Solitons in optical communications

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The history of the proposal that solitons be used for optical fiber communications and of the technical developments toward making soliton transmission practical is reviewed. The causes of bit errors in long-distance soliton transmission are presented and the methods for reducing them are described. A perturbation theory suited for soliton analysis is developed. Current status and future prospects of long-distance repeaterless fiber communications are stated.

CONTENTS

I. INTRODUCTION

The soliton concept is a sophisticated mathematical construct based on the integrability of a class of nonlinear differential equations. Integrable nonlinear differential equations have one feature in common: they are all conservative and are thus derivable from a Hamiltonian. The integration is performed via the method of inverse scattering (Gardner *et al.*, 1967).

The nonlinear Schrödinger equation is one member of the class of integrable equations. In their classic paper, Zakharov and Shabat (1971) presented the mathematical framework, based on inverse scattering theory, for the integration of the nonlinear Schrödinger equation. Soon afterwards, Hasegawa and Tappert (1973) pointed out that solitons could be propagated on glass fibers with negative (anomalous) group velocity dispersion (GVD) under the influence of the optical Kerr effect, in which there is a change of refractive index induced by the optical field. The GVD in glass fibers with a Ge-doped core is controlled mainly by material dispersion, but it can be influenced somewhat by the index (doping) profile. For wavelengths longer than 1.3 μm, standard glass fibers have negative GVD.

Mollenauer, Stolen, and Gordon (1980) demonstrated the propagation of solitons in optical fibers. Extensive experiments were conducted by the group at Bell Laboratories; these confirmed the predictions of the nonlinear Schrödinger equation and proved that the properties of fiber propagation are described, to a remarkable degree, by this equation.

Hasegawa (1984) made the imaginative proposal that solitons could be used for ''repeaterless'' communications over transoceanic distances. Conventional communications using optical fibers at that time (and even today) detect, regenerate electronically, and retransmit optically the bit-stream at every repeater. The repeater distances [120–150 km (Li, 1993)] are chosen to be as large as possible, while still permitting detection and regeneration of the signal with a net bit-error rate of less than 10^{-9}. The ''repeaterless'' transmission concept proposed by Hasegawa would compensate the fiber loss with Raman gain.

One of the authors (H.A.H.) was spending part of a sabbatical in 1984 at Bell Laboratories (renamed AT&T Bell Laboratories after the breakup of AT&T). Learning about the proposal, he and Gordon investigated possible mechanisms that may get in the way of repeaterless communications with solitons. The amplifiers needed to compensate for the loss generate amplified spontaneous emission, and this noise is expected to degrade the soliton signal. This degradation is twofold in nature. The simple effect is that additive noise is generated at each amplifier, which can be combatted by raising the signal level. The more subtle effect is nonlinear, and was analyzed by Gordon and Haus (1986): the noise is, in part, incorporated by the soliton, whose mean frequency is then shifted. A frequency shift leads to a timing shift because pulses of different frequencies have different group velocities due to GVD. This timing shift is a source of errors. It was shown that this mechanism would limit the distance of propagation to a still surprisingly large value of 2800 km for the parameters chosen by Hasegawa. After this limit was established, it was soon shown by Mollenauer (Mollenauer and Smith, 1988) that choice of a dispersion lower than that assumed by Hasegawa could extend the propagation distance to transatlantic distances.

Mollenauer carried out pioneering experiments on long distance optical communications using solitons (Mollenauer and Smith, 1988). He developed the fiber

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loop system into which a bit stream is loaded and recirculated, thereby simulating propagation over arbitrarily large distances using a few hundred kilometers of fiber. Initially he used Raman gain for loss compensation. Meanwhile, however, an erbium-doped fiber amplifier that can be pumped by laser diodes was developed by Desurvire and others (Desurvire *et al.*, 1987; Mears *et al.*, 1987; Laming *et al.*, 1989; Inoue *et al.*, 1989, 1991; Desurvire, Zyskind, and Giles, 1990; Desurvire, Zyskind, and Simpson, 1990; Zyskind *et al.*, 1990; Tachibana *et al.*, 1991). This fiber amplifier is a few meters long and has remarkable properties (to be discussed in greater detail later). M. Nakazawa at Nippon Telegraph and Telephone Company (NTT) recognized their potential for soliton transmission and demonstrated their use in his fiber loop experiments (Nakazawa *et al.*, 1989; Nakazawa, Suzuki, and Kimura, 1990).

It should be mentioned that the replacement of the distributed Raman gain with ''lumped'' amplifiers was an important advance. The soliton concept is associated with a lossless uniform fiber and hence one would expect that the actual loss of the fiber would have to be compensated by distributed gain. This was the reason for the use of Raman gain in which the pumped fiber itself acted as the gain medium. However, a more detailed investigation, which will be outlined further on, shows that lumped gain is permissible provided that the amplifier spacing is not excessive. Mollenauer used 25 km amplifier spacings in his experiments. However, spacings of 33 km, 50 km, and as large as 105 km have also been used by other laboratories (see Table I).

In fiber loop experiments with erbium-doped amplifiers, Mollenauer and his group continued their experiments on long-distance propagation of solitons. The prediction by Gordon and Haus of the amplified spontaneous emission-induced bit errors was confirmed experimentally (Mollenauer and Smith, 1988; Mollenauer, Neubelt *et al.*, 1990; Mollenauer, Nyman *et al.*, 1991). The effect being a nonlinear one, it limits the peak pulse power permissible for transmission. The linear additive noise establishes a lower limit on the peak power. The range of powers permissible for transmission at an acceptable bit-error rate (10^{-9}) turned out to be narrow—too narrow for the system designers, if allowance was to be made for the aging of the amplifiers.

As happens so often, the long-distance soliton communications experiments spurred on the competition. If it is possible to send solitons, which compensate the GVD with fiber nonlinearity, without repeaters over long distances, why not send low-level signals ''linearly'' over long distances on fibers that have zero GVD? This approach has been taken at the AT&T Bell Laboratories (Runge, 1992; Bergano and Davidson, 1995; Forghieri *et al.*, 1995; Hansen *et al.*, 1995; Kerfoot and Runge, 1995), Kokusai Denshin Denwa Company, Ltd. (KDD) (Otani *et al.*, 1995; Taga *et al.*, 1995), Nippon Telegraph and Telephone Company (Kataoka *et al.*, 1994; Murakami *et al.*, 1995), Hitachi (Kikuchi *et al.*, 1995), Northern Telecom (Butler *et al.*, 1994), Alcatel (Clesca *et al.*, 1994; Gautheron *et al.*, 1995), and British Telecom (Gu *et al.*, 1994). The transmission format is nonreturn to zero (NRZ). The signal is composed of roughly rectangular pulses that merge into one block when two or more occur consecutively. ''Linear'' transmission has to overcome serious obstacles of its own. Over the large transoceanic distances with signal levels large enough to dominate over the amplified spontaneous emission, it is impossible to avoid the Kerr nonlinearity, which can lead to serious signal distortion. The effect can be greatly reduced if the fiber is composed of segments with alternating positive and negative GVD; this tends to scramble the phase induced by the nonlinearity. Cables using fiber segments of alternating GVD have already been laid, and are operating at 5 Gbit/s between the U.S. and Europe, and between California and Hawaii. Since this paper is on soliton transmission, we shall not dwell further on this subject. One should note, however, that the eventual implementation of soliton communications can only occur if the soliton system delivers, at comparable cost, better performance than the existing and still developing NRZ systems.

Returning to the soliton story, which we left at around 1990, we note that three major advances occurred in 1991:

(a) Nakazawa and his group propagated solitons over 10^6 km in a fiber loop containing, in addition to the erbium-doped amplifiers, amplitude modulators (Nakazawa, Yamada *et al.*, 1991). The amplitude modulators restored the timing of solitons that had strayed from the center of the bit interval. This work showed that the modulators control the noise-induced timing jitter of the solitons. It also raised the possibility that such loops can maintain a bit stream of ''ones'' and ''zeros'' forever. In a theoretical paper it was shown soon afterwards that a combination of modulators and filters is indeed capable of maintaining the ''ones'' in their assigned time slots and in preventing the amplified spontaneous emission noise from building up in the ''zero'' slots, thus providing signal storage (Haus and Mecozzi, 1992). The storage capability of fiber rings has been demonstrated since then (Doerr *et al.*, 1994; Hall *et al.*, 1995; Moores *et al.*, 1995).

(b) In a closed-loop experiment, the amplitude modulator timing is derived from a common clock. In ''open-loop'' transoceanic transmission, the timing of the modulators must be derived from a local clock recovery, thus complicating the system. Furthermore, if the fiber carries wavelength-division multiplexed bit streams, the timing of each of the channels differs due to GVD. Thus the channels must be demultiplexed, the clock recovered from each channel, each channel modulated separately, and finally the channels must be remultiplexed. This is a complicated procedure that invites simpler alternatives. The discovery of a simpler means was made independently at MIT (Mecozzi *et al.*, 1991), and by Kodama and Hasegawa (1992) at AT&T Bell Laboratories. The erbium-doped amplifiers have a gain bandwidth of the order of 40 nm, while the solitons used in 5 Gbit/s transmission have a bandwidth of the order of 0.05 nm. The filters tend to keep the solitons at the center frequency

TABLE I. Soliton transmission experiments.

Year	Bit rate (Gb/s)	Distance (km)	Distance bit-rate product (Tb/s km)	Amplifier spacing (km)	Company	Reference
1995	20	2 000	40.0	50–70	NTT	(Nakazawa *et al.*, 1995b)[a]
	10	30 000	300.0	30	NTT	(Kawai *et al.*, 1995)[b]
	20	150 000	3000.0	105	France Telecom	(Aubin *et al.*, 1995)
	20	19 000	380.0	35	France Telecom	(Favre and LeGuen, 1995)[c]
	10	2 000	20.0	50–70	NTT	(Nakazawa *et al.*, 1995a)[a]
	20	1 000	20.0	100	KDD	(Edagawa *et al.*, 1995)
	160	200	32.0	25	NTT	(Nakazawa *et al.*, 1995)
	35	205	7.2	50	British Telecom	(Richardson *et al.*, 1995)
	8×2.5	10 000	200.0	30–36	AT&T	(Nyman *et al.*, 1995)
	10	1 000	10.0	105	France Telecom	(Aubin *et al.*, 1995b)[b]
	20	14 000	280.0	46	France Telecom	(LeGuen *et al.*, 1995)[c]
	3.6	94	0.3	105	France Telecom	(Lester *et al.*, 1995)[d]
	10	2 700	27.0	35	France Telecom	(Aubin *et al.*, 1995a)
1994	2×60	154	18.5	30	KDD	(Iwatsuki *et al.*, 1994)
	20	125 000	2500.0	33	British Telecom	(Widdowson and Ellis, 1994)
	80	500	40.0	25	NTT	(Nakazawa, Yoshida *et al.*, 1994)[e]
	20	3 000	60.0	50	NTT	(Nakazawa, Suzuki *et al.*, 1994)
	10	1 000	10.0	70	France Telecom	(Aubin *et al.*, 1994)
	20	11 500	230.0	30	KDD	(Suzuki *et al.*, 1994)[e]
	15	25 000	375.0	26	AT&T	(Mollenauer *et al.*, 1994)[c]
	2.5	17 600	44.0	33	British Telecom	(Widdowson *et al.*, 1994)
	10	7 200	72.0	50	NTT	(Kawai *et al.*, 1994)
	5	6 000	30.0	30	KDD	(Taga *et al.*, 1994)
1993	5	13 100	65.5	30	KDD	(Suzuki *et al.*, 1993)
	40	1 000	40.0	50	NTT	(Nakazawa *et al.*, 1993a)
	20	2 000	40.0	50	NTT	(Nakazawa *et al.*, 1993a)
	40	205	8.2	51	British Telecom	(Ellis *et al.*, 1993)
	2×10	13 000	260.0	26	AT&T	(Mollenauer *et al.*, 1993)[c]
	10	20 000	200.0	26	AT&T	(Mollenauer *et al.*, 1993)[c]
	10	10^6	10000.0	50	NTT	(Nakazawa *et al.*, 1993b)
	80	80	6.4	20–30	NTT	(Iwatsuki *et al.*, 1993)
1992	5	3 000	15.0	33	KDD	(Taga *et al.*, 1992)
	40	65	2.6	10–31	NTT	(Iwatsuki *et al.*, 1992)
	2.5	6 400	16.0	33	KDD	(Taga *et al.*, 1992)
	20	1 020	20.4	50	NTT	(Nakazawa, Suzuki, and Yamada, 1992)
	10	1 200	12.0	50	NTT	(Nakazawa, Suzuki *et al.*, 1992)
	2×5	11 000	110.0	26	AT&T	(Mollenauer, Lichtman *et al.*, 1992)
	5	15 000	750.0	26	AT&T	(Mollenauer, Lichtman *et al.*, 1992)
	32	90	2.9	30	AT&T	(Andrekson *et al.*, 1992)
1991	2.5	14 000	35.0	27	AT&T	(Mollenauer, Neubelt *et al.*, 1991)
	20	350	7.0	25	NTT	(Nakazawa, Suzuki *et al.*, 1991)
	10	10^6	10000.0	50	NTT	(Nakazawa, Yamada *et al.*, 1991)
	10	1 000	10.0	50	NTT	(Yamada *et al.*, 1991)
	5	350	1.8	71	British Telecom	(Ellis *et al.*, 1991)
	2×2	9 000	36.0	30	AT&T	(Olsson *et al.*, 1991)
	2.4	12 000	28.8	25	AT&T	(Mollenauer, Nyman *et al.*, 1991)
1990	0.2	10 000	2.0	25	AT&T	(Mollenauer *et al.*, 1990)
	20	200	4.0	25	NTT	(Nakazawa, Suzuki *et al.*, 1990)
	5	250	1.3	25	NTT	(Suzuki *et al.*, 1990)
	4	136	0.5	30–70	AT&T	(Olsson *et al.*, 1990)
	3.2–5	100	0.5	25–30	NTT	(Nakazawa, Suzuki, and Kimura, 1990)
1988	0.1	4 000	0.4	42	AT&T	(Mollenauer and Smith, 1988)[f]

[a]Field demonstration in a metropolitan optical network.
[b]Signal frequency-sliding technique was demonstrated.
[c]Sliding guiding filters were used.
[d]Distributed erbium-doped fiber amplifiers were used.
[e]Unequal amplitude solitons were transmitted.
[f]Raman gain was used to compensate for losses.

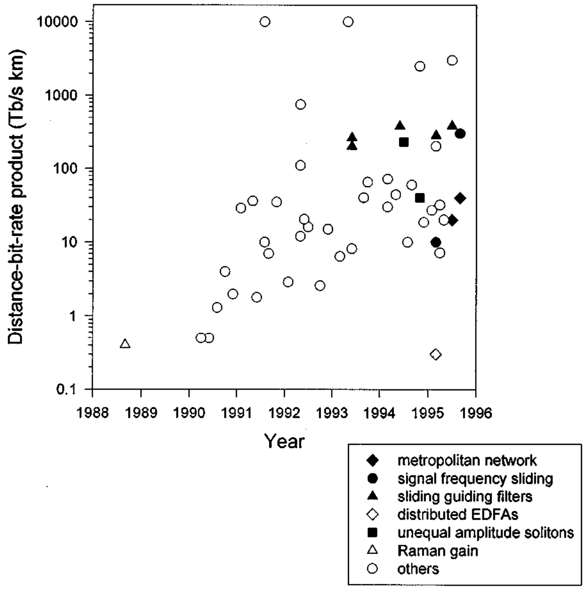

FIG. 1. Plot of soliton transmission capacity versus time: ◆, metropolitan network; ●, signal frequency sliding; ▲, sliding guiding filters; ◇, distributed erbium-doped fiber amplifiers; △, Raman gain; ○, other.

of the filter, thus reducing the timing jitter. The two groups proposed that filters be introduced at each amplifier stage, having roughly ten times the signal bandwidth. The filters could be of the Fabry-Perot type that can accommodate wavelength-division multiplexed transmission. This discovery widened the range of acceptable peak pulse powers. The transmission distance was still limited by the growth of narrow-band noise at the center frequency (wavelength) of the filters, which experiences less loss than the filtered solitons.

(c) Based on this discovery, Mollenauer and coworkers developed the sliding guiding filter concept (Mollenauer, Gordon, *et al.*, 1992; Mollenauer *et al.*, 1993). In this scheme, the center frequency of the filters is displaced from amplifier to amplifier. The soliton carrier frequency follows the filter center frequencies; narrow-band noise, on the other hand, sees filter loss on the average along the total length of transmission.

Recognizing the potential of soliton communication, more than ten research groups worldwide are currently conducting soliton transmission experiments, attempting to push the transmission capacity to higher and higher limits (see Table I and Fig. 1 for a list of soliton transmission experiments and the transmission capacity achieved). Other approaches using signal-frequency sliding (Aubin *et al.*, 1995a) and unequal-amplitude solitons (Suzuki *et al.*, 1994; Nakazawa, Yoshida *et al.*, 1994) have also been implemented.

Following this brief review of the history of soliton communication, we discuss the governing phenomena in soliton fiber communications in some detail. We start with the nonlinear Schrödinger equation and its solution. We study soliton collisions and show complete recovery of the colliding solitons after the collision is completed. This aspect is of importance if solitons are to be used in wavelength-division multiplexed communications. Then we introduce soliton perturbation theory and use it to derive the Gordon-Haus effect. Perturbation theory is extended to predict the shedding of the "continuum" when a soliton encounters a lumped disturbance. This is the kind of phenomenon occurring when solitons are amplified by lumped amplifiers. We end with some estimates as to the advantages of soliton transmission over NRZ transmission.

We do not intend to present a sample soliton communication system, because none has been deployed as yet (practical considerations, such as cost and speed advantages compared with the successfully deployed nonreturning systems, will affect the future deployment of soliton systems). Instead, we present a detailed analytic treatment of some of the salient features of soliton propagation and perturbation, such as may find application in other fields in which solitons represent an important phenomenon. Examples include water-wave solitons associated with of the Korteweg–de Vries equation and self-induced transparency solitons.

II. THE NONLINEAR SCHRÖDINGER EQUATION

By a proper choice of index profile, fibers can be designed so that they support only one electromagnetic mode, with two possible polarizations. Field patterns of greater transverse variation are not guided but are lost to radiation. Only such single-mode fibers are usable in long-distance fiber communications. A single-mode fiber propagates modes of two polarizations. Polarization-maintaining fibers are sufficiently birefringent that a mode launched in one linear polarization along one of the axes of birefringence will remain polarized along this axis. However, polarization-maintaining fibers are expensive and have higher losses than regular fibers, and hence are not used in long-distance fiber communications. Regular fibers still possess birefringence of the order of 10^{-7}. This means that within 10^7 wavelengths, i.e., about 10 m, the polarization changes uncontrollably. We shall discuss depolarization further on. Suffice it to state here that the depolarization length is much shorter than the dispersion length due to GVD, and the length within which the optical Kerr effect produces self-phase modulation. On these larger distance scales, the mode can be treated as a mode of a single (average) polarization (Wai *et al.*, 1991).

A pulse propagating along the fiber experiences GVD and the optical Kerr effect. GVD is called normal, or positive, if the group velocity decreases with increasing frequency; it is anomalous, or negative, if the change is in the opposite direction. GVD acting alone is a linear effect that does not change the spectrum of the pulse. Different frequencies travel with different group velocities and thus a pulse spreads in time under the influence of GVD. The optical Kerr effect is an intensity-induced refractive index change. The Kerr coefficient is defined to be positive if the refractive index increases with increasing intensity. This index change leads to a time-dependent phase shift. Since the time derivative of phase is related to the frequency, the optical Kerr effect usually produces a change of the pulse spectrum. The combination of positive GVD and a positive Kerr coefficient leads to temporal and spectral broadening of the pulse.

A fiber with negative GVD and a positive Kerr coefficient can propagate a pulse with no distortion. This may be surprising, at first, since GVD affects the pulse in the time domain, the Kerr effect in the frequency domain. However, a small time-dependent phase shift added to a Fourier transform-limited pulse does not change the spectrum to first order. If this phase shift is canceled by GVD in the same fiber, the pulse does not change its shape or its spectrum as it propagates. This propagation of an optical soliton is governed by the nonlinear Schrödinger equation, which we now derive.

Consider the propagation of an electromagnetic mode of one polarization along a single-mode optical fiber. The fact that the polarization varies along the fiber due to the natural birefringence of the fiber will be taken up in Sec. X. In the frequency domain, the amplitude $a(z,\omega)$ of the mode obeys the differential equation

$$\frac{d}{dz}a(z,\omega)=i\beta(\omega)a(z,\omega). \tag{2.1}$$

Here $\beta(\omega)$is the propagation constant. The spectrum of $a(z,\omega)$ is assumed to be confined to a frequency regime around ω_0. A Taylor expansion of $\beta(\omega)$ in frequency around the reference frequency ω_0 gives

$$\beta(\omega)\simeq\beta(\omega_0)+\Delta\omega\beta'+\frac{1}{2}\Delta\omega^2\beta''. \tag{2.2}$$

If we introduce the new envelope variable $v(z,\Delta\omega)$, with $\Delta\omega=\omega-\omega_0$,

$$a(z,\omega)=\exp[i\beta(\omega_0)z-i\omega_0 t]v(z,\Delta\omega), \tag{2.3}$$

we obtain the equation for $v(z,\Delta\omega)$

$$\frac{d}{dz}v(z,\Delta\omega)=i\left(\Delta\omega\beta'+\frac{1}{2}\Delta\omega^2\beta''\right)v(z,\Delta\omega). \tag{2.4}$$

Fourier transformation into the time domain

$$v(z,t)=\int_{-\infty}^{+\infty}d\Delta\omega\exp(-i\Delta\omega t)v(z,\Delta\omega) \tag{2.5}$$

gives

$$\frac{\partial}{\partial z}v(z,t)=-\beta'\frac{\partial}{\partial t}v(z,t)-i\frac{1}{2}\beta''\frac{\partial^2}{\partial t^2}v(z,t). \tag{2.6}$$

Transformation of the independent variables z and t into a frame represented by $z'=z,\ t'=t-\beta' z$, comoving with the group velocity $1/\beta'$, removes the first-order time derivative. To keep the notation simple, we shall drop the primes on z and t, getting the simple equation as the result

$$\frac{\partial}{\partial z}v(z,t)=-i\frac{1}{2}\beta''\frac{\partial^2}{\partial t^2}v(z,t). \tag{2.7}$$

This is the linear Schrödinger equation for a free particle in one dimension (with z and t interchanged). If an initial Gaussian wave packet is launched at $z=0$, the group velocity dispersion β'' is responsible for the spreading of this wave packet.

If the fiber is nonlinear, the propagation constant acquires an intensity-dependent contribution; $\beta(\omega)$ of Eq. (2.2) has to be supplemented by the Kerr contribution, which is a change of the refractive index proportional to the optical intensity. The process is known as four-wave mixing, since Fourier components at ω' and ω'' mix with a Fourier component at ω''' to produce a phase shift at $\omega=\omega'''-\omega''+\omega'$. If a distribution of frequencies is involved, a convolution has to be carried out in the frequency domain. The nonlinear index n_2 produces a phase shift proportional to

$$n_2\int_{-\infty}^{+\infty}d\omega'\int_{-\infty}^{+\infty}d\omega'' v(z,\omega')v^*(z,\omega'')v(z,\omega+\omega''-\omega'). \tag{2.8}$$

When transformed back into the time domain, this integral becomes

$$\begin{aligned} n_2\int_{-\infty}^{+\infty}d\omega e^{-i\omega t}\int_{-\infty}^{+\infty}d\omega'\int_{-\infty}^{+\infty}d\omega'' v(z,\omega') \\ \times v^*(z,\omega'')v(z,\omega+\omega''-\omega') \\ =n_2\frac{\partial|v(z,t)|^2}{\partial t}v(z,t). \end{aligned} \tag{2.9}$$

Thus, the Kerr effect is expressed much more simply in the time domain, as long as the Kerr coefficient is frequency independent. The change of amplitude $\Delta\nu$ of the pulse propagating through a differential length of fiber Δz can be derived from standard perturbation theory:

$$\Delta v=i\frac{\omega_0}{c}n_2\frac{|v(z,t)|^2}{A_{\mathrm{eff}}}v(z,t)\Delta z. \tag{2.10}$$

Here n_2 is the Kerr coefficient ($n_2\approx 3\times10^{-20}$ m^2/W in mks units for silica fiber), A_{eff} is the effective mode area, obtained by averaging the phase shift using the mode profile $e(x,y)$ over the fiber, that is, by integrating over the transverse dimensions x and y:

$$\frac{1}{A_{\mathrm{eff}}}=\frac{\int\int|e(x,y)|^4dxdy}{\int\int|e(x,y)|^2dxdy}. \tag{2.11}$$

We have assumed that $|v(z,t)|^2$ is normalized to the power and the mode pattern $e(x,y)$ has been so normalized that $\int\int|e(x,y)|^2dxdy$ is dimensionless. Note that the index change due to the optical Kerr effect is not constant across the fiber. When Eq. (2.7) is supplemented by the Kerr effect, we obtain the nonlinear Schrödinger equation

$$\frac{\partial}{\partial z}v(z,t)=-i\frac{1}{2}\beta''\frac{\partial^2}{\partial t^2}v(z,t)+i\kappa|v(z,t)|^2v(z,t), \tag{2.12}$$

where

$$\kappa=\frac{\omega_0}{c}\frac{n_2}{A_{\mathrm{eff}}}. \tag{2.13}$$

The nonlinearity in this equation can compensate for the dispersion: a pulse need not disperse since it can dig its own potential well, which provides confinement. This happens when $\beta''<0$, when the GVD is negative (anomalous). Indeed, a solitary-wave solution of Eq. (2.12) is

$$v_s(z,t)=A_0\mathrm{sech}\left(\frac{t-\beta''\Delta\omega_0 z}{\tau}\right)\exp(-i\Delta\omega_0 t)\times\exp\left[i\left(\frac{\kappa|A_0|^2}{2}+\frac{\beta''}{2}\Delta\omega_0^2\right)z\right]\exp(i\phi), \quad (2.14)$$

with the constraint that the parameters τ and A_0 obey

$$\frac{1}{\tau^2}=-\frac{\kappa|A_0|^2}{\beta''}. \quad (2.15)$$

This constraint shows that the GVD has to be indeed negative. The physical interpretation of Eq. (2.14) is relatively straightforward. The soliton carrier frequency is detuned from the nominal frequency by $\Delta\omega_0$. Setting $\Delta\omega_0$ to zero, we see that the solution is a simple hyperbolic secant of amplitude A_0. As it propagates, it accumulates a phase delay $\kappa|A_0|^2z/2$, which is due to the Kerr effect produced by the average intensity. The area of the soliton amplitude is fixed at

$$\mathrm{Area}=\int_{-\infty}^{+\infty}dt|v_s(z,t)|=\pi\sqrt{\frac{|\beta''|}{\kappa}}, \quad (2.16)$$

independent of its amplitude. This is the area theorem for solitons of the nonlinear Schrödinger equation. The energy of a soliton is thus proportional to the inverse of its pulse width τ. Detuning by $\Delta\omega_0$ has simple consequences. The propagation constant changes by $\beta''\Delta\omega_0^2/2$ and modifies the phase factor accordingly. The inverse group velocity changes by $\beta''\Delta\omega_0$, thus producing the timing shift in the argument of the sech function. The area in Eq. (2.16) is unaffected by detuning.

It is customary to normalize the distance variable in Eq. (2.12) to a normalizing distance z_n, the time variable to a normalizing time τ_n, and the amplitude $v(z,t)$ to an amplitude A_n. With $|\beta''|z_n/\tau_n^2=1$, $\kappa|A_n|^2z_n=1$, and $v(z,t)/A_n=u(z,t)$, one obtains

$$-i\frac{\partial}{\partial z}u(z,t)=\frac{1}{2}\frac{\partial^2}{\partial t^2}u(z,t)+|u(z,t)|^2u(z,t). \quad (2.17)$$

The normalizing distance is chosen so that the optical Kerr effect, acting alone, would produce one radian of phase shift within unit distance. The normalization of the time variable (choice of normalized bandwidth) is chosen so as to produce equal and opposite effects due to GVD and the optical Kerr effect on a standard pulse of unit width. The purpose of the normalization is to arrive at the standard nonlinear Schrödinger equation, with the exception of the factor of 1/2, which has become customary in fiber soliton theory. In Gordon's notation (Gordon, 1983), the solution in Eq. (2.14) in normalized form becomes

$$u(z,t)=A\,\mathrm{sech}(At-q)\exp(iVt+i\phi), \quad (2.18)$$

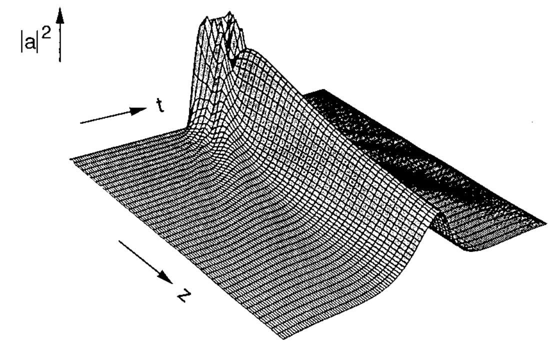

FIG. 2. A square pulse evolves into a first-order soliton.

where

$$\frac{dq}{dz}=AV \quad (2.19)$$

and

$$\frac{d\phi}{dz}=\frac{1}{2}(A^2-V^2). \quad (2.20)$$

Here we retain z for the (normalized) distance variable and t for the time variable. Gordon uses t for the distance variable and x for the time variable to emphasize the nature of Eq. (2.17) as the nonlinear version of the Schrödinger equation.

Gordon's notation has mnemonic value. As defined in (2.18), A is the amplitude of the soliton, V is its velocity, q/A is position, reminding one of the quantum notation for position; ϕ, of course is the phase. Since the velocity of the soliton is caused by a deviation of the carrier frequency of the soliton from the nominal value for which V=0, V also measures frequency deviation. To probe deeper into the theory of optical solitons, interested readers may wish to consult books written by Haus (1984), Hasegawa (1989), Agrawal (1989), Newell and Moloney (1992), and Taylor (1992).

III. PROPERTIES OF SOLITONS

In the preceding section we denoted the solution to the nonlinear Schrödinger equation as a "solitary wave." The term "soliton" is applied, strictly, only to solutions of nonlinear equations that have certain stability properties—for example, in a collision of two such waves, the two components must emerge unscathed. This is the case with the solitary wave solutions of the nonlinear Schrödinger equation, and thus the term "soliton" can be rigorously applied. Before we proceed with the study of collisions, we address the remarkable formation process of solitons.

If an input pulse has an area that lies in the range between $\pi/2$ and $3\pi/2$, a soliton forms from the pulse (Satsuma and Yajima, 1974). This is shown in Fig. 2, which shows the evolution of a soliton from a square pulse whose area obeys this condition. One sees that the soliton "cleans itself out" by shedding continuum. Since the continuum travels away from the soliton in both directions, it has components that are both faster and slower than the soliton. Their frequencies are thus higher and lower, respectively (note the continuum is of low intensity and thus has linear propagation properties). These frequency components are, in part, contained in the original excitation, but they are also generated in the nonlinear processes partaking in the soliton formation.

The formation process described above has been used to generate solitons at high bit rates (Tai *et al.*, 1986; Dianov *et al.*, 1989; Chernikov *et al.*, 1992, 1993, 1994; Swanson *et al.*, 1994; Swanson and Chinn, 1995). The input is a superposition of two continuous waves of equal amplitude offset by a frequency $\Delta\omega_0$. The result is a sinusoidal beat between the two waves. If the intensities are such that the area of the excitation between the two nodes of the beat obeys the soliton formation criterion, a bit-stream of solitons forms after propagating in a fiber of appropriate length. This scheme, followed by an amplitude modulator, has been proposed and demonstrated as a source for soliton communications (Richardson *et al.*, 1995).

Next, consider soliton collisions. These can be described by a higher-order soliton—a soliton of second order, which is also a closed-form solution of the nonlinear Schrödinger equation obtainable by the inverse scattering approach of Zakharov and Shabat (1971). It can be written (Gordon, 1983)

$$u(z,t)=\frac{A_1e^{i\theta_1}(\rho^*\beta e^{-x_2}+\rho\beta^*e^{x_2})-A_2e^{i\theta_2}(\rho^*\beta^*e^{-x_1}+\rho\beta e^{x_1})}{|\rho|^2\cosh(x_1+x_2)+|\beta|^2\cosh(x_1-x_2)-4A_1A_2\cos(\theta_1-\theta_2)}, \tag{3.1}$$

with

$$\begin{aligned} x_j&=A_j-q_j, \quad j=1,2,\\ \theta_j&=V_jt+\phi_j,\\ \rho&=A_1-A_2+i(V_1-V_2),\\ \beta&=A_1+A_2+i(V_1-V_2). \end{aligned} \tag{3.2}$$

The q_j's and ϕ_j's obey the equations [compare with Eqs. (2.19) and (2.20)]

$$\frac{dq_j}{dz}=A_jV_j \tag{3.3}$$

and

$$\frac{d\phi_j}{dz}=\frac{1}{2}(A_j^2-V_j^2). \tag{3.4}$$

The V_j's are normalized carrier frequencies. If they are picked to be different, Eq. (3.1) describes two solitons that are well separated at $t\rightarrow-\infty$, collide, and become again well separated as $t\rightarrow+\infty$. The solitons experience a timing shift and a phase shift, but otherwise recover fully. The smaller the carrier frequency separation between the two pulses, the larger are the shifts (see Appendix A). A collision is shown in Fig. 3. When the two pulse envelopes overlap, beating between the two carrier frequencies is clearly discernible.

The solution in Eq. (3.1) can also be used to study the interaction between two solitons when they are well separated. This was done analytically by Gordon (1983) and verified experimentally by Mitschke and Mollenauer (1987). The interactions are important in optical communications, since they can also introduce errors in a bit-stream of solitons, whose phases may vary randomly. Suffice to state here that some of these interactions are easily understood. If two solitons of equal phase are placed close to each other, the potential well produced in combination is deeper than when they are widely separated. Thus closeness is energetically favored, which leads to an attractive force. The opposite is true when the solitons are in antiphase; they repel each other.

IV. PERTURBATION THEORY OF SOLITONS

Perturbation theories of solitons can be developed from the inverse scattering transform (Kaup, 1976, 1990; Karpman, 1977; Kaup and Newell, 1978; Karpman and Solov'ev, 1981; Kivshar and Malomed, 1989, 1991). An alternate approach is to start with the linearized form of the nonlinear Schrödinger equation and project out the excitations produced by the perturbation using the adjoint functions (Gordon and Haus, 1986; Haus and Lai, 1990). This latter approach provides a more direct insight into the physics of the processes involved. If $u(z,t)$ in Eq. (2.17) is replaced by $u_s(z,t)+\Delta u(z,t)$,

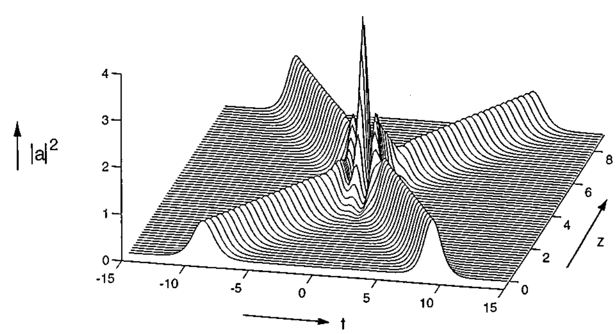

FIG. 3. Two colliding first-order solitons (arbitrary units).

where $\Delta u(z,t)$ is the deviation of the field from the soliton solution $u_s(z,t)$, the equation obeyed by $\Delta u(z,t)$ to first order is

$$-i\frac{\partial}{\partial z}\Delta u(z,t)=\frac{1}{2}\frac{\partial^2}{\partial t^2}\Delta u(z,t)+2|u_s(z,t)|^2\Delta u(z,t)+u_s^2(z,t)\Delta u^*(z,t). \qquad (4.1)$$

This is a linear equation. Linear equations are solved by finding their eigenfunctions and writing the solution as a superposition of the eigenfunction excitations. The amplitudes of the excitations are projected out using the orthogonality of the eigenfunctions, which is assured provided that the system is self-adjoint. Self-adjointness is a natural consequence of energy conservation: solutions with different time dependences must be orthogonal, since if this were not the case, the energy would be time varying. Equation (4.1) is not self-adjoint. Even though the nonlinear Schrödinger equation is derivable from a Hamiltonian, the perturbation equation describes excitations in the presence of a pump $u_s^2(z,t)$, and hence the energy of the perturbations need not be conserved. Orthogonality can be achieved with the solutions of the adjoint equation whose solutions $\Delta\underline{u}(z,t)$ obey cross-energy conservation:

$$\frac{d}{dz}\mathrm{Re}\left[\int_{-\infty}^{+\infty}dt\Delta\underline{u}^*(z,t)\Delta u(z,t)\right]=0. \qquad (4.2)$$

It is easily shown that the system adjoint to Eq. (4.1) is

$$-i\frac{\partial}{\partial z}\Delta\underline{u}(z,t)=\frac{1}{2}\frac{\partial^2}{\partial t^2}\Delta\underline{u}(z,t)+2|u_s(z,t)|^2\Delta\underline{u}(z,t)-u_s^2(z,t)\Delta\underline{u}^*(z,t). \qquad (4.3)$$

Note the sign change in the last term. This sign change corresponds to a 90° phase shift of the pump. In a parametrically pumped system, such a phase shift switches growing solutions to decaying solutions. If a solution of the system is growing, cross-energy is preserved when this solution is paired with the decaying solution of the adjoint system.

We write the perturbation as a superposition of changes in the four soliton parameters and of the continuum [we use notation based on Gordon's form of the solution in Eq. (2.18)]:

$$\Delta u(z,t)=[\Delta A(z)f_A(t)+\Delta\phi(z)f_\phi(t)+\Delta q(z)f_q(t)+\Delta V(z)f_V(t)]e^{iz/2}e^{i\phi}+\Delta u_c(z,t), \qquad (4.4)$$

where $\Delta u_c(z,t)$ is the continuum. The four perturbation functions $f_P(t)$ are derivatives of the soliton solution with respect to its four parameters P, where $P=A$, ϕ, q, and V, evaluated at $z=0$:

$$f_A(t)=\frac{\partial}{\partial A}u_s(0,t)=[1-t\tanh(t)]\mathrm{sech}(t),$$

$$f_\phi(t)=\frac{\partial}{\partial\phi}u_s(0,t)=i\,\mathrm{sech}(t),$$

$$f_q(t)=\frac{\partial}{\partial q}u_s(0,t)=\tanh(t)\mathrm{sech}(t),$$

$$f_V(t)=\frac{\partial}{\partial V}u_s(0,t)=it\,\mathrm{sech}(t). \qquad (4.5)$$

With no loss of generality, the unperturbed soliton solution has been assumed to have $A=1$, $\phi=0$, $q=0$, $V=0$. The adjoint equation has similar solutions. They are orthonormal to the set in Eq. (4.5) and are found to be

$$\underline{f}_A(t)=\mathrm{sech}(t),$$

$$\underline{f}_\phi(t)=i[1-t\tanh(t)]\mathrm{sech}(t),$$

$$\underline{f}_q(t)=t\,\mathrm{sech}(t),$$

$$\underline{f}_V(t)=i\tanh(t)\mathrm{sech}(t). \qquad (4.6)$$

The adjoint functions must be orthogonal to the continuum. Indeed, as $z\to+\infty$, the continuum is completely dispersed and has no overlap with the functions that occupy the region around the soliton. Because of the conservation law, the orthogonality must hold for all θ_z.

When Eq. (4.4) is introduced into the governing equation (1), the second derivative of $f_A(t)$ with respect to time produces a term proportional to $f_\phi(t)$. This simply means that a change of amplitude causes a cumulative change of phase, since the contribution from the Kerr effect has changed. Similarly, the second time derivative of $f_V(t)$ produces a term proportional to $f_q(t)$; a change of carrier frequency causes a cumulative change of displacement due to a change in group velocity. The perturbation parameters are projected out by the four adjoint functions. This results in four equations of motion for the soliton parameters:

$$\frac{d}{dz}\Delta A=S_A(z), \qquad (4.7)$$

$$\frac{d}{dz}\Delta\phi=\Delta A+S_\phi(z), \qquad (4.8)$$

$$\frac{d}{dz}\Delta q=\Delta V+S_q(z), \qquad (4.9)$$

$$\frac{d}{dz}\Delta V=S_V(z), \qquad (4.10)$$

where the sources are given by

$$S_P(z)=\mathrm{Re}\left[\int_{-\infty}^{+\infty}dt\underline{f}_P^*(t)e^{-iz/2}s(z,t)\right], \qquad (4.11)$$

where $s(z,t)$ represents a noise source added to the right-hand side of (4.1). These equations can, and will, be augmented to include filtering. They are sufficient to derive the Gordon-Haus effect. Before we proceed, we take note of the fact that the perturbation analysis permits large changes of ΔA, $\Delta\phi$, Δq, and ΔV, as long as these changes are gradual and the generation of the continuum is not excessive. Phase shifts, displacements, and frequency shifts leave the soliton envelope unchanged. Even large amplitude changes may be incorporated if the projection functions in Eq. (4.5) are generalized to an arbitrary value of the amplitude A, as long as the sources $S_P(z)$ are evaluated at any cross section z consistently with the state of the soliton at that cross section. Then the parameters ΔA, $\Delta\phi$, Δq, and ΔV are allowed to become large. We emphasize this fact by dropping the prefix Δ henceforth and replacing ΔA by $A-1$:

$$\frac{dA}{dz}=S_A(z), \tag{4.7a}$$

$$\frac{d\phi}{dz}=A-1+S_\phi(z), \tag{4.8a}$$

$$\frac{dq}{dz}=V+S_q(z), \tag{4.9a}$$

$$\frac{dV}{dz}=S_V(z). \tag{4.10a}$$

V. AMPLIFIER NOISE AND THE GORDON-HAUS EFFECT

We shall start with distributed gain that compensates for the fiber loss. Lumped gain is, of course, the practical case. We shall then consider the effects that are caused by lumped gain and ways to minimize them.

Suppose the normalized field gain coefficient is γ. If this gain compensates perfectly for the loss, Eq. (2.17) remains unchanged, assuming that noise can be neglected. However, in the case of long-distance propagation with gains in the 100 dB range, amplifier noise cannot be neglected. It must be accounted for by adding a source term $s(z,t)$ Eqs. (2.17) and (4.1)—see Sec. VI for further discussion. If the amplifier bandwidth is much larger than the signal bandwidth, the source is characterized by the correlation function (see Appendix B)

$$\langle s^*(z,t)s(z',t')\rangle=2\gamma N\delta(t-t')\delta(z-z'), \tag{5.1}$$

where γ is amplitude gain per unit length and N is a parameter defined in Appendix B, Eq. (B8). Equation (5.1) is a delta function correlated in time, indicating that the noise is frequency independent. It is also a delta function correlated in space, since the gain and the associated sources are due to gain media in different volume elements. Since the noise is described in terms of a correlation function, the response must be similarly expressed. The responses take the real part of complex projections. This means that only the in-phase or quadrature (out of phase) components of the noise represented by Eq. (5.1) contribute to any one of these projections. Since the noise is stationary, the in-phase and quadrature components have equal intensities, each with correlation functions equal to half the value of Eq. (5.1). The correlation function of the noise source in Eq. (4.10a), driving the velocity V (or normalized frequency deviation), is

$$\begin{aligned}\langle S_V^*(z)S_V(z')\rangle&=\gamma N\int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}dt'\underline{f}_V(t)\underline{f}_V^*(t')\\&\quad\times\delta(z-z')\delta(t-t')\\&=\gamma N\int_{-\infty}^{+\infty}dt\tanh^2(t)\mathrm{sech}^2(t)\delta(z-z')\\&=\frac{2}{3}\gamma N\delta(z-z').\end{aligned}\tag{5.2}$$

The noise source driving the displacement q (note that $A=1$) is

$$\begin{aligned}\langle S_q^*(z)S_q(z')\rangle&=\gamma N\int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}dt'\underline{f}_q(t)\underline{f}_q^*(t')\\&\quad\times\delta(z-z')\delta(t-t')\\&=\gamma N\int_{-\infty}^{+\infty}dt\,t^2\mathrm{sech}^2(t)\delta(z-z')\\&=\frac{\pi^2}{6}\gamma N\delta(z-z').\end{aligned}\tag{5.3}$$

The correlation function of the velocity (or normalized frequency deviation) is

$$\begin{aligned}\langle V^*(z)V(z')\rangle&=\left\langle\int_0^z dz''S_V^*(z'')\int_0^{z'}dz'''S_V(z''')\right\rangle\\&=\begin{cases}\dfrac{2\gamma N}{3}\displaystyle\int_0^{z'}dz'''=\dfrac{2\gamma N}{3}z' & \text{for } z'<z\\[2ex]\dfrac{2\gamma N}{3}z & \text{for } z'>z\end{cases}.\end{aligned}\tag{5.4}$$

The mean square fluctuations of the displacement are produced by frequency fluctuations on the one hand and by a noise source driving the displacement directly on the other hand. Since the two noise sources are independent, the mean-square fluctuations that they produce are additive. Considering first the mean-square fluctuations due to the noise source $S_q(z)$, we have, in analogy with Eq. (5.4),

$$\begin{aligned}\langle q^*(L)q(L)\rangle_q&=\left\langle\int_0^L dz''S_q^*(z'')\int_0^L dz'''S_q(z''')\right\rangle\\&=\frac{\pi^2\gamma N}{6}\int_0^L dz'''=\frac{\pi^2\gamma N}{6}L,\end{aligned}\tag{5.5}$$

where L is the distance of propagation. These mean-square fluctuations grow linearly with L. They correspond to a simple random walk of the displacement variable. The mean-square fluctuations caused by the frequency fluctuations are

$$\langle q^*(L)q(L)\rangle_V=\left\langle \int_0^L dz''V^*(z'')\int_0^L dz'''V(z''')\right\rangle$$
$$=\frac{2\gamma N}{3}2\int_0^L dz''\int_0^{z''} z'''dz'''$$
$$=\frac{2\gamma N}{3}\frac{L^3}{3}. \qquad (5.6)$$

From Eq. (5.4), we see that the frequency fluctuations experience a random walk (linear growth with L), which is translated into a much larger growth of the displacement, since pulses with different carrier frequencies travel at different speeds; this effect becomes severe for large distances of propagation. The mean-square fluctuations grow with the cube of the distance. For large propagation distances they dominate. This is the so called Gordon-Haus effect (Gordon and Haus, 1986). It leads to random displacements of pulses that may end up in neighboring time slots, causing bit errors.

The analysis thus far has been in normalized units. The standard soliton was sech(t), where the normalization time τ_n was equal to the pulse width τ. The mean-square displacement fluctuations $\langle q^*(L)q(L)\rangle$ were normalized to the pulse width. The bit-error rate can be computed from them directly once the pulse-width to bit-interval ratio is chosen. The right-hand side of Eq. (5.6) is converted to physical dimensions as follows:

$$\langle q^*(L)q(L)\rangle_V=\frac{2\gamma z_n N}{3}\frac{L^3/z_n^3}{3}$$
$$=\frac{2\gamma z_n}{3}\frac{\hbar\omega_0\Theta}{|A_n|^2\tau_n}\frac{L^3/z_n^3}{3}; \qquad (5.7)$$

the parameter $\Theta(>1)$ expresses imperfect inversion of the gain medium, and z_n is a normalized distance scale. Using $|\beta''|z_n/\tau_n^2=1$ and $\kappa|A_n|^2 z_n=1$, one can write the above as

$$\langle q^*(L)q(L)\rangle_V=\frac{2\gamma}{3}\hbar\omega_0\Theta|\beta''|\frac{\kappa L^3}{3\tau^3}. \qquad (5.8)$$

The effect is proportional to the Kerr coefficient κ, indicating that the jitter is Kerr-induced. The jitter increases with the cube of the distance and is proportional to the GVD. Reducing the GVD reduces the effect and this fact is used in the design of the fiber. Since the bit rate is proportional to $1/\tau$, we find that the jitter increases with the cube of the bit rate. For given fiber parameters and a fixed bit rate, the energy of the pulse is given by

$$w=2|A_0|^2\tau=2\frac{|\beta''|}{\kappa\tau}. \qquad (5.9)$$

We find that, changing the power level of transmission, the fluctuations scale as the cube of the power (or energy):

$$\langle q^*(L)q(L)\rangle_V=\frac{2\gamma}{3}\hbar\omega_0\Theta|\beta''|\frac{\kappa L^3}{3}\left(\frac{\kappa w}{2|\beta''|}\right)^3. \qquad (5.10)$$

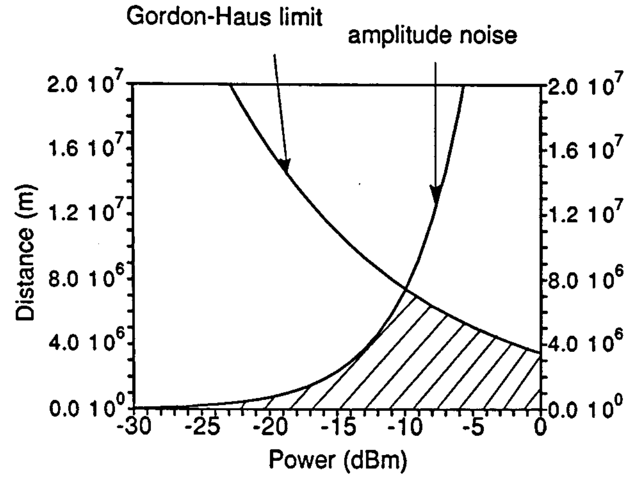

FIG. 4. The distance that can be covered with a bit-error rate of 10^{-9}, dispersion 2 ps/(nm km), and peak soliton intensity 90 mW (after Haus, 1993).

Figure 4 shows a plot of the distance that can be covered with a bit-error rate equal to or better than 10^{-9} at 5 Gbit/s transmission. The shaded area is the allowed range. There are two boundaries: one, denoted amplitude noise, is set by requiring a sufficiently large signal-to-noise ratio so that "ones" would not be mistaken as "zeros" and vice versa. This is due to the additive nature of the amplified spontaneous emission noise of the amplifiers. The other boundary is due to the nonlinear aspect of the noise — timing jitter as a result of random frequency shifts. For the transatlantic distance of 4800 km, the allowed range of signal power is quite narrow. This was an aspect of soliton transmission that had to be improved. The improvement came with the introduction of filters.

VI. CONTROL FILTERS

In work on noise in fiber ring lasers at MIT, it was found that an effect analogous to the Gordon-Haus effect appeared in such systems as well, but the introduction of filters tended to alleviate it. This work was not published until 1993 (Haus and Mecozzi, 1993). The connection was made with long distance pulse propagation, and it was found that introducing filters every so often into repeaterless soliton transmission systems could control the Gordon-Haus effect (Mecozzi *et al.*, 1991) Independently, Kodama and Hasegawa also arrived at the same conclusion (Kodama and Hasegawa, 1992). Let us look at the theory explaining this filter action.

We consider the simple case of a continuous distribution of filters (lumped filters generate continuum and are thus less ideal. Such continuum radiation will be considered later on). The fundamental equation (2.17) is altered

$$-i\frac{\partial}{\partial z}u(z,t)=\frac{1}{2}\frac{\partial^2}{\partial t^2}u(z,t)+|u(z,t)|^2u(z,t)+i\frac{1}{\Omega_f^2}\frac{\partial^2}{\partial t^2}u(z,t)+s(z,t), \tag{6.1}$$

where $1/\Omega_f^2$ expresses the filtering per unit (normalized) distance, and where we have included the noise source $s(z,t)$. The soliton perturbation equation changes accordingly:

$$-i\frac{\partial}{\partial z}\Delta u(z,t)=\frac{1}{2}\frac{\partial^2}{\partial t^2}\Delta u(z,t)+2|u_s(z,t)|^2\Delta u(z,t)+u_s^2(z,t)\Delta u^*(z,t)+i\frac{1}{\Omega_f^2}\frac{\partial^2}{\partial t^2}u_s(z,t)+s(z,t). \tag{6.2}$$

When the ansatz in Eq. (4.4) is introduced into Eq. (6.2), and the projection with the adjoint functions is carried out to first order in the perturbations, the filtering term introduces a damping constant into Eq. (4.10a) for the frequency parameter (velocity) V,

$$\frac{dV}{dz}=-\alpha V+S_V(z), \tag{6.3}$$

where

$$\alpha=\frac{4}{3\Omega_f^2}. \tag{6.4}$$

As the carrier frequency deviates from the center frequency of the filter, parts of the spectrum farther away from the center experience greater attenuation than parts nearer the center. The spectrum is pushed towards the center of the filter response.

The carrier frequency exposed to the noise driving source does not experience a random walk since the filter limits the deviation. First, we compute the correlation function of $V(z)$. Since

$$V(z)=e^{-\alpha z}\int_0^z dz' e^{\alpha z'}S_V(z'), \tag{6.5}$$

we obtain

$$\langle V^*(z)V(z')\rangle=\frac{2}{3}\gamma N e^{-\alpha(z+z')}\int_0^z dz''\int_0^{z'}dz'''\,\delta(z''-z''')e^{\alpha(z''+z''')}$$
$$=\begin{cases}\frac{2}{3}\gamma N e^{-\alpha(z+z')}\int_0^{z'}dz'' e^{2\alpha z''}=\frac{2}{3}\gamma N\frac{[e^{-\alpha(z-z')}-e^{-\alpha(z+z')}]}{2\alpha} & \text{for } z'<z\\ \frac{2}{3}\gamma N\frac{[e^{\alpha(z-z')}-e^{-\alpha(z+z')}]}{2\alpha} & \text{for } z'>z\end{cases}. \tag{6.6}$$

For $\alpha\to 0$, the result checks with Eq. (5.4). When we introduce Eq. (6.6) into the equation for the mean square fluctuations of position, we find that

$$\langle q^*(L)q(L)\rangle_V=\left\langle\int_0^L dz'' V^*(z'')\int_0^L dz''' V(z''')\right\rangle=2\gamma N\int_0^L dz''\int_0^{z''}dz''' e^{-\alpha(z''+z''')}\frac{e^{2\alpha z'''}-1}{2\alpha}$$
$$=\frac{2\gamma N}{3\alpha^3}\left[\alpha L+\frac{1}{2}(1-e^{-2\alpha L})-2(1-e^{-\alpha L})\right]. \tag{6.7}$$

It is clear that as $L\to+\infty$, the position fluctuations grow linearly with L. The contribution of the position noise source remains unchanged. Introducing physical dimensions, Eq. (6.7) becomes, in the limit of large L,

$$\langle q^*(L)q(L)\rangle_V=\frac{\gamma\hbar\omega_0\Theta\kappa|\beta''|L}{\alpha^2\tau^3}, \tag{6.8}$$

where L is the physical length and α is the unnormalized filtering constant.

Figure 5 shows a plot of the range of distances reachable with filters in place. The range of permissible powers for transatlantic and transpacific distances is now greatly increased. The figure also shows the limit imposed by the soliton attraction forces which were mentioned earlier.

It should be noted that the net bandwidth of a cascade of filters spanning transoceanic distances tends to be extremely narrow, so that ''linear'' signal transmission at a high bit rate would be impossible. It is the remarkable stability of solitons that permits them to recover their bandwidth after each filter via the nonlinearity of the fiber; linear signals cannot do so.

Another benefit of the filters is not associated with the noise reduction. Filtering provides stabilization against excessive energy changes of the solitons as they propagate along the fiber cable. An increase of the soliton energy above the design average shortens the soliton and broadens its spectrum. Pulses with a broader spectrum experience excess loss and thus energy increases are reduced by filtering. Energy decreases are similarly

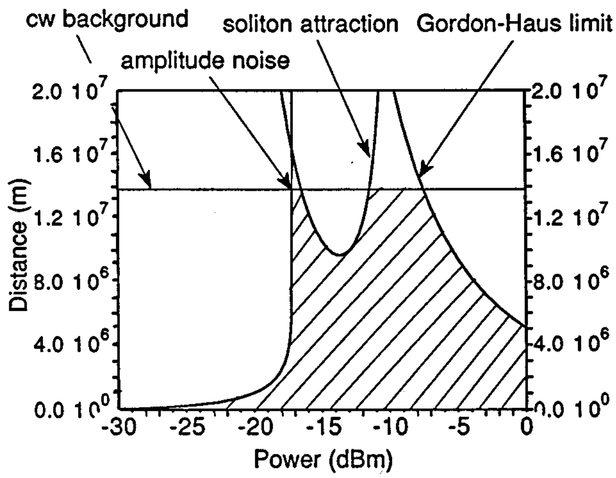

FIG. 5. The distance of propagation for a bit-error rate of 10^{-9} and filter bandwidth 0.72 ps^{-1}. The parameters are the same as for Fig. 4 (after Haus, 1993).

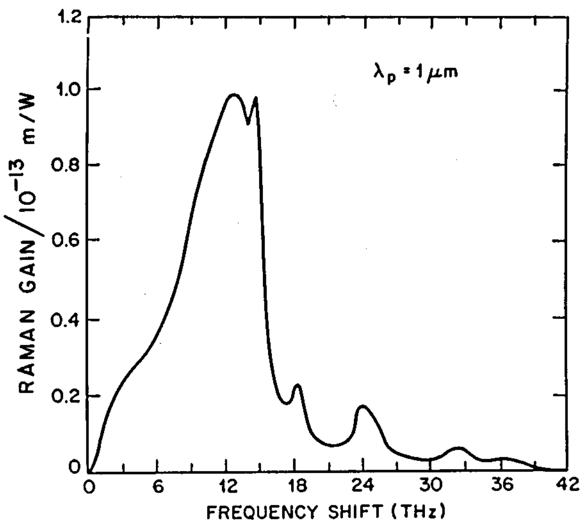

FIG. 6. Raman gain spectrum in silica fiber (after Stolen, 1980).

combatted. This effect is particularly advantageous when solitons are wavelength-division multiplexed. The filters for such an application have periodic passbands, one for each channel. Since the gain varies over the erbium bandwidth, different channels experience slightly different gains. The energy equalization by filtering acts against gain variations.

Filtering, however, is associated with a noise penalty. The solitons require increased gain to compensate for the loss of the filters, which is, per unit length,

$$\frac{\partial}{\partial z}\int_{-\infty}^{+\infty} dt |u_s(z,t)|^2 = \frac{1}{2\Omega_f^2}\int_{-\infty}^{+\infty} dt\left[u_s(z,t)\frac{\partial^2}{\partial t^2}u_s^*(z,t) + u_s^*(z,t)\frac{\partial^2}{\partial t^2}u_s(z,t)\right]$$
$$= -\frac{1}{\Omega_f^2}\int_{-\infty}^{+\infty} dt\left|\frac{\partial}{\partial t}u_s(z,t)\right|^2$$
$$= -\frac{2}{3\Omega_f^2}. \tag{6.9}$$

Noise at the center frequency is not affected by the filters and sees excess gain. This noise eventually limits the propagation distance. Mollenauer and his coworkers (Mollenauer, Gordon *et al.*, 1992) arrived at an ingenious way of eliminating this effect by gradually changing the center frequency of the filters along the cable. In its simplest manifestation, the effect of the sliding guiding filters is incorporated into Eq. (6.3) by noting that V stands for the frequency deviation of the soliton from the soliton carrier frequency ω_0. If the filter center frequencies deviate from ω_0 by the normalized frequency $-V_0(z)$, a function of distance along the fiber, then Eq. (6.3) becomes

$$\frac{dV}{dz} = -\alpha[V - V_0(z)] + S_V(z). \tag{6.10}$$

Solitons adapt to the sliding guiding filters of continuously varying $V_0(z)$ by changing their carrier frequency. Because their carrier follows the center frequency of the filters, their loss is less than the loss of the linear noise which cannot adapt in this way. Offhand, one would expect that upshifting or downshifting of the filter center frequency along the propagation direction would result in the same amount of noise suppression. If Fabry-Pérot type filters are used, so as to permit wavelength-division multiplexed transmission, upshifting leads to better noise suppression due to a subtle effect. It is clear that the soliton carrier frequency will not be situated at the center frequency of the filter passband: since the solitons are continuously forced to change frequency, their carrier frequency lags behind the shift. As they get off the filter center frequency, higher than second-order GVD is experienced by the solitons. The sign of third-order dispersion is opposite for opposite deviations from the filter center frequency. It so happens that upshifting the sliding guiding filters introduces a third-order dispersion term of a sign that is less noxious than that for downshifting (Golovchenko *et al.*, 1995). The solitons change their carrier frequency progressively as they follow the "skewing" of the filter frequency, whereas linear noise sees excess loss over its entire bandwidth. This is the concept of the sliding guiding filters.

VII. STIMULATED RAMAN SCATTERING AND THE RAMAN SELF-FREQUENCY SHIFT

Spontaneous Raman scattering was discovered by Raman (1928), while stimulated Raman scattering was observed 34 years later (Woodbury and Ng, 1962). Incident light is frequency-shifted by the vibration of optical phonons in the medium, producing an upshifted (anti-Stokes) field

$$\omega_a = \omega_p + \Omega \tag{7.1}$$

and a downshifted (Stokes) field

$$\omega_s=\omega_p-\Omega, \tag{7.2}$$

where ω_p is the frequency of the incoming photon and Ω is the optical phonon frequency.

Stolen and his coworkers first observed stimulated Raman scattering in an optical fiber and measured the Raman gain spectrum (Stolen *et al.*, 1972; Stolen and Ippen, 1973). Because silica glass is amorphous, the phonon frequency Ω becomes a continuum and significantly broadens the gain spectrum, which peaks at 13 THz and spans 40 THz (Fig. 6). In the first soliton transmission experiment, Mollenauer used the Raman gain to compensate for losses in the fiber (Mollenauer and Smith, 1988).

Gordon found that for very short solitons, the pulse spectrum can be so broad that the blue part of the spectrum can act as a pump for the stimulated Raman process to amplify the red part of the spectrum. If left unchecked, the mean frequency of solitons is continuously downshifted (Gordon, 1986). This effect, verified experimentally by Mitschke and Mollenauer (1986), is known as the Raman self-frequency shift, which can be modeled by including an additional term in the nonlinear Schrödinger equation (Kodama and Hasegawa, 1987). The Kerr coefficient n_2 becomes frequency dependent when the exciting signal is broadband. If the pumping signals have frequencies ω' and ω'', respectively, n_2 must be replaced by

$$n_2(\omega)=\frac{n_2(0)}{1-i(\omega'-\omega'')\tau_K}, \tag{7.3}$$

where τ_K is the Kerr relaxation rate. Note that $n_2(\omega)$ becomes complex. The system is capable of gain, namely Raman gain: low frequencies are amplified at the expense of high frequencies. Our model for n_2 is adequate to describe Raman gain for frequencies close to the pump frequency. The phase shift in Eq. (2.8) becomes, to first order in frequency difference:

$$n_2(0)\int_{-\infty}^{+\infty}d\omega'\int_{-\infty}^{+\infty}d\omega''[1+i(\omega'-\omega'')\tau_K]$$
$$\times v(z,\omega')v^*(z,\omega'')v(z,\omega+\omega''-\omega'). \tag{7.4}$$

When transformed into the time domain, the result is

$$n_2(0)\left[|v(z,t)|^2-\tau_K\frac{\partial}{\partial t}|v(z,t)|^2\right]v(z,t). \tag{7.5}$$

A perturbation term must be introduced into the normalized nonlinear Schrödinger equation:

$$-i\frac{\partial}{\partial z}u(z,t)=\frac{1}{2}\frac{\partial^2}{\partial t^2}u(z,t)+|u(z,t)|^2u(z,t)$$
$$-\frac{c_R\tau_K}{\tau_n}\frac{\partial|u(z,t)|^2}{\partial t}u(z,t), \tag{7.6}$$

where $c_R\tau_K$ is the effective Raman response time weighted by the shape of the gain spectrum (Gordon, 1986).

In addition to the noise-imparted velocity fluctuations in the Gordon-Haus effect, there are photon-number fluctuations as well, which are commensurate with pulse-width fluctuations as a result of the area theorem. A fluctuation in photon number or in pulse width alters the rate of Raman self-frequency shifting of a soliton. This is analogous to a changing deceleration. When integrating over long distances, it was found that the Raman timing fluctuations grow with the fifth power of distance, and may exceed the Gordon-Haus jitter at high bit rates (Wood, 1990; Nakazawa, Kubota *et al.*, 1991; Moores *et al.*, 1994; Qiu, 1994).

VIII. CONTINUUM GENERATION

A lumped disturbance, such as an amplifier, not only causes a perturbation of the soliton, but also generates continuum. In fact, if one separates Eq. (4.4) into two parts, one associated with the soliton, the other with the continuum, the result is

$$\Delta u(z,t)=[\Delta A(z)f_A(t)+\Delta\phi(z)f_\phi(t)+\Delta q(z)f_q(t)$$
$$+\Delta V(z)f_V(t)]e^{iz/2}e^{i\phi}+\Delta u_c(z,t)$$
$$=\Delta u_s(z,t)+\Delta u_c(z,t), \tag{8.1}$$

in which one can distinguish between the excitation of the continuum Δu_c and the perturbation of the soliton Δu_s. Suppose that a sudden change $\delta u(0,t)$ is caused at $z=0$. Then the continuum part of this change is

$$\Delta u_c(0,t)=\delta u(0,t)-\sum_P \mathrm{Re}\left[\int_{-\infty}^{+\infty}dt\underline{f}_P^*(t)\,\delta u(0,t)\right]. \tag{8.2}$$

The excitation of the continuum is particularly severe when a periodic perturbation approaches phase-matched conditions (Gordon, 1992; Kelly, 1992; Matera *et al.*, 1993). This condition is illustrated in Fig. 7. The ordinate is $\frac{1}{2}\Delta\omega^2\beta''$, the abscissa is the frequency deviation $\Delta\omega$. The parabola shows the dispersion of the linear waves. The propagation constant of the continuum, a linear excitation, follows the parabola. The soliton has a propagation constant corresponding to $\Delta\omega=0$ supplemented by the Kerr phase shift of 1/2 in normalized units. The spectrum of the soliton is shown on the same graph. If the normalized distance of the periodic perturbation is L, then the periodic perturbation can phase match the soliton spectrum to the continuum at two specific frequencies as shown. Excitation of the continuum is suppressed if the phase matching occurs far in the wings of the soliton spectrum. Figure 8 shows a computer simulation of the continuum excitation produced by the perturbation of the bit pattern due to the finite amplifier spacing, *in the absence of filtering*. Filtering tends to suppress the continuum excitation as well.

IX. ERBIUM-DOPED FIBER AMPLIFIERS AND THE EFFECT OF LUMPED GAIN

The amplifying characteristics of an erbium-doped silica fiber amplifier are shown in Fig. 9. Pumping has

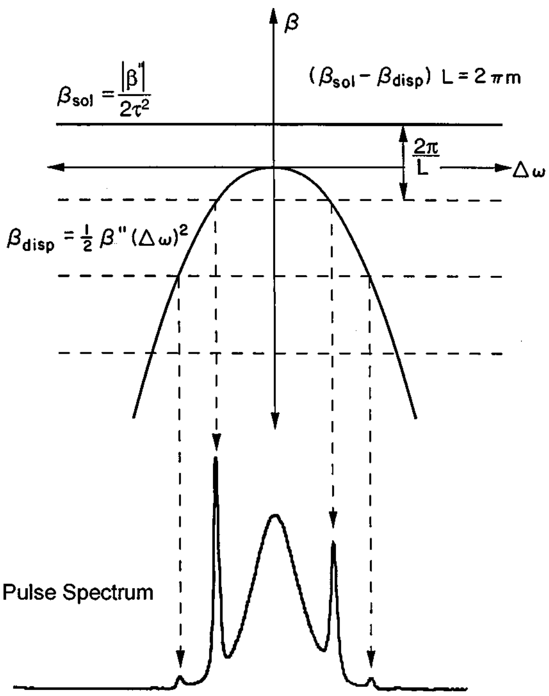

FIG. 7. The phase-matching condition for continuum generation; m is an integer.

been demonstrated at 800 nm, 980 nm, and 1480 nm (Desurvire, 1994). Note that because of the long lifetime of the metastable state, the pump power required to achieve a 30 dB gain is only of the order of 40 mW, easily supplied by a commercially available diode laser. Three typical amplifier configurations, forward pumping, reverse pumping, and bidirectional pumping, are shown in Fig. 10. The amplified spontaneous emission spectrum indicates some gain nonuniformity, which can be mitigated with additional Al doping (Miniscalco, 1991).

The variation of the energy with distance for a 25 km spacing of amplifiers is shown in Fig. 11. Note that the variation is large, and hence one would assume that the

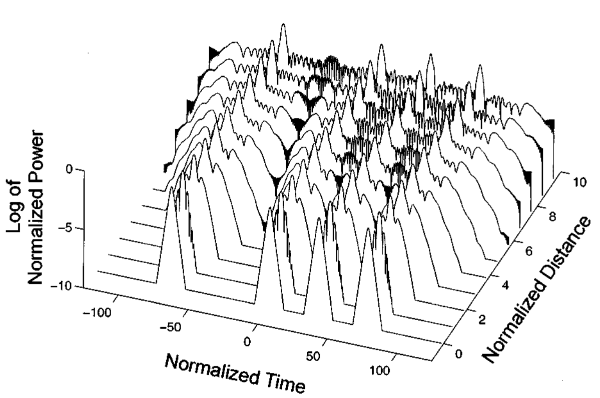

FIG. 8. Computer simulation of continuum generation with a periodic 0101110 bit-pattern. Amplifier spacing = 75 km, soliton period = 78.5 km, dispersion=0.5 ps/(nm km), pulse width = 10 ps.

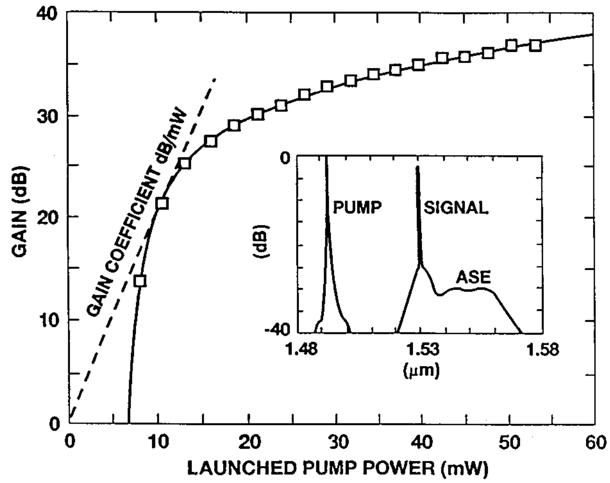

FIG. 9. The amplifying characteristics of an erbium-doped fiber amplifier. ASE—amplified spontaneous emission.

distributed soliton model represented by the nonlinear Schrödinger equation cannot be applied. Fortunately, this is not the case. For typical numbers, the normalization distance Z_n is greater than 200 km. Remember that a soliton experiences a phase shift of 2π over a normalized distance of 8. Thus the nonlinear change of the pulse over 25 km is small. Since the nonlinear change balances the linear dispersive change, the latter is small as well. Thus, the 25 km distance may be considered to

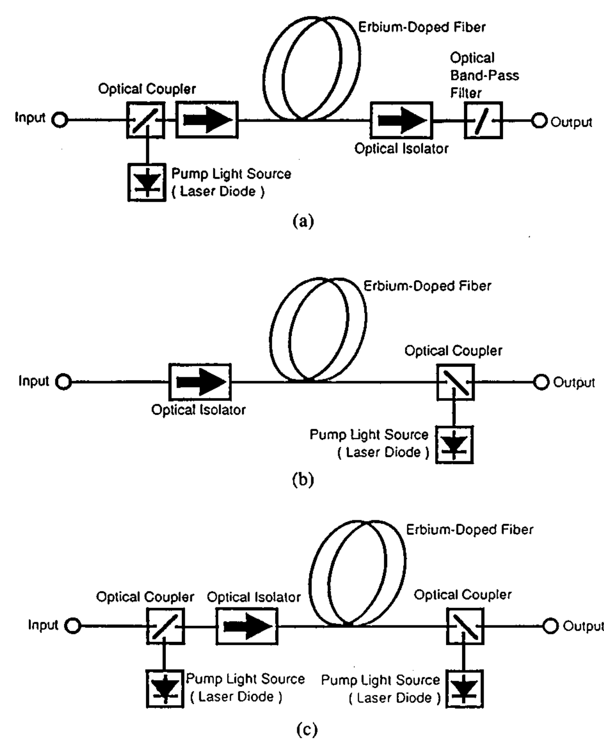

FIG. 10. Various amplifier configurations: (a) forward pumping, (b) backward pumping, and (c) bidirectional pumping (after Nakagawa *et al.*, 1991).

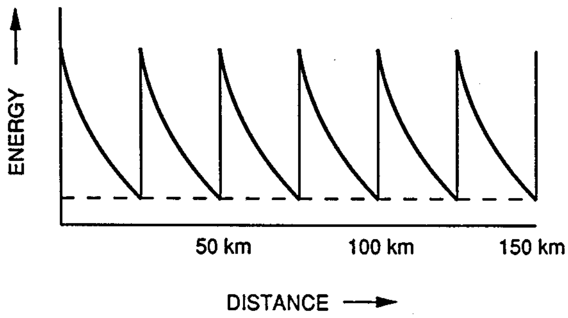

FIG. 11. The variation of pulse energy (arbitrary units) between amplifiers spaced 25 km apart.

be a differential distance. The propagating pulse is an "average" soliton (Mollenauer, Evangelides *et al.*, 1991) or a "guiding center" soliton (Hasegawa and Kodama, 1991). Its average phase shift is computed from the cumulative small phase shifts in each 25 km segment.

Mollenauer used the experimental setup shown in Fig. 12 (Mollenauer, Nyman *et al.*, 1991). A fiber ring with three amplifiers is loaded through a coupler and filled with a pseudorandom sequence of ones and zeros. The excitation is allowed to circulate in the ring and after a chosen number of transits coupled out and detected. The microwave spectrum analyzer is a convenient means for measuring the pulse jitter. Figure 13 shows the experimental results without the use of filters. The jitter tracks the prediction of the Gordon-Haus effect. Figure 14 shows the results of a measurement using the sliding guiding filters. The right-hand graph shows that a propagation distance of 22 000 km has been achieved at a bit-error rate of 10^{-9}. This is a much larger distance than the one predicted from Fig. 5 for filters of identical center frequencies. The change of the bit-error rate with a change of the bit pattern (the left-hand graph) shows that there are contributions to bit-errors other than those attributable to the Gordon-Haus effect. It is believed that these are due to a piezo-optic interaction between solitons (Dianov *et al.*, 1990, 1991).

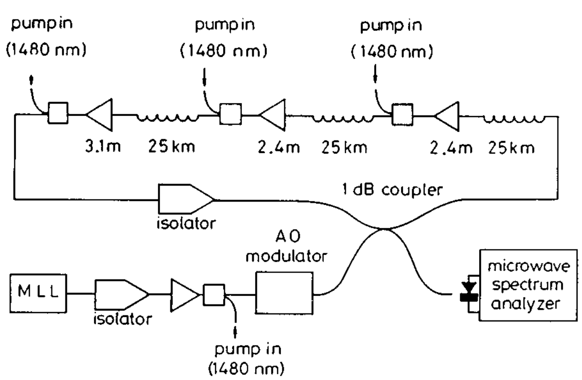

FIG. 12. Mollenauer's experimental recirculating loop (after Mollenauer, Nyman *et al.*, 1991). The acousto-optic (AO) modulator is used to reject signal pulse stream once the loop has been filled.

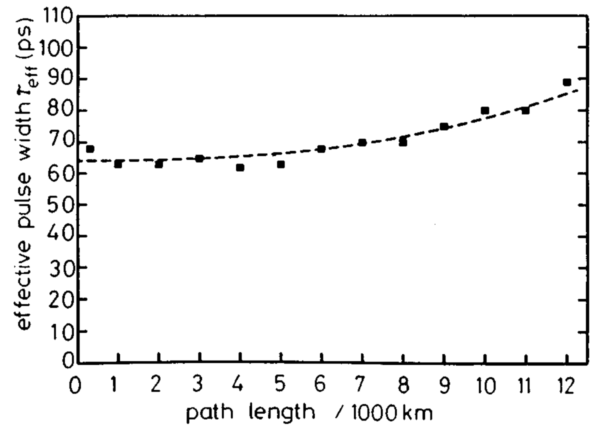

FIG. 13. Experimental confirmation of the Gordon-Haus effect (after Mollenauer, Nyman *et al.*, 1991). The solid squares represent the measured effective pulse width; the dotted line shows the Gordon-Haus limit.

X. POLARIZATION

Thus far we have not discussed the fact that fibers have a natural linear birefringence (of the order of $10^{-6}-10^{-7}$, such that a transformation from one linear polarization to the orthogonal polarization occurs in $10^{6}-10^{7}$ wavelengths). What is the effect of the birefringence on soliton propagation?

If the birefringence were fixed and did not vary randomly along the fiber, the effect would indeed be severe. We have mentioned the remarkable properties of solitons resulting from the integrability of the nonlinear Schrödinger equation. When two polarizations are coupled by birefringence, they are described by two coupled nonlinear Schrödinger equations which are not

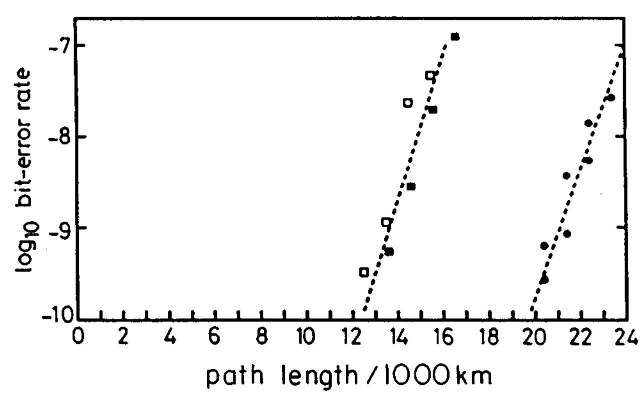

FIG. 14. Experimental results with the sliding guiding filter (after Mollenauer *et al.*, 1993). The logarithm of the bit-error rate is plotted against path length. For varying bit rates: ●, 10 Gbit/s; □, 2×10 Gbit/s wavelength-division multiplexed random bit pattern in interfering channel; ■, 2×10 Gbit/s wavelength-division multiplexed regular pattern in interfering channel. The measured channel always contained a 2^{14}-bit pseudorandom word.

integrable in general. Thus one might expect that soliton propagation would be possible only in polarization-maintaining fiber, which is more expensive than the regular fiber and also possesses higher losses. Fortunately for soliton communications, regular fiber will do for reasons we shall now explain.

The coupled nonlinear Schrödinger equations for x and y polarizations represented by the envelopes $v(z,t)$ and $w(z,t)$ are (Menyuk, 1987)

$$\frac{\partial}{\partial z}v(z,t)=-i\frac{1}{2}\beta''\frac{\partial^2}{\partial t^2}v(z,t)+i\frac{\kappa}{3}\{[3|v(z,t)|^2+2|w(z,t)|^2]v(z,t)+w^2(z,t)v^*(z,t)\} \tag{10.1}$$

and

$$\frac{\partial}{\partial z}w(z,t)=-i\frac{1}{2}\beta''\frac{\partial^2}{\partial t^2}w(z,t)+i\frac{\kappa}{3}\{[3|w(z,t)|^2+2|v(z,t)|^2]w(z,t)+v^2(z,t)w^*(z,t)\}. \tag{10.2}$$

There are cross-coupling terms, both phase-independent and phase-dependent ones. The latter are the so-called coherence terms $w^2(z,t)v^*(z,t)$ and $v^2(z,t)w^*(z,t)$. In the presence of birefringence, phase coherence is not maintained and the effect of the coherence terms averages out to zero. However, the remaining pair of coupled equations is not integrable. Yet, if the nonlinear effects are much weaker than the birefringence effects, the two polarization states wander all over the Poincaré sphere within distances which are short compared to the distance within which soliton effects play a role. Thus if $\langle v(z,t)\rangle$ and $\langle w(z,t)\rangle$ represent average orthogonal polarization states, rather than linear polarizations, the nonlinear phase shift due to each becomes equal on the average (Wai *et al.*, 1991):

$$\frac{\partial}{\partial z}\langle v(z,t)\rangle=-i\frac{1}{2}\beta''\frac{\partial^2}{\partial t^2}\langle v(z,t)\rangle+i\frac{9\kappa}{8}[|\langle v(z,t)\rangle|^2+|\langle w(z,t)\rangle|^2]\langle v(z,t)\rangle, \tag{10.3}$$

$$\frac{\partial}{\partial z}\langle w(z,t)\rangle=-i\frac{1}{2}\beta''\frac{\partial^2}{\partial t^2}\langle w(z,t)\rangle+i\frac{9\kappa}{8}[|\langle w(z,t)\rangle|^2+|\langle v(z,t)\rangle|^2]\langle w(z,t)\rangle. \tag{10.4}$$

This is an equation pair that has been shown by Manakov to be integrable (Manakov, 1973) and that gives rise to solitons of arbitrary polarization (Wai *et al.*, 1991).

Polarization hole burning is another important effect that is related to the saturation properties of erbium-doped amplifiers. Nominally, the amplifier gain is polarization insensitive. However, if one polarization saturates the amplifier, a slight excess gain is left over in the other polarization. Noise can grow in this polarization and affect the bit-error rate. This effect, first observed by Taylor (1993) in the early experiments of open-loop repeaterless systems (namely in experiments in which the pulse stream was propagated over fibers having length equal to the full transoceanic distance), was later explained by Mazurczyk and Zyskind (1994). The effect is circumvented by varying the input polarization at a rate ($>$10 kHz) faster than the relaxation rate of the erbium-doped amplifier (Taylor, 1993, 1994; Heismann *et al.*, 1994).

XI. DISCUSSION

As mentioned before, long-distance optical fiber communication systems using solitons are competing with existing "linear" transmission schemes using the nonreturning format. Solitons are going to prevail only if they offer higher bit rates at no increase of cost per bit. Cost increases with complexity. Thus soliton systems must use sources and components that are not more complex than those currently employed in NRZ transmission.

The current estimate is that solitons could support twice the bit rate per channel of a NRZ system, and that the channels of a soliton system could be spaced three times more closely (Bergano *et al.*, 1995; Nyman *et al.*, 1995). This would give them a sixfold advantage, if and when bit rates much higher than 5 Gbit/s are called for.

With regard to cost, the first issue is the source of the bit stream. Offhand, one would expect that the soliton source would have to emit proper sech-shaped pulses followed by a modulator to represent the ones and zeros. It turns out, however, that a regular NRZ source can be used at an appropriate power level (Mamyshev and Mollenauer, 1995). As the NRZ signal propagates along the fiber equipped with sliding guiding filters, it reshapes itself into the appropriate soliton stream. The continuum generated in the process is eliminated by the filters. Thus no new sources are in fact necessary.

The sliding guiding filter concept is an extremely effective way to increase bit rate and/or distance of transmission. It does imply, however, that the amplifier "pods" that are sunk into the ocean are not identical. This is currently a point of contention with the system designers. Nonlinear fiber loops for the suppression of the narrow-band noise have been proposed (Matsumoto *et al.*, 1994; Smith and Doran, 1994; Yamada and Nakazawa, 1994; Richardson *et al.*, 1995; Rottwitt *et al.*, 1995). Incorporation of such loops would make the pods identical, but this additional component makes soliton transmission less attractive compared with the NRZ format. Other schemes are currently under investigation.

There is also the problem of supervisory control. In a transoceanic cable, one fiber is used for transmission in one direction and another fiber for the opposite direction. At every amplifier stage, a small fraction of the signal propagating in one direction is tapped off and sent in the opposite direction. The bit stream is then amplitude modulated at a very low rate, enabling one to obtain both the information about the state of the amplifiers along the cable, as well as imposing commands for adjustments in the individual pods (Jensen *et al.*, 1994, Mortensen *et al.*, 1995). This simple scheme is not acceptable for soliton communications. Since the effect of amplitude modulation is removed by the filtering, no low-level signal can be returned in the fiber carrying the bit stream in the opposite direction. Thus, the supervisory control in a soliton system is an important issue that has to be settled.

In conclusion one may state that repeaterless propagation of signals via solitons has made enormous progress in recent years. It is an example of a rapid deployment of a sophisticated physical phenomenon for practical use. The work on soliton transmission has spurred on the development of "linear" NRZ repeaterless optical fiber transmission, which has already been deployed. The deployment of soliton optical fiber transmission will be dependent on overcoming the stiff competition presented by the NRZ systems.

ACKNOWLEDGMENTS

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APPENDIX A: SOLITON COLLISION EFFECTS

When two solitons pass each other, they shift each other's position and phase, while their carrier frequencies and amplitudes remain unaffected. This can be shown with the aid of the second order soliton solution in Eq. (3.1):

$$u(z,t)=\frac{A_1e^{i\theta_1}(\rho^*\beta e^{-x_2}+\rho\beta^*e^{x_2})-A_2e^{i\theta_2}(\rho^*\beta^*e^{-x_1}+\rho\beta e^{x_1})}{|\rho|^2\cosh(x_1+x_2)+|\beta|^2\cosh(x_1-x_2)-4A_1A_2\cos(\theta_1-\theta_2)}. \tag{A1}$$

If the velocities satisfy $V_2>V_1$, then the positions obey $q_2>q_1$ for $z\to+\infty$, and $q_2<q_1$ for $z\to-\infty$. With increasing z, soliton (2) starts out ahead of soliton (1) and ends up behind it. One can extricate soliton (1) from Eq. (A1) in the two limits by setting $e^{x_2}=0$ when $z\to+\infty$, and $e^{x_2}=+\infty$ when $z\to-\infty$. One obtains from Eq. (A1),

$$u=\begin{cases}\dfrac{2A_1e^{i\theta_1}\rho^*\beta}{|\rho|^2e^{-x_1}+|\beta|^2e^{x_1}} & \text{for } z\to+\infty\\[2ex] \dfrac{2A_1e^{i\theta_1}\rho\beta^*}{|\rho|^2e^{x_1}+|\beta|^2e^{-x_1}} & \text{for } z\to-\infty\end{cases}. \tag{A2}$$

These two expressions are hyperbolic secants that are mutually shifted by

$$\Delta q=2\ln\frac{|\beta|}{|\rho|}=\ln\frac{(A_1+A_2)^2+(V_1-V_2)^2}{(A_1-A_2)^2+(V_1-V_2)^2}. \tag{A3}$$

The net phase shift is

$$\begin{aligned}\Delta\phi&=2\arg\beta-2\arg\rho\\&=2\tan^{-1}\left(\frac{V_1-V_2}{A_1+A_2}\right)-2\tan^{-1}\left(\frac{V_1-V_2}{A_1-A_2}\right).\end{aligned} \tag{A4}$$

Note that the maximum phase shift is π, when $A_1=A_2$.

APPENDIX B: THE AMPLIFIED SPONTANEOUS EMISSION NOISE SOURCE

Amplified spontaneous emission is represented by a distributed noise source. The propagation of the amplitude $a(z,\omega)$ of a mode in the single-mode fiber experiencing gain is described by

$$\frac{\partial}{\partial z}a(z,\omega)=\gamma a(z,\omega)+n(z,\omega), \tag{B1}$$

where γ is the amplitude gain per unit length and $n(z,\omega)$ is the noise source: this is a delta function correlated from segment to segment, since the noise sources in different volume elements are independent:

$$\langle n^*(z,\omega)n(z',\omega)\rangle=C\delta(z-z'). \tag{B2}$$

The magnitude of C is determined from the well-known result that the amplified spontaneous emission output-power spectral density of an amplifier of length L with power gain $G=\exp(2\gamma L)$ is

$$\langle a^*(L,\omega)a(L,\omega)\rangle=\frac{1}{2\pi}\hbar\omega_0(G-1)\Theta. \tag{B3}$$

Here ω_0 is the nominal center frequency of the amplifier. The gain is assumed to be uniform over the bandwidth of interest and then the noise spectral density is frequency independent. The parameter $\Theta(>1)$ expresses imperfect inversion of the gain medium. In a nondegenerate two level system, $\Theta=n_2/(n_2-n_1)$, where n_1 and n_2 are the populations of the lower and the upper level, respectively. For an erbium-doped fiber amplifer pumped at a wavelength of 980 nm, $\Theta\approx2$.

Returning to Eq. (B1) and integrating it to find the output spectral density, one obtains

$$\langle a^*(L,\omega)a(L,\omega)\rangle = \exp(2\gamma L)\int_0^L dz'\exp(-\gamma z')\int_0^L dz'' \times \exp(-\gamma z'')\langle n^*(z',\omega)n(z'',\omega)\rangle$$
$$= C\exp(2\gamma L)\int_0^L dz'\exp(-\gamma z') \times \int_0^L dz''\exp(-\gamma z'')\delta(z'-z'')$$
$$= C\frac{\exp(2\gamma L)-1}{2\gamma} = \frac{C}{2\gamma}(G-1)$$
$$= \frac{\hbar\omega_0}{2\pi}\Theta(G-1). \tag{B4}$$

In this way the noise source is found to be

$$\langle n^*(z,\omega)n(z',\omega)\rangle = 2\gamma\hbar\omega_0\Theta\frac{1}{2\pi}\delta(z-z'). \tag{B5}$$

In the time domain one obtains for the autocorrelation function

$$\langle n^*(z,t)n(z',t')\rangle = 2\gamma\hbar\omega_0\Theta\,\delta(z-z')\,\delta(t-t'). \tag{B6}$$

If the normalizations of the distance variable, time variable, and amplitude are introduced into Eq. (B6), the autocorrelation of the noise source $s(z,t)$ is

$$\langle s^*(z,t)s(z',t')\rangle = 2\gamma\hbar\omega_0\Theta\frac{z_n}{|A_n|^2\tau_n}\delta(z-z')\,\delta(t-t')$$
$$= 2\gamma z_n\frac{\hbar\omega_0\Theta}{|A_n|^2\tau_n}\delta\left(\frac{z-z'}{z_n}\right)\delta\left(\frac{t-t'}{\tau_n}\right)$$
$$\rightarrow 2\gamma N\,\delta(z-z')\,\delta(t-t'), \tag{B7}$$

where

$$N = Z_n\frac{\hbar\omega\Theta}{|A_n|^2\tau_n} \tag{B8}$$

and where it should be noted that, in the transition from the next-to-last to the last expression, the spatial and time variables have been replaced by their normalized versions without a change of notation (as done in the text). This is indicated by an arrow, rather than an equality sign.

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