The ''friction'' of vacuum, and other fluctuation-induced forces

Mehran Kardar

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Ramin Golestanian

Institute for Advanced Studies in Basic Sciences, Zanjan 45195-159, Iran

The static Casimir effect describes an attractive force between two conducting plates, due to quantum fluctuations of the electromagnetic (EM) field in the intervening space. Thermal fluctuations of correlated fluids (such as critical mixtures, super-fluids, liquid crystals, or electrolytes) are also modified by the boundaries, resulting in finite-size corrections at criticality, and additional forces that affect wetting and layering phenomena. Modified fluctuations of the EM field can also account for the ''van der Waals'' interaction between conducting spheres, and have analogs in the fluctuation-induced interactions between inclusions on a membrane. We employ a path integral formalism to study these phenomena for boundaries of arbitrary shape. This allows us to examine the many unexpected phenomena of the dynamic Casimir effect due to moving boundaries. With the inclusion of quantum fluctuations, the EM vacuum behaves essentially as a complex fluid, and modifies the motion of objects through it. In particular, from the mechanical response function of the EM vacuum, we extract a plethora of interesting results, the most notable being: (i) The effective mass of a plate depends on its shape, and becomes anisotropic. (ii) There is dissipation and damping of the motion, again dependent upon shape and direction of motion, due to emission of photons. (iii) There is a continuous spectrum of resonant cavity modes that can be excited by the motion of the (neutral) boundaries. [S0034-6861(99)00604-2]

CONTENTS

I. OUTLINE

Fluctuation-induced forces are ubiquitous in nature, covering many topics from biophysics to cosmology (Casimir, 1948; Dzyaloshinskii *et al.*, 1961; Mostepanenko and Trunov, 1997; Krech, 1994; Weinberg, 1989). There are two basic ingredients in these phenomena: (i) A fluctuating medium, such as the electromagnetic (EM) field; and (ii) external objects whose presence suppresses (or in some way modifies) the fluctuations, such as dipoles or conductors. The overall strength of the interaction is proportional to the driving energy of fluctuations (k_BT and $\hbar$ for thermal and quantum fluctuations, respectively); its range is related to that of the correlations of the fluctuations. The most interesting cases are when the interactions are long ranged, corresponding to scale-free fluctuations.

To illustrate the first ingredient, i.e., fluctuations, let us consider Feynman's formulation of quantum mechanics. The quantum particle does not exclusively follow its classical trajectory, but can virtually follow any path connecting the beginning and end points; each path is weighted by $e^{iS/\hbar}$, with S being the action. The dominant contributions come from the classical trajectory, for which the action $S_{\rm cl}$ is a minimum, and also from trajectories close enough to the classical path for which the deviation $S-S_{\rm cl}$ is of order of $\hbar$. Such deviations from classical behavior, whose scale is set by $\hbar$, are a manifestation of *quantum fluctuations*. Similarly, a particle in a thermal (noisy) environment undergoes the zig-zag paths of Brownian motion. The locations of the particle in thermal equilibrium are then characterized by a distribution e^{-U/k_BT}, where U is the potential energy at each location. While the most likely configurations are those of minimal energy $U_{\rm min}$, there are also significant contributions from configurations such that $U-U_{\rm min}$ is of the order of k_BT. Such deviations from the minimum energy position, with scale set by k_BT, are due to *thermal fluctuations*. The above picture can be easily generalized to the case of fluctuating fields such as the EM field, in which case the classical physics comes from Maxwell's equations.

The following argument provides a rough impression of how the second ingredient—restriction on fluctua-

tions by external bodies—can lead to interactions. Imagine a field fluctuating freely in infinite space, except for the constraints of vanishing on two parallel plates at a distance H. Each constraint by itself imposes restrictions on fluctuations of the field, increasing the free energy. With two plates, space is partitioned into three domains: Two with continuous modes (less restrictive) and one with discrete modes (more restrictive). The system now favors less separation between the plates so that the restrictive domain is as small as possible. This is an entropic effective attraction between the plates. Let us recall the mattress effect, in which two bodies on an elastic medium are attracted to each other to reduce the energy cost of distortions. While in this case the attraction is due to a reduction in field energy, for fluctuation-induced interactions, there is a reduction in free energy which is of entropic origin.

The goal of this article is to provide a glimpse of the unity and simplicity of fluctuation-induced forces. While we attempt to describe a wide range of phenomena, this selection is by no means exhaustive, and highly biased by subjective interests. In the spirit of a colloquium, we have tried to avoid technical details, preferring to present general arguments and dimensional estimates whenever possible. The interested reader is referred to various sources for calculational details.

The prototype of fluctuation-induced interactions, the Casimir force between conducting plates due to quantum fluctuations of the EM field, is briefly recalled in Sec. II. We then discuss several cases where the source of interaction is the thermal fluctuations of a correlated fluid between the bounding plates. This interaction was originally proposed for a liquid mixture at its critical point, but is also present when long-range correlations appear as a result of symmetry breaking, as in superfluids or liquid crystals. There is even an attractive component to the (mainly repulsive) force between two similarly charged plates, due to fluctuations of counterions in a neutralizing solution. Since the latter connection is seldom made explicit, we expand on its origin in Sec. II and the Appendix. While the reader may skip any one of the subsections in Sec. II without losing general track of the article, we note that experiments on wetting of helium films may provide a beautiful test of these forces.

The van der Waals and London dispersion forces between atoms and molecules can also be attributed to the modified fluctuations of the EM field. As we point out in Sec. III, there are analogous forces between inclusions on a membrane, which may be of relevance to biology. Their origin is the modified surface fluctuations, and they decay more slowly with separation than the standard van der Waals interaction. We use this example to emphasize that non-additivity is an important feature of fluctuation-induced forces: they cannot be obtained from a pairwise sum of two-body potentials.

Several new results are obtained in going beyond the simple geometries of flat plates and spheres, by looking at rough and deformed structures, as in Sec. IV. Our key to implementing the corresponding nonstandard boundary conditions is a functional integral approach, which can also be used in conjunction with a path integral quantization of the EM field. Since, in this relativistic theory, deformations in space and time appear on the same footing, we can then examine the dynamic Casimir effect, which is introduced in Sec. V.

Some of the unexpected phenomena that emerge from quantum fluctuations of the EM field in the presence of moving deformed plates are discussed in Sec. VI. There are corrections to the mass of a plate that depend on its shape. There is also dissipation due to emission of photons (hence, the "friction" in the title of this article). While these effects are typically very small, we believe that they are significant for what they imply about the nature of the quantized EM vacuum. Qualitatively, with the inclusion of quantum fluctuations, the vacuum behaves as a complex fluid that hinders and influences the bodies moving through it.

II. FLUCTUATION-INDUCED FORCES

A. Quantum fluctuations

The standard Casimir effect (Casimir, 1948; Mostepanenko and Trunov, 1997) is a macroscopic manifestation of quantum fluctuations of vacuum. In 1948, Casimir considered the electromagnetic field in the cavity formed by two conducting plates at a separation H. Because the electric field must vanish at the boundaries, the normal modes of the cavity are characterized by wave vectors $\vec{k}=(k_x,k_y,\pi n/H)$, with integer n. Once quantized, these normal modes are harmonic oscillators of frequencies $\omega(\vec{k})=c|\vec{k}|$, each of which in its ground state has energy $\hbar\omega(\vec{k})/2$. While the sum total of the ground state energies is formally infinite, Casimir showed that there is a finite H-dependent contribution

$$\delta\mathcal{E}=-\hbar c\times\frac{A}{H^3}\times\frac{\pi^2}{720}, \tag{1}$$

implying an attractive force, proportional to the plate area A. Thus, by measuring the mechanical force between macroscopic bodies, it is, in principle, possible to learn about vacuum fluctuations.

The predictions of Casimir were followed by experiments on quartz (Abricossova and Deryaguin, 1953) and aluminum (Sparnaay, 1958) plates at separations $H>10^3$ Å. However, these experiments, and others reviewed in (Israelachvili and McGuigan, 1990) provided results that were at best in qualitative agreement with Eq. (1). Recent high precision measurements of the force (using a torsion pendulum) between a gold plate and a gold-plated sphere claim to confirm the accuracy of the theoretical prediction to very high accuracy (Lamoreaux, 1997; Mohideen and Roy, 1998).

B. Thermal fluctuations

While the Casimir interaction is due to the quantum fluctuations of the electromagnetic field, there are several examples in classical statistical mechanics, where

forces are induced by the thermal fluctuations of a correlated fluid. One of the best known examples comes from the finite-size corrections to the free energy at a critical point (Krech, 1994). Fisher and de Gennes (Fisher and de Gennes, 1978; Privman and Fisher, 1984) argued that in a binary liquid mixture, concentrations near a wall are perturbed only over a distance of the order of the correlation length ξ. Any interaction mediated by the concentration fluctuations must also decay with this characteristic length. However, at the critical point where ξ diverges, they suggested an attractive contribution to the free energy of a critical film that varies with its thickness H as

$$\delta\mathcal{F}(H) = -k_B T \times \frac{A}{H^2} \times \Delta. \tag{2}$$

This is to be expected on dimensional grounds, as the free energy comes from thermal fluctuations, hence proportional to $k_B T$, and must be extensive in A. (Similar analysis in d dimensions leads to a dependence as $1/H^{d-1}$.) In two dimensions, exact values for the dimensionless amplitude Δ can be obtained by employing techniques of conformal field theories (Blöte *et al.*, 1986). In higher dimensions, they can be estimated numerically (Nightingale and Indekeu, 1985), and by $\epsilon = 4 - d$ expansions (Krech and Dietrich, 1991).

In analogy to the Casimir energy, we can regard Eq. (2) as due to the modified free energy of concentration fluctuations by the boundaries. However, the force that results from this free energy decays as $1/H^3$. The difference in power of H from Eq. (1) is explained by noting that the fluctuation energy in the latter is quantum in origin, and hence proportional to $\hbar c$, which has dimensions of *energy times length*.

C. Superfluid films

In fact, long-range forces are induced by thermal fluctuations of any *correlated medium*, by which we mean any system with fluctuations that have long-range correlations. The critical system is a very particular example; much more common are cases where long-range correlations exist due to Goldstone modes of a broken continuous symmetry, as in superfluids or liquid crystals.[1] A superfluid is characterized by a complex order parameter, whose phase ϕ may vary across the system. The energy cost of such variations is governed by the Hamiltonian

$$\mathcal{H}[\phi] = \frac{K}{2} \int d^3\mathbf{x} (\nabla\phi)^2, \tag{3}$$

where the stiffness K is related to the superfluid density. There is no characteristic length scale for fluctuations of ϕ, which scale as a power of the observation length. Consequently, we expect power-law finite-size scaling, just as in the case of a critical point. In the Casimir geometry, the free energy resulting from thermal fluctuations of these modes has the form (Li and Kardar, 1991, 1992)

$$\delta\mathcal{F}(H) = -k_B T \times \frac{A}{H^2} \times \frac{\zeta(3)}{16\pi}. \tag{4}$$

Note that the result is universal, i.e., independent of the stiffness K. A similar expression is obtained for the free energy of the electromagnetic field confined between metallic plates at high temperatures $k_B T \gg \hbar c/H$. However, the result is larger by a factor of two (Schwinger *et al.*, 1978), reflecting the two polarizations of the normal modes (photons).

Liquid Helium tends to spread over and wet most metallic surfaces. The thickness of the wetting layer is controlled by the strength of the attractive forces that bind the film to the substrate (Dietrich, 1988), mostly due to van der Waals interactions. In the presence of a chemical potential penalty of $\delta\mu$ per unit volume, the energy of a film of thickness H is

$$\frac{E(H)}{k_B T} = A\left[\frac{\delta\mu}{k_B T} H + \frac{C}{H^2}\right], \tag{5}$$

where C is a *positive* numerical constant. Minimizing this expression leads to a thickness[2]

$$H = \left(\frac{2Ck_B T}{\delta\mu}\right)^{1/3}. \tag{6}$$

When the helium film is in the normal phase, the film thickness is determined solely by the strength of the van der Waals (vdW) force. The numerical value of $C_> = C_{\rm vdW} > 0$ depends on the substrate, and is nonuniversal. However, when the film becomes superfluid, there is an additional attractive fluctuation-induced (FI) force due to Eq. (4), and $C_< = C_{\rm vdW} + C_{FI}$; where $C_{FI} = -\zeta(3)/16\pi \simeq -0.02391$. In the vicinity of the superfluid transition, there is a different attractive contribution to the force due to finite-size scaling (FSS) of the critical fluctuations, as in Eq. (2), and $C_\lambda = C_{\rm vdW} + C_{\rm FSS}$. The best estimates for the finite-size scaling amplitude at criticality in $d = 3$ are $C_{\rm FSS} \simeq -0.03$ from numerical simulations (Indekeu, 1986; Mon and Nightingale, 1987), and $C_{\rm FSS} \simeq -0.024$ from field-theoretic ϵ expansions (Krech, 1994). The parameter C thus takes three different values in the normal fluid, at the λ point, and in the superfluid phase. From Eq. (6) we then expect two jumps in the film thickness, as the temperature is lowered through the superfluid transition. Experiments to monitor the film thickness have been recently

[1] For a study of phonon fluctuation-induced interactions, see Mahale and Cole (1986).

[2] For liquid Helium films with thickness of more than 100 Å, the van der Waals interaction is retarded, and falls off as $1/H^3$, to leading order. In this case, the expression for the film thickness is more complicated, and will not be elaborated upon here.

performed at Pennsylvania State University, in which the corresponding jumps have indeed been observed (Garcia and Chan, 1999).

D. Liquid crystals

Liquid crystals exemplify anisotropic cases of correlated fluids due to broken symmetry, which again lead to fluctuation-induced forces (Mikheev, 1989; Ajdari *et al.*, 1991; Li and Kardar, 1991, 1992). They are also easily accessible, as experiments can be performed at room temperature and require no fine tuning to achieve criticality. A *nematic* liquid crystal is composed of long molecules that are aligned, with an order parameter which is the "director" field $\mathbf{n}(\mathbf{r})$, characterizing the local preferred direction of the long axis of the molecules (de Gennes and Prost, 1993). The energy cost of fluctuations of this field is given by (de Gennes and Prost, 1993)

$$\mathcal{H}_N=\frac{1}{2}\int d^3\mathbf{r}[\kappa_1(\boldsymbol{\nabla}\cdot\mathbf{n})^2+\kappa_2(\mathbf{n}\cdot\boldsymbol{\nabla}\times\mathbf{n})^2+\kappa_3(\mathbf{n}\times\boldsymbol{\nabla}\times\mathbf{n})^2]. \tag{7}$$

Integrating over the nematic fluctuations leads to a free energy contribution

$$\delta\mathcal{E}_N=-k_BT\times\frac{A}{H^2}\times\frac{\zeta(3)}{16\pi}\left(\frac{\kappa_3}{\kappa_1}+\frac{\kappa_3}{\kappa_2}\right). \tag{8}$$

Note that the resulting force does depend on the relative strengths of the elastic coupling constants (reflecting the anisotropy of the system).

In a smectic liquid crystal, the molecules segregate into layers which are fluid like. The deviations of these layers from perfect stacking are described by a scalar deformation $u(\mathbf{x},z)$, which is subject to a Hamiltonian

$$\mathcal{H}_S=\frac{1}{2}\int d^3\mathbf{r}\left[B\left(\frac{\partial u}{\partial z}\right)^2+\kappa(\nabla^2u)^2\right]. \tag{9}$$

The resulting interaction energy

$$\delta\mathcal{E}_S=-k_BT\times\frac{A}{H\lambda}\times\frac{\zeta(2)}{16\pi},\quad\text{with}\quad\lambda\equiv\sqrt{\frac{\kappa}{B}}, \tag{10}$$

falls off as $1/H$, reflecting the extreme anisotropy which has introduced an additional length scale λ into the problem. For potential experimental tests of these forces in surface freezing of liquid crystal films, see Lyra *et al.* (1993).

E. Charged fluids

Interactions between a collection of charged macroions in an aqueous solution of neutralizing counterions,[3] with or without added salt, are in general very complex. The macroions may be charged spherical colloidal particles, charged amphiphilic[4] membranes, stiff polyelectrolytes (e.g., microtubules, actin filaments, and DNA), or flexible polyelectrolytes (e.g. polystyrene sulphonate), and the counterions could be mono- or polyvalent. It is known that, under certain conditions, the accumulation (condensation) of counterions around highly charged macroions can turn the repulsive Coulomb interaction between them into an attractive one. The attractive interaction is induced by the diminished charge-fluctuations close to the macroions (due to the condensation of counterions) (Oosawa, 1968, 1971; Attard, Mitchell, and Ninham, 1988; Marcelja, 1992), and in this sense is related to the effects discussed in the previous sections.

Since the connection between the entropic attraction of charged macroions and fluctuation-induced forces is seldom made explicit, in the Appendix we present a path integral formulation that makes this analogy more transparent. The interaction between macroions can be broken into two parts: Poisson-Boltzmann (PB) free energy, and a fluctuation-induced correction. Specifically, consider two parallel negatively charged 2D plates with densities $-\sigma$, separated by a distance H in $d=3$, in a solution of neutralizing counterions with valence z. The PB equation can be solved exactly in this geometry, and the corresponding PB free energy, in the limit of highly charged plates, is[5]

$$F_{PB}=k_BT\times\frac{\pi}{2}\times\frac{A}{z^2\ell_BH}\left[1+\frac{1}{4\pi^2\ell_B^2z^2\sigma^2H^2}+\cdots\right], \tag{11}$$

in which $\ell_B\equiv e^2/\epsilon k_BT$ is the Bjerrum length. Note that in the limit $\ell_Bz\sigma H\gg1$, the interaction is independent of the charge densities of the plates; i.e., it is universal.

The fluctuation-induced correction involves calculation of a determinant (see the Appendix), which depends on the local charge compressibilities. The true compressibility profile (and the charge density profile) emerging from the solution of the PB equation is generally very complicated. It is usual to simplify the problem by assuming that the surface charge density is so high that the counterions are confined to a layer of thickness $\lambda_{GC}\ll H$, where $\lambda_{GC}=\frac{1}{2}\pi z\ell_B\sigma$ is the Gouy-Chapman length. Then we can use an approximate compressibility profile $m^2(x)=(2/\lambda_{GC})[\delta(x+H/2)+\delta(x-H/2)]$. In the limit $H/\lambda_{GC}\gg1$, we obtain (Attard *et al.*, 1987; Attard, Kjellander, Mitchell, and Jönson, 1988)

$$F_{FI}=-k_BT\times\frac{A}{H^2}\times\frac{\zeta(3)}{16\pi}\left[1+O\left(\frac{\lambda_{GC}}{H}\right)\right], \tag{12}$$

[3]Counterions are free ions in the solutions that are oppositely charged with respect to the macroions. While they may have any valence, their overall number in the solution counterbalances the net charge of the macroions.

[4]The term amphiphilic comes from a Greek word meaning "love on both sides." Here, it refers to molecules with segments of opposite tendencies of attraction to water or oil (Peliti, 1996).

[5]See for example, Attard, Mitchell, and Ninham (1988) and references therein.

for the fluctuation-induced part of the interaction.[6] Note that at large separations, it is asymptotically identical to the Casimir interaction in Eq. (4). In the opposite limit $H/\lambda_{GC} \ll 1$, however, it yields (Pincus and Safran, 1998; Golestanian and Kardar, 1998b)

$$F_{FI} = k_B T \times \frac{A}{\lambda_{GC}^2} \times \Delta_c \times \ln\left(\frac{H}{\lambda_{GC}}\right), \tag{14}$$

where $\Delta_c \simeq 0.0792$ (Golestanian and Kardar, 1998b). A similar analysis in d-dimensions leads to asymptotic dependencies as $1/H^{d-1}$ for large separations, and $1/H^{d-3}$ for small separations, where the crossover is set by a generalized Gouy-Chapman length (Golestanian and Kardar, 1998b). Interestingly, such attractive interactions have recently been suggested to be responsible for DNA bundle formation (Ha and Liu, 1997, 1998), and collapse of stiff polyelectrolytes (Golestanian, Kardar, and Liverpool, 1999) and rigid membranes (Lau and Pincus, 1998).

III. DISPERSION FORCES

A. Van der Waals interactions

In addition to his work on the force between plates, Casimir also realized (Casimir and Polder, 1948) that the van der Waals and London (London, 1930) forces can be understood on the same footing: The presence of the atoms modifies the fluctuations of the electromagnetic field, resulting in an attractive interaction. [For a modern perspective, see the discussion by Kleppner in (Kleppner, 1990).] For example, let us consider two conducting (neutral) spheres of volumes $V_1 = 4\pi a_1^3/3$ and $V_2 = 4\pi a_2^3/3$, at a distance R. The fluctuation-induced interaction is proportional to the product of the excluded volumes, and thus on dimensional grounds we expect a potential

$$\mathcal{V}(R) = -k_B T \times \frac{V_1 V_2}{R^6} \times \Delta_T, \tag{15}$$

due to thermal fluctuations. When the fluctuations are of quantum origin, Eq. (15) is modified to

$$\mathcal{V}(R) = -\hbar c \times \frac{V_1 V_2}{R^7} \times \Delta_Q, \tag{16}$$

with $\Delta_Q = 1287/(256\,\pi^3)$ (Mostepanenko and Trunov, 1997).

Let us compare the above result with the more standard approach to calculating the London force between two neutral *atoms*: While the average dipole for each atom is zero, an instantaneous dipole fluctuation in one can induce a parallel instantaneous dipole in the other, leading to an attraction. Since the direct dipole-dipole interaction decays as $1/R^3$, the induced effect scales as the square, i.e., $1/R^6$. In this regard, it is similar to the result in Eq. (15), except that the characteristic energy is set by a typical atomic excitation energy $\hbar\omega_0$ rather than $k_B T$. The retardation effects are then obtained by taking into account the finite speed of light. For large enough distances, when the signal goes from atom-1 to atom-2 to induce the dipole, and return to atom-1 to induce an attraction, it finds the dipole at atom-1 somewhat misaligned; resulting in a weaker attraction. The characteristic time for electron movements can be estimated from the frequency of the orbit as $\tau = 2\pi/\omega_0$. The crossover occurs when the travel time for the signal is comparable to this characteristic time, namely $R/c \sim \tau$. Hence, taking into account the retardation effect, the interaction is

$$\mathcal{V}_r(R) = -\hbar\omega_0 \times \frac{V_1 V_2}{R^6} \times f\left(\frac{R\omega_0}{c}\right). \tag{17}$$

The crossover function $f(x)$ is a constant for $x \to 0$, and vanishes as $1/x$ for $x \to \infty$. In the latter limit, the dependence on ω_0 vanishes, and the Casimir-Polder result of Eq. (16) is recovered. A similar crossover function between the two forms of interaction for conducting spheres in Eqs. (15) and (16), occurs at a distance $\lambda_T \sim \hbar c/k_B T$.

B. Inclusions on membranes

Dispersion forces are not limited to particles in three dimensional space, but also occur for inclusions on films and membranes, the latter being of potential importance for understanding the interactions between proteins floating on a cell membrane. A membrane is a bilayer of *amphiphilic* molecules, each composed of a *hydrophilic* or polar head, and a *hydrophobic* tail of hydrocarbon chains. The polar heads prefer to be in contact with the water, and in the bilayer structure insulate the "oily" hydrocarbon interior from contact with water. A bilayer that is in equilibrium with amphiphiles in solution can easily change its area by exchanging molecules with this reservoir. This implies that the surface tension is zero (de Gennes and Taupin, 1982; Brochard and Lennon, 1975; David and Leibler, 1991), and the energy cost of deforming the bilayer is entirely due to bending (Canham, 1970; Helfrich, 1973). The "flicker" of the membrane is thus governed by the elastic Hamiltonian

$$\mathcal{H} = \frac{\kappa}{2} \int d^2\mathbf{x} (\nabla^2 h(\mathbf{x}))^2, \tag{18}$$

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[6] The calculation of the determinant using the true compressibility profile for the parallel plate geometry has indeed been carried out (Attard, Mitchell, and Ninham, 1988), with the result

$$F_{FI} = -k_B T \times \frac{A}{H^2} \times \left[\frac{\zeta(3)}{16\pi} + \frac{\pi}{4}\left(\frac{\pi}{4} + \frac{1}{2}\right) + \frac{\pi}{4}\ln\left(\frac{H}{\pi\lambda_{GC}}\right) + O\left(\frac{\lambda_{GC}}{H}\right)\right]. \tag{13}$$

It is interesting to note that the correct result is considerably stronger than that of the simple "Gouy-Chapman" model [Eq. (12)]. For a numerical calculation of higher loop corrections see Coalson and Duncan (1992).

where $h(\mathbf{x})$ is a height function describing deformations of the surface. Typical values of the bending rigidity κ of biological membranes is of the order of $0.1-25k_BT$.

Cell membranes also include proteins that perform various biological functions (e.g., pumps). Each protein inclusion disturbs the lipid bilayer, resulting in interactions between nearby inclusions (cf. Israelachvili 1992, Dan *et al.* 1993, Goulian *et al.* 1993, Mouritsen and Bloom, 1993, and references therein). These disturbances, and the resulting interactions, tend to be short-ranged, falling off exponentially with a characteristic length related to the distance over which the lipid membrane "heals" (Dan *et al.*, 1993). There are also longer-ranged interactions between the proteins: In addition to the standard van der Waals interaction, there are interactions mediated by the disturbed fluctuations of the flickering membrane. As discussed by Goulian *et al.* (1993) and Goulian (1996), such interactions exist as long as the rigidity of the inclusion differs from that of the ambient membrane, and fall off as $1/R^4$. In particular, if the inclusions are much stiffer than the membrane, the fluctuation-induced potential is

$$\mathcal{V}(R)=-k_BT\times\frac{A^2}{R^4}\times\frac{6}{\pi^2}, \tag{19}$$

where A is the area of each inclusion.[7] The interaction is attractive and independent of κ and $\overline{\kappa}$; its energy scale is set by k_BT. A generalization of this result that includes quantum fluctuations of membranes is given in D'Hoker *et al.* (1995).

The form of the interaction depends sensitively on the shapes of the inclusions, as demonstrated by the calculation of the fluctuation-induced interaction between rod-like objects Golestanian, Goulian, and Kardar (1996a, 1996b). The rods are assumed to be sufficiently rigid so that they do not deform coherently with the underlying membrane. They can thus only perform rigid translations and rotations while remaining attached to the surface. As a result, the fluctuations of the membrane are constrained, having to vanish at the boundaries of the rods. Consider the situation depicted in Fig. 1, with two rods of lengths L_1 and L_2 at a separation $R\gg L_i$. The fluctuation-induced interaction is given by

$$V^T(R,\theta_1,\theta_2)=-\frac{k_BT}{128}\times\frac{L_1^2L_2^2}{R^4}\times\cos^2[2(\theta_1+\theta_2)], \tag{20}$$

to the leading order, where θ_1 and θ_2 are the angles between the rods and the line adjoining their centers, as indicated in Fig. 1.

The orientational dependence is the *square* of a quadrupole-quadrupole interaction, with the unusual property of being minimized for both parallel and perpendicular orientations of the rods. The above fluctuation-induced interactions decay less rapidly at large distances than van der Waals forces and may play an important role in aligning asymmetric inclusions in biomembranes. Since orientational correlations are often easier to detect than forces, this result may also be useful as a probe of fluctuation-induced interactions. Finally, this interaction could give rise to novel two-dimensional structures for collections of rodlike molecules. In particular, the resemblance of the orientational part of the interaction to dipolar forces suggests that a suitable way to minimize the energy of a collection of rods is to form chains. (If the rods are not colinear, the interactions cannot be simultaneously minimized.)

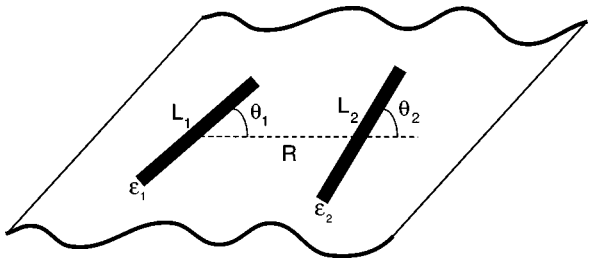

FIG. 1. Two rod-shaped inclusions embedded in a membrane. The rods are separated by a distance R. The ith rod has length L_i, width ϵ_i, and makes an angle θ_i with the line joining the centers of the two rods.

An important property of fluctuation-induced interactions is that they are *non-additive*, and cannot be obtained by adding two-body potentials. For example, consider an interaction, $U(|r_1-r_2|)du_1du_2$, between any two infinitesimal segments of two rods in Fig. 1. If both rods are of length L at a distance $R\gg L$, expanding $|r_1-r_2|$ and integrating over the two rods leads to the interaction

$$\begin{aligned}V(R,\theta_1,\theta_2)=L^2U(R)+\frac{L^4}{6}\left(\frac{U'(R)}{R}+U''(R)\right)\\ -\frac{L^4}{12}\left(\frac{U'(R)}{R}-U''(R)\right)\\ \times(\cos 2\theta_1+\cos 2\theta_2).\end{aligned} \tag{21}$$

The angular dependence is now completely different, and minimized when the two rods are parallel to their axis of separation. Presumably both interactions are present for rods of finite thickness; the additive interaction is proportional to $L^2(L\epsilon/R)^2$, where ϵ is the thickness. The previously calculated interactions are thus larger by a factor proportional to $(R/\epsilon)^2$ and should dominate at large separations.

The case of stiff linear inclusions at close separations ($L\gg R$) is considered in Golestanian (1996). It is shown that a finite rigidity of the linear inclusions leads to a screening out of the Casimir-type fluctuation-induced attraction. The screening length is set by the ratio between the rigidity of the polymer and that of the membrane. This is the length scale below which the polymers are seen as straight parallel lines, hence resulting in a Casimir interaction. Moreover, the attractive interactions

[7]In this formula, the result in Goulian *et al.* (1993) has been corrected by a factor of 1/2 (Golestanian, Goulian, and Kardar, 1996a, 1996b).

could lead to an instability in the shape of the stiff polymers, signalling a major reduction in their rigidity (softening) induced by the membrane fluctuations (Golestanian, 1996).

IV. ROUGH SURFACES

Most computations of Casimir forces are for simple geometries, e.g. between two parallel plates, or perfect spheres. It is natural to consider how these forces are modified by the roughness that is present in most "random" surfaces. There are a number of ways that one can go beyond the simple planar geometry. For example, a multiple scattering approach (Balian and Duplantier, 1978) can be used to compute the interactions for arbitrary geometry in a perturbation series in the curvature. A generalization of the approach due to Dzyaloshinskii, Lifshitz, and Pitaevskii (Dzyaloshinskii *et al.*, 1961) could be developed (Novikov *et al.*, 1990a, 1990b, 1992a, 1992b) to study the Casimir forces for surfaces with roughness. It is possible to use a phenomenological approach (Bordag *et al.*, 1995) in which small deviations from plane parallel geometry are treated by using an additive summation of Casimir potentials. However, as demonstrated in the previous section, fluctuation induced forces are not additive, and additional steps are necessary to correct the result (Bordag *et al.*, 1995). Another perturbative approach is also introduced (Ford and Vilenkin, 1982), which could in principle be used to treat surfaces with roughness, although it is not explicitly carried out in this paper. Most of these approaches suffer from rather cumbersome treatments of the boundary conditions.

A path integral approach, introduced by Li and Kardar (1991, 1992), makes possible relatively simple computations of the fluctuation-induced force. This approach has a number of advantages. First, different manifolds (with arbitrary intrinsic and embedding dimensions) in various correlated fluids can be treated in a similar fashion. Second, the boundary conditions are quite easily implemented, and corrections can be computed perturbatively in the deformations. While this method was originally developed for the study of thermal fluctuations, it can be adapted to quantum fluctuations, as discussed in the next section. In the remainder of this section we calculate the corrections to the thermal Casimir force due to substrate roughness.

Many solid surfaces produced by rapid growth or deposition are characterized by self-similar fluctuations (Kardar, 1996), which grow as

$$\overline{[h(\mathbf{x})-h(\mathbf{y})]^2}=A_S|\mathbf{x}-\mathbf{y}|^{2\zeta_S}, \tag{22}$$

where the overbar denotes an average over the surface profile, and ζ_S is a characteristic roughness exponent. The Casimir force between a flat and a rough surface (with a correlated fluid in between) is calculated in Li and Kardar (1991, 1992). The resulting free energy per unit area is

$$\mathcal{F}(H)=-\frac{k_BT}{H^2}\frac{\zeta(3)}{16\pi}-\frac{k_BTA_SL^{2\zeta_S}}{H^4}\frac{3\zeta(3)}{16\pi}+\frac{k_BTA_S}{H^{4-2\zeta_S}}\frac{C_1}{4}, \tag{23}$$

where C_1 is a numerical coefficient (Li and Kardar, 1991, 1992), and L is the extent (upper cutoff) of the self-affine structure, satisfying $\Delta H\equiv\sqrt{A_S}L^{\zeta_S}\ll H$. (This is the condition that the total width due to roughness, ΔH, is less than the average separation H, so that the plates are not in contact.) As long as $L\gg H\gg\Delta H$, the interactions in Eq. (23) are arranged in order of increasing strength. The largest effect of randomness is to increase the Casimir attraction by an amount proportional to $(\Delta H/H)^2$. There is also another correction term, of the opposite sign, that decays as $1/H^{4-2\zeta_S}$, and in principle can be used to indirectly measure the roughness exponent ζ_S. In Eq. (22), if all lengths are measured in units of an atomic scale a_0 (e.g., the diameter of a surface atom), A_S becomes dimensionless. Using a reasonable set of parameters ($\zeta_S\simeq0.35$, $a_0\simeq5$ Å, $A_S\simeq1$ and $L\simeq300$ Å), we estimate that for surfaces of 1 mm size, and 100 Å apart, the forces generated by the three terms in Eq. (23) are 1.9×10^{-4}, 4.9×10^{-5}, and 3.7×10^{-6} N, respectively (using a reasonable lower cutoff of ~20 Å), which are measurable with the current force apparatus (Lamoreaux, 1997).

V. THE DYNAMIC CASIMIR EFFECT

A. Background

Although less well known than its static counterpart, the dynamical Casimir effect, describing the force and radiation from moving mirrors has also garnered much attention (Moore, 1970; Fulling and Davis, 1976; Calucci, 1992; Jaekel and Reynaud, 1992; Maia Neto and Reynaud, 1993; Law, 1994; Dodonov, 1995; Lambrecht *et al.*, 1996; Meplan and Gignoux, 1996). This is partly due to connections to Hawking and Unruh effects (radiation from black holes and accelerating bodies, respectively), suggesting a deeper link between quantum mechanics, relativity, and cosmology (Weinberg, 1989; Davis, 1996).

The creation of photons by moving mirrors was first obtained (Moore, 1970) for a one-dimensional cavity. It was then demonstrated (Fulling and Davis, 1976) that there is a corresponding force even for a single mirror, which depends on the third time derivative of its displacement. Computations in $(1+1)$-dimensional spacetime take advantage of its conformal symmetries, and cannot be easily generalized to higher dimensions. Furthermore, the calculated force has causality problems reminiscent of radiation reaction forces in classical electron theory (Jaekel and Reynaud, 1992). This is an artifact of the unphysical assumption of perfect reflectivity of the mirror, and is resolved by considering realistic frequency-dependent reflection and transmission from the mirrors (Jaekel and Reynaud, 1992).

Another approach to the problem starts with the fluctuations in the force on a single plate. The fluctuation-

dissipation theorem is then used to obtain the mechanical response function (Maia Neto and Reynaud, 1993), whose imaginary part is related to the dissipation. This method does not have any causality problems, and can also be extended to higher dimensions. The force in $(1+3)$-dimensional space-time depends on the fifth power of the motional frequency. The emission of photons by a perfect cavity, and the observability of this energy, has been studied by different approaches (Calucci, 1992; Law, 1994; Dodonov, 1995; Lambrecht *et al.*, 1996; Meplan and Gignoux, 1996). The most promising candidate is the resonant production of photons when the mirrors vibrate at the optical resonance frequency of the cavity (Davis, 1996).[8] More recently, the radiation due to vacuum fluctuations of a collapsing bubble has been proposed (Schwinger, 1992, 1993, 1994; Eberlein, 1996a, 1996b; Knight, 1996) as a possible explanation for the intriguing phenomenon of sonoluminescence. Subsequent experimental measurements of the duration of the signal (Gompf *et al.*, 1997; Hiller *et al.*, 1998) favor more classical explanations.

A number of authors have further discussed the notion of frictional forces: Using conformal methods in $1+1$ dimensions, a friction term is found (Dodonov *et al.*, 1989) as

$$F_{\text{friction}}(H)=\alpha F_{\text{static}}(H)(\dot{H}/c)^2, \tag{24}$$

for slowly moving boundaries, where α is a numerical constant that only depends on dimensionality. The additional factor of $(v/c)^2$ makes detection of such a force yet more delicate. There are a few attempts to calculate forces (in higher dimensions) for walls that move *laterally*, i.e., parallel to each other. Dielectrics moving laterally seem to experience radiation-reaction-type frictional forces that vanish for the limiting case of perfect mirrors (Barton, 1996). On the other hand, it is found (Levitov, 1989; Mkrtchian, 1995; Pendry, 1997; Eberlein, 1998) that boundaries that are not ideal conductors experience a friction as if the plates are moving in a viscous fluid.[9] This friction has a complicated dependence on the frequency-dependent resistivity of the plates, and vanishes for ideal (nondispersive) conductors. The dominant "dissipation" mechanism for this "friction" is inducing eddy currents in the nonideal conductors, and thus making it distinct from the Casimir effect, although still belonging to the general class of fluctuation-induced forces.

Possible experimental evidence of an electromagnetic contribution to friction has been recently reported (Dayo *et al.*, 1998). The experiment employs a quartz crystal microbalance technique to measure the friction associated with the sliding of solid nitrogen along a lead surface, above and below the superconducting transition temperature of lead. An abrupt drop in friction is reported at the transition point as the substrate enters the superconducting state (Dayo *et al.*, 1998). This signals the presence of an electronic contribution to friction in addition to other mechanisms: The relative motion of the two solids induces currents in the bulk that are dissipated in the normal metal phase. The dissipation disappears in the superconducting phase.

An interesting analog of the dynamic Casimir effect is suggested for the moving interface between two different phases of superfluid ^{3}He (Volovik, 1996). In this system, the Andreev reflection of the massless "relativistic" fermions which live on the A-phase[10] of the interface provides the corresponding mechanism for friction: The interface is analogous to a perfectly reflecting wall moving in the quantum vacuum.

B. Path integral formulation

The path integral methods originally developed for rough surfaces (Li and Kardar, 1991, 1992) can also be applied to the problem of perfectly reflecting mirrors that undergo arbitrary dynamic deformations (Golestanian and Kardar, 1997, 1998a). Consider the path integral quantization of a scalar field ϕ with the action

$$S=\frac{1}{2}\int d^4X\,\partial_\mu\phi(X)\partial_\mu\phi(X), \tag{25}$$

where summation over $\mu=0,\ldots,3$ is implicit. Following a Wick rotation, imaginary time appears as another coordinate, $X_4=ict$, in the four-dimensional space-time. In principle, we should use the electromagnetic vector potential $A_\mu(X)$, but requirements of gauge fixing complicate the calculations, while the final results only change by a numerical prefactor. (We have explicitly reproduced the known result for gauge fields between flat plates by this method, see Golestanian and Kardar, 1998a.) We would like to quantize the field subject to the constraints of its vanishing on a set of n manifolds (objects) defined by $X=X_\alpha(y_\alpha)$, where y_α parametrize the αth manifold. We implement the constraints using delta functions, and write the partition function as

$$\mathcal{Z}=\int \mathcal{D}\phi(X)\prod_{\alpha=1}^{n}\prod_{y_\alpha}\delta(\phi(X_\alpha(y_\alpha)))\exp\left\{-\frac{S[\phi]}{\hbar}\right\}. \tag{26}$$

The delta functions are next represented by integrals over Lagrange multiplier fields. Performing the Gaussian integrations over $\phi(X)$ then leads to an effective action for the Lagrange multipliers which is again Gaussian (Li and Kardar, 1991, 1992). Evaluating $\mathcal{Z}$ is thus reduced to calculating the logarithm of the determinant of a kernel. Since the Lagrange multipliers are defined on a set of manifolds with nontrivial geometry, this calculation is generally complicated. To be specific,

[8]For a review, and more extensive references see Barton and Eberlein (1993).

[9]For an interesting demonstration of an intricate crosstalk between quantum and thermal fluctuations see Polevoi (1990).

[10]There are two different phases (vacua) in superfluid ^{3}He. In the A-phase the relevant excitations (quasiparticles) are chiral and massless, while in the B-phase they are massive (Volovik, 1992).

TABLE I. Behavior of the kernels in different regions denoted in Fig. 2.

	Region I	Region IIa	Region IIb
$A_+^\infty(q,\omega)$	real, finite	imaginary, finite	imaginary, finite
$A_+^H(q,\omega)$	real, finite	real, finite	complex, infinite
$A_-^\infty(q,\omega)$	0	0	0
$A_-^H(q,\omega)$	real, finite	real, finite	complex, infinite

we focus on two parallel $2D$ plates embedded in $3+1$ space-time, and separated by an average distance H along the x_3 direction. Deformations of the plates are parametrized by the height functions $h_1(\mathbf{x},t)$ and $h_2(\mathbf{x},t)$, where $\mathbf{x}\equiv(x_1,x_2)$ denotes the two lateral space coordinates, while t is the time variable. We can then calculate $\ln \mathcal{Z}$ in a perturbative series in powers of the height functions[11] (Li and Kardar, 1991, 1992; Golestanian and Kardar, 1997, 1998a). The resulting expression for the effective action (after rotating back to real time), defined by $S_{\rm eff}\equiv -i\hbar \ln \mathcal{Z}$, and eliminating h-independent terms, is

$$S_{\rm eff}=\frac{\hbar c}{2}\int \frac{d\omega d^2\mathbf{q}}{(2\pi)^3}\{A_+(q,\omega)[|h_1(\mathbf{q},\omega)|^2+|h_2(\mathbf{q},\omega)|^2] - A_-(q,\omega)[h_1(\mathbf{q},\omega)h_2(-\mathbf{q},-\omega) + h_1(-\mathbf{q},-\omega)h_2(\mathbf{q},\omega)]\}+O(h^3). \quad (27)$$

The kernels $A_\pm(q,\omega)$ are closely related to the mechanical response of the system, defined as the ratio between force and displacement in frequency domain (Golestanian and Kardar, 1997, 1998a). They are functions of the separation H, but depend on $\mathbf{q}$ and ω only through the combination $Q^2=q^2-\omega^2/c^2$. The kernels can be decomposed into H-independent (obtained by letting $H\to\infty$) and H-dependent parts: $A_\pm(q,\omega)=A_\pm^\infty(q,\omega)+A_\pm^H(q,\omega)$, whose behaviors are tabulated in Table I. Note that an imaginary response function signifies dissipation of energy (Maia Neto and Reynaud, 1993), presumably by generation of photons (Lambrecht *et al.*, 1996).

VI. CORRUGATED MIRRORS

As a concrete example, let us examine the lateral vibration of plates with fixed roughness, such as two corrugated mirrors. We consider corrugated plates with deformations $h_1(\mathbf{x})=h_2(\mathbf{x})=d\cos(\mathbf{q}\cdot\mathbf{x})$ that are moving laterally with respect to each other. From the frequency–wave-vector dependence of the mechanical response we extract a plethora of interesting results, some of which we discuss next (Golestanian and Kardar, 1997, 1998a).

[11]For two plates separated by H, the perturbation series is in the small parameters h_1/H and h_2/H. For a single plate ($H\to\infty$), the requirement is that the time and space derivatives of the height functions should be small.

A. Mass corrections: Region I

In the limit of $\omega\ll cq$, the response function can be expanded as a series in powers of ω^2. The coefficient of the ω^2 term can be regarded as a correction to the mass of the plate. For a single plate ($H\to\infty$), we find mass corrections that are *anisotropic*: $\delta m_\parallel=(1/288\pi^2)(\hbar/c)Ad^2q^5$, and $\delta m_\perp=0$, where parallel and perpendicular components are defined with respect to $\mathbf{q}$, and A denotes the area of the plates. The mass correction is inherently very small: For a macroscopic sample with $d\simeq\lambda=2\pi/q\simeq 1$ mm, density $\simeq 15$ gr/cm^3, and thickness $t\simeq 1$ mm, we find $\delta m/m\sim 10^{-34}$. Even for deformations of a microscopic sample of atomic dimensions (close to the limits of the applicability of our continuum representations of the boundaries), $\delta m/m$ can only be reduced to around 10^{-10}.

With the second plate at a separation H, the mass renormalization becomes a function of both q and H, with a crossover from the single plate behavior for $qH\sim 1$. In the limit of $qH\ll 1$, we obtain $\delta m_\parallel=\hbar AB q^2d^2/48cH^3$ and $\delta m_\perp=0$, with $B=-0.453$. Compared to the single plate, there is an enhancement by a factor of $(qH)^{-3}$ in $\delta m_\parallel$. While the actual changes in mass are immeasurably small, the hope is that its *anisotropy* may be more accessible, say by comparing oscillation frequencies of a plate in two orthogonal directions.

B. Dissipation: Region IIa

For $\omega\gg cq$ the response function has an imaginary part, and we can define a frequency-dependent effective shear viscosity. This viscosity is also anisotropic, with $\eta_\parallel(\omega)=(1/720\pi^2)(\hbar/c^4)Ad^2q^2\omega^4$, and $\eta_\perp(\omega)=0$. Note that the dissipation is proportional to the fifth time derivative of displacement, and there is no dissipation for a uniformly accelerating plate. However, a freely oscillating plate will undergo a damping of its motion. The characteristic decay time for a plate of mass M is $\tau\simeq 2M/\eta$. For the macroscopic plate of the previous paragraph, vibrating at a frequency of $\omega\simeq 2cq$ (in the 10^{12} Hz range), the decay time is enormous, $\tau\sim 10^{18}$ s. As the decay time scales as the fifth power of the dimension, it can be reduced to 10^{-12} s for plates of order of 10 atoms. However, the required frequencies in this case (in the 10^{18} Hz range) are very large. Also note that for the linearized forms to remain valid in this high frequency regime, we must require very small amplitudes r_0, so that the typical velocities involved, $v\sim r_0\omega$, are smaller than the speed of light. The effective dissipation in region IIa of Fig. 2 is simply the sum of those due to individual plates, and contains no H dependence.

C. Resonant emission: Region IIb

The cavity formed between the two plates supports a continuous spectrum of normal modes for frequencies $\omega^2>c^2(q^2+\pi^2/H^2)$. We find that both real and imagi-

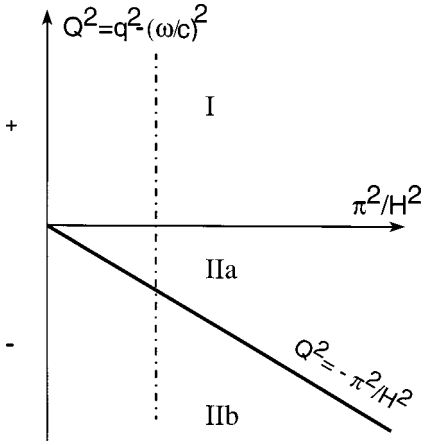

FIG. 2. Different regions of the frequency–wave vector and separation plane. The dependence on frequency and wave vector always appears through the relativistically invariant combination of $Q^2=q^2-\omega^2/c^2$. The dash-dotted line indicates how the three different regions are crossed upon increasing Q^2 for a particular value of the separation H.

nary parts of $A_{\pm}(q,\omega)$ diverge in this regime, which we interpret as resonant dissipation due to excitation of photons in the cavity.[12]

Resonant dissipation has profound consequences for motion of plates. It implies that, due to quantum fluctuations of vacuum, components of motion with frequencies in the range of divergences cannot be generated by any finite external force. The imaginary parts of the kernels are proportional to the total number of excited photons (Lambrecht *et al.*, 1996). Excitation of these degrees of motion must be accompanied by the generation of an infinite number of photons, requiring an infinite amount of energy, and is thus impossible. However, as pointed out in Lambrecht *et al.* (1996), the divergence is rounded off by assuming finite reflectivity and transmissivity for the mirrors. Hence, in practice, the restriction is softened and controlled by the degree of ideality of the mirrors in the frequency region of interest.

As opposed to the examples considered previously, it thus seems possible to create a considerable amount of photons in resonant conditions. However, to make quantitative predictions about the number of emitted photons, we should consider more realistic models than perfectly reflecting mirrors, as described above. We also note that the spectrum of resonant modes is continuous in $1+3$ dimensions, while it is discrete in $1+1$ dimensions (Calucci, 1992; Law, 1994; Dodonov, 1995; Meplan and Gignoux, 1996; Lambrecht *et al.*, 1996; Davis, 1996; Golestanian and Kardar, 1997, 1998a).

[12]The divergence of kernels in IIb comes from integrations over space-time. Given a cutoff L in plate size, and an associated cutoff L/c in time, the kernels diverge as $\exp[(K-2)L/H]/[K(L/H)^3]$, with $K=2QH/\pi$. Some care is necessary in the order of limits for $(L,H)\to\infty$.

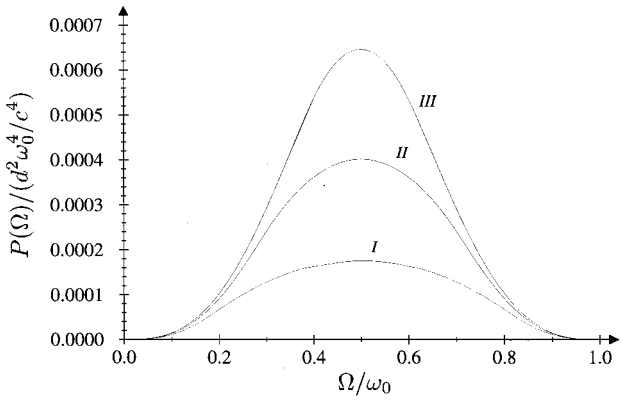

FIG. 3. Spectrum of radiation for different classes. Plot *I* corresponds to $\omega_0/ck_0=5/3$, plot *II* corresponds to $\omega_0/ck_0=5/2$, and plot *III* corresponds to $\omega_0/ck_0=5$.

D. Radiation spectra

Where does the energy go when the plates experience viscous dissipation? When the viscosity is a result of losses in the dispersive boundaries (Levitov, 1989; Mkrtchian, 1995; Pendry, 1997; Eberlein, 1998), the energy is used up in heating the plates. Since we have examined perfect mirrors, the dissipated energy can only be accounted for by the emission of photons into the cavity. The path integral methods can be further exploited to calculate the spectrum of the emitted radiation (Miri and Golestanian, 1999). The basic idea is to relate the transition amplitude from an empty vacuum (at $t\to-\infty$) to a state with two photons (at $t\to+\infty$), to a two-point correlation function of the field, which is then calculated perturbatively in the deformations. From the transition amplitude (after integrating over the states of one photon) we obtain the probability that an emitted photon is observed at a frequency Ω and with a particular orientation.

Specifically, calculations of the angular distribution and spectrum of radiation were performed (Miri and Golestanian, 1999) for a single perfectly reflecting plate, which undergoes harmonic deformations characterized by a height function $h(\mathbf{x},t)=d\cos(\mathbf{k}_0\cdot\mathbf{x}-\omega_0 t)$. Depending on the ratio ω_0/ck_0, it is found that radiation at a frequency Ω is restricted to a particular window in solid angle. The total spectrum of radiation $P(\Omega)$, i.e., the number of photons with frequencies between Ω and $\Omega+d\Omega$ radiated per unit time per unit area of the plate, is found by integrating the angular distribution over the unit sphere, and is shown in Fig. 3. Note that it is a symmetric function with respect to $\omega_0/2$, where it is peaked, that is a characteristic behavior of two-photon processes. The peak sharpens as the parameter ω_0/ck_0 increases, and saturates for $k_0=0$ (Maia Neto and Machado, 1996; Miri and Golestanian, 1999). The interesting problem of the angular distribution and spectrum of radiation between two plates in discussed by Mundarain and Maia Neto (1998).

The connection between the dissipative dynamic Casimir force and radiation of photons is made explicit by

calculating the total number of photons radiated per unit time and per unit area of the plate. The result is identical to the energy dissipation rate calculated from mechanical response considerations (Golestanian and Kardar, 1997, 1998a). No radiation is observed at frequencies higher than ω_0, due to conservation of energy, and also for $\omega_0/ck_0<1$, in agreement with Sec. VI.B above, where no dissipative forces are found in this regime.

VII. CONCLUSION

In this article we presented various examples of fluctuation-induced phenomena, taken from diverse contexts ranging from biophysics to cavity QED. We hope to have illustrated some of the underlying principles that govern these different manifestations. The basic idea is that if fluctuations of a field (whether of quantum or thermal origin) are hampered by the presence of external objects, there is a back reaction felt by these external objects. Due to their entropic nature, the resulting interactions usually depend on geometrical properties of the objects, with scales set by $\hbar$ (quantum fluctuations) or k_BT (thermal fluctuations). There is a great deal of universality, in that the asymptotic limits of the interactions at large distances depend only on the shape of the object, and are independent of microscopic details or energy scales. The dependence on shape, however, can be unexpected and nontrivial; for example, the interaction between extended objects cannot be obtained from a pairwise summation of pair potentials.

Thermal fluctuation–induced interactions are starting to be probed more systematically by experiments on wetting of complex fluids and on colloidal particles. Such interactions may also account for attractions between macroions of like charge, in turn explaining bundle formation and collapse of DNA filaments and other stiff polyelectrolytes. In fact, the very existence of a fluid phase (condensing from a gas) is due to the first historically recognized fluctuation-induced interactions, the van der Waals force.

The nonintuitive world of frictional Casimir forces and radiation from a perturbed vacuum brings about a deeper understanding of a fundamental physical entity, the quantum vacuum. As with any complex fluid, the quantum vacuum interacts with objects moving through it, hindering and modifying their motion. While the dynamic Casimir effect appears more as an academic problem at the moment, it has nevertheless raised the hope among some quantum opticians of making an optical resonator from a mechanical one, i.e., a laser with mechanical pumping!

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APPENDIX: PATH INTEGRAL FORMULATION OF CHARGED FLUIDS

Here, we introduce a systematic path integral formulation to study fluctuation-induced interactions in a charged fluid. Consider n charged manifolds embedded in a d-dimensional aqueous solution of neutralizing counterions, interacting through Coulomb potentials. The manifolds have charge densities $-\sigma_\alpha$ (all assumed to be negatively charged for simplicity), and are described by the functions $R_\alpha(x_\alpha)$, where x_α is a D_α-dimensional internal coordinate, while R_α indicates a position in the d-dimensional solution. There are N_c positively charged counterions of valence z, each described by a position vector R_i, in the d-dimensional solution. The Coulomb Hamiltonian can be written as

$$H_C=\frac{1}{2}\int d^dX d^dX'\,\rho(X)\frac{e^2}{\epsilon|X-X'|^{d-2}}\rho(X'), \quad \text{(A1)}$$

where

$$\rho(X)=-\sum_{\alpha=1}^{n}\int dx_\alpha \sigma_\alpha \delta^d(X-R_\alpha(x_\alpha)) + \sum_{i=1}^{N_c} z\,\delta^d(X-R_i), \quad \text{(A2)}$$

is the number density of the charges. Charge neutrality requires $-\Sigma_{\alpha=1}^{n}\sigma_\alpha A_\alpha+zN_c=0$, where A_α is the D_α-dimensional area of the αth manifold.

A restricted partition function of the Coulomb system, depending upon the shapes and locations of the macroions, is now given by

$$\mathcal{Z}_{N_c}[R_\alpha(x_\alpha)]=\int\prod_{i=1}^{N_c}\frac{d^dR_i}{a^d}e^{-H_C/k_BT}, \quad \text{(A3)}$$

in which a is a short-distance cutoff. Using the Hubbard-Stratanovich transformation of the Coulomb interaction,

$$e^{-H_C/k_BT}=\int\mathcal{D}\phi(X)\exp\left\{-\frac{\epsilon k_BT}{2S_de^2}\int d^dX(\nabla\phi)^2 + i\int d^dX\rho(X)\phi(X)\right\}, \quad \text{(A4)}$$

we can rewrite the partition function as

$$\mathcal{Z}_{N_c}[R_\alpha(x_\alpha)]=\int\mathcal{D}\phi(X)\exp\left\{-\frac{\epsilon k_BT}{2S_de^2}\int d^dX(\nabla\phi)^2 - i\sum_{\alpha=1}^{n}\int dx_\alpha\sigma_\alpha\phi(R_\alpha(x_\alpha))\right\} \times\left(\int\frac{d^dR}{a^d}e^{iz\phi(R)}\right)^{N_c}, \quad \text{(A5)}$$

where S_d is the area of the d-dimensional unit sphere. We can introduce a fugacity y, and a rescaled partition function

$$\mathcal{Z}[R_\alpha(x_\alpha)]=\frac{y^{N_c}}{N_c!}\mathcal{Z}_{N_c}[R_\alpha(x_\alpha)], \tag{A6}$$

which can be rewritten as

$$\begin{aligned}\mathcal{Z}&=\sum_{N=0}^{\infty}\delta_{N,N_c}\frac{y^N}{N!}\mathcal{Z}_N[R_\alpha(x_\alpha)]\\&=\sum_{N=0}^{\infty}\int_0^{2\pi}\frac{d\theta}{2\pi}e^{i\theta(N_c-N)}\int\mathcal{D}\phi(X)\\&\quad\times\exp\left\{-\frac{\epsilon k_BT}{2S_de^2}\int d^dX(\nabla\phi)^2\right.\\&\quad\left.-i\sum_{\alpha=1}^{n}\int dx_\alpha\,\sigma_\alpha\phi(R_\alpha(x_\alpha))\right\}\\&\quad\times\frac{1}{N!}\left(y\int\frac{d^dR}{a^d}e^{iz\phi(R)}\right)^N.\end{aligned} \tag{A7}$$

A shift in the field ϕ by $-\theta$, and use of the neutrality condition renders the θ-integration trivial. We can then sum up the exponential series, and obtain

$$\mathcal{Z}[R_\alpha(x_\alpha)]=\int\mathcal{D}\phi(X)\,e^{-\mathcal{H}[\phi]}, \tag{A8}$$

in which

$$\begin{aligned}\mathcal{H}[\phi]&=\frac{\epsilon k_BT}{2S_de^2}\int d^dX(\nabla\phi)^2\\&\quad+i\sum_{\alpha=1}^{n}\int dx_\alpha\,\sigma_\alpha\phi(R_\alpha(x_\alpha))\\&\quad-\frac{y}{a^d}\int d^dXe^{iz\phi(X)}.\end{aligned} \tag{A9}$$

Note that the fugacity y can be eliminated using the identity $N_c=\partial\ln\mathcal{Z}/\partial\ln y$, which follows from Eq. (A6).

We next evaluate the path integral using a saddle-point approximation. The extremum of Eq. (A8), obtained from $\delta\mathcal{H}/\delta\phi=0$, is the solution of the Poisson-Boltzmann (PB) equation

$$\begin{aligned}&-\nabla^2(z\psi(X))-\kappa^2e^{-z\psi(X)}\\&\quad=-\sum_{\alpha=1}^{n}\int dx_\alpha\frac{S_de^2z\sigma_\alpha}{\epsilon k_BT}\delta^d(X-R_\alpha(x_\alpha)),\end{aligned} \tag{A10}$$

for the (real) field $\psi(X)=-i\bar{\phi}(X)$, in which $\kappa^2=S_de^2yz^2/\epsilon k_BTa^d$ defines the inverse square of the Debye screening length. To study the fluctuations on top of this saddle point, we can set $\phi=\bar{\phi}+\delta\phi$, and expand the Hamiltonian up to quadratic order, to get $\mathcal{H}[\phi]=\mathcal{H}[\bar{\phi}]+(\epsilon k_BT/2S_de^2)\int d^dX[(\nabla\delta\phi)^2+\kappa^2e^{-z\psi(X)}\delta\phi^2]$. The free energy of the system of charged manifolds in the presence of fluctuating counterions now reads

$$F=F_{\rm PB}+\frac{k_BT}{2}\ln\det[-\nabla^2+m^2(X)], \tag{A11}$$

where $F_{\rm PB}=\mathcal{H}[i\psi(X)]$ is the Poisson-Boltzmann free energy, and $m^2(X)=\kappa^2e^{-z\psi(X)}$ is a "mass (or charge compressibility) profile." The PB free energy is known to be generically repulsive (Oosawa, 1968, 1971; Israelachvili, 1992). The fluctuation-induced correction, however, is attractive. For highly charged manifolds, it is indeed reminiscent of the Casimir interactions, but with the boundary constraints smoothed out. To see this, one should note that the mass profile is indeed identical to the density profile of the counterions. Highly charged manifolds accumulate counterions in their vicinity, and consequently the fluctuations of the "potential" field ϕ are suppressed in a region close to the manifolds, but are unconstrained in other regions in the solution, hence leading to a Casimir-type fluctuation-induced attraction.

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