $\Delta_S=1$ TeV and for three different values of the compositeness scale Λ. Solid lines are for the case in which two Higgs doublets are also present at $\mu<\Delta_S$, and dotted lines for the case in which one Higgs doublet is present at $\mu<\Delta_S$. From Bardeen *et al.* (1992).

m_t decreases when $\tan\beta$ decreases (or when Λ increases).

The main reason why the composite minimal SUSY standard-model framework allows acceptably low m_t, in contrast to the fully composite (non-SUSY) type-II 2HDstandard model of Sec. VI.A.3, is the freedom to change (decrease) $\tan\beta$ while still keeping $m_b \approx 5$ GeV. This freedom has its origin in the fact that the SUSY compositeness condition is represented by $g_t(\Lambda)\infty$ and by *no* analogous requirement for g_b. Therefore $g_b(\Lambda)/g_t(\Lambda)=0$ [in fully composite type-II 2HDSM, $g_b(\Lambda)/g_t(\Lambda)\gg 1$]. This is so because no compositeness condition is imposed on $H_1 \equiv H_d$ at $\mu=\Lambda$, since the latter composite superfield already has a canonical kinetic-energy term at Λ (see second paragraph of Sec. VII.A). Even when $\tan\beta=\infty$ ($v_d=0$) is chosen, which is analogous to the minimal framework, the predicted m_t's (Bardeen, Clark, and Love, 1990) are lower than those of the minimal framework. That is, comparing Eq. (7.2) with Eq. (2.16), we see that the extra degrees of freedom of the minimal SUSY standard model in the interval $[\Delta_S,\Lambda]$ weaken the negative contributions of QCD to the slope $dg_t/d\ln\mu$ and enhance the positive contributions of Yukawa interactions, thus lowering the infrared fixed point of g_t.

In later work, Bardeen, Carena, Pokorski, and Wagner (1994), included evolution of the τ Yukawa parameter in the minimal SUSY standard-model framework. They showed that the g_b-g_τ Yukawa unification at $E \sim E_{\rm GUT}$, as predicted by many grand unification schemes, can be implemented in the minimal supersymmetric standard model with acceptable $m_t(m_t)$. They demonstrated that in such cases the value of $m_t(m_t)$ is very near the infrared fixed point (within 10%), thus depending basically only on $\tan\beta \equiv v_u/v_b$. The observation that the g_b-g_τ unification requirement at $E_{\rm GUT}$ drives $m_t(m_t)$ close to the infrared fixed point had already been made somewhat earlier by other authors. However, Bardeen, Carena, *et al.* pointed out that large g_t's at $E_{\rm GUT}$ are suggestive of the onset of nonperturbative physics, which may result in compositeness of $H_2 \equiv H_u$.

It was emphasized by Hill (1997) that the top quark appears to be the only known particle yielding a nontrivial vanishing of the β function of its Yukawa coupling parameter, i.e., that the actual top-quark mass $m_t \sim E_{\rm ew}$ is controlled by the RG infrared fixed point, if the (Landau) pole energies are very large, $\Lambda \gg E_{\rm ew}$. Hill stressed that this is the case not just in the minimal supersymmetric standard model, but also in all those condensation scenarios that have (drastic) fine tuning $\Lambda \gg E_{\rm ew}$, including the Bardeen, Hill, and Lindner scenario of the minimal framework.

Clark, Love, and ter Veldhuis (1991) investigated[49] whether effective ten-dimensional four-quark interactions of the Suzuki type [see Sec. V.E] could substantially change predictions of the minimal SUSY top-condensation model discussed above. These authors also used an $SU(3)\times SU(2)\times U(1)$-invariant softly broken NJLVL-type interaction at a compositeness scale Λ, but added Suzuki terms. They applied the quark-multiplet loop approximation to this effective action in an interval $[\mu_*,\Lambda]$ determined by the "perturbativity breakdown" condition: $g_t^2(\mu_*)/(4\pi)=1$. The ratio μ_*/Λ was dependent on Λ and on the Suzuki parameter ξ, where ξ^2/Λ^4 is the ratio of the coupling parameters of the ten- and six-dimensional four-quark interactions. Then they applied the usual minimal SUSY standard-model RG equations in the interval $[\Delta_S,\mu_*]$, taking into account at μ_* the mentioned condition. In the low-energy interval $[M_Z,\Delta_S]$, minimal-standard-model RG equations were applied. They chose $\Delta_S \sim 1$ TeV or 10 TeV, $\Lambda \sim 10^{16}$ GeV, and $v_u/v_d=4,2,1$. They concluded that, as long as ξ was not unreasonably large ($|\xi|<3$), the predicted m_t varied very little (by 5–6 GeV). This result is similar to those in the non-SUSY framework of Bardeen (1990), Suzuki (1990a), and Hill (1991)—see Sec. V.E.

Froggatt, Knowles, and Moorhouse (1993) applied in their composite minimal SUSY standard-model framework the compositeness condition at Λ for $H_2 \equiv H_u$ [$Z_u(\Lambda)\approx 0$, i.e., $g_t(\Lambda)\gg 1$] and for $H_1 \equiv H_d$ [$Z_d(\Lambda)\approx 0$, i.e., $g_b(\Lambda)\gg 1$] and predicted $m_t \approx 184$ GeV, when $\Lambda \sim 10^{16}$ GeV. However, motivation within the minimal supersymmetric standard model for this second compositeness condition [$g_b(\Lambda)/g_t(\Lambda)\sim 1 \neq 0$] in their work is unclear, in view of the differences in kinetic-energy terms discussed in the second paragraph of this section (VII.A).

Ellwanger (1991) showed that supersymmetric nonlinear sigma models can be the origin of the supersymmetric NJLVL type of interactions, where the latter lead to the DEWSB. As in the works discussed above, the supersymmetry was assumed to be broken explicitly, not dynamically, at a scale $\Delta_S<\Lambda$. Ellwanger investigated in detail a model based on the coset space $E_8/SO(10)\times SU(3)$. The $SU(3)$ family symmetry is broken at $E_{\rm GUT}$ dynamically, simultaneously with the GUT group $SO(10)$. Masses of the lighter quarks and leptons are then generated in this framework through radiative corrections.

Ellwanger's framework is an *effective* one. An attempt was later made by Dawid and Reznov (1996) to construct a supersymmetric *renormalizable* underlying theory leading to a supersymmetric NJLVL model and thus to $\bar{t}t$ condensation. They showed that this is improbable, because exchange of heavy particles with mass M_S ($\equiv\Lambda$) in such a framework would not separate the scales Λ^2 and $G^{-1}\sim\Delta_S^2$ [cf. Eq. (7.1)] and would thus not lead to an effective minimal supersymmetric standard model at $E \lesssim \Lambda$. The authors proposed a framework in which the nonrenormalizable structure is maintained up to $E_{\rm Planck}$, thus arguing that in a supersymmetric framework an underlying theory for $\bar{t}t$ condensation should be connected to supergravity.

[49]That paper was submitted somewhat later than the paper of Bardeen *et al.* (1992), and also contains results showing that m_t decreases to acceptable values when $\tan\beta \equiv v_u/v_d$ decreases.

B. Dynamical left-right symmetry breaking plus electroweak symmetry breaking

Akhmedov *et al.* (1996a, 1996b) studied in detail a fully dynamic scenario of symmetry breaking in a left-right (L-R)-symmetric model. In the model, standard fermions of the third generation and an additional (non-standard) fermion participate in condensation. The model contains no elementary scalars. They started at a certain compositeness scale Λ ($>E_{\rm ew}$) with an NJLVL Lagrangian density $\mathcal{L}^{(\Lambda)}=\mathcal{L}_1^{(\Lambda)}+\mathcal{L}_2^{(\Lambda)}$ respecting gauge symmetry $G\equiv SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$. $\mathcal{L}_1^{(\Lambda)}$ consisted of six NJLVL[50] terms (and their Hermitian conjugates) involving only the third-generation fermions $Q=(t,b)^T$ and $\Psi=(\nu_\tau,\tau)^T$. Terms that might lead to colored condensates were not included. $\mathcal{L}_2^{(\Lambda)}$ consisted of two NJLVL terms involving leptons Ψ and an additional gauge singlet fermion $S_L\sim(1,1,1,0)$. The latter was introduced in order to have a correct dynamical-symmetry-breaking pattern with a spontaneous breaking of parity. The four-fermion coupling parameters $G_k=8\pi^2 a_k/\Lambda^2$ ($k=1,\ldots,8$; $a_k\sim1$) originate from an unspecified underlying physics at $\mu>\Lambda$. If some of them (G_2, G_4, G_5, and G_6) are real, $\mathcal{L}^{(\Lambda)}$ is invariant under the discrete parity symmetry: $Q_L\leftrightarrow Q_R$, $\Psi_L\leftrightarrow\Psi_R$, $S_L\leftrightarrow(S_L)^c$ [$\equiv(S^c)_R$].

Akhmedov *et al.* argued that, in a fully dynamical scenario without the additional fermion S_L, we would have to have in general "strong enough" four-lepton interactions of the Majorana type. That is, in order to arrive at a phenomenologically required dynamical tumbling symmetry-breaking scenario

$$G\equiv SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$$

$$\overset{E_R}{\to} SU(3)_c\times SU(2)_L\times U(1)_Y$$

$$\overset{E_{\rm ew}}{\to} SU(3)_c\times U(1)_{\rm em},$$

these Majorana-type NJLVL interactions would have to be strong enough (critical or supercritical) to lead to the formation of two composite Higgs triplets $\Delta_R\propto\Psi_R^T C\tau_2\vec{\tau}\Psi_R$ and $\Delta_L\propto\Psi_L^T C\tau_2\vec{\tau}\Psi_L$, which transform under the full gauge group G as (1,1,3,2) and (1,3,1,2), respectively. The first part of symmetry breaking (L-R) in the tumbling then occurs dynamically at a right-handed scale $E_R\sim\langle\Delta_R\rangle_0$. Incidentally, such a composite Higgs scenario would lead to the desirable see-saw mechanism and the consequent natural suppression of the lighter neutrino eigenmasses. However, the presence of the two *composite* Higgs triplets would not lead to a phenomenologically viable L-R symmetry-breaking pattern ($\langle\Delta_R\rangle_0>\langle\Delta_L\rangle_0\sim E_{\rm ew}$). The authors argued, in the quark-loop approximation, that this dynamical scenario would give $\langle\Delta_R\rangle_0=\langle\Delta_L\rangle_0$ and hence no parity violation.

On the other hand, $\mathcal{L}_1^{(\Lambda)}$ and $\mathcal{L}_2^{(\Lambda)}$ lead to formation of a composite bidoublet $\phi\sim(1,2,2,0)$, two composite semileptonic doublets $\chi_R\sim(1,1,2,-1)$ and $\chi_L\sim(1,2,1,-1)$, and a singlet scalar σ. The authors showed that this scenario does allow a phenomenologically viable dynamical-symmetry-breaking pattern, with $E_R\sim\langle\chi_R^0\rangle_0\gg E_{\rm ew}\sim\langle\phi_{ij}\rangle_0\sim\langle\chi_L^0\rangle_0$ and $\langle\chi_R^0\rangle_0\gtrsim\langle\sigma\rangle_0$. At $\mu=\Lambda$ these fields are auxiliary, and the authors rewrote the density $\mathcal{L}^{(\Lambda)}=\mathcal{L}_1^{(\Lambda)}+\mathcal{L}_2^{(\Lambda)}$ with them:

$$\mathcal{L}^{(\Lambda)}_{\rm aux.}=-M_0^2(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)-M_1^2\,{\rm tr}(\phi^\dagger\phi)-\frac{M_2^2}{2}{\rm tr}(\phi^\dagger\tilde{\phi}+{\rm H.c.})-M_3^2\sigma^\dagger\sigma \tag{7.3}$$

$$-[Y_1\bar{Q}_L\phi Q_R+Y_2\bar{Q}_L\tilde{\phi}Q_R+Y_3\bar{\Psi}_L\phi\Psi_R+Y_4\bar{\Psi}_L\tilde{\phi}\Psi_R+{\rm H.c.}] \tag{7.4}$$

$$-[Y_5[\bar{\Psi}_L\chi_L(S_L)^c+\bar{\Psi}_R\chi_R S_L]+Y_6(S_L^T C S_L)\sigma+{\rm H.c.}], \tag{7.5}$$

where $\tilde{\phi}\equiv\tau_2\phi^*\tau_2$. The equations of motion show that ϕ is a condensate of third-generation fermions, χ_L and χ_R are condensates of leptons and S_L, and σ is a condensate of S_L,

$$\phi_{ij}\sim\alpha(\bar{Q}_{Rj}Q_{Li})+\beta(\tau_2\bar{Q}_L Q_R\tau_2)_{ij}+\gamma(\bar{\Psi}_{Rj}\Psi_{Li})+\delta(\tau_2\bar{\Psi}_L\Psi_R\tau_2)_{ij}, \tag{7.6}$$

$$\chi_L\sim S_L^T C\Psi_L,\quad \chi_R\sim\bar{S}_L\Psi_R,\quad \sigma\sim\bar{S}_L C\bar{S}_L^T, \tag{7.7}$$

where C is the charge-conjugation matrix. In the dynamical symmetry breaking described above, the first (L-R) breaking occurs due to the nonzero VEV $v_R=\langle\chi_R^0\rangle_0$, and the second (electroweak) breaking due to the bidoublet VEV's $\langle\phi_{11}\rangle_0=\kappa$ and $\langle\phi_{22}\rangle_0=\kappa'$ ($\langle\phi_{12}\rangle_0=\langle\phi_{21}\rangle_0=0$). Minimizing the Higgs potential, the authors obtained zero values for the remaining VEV's: $v_L\equiv\langle\chi_L^0\rangle_0=0$, $\langle\sigma\rangle_0=0$. They assumed the hierarchy $\Lambda\gg v_R\gg E_{\rm ew}$ ($\sim\kappa,\kappa'$). In this phenomenologically viable tumbling dynamical symmetry breaking, the breaking of $SU(2)_R$ and of the discrete parity symmetry at a right-handed scale $E_R\sim v_R$ eventually drives the EWSB at a lower scale $\sim E_{\rm ew}$. The model has nine input parameters: attractive (positive) G_j ($j=1,\ldots,8$) and the compositeness scale Λ. Quark and Dirac neutrino masses are determined by VEV's $\langle\phi_{ij}\rangle_0$ and by corresponding Yukawa parameters Y_j:

$$m_t=Y_1\kappa+Y_2\kappa',\quad m_b=Y_1\kappa'+Y_2\kappa,$$

$$m_{\nu,{\rm D}}=Y_3\kappa+Y_4\kappa',\quad m_\tau=Y_3\kappa'+Y_4\kappa, \tag{7.8}$$

where $\kappa^2+\kappa'^2=v^2/2\approx174^2\,{\rm GeV}^2$. The authors had to abandon the scenario with composite triplets, and thus

[50]NJLVL terms are four-fermion terms without derivatives.

the usual Dirac-Majorana see-saw mechanism, but their model possessed a modified see-saw with two heavy Majorana neutrinos with eigenmasses $M \approx Y_5 v_R$ and a light Majorana neutrino with eigenmass $m_\nu \approx 2Y_6 \langle\sigma\rangle_0 (m_{\nu,\mathrm{D}}/M)^2 - 2Y_5 v_L m_{\nu,\mathrm{D}}/M$. This mass vanishes when $v_L, \langle\sigma\rangle_0 \to 0$ or $M \to \infty$. First Akhmedov *et al.* investigated the described tumbling dynamical symmetry breaking by the effective-potential approach in the approximation of including quark and transverse gauge-boson loops only (no next-to-leading-order scalar "feedback" contributions), and subsequently by the RG approach with compositeness conditions $Z_\phi(\Lambda) = Z_\chi(\Lambda) = Z_\sigma(\Lambda) = 0$. This translated into compositeness conditions for Yukawa and scalar self-coupling parameters. The values $m_t(m_t) \approx 180$ GeV can be achieved in this model for values of the bidoublet VEV ratio $\kappa'/\kappa \approx 1.3$–4 [see Fig. 10 for $m_t(m_t) = 180$ GeV]. As can be seen from Fig. 10, Λ can be low ($\sim 10^2$ TeV), and the right-handed scale $E_R \sim v_R \sim 10$ TeV. For such low Λ and E_R, however, the RG approach is not reliable and gives only qualitative results, since infrared fixed-point behavior is absent. If E_R and Λ are this low, predictions of the model, in particular lepton flavor violation effects (e.g., $\mu \to e\gamma$) as well as light CP-even and CP-odd neutral scalars (with masses as low as 60–100 GeV), could become testable in the foreseeable future.

Among the effective models of dynamical symmetry breaking in which the $\bar{t}t$ condensate plays a central role and which have been investigated so far, the model of Akhmedov *et al.* is probably one of those with the richest phenomenological consequences.

A dynamical-symmetry-breaking model somewhat similar to that of the L-R symmetric framework of Akhmedov *et al.* was proposed by Xue (1997). It contains, in a certain energy region between an energy ε and the cutoff Λ ($E_{\mathrm{ew}} < \varepsilon < \Lambda$), three-fermion right-handed bound states (composite fermions) $T_R \sim (\bar{t}_R t_L) t_R$, $B_R \sim (\bar{b}_R b_L) b_R$. This allows $W^\pm$ to have couplings to the right-handed composite quarks, while at the same time formally respecting the standard-model symmetry. Contributions of such couplings to Dyson-Schwinger equations[51] of the top-mode standard model, in the single-W-exchange approximation, lead to a coupled system for masses $\Sigma_t(\bar{p}^2)$ and $\Sigma_b(\bar{p}^2)$. This system, in contrast to the usual case, does not require the extreme fine tuning $\sim v^2/\Lambda^2$ of the four-quark parameter to its critical value, but a less severe fine tuning $\sim m_b^2/m_t^2$.

C. Other enlarged symmetry groups

To obtain an $m_t^{\mathrm{dyn.}}$ lower than that in the minimal framework, Kuo, Mahanta, and Park (1990) proposed another extended symmetry group, $G \equiv SU(3)_c \times SP(6)_L \times U(1)_Y$, involving the flavor gauge group $SP(6)_L$. They devised a scenario in which the breaking $G \to G_{\mathrm{standard\ model}}$ comes about in two stages: at an intermediate energy Δ ($\Lambda \gg \Delta \gg E_{\mathrm{ew}}$) the $SP(6)_L$ is broken spontaneously to $SU(2)_L$ by two Higgs 14-plets $H^{(\alpha)}$ ($\alpha = 1,2$), which transform under G as (1,14,0). These $H^{(\alpha)}$'s have VEV's and masses $\sim\Delta$. Details of the mechanism were left unspecified. However, the EWSB $SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{em}}$ was dynamical. The starting point was a G-invariant NJLVL Lagrangian density $\mathcal{L}_{4q}^{(\Lambda)} = g \bar{t}_R q_L^i \bar{q}_L^i t_R / \Lambda^2 + \mathrm{H.c.}$ Here the (q_L^i) quark multiplet ($i = 1,\ldots,6$) transforms as **6** under $SP(6)_L$. The top quark t_L is a linear combination of q_L^i's. Color indices are distributed as in the top-mode standard model. For $g > g_{\mathrm{crit}}$, a composite Higgs multiplet $\phi^i \sim \bar{t}_R q_L^i$ [$\sim$(1,6,1/2) under G] is dynamically generated at $\mu \sim \Lambda$. At lower $\mu \sim \Delta$, the mentioned two 14-plets $H^{(\alpha)}$ break $SP(6)_L$ spontaneously to $SU(2)_L$, ϕ^i splits into three $SU(2)_L$ doublets. One of them ($\phi^{(1)}$) is a flavor singlet and plays the role of the minimal-standard-model Higgs doublet at $\mu < \Delta$; it has a low mass ($M_H \sim E_{\mathrm{ew}}$). The other two doublets were assumed to be heavy ($M \sim \Delta$), thus decoupling for $\mu < \Delta$. After devising this scenario, the authors investigated one-loop RG flow for g_t and the scalar self-interaction parameter λ, separately in intervals of (Δ,Λ) and (E_{ew},Δ). In the former interval, the RG equations were based on a G-invariant Lagrangian and included the $SP(6)_L$ coupling parameter g_6 ($= g_2\sqrt{3}$ at $\mu = \Delta$ according to group theory). For $\mu < \Delta$, the RG equations were those of the minimal standard model. Kuo, Mahanta, and Park solved the RG equations, imposing the usual compositeness conditions at $\mu = \Lambda$. For $\Lambda \lesssim 10^9$ GeV and $\Delta \sim 10^4$–10^5 GeV, the resulting m_t and M_H were significantly lower than those of the Bardeen, Hill, and Lindner (1990) minimal framework: $|\Delta m_t| \gtrsim 13$ GeV, $|\Delta M_H| \gtrsim 23$ GeV. However, m_t was still substantially above 200 GeV. For $\Lambda \gtrsim 10^{17}$ GeV, the results were close to those of the minimal framework, lower by only a few GeV. Thus $m_t(m_t) > 210$ GeV for any $\Lambda \lesssim E_{\mathrm{Planck}}$ and $\Delta \gtrsim 10^3$ GeV. This has its origin

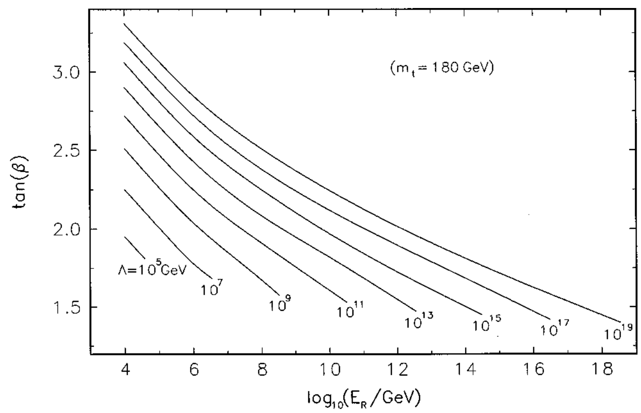

FIG. 10. Values of $\tan\beta \equiv \kappa'/\kappa$ for $m_t(m_t) = 180$ GeV and for various magnitudes of the compositeness scale Λ and the right-handed scale E_R. From Akhmedov *et al.* (1996b).

[51]The Dyson-Schwinger equations employed by Xue are in the quark-loop approximation, with addition of a single-photon-exchange contribution; the corresponding gluon contribution was surprisingly ignored.

in the value of the infrared (IR) fixed point of the RG equation for g_t for $\mu>\Delta$—it is very close to the IR fixed-point value of the minimal-standard-model RG equation for g_t.

Frampton and Yasuda (1991) introduced an $SU(N)_X\times G_{\text{standard model}}$ model with the DEWSB, where $N=2,4,\ldots$ is a so-called sark color. The usual fermions are singlets under $SU(N)_X$. Two additional colorless quarks U^a and D^a (where $a=1,\ldots,N$), called sarks, were introduced. U_L^a and D_L^a transform under $SU(N)_X\times SU(2)_L$ as $(N,2)$, and U_R^a, D_R^a as $(N,1)$. In the model, sarks play the dominant role in the DEWSB, resulting in a degenerate dynamical mass for both sarks and in a sark-composite Higgs isodoublet. The mass m_t is assumed to be substantially lower than the sark masses. Frampton and Yasuda employed one-loop RG analysis similar to that of Bardeen, Hill, and Lindner, and ended up with sark masses in the range of 170–455 GeV and Higgs masses larger than those of sarks by 15–40 %. Sark masses were lower when Λ was higher ($\Lambda \lesssim E_{\text{Planck}}$). When sark color number N was increased, masses increased for high Λ's ($\Lambda>10^{13}$ GeV) and decreased for lower Λ's. A major drawback of the scenario is that it gives no predictions for m_t—a drawback similar to that in the condensation scenarios of the fourth quark generation. With the new type of quarks playing the central role in formation of composite scalars, this framework bears some resemblance to technicolor scenarios in which condensates of new technifermions are responsible for the DEWSB.

VIII. RENORMALIZABLE MODELS OF UNDERLYING PHYSICS

A. Initial remarks

Construction of renormalizable models of the underlying physics leading to the NJLVL-type interactions believed to be responsible for $\bar{t}t$ condensation is in general a formidable project. It entails fulfilling several requirements, among them the following:

(a) anomalies must cancel;

(b) symmetry must be spontaneously broken for the gauge sector believed to be responsible for the effective four-fermion terms; this spontaneous symmetry breaking can also be dynamical;

(c) the effective four-fermion (NJLVL) terms must yield the correct hierarchy of those fermionic dynamical masses predicted by the framework (e.g., $m_t \gg m_b$).

One can well imagine that constructing such models is a difficult task. Concerning the predictability, a drawback of such models is that they are in general not simple, having additional unknown parameters, which can usually be adjusted so that phenomenologically acceptable results are obtained. But such drawbacks are characteristic of most extensions of the minimal framework. There is a large amount of literature on construction of renormalizable models involving $\bar{t}t$ condensation. Some of the models will be described in this review only briefly, without details.

B. Gauge frameworks with additional symmetries as factors of $U(1)$ or $SU(2)$

The standard-model group $G_{\text{standard model}} \equiv SU(3)_c\times SU(2)_L\times U(1)_Y$ can be extended, for example, by a simple Lie group factor. Bönisch (1991) investigated the question of whether an additional factor $U(1)$ or $SU(2)$ could lead to NJLVL terms responsible for $\bar{t}t$ condensation. He followed a general idea that the additional gauge bosons acquire heavy masses $M\sim\Lambda$ by some kind of (usually unspecified) spontaneous symmetry-breaking mechanism, and that exchange of these bosons between fermions leads at $E\lesssim\Lambda$ to effective NJLVL terms.[52] He considered three cases:

(1) An extra $U(1)$: $G\equiv SU(3)_c\times SU(2)_L\times U(1)_S \times U(1)_T$;

(2) An extra $SU(2)$ (L-R symmetric models): $G \equiv SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$;

(3) An extra gauged custodial $SU(2)_V$: $G\equiv SU(3)_c \times SU(2)_L\times U(1)_Y\times SU(2)_V$.

He concluded that in all three cases it is possible to generate NJLVL terms $(\bar{q}_L t_R)(\bar{t}_R q_L)$ and $(\bar{q}_L b_R)(\bar{b}_R q_L)$ $(q=t,b)$. These terms lead to $\bar{t}t$ and $\bar{b}b$ condensation, respectively, if $\kappa_q>\kappa_{\text{crit}}$ [$q=t,b$; cf. Eq. (3.2)]. In addition, he showed that L-R symmetric models naturally lead to the "feed-down" terms $(\bar{t}_L t_R) \times(\bar{b}_L b_R)+\cdots$ [see Eq. (3.2)] that cause the "feed-down" generation of m_b by the $\bar{t}t$ condensate. In all of these scenarios, exchange of heavy colorless gauge bosons leads, by a Fierz rearrangement, to scalar four-quark terms:

$$-\frac{g_X^2}{M_X^2}(\bar{\psi}_{1L}^a\gamma^\mu\psi_{2L}^a)(\bar{\psi}_{3R}^b\gamma_\mu\psi_{4R}^b) = +2\,\frac{g_X^2}{M_X^2}(\bar{\psi}_{1L}^a\psi_{4R}^b)(\bar{\psi}_{3R}^b\psi_{2L}^a), \tag{8.1}$$

where a,b are colors, ψ_k $(k=1,\ldots,4)$ are four-component spinor fields M_X $(\sim\Lambda)$ is the heavy gauge-boson mass, and g_X is its gauge coupling parameter. Bönisch noted that color indices are distributed differently here than they are in the top-mode standard-model terms (2.1), (3.2), suggesting that only terms with $a=b$ contribute to condensation. We shall return to this point shortly.

Bönisch (1991), in his derivation of the effective four-quark interactions, used formulas in which the electroweak gauge bosons W and Z were also taken to have

[52]King and Mannan (1990) were apparently the first to suggest and study heavy gauge-boson exchange as a means of generating four-fermion operators; they did it in a model-independent way (see also end of Sec. III.D).

nonzero masses. In such a picture the entire symmetry breaking $G \to SU(3)_c \times U(1)_{\rm em}$ takes place before the generation of four-quark interactions. However, at that stage, only the breaking $G \to G_{\rm standard\ model}$ should take place (i.e., $M_W = M_Z = 0$) because the four-quark terms generated by the latter breaking are responsible for the subsequent dynamical breaking $G_{\rm standard\ model} \to SU(3)_c \times U(1)_{\rm em}$. Even when one performs the corresponding modifications (i.e., setting $M_W = M_Z = 0$, and changing the mixing angles correspondingly), it appears that all the major qualitative conclusions of Bönisch survive, although some more quantitative conclusions change, e.g., conclusions on when $\bar{t}t$ condensation occurs and $\bar{b}b$ does not, or vice versa.

Lindner and Ross (1992) investigated in greater detail the case of the gauge group $G \equiv SU(3)_c \times SU(2)_L \times U(1)_S \times U(1)_T$. In their framework, symmetry breaking occurs in two stages, as mentioned above. At the first stage, the additional gauge boson $X_\mu \propto (g_S S_\mu - g_T T_\mu)$ obtains a large mass $M_X = g_X v'/2 \sim \Lambda$ through a Higgs mechanism, where v' is a VEV of a remote Higgs field F, which can be either elementary or composite, and $g_X = \sqrt{g_S^2 + g_T^2}$. $B_\mu \propto (g_T S_\mu + g_S T_\mu)$ is the usual $U(1)_Y$ field of the standard model, massless at this stage. The authors argued, on the basis of a one-loop gap equation involving the propagator of the heavy X_μ, that the gauge coupling parameter g_X is very large: $g_X \gg 1$, i.e., $g_X \approx g_S \gg g_T$. Therefore the VEV $\langle F^0 \rangle_0 = v'$ can be much lower than $M_X = g_X v'/2 \sim \Lambda$. This opened the possibility of nondecoupling effects when $M_X \sim \Lambda \to \infty$. Care was taken to ensure cancellation of triangular anomalies. Lindner and Ross conjectured that $M_X \sim \Lambda$ could be pushed down to $\sim$TeV. However, since their scheme suggests a very strongly coupling boson X_μ ($g_X \gg 1$), it is unclear whether their one-loop gap (Dyson-Schwinger) equation has predictive power, a point mentioned by the authors themselves. If Λ is low, they argued that several composite bound states should occur, made up of t_L, t_R, and b_L, among them: color-octet scalars with possibly large decay widths [see also Dai *et al.* (1992); Kundu, De, and Dutta-Roy (1993) in Sec. VI.C], as well as vectors and axial vectors [see also Lindner and Lüst (1991), in Sec. VI.D]. They argued that the masses of the composite particles can be as low as $\sim m_t$. This model, as well as those discussed by Bönisch (see above), yields at energies $E \sim \Lambda$ four-quark terms (8.1), i.e., not those of the top-mode standard model (2.1), (3.2). However, we note that such interactions can be further Fierz-transformed,

$$-(\bar{Q}_L^a \gamma^\mu Q_L^a)(\bar{t}_R^b \gamma_\mu t_R^b) = 2(\bar{Q}_L^a t_R^b)(\bar{t}_R^b Q_L^a)$$
$$= \frac{2}{N_c}(\bar{Q}_L^a t_R^a)(\bar{t}_R^b Q_L^b) + (\bar{Q}_L^a \lambda_{ab}^{(\alpha)} t_R^b)(\bar{t}_R^c \lambda_{cd}^{(\alpha)} Q_L^d). \quad (8.2)$$

Therefore exchange of colorless heavy gauge bosons results in a sum of the top-mode standard-model term (2.1) and a colored sum involving Gell-Mann matrices $\lambda^{(\alpha)}$ ($\alpha = 1,\ldots,8$) employed earlier by Dai *et al.* (1992) and Kundu, De, and Dutta-Roy (1993) [see Eq. (6.19)] and satisfying $\mathrm{tr}(\lambda^{(\alpha)}\lambda^{(\beta)}) = 2\delta_{\alpha\beta}$. The second identity in Eq. (8.2) is a consequence of $\lambda_{ab}^{(\alpha)}\lambda_{cd}^{(\alpha)}/2 = (\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd}/N_c)$. As mentioned in Sec. VI.C, Kundu *et al.* argued that the term involving $\lambda^{(\alpha)}$'s in Eq. (8.2) leads to a composite color-octet isodoublet. It is interesting that Lindner and Ross (1992) also came to the conclusion that such composite states should appear in the framework—without employing Fierz transformations, but rather using arguments involving global symmetry breaking induced by the four-quark terms $[U(6)_L \times U(3)_R \to U(3) \times U(3)]$.

Since the resulting top-mode standard-model term in Eq. (8.2) is $\propto 1/N_c$, one may worry that the $1/N_c$-expansion approach of Sec. IV, as well as the related leading-N_c arguments of Secs. II, III, and V, are in danger in such models with heavy colorless gauge-boson exchange. This is not the case, at least for analysis connected to the appearance of the colorless composites (top-mode standard-model term). That is, the four-quark parameter G of Eq. (2.1), or equivalently κ_t of Eq. (3.2), just has to be reexpressed by the four-quark parameter $G_v \approx -g_X^2/M_X^2$ ($M_X \sim \Lambda$) of the generated vector-vector $(\bar{Q}_L^a \gamma^\mu Q_L^a)(\bar{t}_R^b \gamma_\mu t_R^b)$ term: $G = 2|G_v|/N_c$. Then $1/N_c$ analysis of the colorless composite sector carries through unchanged. For example, the leading-N_c gap equation (4.9) yields again the condition $G\Lambda^2 = \mathcal{O}(N_c^{-1})$ for $\bar{t}t$ condensation, i.e., $|G_v|\Lambda^2 = \mathcal{O}(N_c^0)$.

Martin (1992a) wrote Eq. (8.2) in a normalized form:

$$-(\bar{Q}_L^a \gamma^\mu Q_L^a)(\bar{t}_R^b \gamma_\mu t_R^b) = 2[(\bar{Q}_L \rho^{(0)} t_R)(\bar{t}_R \rho^{(0)} Q_L) + (\bar{Q}_L \rho^{(\alpha)} t_R)(\bar{t}_R \rho^{(\alpha)} Q_L)], \quad (8.3)$$

where $\rho_{ab}^{(0)} = \delta_{ab}/\sqrt{N_c}$ and $\rho^{(\alpha)} = \lambda^{(\alpha)}/\sqrt{2}$ ($\alpha = 1,\ldots,N_c^2 - 1$) are normalized

$$\mathrm{tr}(\rho^{(\beta)}\rho^{(\gamma)}) = \delta_{\beta\gamma}, \quad \beta,\gamma = 0,1,\ldots,N_c^2 - 1. \quad (8.4)$$

He argued that Eq. (8.3) implies that the colorless and the colored channels are equally attractive and could in principle lead not just to a colorless VEV $\langle \bar{t}_L t_R \rangle_0 \neq 0$, but in general also to colored VEV's $\langle \bar{t}_L \lambda^{(\alpha)} t_R \rangle_0 \neq 0$. This would dynamically break $SU(3)_c$ and give gluons masses, thus contradicting experiments. He argued that inclusion of QCD lifts this degeneracy in favor of the first term on the right of Eq. (8.3), because $t_R \sim \mathbf{N}_c$ and $\bar{Q}_L \sim \bar{\mathbf{N}}_\mathbf{c}$ under $SU(N_c)_c$ ($N_c = 3$) and because representations $\mathbf{3}$ and $\bar{\mathbf{3}}$ feel an attractive QCD force to combine into a color singlet ($\bar{Q}_L t_R$) and a repulsive QCD force to combine into a color octet ($\bar{Q}_L \lambda^{(\alpha)} t_R$). The latter arguments are heuristic and might suggest, in addition to $\langle \bar{Q}_L \lambda^{(\alpha)} t_R \rangle_0 = 0$, that $\bar{Q}_L \lambda^{(\alpha)} t_R$ does not become dynamical. On the other hand, Kundu, De, and Dutta-Roy (1993), as well as Lindner and Ross (1992), argued that composite dynamical color-octet scalars may exist, but they must have zero VEV [see Sec. VI.C and the discussion of Lindner and Ross's work earlier in this section (Sec. VIII.B)].

A variation of the described model of Lindner and Ross (1992) was investigated by Martin (1992a), who chose the full gauge group $G \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$, i.e., the additional strong $U(1)_X$ in G was orthogonal to the $U(1)_Y$ group of the standard model. In particular, he independently arrived at conditions for triangular anomaly cancellations and at conditions in which the colorless $\bar{t}t$ condensation channel is the most attractive one. He noted that a possible mixing between the $U(1)$'s, i.e., the case of Lindner and Ross, does not affect anomaly cancellation or the qualitative features of condensation. He stressed that one feature of the model is unfavorable for condensation: if $U(1)_X$ is a separate (not embedded) gauge group, perturbative methods predict that $g_X(\mu)$ decreases when energy μ decreases.

Hošek (1999), working in a framework similar to that of Lindner and Ross, argued that spin-1 $\bar{f}f$ condensates could represent dynamically generated non-Abelian gauge bosons, and that their appearance would solve a host of problems—they could "eat" the large number of unwanted composite Nambu-Goldstone bosons, thus becoming heavy ($M > 1$ TeV), and their presence in radiative corrections would modify the otherwise for condensation unfavorable evolution of $U(1)_X$ coupling parameter to the favorable asymptotically free evolution. Hošek argued that $U(1)_X$ hypercharges assigned to various fermions could account for the hierarchies of (dynamical) fermionic masses.

All in all, the renormalizable frameworks discussed here so far can lead to a rather involved low-energy composite sector, including not just the low-mass scalar isodoublet of the usual top-mode standard-model effective model, but also, among others, low-mass colored scalars. This may pose some problem in view of the fact that no colored particles of any kind have been observed. This problem was briefly addressed by Kundu, De, and Dutta-Roy (1993) (see Sec. VI.C). Lindner and Ross briefly mentioned the possibility that these color-octet particles might have very broad decay widths which would make them effectively invisible.

C. Coloron (topcolor) model

Hill (1991) constructed a renormalizable framework in which, in contrast to the models discussed in Sec. VIII.B, the effective top-mode standard-model terms (2.1) are clearly the dominant ones among the resulting scalar four-quark terms. It is motivated by the following identity:

$$-\left(\bar{Q}_L^a \gamma^\mu \frac{1}{2}\lambda_{ab}^{(\alpha)} Q_L^b\right)\left(\bar{t}_R^d \gamma_\mu \frac{1}{2}\lambda_{de}^{(\alpha)} t_R^e\right)$$
$$=(\bar{Q}_L^a t_R^a)(\bar{t}_R^b Q_L^b) - \frac{1}{N_c}(\bar{Q}_L^a t_R^b)(\bar{t}_R^b Q_L^a) \qquad (8.5)$$
$$=\left(N_c - \frac{1}{N_c}\right)(\bar{Q}_L \rho^{(0)} t_R)(\bar{t}_R \rho^{(0)} Q_L)$$
$$-\frac{1}{N_c}(\bar{Q}_L \rho^{(\alpha)} t_R)(\bar{t}_R \rho^{(\alpha)} Q_L), \qquad (8.6)$$

where $Q_L = (t_L, b_L)^T$; $\lambda^{(\alpha)}$ ($\alpha = 1,\ldots,8$) are Gell-Mann matrices; a,b,d,e are color indices; and isospin indices were suppressed. Identity (8.5) follows from Eq. (8.1) and from $\lambda_{ab}^{(\alpha)}\lambda_{de}^{(\alpha)}/2 = (\delta_{ae}\delta_{bd} - \delta_{ab}\delta_{de}/N_c)$. From Eq. (8.2) follows the identity (8.6), written in the normalized form with notations as in Eqs. (8.3) and (8.4). We see from Eq. (8.6) that the effective term $[-(\bar{Q}_L \gamma^\mu \lambda^{(\alpha)} Q_L)(\bar{t}_R \gamma_\mu \lambda^{(\alpha)} t_R)]$, which can be induced by exchange of $SU(3)$ massive gauge bosons, really leads to a top-mode standard-model-dominated NJLVL model, thus resulting in a low-mass composite scalar isodoublet. Low-mass colored composite scalars in general would not appear, because terms with $\lambda^{(\alpha)}$'s in Eq. (8.6) have a negative factor (are repulsive).

Hill (1991) therefore extended $G_{\text{standard model}}$ by a factor of $SU(3)$, i.e., the gauge group at scales $\mu > \Lambda$ was $G \equiv U(1)_Y \times SU(2)_L \times SU(3)_1 \times SU(3)_2$. In the first version of such *topcolor* models (1991), he required that t behave under $SU(3)_1 \times SU(3)_2$ in a manifestly different way from other fermions. The motivation behind this requirement was the hierarchy $m_t \gg m_b$. Hill required that t_R, t_L, and b_L (relevant degrees of freedom in $\bar{t}t$ condensation) couple to the strong $SU(3)_2$, and the other quarks to the weaker $SU(3)_1$. Consequently he assigned to fermions the following representations under $SU(3)_1 \times SU(3)_2$: all right-handed quarks except t_R are (3,1) while t_R is (1,3); $(u,d)_L$ and $(c,s)_L$ are (3,1) while $(t,b)_L$ is (1,3); all leptons are singlets [$\sim$(1,1)]. He extended the quark sector in the simplest way to ensure triangular anomaly cancellation—with an electroweak-singlet quark q of electric charge $-1/3$ and transforming under $SU(3)_1 \times SU(3)_2$ as $q_R \sim (1,3)$, $q_L \sim (3,1)$.

In order to provide a framework for the first stage of the symmetry breaking [$G \to G_{\text{standard model}}$, i.e., $SU(3)_1 \times SU(3)_2 \to SU(3)_c$], Hill introduced a scalar isosinglet Higgs field Φ_b^a which transforms as $(3,\bar{3})$ under $SU(3)_1 \times SU(3)_2$. The nature of Φ_b^a was left unspecified—it could be either elementary or composite. $SU(3)_1 \times SU(3)_2 \to SU(3)_c$ was then assumed to occur at an energy $\mu \sim \Lambda$ spontaneously through the Higgs mechanism characterized by development of a VEV $\langle \Phi_b^a \rangle_0 = \text{diag}(M,M,M)$ ($M \sim \Lambda$). This VEV matrix, while breaking $SU(3)_1 \times SU(3)_2$ to the QCD gauge group $SU(3)_c$ with massless gluons, gives large degenerate masses to eight other gauge bosons (colorons) $B_\mu^{(\alpha)}$:

$$m_B(=\Lambda) = M\sqrt{h_1^2 + h_2^2} = \frac{g_3}{\sin\theta\cos\theta}M, \qquad (8.7)$$

where h_1 and h_2 are gauge coupling parameters of $SU(3)_1$ and $SU(3)_2$, respectively, and θ is the mixing angle of the orthonormal transformation between the original massless gauge bosons $A_{1\mu}^{(\alpha)}$ and $A_{2\mu}^{(\alpha)}$ of $SU(3)_1$

and $SU(3)_2$, on the one hand, and massless gluons $A_\mu^{(\alpha)}$ and massive colorons $B_\mu^{(\alpha)}$, on the other hand ($\tan\theta = h_1/h_2 \ll 1$, $h_1\cos\theta = g_3$, $h_2\sin\theta = g_3$). The QCD coupling parameter g_3 in Eq. (8.7) is $g_3(\mu)$ at $\mu\approx\Lambda$. The hierarchy $h_2 \gg h_1$ ($\theta \ll 1$) is a reflection of the fact mentioned above, that $SU(3)_2$ must be stronger than $SU(3)_1$ so that t will be favored over other quarks in condensation. The resulting interaction of massive colorons with fermions at an energy μ is then

$$\mathcal{L}^{(\mu)}(\text{colorons-quarks})$$
$$=\frac{1}{2}g_3(\mu)\cot\theta B_\mu^{(\alpha)}[\bar{t}_R\gamma^\mu\lambda^{(\alpha)}t_R+\bar{t}_L\gamma^\mu\lambda^{(\alpha)}t_L + \bar{b}_L\gamma^\mu\lambda^{(\alpha)}b_L+\bar{q}_R\gamma^\mu\lambda^{(\alpha)}q_R]+\mathcal{O}(g_3\theta), \qquad (8.8)$$

where the negligible terms $\sim g_3\theta$ contain interactions with quarks other than t. At low energies, $\mu \lesssim \Lambda = M_B$, Eq. (8.8) leads to an effective four-quark term

$$-\frac{g_3^2(\Lambda)\cot^2\theta}{M_B^2}\left(\bar{Q}_L\gamma^\mu\frac{1}{2}\lambda^{(\alpha)}Q_L\right)\left(\bar{t}_R\gamma_\mu\frac{1}{2}\lambda^{(\alpha)}t_R\right),$$
$$\text{where} \quad Q_L=(t_L,b_L)^T.$$

This term, in view of Eqs. (8.5) and (8.6), leads to the usual top-mode standard-model interaction (2.1) with $G=[g_3^2(\Lambda)\cot^2\theta/\Lambda^2][1+\mathcal{O}(N_c^{-2})]$, if the compositeness scale is identified naturally as $\Lambda = M_B$ ($\approx g_3 M/\theta$). Hill also verified that the additional quark q in general decouples from the low-energy physics—it obtains a large mass $m_q \gg m_t$ through Yukawa coupling to the $(3,\bar{3})$ Higgs Φ_b^a and therefore does not form condensates $\bar{t}_L q_R$, $\bar{b}_L q_R$.

Martin (1992a, 1992b) raised a point of criticism with the original coloron model proposed by Hill (1991) and described above. Martin noted that in a basis of left-handed two-component Weyl fields, the additional quark q_L and $(b^c)_L$ transform[53] under the full group $G \equiv U(1)_Y \times SU(2)_L \times SU(3)_1 \times SU(3)_2$ as $q_L \sim (-2/3,1,3,1)$, $(b^c)_L \sim (+2/3,1,\bar{3},1)$. Therefore q_L and $(b^c)_L$ form a real representation of G even before the spontaneous symmetry breaking $G \to G_{\text{standard model}}$. This would allow them in general to pair up and form large-bare-mass terms (in general with mass$>\Lambda$) and thus decouple. In the low-energy region, $q_R \sim (-2/3,1,1,3)$ would then play the role of b_R. However, since it feels the strong $SU(3)_2 (h_2 \gg h_1)$, just as $Q_L=(t_L,b_L)^T$ and t_R do, a condensate $\bar{Q}_L q_R$ will form alongside the condensate $\bar{Q}_L t_R$, with equal strength. This would give $m_t^{\text{dyn}} \approx m_b^{\text{dyn}}$, rendering the scenario unacceptable. However, Martin (1992a, 1992b) did mention that the unfavorable decoupling of q_L and $(b^c)_L$ could be avoided, and hence the first coloron model of Hill does work, if one assumes that large-bare-mass terms are prevented from forming, for example, by an (unspecified) global symmetry.

Furthermore, Martin (1992a) constructed a renormalizable model that is based on the same group G as Hill's original coloron model and that contains the same (elementary) $(3,\bar{3})$ Higgs Φ_b^a, but does not contain the problem of possible $m_b^{\text{dyn}} \approx m_t^{\text{dyn}}$. To achieve this and to avoid the potential danger of breaking $U(1)_{\text{em}}$, he introduced a more involved additional spectrum of fermions, two of them isodoublets and two of them isotriplets. Each pair forms a real representation of $G_{\text{standard model}}$ and forms mass terms with masses $\sim\langle\Phi_a^a\rangle_0 = M$, by coupling to Φ_b^a. These fermions transform under $SU(3)_1 \times SU(3)_2$ either as (3,1), $(\bar{3},1)$, (1,3), or $(1,\bar{3})$. The model contains no triangular anomalies. The physical t (i.e., with $m_t^{\text{dyn}} \neq 0$) is a linear combination of the original t from the unbroken theory and of extra fermions. Admixtures of these extra fermions are weak when Yukawa parameters between them and Φ_b^a are large (~ 1). Since b_R does not participate in condensation in this scenario, $m_b = 0$.

Later, Martin (1992b) constructed, on the basis of the above-mentioned coloron group G, a fully dynamical scenario of symmetry breaking $G \to G_{\text{standard model}} \to U(1)_{\text{em}}$—the $(3,\bar{3})$ Higgs field Φ_b^a responsible for the first part of the symmetry breaking was also composite. He achieved this by introducing, in addition to the standard-model fermions, isosinglet quarks that are (3,3), $(1,\bar{6})$, or $(\bar{6},1)$ under $SU(3)_1 \times SU(3)_2$. In this tumbling scenario of condensation, (3,3) and $(1,\bar{6})$ fermions participated in forming the composite Φ_b^a (note, $h_2 \gg h_1$). The other additional fermions $(\bar{6},1)$ were present to ensure triangular anomaly cancellations. The scenario yielded four axionlike neutral pseudo–Nambu-Goldstone bosons that could be dangerously light. Numerical calculations were not performed. The author employed sophisticated group-theoretical arguments to explain qualitatively the tumbling scheme of dynamical symmetry breaking. Among the arguments employed were the dynamical assumptions of Raby, Dimopoulos, and Susskind (1980). These assumptions, based on the approximation of a single gauge-boson exchange, lead to an algorithm determining the "most attractive scalar channel" in which the condensate is supposed to appear.

It should be noted that a modification of the original topcolor model (Hill, 1991) was later introduced by Hill and Parke (1994). They performed an analysis of the effects of dynamical-symmetry-breaking scenarios on the production of top quarks. The modified model was later also used by Hill and Zhang (1995) in analyzing $Z \to b\bar{b}$ (see Sec. IX.A.3). The modified assignment of quantum numbers to fermions in this topcolor variant is much along the lines of the "topcolor-assisted technicolor" framework (topcolor I) to be discussed in Sec. VIII.F in which b transforms under $SU(3)_1 \times SU(3)_2$ the same way as does t: $(t,b)_L \sim (1,3)$; t_R, $b_R \sim (1,3)$. Other fermions transform the same way as in the original model. No electroweak-singlet quark q is introduced, because the assignment is already anomaly free. As a result, the dangerous decoupling of $(b^c)_L$ and q_L

[53]The superscript c denotes here that the field is charge conjugated; therefore $(b^c)_L = (b_R)^c$.

criticized by Martin (1992a, 1992b) does not occur here. In this model, the different $U(1)_Y$ coupling strengths of t_R and b_R modify the coloron-exchange-induced four-quark parameters so that the latter lead to $m_t^{\rm dyn} \neq 0$ and $m_b^{\rm dyn}=0$ ($\kappa_t^{\rm (eff)} > \kappa_{\rm crit}$ and $\kappa_b^{\rm (eff)} < \kappa_{\rm crit}$). Therefore, despite the fact that the coloron exchange alone would imply in this model $m_t^{\rm dyn} \approx m_b^{\rm dyn}$, the weak $U(1)_Y$ interaction can cause the hierarchy $m_t \gg m_b$—see a related discussion in Sec. VI.A.4.

D. A renormalizable model with fine tuning of gauge couplings

Luty (1993) constructed another renormalizable gauge theory leading, via dynamical symmetry breaking at $\mu \sim \Lambda$, to an NJLVL model with F fermion flavors and N colors:

$$\mathcal{L}_{4f}^{(\Lambda)} = \frac{8\pi^2}{N} \frac{g}{\Lambda^2} (\overline{\psi}_L^{ia} \psi_R^{ja})(\overline{\psi}_R^{jb} \psi_L^{ib}), \tag{8.9}$$

where $i,j=1,\ldots,F$ are flavor indices and $a,b=1,\ldots,N$ color indices. If $g > g_{\rm crit} \sim 1$, this leads to fermion condensation and an effective $U(F)\times U(F)$ linear σ model (at $\mu < \Lambda$) with a composite scalar Φ_{ij} ($i,j=1,\ldots,F$). As in Hill's coloron model, it was assumed that the above effective terms are generated by an exchange of non-Abelian, in general colored, gauge bosons. Luty considered the gauge group $G \equiv SU(N)_{UC} \times [SU(K) \times SU(K)]_I \times SU(N)_{C'}$ (made up of asymptotically free factors) and the following fermion content including additional fermions $\chi_L, \chi_R, \xi_L, \xi_R$: $\psi_L, \psi_R \sim (N,1,1,1)(F)$; $\chi_L \sim (N,K,1,1)$; $\chi_R \sim (1,K,1,N)$; $\xi_R \sim (N,1,K,1)$; $\xi_L \sim (1,1,K,N)$. K is here an (unspecified) integer. The first three group factors were considered strong, leading via dynamical symmetry breaking to fermion condensates, while $SU(N)_{C'}$ was considered weak in the energy range of interest. The reason for introduction of $SU(N)_{C'}$ was to ensure triangular anomaly cancellations and to avoid emergence of (too) light pseudo–Nambu-Goldstone bosons. Luty set Λ_{UC} and Λ_I as the energy scales at which the respective gauge couplings of $SU(N)_{UC}$ and $[SU(K)\times SU(K)]_I$ become strong enough to trigger dynamical symmetry breaking [couplings of the two $SU(K)$'s were equally strong]. He then considered two different limiting cases:

(1) $\Lambda_I \gg \Lambda_{UC}$: In this case, due to asymptotic freedom, $SU(N)_{UC}$ is relatively weak at $\mu \sim \Lambda_I$, and $[SU(K)\times SU(K)]_I$ interactions are strong there. They trigger dynamical symmetry breaking and $\langle \overline{\chi}_L \chi_R \rangle_0, \langle \overline{\xi}_L \xi_R \rangle_0 \sim \Lambda_I^3$ appear. This leads to the dynamical symmetry breaking $SU(N)_{UC} \times SU(N)_{C'} \to SU(N)_C$. Of the total of $2(N^2-1)$ Nambu-Goldstone bosons, half are absorbed by gauge bosons of the broken sector and give them heavy masses $\sim g_{UC}\Lambda_I$, while the other half transform in the adjoint representation of the unbroken weak $SU(N)_C$ and acquire, via loops containing $SU(N)_C$ gauge bosons, lighter masses $\sim g_C \Lambda_I$ (note, $g_C \approx g_{C'}$). The effective theory at $\mu \ll g_C \Lambda_I$ contains NJLVL terms suppressed by Λ_I^2, and the fermions ψ in them remain massless.

(2) $\Lambda_{UC} \gg \Lambda_I$: In this case, $SU(N)_{UC}$ is strong at $\mu \sim \Lambda_{UC}$. It triggers dynamical symmetry breaking resulting in $\langle \overline{\psi}_L \psi_R + \overline{\chi}_L \xi_R \rangle_0 \sim \Lambda_{UC}^3$ and in the dynamical symmetry breaking $[SU(K)\times SU(K)]_I \to SU(K)_{I'}$. There are $(F+K)^2-1$ (potential) Nambu-Goldstone bosons. Of these, K^2-1 are absorbed by gauge bosons of the broken sector and give them masses $\sim g_I \Lambda_{UC}$, $2FK$ get masses $\sim g_{I'}\Lambda_{UC}$ from $SU(K)_{I'}$ gauge-boson loops (note: $g_{I'} \sim g_I$), but F^2 remain massless. Fermions χ_R and ξ_L remain massless, transforming in the fundamental representation of the unbroken $SU(K)_{I'}$. The latter group becomes strong at a lower scale $\Lambda_{I'}$ and triggers $\langle \overline{\xi}_L \chi_R \rangle_0 \sim \Lambda_{I'}^3$. As a result, N^2-1 Nambu-Goldstone bosons appear and get masses $\sim g_{C'}\Lambda_{I'}$ from $SU(N)_{C'}$ gauge-boson loops.

Let us summarize: in the first limiting case ($\Lambda_I \gg \Lambda_{UC}$), the order parameter $\langle \overline{\psi}_L \psi_R \rangle_0$ remains zero and ψ's are massless, while in the second case ($\Lambda_{UC} \gg \Lambda_I$) $\langle \overline{\psi}_L \psi_R \rangle_0$ is in general very large $\sim \Lambda_{UC}^3$ and ψ's acquire large[54] $m_\psi^{\rm dyn} \sim \Lambda_{UC}$. Luty then assumed that the transition between the two limiting cases is continuous (second order), i.e., VEV's of condensates and $m^{\rm dyn}$ change continuously when Λ_{UC} and Λ_I, or equivalently $g_{UC}(\Lambda)$ and $g_I(\Lambda)$, are adjusted (chosen) accordingly. In an intermediate, "fine-tuned" case, when $\Lambda_{UC} \sim \Lambda_I \sim \Lambda$, fermions ψ have $m_\psi^{\rm dyn} \ll \Lambda$, and the VEV's of their condensates are $\langle \overline{\psi}_L \psi_R \rangle_0 \sim m_\psi^{\rm dyn} \Lambda^2 \ll \Lambda^3$.

Having in mind the choice $m_\psi^{\rm dyn} \sim E_{\rm ew}$, Luty then conjectured that such a fine-tuned dynamical-symmetry-breaking scenario would lead to an effective NJLVL picture of strong enough four-ψ interactions (8.9) at $\mu \sim \Lambda$, with a fine-tuned $g \gtrsim g_{\rm crit}$. These terms have their origin in exchanges of the heavy $SU(N)_{UC}$ gauge bosons between ψ's. In this picture, Eq. (8.9) leads via dynamical symmetry breaking (similar to the DEWSB) to a linear σ model with a dynamical light composite scalar Φ_{ij} ($i,j=1,\ldots,F$) at low energies $\mu < \Lambda$. Luty then employed a Dyson-Schwinger gap equation for the running $m_\psi^{\rm dyn}(p^2) \equiv \Sigma(p^2)$, based on the approximation of one $SU(N)_{UC}$ gauge-boson exchange. Using this Dyson-Schwinger equation, he showed that a specific function of the mass M ($=\Lambda$) of these gauge bosons has to be fine tuned to the value 1 up to the order of $(m_\psi^{\rm dyn}/M)^2$ for this dynamical symmetry breaking to occur. He showed that, in this dynamical symmetry breaking, light dynamical composite scalars occur, and that the decay constants of the resulting Nambu-Goldstone bosons are $f \sim m_\psi^{\rm dyn}$ ($\sim E_{\rm ew}$).

When one takes $F=2$, $\psi_1 = t$, and $\psi_2 = b$, the model

[54] In general $\langle \overline{\psi}\psi \rangle_0 \sim m_\psi^{\rm dyn} \Lambda^2$, where Λ is the relevant compositeness scale (in this case Λ_{UC}). In the general, non-fine-tuning case we have $m_\psi^{\rm dyn} \sim \Lambda$.

leads to a top-mode standard-model scenario with equally strong $\overline{t}t$ and $\overline{b}b$ condensates ($\Rightarrow m_t = m_b$). However, as discussed in Sec. VI.A.4 and at the end of Sec. VIII.C, $U(1)_Y$ interactions can change this situation so that $m_t \gg m_b$.

E. Models possessing simultaneously horizontal and vertical gauge symmetries

A gauge theory with broken horizontal symmetries, in addition to the left-right (L-R) symmetric gauge group (a "vertical symmetry"), as an underlying physics for the DEWSB, has been introduced by Nagoshi, Nakanishi, and Tanaka (1991). These authors assumed that the full gauge group contains, in addition to the L-R symmetric group $G_{L-R} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, horizontal $SU(N)$ gauge groups with the following horizontal interactions over N generations of leptons and quarks ($N=3$):

$$\mathcal{L}_{\text{hor}} = if \sum_{k=1}^{N^2-1} \overline{\psi} \gamma_\mu T_k H_k^\mu \psi. \tag{8.10}$$

Here ψ stands for any N-component sector of quarks or leptons (up-type quarks, down-type quarks, charged leptons, neutrinos); H_k^μ are the horizontal gauge fields; T_k are generators of $SU(N)$: $T_k = \lambda^{(k)}/2$, where $\lambda^{(k)}$ are Gell-Mann matrices, for $N=3$. As can be seen from Eq. (8.10), horizontal interactions, in a certain sense, treat all generations equally. Nagoshi *et al.* assumed that horizontal symmetries are broken by an unspecified mechanism at an energy scale $\mu = \Lambda_H$, while the L-R symmetry is left unbroken. The resulting masses of the $N^2-1 = 8$ gauge bosons were assumed to satisfy a certain hierarchy, for example, $M^2_{4,5,6,7} (\sim \Lambda^2) \gg M^2_{1,2,3,8}$ $(\sim \Lambda_H^2)$, where Λ_H satisfies $10^2\,\text{TeV} \lesssim \Lambda_H \ll E_{\text{GUT}}$. The lower bound was needed to avoid possibly too large induced FCNC's. This breaking of the horizontal symmetry, and the resulting heavy gauge-boson exchange processes between fermions, leads to a pattern of dynamical symmetry breaking of the L-R gauge symmetry and eventually to the DEWSB. The authors obtained one gap equation for each of the two quark sectors, based on the exchange of horizontal gauge bosons with mass $M_k \sim \Lambda_H$ between the quarks. The UV cutoff in the gap equations was taken to be $\Lambda (\sim M_{4,5,6,7})$. For the neutrino sector, they obtained similarly two coupled gap equations for the Dirac $\mathcal{M}_D$ and Majorana $\mathcal{M}_M$ mass matrices. If the hierarchy $\mathcal{M}_D^2 \ll \mathcal{M}_M^\dagger \mathcal{M}_M \ll \Lambda_H$ is satisfied, the see-saw mechanism can give phenomenologically acceptable eigenmasses of neutrinos, and the authors suggested that such a framework would lead to a dynamical L-R gauge symmetry breaking.[55] The gap equation gave $m_t^{\text{dyn}} \neq 0$ and $m_b = 0$, due to the mentioned gauge-boson mass hierarchy. However, the authors pointed out that m_b would become nonzero and $m_b \ll m_t$, through a kind of m_t-induced "feed-down" effect: b obtains a small $m_b^{(0)} \neq 0$ through mixing between W_L and W_R of the L-R gauge sector and the large m_t^{dyn}, as illustrated in Fig. 11. This $m_b^{(0)}$ gets enhanced by horizontal gauge interactions (through the gap equation) to a value $m_b \gg m_b^{(0)}$, but still remains much smaller than m_t as long as the mixing angle between W_L and W_R is small enough. Stated differently, the tiny $m_b^{(0)}$, generated by the process of Fig. 11, effectively increases the four-quark parameter κ_b of the NJLVL interaction (3.2) from a slightly subcritical to a slightly supercritical value.

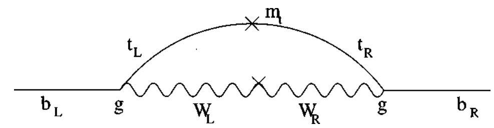

FIG. 11. Generation of a small mass $m_b^{(0)}$ out of a large m_t^{dyn} and with the help of W_L-W_R mixing. From Nagoshi, Nakanishi, and Tanaka (1991).

Nakanishi and Tanaka (1992) subsequently applied the described framework in a more detailed investigation of the spectrum of dynamical neutrino masses. The requirement of having three nearly massless generations of neutrino eigenstates led them naturally to assume the existence of higher generations of quarks and leptons. Nagoshi, Nakanishi, and Tanaka (1993), and later Nagoshi and Nakanishi (1995), investigated within the described framework the dynamical generation of the mass hierarchy between and within the up- and down-type quark sectors. The role of the vertical gauge interactions was played this time by QED or $U(1)_Y$. This interaction was important in generating the mass hierarchy between the up- and down-type sectors, while the mass hierarchy within the sectors was explained by a suitable breaking pattern of horizontal gauge symmetries. In a simplified scheme, they showed that generated flavor mixings can involve CP violation. The authors pointed out that two factors are crucial for dynamical generation of flavor mixings in their framework, i.e., for generation of CKM mixing matrix elements through the DEWSB: (a) radiative corrections from vertical gauge interactions; (b) noncommutativity of those generators of horizontal symmetries associated with the lighter horizontal gauge bosons.

Another type of renormalizable model involving horizontal symmetries and leading to the DEWSB was constructed by King (1992) and further investigated by Elliott and King (1992) and Evans, King, and Ross (1993). It is based on the horizontal gauge-symmetry group $G \equiv SU(3)_f \times SU(3)_L \times SU(3)_R$, in addition to $G_{\text{standard model}}$. The model does not involve leptonic degrees of freedom in dynamical symmetry breaking. Left-handed Weyl quarks of the standard model transform under G: $u_L \sim (1,3,1)$, $(u_R)^c \sim (1,1,\overline{3})$. The framework contains additional Weyl quarks a, b, c, and d, which transform nontrivially under $SU(3)_f$. It was assumed that condensates form $\langle ad \rangle_0 \neq 0$ and $\langle bc \rangle_0 \neq 0$. This sym-

[55]The L-R dynamical-symmetry-breaking framework as constructed and investigated in detail by Akhmedov *et al.* (1996) (see Sec. VII.B), is different; among other things, it involves a modified see-saw mechanism.

metry breaking gave heavy masses $M_A > M_B \sim \Lambda$ to the corresponding gauge bosons of the horizontal group and preserved global flavor symmetries of the standard model. Therefore the resulting model at $\mu \lesssim \Lambda$ exhibits the Glashow-Iliopoulos-Maiani mechanism and has suppressed FCNC's even for low $\Lambda \sim 1$ TeV. At $\mu \sim \Lambda$, the model is, in fact, the truncated top-mode standard model (2.1), generated by exchange of heavy gauge bosons. Elliott and King (1992) studied dynamics of $\bar{t}t$ condensation (and DEWSB) in this framework, taking into account the dynamical structure (momentum dependence) of propagators of heavy gauge bosons involved in the exchange. The Dyson-Schwinger (gap) equation was investigated numerically under inclusion of QCD in the dressed ladder approximation, and the Pagels-Stokar relations were included in the analysis, too (see Sec. III on the Dyson-Schwinger +Pagels-Stokar approach). The new high-energy dynamics reduced m_t somewhat in comparison with the minimal top-mode standard model, but m_t remained still too high. Later, Evans, King, and Ross (1993) discussed vacuum alignment in a fully dynamical scenario of the model. They considered a scenario that produces condensates $\langle ad \rangle_0 \neq 0$ and $\langle bc \rangle_0 \neq 0$. These condensates form when gauge interactions associated with $SU(3)_f$ become strong at a scale $\Lambda_f \gtrsim 1$ TeV.

F. Topcolor-assisted technicolor

Several authors have proposed combining the $\bar{t}t$ condensation mechanism with the technicolor and extended-technicolor mechanisms. Their purpose was to obtain a phenomenologically acceptable value of m_t, i.e., a somewhat smaller m_t than the minimal top-mode standard-model scenario gives, and a reduced compositeness scale Λ for the $\bar{t}t$ condensate. Furthermore, such combined (topcolor-assisted technicolor) scenarios, being more complex than the minimal top-mode standard-model framework, may also explain the lighter masses of fermions other than t, flavor mixings, CP violation, and suppression of the flavor-changing neutral currents (FCNC's). It is interesting that such combined mechanisms tend to cure simultaneously the well-known deficiencies of the pure $\bar{t}t$ condensation mechanism (or of the pure topcolor scenario leading to the minimal framework of $\bar{t}t$ condensation) and those of the technicolor +extended-technicolor mechanism:

(1) the minimal framework requirement that the $\bar{t}t$ condensate be fully responsible for the EWSB [i.e., the corresponding Nambu-Goldstone bosons have $F_\pi = v$ (=246 GeV) leading to M_W, M_Z] leads to too large m_t (>220 GeV); minimal m_t (≈220 GeV) is obtained when Λ is exceedingly high ($\sim E_{\rm Planck}$); some people regard such high Λ's and the related fine tuning $\kappa_t \approx \kappa_{\rm crit}$ of the four-quark coupling parameter of Eq. (3.2) as unnatural;

(2) technicolor+extended technicolor predict m_t substantially lower than the measured value ≈175 GeV, while technicolor can account for the full (dynamical) EWSB and extended technicolor for light quark and lepton masses, at low cutoffs $\Lambda_{\rm technicolor} \sim 10^2 - 10^3$ GeV and $\Lambda_{\rm ETC} \sim 10^5$ GeV.

Furthermore, such combined frameworks offer the possibility of a *fully dynamical* symmetry-breaking scenario, in which even the masses $M \sim \Lambda$ of new heavy gauge bosons (whose exchanges lead to top-mode standard-model-type four-quark terms responsible for $\bar{t}t$ condensation) can be explained dynamically via dynamical symmetry breaking induced by condensation of technifermion pairs.

A renormalizable framework incorporating these ideas of "topcolor-assisted technicolor" was first constructed by Hill (1995) and later investigated by Buchalla, Burdman, Hill, and Kominis (1996), who called it the topcolor I model. The model builds partly on Hill's previous idea of topcolor—see Sec. VIII.C (Hill, 1991). At energies $\mu < \Lambda_{\rm ETC} \sim 10^2$ TeV, the unbroken gauge group is assumed to be a product of a technicolor group $G_{\rm technicolor}$ and a variation of the topcolor (coloron) group $G_{\rm cln} \equiv [SU(3)_1 \times U(1)_{Y1}] \times [SU(3)_2 \times U(1)_{Y2}]$ and the standard-model isospin group $SU(2)_L$. However, now the assignment of $G_{\rm cln}$ quantum numbers is different: the third generation of quarks (*including* b_R) preferentially couples to the strong[56] $SU(3)_1 \times U(1)_{Y1}$, first and second generations couple to the weaker $SU(3)_2 \times U(1)_{Y2}$. The model treats t and b equally under $SU(3)_1 \times SU(3)_2$. This is in contrast to the original topcolor model (Hill, 1991) and similar to a modified version proposed by Hill and Parke (1994). Leptons are assigned quantum numbers under $G_{\rm technicolor} \times G_{\rm cln}$ such that the model has cancellation of triangular anomalies. Strong $U(1)_{Y1}$ was introduced to ensure that the generated NJLVL terms leading to the $\bar{t}t$ condensate would be stronger ($\kappa_t \gtrsim \kappa_{\rm crit}$) than those leading to the $\bar{b}b$ condensate ($\kappa_b \lesssim \kappa_{\rm crit}$). Stated differently, strong $U(1)_{Y1}$ was introduced to ensure an isospin hierarchy $m_t \gg m_b$. In this context, we recall that quarks t_L, b_L, t_R, b_R couple to the $U(1)_{Y1}$ gauge boson with strengths $q_1 Y/2$, where q_1 is the strong $U(1)_{Y1}$ coupling parameter and Y the usual electroweak hypercharge: $Y = +1/3$ for t_L and b_L; $Y = +4/3, -2/3$, for t_R, b_R, respectively. Quarks of the first two generations couple analogously to $U(1)_{Y2}$, with strengths $q_2 Y/2$ ($q_2 \ll q_1$).

In one of the scenarios proposed by Hill, there are techniquarks Q and T transforming nontrivially under subgroups $SU(3)_{TCQ}$ and $SU(3)_{TCT}$ of the technicolor gauge group $G_{\rm technicolor} \equiv SU(3)_{TCQ} \times SU(3)_{TCT}$, respectively, and transforming nontrivially under the coloron group $G_{\rm cln}$. They condense due to the strong and confining technicolor interaction ($\Lambda_{\rm technicolor} \sim 1$ TeV).

[56] Hill (1995) used a different notation than in his earlier work (1991). $SU(3)_1$ and $U(1)_{Y1}$ are now the stronger groups; in the earlier work, $SU(3)_1$ was the group with the weaker coupling.

The condensate $\langle \bar{Q}Q\rangle_0$ is responsible for the first stage of dynamical symmetry breaking $G_{\rm cln}\times SU(2)_L \to G_{\rm standard\ model}$; the condensate $\langle \bar{T}T\rangle_0$ is responsible for the second stage (i.e., the DEWSB): $G_{\rm standard\ model} \to SU(3)_c\times U(1)_{\rm em}$. After the first stage, a residual global symmetry $SU(3)'\times U(1)'$ is left, and as a result a degenerate massive color octet of gauge bosons called *colorons* $B_\mu^{(\alpha)}$ appears, as in the earlier topcolor model (Hill, 1991). In addition, a color-singlet heavy gauge boson Z' appears, corresponding to the global $U(1)'$. The masses of these gauge bosons satisfy the hierarchy $E_{\rm ew} \sim 10^2$ GeV$<M_B\sim M_{Z'}\sim 10^3$ GeV$<\Lambda_{\rm ETC}\sim 10^5$ GeV. Exchange of these bosons between the standard-model quarks leads to strong effective NJLVL interactions at $\Lambda\sim M_B\sim M_{Z'}$ ($\sim$1 TeV), involving $t_{L,R}$ and $b_{L,R}$. Due to the mentioned couplings of these quarks to the $U(1)_{Y1}$ factor in the original $G_{\rm cln}$, the effective four-quark parameters in the $\bar{b}b$ and $\bar{t}t$ channels may satisfy $\kappa_b^{\rm (eff)}<\kappa_{\rm crit}<\kappa_t^{\rm (eff)}$. In this case, only t condenses *and* obtains a large $m_t^{\rm dyn}\neq 0$; $m_b^{\rm dyn}=0$, although b may still condense when $\kappa_b^{\rm (eff)}$ is close (from below) to $\kappa_{\rm crit}$ [see Eq. (6.9)].

The $\bar{t}t$ condensation in this framework is, to a large degree, only a spectator to the technicolor-driven dynamical electroweak symmetry breaking. This means that $\bar{t}t$ condensation contributes only a small part to the electroweak VEV v, and hence only a small part to M_W and M_Z, in contrast to the technicolor mechanism. To understand this statement in a bit more quantitative way, we recall the Pagels–Stokar formula in the quark-loop approximation (3.13) for the decay constant of the Nambu-Goldstone bosons originating from the pure $\bar{t}t$ condensation scenario:

$$F^2_{\pi^\pm}({\rm q.l.})=\frac{N_{\rm c}}{8\pi^2}m_t^2\left[\ln\frac{\Lambda^2}{m_t^2}+\frac{1}{2}\right],$$

$$F^2_{\pi^0}({\rm q.l.})=\frac{N_{\rm c}}{8\pi^2}m_t^2\ln\frac{\Lambda^2}{m_t^2}, \qquad (8.11)$$

where $m_t\approx 175$ GeV. If $\bar{t}t$ condensation were entirely responsible for the DEWSB, then $F_\pi\approx v=246$ GeV. For this we would need, as suggested by Eq. (8.11), a huge $\bar{t}t$ compositeness scale $\Lambda\sim 10^{13}$–10^{14} GeV (see also Table II in Sec. III.C). However, since in the discussed topcolor-assisted technicolor framework Λ has much lower (and hence more attractive) values, $\Lambda\sim M_B \sim M_{Z'}\sim 1$ TeV, Eq. (8.11) suggests that $\bar{t}t$ condensation cannot be responsible for most of the needed electroweak VEV $v\approx 246$ GeV: for $\Lambda=1$ TeV and $m_t=175$ GeV, Eq. (8.11) yields F_π ($\equiv f_\pi\sqrt{2}$)$\approx$64–68 GeV. Therefore the actual massless Nambu-Goldstone bosons, subsequently eaten by W's and Z to provide the latter with masses, are linear combinations of composite scalars provided by the technicolor mechanism and those provided by $\bar{t}t$ condensation; the latter components are only a small admixture in the actual eaten Nambu-Goldstone bosons. Stating this in an even more simplified (approximate) way: the actual (composite) electroweak Nambu-Goldstone bosons are basically provided by the technicolor mechanism.

Therefore an additional triplet of (uneaten) pseudo–Nambu-Goldstone bosons $\tilde{\pi}^a$ occurs in the course of $\bar{t}t$ condensation. Techniquarks (denoted generically as Q_i), which had condensed by strong confining technicolor interactions, have masses $\sim$500 GeV and can be neglected at the electroweak scale. As a result, the dangerous technicolor-breaking condensates $\langle \bar{Q}t\rangle_0$ do not form. Extended technicolor interactions in this scenario (or an elementary Higgs) generate masses of light fermions, and contribute ϵm_t to the full m_t. This contribution is much smaller than the $m_t^{\rm dyn}=(1-\epsilon)m_t$ obtained via the described $\bar{t}t$ condensation. Typical expected values for ϵ are $\epsilon\sim 10^{-2}$–10^{-1}. The mass m_b originates partly from extended technicolor and partly from instantons in $SU(3)_1$. Other light quarks obtain masses from extended technicolor. Consequently, a CKM mixing matrix may be generated.

A small extended-technicolor-induced ϵm_t induces a nonzero mass of the mentioned triplet of pseudo-Nambu-Goldstone bosons $\tilde{\pi}^a$, in the range $m_{\tilde{\pi}} \approx 180-240$ GeV, thus ameliorating the problem of having dangerously light scalars. Their decay lengths are $f_{\tilde{\pi}}$ ($\equiv F_{\tilde{\pi}}/\sqrt{2}$)$\approx$50 GeV, according to Eq. (8.11). Hill called them top-pions, and argued that their appearance is a general feature in scenarios where $\bar{t}t$ condensation is accompanied by some additional mechanism leading to the EWSB and to the light quark masses (e.g., technicolor+extended technicolor or elementary Higgs).

Buchalla, Burdman, Hill, and Kominis (1996), in a long paper on the theory and phenomenology of topcolor scenarios, further discussed the above topcolor-assisted technicolor framework, which they called topcolor I. In addition, they proposed and also investigated a variant of Hill's original coloron model (see Sec. VIII.C), calling it topcolor II. It has an additional (technicolor) group factor $SU(3)_q$, under which only the additional quark (q_L,q_R) transforms nontrivially—as a triplet. Topcolor II is therefore based on the gauge group $G\equiv SU(3)_q\times SU(3)_1\times SU(3)_2\times U(1)_Y \times SU(2)_L$, where $U(1)_Y$ is the conventional weak group which tilts condensation in the $\bar{t}t$ flavor direction. The model is anomaly free. Among other things, $(c,s)_{L,R}$ are assigned different quantum numbers under $SU(3)_1\times SU(3)_2$: $(c,s)_L\sim(3,1)$ feels strong[57] $SU(3)_1$ and participates in t_R-dominated condensation, resulting in a heavy $m_t^{\rm dyn}$; $c_R,s_R,b_R\sim(1,3)$ couple only to weak $SU(3)_2$ and cannot lead to dynamical m_c,m_s,m_b. $SU(3)_q$ is confining and forms a $\bar{q}q$ condensate that acts like the Φ_b^a effective scalar, breaking the coloron group down to QCD dynamically. The problem of $(b^c)_L$ and q_L pairing, as discussed by Martin (1992a, 1992b; see

[57]Buchalla *et al.* had a different notation from that of Hill in his original coloron model (1991)—for them, $SU(3)_1$ is strong and $SU(3)_2$ weak ($h_1\gg h_2$).

Sec. VIII.C), does not occur here, since these two quarks transform as a singlet and a triplet, respectively, under the technicolor group $SU(3)_q$. The EWSB $G_{\text{standard model}} \to SU(3)_c \times U(1)_{\text{em}}$ is assumed to occur via the Higgs mechanism, by an elementary or composite Higgs other than $\bar{t}t$. Consequently, a rich structure of pseudo–Nambu-Goldstone bosons emerges, leading to a rich phenomenology, as investigated by these authors (see Sec. IX.A.3). In topcolor II, just as in topcolor I (topcolor-assisted technicolor), $\bar{t}t$ condensation is just a spectator to the (D)EWSB.

Soon after the appearance of Hill's work (1995) on topcolor-assisted technicolor, a series of papers by other authors appeared (Chivukula, Dobrescu, and Terning, 1995; Lane and Eichten, 1995; Chivukula and Terning, 1996; Lane, 1996), discussing some problems in topcolor-assisted technicolor scenarios and solutions to these problems. Chivukula, Dobrescu, and Terning (1995) argued that topcolor-assisted technicolor scenarios are likely to have a problem in the incompatibility of the following two requirements: (a) "naturalness" $M_{\text{coloron}} \sim 1$ TeV to avoid the fine tuning (recall $M_{\text{coloron}} \equiv \Lambda$, where Λ is $\bar{t}t$-compositeness scale); (b) a phenomenologically acceptable $\delta\rho$ $[\rho \equiv M_W^2/(M_Z^2 \cos^2 \theta_W)]$. They argued that the up-type and down-type technifermions (their right-handed components) are likely to have different couplings to the strong $U(1)_{Y1}$ and violate custodial symmetry, thus leading to a large $\delta\rho$. To prevent this, $U(1)_{Y1}$ couplings must be relatively small. However, the latter are responsible within topcolor-assisted technicolor scenarios for bringing the effective $(\bar{t}t)(\bar{t}t)$ parameter above κ_{crit} to enable the condensation to occur $[\Rightarrow \text{large}\, m_t^{\text{dyn}} = (1-\epsilon) m_t]$, while at the same time keeping (pushing) the $(\bar{b}b)$ parameter below κ_{crit} to prevent a (large) m_b^{dyn}. Therefore the required weakening of $U(1)_{Y1}$, stemming from the $\delta\rho$ restriction, results in a fine tuning of the $\bar{t}t$ condensation mechanism and consequently in a large ("unnatural") $\bar{t}t$ condensation scale $\Lambda \equiv M_{\text{coloron}} \gtrsim 4.5$ TeV.

Lane and Eichten (1995) and Lane (1996) constructed topcolor-assisted technicolor scenarios that overcome these problems. In particular, in their anomaly-free scenarios, $U(1)_{Y1}$ couplings to technifermions are isospin symmetric and preserve custodial $SU(2)$, allowing acceptably small $\delta\rho$ and at the same time strong $U(1)_{Y1}$—consequently "naturalness": $M_{\text{coloron}} \approx 1$ TeV.

The phenomenological implications of topcolor-assisted technicolor models will be discussed in Sec. IX.A.3.

We do not attempt to go into detail in explaining the technicolor and extended-technicolor mechanisms, since this would take us beyond the scope of this review article. It should be stressed that, earlier, other authors (King and Mannan, 1991b; Mendel and Miransky, 1991; Miransky, 1992) had proposed the idea that $\bar{t}t$ condensation be combined with (extended) technicolor to bring down[58] m_t^{dyn} and the compositeness scale Λ, when compared with the minimal framework. In contrast to the renormalizable topcolor-assisted technicolor framework of Hill (1995) described above, they used as a starting point effective NJLVL terms involving third-generation quarks and technicolor quarks. In their investigation, they employed Dyson-Schwinger equations in the dressed ladder approximation. They also predicted the emergence of a triplet of quark-antiquark pseudo–Nambu-Goldstone bosons (called top-pions by Hill). King and Mannan argued that the masses of these pseudo–Nambu-Goldstone bosons in their framework are $\sim$20 GeV, while Mendel and Miransky (1991) obtained masses of several hundred GeV. Subsequently, Miransky argued that these heavy pseudo-Nambu-Goldstone bosons could lead to important enhancements of FCNC's in B-meson physics and could be detected at the Superconducting Super Collider (SSC).

Martin (1993) proposed a dynamical scenario similar to topcolor-assisted technicolor: a self-breaking technicolor combined with topcolor. The usual technicolor is like a scaled-up QCD, non-self-breaking, gauge bosons (technigluons) of the technicolor group remain massless, although their coupling to technifermions is strong enough to make the latter condense and thus (in topcolor-assisted technicolor scenarios) to make them break the topcolor (coloron) sector and possibly also the standard-model sector of the gauge group. However, Martin proposed that strong technicolor interactions break the technicolor group itself and also $G_{\text{standard model}}$ (DEWSB). He proposed a full gauge group

$$G \equiv SU(4)_{\text{technicolor}} \times SU(3)_{C'} \times SU(3)_{C''} \times SU(2)_L \times U(1)_{Y'}$$

with a complicated anomaly-free content of technifermions and other technisinglet fermions (in addition to standard-model fermions). Three types of condensates, assumed to appear simultaneously at $\Lambda \sim 1$ TeV, are responsible for the full breaking $G \to SU(3)_c \times U(1)_{\text{em}}$. One type of condensate contains a pair of technifermions in different technicolor representations, and therefore breaks technicolor group $SU(4)_{\text{technicolor}}$ dynamically: $SU(4)_{\text{technicolor}} \times SU(3)_{C'} \times U(1)_{Y'} \to SU(3)_{C'''} \times U(1)_Y$. It thus gives heavy mass to "technigluons" and leads to the usual coloron model (Sec. VIII.C), with $SU(3)_{C'''}$ being the strong and $SU(3)_{C''}$ the weak coloron group factor. The second type of condensate, made up of pairs of technicolor-singlet and $SU(2)_L$-singlet fermions (quix-antiquix pairs), then breaks $SU(3)_{C'''} \times SU(3)_{C''}$ to QCD $SU(3)_c$, giving heavy mass to the eight colorons. The third type of condensate, made up of technifermions, then breaks $SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}$. The $\bar{t}t$ condensate (and hence m_t) is generated by strong interactions mediated by heavy colorons just as in the coloron (topcolor) model, and is largely a

[58]The idea that a full strong dynamics could lower m_t^{dyn} and account for M_W and M_Z was also mentioned by Hill (1991).

"spectator" to the DEWSB. The mentioned technicolor condensates of the third type are largely responsible for the DEWSB. The model behaves similarly to topcolor I.

It should be mentioned that the broken technicolor helps reduce the electroweak S parameter (Peskin and Takeuchi, 1992), making it more compatible with experimental evidence and thus avoiding a potentially dangerous problem of too large values of S in the usual scaled-up QCD-like technicolor theories, as pointed out by Hill, Kennedy, Onogi, and Yu (1993).

Apart from technicolor scenarios, in which condensates $\bar{Q}Q$ of technifermions are largely responsible for the DEWSB, there exists a somewhat related dynamical scenario in which the DEWSB is brought about by a condensate of new heavy vector bosons: $\langle B_\mu^{(0)\dagger} B^{(0)\mu} \rangle_0 \propto v^2$ (Cynolter, Lendvai, and Pócsik, 1997, and references therein).

G. Other renormalizable scenarios with $\bar{t}t$ condensation

Bando, Kugo, and Suehiro (1991) considered E_6 grand unified theory as the underlying theory leading to $\bar{t}t$ condensation with a high compositeness scale $\Lambda \sim E_{\rm GUT} \sim 10^{16}$ GeV. Their main motivation was to construct a dynamical scheme that would allow, in a natural way, the large isospin breaking $m_b \ll m_t$. The authors emphasized that E_6 is apparently the only gauge group to have, simultaneously, the following properties: (a) it is free of anomalies; (b) all fermions of each generation are combined into an irreducible representation; (c) all the symmetry breaking can be realized by using only those Higgs representations that give masses to fermions. Property (c) implies that all stages in the breaking $E_6 \to SU(3)_c \times U(1)_{\rm em}$ could in principle be realized dynamically. The authors considered the breaking pattern $E_6 \to SO(10) \to SU(5) \to G_{\text{standard model}}$, assuming that it takes place via (unspecified) Higgs mechanisms. In the course of the breaking, part of the $SO(10)$-spinor gauge bosons, as well as fermions of those irreducible representations in $SO(10)$ not containing standard-model fermions, acquire masses $M \sim E_{\rm GUT}$. Then a box diagram involving these fields leads to NJLVL interactions ($\propto \alpha_{\rm GUT}^2/M^2$), including terms containing simultaneously t_R and b_R, i.e., the terms $(\bar{t}_L t_R)(\bar{b}_L b_R)$, $(\bar{b}_L t_R)(\bar{t}_L b_R)$, and their Hermitian conjugates. Therefore,[59] such terms lead to the "feed-down" effect mentioned in Sec. III.B: large $m_t^{\rm dyn}$ ($\Leftrightarrow \langle \bar{t}_L t_R \rangle_0 \neq 0$) induces a tiny $m_b^{(0)} \neq 0$ (mass term $\propto \bar{b}_L b_R$). The four-quark parameter $\kappa_b^{(\rm eff)}$ of $(\bar{b}_L b_R)(\bar{b}_R b_L)$ is below $\kappa_{\rm crit}$ [due to $U(1)_Y$], thus preventing a nonzero $m_b^{\rm dyn}$ from emerging from its initial zero value. However, the tiny "feed-down"-induced $m_b^{(0)}$ brings this $\kappa_b^{(\rm eff)}$ above $\kappa_{\rm crit}$, and solution of the modified gap equation for the dynamical m_b acquires a value $m_b \gg m_b^{(0)}$. This mechanism, known as Nagoshi-Nakanishi-Tanaka (NNT) enhancement, was earlier investigated and explained by these authors (Nagoshi, Nakanishi, and Tanaka, 1991) within a dynamical framework with horizontal and L-R gauge symmetries (see Sec. VIII.E). However, also Bando *et al.* can easily ensure in their E_6-based framework that the obtained NNT-enhanced m_b is still small enough: $m_b \ll m_t$.

Yoshida (1996) performed a detailed analysis of a renormalizable model stemming from the fine-tuning framework of Luty (1993; see Sec. VIII.D) and the topcolor frameworks of Hill (1991, 1995; see Secs. VIII.C, VIII.F). Yoshida considered the gauge group $G \equiv SU(N_A) \times SU(N_B) \times SU(2)_L \times U(1)_Y$ (with $N_B = 3$) as leading to a tumbling dynamical symmetry-breaking scenario. He first discussed, in analogy with Luty, two extreme cases: $\Lambda_A \gg \Lambda_B$ and $\Lambda_A \ll \Lambda_B$. In the first case, strong $SU(N_A)$ drives a huge fermion condensate VEV ($\sim \Lambda_A^3$), which breaks $SU(N_B)$ dynamically and gives its gauge bosons ("colorons") large masses M_B, while gauge bosons of $SU(N_A)$ remain massless $M_A = 0$. In the second case, the roles in the dynamical symmetry-breaking scenario are simply inverted. He then investigated the generic case $\Lambda_A \sim \Lambda_B$, employing the formalism of the dressed ladder Dyson-Schwinger equation[60] plus the (generalized) Pagels-Stokar relations. Like Luty, he required that the transition between the two cases (phases) not have discontinuity, i.e., that the phase transition be second order, to ensure the hierarchy $m_t \ll \Lambda$ ($\sim \Lambda_A \sim \Lambda_B$) should DEWSB eventually occur: $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$. He found out that a second-order phase transition occurs only if $N_A \geq 9$ ($N_B = 3$ was taken). After the described dynamical symmetry breaking, a broken $SU(3)$ appears, a kind of a coloron group with gauge-boson masses $M \sim M_B$ ($\sim \Lambda$). These bosons then mediate four-quark interactions at energies $\mu < M$, leading to $\bar{t}t$ condensation and DEWSB.

Triantaphyllou (1994) and Blumhofer and Hutter (1997), investigated the possibility of explaining fermion family mass hierarchies dynamically *without* introducing horizontal (plus vertical) gauge interactions, in contrast to the models described in Sec. VIII.E.

Triantaphyllou (1994) investigated a scenario within a technicolor+extended-technicolor framework. In the scenario, the Dyson-Schwinger (gap) equation allows several solutions that may correspond to the dynamical masses of a three-generation sector of fermionic particles (e.g., of up-type quarks). Each solution represents another (local) minimum of the effective potential. This means that fermions (of a given sector) from different generations reside in different vacua. The CKM matrix could in principle be calculated by considering instanton transitions between different vacua. The familiar standard model is then an effective low-energy description of this model.

[59]Other NJLVL terms also emerge, e.g., those without b_R ($\Rightarrow$ top-mode standard model).

[60]"Dressed" means that the one-loop evolution of $SU(N_A)$ and $SU(N_B)$ coupling parameters was taken into account.

Blumhofer and Hutter (1997), on the other hand, investigated whether the Dyson-Schwinger equation could possess a fermion propagator solution $i[\not{k}-B(k^2)]^{-1}$ which has three poles $k^2=m_n^2$ $(n=1,2,3)$. Indeed, they found such a possibility, with a phenomenologically acceptable mass spectrum $m_n \approx m_1 e^{(n-1)\alpha}$. A one-loop Dyson-Schwinger equation giving such a solution can be realized in an unusual and peculiar theory containing a light "hidden" gauge-boson sector with a k^2-dependent coupling strength to the fermion, the origin of this k^2 dependence being predominantly mass-threshold effects. Stated differently, the Dyson-Schwinger equation at one loop was generated by exchange of gauge bosons whose masses are $\sim E_{\rm ew}$ or less, and these gauge bosons are "hidden" in the sense that they manifest themselves in interactions only at the loop, not at the tree level. The authors also showed that the fermion particle spectrum is stable. The three fermion particles of a given sector (e.g., up-type quarks) have in this scenario just *one* propagator $i[\not{k}-B(k^2)]^{-1}$. The authors showed that at tree level this is equivalent to having the usual picture of three fermions with three propagators $i[\not{k}-m_n]^{-1}$. Only at loop level do deviations from the standard model arise.

IX. SOME PHENOMENOLOGICAL AND THEORETICAL ASPECTS AND QUESTIONS

A. Phenomenological predictions

1. Phenomenology of general strong-dynamics frameworks with dynamic electroweak symmetry breaking

Chesterman and King (1992) investigated how a relatively low compositeness scale $\Lambda \lesssim 10\,\text{TeV}$ might be *directly* tested at the Superconducting Super Collider (SSC) or at the CERN Large Hadron Collider (LHC). They pointed out that the most promising channels are the scattering processes $V_L V_L \to V'_L V'_L$ of the longitudinal components of electroweak gauge bosons $(V_L, V'_L = W_L^{\pm}, Z_L)$. Such processes can be calculated by replacing $W_L^{\pm}, Z_L$ in amplitudes by the (composite) Nambu-Goldstone bosons $\mathcal{G}^{(\pm)}, \mathcal{G}^{(0)}$. The resulting errors are $\sim M_Z/M$, where $M \equiv \sqrt{s}$ is the invariant mass of the gauge-boson pair. Thus, by probing such processes, one can test the compositeness of Nambu-Goldstone bosons. For example, in the minimal (top-mode standard-model) framework, the composite structure of the Nambu-Goldstone bosons is, according to Eq. (2.2), $\mathcal{G}^{(0)} \sim \bar{t}\gamma_5 t$, $\mathcal{G}^{(+)} \sim \bar{b}(1+\gamma_5)t$. The scalar sector of the minimal standard model is described by the density $\mathcal{L}^{\sigma}(H,\pi)$ of the Gell-Mann–Levy σ model. The authors parametrized deviations of Nambu-Goldstone boson interactions from the minimal-standard-model behavior by adding to $\mathcal{L}^{\sigma}$ an additional nonrenormalizable contact term $\Delta\mathcal{L}^{\sigma} = -g^2[(\vec{\pi}\partial^{\mu}\vec{\pi}) + (H\partial^{\mu}H)]^2/(4\Lambda^2)$, where $\vec{\pi}\partial^{\mu}\vec{\pi} = Z_L\partial^{\mu}Z_L + (W_L^+\partial^{\mu}W_L^- + \text{H.c.})$. Here, g^2/Λ^2 is the top-mode standard-model four-quark parameter. They calculated cross sections $d\sigma/dM$ for the processes $pp \to W_L^+W_L^- \to W_L^+W_L^-$ and $pp \to Z_L Z_L \to W_L^+W_L^-$ and concluded that appreciable deviations can be found at LHC or SSC energies, as long as $\Lambda < 2$ TeV at the LHC or 5 TeV at the SSC. These limits are not high when compared with the c.m. system energies of the LHC or SSC. The results may hence seem to be somewhat disappointing.

Such results are in accordance with the nature of the compositeness in, for example, $\bar{t}t$ condensation. As stressed by Miransky (1990, 1991), Higgs and Nambu-Goldstone bosons in top-mode-type models are tightly bound states, and their composite structure can hence be discerned only at probe energies μ close to Λ and well above $E_{\rm ew} \sim 10^2$ GeV. This is in stark contrast with the dynamical symmetry breaking in QCD (or dynamical symmetry breaking in a confining technicolor) where bound states are less tight and their structure starts to be seen at $\mu \sim \Lambda_{\rm QCD} \sim 10^{-1}$ GeV. This is well below the "effective" cutoff $\Lambda \sim 1$ GeV where the nonperturbative and perturbative QCD meet. This difference is reflected also in the fact that QCD is confining (gluons remain massless despite light-quark condensation), while top-mode-type models are not (top-mode standard-model terms are usually assumed to be mediated by massive gauge bosons). Miransky pointed out that this difference can be understood by looking also at the amputated Bethe-Salpeter wave function $\chi(\bar{q}^2) = -i\Sigma(\bar{q}^2)/F_{\pi}$ of the composite pseudoscalar, where $\Sigma(\bar{q}^2)$ is the dynamical mass of the constituent fermion. $\Sigma(\bar{q}^2)$ in the asymptotic region ($|\bar{q}| \gg v$ in top-mode standard model, $|\bar{q}| \gg \Lambda_{\rm QCD}$ in QCD) falls off as $\Sigma(\bar{q}) \sim \bar{q}^{\gamma_m - 2}$, where γ_m is called an anomalous dimension. In the top-mode standard-model type of binding, $\gamma_m \approx 2$ and $\chi(\bar{q}^2)$ falls off very slowly (at least in the leading-N_c); in QCD, $\gamma_m = 0$ and $\chi(\bar{q}^2)$ falls off fast for $\bar{q}^2 > \Lambda_{\rm QCD}^2$.

2. Phenomenology of $\bar{t}t$ condensation models

Phenomenological aspects of $\bar{t}t$ condensation models with color-octet isodoublet composite scalars (see Sec. VI.C) were investigated by Kundu, De, and Dutta-Roy (1994a, 1994b) and by Kundu, Raychaudhuri *et al.* (1994). The first work investigated K-$\bar{K}$ and B_d-$\bar{B}_d$ mixing, the CP-violating ϵ parameter, and $\Gamma(b \to s\gamma)$. The contribution of colored scalars $\chi^{\pm}$ to K-$\bar{K}$ mixing was shown to be small. B_d-$\bar{B}_d$ mixing and ϵ yielded important constraints on two parameters of the framework—on mass $m_{\chi^{\pm}}$ and on the Yukawa parameter g'_t (of χ with t_R). For $m_{\chi^{\pm}} \sim 10^2$ GeV, these constraints gave an upper bound $g'_t(m_t) \lesssim 0.8$. For lower $m_{\chi^{\pm}}$, the upper bound $(g'_t)_{\max}$ decreases. Such restrictive bounds on g'_t are still marginally acceptable as to the mass m_t, because the measured m_t is rather high (≈ 180 GeV). For example, a fixed-point analysis of one-loop RG equations for g_t and g'_t (see Sec. VI.C) yielded $g'_t(m_t) \approx 1.06$ for $m_t = 150$ GeV, and $g'_t(m_t) \approx 0.78$ for $m_t = 190$ GeV. The authors emphasized that the analysis was accompanied by large uncertainties connected mostly to experimental uncertainties in the B_d-$\bar{B}_d$ mixing parameter x_d, the CKM parameter ratio $q = s_{13}/s_{23}$, the CP-violating CKM phase δ, and especialy to the large theoretical uncertainty of the hadronic bag parameter $f_B\sqrt{B_B}$. They showed that the presence of $\chi^{\pm}$ enhances $\Gamma(b \to s\gamma)$ and that the resulting lower bound $(m_{\chi^{\pm}})_{\rm min}$ is several hundred GeV, compatible with constraints on $m_{\chi^{\pm}}$ obtained from B_d-$\bar{B}_d$ and from the value of ϵ. An analysis of $R_b \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ by Kundu, Raychaudhuri, *et al.* gave a negative contribution of colored scalars. This was regarded as unfavorable at that time, when the measured R_b was above the minimal-standard-model value by about 3σ. However, newer data (Dawson, 1996) show that the world average is $R_b = 0.2178 \pm 0.0011$, which is less than 2σ above the minimal-standard-model value. Kundu, Raychaudhuri, *et al.* imposed the requirement that the (negative) new contribution not exceed typical values $\sigma \approx 0.001$, and obtained rather restrictive (high) values $(m_{\chi^{\pm}})_{\rm min} \gtrsim 500-900\,\text{GeV}$, for $m_t = 160-180\,\text{GeV}$.

B_d-$\bar{B}_d$ mixing was also studied within a modified minimal framework by Kimura *et al.* (1992). Their calculation was performed in the quark-loop (bubble) approximation. The authors took special care to modify the minimal (truncated) top-mode standard model (2.1) so that the phenomenological introduction of CKM mixing remained consistent with gauge invariance. They imposed the requirement that the W propagator have a pole at $M_W \approx 80\,\text{GeV}$ and that the quark-loop relation (2.13) be satisfied. This led them to the nowadays unacceptable values $m_t \approx 300\,\text{GeV}$ and $\Lambda \sim 10^6\,\text{GeV}$. Quark-loop modified propagators of $W^{\pm}$ and of Nambu-Goldstone bosons $\mathcal{G}^{\pm}$ in the relevant box diagrams for B_d-$\bar{B}_d$ mixing resulted in sizable modifications of the minimal-standard-model Inami-Lim functions—by 1–10 %. The modification was larger for smaller m_t. Due to uncertain values of the bag parameter $f_B\sqrt{B_B}$, they refrained from discussing the subject further.

Peccei, Peris, and Zhang (1991), motivated by the case of low-energy QCD, considered the possibility that appreciable residual (nonuniversal) interactions between t and the composite Nambu-Goldstone bosons arise in the course of DEWSB, thus leading to extra interactions between t and gauge fields. Large additional axial $Z\bar{t}t$ couplings would generate additional radiative corrections to low-energy physical quantities. Using data from LEP and deep-inelastic scattering experiments at that time, the authors showed that there was enough room in the parameter space for deviations of up to 10% for the strength of the axial $Z\bar{t}t$ coupling.

Zhang (1995) proposed a scenario in which the constituent t of the broken phase, arising after DEWSB, is a topological color soliton. This was motivated by the work of Kaplan (1991), who had shown, within the framework of a Skyrme model, that constituent quarks could be considered as color solitons ("qualitons"). In this picture, the constituent top quark is a valence (current) top quark surrounded by a deformed $\langle \bar{t}t \rangle_0$ background, where t and $\bar{t}$ interact through exchanges of Nambu-Goldstone bosons. Berger *et al.* (1996) carried out calculations in this scenario, based on the color chiral $SU(3)_L \times SU(3)_R$ symmetric four-quark interaction represented by the second term (sum) on the right of Eq. (8.3) ($\alpha = 1,\ldots,8$). Soliton solutions were obtained by embedding an $SU(2)$ "hedgehog" *Ansatz* for the top-pion color octet and minimizing the classical energy functional. The authors showed that the top-quark soliton can acquire values that would lead to predictions for the production cross section $\sigma_{t\bar{t}}$ in accordance with the data at the Collider Detector at Fermilab (CDF).

3. Phenomenology of coloron (topcolor) and topcolor-assisted technicolor frameworks

Hill and Parke (1994) studied, in dynamical symmetry-breaking schemes, the production cross section $\sigma(\bar{p}p \to \bar{t}t) \equiv \sigma_{t\bar{t}}$ and top-quark distributions in $\bar{t}t$ production at the Tevatron Collider, at the center-of-mass (c.m.) system energy $\sqrt{s} = 1.8\,\text{TeV}$. They investigated vector color-singlet (see Sec. VIII.B) and vector color-octet (coloron) channels (see Sec. VIII.C). For both types of channels, they concluded that $\sigma_{t\bar{t}}$ could be more than doubled in comparison with the pure QCD case, if the compositeness scale were $\Lambda \lesssim 1$ TeV. Lane (1995) subsequently investigated, among other models, predictions of a variant of the coloron (topcolor) model introduced by Hill and Parke (see the end of Sec. VIII.C). He calculated the invariant-mass distribution $d\sigma_{t\bar{t}}/d\mathcal{M}_{t\bar{t}}$ and the c.m. angular distribution $d\sigma_{t\bar{t}}/d\cos\theta$ of the top quark at the Tevatron at $\sqrt{s} = 1.8\,\text{TeV}$. His results also imply that the coloron resonances, via the process $q\bar{q} \to V_8 \to t\bar{t}$, may easily double $\sigma_{t\bar{t}}$ at $\sqrt{s} = 1.8\,\text{TeV}$. Such an enlarged $\sigma_{t\bar{t}}$, in comparison with the pure QCD case, may be in accordance with the evidence at the Collider Detector at Fermilab (CDF). Hill and Parke (1994), and Lane (1995), apparently employed the lowest-order calculation at the parton level.

Hill and Zhang (1995) later investigated the two types of framework (see Secs. VIII.B and VIII.C) as to the process $Z \to \bar{b}b$. They found out that the topcolor variant introduced and applied by Hill and Parke predicts a substantially increased $R_b \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ (by up to 2σ from $R_b^{\text{standard model}}$) if the coloron mass is low: $M_B \lesssim 600$ GeV. However, in such a case $\sigma_{t\bar{t}}$ appears to be too large—at least four times larger than the standard QCD prediction. On the other hand, $M_B \gtrsim 800$ GeV seems to be compatible with $\sigma_{t\bar{t}}$. In such a case, R_b is within 1σ of the minimal-standard-model prediction. In this context, we mention that the measured R_b world average has recently decreased to the values $R_b = 0.2178 \pm 0.0011$, which are now about 1.8σ above the standard-model value (see Dawson, 1996).

While Hill and Zhang (1995) considered only contributions of colorons (top gluons) to ΔR_b [$(\Delta R_b)_{\rm coloron} > 0$], Burdman and Kominis (1997) also included in their analysis top pions $\tilde{\pi}^a$ that arise in topcolor-assisted technicolor frameworks (see Sec. VIII.F). They found that $\tilde{\pi}$ contributions to ΔR_b are negative and can be dangerously strong for low $m_{\tilde{\pi}}<300$ GeV and low decay constants $f_{\tilde{\pi}}\approx 60$ GeV. However, Hill (1997) pointed out that their results are quite sensitive to the precise values of $m_{\tilde{\pi}}$ and $f_{\tilde{\pi}}$. For example, for $f_{\tilde{\pi}}\approx 120$ GeV, all $m_{\tilde{\pi}}$'s are compatible with measured values of R_b; the ranges $f_{\tilde{\pi}}\gtrsim 100$ GeV and $m_{\tilde{\pi}}\gtrsim 300$ GeV are also compatible. He also argued that the ϵ factor (ϵm_t is the small extended-technicolor-generated component of m_t) in general gets enhanced by radiative corrections of the topcolor $SU(3)_1$ and $U(1)_{Y1}$ sectors by a factor $\sim 10^1$, and therefore the fermion-loop-induced mass $m_{\tilde{\pi}}$ ($\propto \epsilon$) gets enhanced, too.

Chivukula, Cohen, and Simmons (1996) constructed a flavor-universal variant of the coloron model of Hill and Parke (see the end of Sec. VIII.C on the latter). With such a variant they were able to explain recent results for $\bar{p}p$ collisions at $\sqrt{s}=1.8$ TeV from CDF (Abe *et al.*, 1996), which indicate that the inclusive cross section for jets with $E_T>200$ GeV is substantially higher than that predicted by QCD. The model does not appear to lead to any contradiction with other experimental evidence. In contrast to Hill and Parke, they assigned *all* quarks to triplet representations of the strong $SU(3)_2$ group appearing in the full coloron group $SU(3)_1\times SU(3)_2$. Subsequently, Simmons (1997) continued the analysis of the flavor-universal coloron model and showed that constraints from searches for new particles decaying to dijets and from measurements of the electroweak ρ parameter imply that colorons in the framework must have masses $M_B\gtrsim 900$ GeV.

Kominis (1995) investigated questions of flavor-changing neutral currents (FCNC's) in Hill's topcolor I model (see Sec. VIII.F, on topcolor-assisted technicolor). He pointed out that the model, if without additional constraints, may lead to dangerously light composite scalars of the b type: $\Phi_d\sim \bar{Q}_L b_R$ [$Q_L\equiv(t_L,b_L)^T$]. This may occur when the strong $U(1)_{Y1}$ is not strong enough to push the four-quark parameter $\kappa_b^{(\mathrm{eff})}\lesssim\kappa_{\mathrm{crit}}$ substantially below κ_{crit}. Then the isodoublet Φ_d ($\langle\Phi_d\rangle_0=0$) may become dynamical (detectable) in the sense that its mass is substantially below the cutoff $\Lambda\approx 1$ TeV [see also Eq. (6.9)]. Such scalars can become relatively light ($\sim 10^2$ GeV) when $\kappa_b^{(\mathrm{eff})}$ approaches κ_{crit} from below—they are dangerous because their exchanges result in too strong B-$\bar{B}$ mixing (an FCNC effect).

Effects of these scalars were subsequently taken into account by Buchalla, Burdman, Hill, and Kominis (1996), who investigated two types of topcolor scenarios (topcolor I, II), as already mentioned in Sec. VIII.F. A substantial part of their work was devoted to the low-energy phenomenological implications of these models. They showed that these frameworks offer a natural way of suppressing dangerous FCNC's, in particular the dangerous B-$\bar{B}$ mixing contributions induced by exchanges of relatively light composite scalars. Suppression can be achieved through the chiral-triangular texture of the mass matrices. This texture is a consequence of the fact that in these models gauge quantum numbers distinguish generations. Topcolor I models contain two $U(1)$ group factors and consequently a massive isosinglet color singlet Z'_μ. Buchalla *et al.* showed that exchanges of Z'_μ in topcolor I can lead to substantial deviations in several semileptonic processes: $\Upsilon(1S)\to l^+l^-$; $B_s\to l^+l^-$; $B\to X_s l^+l^-$; $B\to X_s\nu\bar{\nu}$; and $K^+\to\pi^+\nu\bar{\nu}$. Future experiments involving these processes could give some important clues as to the nature of new physics at the onset scale $\Lambda\sim M_{Z'}\sim 1$ TeV, once they show any significant deviation from the standard-model values. On the other hand, topcolor II gives no novel effects in semileptonic processes and only a few such effects in low-energy nonleptonic processes, e.g., in D^0-$\bar{D}^0$ mixing. Since low-energy nonleptonic effects are hard to disentangle from standard-model physics, the authors suggested that high-energy experiments ($\mu\gtrsim E_{\mathrm{ew}}$) are the most promising for searching for the new physics.

Chivukula and Terning (1996) performed global fits to precision data for three examples of natural topcolor-assisted technicolor scenarios proposed by Lane and Eichten (1995; Lane, 1996; see Sec. VIII.F) and came to the conclusion that the mass of the new massive Z' bosons [originating from the $U(1)_{Y1}\times U(1)_{Y2}\to U(1)_Y$ breaking] must be larger than roughly 2 TeV. Recently, Su, Bonini, and Lane (1997) concluded that the topcolor-assisted technicolor models proposed by Lane (1996) must have in general an even stronger fine tuning[61] $m_t\ll\Lambda$ as a result of stringent bounds coming from data on Drell-Yan processes at the Tevatron. Hill (1997) stressed that the analysis of Chivukula and Terning (1996), and of Chivukula, Dobrescu, and Terning (1995; see Sec. VIII.F) should be extended to include, in addition to heavy gauge bosons, top pions and possibly other low-mass bound states.

Eichten and Lane (1996a) pointed out that conventional multiscale technicolor models in general result in light technipions $\tilde{\pi}_T$ of mass $\simeq 100$ GeV and lead in such scenarios to large decay rates, $\Gamma(t\to\tilde{\pi}_T^+ b)$, due to the large coupling of $\tilde{\pi}_T$ in these decays: $m_t\sqrt{2}/F_{\tilde{\pi}_T}$, where the decay width $F_{\tilde{\pi}_T}\approx v\approx 246$ GeV. The authors stressed that topcolor-assisted technicolor scenarios can in general suppress this decay to phenomenologically tolerable levels, since in such scenarios the relevant coupling is substantially smaller: $\epsilon m_t\sqrt{2}/F_{\tilde{\pi}_T}$, where $\epsilon m_t\approx(0.01-0.1)m_t$ is the extended-technicolor-induced part of m_t (see Sec. VIII.F).

Balaji (1997) further investigated this question, within a natural topcolor-assisted technicolor introduced previously by Lane and Eichten (1995; see Sec. VIII.F). Top pions $\tilde{\pi}_t$ (originating from $\bar{t}t$ condensation in topcolor-assisted technicolor models; see Sec. VIII.F) in the framework are found to have $m_{\tilde{\pi}_t}>200$ GeV. If there were no mixing between the heavy $\tilde{\pi}_t$ and the light technipion $\tilde{\pi}_T$, there would be no decays $t\to\tilde{\pi}_t^+ b$. However, there is mixing due to the (walking) extended techni-

[61]Such fine tuning was regarded by the authors as unacceptable.

color, leading to the existence of pseudo–Nambu-Goldstone bosons, which are mass eigenstates with masses possibly lower than m_t. This introduces the dangerous possibility that t could decay into such a pseudo-Nambu-Goldstone boson (plus b) at an unacceptably high rate. Balaji showed that the decay rate in such frameworks can be within phenomenologically acceptable limits due to the large mass of $\tilde{\pi}_t$'s and their weak mixing with $\tilde{\pi}_T$'s.

Eichten and Lane (1996b), and Lane (1997), discussed possible signatures of topcolor-assisted technicolor at the Tevatron and at the Large Hadron Collider—the detection of top pions $\tilde{\pi}_t^+$, massive colorons ($M \approx 0.5$–1 TeV), and color singlets Z'_μ ($M_{Z'} \approx 1$–3 TeV). If $m_{\tilde{\pi}_t} \gtrsim 150$ GeV, $\Gamma(t \to \tilde{\pi}_t^+ b)$ is phenomenologically acceptable and can be sought with high luminosity at the Tevatron and moderate luminosity at the LHC. Colorons may appear as resonances in $b\bar{b}$ and $t\bar{t}$ production. Heavy Z'_μ, due to its strong couplings to fermions, can lead to an excess of jets at high E_T and at high $\sqrt{s}$.

Wells (1997), on the other hand, discussed the prospects for detecting signatures of the scalar degrees of freedom relevant to EWSB (e.g., originating from technicolor), in scenarios where $\bar{t}t$ condensation is mainly a spectator to the EWSB ($F_t \equiv f_t\sqrt{2} \ll v \approx 246$ GeV), for example, in topcolor-assisted technicolor scenarios. He parametrized such scalar degrees of freedom by an unspecified, possibly composite Higgs field $h^0_{\rm ew}$: $\langle h^0_{\rm ew}\rangle_0 \approx v$. Since in such scenarios most of the m_t comes from $\bar{t}t$ condensation, Yukawa coupling of $h^0_{\rm ew}$ to t is weak ($g_t \ll 1$). He argued that possible detection of $h^0_{\rm ew}$ at the LHC would come largely from the production cross section due to top-quark Yukawa processes and thus might be very difficult because $g_t \ll 1$. On the other hand, the search capabilities for such a $h^0_{\rm ew}$ at the Tevatron and LEPII should be similar to those for the minimal-standard-model Higgs, since the searches there do not rely mainly on g_t.

Delépine, Gérard, González Felipe, and Weyers (1997) considered a model that is a version of the topcolor I of Hill (1995) and Buchalla *et al.* (1996). They assumed that the strong $SU(3)_1$ of this framework also involves the strong topcolor CP phase θ arising from topcolor instantons (this possibility was also mentioned by Buchalla *et al.*, 1996). Their framework involves the usual scalar sector of topcolor I—an elementary Higgs doublet H and two composite doublets $\tilde{\phi}_1 \sim \bar{t}_R \Psi_L$ and $\phi_2 \sim \bar{b}_R \Psi_L$. In contrast to topcolor I, the model does not contain a strong $U(1)_{Y1}$, which would discriminate between b_R and t_R and would thus prevent a nonzero $\langle \bar{b}b\rangle_0$ from appearing. Thus Delépine *et al.* allowed $\langle \phi_2\rangle_0 \neq 0$. They showed that in such a scenario there is a possibility that the θ term will simultaneously trigger the mass hierarchy $(0 \neq) m_b \ll m_t$ and a large CP-violating phase in the CKM matrix required phenomenologically by K-$\bar{K}$ physics. They did not investigate the dynamical role of the condensation scale Λ and the question of whether their framework could account for the full EWSB.

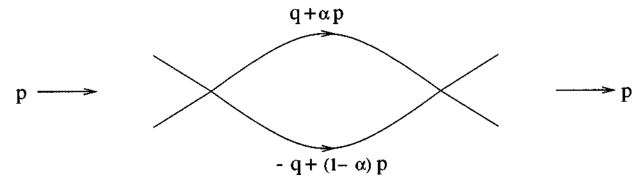

FIG. 12. Momentum routing ambiguity in a quark loop. The routing parameter α is in principle arbitrary; it influences predictions for M_H/m_t when a spherical covariant cutoff is employed for the quark momenta.

B. Some theoretical issues and questions about $\bar{t}t$ condensation frameworks

There are several outstanding theoretical questions about the NJLVL type of $\bar{t}t$ condensation. In the early investigations of $\bar{t}t$ condensation (1989–1991), these questions were either left unanswered or were not addressed. Some were later clarified, while others remain open. The effects of the next-to-leading terms in the $1/N_{\rm c}$ expansion (see Sec. IV), and of higher than six-dimensional quark contact operators (see Sec. V.E), were discussed in previous parts of this review. Below we briefly discuss some other theoretical points.

When one uses a simple spherical covariant cutoff in the quark-loop (bubble) approximation of the top-mode standard model, a momentum routing ambiguity appears, as noted by Willey (1993). This is shown in Fig. 12 for the case of a four-point function. When the otherwise arbitrary routing parameter α is set to zero, the pole of the scalar channel in the bubble sum of such four-point functions gives the known top-mode standard-model result $\bar{p}^2_{\rm pole} = 4m_t^2$, i.e., $M_H = 2m_t$. Willey pointed out that $M_H/(2m_t)$ depends on the routing parameter α and can thus acquire any value. Therefore, he argued, the top-mode standard model (2.1), or any similar NJLVL model, is not predictive.

Gherghetta (1994) investigated the compatibility of the $SU(2)_L \times U(1)_Y$ gauge invariance with the necessary existence of quadratic (Λ^2) terms in the top-mode standard-model gap equation in the quark-loop (bubble) approximation. He showed that the two requirements are incompatible when using the usual spherical covariant cutoff for the (quark) loop momenta. Gauge invariance is destroyed by Λ^2 terms in propagators of electroweak gauge bosons. He observed that Bardeen, Hill, and Lindner (1990), in their calculation of various relations in bubble approximation, used, somewhat inconsistently, the spherical covariant cutoff in quark loops for the gap equation and dimensional regularization in the quark loops for propagators of electroweak gauge bosons. Dimensional regularization ignores Λ^2 terms. The gap equation without Λ^2 terms does not lead to dynamical symmetry breaking at the quark-loop level. Gherghetta, motivated partly by the work of Nambu and

Jona-Lasinio (1961), demonstrated that use of dispersion relations to regulate quark loops leads to a satisfactory solution of this problem. Such a regularization maintains gauge invariance and does not possess any routing ambiguity. Therefore it also solves the routing ambiguity issue raised by Willey. As a matter of fact, such regularization had already been used by Nambu and Jona-Lasinio (1961) to investigate poles of various channels in the four-quark functions. Gherghetta also applied this same regularization procedure to the W and Z propagators and showed that they then do not have any routing ambiguity and do not contain any Λ^2 terms, i.e., they respect gauge invariance. It appears, however, that the gap equation itself cannot be regularized by this method. Gherghetta therefore took the position of Nambu and Jona-Lasinio (1961) who had declared that the requirement of masslessness for the pole in the pseudoscalar channel of the four-quark function is the gap equation. Stated differently, imposition of the Goldstone theorem at the quark-loop level is then regarded to be the gap equation. Interestingly, this led to a relation very similar to the usual gap equation in the spherical covariant cutoff approach (4.9). The two relations in fact become identical in the high-Λ limit $m_t/\Lambda \to 0$.

A related question is: when beyond the leading-N_c (quark-loop) approximation can we find a regularization procedure that regularizes all quark-loop momenta at the leading N_c and at the next-to-leading order (NTLO) in a mutually consistent manner *and* that possesses no routing ambiguities? It has been pointed out (Cvetič, 1997) that the proper time-regularization procedures, when applied within the approach of the effective potential (V_{eff}), satisfy these two requirements. Such procedures are invariant under translation in the quark momentum space and can probably be applied even when transverse gauge-bosonic degrees of freedom are included. On the other hand, it is unclear how to regularize NTLO contributions to V_{eff} and to the gap equation with the method of dispersion relations.

Chivukula, Golden, and Simmons (1993) studied the question of whether and when we can have, in models of dynamical symmetry breaking (those containing more than one quartic self-coupling of scalars, like nonminimal $\bar{t}t$ condensation and strong extended-technicolor frameworks), a fine tuning leading to the hierarchy $v/\Lambda \ll 1$, where v is the VEV of the composite scalar at low energies $\mu(\sim v \ll \Lambda)$: $v/\sqrt{2} \equiv \langle \Phi_0^{(\mu)} \rangle_0$. A point emphasized by them was that quantum fluctuations may drive the transition between the symmetric and the broken phase to become sudden and violent ("first order"). Such a transition in dynamical-symmetry-breaking scenarios represents the following behavior: The VEV $v(\mu)/\sqrt{2} \equiv \langle \Phi_0^{(\mu)} \rangle_0$ and the corresponding $m_f^{\text{dyn}}(\mu)$ of fermions at low energies μ ($\ll \Lambda$) experience a sudden (discontinuous) jump from zero to large values $\sim \Lambda$ when the corresponding four-fermion parameters $\kappa_f(\Lambda)$ of the (NJLVL-type) theory at high-energy Λ cross the critical values from below. This phenomenon, although not present in the minimal framework, may occur in theories that have more than one quartic self-coupling of composite scalars at low μ, including, for example, NJLVL-type effective models leading to the composite two-Higgs-doublet scenarios discussed in Sec. VI.A. Chivukula, Golden, and Simmons specifically considered an effective composite (Ginzburg-Landau) theory with chiral symmetry $U(N_{\text{f}})_L \times U(N_{\text{f}})_R$, i.e., with N_{f} left- and right-handed fermion flavors $\Psi_L^{(j)}, \Psi_R^{(j)}$, ($j = 1,\ldots,N_{\text{f}}$) in an N_{c}-dimensional representation and with the $(\bar{N}_{\text{f}}, N_{\text{f}})$ order parameter $\Phi_{ij} \sim \bar{\Psi}_R^{(j)} \Psi_L^{(i)}$:

$$\begin{aligned}\mathcal{L}^{(\mu)} = {} & \bar{\Psi} i \partial\!\!\!/ \Psi + \frac{\pi y(\mu)}{N_{\text{f}}^{1/2}} (\bar{\Psi}_L \Phi \Psi_R + \text{H.c.}) \\ & - M^2(\mu) \text{tr}(\Phi^\dagger \Phi) + \text{tr}(\partial^\nu \Phi^\dagger \partial_\nu \Phi) \\ & - \frac{\pi^2}{3} \frac{\lambda_1(\mu)}{N_{\text{f}}^2} (\text{tr}\, \Phi^\dagger \Phi)^2 - \frac{\pi^2}{3} \frac{\lambda_2(\mu)}{N_{\text{f}}} \text{tr}(\Phi^\dagger \Phi)^2.\end{aligned} \tag{9.1}$$

Superscripts (μ) at the fields, where μ is a finite effective UV cutoff, are omitted. Such a theory can arise from a flavor-democratic NJLVL term at $\mu = \Lambda$,

$$\mathcal{L}_{4\text{f}}^{(\Lambda)} = \bar{\Psi} i \partial\!\!\!/ \Psi + \frac{\alpha}{\Lambda^2} (\bar{\Psi}_L^{(i)} \Psi_R^{(j)})(\bar{\Psi}_R^{(j)} \Psi_L^{(i)}). \tag{9.2}$$

Applying the equations of motion for Φ and $\Phi^\dagger$ to the density consisting of the first line in Eq. (9.1), i.e., when Φ is an auxiliary field (at $\mu = \Lambda$), leads to $\Phi_{ij} \propto \bar{\Psi}_R^{(j)} \Psi_L^{(i)}$ and to the NJLVL term (9.2). The other terms on the right of Eq. (9.1) appear after inclusion of quantum effects in the energy interval (μ, Λ) and after an appropriate rescaling of composite fields Φ, so that the induced kinetic-energy term of Φ has a factor 1 in front of it.

For a large hierarchy, we need $\langle \Phi^{(\mu)} \rangle_0 \sim E_{\text{ew}}$, i.e., $\langle \Phi^{(\mu)} \rangle_0 \ll \Lambda$ at $\mu \sim E_{\text{ew}}$. Chivukula, Golden, and Simmons pointed out that such a prediction can be spoiled by the Coleman-Weinberg phenomenon (Coleman and Weinberg, 1973). Specifically, when Chivukula, Golden, and Simmons calculated trajectories of $\lambda_1(\mu)$ and $\lambda_2(\mu)$ by one-loop RG equations [taking $N_{\text{f}} = 2$, $N_{\text{c}} = 3$, and the Yukawa parameter $y(\mu) = \text{const}$], they found that for initial choices of small $\lambda_1(\Lambda)$ and large $\lambda_2(\Lambda)$ these parameters evolved very quickly and, after a small change in μ [$\ln(\Lambda/\mu) \sim 1$], acquired values such that the VEV $\langle \Phi \rangle_0$ changed from zero to a large value $\sim \mu$. Stated differently, quantum fluctuations do not allow a fine tuning of parameters at high energies Λ so as to obtain a small nonzero VEV $\langle \Phi^{(\mu)} \rangle_0 \sim E_{\text{ew}} \ll \Lambda$ at low energies $\mu \ll \Lambda$. Since the initial values for $\lambda_i(\Lambda)$ $(i = 1,2)$ were chosen so as to correspond approximately to the compositeness condition at Λ, the authors argued that the discussed dynamical framework does not allow large hierarchies $E_{\text{ew}}/\Lambda \ll 1$. They also argued that the problem may appear generally in models of DEWSB with more than one Φ^4 coupling.

Since coupling parameters at Λ are large, several authors have performed calculations in the model (with $N_{\text{f}} = 2$, $N_{\text{c}} = 3$) aimed at improving the (one-loop) perturbative approach of Chivukula, Golden, and Simmons, primarily by employing various nonperturbative approximations in the energy region $\mu\sim\Lambda$ (Shen, 1993; Bardeen, Hill, and Jungnickel, 1994; Clark and Love, 1995; Khlebnikov and Schnathorst, 1995).

Bardeen *et al.* (1994) calculated in the NJLVL $U(2)_L\times U(2)_R$ framework (9.2) the explicit leading-N_c solution. They applied this solution [$\lambda_1(\mu)=0$, $\lambda_2(\mu)=16/\ln(\Lambda/\mu)$, $y^2(\mu)/\lambda_2(\mu)=1/3$] to a nonperturbative region (logarithmically) close to Λ and matched it to the perturbative two-loop RG equations at a scale μ_i, where the perturbative approach is about to break down. They chose $\mu_i/\Lambda\approx 0.05$, although the results were reasonably insensitive to the precise value of this ratio. The Yukawa parameter was evolving, too. They showed that Coleman-Weinberg instabilities occur at substantially lower energies μ_{trans} as a result of the leading-N_c compositeness condition alone. Further, inclusion of two-loop effects in the RG equations (for $\mu<\mu_i$) decreases μ_{trans} by several orders of magnitude, and inclusion of QCD in the RG equations tends to eliminate such μ_{trans} completely. Therefore the authors argued that the framework in general does allow fine tuning hierarchies for NJLVL-motivated choices of the bare parameters [i.e., for the "strong Yukawa coupling" choice: $y^2(\Lambda)/\lambda_2(\Lambda)\sim 1$].

Shen (1993,1994) investigated the case of a negligible Yukawa parameter ($y^2=0$, thus ignoring fermions), by employing a lattice Monte Carlo technique to deal with the nonperturbative compositeness region. He found that the framework turns first order and hence does not allow hierarchies $v/\Lambda\ll 1$. Clark and Love (1995), on the other hand, investigated the case of weak but non-negligible bare Yukawa parameters [$y^2(\Lambda)/\lambda_2(\Lambda)\sim 10^{-1}$]—i.e., the case intermediate between the lattice bosonic case of Shen and the (relatively) strong Yukawa coupling case of Bardeen *et al.* Clark and Love employed a nonperturbative continuous Wilson renormalization group equation approach near Λ, including the chiral fermions. Since the Wilson RG formalism is in general very complicated, they worked in a so-called local-action approximation (which ignores anomalous dimensions and derivative interactions), neglected operators higher than bilinear in fermion fields, and restricted themselves to a fixed Yukawa parameter. In general, they found that the phase transition turns first order near Λ and does not allow hierarchies $v/\Lambda\ll 1$. Further, Khlebnikov and Schnathorst (1995) considered the choice of very (infinitely) large y, employed expansion in inverse powers of y, and found out that the model resembles closely the case of purely bosonic theory (cf. Shen, 1993, 1994) and that it consequently also exhibits first-order phase transitions, i.e., the appearance of Coleman-Weinberg instability, which does not allow a hierarchy $\Lambda\gg E_{\text{ew}}$.

All in all, the above-mentioned investigations of the nature of phase transitions in the chiral $U(2)_L\times U(2)_R$ model suggest that in models involving more than one Φ^4 coupling the appearance of Coleman-Weinberg (quantum) effects in general tends to drive the transition first order and thus to prohibit hierarchies $\Lambda/v\gg 1$, except possibly in models where new physics above Λ results primarily in NJLVL-type effective interactions [e.g., in Eq. (9.2); in nonminimal NJLVL $\bar{t}t$ condensation; in some strong extended-technicolor models]. These investigations rely on RG analyses or on other RG-related nonperturbative methods, and in one case partly on a *leading-N_c* approximation. Such methods imply that the minimal (top-mode standard-model) $\bar{t}t$ condensation model, now generally believed to be phenomenologically untenable, does not exhibit Coleman-Weinberg instabilities and its dynamical symmetry-breaking phase transition is second order. It is interesting that the latter conclusion for the top-mode standard model is also suggested by a pure $1/N_c$ expansion approach at the next-to-leading level (see Sec. IV.D), although this approach is essentially different from the RG-related ones.

Martin (1992a) investigated the following problem, which we face when dealing with NJLVL four-fermion interactions: For any given (set of) NJLVL term(s) with given coupling strength(s), how can we find out which fermions actually participate in condensation, and what kind of composite scalars with nonzero VEV (dynamic symmetry breaking) such a condensation would lead to? Martin applied two approaches to this problem:

(1) A direct comparison of the strengths of relevant attractive four-fermion channels (see Sec. VIII.B) in a given NJLVL Lagrangian density. For this, care must be taken to normalize the four-fermion operators appropriately [for example, $\rho^{(\beta)}$ in Eqs. (8.3) and (8.4)].

(2) A corresponding fully renormalizable underlying gauge theory. The two fermions (left-handed two-component Weyl fermions) involved in condensation, and the resulting condensate, transform in this gauge theory in irreducible representations R_1, R_2, and R_3, respectively. Thus a decomposition of the direct product $R_1\times R_2$ into a direct sum holds: $R_1\times R_2\equiv R_3+\cdots$. In the single-boson-exchange approximation, the condensate appears in the most attractive channel R_3 for which the difference $V=C_3-C_1-C_2$ is the most negative (C_j is the quadratic Casimir invariant for R_j). The approach is based on work by Raby, Dimopoulos, and Susskind (1980).

Dudas (1993) focused on a somewhat related question. He investigated an $SU(2)$-invariant NJLVL model with two $SU(2)$ doublets ψ_1 and ψ_2,

$$\mathcal{L}=\bar{\psi}_1 i\partial\!\!\!/\psi_1+\bar{\psi}_2 i\partial\!\!\!/\psi_2+\frac{g}{4\Lambda^2}(\bar{\psi}_1\tau_\alpha\psi_2)(\bar{\psi}_2\tau_\alpha\psi_1), \quad (9.3)$$

where τ_α ($\alpha=1,2,3$) are Pauli matrices. He wished to determine which way fermions would condense and result in nonzero VEV's (dynamic symmetry breaking). The model possesses the symmetries $SU(2)$; $U(1)_A$ [$\psi_1\to e^{i\beta\gamma_5}\psi_1,\psi_2\to e^{-i\beta\gamma_5}\psi_2$]; $U(1)_1$ [$\psi_1\to e^{i\beta}\psi_1,\psi_2\to\psi_2$]; $U(1)_2$ [$\psi_1\to\psi_1,\psi_2\to e^{i\beta}\psi_2$]. Dudas applied the method of the effective potential (V_{eff}), at quark-loop level and made two choices for the auxiliary field: $\varphi^{(\alpha)}$

$\propto\bar{\psi}_2\tau_\alpha\psi_1$, and $(\phi_1,\phi_2,\phi_3)\propto(\bar{\psi}_2\psi_1,\bar{\psi}_2\psi_2,\bar{\psi}_1\psi_1)$. By minimizing the corresponding $V_{\rm eff}$, he obtained in the first case a VEV $\langle\bar{\psi}_2\tau_3\psi_1\rangle_0\neq 0$, which breaks $SU(2)\times U(1)_{1-2}$, and in the second case VEV's $\langle\bar{\psi}_1\psi_1\rangle_0=-\langle\bar{\psi}_2\psi_2\rangle_0\neq 0$, which break $U(1)_A$. By comparing the energy densities at the minimum,[62] he concluded that the first case gives a more negative value and is thus the actual vacuum. Since this vacuum does not break $U(1)_A$, fermions remain massless in it. Dudas also applied the Dyson-Schwinger equation for the fermionic self-energy Σ in the quark-loop approximation. He could reproduce in this way the second case of dynamic symmetry breaking, but not the first one. This point may appear to be disconcerting, in view of the fact that precisely the first case was the energetically preferred dynamical-symmetry-breaking scenario. Concerning this point, Dudas mentions only that the actual dynamical symmetry breaking cannot be obtained by the Dyson-Schwinger approach because there is no ψ_1-ψ_2 mixing at the tree level in the Lagrangian density. However, it seems important to emphasize that the Dyson-Schwinger approach has no inherent deficiencies when compared with the $V_{\rm eff}$ approach. Indeed, it is even more powerful than the latter, once we go beyond the quark-loop approximation, because it gives us information on the momentum dependence of the dynamical quark mass $\Sigma(\bar{p}^2)$ in an inherently nonperturbative way (see Sec. III). The reason the Dyson-Schwinger approach did not give the vacuum in the model is that in the specific vacuum the fermions remain massless, while the Dyson-Schwinger approach searches for nonzero dynamical masses of fermions.

C. Suppression of Λ^2 terms in the composite Higgs self-energy

Blumhofer (1994, 1995) investigated the question of whether radiative Λ^2 contributions that appear in the coefficient of the Higgs $\Phi^\dagger\Phi$ term (Higgs self-energy) could be suppressed in $\bar{t}t$ condensation frameworks. The minimal standard model is plagued by such terms at the one-loop level, where Λ is a large UV cutoff of the theory. It is sometimes postulated that these contributions cancel (Decker and Pestieau, 1979, 1989, 1992, Veltman, 1981; Nambu, 1989). This condition, however, requires an unnatural fine tuning of otherwise independent masses of particles in the loops $(t,b,\tau,\ldots;\ \mathcal{H},\mathcal{G}^0,\mathcal{G}^\pm;\ Z,W^\pm)$. It is not enforced by any known symmetries; and it is scale dependent (Al-sarhi, Jack, and Jones, 1990; Chaichian, Gonzalez Felipe, and Huitu, 1995).

The same type of Λ^2 contributions also appear in the top-mode standard model (2.1), where Λ has the meaning of compositeness scale. Blumhofer raised the question of whether the *dynamics* of the top-mode standard model (2.1), without invoking any additional symmetry, leads to cancellation or at least a strong suppression of such terms in the Higgs self-energy. This question is motivated, in a picture in which the underlying physics is far away (e.g., $\Lambda\sim\Lambda_{\rm GUT}$), by our desire to avoid excessive *ad hoc* fine tuning of parameters. Composite scalars in this framework have the following form of propagator at quark-loop level (leading-N_c with no QCD):

$$D_{\mathcal{H}}=\zeta^{-1}\frac{i}{p^2-4m_t^2},\quad D_{\mathcal{G}^0}=D_{\mathcal{G}^\pm}=\zeta^{-1}\frac{i}{p^2},$$

$$\text{where}\quad \zeta=\frac{N_c g_t^2}{(4\pi)^2}\left[\ln\frac{\Lambda^2}{p^2}+\mathcal{O}(1)\right]. \quad (9.4)$$

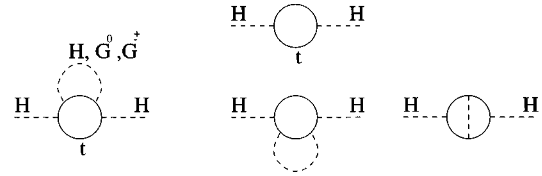

FIG. 13. Diagrams for the self-energy of composite Higgs bosons in the top-mode standard model: at the leading N_c (first graph), and at next-to-leading order (all four graphs).

These propagators are obtained by adding the bubble diagrams of Fig. 5(b) (in Sec. IV), $g_t=M_0\sqrt{G}$ of the top-mode standard model (2.3), and the mass $m_t^{(0)}$ in the bubbles is given by the quark-loop gap equation (4.8) and (4.9) (with $\Lambda_f=\Lambda$). The Λ^2 term in the composite Higgs self-energy in the quark-loop approximation is in fact still there and is $\propto 4N_c m_t^2\Lambda^2/3$, just as in the minimal standard model. Blumhofer (1994) noted the following interesting property: if we also include in the composite Higgs self-energy corrections from exchange of the composite scalars with leading-N_c propagators (9.4), as depicted in Fig. 13, *and* set $N_c=3$, the Λ^2 terms cancel

$$\left[\frac{4N_c}{3}m_t^2-M_H^2\zeta^{-1}\zeta\left(\frac{2m_t}{M_H}\right)^2\right]\Lambda^2 \quad (\mapsto 0 \quad \text{for } N_c=3). \quad (9.5)$$

The first term is the formally leading N_c $[\mathcal{O}(N_c^{\,1})]$ and the second the next-to-leading order [NTLO, $\mathcal{O}(N_c^{\,0})$] in the $1/N_c$ expansion. For $N_c\neq 3$ the two terms do not cancel. In Sec. IV, where the NTLO contributions to the gap equation were discussed, this effect was not investigated, because only leading-N_c self-energies (propagators) of the composite scalars contributed to the NTLO terms of the gap equation. As can be seen from Fig. 13, the diagrams corresponding to the NTLO term in Eq. (9.5) in fact represent an effective one-loop-induced four-scalar vertex with two of the four external legs connected into a (leading-N_c) propagator. Blumhofer then showed in his subsequent work (1995) that the full four-scalar vertex is induced just by the top quark loop (see Fig. 1, where the top quark has $m_t\neq 0$), i.e., that all higher-loop (higher in $1/N_c$) corrections cancel out. Consequently, all higher-than-one-loop (higher than NTLO) corrections in the scalar self-energy also cancel

[62]The VEV-independent undetermined additive constants in both energy densities were fixed so as to give zero energy density for zero VEV's.

out. Blumhofer showed this by means of a complicated system of Dyson-Schwinger-type equations for the full induced four-scalar vertices. This solution existed only for $N_c=3$ and $N_L=2$, where N_L is the isospin number of the generalized weak-isospin group $SU(N_L)_L$. Therefore the reason for cancellation of Λ^2 terms here is fundamentally different from that in supersymmetry. It reminds one of the conditions for the triangular anomaly cancellations. The cancellation solution was obtained by using a specific *Ansatz*, but the hope is that it represents the physical, i.e., the energetically most favored, solution. Transverse degrees of freedom of the electroweak gauge bosons were not included in the analysis. Even if gauge bosons break cancellation of Λ^2 terms, these terms can appear only as radiative corrections and are therefore suppressed by factors g_1^2 and/or g_2^2. Hence, even in such a case, the hierarchy problem is substantially tamed and $\Lambda > \mathcal{O}(1\ \text{TeV})$ could still appear without excessive fine tuning.

The described cancellation phenomenon of Blumhofer suggests that the $1/N_c$ expansion approach in the top-mode standard model should be regarded with caution. Among other things, it appears to suggest that the NTLO effects are crucial and cannot be ignored. Results discussed in Sec. IV offer a similar warning. The analysis of Blumhofer suggests that, for at least some predictions, it is wise to include the leading-N_c and NTLO contributions together, and it holds out the prospect that contributions beyond the NTLO level could be negligible. The implications of the discussed cancellation mechanism for $1/N_c$ expansions in condensation frameworks deserve further investigation. This type of cancellation and/or suppression may also occur in some other—nonminimal—frameworks, specifically those with an extended composite scalar sector. Unfortunately, investigations in this direction have not been carried out.

X. SUMMARY AND OUTLOOK

The author hopes that this review has provided an outline of research activities in the physics of $\bar{t}t$ condensation. It should be clear that there are two main research directions:

(1) One direction focuses on effective four-fermion (NJLVL) interactions at a compositeness scale Λ. If some of these interactions are strong enough, fermion-antifermion condensation and dynamical generation of fermion (heavy-quark) masses as well as dynamical symmetry breaking take place.

(2) The other, more ambitious, approach focuses on constructing and investigating renormalizable models of the underlying physics above Λ, which effectively lead at "low energies" $E \sim \Lambda$ to four-fermion interactions and thus to condensation.

The first group includes the initially proposed minimal framework (e.g., modeled as a truncated top-mode standard model) in which the $\bar{t}t$ condensation is fully responsible simultaneously for the mass m_t and for the full (dynamical) EWSB, thus leading to an effective minimal standard model with one Higgs $\mathcal{H} \sim \bar{t}t$. This minimal framework appears to be ruled out experimentally, by the measurement of $m_t \approx 175\ \text{GeV}$. That is, the minimal framework predicts too high an $m_t^{\text{dyn}} > 200\ \text{GeV}$ when the full DEWSB is implemented, as various calculation methods indicate.

To accommodate an acceptable value for m_t^{dyn} (≈ 175 GeV) while still saturating the EWSB ($v \approx 246$ GeV), various extensions of the minimal framework involving additional scalars with nonzero VEV's have been suggested (see Secs. VI–VIII). The price one must pay is that the extended models become more speculative and/or less predictive. This is certainly the case for those extensions of the minimal (top-mode standard-model) framework which do not involve an extension of the gauge symmetry (see Sec. VI). However, when the symmetry is enlarged and the effective four-fermion NJLVL framework is retained (see Sec. VII), the physics becomes very rich and can dynamically accommodate interesting scenarios, such as a fully dynamical scheme for a L-R symmetric model.

The second major group of studies incorporating $\bar{t}t$ condensation represents more ambitious extensions of the minimal framework—those embedding $\bar{t}t$ condensation in fully renormalizable models of the underlying physics at energies above the condensation scale Λ (see Sec. VIII). Many of these extensions assume that the effective four-quark coupling terms responsible for m_t^{dyn} emerge as a result of the exchange of very massive gauge bosons ($M \sim \Lambda$). While they explain the large m_t and possibly the EWSB dynamically, they usually leave open the question of what mechanism is responsible for the large masses of new gauge bosons. Hence such models are not fully dynamical. Further, most of these models do not address the issue of too large m_t^{dyn} when EWSB is fully saturated by the $\bar{t}t$ condensation. It appears that topcolor-assisted technicolor models (see Secs. VIII.F and IX.A.3) are at this time the only renormalizable frameworks that have been shown to cure that problem satisfactorily. At the same time they are more dynamical in the sense that DEWSB is induced primarily by condensates (of technifermions) bound by exchanges of *massless* new gauge bosons (technigluons). Topcolor-assisted technicolor models combine renormalizable models incorporating $\bar{t}t$ condensation [topcolor models; see Sec. VIII.C] with technicolor and extended technicolor, curing simultaneously some of the unsatisfactory features of the technicolor+extended-technicolor models and of the minimal $\bar{t}t$ framework. Topcolor-assisted frameworks offer even scenarios explaining the masses $M \sim \Lambda$ of the new heavy gauge bosons[63] *dynamically* via dynamical symmetry breaking

[63]Their exchanges lead to the top-mode standard-model four-quark interactions responsible for $\bar{t}t$ condensation.

induced by condensation of technifermion pairs. Apparently the most intensive investigations of resulting low-energy phenomenology are being carried out precisely for various topcolor-assisted technicolor models (see Sec. IX.A.3), and some of these models appear able to accommodate experimental phenomenology (including the otherwise problematic FCNC suppression constraints). There are other renormalizable models that aim at explaining dynamically the mass hierarchies of standard-model fermions and DEWSB *without* involving technicolor+extended technicolor, among them models possessing simultaneously horizontal and vertical gauge symmetries (see Sec. VIII.E). However, it is not yet clear whether they can be compatible with available (low-energy) phenomenology.

Apart from the aforementioned division of condensation models into effective and renormalizable frameworks, there are also distinct classes of calculational approaches used in these models. The applicability of the perturbative RG equations plus compositeness conditions (see Sec. II) in effective frameworks is in general restricted to cases of high cutoff values Λ because it relies on an infrared fixed-point behavior. This method, although somewhat indirect, can be used for any particular realization of an effective strong attraction at a large Λ, not just the top-mode standard model or other NJLVL four-quark interactions. On the other hand, the approach with Dyson-Schwinger and Bethe-Salpeter equations ($1/N_{\rm c}$ expansion) deals with the strong dynamics directly—e.g., with NJLVL interactions in effective frameworks, or with heavy-particle exchanges in renormalizable frameworks. This method has been generally applied to models with $\bar{t}t$ condensation only in the leading-$N_{\rm c}$ approximation (see Sec. III), employing instead of the leading-$N_{\rm c}$ Bethe-Salpeter equations their approximate sum rules, called Pagels-Stokar relations. The main drawback of the Dyson-Schwinger+Bethe-Salpeter approach is the problem of gauge noninvariance (or viability of the choice of the Landau gauge; see Sec. III.E), and possibly a restricted application range for the $1/N_{\rm c}$ expansion. This expansion may not be predictive (if $\Lambda \gg m_t$), or the next-to-leading (NTLO) effects might be reasonable but still very strong (for $\Lambda \sim 1$ TeV)—see Sec. IV. A full NTLO analysis would have to include an NTLO version of the Bethe-Salpeter equation for the composite Nambu-Goldstone boson, but this is still lacking. For larger Λ's (10^3 GeV$<\Lambda<10^8$ GeV), the analysis could either use, at NTLO, the Dyson-Schwinger+Bethe-Salpeter equation in its functional form beyond the "hard-mass" framework $[\Sigma_t \equiv m_t(\Lambda) \mapsto \Sigma_t(\bar{p}^2)]$, or combine NTLO gap equation with perturbative RG evolution. In principle, the viability of the minimal framework for low Λ's, although unlikely, is still an open question, since a consistent nonperturbative analysis applicable to the range of low Λ's ($\Lambda < 10^8$ GeV) has not been carried out. In this respect, a recently proposed alternative method—an approximation scheme for the nonperturbative renormalization-group approach (see Sec. II.E)—is also promising, since it appears to avoid many of the deficiencies of the $1/N_{\rm c}$ expansion and of the perturbative RG approaches.

In conclusion, the author wishes to reemphasize the importance of studying renormalizable and effective models involving $\bar{t}t$ condensation. The challenge remains to find a framework fully consistent with experimental data while offering a dynamical explanation for those sectors of the standard model that are at the moment the most mysterious—the electroweak symmetry breaking (EWSB) and the mass hierarchies of fermions, including possibly the structure of CKM mixing. Of course, such investigations will gain additional motivation if a Higgs particle is detected *and* direct or indirect experimental indications for a composite nature of the Higgs are found. The author also wishes to stress the need for systematic nonperturbative calculational methods, which would be applicable to cases in which the onset scale Λ of new physics responsible for the condensation is not exceedingly far above the electroweak scale, e.g., when $\Lambda < 10^8$ GeV.

Note added. Here we briefly mention some work that appeared after the present article was accepted for publication.

Blumhofer, Dawid, and Manus (1998) investigated, using the RG approach, the role of higher than six-dimensional quark operators as well as the related role of heavy scalar and heavy vector exchanges in $\bar{t}t$ condensation models. Dawid (1988) applied a similar approach in studying a condensation mechanism in supersymmetry such that the scalar composites are made entirely of sfermions.

Following the Dyson-Schwinger equation and using a Ward-Takahashi identity, Hashimoto (1998a) derived a formula for the mass of the composite Higgs particle in terms of the fermionic $m_f^{\rm dyn}$. Further, he (1998b) employed the Dyson-Schwinger+Pagels-Stokar approach (see Sec. III) to calculate $m_t^{\rm dyn}$ in the minimal $\bar{t}t$ condensation framework, this time by including composite Higgs boson loop effects in the Dyson-Schwinger equation (next-to-leading effects in $1/N_c$) and by employing simultaneously the Pagels-Stokar equation (i.e., leading-N_c effects only).

Holdom and Roux (1998) investigated the next-to-leading-order (in α) contributions to the gap equation when the exchanged gauge boson is massive; they found that these contributions are large and thus throw doubt upon the expansion, as well as upon the "most attractive channel" hypothesis (see Sec. VIII.C). When the gauge boson is massless, these contributions are small (Appelquist *et al.*, 1988; see Sec. IV.E).

Dobrescu and Hill (1998) and Chivukula, Dobrescu, Georgi, and Hill (1999) introduced and studied a framework in which dynamical EWSB is fully driven by a condensate involving the top quark and an additional isosinglet quark χ; and observed (i.e., low enough) m_t is then still allowed due to a seesaw mechanism between the top quark and χ.

Chivukula and Georgi (1998a, 1998b) studied topcolor and topcolor-assisted technicolor models by going beyond the large-N_c and the NJLVL approximations; they concluded that the strength of the "tilting" $U(1)$ gauge interaction has an upper bound and that, as a consequence, the topcolor coupling must be equal to the critical value to within a few percent.

Lindner and Triantaphyllou (1998) studied a left-right symmetric framework in which the $\bar{t}t$ condensate is assisted in the dynamical EWSB by heavy mirror fermions, which in turn keep the electroweak parameters S and T under control.

Andrianov, Andrianov, Yudichev, and Rodenberg (1997) developed further the two-Higgs-doublet model in which the two composite Higgs particles result from four-quark interactions which include derivatives (see also Sec. VI.A.4).

Production rates and detection of neutral scalars and charged (pseudo)scalars at colliders, as predicted by various $\bar{t}t$ condensation frameworks (two-Higgs-doublet standard model, topcolor, and topcolor-assisted technicolor), were studied by Balázs, He, and Yuan (1998), Balázs, Diaz-Cruz, *et al.* (1999), Choudhury, Datta, and Raychaudhuri (1998), Diaz-Cruz, He, Tait, and Yuan (1998), He and Yuan (1998), and Spira and Wells (1998). Contributions, in such models, of light scalars to the $\bar{t}t$ production in photon colliders were investigated by Zhou *et al.* (1998).

ACKNOWLEDGMENTS

The author is grateful for the suggestion of E. A. Paschos that he write this review article. He acknowledges useful discussions with T. Hambye and U. Sarkar related to chiral perturbation theory as low-energy QCD, and aspects of supersymmetry, respectively. During the middle stages of this work, the author was supported in part by the Bundesministerium für Bildung, Wissenschaft, Forschung, und Technologie, Project No. 057DO93P(7).

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