Nucleon-nucleon elastic scattering and total cross sections

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An up-to-date review of the nucleon-nucleon scattering situation is presented in the intermediate-energy region (100 MeV up to a few GeV). Total cross-section measurements are discussed, but the main emphasis is on the spin physics. Technical advances for polarized beams and targets are presented. The most recent spin-dependent *NN* data are reviewed and their influence in phase-shift analyses is discussed. The direct reconstruction of the scattering amplitudes is studied and results are compared to those obtained by phase-shift analyses. Finally future plans and expected improvements are detailed.

CONTENTS

I. INTRODUCTION

It is essential to have a clear understanding of the nucleon-nucleon interaction, as it bears directly on a variety of topics in both particle and nuclear physics. For instance, a knowledge of the nucleon-nucleon interaction is essential in calculating the scattering of nucleons from nuclei. Elastic *NN* scattering is the basic reaction necessary to understand the nuclear force. Spin effects of the nucleon constituents can be investigated in great detail. All these factors justify the effort that has gone into the study of *NN* interactions for more than 30 years, from both the theoretical (Myhrer and Wroldsen, 1988), and the experimental side. In the latter, the greatest progress has been achieved within the last 10 to 15 years. Major advances came from free-neutron scattering and, most recently, within the last 5 years from the use of pure, intense $\bar{p}$ beams to study $\bar{p}p$ interactions complementary to *NN* scattering.

All these recent results have been obtained thanks to technical advances: (1) several accelerators have successfully produced polarized beams of protons and/or deuterons as well as polarized neutrons over a wide range of energies. Antiproton beams have also been produced in different laboratories, but we do not know at present how to polarize antiprotons (Martin *et al.*, 1988; LEAR Workshop, 1988). (2) New target materials have been developed providing high target polarization. Many polarized targets are presently working in a frozen-spin mode. (3) Detection techniques and event acquisition have been improved.

These recent results have also been incorporated in different phenomenological analyses, in particular in phase-shift analyses (PSA) resulting in a great improvement in our ability to determine the different phase shifts. In several cases, a direct reconstruction of the elastic-scattering amplitudes has even been carried out successfully.

The aim of the present article is to give an up-to-date review of new developments, of recent results, and of the status of different analyses. To cover the *NN* as well as the $\bar{N}N$ interaction in one single paper is impossible. We shall restrict ourselves to problems concerning nucleon-nucleon elastic scattering and total cross sections in the intermediate and high-energy regions, starting from about 100-MeV beam kinetic energy. Specific attention is devoted to spin-dependent experiments. Note that, in the inelastic *NN* channels, a large number of new data has become available. This subject necessitates a separate treatment. Problems concerning $\bar{N}N$ scattering are also quite specific. Extensive data have become available mainly from LEAR (Rapin, 1991), but measurements of spin-dependent observables are mostly limited to the analyzing power in different reaction channels. Their theoretical interpretation is far from being understood (Amsler and Myhrer, 1991).

In Sec. II we review the NN (and $\bar{N}N$) scattering formalism, relations between amplitudes and observables, and optical theorems. Section III is devoted to polarized beams of protons, deuterons, and neutrons and some of their methods of production. A brief review of the improvements in polarized-target techniques is given in Sec. IV. The experimental examples here will be taken mostly from our own experience. Section V reviews experimental results recently measured or still unpublished. Most of these results concern np interactions. Section VI shows some interesting features deduced from existing data. The method of phase-shift analysis is discussed in Sec. VII. Direct reconstruction of scattering amplitudes and results for pp and np scattering are given in Sec. VIII. Conclusions are drawn in Sec. IX.

Experimental problems related to different types of detectors, methods of instrumentation, data acquisition, on-line computing, and off-line data analysis cannot be treated here, since they are related to the specific apparatus used and cannot be generalized. In the planning of any experiment, experimental constraints impose limitations and play a major role.

We believe that the problems related to dibaryonic resonances have already been well treated in many papers (Huber, 1990), so we shall not review them again here. Concerning tests of fundamental laws, which represent an interesting and very important part of NN study, we mention a few experiments without discussing related problems in detail.

The present paper is written from the point of view of an experimentalist, given the much more developed state of that sector as contrasted to theory. The large volume of new data and results from different analyses, however, does not allow us to cite all of them and requires us to make a selection. Our choices do not imply that omitted results are any less important. For the same reason references were also reduced to the necessary minimum. References to the major original papers are either given here or can be found in review articles listed herein.

II. NUCLEON-NUCLEON FORMALISM AND PHENOMENOLOGY

Assuming parity conservation, time-reversal invariance, the Pauli principle, and isospin invariance, we can write the nucleon-nucleon scattering matrix in terms of only five invariant amplitudes, a,b,c,d,e (out of 16 possible amplitudes; (Oehme, 1954; Goldberger *et al.*, 1960; Bilenky, Lapidus, and Ryndin, 1964; Bystricky, Lehar, and Winternitz, 1978c):

$$\begin{aligned}M(\mathbf{k}',\mathbf{k})=\frac{1}{2}\{&(a+b)+(a-b)(\boldsymbol{\sigma}_1,\mathbf{n})(\boldsymbol{\sigma}_2,\mathbf{n})\\&+(c+d)(\boldsymbol{\sigma}_1,\mathbf{m})(\boldsymbol{\sigma}_2,\mathbf{m})\\&+(c-d)(\boldsymbol{\sigma}_1,\mathbf{l})(\boldsymbol{\sigma}_2,\mathbf{l})+e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2,\mathbf{n})\}\ ,\end{aligned} \tag{2.1}$$

where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli 2×2 matrices acting on the first and second nucleon wave functions, and $\mathbf{k}$ and $\mathbf{k}'$ are unit vectors in the direction of the incident and scattered particles, respectively. The center-of-mass (c.m.) basis vectors are given by

$$\mathbf{l}=\frac{\mathbf{k}'+\mathbf{k}}{|\mathbf{k}'+\mathbf{k}|},\quad \mathbf{m}=\frac{\mathbf{k}'-\mathbf{k}}{|\mathbf{k}'-\mathbf{k}|},\quad \mathbf{n}=\frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}\times\mathbf{k}'|}\ . \tag{2.2}$$

For the pp,nn, and np interactions, one can write the scattering matrix in terms of two matrices M_0 and M_1 having the same form as Eq. (2.1),

$$M(\mathbf{k}',\mathbf{k})=\tfrac{1}{4}M_0[1-(\boldsymbol{\tau}_1,\boldsymbol{\tau}_2)]+\tfrac{1}{4}M_1[3+(\boldsymbol{\tau}_1,\boldsymbol{\tau}_2)]\ , \tag{2.3}$$

where $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are the nucleon isospin matrices and M_0,M_1 are isosinglet and isotriplet scattering matrices. Ignoring the electromagnetic interaction, we can then write

$$\begin{aligned}&M(pp\Longrightarrow pp)=M(nn\Longrightarrow nn)=M_1\ ,\\&M(np\Longrightarrow np)=M(pn\Longrightarrow pn)=\frac{1}{2}(M_1+M_0)\ ,\\&M(np\Longrightarrow pn)=M(pn\Longrightarrow np)=\frac{1}{2}(M_1-M_0)\ .\end{aligned} \tag{2.4}$$

The generalized Pauli principle for nucleons implies the following symmetry relations for the amplitudes (the indices 1 or 0 refer to the isospin value):

$$\begin{aligned}&a_1(\theta)=-a_1(\pi-\theta),\quad a_0(\theta)=a_0(\pi-\theta)\ ,\\&b_1(\theta)=-c_1(\pi-\theta),\quad b_0(\theta)=c_0(\pi-\theta)\ ,\\&c_1(\theta)=-b_1(\pi-\theta),\quad c_0(\theta)=b_0(\pi-\theta)\ ,\\&d_1(\theta)=d_1(\pi-\theta),\quad d_0(\theta)=-d_0(\pi-\theta)\ ,\\&e_1(\theta)=e_1(\pi-\theta),\quad e_0(\theta)=-e_0(\pi-\theta)\ .\end{aligned} \tag{2.5}$$

In the forward direction at $\theta=0$, total angular momentum conservation implies that $e=0$ and $a-b=c+d$. Similarly at 180° one obtains $e=0$ and $a-b=c-d$.

If isospin invariance is not assumed, the scattering matrix of two nonidentical particles contains a sixth term, namely,

$$f(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2,\mathbf{n})\ . \tag{2.6}$$

The formalism with six scattering amplitudes is treated by LaFrance and Winternitz (1980), and the one with eight amplitudes (time-reversal invariance is not assumed) by Bystricky, Lehar, and Winternitz (1984b). The formalism for $\bar{N}N$ is the same as for NN scattering: in order to check C,P,CP, and CPT invariances, one must take all 16 amplitudes into account. This type of formalism is treated in LaFrance *et al.*, 1992. For some of the original work and previous reviews on the formalism, see Dalitz, 1952; Wolfenstein and Ashkin, 1952; Oehme, 1954; Wolfenstein, 1954, 1956a, 1956b; Goldberger, Nambu, and Oehme, 1957; Puzikov, Ryndin, and Smorodinskii, 1957; Stapp, Ypsilantis, and Metropolis, 1957; Faissner, 1959; Goldberger *et al.*, 1960; MacGregor, Moravcsik, and Stapp, 1960; Wilson, 1963; Bilenky, Lapidus, and Ryndin, 1964; Lacombe, 1964; Lehar and Winternitz, 1967; Hoshizaki, 1968; Csonka and Moravcsik, 1970; Halsen and Thomas, 1974; Reid and Moravcsik, 1974).

Throughout this article we shall use a four-subscript notaton X_{srbt} for experimental quantities as given in Table I (Puzikov, Ryndin, and Smorodinskii, 1957). Subscripts s, r, b, and t refer to the polarization components of the scattered, recoil, beam, and target particles, respectively. If an initial particle is unpolarized or a final particle polarization not analyzed, the corresponding subscript is set equal to zero. In principle, 256 "pure" experimental quantities exist, which can be defined as components of various tensors. A "pure experiment" is by definition one that involves only spin projections onto certain basis vectors in momentum space. Due to the symmetry principles detailed above, there remain only 25 linearly independent pure experimental quantities.

For any c.m. observable X_{pqik}, the following expression holds:

$$d\sigma/d\Omega\, X_{pqik}=\tfrac{1}{4}\mathrm{Tr}(\sigma_{1p}\sigma_{2q}M\sigma_{1i}\sigma_{2k}M^{+})\ , \tag{2.7}$$

where

$$d\sigma/d\Omega=I_{oooo}=\tfrac{1}{4}\mathrm{Tr}(MM^{+}) \tag{2.8}$$

is the unpolarized differential cross section.

Equation (2.9), given below, is the most general formula for the correlated nucleon-nucleon scattering cross section Σ. It contains all possible experimental quantities (see Table I) and does not change whether the fundamental conservation laws are applied or not. It can be adapted to each specific experimental condition, provided by the rescattering of the scattered particle labeled 1 (and/or recoil particle labeled 2) on an analyzer:

$$\begin{aligned}\Sigma(P_B,P_T,P_1,P_2)=I_1I_2\,d\sigma/d\Omega\,\{&[1+A_{ooio}P_{Bi}+A_{oook}P_{Tk}+A_{ooik}P_{Bi}P_{Tk}]\\&+P_1[P_{pooo}+P_{Bi}D_{poio}+P_{Tk}K_{pook}+P_{Bi}P_{Tk}M_{poik}]n_{1p}\\&+P_2[P_{oqoo}+P_{Bi}K_{oqio}+P_{Tk}D_{oqok}+P_{Bi}P_{Tk}N_{oqik}]n_{2q}\\&+P_1P_2[C_{pqoo}+P_{Bi}C_{pqio}+P_{Tk}C_{pqok}+P_{Bi}P_{Tk}C_{pqik}]n_{1p}n_{2q}\}\ .\end{aligned} \tag{2.9}$$

A summation is implicit over the indices p,q,i,k. Here P_B,P_T are the beam and target polarization, I_1 (I_2) and P_1 (P_2) are the cross section and the analyzing power for the analyzer 1 (2). If there is no rescattering, we put $I_i=1, P_i=0$. Unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are along the direction of the normals to the first and second analyzing planes, respectively, and $\mathbf{n}$ is the normal to the scattering plane. The scalar products $(\mathbf{n},\mathbf{n}_1)$ and $(\mathbf{n},\mathbf{n}_2)$ determine the components n_{1p} and n_{2q} for different directions of $\mathbf{n}_1$ and $\mathbf{n}_2$.

TABLE I. Experimental quantities in the scattering of two spin-$\frac{1}{2}$ particles. From Puzikov, Ryndin, and Smorodinskii (1957).

Analyzed particles		Unpolarized beam, unpolarized target *A*	Polarized beam, unpolarized target *B*	Unpolarized beam, polarized target *C*	Polarized beam, polarized target *D*
No particle	1	I_{oooo}	A_{ooio}	A_{oook}	A_{ooik}
Scattered particle	2	p_{pooo}	D_{poio}	K_{pook}	M_{poik}
Recoil particle	3	P_{oqoo}	K_{oqio}	D_{oqok}	N_{oqik}
Scattered and recoil particle	4	C_{pqoo}	C_{pqio}	C_{pqok}	C_{pqik}

Equation (2.9) is valid in any reference frame, but we shall next use it in the laboratory system where the basis unit vectors are

(1) for the initial state particles $\mathbf{k}$, $\mathbf{n}$, and $\mathbf{s}'=[\mathbf{n}\times\mathbf{k}']$,

(2) for the scattered particle $\mathbf{k}'$, $\mathbf{n}$, and $\mathbf{s}'=[\mathbf{n}\times\mathbf{k}']$,

(3) for the recoil particle in $\mathbf{k}''$, $\mathbf{n}$ and $\mathbf{s}''=[\mathbf{n}\times\mathbf{k}'']$.

The unit vectors $\mathbf{k}$, $\mathbf{k}'$, and $\mathbf{k}''$ are now oriented along the direction of the incident, scattered, and recoil particle momenta.

It must be kept in mind that, in the absence of a magnetic field, the scalar products $n_{1k'}$ and $n_{2k''}$ are zero, since the vectors $\mathbf{k}'$ and $\mathbf{k}''$ lie in the first and second analyzing planes, respectively. Thus all components of polarization tensors involving k' and k'' subscripts have vanished from the measured distributions. A magnetic field between the target and the analyzer 1 (2) along the direction $\mathbf{s}'$ ($\mathbf{s}''$) will rotate the polarization of the scattered (recoil) particle in the $(\mathbf{k}',\mathbf{n})$ [or $\mathbf{k}'',\mathbf{n})$]. The scalar products n_{1n} and $n_{1k'}$ (n_{2n} and $n_{2k''}$) are then to be understood as cosines of the angles between the normals $\mathbf{n}_1$ ($\mathbf{n}_2$) and the direction to which the n and k' (n and k'') of the scattered (recoil) particle polarization have been rotated by the magnetic field (after the scattering under consideration). Note that, in any experiment, residual components of the beam and target polarizations in nondominant directions may exist. The target magnetic field bends the charged particles and rotates the spins of all incoming and outgoing particles. This may result in combinations of "pure observables."

The notation for observables often differs. In older papers dealing with the low-energy region, nonrelativistic relations or expressions of the observables in the c.m. frame are used. Some authors simply omit the zero-spin labels. These approaches introduce ambiguities requiring additional explanations, and we strongly recommend avoiding them. Two existing notations are unambiguous: one (X) is used throughout this paper, the second (Y) (see, for example, Auer *et al.*, 1985) changes the order of the initial and final spin-state labels. These two notations are related by $X_{srbt}=Y[B,T;S,R]$ if the frames are the same. Capitals denoting the kind of observable may also differ, but are irrelevant for the transformation from one system to another one. For the notation (Y), the labels N, S, and L, instead of n, s, and k are used, and the frames for the initial, scattering, and recoil particles are implicitly determined by the position of a given label. Both notations are compared by Lapidus (1984).

The choice of frame and the definition of the normals can be different from one paper to another. The so-called Basle Convention (1960) is not unambiguous for target and recoil particles, so that some observables may differ in sign for different basis-vector definitions (for example, $K_{os''ko}=-K[L,O;O,S]$, $K_{0k''so}=-K[S,0;0,L]$, etc.). Some definitions of observables are inconsistent with any laboratory frame, as is the case for $A(Z,X)=-A_{oosk}=-A_{ooks}$ (Arndt, 1989).

All scattering observables can be expressed as bilinear combinations of the amplitudes [except the total cross section; see Eqs. (2.11) and (2.12)]. We shall limit ourselves to observables with zero, one, two, and three spin indices, which have been most frequently measured (observables are denoted as in Table I). In Table II, we give the explicit relativistic relations between these observables and the five invariant amplitudes. Many of these relations have been discussed in original papers (Raynal, 1961; Bilenky, Lapidus, and Ryndin, 1964; Winternitz, Lehar, and Janout, 1967; Bourrely and Soffer, 1975; Delaney and Gammel, 1975). A complete review is given in Bystricky, Lehar, and Winternitz, 1978c, which includes references to original papers as well.

The choice of amplitude representation in the present paper is quite arbitrary; an infinite number of representations exists. It may be worthwhile to adapt the representation used to the given set of measured observables. Relations between any two different amplitude representations are linear. Several representations are often used, for instance Wolfenstein's original amplitudes (Wolfenstein and Ashkin, 1952; Wolfenstein, 1954, 1956a, 1956b), Hoshizaki's amplitudes (Hoshizaki, 1968), singlet-triplet representation (Stapp, Ypsilantis, and Metropolis, 1957), the helicity amplitudes (Jacob and Wick, 1959), the exchange amplitudes (Leader and Slansky, 1966; Halsen and Thomas, 1974), the transversity amplitudes (Kotanski, 1966a, 1966b, 1970), etc. Table III gives the transformation relations between the amplitudes listed above and the invariant amplitudes a to e. We stress that the present transformation relations for the helicity amplitudes (Bystricky, Lehar, and Winternitz, 1978c) are based on the formalism used in Goldberger *et al.*, 1960, Hoshizaki, 1968, Jacob and Wick, 1959, and Martin and Spearman, 1970. Other authors (e.g., Cohen-Tannoudji, Morel, and Navelet, 1968; Kotanski, 1966a, 1966b, 1970) use somewhat different phase conventions. Many transformation formulas are given in Bystricky, Lehar, and Winternitz (1987c), LaFrance and Winternitz (1980), and Lapidus (1984). The most complete list can be found in Moravcsik, Pauschenwein, and Goldstein, 1989.

The general expression for the total cross section of a polarized nucleon beam transmitted through a polarized proton target, with arbitrary directions of beam and target polarizations, was first written (Bilenky and Ryndin, 1963; Phillips, 1963), as

$$\sigma_{\mathrm{tot}}=\sigma_{0\,\mathrm{tot}}+\sigma_{1\,\mathrm{tot}}(\mathbf{P}_B,\mathbf{P}_T)+\sigma_{2\,\mathrm{tot}}(\mathbf{P}_B,\mathbf{k})(\mathbf{P}_T,\mathbf{k}) \ , \quad (2.10)$$

where $\mathbf{k}$ is the unit vector in the incident-beam direction, $\sigma_{0\,\mathrm{tot}}$ is the spin-independent total cross section, and $\sigma_{1\,\mathrm{tot}}$ and $\sigma_{2\,\mathrm{tot}}$ are the spin-dependent contributions. The difference between σ_{tot} measurements with parallel or antiparallel beam and target polarization directions, both oriented perpendicular ($\uparrow$) with respect to $\mathbf{k}$, is

$$\{\sigma_{\mathrm{tot}}(\uparrow\uparrow)-\sigma_{\mathrm{tot}}(\downarrow\uparrow)\}/P_BP_T=2\sigma_{1\,\mathrm{tot}}=-\Delta\sigma_T \ . \quad (2.11a)$$

The same difference obtained with longitudinally ($\rightarrow$)

TABLE II. Most frequently measured scattering observables with 0, 1, 2, and 3 spin indices. We denote by θ the c.m. scattering angle and by θ_1 and θ_2 the laboratory angles of scattered and recoil particles, respectively. The notation of experiments is the same as in Table I (e.g., B3). k is the wave number. Throughout this table the symbol σ for the differential cross section is used instead of $d\sigma/d\Omega$, in order to simplify the formulas.

Differential cross section:

$$\sigma \equiv d\sigma/d\Omega = \tfrac{1}{2}\{|a|^2+|b|^2+|c|^2+|d|^2+|e|^2\} \tag{T2.1}$$

Polarizations and analyzing powers (A2, A3, B1, C1):

$$\sigma P_{nooo} = \sigma A_{onoo} = \sigma A_{oono} = \sigma A_{ooon} = \mathrm{Re}\, a^* e \tag{T2.2}$$

Spin correlations (D1):

$$\sigma A_{oonn} = \tfrac{1}{2}\{|a|^2-|b|^2-|c|^2+|d|^2+|e|^2\} \tag{T2.3a}$$

$$\sigma A_{ooss} = \mathrm{Re}\, a^* d \cos\theta + \mathrm{Re}\, b^* c - \mathrm{Im}\, d^* e \sin\theta \tag{T2.3b}$$

$$\sigma A_{oosk} = -\mathrm{Re}\, a^* d \sin\theta - \mathrm{Im}\, d^* e \cos\theta \tag{T2.3c}$$

$$\sigma A_{ookk} = -\mathrm{Re}\, a^* d \cos\theta + \mathrm{Re}\, b^* c + \mathrm{Im}\, d^* e \sin\theta \tag{T2.3d}$$

Depolarization tensor for polarized beam (B2):

$$\sigma D_{nono} = \tfrac{1}{2}\{|a|^2+|b|^2-|c|^2-|d|^2+|e|^2\} \tag{T2.4a}$$

$$\sigma D_{s'oso} = \mathrm{Re}\, a^* b \cos(\theta-\theta_1) + \mathrm{Re}\, c^* d \cos\theta_1 - \mathrm{Im}\, b^* e \sin(\theta-\theta_1) \tag{T2.4b}$$

$$\sigma D_{s'oko} = -\mathrm{Re}\, a^* b \sin(\theta-\theta_1) - \mathrm{Re}\, c^* d \sin\theta_1 - \mathrm{Im}\, b^* e \cos(\theta-\theta_1) \tag{T2.4c}$$

$$\sigma D_{k'oso} = \mathrm{Re}\, a^* b \sin(\theta-\theta_1) - \mathrm{Re}\, c^* d \sin\theta_1 + \mathrm{Im}\, b^* e \cos(\theta-\theta_1) \tag{T2.4d}$$

$$\sigma D_{k'oko} = \mathrm{Re}\, a^* b \cos(\theta-\theta_1) - \mathrm{Re}\, c^* d \cos\theta_1 - \mathrm{Im}\, b^* e \sin(\theta-\theta_1) \tag{T2.4e}$$

Depolarization tensor for polarized target (C3):

$$D_{onon} = D_{nono} \tag{T2.5a}$$

$$\sigma D_{os''os} = -\mathrm{Re}\, a^* b \cos(\theta+\theta_2) - \mathrm{Re}\, c^* d \cos\theta_2 + \mathrm{Im}\, b^* e \sin(\theta+\theta_2) \tag{T2.5b}$$

$$\sigma D_{os''ok} = \mathrm{Re}\, a^* b \sin(\theta+\theta_2) - \mathrm{Re}\, c^* d \sin\theta_2 + \mathrm{Im}\, b^* e \cos(\theta+\theta_2) \tag{T2.5c}$$

$$\sigma D_{ok''os} = -\mathrm{Re}\, a^* b \sin(\theta+\theta_2) - \mathrm{Re}\, c^* d \sin\theta_2 - \mathrm{Im}\, b^* e \cos(\theta+\theta_2) \tag{T2.5d}$$

$$\sigma D_{ok''ok} = -\mathrm{Re}\, a^* b \cos(\theta+\theta_2) + \mathrm{Re}\, c^* d \cos\theta_2 + \mathrm{Im}\, b^* e \sin(\theta+\theta_2) \tag{T2.5e}$$

Polarization transfer from beam to recoil particle (B3):

$$\sigma K_{onno} = \tfrac{1}{2}\{|a|^2-|b|^2+|c|^2-|d|^2+|e|^2\} \tag{T2.6a}$$

$$\sigma K_{os''so} = -\mathrm{Re}\, a^* c \cos(\theta+\theta_2) - \mathrm{Re}\, b^* d \cos\theta_2 + \mathrm{Im}\, c^* e \cos(\theta+\theta_2) \tag{T2.6b}$$

$$\sigma K_{os''ko} = \mathrm{Re}\, a^* c \sin(\theta+\theta_2) - \mathrm{Re}\, b^* d \sin\theta_2 + \mathrm{Im}\, c^* e \cos(\theta+\theta_2) \tag{T2.6c}$$

$$\sigma K_{ok''so} = -\mathrm{Re}\, a^* c \sin(\theta+\theta_2) - \mathrm{Re}\, b^* d \sin\theta_2 - \mathrm{Im}\, c^* e \cos(\theta+\theta_2) \tag{T2.6d}$$

$$\sigma K_{ok''ko} = -\mathrm{Re}\, a^* c \cos(\theta+\theta_2) + \mathrm{Re}\, b^* d \cos\theta_2 + \mathrm{Im}\, c^* e \sin(\theta+\theta_2) \tag{T2.6e}$$

Polarization transfer from target to scattered particle (C2):

$$K_{noon} = K_{onno} \tag{T2.7a}$$

$$\sigma K_{s'oos} = \mathrm{Re}\, a^* c \cos(\theta-\theta_1) + \mathrm{Re}\, b^* d \cos\theta_1 - \mathrm{Im}\, c^* d \sin(\theta-\theta_1) \tag{T2.7b}$$

$$\sigma K_{s'ook} = -\mathrm{Re}\, a^* c \sin(\theta-\theta_1) - \mathrm{Re}\, b^* d \sin\theta_1 - \mathrm{Im}\, c^* e \cos(\theta-\theta_1) \tag{T2.7c}$$

$$\sigma K_{k'oos} = +\mathrm{Re}\, a^* c \sin(\theta-\theta_1) - \mathrm{Re}\, b^* d \sin\theta_1 - \mathrm{Im}\, c^* e \cos(\theta-\theta_1) \tag{T2.7d}$$

$$\sigma K_{k'ook} = \mathrm{Re}\, a^* c \cos(\theta-\theta_1) - \mathrm{Re}\, b^* d \cos\theta_1 - \mathrm{Im}\, c^* e \sin(\theta-\theta_1) \tag{T2.7e}$$

Contribution to the polarization of scattered particle from beam and target polarization (D2):

$$M_{nonn} = P_{nooo} \tag{T2.8a}$$

$$\sigma M_{noss} = -\sigma M_{nokk} = \mathrm{Re}\, d^* e \cos\theta + \mathrm{Im}\, a^* d \sin\theta \tag{T2.8b}$$

TABLE II. (*Continued*)

$$\sigma M_{nosk} = -\mathrm{Re}\,d^*e\sin\theta + \mathrm{Im}\,a^*d\cos\theta - \mathrm{Im}\,b^*c \tag{T2.8c}$$

$$\sigma M_{noks} = -\mathrm{Re}\,d^*e\sin\theta + \mathrm{Im}\,a^*d\cos\theta + \mathrm{Im}\,b^*c \tag{T2.8d}$$

$$\sigma M_{s'osn} = \mathrm{Re}\,b^*e\cos(\theta-\theta_1) + \mathrm{Im}\,a^*b\sin(\theta-\theta_1) - \mathrm{Im}\,c^*d\sin\theta_1 \tag{T2.8e}$$

$$\sigma M_{s'okn} = -\mathrm{Re}\,b^*e\sin(\theta-\theta_1) + \mathrm{Im}\,a^*b\cos(\theta-\theta_1) - \mathrm{Im}\,c^*d\cos\theta_1 \tag{T2.8f}$$

$$\sigma M_{s'ons} = \mathrm{Re}\,c^*e\cos(\theta-\theta_1) + \mathrm{Im}\,a^*c\sin(\theta-\theta_1) - \mathrm{Im}\,b^*d\sin\theta_1 \tag{T2.8g}$$

$$\sigma M_{s'onk} = -\mathrm{Re}\,c^*e\sin(\theta-\theta_1) + \mathrm{Im}\,a^*c\cos(\theta-\theta_1) - \mathrm{Im}\,b^*d\cos\theta_1 \tag{T2.8h}$$

$$\sigma M_{k'osn} = \mathrm{Re}\,b^*e\cos(\theta-\theta_1) - \mathrm{Im}\,a^*b\sin(\theta-\theta_1) - \mathrm{Im}\,c^*d\sin\theta_1 \tag{T2.8i}$$

$$\sigma M_{k'okn} = -\mathrm{Re}\,b^*e\cos(\theta-\theta_1) - \mathrm{Im}\,a^*b\sin(\theta-\theta_1) - \mathrm{Im}\,c^*d\sin\theta_1 \tag{T2.8j}$$

$$\sigma M_{k'ons} = \mathrm{Re}\,c^*e\sin(\theta-\theta_1) - \mathrm{Im}\,a^*c\cos(\theta-\theta_1) - \mathrm{Im}\,b^*d\cos\theta_1 \tag{T2.8k}$$

$$\sigma M_{k'onk} = \mathrm{Re}\,c^*e\cos(\theta-\theta_1) + \mathrm{Im}\,a^*c\cos(\theta-\theta_1) + \mathrm{Im}\,b^*d\cos\theta_1 \tag{T2.8l}$$

Contribution to the polarization of recoil particle from beam and target polarization (D3):

$$N_{onnn} = P_{nooo},\quad N_{onss} = -N_{onkk} = M_{noss}, \tag{T2.9a}$$

$$N_{onsk} = M_{noks},\quad N_{onks} = M_{nosk}$$

$$\sigma N_{os''sn} = -\mathrm{Re}\,c^*e\cos(\theta+\theta_2) - \mathrm{Im}\,a^*c\sin(\theta+\theta_2) - \mathrm{Im}\,b^*d\sin\theta_2 \tag{T2.9b}$$

$$\sigma N_{os''kn} = \mathrm{Re}\,c^*e\sin(\theta+\theta_2) - \mathrm{Im}\,a^*c\cos(\theta+\theta_2) + \mathrm{Im}\,b^*d\cos\theta_2 \tag{T2.9c}$$

$$\sigma N_{os''ns} = -\mathrm{Re}\,b^*e\sin(\theta+\theta_2) - \mathrm{Im}\,a^*b\cos(\theta+\theta_2) - \mathrm{Im}\,c^*d\cos\theta_2 \tag{T2.9d}$$

$$\sigma N_{os''nk} = \mathrm{Re}\,b^*e\sin(\theta+\theta_2) - \mathrm{Im}\,a^*b\cos(\theta+\theta_2) + \mathrm{Im}\,c^*d\cos\theta_2 \tag{T2.9e}$$

$$\sigma N_{ok''sn} = -\mathrm{Re}\,c^*e\sin(\theta+\theta_2) + \mathrm{Im}\,a^*c\cos(\theta+\theta_2) + \mathrm{Im}\,b^*d\cos\theta_2 \tag{T2.9f}$$

$$\sigma N_{ok''kn} = -\mathrm{Re}\,c^*e\cos(\theta+\theta_2) - \mathrm{Im}\,a^*c\sin(\theta+\theta_2) + \mathrm{Im}\,b^*d\sin\theta_2 \tag{T2.9g}$$

$$\sigma N_{ok''ns} = -\mathrm{Re}\,b^*e\sin(\theta+\theta_2) + \mathrm{Im}\,a^*b\cos(\theta+\theta_2) + \mathrm{Im}\,c^*d\cos\theta_2 \tag{T2.9h}$$

$$\sigma N_{ok''nk} = -\mathrm{Re}\,b^*e\cos(\theta+\theta_2) - \mathrm{Im}\,a^*b\sin(\theta+\theta_2) + \mathrm{Im}\,c^*d\sin\theta_2 \tag{T2.9i}$$

oriented beam and target polarizations yields

$$\{\sigma_{\mathrm{tot}}(\rightrightarrows) - \sigma_{\mathrm{tot}}(\leftrightarrows)\}/P_B P_T = 2(\sigma_{1\,\mathrm{tot}} + \sigma_{2\,\mathrm{tot}}) = -\Delta\sigma_L\ . \tag{2.11b}$$

The equations given above have been discussed in detail by Bystricky, Lehar, and Winternitz (1978) and Perrot *et al.* (1986). The negative sign for $\Delta\sigma_T$ and $\Delta\sigma_L$ in Eqs. (2.11) corresponds to the usual, although unjustified, convention used in past literature. The three optical theorems are related to the three independent forward-scattering amplitudes via the total cross sections as given below:

$$\sigma_{0\,\mathrm{tot}} = (2\pi/k)\mathrm{Im}[a(0)+b(0)]\ , \tag{2.12a}$$

$$\sigma_{1\,\mathrm{tot}} = (2\pi/k)\mathrm{Im}[c(0)+d(0)]\ , \tag{2.12b}$$

$$\sigma_{2\,\mathrm{tot}} = -(4\pi/k)\mathrm{Im}\,d(0)\ , \tag{2.12c}$$

where k is the wave number.

III. POLARIZED BEAMS

Polarized proton and neutron beams are now widely available. At low energy, polarized protons and deuterons can be accelerated, or polarized nucleons can be produced in nuclear reactions on unpolarized particles. Several accelerators in the intermediate and high-energy regions can provide intense polarized proton beams; examples are IUCF, TRIUMF, PSI, LAMPF, SATURNE II, KEK, and BNL-AGS. At the BNL-AGS accelerator, at 21 GeV, 40% polarized proton beam can be produced. The ANL-ZGS was unfortunately closed in 1979, and the polarized proton beam line at PSI in 1985, resulting in the interruption of important measurements. Some injectors of these accelerators may also be used as sources of polarized particles, as is the case with PSI. Accelerator physicists have even proposed polarized beams at the high energy of TEVATRON, UNK, or even colliders such as the SSC (Montague, 1984; Teng, 1988; Terwilliger, 1988; Ado *et al.*, 1989). None of these proposals has yet been realized. The highest-energy beam has been

TABLE III. Transformation relations between different amplitude representations and invariant amplitudes a,b,c,d, and e. θ is the c.m. angle.

Type of amplitude	Relations with invariant amplitudes
Wolfenstein amplitudes (Wolfenstein and Ashkin, 1952; Wolfenstein, 1954, 1956a, 1956b)	$B=b-c,\ C=\frac{1}{2}e,$ $N=a,\ G=a+b+c,\ H=d.$
Hoshizaki amplitudes (Hoshizaki, 1968)	$a_H=\frac{1}{2}(a+b), c_H=\frac{1}{2}e,$ $m_H=\frac{1}{2}(a-b), g_H=\frac{1}{2}c, h_H=-\frac{1}{2}d$
Singlet-triplet amplitudes (Stapp, Ypsilantis, and Metropolis, 1957)	$M_{ss}=b-c, M_{00}=a+d\cos\theta,$ $M_{11}=\frac{1}{2}(a+b+c-d\cos\theta),$ $M_{10}=-(d\sin\theta+ie)/\sqrt{2},\ M_{01}=-(d\sin\theta-ie)/\sqrt{2},$ $M_{1-1}=\frac{1}{2}(-a+b+c+d\cos\theta)$ $M_{-1-1}=M_{11}, M_{-11}=M_{1-1},$ $M_{0-1}=-M_{01}, M_{-10}=-M_{10}$
Helicity amplitudes Φ_1 (Jacob and Wick, 1959)	$\Phi_1=\frac{1}{2}(a\cos\theta+b-c+d+ie\sin\theta)$ $\Phi_2=\frac{1}{2}(a\cos\theta-b+c+d+ie\sin\theta)$ $\Phi_3=\frac{1}{2}(a\cos\theta+b+c-d+ie\sin\theta)$ $\Phi_4=\frac{1}{2}(-a\cos\theta+b+c+d-ie\sin\theta)$ $\Phi_5=\frac{1}{2}(-a\sin\theta+ie\cos\theta)$
Exchange amplitudes, (Halsen and Thomas, 1974; Leader and Slansky, 1966)	$N_0=\frac{1}{2}(\Phi_1-\Phi_3)=\frac{1}{2}(a\cos\theta+b+ie\sin\theta)$ $N_1=\Phi_5=\frac{1}{2}(a\sin\theta+ie\cos\theta)$ $N_2=\frac{1}{2}(\Phi_4-\Phi_2)=\frac{1}{2}(-a\cos\theta+b-ie\sin\theta)$ $U_0=\frac{1}{2}(\Phi_1-\Phi_3)=\frac{1}{2}(-c+d)$ $U_2=\frac{1}{2}(\Phi_4+\Phi_2)=\frac{1}{2}(c+d)$
Transversity amplitudes (Kotanski, 1966a, 1966b, 1970)	$T_1\equiv T_{++++}=\frac{1}{2}(\Phi_1+\Phi_2+\Phi_3-\Phi_4-4i\Phi_5)$ $=(a+e)\exp(i\theta)$ $T_2\equiv T_{----}=\frac{1}{2}(\Phi_1+\Phi_2+\Phi_3-\Phi_4+4i\Phi_5)$ $=(a-e)\exp(-i\theta)$ $T_3\equiv T_{--+-}=\frac{1}{2}(\Phi_1-\Phi_2+\Phi_3-\Phi_4)=b$ $T_4\equiv T_{++--}=\frac{1}{2}(-\Phi_1-\Phi_2+\Phi_3-\Phi_4)=-d$ $T_5\equiv T_{+--+}=\frac{1}{2}(\Phi_1-\Phi_2-\Phi_3-\Phi_4)=-c$

built at the Fermilab TEVATRON, where highly polarized protons and antiprotons are produced by Λ or $\overline{\Lambda}$ decay, respectively (Grosnick *et al.*, 1990).

A. Accelerated polarized proton and deuteron beams

Problems with the acceleration of polarized protons or deuterons in any ring-type accelerator are mainly related to depolarizing resonances (Froisart and Stora, 1960). This subject cannot be treated here in detail, but we refer the interested reader to Froisart and Stora (1960), Grorud, Laclare, and Leleux (1979), Courant and Ruth (1980), Arvieux (1982), Grorud *et al.* (1982), Nakach (1989).

There are basically two types of depolarizing resonances: the first is the so-called "closed-orbit resonance" (Type-I resonance), often referred to as an "imperfection resonance," which affects all beam particles equally; the second (Type-II) resonance depends on the position of each beam particle. Type-II resonances are due to the focusing elements in the ring and to the nonzero emittance of the accelerated particles. They are characterized by

$$\gamma G=\pm k \tag{3.1}$$

and

$$\gamma G=\pm q\pm m\nu_x\pm n\nu_z \tag{3.2}$$

for Type I and Type II, respectively, where $G=(g/2-1)$ is the gyromagnetic anomaly, ($G_p=1.792\,845\,6$ and $G_d=-0.1427$ in nuclear magneton units), k,q,m,n are positive integers, ν_x,ν_z are the betatron horizontal and vertical wave numbers, and γ is the relativistic energy factor.

According to Eq. (3.1), Type-I resonances will have almost equal spacing in kinetic energy: for protons with

$T_{\rm kin} \gg M_p$, this energy spacing $\Delta T_{\rm kin}$ ($=M_p/G_p$) equals 523 MeV. For example, at SATURNE II ($2 \le k_p \le 7$) the resonance lines are positioned at 108, 632, 1155, 1678, 2202, and 2725 MeV. For the TEVATRON, at 800 GeV, k_p(max)=1530, while for UNK (3 TeV) it is equal to 5734!. For deuterons these resonances have a much wider energy spacing due to the small deuteron anomalous magnetic moment; for $T_{\rm kin} \gg M_d$, $\Delta T_{\rm kin} = 13.14$ GeV. Consequently these resonances are not present at SATURNE II or at Dubna.

Equation (3.2) is the most general formula for Type-II resonances. Usually both wave numbers ν_x, ν_z are equal, but can be independently changed (for example at SATURNE II they can be changed from 3.5 to 4.0). A special case occurs when $q = r \times N$, where $r = 0,1,2,\ldots$ integer and N is the lattice periodicity of the accelerator elements. Such depolarizing resonances are called "intrinsic resonances." At SATURNE II, $N=4$ and two intrinsic resonances depending on ν_z exist: $\gamma G = \nu_z$ and $\gamma G = 8 - \nu_z$. A summary of γG =integer and quadrupolar resonance lines for vertical oscillations at SATURNE II is shown in Fig. 1. The first two quadrupolar lines are very weak and remain uncorrected. The sextupolar lines are absent during the acceleration procedure, since sextupoles are used for beam extraction only. In principle, the quadrupolar resonance lines shown in Fig. 1 exist for horizontal oscillations, but only the line $\gamma G = \nu_x$ (intrinsic resonance) needs corrections. All other ν_x lines may be neglected.

Much effort has been put into the crossing of depolarizing resonances (Khiari *et al.*, 1989; Nakach, 1989). Theoretical calculations carried out by Grorud, Laclare, and Leleux (1975) predicted that a complete spin flip should occur during the crossing of most of the resonances encountered. This adiabatic spin-flip phenomenon has indeed been observed and is nowadays used for crossing strong depolarizing resonances (Type I except $\gamma G = 2$, and some of Type II at SATURNE II). This leads to almost no polarization loss and a change in beam polarization by 180°. But in general pulsed correctors are required to change the wave number rapidly. Methods for passage through resonances are different for each accelerator and will not be treated here.

Another interesting approach to crossing resonances has been developed by Derbenev and Kondratenko (1977; Derbenev *et al.*, 1978), resulting in the so called "Siberian Snake" technique. The "snake" represents a set of dipoles, in which the axes of the magnetic dipole elements differ from the common bending field axis (e.g., by ±45° or 90°; Lee and Courant, 1991). The "full" snake rotates particle spins by 180° during one turn, but conserves the beam closed orbit. Consequently the absolute value of the beam polarization remains unchanged during acceleration (Lee and Trepikian, 1986; Terwilliger, 1988; Lee, 1989; Derbenev, 1991; Krisch, 1991). Full snakes correct Type-I as well as Type-II resonances, whereas "partial" snakes (Ratner, 1991), rotating particle spins by 180° during several turns, Type-I resonances only.

The monitoring of the beam polarization is extremely important, as all experimental results are affected by this knowledge. The polarization of the extracted proton beam is oriented in the vertical direction and is usually monitored by a polarimeter (Spinka *et al.*, 1983; Bystricky, Derégel, *et al.*, 1985b) which measures the left-right asymmetry in *pp* elastic scattering. This beam polarization can be rotated around the beam axis or precessed into the beam direction. One possible method for checking the longitudinal beam polarization is to measure its residual components in the transverse planes (Bystricky, Derégel, *et al.*, 1985b).

The determination of the beam polarization depends on our knowledge of the analyzing power and vice versa,

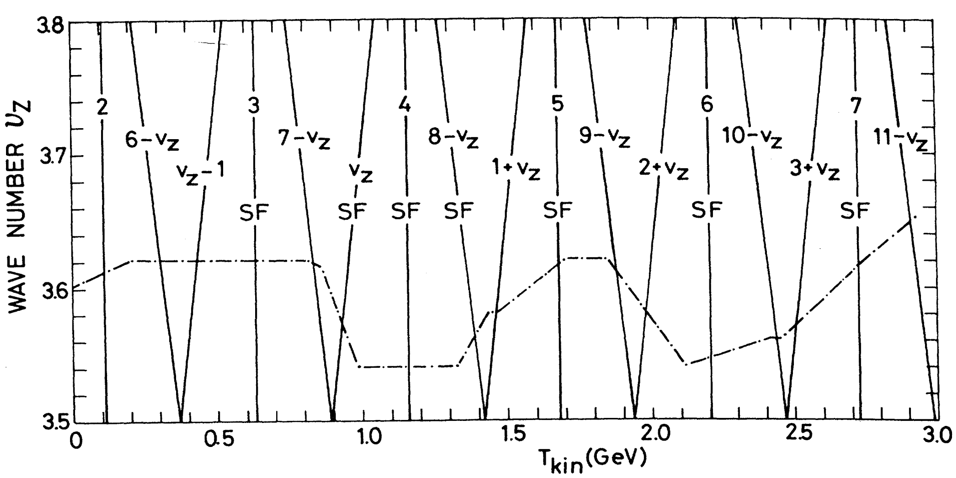


FIG. 1. Summary of γG =integer and quadrupolar resonance lines for vertical oscillations ν_Z at SATURNE II. The resonance lines are parametrized in γG units. A symbol SF means that the line is crossed by the spin flip. The dot-dashed curve is the energy dependence of ν_Z for a beam extraction at 2.91 GeV.

since the measured asymmetry ε is given by the product $P_B A_{oono}$. In *pp* scattering, the relation $A_{oono}=A_{ooon}$ always holds, and the analyzing power may be checked by measurements with a polarized proton target. Another method is worth mentioning since it allows the determination of the beam polarization at any energy once it has been measured at a single energy. One first measures $\varepsilon_1=P_1(T_1)A_{oono}$ with an extracted beam accelerated up to energy T_1. Then the beam is accelerated to energy T_2, eventually traversing depolarizing resonances. The beam polarization P_2 is unknown. Then, without extraction, the beam is decelerated back to its original energy T_1, at which point it is extracted and a new asymmetry measured, $\varepsilon_3=P_3(T_1)A_{oono}$. The beam polarization at energy T_2 (in the first approximation) will then be

$$P_2=(\varepsilon_1+\varepsilon_3)/2A_{oono} . \tag{3.3}$$

Here one assumes that the same kind of depolarization occurs during acceleration and deceleration, an assumption that has been proven to be valid (Bystricky, Lehar, *et al.*, 1985). But the method requires extraction of the beam during deceleration, a scheme that is not always available (as at SATURNE II).

B. Polarized beams of free neutrons

In this section we discuss three possible ways of obtaining highly polarized neutron beams. A summary of the different available polarized neutron beams is presented in Table IV.

A correct and precise knowledge of the neutron beam polarization is a critical point (as will be discussed below). It requires dedicated measurements and can represent a significant part of an experiment. But it is absolutely essential to know it accurately, as it directly affects all results. People have sometimes underestimated this difficulty, leading to painful renormalization problems.

1. Deuteron breakup

The best way to obtain a polarized neutron beam with well-defined orientation and a small energy spread is to use the breakup of polarized deuterons on a production target. This technique is very appealing, since the deuteron vector polarization is almost totally transferred to the outgoing neutrons and protons (the small amount lost is due to the deuteron *D* state, which contributes about 6%; Boudard, 1988). As the maximum vector polarization of deuteron beam is $\frac{2}{3}$, one can therefore anticipate about 60% polarized neutron beams. Moreover, the neutron beam polarization is independent of the deuteron energy below the first depolarizing resonance.

At SATURNE II, "breakup" neutron beams were first built for the IKAR experiment (Korolev *et al.*, 1985; Silverman *et al.*, 1989; see Sec. V.F) and later for the nucleon-nucleon (*NN*) program (Lehar, deLesquen, van Rossum, *et al.*, 1987). The characteristics of both beams are similar. Vector-polarized deuterons are broken up on a 20-cm-thick Be target. For the *NN* beam line (Ball *et al.*, 1988; Lehar, de Lesquen, Meyer, 1987) breakup protons are swept away by a magnetic field, while the remaining neutron beam is allowed to pass through a spin-rotating solenoid and precessing magnets. This allows the neutron polarization to be oriented along any of

TABLE IV. Comparison between polarized neutron facilities.

Accelerator	LAMPF Bhatia *et al.*, 1981	TRIUMF Axen *et al.*, 1980	SATURNE II Ball *et al.*, 1988	PSI NA2 Gaillard, 1990
Primary beam intensity	0.25–0.5 μA $p\uparrow$	1–5 μA $p\uparrow$	3×10^{11} $d\uparrow$/spill	2–10 μA $p\uparrow$
Neutron production	LD2	LD2	$d\uparrow$breakup on Be	Carbon
Neutron flux $10^5 n/(\mathrm{s\,cm}^2)$	1	0.9–5	25[a]	10–50
Neutron Energy (MeV)	484–788	180–500	300–1150	200–580
Neutron Polarization	0.4–0.5	0.5–0.6	0.59	0.4–0.5
Resolution FWHM MeV	20	15	60–40	11–50

[a]This is intensity/spill. For 0.5 sec spill length the repetition time increases from 1 sec at 0.5 GeV to 3.8 sec at 1.15 GeV.

the three orthogonal directions, **n** (vertical), **s** (horizontal), and **k** (longitudinal). The neutron beam is defined by 8 meters of collimators inserted into the 17.5-m-long neutron beam line. The beam spot at the target is either 20 or 30 mm in diameter. The MIMAS booster has allowed an increase in the SATURNE II polarized beam intensity up to 3×10^{11} deuterons/spill providing more than 10^7 neutrons/spill/cm^2 on the target.

Boudard and Wilkin, 1987, calculated the polarization of the "breakup" neutrons in the forward direction, using the results from Arvieux *et al.*, 1984 and 1988, and Bystricky, Derégel, *et al.*, 1985a. They predicted a value of 0.59 at SATURNE II. Experimentally it can be obtained by comparing the beam analyzing power A_{oono} and the target analyzing power A_{ooon}, measured with a polarized neutron beam and unpolarized proton target and vice versa. The slope of the angular dependence of A_{oono} near the zero-crossing point must be equal to that of A_{ooon}, which is independent of the beam polarization. The comparison was made at 447 MeV, i.e., the very energy at which a precise test of isospin invariance $[A_{oono}(np)=A_{ooon}(np)]$ was carried out at TRIUMF (Bandyopadhyay *et al.*, 1989). Both the TRIUMF and the SATURNE II results are shown in Fig. 2. This comparison gives 0.59 ± 0.02 for the "breakup" neutron beam polarization (in perfect agreement with the calculations of Boudard and Wilkin, 1987).

One of the advantages of this method is a high neutron beam intensity. A serious drawback to this technique is that the maximum neutron energy is limited to half the deuteron energy. Taking into account absorption of deuterons in the production target, the maximum attainable energy at SATURNE II is only 1.13 GeV. The only accelerator providing polarized deuteron beams at higher energy is the Dubna JINR-HEL synchrophasotron. The variable momentum of the Dubna accelerator reaches 9.5 GeV/*c*, i.e., 7.8 GeV deuteron kinetic energy with 10^9 deuterons/spill resulting in more than 10^5 neutrons/spill on the target with an energy up to 3.9 GeV. This energy limitation is compensated for by the absence, or by a small number, of depolarizing resonances, which makes possible the use of the breakup method at any energy. In principle, a polarized deuteron beam could be accelerated at KEK and at Brookhaven National Laboratory AGS. In the latter case the energy of polarized neutrons (as well as protons) could reach ~ 15 GeV. It would be interesting to examine this possibility at higher energies, in particular at UNK.

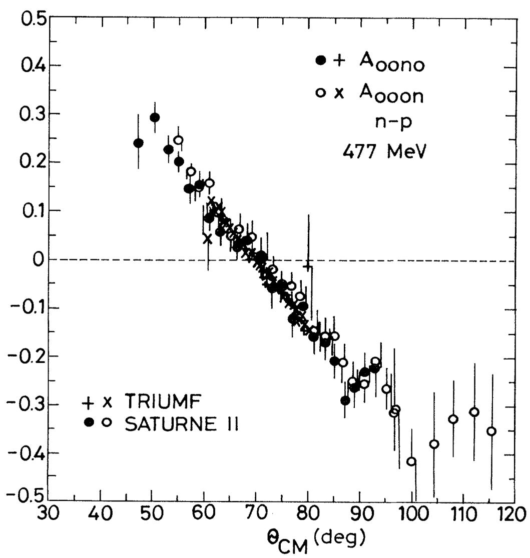


FIG. 2. $A_{oono}(np)$ (●, +) and $A_{ooon}(np)$ (○, ×) energy dependence at 477 MeV. Data denoted by + and × were measured at TRIUMF (Bandyopadhyay *et al.*, 1989), data ● and ○ were obtained at SATURNE II (Lehar, 1990, 1991a).

Note that the deuteron breakup technique has been used in many laboratories in order to obtain unpolarized neutron beams. For example, in 1960 at Dubna a measurement of the 200-MeV *np* differential cross section was carried out using the breakup of 400-MeV deuterons (Kazarinov and Simonov, 1962). In 1975 a similar method was used at SATURNE I (Bizard *et al.*, 1975), and this experiment gave the most detailed information about the breakup mechanism. The IKAR experiment at SATURNE II also started with an unpolarized deuteron beam (Silverman *et al.* 1989), where neutrons were used for differential cross-section measurements.

2. *p-n* charge exchange

Most of the existing accelerators cannot accelerate deuterons. In such cases one possible method of obtaining highly polarized neutrons is to use the backward scattering (charge exchange) of polarized protons on a liquid deuterium target via the inclusive reaction $p\uparrow+d\Longrightarrow n\uparrow+X$. The choice of the neutron production angle is subject to different requirements: at 180° c.m. the resulting neutrons are certainly optimized with respect to intensity, maximum energy of outgoing neutrons, and energy spread. A 180° angle also has the advantage of providing a neutron beam polarized along only one basis vector (**n**, **s**, or **k**). As far as maximizing neutron polarization is concerned, however, this choice is not necessarily the best, so that some other angle might be more suitable. Taking all these considerations into account, Axen *et al.* (1980) and Bhatia *et al.* (1981) have constructed and successfully used quasi monoenergetic polarized neutron beams of this type at TRIUMF and LAMPF, respectively.

Production of a neutron beam polarized along one single direction at scattering angles other than 180° c.m. is possible only if the protons are polarized along the normal to the scattering plane. The resulting neutron beam is then polarized also along the normal. Here the polarization transfer K_{onno} at energies between 400 and 800 MeV is rather large around 160° c.m. but changes rapidly with angle and becomes small at 180° c.m.

A highly polarized neutron beam can be obtained from protons oriented in the s direction by using the spin-transfer parameter $K_{os''so}$. Between 200 and 600 MeV the neutron beam polarization will have an s component between 0.9 and 0.7 in a small angular region around 160° c.m. The k component, due to the influence of the parameter $K_{ok''so}$ at the same angle, is small. Above 600 MeV the value of the s component decreases and that of the k component increases. A small n component is always present due to the analyzing power. This technique has been used at TRIUMF to provide a 50–60 % transverse polarized neutron beam with 9×10^3 n/sec cm^2 on target per 100 nA incident polarized protons. Similarly at LAMPF, a proton beam oriented in the **k** direction provides an important k component to the resulting neutron beam up to 800 MeV at scattering angles close to 180° c.m. The s and n components of the neutron beam polarization are negligible. Unfortunately, the neutron beam intensity is rather low, due to the low intensity of incident protons, as shown in Table IV.

It was recently established by McNaughton (1992) that at 800 MeV this neutron beam polarization was 13% higher than experimentalists have previously believed; this is based on a "direct" measurement at 800 MeV of the polarization transfer parameter $K_{ok''ko}$ in $d(p\uparrow,n\uparrow)$ scattering. Previous results (Riley *et al.*, 1981; Chalmers *et al.*, 1985) were obtained via a renormalization to the np elastic analyzing power (Newsom *et al.*, 1989), which was not correctly known. McNaughton, 1992, consequently recommends that several previous data sets be renormalized (Ransome *et al.*, 1982; Burleson *et al.*, 1987; Rawool, 1988; Garnett, 1989; Garnett *et al.*, 1989; Nath *et al.*, 1989; Newsom *et al.*, 1989; Beddo, 1990; Beddo *et al.*, 1991). From phase-shift analyses (Bystricky, Lechanoine-LeLuc, and Lehar, 1987b, Bugg, 1990) similar normalization factors were independently found for Newsom's data, making these renormalization factors more solid. Whenever fitting the LAMPF data, great care should be taken in the normalization. A total clarification should come from LAMPF experimentalists.

At higher energies the polarization transfer parameters are badly known, and intensities of all existing accelerators are too small for this type of polarized neutron beam production.

3. *p-n* reactions on light nuclei

It is also possible to use targets consisting of light nuclei (Li, B, Be, C) to produce polarized neutron beams. The energy distribution of these neutrons is continuous, with a quasielastic peak at the higher-energy end of the spectrum, as illustrated in Fig. 3(a). Therefore the energy of each incoming neutron has to be measured using its time of flight relative to the accelerator radio frequency signal. This method again necessitates intense incident-proton beams, but its advantage is to provide considerably higher intensity of polarized neutrons with respect to the pn charge-exchange method. As in the previous case, this kind of beam is difficult to build for neutron energies above 1 GeV.

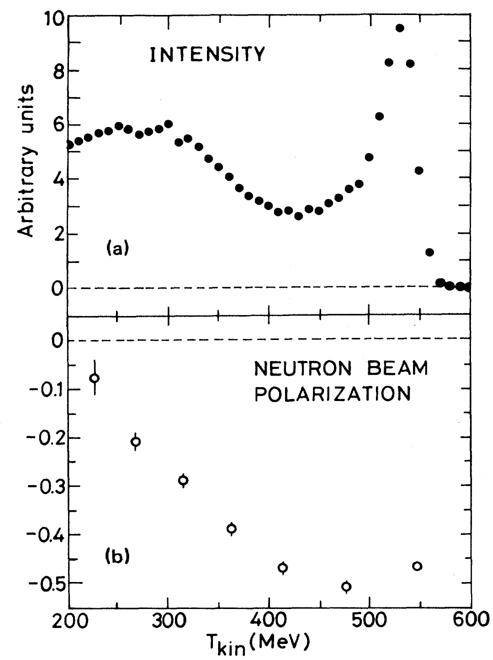


FIG. 3. Neutron beam produced by inclusive pn charge exchange in carbon: (a) Continuous neutron energy spectrum at PSI (NE1). (b) Energy dependence of the longitudinal neutron beam polarization (Binz *et al.*, 1989).

At LAMPF different production targets were tested; beryllium (Riley *et al.*, 1981), ^{6}Li, and ^{7}Li (McNaughton, Spinka, and Shimizu, 1986) at 0° and 800 MeV. Rather large values of the longitudinal polarization transfer were measured for all three targets, demonstrating that this transfer mechanism is a useful source of polarized neutrons. Moreover thin lithium targets would provide a narrow high-energy neutron peak ($\sim$1 MeV FWHM instead of 10 MeV with liquid deuterium).

At PSI a polarized neutron beam (NE1, now dismantled) was obtained by scattering polarized protons on carbon (Binz *et al.*, 1989, 1991) at 3.4° lab. It was also found that, in the energy region from 300 to 600 MeV, longitudinal transfer of polarization to the neutron was more efficient than vertical. The resulting longitudinal neutron beam polarization is shown in Fig. 3(b). It is worth insisting on the fact that the energy dependence observed in the polarization transfer parameter $K_{ok''ko}$ and K_{onno} for the $p+\mathrm{C}\Longrightarrow n+X$ inclusive reaction is surprisingly similar to that predicted by phase-shift analysis for elastic np scattering (Binz *et al.*, 1989). This interesting observation is still not understood. A new polarized neutron beam line (NA2) has been operational at PSI since December 1991 (Ahmidouch *et al.*, 1991). An intensity of $1-5\times10^6$ neutrons/(sec cm^2) was observed at 0° production angle. This beam is one of the best polarized neutron beams, as illustrated in Table IV. Details about the NA2 beam line can be found in Gaillard *et al.*, 1989 and Gaillard, 1990.

IV. POLARIZED TARGETS

Many spin-dependent NN experiments require the use of polarized proton targets (PPT) or polarized deuteron targets (PDT). For the nonspecialist we give some basic notions concerning polarized targets and refer the interested reader to the reviews of Abragam and Goldman (1982), de Boer (1974) and Jeffries (1991).

The orientation of a system of spins S along an axis Oz can be described by the so-called "orientation parameters." Here we shall consider only the vector polarization (we omit the target index T in this section)

$$P=(S_z)/S \tag{4.1}$$

and the tensor polarization or alignment A, defined as

$$A=\{3(S_z)^2-S(S+1)\}/S^2 . \tag{4.2}$$

Orientation parameters of higher order in S_z are usually zero or very small. For $S=\frac{1}{2}$ the alignment is always zero. If a spin is subjected to a magnetic field H parallel to the Oz direction, the Zeeman effect establishes a set of $(2S+1)$ sublevels for electrons as well as for nuclei. If the spins are in thermal equilibrium at a temperature T, the polarization for arbitrary spin value is given by the Brillouin formula

$$P=M\coth(M\mu H/kT)-N\coth(N\mu H/kT) , \tag{4.3}$$

where $M=(2S+1)/2S$, $N=1/2S$, μ is the magnetic dipole moment, and k is Planck's constant. In the special case of $S=\frac{1}{2}$, we have

$$P=\tanh(\mu H/kT) . \tag{4.4}$$

The brute force method consists simply in cooling the sample in an external magnetic field and waiting for thermal equilibrium. For example, with protons, an $H=2.5$ Tesla and $T=0.5$ K yield $P=0.0051$. This so-called natural polarization can be amplified by dynamical methods. Most dynamic methods make use of transitions between different energy levels, which can be induced by hyperfrequency waves produced by a carcinotron around the electron Larmor frequency. Starting at paramagnetic centers, which dope the target, this induced polarization is transmitted to other electrons of the sample and then to the nuclei by a dipolar coupling between electron and nucleus spins (Abragam and Proctor, 1958). The spin-spin interaction between different spin species in a sample is always present, but only becomes dominant at low temperatures. Below 100 mK the relaxation time increases considerably, so that the target can work in the so-called frozen-spin mode, where continuous dynamic polarization is no longer necessary. It has been observed that the optimal density of paramagnetic centers depends on the given magnetic-field value (Trentalange *et al.*, 1991).

A. Polarized proton targets (PPT)

Polarized-target technique has been considerably improved over the last ten years. A few targets are currently operated in the frozen mode (Niinikoski and Udo, 1976) using ^{3}He-^{4}He dilution refrigerators. In this case, once the target is polarized by dynamic nuclear polarization, the hyperfrequency is switched off. Heating is thus considerably reduced, so that the temperature decreases to a value between 90 and 20 mK. The target polarization can then be held in a low magnetic holding field ($0.2-0.5$ Tesla). Without beam heating, relaxation times reach about 40 days at 50 mK and 0.3 Tesla holding field.

Use of superconducting magnetic elements in the target design gives access to a large free solid angle, necessary for scattering experiments (Bernard *et al.*, 1986; Chaumette *et al.*, 1991). This construction allows orientation of the target polarization in three orthogonal directions simply by changing the orientation of the holding coil. The holding field also permits a fast reversal ($\sim$20 min) of the polarization sign. In practice this has been used in the case of the longitudinal polarization only, when the magnetic holding field has a cylindrical symmetry along the beam axis, which neither bends beam charged particles nor affects the background distribution symmetry. All these improvements facilitate particle track reconstruction and increase the number of measurable spin-dependent observables.

Concerning target materials, different alcohols (propanediol, butanol, pentanol) are commonly in use. Their hydrogen content is rather low (12 to 14.5 %), but target preparation is relatively easy. Traces of paramagnetic centers (e.g., Cr^V) are introduced chemically into these materials. Moreover, apart from hydrogen, only carbon and oxygen are present, elements for which the quantity of nonzero spin isotopes (e.g., ^{13}C) is small. A new material in use is ammonia (NH_3), in which the ratio of free protons to bound nucleons is 21% (Althoff *et al.*, 1991; Crabb, Higley, *et al.*, 1990). On the other hand, NH_3 needs to be previously irradiated in an intense electron beam in order to create paramagnetic centers. Nitrogen is a nonzero spin nucleus and may, in principle, be polarized.

Target dimensions vary from one experiment to another. In the EMC and SMC experiments at CERN, the target volume reaches 2000 cm^3 (Rijllart *et al.*, 1991), while special thin targets (5 mm thick) have been used at LEAR for antiproton-proton scattering.

Improvements have been made in target polarization measurements, which are mainly done by the NMR method. The NMR signals of different nuclei are usually detected by continuous magnetic resonance, using "Q meters" (Petricek and Odehnal, 1967; Petricek, 1968; Bazhanov and Kovaljov, 1991). An NMR signal amplitude spectrum for a polarized proton target (in a fixed homogeneous magnetic field) as a function of frequency is close to Gaussian in shape. The integral over the frequency is proportional to the target polarization. By normalizing the final PPT polarization integral to the integral of its thermal equilibrium signal (natural polarization), one obtains the absolute value of the target polarization. Different calibrations have shown that the errors in the PPT polarization are $\sim 3\%$ and mainly due to systematics in the NMR measurements only. These errors may be decreased if scattering calibration measurements are used as well.

B. Polarized deuteron targets (PDT)

In NN interaction studies, deuterated targets are needed for scattering on polarized neutrons. Such a target, using deuterated butanol ($P_T \geq 0.4$) was developed at CERN by Borghini's group (de Boer *et al.*, 1974) and has been used in experiments since 1975. Recent examples are the KEK (Ishiomoto *et al.*, 1989) or JINR-Dubna targets (Borisov *et al.*, 1988), which both use propanediol as target material ($0.4 \leq P_T \leq 0.5$). Targets containing deuterated ammonia represent an improvement. The most promising target material is ^{6}LiD, which is composed of two nuclei having weak nucleon binding energy and an equal number of protons and neutrons. The ^{6}Li and D resonant frequencies are very close. Assuming that ^{6}Li behaves as ^{4}He+D, the expected polarization of ^{6}Li and D should be equal. This has been observed in experiment (Ritt *et al.*, 1991). ^{6}LiD targets open new possibilities for $p-n$ scattering experiments.

A "Factor of Merit" F, defined as

$$F=(P_{\max})^2 f_D \rho \ , \tag{4.5}$$

is used to compare the quality of different deuteron targets. $P_{\max}$ is the maximum deuteron vector polarization, f_D is the deuteron fraction in the target material, and ρ is the target density at 0.1 K (by convention). Since $P_{\max}$ and ρ are almost the same for different target materials, f_D is the dominant factor. For fully deuterated targets, it is 0.14 to 0.16 for alcohols, 0.43 for ND_3, and 0.5 for ^{6}LiD (assuming D+^{4}He structure of ^{6}Li). Since alcohols are often partially deuterated, whereas ^{6}Li is deuterated to 0.96–0.99, the difference between f_D factors may increase.

NMR measurement of a polarized deuteron target is troublesome for deuterated alcohols, NH_3, and for any amorphous solid, due to the shape of the NMR line. Details are given in de Boer, 1974. For a deuteron spin system ($S=1$) in an external magnetic field H, the transitions between the three magnetic substates depend not only on the deuteron Larmor frequency (16.34 MHz in a 2.5 Tesla magnetic field), but also on the deuteron quadrupolar moment and on the electrical field gradient between carbon and deuteron nuclei (the CD bond). In an amorphous solid all orientations of CD bonds are equally probable, and the magnetic levels are shifted depending on the angle between the external magnetic field and the CD bonds. The presence of the quadrupolar moment in-

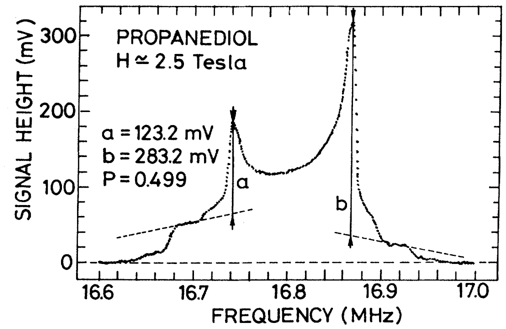


FIG. 4. NMR line for deuterated propanediol. Signal heights between the backgrounds and the tops (a and b) and formula (4.6) are used to evaluate $P(D)$ (Trentalange *et al.*, 1991).

duces two simultaneously excited transitions that overlap. The NMR line is then a superposition of these two, as shown in Fig. 4 (Trentalange *et al.*, 1991). For an accurate calculation of the polarization and the alignment, it is necessary to subtract the background and separate the contributions from each of the two transitions (surface integrals J^+ and J^-). At high polarizations the integrals J^+ and J^- differ considerably and the ratio $R=J^+/J^-$ (called "asymmetry") is related to $P(D)$ by

$$|P(D)|=(R^2-1)(R^2+R+1) \ . \tag{4.6}$$

A convolution of the NMR line with two transitions and background contributions is complicated. Under the assumption that the shape of surfaces J^+ and J^- are similar, their bases are the same and differ only in signal heights, $P(D)$ determination is then straightforward. If the backgrounds are extrapolated under both peaks, the differences [(a) and (b) in Fig. 4] between maxima and backgrounds are proportional to corresponding surfaces, and the ratio R is easy to calculate. No natural polarization measurements are necessary. This method is often used in practice.

One can clearly see why the errors become considerably larger for polarized deuteron targets than those for polarized proton targets (e.g., the background extrapolation procedure). Fortunately, in pn measurements, there exists another calibration method for target polarization. It consists of simultaneous measurements of pp and pn scattering observables. The quasielastic pp quantities (at least, the measured ones) have always been found to be equal to the free pp observables. The same normalization for pp and np quasielastic data must then be applied.

The problem of target polarization measurement does not arise with ^{6}LiD, in which no quadrupolar moment contribution exists, due to the crystalline structure of the material (face-centered cubic). The NMR signal distribution has the same Gaussian-like shape as that for proton polarization, and the natural polarization is rather easily measured.

The preceding discussion stresses the importance of ^{6}LiD targets. A world-wide collaborative effort in this field was initiated at the Bonn International Conference of 1990, involving a large number of experts from many laboratories, including Bonn, CERN, Gatchina, JINR, Dubna, Kharkov, Kyoto, LAMPF, Prague, PSI, Saclay, and TRIUMF. Tests with this type of target were carried out at Saclay (Chaumette *et al.*, 1989) and at PSI (van den Brandt *et al.*, 1991), where a first experiment, studying spin effects on ^{6}Li, has already been performed (Ritt *et al.*, 1991).

C. General comments on the polarized-target technique

From the user's point of view, the main drawback of the polarized-target technique is that the sign of the polarization cannot be changed very frequently. Delays between two measurements with opposite target polarizations introduce possible systematic errors due to the variations in some other detector efficiencies [mainly multiwire proportional chambers (MWPC's)]. One of the possible ways of avoiding this type of systematic error is to make a simultaneous measurement with two targets, one polarized and one unpolarized. The distance between the two targets must be small enough that measurements can be done with the same apparatus under approximately the same conditions. Since scattering on the unpolarized target depends only on the beam characteristics, the counting rates and asymmetries from the PPT (PDT) may be normalized to those of the unpolarized target for the same beam polarization state. This method suppresses a major part of the systematic instrumental errors (Perrot *et al.*, 1987).

The same reason may justify the development of polarized jets, whose polarization may be reversed as quickly as that of a polarized-ion source. They may be built into the internal beam line and can work during simultaneous runs with other experiments. Physicists have already worked with unpolarized jet targets (e.g., JINR synchrophasotron, CERN-ISR, UA6, JETSET at LEAR, etc.). But the main difficulty with polarized jets is to increase the intensity by a factor of ~ 100 in order to reach convenient luminosity. Development of polarized atomic jets is being pursued by the NEPTUN (UNK) and HELP (CERN) collaborations. Use of a storage cell to accumulate atoms in the tube around the stored beam, in order to increase the target thickness by about one order of magnitude, is being developed by different groups [e.g., HERMES (HERA), COSY, FILTEX (LEAR)].

V. RECENT EXPERIMENTAL RESULTS

A large number of new data for *NN* elastic scattering and transmission experiments above 100 MeV have appeared recently. The quality of the different *pp* and *np* sets of observables is discussed below. Mainly unpublished or recently published results are treated here. For older results, we refer the reader to compilations by Bystricky and Lehar, 1978, 1981, Bystricky, Carlson, *et al.*, 1980, Bystricky, Lechanoine-LeLuc, and Lehar, 1987, 1990, and Lechanoine-LeLuc *et al.*, 1987.

A. Proton-proton total cross sections

No new data for the spin-independent total cross sections $\sigma_{0\,\mathrm{tot}}$ and total elastic cross sections $\sigma_{\mathrm{tot}}(\mathrm{elastic})$ have been measured recently for *pp* or for *nn* scattering. What quantities there are for *pp* are shown in Fig. 5. Figure 6 shows the energy dependence of *pp* σ_{tot} (inelastic).

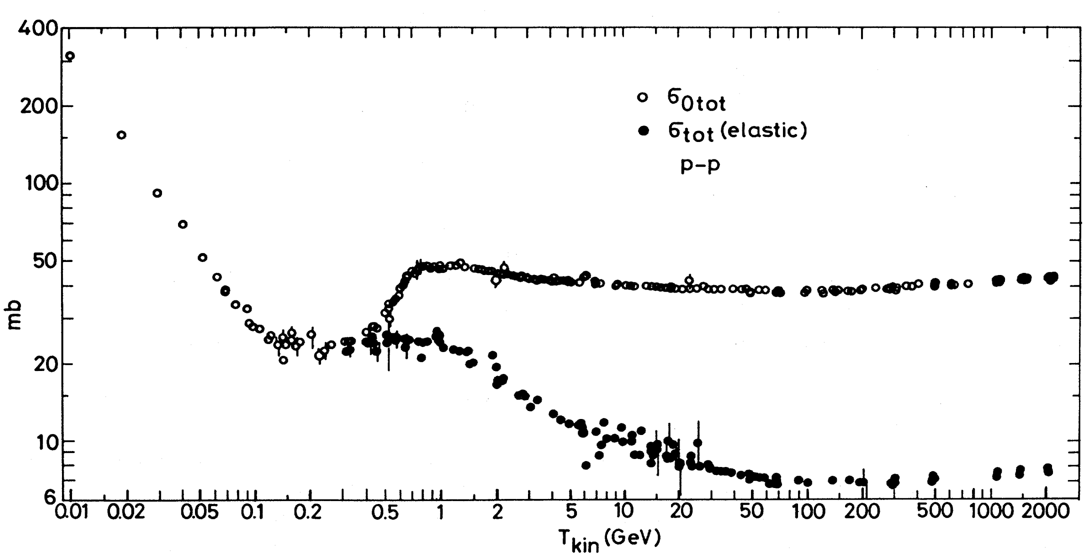


FIG. 5. Energy dependence of *pp* spin-independent total cross section $\sigma_{0\,\mathrm{tot}}$ (open circles) and total elastic cross section σ_{tot} (solid curves).

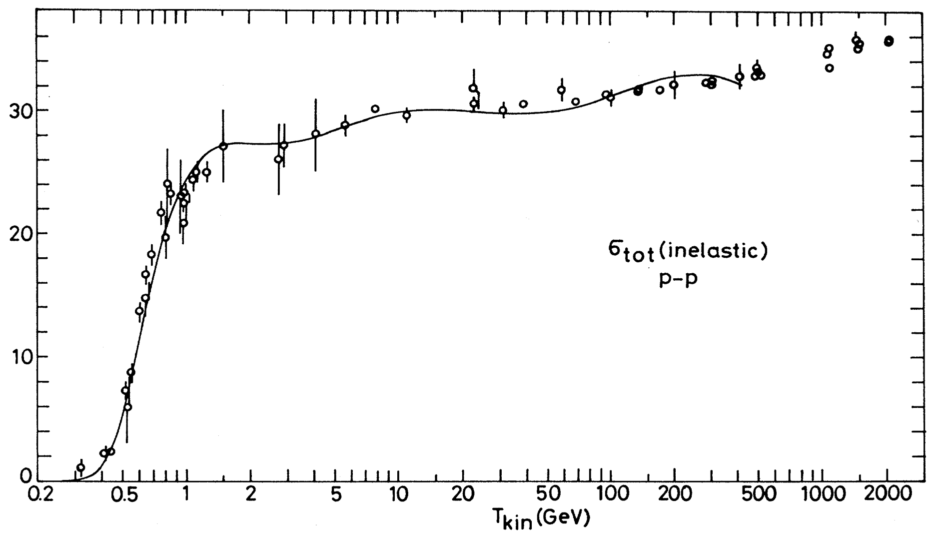


FIG. 6. Energy dependence of pp total inelastic cross section σ_{tot} (inelastic). ○, all existing "direct" results, ——, fit from Bystricky, LaFrance, *et al.*, 1987.

Few so-called "direct" measurements of the inelastic cross-section data are published (shown as open circles in Fig. 6); these were deduced from the difference $\sigma_{0\ tot}-\sigma_{tot}$ (elastic), for the most part using bubble-chamber data. The total inelastic cross section can, however, also be determined by summing the total cross sections over all inelastic channels (Bystricky, Lechanoine-LeLuc, and Lehar, 1987). Results from this method are shown as the solid line in Fig. 6. For nn $\sigma_{0\ tot}$, measurements are available between 80 MeV and 8.3 GeV.

Note that the pp total cross sections are less accurately measured than the $\bar{p}p$ ones. At low energy, for example, pp transmission experiments for $\sigma_{0\ tot}$ have been done around 100 MeV (integrated differential cross sections are known at lower energies), whereas for the $\bar{p}p$ interaction, such measurements exist down to 10 MeV (Hamilton *et al.*, 1980; Bugg *et al.*). At higher energy, pp data have been measured up to 2 TeV and $\bar{p}p$ data up to 1700 TeV. A similar statement is also valid for the elastic, inelastic, and individual reaction channel total cross sections.

B. The pp polarized total cross-section differences

The available data for the pp total cross-section differences, measured with polarized beam and target,

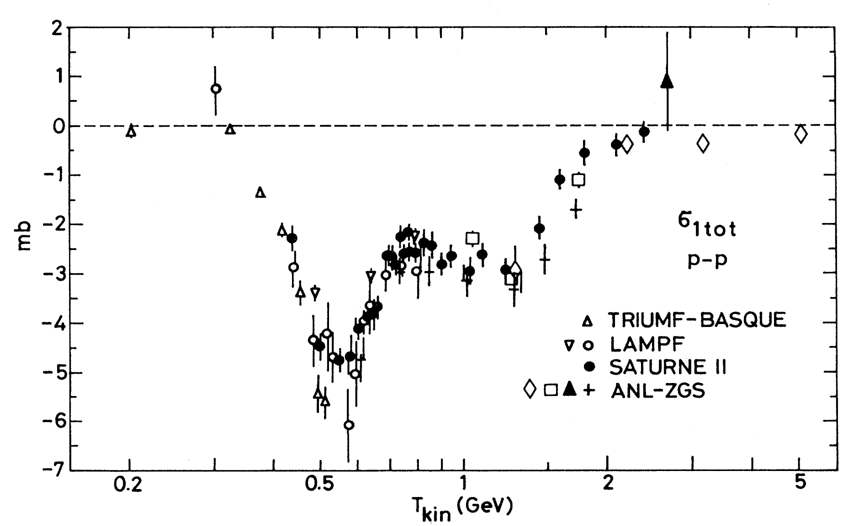


FIG. 7. $\sigma_{1\ tot}(pp)=-\frac{1}{2}\Delta\sigma_T(pp)$ energy dependence. BASQUE (Axen *et al.*, 1981), LAMPF (Ditzler *et al.*, 1983, Madigan *et al.*, 1985), SATURNE II (Perrot *et al.*, 1987), ANL-ZGS (Parker *et al.*, 1973; de Boer *et al.*, 1975; Biegert *et al.*, 1978).

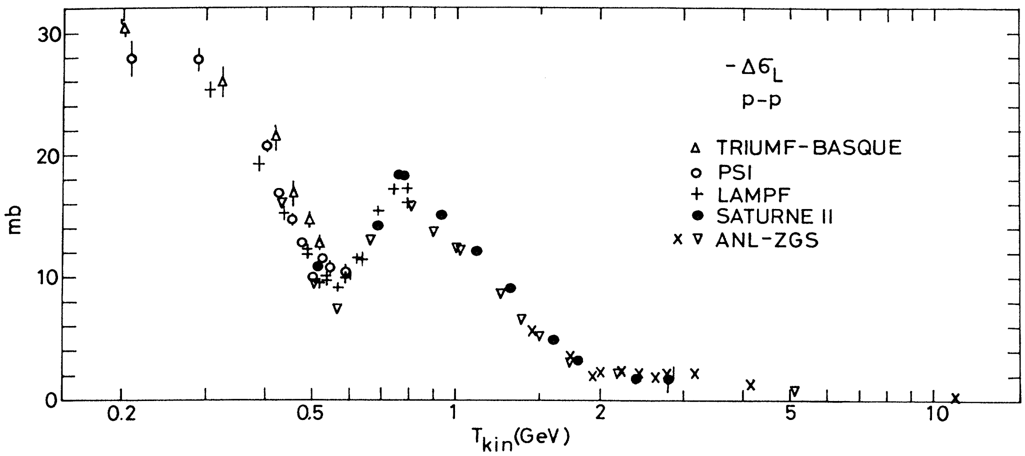


FIG. 8. $-\Delta\sigma_L(pp)$ energy dependence. BASQUE (Axen *et al.*, 1981), PSI (Aprile-Giboni *et al.*, 1984), LAMPF (Auer *et al.*, 1984, 1986), SATURNE II (Bystricky Chaumette, *et al.*, 1984), ANL-ZGS (Auer, Beretvas, *et al.*, 1977; Auer, Colton, *et al.*, 1977, 1978, 1989).

$\Delta\sigma_T$ $(-2\sigma_{1\ \mathrm{tot}})$ and $\Delta\sigma_L$, are shown in Figs. 7 and 8. The dip at $\sim$600 MeV, observed in the $\Delta\sigma_L$ energy dependence, was first interpreted as a manifestation of a possible dibaryonic resonance in the $^1D_2$ partial wave and the maximum at 750 MeV as a resonance in the $^3F_3$ wave. Note that $\Delta\sigma_L$ contains singlet, uncoupled, and coupled triplet partial waves, whereas $\sigma_{1\ \mathrm{tot}}$ contains only singlet and coupled triplet waves. The fact that the two structures at 600 MeV and 750 MeV are observed in both $\sigma_{1\ \mathrm{tot}}$ and $\Delta\sigma_L$ tends to confirm the hypothesis of a $^1D_2$ resonance but disprove that of an uncoupled triplet $^3F_3$. In Sec. V.G. these hypotheses will be discussed once more, where we compare spin-dependent total cross sections for $I=1$ and $I=0$ and where even the $^1D_2$ structure becomes dubious.

The ANL-ZGS measurements at energies between 2 and 12 GeV (Auer *et al.*, 1989) show a possible structure around 2.7 GeV mass ($T_{\mathrm{kin}}\sim 2.1$ GeV). These results, together with the Saclay data (Bystricky, Chaumette, *et al.*, 1984) above 2 GeV/c, are shown in Fig. 9 with their total errors obtained by combining statistical and systematic errors. The observed structure is of the same size as the systematic errors; it would be highly desirable to repeat this experiment with better statistics and systematics. This structure has been interpreted by Gonzales and Lomon (1986), LaFrance and Lomon (1986), and Gonzales, LaFrance, and Lomon (1987) as a possible dibaryonic resonance in the S state.

At very high energy, $\Delta\sigma_L$ has recently been measured at 200 GeV for pp and $\bar{p}p$ scattering at FERMILAB. Preliminary results show that the absolute value of $\Delta\sigma_L$ is about 0.1 mb for both interactions.

C. Elastic *pp* scattering

The analyzing power and spin correlation A_{oonn} (Vovchenko *et al.*, 1986) were recently measured at the LIYaF-Gatchina 1-GeV synchrocyclotron. Figures 10(a) and 10(b) show these Gatchina data at 0.95 GeV and compare them to the SATURNE II data at 0.934 GeV (Bystricky, Chaumette, *et al.* 1985). The A_{oonn} angular dependence of both sets is similar, but they differ slightly in absolute value. This is in contrast to the preliminary

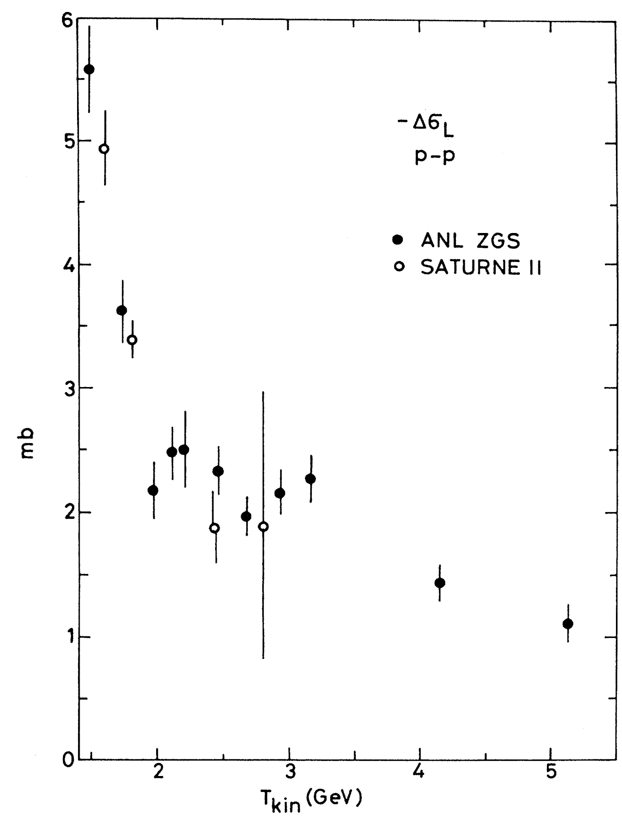


FIG. 9. $-\Delta\sigma_L(pp)$ energy dependence between 1.4 and 5.5 GeV. ●, ANL-ZGS (Auer, Colton, *et al.*, 1989); ○, SATURNE II, (Bystricky, Chaumette, *et al.*, 1984).

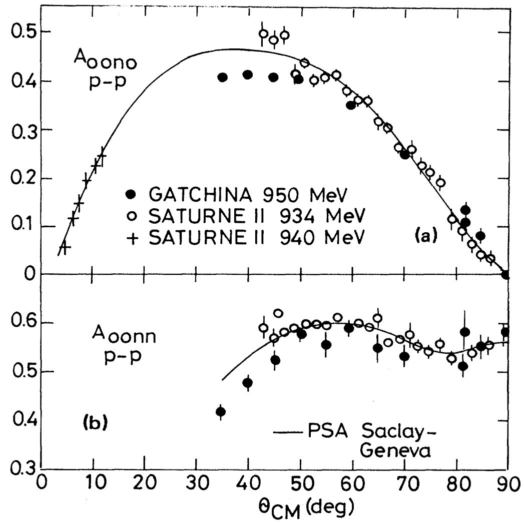


FIG. 10. $A_{oono}(pp)$ (10a) and $A_{oonn}(pp)$ (10b) angular dependence. ●, 0.940 GeV Gatchina (Vovchenko *et al.*, 1986); ○, 0.934 GeV SATURNE II (Bystricky, Derégel, *et al.*, 1985b); +, 0.940 GeV SATURNE II (Dalla Torre-Colautti *et al.*, 1989); the solid line is from Saclay-Geneva phase-shift analysis (Bystricky, Lechanoine-LeLuc, and Lehar, 1990).

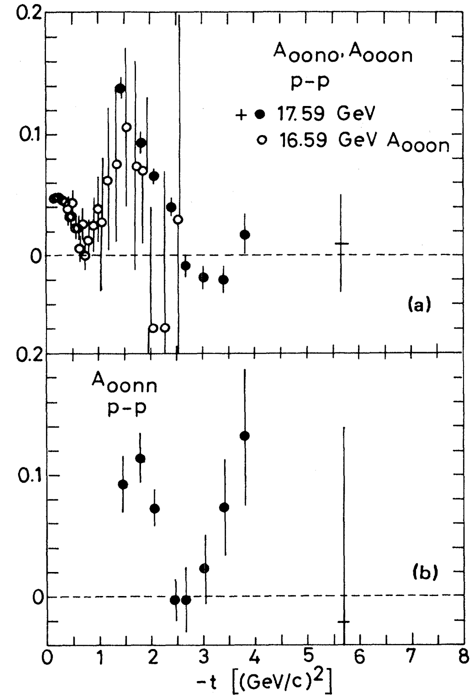


FIG. 11. $A_{oono}(pp)$ (11a) and $A_{oonn}(pp)$ (11b) t dependence. ●, 17.59 GeV (A_{oono}, A_{ooon}) BNL-AGS (Crabb *et al.*, 1988); +, 17.59 GeV (A_{oono}, A_{ooon}) BNL-AGS (Court *et al.*, 1986); ○, 16.59 GeV (A_{ooon}) CERN (Borghini *et al.*, 1972).

LIYaF data (Prokofiev, 1984), which were in excellent agreement with the Saclay results. The present small disagreement is probably due to the renormalized value of the proton beam polarization introduced in the latest Gatchina measurements. Figure 10(a) also shows the A_{oono} data at small angles (Dalla Torre-Colautti *et al.*, 1989) measured by the Trieste and Annecy groups at SATURNE II. Solid lines in Fig. 10 are predictions from the recent Saclay-Geneva phase-shift analysis (Bystricky, Lechanoine-Leluc, and Lehar, 1990). At Gatchina the spin-transfer parameter K_{noon} was also measured at 0.80, 0.85, and 0.90 GeV (Borisov *et al.*, 1986), as well as the three-index parameter $M_{s'okn}$ at 0.95 GeV (Bazhanov *et al.*, 1988).

At KEK the analyzing power A_{oono} was measured for a fixed backward laboratory angle (68°) at kinetic energies between 0.44 and 2.2 GeV (Shimizu *et al.*, 1989) using an internal "wire" CH_2 target. The measurements were performed during acceleration cycles. The data below 1.4 GeV show two maxima at 0.66 and 0.73 GeV and a minimum at 0.85 GeV. The authors have suggested a possible relation with narrow dibaryonic resonances at 2.160, 2.192, and 2.242 total masses, respectively. Such effects were not observed in the SATURNE II results, measured in small energy steps between 0.5 and 1.1 GeV (Garçon *et al.*, 1987) in the angular range 40°–90°. Numerical values of the KEK data are not yet available.

A large number ($\sim$3000) of new pp data were obtained by the nucleon-nucleon group at SATURNE II in the energy range from 0.5 to 2.7 GeV. At eleven energies between 0.83 and 2.7 GeV, the number of different spin observables (between 11 and 15 depending on energy) is

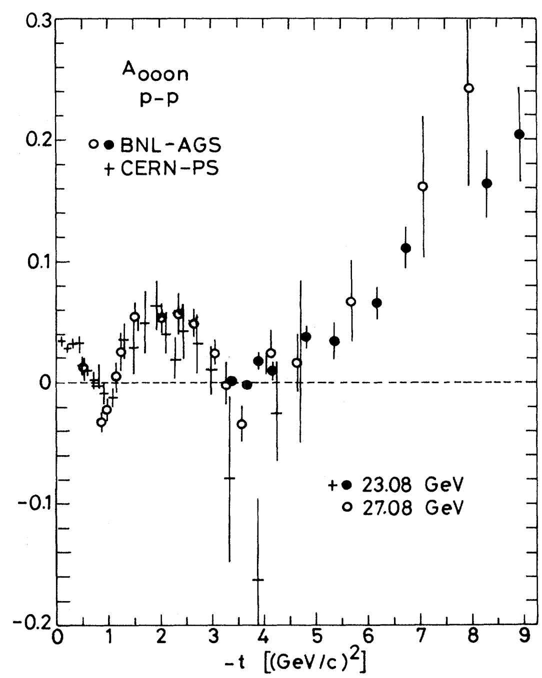


FIG. 12. $A_{ooon}(pp)$ t dependence. ●, 23.08 GeV BNL-AGS (Crabb, Kaufman, *et al.*, 1990); +, 23.08 GeV CERN-PS (Antille *et al.*, 1981); ○, 27.08 GeV BNL-AGS (Cameron *et al.*, 1985).

large enough to allow a direct reconstruction of the pp elastic-scattering matrix (see Sec. VIII). All data points are now published (see the most recent references listed by Bystricky, Lechanoine-LeLuc, and Lehar, 1990; Lac *et al.*, 1989a, 1989b, 1989c, 1989d; and Fontaine *et al.*, 1989).

At the BNL-AGS, the analyzing power A_{oono} and spin-correlation parameter were measured at 17.59 GeV (Crabb *et al.*, 1988) using an accelerated polarized proton beam (Khiari *et al.*, 1989) and a NH_3 polarized proton target (Crabb, Higley, *et al.*, 1990). The results are shown in Figs. 11(a) and 11(b) together with previous measurements (Court *et al.*, 1986). Analyzing power is also compared to that of the CERN-PS data (Borghini *et al.*, 1972). The analyzing power A_{ooon} was also measured at 23.08 GeV (Crabb, Kaufman, 1990) with the NH_3 polarized proton targets Figure 12 compares these data with previous results at 23.08 GeV (Antille *et al.*,

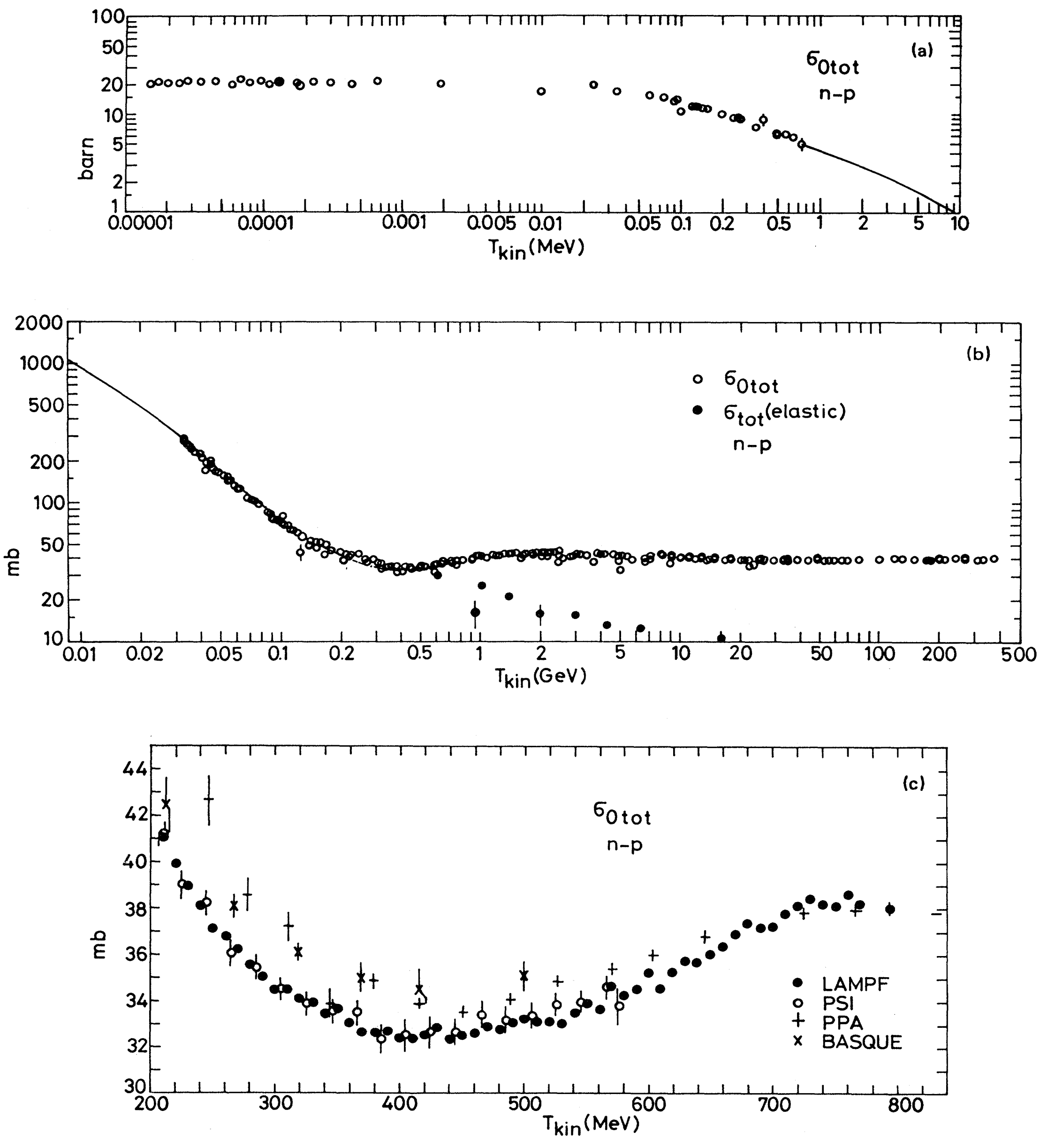


FIG. 13. Total neutron-proton cross sections: (a) np spin-independent total cross section $\sigma_{0\,\mathrm{tot}}$ energy dependence from 10 eV to 10 MeV. ●, Dilg, 1975; ○, all existing data; ——, Cierjacks *et al.*, 1969. (b) $\sigma_{0\,\mathrm{tot}}$ and σ_{tot} (elastic) energy dependence for np scattering from 9 MeV to 500 GeV. ○, all existing data for $\sigma_{0\,\mathrm{tot}}$; ●, all existing data for σ_{tot} (elastic); Cierjacks *et al.*, 1969. (c) $\sigma_{0\,\mathrm{tot}}(np)$ energy dependence from 200 MeV to 800 MeV. ●, LAMPF (Lisowski *et al.*, 1982); ○, PSI (Grundies *et al.*, 1985); ×, BASQUE (Keeler *et al.*, 1982); +, PPA (Devlin *et al.*, 1973).

1981) and 27.08 GeV (Cameron *et al.*, 1985). All new BNL data confirm the t-dependent structures observed at all energies (see Sec. VI).

D. Neutron-proton total cross sections

Total cross sections $\sigma_{0\,\mathrm{tot}}(np)$ have been measured over a large energy range (1 eV up to several hundred GeV). The data at very low energy show that the total cross section is constant up to ~ 200 eV, then decreases up to 400 MeV. The generally accepted value at zero kinetic energy has been most accurately measured by Dilg (1975) and is (20.491 ± 0.014) barn, corresponding to an np scattering length a_s of (-23.749 ± 0.009) fm.

Figure 13(a) shows the energy dependence of $\sigma_{0\,\mathrm{tot}}$ from 10 eV to 10 MeV. The black dot is the value from Dilg, 1975, while the solid line represents the Karlsruhe data (Cierjacks *et al.*, 1969). Figure 13(b) shows the data from 9 MeV to 500 GeV. Numerous data, from Cierjacks *et al.*, 1969, are also shown as a solid line. A considerable disagreement (up to 2 mb at 400 MeV) is observed between LAMPF (Lisowski *et al.*, 1982) and PSI (Grundies *et al.*, 1985) data on the one hand and TRIUMF-BASQUE (Keeler *et al.*, 1982) and PPA (Devlin *et al.*, 1973) data on the other, as illustrated in Fig. 13(c). The other existing points (Bystricky and Lehar, 1978, 1981; Bystricky, Carlson, *et al.*, 1980) cannot help to resolve this discrepancy due to their large errors. The fact that the PSI data (Grundies *et al.*, 1985) have confirmed the energy dependence observed at LAMPF (Lisowski *et al.*, 1982) and that phase-shift analyses (Bystricky, Lechanoine-LeLuc, and Lehar, 1987; Arndt, 1989) describe these two sets of data well give them more credence. There exist only a few directly measured np total elastic cross sections. The energy dependence of this quantity is plotted in Fig. 13(b) as a group of black dots.

To determine the total np inelastic cross σ_{tot} (inel), Bystricky, LaFrance, *et al.* (1987) used a method similar to that described in Sec. V.A for pp. In contrast with the case for pp scattering, it was hard to use np topological cross sections at high energies, since one-prong np events were often measured for elastic and inelastic scattering together. At low energies, mainly exclusive np reaction channels are known, and in these cases the np σ(inelastic) energy dependence can be calculated only up to 4.2 GeV. In the energy range below 0.8 GeV, the following four channels dominate:

$$np \Longrightarrow d\pi^0 \,, \tag{5.1}$$

$$np \Longrightarrow np\pi^0 \,, \tag{5.2}$$

$$np \Longrightarrow pp\pi^- \,, \tag{5.3}$$

$$np \Longrightarrow nn\pi^+ \,. \tag{5.4}$$

The reaction (5.1) has often been measured for np scattering, but almost always normalized to $\frac{1}{2}\sigma_{\mathrm{tot}}(pp \Longrightarrow d\pi^+)$ assuming isospin invariance. Consequently no serious check of this symmetry can be performed by comparing $pp \Longrightarrow d\pi^+$ to $np \Longrightarrow d\pi^0$ total cross sections. The reactions (5.3) and (5.4) are related by charge symmetry conservation. Here again no check of this symmetry is possible, as the existing data for each reaction have not been measured at the same energy. The total cross section for the reaction (5.2) is hard to measure and was deduced mainly from the difference between pp and pd scattering, corrected for the deuteron bound state. Assuming isospin invariance, one does not need to take the reaction $np \Longrightarrow np\pi^0$ into account. This is demonstrated by the relation $A = B$ where

$$A = 2\sigma_{\mathrm{tot}}(np \Longrightarrow pp\pi^-) + \sigma_{\mathrm{tot}}(pp \Longrightarrow pn\pi^+) \,, \tag{5.5a}$$

$$B = 2\sigma_{\mathrm{tot}}(pp \Longrightarrow pp\pi^0) + 2\sigma_{\mathrm{tot}}(np \Longrightarrow np\pi^0) \,, \tag{5.5b}$$

assuming

$$\sigma_{\mathrm{tot}}(np \Longrightarrow pp\pi^-) = \sigma_{\mathrm{tot}}(np \Longrightarrow nn\pi^+) \,. \tag{5.6}$$

In Bystricky *et al.*, 1987a, it was found that the relation $A = B$ does not hold even below 670 MeV where data are sufficient: the ratio $(A - B)/(A + B)$ was found to be ~ 0.06 at 600 MeV. Obviously one obtains different $\sigma_{\mathrm{tot}}(\mathrm{inel})$ values assuming $A = B$ or not. This is one important source of disagreement between different phase-shift analyses using the same database. The energy dependence of $\sigma_{\mathrm{tot}}(\mathrm{inel})$ is shown in Fig. 14(a), where the equivalence $A = B$ is not assumed. Using the fit for $\sigma_{\mathrm{tot}}(\mathrm{inel})pp$, the $I = 0$ part is obtained as shown in Fig. 14(b).

E. The *np* polarized total cross-section differences

The np total cross-section differences $\Delta\sigma_T(np)$ and $\Delta\sigma_L(np)$, using free polarized neutrons, were first measured at SATURNE II yielding four points with relatively large errors (Lehar, de Lesquen, van Rossum, *et al.*, 1987). These results have been completed by new measurements at 9 to 10 energies for each observable (Fontaine *et al.*, 1991; Ball, 1992). The Saclay results were soon followed by PSI measurements in the energy region from 140 to 590 MeV. The latter used a similar counter setup, but completely different electronics. A polarized neutron beam with a continuous energy spectrum (Binz *et al.*, 1989) as described in Sec. III.B.3 was used, so that $\Delta\sigma_T$ or $\Delta\sigma_L$ data could be collected simultaneously over the entire energy range 140–590 MeV (Binz *et al.*, 1991).

The observable $\Delta\sigma_L$ was also measured at five energies at LAMPF (Beddo, 1990; Beddo *et al.*, 1991). These measurements were made with a quasi-monoenergetic polarized neutron beam produced in $pd \Longrightarrow n + X$ scattering of longitudinally polarized protons (see Sec. III.B.3). A large neutron counter hodoscope had to be used because of the small neutron beam intensity. Figures 15(a) and 15(b) show the energy dependence of all these measurements for $\sigma_{1\,\mathrm{tot}}(np)$ and $-\Delta\sigma_L(np)$, respectively. As can be seen, there is excellent agreement between all data sets. A dip at ~ 600 MeV for both observables is less

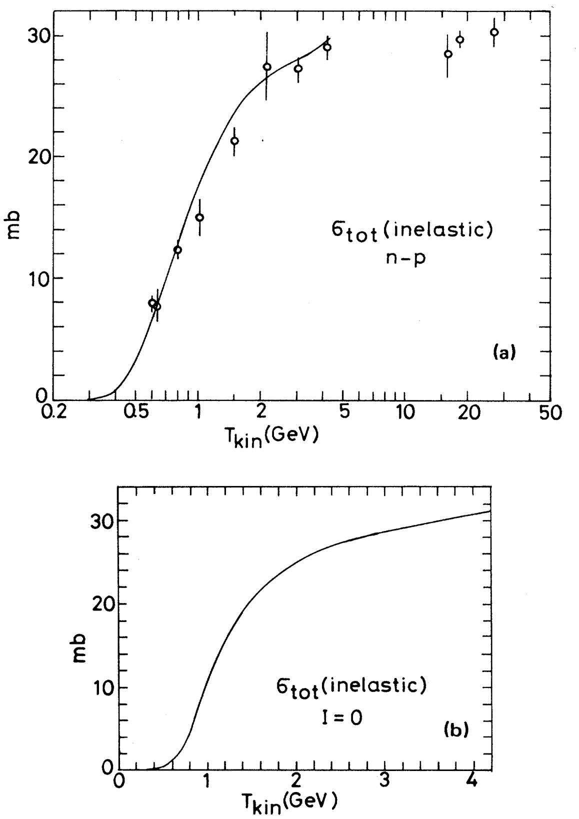


FIG. 14. Inelastic part of nucleon-nucleon total cross sections for different isospin states: (a) σ_{tot} (inelastic) energy dependence for *np* scattering. ○, all existing "direct" results; ——, fit from Bystricky, LaFrance, *et al.*, 1987. (b) σ_{tot} (inelastic) for the $I=0$ isospin state (Bystricky, Lechanoine-LeLuc, and Lehar, 1987).

pronounced than in the analogous *pp* quantities. A very broad maximum can be observed at ~0.9 GeV in the $-\Delta\sigma_L$ energy dependence.

In fact, the first $\Delta\sigma_L$ results were obtained in 1981 in a quasifree $p-n$ transmission measurement at the ANL-ZGS (Auer *et al.*, 1981). This experiment measured $\Delta\sigma_L(pd)$ and $\Delta\sigma_L(pp)$ by transmission of polarized protons through a partially deuterated polarized target. Taking a simple difference between *pd* and *pp* results, corrected only for beam and target polarizations and for Coulomb-nuclear rescattering including deuteron breakup, yields data in fairly good agreement with the Saclay, PSI, and LAMPF results. This is demonstrated in Fig. 16(a), where the solid line represents the hand-drawn energy dependence of $-\Delta\sigma_L$ and the black dots are the "uncorrected" ANL-ZGS results. The data corrected for Glauber-type rescattering (Spinka and Underwood, 1987) are represented in Fig. 16(b) by open circles. Between 0.8 and 1.1 GeV these results disagree with the free-*np* results. In 1982 and ANL-ZGS data were corrected by Kroll (1982), for Glauber-type rescattering, including final three-body state interactions (Grein and Kroll, 1982). This correction increases the disagreement, as can be seen in Fig. 16(c) (open squares).

We might mention that in the past no serious difference has ever been found between elastic and quasi-elastic scattering observables and that all attempts to correct quasifree scattering have failed. This is illustrated by the results obtained at PSI (Binz *et al.*, 1989; see Sec. III.B.3).

F. Elastic *np* scattering

Differential cross sections at medium and large scattering angles have been accurately measured for many years now in different laboratories. For energies below 100 MeV, we refer the reader to the review talks of Doll *et al.* (1991) and Ronnquist *et al.* (1991). Above 100

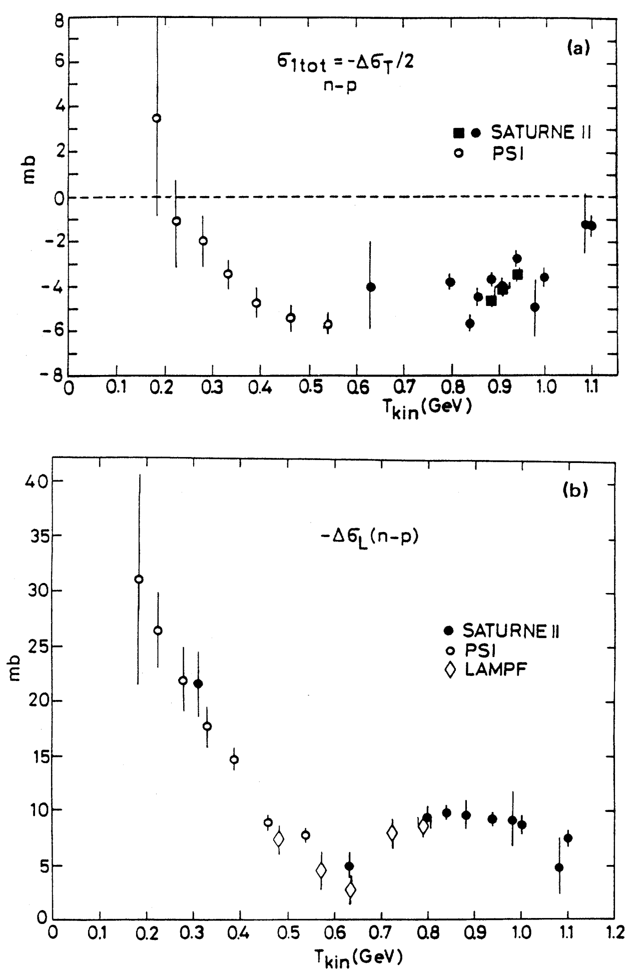


FIG. 15. Spin-dependent neutron-proton total cross section differences: (a) $\sigma_{1\,tot}(np) = -\frac{1}{2}\Delta\sigma_T(np)$ energy dependence. ○, PSI (Binz *et al.*, 1991); ●, SATURNE II (Fontaine *et al.*, 1991); ■, SATURNE II (Ball, 1992). (b) $-\Delta\sigma_L(np)$ energy dependence. ○, PSI (Binz *et al.*, 1991); ◇, LAMPF (Beddo *et al.*, 1991); ●, SATURNE II (Fontaine *et al.*, 1991).

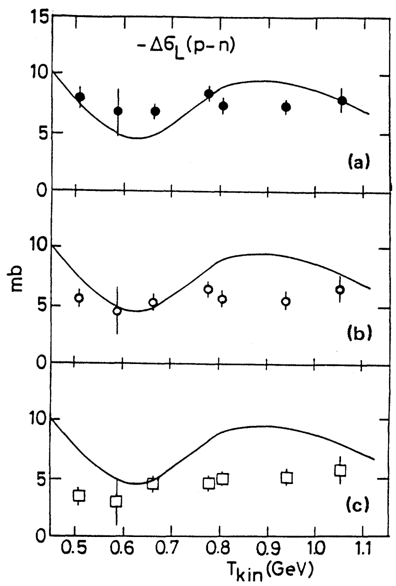
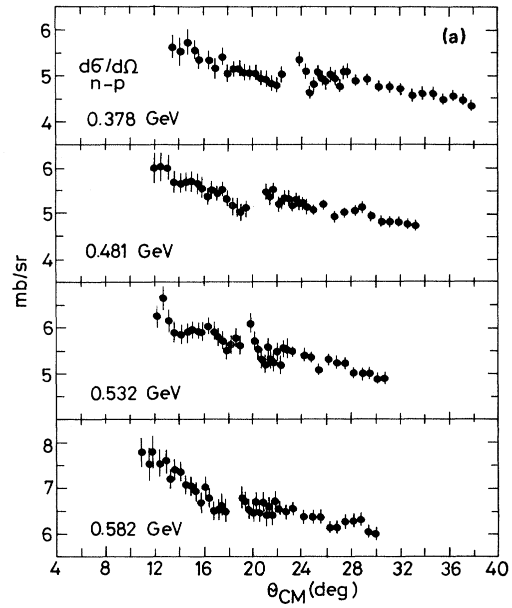


FIG. 16. Free np $-\Delta\sigma_L$ energy dependence compared with the ANL-ZGS quasielastic transmission measurements. ——, hand-drawn energy dependence for free np transmission; (a) ●, uncorrected data (Auer *et al.*, 1981); (b) ○, ANL corrections (Spinka and Underwood, 1987); (c) □, corrections from Kroll, 1982.

MeV, however, measurements at small angles have been scarce. At SATURNE II, the small-angle elastic np differential cross section at 378, 481, 532, 582, 633, 683, 708, 784, 834, 884, 934, 985, 1085, and 1135 MeV was measured by Saclay and Gatchina groups (Silverman *et al.*, 1989), resulting in 585 experimental points. A selective choice of results at six energies is shown in Figs. 17(a) and 17(b). In these experiments, neutrons were scattered at small angles inside a high-pressure drift chamber IKAR, which measured the slow recoil proton angle and path length in the gas. For A_{oono} measurements at small angles, the experimental setup was completed by two large neutron counters in order to determine the left-right asymmetry. These measurements were performed at 0.633, 0.784, 0.834, 0.934, and 0.985 GeV (Korolev *et al.*, 1985; Silverman *et al.*, 1989).

At IUCF the analyzing powers A_{oono} and A_{ooon}, as well as the spin correlation parameter A_{oonn}, were measured at 181 MeV (Sowinski *et al.*, 1987). One of the aims of this measurement was a check of isospin invariance (Knutson *et al.*, 1991) similar to a previous TRIUMF experiment at 477 MeV (Abegg *et al.*, 1986). At TRIUMF very precise analyzing power at four energies and spin correlation A_{oonn} at three energies were also measured, between 225 and 477 MeV, in a large angular region (Bandyopadhyay *et al.*, 1989). Figure 18 illustrates the A_{oonn} results around 200 MeV, which show a rapid energy variation.

Comparing the TRIUMF analyzing power data (Bandyopadhyay *et al.*, 1989) with LAMPF measurements



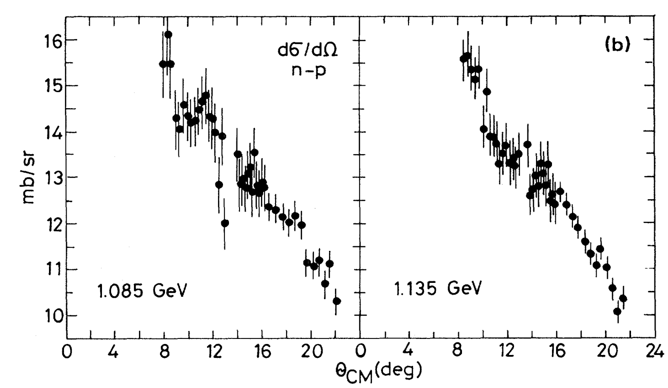


FIG. 17. np elastic differential cross section at small angles (Silverman *et al.*, 1989): (a) Data at 0.378, 0.481, 0.532, and 0.582 GeV; (b) Data at 1.085 and 1.135 GeV.

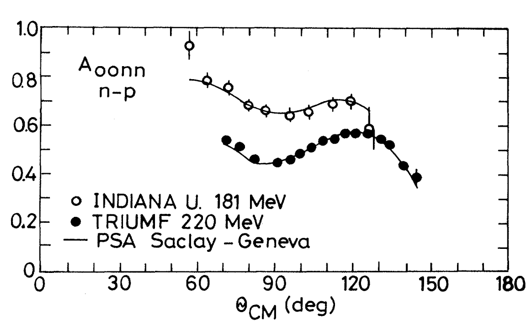


FIG. 18. $A_{oonn}(np)$ angular dependence at 181 MeV (Sowinski *et al.*, 1987) and 220 MeV (Bandyopadhyay *et al.*, 1989). Solid line is from Saclay-Geneva phase-shift analysis (Bystricky, Lechanoine-LeLuc, and Lehar, 1987).

(Newsom *et al.*, 1989) at 425 MeV, one observes a similar angular dependence, so that both data sets can easily be renormalized. In fact, as already discussed in Sec. III.B.2, McNaughton *et al.* (1992) recommend multiplying the data by 0.89; independently, phase-shift analyses (Bystricky, Lechanoine-LeLuc, and Lehar, 1987; Bugg, 1990) quote similar numbers (0.91 for Bystricky, Lechanoine-LeLuc, and Lehar, 1987). On the other hand, the BASQUE data (Clough *et al.*, 1980) are incompatible, as can be seen in Fig. 19. From this incompatibility, now definitely confirmed (Bugg, 1990), one important conclusion emerges: it is also necessary to check the two-index rescattering polarization observables measured by the BASQUE group, since these experimental quantities may also be affected. This can be done either by remeasuring the same quantities or by measuring other observables that provide constraints. Such measurements will be done at PSI on the NA2 new $n\uparrow$ line. The TRIUMF 477 MeV A_{oono} and A_{ooon} data (Bandyopadhyay *et al.*, 1989) agree with the SATURNE II measurement at the same energy, as was shown in Fig. 2. Above 600 MeV one observes a smooth connection between the results at small and medium angles and an excellent agreement between the elastic and quasielastic data.

At PSI measurements of spin correlation parameters A_{oonn}, A_{ookk}, A_{ooss}, and A_{oosk} between 90° and 170° have been performed with the *NE*1 polarized neutron beam discussed above (Sec. III.B.2). In addition, using a dedicated apparatus, Binz *et al.* (1990; Binz, 1991) have measured A_{oonn} and A_{ookk} observables close to 180°. Similar 180° experiments have been performed at SATURNE II at discrete energies (Binz, 1991).

At LAMPF, spin-correlation parameters A_{ooss} and A_{ooks} were measured using a one-arm spectrometer, between 30° and 180° c.m. at 484, 634, 720, and 788 MeV (Rawool, 1988; Garnett, 1989; Garnett *et al.*, 1989) and the spin correlations A_{ookk} and A_{oosk} between 80° and 180° at 484, 634, and 788 MeV (Ditzler *et al.*, 1992). The observable A_{oonn} was measured at 790 MeV between 48° and 99° c.m. using a two-arm spectrometer (Nath *et al.*, 1989). Polarization transfer parameters $K_{os''so}$, $K_{ok''ko}$, $K_{os''ko}$, and $K_{ok''so}$ were measured at 484 and 788 MeV using a free polarized neutron beam and a liquid-hydrogen target (McNaughton *et al.*, 1991). The same observables were measured at 634 and 720 MeV in 1991. Plans exists for K_{onno} measurements at 484, 634, 720, and 788 MeV in the future.

At SATURNE II, in the nucleon-nucleon program, 11 spin-dependent *np* observables were measured at 840, 880, 940, 1000, and 1100 MeV, in order to determine directly the *np* scattering amplitudes (see Sec. VIII). A small fraction of the data were obtained in quasielastic scattering of protons and neutrons using accelerated polarized deuterons on a polarized proton target. The quasielastic *np* and *pp* data (denoted by QE in the figures; Bystricky, Derégel, *et al.*, 1985a; de Lesquen *et al.*, 1988) are in excellent agreement with the free-nucleon results. A major part of the *NN* results is still unpublished, and we refer the reader to the preliminary summaries given by Lehar (1991a, 1991b, 1992) and Ball (1992).

It is impossible to illustrate all these data, so we have chosen to show the experimental situation around 800 MeV. At this energy the data are most complete and involve overlapping results from SATURNE II, Gatchina, and mainly LAMPF [Fig. 20(a) to 20(j)].

There are a few remarks to be made about the energy dependence of the spin-correlation parameters:

(1) For A_{oonn}: below 800 MeV this parameter stays almost positive and crosses zero at $\theta_{\mathrm{c.m.}} \sim 150°$. Above this energy, the A_{oonn} angular distribution changes shape for $\theta_{\mathrm{c.m.}} > 80°$ and becomes large and negative. A pronounced minimum is observed around 110°.

(2) For A_{ookk}: At 630 MeV, the values measured in the forward hemisphere, seem to be larger than the values predicted by different phase-shift analyses (Lechanoine-Leluc *et al.*, 1987). Close to 180° c.m., the observable changes sign and becomes largely negative [see Fig. 20(d)].

(3) A_{oosk} stays small over a large energy domain [Fig. 20(e)].

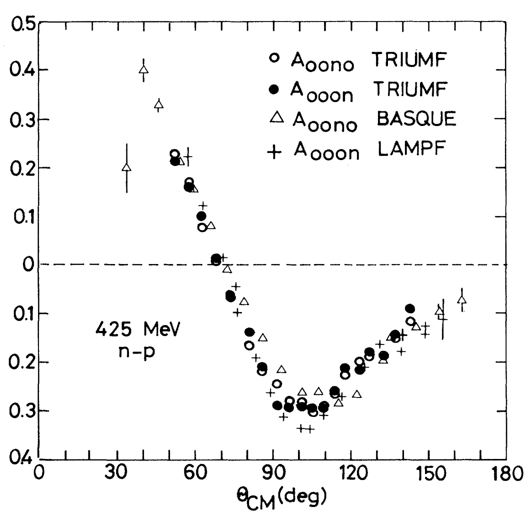


FIG. 19. A_{oono} and A_{ooon} data at 425 MeV. ○, A_{oono} TRIUMF (Bandyopadhyay *et al.*, 1989); ●, A_{ooon} TRIUMF (Bandyopadhyay *et al.*, 1989); △, A_{oono} BASQUE (Clough *et al.*, 1980); +, A_{ooon} LAMPF (Newsom *et al.*, 1989).

G. Isospin $I=0$ total cross sections

From the measured $\sigma_{0\,\mathrm{tot}}$, $\Delta\sigma_T$ and $\Delta\sigma_L$ for *pp* and *np* scattering, it is possible to determine the $I=0$ total cross sections, as illustrated in the following equations:

$$\begin{aligned}
&\sigma_{0\,\mathrm{tot}}(I=0)=2\sigma_{0\,\mathrm{tot}}(np)-\sigma_{0\,\mathrm{tot}}(pp)\ ,\\
&\sigma_{1\,\mathrm{tot}}(I=0)=2\sigma_{1\,\mathrm{tot}}(np)-\sigma_{1\,\mathrm{tot}}(pp)\ ,\\
&\Delta\sigma_L(I=0)=2\Delta\sigma_L(np)-\Delta\sigma_L(pp)\ .
\end{aligned} \tag{5.7}$$

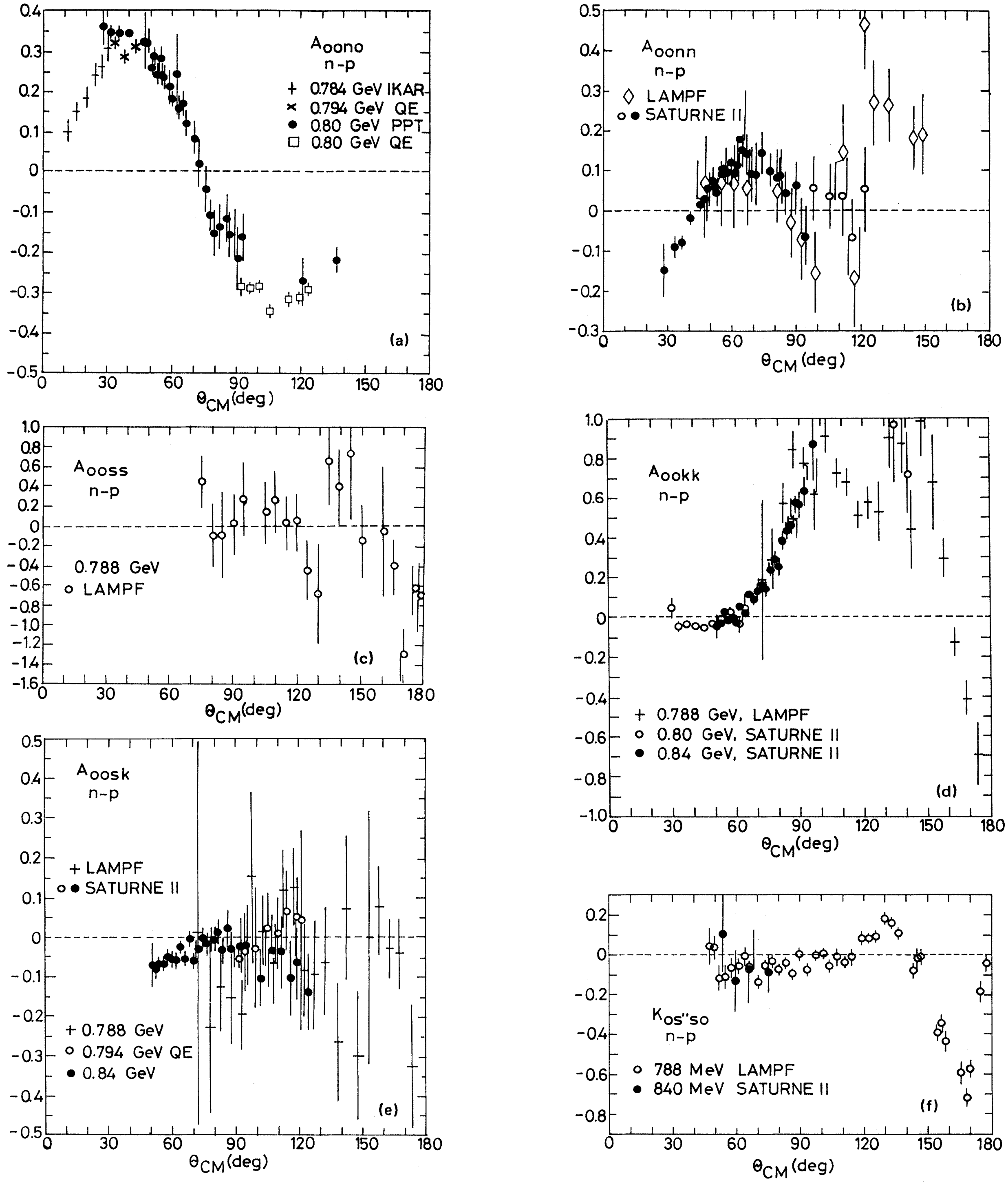


FIG. 20. Spin-dependent neutron-proton elastic scattering observables. (a) $A_{oono}(np)$ angular dependence around 0.8 GeV at SATURNE II. +, IKAR, Korolev *et al.*, 1985, Silverman *et al.*, 1989; ×, Bystricky, Derégel, *et al.*, 1985; □, de Lesquen *et al.*, 1988; •, Ball, 1992; *QE*, deuteron beam. (b) $A_{oonn}(np)$ at ~0.8 GeV. ◇, 790 MeV, LAMPF (Nath *et al.*, 1989), ○, 794 MeV, SATURNE II (de Lesquen *et al.*, 1988); •, 800 MeV, SATURNE II (Ball, 1992); QE, deuteron beam. (c) $A_{ooss}(np)$ angular dependence at 0.788 GeV measured at LAMPF (Garnett *et al.*, 1989; Rawool, 1988). (d) $A_{ookk}(np)$ at ~0.8 GeV. ○, LAMPF (Ditzler *et al.*, 1990); •, SATURNE II (Ball, 1992). (e) $A_{oosk}(np)$ at ~0.8 GeV. ○, LAMPF (Ditzler *et al.*, 1990); •, SATURNE II (Ball, 1992). (f) $K_{os''so}(np)$ at ~0.8 GeV. ○, LAMPF (McNaughton *et al.*, 1991); •, SATURNE II (Lehar, 1991a). (g) $K_{os''ko}(np)$ at ~0.8 GeV. ○, LAMPF (McNaughton *et al.*, 1991); •, SATURNE II (Lehar, 1991a). (h) $K_{ok''so}(np)$ at 0.788 GeV measured at LAMPF (McNaughton *et al.*, 1991). (i) $K_{ok''ko}(np)$ at 0.788 GeV measured at LAMPF (McNaughton *et al.*, 1991). (j) D_{onon}, $D_{os''ok}$, K_{onno}, N_{onkk}, and N_{onsk} data at 0.84 GeV measured at SATURNE II in *np* scattering (Lehar, 1991a).

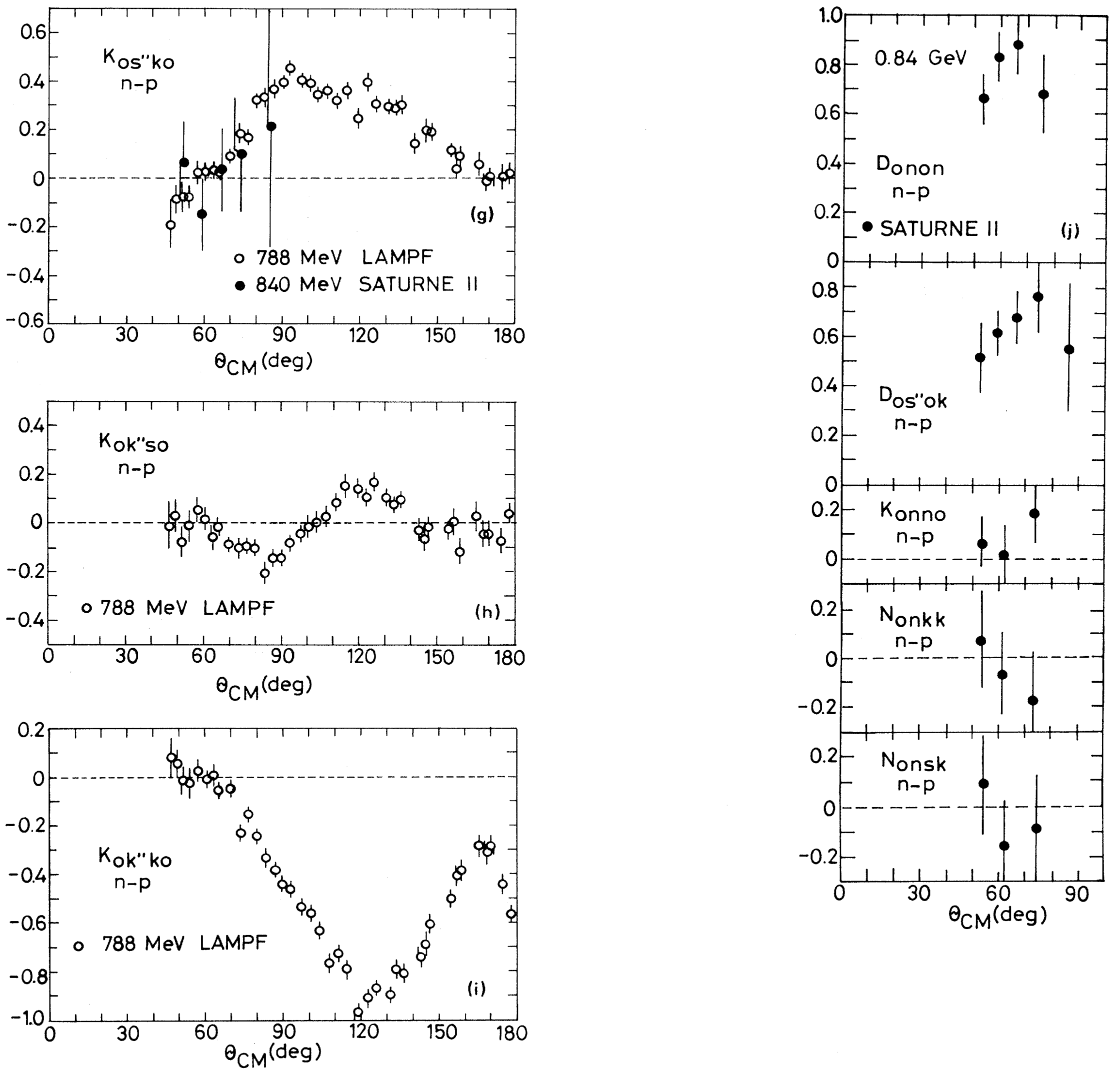


FIG. 20. (*Continued*).

Figures 21(a) and 21(b) show the energy behavior of $\sigma_{1\,\mathrm{tot}}(I=0)$ and $-\Delta\sigma_L(I=0)$, respectively. The shape of $\sigma_{1\,\mathrm{tot}}$ and $-\Delta\sigma_L$ observables for the $I=0$ state is similar to that observed for the corresponding *np* observables. Note that the $^1D_2$ partial wave is not present in $I=0$ amplitudes. Therefore one cannot say that the energy dependence of $I=0$ observables supports the hypothesis attributing the origin of the $\sigma_{1\,\mathrm{tot}}$ and $-\Delta\sigma_L$ behavior in *pp* and *np* scattering only to the $^1D_2$ partial wave (see Sec. V.B).

H. Amplitudes in the forward direction

At zero degrees, only three independent complex scattering amplitudes remain (see Sec. II); therefore at least six independent experimental quantities are required to directly compute the amplitudes. But only three total cross sections, $\sigma_{0\,\mathrm{tot}}$, $\Delta\sigma_T$, $\Delta\sigma_L$, can be measured at 0°. As the knowledge of this forward amplitudes is important, different approaches have been used to access this information. Let us review them briefly.

(1) Optical theorems, explicitly written in Eq. (2.12), give access to the imaginary parts of $(a+b)$, c, and d via measurements of $\sigma_{0\,\mathrm{tot}}$, $\Delta\sigma_T$, and $\Delta\sigma_L$. Using Eqs. (5.7), one can extract separately the $I=0$ and $I=1$ parts of these amplitudes. This is represented in Figs. 22(a) and 22(b), respectively.

(2) A quantity often discussed at 0° is the ratio of the real to the imaginary part of the spin-independent amplitudes $(a+b)$, that is, $\rho=\mathrm{Re}(a+b)\mathrm{Im}(a+b)$. Writing the decomposition of the differential cross section measured at small angles into Coulomb (C), nuclear (N), and interference contributions, we have

$$(d\sigma/dt)^N=(\pi/4h^2)\sigma_{0\,\mathrm{tot}}(1+\rho^2)(1+\eta^2)\exp(-bt)\ , \quad (5.8)$$

where t is the four-momentum transfer, η expresses the spin dependence of the forward nuclear amplitudes, and b is the slope of the forward nuclear spin-independent amplitude. The value of the parameters $b(pp)$ rises rapidly up to 10.5 $(\mathrm{GeV}/c)^{-2}$ between 0 and 20 GeV and then increases slowly up to ~ 12 $(\mathrm{GeV}/c)^{-2}$ at 2 TeV. For np scattering, $b(np)$ is smaller and poorly known. It reaches ~ 9 $(\mathrm{GeV}/c)^{-2}$ at 25 GeV.

(3) Another theoretical approach is to calculate the real parts of all three amplitudes from their imaginary parts, through a dispersion-relation analysis (Grein and Kroll, 1982). This old-fashioned kind of analysis was recently revived due to an improved knowledge of the imaginary parts up to high energy, from the precise $\Delta\sigma_T$, $\Delta\sigma_L$ measurements. The dispersion-relation analysis is interesting also as it constitutes a powerful tool for exploring unphysical regions, e.g., the virtual meson-nucleon exchange. Figures 23(a), 23(b), and 23(c) show the energy dependence of the ρ parameter as calculated by analysis (Kroll, 1981) for pp, np, and $I=0$ scattering separately. For pp, a comparison with recent PSA predictions (Bystricky, Lechanoine-LeLuc, and Lehar, 1990) up to 1.8 GeV is shown (see Sec. VII). At higher energy, $\rho(pp)$ remains negative and becomes positive above ~ 250 GeV; $\rho(np)$ is poorly known.

(4) It is worth noting the interesting approach developed at PSI (Binz *et al.*, 1990; Binz, 1991) and SATURNE II, which allows a model-independent determination of the $I=0$ scattering amplitudes. One only assumes that the $I=1$ amplitudes are well known. Due to different symmetry relations around 90° c.m. for $I=0$ and $I=1$ scattering amplitudes [see Eq. (2.5)], np measurements performed at 180° c.m. can be used to determine the $I=0$ scattering amplitudes at 0°.

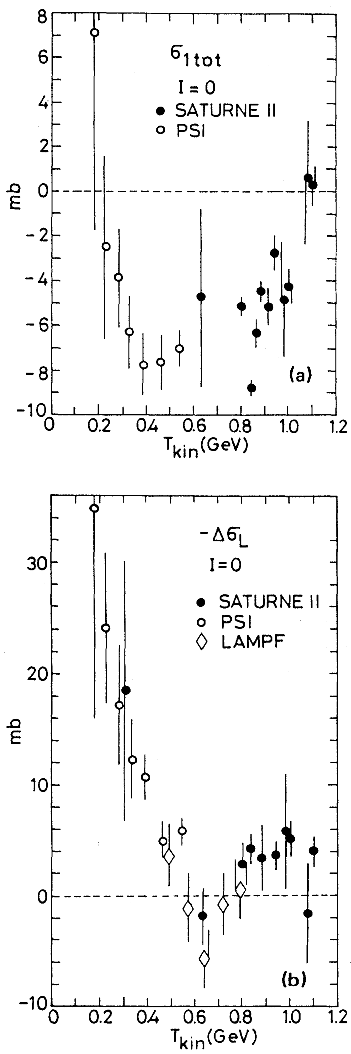


FIG. 21. Spin-dependent total cross section differences deduced from pp and np data. (a) Energy dependence of $\sigma_{1\,\mathrm{tot}} = -\frac{1}{2}\Delta\sigma_T$ for the $I=0$ state. Statistical errors are shown. ○, PSI (Binz *et al.*, 1991); ●, SATURNE II (Fontaine *et al.*, 1991). (b) Energy dependence of $-\Delta\sigma_L$ for $I=0$ state. Statistical errors are shown. ○, PSI (Binz *et al.*, 1991); ◊, LAMPF (Beddo *et al.*, 1991); ●, SATURNE II (Fontaine *et al.*, 1991).

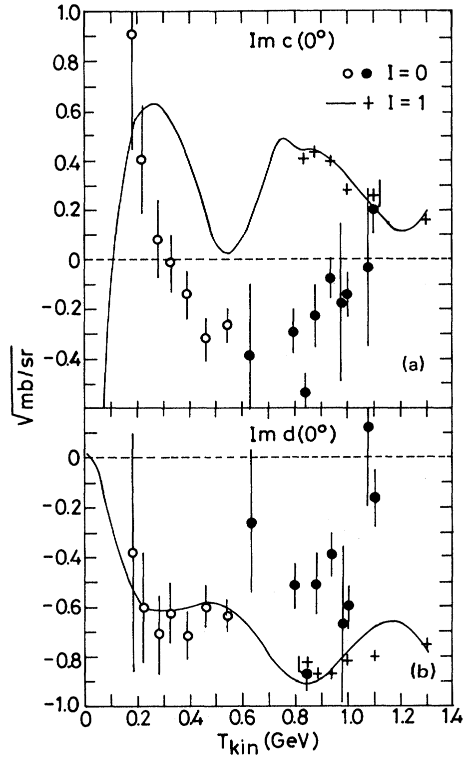


FIG. 22. Energy dependence of the imaginary parts of amplitudes c and d at 0° for $I=0$ and $I=1$ isospin state in $\sqrt{\mathrm{mb/sr}}$ units. Statistical errors are shown. ○, PSI (Binz *et al.*, 1991); ●, SATURNE II (Fontaine *et al.*, 1991); +, PSA Saclay-Geneva (Bystricky, Lechanoine-LeLuc, and Lehar, 1990); ——, PSA Saclay-Geneva (Bystricky, Lechanoine-LeLuc, and Bystricky, 1987).

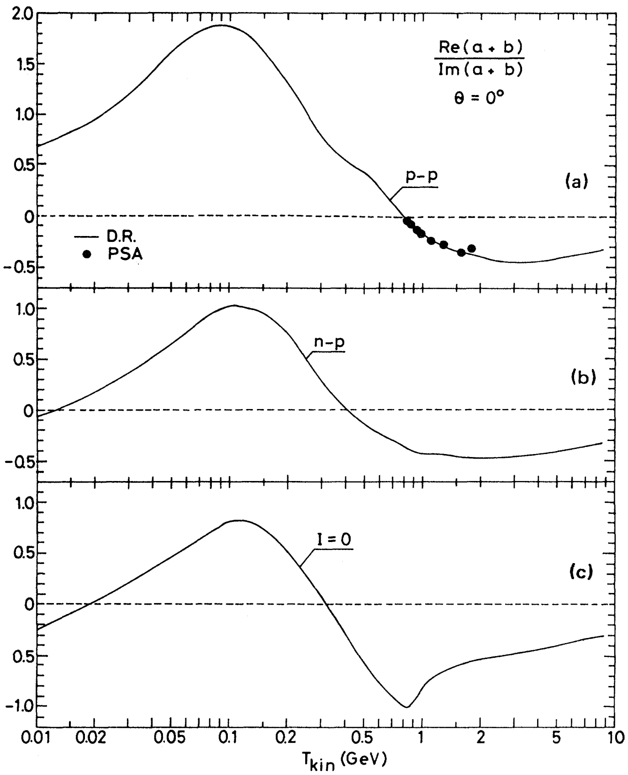


FIG. 23. Energy dependence of the ρ parameter for *pp* (a), *np* (b), and $I=0$ (c) interactions. ——, Kroll, 1981; ●, Bystricky, Lechanoine-LeLuc, and Lehar, 1990).

VI. INTERESTING FEATURES OF NUCLEON-NUCLEON OBSERVABLES

The nucleon-nucleon total cross-section energy dependence above 100 GeV has shown unexpected behavior that still remains unexplained (Amaldi *et al.*, 1973, 1976, 1977, 1978; Amendolia *et al.*, 1973, 1973b; Eggert *et al.*, 1975). An increase of the *pp* total and total elastic cross sections was first observed and a similar increase of the *pp* total inelastic cross section was then deduced. Similar behavior was later observed for $\bar{p}p$ quantities.

An interesting behavior was observed in the *t* dependence of the *pp* analyzing power A_{oono} over a very broad energy domain (Lehar, de Lesquen, Perrot, and van Rossum, 1987). Let us consider this observable between 0° and 90° c.m. where $A_{oono}(pp)$ crosses zero. A minimum at $-t=\sim 1.0$ $(\mathrm{GeV}/c)^2$ followed by a maximum at $-t=\sim 1.7$ $(\mathrm{GeV}/c)^2$ was observed for all scattering energies. The A_{oono} values in the minimum may be positive or negative at different energies. Only positive values in each maximum were observed. This behavior is shown in Fig. 24. Minima and maxima appear at fixed *t*. Similar plots have been made for A_{oonn} (Lehar, de Lesquen, Meyer, *et al.*, 1987) and A_{ookk} (Lehar *et al.*, 1988), but this observation requires more data.

It is worth stressing the fact that structures in nucleon-nucleon scattering are observed only in spin-dependent observables. No significant structures in the energy dependence of the *pp* total cross sections are found. Total elastic or total inelastic cross sections are determined with insufficient precision to draw any conclusion. Structures may be better seen in integrated cross sections of inelastic channels (Yonnet *et al.*, 1990).

Finally we note that, while a structure in an observable does not necessarily imply a dibaryon resonance, the absence of structure does not exclude one either. For example, consider an analyzing power that is related to the scattering amplitudes by $(d\sigma/d\Omega)A_{oono}=\mathrm{Re}(a^*e)$ [Eq. (T2.2) in Table II]. Suppose that only one resonant triplet "partial wave" is present, so that it contributes to both amplitudes. The resonant parts $\mathrm{Re}[a^*(\mathrm{res})e(\mathrm{res})]$ will be identically equal to zero because $a^*(\mathrm{res})$ and $e(\mathrm{res})$ are orthogonal functions at the resonant energy, and only combinations of resonant-background and background-background parts of *a* and *e* survive. It is not obvious, therefore, that a resonance in nucleon-nucleon scattering will manifest itself as a structure in the differential cross-section energy dependence, due to the large number of possible partial waves.

VII. PHASE-SHIFT ANALYSES

A. General philosophy

As discussed in Sec. II, *NN* observables can be expressed in terms of the scattering matrix elements given in Eq. (2.1). This matrix relates the initial states of the two particles to the final state at one energy and angle through the scattering amplitudes *a*, *b*, *c*, *d*, and *e*. In order to take into account the angular dependence of the observables, the scattering amplitudes may be developed as a series of Legendre polynomials and partial-wave amplitude S_J, S_{JJ}, $S_{J-1,J}$, $S_{J+1,J}$, and S^J, which are independent of the scattering angle (Stapp, Ypsilantis, and Metropolis, 1957). The partial-wave elastic-scattering matrix is denoted by *S*. The partial-wave amplitudes contain phase shifts δ_{LJ} usually labeled with orbital and total momentum subscripts, according to spectroscopic notation (waves *S,P,D,F,G*, etc.). For singlet and uncoupled triplet amplitudes ($L=J$), we have

$$S_J=\exp(2i\delta_J),\quad S_{JJ}=\exp(2i\delta_{JJ}) . \tag{7.1a}$$

Coupled triplet partial-wave amplitudes also contain mixing parameters ε_J, which relate phase shifts with the orbital momenta $J=J\pm 1$:

$$S_{J\pm 1,J}=\cos 2\varepsilon_J \exp(2i\delta_{J+1,J})-1 , \tag{7.1b}$$

$$S^J=i\sin 2\varepsilon_J \exp[i(\delta_{J+1,J}+\delta_{J-1,J})] . \tag{7.1c}$$

Legendre polynomials and their derivatives are practical to use, as the polynomial degree has the significance of orbital momentum *L*, and also because of their orthogonality properties. This infinite polynomial series must be cut at a well-chosen L_{max}. Residual terms from L_{max} to infinity are then replaced by the peripheral part of the interaction, e.g., by a virtual one-pion-exchange (OPE) contribution. Scattering amplitudes a,b,c,d,e must be corrected for electromagnetic effects, depending on interacting particles. Note that the aim of phase-shift analysis is to determine the nuclear part of the interaction, namely "nuclear bar phase shifts."

Below the one-pion production threshold (T_{Thr}) the phase shifts δ and the mixing parameters ε_J are real, and the unitarity condition automatically holds for the S matrix. Above T_{thr}, inelasticities are present and the situation changes due to unitarity problems. These changes are minor for singlet and uncoupled triplet partial waves [Eqs. (7.1)], where the phase shifts simply become complex. A first representation fully consistent with unitarity was obtained by Bryan (1981).

A classical method is to define the partial-wave elastic-scattering matrix for coupled states with angular momenta $J=L\pm1$ as

$$S_{L,J}=\begin{bmatrix} S_{J-1,J} & S^J \\ S^J & S_{J+1,J} \end{bmatrix}, \quad (7.2)$$

where

$$S^J=i\sin 2\varepsilon_J \exp[i(\delta_{J+1,J}+\delta_{J-1,J}+\alpha_J)] . \quad (7.3)$$

The phase shifts $\delta_{J\pm1}$ are complex and ε_J is real. The unitarity condition requires $\text{Im}\delta\geq0$. The so-called "sixth parameter" α_J can be understood as the imaginary part of the mixing parameter ε_J. Theorists have always insisted on having this quantity calculated, as they claimed that its introduction would impove the convergence. Until now all practical trials have shown that this parameter is not important. This is due to the fact that the "effect" of the sixth parameter is multiplied by $\sin\varepsilon_J$, and all ε are small. For example, at 1.8 GeV, $\varepsilon_2=-3.5°$, $\alpha_2=6°\pm18°$, and varying α_2 only decreases the χ^2 value of the fit by 0.1 (Bystricky, Lechanoine-LeLuc, and Lehar, 1990).

One can use the complex K-matrix approach as developed by Arndt and Roper (1982; see also Arndt *et al.*, 1983), in which the S matrix is written as

$$S=(1+iK)/(1-iK) , \quad (7.4)$$

$$K=\begin{bmatrix} K_+ & K_0 \\ K_0 & K_- \end{bmatrix} \quad (7.5)$$

with the real and imaginary parts of the K-matrix elements given by ($2s=\delta_{+1,J}, 2d=\delta_{J-1,J}$):

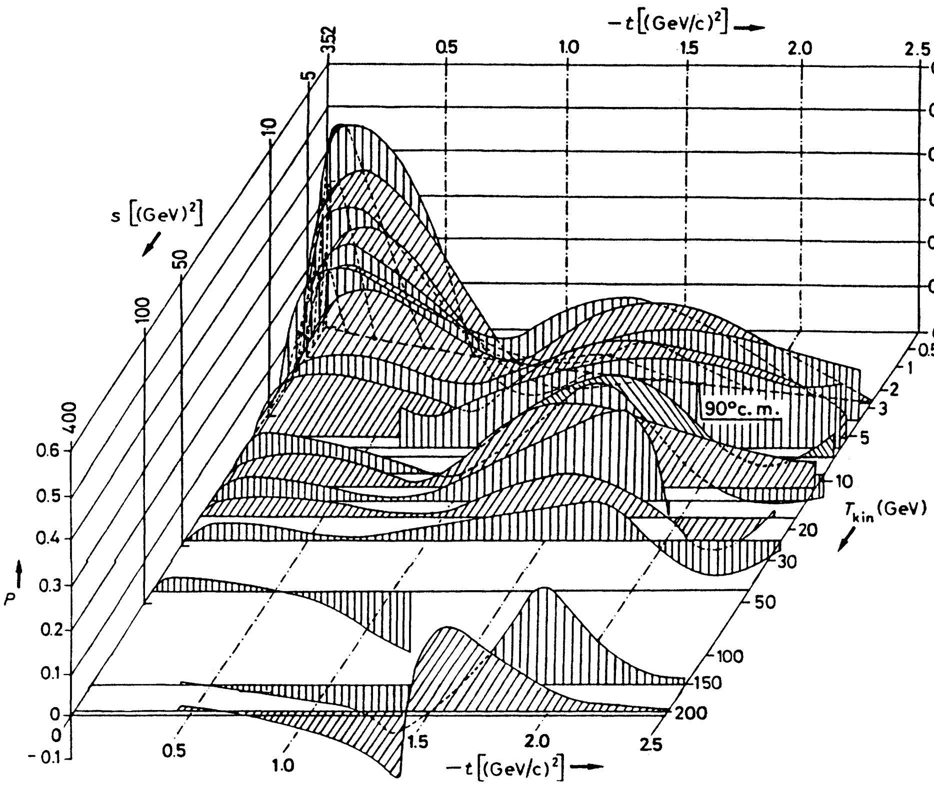


FIG. 24. The s and t dependences of the polarization P in pp elastic scattering. The curve "90° c.m." represents the 90° limit where polarization must vanish (Lehar, de Lesquen, Perrot, and van Rossum, 1987).

$$\mathrm{Re}K_{\pm}=\frac{\sin 2s \pm \cos 2\varepsilon \sin 2d}{\cos 2s+\cos 2\varepsilon \cos 2d} , \tag{7.6a}$$

$$\mathrm{Re}K_0=\frac{\sin 2\epsilon}{\cos 2s+\cos 2\varepsilon \cos 2d} , \tag{7.6b}$$

$$\mathrm{Im}K_{\pm}=\tan^2\rho_{\pm}, \quad \mathrm{Im}K_0=\tan\rho_+\tan\rho_-\cos\mu , \tag{7.6c}$$

where δ and ε are real and ρ_+, ρ_-, and μ are the inelasticity parameters. Of course, δ and ε in Eqs. (7.6) are not identical to those of Eq. (7.1) except in the absence of inelasticity. The K-matrix approach introduces implicitly all the requirements of unitarity. The two different approaches are discussed in detail by Sprung and Klarsfeld (1988).

The phase shifts may further be parametrized in terms of energy. This parametrization is more delicate than the angular development, as there is almost no theoretical guidance except for the energy threshold behavior of the real (at $T_{\mathrm{kin}}=0$) and imaginary ($T_{\mathrm{kin}} \geq T_{\mathrm{Thr}}$) phase shifts. One must be careful not to suppress existing structures in the data or phase shifts by the choice of the order of the polynomial. On the other hand, one cannot use high-degree series, as this rapidly increases the number of free parameters.

If pion production can be neglected, an analysis in terms of potentials may be carried out. One solves the Schrödinger equation with the potential split into different terms, corresponding to the central part, spin-orbit part, spin-spin part, etc., for repulsive and attractive forces. The electromagnetic interaction can be introduced accurately, since its potential representation is well known. Each part of the potential may be developed, for instance, as a superposition of Yukawa potentials guaranteeing the appropriate analyticity and the threshold condition requirement (Feshbach and Lomon, 1964). The coefficients are free parameters, and the energy dependence is then a function of the wave number (Gammel and Thaler, 1957a, 1957b; Signell and Marshak, 1957, 1958, Signell, Zinn, and Marshak, 1958). There are various potential models that employ a hard core (infinitely repulsive) (Gammel and Thaler, 1957a, 1957b, Signell and Marshak, 1957, 1958, Signell, Zinn, and Marshak, 1958, Hamada and Johnston, 1962, Lassila *et al.*, 1962), or a Hamada-Johnston-type potential with a soft core (Bystricky, Lehar, and Ulehla, 1966; Reid, 1968; Ulehla *et al.*, 1969). Note that the hard-core potentials badly describe the angular and energy dependences of any spin-dependent observable.

The dominant term may be chosen as the one-pion-exchange potential (long-range part of the interaction) which contains pion-nucleon coupling constants f^2. Other potential terms for the short-range part (described by nuclear phase shifts in phase-shift analysis) may be considered as corrective terms to the OPEP term. The constant f^2 is often taken from πp scattering experiments with charged pions. This constant is, in principle, different for the exchange of charged or neutral pions. Note that, at low energies, the potential approach may give more accurate results than a phase-shift analysis. It may be a better check of the f^2 value in NN scattering due to the "good" extrapolation of amplitudes into the nonphysical region (Bergervoet *et al.*, 1990). On the other hand, the potential approach is strongly model dependent. Indeed, not only must a cut in $L_{\max}$ be applied as in a phase-shift analysis, but the number of constituent potentials is arbitrary and their shape (radius dependence) must be imposed. As a consequence this approach is limited to the low-energy domain (which is not the subject of this article).

Some potentials make use of the known properties of the various boson fields (π,ρ,ω,η). For examples we refer the reader to Bryan and Scott (1965), Lomon and Feshbach (1968), and Scotti and Wong (1965). Cottingham and Vinh Mau (1964), relate the nucleon-nucleon potential to pion-nucleon and pion-pion amplitudes by employing the Cini and Fubini (1960) representation of Mandelstam amplitudes.

A different approach at intermediate energies, above the pion production threshold, is based on dispersion relations and mesonic exchange models. This is the approach of the Paris Group, for example (Lacombe *et al.*, 1980; Coté *et al.*, 1984; Loiseau, 1984) in determining the high-L imaginary phase shifts. Their calculation provides a good theoretical initial condition for purely phenomenological phase-shift analyses, which are often compared with the final phase-shift analysis results. Another example is the Bonn potential (Machleidt, Holinde, and Elster, 1987; Elster *et al.*, 1988). The mesonic exchange model is also used by Green, Niskanen, and Sainio (1978), Green and Sainio (1979), Araki, Koike, and Ueda (1980, 1982), Kloet and Silbar (1980a, 1980b, 1981, 1984), Araki and Ueda (1982), van Faasen and J. A. Tjon (1983, 1984, 1986), Hultage and Myhrer (1984), Kloet and Tjon (1984), Ueda (1984, 1986a, 1986b), Lamot *et al.* (1987), and Mizutani, *et al.* (1987).

Determination of a "classical" potential directly from observables is based on inverse scattering theory (see, for example, Gelfand and Levitan, 1955; Agranovitsch and Marcheko, 1963). The calculation of the "inverse scattering potential" is extremely cumbersome. This is due to the fact that all data, from the lowest to the highest energy, must be treated together in order to describe the entire r dependence of the different potential components. Moreover, precise spin-dependent data at low energy do not exist. It is worthwhile to mention an "intermediate" phenomenological method, developed by von Geramb's group. It consists in calculating potentials for each phase shift separately, using the inverse scattering theory. The phase shifts at different energies, determined from phase-shift analysis, represent the "experimental" input. Existing results (Kirst *et al.*, 1989; von Geramb and Amos, 1990) successfully reproduce the real parts of the phase shifts at any energy below 800 MeV. This method will soon be extended to the imaginary parts. This kind of NN interaction presentation may easily be applied to a description of nucleon-nucleus scattering.

B. Phase-shift analysis

It is not possible to show all the available phase-shift analysis results. We shall show figures only for the most recent *pp* analyses between 800 and 1800 MeV. In Fig. 25 the energy dependence of six *pp* phase shifts is plotted for the energy-dependent phase-shift analysis of Arndt, 1989 (dot-dashed lines for solutions SP89 and SM89) and for the new fixed-energy Saclay-Geneva phase shift analysis (Bystricky, Lechanoine-LeLuc, and Lehar, 1990) (black dots for solution *A* and open circles for solution *B* at 1.3 GeV, rectangles for spreads). Results are com-

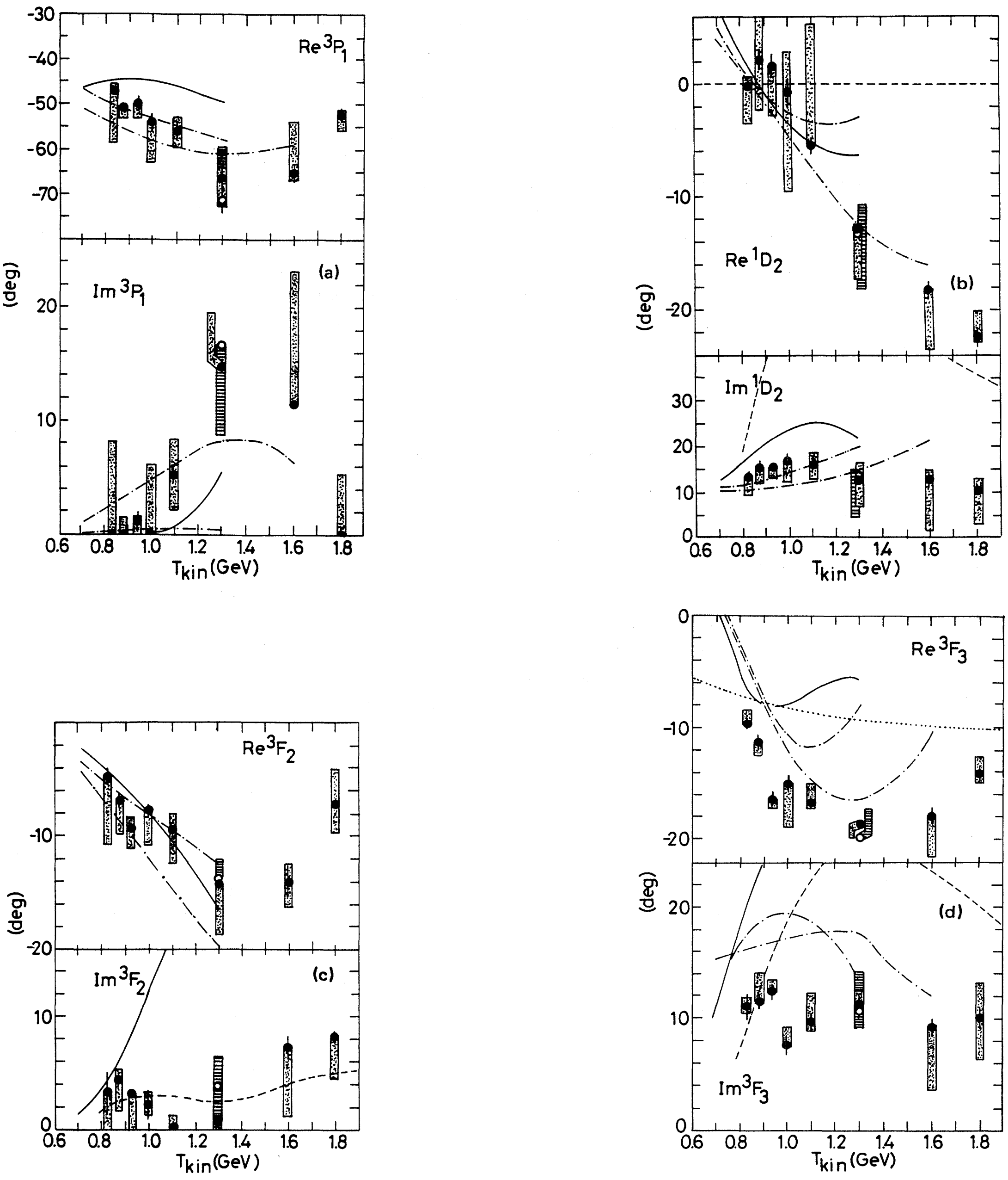


FIG. 25. Proton-proton phase shifts. (a) Real and imaginary parts of $^3P_1$. Long rectangles show phase-shift spreads. ●, Solution *A* (Bystricky, Lechanoine-LeLuc and Lehar, 1990); ○, Solution *B* at 1.3 GeV (Bystricky, Lechanoine-LeLuc, and Lehar, 1990); ——, PSA (Lehar, Lechanoine-LeLuc, and Bystricky, 1987); —·—·—·, Solutions SP89 and SM89 (Arndt, 1989); — — —, Paris Group (Côté *et al.*, 1984; Lacombe *et al.*, 1980; Loiseau, 1984); · · · ·, One-pion-exchange predictions. (b) Real and imaginary parts of $^1D_2$. Symbols as in (a). (c) Real and imaginary parts of $^3F_2$. (d) Real and imaginary parts of $^3F_3$. (e) Real and imaginary parts of $^3F_4$. (f) Real and imaginary parts of $^1G_4$.

pared with the previous energy-dependent Saclay-Geneva phase-shift analyses (Lehar, Lechanoine-LeLuc, and Bystricky, 1987) (solid line), with OPE predictions (dotted line for the real parts) and with calculations of the Paris Group (Lacombe *et al.*, 1980; Coté *et al.*, 1984; Loiseau, 1984) (dashed line for the imaginary parts).

There often exists a misunderstanding concerning the errors quoted on phase shifts, as well as the errors on predicted observables. How is it possible that after the addition of new data phase-shift predictions change outside the previously calculated error bars? The errors in a phase-shift analysis are calculated from the error matrix and represent errors of a fit with a given number of free parameters to a given data set. For fixed-energy phase-shift analyses they are very small and often quoted, since they are closer to the so-called confidence level 1σ. For variable-energy analyses the errors are also very small, but their interpretation is not so direct due to strong correlations. Note that there exist several nonstatistical criteria for a "good" PSA solution: analyticity, correct behavior at "zero energy," and stability of the solution. The last property means that the addition of new free parameters in a "statistically good" solution must preserve low-J phase shifts. In fact, such an addition of free parameters changes all phases inside intervals that are, in general, nonsymmetric around the value of the statistically good solution, called the "Primary Solution." In the recent Saclay-Geneva phase-shift analysis (Bystricky, Lechanoine-LeLuc, and Lehar, 1990) it was found, using all existing world data, that the spreads for several phases are still important. Phase-shift spreads, shown in Fig. 25, were studied up to 1.8 GeV by a gradual increase of phase-shift number. The authors expect that phase shifts will move through the spread intervals when new data become available.

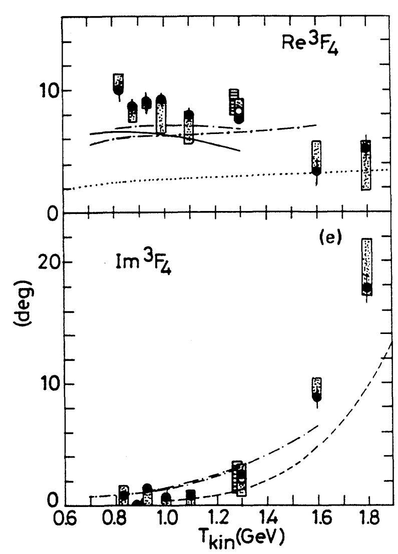
At present it is not possible to show any final *np* phase shifts for three reasons: (1) definite renormalization factors have to be worked out for some of the earlier LAMPF data, not only at 800 MeV, but also at lower energies (see Sec. III.B.2). (2) above 580 MeV there is a continuous flow of data coming from LAMPF and SATURNE II, and one should wait for final results to perform complete new analyses. (3) below 580 MeV, as stated in Sec. V.F, it is necessary to measure more spin-dependent observables to ensure determination of the phase shift. To be complete, one should mention the most recent fixed-energy analysis published by Bugg (1990), and those of Arndt (fixed and variable energies), which are being continuously updated.

It is hard to give an upper limit in energy for the validity of a phase-shift analysis. This depends on the quality and quantity of available data. This validity may be extended if theoretical input in future analyses is improved. The theoretical input furnishes precise peripheral phase shifts, real and imaginary parts, and predictions of specific observables in the angular region inadequately covered by experiments (ratio of real to imaginary parts of amplitudes, for example). A complementary experimental input may consist of the determination of important quantities from other than elastic scattering measurements (e.g., total inelastic cross sections).

C. What may we expect in the future?

(i) *pp* at $T_{\rm kin} \leq 1300$ MeV: As all *pp* data have now been published, one hopes that different analyses will reach good agreement.

(ii) *pp* at $T_{\rm kin} \geq 1300$ MeV: It is still hard to do energy-dependent analyses. Fixed-energy phase-shift analyses are possible up to 2.7 GeV.



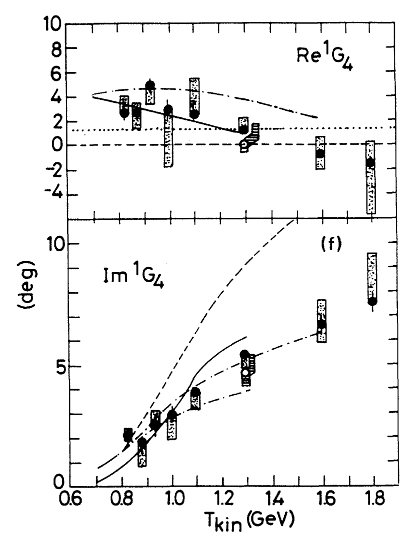


FIG. 25. (*Continued*).

(iii) *np* at $T_{\rm kin} \leq 580$ MeV: More data are needed.

(iv) *np* at 580 MeV $\leq T_{\rm kin} \leq 1100$ MeV: Energy-dependent analyses will be made when all data are available.

(v) *np* at $T_{\rm kin} \geq 1100$ MeV: Fixed-energy analyses using quasielastic *pn* measurements are planned at SATURNE II for $T_{\rm kin} \leq 2.7$ GeV. A free polarized neutron beam was recently extracted at Dubna up to 3.9 GeV.

To conclude Sec. VII, we can state that even though the phase-shift method is now more than 30 years old it has survived competition with all other types of analysis. Phase-shift analyses are very useful for predicting spin observables and making comparisons with new experimental results. They are also very helpful in selecting the most efficient polarization transfer mechanism for creating a polarized neutron beam from a polarized proton beam. Theorists often calculate phase shifts and not spin observables, so phases-shift analysis provides a link between theory and experiment. Last but not least, phase-shift analyses are used in any model for calculating nucleon-nucleus scattering. For all these reasons it is important to have reliable and precise phase-shift analyses.

VIII. DIRECT RECONSTRUCTION OF SCATTERING AMPLITUDES

A. Complete experiment

Puzikov, Ryndin, and Smorodinskii (1957) introduced the concept of the complete experiment for the case of *NN* scattering. They proposed a certain ensemble of observables to be measured, which they called complete if it contained sufficient information for a complete and exhaustive description of the interaction. This is to be contrasted with single observables, such as the differential cross section or the polarization parameter, which provide only very specific information about the process considered. Since theorists have introduced the scattering matrix, which is unfortunately not directly accessible to experiment, but convenient for describing the entire interaction in all its aspects, a complete experiment may be defined as a set of observables that allows a direct and unambiguous reconstruction of the scattering matrix.

Assuming that all observables are determined without error, one can ask: how many observables have to be measured? The minimum number of observables needed for a set to be complete is a rather mathematical problem. Goldstein, Moravcsik, and Bregman (1974) provide a solution and derive a completely general prescription for necessary and sufficient conditions for reactions with arbitrary spin. If N is the number of independent amplitudes, a complete knowledge of the scattering matrix up to an overall phase requires only $(2N-1)$ real functions, since there are $(N-1)^2$ independent nonlinear relations between the set of N^2 observables. Once the $(2N-1)$ measurements are done, the amplitudes are extracted by solving the set of $(2N-1)$ simultaneous quadratic equations in the amplitudes.

The above is in principle a complete scheme for determining the scattering matrix from experiment. If, for example, we apply it to *pp* scattering that is described by five complex amplitudes, we find that there are 25 linearly independent experiments, which are related by 16 independent quadratic equations. Therefore a minimum set of nine well-chosen experiments is sufficient to extract the amplitudes. But this is of a rather academic interest, in fact, since the measured set of observables is dictated by the experimental conditions, i.e., feasibility of beam and target spin orientations, possibility of analyzing different components of the final-state particle polarizations, etc. Moreover, no attention has been given to the actual numerical values of the observables and to their experimental errors. Measurements are carried out in the laboratory frame, of course, but analytic solutions for the amplitudes are carried out in the c.m. frame. This implies relativistic transformations which further complicate the system. For these reasons the number of observables has to be larger than $(2N-1)$. The analytic reconstruction of scattering amplitudes from laboratory-frame observables is treated in Bystricky, Lehar, Patera, and Winternitz, 1978, for example.

In order to take into account experimental errors, statistical methods are usually employed. The errors in the amplitudes depend on experimental errors of the observables and decrease with the degree of the observable set overdetermination. One efficient and reliable method is that proposed by Besset *et al.* (1978). This is based on a Monte Carlo simulation of possible experimental values for the observables, distributed around predicted phase-shift analysis values with assumed realistic experimental errors. This is an efficient way of treating the influence of experimental errors and understanding their role in the resolution of ambiguities. It also allows calculations to be made of the expected amplitude errors.

B. Choice of observables and amplitude representation

Any set of observables contains the differential cross section, since this quantity gives the absolute normalization of amplitudes. All the different amplitude reconstructions performed so far are listed in Tables V and VI. One immediately notices that not two, but three spin observables have been measured over a large angular and energy domain. One notices also that different choices have been made for analyzing the final-state particle polarization. In order to assure a high pC analyzing power, some experiments have analyzed the recoil and some the scattered-particle polarization. Some experiments have deliberately eliminated the option of measuring the polarization for the k' (k'') final-state particle orientation. This option necessitates an additional spin rotator and moreover mixes observables in the magnetic field. Dominant observables (except $A_{oono} = A_{ooon}$) are listed in Tables V and VI. In practice, many observables were determined as combinations of "pure" quantities.

TABLE V. List of available amplitude reconstructions from *pp* scattering observables. $A_{oono} = A_{ooon}$ was always measured and is not listed. Only dominant observables are given. In many experiments combinations of "pure" observables were determined. The * in the number of observables means that the experiments are not all independent.

Laboratory	T_{kin} (MeV)	Angular Range (c.m. deg)	Number of Angles	Number of Obs.	Spin observables	Ref.
PSI	447	38–58	6	11	A_{oonn}, A_{ooss}, $A_{ookk'}$, A_{oosk}, D_{nono}, K_{onno}, $D_{s'oso'}$, $D_{s'oko'}$ $M_{s'osn'}$, $M_{s'okn}$	Aprile *et al.* 1981; Hausammann *et al.*, 1989.
	497	"	"	"		
	517	"	"	"		
	539	"	"	"		
	579	"	"	"		
	447	62–90	8	15	All preceding and $K_{os''so}$, $K_{os''ko'}$ $N_{os''sn}$, $N_{os''kn}$	
	497	"	"	"		
	517	"	"	"		
	539	"	"	"		
	579	"	"	"		
LAMPF	730	38–72	12	14*	A_{oosk}, A_{ookk}, D_{nono}, $D_{s'oso}$, $D_{s'oko}$, $D_{k'oso}$, $D_{k'oko}$, $K_{s'ook}$, $K_{k'ook}$, M_{nosk}, M_{nokk} $M_{s'onk}$, $M_{k'onk}$	McNaughton *et al.*, 1990.
	800	46–90	5	10	A_{oonn}, D_{nono}, $D_{s'oso}$, $D_{s'oko}$, $D_{k'oko}$, K_{noon}, $K_{s'oos}$, $K_{s'ook}$, $K_{k'ook}$,	Moravcsik, Arash, and Goldstein, 1985.
SATURNE II	834	50–82	6	12	A_{oonn}, A_{oosk}, A_{ookk}, D_{nono}, $D_{os''ok}$, K_{onno}, $K_{os''so}$, $K_{os''ko}$, N_{onkk}, N_{nosk}, $N_{os''nk}$, $N_{os''kn}$,	Lac *et al.*, 1990
	874	46–83	7	12		
	934	51–80	5	12		
	995	51–82	6	12		
	1095	41–83	7	12		
	1295	38–85	8	12		
	1539	34–84	10	10–12		
	1796	33–87	11	10–12		
	2096	31–85	10	10–12		
	2396	29–83	4	12		
	2796	29–83	3	12		
ANL	5135	19–38	6	20–21*	A_{oonn}, A_{ooss}, A_{ooks}, A_{ookk}, D_{onon}, $D_{os''os}$, $D_{os''ok}$, $D_{ok''os}$, $D_{ok''ok}$, K_{onno}, $K_{os''so}$, $K_{os''ko}$, $K_{ok''so}$, $K_{ok''ko}$, $N_{os''sn}$, $N_{os''ns}$, $N_{os''nk}$, N_{onsk}, N_{onss}, N_{onks}, N_{onsk}	Matsuda, Suemitsu, and Yonezawa, 1986.

TABLE VI. List of available amplitude reconstruction from *np* scattering observables.

Laboratory	T_{kin} (MeV)	Angular range (c.m. deg)	Spin observables No.	Spin observables Quantity	Ref.
SATURNE II	840	38–74	11	A_{oono}, A_{oonn}, A_{ookk}, A_{oosk}, D_{nono}, K_{onno}, $D_{os''ok}$, $K_{os''ko}$, $K_{os''so}$, N_{onkk}, N_{onsk}	Ball, 1992.
	880	"	"		
	940	"	"		
	1000	"	"		
	1100	"	"		

In order to be independent of the undetermined overall phase, one usually defines the amplitude e as real and positive. This choice is arbitrary, but is justified by the fact that the absolute value of e stays large at all angles over a measured energy range. The polar representation used is

$$A = |A| \exp(i\varphi_A) , \tag{8.1}$$

where $A = a,b,c,d$ and φ_A is the phase of A relative to e. In this way, the undetermined common phase is set into the phase of e. The overall phase (which multiplies all the amplitudes) is not experimentally measurable. This is clearly seen in the equations given in Table II. Any change in the common phase would not be seen in the observables, as the amplitudes always appear as a modulus square or a product of conjugates. In phase-shift analysis the common phase is fixed, due to Coulomb-nuclear interference, and the amplitudes are rotated in the Gaussian plane.

C. Status of amplitude reconstruction

The pp elastic-scattering amplitudes ($I = 1$) have been determined in a model-independent analysis over part of the scattering angle range at 19 energies between 447 MeV up to 5135 MeV (6 GeV/c) as detailed in Table V. The first analysis was performed on PSI data at 579 MeV (Aprile *et al.*, 1981), then complemented at four lower momenta by Hausammann *et al.*, 1989. Similar reconstructions have been performed on LAMPF data at 730 MeV (McNaughton *et al.*, 1990). The present number of existing measurements could also permit such analyses at two or three more LAMPF energies, such as 650 and 800 MeV. At higher energy, a strong effort has been made by the nucleon-nucleon group at SATURNE II (Lac *et al.*, 1990); this group has completed reconstructions at 11 momenta between 830 and 2700 MeV. Original work was done by Arash, Moravcsik, and Goldstein (1985) in the transversity frame on the PSI data at 579 MeV and on the LAMPF data at 800 MeV (Moravcsik, Arash, and Goldstein, 1985). To be complete, one must cite the work done at the higher energy of 6 GeV/c (Auer *et al.*, 1985), for which an amplitude analysis was also performed (Matsuda, Suemitsu, and Yonezawa, 1986), but this approach is somewhat different, as at 6 GeV/c the spin parameters are smaller; moreover, they had less precision. For this reconstruction 20 to 21 different spin-dependent observables were used, and relations between different experimental quantities were added as further information. This reconstruction is not as straightforward as at medium energy.

In contrast, as illustrated in Table VI, the $I = 0$ amplitudes are very poorly known at present. Nowadays only the SATURNE II data make this reconstruction possible at five energies between 840 and 1100 MeV. Preliminary results for the np scattering amplitudes were presented at the 1990 Bonn Conference (Lehar, 1991a) at 840 and 880 MeV for only three different angles (54°, 61°, and 73° c.m.). Results for four different angles (49°, 54°, 62° and 74° c.m.) at 840, 880, 940, 1000, and 1100 MeV are given by Ball (1992). The results are promising at this stage of the analysis. In Fig. 26 are shown the results for four scattering angles at 840 MeV. Three different solutions have been found at 49°, 54°, and 74°, while four solutions exist at 62°. All solutions are statistically independent. It is still uncertain whether a reduction of the experimental error bars will be sufficient to lead to a unique solution. Data already collected at PSI and the forthcoming approved experimental program on the NA2 beam line will allow such a np reconstruction at four or five additional energies between 300 and 560 MeV with a high degree of precision (Gaillard *et al.*, 1989; Gaillard, 1990), but results are not likely to be available before 1993. Quasielastic scattering spin observables will be obtained at SATURNE II using a polarized ^{6}LiD target, which is under construction; so far no experimental results prove that free elastic spin observables are different from quasi-

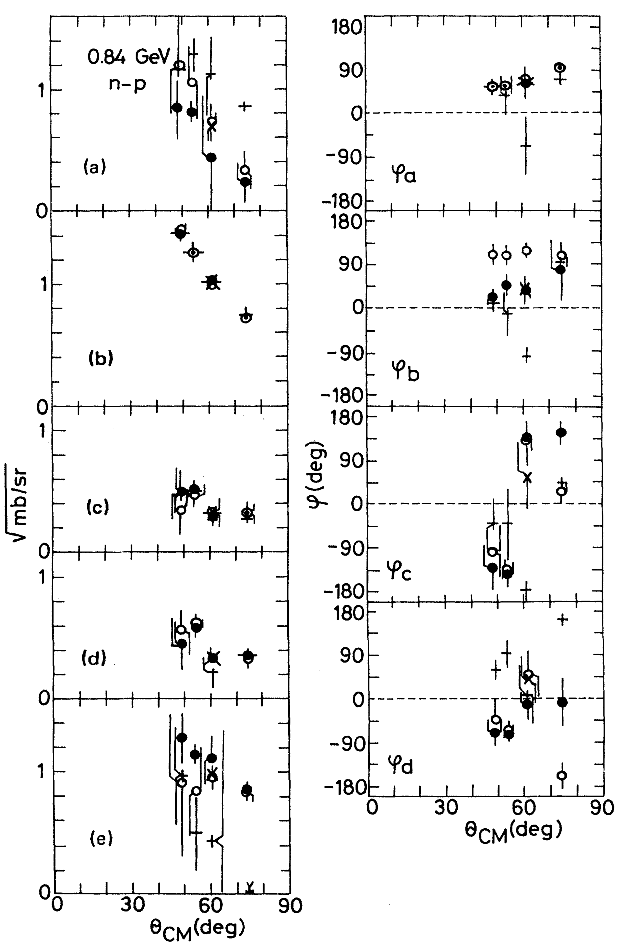


FIG. 26. Direct reconstruction of the np scattering matrix at 0.84 GeV. Absolute values and phases relative to e of the five invariant amplitudes a to e at four scattering angles are shown (Ball, 1992). Symbols ●, ○, +, and × denote statistically independent solutions.

free ones (Lehar, 1990, 1991a; Ball, 1992; see Sec. V.E).

The use of *np* amplitude analysis will enable us to determine $I=0$ amplitudes, which are very poorly known at present. For a given c.m. scattering angle and $T_{\rm kin}$, $I=0$ amplitudes can be obtained using

$$\mathrm{Amp}(I=0)=2\,\mathrm{Amp}(np)-\mathrm{Amp}(I=1) \tag{8.2}$$

if the $I=1$ amplitudes are known from *pp* scattering. The factor of 2 in Eq. (8.2) implies that the $I=0$ amplitudes will generally be more poorly determined than the $I=1$ amplitudes, assuming that the *pp* and *np* spin observables are measured to the same accuracy. The preliminary results for $I=0$ amplitudes, using the solution (●) from Fig. 26 and the *pp* amplitudes (Lac *et al.*, 1990), are shown in Fig. 27.

In general one observes excellent agreement between phase-shift analyses and amplitude analyses whenever both have been performed on the same data basis. A disadvantage of the amplitude reconstruction approach with respect to the phase-shift analysis method is a significant loss of input data. This is due to the fact that model-independent amplitude analysis can be performed at one angle and one energy at a time, and only at those coordinates, where a complete set of observables exists.

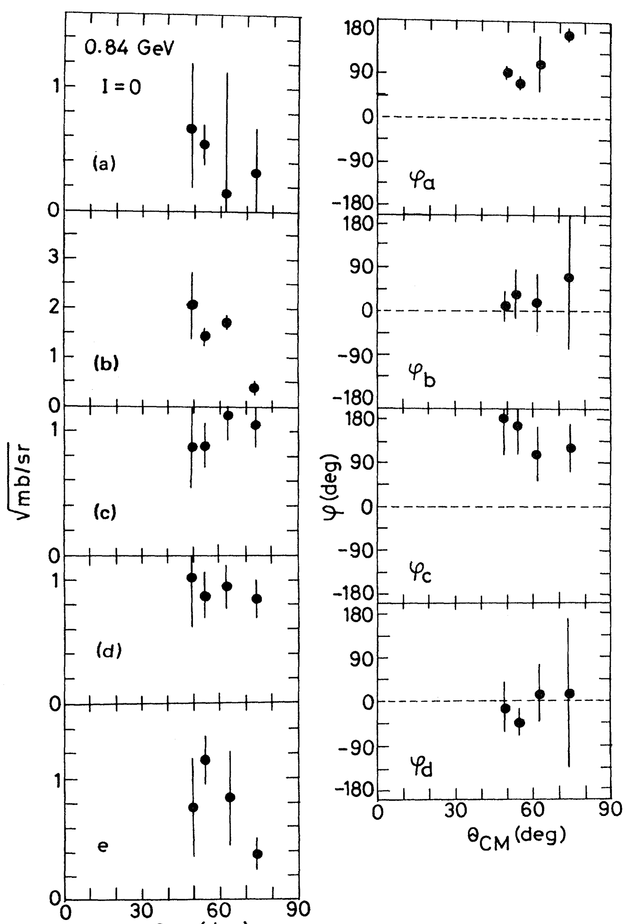


FIG. 27. Direct reconstruction of the scattering matrix at 0.84 GeV for isospin $I=0$. Solution (●) from Fig. 26 (Ball, 1992) and *pp* amplitudes (Lac *et al.*, 1990) were used. Absolute values and phases relative to *e* of the five invariant amplitudes *a* to *e* at four scattering angles are shown.

IX. CONCLUSIONS

Thanks to the many accelerators providing polarized beams and to improvements in polarized targets and experimental technique, as well as to the development of computing, the lack of nucleon-nucleon data so critical several years ago is now alleviated. Measurements at intermediate and high energies have uncovered some interesting general features of spin observables. The data available permit a comparison of model predictions to the results of phenomenological analyses. The direct reconstruction of scattering amplitudes is also beginning to compete with the PSA method.

As a consequence, "superficial" experiments or experiments providing only rough information are now practically useless. New results must be more accurate, energy dependent, and measured over a large angular region. Selected nucleon-nucleon experiments, spin-dependent or independent, elastic or inelastic, as well as studies of fundamental laws at any energy will improve our knowledge of phase shifts and scattering amplitudes and thus our ability to make observable predictions. This statement is self-evident for *np* scattering, but the phase-shift spreads for *pp* scattering above 800 MeV show how far we are from a "perfect" *pp* phase-shift analysis.

Structures in nucleon-nucleon scattering must be studied not only on the basis of the energy and angular dependence of the observables, but also on the basis of the scattering amplitudes. In this case the number of predicted dibaryonic resonances will be considerably reduced. To solve a dibaryon problem it will be useful to perform measurements allowing one to obtain complete sets of observables and the consequent reconstruction of the scattering matrix. Such a "global" picture may help to stimulate experiments for a dedicated check of possible resonances.

The quality of the angular and energy dependence of available *np* scattering observables does not allow one to draw any conclusion about possible structures. Comparing different spin-dependent observables (analyzing power and spin correlations), one can state that no difference has been found between *np* elastic-scattering data and *np* or *pn* quasielastic data using deuteron beams or targets.

ACKNOWLEDGMENTS

We are grateful to J. Arvieux, J. Ball, A. Boudard, P.-A. Chamouard, H. V. von Geramb, R. Hess, W. R. Leo, B. Loiseau, M. McNaughton, A. Nakach, and H. Spinka for helpful discussions and comments on different subjects of this paper.

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