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Gyromagnetic and Electron-Inertia Effects

S. J. BARNETT, *University of California at Los Angeles and California Institute of Technology*

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Part I. Gyromagnetic Effects[1]

Introduction

THIS paper has to do with two closely related classes of phenomenon: (1) Gross magnetic or dynamical phenomena which are due to the behavior of the elementary magnet as the rotor of a gyroscope and which are known as *gyromagnetic* or *magnetomechanical* phenomena; and (2) Gross mechanical or electrical phenomena which must be attributed to the inertia of the free electrons in conductors or bound electrons in insulators. Gyromagnetic phenomena will be treated in Part I of this article, the others in Part II.

A. Introductory, Historical and General

§1. The fundamental basis of the effects

Everyone who has predicted the possible discovery of any gyromagnetic effect has based the prediction on the assumption of the validity of the celebrated hypothesis of Ampère and Weber,[2] according to which the magnetic element in a magnetic substance consists of a permanent molecular or intramolecular whirl of electricity endowed with mass or inertia. On this hypothesis the magnetic element must have both angular momentum and magnetic moment, unless it is constituted of both positive and negative electricities rotating in opposite directions. In this case it is obvious that a definite magnetic moment might be accompanied by no angular momentum, or a definite angular momentum might be accompanied by no magnetic moment. Otherwise the magnetic element must behave both as a magnet and as the rotor of a gyroscope.

§2. A simple gyroscopic model

It will aid in the discussion of all the gyromagnetic phenomena hitherto looked for, of which there are four, if we consider at this point the behavior of the gyroscopic model illustrated in Fig. 2–1 and first used some years ago.[3] It differs from a common type of gyroscope only in the addition of two springs *SS*, conveniently in the form of rubber bands, and the arrangement for their attachment. The wheel, pivoted in a ring, can be rotated rapidly about its axis *A*. Except for the action of the springs, the ring and the axis *A* are free to move in altitude about a horizontal axis *B*, the axis *A* making an angle θ with the vertical *C*; while the axis *B*, together with the wheel and the framework supporting it and the springs, can be rotated about the vertical axis *C*. If the wheel is spun rapidly about the axis *A*, and the instrument then rotated about the vertical *C* slowly,

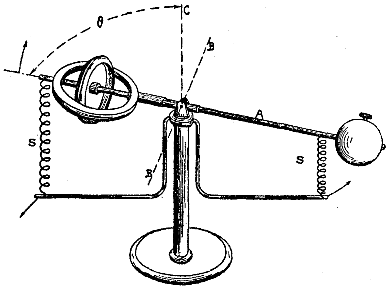

Fig. 2–1.

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[1] Earlier and less complete reviews of work on gyromagnetic phenomena have been given by the author in the Bull. Nat. Research Coun. **3**, 235, August (1922). (Translation by J. Würschmidt, after revision by the author, in Die Wissenschaft **74**, 270 (1925)); in Physica **13**, 241 (1933); and in the Physik. Zeits. **35**, 203 (1934). The last paper goes fully into the matter of priority, about which very numerous errors have been made in the literature ever since 1915.

[2] For Weber's ultimate ideas, which greatly resemble those in vogue in recent years, see Abhandlungen d. K. Sächs. Ges. d. Wiss. **10**, 1871, §17; or W. Weber's *Werke* (Berlin, J. Springer) **4**, p. 281.

[3] See S. J. Barnett, Science **48**, 303 (1918).

so that the centrifugal torque is negligible, the wheel tips up or down so as to make the direction of its rotation coincide more nearly with the direction of the impressed rotation about C, or so as to diminish the angle θ. The greater the rotary speed about C the greater is the tip of the wheel; it would tip until the axes A and C became coincident if it were not for the springs and the mechanical obstructions due to the form of the instrument (centrifugal torque being still supposed negligible).

§3. The four gyromagnetic effects hitherto looked for

Only qualitative discussions of these effects will be given in this section, together with references to the more detailed treatment presented below.

(1) *The gross magnet as a gyrostat* (*Maxwell's experiment, about 1861*) (see §7). If all the magnetic elements in a mass of iron (or other magnetic body) are alike, and each has angular momentum, then the whole mass, when magnetized in any direction, must have (concealed) angular momentum about this direction, and thus when rotated about an axis in any other direction must act like the gyroscope of §2, the mass striving, as it were, to change its orientation in such a way as to make the (concealed) momentum coincide in direction with the direction of the impressed rotation. This experiment, apparently the first gyromagnetic experiment to be suggested or tried, was made by Clerk Maxwell, with a negative result, in or before 1861.[4]

(2) *Magnetization by rotation* (*Barnett effect, 1914*) (see §§8–23). It does not seem to have occurred to Maxwell to make an experiment in which every one of the countless multitude of magnetic elements in a magnetic body should simultaneously replace his magnet, and to measure the gross change in the orientation of all these elements by a magnetic method.

The first experiment based upon this idea appears to be one made more than forty years ago by John Perry,[5] who tried, but without success, to detect magnetization in an iron rod produced by rotating it. In 1909, the same idea occurred to the author of this paper,[6] who, with the help of L. J. H. Barnett, then began experiments[7] which were first successful in 1914, when they were published by presentation to the *Ohio Academy of Sciences* and the *American Physical Society*.[8] These were the *first successful experiments in the whole field of gyromagnetic phenomena, as well as the first published* (by presentation to scientific bodies), and *have been fully confirmed both qualitatively and quantitatively* by much later work on both this effect and its converse (see below).

The qualitative classical theory of these experiments is as follows: When the body of which it is a part is set into rotation about any axis the magnetic element, if it has angular momentum, will behave like the wheel of our gyroscope in §2, and will thus change its orientation so as to make its direction of rotation coincide more nearly with the direction of impressed rotation; the coincidence would finally become exact if it were not for the action of the rest of the body.

In an ordinary ferromagnetic body in the usual state with which we are familiar only a slight change of orientation can occur on account of the torques due to adjacent elements, which perform the function of the springs in our gyroscopic model. The rotation causes each element to contribute a minute angular momentum, and thus also a minute magnetic moment, parallel to the axis of impressed rotation; and thus the body, whose magnetic elements originally pointed in all directions equally, becomes magnetized along the axis of impressed rotation.

If the whirling electricity of the magnetic elements is all positive, the body will thus become magnetized in the direction in which it

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[4] Maxwell, *Electricity and Magnetism*, §575.

[5] John Perry, *Spinning Tops*, 1890, p. 112, footnote. In 1916 I learned from Professor Henry Crew that the late Professor H. A. Rowland was desirous of making experiments on this subject, and said that if he possessed the Koh-i-noor diamond he would cut it into halves and make two bearings in which he would rotate a rod of iron. In Nature **112**, 186 (1923) Lodge reports that he had made experiments of this kind; and Schuster appears to have had an investigation of this sort in progress in 1912 (A. Schuster, Proc. Phys. Soc. **24**, 121 (1911–12)).

[6] Perhaps as a result of subconscious mentation on Perry's footnote, which I had very probably read twelve years before, but which, in that case, I had long forgotten and did not rediscover until 1918, when I called attention to it in print.

[7] S. J. Barnett, Science **30**, 413 (1909).

[8] S. J. Barnett, Phys. Rev. **6**, 239 (1915); also Phys. Rev. **6**, 171 (1915); and Science **42**, 163, 459 (1915).

would be magnetized by an electric current flowing around it in the direction of the angular velocity imparted to it. If it is all negative, or if the action on the negative elements is preponderant, the body will be magnetized in the opposite direction. This latter is what actually happens.

(3) *Rotation by magnetization*[9] (*Einstein and de Haas effect, 1915–1916*). If, as suggested by Maxwell, a rod of iron magnetized along its axis has (concealed) angular momentum about this axis (the resultant of the momenta of its aligned magnetic elements) it follows that an alteration of the magnetization must be accompanied by an alteration in its concealed momentum, and must thus, in view of the third law of motion, impart an angular momentum of the same magnitude in the opposite direction to the rod (and the system which magnetizes it). This idea was suggested by O. W. Richardson[10] in 1907. In that year he worked out the detailed theory and also tried experiments which, however, were unsuccessful. The first successful experiments on the approximate magnitude of this effect, and on its sign, were made in 1915 and 1916, respectively, by Einstein and de Haas,[11] who, until late in 1915, were not aware of the author's previous success with the converse experiment. They were also unacquainted with Richardson's work. See further §§24–44.

(4) *Gyroscopic magnetization by the rotation of a magnetic field. Experiments of Fisher*[12] (*1922, 1924*) *and of Barnett* (*1926, 1933*).[13] In these experiments a rod (or toroid) of magnetic substance, as free from residual magnetization as practicable, was crossed normally by a magnetic field. This field (and the magnetization which it produced) were rapidly rotated, and a change in the longitudinal magnetization of the substance was looked for. Such a change should not result unless the magnetic elements participate in the rotation of the vector which specifies the intensity of magnetization, and might even then be impossible to detect except with sensitivities not hitherto attained. Null effects were obtained in all the experiments. See §45.

§4. The gyromagnetic ratio

The most important quantities associated with the elementary magnet are its magnetic moment μ_0, its angular momentum M_0, and the ratio of the second to the first, which has come to be known as the *gyromagnetic*, or *magnetomechanical, ratio.* Thus the gyromagnetic ratio, which will be designated by ρ, is given by the equation

$$\rho = M_0/\mu_0. \qquad (4\text{–}1)$$

As we shall see, the quantity ρ has now been determined with considerable precision for a number of ferromagnetic and paramagnetic substances. It is the chief quantity involved in all experiments on gyromagnetic effects.

§5. The gyromagnetic ratios for several types of magnetic elements

(1) *Electron orbit* (W. Weber, Rutherford, Bohr). Suppose the element to consist of an electron, with mass m_0, and charge e, revolving in a circular orbit of radius r with constant angular velocity ω radians per second (and areal velocity $\alpha = \frac{1}{2}\omega r^2$) about a much more massive, and fixed, nucleus with charge $-e$. In this case, we have

$$\mu_0 = e\alpha, \quad M_0 = m_0\omega r^2 = 2m_0\alpha, \quad \text{and}$$

$$\rho (\equiv M_0/\mu_0) = 2m_0/e = \rho_0. \qquad (5\text{–}1)$$

If the orbit is elliptical instead of circular, it is easy to show that the same ratio holds for the mean angular momentum and the mean magnetic moment.

(2) *Charged solid in rotation.* Voigt[14] has ex-

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[9] It is a curious and interesting fact that the general idea of rotation by magnetization is as old as the *De Magnete* of William Gilbert, who, in Book VI of that celebrated treatise, attributed the earth's rotation at least chiefly to its magnetization. Gilbert gave no reason for his conviction, and apparently made no experiments upon the subject; and there has, of course, long been conclusive evidence that the angular momentum of the earth is far too great to have been produced in this way.

[10] O. W. Richardson, Phys. Rev. **26**, 248 (1908). The theory developed by Richardson was somewhat complex. The very simple theory of this effect, equally rigorous, which is given in this paper, was presented by the author to the American Physical Society in 1914 (reference 8); and independently, a few months later, by Einstein and de Haas to the German Physical Society (Reference 11.)

[11] A. Einstein and W. J. de Haas, Verh. d. D. Phys. Ges. **17**, 152 (1915); A. Einstein, Verh. d. D. Phys. Ges. **18**, 173 (1916); W. J. de Haas, Verh. d. D. Phys. Ges. **18**, 423 (1916).

[12] J. W. Fisher, Proc. Phys. Soc. **34**, 177 (1922); Proc Roy. Soc. **A109**, 7 (1925).

[13] See S. J. Barnett, Phys. Rev. **27**, 115 (1926); and Proc. Am. Acad. **68** (7), 229 (1933).

[14] W. Voigt, Ann. d. Physik **9**, 130 (1902).

amined the behavior in a magnetic field of magnetic elements consisting of homogeneous uniformly charged solids in rotation. No account is taken of the electromagnetic origin of the mass, but the mass density is taken as everywhere proportional to the electric density. For this type of element it is easy to show that, as in the case of an electron orbit,

$$\rho=2m_0/e=\rho_0. \qquad (5\text{–}1)\ \textit{bis}$$

(3) *Spinning electron.* M. Abraham[15] has considered the behavior in a magnetic field of a spherical electron in rotation and uniformly charged either over the surface or throughout the volume, and has calculated the angular momenta on the assumption that the masses and momenta are purely electromagnetic.

The masses, m_s and m_v, of the electron or magnetic element for surface and volume charge, respectively, are

$m_s=\frac{2}{3}e^2/a$ [16] and $m_v=\frac{4}{5}e^2/a$ (5–2)

if e denotes the charge of the electron and a its radius.

For the angular velocity ω radians per second the corresponding angular momenta are

$$M_s=\tfrac{1}{3}m_sa^2\omega \quad \text{and} \quad M_v=\tfrac{1}{7}m_va^2\omega \qquad (5\text{–}3)$$

and the corresponding magnetic moments are

$$\mu_s=\tfrac{1}{3}ea^2\omega \quad \text{and} \quad \mu_v=\tfrac{1}{5}ea^2\omega. \qquad (5\text{–}4)$$

Thus the corresponding values of the ratio $\rho=M/\mu$ are

$$\rho_s=\frac{m_s}{e} \quad \text{and} \quad \rho_v=\frac{5}{7}\frac{m_v}{e}=\frac{6}{5}\frac{m_s}{e}. \qquad (5\text{–}5)$$

The first[17] of these is just one-half the value for an electron orbit.

(4) *Ions and atoms with electron orbits and electron spins.* If Landé's "splitting factor" or "g-factor" is designated by g, the ion or atom as magnetic element has the gyromagnetic ratio $\rho=2(m/e)/g=\rho_0/g$. This is equivalent to

$$g=2(m/e)/\rho=\rho_0/\rho. \qquad (5\text{–}6)$$

That is, the splitting factor is equal to the reciprocal of our ρ expressed in terms of ρ_0, the gyromagnetic ratio for an electron orbit.

(5) *Complex elements.* Complex elements in which both nuclei and electrons participate have been considered by O. W. Richardson[18] and others.[19]

B. THE GROSS MAGNET AS A GYROSTAT. MAXWELL'S EXPERIMENT

§6. The relation between the magnetic moment and the internal angular momentum of a gross magnet

Assume the magnet symmetrically magnetized about the geometric axis, and the magnetic elements all alike. If θ denotes the angle between the axis of a magnetic element and the intensity of magnetization I in its neighborhood, we have

$$I=\sum\mu_0\cos\theta,$$

the summation extending over the unit of volume. The internal or concealed angular momentum j per unit volume will then be

$$j=\sum M_0\cos\theta=\rho\sum\mu_0\cos\theta=\rho I. \qquad (6\text{–}1)$$

Thus if I denotes the mean intensity of magnetization along the axis of the magnet, V its volume, and IV its magnetic moment, its total internal, or concealed, angular momentum will be $M=\rho IV$. Thus ρ can be determined from M and IV.

§7. Maxwell's experiment (see §3, (1))

We are now in a position to consider the quantitative theory of the first gyromagnetic experiment on record, *viz.*, that of Maxwell. This experiment was made with apparatus somewhat similar to that of Fig. 2–1, but with the springs, which will at first be assumed present, removed. The wheel and its supporting framework are replaced by a coil of wire traversed by an electric current, or by an electromagnet, at

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[15] M. Abraham, Ann. d. Physik **10**, 151, 169, 171 (1903).

[16] J. J. Thomson, Phil. Mag. **11**, 234 (1881), gave the first calculation of m_s, but obtained the factor 4/15 instead of the factor 2/3. See also his *Electricity and Matter*, 1912, p. 21, where the result quoted here is given.

[17] A very simple derivation of this result is given by E. H. Kennard, Phys. Rev. **19**, 420 (1922).

[18] O. W. Richardson, reference 10; also Proc. Roy. Soc. **A102**, 538 (1922); and Nature **117**, 652 (1926).

[19] See W. Braunbeck, Physik. Zeits. **23**, 307 (1922); K. Honda, Tohoku Sci. Rep. **19**, 745 (1930).

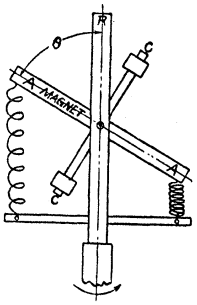

Fig. 7–1.

will, with axis along the axis A and the center of mass on the axis B (Figs. 2–1 and 7–1). This body, if it possesses (concealed) angular momentum, should behave like the gyroscope of §2. The centrifugal torque will not, however, in general be negligible, especially as great speeds about the vertical will be necessary to make the gyroscopic effects appreciable. This torque may be altered at will, or made to vanish, by adjusting suitable weights attached to the body along an axis CC intersecting A and B normally (Fig. 7–1) and centrally.

Let A, B, C, respectively, denote the moments of inertia of the body about the axis of the magnet, the horizontal axis B, and the axis CC normal to the two other principal axes. Let θ denote the angle between the axis A of the magnet and the vertical (R, Fig. 7–1, C, Fig. 2–1), Ω the impressed angular velocity about the vertical, J the total angular momentum of the body, M the concealed angular momentum, and β the angle between J and the axis A.

Let us suppose that under the action of the springs, producing a torque T in the direction of increase of θ, the angular velocity Ω and the angle θ are maintained constant. J can be resolved into two rectangular components: one parallel to the axis of the impressed rotation, $J\cos(\theta-\beta)$, which is constant; and one perpendicular to this axis, $J\sin(\theta-\beta)$, which has the constant rate of change $T=\Omega J\sin(\theta-\beta)$. Now $J\cos\beta = A\Omega\cos\theta+M$, and $J\sin\beta=C\Omega\sin\theta$. Hence

$$T=(A-C)\Omega^2\sin\theta\cos\theta+M\Omega\sin\theta. \qquad (7\text{–}1)$$

If C is somewhat greater than A the applied torque T necessary to maintain the motion constant will vanish and the motion (under the action of the centrifugal torque) will be stable with the springs removed when

$$\cos\theta=M/(C-A)\Omega. \qquad (7\text{–}2)$$

By means of two nuts movable symmetrically along the axis C on screws, the axis B was maintained a principal axis, while C was made just to exceed A, so that the instrument was very sensitive. On account of disturbances due to the earth's magnetic field the results were very rough, but no change in θ with reversal of M or Ω could be detected even when an iron core was inserted in the coil.

Maxwell concludes that if a magnet, or a coil of wire carrying an electric current, contains matter in concealed motion the angular momentum due to this motion must be very small in comparison with any quantity which we can measure.

By calculating M as the product of the constant ρ, determined by experiments on other gyromagnetic effects, and the magnetic moment of the magnet, and by taking account of the fact that Eq. (7–1) holds only if the horizontal axis about which rotation is possible passes exactly through the center of mass of the magnet, W. J. de Haas and G. L. de Haas-Lorentz[20] have shown that a change in θ would hardly be perceptible in the most favorable conditions possible even today.

By making $A-C$ very small and θ very nearly 90°, and measuring T and Ω, M could also be determined from Eq. (7–1).

C. MAGNETIZATION BY ROTATION (BARNETT EFFECT)

§8. Theory of magnetization by rotation[21] (see §3, (2))

The magnetic element will at first be assumed to consist of a symmetrical electrical system rotating with angular velocity ω, with magnetic moment μ_0, and with angular momentum M_0

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[20] W. J. de Haas and G. L. de Haas-Lorentz, Proc. K. Akad, Amsterdam **19**, 248 (1915).

[21] See S. J. Barnett, references 7, 8; Einstein and de Haas, reference 11; S. J. Barnett, Bull. Nat. Res. Council **3**, Part 2, 235 (1922).

$=\rho\mu_0$ about the axis of symmetry, the electrical charges in rotation being all of one sign.

The vectors representing M_0 and μ_0 are in the same direction or opposite directions according as the moving charge is positive or negative.

Let A denote the moment of inertia of the magnetic element about its axis of rotation, and suppose $B=C$ the (mean) moment of inertia about any central axis normal to the axis of symmetry.

If now the body of which this element is a part is set into rotation with angular velocity Ω about an axis C, the element, behaving[22] like the wheel of a gyroscope, will strive, as it were, to take up a position with its axis of revolution coincident with that of the impressed rotation, but it will be prevented from turning so far by a torque T due to the action of the rest of the body and brought into existence by the displacement. In a minute time a steady state will be reached, and the axis of the magnetic element will then continuously trace out a cone making a constant angle θ with a line through its center parallel to the axis C of the impressed rotation. When this state has been reached T will be given by Eq. (7–1) above, which may be written

$$T=[M_0\Omega+(A-C)\Omega^2\cos\theta]\sin\theta. \quad (8\text{–}1)$$

Now imagine the body, instead of being rotated, to be placed in a uniform magnetic field whose intensity H is directed along the previous axis of rotation, and consider the same magnetic element, whose magnetic axis, after displacement by the field, makes the angle θ with H. The element would keep on turning under the action of the field until its axis coincided with H, but is prevented from doing so by the torque T' upon it due to the action of the rest of the body and brought into existence by the displacement. This torque is well known to be

$$T'=\mu_0 H\sin\theta. \quad (8\text{–}2)$$

To find, therefore, the magnetic intensity which would produce the same effect on the orientation of the magnetic element as would be produced by rotating the body at the angular velocity Ω, all we have to do is to equate T and T'. This gives

$$\mu_0 H\sin\theta=[M_0\Omega+(A-C)\Omega^2\cos\theta]\sin\theta$$

or

$$H=(M_0\Omega/\mu_0)[1+(A-C)\Omega/A\omega\cos\theta]. \quad (8\text{–}3)$$

The values of Ω experimentally attainable are so small in comparison with any possible values of ω in the case of any magnetic element of probable type that the last term is negligible. Hence we have for any magnetic element in the body, whatever its orientation, with all sufficient exactness,

$$H=M_0\Omega/\mu_0=\rho\Omega=2\pi\rho\nu=\lambda\nu, \quad (8\text{–}4)$$

where ν is the impressed angular velocity in revolutions per second, and $\lambda\equiv 2\pi\rho$ is the quantity which in 1914 was named the *intrinsic magnetic intensity of rotation.*

From what precedes it follows that if all the magnetic elements in a body are alike, rotating it at an angular velocity of ν r.p.s. will produce the same intensity of magnetization in it as placing it in a field of strength $2\pi\rho\nu=\lambda\nu$ gauss.

It is clear that the magnetic element is undergoing regular precession under the action of the disturbance field. If we designate by $\delta H(=-H)$ the change in the magnetic intensity which acts on the element under the influence of the displacement produced by the rotation, we see at once that Eq. (8–4) becomes

$$\nu=H/2\pi\rho=-\delta H/2\pi\rho, \quad (8\text{–}5)$$

which gives the classical change of frequency in the Zeeman effect due to a field with intensity δH, provided $\rho=\rho_0$. The element is simply executing Larmor precession with the frequency ν of the rotating body under the action of the disturbance-field with intensity δH.

If the magnetic elements in a body are of two kinds, positive and negative, with constants ρ_1 and ρ_2, rotating the body will have the same effect as if a magnetic intensity $H_1=\rho_1\Omega$ were applied to the positive elements and an intensity $H_2=\rho_2\Omega$ were applied to the negative elements.

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[22] *i.e., on the classical theory.* If we assume with the *quantum theory,* that when the body is rotated, or when it is placed in a weak field, the magnetic element *jumps* from one orientation to the reverse, with angle θ, instead of being *turned* to the angle θ, the formulae are precisely the same. Hence experiments on magnetization by rotation can give us no information on the nature of the *process of magnetization,* but only on the *nature of the elements.* Indeed, Langevin has given a derivation of the formula which is independent of any model. See P. Weiss, Rapport Inst. internat. de Physique Solvay, 1930, p. 73.

If the effect on the negative elements is preponderant, the rotation will thus produce an intensity of magnetization in the direction of H_2 but of magnitude less than that which would be produced by the intensity $\rho_2\Omega$ if all the elements were negative.

In very weak fields magnetic bodies all receive magnetic moments proportional to the intensities of the fields applied. Similarly, since $\rho\Omega$ is equivalent to a very small value of H for even the greatest speeds practicable, these bodies must be magnetized by rotation proportionally to the speed.

If, however, in either case we start with a ferromagnetic substance not in or near the neutral state but at a steep portion of the magnetization curve, the application of either a small increment of magnetic intensity or of a small speed may be sufficient to produce a considerable and irreversible change in magnetization.

§9. Types of experiments on magnetization by rotation

Two types of experiment have been made, very different from one another. The first successful experiments were made in 1914 (and repeated in 1915) on large iron rods—nearly a meter long and 7 cm in diameter—by a method depending on the principles of electromagnetic induction;[23] the second, on much smaller rods of iron, cobalt, nickel and many alloys, by magnetometer methods.[24]

In both methods the rods were mounted with their axes horizontal and in the magnetic prime vertical in a region in which, in general, the earth's magnetic field had been neutralized.

One major object of neutralizing the earth's field is to prevent the generation of eddy currents in the rotating body. On account of symmetry, the effect of these currents vanishes, or nearly vanishes, in the method of electromagnetic induction used; and the earliest successful experiments, in the first half of 1914, were made in the earth's field. In the experiments made in the latter half of 1914 and in 1915, and in the later work, the earth's magnetic intensity was in general compensated.

§10. Experiments by the method of electromagnetic induction

In the method of electromagnetic induction the intrinsic magnetic intensity of rotation (equal to $2\pi\rho\nu$) was determined by comparing the change of flux through the rod under investigation, produced by rotation about its axis at measured speed, with the change of flux produced through the same rod by the application, parallel to the axis of rotation, of a uniform magnetic field of known intensity. The changes of flux are proportional to the intensities, if small. The changes of flux were measured ballistically, with a galvanometer of the type known as a fluxmeter, a coil of wire surrounding the rod being in the fluxmeter circuit. If D is the deflection produced by reversing the direction of rotation at the frequency ν, and D_0 the deflection produced by reversing the standardizing intensity H_0, we have

$$2\pi\rho\nu/H_0 = D/D_0, \text{ or } \rho = (D/D_0)H_0/2\pi\nu. \quad (10-1)$$

In the actual experiments *two similar* rods, parallel to one another, were used, and *two similar* coils of insulated copper wire were mounted symmetrically about their centers, as shown in Fig. 10–1. These coils were placed in series with one another and with the fluxmeter and were oppositely connected so that any fluctuations in the intensity of the earth's magnetic field, which act in the same way on both rods, might produce no effect on the fluxmeter. One of the rods, the

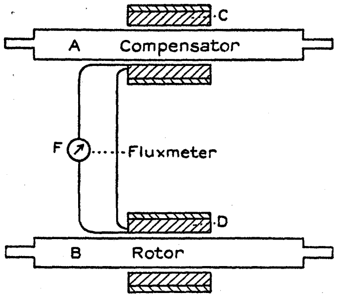

FIG. 10–1.

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[23] S. J. Barnett, Phys. Rev. 6, 239 (1915). Earlier reports in Phys. Rev. 6, 171 (1915); Science 42, 163, 459 (1915).

[24] S. J. Barnett, Phys. Rev. 10, 7 (1917); J. Wash. Acad. Sci. 11, 162 (1921). Also, S. J. Barnett and L. J. H. Barnett, Phys. Rev. 17, 404 (1921); Phys. Rev. 20, 90 (1922); and especially Proc. Am. Acad. 60, No. 2, 125 (1925).

compensator, A, remained at rest; while the other, the *rotor*, B, was alternately rotated in opposite directions, the change of its magnetization being determined as it came to rest. For use in the standardizing experiments the rods A and B were uniformly wound with coils of insulated copper wire; and two long wooden rods of the same diameter and similarly wound were provided for attachment to the ends of the rotor, in order to make reductions to a strictly uniform field. The earth's field, in the region occupied by the rotor, was, in general, neutralized in order to prevent eddy currents and changes in its axial flux due to alterations in the shape or position of the rod, including slight alterations of axial orientation caused by the rotation. In successive experiments the rod was rotated in opposite directions at the same speed in order to eliminate the effects of changes of magnetization due to centrifugal expansion and other distortions and to heating at the bearings. Experiments were made with the axis of the rotor turned in both directions, chiefly to eliminate the possible magnetic effects of the twist in the rods, which were driven from one end.

§11. Results of the experiments by the method of electromagnetic induction

The great symmetry of the apparatus, and the precautions taken, were such that no systematic errors remained which were of consequence in view of the accidental error of about 12 percent. The magnetization was found to be *proportional to the speed*, as indicated by the theory. With reference to the kind of electricity involved, the rotation was found always to magnetize the iron in the direction opposite to that in which it would be magnetized by a current flowing around it in the direction of the rotation, which proves, as we have seen above, that *the electricity in the whirls of Ampère is negative.*

The numerical results of 1914 give for the gyromagnetic ratio the value $=1.01\times m/e$. The somewhat better results obtained in 1915 by the same method, with improvements, give $0.95 \times m/e$. Within the limits of the experimental error, both results are only *one-half the value, viz.,* $2m/e$, *to be expected on the basis of the orbital theory.* The quantity m/e is here assumed to have the value 1.757×10^7 e.m.u.

The investigation gave a *direct proof* (and the *first proof*) of the *actual existence in iron of the molecular currents of Ampère*, before hypothetical; it proved that the electricity in these currents is *negative*, and has mass or inertia; and it revealed a *second and entirely new method of magnetizing bodies*.

The magnetization produced, however, by speeds experimentally attainable is exceedingly minute. Thus, as the insertion in the formula of the result given above would show, rotating a body at 100 revolutions a second is equivalent to putting it in a magnetic field which is only about one fifteen-thousandth as intense as the earth's field in Los Angeles.

§12. The gyromagnetic anomaly and the nature of the magnetic element

One of the most important results of the investigation is the magnitude obtained for the gyromagnetic ratio, which, as stated above, is about *one-half* the value calculated for an electron moving in an orbit. This discrepancy has become known as the *gyromagnetic*, or *magneto-mechanical, anomaly;* and the result indicates, with great probability, according to the data of §5, that the magnetic element in iron consists primarily of a *Lorentz electron spinning on a diameter*, and not of an electron moving in an orbit.[25] More precise results obtained by the author and L. J. H. Barnett later show that in general the orbit also participates to some extent, the gyromagnetic ratio for ferromagnetic substances being, in general, somewhat greater than m/e. See §§22, 23, 41 below.

§13. Experiments by magnetometer methods.[26] Arrangement of apparatus

Of the later experiments, i.e., those made by magnetometer methods, the first, chiefly on iron, cobalt, and nickel, were published in 1917; the most extensive and precise were the last series, made in 1923–24.

In the magnetometer methods an astatic magnetometer is mounted with the center of its

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[25] As suggested by O. W. Richardson, Inst. internat. de Phys. Solvay, 1921, p. 220; also, a little later, by E. H. Kennard (reference 17). A. L. Parson and others had suggested long before that the electron itself possesses a magnetic moment.

[26] See reference 24.

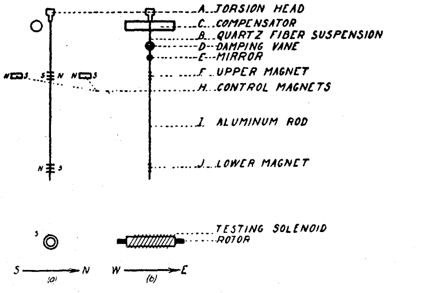

FIG. 13–1. Diagram of magnetometer, etc., as seen (a) from east; (b) from south.

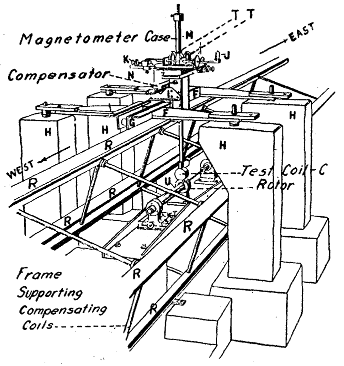

FIG. 13–2.

lower magnet system on the polar axis, or, preferably, in the equatorial plane, of the rod, or rotor, under investigation. The magnetometer deflection produced by reversing the direction of rotation of the rotor, driven at a measured speed, is compared with the deflection produced by reversing a known magnetic intensity in the rotor parallel to its axis. The deflections are proportional to the changes in the magnetic moments in the two cases, and these are proportional to the intrinsic intensity of rotation and the standardizing field intensity, Eq. (10–1) being valid here also.

It is more difficult to eliminate sources of error in a magnetometer method than in the method already described, but it has the great advantages that it can be made more sensitive and does not necessitate such large samples of the materials to be studied.

A greatly simplified diagram of the principal arrangement used in the investigation is given in Fig. 13–1. A light vertical rod of aluminum, *I*, carries two groups, *F* and *J*, of very small horizontal magnets constituting a carefully adjusted astatic system. The rod *I*, with its magnets, is hung on a fine thread *B* of fused quartz from a torsion head *A*. It carries a thin metal damping vane *D*, situated between two adjustable parallel plates not shown in the figure, and a small mirror *E*, by means of which and a remote lamp and scale the deflections are read in the usual way. To prevent disturbances from air currents, the whole suspended system is inclosed in a tight metal case provided with glass windows and heavily wrapped with cotton or other insulating material. The case contains a small amount of a radioactive salt to prevent electrification of the moving parts.

H indicates two small control magnets producing fields with intensities parallel and normal, respectively, to the axis of the upper magnets of the astatic system, by which the zero of the needle and the sensibility of the instrument can be controlled independently of one another.[27] Only one of these magnets, whose axes are perpendicular to one another, is shown in each part of the figure. Without such a system, devised for this and other exacting work, or an equivalent system in which electric coils replace magnets, the precise investigation would have been almost, if not quite, impossible.

The rod under test, or rotor, is mounted with its axis magnetic east and west and generally directly beneath the lower magnetometer magnet; and a nearly similar rod *C*, the compensator, is mounted nearly parallel to it and at almost the same distance above or below the upper and oppositely directed magnet, but, for convenience, a little south or north. By slight adjustments of this compensator, for which much better provision was made in the later part of the work than in the earlier, the effect on the rotor of the continual fluctuations in the earth's field, and

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[27] S. J. Barnett, *A Sine Galvanometer for Determining in Absolute Measure the Horizontal Intensity of the Earth's Magnetic Field,* Researches of the Dept. of Terrestial Magnetism of the Carnegie Institution of Washington **4**, 373 (see p. 388) (1921).

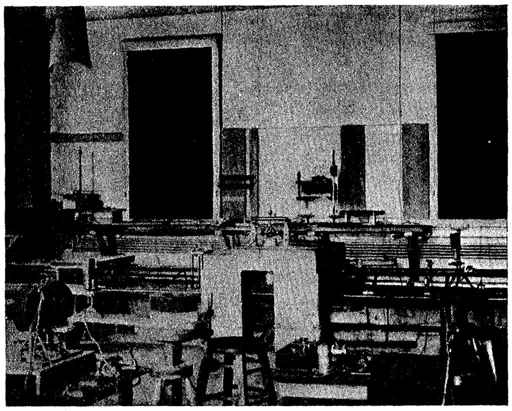

Fig. 13–3.

the effect of this on the magnetometer, can be very largely compensated.

These arrangements and adjustments are quite necessary to prevent large accidental errors in the measurements. Indeed, in spite of these precautions the most difficult part of the work had to be performed late at night (as was the case in the earlier experiments, by the method of electromagnetic induction) when the disturbing effect of the sun on the earth's magnetic field is much less than in the daytime; and even after two o'clock in the morning, when magnetic disturbances from the street car traffic were a minimum. (It was fortunate that just as the most precise work had to be done, one of the sunspot minima arrived.)

Small Helmholtz coils centered on the lower magnet in the early part of the work and on the upper magnet in the later part, and the solenoid *S*, slipped over the rotor when necessary, were used in the standardizing process.

A drawing of the principal parts of the apparatus, but not in its final state, is given in Fig. 13–2.

The framework *RR* carried the electric coils which neutralized the main part of the earth's magnetic field. Extra coils *TT*, centered on the upper magnet, adjustable in distance apart and in azimuth, and carrying the same current traversing the coils supported by *RR*, made the zero of the magnetometer and the sensitivity nearly independent of this current. The rotor in its bed-plate is shown at *U*.

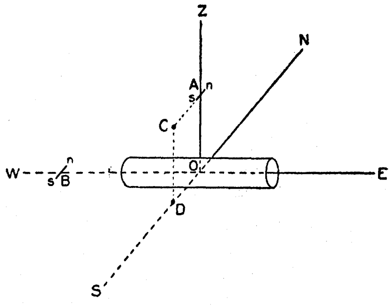

Fig. 13-4.

A photograph of this part of the apparatus in its final state is shown in Fig. 13–3, where the magnetometer supports are shown heavily imbedded in concrete, to prevent vibration, and where additional electric coils are shown whose function it was to compensate those parts of the magnetic field at the rotor which were not neutralized when the main coil shown in Fig. 13–2 was in operation alone.

In all the work the lower magnetometer magnet was placed either on the axis of the rotor (produced) (axial position), or in or near the equatorial plane passing through the center of the rotor (equatorial position). In the last work, the magnets were always in the vertical plane through the axis of the rotor, as indicated in Fig. 13–4, where the lower magnet is shown at B and A for the two positions.

All but the earliest magnetometer work was done in a small nonmagnetic building in which the earth's magnetic field was very nearly uniform.

§14. The processes involved in calibration. Slow rotation

The method of calibration in nearly all the work involved three processes, (1), (2) and (3), requiring the use of three coils as primary, secondary and tertiary standards, respectively.

In process (3) it is necessary for convenience and precision that the rotor be in uniform rotation, so that a mean effect is obtained. When the earth's magnetic field is just compensated this necessity arises chiefly from the fact that the field about the magnetometer magnets due to the residual magnetism of the rotor, in both direction and magnitude, is in general a function of the position angle. In addition, when the earth's field is not entirely compensated, there is a similar effect on the magnetometer due to the fact that the rotor's susceptibility is not in general symmetrical about its axis.

In processes (2) and (1), where only ratios of sensibilities are involved, slow rotation, when necessary, is required only on account of a rotor's lack of axial symmetry. Even with unsymmetrical rotors no appreciable error is made in process (1), and only minute errors in process (2), if the rotor is kept at rest.

Throughout nearly all of the work described here the rotor, in processes (3) and (2), was rotated slowly and uniformly, but fast enough to keep the magnetometer mirror steady. In some cases, for special reasons, the calibrations were made at the speeds of the principal experiments.

§15. The calibration

(A). *Process* (*3*). *The magnetometer or absolute sensitivity.* In process (3) the sensitivity of the magnetometer itself, almost independent of the particular rotor and compensator in use, was determined by reversing a minute known current I through the small *tertiary standard* coils C centered on the upper or lower magnet, and adjusting the sensitivity control magnet until the desired deflection c was obtained. This is the *magnetometer* or *absolute* sensitivity. In this process, as also in (2) and (1), observations were made on a regular time schedule, reversals being made at equal intervals.

(B) *Process* (*2*). In this process the *ratio* of the *rotor sensitivity* to the *magnetometer sensitivity* was obtained. The *secondary standard* solenoid B (S in Fig. 13–1) was slipped over the rotor and held in place coaxially by suitable supports, and the deflection b was found which would be produced by the reversal through this solenoid of the same current I as that used in process (3). This is the *rotor sensitivity*, b.

The ratio $Q=b/c$ is fixed for a given rotor with a fixed position and condition of the magnetometer. For precise work, especially when the coil C is on the upper magnet, it is necessary to have the field neutralized, as in the chief experiments, because Q depends on the ratio of the moments of the two magnetometer magnets, and this is different with and without compensation—by about one-third percent.

(C) *Process* (*1*). *Reduction to a uniform field.* If the calibrating solenoid B (secondary standard), which was only slightly longer than the rotors, were infinite in length, it would, for equatorial positions of the magnetometer, have *no effect* on the magnetometer and would produce a *uniform axial field* throughout the rotor, like that of the intrinsic intensity of rotation. The shortness of the solenoid B, as well as work with the magnetometer in axial positions, necessitated a third process (1) in which for each rotor and each position of the magnetometer, a determination was made of the *rotor sensitivity*, a, *as it would be if the solenoid were infinitely long and had no effect on the magnetometer.* This was accomplished with the aid of the *primary standard solenoid* A, exactly similar to B, but much longer and provided with leads attached to the ends of a central part exactly similar to B, as well as with leads attached to the extreme ends. In what follow the ratio $(b-a)/b$ will be denoted by K. For the rotors used in the principal part of the work, K ranged from 0.0 percent (Permalloy) to 5.1 percent (Hopkinson's Fe-Ni alloy).

§16. Sets of observations in the main experiments

In order to reduce the accidental errors due to fluctuations of the earth's field and other causes it was of course necessary in the principal experiments to make many magnetometer readings, and to make them on a regular time schedule, with equal intervals, as in processes (1), (2) and (3); and to eliminate various systematic errors (see below §§19, 20) it was necessary to spin the rotor alternately in opposite directions at the same speed. In the later work observations were usually made in sets of 12 with the time interval 30 seconds. To obtain the mean double deflection for the set, these were combined in the usual way.

§17. The principal equation for the experimental determination of λ and ρ

Let $\lambda\nu=2\pi\rho\nu$ denote the intrinsic magnetic intensity of rotation of a rotor for the angular velocity ν revolutions per second, and d the magnetometer deflection produced by reversal of the direction of rotation. Also, let H_0 denote the uniform magnetic intensity which would be produced in the region occupied by the rotor by a minute electric current I traversing the calibrating solenoid B if infinite in length, and a the deflection produced by reversal of the magnetization it would cause. And let b denote the deflection produced by reversal of I through the actual calibrating solenoid with the rotor coaxially inside. There we have from Eq. (10–1) and §15

$$\lambda\nu/H_0=d/a=d/b(1-K)$$

or

$$\lambda(=2\pi\rho)=dH_0/b(1-K)\nu \text{ gauss/r.p.s.} \qquad (17\text{–}1)$$

H_0 was determined, as $4\pi nI$, from the standard current I and the pitch n of the solenoids, and b from the magnetometer sensitivity c and the ratio $Q=b/c$. The frequency ν was obtained from the frequency N of the driving motor and the equation $\nu=GN$, where G is the gear-ratio of a gear-box connecting the motor to the rotor shafts. Thus we have

$$\rho\left(=\frac{\lambda}{2\pi}\right)=\frac{2nId}{Qc(1-K)GN}\,\frac{\text{gauss}}{\text{radian per sec.}} \qquad (17\text{–}2)$$

or

$$\rho/m/e=\frac{2nId}{Qc(1-K)GN}\,\frac{e}{m}. \qquad (17\text{–}3)$$

The results obtained in this way are in good agreement with those calculated from the standard observations by means of Eq. (17–3). See Table 23–1.

§18. Equation for the determination of ρ by comparison with ρ_s for a standard rotor s

As a check on the results obtained directly from Eq. (17–3), another process was used as follows: (1) The quantity ρ_s was obtained from the direct observations on the rotor Steel III. These were considered particularly trustworthy not only on account of the consistent behavior of this rotor in the last extended series of rotation observations, but also because the quantity Q for this rotor, *viz.*, Q_s, was determined, with consistent results, by means of three sets of coils, one of them wound and tested especially for use with this method. (2) The ratio $R=Q/Q_s$, that is, the ratio of the quantity Q for each of the other rotors to Q_s, its value for Steel III, was obtained from four (in a few cases, three) independent and concordant series of observations made especially for this purpose. (3) The quantity ρ for each rotor was then calculated from the formula:

$$\rho=\rho_sQ_s/Q=\rho_s/R. \qquad (18\text{–}1)$$

§19. Systematic errors. Errors of Class A (with origin independent of the magnetization of the rotor)

In the course of the work it was necessary to investigate and eliminate many sources of systematic error. Aside from errors in the standards, which were all proved to be small and all but one of them entirely negligible, the principal systematic errors which had to be investigated or overcome may be divided into two classes, as follows: (A) *those with origin independent of the magnetization* of the rotor, and (B) *those with origin dependent on the magnetization* of the rotor, either permanent or induced. The chief errors arising from these sources will now be briefly considered, those of class A first.

(1) *Eddy currents in the rotor due to incomplete compensation of* (*a*) *the uniform and* (*b*) *the nonuniform part of the magnetic field.* These were inappreciable in the experiments by the method of electromagnetic induction, on account of the high degree of symmetry of the apparatus, and

the relatively small precision required. In the magnetometer work they were much more serious, and a large amount of time and labor were necessary to study and eliminate them. Their effects were rendered negligible, or very small, by arranging electric coils traversed by steady currents to neutralize the magnetic fields as far as practicable, by placing and orienting the magnetometer magnets in such positions that the effects of eddy currents on them would be small, by using for the magnetometer and the controls magnets and coils with very small moments, and by keeping the controls and the compensator (itself given a small moment) at sufficient distances from the rotor. On account of the continuous fluctuations of the earth's magnetic field, it was necessary, in order to give the currents in the neutralizing coils proper values, to make numerous test-coil measurements and to install three variometers to measure the changes in the N–S, E–W, and vertical components of the intensity of the earth's magnetic field. The effects of the residual field were proved to be negligible or small in special experiments, chiefly by means of rotating a copper rotor.

(2) *Electric currents due to thermal effects at the bearings.* The objectionable effects of these were definitely observed only with the copper rotor. The effect was avoided in all the later work by sufficient insulation of all the bearings within a considerable distance of the magnetometer.

(3) *Electric currents due to thermal effects produced by the air driven against the bed-plate etc., by the moving rotor.* These were not certainly observed, but in most of the final work the possibility was prevented by screening the rotor from the bed-plate, by means of a coaxial cardboard tube, or otherwise.

(4) *Eddy currents and other electrical and magnetic effects in the motor, starter and other driving apparatus.* In the final work these were made inappreciable by sufficient neutralization of the magnetic field in which all but the more remote rotating parts were placed, and by using an alternating-current electric motor without commutator and at a considerable distance.

(5) *Thermal effects on the magnetometer due to air thrown against it by the motion of the rotor.* Such effects have long been known, or at least suspected, to be far from negligible, and would evidently be much greater for equatorial than for axial positions of the magnetometer. The thorough wrapping of all but the uppermost part of the magnetometer case with cotton and paper (necessary for other reasons) apparently sufficed (if indeed it was necessary) to prevent this effect, even without the cardboard screen already mentioned.

(6) *Vibrations of the magnetometer case, affecting the suspension differently for the two directions of rotation.* Everything practicable was done to make the vibration as small as possible, by such balancing of the rotors as was practicable, and by such balancing and straightening of all but the most remote driving parts as was practicable, by bolting securely to heavy concrete piers the rotor bed-plate, the countershaft bearings, the motor and the gear-box, by keeping the motor and gear-box remote from the magnetometer, and by surrounding the magnetometer supports with heavy concrete. Even before the later precautions were taken the magnetometer image, with well demagnetized rotors, showed no effect of vibration, except in certain special cases, as when one corner of the motor became loose.

§20. Errors of Class B (with origin dependent on the magnetization of the rotor)

Sources of error dependent on the magnetization of the rotor, which would vanish if it were possible to demagnetize it completely, are as follows:

(1) *Torsion of the rotor*, which is driven from one end and moves with friction in a bearing at the other. Torsional effects were negligible in the work by the method of electromagnetic induction, with large rods of steel, but were often manifest in the later magnetometer work, in which the range and precision were greatly increased. They are of two kinds, according as the magnetization is residual or induced. The first vanishes with the residual magnetization, the second with the residual field. For very small twists, at least, the first is proportional to the torque, and changes sign with the direction of rotation; the second is proportional to the square of the torque and independent of the direction of rotation, but there is no evidence of its appearance in this work. If the two journals and the two bearings are exactly alike, the rotor balanced and its

magnetization stable, and the journal friction independent of the direction of rotation, the first effect can be eliminated by observing first with the magnetic axis in one direction, then with the axis turned through 180° by reversing the rotor in its bearings. Great pains were taken to make the journals and bearings as nearly alike as possible, and in most of the later work the diameters of the journals were reduced to one-eighth or three-sixteenths inch, and bearings of agate provided. They were lubricated with clock oil. Since the journal friction is nearly independent of speed, the residual error, after reversal, is much less at high than at low speeds.

(2) *Thermal effects on the rotor due to journal friction.* These in general introduce systematic error because they are different for the two directions of rotation. Under the conditions mentioned in (1) an error due to this source, like that due to torsion, is eliminated by reversing the rotor in its bearings. It is made as small as practicable by the type of construction mentioned in (1), and by separating the journals from the magnetic part of the rotor by material which is a good thermal insulator.

(3) *Error from thermal effect of air acting on rotor.* A similar error may be introduced by air currents due to the motion, which, on account of asymmetry, heat the rotor differently for the two directions of rotation. This error was guarded against in much of the work by the use of the coaxial cardboard tube surrounding the rotor which is mentioned in §19, (3). The error is eliminated by reversing the rotor, when stable.

(4) *Error from centrifugal expansion of the rotor, or other strain, aside from torsion, produced by rotation.* This is eliminated by observing at the same speed for both directions of rotation. If there is a systematic difference between the speeds for the two directions of rotation, error due to the permanent part of the magnetization can be eliminated by reversing the rotor. If there is axial induced magnetization, however, error from it and the speed differences cannot be eliminated this way. The application, however of considerable axial intensities—greater than those which could possibly have existed in the principal experiments—was found to be without appreciable effect.

(5) *Error from axial displacement of the rotor.* A certain amount of axial displacement always occurs when a rotor is set into motion, and if this depends on the direction of rotation a systematic error may thus be introduced. This may happen in three ways: (1) Eddy currents due to the residual field will affect the magnetometer differently for the two directions of rotation; (2) the effect on the magnetometer of the rotor's permanent magnetization will be different for the two directions; and (3) if the residual axial intensity has a gradient, the induced magnetization of the rotor, and its effect on the magnetometer, will depend on the direction of rotation. All these effects were made to vanish in this work. Effect (2), if appreciable, would be eliminated by reversal of the rotor.

(6) *Error from change in the rotor's azimuth or altitude.* If the (minute) angle between the normal to the axis of the rotor and the component ΔX of the residual magnetic intensity lying in the meridian (practically normal to the rotor) changes by an amount $\Delta\alpha$ when the rotation is reversed, the effect will be equivalent to reversing in the rotor an axial intensity $f=\frac{1}{2}\Delta\alpha\cdot\Delta X$, a quantity which was proved to be negligible in these experiments.

(7) *Error from the Elihu Thomson repulsion effect.* Since the rotor magnetization is never entirely symmetrical about its axis, the rotor produces an alternating magnetic field which induces currents of its own frequency in the magnetometer core and the suspension itself, and thus, on account of lack of perfect symmetry, may produce an appreciable torque on the suspended system, different for different azimuths, and thus different for the two directions of rotation. The effect was proved to be inappreciable in these experiments.

(8) *Errors from mechanical disturbances.* By taking many precautions the degree of mechanical perfection of the apparatus was made such that, in general, no appreciable error came from mechanical disturbances.

§21. Rotor construction, rotor bed-plate, driving mechanism and speed

The magnetic parts of the rotors were about 30–31 cm in length and varied in diameter from 2.26 cm to 3.34 cm. The types of construction used in the later work are shown in Fig. 21–1,

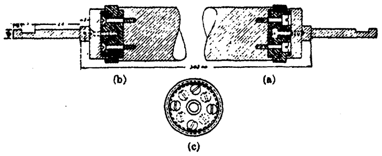

FIG. 21-1.

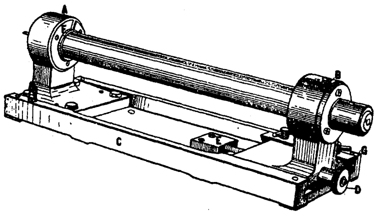

FIG. 21-2.

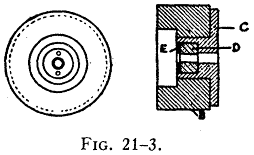

FIG. 21-3.

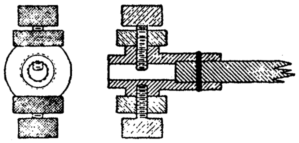

FIG. 21-4.

a–c, and b–c. In nearly all the latest work the type b–c was used. The journal piece is of phosphor bronze, and is separated by a thick disk of Bakelite from the magnetic material. A photograph of one of the rotors in the bed-plate, as used in the last work, is given in Fig. 21–2, and drawings of the bearings and driving arrangement are given in Figs. 21–3 and 21–4.

The rotor was direct-driven by a series of carefully balanced countershafts connected through a spiral-gear-box to an alternating-current motor, 6.7 meters from the magnetometer in the final work. The speeds obtainable were 2/7, 1/2, 1/1, and 2/1 times that of the motor—the lowest and highest speeds being about 9 and 61 r.p.s., respectively. The speeds were very nearly constant, and when different for the two directions of rotation such corrections were made as were necessary to avoid systematic errors which would have been automatically eliminated for exactly equal speeds in opposite directions.

§22. Mean results of the principal magnetometer observations

Only the results of the last two groups of observations, referred to in the original paper as Series A and Series B, will be quoted here, as they are far superior to those made earlier. The last group, Series B, is far superior to Series A, as the final improvements in the apparatus were incorporated for this series. The mean results for the highest two speeds, calculated from (17–3) are given in Table 22–1. The results for the lower speeds are not quoted, as the errors are better eliminated in the work at higher speeds. In Series B the means were in close agreement for all speeds.

As indicated above, the theory requires that

TABLE 22–1. *Magnetization by rotation.*
Mean values of $\rho e/m$ from Series A and Series B (e/m assumed 1.757×10^7 e.m.u.).

Series	Rotors	Gear-ratio	No. of sets	$\rho e/m$	Approximate ave. error
A	Steels II & III, Heusler alloy II, Permalloy, Cobalt-iron, Cobalt II	1–1*	100	1.063	±0.040
	As above + Armco iron	2–1†	150	1.057	±0.020
B	Steels I, III, IV; Electrolytic iron; Armco iron; Nickels I, III; Cobalt II; Heusler alloy I; Permalloy; Nickel-iron (Hopkinson); Cobalt-iron; Cobalt-nickel	1–1*	163	1.060	±0.030
	As above + Norway iron	2–1†	159	1.051	±0.015

* *Ca.* 31 r.p.s.
† *Ca.* 61 r.p.s.

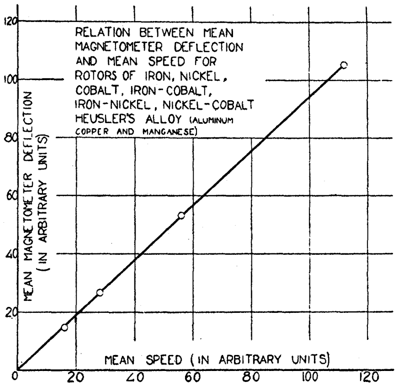

FIG. 22–1.

the deflection in the case of each rotor be proportional to the speed. The experimental relation between speed and reduced deflection for all the observations in Series B is given in Fig. 22–1. The reduced deflection for each rotor is the actual deflection reduced to a fixed magnetometer sensitivity divided by the change of moment produced by a standard weak axial magnetic intensity. The relation is seen to be strictly linear.

§23. Results for the individual rotors from Series B. The final mean

The data are given in Table 23–1 for the observations at the highest speed, which, as indicated above, are the most reliable, though the mean results for the lower speeds differ but little from them. The values of $\rho e/m$ in column (3) are calculated directly from the observations (including the calibrations) of Series B by the formula (17–3). The values given in column 4 are calculated by the methods of §18, from the formula (18–1), $\rho e/m$ for Steel III, as obtained from Series B at the gear-ratio 2–1, being taken as standard. Column (5) gives the means of the two calculations. There is no systematic difference between the numbers in columns (3) and (4), the mean for column (4) exceeding that for column (3) by only +0.002; while the average difference without regard to sign is ±0.015.

There can be little question of the precision of the *means* given in Table 22–1, which agree closely for the two series of observations and for the two speeds. It would be unsafe to conclude, however, from the results given in Table 23–1, that the values of ρ for the different materials are *certainly* different, since the discrepancies are as great or nearly as great for different rotors of nearly the same material as for rotors of different materials. The means and a number of the individual values, however, agree well with the results obtained by the author on the converse effect, and given in Table 27–1 below, which show unmistakable differences for different materials.

TABLE 23–1. *$\rho e/m$ for the individual rotors.*
Series B—Gear-ratio 2–1 (ca. 61 r.p.s.)
(e/m assumed 1.757×10^7 e.m.u.)

(1) Rotor	(2) No. Sets	(3) $\rho \cdot e/m$	(4) $\rho_Q e/m$	(5) Mean of (3) and (4)
El. iron II*	18	1.067	1.092	1.080
Armco iron	10	1.026	1.022	1.024
Norway iron	6	1.032	1.052	1.042
Steel III	15	1.050	(1.050)	(1.050)
Steel IV*	21	1.054	1.049	1.052
Steel I	9	1.049	1.044	1.046
Nickel I	9	1.049	1.020	1.034
Nickel III	10	1.014	1.003	1.008
Cobalt II	13	1.073	1.109	1.091
Heusler alloy I	10	1.012	1.031	1.022
Permalloy	16	1.057	1.036	1.046
Nickel-iron	10	1.015	1.016	1.016
Cobalt-iron	6	1.071	1.060	1.066
Cobalt-nickel	4	1.070	1.077	1.074

* The value of ρ for this rotor has been altered from the value previously given on account of the rejection of one calibration because a new examination of the data indicates, with great probability, that one of the coils was defective when this calibration was made.

To a high degree of probability the mean value of $\rho \cdot e/m$ obtained from the highest speeds in Series B, *viz.*, 1.051, is correct within less than 2 percent.

D. ROTATION BY MAGNETIZATION (EINSTEIN AND DE HAAS EFFECT)

§24. Rotation by magnetization (§3, (3)). General experimental methods and theory

In all experiments on this effect[28] a round cylinder of the substance under investigation is hung with its axis vertical in a fixed framework by means of a vertical wire or fiber along the axis extended, or by two wires or fibers, one above and one below. Surrounding the cylinder and

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[28] Except certain experiments by E. Beck (reference 44) in which a bundle of round cylinders was used.

fixed either to the cylinder or to the framework, is a coaxial magnetizing coil of insulated wire. The motion of the cylinder produced by changes in axial magnetization is studied.

When the axial magnetic moment μ changes at the rate $\dot{\mu}$ the concealed angular momentum $M=\rho\mu$ changes at the rate $\dot{M}=\rho\dot{\mu}$; hence the angular momentum J of the rotor will change at the rate

$$g=\dot{J}=-\dot{M}=-\rho\dot{\mu}, \qquad (24\text{–}1)$$

which is the *gyromagnetic torque* on the *rotor and magnetizing coil together.*[29] In most work it has been assumed, on the basis of probable theory, that the torque on the magnetizing coil vanishes. The frictional torque is taken (consistently with experiment) as proportional to the angular velocity of the rotor.

If disturbing torques, also producing changes in J, are present, Eq. (24–1) will of course not be valid. In what follows such disturbing torques, all of which can be eliminated or corrected by suitable methods, will at first be assumed absent.

Two general methods of investigating the effect have been tried, *ballistic* methods and *alternating-current resonance* methods. Resonance methods are of two kinds, *single torque* and *multiple torque.*

In most of the work done by resonance methods flat-topped, or approximately flat-topped, current waves have been used, the first harmonic of the magnetization being thus brought into phase, or approximately into phase, with the first harmonic of the current, and also being readily determinable in amplitude. In some of the work, including the first investigation by Einstein and de Haas, this condition has been brought about by using large amplitudes of magnetic intensity, producing approximate saturation early in each half cycle. In much other work flat-topped waves have been produced, even in the case of very weak fields, by using a battery and commutator, or equivalent devices.

§25. Ballistic methods (Richardson,[10] J. Q. Stewart,* Chattock and Bates†)

In ballistic methods Eq. (13–1) is used in the integral form

$$\delta J=\int g\, dt=-\rho\delta\mu, \qquad (25\text{–}1)$$

the rotor being initially at rest. That is, the change δJ in the angular momentum of the rotor is determined for a given change $\delta\mu$ in the magnetic moment produced by altering the current[30] in the magnetizing coil. The quantity $-\rho$ is then calculated as $(\delta J=J)/\delta\mu$. J is obtained from the standard formula

$$J=(AK)^{\frac{1}{2}}\cdot\theta, \qquad (25\text{–}2)$$

where θ is the throw which would be produced by $\delta\mu$ if there were no damping, K the moment of inertia of the suspended system, and A the torsional constant. According to the standard treatment, θ may be obtained from θ_0, the observed throw, by means of the relation

$$\theta=\theta_0(d_1/d_2)^{(1/\pi)\tan-1[\pi/(\log d_1/d_2)]}, \qquad (25\text{–}3)$$

where d_1/d_2 is the ratio of successive deflections in opposite directions. This may be written

$$\theta=\theta_0(1+\lambda), \qquad (25\text{–}4)$$

where λ = one-half the logarithmic decrement when the damping is very small. Probably because of the inertia of the air which is dragged along by the rod in its own motion, however, this formula is sometimes far from valid. See §39.

In order to avoid as far as possible the disturbing action of the field of the magnetizing coil on the rotor, Stewart brought the galvanometer to rest when the field was annulled but the residual magnetic moment of the rod was large, and then observed the throw when such a (weak)

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[29] The process by which the equal and opposite momenta are produced is doubtless somewhat as follows: For definiteness assume the rotating electricity in the elements to be positive, and assume the field to be impressed in the positive direction along the axis of the rod. Then the application of the field (or its increase) will produce a torque on each element which makes it precess around the direction of the field with a *negative* angular velocity. Interaction with the adjacent elements (retarding the precession) thus produces a *negative* axial torque upon the rest of the body and a *positive* axial torque upon the particular element under consideration. This latter increases the degree of alignment of the axis of spin (and magnetic moment) of the element with the direction of the field. In this way, as the field increases in strength, the total axial momentum of the elements and the axial momentum of the rod increase together in opposite directions.

* See reference 43.

† See reference 46.

[30] Or in any other manner. For example, Professor Kerr Grant once suggested to me that the magnetization might be destroyed by an intense beam of radiation.

field was applied in the opposite direction as to reduce the moment to zero. Chattock and Bates obtained the throw due to the *reversal* of the residual moment. The essential part of this idea is due to Einstein.[31]

In Stewart's investigation observations were made systematically for both directions of the residual magnetization; and in that of Chattock and Bates, observations were made systematically for both directions of the *reversal* of the residual magnetization. These procedures should eliminate the effect of magnetostriction (§37) from the mean result provided the permanent part of the vertical magnetic moment of the specimen, and also the part induced by the vertical intensity of the earth's field if not compensated, are negligible in comparison with that produced by the magnetizing coil. And since the magnetostriction torque may either aid or oppose the gyromagnetic torque, errors due to this source tend to vanish from a mean derived from many observations with the rotors repeatedly reset in slightly different orientations at random.

In the investigation of Chattock and Bates observations were also systematically made for two azimuths of the rotor differing by 180°. This process tends to eliminate the systematic errors due to a residual horizontal intensity of the earth's magnetic field.

§26. Single-torque resonance methods (Einstein and de Haas[32])

In these methods the only torque deliberately impressed on the system is the gyromagnetic torque $g=-\rho\dot{\mu}$.

Method (1). The system, whose natural frequency will be denoted by ν_0, is magnetized by an alternating current, the frequency ν of whose first harmonic of constant amplitude, is adjustable over a small range including ν_0. The semi-angle α through which the system vibrates is determined as a function of ν. From α_0 (the value of α at resonance), K, the logarithmic decrement λ, and ν_0, J, the amplitude of the angular momentum at resonance, can be obtained by means of the standard formula

$$J=\pi\lambda K\nu_0\alpha_0\,; \qquad (26\text{–}1)$$

and from $J(=\delta J)$ and $\mu(=\delta\mu)$, the amplitude of the first harmonic of the magnetic moment, $|\rho|$ can be calculated by means of Eq. (25–1).

Method (2). By measuring additional values of α on the resonance curve beside α_0, together with the corresponding values of ν, λ can be eliminated and its measurement therefore made unnecessary.

If the frequency is very low, as in the later investigations of Einstein and de Haas, the phase relations between the current and the rotor displacement can be immediately determined by visual observations, and thus the sign (along with the magnitude) of ρ can be determined. If the frequency is high, this is impossible, and some sort of oscillographic arrangement must be used to supplement the method, as in the work of Beck and the first work of Einstein and de Haas.

§27. Multiple torque resonance methods (de Haas,[33] Chattock,[34] Sucksmith and Bates,[34] Barnett[35])

In addition to the gyromagnetic torque, one or more additional torques, in synchronism with it, may be deliberately applied in order to annul or alter the effects of the gyromagnetic torque, or torques in quadrature with it, or in phase or opposition.

The first to use such a method was de Haas, who, in 1916, attached to the rigid vibrating system, on a small rod forming an extension of the magnetic substance under investigation, a minute permanent magnet with its axis horizontal. A small fixed coil of wire with its turns parallel to the magnet and its axis passing through the magnet, was connected in series with the magnetizing coil, and was provided with a shunt by which its field could be adjusted in strength. The torque produced by the coil could also be altered by changing its distance from the magnet. Flat-topped waves of current being used, this coil produced a torque in quadrature with the gyromagnetic torque, and could thus be used to annul the effects of quadrature torques otherwise produced.

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[31] A. Einstein, Verh. d. D. Phys. Ges. **18**, 173 (1916).

[32] See reference 11.

[33] W. J. de Haas, Verh. d. D. Phys. Ges. **18**, 423 (1916).

[34] A. P. Chattock, quoted by W. Sucksmith and L. F. Bates, Proc. Roy. Soc. **A104**, 499 (1923).

[35] S. J. Barnett, Proc. Am. Acad. **66** (8), 273 (1931); **69** (2), 119 (1934).

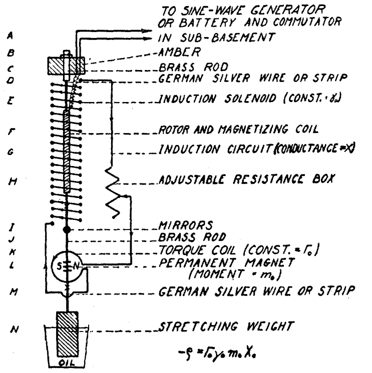

FIG. 27–1.

In some of the author's work a quadrature torque coil and magnet were used in much the same way, except that the coil was fixed, with the magnet at its center, and was connected, through an adjustable high resistance box, either to the terminals of the commutator operating on the magnetizing circuit, or else to adjustable points on a noninductive rheostat in series with this circuit.

If the extra coil operating on the permanent magnet, instead of being connected with the magnetizing circuit, as above, is connected through a considerable resistance in series with a solenoid or other coil surrounding the rotor, the torque it produces will, as shown below, be in phase with or in opposition to the gyromagnetic torque, according to the way in which the connections are made. The total torque, except for quadrature components, may then be made zero, and a null method results. This idea was suggested by Chattock, and first used, in his laboratory, by Sucksmith and Bates. It has since been used, with various modifications, by the author of this paper, sometimes in conjunction with the quadrature coil mentioned above.

In the null method, as well as in its modifications, there is attached to the lower end of the rotor, F, Fig. 27–1, a vertical, nonmagnetic rod, J, which carries a pair of small parallel mirrors I on opposite sides, and, below the mirrors, a small permanent magnet L (moment m_0) with its axis horizontal. A heavy brass or copper weight N hangs from the lower end of the rod J on a suspension of German silver or other material and in a vessel containing oil, or other damping fluid. In order to eliminate certain systematic errors (see below §§31 ff.) observations are made with the torsion head, and the complete suspended system, set in two azimuths 180° apart. Coils of wire surrounding the apparatus and traversed by suitable electric currents neutralize the intensity of the earth's magnetic field (all components in the author's experiments) almost completely, when desired, in the region occupied by the rotor. The rotor is driven at the resonance frequency ν_0 by the gyromagnetic torque as before. By means of the small coil of wire K (the principal torque coil, constant Γ) surrounding the permanent magnet, and with axis normal thereto, and traversed by a current i with frequency ν_0, an extra torque c may be applied to the vibrating system. By a suitable arrangement the current i is given any value desired. In the original (null) method, c is made equal in magnitude and opposite in phase to g, and the amplitude of the vibration is thus made to vanish, provided no disturbing torques are present.

The coil surrounding the rotor for this purpose is, in the simplest arrangement, as adopted by the author, a much longer coaxial solenoid E fixed to the earth, the *induction solenoid*, whose constant is nearly uniform, with mean value γ, in the space occupied by the rotor. In the work of Sucksmith and Bates a solenoid equal in length to that of the rotor was used, an arrangement necessitating a large correction to infinite length, as determined by extra experiments for each rotor. The induction solenoid is connected in series with the torque coil through an adjustable resistance box H, the total conductance of the circuit being X. If the connections between the two coils are properly made (depending on the sign of ρ), and if X is given a suitable value X_0, the amplitude of vibration (provided no disturbing torques are present) will vanish, as shown in the next article.

In nearly all of the author's work the magnetizing coil was wound directly upon the rotor F, as in the work of de Haas, and as shown in Fig. 27–1. In the remainder of the author's work the rotor was magnetized by a long fixed coaxial solenoid located inside the induction solenoid. In the work of Sucksmith and Bates the magnetizing coil was fixed outside the induction solenoid and coaxial with the rotor.

§28. Theory of multiple-torque null method

Let φ denote the magnetic flux through the induction circuit produced by μ (that due to the magnetizing coil being supposed negligible or compensated by extra mutual inductance coils). Then $\varphi=\mu\gamma$; and the current in the induction circuit, if X is sufficiently small in comparison with the reactance, which can always be made the case in practice, will be $-X\dot{\varphi}=-\gamma X\dot{\mu}$. Thus the coil torque will be

$$c=-\Gamma\gamma m_0 X\dot{\mu}. \tag{28–1}$$

The two torques c and g (24–1) will thus be equal in magnitude and opposite in phase, giving an amplitude of vibration O, for the value X_0 of X determined by the equation

$$-\rho=\Gamma\gamma m_0 X_0. \tag{28–2}$$

If in an independent experiment a steady current i is sent through the torque coil it will deflect the mirror through an angle θ such that $\Gamma m_0 i=A\theta$, where A is the torsion constant of the suspension. Thus by measuring θ, A and i, the quantity Γm_0 may be eliminated from (28–2), which then becomes

$$-\rho=\gamma X_0 A\theta/i. \tag{28–3}$$

In the experiments of Sucksmith and Bates Γm_0 was eliminated by this process; in those of the author Γ and m_0 were both measured with precision.

If the magnetizing coil is wound on the rotor, as in the figure, and is not compensated by extra mutual inductance, we must substitute for (28–2) the equation[36]

$$-\rho=\Gamma\gamma m_0 X_0\frac{1+\mu'/\mu}{1+\rho'\mu'/\rho\mu}, \tag{28–4}$$

where μ' is the moment the winding would have in air and ρ' the gyromagnetic ratio for electrons moving in a circle (*viz.*, $2m/e$).

In most of the author's work, *deflection methods* have been used, and to discuss them properly it will be convenient to transform and add to what precedes as in the following section.

§29. Theory for multiple-torque deflection methods

If the first harmonic of the magnetic moment of the rotor is $\mu=\mu_0\sin\omega t$, the first harmonic of the gyromagnetic torque $-\rho\dot{\mu}$ will be

$$g=-\rho\omega\mu_0\cos\omega t=G\cos\omega t. \tag{29–1}$$

The variation of μ induces an electromotive force $\psi=-\omega\mu_0\gamma\cos\omega t$, and thus a current $\psi X=-\omega\mu_0\gamma X\cos\omega t$, in the induction circuit. This current, traversing the torque coil, produces a torque

$$c=-\omega\mu_0\gamma X\Gamma m_0\cos\omega t=C\cos\omega t \tag{29–2}$$

on the rotor system. The total impressed torque τ of frequency $\nu=\omega/2\pi$ on this system is thus

$$\tau=(G+C)\cos\omega t. \tag{29–3}$$

The frequency ν of the first harmonic of the impressed electromotive force is, in general, made equal, or very nearly equal, to the natural frequency ν_0 of the vibrating system;[37] thus the amplitude A of the vibration may be written

$$A=\beta(G+C), \tag{29–4}$$

where β is a constant; or

$$A=\beta(G-C), \tag{29–5}$$

if, as in practice, the torque coil is so connected

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[36] The corrected formula is derived as follows: In place of the gyromagnetic torque g we now have g', the sum of the gyromagnetic torque $-\rho\dot{\mu}$ and the electron inertia torque $-\rho'\dot{\mu}'$, where μ' is the magnetic moment of the magnetizing coil and $\rho'(=2m/e)$ the gyromagnetic ratio for an electron moving in a circle. Thus $g'=-\rho\dot{\mu}-\rho'\dot{\mu}'$. The flux through the induction solenoid is now $\varphi'=\gamma(\mu+\mu')$, so that the current is

$$q'=-X\dot{\varphi}'=-\gamma X(\mu+\mu')$$

and the torque due to the current,

$$c'=\Gamma m_0 q'=-\Gamma\gamma m_0 X(\dot{\mu}+\dot{\mu}').$$

For a certain value X_0 of X the total torque $g'+c'$ vanishes. We have then

$$-\rho\dot{\mu}-\rho'\dot{\mu}'=\Gamma\gamma m_0 X_0(\dot{\mu}+\dot{\mu}')$$

whence

$$-\rho=\Gamma\gamma m_0 X_0\,\frac{1+\mu'/\mu}{1+\rho'\mu'/\rho\mu}.$$

With sufficient precision this equation may be written

$$-\rho=\Gamma\gamma m_0 X_0[1+(\mu'/\mu)(1-\rho'/\rho)]. \tag{28–5}$$

[37] The rotor will also resound for the impressed frequencies $\nu=3\nu_0$, $5\nu_0$, etc., but with continually diminishing amplitudes for a given amplitude of impressed electromotive force, as suggested to me by Dr. V. Hoover.

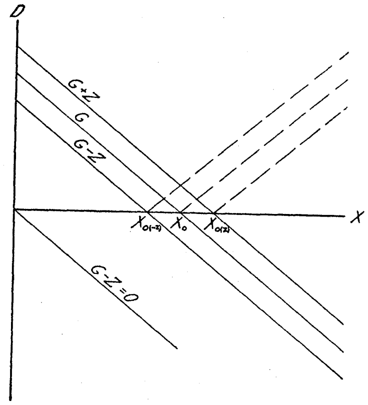

FIG. 29–1. Vibration amplitude (*d*) *versus* conductance (X) with and without disturbing torques in phase with or in opposition to g.

in the circuit that c and g have opposite phases. In this case A vanishes for such a value X_0 of X that $G=C$; that is, when $\Gamma\gamma m_0 X_0=-\rho$.

The equation $A=\beta(G-C)$ may be written

$$A=\beta(G-\alpha X), \tag{29–6}$$

the relation between A and C, or A and X, thus being linear, as shown in Fig. 29–1, curve G. The phases of the motion and of the torque change sign together when $X=X_0$; but if we consider only magnitudes, the relation between A and X is given by the two straight lines meeting at $X=X_0$ (corresponding to AF and FB of Fig. 29–2).

If there is an extraneous torque with amplitude Z in phase with g or in the opposite phase, the lines G will be replaced by the lines $G+Z$ or $G-Z$, meeting the axis of abscissae at $X_{0(z)}=X_0+\delta X_0$ or at $X_{0(-z)}=X_0-\delta X_0$. If such a torque is present, and its phase can be reversed without altering its amplitude, the true value of X_0 can thus be found by taking the mean of $X_{0(z)}$ and $X_{0(-z)}$.

If there is a torque in quadrature with g, the straight lines meeting at $X=X_0$ will be replaced by the symmetrical curved line CED, Fig. 29–2, with a *minimum* at $X=X_0$. Thus no systematic error is introduced by the presence of such a torque in determining X_0, but unless the minimum amplitude (EF) is small in comparison with that produced by g, the curve near the minimum will be so flat that X_0 cannot be determined with precision.

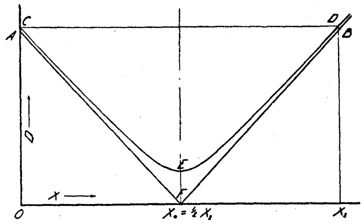

FIG. 29–2. Vibration amplitude (D) *versus* conductance (X) with and without disturbing torque in quadrature with g.

§30. Various multiple-torque deflection methods

A. Graphical method. The observed amplitude A is plotted as a function of the conductance X, and $X_0 \pm \delta X_0$ obtained from the location of the minimum (Fig. 29–2), or from the intersection with the axis of abscissae of the *symmetrical straight line* best representing the observations and drawn across the axis of abscissae, as in Fig. 29–1. In case of quadrature torques the points will of course lie off the straight line near the intersection.

B. Large deflection methods (a). In much of the author's work $X_0+\delta X_0$ and $X_0-\delta X_0$ were determined by measuring *accurately* the amplitudes A and A_2, respectively, for $X=0$, and $X=$ approximately $2(X_0 \pm \delta X_0)=X_2$; and by measuring *approximately* the amplitudes A_0 for $X=X_0 \pm \delta X_0$; and inserting in the formula

$$X_0 \pm \delta X_0 = X_2 \frac{(A^2-A_0^2)^{\frac{1}{2}}}{(A^2-A_0^2)^{\frac{1}{2}}+(A_2^2-A_0^2)^{\frac{1}{2}}}, \tag{30–1}$$

which is easily developed.

(b) In other work of the author's $X_0+\delta X_0$ was obtained by taking one-half the value of X for which the precisely measured amplitude (beyond the minimum) was *equal* to the amplitude (also measured with precision) for which $X=0$. The first value was of course not observed directly, but was obtained by interpolation or extrapolation for values of X near $2(X_0+\delta X_0)$; and $X_0-\delta X_0$ was obtained in a similar way.

(c) In still another method, not so precise and but little used, a comparison is made between the amplitude A_G of the vibration produced by the gyromagnetic torque when there is no current in the torque coil ($C=0$), and the amplitude A_C of the vibration produced by an independent current $c=C\cos(\omega t+\delta)$ in the torque coil when there is no current in the magnetizing coil ($G=0$). If the observed amplitudes in the two cases are A and A', respectively, we have

$$A_G=(A^2-A_0^2)^{\frac{1}{2}} \quad \text{and} \quad A_C=A',$$

and

$$A_G/A_C=G/C=|\rho\omega\mu/\Gamma m_0 C|, \qquad (30\text{–}2)$$

from which

$$|\rho|=(\Gamma m_0 C/\omega\mu_0)\cdot A_G/A_C. \qquad (30\text{–}3)$$

The method is applicable only in case A_0 is known to be negligible, or is determined by some other method.

Most of the methods discussed in §§27–30 offer great advantages over the others for the detection and elimination of systematic errors due to disturbing torques, and for determining the sign of ρ.

§31. Disturbing torques

In practice the gyromagnetic torque never exists alone, or its measurement would be a comparatively simple matter. Most of the sources of error and methods of eliminating them were pointed out in the early papers of Einstein and de Haas. Other errors and methods were described in the papers of Richardson, Stewart, Chattock and Bates, Sucksmith, Coeterier and Scherrer, and the author.

Most of the disturbing torques would vanish if the vibrating body, and the magnetizing coil in case it is fixed to the framework, could be made exactly symmetrical, geometrically, mechanically and magnetically, about a vertical through the point of suspension; and in all the work which has been done a great deal of attention has been devoted to securing such conditions. As the multiple-torque alternating-current method offers advantages over the others for the detection and elimination of systematic errors, and as the greater part of the more precise work has been done by it, most of the discussion here will apply primarily to this method. As will be obvious, much of the discussion will apply, *mutatis mutandis*, to the other methods.

The alternating vertical and horizontal magnetic moments of the rotor will be assumed to be

$$\mu=\mu_0\sin\omega t \qquad (31\text{–}1)$$

and

$$\nu=\nu_0\sin(\omega t-\alpha) \qquad (31\text{–}2)$$

respectively. The gyromagnetic torque may then be written

$$g=G\cos\omega t, \qquad (31\text{–}3)$$

as in (29–1); and any disturbing torque in phase with it, or in opposition, may be written

$$p=P\cos\omega t, \qquad (31\text{–}4)$$

while a quadrature torque may be written

$$q=Q\sin\omega t. \qquad (31\text{–}5)$$

The total torque may be written

$$t=g+p+q=((G+P)^2+Q^2)^{\frac{1}{2}}\cos(\omega t-\delta)$$

$$=T\cos(\omega t-\delta), \qquad (31\text{–}6)$$

where

$$\tan\delta=Q/(G+P). \qquad (31\text{–}7)$$

As the Eq. (31–6) shows, the total torque, and thus, in general, the amplitude, is increased by the presence of a quadrature torque. In §43 a special method of experimentation is described in which quadrature torques produce no effect on the motion.

The most serious disturbing torques belong in four classes as described in the next sections.

§32. Torques due to the earth's magnetic field

This field can be approximately neutralized by means of electric coils or magnets placed about the apparatus; but it is doubtful whether the neutralization has been so good in any investigation as to make all of these torques negligible. All of them, however, can be eliminated or made negligibly small.

Let ΔH denote the horizontal intensity of the residual magnetic field in which the rotor is placed, and ΔX and ΔY its northerly and easterly components; also let ΔZ denote the vertical intensity of the residual field. Then

(a) There will be an axial torque on the rotor due to ΔH which may be written

$$e_h=h\cdot\Delta H\cdot\nu=E_h\sin(\omega t-\alpha). \qquad (32\text{–}1)$$

If the suspended system is turned through 180° about its axis, this torque will remain unaltered in magnitude but will be reversed in sign, and thus can be eliminated by the process (§29)

(b) ΔH, acting on the vertical moment, will produce a torque $l=L\sin\omega t$ about a horizontal axis. If the system is not axially symmetrical, this torque, together with that due to the relative displacement of the suspension and rotor which it entails, will give rise to an axial torque

$$b=B\sin(\omega t-X), \tag{32–2}$$

which, like e_h, is reversed by changing the azimuth 180°.

(c) The residual vertical intensity ΔZ, acting on ν, may produce a resultant effect similar to that of (b), but it does not change sign when the change of azimuth is made, and can be directly eliminated only by making ΔZ or ν negligible. In ballistic methods this torque will either aid or oppose the gyromagnetic torque, hence the error due to it will tend to vanish from a mean result obtained from many observations in which the rotor is repeatedly reset in slightly different orientations at random. In resonance methods this process will eliminate the error only if the effect of the quadrature torque is annulled or negligible.

(d) If the magnetizing coil is wound upon the rotor, there will be a torque

$$i=I\sin(\omega t+\gamma) \tag{32–3}$$

where γ is very small for rectangular waves, upon the system due to the action of the residual field upon the wire. This torque also keeps its magnitude and changes its sign when the system is turned through 180°.

§33. Torques due to the action of the electric coils on the movable system

(a) If the magnetizing coil is fixed to the earth, and if the rotor has a permanent horizontal moment ξ, there will, in general, be an axial torque d upon the rotor due to the horizontal intensity of the fixed coil, the direction of whose field is not, in general, parallel to ξ. This torque, with the commutator producing the alternating current, will be nearly in phase with μ. Thus

$$d=D\sin(\omega t+\delta), \tag{33–1}$$

where δ is small. This torque also is reversed in sign by changing the azimuth 180°. There is a similar torque upon the fixed magnets carried by the suspension.

(b) In this same case there is an axial torque due to the action of the horizontal intensity of the alternating field on the alternating horizontal moment ν; but if the half-cycles of current and magnetization are equal, its fundamental frequency is twice that of the current, and so its effect is negligible. If the half-cycles are not equal, there may also be a residual torque, with the frequency of the current; but it is in quadrature with g, as stated by de Haas.

(c) In the same case, also, there is a torque on the rotor about a horizontal axis due to the action of the alternating vertical intensity on the permanent horizontal moment ξ. This entails a relative displacement of the rotor and suspension, and, because of lack of symmetry about the vertical, results in a torque about the vertical which may have any phase relation with g. It has the frequency of the current, and does not change sign or magnitude by reversal of the system in azimuth.

(d) The torque coil produces a magnetic field extending upward into the region occupied by the rotor, where its lines of intensity are horizontal. Thus, if the rotor's magnetization is not symmetrical about the axis of suspension, an axial torque may be produced. If the magnetization is strictly alternating (and half-cycles equal) this torque will have twice the frequency of the current, and may be left out of account in resonance experiments. If there is a residual moment, however, normal to the axis of the coils, and produced by an uncompensated part of the earth's field, or otherwise, there will result an axial torque of the frequency of the current and in phase with (or in opposition to) the gyromagnetic torque. In the author's work described here it was negligible. It is reversed by reversing the suspended system in azimuth.

(e) If the induction solenoid is not exactly vertical, but displaced from the vertical through a small angle α, the intensity of its field will have a horizontal component $h=\alpha\gamma$ per unit current, and may exert torques on the rotor due to ν and ξ. The effect of one of these torques disappears because of its double frequency, that of the other is eliminated by reversing the rotor in azimuth. A torque similar to the last may originate in the action of the induction circuit upon the small magnet with moment m_0.

§34. Torque due to leak between magnetizing and induction circuits

If there is such a leak there will be an axial torque

$$\lambda = \Lambda \sin(\omega t + \gamma) \qquad (34\text{–}1)$$

on the vibrating system. The coefficient Λ is made very minute by using high insulation, and γ is very small when rectangular waves are used. The effect is thus both minute and also in quadrature with g.

§35. Torques due to mutual induction between magnetizing coils and induction solenoid

These can always be made negligible by the use of a mutual inductance compensator, or their effects can be calculated and the proper corrections applied. In the author's work with the magnetizing coil fixed, compensation was used and was proved to be exact. In the experiments with the magnetizing coil used on the rotor, the (small) effect of the coil was calculated and allowed for.

§36. Torques due to electron-inertia in the rotor and rotor coil[38]

When the current in the coil, if wound upon the rotor, and therefore the angular momentum of the free electrons, changes, an equal and opposite change occurs in the angular momentum of the rotor. Thus there is an electron-inertia torque upon the rotor. It can be shown to be in phase with the gyromagnetic torque g, and its amplitude T can be shown to bear to G, the amplitude of the gyromagnetic torque, the ratio

$$T/G = (2m/e \cdot \mu')/\rho\mu. \qquad (36\text{–}1)$$

This is readily calculated and allowed for (see above, §28). Electron-inertia torques due to the induction of currents in the rotor can be shown to be negligible.

§37. Torque due to magnetostriction

Magnetostriction gives rise to vertical motions which, through asymmetry, or variation of the suspension twist with tension, are partially converted into axial vibrations. Since the change in rotor length due to magnetization is independent of the direction of magnetization, no magnetostrictive torque of the same period as that of the first harmonic of the magnetization can arise if the two half-cycles of magnetization are similar. If they are dissimilar, however, either because of a residual vertical magnetic moment, or because the two half-cycles of magnetizing current are dissimilar, there will be an elongation with a first harmonic of the period of the magnetization. It may have any phase relation to the gyromagnetic torque. By interchanging the half-cycles of current producing positive and negative magnetization of the rotor, as by interchanging the connections between the terminals of the magnetizing coil and the rest of the circuit, the torque is reversed in sign without changing its magnitude. In the multiple-torque resonance method, in which the error from quadrature torque is made to vanish, its effect can thus be eliminated from a single set by taking observations in which such reversals are periodically made. In ballistic methods this torque will either aid or oppose the gyromagnetic torque, hence the error due to it will tend to vanish from a mean result obtained from many observations in which the rotor is repeatedly reset in slightly different orientations at random. In resonance methods this process will eliminate the error only if the quadrature torque is annulled, or its effect on the amplitude negligible.

§38. Early experiments on rotation by magnetization[39]

The first experiments to yield any results on this effect were made on iron by A. Einstein and W. J. de Haas[40] early in 1915 by the methods of resonance (1) and (2), §26. They obtained for the magnitude of ρ a value close to $2m/e$, which they judged correct to within some ten percent; and they thought they had proved its sign to be negative. But Lorentz[41] soon pointed out that the experiments were not sufficient to determine the sign. In 1916 Einstein[42] and de Haas,[42] working

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[38] See S. J. Barnett, Phil. Mag. **12**, 349 (1931) (*A New Electron-Inertia Effect and the Determination of m/e for the Free Electrons in Copper*).

[39] e/m is assumed as 1.77×10^7 e.m.u.

[40] A. Einstein and W. J. de Haas, reference 11.

[41] See A. Einstein, Verh. d. D. Phys. Ges. **17**, 203 (1915).

[42] A. Einstein, Verh. d. D. Phys. Ges. **18**, 173 (1916); W. J. de Haas, Verh. d. D. Phys. Ges. **18**, 423 (1916).

separately, made resonance experiments, chiefly qualitative, and at very low frequencies, of the order of one per second or two, in which they found ρ negative and of the order of magnitude determined in their joint work.

The work of Einstein and de Haas was limited to comparatively few experiments on iron. The first extensive investigation of their effect, on both iron and nickel, was made by the ballistic method in 1918 in Richardson's former laboratory at Princeton by J. Q. Stewart,[43] who found for iron $\rho = m/e \times 1.02$ and for nickel $\rho = m/e \times 0.94$, with a mean error of about 15 percent—values equal, within the experimental errors, to that found in 1914 by the author of this paper in the work on magnetization of iron by rotation, and equal to half the value found by Einstein and de Haas. This investigation was a thorough one, and one of the few in which the vertical intensity of the earth's magnetic field was annulled, and in which a deliberate effort was made to eliminate the effects of magnetostriction.

In 1919 E. Beck,[44] by resonance methods (1) and (2), found $\rho = 1.06 \times m/e$ for iron and $\rho = 1.14 \times m/e$ for nickel; but the errors were such that he considered the excess of the numbers above m/e to be meaningless. In the same year Arvidsson,[45] by resonance method (2), and annulling all the components of the earth's field, found for iron $\rho = 0.94 \times m/e$.

§39. Later experiments on the rotation of ferromagnetic substances by magnetization. Experiments of Chattock and Bates

A more elaborate and precise investigation than any one of those mentioned in the last article (§38), by the ballistic method, was published in 1923 by Chattock and Bates.[46] A great deal of attention was devoted to the elimination of possible systematic errors, though the vertical intensity of the earth's field was not annulled. The mean values of $\rho e/m$[47] obtained for iron and nickel are 1.005 and 1.01, respectively. The value of ρ calculated from the formula (25–3) for a given moment of inertia, was found to be a linear function of $1+\lambda$, as shown in Fig. 39–1. The upper curve B gives the observations when the moment of inertia of the system was large (proportional to the numbers 7 to 15 below the curve); the lower curve A, when the moment of inertia was much smaller (from 1 to 3.6 on the same scale). The true values of ρ were assumed to be those given by extrapolation from the curves to $\lambda = 0$. The observations were numerous, and on the average each point in the diagram corresponds to about 105 throws (more than twenty-five to a set) for adjacent values of $1+\lambda$. The average departure from its curve for a point in group B is about 2 percent; for a point in group A about 1.5 percent. Discrepancies between the individual observations were very much greater. The values obtained for ρ were found to be independent of the intensity of the magnetizing field, as in other investigations.

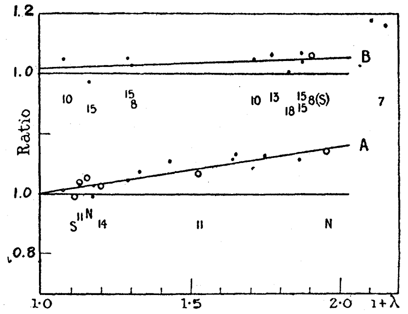

Fig. 39–1.

§40. Experiments of Sucksmith and Bates, and of Sucksmith

The work described in the last section was followed in the same year by another elaborate investigation on iron, nickel and Heusler's alloy, in which, by Chattock's null method, Sucksmith and Bates[48] found for the three substances, respectively, $\rho = 1.006 \times m/e$, $\rho = 1.002 \times m/e$, and $\rho = 1.002 \times m/e$. In 1925 Sucksmith[49] found by the same method for cobalt and magnetite, respectively, $\rho = 1.03 \times m/e$ and $\rho = 0.99 \times m/e$. The authors believe that in the first of these

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[43] J. Q. Stewart, Phys. Rev. **11**, 100 (1918).

[44] E. Beck, Ann. d. Physik **60**, 109 (1919).

[45] G. Arvidsson, Physik. Zeits. **21**, 88 (1920).

[46] A. P. Chattock and L. F. Bates, Phil. Trans. Roy. Soc. **A233**, 257 (1923).

[47] e/m is assumed as 1.77×10^7 e.m.u.

[48] W. Sucksmith and F. L. Bates, Proc. Roy. Soc. **A104**, 499 (1923).

[49] W. Sucksmith, Proc. Roy. Soc. **A108**, 638 (1925).

TABLE 40–1. *Gyromagnetic ratios from the work of Sucksmith and Bates, and of Sucksmith*†
(e/m assumed as 1.77×10^7 e.m.u.)

Material	Freq. (cycles/sec.)	Approx. impressed max. mag. int. (gauss)	No. sets	Range of $\rho\cdot e/m$ (e.m.u.)	$\rho\cdot e/m$ (e.m.u.)	Approx. vibration double ampl. (mm)
Iron	31–78	90	16	0.974–1.028	1.006±0.012	10–20
Nickel	38–50	45–176	6	0.988–1.010	1.002±0.004	5–10
Heusler alloy	21–40	176	14	0.988–1.014	1.002±0.006	5–8
Cobalt	33–46	225	9	0.916–1.160	1.030±0.064	1
Magnetite	27–59	225 (?)	10	0.894–1.146	0.990±0.048	0.7

† Sucksmith has recently found (Nature **134**, 936 (1934)) for three nickel alloys (*ca.* 56 percent Ni) above the Curie point the value $\rho.e/m=1.05$ ±10 percent.

investigations the results are correct to within one percent and that all indicate for $\rho\cdot e/m$ the value 1.00. The chief results are collected in Table 40–1.

The rods, of compressed powder in the last two cases, were 15.2 cm long and varied in diameter from 1.65 mm to 3.4 mm.

As in the work of Chattock and Bates, a great deal of care was taken to eliminate systematic errors, though the vertical intensity of the earth's magnetic field was not compensated and the effects of magnetostriction were not considered. As the author's work has abundantly proved, these are often, but not always, of much importance. The data published with respect to some of the constants and measurements are insufficient to enable the reader to form an adequate independent judgment of the magnitude of the errors.

§41. Barnett's experiments on rotation by magnetization

On account of the discrepancies of several percent between the results of the work just described and those of S. J. and L. J. H. Barnett on magnetization by rotation, the author, several years ago, undertook to prosecute further an investigation of the Einstein and de Haas effect which had been started many years before, but temporarily abandoned on account of more pressing work.

In this work,[50] which is very elaborate, multiple-torque resonance methods have been used. Great pains have been taken to discover and eliminate systematic errors. The resulting values of $\rho\times e/m$ from the most reliable series of observations are given in Table 41–1.

Each set of observations contained, in addition to the numerous readings necessary to determine the proper values of the currents to annul the earth's magnetic intensity, and other subsidiary observations, either 48, 72 or 96 determinations of amplitude (according to the exact method used), all made on a strict time schedule and in such a way as to eliminate all the systematic errors discussed above as nearly completely as the methods described permitted.

The mean value of $\rho\cdot e/m$ for all the materials is 1.047, which agrees remarkably with the mean results on the Barnett effect given in §§22 and 23.

Forty-one earlier sets on permalloy and iron made in nearly the same way but without so strict a time schedule gave nearly the same mean results ($\rho\times e/m=1.050\pm0.013$ for permalloy, and 1.034 ± 0.008 for iron). Also, more than 150 sets obtained before deliberate precautions were taken to eliminate the effects of cycle asymmetry gave nearly the same mean results.

So far as the error in ρ is determined by errors in the knowledge of the constants involved, it is estimated as much less than one-half percent. All the constants have been repeatedly determined.

In nearly all of the work whose results are quoted, and in most of the others, the magnetizing solenoid was wound rigidly on the rotor, so that the total momentum was certainly measured. The experiments made on electrolytic iron with the magnetizing coil fixed to the earth agree

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[50] S. J. Barnett, Proc. Am. Acad. **66** (8), 273 (1931); Phys. Rev. **42**, 147 (1932). Earlier reports in Phys. Rev. **30**, 964 (1927); Phys. Rev. **31**, 1116 (1928); Phys. Rev. **36**, 789 (1930); also Proc. Am. Acad. **69**, No. 2, 119 (1934).

TABLE 41–1. *Gyromagnetic ratios from Barnett's work on rotation by magnetization.* ($e/m = 1.757 \times 10^7$ e.m.u.)

Material	Freq. (approx.)	Impressed mag. int. (approx.)	No. sets	$\rho \times e/m$ (e.m.u.)
Armco iron	$10 \frac{\sim}{\text{sec.}}$	15–30 gauss	25	1.031 ± 0.003
Electrolytic iron*	9	20	7	1.031 ± 0.007
	9	20	2†	1.028 ± 0.002
Nickel	5.4 to 12.8	40	60	$1.05 + \pm < 0.01$
Permalloy	10	20	17	1.043 ± 0.003
(Ca 80% Fe, 20% Ni)	19	35	10	1.041 ± 0.005
Iron-nickel (75% Fe, 245% Ni)*	5	40	8	1.015 ± 0.012
H. K. Cobalt*	10	40	5	$1.096 + 0.002$
(99.9% pure)	10	40	5	$1.081 + 0.016$
W. E. Cu-cobalt (92.4% Co)	5.5	40	12	$1.068 + 0.006$
Cobalt-nickel* (Ca 54% Co, 45% Ni)	9	20	4	$1.087 + 0.038$
Cobalt-iron* (Ca 34% Co)	3	20 and 40	5	$1.009 + 0.007$

The double-amplitudes ranged from 7 mm for pure cobalt to over 10 cm for Armco iron, and 13 cm for Cobalt-iron with and intensity of 40 gauss. In some of the earlier work on Armco iron the double amplitude reached 17 cm.

* Data not hitherto published. The electrolytic iron, cobalt-nickel, and cobalt-iron rods were cut from the material used in the researches of Barnett and Barnett.

† Made with the magnetizing coil fixed to the earth.

nearly with the others. An additional series of 31 sets, with four different values of the current, made especially for this purpose, with the coil in alternate sets moving and fixed, gave a small systematic difference not yet accounted for by any error, the ratio for the moving coil being consistently about 1 percent greater than for the fixed coil. A more recent, but much shorter, series gave a similar unexplained difference in the opposite direction. Numerous other observations, including many on iron and permalloy, made in the early part of the investigation with the magnetizing coil fixed to the earth, agree nearly with the others; but they are certainly less free from errors, since they were made before precautions were taken to eliminate systematically from every set the effects of cycle asymmetry.

Nearly all these measurements were made after midnight, when disturbances of all kinds were least. The apparatus was mounted on springs and provided with suitable damping devices.

The construction of most of the rotors used is illustrated in Fig. 41–1. The magnetic rods were always either 20 cm or 27.5 cm (approximately) in length, and ranged from 1/16 inch to 1/8 inch in diameter. Some of the rotors were similar, but without windings. A third, and more complex, type, used in a few cases to reduce magnetostrictive effects, is illustrated in Fig. 41–2. The magnetic rod was mounted coaxially inside a brass tube of somewhat greater length. Three small brass rings with internal diameter just greater than the diameter of the rod, and external diameter just less than the internal diameter of the tube, were soldered around the rod, coaxially, at the center and ends. The central ring was soldered to the tube also. The center of the rod was thus fastened rigidly to the tube, while the ends were free to slide longitudinally in the tube. The magnetizing coil was wound on the brass tube. This type of construction was adopted with the hope that when the magnetostrictive oscillations occurred, the center of mass of the rod would remain fixed and the disturbance would not be communicated to the brass tube and suspension. In at least one case the behavior of the rod under investigation was very greatly improved by adopting this type of construction.

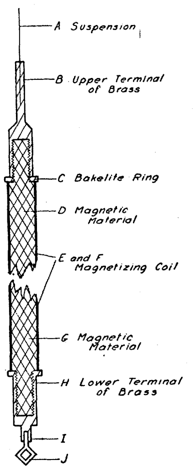

FIG. 41–1. Illustrating construction of typical wound rotor. In most of the rotors the ring C was of brass continuous with 13.

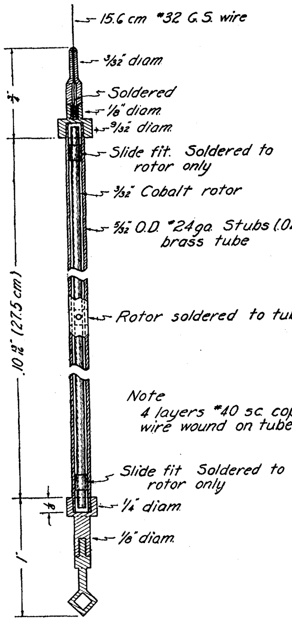

FIG. 41–2. Illustrating construction of special rotors designed to eliminate the effects of magnetostriction.

§42. Sucksmith's experiments on paramagnetic substances

Resonance method (1) has been applied by Sucksmith[51] to salts (*viz.*, Cd_2O_3, Nd_2O_3, Eu_2O_3, Dy_2O_3) of some of the elements of the rare earth group and to salts (*viz.*, $FeSO_4$, $CoCl_2$, $CoSO_4$, $CrCl_2$, $MnCO_3$, $MnSO_4$) of most of the elements of the iron group. The magnetic moments acquired by the specimens even in the strong fields used (up to 1200 gauss) were so small that it was necessary to reduce the frictional torques by using very low frequencies (down to 1/3 cycle per second, flat-topped waves), by making the suspension of fused quartz, and by inclosing the vibrating body in an evacuated tube. Even then the double amplitudes of vibration were very small, ranging (approximately) from 1 mm to $4\frac{1}{2}$ mm. In addition to the precautions which were taken in the investigations by Sucksmith,[49] and Sucksmith and Bates,[48] in their work on ferromagnetic bodies care was taken to eliminate the effects of ferromagnetic impurities and of electric charges.

From the experiments Sucksmith obtained the values of ρ, and thus values of Landé's splitting factor $g=(2m/e)\cdot 1/\rho$. He then compared these with the values of g calculated from the theories which were found to be in best agreement with them, on the most probable assumptions with reference to the states of the ions. The state is doubtful for one ion in each group. For the others, the agreement of experiment with Van Vleck's[52] modification of Hund's theory in the case of rare earths, and with Stoner's[53] theory in the case of the iron group, is within the limits of the experimental error of (usually) five to ten percent.

§43. Coeterier :and Scherrer's experiments on pyrrhotite and iron[54]

In these experiments flat-topped waves with long periods—of the order of 5 seconds—were used,

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[51] W. Sucksmith, Proc. Roy. Soc. **A128**, 276 (1930); Proc. Roy. Soc. **A133**, 179 (1931); Proc. Roy. Soc. **A135**, 1932, p. 276.

[52] J. H. Van Vleck, Phys. Rev. **31**, 587 (1928); Van Vleck and Frank, Phys. Rev. **34**, 1494, 1625 (1929).

[53] E. C. Stoner, Phil. Mag. **8**, 250 (1929).

[54] F. Coeterier and P. Scherrer, *Eine neue Methode zur Messung des Einstein-de Haas-Effektes.* Helv. Physika Acta **5**, 217 (1932); F. Coeterier, *Einstein-de Haas-Effekt an Pyrrhotin*, Helv. Physika Acta **6**, 483 (1933).

and sharp resonance was automatically produced and maintained with the help of a photoelectric cell device and a polarized relay. The cell was operated by a very narrow beam of light reflected from a mirror attached to the vibrating rod and striking the cell only at the instants at which the rod was in the center of its vibration. Through the relay the cell reversed the current at these instants. The arrangement has not only the advantage of maintaining sharp resonance but also that of automatically eliminating the effect of torques in quadrature with the gyromagnetic torque. Few details of the experiments are given. The specimens were hung in quartz fibers and vibrated in vacuum.

Thirty measurements on four specimens of pyrrhotite were made. They were prepared by packing the powdered substance in small tubes placed in a magnetic field parallel to the intensity so that the particles might align themselves along the direction of easiest magnetization. The substance is paramagnetic except in one plane, as Weiss showed long ago. The four results are $g=0.62$; 0.63; 0.63; 0.64.

This result may be accounted for as follows:[55] Assume one d electron, or several independent d electrons, per atom, with fairly large spin-orbit interaction. Ferromagnetic saturation being assumed, the lowest energy states will be those in which the orbital momenta are oriented antiparallel to the electron spins. Each m_s is then $-1/2$, and each m_l is 2; hence the value of $g=(2m_s+m_l)/(m_s+m_l)=2/3$. Incomplete orientation of the orbits would decrease the value of g, thus accounting for Coeterier's $g=0.63$. The brief published data are insufficient to show whether this deviation is real or due to experimental error.

Coeterier and Scherrer give as the result of a provisional measurement on iron, $g=2.01$, or $\rho=m/e\times 0.995$; and Coeterier[56] has obtained for iron powder the result $\rho=1.01\times m/e$.

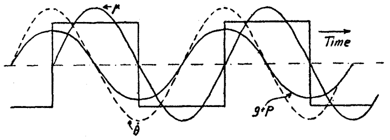

FIG. 43–1. Magnetic moment, torque and angular velocity of rotor as functions of the time.

In Fig. 43–1 the magnetic moment as a function of the time is represented by the flat-topped wave, and its first harmonic by the sine curve marked μ. The gyromagnetic torque, or this torque plus any additional in-phase or opposition torques, is represented by the curve marked $(g+p)$ which leads μ by a quarter period. The damping is assumed, consistently with experiment, to be proportional to the angular velocity $\dot{\theta}$ of the rotor. Then we have

$$\dot{\theta}=\alpha(g+p)=\Theta\cos\omega t$$

in phase with $(g+p)$. In the experiments of Coeterier and Scherrer this relation between $\dot{\theta}$ and $(g+p)$ is clearly maintained by the process described. Hence the mean value of the activity of any quadrature torque $q=Q\sin\omega t$ is

$$\overline{q\dot{\theta}}=Q\Theta\,\overline{\sin\omega t\cos\omega t}=(Q\Theta/2)\,\overline{\sin 2\omega t}=0.$$

Thus the quadrature torque produces no effect on the motion.

§44. Ray-Chandhuri's experiments on iron oxides[57]

These were made by resonance method (1) as modified by Sucksmith for work on paramagnetic substances (see §42). The substances were in powdered form and were packed in thin glass tubes from 1.3 mm to 2.3 mm in diameter and 6 cm long. The cylinders were vibrated in a vacuum in a magnetic field with intensity 230 gauss. The thinner cylinders gave vibration bands 3 cm wide at 1.3 m scale distance. The errors are estimated as probably not greater than 2 percent. Few experimental details are given, and it is not possible to make an independent estimate of the error. The results obtained for $\rho\times e/m$ are as follows: Fe_3O_4 (precipitated), 1.008; Fe_2O_3 (ferromagnetic), 1.016; NiO Fe_2O_3, 1.022.

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[55] D. R. Inglis, Phys. Rev. **45**, 118 (1934).

[56] *Amsterdam Thesis,* 1933; quoted by L. F. Bates, Nature **134**, 50 (1934). In the quotation no details are given on which an estimate of the error can be based.

[57] D. P. Ray-Chandhuri, Nature **130**, 891 (1932).

E. GYROSCOPIC MAGNETIZATION BY ROTATING FIELDS

§45. Gyroscopic magnetization by rotating fields (see §3, (4))

The gyromagnetic effects considered in what precedes, while they give most fundamental information on the nature of the magnetic elements, show nothing about the *process* of magnetization, which is still obscure and uncertain. There are indications that in the early part of the process, i.e., in weak fields, the magnetization proceeds by quantum *jumps*, or *reversals*, of the elements, and not by continuous changes in direction, as required by the classical theories; and on this basis Einstein predicted a null effect. The investigations by Fisher and by the author, referred to in §3, and made with weak fields, are consistent with this hypothesis. It was shown by the author in 1925,[58] however, on the hypothesis most favorable to an explanation of the results by rotation of the elements, that Fisher's expectation of the magnetization which would result if rotation occurs should be multiplied by a small factor equal, as to order of magnitude, to 3/2 times the ratio of the cross-magnetization to the saturation magnetization of the material. Fisher's combined errors were of the same order as this reduced result. The expectation on the rotation hypothesis should probably be reduced much further because the torques opposing axial alignment of the elements are probably much greater (as Epstein has suggested) when they are rotated with respect to the remaining atomic structure than when they and the rest of the structure all rotate together. In the (very rough) calculation referred to the torques opposing alignment were assumed identical in the two cases.

Fisher's chief experiments were made on powdered iron and magnetite and solid magnetite; the author's chief experiments on Bell Telephone compressed iron dust and permalloy dust. Both were made by magnetometer methods analogous to that of the author's later work on magnetization by rotation.

In the author's work, with impressed fields of about 15 gauss rotating about 15,000 revolutions per second, the mean magnetometer deflection on reversal of the direction of rotation (all corrections being applied) was about 0.2 mm (in the wrong direction), with a mean error of about 1 mm; while the deflections predicted from the modified formula mentioned above are about 2.0 cm and 0.7 cm for permalloy and iron, respectively. Fisher's rotation hypothesis formula (or the Barnett effect) would give at the same frequency about 5 meters and $3\frac{1}{2}$ meters, respectively. Similar results were obtained with permalloy dust at the frequency 21,000 per second. Systematic errors due to residual magnetization of the rods, to heat, to leakage between the plate circuits of the vacuum tube generator and the magnetizing coils (linked together with transformers), and to eddy currents were all eliminated.

Part II. Electron-Inertia Effects[59]

A. INTRODUCTORY AND HISTORICAL

§46. The work of Maxwell and his followers

In Maxwell's *Treatise on Electricity and Magnetism* (§§574, 575 and 577) he describes three inertia effects which should exist in conductors if the electric current is due to the motion of one kind of electricity only and if this electricity has inertia. Special cases, as follows, will suffice to illustrate these effects, on all of which Maxwell appears to have tried experiments, or to have known that such experiments had been tried by others:

(1) If the current in a circular conductor or a circular cylindrical coil of wire free to move about its axis is altered, the free electricity will be accelerated and the coil itself will be accelerated in the opposite direction, the changes of angular momenta being equal in magnitude and opposite in sign. After Maxwell, this effect was looked for

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[58] See reference 13.

[59] For some remarks on the relation between the phenomena discussed here and the phenomena of self-induction, which are evidently closely connected, reference may be made to Lorentz's *The Theory of Electrons*, 1909, §36 and Note 17. See also H. A. Lorentz, *Collected Papers*, vol. 8, pp. 316, 410.

by Sir Oliver Lodge[60] in or before 1892; but it was first discovered in an investigation by the author[61] in 1930. See §§55–58.

(2) If the coil of wire is traversed by a steady electric current the electricity has a constant angular momentum about the axis, and the coil should exhibit the properties of a gyroscope. Maxwell looked for this effect in or about 1861, but without success because of great experimental difficulties (see above, §§3, 7). The author's experiments on magnetization of iron by rotation in 1914 (see above §3 and §§8–23) established the same effect for the individual whirls of Ampère, each of which behaves as Maxwell's coil would behave if suitable conditions could be obtained.

(3) If the coil of wire is accelerated about its axis the free electricity will be differently accelerated, lagging behind when the coil's speed is increased, and going ahead when the speed is decreased. Thus the acceleration of the coil gives rise to an electric current in it. This is the effect discovered by Tolman and Stewart in 1916 and investigated in four papers by Tolman and Stewart,[62] Tolman, Karrer and Guernsey[63] and Tolman and Mott-Smith.[64] See §§49–53.

Before the discovery of effect (3) by Tolman and Stewart it was looked for by centrifugal methods in two investigations, one by P. Lébedèw (§47), the other by E. F. Nichols (§48).

B. CENTRIFUGAL EXPERIMENTS OF LÉBEDÈW AND OF E. F. NICHOLS

§47. Experiments of P. Lébedèw[65]

In these experiments, made in Lébedèw's last investigation, and intended as preliminary to more extensive work, prevented by his death, toroidal rings of ebonite, brass, water and benzol, all nonmagnetic, were rotated about their axes at a frequency of about 500 r.p.s. The rings were 2 cm thick and had inner and outer diameters of 3 cm and 6 cm, respectively. Lébedèw assumes that the positive parts of the atoms remain radially fixed, while the electrons are centrifugally displaced; also that the radial electron displacement is proportional by a constant K, depending on the nature of the material, to the radius and the square of the frequency. Hence he calculates the distribution of the electric convection current in the toroid. A "current model," of the same size and shape as the rings, was so wound with a continuous wire that when traversed by an electric current this current would have approximately the same distribution as the convection current in the toroid. The ring and the model were so placed that their magnetic fields would act in the same way on an astatic magnetometer, and the deflections due to the rotation and to the current in the model were compared.

Making use of his own centrifugal formula, and of Kelvin's similarity theorem, Lébedèw shows that if on two similar rotating bodies of the same material two corresponding points have the same linear velocity, the magnetic intensities they produce at two corresponding points in space will be identical and proportional to the cube of this velocity.

From this it is shown that at the equator of a sphere 6 cm in diameter and making 500 revolutions per second the magnetic intensity should be about one-hundredth the intensity at the earth's equator—*provided* that the material of the sphere is identical with that of the earth, and *provided* that K is the same for the sphere as for the earth, although its equatorial centripetal acceleration is about 10^7 times as great. Lébedèw considers that actually K for the sphere is *very much smaller* on account of the tremendous difference between the accelerations.

It was found that a current of 0.1 ampere in the current model would produce a magnetic intensity equal to that due to the earth. The linear velocity at the equator of the ring was about 0.2 that at the earth's equator. Thus the magnetic intensity due to the rotating ring, on the above theory, should be equal to that produced by a current $0.1\times(0.2)^3$ ampere = *ca.* 0.001 ampere, i.e., about one-hundredth of the earth's intensity. This current in the model actually produced a magnetometer deflection of 10 scale divisions, while no appreciable deflection resulted from the rotation of any one of the rings.

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[60] O. J. Lodge, *Modern Views of Electricity*, 2nd ed., 1892, p. 97.

[61] S. J. Barnett, Phil. Mag. **42**, 349 (1931).

[62] R. C. Tolman and T. Dale Stewart, Phys. Rev. **8**, 97 (1916); Phys. Rev. **9**, 164 (1917).

[63] Tolman, Karrer and Guernsey, Phys. Rev. **21**, 525 (1923).

[64] Tolman and Mott-Smith, Phys. Rev. **28**, 794 (1926).

[65] P. Lébedèw, Ann. d. Physik **39**, 840 (1912).

§48. Experiments of E. F. Nichols[66]

If a metal disk is rotated about its axis with a frequency ν r.p.s. the mobile electricity, with charge e and mass m per particle, will be driven outward, until equilibrium ensues when at a distance r from the center an electric intensity E (reckoned positive toward the center) is developed such that

$$E=(m/e)\cdot 4\pi^2\cdot \nu^2 r. \tag{48–1}$$

Integrating this intensity from the edge of the disk (radius R) to the center we obtain for the potential difference from edge to center (equal to the intrinsic electromotive force from center to edge)

$$V=(m/e)2\pi^2R^2\nu^2. \tag{48–2}$$

In Nichols' experiments the disk was of aluminum, R was about 10 cm, and ν was 100 r.p.s. or more. Thus (48–2) gives for electrons as carriers $V=ca.\ 10^{-8}$ volt *outward;* for protons as carriers, about 2×10^{-5} volt *inward.* With a galvanometer, and brushes applied near the edge and the center of the disk, attempts were made to measure the potential difference. Effects of the earth's magnetic field were at least largely eliminated. About 2000 scale divisions should have been obtained for protons, about 1 division for electrons. Irregular deflections of 500 scale divisions and more were obtained, the chief difficulties coming from thermal electromotive forces and contacts. The experiments indicate that the magnitude of m/e for the carriers is less than that of protons.

C. Ballistic Experiments of Tolman and Stewart[67]

§49. Details of the experiments

In these experiments a circular coil of wire, wound tightly on a rigid nonmagnetic bobbin, was rotated at high speed about its (vertical) axis, and was quickly brought to rest with suitable brakes. The ends of the coil were connected through long twisted wires to a galvanometer, the great difficulties involved in using sliding contacts thus being avoided. In circuit with the coil to be tested was another coil wound to have the same total area and placed parallel, but oppositely connected, so that fluctuations of the earth's magnetic field had a minimum effect on the galvanometer. The rotating coil was surrounded with a coil of wire so constructed and so oriented as approximately to neutralize, when traversed by a suitable electric current, the vertical intensity of the earth's field in the region occupied by the coil. This was desirable in order to eliminate the electromotive force which would have been induced in the coil by its centrifugal expansion across the vertical intensity. This electromotive force could also be eliminated by rotating the coil in both directions, since it is independent of the direction of rotation, while the effect under investigation is reversed with this direction. Both processes were used, for greater security.

In making an experiment the coil was first brought up to the full speed, and the galvanometer zero reading taken. Then the brakes were applied and the coil brought to rest (in a fraction of a second). Then the throw of the galvanometer was determined.

§50. Theory and results

A diagram of the circuit is given in Fig. 50–1. ABC is the rotating coil, D the galvanometer, and EFG the compensating coil. The arrowheads indicate the direction around the circuit here chosen as positive. Let l denote the length, and dl an element of length, of the uniform conductor ABC; s its cross section; V its linear speed (relative to fixed coordinates); c the current density; k the conductivity of the material; and $i=cs$ the current; v the velocity of an electron in ABC.

Then in any element dl of the rotating coil an electron with mass m and charge e will be acted upon by two forces as follows: one, Xe, due to the total electric field intensity X; the other, $-ce/k$, due to friction. The equation of

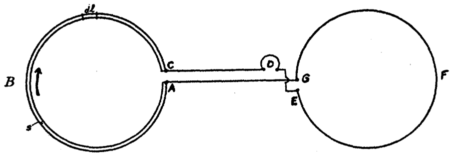

Fig. 50–1.

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[66] E. F. Nichols, Physik Zeits. 7, 640 (1906).
[67] See reference 62.

motion of the electron is thus

$$Xe - ce/k = m\,dv/dt. \qquad (50\text{–}1)$$

Dividing by e and integrating with respect to l from A through B to C, we obtain, on changing signs throughout,

$$cl/k - \int X\,dl = -(ml/e)(dv/dt). \qquad (50\text{–}2)$$

Let R_c and L_c denote the resistance and inductance of the coil ABC; R_g and L_g the resistance and inductance of CFA; and let U denote the potential difference from A to C through B. U is also the potential difference for A to C through F, so that we have

$$\begin{aligned} -X\,dl &\equiv -(U - L_c(di/dt)) \\ &= R_g i + L_g(di/dt) + L_c(di/dt). \qquad (50\text{–}3) \end{aligned}$$

The first term of (50–2) is evidently equal to $R_c i$. Thus this equation may be written

$$(R_c + R_g)i + (L_c + L_g)\frac{di}{dt} = -\frac{ml}{e}\frac{dv}{dt}. \qquad (50\text{–}4)$$

If the conductor were maintained at rest, and the same values of i and di/dt were produced by means of a battery or other generator with intrinsic electromotive force E, we should have in place of (50–4) the equation

$$(R_c + R_g)i + (L_c + L_g)di/dt = E. \qquad (50\text{–}5)$$

Thus the acceleration of the electrons in the conductor produces in it an intrinsic electromotive force

$$E = -(ml/e)(dv/dt). \qquad (50\text{–}6)$$

The time integral of Eq. (50–4) from an instant at which the coil and its electrons have the steady linear velocity $v = V = V_0$ to the time at which the coil has come to rest and the current has ceased ($v = V = 0$) is evidently

$$RQ = (ml/e)V_0, \qquad (50\text{–}7)$$

where R is written for $R_c + R_g$, the total resistance in the circuit, and Q for $\int i dt$, the complete discharge through the galvanometer. For m/e we thus obtain

$$m/e = RQ/lV_0. \qquad (50\text{–}8)$$

Six hundred and twenty-four runs were made on a number of different coils, with wire of three different materials (copper, aluminum and silver) and two different sizes, and with two different kinds of insulating binder to hold the coils in place. The resistance in the circuit varied from 27 ohms to 63 ohms; the length of the wire in the coil, from 285 m to 529 m; the linear velocity, from 19.8 m/sec. to 56.4 m/sec. Many runs were made for each direction of rotation. The electric pulse through the circuit was invariably in the direction predicted on the basis of the negative electron as carrier and concordant results were obtained for each substance by calculation from the above formula. For copper, aluminum and silver, respectively, m/e was found to have the standard value for free electrons multiplied by 1.11, 1.16 and 1.20. Inasmuch as the excess of m/e above the standard value for free electrons was found to be less for different coils the more rigidly they were bound to the bobbin, an obscure systematic error, arising from lack of rigidity, and in such a direction as to render m/e too large, was suspected. See below, §53, for experiments which appear to confirm this suspicion.

D. Experiments of Tolman, Karrer and Guernsey,[63] and of Tolman and Mott-Smith[64]

§51. General plan and theory

In each of these investigations, of which the second is an extension of the first, two principal operations were performed: (1) A circular hollow copper cylinder, mounted with its axis practically parallel to the earth's magnetic intensity, was harmonically oscillated about this axis with a steady amplitude A and frequency $\nu = \omega/2\pi$, and an alternating current thus produced therein on account of electron-inertia. Around the copper cylinder and coaxial with it was a fixed cylindrical coil of many turns of fine wire. The electromotive force E_a induced in this coil by the electron-inertia current was measured by processes described below. (2) The same copper cylinder, inside the same coil and again coaxial with it, was mounted with its axis normal to the earth's intensity, and was oscillated about an axis normal to both with the same frequency ν and with amplitude B. The electromotive force E_b in the coil due to the current produced in the cylinder by electromagnetic induction was measured as in (1).

It is clear that the current density will have exactly the same distribution in both cases, and likewise the electric intensity, also that the streamlines and lines of intensity will be circles coaxial with the cylinder and coil. Imagine the cylinder divided up into a series of coaxial tubes each of very small thickness dr, and consider any one of these tubes, with mean radius r. In such an operation as (2) the oscillation will produce around every circle of radius r the electromotive force $E=-d\phi/dt$, where $d\phi/dt$ is the rate of increase of the magnetic flux ϕ through the tube due to the motion in the earth's field. If i denotes the (circular cylindrical) current in the tube, R its resistance, and L its inductance, the total force on an electron, with mass m and charge e will be

$$(E-Ldi/dt-Ri)e/2\pi r=mdv/dt. \quad (51\text{–}1)$$

If n is the number of mobile electrons per unit volume and a the area of the cross section of the conductor, $i=neav$; and the above equation may be written

$$E=Ri+(L+2\pi rm/ne^2a)di/dt. \quad (51\text{–}2)$$

In operation (1), in which the tube just considered is oscillating about its axis, $E=-d\phi/dt=0$; and we shall have, in place of (51–1), *for the same current i and the same rate of change di/dt*,

$$(-Ldi/dt-Ri)e/2\pi r=mdv'/dt, \quad (51\text{–}3)$$

where v' is now the electron velocity. In this case, however, since the tube is moving with the tangential velocity V, we have $i=nea(v'-V)$. Eliminating v' by this relation, we obtain in place of (51–3) the equation

$$\frac{-2\pi rm}{e}\frac{dV}{dt}=Ri+\left(L+\frac{2\pi rm}{ne^2a}\right)\frac{di}{dt}. \quad (51\text{–}4)$$

A comparison of this equation with (51–2) shows that the acceleration of the conductor produces an electromotive force

$$E_e=-(2\pi rm/e)dV/dt \quad (51\text{–}5)$$

proportional to the acceleration, in phase with it if e is negative, and in opposite phase if e is positive.

Let us now apply operations (1) and (2) to this tube. In (1), if the angular displacement is written $\theta=A\sin\omega t$, we obtain

$$E_e=-\frac{2\pi rm}{e}\frac{dV}{dt}=\frac{2\pi r^2\omega^2mA}{e}\sin\omega t. \quad (51\text{–}6)$$

In (2), if H is the earth's total magnetic intensity, and if the angular displacement is written $\psi=B\sin(\omega t+\delta)$, we obtain

$$\varphi=\pi r^2HB\sin(\omega t+\delta)$$

and

$$E=-d\phi/dt=-\pi r^2HB\omega\cos(\omega t+\delta). \quad (51\text{–}7)$$

Thus, for the tube of radius r, we obtain, at any time t,

$$\frac{E_e}{E}=\frac{2A}{B}\frac{m}{e}\frac{\omega}{H}\frac{\sin\omega t}{\sin(\omega t+\delta-90°)}. \quad (51\text{–}8)$$

Since this expression is independent of r, it is identical for all the elementary tubes into which we imagine the cylinder divided, and is therefore the ratio of the complete electromotive forces E_a and E_b induced in the coil in the two operations by the processes occurring in the *complete* oscillating body.

For the ratio R of the maximum value of E_a to that of E_b we have

$$R=E_a(\max)/E_b(\max)=2Am\omega/BeH. \quad (51\text{–}9)$$

§52. Details of the experiments

In the experiments of Tolman, Karrer and Guernsey, the coil was connected through a vacuum tube amplifier and transformer to a vibration galvanometer tuned to the frequency of the cylinder, and R, and with it m/e, determined from the ratio of the vibration amplitudes in the two operations. The sign of e was not determined. In the investigation of Tolman and Mott-Smith both the magnitude and the sign of m/e were determined. In this latter work (see Fig. 52–1) an earth inductor, connected in series through slip rings and brushes with the terminals of a fixed uniform wire provided with two sliding contacts, was driven in synchronism with the coil in both operations (1) and (2).

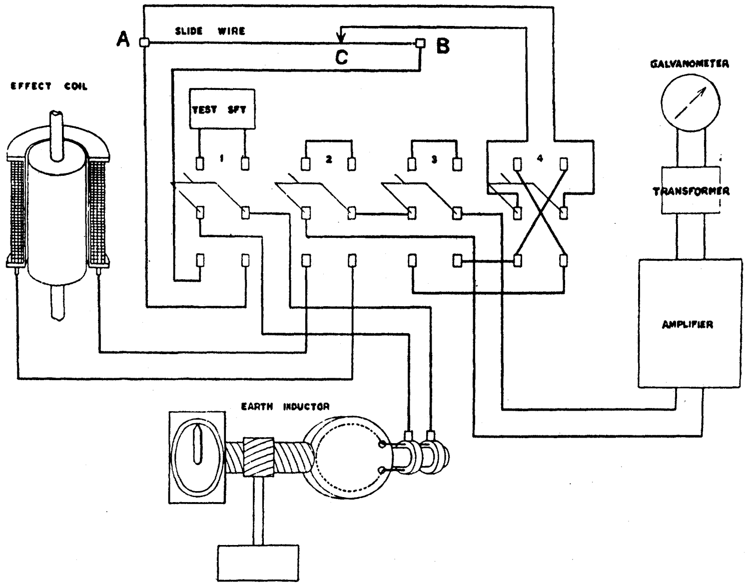

FIG. 52-1.

In each operation one end of the coil was connected to one of the sliding contacts, the other end of the coil and the other sliding contact to the input of the amplifier. By a spiral gear device the phase of the alternating potential difference along the rheostat relative to that produced in the coil by the oscillating cylinder could be varied at will and continuously while operations were in progress; likewise the magnitude of the potential difference between the sliding contacts could be varied at will by simply changing the distance between them. In each operation the two adjustments were made together until the vibration amplitude of the galvanometer was annulled. The lengths of the wires at balance were proportional to E_a and E_b; and the phase relations between the electromotive forces produced by the earth inductor and the vibrating cylinder in operation (2) were known. Hence, since the phase relation between dV/dt, the acceleration of the osclliating cylinder in operation (1), and the electromotive force in the earth inductor circuit was known, the relation between the phases of E_a and $-dV/dt$ became known.

§53. Systematic errors and their elimination

All the systematic errors which the authors could think of were carefully studied and were shown to be either negligible or very nearly so. Many of the possible errors are due to the presence of the earth's magnetic field; and most of these would vanish completely if the axes of the vibrating cylinder and detecting coil were exactly parallel to the earth's intensity. It is shown in the paper that the possible deviations from parallelism due to imperfections in mounting and construction, to vibration of coil support, to play in cylinder bearings, and to asymmetry of the vibrating cylinder produce either negligible or small effects. Expansion of the cylinder due to centrifugal action in the earth's field also produces a small electromotive force (about 5 percent of the electromotive force being looked for) but of twice the frequency of the cylinder, and without effect on the galvanometer. The phase relations predicted from the mechanical connections of the apparatus and well-established theory, and also the simple-harmonic character of the vibrations, were verified by optical methods. By special earthing tests it was shown that any electric charges present had no effect. Suitable precautions were taken to eliminate the effects of troublesome extraneous electric circuits in the neighborhood. As a further check on eddy current effects tests were made with a laminated cylinder constructed of copper rings separated by thin rings of Bakelite micarta, but the results proved very unreliable. It was concluded that the interaction between the rings was more likely to increase the electromotive force than to decrease it, and that in this way the large values of m/e obtained in the investigation by Tolman and Stewart might perhaps be explained. (See §50.)

§54. The results

The investigation of Tolman, Karrer and Guernsey gave a mean value of m/e 8 percent less than the standard value for free electrons; the more elaborate investigation of Tolman and Mott-Smith, a value 19 percent less than the standard value; while that of Tolman and Stewart gave a value 15 percent greater than the standard value (see §50). In the work of Tolman and Mott-Smith the phase of the electromotive force due to electron-inertia lagged about 10^0 behind that of $-dV/dt$, while the two quantities should be cophasal according to the theory. No explanation of this discrepancy was found.

E. BARNETT'S EXPERIMENTS ON ELECTRON-INERTIA[68]

§55. The experimental methods (see §§27, 41)

These experiments were made by methods almost exactly similar to those of the author's experiments on rotation by magnetization in which the magnetizing coil is wound rigidly on the rotor. The magnetic cylinder is simply replaced by brass or glass, preferably the latter in order to eliminate possible electron-inertia effects in the cylinder. The author's principal experiments were made with a copper coil wound on glass, the whole rotor being of the same length as the majority of the magnetic rotors, but of somewhat greater diameter than those of the largest magnetic rotors.

§56. Theory of the null method

When the vibration is *annulled*, we have, exactly as in §28, Part I,

$$-\rho=\Gamma\gamma^{*}m_0X_0, \tag{56–1}$$

where now ρ is the ratio of the angular momentum of the free electricity in the coil (with circular turns) to the magnetic moment of the coil. For a free electron moving in a circle the ratio is $2m/e$, as seen above (Part I, §5).

There may, of course, be disturbing torques in quadrature or in phase with (or in opposition to) the electron-inertia torque, t. In the first case the amplitude can be reduced to a minimum, but not to zero, by changing the conductance, X; in the second it will come to zero, but the value X_0 inserted in Eq. (56–1) will give not ρ but $\rho+\Delta\rho$, where $\Delta\rho$ is the error.

If the amplitude A is observed as a function of X for a number of conductances on both sides of X_0, and reckoned negative on one side and positive on the other (since the torque changes sign at this value of X), and if A is plotted as a function of X, it is easy to show that the result will be a straight line, provided there are no disturbing torques in quadrature with t and that the effect of amplitude, discussed in the next paragraph, is negligible. If there is a torque in phase with t we shall have

$$-\rho+\Delta\rho=\Gamma\gamma m_0X_0,$$

where $\Delta\rho$ is the resulting error in ρ, as above. If the experiments can be repeated with the phase of the disturbing torque reversed, we shall have

$$-\rho-\Delta\rho=\Gamma\gamma m_0X_0',$$

where X_0' is the new value of the conductance for zero amplitude. Then

$$-\rho=\Gamma\gamma m_0((X_0+X_0')/2). \tag{56–2}$$

If there are torques in quadrature with t, a minimum amplitude not zero will result, as stated above, and the line will curve on each

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[68] S. J. Barnett, *A New Electron-Inertia Effect and the Determination of m/e for the Free Electrons in Copper*, Phil. Mag. **42**. 349 (1931).

[*] Some of the observations were made with a small bunched winding around the rotor near its center replacing the long induction solenoid. With this arrangement γ in the above formula must be replaced by $4\pi Z_2/l$, where Z_2 is the number of turns in the bunched winding and l is the length of the rotor coil.

side near the minimum, where it becomes discontinuous.

If the induction coil with bunched winding is used, in place of the induction solenoid, it is easy to show that the constant γ must be replaced by the constant $\gamma' = 4\pi Z_2/l$, where Z_2 is the total number of turns in the bunched winding and l is the axial length of the uniform winding on the rotor.

§57. General theory

To obtain a more general theory we may proceed as follows: Suppose the rotor coil to contain Z turns of thin wire with cross section a and mean radius r, and to be traversed by a current

$$i = I \sin \omega t. \qquad (57\text{–}3)$$

Let the number of free electrons per unit volume be denoted by n, and the angular amplitude of the rotor's vibration by Θ, its angular velocity $\dot{\theta}$ (in phase with the angular momentum j) being given by the equation

$$\dot{\theta} = \omega\Theta \sin (\omega t + \beta). \qquad (57\text{–}4)$$

If v denotes the electron velocity and V the velocity of the wire, the current in the coil will be $i = nea(v - V)$, so that

$$v = i/nea + V = i/nea + r\dot{\theta} \qquad (57\text{–}5)$$

and

$$\dot{v} = \dot{i}/nea + r\ddot{\theta}. \qquad (57\text{–}6)$$

Thus the torque on one electron, reckoned about the axis of symmetry of the coil, will be

$$rm\dot{v} = rm\dot{i}/nea + r^2 m\ddot{\theta} \qquad (57\text{–}7)$$

and the torque on all the electrons in the Z turns will be equal to this quantity $\times 2\pi ranZ$. Hence the total inertia torque on the rotor, equal in magnitude and opposite in sign to that on the electrons, will be

$$t = -(2m/e)\pi r^2 Z\dot{i} - (2m/e)\pi r^2 Z\, near\, \ddot{\theta}. \qquad (57\text{–}8)$$

In conformity with (57–3) and (57–4) this equation may be written

$$t = -(2m/e)\pi r^2 ZI\omega \cos \omega t - (2m/e)\pi r^2 Z\, near\, \omega^2\Theta \cos (\omega t + \beta) = T_1 \cos \omega t + T_2 \cos (\omega t + \beta). \qquad (57\text{–}9)$$

In the null method T_2 vanishes, and (57–9) gives the result already obtained (56–1).

In these experiments the ratio $T_2/T_1 = near\,\omega\Theta/I$ was very small for the amplitude produced by the inertia effect, *viz.*, about $\frac{1}{2} \times \frac{1}{2} \times 1/20 \div 460$ radian. Thus if we take $n = 10^{23}$, $e = 1.6 \times 10^{-20}$, $a = 5 \times 10^{-5}$, $r = 0.27$, $\omega = 2\pi \times 14.6$, $I = 0.01$, all in c.g.s. and e.m.u., and Θ as above, we obtain for the ratio about 5×10^{-3}. For the largest amplitudes used in the experiments (in the calibrating part of the deflection method), *viz.*, amplitudes of the order of 150 times that produced by the inertia effect, the ratio is of the order of unity.

When Θ or T_2 does not vanish it is easy to show that the relation

$$\beta = \cot^{-1}(T_2/T_1) \qquad (57\text{–}10)$$

holds approximately, so that t_2 is nearly in quadrature with t_1.

When the amplitude is small T_2 is negligible in comparison with T_1, and the null method may be replaced by an amplitude or deflection method, as in the case of the gyromagnetic experiments on rotation by magnetization.

§58. The results

Three sets of experiments made by the null-graphical method (§30 A, Part I) in favorable conditions gave $-10^7\rho = 1.10 \pm 0.03$, while the standard value of $-10^7\rho$ is 1.13. Three sets made by the deflection method of §30, *B*, (a), Part I, gave $|10^7\rho| = 0.87 \pm 0.05$; but they are considered much less trustworthy.

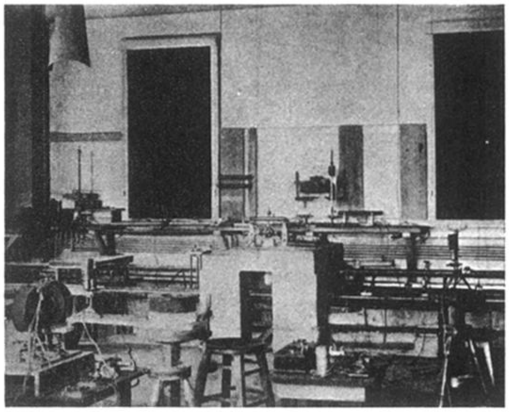

Fig. 13–3.